# Ricardo Alonso
## Strategic control and strategic communication

## Working paper

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# Shared Control and Strategic Communication[*]

Ricardo Alonso

University of Southern California

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## Abstract

This paper studies the optimal allocation of decision rights between an uninformed principal and an informed agent when interdependent activities need to be adapted to local conditions. While the principal cares only about overall profits the agent may favor higher, or lower, levels for each activity than the profit-maximizing level. The principal has limited commitment power and can only commit to an ex-ante allocation of decision rights. Whenever the principal retains some (or all) decision rights the agent communicates his information strategically, i.e. via cheap talk. We show that if activities are complementary the principal can always improve the informativeness of communication by sharing control with the agent, while sharing control over activities that interact as substitutes always worsens communication. As a result of this communication advantage, sharing control over complementary activities can be optimal, while control over activities that interact as substitutes is optimally allocated to the same party.

Keywords: authority, cheap talk, incomplete contracts

JEL classifications: D23, D83, L23

# 1 Introduction

The allocation of decision rights is a key determinant of the performance of organizations. To allocate decision rights efficiently, organizations must resolve a trade-off between information and control: on the one hand, the efficient utilization of local information requires that decision rights are allocated to the best informed party (Jensen and Meckling 1992) but, on the other hand, doing so may expose the organization to the costs of biased decision making (Jensen and Meckling 1976, Jensen 1986). The resolution of this trade-off is made more difficult when decisions are interdependent. In particular, the organization must then take into account how the allocation of control over one decision affects the incentives of agents to communicate information relevant for other decisions. In this paper I analyze the optimal allocation of decision rights between an uninformed principal and an informed but biased agent in the light of such interdependencies. More specifically, I analyze organizational design in a setting in which the only formal mechanisms are the allocation of control rights and the informed agent communicates via cheap talk.

The central result in this paper is that the principal may be willing "to trade control for information". In particular, we show that when two activities $A_1$ and $A_2$ are complementary, the principal can induce the agent to communicate more information relevant for activity $A_1$ if she gives the agent control over activity $A_2$. This communication advantage can be sufficiently large to compensate the principal for her loss of control when delegating activity $A_2$. As a result, sharing some control rights with the agent can be optimal when activities are complementary. In contrast, sharing control over substitutive activities worsens communication, and thus sharing control is never optimal.

There is ample evidence that organizations assign control rights over interdependent activities to different decision makers. For instance, Taggart (1987) reports that investments decisions on new product lines and expansions of existing product lines are often made by different levels of a hierarchy (see Harris and Raviv 2002 for a discussion of these findings). Likewise, the authority to deploy new production facilities typically lies with the CEO or division manager while investments in plant expansion, upgrades and maintenance are made at the technical or plant manager level.[1] In many multidivisional firms corporate headquarters has active oversight over certain functional activities, while divisions have wide discretion in making operational decisions.[2] A concrete ex-

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[1] Also, control over capital decisions are sometimes disassociated from related human resource policies (e.g. staffing, training or job assignment). For instance, in the context of Italian manufacturing firms, this separation of control over capital and labor decisions has been reported by Colombo and Delmastro (2004).

[2] Alternative reasons for centralizing certain activities involve gains in coordination, when these activities have a firm-wide impact, and exploitation of economies of scale (i.e. when headquarters acts as a "shared service" provider, e.g. in centralizing payroll services).

ample is provided by Information Technology (IT), where corporations have alternated between centralizing IT-investment decisions (giving rise to the position of the Chief Information Officer (CIO)), or allowing their business units freedom to pursue their own IT projects subject to general budgetary constraints, often with mixed results (see HBR 2000). The difficulty in finding the efficient governance structure for IT related decisions may stem from the observation that these investments are complementary to, for instance, investments in product variety and design, and human resources policies (Brynjolfsson and Hitt 2000, Bresnahan, Brynjolfsson and Hitt 2002, Gao and Hitt 2004). In this paper we argue that one benefit of letting a corporate CIO have authority over IT decisions is the improved communication of the information held by the business units.

To study how the allocation of control affects the quality of communication between informed and uninformed parties I analyze a model with the following characteristics: (i) An organization that consists of a principal and agent must select the levels of two interdependent activities $A_1$ and $A_2$. Activities $A_1$ and $A_2$ may interact as complements, when the marginal return to each activity increases with the level of the other activity, or interact as substitutes, when the marginal return to each activity decreases in the level of the other activity.[3] (ii) Principal and agent have conflicting preferences in that their preferred levels for each activity depend on the realization of the state of the world and do not in general coincide. (iii) Contracts are highly incomplete and the principal can only commit to an ex-ante allocation of control over each activity. In particular, the principal can control both activities (P-Authority), delegate both activities to the agent (A-Authority), or transfer control over one activity to the agent while retaining the right to set the level of the other activity (S-Authority). (iv) After the initial allocation of control, the agent learns the realization of the state. If the principal has control over some (or all) activities the agent communicates his private information strategically, i.e. communication between principal and agent takes the form of "cheap talk".

The impossibility of paying the agent according to the information revealed implies that in equilibrium communication is noisy and information is lost. As in Crawford and Sobel (1982) (henceforth CS 1982) and Melumad and Shibano (1991) (henceforth MS 1991), cheap talk equilibria under P-Authority and S-Authority are characterized by a partition of the state space into intervals, where the only substantive information transmitted is the interval where the actual realization of the state lies.[4] A natural question then is if an informed agent is willing to share more precise

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[3]See Milgrom and Roberts (1990).

[4]We will posit that if the agent controls both activities no communication takes place. Indeed, since in this case the principal makes no decisions the agent may fully reveal his private information but will have no impact on final outcomes.

information with his principal when he enjoys more control rights. We show that the agent may reveal *more* or *less* information when he gains control over one activity, where the informativeness of the agent's message depends on the nature of the interaction in a simple manner: while the principal can always *improve* the quality of communication by sharing control over *complementary* activities, communication *worsens* when control is shared over *substitutive* activities.

In essence, by granting formal authority over one activity to the agent, say $A_2$, joint decision making will feature a game of strategic complements (if activities are complementary) or strategic substitutes (if activities are substitutes), where the agent's message conveys information both about the state and his intended decision. In turn, the principal will react to the anticipated decision of the agent by adjusting the level of $A_1$ away from her preferred level if she controlled both activities. Therefore, by sharing control the principal can commit to act on the agent's information differently than if she centralized decision making. We show that this change in the responsiveness of the principal dampens the agent's incentive to misrepresent his private information (relative to the centralized case) when activities are complementary, but intensifies the agent's incentive to misrepresent his information when activities interact as substitutes. Thus, in our specification, shared control over complementary activities exhibits a communication advantage over centralization, while the reverse is true if activities interact as substitutes.

The main finding is that this communication advantage may result in the optimality of sharing control over complementary activities. In order to shed light on this result we first consider as a benchmark the case in which communication is imperfect, but non-strategic, and does not vary with the allocation of decision rights. Given the symmetry across activities in our specification, we show that sharing control is never optimal in this case: an enhanced willingness to share control also makes A-Authority (i.e. complete delegation) very attractive.

The fact that the allocation of control endogenously determines the quality of communication drives the main result: not only can the principal improve communication by sharing control over complementary activities, but she is willing "to trade control for information" whenever the conflict of interest is mild. The analysis considers both the case in which the agent favors higher decisions than the principal, and thus has an incentive to overstate his information, and the case that the agent favors lower decisions, resulting in his incentive to understate his information. While the comparison between the communication performance of P-Authority and S-Authority does not qualitatively depend on the type of agent, we show that communication is comparatively worse in the second case, i.e. when the agent's incentive is to understate his information. This implies that in our model, the principal shares control with her agent only if the latter has an incentive to

overstate his information.

The rest of the paper is structured as follows. In the following section we discuss the related literature. Section 3 presents the model which is solved in Section 4. Section 5 compares the quality of communication under centralization to the quality of communication when the principal shares control. Section 6 describes the performance of the different organizational arrangements and Section 7 determines under which conditions the principal selects each arrangement. Finally, I conclude in Section 8.

## 2  Related Literature

*Allocation of Authority in Organizations:* There is a vast literature that analyzes the optimal allocation of decision rights in the presence of incentive conflicts when information is distributed throughout an organization.[5] A substantial body of work has considered the complete contracting case that, following a mechanism design approach, analyzes optimal decision rules when contracts can specify what decisions are made under every contingency. Under complete contracting, however, the allocation of authority is often indeterminate: a given decision rule maybe implemented by granting authority to the agent and conditioning his compensation on the decisions he makes, or by centralizing decision making and compensating the agent according to the information transmitted.

Following Grossman and Hart (1986), when contracts are incomplete the allocation of authority does affect the set of attainable decision rules. In particular, our assumption that decisions are ex-ante and ex-post non-contractible is in line with a number of recent papers that study organizational design with ex-post asymmetric information when only the allocation of decision rights can be specified. The literature so far has considered different degrees of contractual incompleteness: (i) decision rights can be contracted ex-ante and ex-post, (ii) decision rights can only be contractually specified ex-ante.

When decision rights can be contractually specified both ex-ante and ex-post, the organizational designer can employ revelation mechanisms that specify an allocation of control and a set of transfers for every set of messages that informed parties might send. This contracting over control has been considered, for instance, by Aghion, Dewatripont and Rey (2004) and Bester (2005).[6] In a

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[5]The literature on team theory also analyzes decision making in organizations when information is dispersed and physical constraints impose limits on communication (e.g. Marshak and Radner 1972, Radner 1993, Bolton and Dewatripont 1994, Van Zandt 1999). This literature, however, abstracts from incentive considerations. In contrast, in our paper, the quality of information flows is determined by the inherent incentive conflict between principal and agent. In Section 7.1 we briefly compare our results to a setting where communication is imperfect but unaffected by incentive conflicts.

[6]See also Aghion and Bolton (1992), Aghion and Rey (2002), Hart and Moore (2005), Hart and Holmstrom (2002) and Krähmer (2006).

dynamic setting, Aghion, Dewatripont and Rey (2004) show that a principal can induce truthful revelation of the agent's information by making delegation of a first period decision contingent on the agent's message.[7] Bester (2005) identifies more general situations in which truthful revelation of information can be achieved. It follows that, in these cases, asymmetric information does not impose an efficiency loss with respect to the full information case, in which control rights are allocated contingent on the realized state.

