ISSN 2042-2695

CENTRE *for* ECONOMIC PERFORMANCE

CEP Discussion Paper No 1497

September 2017

Robot Arithmetic: Can New Technology Harm All Workers or the Average Worker?

Francesco Caselli
Alan Manning

THE LONDON SCHOOL OF ECONOMICS AND POLITICAL SCIENCE

E·S·R·C ECONOMIC & SOCIAL RESEARCH COUNCIL

Abstract
It is well-established that new technology can cause large changes in relative wages and inequality. But there are also claims, based largely on verbal expositions, that new technology will harm workers on average or even all workers. Using formal models (which impose logical consistency and clear links between assumptions and conclusions) we show – under plausible assumptions - that new technology will cause average wages to rise if the prices of investment goods fall relative to consumer goods (a condition supported by the data) and if the new technologies do not lead to a fall in market competition. Some groups of workers must gain but others may be harmed. However, if workers can freely choose their occupation, or redistribution among workers is possible, all workers can gain.

Keywords: technology, wages
JEL: J31; O33

This paper was produced as part of the Centre's Labour Markets Programme. The Centre for Economic Performance is financed by the Economic and Social Research Council.

Francesco Caselli, Centre for Macro Economics, CEPR and London School of Economics. Alan Manning, Centre for Economic Performance and London School of Economics.

Published by
Centre for Economic Performance
London School of Economics and Political Science
Houghton Street
London WC2A 2AE

Introduction

There are widespread concerns about the current and likely future impact of new technology (mostly robots and artificial intelligence) on the demand for labor (Erik Brynjolfsson and Andrew McAfee, 2014, Martin Ford, 2015, Carl Benedikt Frey and Michael A Osborne, 2017, Richard Susskind and Daniel Susskind, 2015). Commentators from many different backgrounds including science, philosophy, business as well as economics have speculated on how new technology will affect the labor market and wider society (Nick Bostrom, 2014, White House, 2016, Peter Stone et al., 2016). The existing literature establishes that new technology can cause large changes in relative wages and inequality but stronger claims are sometimes made, that average wages will fall or even that all workers will lose. There is also an empirical literature on the impact of new technology and robots on the labor market (Daron Acemoglu and Pascual Restrepo, 2017a, D. H. Autor and D. Dorn, 2013, Maarten Goos et al., 2014, Georg Graetz and Guy Michaels, 2015).

Fears about the impact of new technology on workers are not new, although the technology feared has varied over time (D. H. Autor, 2015, Harold R. Bowen, 1966). Past fears proved unfounded, but it is argued (not for the first time) that 'this time is different', and that the impact of past technologies can be no guide to the impact of future technologies.

Most analyses of the impact of new technology identify some jobs currently done by humans that are likely to be replaced by machines/robots/AI etc (Carl Benedikt Frey and Michael A Osborne, 2017). Past predictions have often been accurate about the impact on specific occupations but have been inaccurate as a prediction about the impact on the overall demand for labor and real wages, because they fail to recognize that job destruction cannot be the only impact of new technology on the labor market. A simple example makes clear why. Firms replace humans with machines if they lower costs by doing so. If costs fall then it is likely that prices of the affected goods fall as well. As prices fall, consumers find they can buy what they did before and still have some money left over. Consumers spend this extra disposable income on many different types of goods and services and, as they do so, they create jobs among the producers of those goods and services. These will not just be jobs in 'new' occupations, they will also be jobs in 'old' occupations. This chain of reasoning might not be watertight but it represents one possibility that should not be ignored.

Most of the claims of a dire future for workers are based on verbal analysis. But, to work out the possible scenarios for the impact of new technology on the demand for labor it is useful to have a model of the economy as a whole. One should not think of these models as being true descriptions of the world but they are nonetheless useful. A model requires explicit assumptions and forces analysis to be logically consistent. If a model predicts a conclusion that one finds implausible it must be because one of the assumptions is not satisfied. The use of a model then makes very clear the assumptions being made and the conclusions drawn in a way which most existing discussions of the impact of new technology do not.

We present a variety of economic models about the impact of new technology on the demand for labor. Our main target audience is scientists concerned about the impact of new technology but who are largely unfamiliar with economic models. But we also think that some of our conclusions might be of interest to economists. Most of the existing economic literature focuses on how new technology can change relative wages and inequality (D. Acemoglu and D. Autor, 2011, Daron Acemoglu and Pascual Restrepo, 2017b, Daniel Susskind, 2017). This is consistent with our analysis but we focus on the circumstances in which average wages rise or fall, and whether it is possible that new technology harms all workers. We argue that there are reasons to think that workers as a whole might gain whatever the form new technology takes.

We work through a variety of models that differ in the number of types of labor and goods they have. We start with the most general model and then present simpler models as the workings of simpler models are often more transparent.

Benchmark Model and Main Results

We assume there are many types of labour in the economy and denote the supply of them by a (row) vector L. For the moment we assume the supply of each type of labor is fixed but we return to this below. We denote wages by a (column) vector w and assume all workers supply labour inelastically and will work for any non-zero wage.

There are many produced goods in the economy which can be used for consumption, as intermediate inputs into production and as capital goods. The difference between intermediate goods and capital goods is that the former are assumed to be entirely used up in the production process while capital goods retain their usefulness over many periods - though not indefinitely. Denote the price vector of consumer and/or intermediate goods as p and the rental price of capital goods as p^K.

We follow standard microeconomic theory (Hal R. Varian, 2010) and represent the technology for producing goods by a cost function, a function of prices, wages, output and the level of technological efficiency (denoted by θ), which represents the least cost way of producing a given level of output of a good. We make the following assumption about technology in all sectors.

