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The climate beta ☆

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ABSTRACT

How does climate-change mitigation affect the aggregate consumption risk borne by future generations? In other words, what is the 'climate beta'? In this paper we argue using a combination of theory and integrated assessment modelling that the climate beta is positive and close to unity for maturities of up to about one hundred years. This is because the positive effect on the climate beta of uncertainty about exogenous, emissions-neutral technological progress overwhelms the negative effect on the climate beta of uncertainty about the carbon-climate-response, particularly the climate sensitivity, and the damage intensity of warming. Mitigating climate change therefore has no insurance value to hedge the aggregate consumption risk borne by future generations. On the contrary, it increases that risk, which justifies a relatively high discount rate on the expected benefits of emissions reductions. However, the stream of undiscounted expected benefits is also increasing in the climate beta, and this dominates the discounting effect so that overall the net present value of carbon emissions abatement is increasing in the climate beta.

Introduction

Because most of the benefits of mitigating climate change arise in the distant future, the choice of the rate at which these benefits should be discounted is a crucial determinant of our collective willingness to reduce emissions of greenhouse gases. The discount-rate controversy that has emerged in economics over the last two decades shows that there is still substantial disagreement about the choice of this parameter for cost-benefit analysis. One source of controversy comes from the intrinsically uncertain nature of these benefits. It is a tradition in economic theory and finance to adapt the discount rate to the risk profile of the flow of net benefits generated by the policy under scrutiny. The underlying intuition is simple. If a policy tends to raise the collective risk borne by the community of risk-averse stakeholders, this policy should be penalised by increasing the discount rate by a risk premium specific to the policy. On the contrary, if a policy tends to hedge collective risk, this insurance benefit should be acknowledged by reducing the rate at which expected net benefits are discounted, i.e. by adding a negative risk premium to the discount rate.

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This simple idea can easily be implemented through the Consumption-based Capital Asset Pricing Model (CCAPM) of Lucas (1978). An investment raises intertemporal social welfare if and only if its Net Present Value (NPV) is positive, where the NPV is obtained by discounting the expected cash flow of the investment at a risk-adjusted rate. This investment-specific discount rate is written as

$$r = r_f + \beta\pi,$$

where r_f is the risk-free rate, π is the systematic risk premium and β is the CCAPM beta of the specific investment under scrutiny. It is defined as the elasticity of the net benefit of the investment with respect to a change in aggregate consumption. This means that a marginal project, whose net benefit is risky but uncorrelated with aggregate consumption, should be discounted at r_f, because implementing such a project has no effect at the margin on the risk borne by the risk-averse representative agent. A project with a positive β raises collective risk and should be penalised by discounting its flow of net benefits at a higher rate, and *vice versa* for a project with a negative β.

The objective of this paper is not to offer a new contribution to the debate about the choice of the risk-free rate, or of the systematic risk premium: there have been many of these in the recent past (see Kolstad et al., 2014, for a recent summary). Rather, the aim of this paper is to discuss the CCAPM β that should be used to value climate-mitigation projects. This 'climate β' should play an important role in the determination of the social cost of carbon (i.e. the present social value of damages from incremental carbon emissions), just as an asset β is known to be the main determinant of the asset price. Indeed, in the United States over the last 150 years, financial markets have exhibited a real risk-free rate of around 1.6% and a systematic risk premium of around 4.8 percentage points. Thus assets whose CCAPM betas are respectively 0 and 2 should be discounted at very different rates of 1.6% and 11.2% respectively.[1]

Howarth (2003) was one of the first to examine this question. He pointed out that the net benefits of climate-mitigation projects should be discounted at r_f, provided those net benefits are certainty equivalents (thereby containing a risk premium). He went on to suggest that the climate β is negative, but did not offer detailed analysis to back up the suggestion.[2] Weitzman's Weitzman (2007a) *Review of the Stern Review* also emphasised that the appropriate discount rate for climate-mitigation projects depends on the correlation between mitigation benefits and consumption, although he did not offer detailed analysis of this correlation either. He was contributing to a debate about discounting in the wake of the *Stern Review* (Stern, 2007), in which some scholars' views of what is an appropriate rate at which to discount mitigation benefits were in effect anchored against r_f, while others were anchored against r for standard investments, such as a diversified portfolio of equities. As Weitzman pointed out, there is no guarantee the features of climate mitigation match either of these cases.

Sandsmark and Vennemo (2007) provided the first explicit investigation of the climate β. They constructed a simplified climate-economy model, in which the only stochastic parameter represents the intensity of damages – the loss of GDP – associated with a particular increase in global mean temperature. Given this set-up, large damages are simultaneously associated with low aggregate consumption and a large benefit from mitigating climate change. Hence this model yields a negative climate β. Weitzman (2013) extended the idea that emissions abatement is a hedging strategy against macroeconomic risk, invoking potential catastrophic climate change and its avoidance, while Daniel et al. (2015) also find a negative climate β in the more general context of Epstein-Zin preferences, since their estimation of the social cost of carbon is increasing in the degree of risk aversion of the representative agent.[3]

On the other hand, an alternative channel driving the climate β may exist. Nordhaus (2011) concludes from simulations with the RICE-2011 integrated assessment model (IAM) that "*those states in which the global temperature increase is particularly high are also ones in which we are on average richer in the future*." This conclusion implicitly signs the climate β and is compatible with the following scenario. Suppose that the only source of uncertainty is exogenous, emissions-neutral technological progress, which determines economic growth. In this context, as long as growth is in some measure carbon-intensive, rapid technological progress yields at the same time more consumption, more emissions, more warming and, under most circumstances, a larger marginal benefit from reducing emissions. This would yield a positive correlation between consumption and the benefits of mitigation, i.e. a positive climate β. This channel is present in neither Sandsmark and Vennemo (2007) nor Daniel et al. (2015), because they assume a sure growth rate of pre-climate-damage production and consumption.

In this paper, we provide an overarching analysis of the sign and size of the climate β, which encompasses the aforementioned two stories, as well as other drivers. Our analysis is in two complementary parts. First, we explore analytical properties of the climate β in a simplified model. As well as serving to develop intuition, the model allows us to explore the role of the structure of climate damages, in particular whether they are multiplicative, as standardly assumed, or additive. We then estimate the climate β numerically using a dynamic IAM with investment effects on future consumption. We perform Monte Carlo simulations of the DICE model, introducing ten key sources of uncertainty about the benefits of climate mitigation and future consumption. We use these simulations to estimate the climate β for different maturities of our immediate efforts to reduce emissions. We find that in our version of DICE the positive effect on β of uncertain technological

[1] See Shiller's dataset: http://www.econ.yale.edu/~shiller/data.htm.

[2] Aalbers (2009) situated the climate β within a broader set of theoretical conditions, according to which climate-mitigation investments might be discounted at a lower rate than other investments.

[3] Our paper sits within a large literature on uncertainty and climate policy (see Heal and Millner, 2014, for a review). Recent papers relevant to our analysis include Bansal et al. (2015) and Lemoine (2015).

progress dominates the negative effect on β of uncertain climate sensitivity and damages. Put another way, emissions reductions actually increase the aggregate consumption risk borne by future generations. This is in line with Nordhaus (2011), but our analysis advances the literature by quantifying the climate β explicitly. We also extend Nordhaus' analysis in several ways: we treat TFP growth as a first-order autoregressive process, consistent with historical data; we treat the income elasticity of damages as uncertain, so damages are not necessarily multiplicative; and we include the possibility of catastrophic damages.

In the next section we review β in the context of Lucas' CCAPM and clarify how it relates to the NPV of a project. *A simple analytical model of the climate beta* describes our analytical model and its results. *Estimating beta with DICE* describes how we set up and run the DICE model in order to estimate the climate β. *Results* sets out the results from our DICE simulations. The subsequent sections provide a discussion and some concluding comments.

The CCAPM beta

In this section, we derive the standard CCAPM valuation principles as in Lucas (1978) and obtain an important result, which means that the relationship between the climate β and the NPV of climate mitigation is very likely to be positive, the opposite of what one might have expected.

Consider a Lucas-tree economy with a von Neumann-Morgenstern representative agent, whose utility function u is increasing and concave and whose rate of pure preference for the present is δ. Her intertemporal welfare at date 0 is

$$W_0 = \sum_{t=0}^{\infty} e^{-\delta t}\mathbb{E}\left[u(c_t)\right], \tag{1}$$

where c_t measures her consumption at date t. Because c_t is uncertain from date 0, it is a random variable. We contemplate an action at date 0, which has the consequence of changing the flow of future consumption to $c_t + \epsilon B_t$, $t = 0, 1, \ldots$, where B_t is potentially random and potentially statistically related to c_t. For small ϵ, the change in intertemporal welfare generated by this action is equivalent to an immediate increase in consumption by ϵNPV, where NPV can be measured as follows:

$$\text{NPV} = \sum_{t=0}^{\infty} e^{-\delta t}\mathbb{E}B_t\frac{u'(c_t)}{u'(c_0)} = \sum_{t=0}^{\infty} e^{-r_t t}\mathbb{E}B_t, \tag{2}$$

with

$$r_t = \delta - \frac{1}{t}\ln\frac{\mathbb{E}B_t u'(c_t)}{u'(c_0)\mathbb{E}B_t}. \tag{3}$$

The right-hand side of Eq. (2) can be interpreted as the NPV of the action, where, for each maturity t, the expected net benefit $\mathbb{E}B_t$ is discounted at a risk-adjusted rate r_t, which is in turn defined by Eq. (3). In order to simplify Eq. (3), we make three additional assumptions, which are in line with the classical calibration of the CCAPM model:

1. For all states of nature, the elasticity of the net conditional benefit at date t with respect to a change in consumption at t is constant, so that there exists $\beta_t \in \mathbb{R}$ such that $\mathbb{E}\left[B_t|c_t\right] = c_t^{\beta_t}$.
2. Consumption follows a geometric brownian motion with drift μ and volatility σ, so that $x_t = \ln c_t/c_0 \sim N(\mu t, \sigma^2 t)$.
3. The representative agent has constant relative risk aversion γ, so that $u'(c_t) = c_t^{-\gamma}$.