A second strand of the literature argues that contracting over control is not feasible ex-post (for instance, because once the agent reveals his private information the principal may have an incentive to renege on his promise and make decisions herself), and contracts can only establish who has formal authority over each decision.[8] We distinguish two cases: (i) control can be transferred ex-post if it is in the interest of the party with authority to do so,[9] (ii) the initial allocation of control rights determines who ultimately makes each decision.[10]

In the first case, Aghion and Tirole (1997) make the distinction between "formal" and "real" authority, i.e. the difference between who has formal control over decisions and who actually makes decisions, to highlight the possibility that control may be credibly transfered even when the parties cannot contractually commit to do so.[11] More related to our paper, Aghion, Dewatripont and Rey (2002, 2004) and Gautier and Paolini (2002) analyze in a dynamic setting the notion of "transferable control" where, by delegating control in the first period to the agent, the principal can infer the agent's type through his equilibrium action. This is beneficial if the principal can use this information to make decisions herself in later stages. Thus, as in our paper, partial delegation of authority may enhance the information that the principal obtains from the agent. In these papers information revelation operates through a "signaling mechanism": delegation in the first period allows the agent to credibly signal his type through his choice of action. In contrast, in our paper information transmission does not rely on the ability of the agent to credibly signal his type: when control is shared, decisions are made simultaneously.

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[7]In the case of Aghion, Dewatripont and Rey (2004) the state of the world corresponds to the agent's willingness to cooperate in the future.

[8]Even when neither decisions nor control over decisions can be credibly allocated, an uninformed principal can still use the power of message games to elicit information from an informed agent. This ex-post contracting over information has been considered by Krishna and Morgan (2005), who, for the leading example of Crawford and Sobel (1982) and when the agent is protected by limited liability, find that although the principal can induce full revelation of information by the agent this is never optimal, i.e. the optimal contract involves noisy communication by the agent. In contrast, in our setup the principal has limited commitment to shape the communication protocol and thus "paying for information" is not credible.

[9]See Aghion and Tirole (1997), Gautier and Paolini (2002) and Aghion, Dewatripont and Rey (2002, 2004).

[10]See Dessein (2002), Ottaviani (2000) and Marino and Matsusaka (2005).

[11]Aghion and Tirole (1997) introduce this distinction to study how the allocation of formal authority affects the incentives of principal and agent for information acquisition.

Our paper follows the strand of the literature that assumes that the limited commitment of the principal restricts her contractual choices to an ex-ante allocation of decision rights, i.e. only the party originally in charge can exercise its right to make a decision. In this sense our paper is closest to Dessein (2002) that explores a principal's choice between delegation to an agent and centralization of decision rights when the agent communicates his information via "cheap talk". A central insight of Dessein (2002) is that despite the increased informativeness of communication when the conflict of interest reduces, the principal prefers to delegate control rather than engage in communication precisely when the conflict of interest is not too pronounced. Dessein (2002) considers a single decision and thus the allocation of authority does not affect the quality of communication. In contrast we explore a setting with multiple, interdependent decisions where different decisions can be allocated to different parties. Thus we are able to show that the quality of communication does indeed depend on the allocation of control, and derive its relation to the nature of interaction between decisions.[12]

*Cheap Talk:* In our setup the agent communicates via "cheap talk". Therefore this paper is related to the literature that follows CS (1982). The specification of the preferences of principal and agent, however, are closer to MS (1991) since we allow for the possibility that under certain realizations of the state the agent has no incentive to distort his private information (see also Stein 1989, Alonso, Dessein and Matouschek 2006 and Rantakari 2006). The point of departure in our analysis is that by contrasting the case in which an agent has an incentive to overstate his private information to the case in which he has an incentive to understate his private information, we show that "cheap talk" equilibria are qualitatively different: while an infinite number of equilibria are possible if the agent's incentives are to overstate his private information, only a finite number of equilibria are possible if he would like the principal to believe that the realization of the state is lower than its actual value.[13]

Under shared control both principal and agent select a decision after the agent has communicated his private information. In this sense this paper also follows the literature that considers pre-play communication between players when there is one-sided asymmetric information (e.g. Far-

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[12]In our model information is "localized", i.e. the agent acquires all relevant information prior to decision making. Other papers consider situations in which information is dispersed throughout the organization (i.e. no party can make a fully informed decision without engaging in communication) and thus the problem of information aggregation is central to organizational design. See Stein (2002), Harris and Raviv (2005), Friebel and Raith (2006), Dessein, Garicano and Gertner (2005), Alonso, Dessein and Matouschek (2006) and Rantakari (2006).

[13]Our paper also differs from Alonso, Dessein and Matouschek (2006) and Rantakari (2006) in that they consider multiple senders that observe independent pieces of information (see Battaglini 2004 for the case in which the information of senders is imperfectly correlated) while in our setup one party (the agent) has all the relevant information for decision making. Also Battaglini (2002) and Krishna and Morgan (2001) analyze cheap talk equilibria with multiple senders that observe the same piece of information.

rell 1987, Baliga and Morris 2002). For instance, Baliga and Morris (2002) provide conditions for the existance of a full revelation equilibrium of the cheap talk stage and, alternatively, conditions under which pre-play communication does not expand the set of equilibrium outcomes, i.e. communication is uninformative. In contrast, in our setup communication by the agent under shared control is noisy but informative, unless the conflict of interest is extreme.

## 3  The Model

An organization is composed of a principal (she) and an agent (he) that need to choose the levels $y_1 \in \mathbf{R}^+$ and $y_2 \in \mathbf{R}^+$ of two interdependent activities $A_1$ and $A_2$. The profits of the organization $\Pi(y_1, y_2, \theta)$ depend on $y_1$ and $y_2$ and a random variable $\theta$ in the following fashion:

$$\Pi(y_1, y_2, \theta) = K_P - (y_1 - \theta)^2 - (y_2 - \theta)^2 + 2\beta y_1 y_2, \tag{1}$$

where the state of the world $\theta$ denotes the conditions under which the organization operates, $\beta$ captures the nature and degree of interaction between both activities and $K_P \gg 0$ is some constant level of profits. This simple specification of profits captures the two sources of interdependence in our model. First, for a given state $\theta$ the marginal return to each activity will depend on the level of the other activity. [14] In particular, if $\beta > 0$ activities are complementary, in the sense that the marginal return to each activity increases with the level of the other activity. Similarly, if $\beta < 0$ activities are substitutes since the marginal return to each activity decreases with the level of the other activity. Activities are independent if $\beta = 0$. Second, for a fixed $y_j$, the marginal return to activity $A_i$, $i \neq j$, will depend on the realized state $\theta$. In this symmetric specification an increase in $\theta$ will produce the same variation in the marginal return to each activity. In summary, to optimize performance, the organization needs to simultaneously adapt activities to each other and also to its underlying environment.

*Preferences:*  We assume that the principal cares about the overall performance of the organization, i.e. the utility of the principal is $u_P(y_1, y_2, \theta) = \Pi(y_1, y_2, \theta)$. From (1) it follows that the levels $y_1^P(\theta)$ and $y_2^P(\theta)$ that maximize profits are given by

$$y_i^P(\theta) = \left(\frac{1}{1 - \beta}\right)\theta, \quad i \in \{1, 2\}. \tag{2}$$

In contrast, the agent's utility is given by

$$u_A(y_1, y_2, \theta) = K_A - (y_1 - (1 + a)\theta)^2 - (y_2 - (1 + a)\theta)^2 + 2\beta y_1 y_2, \tag{3}$$

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[14] With a slight abuse of notation we will denote the random variable and its realization by $\theta$. To avoid confusion the exposition will make clear exactly what object we have in mind.

where the *adaptation bias* $a > -1$ captures the discrepancy between the need of principal and agent to adapt decision $y_i$ to the state, and $K_A \gg 0$ is some constant level of utility.[15] We will make the following assumptions:

*Assumption 1* (existence of equilibrium): $|\beta| < 1$.

*Assumption 2* (desirability of information): $|a| < \frac{1+\beta}{|\beta|}$.

*Assumption 3* (desirability of control): $\beta > -\frac{1}{2}$.

Assumption 1 simply ensures that optimal decisions exist for principal and agent. Assumption 2 is made for convenience in the exposition and ensures that the principal benefits from improved communication when sharing control with her agent.[16] Finally, Assumption 3 guarantees that the principal always benefits from controlling more activities.

Expressions (1) and (3) imply that $2(\theta + \beta y_j)$ and $2((1+a)\theta + \beta y_j)$ describe the marginal benefit of increasing $y_i$ to principal and agent, respectively. Thus, if the principal knows the realization of $\theta$ and has control over $A_i$ she would set $y_i$ according to the decision rule $r_i^P(y_j, \theta)$ where

$$r_i^P(y_j, \theta) = \theta + \beta y_j \quad i \in \{1, 2\}, i \neq j. \tag{4}$$

If the agent has control over $A_i$, however, he would set $y_i$ according to the decision rule $r_i^A(y_j, \theta)$ where

$$r_i^A(y_j, \theta) = (1+a)\theta + \beta y_j \quad i \in \{1, 2\}, i \neq j. \tag{5}$$

Clearly if $a = 0$ principal and agent are completely aligned: for given $\theta$ both would make the same choice if they had control over activity $A_i$, $i \in \{1, 2\}$. If $a \neq 0$ the principal and her agent differ in their preferred level of activity $A_i$. When $a > 0$ an agent is *reactive* in the sense that he would tend to overadapt to the environment by selecting higher levels than the principal for each activity, i.e. $r_i^A(y_j, \theta) > r_i^P(y_j, \theta)$. Conversely, if $a < 0$ the agent is *passive* in that he has an incentive to select lower levels of activity $A_i$, $i \in \{1, 2\}$, than the principal, i.e. $r_i^A(y_j, \theta) < r_i^P(y_j, \theta)$.

There may be several reasons why the principal and agent perceive a different productivity for a given activity. For example, it has often been voiced that managers have a propensity to engage in "empire building," are shortsighted with respect to the future benefits of their choices, or have sunk their human capital in certain technologies making them averse to changes brought about by innovation in the product lines that they manage. Another reason is that observed

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[15] The fact that the state $\theta$ has an equal impact on the marginal return to each activity for the principal and the agent is not an essential feature of our model. Indeed, one could always translate a specification with differing marginal returns per activity to the symmetric specification (1) and (3) as long as the ratio of the coefficient in the linear terms in $\theta$ for principal and agent is the same across activities.

[16] In Section 7 it will shown that if Assumption 2 is violated the conflict of interest is so extreme relative to the interaction between decisions that the principal optimally retains control over both activities.

incentives, in the form of pay for performance, may not completely align the preferences of owners and managers over $(y_1, y_2)$. For instance, Athey and Roberts 2000 argue that providing incentives to elicit effort by rewarding division managers upon a measure of divisional performance may distort their investment choices when investments have firm-wide repercussions.[17] We can interpret (1) and (3) as the reduced-form formulation of a situation in which the agent's choice of $(y_1, y_2)$ imposes an (state-dependent and additive) externality on the organization that he does not fully internalize. For instance, a passive agent's choice of $(y_1, y_2)$ may impose a positive externality on the rest of the organization, resulting in underinvestment in both $A_1$ and $A_2$ relative to (2), or a reactive agent's may impose a negative externality on the rest of the organization, resulting in overinvestment in both $A_1$ and $A_2$ relative to (2).

In summary, we will consider four cases in our analysis, based on whether activities are complementary ($\beta > 0$) or substitutes ($\beta < 0$), and whether the agent is passive ($a < 0$) or reactive ($a > 0$).