Assumption CRS: The production function has constant returns to scale so that a doubling of all inputs leads to a doubling of output.

The usual argument for CRS is that one can always replicate a production process so that one cannot do worse than constant returns to scale – we discuss the consequence of relaxing this assumption later.

CRS also implies that a doubling of output leads to a doubling of costs. This means the cost function can be summarized by a unit cost function, the least cost way of producing one unit of output. For consumption goods denote this unit cost function by the vector $c\left(w, p, p^K, \theta\right)$ and for investment goods by $c^i\left(w, p, p^K, \theta\right)$. This set-up allows for the possibility that, for some levels of technology θ, some goods may be impossible to produce. This can be modelled as them having an infinite unit cost. So this set-up does not assume a fixed menu of either consumption or investment goods. A doubling of all wages and prices also leads to a doubling of costs so that the unit cost function is homogeneous of degree 1 in prices and wages. Finally, improvements in technology are captured by assuming that the cost function must be non-increasing in θ for all goods (both consumption and investment) – for a new improved technology means that costs of production cannot rise for a fixed set of wages and prices i.e. we must have:

$$c_\theta \equiv \frac{\partial c\left(w, p, p^K, \theta\right)}{\partial \theta} \leq 0, \qquad c^i_\theta \equiv \frac{\partial c^i\left(w, p, p^K, \theta\right)}{\partial \theta} \leq 0 \tag{1}$$

with a strict inequality for at least one good. That new technology must satisfy the conditions laid out in (1) is uncontroversial.

In what follows we simply compare steady-states with constant levels of technology, asking whether wages are higher or lower in economies with more advanced technology. This approach allows us to be as general as possible in assumptions about the way in which technology affects production opportunities. But there is a cost – we do not model the transition from one steady-state to another, nor do we model an economy in which technology is changing over time. There are economic models that do this but they typically have to make quite restrictive assumptions about how new technology affects productive opportunities, often to have a model that is analytically tractable and displays balanced growth (Daron Acemoglu, 2008, Gene M Grossman et al., 2017, Hirofumi Uzawa, 1961).

Shephard's Lemma (Hal R. Varian, 2010) tells us that the demand for labor and other inputs is the derivative of the unit cost function multiplied by the level of output so that, if X is the vector of outputs of consumption goods and I is the vector of outputs of investment goods, we have that the demand for labor is given by:

$$L^d = X\frac{\partial c\left(w,p,p^K,\theta\right)}{\partial w} + I\frac{\partial c^i\left(w,p,p^K,\theta\right)}{\partial w} \equiv Xc_w + Ic_w^i, \tag{2}$$

where the derivatives are matrices such that the element (i,j) is the derivative of the unit cost of good i with respect to the wage for labor of type j. Looking at this expression one might think that whether new technology raises or reduces the demand for labor depends on the cross-partial of the unit cost functions with respect to wages and θ. Since there is no particular reason to think this sign is negative or positive one would then be led to the conclusion that the impact of new technology on the demand for labor depends on the nature of the technology. But one point that this paper makes is that this is a misleading analysis – it is based on assuming that prices and wages and outputs are fixed. However new technology will cause these to change and these changes potentially lead to different conclusions.

We need to make some assumptions about how prices are determined.

Assumption RK: There are financial assets paying an interest rate r, and (for now) this interest rate is constant.

We also assume that capital goods depreciate at a constant rate δ. Then, a conventional no-arbitrage argument yields $p^K = \left(r+\delta\right)p^i$,

where p^i is the price of investment goods.

About the nature of markets we assume:

Assumption PC: Output and input markets are perfectly competitive.

PC implies that prices must be equal to unit costs so that we have:

$$p = c\left(w,p,\left(r+\delta\right)p^i,\theta\right) \tag{3}$$

$$p^i = c^i\left(w,p,\left(r+\delta\right)p^i,\theta\right) \tag{4}$$

Finally, we make the following assumption, largely for tractability

Assumption HOM: Consumers' preferences are homothetic so there is a unique consumer price index, denoted by $e\left(p\right)$.

This means that any differential impact of new technology on different types of workers must come from a differential impact on wages, not prices.

The first result of this paper is the following (all proofs in the Supplementary Material).

Result 1: Improvements in technology raise the average real wage of workers if the price index of investment goods does not increase relative to the price index of consumption goods.

The intuition for the result is that new technology allows more output to be produced than before. This extra output might go to labor or owners of capital. But if the impact of the new technology is to reduce the price of investment goods relative to consumption goods, then the return to existing capital must fall, causing a rise in the overall return to labor. And any additional capital must be paid its marginal product so its return cannot be at the expense of labor. This does not mean that the labor share of national income rises (Loukas Karabarbounis and Brent Neiman, 2013) because the stock of capital might increase enough to more than off-set the fall in relative investment goods prices.

The result is about the impact on the average wage of workers – it does not say anything about whether all or most workers benefit – the existing literature already contains many examples where some types will experience falls in their wages. It is also possible that the majority of workers will lose or that there will be no demand for some types of workers even if their wages fell to zero. So Result 1 does not say that new technology will not have serious consequences for inequality in labor income.