This allows us to rewrite Eq. (3) as follows:

$$r_t = \delta - \frac{1}{t}\ln\frac{\mathbb{E}\left[e^{(\beta_t-\gamma)x_t}\right]}{\mathbb{E}\left[e^{\beta_t x_t}\right]}. \tag{4}$$

We now use the well-known property that if $x \sim N(a, b^2)$, then for all $k \in \mathbb{R}$, $\mathbb{E}\left[\exp(kx)\right] = \exp\left(ka + 0.5k^2b^2\right)$. Applying this result twice in the above equation implies that

$$r_t = \delta + \left(\beta_t\mu + 0.5\beta_t^2\sigma^2\right) - \left[(\beta_t - \gamma)\mu + 0.5(\beta_t - \gamma)^2\sigma^2\right] = r_f + \beta_t\pi, \tag{5}$$

where the risk-free rate r_f equals

$$r_f = \delta + \gamma\mu - 0.5\gamma^2\sigma^2, \tag{6}$$

and the systematic risk premium equals

$$\pi = \gamma\sigma^2. \tag{7}$$

Observe that both the risk-free rate r_f and the systematic risk premium π have a flat term structure in this framework. However, the risk-adjusted discount rate r_t may have a non-constant term structure, which is homothetic in the term structure of β_t. Therefore later in the paper we shall be interested in estimating the term structure $(\beta_1, \beta_2, \ldots)$ of the climate β. This can be done by observing that if $\mathbb{E}\left[B_t|c_t\right] = c_t^{\beta_t}$, then β_t is nothing other than the regressor of $\ln B_t$ with respect to $\ln c_t$:

$$\ln B_t = \beta_t \ln c_t + \xi_t, \tag{8}$$

where c_t and ξ_t are independent random variables. We take 50,000 draws from a Monte-Carlo simulation of the DICE model to generate, for each maturity t, a series $(\ln B_{it}, \ln c_{it})$, $i = 1, 2, \ldots, 50, 000$, from which the OLS estimate of $\ln B_t$ on $\ln c_t$ gives us the climate β associated with that maturity.

Before turning to the modelling proper, we show an important result. Although a larger β implies a higher discount rate on project benefits, a larger β also raises the expected benefit $\mathbb{E}B_t$ to be discounted. Given the assumptions just set out,

$$\mathbb{E}B_t = c_0^{\beta_t}\mathbb{E}e^{\beta_t x_t} = c_0^{\beta_t}e^{\left(\beta_t\mu+0.5\beta_t^2\sigma^2\right)t}. \tag{9}$$

With constant β, $\mathbb{E}B_t$ is exponentially increasing in t when trend growth μ is positive. Moreover, the larger is β_t, the larger is the growth rate of the expected benefit. The intuition is as follows. The elasticity of benefits with respect to changes in consumption has two reinforcing effects on $\mathbb{E}B_t$. First, if trend growth is rapid, highly elastic investments will benefit more from economic growth. Second, the benefit is a convex function of the growth rate x_t of consumption. By Jensen's inequality, the uncertainty affecting economic growth raises the expected benefit. Because this convexity is increasing in the elasticity β_t, this effect is increasing in β_t. The combination of these two effects may dominate the discounting effect. Indeed, combining Eqs. (5) and (9) implies that

$$\mathrm{NPV} = \sum_{t=0}^{\infty} c_0^{\beta_t}\exp\left[\left(-r_f + \beta_t\left(\mu - \gamma\sigma^2\right) + 0.5\beta_t^2\sigma^2\right)t\right].$$

This is increasing in β_t if β_t is larger than $\gamma - (\mu/\sigma^2)$. This result is summarised in the following proposition:

Proposition 1. *Consider an asset with maturity-specific constant betas, i.e., an asset whose future benefit $B_t|_{t\geq 0}$ is related to future aggregate consumption $c_t|_{t\geq 0}$ in such a way that for all t there exists $\beta_t \in \mathbb{R}$ such that $\mathbb{E}\left[B_t|c_t\right] = c_t^{\beta_t}$. Under the standard assumptions of the CCAPM, the value of this asset is locally increasing in β_t if it is larger than the difference between relative risk aversion and the ratio of the mean by the variance of the growth rate of consumption.*

In the United States over the last century, we observed $\mu \approx 2\%$ and $\sigma \approx 4\%$ (Kocherlakota, 1996; Mehra, 2012). If we take $\gamma = 2$, this implies that $\gamma - (\mu/\sigma^2) \approx -10.5$. Alternatively, to acknowledge the equity premium puzzle, we might take $\gamma = 10$, so that we obtain $\gamma - (\mu/\sigma^2) \approx -2.5$. Because most actions yield β_t larger than either of these two numbers, we conclude that the NPV of most investment projects is increasing in their CCAPM β. The intuition is that the mean growth rate of consumption has been so much larger than its volatility in the past that the effect of a larger β on the expected benefit is much larger than its effect on the discount rate, thereby generating a positive effect on NPV.

A simple analytical model of the climate beta

In this section we derive the climate β from a simple analytical model. As well as helping to formalize notions of what determines the climate β, we also use the model to make an important point about the role of the structure of climate damages, specifically what difference it makes to the climate β that damages are multiplicative in most models such as standard DICE, as opposed to additive.

Let us consider any specific future date t, and let Y represent global economic output within the period $[0, t]$ in the absence of climate damages. Over timescales from a decade to centuries, important recent papers in climate science have shown that (a) the increase in the global mean temperature T is approximately linearly proportional to cumulative carbon dioxide emissions (Allen et al., 2009; Matthews et al., 2009; Zickfeld et al., 2009; Goodwin et al., 2015) and (b) the warming response to an emission of carbon dioxide is virtually instantaneous, and then constant as a function of time (Matthews and Caldeira, 2008; Shine et al., 2005; Solomon et al., 2009; Eby et al., 2009; Held et al., 2010; Ricke and Caldeira, 2014). This enables us to write

$$T = \omega_1 E, \tag{10}$$

where E stands for cumulative industrial CO_2 emissions from 0 to t and ω_1 is a parameter called the carbon-climate response (CCR)[4], combining the response of the carbon cycle to emissions and the temperature response to atmospheric carbon. More complex models like DICE deal with these components separately. Emissions of CO_2 are themselves proportional to pre-damage

[4] The Intergovernmental Panel on Climate Change has also called it the Transient Climate Response to Cumulative Carbon Emissions or TCRE (Collins et al., 2013).

production, so that

$$E = \omega_2 Y - I_0, \tag{11}$$

where $\omega_2 \in [0, 1]$ parameterises the carbon intensity of production, and I_0 is an investment to reduce emissions at the margin.

We assume the damage index D is proportional to increased temperature T at some power k:

$$D = \alpha T^k, \tag{12}$$

where α calibrates the damage function. Parameter k turns out to play an important role in the determination of the climate β in this model. It is widely believed that there is a convex relationship between climate damages and warming, i.e. $k > 1$.

At this stage, let us remain quite general about the way to model the interaction between the damage index D and the index of economic development Y:

$$Q = q(Y, D), \tag{13}$$

where Q is post-damage aggregate output and q is a bivariate function, which is increasing in Y and decreasing in D, with $Q(Y, 0) = Y$ for all Y. If $c \in (0, 1]$ is the propensity to consume output in period t, then the model yields the following reduced form:

$$C(I_0) = cq\left[Y, \alpha\omega_1^k(\omega_2 Y - I_0)^k\right]. \tag{14}$$

We consider the β of a marginal emissions reduction project. The benefit or cash flow of the project is

$$B \equiv \left.\frac{\partial C}{\partial I_0}\right|_{I_0=0} = -c\omega_2^{-1} h Y^{k-1} q_D(Y, hY^k), \tag{15}$$

with $h = \alpha\omega_1^k\omega_2^k$. To sum up, our model characterises the statistical relationship between future consumption $C = C(0)$ and future benefits B as a function of a set of uncertain parameters, such as Y and ω_1. This system is given by the following two equations:

$$\begin{aligned} \ln B &= \ln\left(c\omega_2^{-1}h\right) + (k-1)\ln Y + \ln\left[-q_D(Y, hY^k)\right], \\ \ln C &= \ln c + \ln q\left(Y, hY^k\right). \end{aligned} \tag{16}$$

How does β respond to the various uncertainties in this model? We proceed one by one through each of the key sources of uncertainty.[5]

The climate β when the main source of uncertainty is related to exogenous economic growth

Suppose the only source of uncertainty is exogenous, emissions-neutral technological progress, captured in this simplified model by pre-damage production Y. Then a local estimation of β can be obtained by differentiating the system (16) with respect to Y:

$$\beta \approx \frac{d\ln B/dY}{d\ln C/dY} = \frac{q}{q_D}\frac{(k-1)q_D + Yq_{YD} + Dq_{DD}}{Yq_Y + kDq_D}, \tag{17}$$

where q and its partial derivatives appearing in this equation are evaluated at $\left(Y, hY^k\right)$. The approximation is exact when the uncertainty affecting Y is small.