*Information:* At the time of contracting it is common knowledge that the state $\theta$ is uniformly distributed in $\Omega = [0, s]$. Thus the parameter $s$ captures the extent of ex-ante uncertainty over the environment faced by the organization. After the initial contract has been signed by both parties the agent costlessly learns the realization of $\theta$. An important assumption of our model is that information is "soft, " i.e. $\theta$ is non-verifiable or, alternatively, can be made verifiable at a rather high cost.

*Contracts and Communication:* We follow the incomplete contracting literature (Grossman and Hart 1986, Hart and Moore 1990 and Hart 1995) in assuming that the principal has available a limited number of institutions to regulate decision making. In particular, the principal can only commit to an ex-ante (deterministic) allocation of decision rights. Thus the principal's limited commitment implies that she is unable to contract over decisions, either ex-ante or ex-post, nor can she compensate the agent differently depending on the information transmitted. Furthermore, once the agent is informed authority over decisions cannot be credibly transferred.

We consider three possible organizational regimes that differ on the identity of the party with authority over activity $A_i$, $i \in \{1, 2\}$. Under *P-Authority* (which we occasionally refer to "centralization") the principal retains control over both activities $A_1$ and $A_2$, and asks the agent to report his private information prior to the decision making stage. Under *A-Authority* ("delegation") the principal transfers control over $A_1$ and $A_2$ to the agent, who makes decisions $y_1$ and $y_2$ after learning $\theta$. Finally, under *S-Authority* ("shared control") the principal retains control over $A_1$ and transfers

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[17]See also Dessein, Garicano and Gertner (2005) and Friebel and Raith (2006).

activity $A_2$ to the agent.[18] Prior to decision making the agent reports his private information after which decisions are made simultaneously.[19]

Whenever the principal controls some (or all) activities, the agent communicates his private information through an informal mechanism: cheap talk. In this sense the message sent by the agent is "unmediated, nonbinding and payoff-irrelevant" (Aumann and Hart 2002). It is well known in the cheap-talk literature[20] that several rounds of communication can expand the set of equilibrium outcomes: more can be achieved through long conversations than with a single message sent by the informed party. It is still an open question, however, what the "optimal communication protocol" would be in settings like the ones studied in CS (1982) and MS (1991). We follow Alonso, Dessein and Matouschek (2006) in arguing that given the limited commitment of the principal to structure the relation with the agent, it seems reasonable to focus on the simplest informal communication mechanism.

The timing of the game is as follows. First, the principal decides on an allocation of decision rights between herself and the agent. Second, the agent learns the realization of the state $\theta$. If control over both activities is transferred to the agent (A-Authority), he makes decisions $y_1$ and $y_2$. If both activities are centralized or control is shared, the agent sends a single, unmediated, and costless message $m$. Under P-Authority, upon reception of the message $m$ the principal makes decisions $y_1$ and $y_2$. Under S-Authority, the principal and agent simultaneously select $y_1$ and $y_2$, respectively. For all possible allocations of decision rights, once decisions are made payoffs are realized and the game ends.

## 4    Equilibrium Analysis

In this section we define the equilibrium solution that will be used and analyze equilibrium behavior for each possible allocation of control. Since, in our framework, whenever the principal transfers control of both activities to the agent the latter does not engage in communication, the equilibrium under A-Authority will be determined by the pair of decision rules $\left\{ y_1^A\left(\theta\right), y_2^A\left(\theta\right) \right\}$ that maximize

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[18] Given the symmetry of activities in our model it is clear than the principal's expected utility would be unchanged if instead she retained control over $A_2$ and delegated $A_1$ to the agent.

[19] Our model affords an alternative "delegation" interpretation in which the agent is delegated both decisions at the outset and the principal can potentially reverse both decisions (P-Authority), reverse only one decision (S-Authority), or commits not to interfere in the agent's choice (A-Authority). To make the analogy complete, under S-Authority the principal only observes the agent's choice over the decision that she can reverse. Anticipating that the principal will reverse any decision that is not ex-post optimal given the information revealed by the agent's choice, the agent will select decisions that he is sure to be rubberstamped by the principal. In this setting, equilibrium outcomes would be equivalent to our model with communication.

[20] See Aumann and Hart (2002) and Krishna and Morgan (2004).

the agent's utility for each possible realization of the state $\theta$. Whenever the principal retains some control rights, however, the agent communicates with her prior to decision making. Although the equilibrium strategies are different if the principal centralizes all decisions or shares control with her agent, we shall refer to both cases as a communication equilibrium.

Formally, a communication equilibrium under P-Authority and S-Authority is characterized by (i.) a communication rule for the agent, (ii.) decision rules for the decision makers (principal under P-Authority and principal and agent under S-Authority), and (iii.) a belief function for the principal. The communication rule for the agent specifies the probability of sending message $m \in M$ conditional on observing state $\theta$, and we denote it by $\mu(m \mid \theta)$. Under P-Authority the decision rule maps the message $m$ into decisions $y_1 \in \mathbf{R}^+$ and $y_2 \in \mathbf{R}^+$ which we denote by $\{y_1^P(m), y_2^P(m)\}$. Under S-Authority the decision rule for the principal maps the message $m$ into a decision $y_1 \in \mathbf{R}^+$, denoted by $y_1^S(m)$, while the decision rule for the agent maps the message $m$ and the state $\theta$ into a decision $y_2 \in \mathbf{R}^+$, denoted by $y_2^S(m, \theta)$. Finally, the belief function is denoted by $p(\theta \mid m)$ and states the posterior probability assigned by the principal to state $\theta$ after receiving message $m$.

We focus on Perfect Bayesian Equilibria of the communication game which require that (i.) the communication rule is optimal for the agent given the decision rules, (ii.-a) under P-Authority the decision rule of the principal is optimal given the belief function, (ii.-b) under S-Authority the decision rule for each decision maker is optimal given the equilibrium decision rule of the other player and the belief function, and (iii.) the belief function is derived from the communication rule using Bayes' rule whenever possible.

We will solve for an equilibrium by backward induction and thus we start by characterizing equilibrium decision rules under A-Authority, P-Authority and S-Authority. In Section 4.2, we study how the incentives of the agent to strategically misrepresent his information vary with the activities under the control of the principal. These incentives will be captured by the *communication bias* that describes the relative desire of a reactive agent to overstate, and a passive agent to understate, his private information. We then characterize all PBE of the communication game in Section 4.3. The proofs of all lemmas and propositions are relegated to the Appendix.

## 4.1 Decision Making

The next lemma characterizes equilibrium decision making under A-Authority, P-Authority and S-Authority.

LEMMA 1 (Decision Making) i- A-Authority: *Under A-Authority equilibrium decisions satisfy*

$$y_i^A(\theta) = \left(\frac{1}{1-\beta} + \frac{a}{1-\beta}\right)\theta, \quad i \in \{1, 2\}. \tag{6}$$

ii- P-Authority: *Consider a PBE of the communication game under P-Authority and suppose that the principal receives message m. Then equilibrium decisions satisfy*

$$y_i^P(m) = \frac{1}{1-\beta}\mathrm{E}\left[\theta\,|m\,\right], \quad i \in \{1, 2\}. \tag{7}$$

iii- S-Authority: *Consider a PBE of the communication game under S-Authority and suppose that the principal receives message m. Then equilibrium decisions satisfy*

$$
\begin{aligned}
y_1^S(m) &= \left(\frac{1}{1-\beta} + \frac{a\beta}{1-\beta^2}\right)\mathrm{E}\left[\theta\,|m\,\right], \\
y_2^S(\theta, m) &= \left(\frac{1}{1-\beta} + \frac{a}{1-\beta^2}\right)\mathrm{E}\left[\theta\,|m\,\right] + (1+a)\left(\theta - \mathrm{E}\left[\theta\,|m\,\right]\right).
\end{aligned}
\tag{8}
$$

Under A-Authority the agent has full flexibility in adapting each activity both to the realization of $\theta$ and to the level of the other activity. Therefore decision making fully utilizes the agent's private information. Whenever $a \neq 0$, however, the agent's choice (6) will differ from the principal's preferred decision, as given by (2), i.e. delegation to the agent induces biased decision making if preferences are dissonant. Furthermore, this bias $\left|y_i^A(\theta) - y_i^P(\theta)\right| = |a|\,\theta/\left(1-\beta\right), i \in \{1, 2\}$, increases whenever the adaptation bias $|a|$ or $\beta$ increases.

Under P-Authority the principal can avoid the previous distortion in decision making by making both decisions herself. Nevertheless, whenever the message $m$ does not perfectly reveal $\theta$, decision making does not incorporate all available information in the organization, and as a result decisions are poorly adapted to the environment. In the extreme, when the message is completely uninformative, the principal's choice is constant over the state space, i.e. $y_i^P(m) = \frac{1}{1-\beta}\mathrm{E}\left[\theta\right], i \in \{1, 2\}$. In this case the quality of communication is so poor that the principal cannot improve with respect to the situation in which she bases decisions solely on her prior.

Finally, under S-Authority both principal and agent will adjust the decisions under their control based on the information transmitted by the agent. To understand (8), let $\Omega\left(m\right) \subset \Theta$ denote the subset of the state space where the agent sends message $m$. As shown in (5), for a fixed $y_1$ the agent's optimal choice of $y_2$ depends linearly in $\theta$, thus $y_2^S(\theta, m)$ incorporates a linear term $(1+a)\left(\theta - \mathrm{E}\left[\theta\,|m\,\right]\right)$. Furthermore, the principal will optimally choose $y_1$ based on $\mathrm{E}\left[\theta\,|m\,\right]$ and the average decision made by the agent in $\theta \in \Omega\left(m\right)$. As a result we have that *average* equilibrium

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decisions for $\Omega(m)$ are given by[21]

$$
\begin{array}{rcl}
y_1^S(m) & = & \left( \dfrac{1}{1-\beta} + \dfrac{a\beta}{1-\beta^2} \right) \mathrm{E}\left[\theta \,|m\right], \qquad\qquad (9)\\[2ex]
\mathrm{E}\left[y_2^S(\theta, m)\,|m\right] & = & \left( \dfrac{1}{1-\beta} + \dfrac{a}{1-\beta^2} \right) \mathrm{E}\left[\theta \,|m\right].
\end{array}
$$

This expression allows us to give an alternative interpretation of (8) by noting that (9) would be the outcome of joint decision making if *both* principal and agent share the *same* posterior $\mathrm{E}\left[\theta \,|m\right]$ over the state, and simultaneously make decisions $y_1$ and $y_2$, respectively. Therefore, (8) reflects the fact that equilibrium decisions under shared control are adapted to each other only on the basis of the information shared by the agent, while the agent is able to use his precise knowledge of $\theta$ to further adjust $y_2$ to the environment.