But there are policies that could ensure all workers gain. With the assumption that the number of different types of workers is fixed, one can simply tax the winners and distribute to the losers without affecting any production decisions. Note that one can achieve this by taxing only labor – one does not need to tax 'robots' as has been suggested by, among others, Bill Gates (though see (Joao Guerreiro et al., 2017) for a different model and conclusion). This process of redistribution may be politically difficult - especially if the winners and losers are in different countries – and one should not be complacent about the ability of political processes to restrain rises in inequality but it is important to understand that there is a simple policy to ensure that all gain. Later we also show that growth is likely to be more inclusive if workers can choose what type of worker to become.

A corollary of Result 1 is that if there is only one type of good that can be used as both consumption and capital then new technology must raise the real wage of workers.

It is obviously important to consider whether the condition that investment good prices fall relative to consumption goods is likely to be satisfied in practice. Most data suggests that it is (IMF, 2017, Charles I Jones, 2016, Per Krusell et al., 2000). But one implication is that it is possible to come up with an example in which the average wage of workers falls - this is done in the Supplementary Material.

Result 1 provides a sufficient condition for new technology to raise average real wages which also implies that the real wage must rise for at least one type of worker. But it is also possible to show that, even if the condition of Result 1 is not satisfied, new technology must raise the real wage of at least one type of worker i.e. it is impossible for all types of worker to lose.

Result 2: Improvements in technology must raise the real wage of at least one type of worker.

One corollary of Result 2 is that if there is only one type of labor, then new technology of any form satisfying (1) must raise the real wage of that type of worker -- which says that all workers gain. The case with one type of worker may seem to be of little use because there are surely many different types of labour. But the next section shows that one interesting case can be thought of as equivalent to the one type of labor case.

One or Many Types of Workers?

The models used so far have assumed that the supply of different types of workers is fixed. In the long-run that is not a plausible assumption - think of types of labor as occupations and that workers can choose their occupation at the start of their careers. It is plausible that the numbers of workers choosing different careers depends on the wages on offer, the costs of training for different occupations, and how pleasant or unpleasant is the nature of the work. One prominent case is that the labor supply to different occupations is perfectly elastic, which means that relative wages are fixed - occupations which require longer periods in education or are more unpleasant have to be compensated by higher wages.

The perfect elasticity model may seem extreme but is not a bad approximation to the data – over time there have been huge changes in the level of employment in different occupations but relatively modest changes in relative wages.

Result 3: If labor of different types is in perfectly elastic supply, then workers of all types must gain from technological progress.

The intuition for this result is that perfectly elastic labor supply between occupations means that wage differentials are fixed, so that all wages must go up or down together, reducing the model effectively to one with only one type of labor.

The Role of the Assumptions

As indicated in the introduction, these models are only as good as their assumptions. Here we indicate how the results can change if the assumptions are violated. In clarifying the role of the assumptions it is useful not to use an approach based on production functions rather than cost functions, and to reduce the number of goods and labor to one. Results 1 and 2 then imply that workers must gain from new technology if the assumptions CRS, RK and PC are satisfied. There is a straightforward proof of this for this simple case.

Represent the gross output of the economy through the use of a production function – this tells us the maximum amount of gross output, Y, that can be produced given inputs (labor, L, intermediate inputs, X, and capital, K) and the state of technology that we denote by θ. Write the production function as:

$$Y = F\left(L, X, K, \theta\right) \tag{5}$$

If there are constant returns to scale then the production function will be homogeneous of degree one in the inputs. In this one-good model an improvement in technology can only be represented by the assumption that more output can now be produced with the same inputs. This amounts to the assumption:

$$\frac{\partial F\left(L, X, K, \theta\right)}{\partial \theta} > 0 \tag{6}$$

With one good it is convenient to normalize the price of that good to 1. Denote the wage paid to labor by w - the question of whether workers benefit or are harmed by new technology can then be expressed as the question of whether w rises or falls with new technology.

The cost of intermediate inputs will also be equal to 1 and the cost of capital will be $(r+\delta)$. PC means that the wage of workers will be equal to their marginal product, which is given by:

$$w=\frac{\partial F\left(L,X,K,\theta\right)}{\partial L} \tag{7}$$

Just like labor, intermediate goods and capital will be used up to the point where their marginal product equals their cost i.e. that:

$$\frac{\partial F\left(L,X,K,\theta\right)}{\partial X}=1, \qquad \frac{\partial F\left(L,X,K,\theta\right)}{\partial K}=\left(r+\delta\right) \tag{8}$$

It is a well-known result that with constant returns to scale the total payment to inputs exhausts total output. So total payments to labor can be written as:

$$wL=F\left(L,X,K,\theta\right)-X-\left(r+\delta\right)K \tag{9}$$

i.e. gross output net of the intermediate goods used and the payments to the owners of capital. Differentiating (9) with respect to new technology leads to:

$$L\frac{\partial w}{\partial \theta}=\frac{\partial F}{\partial \theta}+\left[\frac{\partial F}{\partial X}-1\right]\frac{\partial X}{\partial \theta}+\left[\frac{\partial F}{\partial K}-\left(r+\delta\right)\right]\frac{\partial K}{\partial \theta}=\frac{\partial F}{\partial \theta}>0 \tag{10}$$

Where the second equality follows because the terms involving X and K cancel under the assumption that these inputs are paid their marginal product, (8). This is a very quick route to the main result for the special case of one good and one type of labor.

Now consider what happens when we change the assumptions.