We calibrate this equation by considering two alternative damage models. In IAMs like standard DICE, damages are assumed to be multiplicative – proportional to Y – which implies that for instance doubling income also doubles absolute climate damages, all else being equal. We can represent this class of model with the function

$$q(Y, D) = Y(1 - D),$$

where D is expressed in percentage points of aggregate income. In this context, (17) simplifies to

$$\beta \approx \frac{k(1-D)}{1-(k+1)D}. \tag{18}$$

[5] It can be seen that, in fact, the CCAPM climate β is not constant in this model. In other words, log climate damages are not linear in log consumption, plus white noise (8). Therefore the risk-adjusted discount rate $r = r_f + \beta\pi$ holds only as an approximation. In reality, the true climate β is stochastic and correlated with economic growth. Recent developments in the finance literature initiated by Jagannathan and Wang (1996) have focused on the impact of stochastic betas on equilibrium asset prices, however the literature is yet to reach the stage where such an extension could be implemented here. In our numerical modelling with DICE, we allow the climate β to be sensitive to maturity, and we are also able to show that at a given date t the relationship between log benefits and log consumption in DICE is linear (data available from the authors on request).

Table 1
Calibration of the climate β using Eq. (18) when the source of uncertainty is exogenous emissions-neutral technological progress. If instead Eq. (20) is used, subtract one from all cells.

	$k = 0.5$	$k = 1$	$k = 2$	$k = 3$
$D = 1\%$	0.50	1.01	2.04	3.09
$D = 3\%$	0.51	1.03	2.13	3.31
$D = 5\%$	0.51	1.06	2.24	3.56
$D = 10\%$	0.53	1.13	2.57	4.50
$D = 20\%$	0.57	1.33	4.00	12.00

In Table 1, we compute the climate β derived from this formula for reasonable values of k and D. It is uniformly positive. Moreover, observe that for damage of less than 5% of GDP,[6] the climate β can be approximated by k. In other words, when the main source of uncertainty is emissions-neutral technological progress, the climate β is approximately equal to the elasticity of climate damage with respect to the increase in global mean temperature. The consensus in the damages literature is that $k > 1$, which implies that the climate $\beta > 1$, based on this source of uncertainty. What is the intuition behind this result? It is simply that faster technological progress serves as a positive shock to output and consumption, which in turn leads to higher emissions (assuming $\omega_2 > 0$, i.e. provided production is not carbon-free), higher total damages from climate change and higher marginal damages, thus higher benefits from emissions abatement. Future climate benefits of mitigation and future consumption are positively correlated.

Obviously, the fact that damages are assumed to be proportional to pre-damage aggregate income Y plays an important role in this calibration. It is a built-in mechanism towards a positive β. Let us therefore consider an alternative, additive damage structure with

$$q(Y, D) = Y - D,$$

where D measures the absolute level of damages expressed in consumption units.[7] In other words, for given warming, doubling pre-damage income has no effect on absolute climate damage. However, the above intuition still applies: increasing income/production results in an increase in emissions as long as $\omega_2 > 0$, which in turn increases temperature and marginal climate damages, if the damage function (12) is convex. So the benefit of mitigation is increased accordingly. What difference then does the additive structure make? When the only source of uncertainty is Y,

$$\beta \approx \frac{(k-1)(Y-D)}{Y-kD}. \tag{19}$$

It is interesting to compare Eqs. (18) and (19), i.e. our estimates of β under multiplicative and additive damages respectively. These two equations are not immediately comparable in fact, because D is expressed in percentage points in the former and in consumption units in the latter. If we express the damage in Eq. (19) in percentage points, $D^{\%} = D/Y$, it can be rewritten as

$$\beta \approx \frac{(k-1)(1-D^{\%})}{1-kD^{\%}}. \tag{20}$$

Eq. (20) is now directly comparable with Eq. (18) and it is clear that the difference lies in replacing k in (18) with $k - 1$ in (20). Thus, the numbers in Table 1 also apply in the additive case, except that *all betas appearing in this table should be reduced by 1*. This means that $\beta < 0$ when k=0.5. We summarise these results in the following proposition:

Proposition 2. *Suppose that the main source of uncertainty is emissions-neutral technological progress, and that climate damages are small ($D \le 5\%$). Then in (a) the multiplicative case, the climate β can be approximated by k, the elasticity of climate damages with respect to warming. In (b) the additive model, the climate β can be approximated by $k - 1$.*

Conversely when climate damages are large, there is no short-cut to using Eqs. (18) and (20) in the multiplicative and additive cases respectively to estimate the climate β. Either way, our analysis shows the classical multiplicative model of climate damages has a built-in mechanism towards producing a positive climate β, which is dampened in the additive model. In fact, our analysis shows that there are two independent channels that generate a positive β in the multiplicative case:

- *(convexity effect)* An increase in Y results in higher cumulative emissions E. This in turn increases marginal climate damage – thus the marginal benefit of mitigation – if the damage function (12) is convex, i.e. if $k > 1$;
- *(proportionality effect)* An increase in Y raises damages directly if damages are proportional to Y.

[6] The literature on the total economic cost of climate change indicates that it might be at most 5% of GDP when T = 3degC (Tol, 2009; IPCC, 2014).
[7] The damage function (12) parameter α would need to be recalibrated in order to yield the same absolute damages as in the multiplicative case, for given warming.

We believe that these two explanations for a positive β in this context have their own merit. The bottom line is that the climate β is positive in this context.

The climate β when the main source of uncertainty is related to the carbon-climate-response and/or the damage intensity of warming

By contrast, let us now suppose that the only source(s) of uncertainty are the CCR parameter ω_1 and/or the damage intensity of warming α. Differentiating the system (16) with respect to ω_1 we obtain

$$\beta \approx \frac{d\ln B/d\omega_1}{d\ln C/d\omega_1} = \frac{d\ln B/d\alpha}{d\ln C/d\alpha} = \frac{q}{q_D}\frac{q_D + Dq_{DD}}{Dq_D}, \tag{21}$$

where q and its partial derivatives appearing in this equation are again evaluated at (Y, hY^k). The approximation is exact when the uncertainty affecting ω_1 is small. Exactly the same expression for β is obtained when assuming that α rather than ω_1 is uncertain, as examined by Sandsmark and Vennemo (2007) and Daniel et al. (2015). Therefore Eq. (21) shows how uncertainty about the CCR and the damage intensity of warming affect the climate β.

Observe that in both the multiplicative and additive models, $q_{DD} = 0$, so that this equation simplifies to

$$\beta \approx \frac{q}{Dq_D}, \tag{22}$$

which is unambiguously negative. The intuition for this result is that a higher CCR results in more warming for given cumulative carbon emissions, which in turn yields at the same time higher marginal damage and lower aggregate consumption. Therefore the uncertainty affecting the CCR results in a negative correlation between B and C, and a negative climate β. Similarly, a higher damage intensity of warming results in greater damages for given emissions, and so on.

Proposition 3. *The climate β is unambiguously negative when the main sources of uncertainty are the carbon-climate response and/or the damage intensity of warming.*

This result is independent of whether climate damages are additive or multiplicative in relation to aggregate consumption. For example, in the multiplicative case $q = Y(1 - D)$, the climate β is approximately equal to $-(1 - D)/D$. The same approximation holds in the additive case.[8] If we expect climate damage of around 5% of GDP, we should use a climate β of around -19. There is also an explanation for why the climate β is so large in absolute value in this context. Take the limiting case $\omega_1 = 0$ as a benchmark, and examine the impact of a marginal increase in its value. This will have a marginal (negative) effect on log consumption, but an unbounded effect on the marginal log benefit, since the initial benefit is zero. In other words, fluctuations in ω_1 yield limited relative fluctuations in consumption, but wild relative fluctuations in marginal benefits. This yields a large β in absolute value.

Overall, this analysis illustrates that uncertainty about technological progress on the one hand and about the carbon-climate response and damage intensity of warming on the other hand most likely have contrasting effects on the climate β, the former positive, the latter two negative. This explains the contradictory conclusions that can be found in the literature. Sandsmark and Vennemo (2007) and Daniel et al. (2015) propose models, in which there is no macro-economic uncertainty independent of climate change. Sandsmark and Vennemo (2007) concluded that fighting climate change has a negative CCAPM β. Daniel et al. (2015) corroborate the result of Sandsmark and Vennemo (2007), by showing that the social cost of carbon is increasing in risk aversion in their model. But Nordhaus (2011) contradicts these conclusions by modelling benefits of mitigation that are positively correlated with aggregate consumption. We propose that this contradiction rests in the fact that the Monte-Carlo simulations in Nordhaus (2011) include a source of uncertainty about emissions-neutral technological progress, and it can also be attributed in part to the fact that DICE/RICE deploys a multiplicative damage structure.

Estimating beta with DICE

We now develop estimates of the β of CO_2 emissions abatement using a modified version of William Nordhaus' well-known DICE model. The advantages of using an IAM include: we can obtain more empirically grounded estimates of the climate β, albeit the empirical basis of IAMs has been criticised (e.g. Stern, 2013; Pindyck, 2013); we can obtain estimates of the term structure of β; and DICE can incorporate a broader range of uncertainties than our analytical model. Another advantage is that DICE is a dynamic model, in which future consumption depends in part on current output through current savings and investment. This introduces a new set of effects on the β, which we describe below. We can also generalise the form of the damage function, so that we can consider the pure multiplicative and additive cases, as well as cases between and beyond these. Naturally the disadvantage of using an IAM is that the workings of the model are less transparent.

[8] Indeed, assuming $q = Y - D$, Eq. (21) yields $\beta \approx -(Y - D)/D$. This is equal to $-(1 - D^{\%})/D^{\%}$, where $D^{\%} = D/Y$ is the damage expressed as a fraction of Y.

Table 2
Uncertain parameters for simulation of modified DICE-2013R.