An immediate implication of (9) is that both decisions will be biased in expectation with respect to the principal's preferred choice. Indeed, since activities interact, decision making under S-Authority will feature a game of strategic complements if $\beta > 0$, and strategic substitutes if $\beta < 0$. As a result, for each $m$ the principal will react to the agent's expected choice of $y_2$ by adjusting $y_1$ away from her first best choice by an amount $y_1(m) - \mathrm{E}\left[y_1^P(\theta)\,|m\right] = \frac{a\beta}{1-\beta^2}\mathrm{E}\left[\theta\,|m\right]$, while on average the agent will alter $y_2$ with respect to the case of A-Authority by $\mathrm{E}\left[y_2^A(\theta) - y_2^S(\theta, m)\,|m\right] = \frac{a\beta}{1-\beta^2}\mathrm{E}\left[\theta\,|m\right]$.[22] As a result, the indirect effect of centralizing activity $A_1$ on the agent's average choice of $y_2$ will be a reduction in the extent by which the agent biases $y_2$ if activities are complementary, while it will increase the difference $\left|\mathrm{E}\left[y_2(\theta, m) - y_2^P(\theta)\,|m\right]\right|$ when activities are substitutes. Finally, as was true under A-Authority, average decision making under S-Authority will be further away from the principal's ideal choice whenever the adaptation bias $|a|$ or $\beta$ increases.

Given equilibrium decisions we now turn to the incentives of the agent to strategically misrepresent his information. Since, as mentioned earlier, communication takes place only if the principal retains some control rights, we will focus in the next two subsections on P-Authority and S-Authority.

## 4.2 Incentives to Strategically Misrepresent Information.

Whenever a principal relies on the information of an agent for decision making the latter has an incentive to distort his message to accommodate decisions to his self-interest. In the context of our

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[21] Note that the agent could conceivably use the possibility of communication with the principal in order to commit to different decision rules. However since communication is costless in our setting this commitment device would not be credible. See Baliga and Morris (2002) for a general discussion of this issue when both sender and receiver play a game after the cheap talk stage.

[22] In order to compare the bias in decision making we are implicitly assuming that the message $m$ generates the same posterior under P-Authority and S-Authority.

model, a passive agent would want the principal to believe that the state is lower than is true value, while a reactive agent would want her to believe a higher realization of the state. In this section we seek to quantify the incentive of a reactive agent to overstate, and, conversely, of a passive agent to understate, his private information as a function of the activities under the control of the principal. We start with P-Authority.

*P-Authority* : Suppose that when the realized state is $\theta$ the agent is able to convince the principal that the true state is actually $\widehat{\theta}_P$. What would be the optimal $\widehat{\theta}_P$ from the perspective of the agent? If the principal makes decisions according to the rule $\left(y_1^P(\cdot), y_2^P(\cdot)\right)$ the agent would select $\widehat{\theta}_P(\theta)$ such that

$$\widehat{\theta}_P(\theta) \in \arg\max_{\theta'} - (y_1^P(\theta') - (1+a)\,\theta)^2 - (y_2^P(\theta') - (1+a)\,\theta)^2 + 2\beta y_1^P(\theta')y_2^P(\theta')). \qquad (10)$$

Since in equilibrium the principal acts on given information according to (7), we can solve (10) to find that

$$\widehat{\theta}_P(\theta) = (1 + b_P)\theta,$$

where $b_P$ is the *communication bias under P-Authority* that satisfies

$$b_P = a. \qquad (11)$$

First, we have that for a reactive agent $b_P > 0$, and for a passive agent $b_P < 0$, thus corroborating our previous claim that a reactive agent, who favors higher decisions than the principal, has an incentive to overstate his private information while a passive agent, who favors lower decisions than the principal, has an incentive to understate his private information. Second, the relative desire to overstate or understate $\theta$, as given by $\left|\widehat{\theta}_P(\theta) - \theta\right| = |b_P|\,\theta$, is linear in $\theta$. In particular, for $\theta = 0$ principal and agent's preferred decisions coincide and thus the agent has no incentive to misrepresent his private information. This incentive increases, however, as the state becomes more extreme.[23]

Second, changes in the adaptation bias $|a|$ that increase the conflict of interest between principal and agent also exacerbate the agent's incentive to misrepresent his private information. Intuitively, a higher conflict of interest leads to a higher discrepancy between the preferred decisions of principal

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[23] In Section 4.3 we will show that this does not necessarily imply that communication corresponding to higher states of the world is less informative. Indeed, we will see that messages corresponding to higher states are less informative only for a reactive agent.

and agent. Since decision making under P-Authority is unaffected by the agent's preferences, a more "biased" agent would be compelled to magnify the distortion in his message.

*S-Authority:* We now turn to the incentives of the agent to misrepresent his information if the principal shares control with him. We proceed as before by inquiring what $\widehat{\theta}_S$ would the agent want the principal to believe that is the value of his private information. If the principal believes that the state is $\widehat{\theta}_S$, and given that the agent will react by selecting $y_2$ according to (8), the principal will set $y_1\left(\widehat{\theta}_S\right)$ according to (8) with $\mathrm{E}\left[\theta\left|m\right.\right]=\widehat{\theta}_S$. Since the agent has full flexibility in selecting $y_2$, however, he will jointly determine $\left(\widehat{\theta}_S,y_2\right)$ such that

$$\left(\widehat{\theta}_S,y_2\right)\in\arg\max_{\theta',y_2}-\left(y_1(\theta')-(1+a)\,\theta\right)^2-\left(y_2-(1+a)\,\theta\right)^2+2\beta y_1(\theta')y_2. \tag{12}$$

The solution to (12) is direct: since it follows from (8) that the agent can potentially induce any decision $y_1$ through his choice of $\widehat{\theta}_S$, he can implement his first-best choice of $(y_1,y_2)$, as given by (6), by setting $\widehat{\theta}_S$ according to

$$\widehat{\theta}_S=(1+b_S)\,\theta,$$

where the *communication bias under* S-*Authority* $b_S$ satisfies

$$b_S=\frac{a}{(1+\beta\,(a+1))}. \tag{13}$$

In essence, under S-Authority the agent can always set $y_2$ to his preferred level given the true state $\theta$, as given by (6). This leaves him with the sole purpose of inducing the principal to set $y_1$ at the agent's preferred level, as given by (6). Joint decision making, however, implies that the principal's choice of $y_1$ will no longer concord with her preferred choice under P-Authority, as given by (7), but rather by the equilibrium level when she reacts to the agent's anticipated choice of $y_2$. As we discuss in Section 5 it is this responsiveness of the principal that results in the amount of information transmitted by the agent differing significantly if decisions are complements or substitutes. Finally, the qualitative nature of communication and the comparative statics under S-Authority are similar to the case of P-Authority.[24]

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[24] First, the qualitative nature of the communication bias in our model does not depend on the allocation of decision rights: a passive agent would have an incentive to understate his private information and, conversely, a reactive agent to overstate his private information, irrespective of the decisions controlled by the principal. Also, whenever $\theta=0$ the agent has no incentive to distort his message, and the extent of the distortion, as measured by $\left|\widehat{\theta}_S\left(\theta\right)-\theta\right|=|b_S|\,\theta$, increases with $\theta$. Thus, from a communication perspective, the interests of principal and agent are perfectly aligned at $\theta=0$ and diverge at a constant rate as the state increases. Second, as preferences become more dissonant the agent's incentive to misrepresent his private information increases. Formally this follows from $\partial\left[b_S\right]/\partial a=(1+\beta)/(\beta+a\beta+1)^2>0$.

We defer to Section 5 a full comparison of the impact of the allocation of authority on the strategic transmission of information by the agent after we analyze in the next subsection how the equilibrium quality of communication relates to the communication bias of the agent.

## 4.3   Communication Equilibria

We now turn to the characterization of the communication equilibria under P-Authority and S-Authority. We first show that, as in CS (1982) and MS (1991), all communication equilibria are interval equilibria in the sense that the state space is partitioned into intervals and the agent only reveals the interval in which the true state lies.[25] Thus communication is noisy and information is lost in decision making, where it will be shown that this loss depends directly on the corresponding communication bias $b_l$, $l \in \{P, S\}$ as defined in (11) and (13), respectively. We also show that communication is *qualitatively* different if the agent has an incentive to overstate or understate his information. When the agent's incentive is to overstate his private information there are infinitely many communication equilibria which differ in the amount of information transmitted. In contrast, we will show that only a finite number of equilibria are possible if the agent has an incentive to understate his information. This will imply, among other things, that strategic communication is comparatively worse with a passive agent.

Let $a^N \equiv (a_0, ., a_i, .., a_N)$, with $a_{i-1} < a_i$, be a partition of $[0, s]$ into $N$ disjoint intervals, where $a_0 = 0$, and $a_N = s$.[26] Since the nature of strategic communication is qualitatively different if $b_l > 0$ or $b_l < 0$ we discuss each case separately.

### 4.3.1   Communication with a Reactive Agent.

When the agent has an incentive to (weakly) overstate his private information, and interests coincide at some state, there are an infinite number of equilibria of the communication game.[27] The following proposition characterizes all *finite* equilibria of the communication game for $b_l > 0$, $l \in \{P, S\}$.

PROPOSITION 2 (Communication Equilibria $b_l > 0$). *If $b_l > 0$ then for every positive integer $N$ there exists at least one equilibrium $(\mu(\cdot), y_1^l(\cdot), y_2^l(\cdot), p(\cdot))$, where*

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[25] One difference between CS (1982) and MS (1991) and this paper is that under S-Authority parties play a game after the cheap talk stage, instead of autonomously making decisions based on the message received. In spite of this different structure, the proofs of Propositions 2 and 4 show that the agent's equilibrium communication rule will take the form of an interval equilibrium.

[26] As all communication equilibria will be shown to be interval equilibria for expositional brevity we will refer to a given equilibrium by the partition $a^N$ induced in the state space.

[27] This is also a feature of the cheap talk equilibria considered in Stein (1989), MS (1991), Alonso, Dessein and Matouschek (2006), and Rantakari (2006).

  i.  $\mu(m \mid \theta)$ *is uniform, supported on* $[a_{i-1}, a_i]$ *if* $\theta \in (a_{i-1}, a_i)$.

  ii.  $p(\theta \mid m)$ *is uniform, supported on* $[a_{i-1}, a_i]$ *if* $\theta \in (a_{i-1}, a_i)$.

  iii.  *P-Authority:*   $\{y_1^l(\cdot), y_2^l(\cdot)\} = \{y_1^P(m), y_2^P(m)\}$, *as given by (7).*

     *S-Authority:*   $\{y_1^l(\cdot), y_2^l(\cdot)\} = \{y_1^S(m), y_2^S(m, \theta)\}$, *as given by (8).*

  iv.  *The partition* $a^N$ *satisfies* $a_{i+1} - a_i = a_i - a_{i-1} + 4b_l a_i$.

*Moreover, all other finite equilibria induce decisions as a function of the state that are equivalent, almost everywhere, as those in this class for some value of* $N$.