Non-constant returns to scale

Suppose we modify the production function so that it is homogeneous of degree α in the inputs. A well-known result then says that the sum of the factor returns is a share $\alpha<1$ of gross output so that (9) becomes:

$$wL=\alpha F\left(L,X,K,\theta\right)-X-\left(r+\delta\right)K \tag{11}$$

i.e. gross output net of the intermediate goods used and the payments to the owners of capital. Differentiating (11) with respect to new technology leads to:

$$\begin{aligned} L\frac{\partial w}{\partial \theta}&=\alpha\frac{\partial F}{\partial \theta}+\left[\alpha\frac{\partial F}{\partial X}-1\right]\frac{\partial X}{\partial \theta}+\left[\alpha\frac{\partial F}{\partial K}-\left(r+\delta\right)\right]\frac{\partial K}{\partial \theta} \\ &=\alpha\frac{\partial F}{\partial \theta}-\left(1-\alpha\right)\left[\frac{\partial X}{\partial \theta}+\left(r+\delta\right)\frac{\partial K}{\partial \theta}\right] \end{aligned} \tag{12}$$

Which can no longer be unambiguously signed. A very simple example of a production function where the wage can fall is $\left(L+\theta X\right)^{\alpha}$ for $\alpha<1$.

Decreasing returns to scale is often thought to result from an 'omitted' fixed factor. So this result could be interpreted to say that new technology increases the returns to fixed factors as a whole: it is just that labor is not the only fixed factor. Although it is a common and plausible assumption that labor is currently the main fixed factor, it is possible that some other fixed factor comes to be important, e.g. if robots required some rare earth in their manufacture. In that case it is possible that the benefits from new technology go to the owners of that scarce factor and not to labor. But this is a different argument from most accounts of the impact of new technology.

One might wonder what happens if there are increasing returns to scale in production. This is not compatible with perfect competition so requires a discussion of a relaxation of assumption PC.

Imperfect Competition

If there is imperfect competition then prices will be a mark-up on marginal costs. Mark-ups do not necessarily cause the results outlined above to fail. For example, Result 2 will still apply if one inserts mark-ups (possibly different for different goods) into the model of pricing (3) and (4), as long as mark-ups are constant. But it is conceivable that technical change causes mark-ups to rise for some goods in which case it is possible for wages to fall. Some concern about rising mark-ups has been expressed (David Autor et al., 2017, Jan De Loecker and Jan Eeckhout, 2017), but, even if relevant, it is less about the direct impact of technology and more about the way technology affects market competition.

Imperfect competition also allows for increasing returns to scale in production. The Supplementary Material presents a simple model where each individual firm has increasing returns to scale and some market power, but there is free entry of firms into industries (this is a model of what is called monopolistic competition). It shows this is isomorphic to the models already considered if the fixed and variable costs of firms use inputs in the same proportions.

Rising Interest Rate

Assumption RK is that the interest rate is constant. If new technology causes the interest rate to rise then this causes a rise in the return to capital and possible falls in real wages. To see this note that (10) becomes in this case:

$$L\frac{\partial w}{\partial \theta}=\frac{\partial F}{\partial \theta}-K\frac{\partial r}{\partial \theta} \tag{13}$$

In most standard economic models the interest rate is a function of the underlying growth rate (zero is our steady-state) and the rate of time preference. There is no particular reason why new technology would affect the rate of time preference so the mechanism for why interest rates might rise are not clear to us but we outline it as a hypothetical possibility.

Non-Steady States

Our comparison of steady states allows us to be relatively general about the way that new technology affects production, but does come at the cost that we do not analyse the transition from one steady-state to another, and do not analyse an economy in which technology changes over time. One way in which our comparison of steady-states may be limited is in its analysis of a singularity if it becomes possible to produce robots that are identical to (or better than) people. If this is the case then labor is no longer effectively a fixed factor. In a steady-state this is a situation in which wages would fall to zero and prices of all goods would also fall to zero if there is perfect competition. This would be an economy of total abundance because there is no longer a natural limit to the level of production caused by the existence of labor as a fixed factor. But one could not get to the point of total abundance instantaneously so a model of transition would be needed.

Supplementary Material: Proof of Results and Specific Examples

Proof of Result 1

Given assumption HOM the expenditure function for workers can be written as $e(p)u^w$ where $e(p)$ is the price index and u^w is the (column) vector of utilities for each type of worker. In equilibrium total expenditure must equal total income for each type of worker obtainable from labor income which gives us:

$$w = e(p)u^w \tag{14}$$

The total utility of workers will be Lu^w, which can be interpreted as (total) real wages. Using (14) and taking logs we have that:

$$\log Lu^w = \log Lw - \log e(p) \tag{15}$$

Now consider a change $d\theta$ in technology. From (15) we have that:

$$\begin{aligned}\frac{Ldu^w}{Lu^w} &= \frac{Ldw}{Lw} - \frac{e_p(p)dp}{e(p)} = \frac{Ldw}{Lw} - \frac{X^w dp}{Lu^w e(p)} \\ &= \frac{1}{Lw}\left[Ldw - X^w dp\right]\end{aligned} \tag{16}$$

Where X^w is the vector of consumption demands by workers, and we have used Shephard's Lemma to substitute out for e_p.

Given the assumption that existing capital all depreciates at a rate δ, to maintain capital stocks of K in a steady-state requires investment of $I = \delta K$. Capital-owners have total income per period of $(r+\delta)p^i K$ but have to spend $\delta p^i K$ on maintaining their capital holdings so have total consumption expenditure of $rp^i K$.