Parameter	Functional form	Mean	Standard deviation	Source	Effect on β (likely)
Initial trend growth rate of TFP (per year) g_0^A	Normal	0.016	0.009	Maddison project and other sources (see text)	+
TFP shock (per five years) ε	Normal	0	0.06	Maddison project and other sources (see text)	+
Asymptotic global population (millions)	Normal	10,854	1368	United Nations (2013)	–
Initial rate of decarbonisation (per year)	Normal	−0.0102	0.0064	IEA (2013)	(+)
Price of back-stop Technology in 2050 US$/$tCO_2$(2010 prices)	Log-normal	260	51	Edenhofer et al. (2010)	+
Uptake of atmospheric carbon by the upper ocean and biosphere (per five years)	Normal*	0.06835	0.0202	Ciais et al. (2013)	(–)
Climate sensitivity °C per doubling of atmospheric CO_2	Log-logistic**	2.9	1.4	IPCC (2013)	(–)
Damage function coefficient α_2 (% GDP)	Normal	0.0025	0.0006	Tol*** (2009)	(–)
Damage function coefficient α_3 (% GDP)	Normal	0.082	0.028	Dietz and Asheim (2012)	(–)
Income elasticity of damages ξ	Normal	1	0.33	Anthoff and Tol (2012)	(+)

*Truncated from above at 0.1419. ***Truncated from below at 0.75. ***Including corrigenda published in 2014.

DICE couples a neoclassical growth model to a simple climate model. Output of a composite good is produced using aggregate capital and labour inputs, given exogenous total factor productivity (TFP). However, production also leads to CO_2 emissions, which are an input to the climate model, resulting in an increase in the atmospheric concentration of CO_2, radiative forcing of the atmosphere and an increase in global mean temperature. The climate model is coupled back to the economy via a damage function, which is a reduced-form polynomial equation associating an increase in temperature with a loss in utility, expressed in terms of equivalent output.

Our analysis is based on the 2013 version of the model (Nordhaus and Sztorc, 2013). We randomise ten parameters to estimate the climate β (details in Table 2). These parameters represent key uncertainties at all stages in the (circular) chain of cause and effect that links baseline economic and population growth with CO_2 emissions, the climate response to emissions, damages and the costs of emissions abatement. Our parameter selection is informed by, but extends, past studies with DICE, which provide evidence on the most important uncertainties (Nordhaus, 2008; Dietz and Asheim, 2012; Anderson et al., 2014).

We implement a CO_2 emissions reduction project by removing one unit of industrial emissions in 2015.[9] For reasons of computational tractability, we assume that the marginal propensity to save is exogenous and we use Nordhaus' (2013) time series of values, whereby the savings rate is always c. 0.23 – 0.24. Previous research (e.g. Golosov et al., 2014; Jensen and Traeger, 2014), as well as our results below, indicate that endogenous savings decisions would not have a major effect on the results. We take a large Latin Hypercube Sample of the parameter space, which has the advantage of sampling evenly from the domain of each probability distribution, with 50,000 draws. The parameter distributions are assumed independent.

Most of the technical details of the parameter scheme are relegated to the Appendix. However, we make two changes to the structure of standard DICE that are worth detailing here.

[9] This amounts to one gigatonne of CO_2 (Gt CO_2). Since the atmospheric concentration of CO_2 in 2015 is estimated by DICE to be c. 3167Gt CO_2, it may indeed be regarded as a marginal reduction, consistent with the definition of β given above.

TFP growth As a neoclassical growth model, DICE allocates to TFP the portion of output that cannot be explained by capital and labour inputs at their assumed elasticities (0.3 and 0.7 respectively). It follows that TFP growth plays a very significant role in determining GDP growth and therefore future consumption and CO_2 emissions (Kelly and Kolstad, 2001). As discussed in *the climate beta*, the effect on β of variation in TFP growth should be positive.

In DICE, the equation of motion for TFP is

$$A_{t+1} = A_t(1 + g_t^A)$$

where A is TFP and g^A is the growth rate of TFP.

We depart from standard DICE, however, in how we specify the evolution of g_t^A, so that we can distinguish two sources of TFP uncertainty. In particular, we assume that g_t^A evolves according to a transformed first-order autoregressive process with an uncertain trend:

$$g_t^A = \left[(1 - \psi) g_0^A + \psi g_{t-1}^A + \varepsilon_t \right] \left(1 + \delta^A \right)^{-t}, \tag{23}$$

where g_0^A is the uncertain trend growth rate, ε is an independent and identically distributed (i.i.d.) normal shock and ψ is the coefficient of persistence of shocks, which is assumed certain/fixed. This AR(1) process is multiplied by the factor $\left(1 + \delta^A\right)^{-t}$, which is a feature of standard DICE. The parameter δ^A is an assumed rate of decline of TFP growth. It is several times smaller than the expected value of g_0^A.[10]

We estimate g_0^A, ψ and ε using data on historical TFP growth. Since we are forecasting more than two centuries into the future, we want a very long-run series of historical TFP growth, so we use data from the US and UK over the period 1820–2010, compiled from multiple sources.[11] The coefficient of persistence in this time series is $\psi = 0.42$. The estimates of g_0^A and ε can be found in Table 2.[12]

Damage function Damages are one of the most contestable elements of IAMs. By virtue of its accessibility and simplicity in this regard, DICE has become the common means to give expression to competing views. Much of the debate stems from the inability to constrain a reduced-form damage function at warming of more than 3degC, due to the lack of underlying studies. Antipodes in the literature are given by the traditional quadratic form of Nordhaus (2008; 2013) and the damage function proposed by Weitzman (2012), in which damages are much more convex with respect to warming. However, the curvature of the damage function is not the only issue. As the previous section showed, the climate β also depends on the income elasticity of damages.

Our damage function takes the following flexible form:

$$D_t = Y_t \left[1 - \frac{1}{1 + \alpha_1 T_t + \alpha_2 T_t^2 + (\alpha_3 T_t)^7} \right] \left(\frac{Y_t}{Y_0} \right)^{\xi - 1},$$

where D is damages as a percentage of GDP, Y is pre-damage output, α_i, $i \in \{1, 2, 3\}$, are coefficients and ξ is the income elasticity of damages (following the specification in van den Bijgaart et al., 2016). If $\xi = 1$ then the damage function is multiplicative like standard DICE, whereas if $\xi = 0$ it is additive.

We specify both α_2 and α_3 as random parameters ($\alpha_1 = 0$ as usual). The former coefficient enables us to capture uncertainty about damages that is represented by the spread of existing estimates at warming of 2-3degC (summarised in Tol, 2009).[13] The coefficient α_3 may be calibrated so as to capture the difference in subjective beliefs of modellers about how substantial damages may be at higher temperatures (given there are virtually no existing estimates). We follow Dietz and Asheim (2012) in specifying a normal distribution for α_3 that spans existing suggestions: at three standard deviations above the mean total damages approximate Weitzman (2012), while at three standard deviations below the mean they approximately reduce to standard quadratic damages.

Empirical evidence to directly inform ξ is limited to a study by Anthoff and Tol (2012), which used the FUND IAM to estimate ξ disaggregated by region and impact type. Other IAMs like standard DICE cannot be used to estimate ξ, because of course they assume a multiplicative structure. The estimates in Anthoff and Tol (2012) suggest that ξ is normally distributed and centred around the multiplicative case ($\xi = 1$).

Since α_2 and α_3 determine the damage intensity of warming, the main effect of an increase in one or both will be a decrease in β, for a given path of output (cf. Proposition 3). However, unlike the simple model of the previous section, the

[10] Standard DICE simply assumes that $g_t^A = g_0^A\left(1 + \delta^A\right)^{-t}$. Nordhaus (2008) and Dietz and Asheim (2012) randomised g_0^A in this structure, meaning that all the uncertainty about future TFP stems from the initial trend and that this uncertainty is very large.

[11] Bolt and van Zanden (2013); US Census Bureau; US Bureau of Economic Analysis; Feinstein and Pollard (1988); Matthews et al. (1982). We would like to acknowledge the help of Tom McDermott and Antony Millner in collecting these data, although the resulting estimates are our responsibility.

[12] In terms of whether the historical time series conforms with an AR(1) process, we fail to reject the null hypothesis that there is no serial correlation in ε_t, using both Durbin's alternative test and the Breusch-Godfrey test. Based on the Ljung-Box portmanteau test, we reject the null hypothesis that ε_t is white noise, however further inspection of the time-series of ε_t indicates that the heteroskedasticity is caused by noisy data around World War II, rather than a secular trend.

[13] α_2 is also equivalent to the stochastic parameter in the model proposed by Sandsmark and Vennemo (2007).

path of output is not given in a dynamic economy like that of DICE. Instead, when higher damages at time t reduce output at t, there is a knock-on, negative effect on investment at t, which reduces pre-damage output at future times.[14] All else being equal, this negative effect on future pre-damage output will reduce future emissions, damages and the benefits of mitigation. Therefore the direction of the overall main effect of an increase in α_2 and α_3 on β cannot be determined *a priori*. Nonetheless, we might suppose the direct negative effect on β dominates. In addition to the main effect of α_2 and α_3 on β, they likely interact with other uncertainties. In particular, the previous section showed that the effect of uncertainty about emissions-neutral technological progress on β is more positive, the higher is the curvature of the damage function.

The main effect on β of variation in ξ is similar. For a given output path, an increase in ξ results in an increase in damages, hence a decrease in consumption and an increase in the benefits of mitigation. This decreases β, but again the output path is not given. The previous section showed that ξ has an important interaction effect too: we would expect the positive effect of TFP uncertainty on β to be larger, the higher is ξ.

Effect on β of remaining uncertainties In addition to our treatment of TFP growth and damages, here is a brief summary of how each of the other uncertain parameters in Table 2 is expected to affect the climate β.