  Part (i) and (ii) of Proposition 2 ratifies our previous claim that communication equilibria are interval equilibria in which the only substantive information transmitted is the interval in which the true state lies. Now let the precision of a given message $m$ be the inverse of the residual variance given $m$, i.e. $1/\mathrm{E}\left[(\theta - \mathrm{E}[\theta|m])^2 \mid m\right]$. Then part (iv) establishes that the size of the intervals $\Delta_{i+1} = a_{i+1} - a_i$ increases with $\theta$, and thus the precision of a message $m \in [a_{i+1}, a_i]$ decreases as the state becomes more extreme. To see this, note that the principal's decision(s) will be based on the conditional expectation of the state $\theta$ given the agent's message $m$, i.e. on $\mathrm{E}[\theta|m]$. Since the agent has an incentive to overstate $\theta$, the agent's indifference at state $a_i$ between consecutive intervals requires that the distance to consecutive conditional expectations must increase. Given a uniform distribution this is only possible if the size of consecutive intervals increases as well. As a result, the informativeness of communication with a reactive agent worsens as the agent's incentive to misrepresent his information, given by $b_l\theta$, increases. Moreover, since the amount by which the agent exaggerates his private information is proportional to the state it is then intuitive that the rate at which communication deteriorates also increases with $\theta$.

  We now turn to the equilibrium effectiveness of communication which we describe by the residual variance of the message $\mathrm{E}\left[(\theta - \mathrm{E}[\theta|m])^2\right]$. We will also occasionally refer to the variance of communication for each organizational regime $\sigma_l^2 = \mathrm{E}\left[(\mathrm{E}[\theta|m] - \mathrm{E}[\theta])^2\right]$, $l \in \{P, S\}$, where clearly $\mathrm{E}\left[(\theta - \mathrm{E}[\theta|m])^2\right] = \sigma^2 - \sigma_l^2$. Proposition 2 establishes that there is an infinite number of communication equilibria.[28] We next show that the limit of strategy profiles and beliefs as the number of intervals grows indefinitely constitutes an equilibrium of the communication game, and that this equilibrium is *ex-ante* preferred by both principal and agent under P-Authority and S-Authority to any other equilibrium.

PROPOSITION 3 (Infinite Equilibrium). *The limit of strategy profiles and beliefs* $(\mu(\cdot), y_1^l(\cdot), y_2^l(\cdot), p(\cdot))$ *as* $N \to \infty$ *is a Perfect Bayesian Equilibrium of the communication game. This equilibrium*

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[28]This also implies that there is no uniform upper-bound on the precision of the message sent by the agent. Indeed, there is always a message $m$, associated with a partition close to the state $\theta = 0$ where the interests of principal and agent coincide, such that the precision of message $m$ exceeds any given number $M$.

*is the most informative equilibrium and, given Assumption 2, ex-ante Pareto dominates any finite equilibrium. In particular the residual variance of communication in this equilibrium is given by*

$$\sigma^2 - \sigma_l^2 = \frac{4b_l}{4b_l + 3}\sigma^2 \qquad l \in \{P, S\}. \tag{14}$$

This proposition establishes that there is an upper-bound on the quality of communication with the agent which is attained in the equilibrium with an infinite number of intervals. Moreover, the quality of communication varies with the communication bias in a simple manner, where it is readily seen that the residual variance increases whenever $b_l$ increases. This confirms the intuitive result that for any allocation of decision rights communication worsens whenever the agent's incentive to distort his information increases. Communication is perfect if there is complete alignment between principal and agent $(a = 0)$ and breaks down (i.e. is completely uninformative) as $b_l$ becomes arbitrarily large.

As it is often the case in cheap talk models, the presence of multiple equilibria makes equilibrium selection a delicate matter. Nevertheless, Proposition 2 indicates that, for the case of a reactive agent, all finite equilibria are ex-ante Pareto dominated by the equilibrium described in Proposition 2 and it is thus natural that we focus on the latter when assessing the expected utility of the principal under each organizational arrangement.

### 4.3.2 Communication with a Passive Agent.

The following proposition characterizes all communication equilibria under P-Authority and S-Authority whenever $b_l < 0$, $l \in \{P, S\}$.

PROPOSITION 4 (Communication Equilibria $b_l < 0$). *If $b_l < 0$ then there exists a positive integer $N(b_l)$ such that, for every $N$ with $1 \leq N \leq N(b_l)$, there exists at least one equilibrium $(\mu(\cdot), y_1^l(\cdot), y_2^l(\cdot), p(\cdot))$, where*

    *i.   $\mu(m \mid \theta)$ is uniform, supported on $[a_{i-1}, a_i]$ if $\theta \in (a_{i-1}, a_i)$.*

    *ii.  $p(\theta \mid m)$ is uniform supported on $[a_{i-1}, a_i]$ if $\theta \in (a_{i-1}, a_i)$.*

    *iii. P-Authority:  $\{y_1^l(\cdot), y_2^l(\cdot)\} = \{y_1^P(m), y_2^P(m)\}$, as given by (7).*

              S-Authority:  $\{y_1^l(\cdot), y_2^l(\cdot)\} = \{y_1^S(m), y_2^S(m, \theta)\}$, as given by (8)*

    *iv.  The partition $a^N$ satisfies $a_i - a_{i-1} = a_{i+1} - a_i - 4b_l a_i$.*

*Moreover, all other equilibria induce decisions as a function of the state that are equivalent, almost everywhere, as those in this class for some value of $N$.*

A key insight of Proposition 4 is that strategic transmission of information by a passive agent always leads to a bounded number of equilibria, in spite of the interests of principal and agent coinciding at state $\theta = 0$. CS 1982 already showed that if the principal's and agent's preferred

decisions never coincide it must be the case that only a finite number of different communication equilibria are possible. Therefore Proposition 4 states that this could also be the case if the interests of both parties coincide at some state, as long as the agent has an incentive to understate his information. To see why this is the case, we analyze the indifference condition in part (iv..). Since the agent would want the principal to believe that the state is lower than its true value, for the agent to be indifferent at $a_i$ between sending two consecutive messages it must be that the difference between consecutive conditional expectations must decrease as $\theta$ increases. Given a uniform distribution this is possible only if the size of the intervals also decreases with $\theta$, *raising the informativeness of the message $m$ as the state becomes more extreme*. However, if there is an equilibrium with an infinite number of partitions it must be that the informativeness of the message increases as partitions approach the state in which the principal's and agent's preferred decisions coincide, i.e. $\theta = 0$, which is inconsistent with the decrease in informativeness of the message as $\theta \to 0$.

We now turn to the analysis of the quality of communication with a passive agent. The next proposition describes the quality of communication for the equilibrium with the highest number of partitions $N(b_l)$.

PROPOSITION 5 (Residual Variance $b_l < 0$). (i) *The equilibrium with the highest number of partitions $a^{N(b_l)}$ is the most informative equilibrium. In particular, whenever $b_l > -1/2$ the residual variance of communication is given by*

$$\sigma^2 - \sigma_l^2 = \frac{4\,|b_l|}{4b_l + 3}\left[\frac{3\,(1 + b_l)}{\sin^2 N(b_l)\,\varphi} - 1\right]\sigma^2 \qquad l \in \{P, S\}\,, \tag{15}$$

*where* $\varphi = \tan^{-1}\frac{2\sqrt{|b_l|(b_l+1)}}{2b_l+1}$ *and* $N(b_l)$ *satisfies* $\frac{\pi}{2|\varphi|} - \frac{1}{2} \le N(b_l) \le \frac{\pi}{2|\varphi|} + \frac{1}{2}$. *For* $b_l \le -1/2$, $\sigma_l^2 = 0$.

(ii) $\sigma^2 - \sigma_l^2$ *is weakly increasing in* $|b_l|$.

(iii) *Given Assumption 2, the principal and agent prefer the equilibrium $a^{N(b_l)}$ to any other equilibrium.*

The comparative statics for the residual variance are as expected: communication is less informative as the conflict between principal and agent intensifies. In particular, for $b_l < -1/2$ the only possible equilibrium is the babbling equilibrium in which communication is uninformative. Also, we can substantiate our claim that communication with a passive agent is comparatively worse than with a reactive agent. By comparing (14) and (15) we see that for two agents $B_1$ and $B_2$, and a fixed allocation of decision rights, if the communication biases satisfy $b_l^{B_1} + b_l^{B_2} = 0$ with $b_l^{B_1} > 0$,

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then $\sigma_l^2\left(b_l^{B_1}\right) > \sigma_l^2\left(b_l^{B_2}\right)$. This difference stems from the finiteness of communication equilibria for a passive agent and, as we will see in Section 7, will render allocations of authority that rely on communication comparatively less desirable when the agent is passive. Finally, given Assumption 2, all equilibria are ex-ante Pareto dominated by the most informative equilibrium and, as in the case of a reactive agent, it is natural that we focus on the latter when assessing the expected utility of the principal under each organizational arrangement.

## 5    Allocation of Authority and the Quality of Communication

In this section we derive our main results concerning the question of if and when the agent will reveal more information to the principal when he enjoys some control over activities. We will show that: (i) the agent may reveal *more* or *less* information when the principal shares control with him, (ii) whether the principal can improve communication by sharing control will be determined by the nature of the interaction between activities: if activities are complementary the principal always improves communication with respect to the centralized case by delegating one activity to the agent, while if activities interact as substitutes communication actually worsens if control over activities is shared.

It may seem odd that an agent who gains control over one activity will be less forthcoming in revealing his private information. In principle, if under symmetric information moving from centralized to shared control leads to equilibrium decisions that make the agent better off, it follows that decisions under shared control are "closer" to the agent's preferred choices than under centralization, and he would thus reduce the distortion in his message. Indeed, if the principal could commit to making one decision, say $y_2$, *according to the agent's decision rule (4)*, the agent would be willing to share *more* information with the principal regardless of the nature of the interaction between activities.[29] Intuitively, the agent is more willing to communicate precise information if the principal takes his interests into account. If the agent has control over $A_2$, however, he is no longer

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[29] This is true given our assumptions except for the case of an extremely biased passive agent and substitute activities in which $a + \beta < -1$. To see this, note that now the principal's choice satisfies

$$y_1(y_2, \theta) = \theta + \beta y_2$$
$$y_2(y_1, \theta) = (1 + a)\theta + \beta y_1.$$

Clearly, equilibrium decisions in this case are equivalent to average equilibrium decisions under shared control, as given by (9), for the same posterior mean of the state. We can compute the communication bias $\widetilde{b}_P$ under P-Authority in this case to obtain

$$\widetilde{b}_P = \frac{a\left(1 + \beta\left(a + 1\right)\right)}{\left(a + 1 + \beta\right)^2 + \left(1 - \beta^2\right)}.$$

As long as $1 + a + \beta > 0$ we have that $\left|\widetilde{b}_P\right| < |b_P|$.

concerned in affecting decision making in both decisions, since he has full flexibility in adjusting $y_2$ to his ideal choice $(1 + a)/(1 - \beta)\theta$, but in inducing the principal to set $y_1$ to the agent's ideal choice for $A_1$, i.e. inducing the principal to set $y_1 = (1 + a)/(1 - \beta)\theta$.