Now consider change $d\theta$ in technology. Since the prices of consumption goods equal their unit costs, the change in prices can be written as:

$$\begin{aligned}dp &= c_w dw + c_p dp + c_k dp^k + c_\theta d\theta \\ &= c_w dw + c_p dp + (r+\delta)c_k dp^i + c_\theta d\theta\end{aligned} \tag{17}$$

And the change in the price of investment goods can be written as:

$$\begin{aligned}dp^i &= c_w^i dw + c_p^i dp + c_k^i dp^k + c_\theta^i d\theta \\ &= c_w^i dw + c_p^i dp + (r+\delta)c_k^i dp^i + c_\theta^i d\theta\end{aligned} \tag{18}$$

From Shephard's Lemma, total demands for intermediate goods, X^d can be written as:

$$X^d = Xc_p + Ic_p^i = Xc_p + \delta K c_p^i \tag{19}$$

There is also an equivalent equation for the demand for capital goods:

$$K^d = Xc_k + Ic_k^i = Xc_k + \delta K c_k^i \tag{20}$$

And total demands for labor can be written as:

$$L^d = Xc_w + Ic_w^i = Xc_w + \delta K c_w^i \tag{21}$$

In equilibrium, we must have the complementary slackness condition $\left(L - L^d\right)w = 0$. This implies that if wages for a particular type of labor is positive then demand for that type of labor must equal supply. But if the wages for a particular type of labor are zero then it is possible that demand is less than supply and there is some unemployment of that type of labor. For the moment assume that $L = L^d$ for all types of labor as this makes the algebra simpler. But we discuss the other case at the end of the proof – it does not alter the result.

Now pre-multiply (17) by X, the total vector of consumption goods (some of which are used as intermediate goods) and (18) by I and sum them to have:

$$\begin{aligned} &Xdp + Idp^i \\ &\quad = Xc_w dw + Ic_w^i dw + Xc_p dp + Ic_p^i dp + \left(r+\delta\right) Xc_k dp^i + I\left(r+\delta\right)c_k^i dp^i + \left(Xc_\theta + Ic_\theta^i\right)d\theta \end{aligned} \tag{22}$$

Using (19)-(21) this can be written as:

$$\left[X - X^d\right]dp + Idp^i = Ldw + \left(r+\delta\right)Kdp^i + \left[Xc_\theta + Ic_\theta^i\right]d\theta \tag{23}$$

Now $X - X^d = X^w + X^k$ where X^w is consumption of workers and X^k is consumption of capitalists, and $I = \delta K$ in steady-state in which case we have:

$$\left[Ldw - X^w dp\right] + \left[rKdp^i - X^k dp\right] = -\left[Xc_\theta + Ic_\theta^i\right]d\theta > 0 \tag{24}$$

The first term in square brackets is, from (16), the change in the total utility of workers. The second term in square brackets is related to the change in the total utility of capitalists. The sum of these terms must be positive saying that the gains from new technology must flow either to workers or capitalists. But this does not say that workers must get some share of the gains. But (24) can be written as:

$$\begin{aligned} Ldu^w &= \left[Ldw - X^w dp\right] = \left[X^k dp - rKdp^i\right] - \left[Xc_\theta + Ic_\theta^i\right]d\theta \\ &= \left[\left(p \circ X^k\right)\frac{dp}{p} - r\left(p^i \circ K\right)\frac{dp^i}{p^i}\right] - \left[Xc_\theta + Ic_\theta^i\right]d\theta \end{aligned} \tag{25}$$

Where $\circ$ denotes a Hadamard product and dp / p is vector of proportional changes in prices. (25) can then be written as:

$$\begin{aligned} Ldu^w &= pX^k \left[\frac{\left(p \circ X^k\right)\frac{dp}{p}}{pX^k} - \frac{\left(p^i \circ K\right)\frac{dp^i}{p^i}}{p^i K}\right] - \left[Xc_\theta + Ic_\theta^i\right]d\theta \\ &= pX^k \left[\frac{d\tilde{p}}{\tilde{p}} - \frac{d\tilde{p}^i}{\tilde{p}^i}\right] - \left[Xc_\theta + Ic_\theta^i\right]d\theta \end{aligned} \tag{26}$$

Where the first line uses the fact that from the capitalists' budget constraint $pX^k = rp^i K$ and $\tilde{p}$ is the consumer price index and $\tilde{p}^i$ the investment goods price index. The term in the difference in inflation rates is positive if investment goods prices fall faster than consumer goods prices (e.g. because consumer goods involve more labour-intensive services), proving the result.

Now consider the case where the wages of some types of labor fall to zero and there is possibly some unemployment for those types. This case can be allowed for in the following way. Remove these types of workers from the cost functions as the zero wage allows us to do this. The result above then goes through for the set of workers with non-zero wages. But the average wage result applies to all workers if we include the unemployed as having zero earnings. The set of types of workers with zero wages may change with the technology but the formulae above remain valid even for this case.

If all types of workers have zero wages then we are in a situation where all prices (as well as wages) will be zero, i.e. this is a situation of total abundance. Our static analysis is not well-suited to this case.