- Since DICE has a neoclassical (Cobb-Douglas) production function, an increase in **population growth** reduces capital intensity and hence pre-damage output per capita. But although β depends on consumption and benefits measured on a per-capita basis (see *The CCAPM beta*), the effect of population growth on the aggregate scale of the economy also matters. A faster-growing population means a bigger economy on aggregate, higher emissions and higher total and marginal damages. This reduces post-damage consumption per capita and raises the benefits of mitigation. Therefore population growth should have a negative effect on β.
- While growth in CO_2 emissions is proportional to growth in GDP in IAMs like DICE, the proportion is usually assumed to decrease over time due to structural change away from carbon-intensive production sectors and decreases in emissions intensity in a given sector. These are baseline trends, i.e. achieved without the imposition by a planner of a price/quantity constraint on emissions. *A priori*, variation in the **rate of decarbonisation** has an ambiguous effect on β. For a given path of output, an increase in the rate of decarbonisation reduces the benefits of mitigation, because it lowers emissions and hence total and marginal climate damages. But lower damages increase current income and hence they increase capital investment, future consumption, emissions and damages. So while there is no doubt that an increase in the rate of decarbonisation increases consumption, what happens to the benefits of mitigation depends in principle on the balance between the negative effect on marginal damages of a reduction in emissions intensity and the positive effect on marginal damages of an expansion in production.
- While β is a measure of the correlation of the marginal benefits of emissions abatement with consumption, and therefore **abatement costs** do not play a direct role in its calculation, they nonetheless play an indirect role, since the emissions scenario on which the mitigation project is undertaken may involve abatement. Variation in abatement costs increases β: an increase in abatement costs, for a given quantity of abatement, decreases income/consumption, but by decreasing income it also decreases industrial emissions in the long run, through the investment channel. This reduces the benefits of mitigation.
- There are numerous uncertainties, many of them large, about the behaviour of the climate system in response to carbon emissions (e.g. IPCC, 2013). In the structure of DICE's simple climate model, these can be grouped into two types. The first type is uncertainties about the carbon cycle, which render estimates of the atmospheric stock of CO_2 for a given emissions scenario imprecise. We focus on variation in the **uptake of atmospheric carbon by the upper ocean and biosphere**, which also has an ambiguous *a priori* effect on β. Consider a decrease in this uptake, which means that more CO_2 emissions remain in the atmosphere. Under these circumstances, if the path of pre-damage output is taken as given, then more atmospheric CO_2 means increased total damages, hence consumption is reduced and the marginal benefits of mitigation are increased. This reduces β. However, to reiterate, the investment effect means that the path of pre-damage output is not given; reduced income at a particular date due to greater damages results in lower investment, which depresses future output. This reduces future consumption too, but because it reduces future CO_2 emissions there is a countervailing, negative effect on the benefits of mitigation. Again, we might expect the direct effect to dominate, so variation in the uptake of atmospheric carbon should reduce β.
- The second type of uncertainty about the climate system is about the relationship between the stock of atmospheric CO_2 and global mean temperature.[15] Studies that deploy stochastic versions of DICE have overwhelmingly fixed on the **climate sensitivity** parameter as a means of rendering uncertain the temperature response to atmospheric CO_2. Climate sensitivity is the increase in global mean temperature, in equilibrium, that results from a doubling in the atmospheric stock of CO_2 from the pre-industrial level. In simple climate models, it is indeed critical in determining how fast and how far the planet is forecast to warm in response to emissions. Variation in climate sensitivity has an ambiguous – but likely negative – effect on β, with the causal mechanisms being very similar to those at play in the carbon cycle. Higher climate sensitivity means higher damages, lower consumption and higher benefits of mitigation for given output, but with lower income comes lower investment, lower future output and therefore a counter-balancing negative effect on future emissions that tends to reduce the benefits of mitigation.

[14] We can be sure of this, since the marginal propensity to save is exogenous.

[15] Note that together these two types of uncertainty make up the carbon-climate response in the previous section.

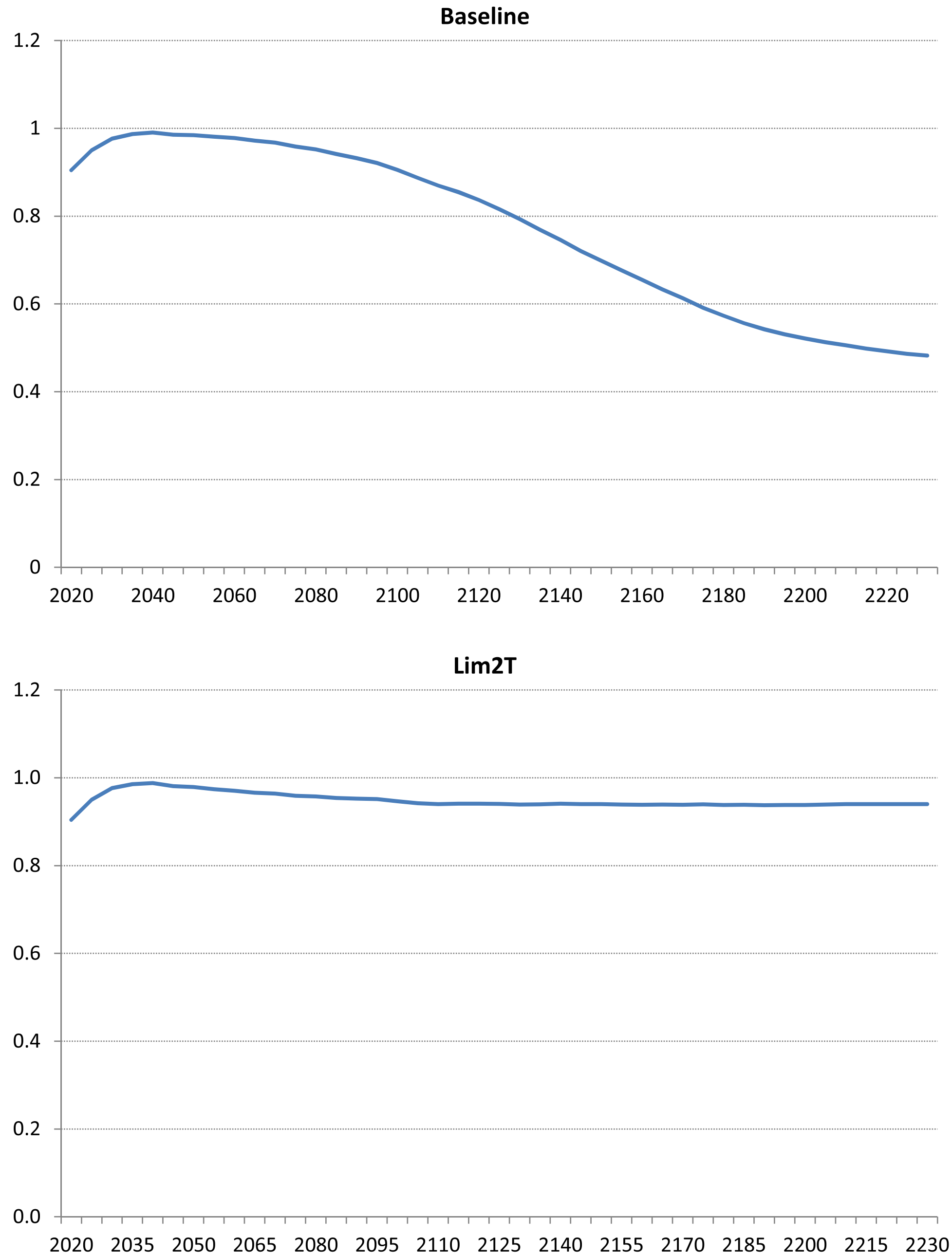

Fig. 1. The term structure of β_t for two contrasting emissions scenarios.

Results

Using the 50,000 draws of the Monte Carlo simulation as the source of variation, we can calculate the instantaneous consumption β of CO_2 emissions abatement. As a function of time, we can then plot its term structure.

Define the benefits of emissions abatement as its avoided damages, in particular as the difference in consumption per capita with and without removing 1Gt CO_2. The benefits of abatement B are then given by

$$B_t = c_t - c_t^{REF}$$
$$B_t = (1 - s_t)(1 - D_t)y_t - (1 - s_t)\left(1 - D_t^{REF}\right) y_t^{REF}$$

where c is consumption per capita, y is pre-damage output per capita, REF denotes reference outcomes before 1Gt CO_2 is removed and s is the savings rate. Note that output here is net of abatement costs.

Table 3
Estimates of β_t on the baseline scenario in selected years, for different subsets of uncertain parameters.