In our model, the mechanism by which the agent's incentives to misrepresent his information vary with the allocation of decision rights lies in the differing responsiveness of the principal to the information communicated by the agent. In essence, delegating activity $A_2$ to the agent allows the principal to commit to respond to the agent's information differently than if she centralized decision making. For example, when activities are complementary, transferring control of $A_2$ to a reactive agent will make the principal bias her decision upwards by an amount $a\beta/\left(1 - \beta^2\right)$ in response to the agent's expected choice of $y_2$.[30] This increase in the *responsiveness* of the principal implies that in this case the agent would need to overstate his information less to induce a choice of $y_1 = (1 + a)/(1 - \beta)\theta$. The opposite is true if decisions are substitutes: if a reactive agent controls $A_2$ the principal will respond to the agent's message by biasing her decision downwards by an amount $a\left|\beta\right|/\left(1 - \beta^2\right)$. As a result the agent would need to overstate his information more to induce his preferred choice of $y_1$. The following proposition states that the principal can always improve communication by sharing control over complementary activities, while communication worsens when control is shared over activities that interact as substitutes.

PROPOSITION 6 (Communication Comparison). *Suppose $a \neq 0$ and let $\sigma_P^2$, $\sigma_S^2$, be the variance of communication under P-Authority and S-Authority, respectively.*

    *i.    If activities are complements $(\beta > 0)$ then $\sigma_P^2(a, \beta) \leq \sigma_S^2(a, \beta)$ and*
        *$\sigma_S^2(a, \beta) - \sigma_P^2(a, \beta)$ increases in $\beta$.*

    *ii.   If activities are substitutes $(\beta < 0)$ then $\sigma_P^2(a, \beta) \geq \sigma_S^2(a, \beta)$ and*
        *$\sigma_P^2(a, \beta) - \sigma_S^2(a, \beta)$ increases in $\left|\beta\right|$.*

Figure 3 depicts the variance of communication $\sigma_S^2(a, \beta)$ and $\sigma_P^2(a, \beta)$ for $\left|\beta\right| = 0.3$. The proposition shows that a commitment to be more responsive to the agent's message can lead to better communication only when activities are complementary and that the communication advantage of S-Authority over P-Authority for complementary activities increases with the strength of the complementary of activities, while the communication advantage of P-Authority over S-Authority increases with the strength of interaction for substitute activities. To see this, note that in our model communication under P-Authority is unaffected by $\beta$: since both principal and agent perceive the same interaction between activities they would both scale their preferred choices by the same amount in response to a change in $\beta$. However, the strength of the interaction does affect the responsiveness of the principal under shared control. This can be seen from (7) and (8) by noting

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[30]See our discussion in Section 4.1.

that $\left|y_1^P(m) - y_1^S(m)\right| = |a| \beta \mathrm{E}\left[\theta\,|m\right] / \left(1 - \beta^2\right)$ which increases in $\beta$. Intuitively, the principal will react more to the expected choice of the agent the higher her perceived interaction between $A_1$ and $A_2$. This results in improved communication if she shares control over complementary activities and worse communication if she shares control over substitute activities.

In summary, the allocation of decision rights and the nature of the interaction between activities determines the amount of information that reaches the principal. Moreover, granting the agent formal authority over some decisions does not result in improved communication: if activities are substitutes more information will reach her if she centralizes all decision making.[31] In the next section we analyze how the informativeness of communication and the lack of congruence between principal and agent affect the expected utility of the principal under each organizational arrangement.

## 6    Organizational Performance

When assessing each allocation of decision rights the principal compares the *loss of control* from biased decision making and the *loss of information* when decision making does not incorporate all available information to the organization. Under P-Authority, the principal ensures unbiased decision making, but she must rely on a noisy estimate of the state to make decisions. This lack of precise information may cause equilibrium decisions to be far from the principal's first-best choice. Under A-Authority control is transferred to the inherently best informed party, and all pertinent information is employed in decision making. However, the conflicting goals of principal and agent lead to equilibrium decisions that are suboptimal for the principal. An intermediate solution, S-Authority, leads to both losses, albeit in different magnitude. First, under S-Authority the principal reacts to the agent's expected choice of $y_2$ by adjusting $y_1$ away from her first-best choice. As a result, shared control induces biased decision making in both decisions. Second, as a result of noisy communication joint decision making will take place under asymmetric information, implying that (i) both $y_1$ and $y_2$ will be adapted to each other only on the basis of the posterior mean of the state based on the message of the agent, and (ii) the agent's ex-post informational advantage will allow him to locally adjust $y_2$ to his precise knowledge of $\theta$. Thus, information is incorporated into equilibrium decisions asymmetrically, where the decision controlled by the agent is more sensitive to the underlying environment than the decision under the control of the principal.

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[31] We can also cast this result under the "delegation" interpretation introduced in Footnote 20: if the principal can commit not to interfere in one decision (S-Authority), she is able to "rubberstamp" more decisions if activities are complementary than if she cannot commit not to interfere in both decisions (P-Authority), while the reverse is true when activities interact as substitutes.

In this section we characterize the performance of each organizational regime, and in the next section we analyze the principal's optimal choice for the different parameter values of our model.[32] To this end, let $L^l = E_\theta \left[ u_P \left( y_1^P(\theta), y_2^P(\theta), \theta \right) \right] - E_\theta \left[ u_P \left( y_1^l(\cdot), y_2^l(\cdot), \theta \right) \right], l \in \{A, P, S\}$, where $y_i^P(\theta)$, $i \in \{1, 2\}$, is given by (2) and $\{y_1^l(\cdot), y_2^l(\cdot)\}$ are given in Propositions 2 and 4, denote the loss in the expected utility of the principal under organizational regime $l$ when compared to first-best decision making. The next proposition characterizes this loss for each allocation of decision rights.

PROPOSITION 7 (Organizational Performance). *The expected loss to the principal* $L^l$ *for each organizational regime* $l \in \{A, P, S\}$ *is given by*

P-Authority:  $L^P = L_X \left( \sigma^2 - \sigma_P^2 \right)$ *where* $L_X = \frac{2}{1-\beta}$.

A-Authority:  $L^A = 4L_C^A \sigma^2$ *where* $L_C^A = \frac{2}{1-\beta} a^2$.

S-Authority:  $L^S = 4L_C^S \sigma^2 - L_C^S \left( \sigma^2 - \sigma_S^2 \right) + \left( L_X + L_L \right) \left( \sigma^2 - \sigma_S^2 \right)$,

  *where* $L_C^S = \frac{a^2}{1-\beta^2}$ *and* $L_L = a^2 - 1$.

We analyze each of these losses in detail.

*P-Authority*: Decision making under centralization is hindered by the agent's imperfect revelation of his private information. This loss of information depends on two factors: (i) the quality of communication under P-Authority $\left( \sigma^2 - \sigma_P^2 \right)$, and (ii) the principal's relative value of information, as given by $L_X = 2/(1 - \beta)$. The relative value of information to the principal increases with the strength of the interaction for complementary activities, and decreases with the strength of the interaction if activities are substitutes. Intuitively, when activities are complementary a slight variation in the underlying environment leads to changes in the marginal returns to each activity that reinforce each other, resulting in substantial variations in the optimal levels for both activities. The opposite is true when activities are substitutes: slight changes in the underlying conditions may have little impact on optimal levels since substitutability of activities tends to be self-correcting.[33] Thus relying on an imprecise signal of the state is more costly when activities are complements, since the optimal level for both activities may be quite far from the average optimal level. Finally, since decision making under P-Authority is unaffected by the agent's preferences, the adaptation bias of the agent affects performance only through its impact on the quality of communication. As shown in Section 4.2 an increase in $|a|$ worsens communication, thereby reducing the desirability

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[32] Judging the performance of each organizational regime on the basis of the principal's expected utility follows from our assumption that contracts only specify an ex-ante allocation of decision rights and transfers are not allowed. Allowing for (unconstrained) transfers between the two parties would shift the focus to allocations that maximize expected joint surplus. In this case, decision making under shared control leads to intermediate decisions between the principal's and agent's ideal points and may yield higher joint surplus, even under perfect information, than P-Authority or A-Authority. However, if the weight of the principal's utility on joint surplus is sufficiently high, the results and insights in Section 7 will remain valid if ex-ante contracting over decision rights is feasible.

[33] See Siggelkow (2002) for a discussion of this topic in the context of misperceptions of interactions between complementary and substitute activities.

of centralizing decision making.

*A-Authority*: Under A-Authority the agent enjoys full control over activities and thus decision making makes full use of his private information. The implied cost of delegating all activities to the agent is that equilibrium decisions will be sub-optimal from the principal's perspective. As expected, this loss of control under delegation increases with $|a|$. Therefore, the same changes in the preferences of the agent that reduce the expected utility of the principal under centralization also reduce her expected utility under delegation. This is consistent with Dessein 2002 in that variations in the conflict of interest between principal and agent that make delegation to an agent less desirable also have an adverse effect if the principal retains control over decisions and relies on the information communicated by the agent.

*S-Authority*: Under S-Authority the principal incurs both types of losses: biased decision making and under-utilization of the agent's information. First, the loss of control is given by $4L_C^S \sigma^2 - L_C^S \left( \sigma^2 - \sigma_S^2 \right)$. In particular, if communication is perfect $\left( \sigma_S^2 = \sigma^2 \right)$ the loss under S-Authority will be $L^S = 4 \frac{a^2}{1-\beta^2} \sigma^2$. As in the case of A-Authority this loss increases at an increasing rate with the adaptation bias $|a|$. Also, by noting that $L_C^A / L_C^S = 2 \left( 1 + \beta \right)$, and recalling that there is no loss of control under P-Authority, it can be seen that the principal experiences *decreasing returns to control* if activities are complements and *increasing returns to control* if activities are substitutes.[34] Essentially, the indirect effect of sharing control on the agent's choice has a disciplining effect when activities are complementary, by forcing his choice of $y_2$ to move closer to the principal's ideal choice, but exacerbates the bias in $y_2$ when decisions interact as substitutes.[35]

Second, consistent with the fact that information is incorporated asymmetrically into decision making, the loss of information $\left( L_X + L_L \right) \left( \sigma^2 - \sigma_S^2 \right)$ has two components. First, $L_X \left( \sigma^2 - \sigma_S^2 \right)$ captures the loss in communication resulting from activities being adapted to each other *only* on the basis of the information communicated by the agent. Each factor in this loss of communication has the same interpretation as in the case of P-Authority. Second, there is a gain (or loss if $a > 2$) in local adaptation $L_L \left( \sigma^2 - \sigma_S^2 \right)$ since the agent can use his ex-post informational advantage to

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[34]It should be noted that in our model the principal always reduces the loss of control by centralizing one activity. This follows from Assumption 3 that states that, absent informational considerations, more control is always desirable.