Proof of Result 2

Stack the prices of consumption and investment goods into a single vector ρ. Combine the cost functions into a single vector as well – continue to denote this by c. Without loss of generality we can normalize the wage to be constant as technology changes so we simply have to see whether prices rise or fall to judge whether real wages go down or up. Write the stacked prices as:

$$\rho = c(w, \rho, \theta) \tag{27}$$

Taking logs and differentiating leads to:

$$\frac{d \log \rho}{d\theta} = \Lambda^{\rho} \frac{d \log \rho}{d\theta} + \Lambda^{w} \frac{d \log w}{d\theta} + \frac{\partial \log c}{\partial \theta} \tag{28}$$

Where Λ^{ρ} is a non-negative matrix whose ijth element, γ_{ij}^{ρ}, is given by:

$$\gamma_{ij}^{\rho} = \frac{\rho_j}{c_i} \frac{\partial c_i}{\partial \rho_j} \tag{29}$$

From Shephard's Lemma we know that the derivative of the cost function with respect to a price is the per output demand for that input. Hence γ_{ij}^{ρ} is the share of the cost of input j in the production of good i. Similarly, Λ^{w} is a non-negative matrix whose ijth element, γ_{ij}^{w}, is given by:

$$\gamma_{ij}^{w} = \frac{w_j}{c_i} \frac{\partial c_i}{\partial w_j} \tag{30}$$

γ_{ij}^{w} is the share of the cost of type of labor j in the production of good i. The ith row of Λ^{ρ} must sum to one minus the share of labor costs in the production of good i, and the ith row of Λ^{w} must sum to the share of labor in the production of good i. Denote the vector of shares of labor costs by s.

Denote the maximum goods price change as $\dfrac{\partial \log \rho^{\max}}{\partial \theta}$ and the maximum wage change as $\dfrac{\partial \log w^{\max}}{\partial \theta}$. Then (28) implies that, for all goods, we must have:

$$
\begin{aligned}
\frac{d\log\rho}{d\theta} &\le \Lambda^{\rho}\frac{d\log\rho^{\max}}{d\theta}+\Lambda^{w}\frac{d\log w^{\max}}{d\theta}+\frac{\partial\log c}{\partial\theta} \\
&=(1-s)\frac{d\log\rho^{\max}}{d\theta}+s\frac{d\log w^{\max}}{d\theta}+\frac{\partial\log c}{\partial\theta}
\end{aligned}
\tag{31}
$$

With equality only for goods which are produced only use goods and labor with the maximum price and wage changes. (31) applies for all goods, including goods with prices increasing at the fastest rate. For these goods we can re-arrange (31) to yield:

$$
\frac{d\log\rho^{\max}}{d\theta}\le\frac{d\log w^{\max}}{d\theta}+\frac{1}{s}\frac{\partial\log c}{\partial\theta}\le\frac{d\log w^{\max}}{d\theta}
\tag{32}
$$

Where s is the labor share for that good. This proves the result but is only valid if $s>0$. What happens if the good with the highest price increase is produced using no labor? If this good is produced using some goods with price increases below the maximum then it cannot be the good with the highest price index as (31) will be a strict inequality leading to a contradiction if $s=0$. If it is only produced using goods with the highest price increase, this is a contradiction if there is any technical change in that sector. If there is not, there is a set of goods with no technical change produced with no labor and only each other as intermediate or capital goods. Because these goods are produced without fixed factors, there is no limit to the supply of them so the price of them will always be zero, contradicting the fact that they have the highest price index.

Ultimately workers are only interested in the price of consumption goods and this result seems to leave open the possibility that prices only fall for investment goods. But if these investment goods are used, directly or indirectly (meaning it might only be used to produce investment goods but those investment goods are ultimately used in the production of consumption goods through some chain), this must be transmitted to the price of some consumption good. There will be no production, in equilibrium, of a set of investment goods only used to produce themselves in which case the result says that technological change in goods that are not produced will have no benefit.

An example where the average wage of workers falls

The example outlined here shows how the real wage of workers can fall if new technology causes the price of investment goods to rise relative to consumption goods i.e. the condition of Result 1 is not satisfied. Such an example must have at least two types of goods (to allow relative prices to change) and two types of labor (otherwise Result 2 would apply).

Assume that there are two sectors, a consumption good sector and an investment good sector. The consumption good is assumed to be produced by one type of labor – call it c-labor – and capital goods, according to the production function:

$$
X=L_c f\left(\frac{K_c}{L_c},\theta\right)=L_c f\left(k_c,\theta\right)
\tag{33}
$$

The investment good is assumed to be produced by a different type of labor – call it i-labor – and capital goods according to the production function:

$$
I=L_i g\left(\frac{K_i}{L_i},\theta\right)=L_i g\left(k_i,\theta\right)
\tag{34}
$$

Assume the price of consumption good is numeraire – set it equal to 1.

The wage of c-labour will be given by:

$$w_c = f - k_c f_k \tag{35}$$

And the demand for capital in consumption good sector will be given by:

$$(r+\delta)\, p_i = f_k \tag{36}$$

The wage of i-labour will be given by:

$$w_i = p_i \left[g - k_i g_k \right] \tag{37}$$

And the demand for capital in the i-sector will be given by:

$$(r+\delta)\, p_i = p_i g_k \tag{38}$$

Note that (38) implies that the capital-labour ratio in the i-sector solves the equation:

$$g_k(k_i, \theta) = (r+\delta) \tag{39}$$

Which, conveniently, is independent of prices. Given the inelastic supply of i-labour this also fixes the amount of i-capital. Now the total supply of c-capital must satisfy the equation

$$\delta(K_c + L_i k_i) = L_i g \tag{40}$$

Which can be re-arranged to give:

$$K_c = L_i \frac{g - \delta k_i}{\delta} \tag{41}$$

Which implies that the amount of c-capital can also be solved for independent of prices. This then implies that the total capital stock is given by:

$$K = L_i \frac{g}{\delta} \tag{42}$$

Note that the production function for the consumer good plays no role in determining the total level of capital or its allocation across sectors.