Uncertain parameters	2025	2065	2115	2165	2215
TFP shocks	1.02	1.06	1.06	1.05	1.05
TFP shocks + initial trend growth rate of TFP	1.02	1.06	1.06	1.05	1.05
TFP shocks + asymptotic global population	1.02	1.06	1.06	1.05	1.05
TFP shocks + initial rate of decarbonisation	1.02	1.06	1.06	1.05	1.05
TFP shocks + price of back-stop technology in 2050	1.02	1.06	1.06	1.05	1.05
TFP shocks + uptake of atmospheric carbon by the upper ocean and biosphere (per five years)	1.02	1.05	1.05	1.04	1.04
TFP shocks + climate sensitivity	1.00	1.01	0.93	0.85	0.78
TFP shocks + damage function coefficient α_2	0.95	1.04	1.04	1.03	1.03
TFP shocks + damage function coefficient α_3	1.02	1.06	1.09	1.10	1.10
TFP shocks + income elasticity of damages	1.01	1.05	1.03	1.01	1.00
TFP shocks + climate sensitivity + $\alpha_2 + \alpha_3$ + income elasticity of damages	0.95	0.98	0.86	0.67	0.55
All	0.95	0.97	0.85	0.63	0.49

β_t is then the covariance between $\ln c_t^{REF}$ and $\ln B_t$, divided by the variance of $\ln c_t^{REF}$:

$$\beta_t = \frac{\mathrm{cov}\left[\ln c_t^{REF}, \ln B_t\right]}{\mathrm{var}\left[\ln c_t^{REF}\right]} \tag{24}$$

The discussion above gives us reason to suppose that, in a dynamic model, the β of CO_2 emissions abatement might depend on the path of growth and emissions. Many of the parameter choices we have already described will impact on this, for instance the various determinants of TFP growth, and the initial rate of decarbonisation. But one set of exogenous variables that we must still choose is the emissions reductions imposed by the planner. Therefore in Fig. 1 we plot the term structure of β for two different emissions control scenarios. The first scenario corresponds to the baseline in DICE-2013R, which is a representation of 'business as usual'. According to this scenario, emissions reductions rise gradually from 4% of uncontrolled industrial emissions in 2015 to 14% in 2100 and 54% in 2200. Hence emissions abatement is non-trivial even in the baseline.[16] The second scenario is an example of a path in which emissions reductions are deep: it is the so-called 'Lim2T' scenario from DICE-2013R, in which the planner seeks to limit global warming to no more than 2degC. In Lim2T, emissions reductions are already 33% in 2015 and they hit the maximum 100% in 2060.[17]

The headline result is that on both emissions scenarios β is positive. Overall, given the various uncertainties we specify, there is a positive correlation between consumption and the benefits of emissions abatement. Indeed, over the remainder of this century, the magnitude of β is quite similar on what are two very different emissions paths; it is between 0.9 and 1. However, the term structure of β on the two emissions paths is different and this difference starts to matter after 2100. In

[16] Which illustrates why abatement costs might affect β even in the baseline scenario.
[17] While the changes we have made to DICE-2013R in this study mean that Lim2T is no longer guaranteed to deliver warming equal to 2degC, for the purpose of estimating β it is a perfectly good example of a stringent mitigation scenario.

the baseline scenario, β falls monotonically to 0.48 in 2230. In the Lim2T scenario, β remains between 0.9 and 1 throughout.

What is behind these results? To answer this question, we perform repeated Monte Carlo simulations of the baseline scenario with subsets of the uncertain parameters and re-estimate the term structure of β. The results can be found in Table 3 for selected years. Where a parameter is treated as certain, it is fixed at its mean value. First, we treat all the model parameters as certain, except for the TFP shocks ε_t. Then we run through the remaining uncertain parameters, one at a time, and combine each with the TFP shocks.

What emerges clearly from Table 3 is that the driver of positive β is uncertainty about TFP growth. Moreover it is specifically the transitory shocks to TFP, allied with their moderate persistence, that do it, rather than uncertainty about trend TFP growth. If we run the model just with TFP shocks, $\beta = 1.02$ in 2025, 1.06 in 2115 and 1.05 in 2215. Most of the remaining uncertainties make no discernible difference to β when combined individually with TFP shocks: trend TFP growth; population growth; the rate of decarbonisation; abatement costs; and uptake of atmospheric CO_2. Including uncertainty about the damage function coefficient α_2 or the income elasticity of damages reduces β very slightly, while including uncertainty about the damage function coefficient α_3 increases it very slightly.[18]

The one source of uncertainty that does have a significant effect on the β obtained with TFP shocks alone is the climate sensitivity. The effect is negative. However, this negative effect is not enough to pull β much below unity this century, so the effect of TFP shocks dominates. When the model is run with TFP shocks and uncertain climate sensitivity, $\beta = 0.93$ in 2115 and 0.78 in 2215.

At the foot of the table we reproduce the simulation in which all parameters are uncertain. In this simulation, β does fall to 0.63 in 2165 and eventually 0.49 in 2215. The penultimate simulation in the table shows that this is mostly accounted for by combining just five uncertainties: TFP shocks; climate sensitivity; α_2; α_3; and the income elasticity of damages. These analyses also help us explain why β has a different term structure on the Lim2T emissions scenario than it has on the baseline. On Lim2T the atmospheric concentration of CO_2 is much lower than on the baseline, so the negative effects on β of the climate sensitivity and damage parameters are lower. Consequently β does not decline after the beginning of the next century.

Discussion

In this paper we have studied the sign and size of the climate β, using both a simple analytical model and an empirically grounded Monte Carlo simulation of the DICE model. Using the DICE model also enabled us to take into account the effects on the climate β of investment, as well as generalising the form of the damage function. Our results strongly suggest that the climate β is positive. In particular, our numerical modelling with DICE suggests it is positive and close to unity for maturities of up to about one hundred years. Beyond that, the climate β depends more strongly on the emissions path. On business as usual it falls to about 0.5 for maturities of two hundred years or more, while it remains close to unity on a path of deep emissions cuts that aims to limit warming to 2degC. One might think that reality will turn out to be somewhere between these two extreme cases (e.g. UNEP, 2015), hence the climate β for very long maturities is somewhere between 0.5 and 1.

The overwhelming driver of these results is uncertainty about exogenous, emissions-neutral technological progress in the shape of transitory but moderately persistent shocks to TFP. Positive TFP shocks are simultaneously associated with higher marginal benefits of emissions reductions and higher consumption. Uncertainty about climate sensitivity and the damage intensity of warming provide a countervailing effect that tends to reduce β, but it is outweighed by the effect of TFP shocks. It is important to remember that we allow for fat-tailed climate sensitivity and large convexity of the damage function, two of the principal sources of risk of catastrophic climate damages, which have been claimed to give rise to a negative β.

Naturally the validity of our numerical estimates is affected by the well-known weaknesses shared by all IAMs (e.g. Pindyck, 2013; Stern, 2013). In addition, we face the particular issue of whether and to what extent damages are proportional to output. The basic assumption embodied in a multiplicative damage structure is that damages are a constant fraction of output, for given warming and damage intensity. By contrast, in an additive structure the share of damages in output decreases as output increases, and *vice versa*. Therefore it is related to the so-called 'Schelling conjecture' that developing countries "best defense against climate change may be their own continued development" (Schelling, 1992, p6). *A simple analytical model of the climate beta* made clear that if climate damages are better represented by an additive structure, then the conditions required for a positive climate β are stricter. However, the empirical evidence we used to calibrate the income elasticity of damages in DICE does not support this (Anthoff and Tol, 2012). Rather, it suggests that the income elasticity of damages in most regions at most times is greater than zero and often greater than one, without strong support for a central value other than one. The worry is that the empirical evidence is currently very thin, and more research is clearly required on this issue.

Understanding the implications of our findings for climate mitigation requires understanding the dual role played by β in determining the NPV of mitigation. It is most straightforward to observe that positive β implies the future benefits of emissions abatement should be discounted at a relatively higher rate. How much higher?

[18] This implies that when the only sources of uncertainty are TFP shocks and α_3, the positive interaction between α_3 and the TFP shocks dominates the main, negative effect of α_3 on β.

Two approaches can be followed to answer this question, with radically different conclusions. Both approaches use the CCAPM rule $r = r_f + \beta\pi$. The first approach consists in using the systematic risk premium π that has been observed in markets, for instance in the United States over the last century, where it has been around 5% (see Gollier (2012), chapter 12). For a project with a unit β, this means the efficient discount rate for that project should be five percentage points higher than the risk-free rate. The second approach is model-based rather than market-based; one uses the CCAPM formula $\pi = \gamma\sigma^2$ to estimate the risk premium, where σ^2 is the volatility of consumption growth estimated in DICE. According to our simulations, $\sigma^2 = 0.1\%$ with respect to average growth over the period 2015-2230, so we obtain a risk premium of only 0.2 percentage points if we accept a coefficient of relative risk aversion $\gamma = 2$, which much of the existing literature would suggest (Kolstad et al., 2014). This leads to a much smaller impact of the positive climate β on the risk-adjusted climate discount rate.

The large discrepancy between these two recommendations may be seen as a manifestation of the well-known "equity premium puzzle". Three decades of research on this financial puzzle suggests that the model-based CCAPM approach fails to capture many dimensions of the real world, in particular the existence of structural uncertainties and fat tails (Weitzman, 2007b). Although including these dimensions in our model is beyond the reach of this paper – a new concept of β will need to be developed to accommodate these features – we are inclined to accept this position. We then conclude that a large positive climate β is important for discounting the future benefits of mitigating climate change.

But this is not the end of the story. *The CCAPM beta* showed that the NPV of climate mitigation is increasing in β if β is larger than $\gamma - \left(\mu/\sigma^2\right)$, which is at most of the order of -2.5. Since our estimates are clearly larger than that, it can be concluded that the NPV of climate mitigation is indeed increasing in β. More broadly, this shows that the implications of our work do not just concern the discount rate. It would be wrong to discount the future benefits of emissions abatement at a risk-adjusted rate with unit β, unless the *undiscounted* future benefits have been calculated in a way that properly factors in, implicitly or explicitly, how they scale with economic growth.

Conclusion

Because a large fraction of the climate damages generated by greenhouse gases emitted today will not materialise until the distant future, the choice of the rate at which these future damages should be discounted plays a critical role in the determination of the social cost of carbon. Most of the recent literature on climate discounting implicitly assumes that these damages are uncorrelated with aggregate consumption, so that they should be discounted at the risk-free rate. This justifies using either the Ramsey rule or the observed interest rate to estimate the climate discount rate. However, we show in this paper that the climate β, i.e. the elasticity of climate damages with respect to a change in aggregate consumption, is close to one, at least for maturities of up to one hundred years. This is mainly due to the role of exogenous, emissions-neutral technological progress in raising consumption, emissions, atmospheric carbon and marginal damages. This implies that mitigating climate change raises the risk borne by future generations, which justifies using a climate discount rate that is larger than the risk-free rate. How much larger depends on our evaluation of the equity premium puzzle in finance. That the climate β is relatively large should induce climate economists to change the focus of long-term discounting from safe to risky claims.