[35]Intuitively, the principal's incentive to select a lower $y_1$ than a reactive agent indirectly reduces the agent's marginal return to $A_2$ when activities are complements, dampening the agent's relative desire to select a high $y_2$. Conversely, the principal's incentive to select a higher $y_1$ than a passive agent increases the agent's marginal return to $A_2$, inducing him to select a higher $y_2$. This gain from indirectly controlling the agent's behavior is largest when the agent's decisions are farthest away from the principal's preferred decisions. As a result, the reduction in the loss of control is larger when the principal moves from A-Authority to S-Authority than if the principal further centralizes all decision making. If activities interact as substitutes, however, a reactive agent will face a higher marginal return from $A_2$ when the principal controls $A_1$, leading him to select an even higher level of $y_2$. The principal can avoid this increase away from first best in the agent's choice of $y_2$ by centralizing $A_2$. Thus the reduction in the loss of control is larger in this case when the principal moves from S-Authority to P-Authority.

further adjust $y_2$ to his precise knowledge of $\theta$. This gain (or loss if $a > 2$) is proportional to the quality of communication. Intuitively, the higher $\sigma_i^2$ the smaller the informational advantage of the agent and, consequently, the smaller the gain from adjusting $y_2$ according to the exact value of $\theta$. Also, the gain/loss $\mathrm{L}_L$ does not depend on the strength of the interaction between activities: for a given message, and thus a fixed decision by the principal, the agent's choice of $y_2$ varies with $\theta$ according to $(1 + a)\,\theta$. This has one important implication in our analysis: as $\beta$ increases, the value to the principal of incorporating information symmetrically to both decisions (given by $\mathrm{L}_X\left(\sigma^2 - \sigma_i^2\right)$) increases over the gain of local adaptation $\mathrm{L}_L\left(\sigma^2 - \sigma_S^2\right)$. In particular, as the strength of the interaction between complementary activities increases, the principal will favor the allocation of decision rights that elicits more information from the agent.[36]

The fact that the principal's loss under S-Authority reduces whenever communication is more informative also implies that the performance of S-Authority decreases with the adaptation bias of the agent $|a|$. As a result under all three organizational arrangements, a higher conflict of interest reduces the expected utility of the principal.

Finally, Proposition 7 together with Propositions 3 and 5, show that the loss $\mathrm{L}^l, l \in \{A, P, S\}$, is proportional to the underlying uncertainty $\sigma^2$ faced by the principal at the contracting stage. This implies that the optimal allocation of decision rights will not vary with variations in $\sigma^2$. This feature of our model is not robust, however, to changes in the underlying preferences of principal and agent. For example, if instead of preferences being completely aligned when the state is zero one allows for some misalingment at that state[37] then the optimal allocation of authority will indeed vary with $\sigma^2$. Nevertheless, the comparative analysis of each governance structure will be complicated by the fact that the quality of communication will in general not lend itself to simple algebraic manipulations.

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[36] We also note that the quality of communication $\left(\sigma^2 - \sigma_S^2\right)$ affects all components in the loss function under S-Authority. For instance, the loss of control $4\mathrm{L}_C^S\sigma^2 - \mathrm{L}_C^S\left(\sigma^2 - \sigma_S^2\right)$ increases and the gain in local adaptation $\mathrm{L}_L\left(\sigma^2 - \sigma_S^2\right)$ reduces whenever the agent communicates more precise information. It is therefore not immediate that the principal would be better off under S-Authority if the quality of communication improves. However, from Assumption 2 it follows that $\mathrm{L}_X - \mathrm{L}_C^S + \mathrm{L}_L > 0$, i.e. the principal always benefits from better communication as long as $|a| < \left|1 + \frac{1}{b}\right|$ (It will become apparent in Section 7.2 the next section that if $|a| > \left|1 + \frac{1}{b}\right|$ the conflict is so severe that the principal optimally centralizes all decision making). In summary, if the conflict of interest is not extreme, the reduction in the loss of communication whenever the agent reveals more information outweighs the increase in the loss of control under S-Authority and the reduction in the gain from local adaptation.

[37] For instance, by considering the utility function with non-zero constant biases $u_A\left(y_1, y_2, \theta\right) = -(y_1 - (1 + a_1)\,\theta - d)^2 - (y_2 - (1 + a_2)\,\theta - d)^2 + 2by_1y_2)$

# 7  Principal's Delegation Choice

This section analyzes the principal's optimal assignment of decision rights as a function of the adaptation bias $a$ and the degree of interaction between activities $\beta$. Our analysis proceeds in two steps. First, in Section 7.1 we introduce a benchmark case in which the amount of information that reaches the principal under P-Authority or S-Authority is exogenously given. The analysis of this case will make it clear that the only possible rationale in our model for a principal to share control with her agent is to improve the quality of communication. In Section 7.2 we proceed to characterize the principal's optimal delegation choice when information transmission is strategic. We will then see that the communication advantage of S-Authority over P-Authority when activities are complementary, as stated in Proposition 6, leads the principal to optimally share control with a reactive agent for intermediate levels of the conflict of interest, while if activities interact as substitutes or the agent is passive, control over activities is optimally allocated to the same party.

## 7.1  Benchmark: Exogenous Communication.

Suppose that communication is non-strategic and does not vary with the allocation of decision rights. This benchmark is interesting in its own right since it covers the case in which the agent's information is too costly to communicate or process by the principal and as a result no effective communication takes place (i.e. the agent has "specific knowledge " in the sense of Jensen and Meckling 1992), and also the case in which information manipulation and disclosure is not an issue for the organization but the communication channel is inherently noisy.[38]

To this end define a partition $E$ of $\Omega$ into intervals and suppose that the principal only knows to what interval the true state belongs. Furthermore, the interval that the principal observes is common knowledge among the parties. Let $\left(\sigma^2 - \sigma_E^2\right)$ be the residual variance associated with partition $E$ , where we explicitly allow the cases $\sigma_E^2 = 0$ (i.e. the agent has "specific knowledge"), $\sigma_E^2 = \sigma^2$ (i.e. communication perfectly reveals the state to the principal) and the intermediate cases of a noisy communication channel $0 < \sigma_E^2 < \sigma^2$. The loss to the principal under each organizational arrangement with an exogenously given quality of communication is given by Proposition 7 where $\sigma_P^2$ and $\sigma_S^2$ are replaced by $\sigma_E^2$. The following proposition describes the optimal allocation of authority in this case.

PROPOSITION 8 (Allocation of Control: Exogenous Communication). *Suppose that communication is non-strategic and does not vary with the allocation of decision rights. Let $\left(\sigma^2 - \sigma_E^2\right)$ be the*

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[38]See, for instance, Cremer, Garicano and Prat (2006).

*residual variance of the message received by the principal. Then,*[39]

    i.    *Irrespective of the value of $\beta$, S-Authority is never strictly optimal.*

    ii.   *Whenever $\left(\left(\sigma^2 - \sigma_E^2\right)/\sigma^2\right) < 4a^2$ the principal centralizes decision making and*

           *delegates all decisions to the agent if $\left(\left(\sigma^2 - \sigma_E^2\right)/\sigma^2\right) \geq 4a^2$, regardless of the value of $\beta$.*

Figure 4 illustrates the optimal allocation of control when communication is non-strategic and $\sigma_E^2 = (3/4)\,\sigma^2$. To understand why sharing control can never be optimal, note that when the allocation of control does not affect communication, the only benefit from shared control over centralization resides in the gain from local adaptation, while the principal must incur a loss of control when she partially delegates to her agent. Therefore S-Authority is preferred to P-Authority only if the conflict of interest is sufficiently small relative to the informativeness of communication, as given by the following condition

$$\frac{\left(\sigma^2 - \sigma_E^2\right)}{\sigma^2} > m(a, \beta), \tag{16}$$

where $m(a, \beta) = \frac{4a^2}{1+\beta^2(a^2-1)}$. However, a reduction in the conflict of interest makes complete delegation to the agent more attractive. For shared control to be preferred over delegation it must be that the reduction in the loss of control $4\mathrm{L}_C^A\sigma^2 - \left(4\mathrm{L}_C^S\sigma^2 - \mathrm{L}_C^S\left(\sigma^2 - \sigma_E^2\right)\right)$ outweighs the loss of information $(\mathrm{L}_X + \mathrm{L}_L)\left(\sigma^2 - \sigma_E^2\right)$. Therefore, for S-Authority to dominate A-Authority it must be the case that this loss of information is small relative to the gains in control, as given by the following condition

$$\frac{\left(\sigma^2 - \sigma_E^2\right)}{\sigma^2} < n(a, \beta), \tag{17}$$

where $n(a, \beta) = \frac{4a^2+8a^2\beta}{1-\beta^2(a^2-1)+2\beta}$. In the proof of the proposition it is shown that $m(a, \beta) \geq n(a, \beta)$ implying that (16) and (17) cannot be satisfied simultaneously, and shared control is never strictly optimal. In summary, when the allocation of decision rights does not affect the quality of communication the principal may be willing to partially delegate to the agent if the quality of communication is sufficiently low relative to the conflict of interest. If this is the case, however, the principal always gains by further relinquishing all decision rights to her agent, i.e. by engaging in A-Authority. Thus in this symmetric model with exogenous communication, where the agent has the same adaptation bias for each activity, there is no reason for the principal to delegate one activity while retaining control over the other. Finally, part ii of Proposition 6 simply captures the idea that whenever the

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[39] We take the stance that if the principal is indifferent between different allocations, she selects the allocation that maximizes the number of decision rights transferred to the agent.

agency costs of delegation are small relative to the informational advantage of the agent, decision rights should be assigned to the best informed party.[40]

## 7.2  Optimal Allocation of Decision Rights when Communication is Strategic

If communication is strategic, the quality of communication will depend on the congruence of preferences, nature of interaction between activities and the allocation of decision rights. The following proposition characterizes the optimal allocation of decision rights in this case.

PROPOSITION 9 (Allocation of Control: Strategic Communication) *Suppose* $a \neq 0$, *then* [41]

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(i)-*Complements: If* $\beta > 0$, *there exist* $a_P < 0$ *and* $0 < \underline{a}_R(\beta) < \overline{a}_R(\beta)$ *such that*

    a.- *If* $a \in [a_P, \underline{a}_R(\beta)]$ *A-Authority is optimal.*

    b.- *If* $a < a_P$ *or* $a > \overline{a}_R(\beta)$ *P-Authority is optimal.*

    c.- *If* $a \in [\underline{a}_R(\beta), \overline{a}_R(\beta)]$, *S-Authority is optimal.*

    d.-*The range of values* $\overline{a}_R(\beta) - \underline{a}_R(\beta)$, *increases in* $\beta$.