Total income to workers must be the difference between the production of consumption goods and the consumption of capitalists which is the part of their income not used to cover depreciation i.e. $rp^i K$ of capitalists. This implies:

$$\begin{aligned} Lw = L_c w_c + L_i w_i &= L_c f - rp^i K \\ &= L_c f - \frac{r}{r+\delta} f_k K = L_c f - \frac{r}{r+\delta} \frac{g}{\delta} f_k L_i \end{aligned} \tag{43}$$

Now suppose that the nature of the new technology is that it does not affect production of the investment good (this is an example so this is not meant to be plausible). In this case g is fixed and the capital-labor ratios in the two sectors are unaffected by the new technology. Differentiating (43) we have that:

$$\frac{d(Lw)}{d\theta} = L_c f_\theta - \frac{r}{r+\delta} \frac{g}{\delta} f_{k\theta} L_i \tag{44}$$

The first term is positive but the second term can outweigh it if new technology heavily raises the marginal product of capital in the consumption goods sector.

Note the link to the condition in Result 1. (36) implies that the relative price of investment goods rises (resp. falls) if $f_{k\theta} > (<) 0$. If $f_{k\theta} < 0$ the relative price of investment goods falls and (44) says that average wages must rise, consistent with Result 1. But if $f_{k\theta} > 0$ then the relative price of investment goods rises and and (44) says that average wages can fall.

Proof of Result 3

If there are many types of labor but they are in perfectly elastic supply then relative wages are constant. The cost functions can be written as a function of the wage of one type of labor chosen as numeraire and the relative wages which are exogenous. The model is then reduced to one in which there is only one wage and the corollary of result 2 that if there is only one type of labor, real wages must rise can then be applied.

A Model of Imperfect Competition

Many current models of the economy assume that individual firms have increasing returns to scale. This section considers what happens if that is the case. Continue to use c to denote marginal costs but now assume that firms have to pay a fixed cost $c^f\left(w,\rho,\theta\right)$ to enter an industry – for simplicity here we use the stacked price approach of Result 1 rather than distinguish between consumption and investment goods.

Increasing returns at the individual firm level is not compatible with perfect competition so we assume that price is a mark-up, μ , possibly varying across sectors, on marginal costs i.e. we have:

$$\rho = \left(1+\mu\right)c \tag{45}$$

We treat μ as exogenously given though it is usually derived from other parameters in the model – for our purpose this is not important.

Free entry into an industry implies that total revenue of the industry must equal total costs which can be written as:

$$\left(\rho - c\right)X = Nc^f \tag{46}$$

Where X is gross output and N is the number of firms. Using (45) this can be written as:

$$N = X\frac{\mu c}{c^f} \tag{47}$$

Now consider input demands. Total demand from this sector for input j can be written as:

$$X\frac{\partial c}{\partial p_j} + N\frac{\partial c^f}{\partial p_j} \tag{48}$$

One can derive a similar expression for the demand for a type of labor replacing the price with the wage. Using (47), (48) can be written as:

$$X\frac{\partial c}{\partial p_j}+X\mu c\frac{\partial \log c^f}{\partial p_j}=X\frac{\partial c}{\partial p_j}\left[1+\mu\frac{\frac{\partial \log c^f}{\partial p_j}}{\frac{\partial \log c}{\partial p_j}}\right] \tag{49}$$

If marginal costs and fixed costs using inputs in the same proportions this implies that total factor demands can be written as:

$$X\frac{\partial c}{\partial p_j}\left[1+\mu\right] \tag{50}$$

Which is completely isomorphic to our standard model using $\left(1+\mu\right)c$ as the cost function.

This leaves open the possibility that technology might harm workers if it affects fixed costs in a different way to marginal costs. And it does assume that all firms within an industry are identical – many models assume heterogeneity which gives rents to the more productive firms. It is possible that new technology might disproportionately advantage these firms. The analysis of these models is left for later research.

References and Notes

Acemoglu, D. and D. Autor. 2011. "Skills, Tasks and Technologies: Implications for Employment and Earnings." *Handbook of Labor Economics, Vol 4b*, 4, 1043-171.
Acemoglu, Daron. 2008. *Introduction to Modern Economic Growth*. Princeton University Press.
Acemoglu, Daron and Pascual Restrepo. 2017a. " The Race Between Man and Machine: Implications of Technology for Growth, Factor Shares and Employment ," *Working Paper.* NBER,
Acemoglu, Daron and Pascual Restrepo. 2017b. "Robots and Jobs: Evidence from US Labor Markets."