A large climate β not only implies a large climate discount rate. Indeed, the climate β measures the sensitivity of monetized climate damages to a change in consumption of other goods and services in the economy. In a growing economy, a large climate β also implies large expected damage in the long run. We have shown that an increase in the climate β increases expected damages more than it reduces the discount factor, so that in fact the social cost of carbon is increasing in the climate β.

Appendix A. Appendix. Further details of random parameters in DICE

Asymptotic global population In DICE population grows according to the following equation of motion:

$$L_{t+1} = L_t\left(\frac{L_\infty}{L_t}\right)^{g^N},$$

where L is the population, which converges to the asymptotic global population L_∞ according to the growth rate g^N.

We use the global population projections of the United Nations United Nations (2013) to calibrate a probability distribution over L_∞. According to these projections, the world population will be at an approximate steady state of 10.85 billion in 2100 on the medium (fertility) variant, within a range of 6.75 billion on the low variant to 16.64 billion on the high variant. This is a non-probabilistic range, which can be set against an emerging – though not uncontested (Lutz et al., 2014) – field of probabilistic population forecasting based on Bayesian methods (Raftery et al., 2012). According to these forecasts, the UN's low and high variants are very unlikely to eventuate (i.e. they are suggested to be well outside the 95% confidence interval: Gerland et al. (2014)), because they assume fertility is systematically different to the medium scenario in all countries. Taking this perspective into account, we fit a normal distribution to the UN population projections, such that the low variant is three standard deviations away from the mean, with the result that the high variant is even further from the mean.

Initial rate of decarbonisation In DICE, autonomous decarbonisation is achieved by virtue of a variable representing the ratio of emissions/output, which decreases over time as a function of a rate-of-decarbonisation parameter:

$$E_t^{IND} = \sigma_t(1 - \mu_t)Y_t, \tag{25}$$

where E^{IND} represents industrial CO_2 emissions, μ is the control rate of emissions set by the planner, Y is pre-damage output and σ is the ratio of uncontrolled emissions to output, given by

$$\sigma_{t+1} = \sigma_t(1 + g_t^{\sigma}),$$

where $g^{\sigma} < 0$ is the rate of decline of emissions to output, given by

$$g_t^{\sigma} = g_0^{\sigma}\left(1 + \delta^{\sigma}\right)^t,$$

with the initial rate of decline of emissions to output being g_0^{σ}, subject itself to a rate of decline of $\delta^{\sigma} < 0$. Similar to TFP, δ^{σ} is around an order of magnitude smaller than g_0^{σ}, so the latter is key in driving long-run uncertainty about declining emissions intensity.

To calibrate a distribution over g_0^{σ} we use data from the International Energy Agency (IEA, 2013), which provides the ratio of global CO_2 emissions from fossil fuels to real global GDP for the period 1971-2011, a period in which planned emissions reductions (i.e. through μ) were trivially small at the global level. We partly smooth annual fluctuations by taking a five-year rolling average. The resulting data are fit best by a normal distribution with mean and standard deviation as reported in Table 2.

Price of the backstop technology In DICE the total cost of abatement as a percentage of annual GDP, Λ, is determined by

$$\Lambda_t = \theta_{1,t}\mu_t^{\theta_2},$$

where θ_1 and θ_2 are coefficients. The time-path of θ_1 is set so that the marginal cost of abatement at $\mu_t = 1$ is equal to the backstop price at t. Hence randomising the backstop price is a way to introduce uncertainty into abatement costs.

We use the findings of an inter-model comparison study by Edenhofer et al. (2010) to update and characterise uncertainty over the backstop price. Edenhofer et al. (2010) assess the cost of limiting warming to below 2degC in five global energy models. A scenario that stabilises the atmospheric stock of CO_2 at 400 ppm requires zero emissions by around 2050, so we can use the models' estimates of marginal abatement costs in 2050 as a measure of the backstop price at that time. Marginal costs range from \$150/tCO_2 to \$500, with an average of \$260, all at today's prices. Since the distribution of cost estimates is asymmetric, we use a log-normal distribution. We set the mean to \$260 and posit that the probability of the lowest and highest estimates is 1/1000. We use a comparable emissions scenario in DICE to retrieve, for each value of the backstop price in 2050, the value of the backstop price in 2010, the initial period.

Uptake of atmospheric carbon by the upper ocean and biosphere The atmospheric stock of carbon in DICE is driven by the sum of industrial emissions from (25) and exogenous emissions from land-use. A system of three equations represents the cycling of carbon between three reservoirs, the atmosphere M^{AT}, a quickly mixing reservoir comprising the upper ocean and parts of the biosphere M^{UP}, and the lower ocean M^{LO}:

$$\begin{aligned}
M_{t+1}^{AT} &= E_{t+1} + \phi_{11}M_t^{AT} + \phi_{21}M_t^{UP}, \\
M_{t+1}^{UP} &= \phi_{12}M_t^{AT} + \phi_{22}M_t^{UP} + \phi_{32}M_t^{LO}, \\
M_{t+1}^{LO} &= \phi_{23}M_t^{UP} + \phi_{33}M_t^{LO},
\end{aligned}$$

where total emissions of CO_2 to the atmosphere are E, and the cycling of CO_2 between the reservoirs is determined by a set of coefficients ϕ_{jk} that govern the rate of transport from reservoir j to k per unit of time. We follow Nordhaus (2008) uncertainty analysis by randomising ϕ_{12}, the coefficient for the transfer of carbon from M^{AT} to M^{UP}. However, we make use of the latest scientific findings from the IPCC's *Fifth Assessment Report* (Ciais et al., 2013) to calibrate ϕ_{12}. In particular, ϕ_{12} may be calibrated by inspecting evidence on the percentage of a pulse of CO_2 emissions that remains in the atmosphere after 100 years. According to the standard parameterisation of DICE-2013R, this would be c. 36%, but the evidence from multiple climate models collected by Ciais et al. (2013) suggests a mean of 41%, with 54% at +2 standard deviations and 28% at -2 standard deviations. We calibrate ϕ_{12} accordingly, however to ensure the DICE carbon cycle maintains physically consistent behaviour at all values of ϕ_{12}, we must set the lower bound at 31% removed. Table 2 provides details.

Climate sensitivity The equation of motion of temperature in DICE is given by:

$$T_{t+1} = T_t + \kappa_1\left[F_{t+1} - \frac{F_{2\times CO_2}}{S}(T_t) - \kappa_2\left(T_t - T_t^{LO}\right)\right],$$

where F_{t+1} is radiative forcing, which depends on the atmospheric stock of CO_2, $F_{2\times CO_2}$ is the radiative forcing resulting from a doubling in the atmospheric stock of CO_2 from the pre-industrial level, S is climate sensitivity, T^{LO} is the temperature of the lower ocean, κ_1 is a parameter determining speed of adjustment and κ_2 is the coefficient of heat loss from the atmosphere to the oceans. Calel et al. (2015) contains a detailed explanation of the physics behind this equation.

The latest IPCC report (IPCC, 2013) provides a subjective probability distribution for the climate sensitivity, which is the consensus of the panel's many experts. According to this distribution, *S* is 'likely' to be between 1.5 and 4.5degC, where likely corresponds to a subjective probability of anywhere between 0.66 and 1. It is 'extremely unlikely' to be less than 1degC, where extremely unlikely indicates a probability of ≤ 0.05, while it is 'very unlikely' to exceed 6degC, where this denotes a probability of ≤ 0.1. Dietz and Stern (2015) find that a log-logistic function has the appropriate tail shape to fit these data[19] (taking the midpoints of the IPCC ranges), and set the scale and shape parameters of the distribution such that the mean *S* is 2.9degC, and the standard deviation is 1.4degC. In addition, we truncate the distribution from below at 0.75degC in order to again ensure that the DICE climate model exhibits physically consistent behaviour.

References

Aalbers, R., 2009. Discounting investments in mitigation and adaptation: a dynamic stochastic general equilibrium approach of climate change. Tech. rep., CPB Netherlands Bureau for Economic Policy Analysis.

Allen, M.R., Frame, D.J., Huntingford, C., Jones, C.D., Lowe, J.A., Meinshausen, M., Meinshausen, N., 2009. Warming caused by cumulative carbon emissions towards the trillionth tonne. Nature 458, 1163–1166.

Anderson, B., Borgonovo, E., Galeotti, M., Roson, R., 2014. Uncertainty in climate change modeling: can global sensitivity analysis be of help? Risk Anal. 34, 271–293.

Anthoff, D., Tol, R.S.J., 2012. Schelling's conjecture on climate and development: a test, in Climate Change and Common Sense: Essays in Honour of Tom Schelling. ed. by Hahn, R.W., Ulph, A., Oxford: Oxford University Press, pp. 260–274.

Bansal, R., Kiku, D., Ochoa, M., 2015. Climate Change and Growth Risks. Duke University.

Bolt, J., van Zanden, J.L., 2013. In: Proceedings of the first update of the Maddison Project: re-estimating growth before 1820. Maddison-project working paper wp-4, University of Groningen.

Calel, R., Stainforth, D.A., Dietz, S., 2015. Tall tales and fat tails: the science and economics of extreme warming. Clim. Change 132, 127–141.