Autor, D. H. 2015. "Why Are There Still So Many Jobs? The History and Future of Workplace Automation." *Journal of Economic Perspectives*, 29(3), 3-30.
Autor, D. H. and D. Dorn. 2013. "The Growth of Low-Skill Service Jobs and the Polarization of the Us Labor Market." *American Economic Review*, 103(5), 1553-97.
Autor, David; David Dorn; Lawrence F Katz; Christina Patterson and John Van Reenen. 2017. "The Fall of the Labor Share and the Rise of Superstar Firms," Centre for Economic Performance, LSE,
Bostrom, Nick. 2014. *Superintelligence: Paths, Dangers, Strategies*. OUP Oxford.
Bowen, Harold R. 1966. *Report of the National Commission on Echnology, Automation and Economic Progress, Volume 1*. Washington: US Government Printing Office
Brynjolfsson, Erik and Andrew McAfee. 2014. *The Second Machine Age: Work, Progress, and Prosperity in a Time of Brilliant Technologies*. WW Norton & Company.
De Loecker, Jan and Jan Eeckhout. 2017. "The Rise of Market Power and the Macroeconomic Implications," National Bureau of Economic Research,
Ford, Martin. 2015. *Rise of the Robots: Technology and the Threat of a Jobless Future*. Basic Books.
Frey, Carl Benedikt and Michael A Osborne. 2017. "The Future of Employment: How Susceptible Are Jobs to Computerisation?" *Technological Forecasting and Social Change*, 114, 254-80.
Goos, Maarten; Alan Manning and Anna Salomons. 2014. "Explaining Job Polarization: Routine-Biased Technological Change and Offshoring." *The American Economic Review*, 104(8), 2509-26.
Graetz, Georg and Guy Michaels. 2015. "Robots at Work," *Discussion Paper.* Centre for Economic Performance, London School of Economics,
Grossman, Gene M; Elhanan Helpman; Ezra Oberfield and Thomas Sampson. 2017. "Balanced Growth Despite Uzawa." *The American Economic Review*, 107(4), 1293-312.
Guerreiro, Joao; Sérgio Rebelo and Pedro Teles. 2017. "Should Robots Be Taxed?," CEPR Discussion Papers,
House, White. 2016. "Artificial Intelligence, Automation, and the Economy," *Executive office of the President. https://obamawhitehouse. archives. gov/sites/whitehouse. gov/files/documents/Artificial-Intelligence-Automation-Economy. PDF.*
IMF. 2017. *World Economic Outlook, April 2017: Gaining Momentum?* Washington: IMF.
Jones, Charles I. 2016. "The Facts of Economic Growth." *Handbook of Macroeconomics*, 2, 3-69.
Karabarbounis, Loukas and Brent Neiman. 2013. "The Global Decline of the Labor Share." *The Quarterly Journal of Economics*, 129(1), 61-103.
Krusell, Per; Lee E Ohanian; Jose-Victor Rios-Rull and Giovanni L Violante. 2000. "Capital-Skill Complementarity and Inequality: A Macroeconomic Analysis." *Econometrica*, 68(5), 1029-53.
Stone, Peter; Rodney Brooks; Erik Brynjolfsson; Ryan Calo; Oren Etzioni; Greg Hager; Julia Hirschberg; Shivaram Kalyanakrishnan; Ece Kamar and Sarit Kraus. 2016. "Artificial Intelligence and Life in 2030." *One Hundred Year Study on Artificial Intelligence: Report of the 2015-2016 Study Panel*.
Susskind, Daniel. 2017. "A Model of Technological Unemployment," *Economics Department Working Paper.* University of Oxford,
Susskind, Richard and Daniel Susskind. 2015. *The Future of the Professions: How Technology Will Transform the Work of Human Experts*. Oxford University Press, USA.

Uzawa, Hirofumi. 1961. "Neutral Inventions and the Stability of Growth Equilibrium." *The Review of Economic Studies*, 28(2), 117-24.
Varian, Hal R. 2010. *Intermediate Microeconomics : A Modern Approach*. New York: W.W. Norton & Co.

CENTRE FOR ECONOMIC PERFORMANCE
Recent Discussion Papers

1496	Nicholas Bloom Charles I Jones John Van Reenen Michael Webb	Are Ideas Getting Harder to Find?
1495	Christian Krekel	Can Rising Instructional Time Crowd out Student Pro-Social Behaviour? Unintended Consequences of a German High School Reform
1494	Lorenzo Caliendo Luca David Opromolla Fernando Parro Alessandro Sforza	Goods and Factor Market Integration: A Quantitative Assessment of the EU Enlargement
1493	Andrew E. Clark Sarah Flèche Warn N. Lekfuangfu	The Long-Lasting Effects of Family and Childhood on Adult Wellbeing: Evidence from British Cohort Data
1492	Daniel Paravisini Veronica Rappoport Philipp Schnabl	Specialization in Bank Lending: Evidence from Exporting Firms
1491	M.A. Clemens J. Hunt	The Labor Market Effects of Refugee Waves: Reconciling Conflicting Results
1490	V. Bhaskar Robin Linacre Stephen Machin	The Economic Functioning of Online Drug Markets
1489	Abel Brodeur Warn N. Lekfuangfu Yanos Zylberberg	War, Migration and the Origins of the Thai Sex Industry
1488	Giuseppe Berlingieri Patrick Blanchenay Chiara Criscuolo	The Great Divergence(s)

1487	Swati Dhingra Gianmarco Ottaviano Veronica Rappoport Thomas Sampson Catherine Thomas	UK Trade and FDI: A Post-Brexit Perspective
1486	Christos Genakos Tommaso Valletti Frank Verboven	The Compact City in Empirical Research: A Quantitative Literature Review
1485	Andrew E. Clark Sarah Flèche Richard Layard Nattavudh Powdthavee George Ward	The Key Determinants of Happiness and Misery
1484	Matthew Skellern	The Hospital as a Multi-Product Firm: The Effect of Hospital Competition on Value-Added Indicators of Clinical Quality
1483	Davide Cantoni Jeremiah Dittmar Noam Yuchtman	Reallocation and Secularization: The Economic Consequences of the Protestant Reformation
1482	David Autor David Dorn Lawrence F. Katz Christina Patterson John Van Reenen	The Fall of the Labor Share and the Rise of Superstar Firms
1481	Irene Sanchez Arjona Ester Faia Gianmarco Ottaviano	International Expansion and Riskiness of Banks
1480	Sascha O. Becker Thiemo Fetzer Dennis Novy	Who voted for Brexit? A Comprehensive District-Level Analysis

The Centre for Economic Performance Publications Unit
Tel: +44 (0)20 7955 7673 Email info@cep.lse.ac.uk
Website: http://cep.lse.ac.uk Twitter: @CEP_LSE