Ciais, P., Sabine, C., Bala, G., Bopp, L., Brovkin, V., Canadell, J., Chhabra, A., DeFries, R., Galloway, J., Heimann, M., Jones, C., Qur, C.L., Myneni, R., Piao, S., Thornton, P., 2013. Carbon and other biogeochemical cycles, in Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, ed. by Stocker, T., Qin, D., Plattner, G.-K., Tignor, M., Allen, S., Boschung, J., Nauels, A., Xia, Y., Bex, V., Midgley, P. Cambridge, United Kingdom and New York, NY, USA: Cambridge University Press.

Collins, M., Knutti, R., Arblaster, J., Dufresne, J.-L., Fichefet, T., Friedlingstein, P., Gao, X., Gutowski, W., Johns, T., Krinner, G., Shongwe, M., Tebaldi, C., W.A.J., Wehner, M., 2013. Long-term climate change: projections, commitments and irreversibility, in Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, ed. by Stocker, T., Qin, C., Plattner, G.-K., Tignor, M., Allen, S., Boschung, J., Nauels, A., Xia, Y., Bex, V., Midgeley, P. Cambridge, UK: Cambridge University Press.

Daniel, K., Litterman, R., Wagner, G., 2015. Applying asset pricing theory to calibrate the price of climate risk. Columbia Business School.

Dietz, S., Asheim, G., 2012. Climate policy under sustainable discounted utilitarianism. J. Environ. Econ. Manag. 63, 321–335.

Dietz, S., Stern, N., 2015. Endogenous growth, convexity of damages and climate risk: how Nordhaus' framework supports deep cuts in carbon emissions. Econ. J. 125, 574–620.

Eby, M., Zickfeld, K., Montenegro, A., Archer, D., Meissner, K., Weaver, A., 2009. Lifetime of anthropogenic climate change: millennial time scales of potential CO_2 and surface temperature perturbations. J. Clim. 22, 2501–2511.

Edenhofer, O., Knopf, B., Barker, T., Baumstark, L., Bellevrat, E., Chateau, B., Criqui, P., Isaac, M., Kitous, A., Kypreos, S., et al., 2010. The economics of low stabilization: model comparison of mitigation strategies and costs. Energy J. 31, 11–48.

Feinstein, C.H., Pollard, S., 1988. Studies in Capital Formation in the United Kingdom. Oxford University Press1750–1920.

Gerland, P., Raftery, A.E., Ševčíková, H., Li, N., Gu, D., Spoorenberg, T., Alkema, L., Fosdick, B.K., Chunn, J., Lalic, N., et al., 2014. World population stabilization unlikely this century. Science 346, 234–237.

Gollier, C., 2012. Pricing the Planet's Future: the Economics of Discounting in an Uncertain World. Princeton University Press.

Golosov, M., Hassler, J., Krusell, P., Tsyvinski, A., 2014. Optimal taxes on fossil fuel in general equilibrium. Econometrica 82, 41–88.

Goodwin, P., Williams, R.G., Ridgwell, A., 2015. Sensitivity of climate to cumulative carbon emissions due to compensation of ocean heat and carbon uptake. Nat. Geosci. 8, 29–34.

Heal, G., Millner, A., 2014. Uncertainty and decision making in climate change economics. Rev. Environ. Econ. Policy 8, 120–137.

Held, I.M., Winton, M., Takahashi, K., Delworth, T., Zeng, F., Vallis, G.K., 2010. Probing the fast and slow components of global warming by returning abruptly to preindustrial forcing. J. Clim. 23, 2418–2427.

Howarth, R.B., 2003. Discounting and uncertainty in climate change policy analysis. Land Econ. 79, 369–381.

IEA, 2013. CO_2 Emissions from Fuel Combustion 2013 - Highlights. Tech. rep., IEA, Paris.

IPCC, 2013. Working Group I Contribution to the IPCC Fifth Assessment Report: Summary for Policymakers. in Climate Change 2013: The Physical Science Basis, IPCC.

IPCC, 2014. Summary for Policymakers. in Climate Change 2014: Impacts, Adaptation, and Vulnerability. Contribution of Working Group II to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, IPCC.

Jagannathan, R., Wang, Z., 1996. The conditional CAPM and the cross-section of expected returns. J. Financ. 51, 3–53.

Jensen, S., Traeger, C.P., 2014. Optimal Climate Change mitigation under long-term growth uncertainty: stochastic integrated assessment and analytic findings. Eur. Econ. Rev. 69, 104–125.

Kelly, D.L., Kolstad, C.D., 2001. Malthus and Climate Change betting on a stable population. J. Environ. Econ. Manag. 41, 135–161.

Kocherlakota, N.R., 1996. The equity premium: it's still a puzzle. J. Econ. Lit. 34, 42–71.

Kolstad, C., Urama, K., Broome, J., Bruvoll, A., Cario, M., Olvera, D. Fullerton, C. Gollier, W.M. Hanemann, Hassan, R., Jotzo, F., Khan, M.R., Meyer, L., Mundaca, L., 2014. Social, economic and ethical concepts and methods. in Climate Change 2014: Mitigation of Climate Change. Contribution of Working Group III to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change, ed. by Edenhofer, O., Pichs-Madruga, R., Sokona, Y., Farahani, E., Kadner, S., Seyboth, K., Adler, A., Baum, I., Brunner, S., Eickemeier, P., Kriemann, B., Savolainen, J., Schlmer, S., von Stechow, C., Zwickel, T., Minx, J., Cambridge, United Kingdom and New York, NY, USA: Cambridge University Press.

Lemoine, D., 2015. The climate risk premium: how uncertainty affects the social cost of carbon. University of Arizona.

Lucas, R.E., 1978. Asset prices in an exchange economy. Econometrica 46, 1429–1445.

Lutz, W., Butz, W., Sanderson, W., Scherbov, S., et al., 2014. Population growth: peak probability. Science 346, 561.

Matthews, H.D., Caldeira, K., 2008. Stabilizing climate requires near-zero emissions. Geophys. Res. Lett. 35, L04705.

Matthews, H.D., Gillett, N.P., Stott, P.A., Zickfeld, K., 2009. The proportionality of global warming to cumulative carbon emissions. Nature 459, 829–832.

Matthews, R.C.O., Feinstein, C.H., Odling-Smee, J.C., 1982. British Economic Growth. Stanford University Press1856–1973.

[19] That is, the log-logistic function has the lowest root-mean-square error of any distribution fitted.

Mehra, R., 2012. Consumption-based asset pricing models. Annu. Rev. Financ. Econ. 4, 385–409.
Nordhaus, W.D., 2008. A Question of Balance: Weighing the Options on Global Warming Policies. Yale University Press.
Nordhaus, W.D., 2011. Estimates of the social cost of carbon: background and results from the RICE-2011 model. Tech. rep., National Bureau of Economic Research.
Nordhaus, W.D., 2013. The Climate Casino: Risk, uncertainty, and Economics for a Warming World. Yale University Press, New Haven, CT.
Nordhaus, W.D., Sztorc P., 2013. DICE 2013R: Introduction and User's Manual. Tech. rep., Yale University.
Pindyck, R.S., 2013. Climate Change policy: what do the models tell us? J. Econ. Lit. 51, 860–872.
Raftery, A.E., Li, N., Ševčíková, H., Gerland, P., Heilig, G.K., 2012. Bayesian probabilistic population projections for all countries. Proc. Natl. Acad. Sci. 109 (1), 13915–13921.
Ricke, K.L., Caldeira, K., 2014. Maximum warming occurs about one decade after a carbon dioxide emission. Environ. Res. Lett. 9, 124002.
Sandsmark, M., Vennemo, H., 2007. A portfolio approach to climate investments: CAPM and endogenous risk. Environ. Resour. Econ. 37, 681–695.
Schelling, T., 1992. Some economics of global warming. Am. Econ. Rev. 82, 1–14.
Shine, K.P., Fuglestvedt, J.S., Hailemariam, K., Stuber, N., 2005. Alternatives to the global warming potential for comparing climate impacts of emissions of greenhouse gases. Clim. Change 68, 281–302.
Solomon, S., Plattner, G.-K., Knutti, R., Friedlingstein, P., 2009. Irreversible climate change due to carbon dioxide emissions. Proc. Natl. Acad. Sci. 10 (6), 1704–1709.
Stern, N., 2007. The Economics of Climate Change: The Stern Review. Cambridge University Press.
Stern, N., 2013. The structure of economic modeling of the potential impacts of climate change: grafting gross underestimation of risk onto already narrow science models. J. Econ. Lit. 51, 838–859.
Tol, R.S., 2009. The economic effects of climate change. J. Econ. Perspect. 23, 29–51.
UNEP, 2015. The Emissions Gap Report 2015: A UNEP Synthesis Report, Tech. rep., United Nations Environment Programme (UNEP).
United Nations, 2013. World Population Prospects: the 2012 Revision. Tech. rep., United Nations.
van den Bijgaart, I., Gerlagh, R., Liski, M., 2016. A simple formula for the social cost of carbon. J. Environ. Econ. Manag. 77, 75–94.
Weitzman, M., 2007a. A review of the Stern Review on the economics of climate change. J. Econ. Lit. 45, 703–724.
Weitzman, M.L., 2007b. Subjective expectations and asset-return puzzles. Am. Econ. Rev. 97, 1102–1130.
Weitzman, M.L., 2012. GHG targets as insurance against catastrophic climate damages. J. Public Econ. Theory 14, 221–244.
Weitzman, M.L., 2013. Tail-hedge discounting and the social cost of carbon. J. Econ. Lit. 51, 873–882.
Zickfeld, K., Eby, M., Matthews, H.D., Weaver, A.J., 2009. Setting cumulative emissions targets to reduce the risk of dangerous climate change. Proc. Natl. Acad. Sci. 106 (1), 16129–16134.