(ii)-*Substitutes: If* $\beta > 0$, *there exist threshold values* $c_P < 0 < c_R$ *such that*

    a.- *If* $a \in [c_P, c_R]$ *A-Authority is optimal.*

    b.- *If* $a < c_P$ *or* $a > c_R$ *P-Authority is optimal.*

    c.- *Shared control is never optimal.*

The proposition is illustrated in Figure 5. Part (i-c) of the proposition captures our key result: if communication is strategic, sharing control of complementary activities can be optimal. As shown in Proposition 6, the principal can always improve the quality of communication by sharing control over complementary activities. Thus part c of the proposition states the she is willing "to trade control for information" only when the agent is reactive and the conflict of interest is mild. The fact that for a passive agent shared control is never optimal, despite the improved communication, follows from the observation in Section 4.3 that communication with a passive agent is comparatively worse than with a reactive agent. Thus, the qualitative difference in the communication equilibria between a passive and a reactive agent impacts the principal's optimal assignment of control rights. In particular, with a passive agent communication deteriorates in such a way that it is not worthwhile for the principal to render some control over to the agent to improve the informativeness of his message.

Furthermore, part i-d of the proposition states that the principal would tend to share control more often with a reactive agent whenever the strength of the complementarity between decisions intensifies. To see this note that part (i-c) of Proposition 9 also states that the likelihood of sharing

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[40]See Jensen and Meckling (1992) and Milgrom and Roberts (1992).

[41]The values of $\underline{a}_R(\beta)$ and $\overline{a}_R(\beta)$, as well as $a_P$, $c_P$ and $c_R$ are explicitly derived in the proof of the proposition.

control increases with the strength of the interaction between activities $\beta$. This is driven by three effects: (i) as $\beta$ increases the quality of communication under shared control increases , while the quality of communication under centralization remains unchanged, (ii) the incremental value of adapting both decisions to the state increases, i.e. the impact of the level of shared information on performance is more pronounced, and (iii) the returns to control from centralizing one activity increase. Therefore the communication advantage of shared control *and* the principal's returns from improved communication increase with the strength of the complementarity between activities, raising the relative benefit of shared control over centralization. Furthermore, the increase in the returns to control increase the desirability of sharing control over delegation. The net effect is an increase willingness of the principal to share control whenever the strength of the interaction increases.

The proposition indicates that the principal never finds it optimal to share control with her agent if activities are substitutes. This follows from three observations: (i) As discussed in Section 6, under S-Authority the principal always benefits from improved communication, (ii) following Proposition 8, if the quality of communication under P-Authority and S-Authority is the same, S-Authority is never optimal, and (iii) Proposition 6 states that P-Authority has a communication advantage over S-Authority whenever activities are substitutes.

Parts i-a and ii-a of the proposition state that whenever the adaptation bias of the agent is sufficiently small, the principal optimally transfers all decision rights to her agent. In essence, when information transmission is strategic the loss under each allocation of control decreases as the preferences of principal and agent become more congruent, and eventually vanishes when $a = 0$. It follows that all organizational arrangements achieve the first-best decision making when the agent and principal are perfectly aligned. Introducing a small conflict $|\Delta a|$ in this case, however, will have a zero first order effect on the quadratic loss of control under A-Authority and S-Authority but it would yield a strictly positive reduction of the quality of communication both under P-Authority and S-Authority. This follows from the observation that $\lim_{|a| \to 0} \left( a^2 / \left( \sigma^2 - \sigma_k^2 \right) \right) = 0$, $l \in \{P, S\}$.[42] In consequence, when the agent's preferences are sufficiently aligned the principal optimally avoids communication and transfers control over both activities to the agent.

Finally, when the conflict of interest is extreme the principal retains all decision rights, i.e. P-Authority is optimal. This is stated in parts (i-b) and (ii-b) of the proposition. To see this, note

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[42]This result mirrors a main insight in Dessein (2002) that the ratio of the quality of communication to the loss under delegation grows indefinitely as the preferences become perfectly congruent. In contrast to our present setup, Dessein (2002) derives this result for the case where the difference between principal's and agent's preferred choices is constant and for general distributions of the state space.

that the principal can always bound her loss by making both decisions herself without recourse to the information of the agent. This ensures her a maximum loss in decision making of $L_X \sigma^2$. Since delegating some (or all decisions) to the agent incurs in a loss proportional to $a^2$, when $|a|$ is sufficiently large the loss of control outweighs the benefit of incorporating information into decision making through A-Authority or S-Authority, and as a result P-Authority is optimal.

# 8    Conclusions

This paper aims at furthering our understanding of the determinants of authority in organizations by analyzing the optimal allocation of decision rights between an uninformed principal and an informed but biased agent when interdependent activities need to be adapted to local conditions. A key insight of the analysis is that the assignment of control rights impacts the informativeness of communication between principal and agent when information transmission takes place under cheap talk. In the simple framework analyzed, when activities are complementary the principal can always obtain more precise information by putting the agent in charge of one activity, that is shared control of complementary activities leads to better communication. Conversely, letting the agent control one activity worsens communication when activities interact as substitutes.

The central result in this paper is that the principal finds it optimal to share control over complementary activities solely because of the improved communication, i.e. the principal optimally "trades control for information". This is true in our model if cheap talk is sufficiently informative, which is the case when the agent has an incentive to overstate his private information. When the agent's incentive is to understate his private information, the communication advantage of shared control over complementary activities is not sufficient to offset the principal's loss from biased decision making.

When an organization needs to make a single decision Dessein (2002) showed that the principal is in general better off delegating control rather than engaging in communication, specially if the conflict of interest is not too pronounced. While this insight remains true in the present model, I also find that the principal may delegate some control to the agent precisely to improve communication. Thus, the comparison between delegation and centralization with multiple, interdependent decisions showcases a new effect: the principal may want to put the agent in charge of one activity because she obtains better information to make decisions on other activities.

The basic model analyzed in this paper could be extended in several directions. First, the model posits a symmetric conflict of interest across activities. This implies, for instance, that the performance of the organization under shared control is independent of the activity delegated to

the agent and that shared control when communication is non-strategic is never optimal. While it is clear that both features of our model are not robust to the possibility of asymmetric agency costs across activities, the communication advantage of shared control over complementary activities is likely to hold in more general cases.[43] Second, while the analysis assumes that some level of congruence between principal and agent exists, possibly through the use of some incentive scheme, I do not endogenize the design of such scheme. I leave the organizational design problem of jointly determining the allocation of decision rights and the provision of incentives for future research.

## 9    Appendix

**Proof of Lemma 1.:** *i- A-Authority*: Follows immediately by solving for $y_i^A(\theta)$, $i \in \{1, 2\}$, in the decision rules (5).

*ii- P-Authority*: Letting $p(\theta \mid m)$ denote the posterior belief of the principal after receiving message $m$, the principal will set $\left(y_1^P(m), y_2^P(m)\right)$ according to

$$\left(y_1^P(m), y_2^P(m)\right) \in \arg \max_{\left(y_1', y_2'\right)} \mathrm{E}\left[-(y_1' - \theta)^2 - (y_2' - \theta)^2 + 2\beta y_1' y_2' \,\middle|\, m\right],$$

which implies that

$$y_i^P(m) = \frac{1}{1-\beta}\mathrm{E}\left[\theta \,\middle|\, m\right], \quad i \in \{1, 2\}.$$

*iii- S-Authority.* Let $y_2(\theta, m)$ be an arbitrary integrable decision rule for the agent, and let $p(\theta \mid m)$ be the posterior belief of the principal after receiving message $m$. The principal's optimal response $y_1(m)$ to $y_2(\theta, m)$, defined as

$$y_1(m) \in \arg \max_{\left(y_1'\right)} \mathrm{E}\left[-(y_1' - \theta)^2 - (y_2(\theta, m) - \theta)^2 + 2\beta y_1' y_2(\theta, m) \,\middle|\, m\right],$$

satisfies the first order condition

$$\mathrm{E}\left[-2(y_1' - \theta) + 2\beta y_2(\theta, m) \,\middle|\, m\right] = 0.$$

Therefore $y_1(m) = \mathrm{E}\left[\theta \,\middle|\, m\right] + \beta\mathrm{E}\left[y_2(\theta, m) \,\middle|\, m\right]$. The agent's optimal choice given the principal's strategy $y_1(m)$ and his message $m$ is simply given by his reaction curve (5), i.e. $y_2(\theta, m) = (1 + a)\theta + \beta y_1(m)$. In equilibrium the agent's strategy must satisfy

$$y_2(\theta, m) = (1 + a)\theta + \beta\mathrm{E}\left[\theta + \beta y_2(\theta, m) \,\middle|\, m\right]. \tag{18}$$

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[43]This communication advantage would need to be rephrased with respect to Proposition 6 in the following way: when activities are complementary, the principal can always find an activity that if delegated to the agent improves communication.

Taking expectations in (18) it follows that $\mathrm{E}\left[y_2(\theta,m)\,|m\right] = \left[(1+a+\beta)/\left(1-\beta^2\right)\right]\mathrm{E}\left[\theta\,|m\right]$, and thus the principal's and agent's optimal decision are given by (8). ∎

The proof of propositions 2 and 4 will make use of the following lemma that establishes that all communication equilibria are interval equilibria, and derives the relation between the end points of consecutive intervals.

**LEMMA 2** (Interval Equilibria and Arbitrage Condition). *Every communication equilibria $a^N$ under P-Authority or S-Authority is an interval equilibria, where consecutive intervals satisfy*

$$a_{i+1} - a_i = a_i - a_{i-1} + 4b_l a_i, \qquad l \in \{P,S\}. \tag{19}$$

**Proof**: a-*Interval Equilibria.* We discuss the case of P-Authority and S-Authority separately.

**P-Authority:** Under P-Authority the agent can only affect decision making through the posterior belief induced on the principal. Thus the agent can only induce decisions in the set

$$P = \left\{(y_1, y_2) : y_1 = y_2 = \frac{1}{1-\beta}\widetilde{\theta}, \widetilde{\theta} \in [0, s]\right\}$$

Now define $\widetilde{u}_A\left(\widetilde{\theta},\theta\right) = u_A(\frac{1}{1-\beta}\widetilde{\theta}, \frac{1}{1-\beta}\widetilde{\theta}, \theta)$. It readily follows that $\frac{\partial^2}{\partial\widetilde{\theta}\partial\theta}\widetilde{u}_A\left(\widetilde{\theta},\theta\right) = 4\frac{1+a}{1-\beta} > 0$ and $\frac{\partial^2}{\partial^2\widetilde{\theta}}\widetilde{u}_A\left(\widetilde{\theta},\theta\right) = -\frac{4}{1-\beta}$, i.e. $\widetilde{u}_A\left(\widetilde{\theta},\theta\right)$ is concave in its first argument and supermodular. To show that all equilibria are interval equilibria, it suffices to prove that when the state increases the agent wants to induce a (weakly) higher posterior mean on the principal. Suppose that this is not the case, i.e. there exist two states $\theta_2 > \theta_1$ and two expected posteriors $\widetilde{\theta}_2 > \widetilde{\theta}_1$ such that the agent at state $\theta_1$ prefers to induce $\widetilde{\theta}_2$ and at state $\theta_2$ prefers to induce $\widetilde{\theta}_1$. This implies that $\widetilde{u}_A\left(\widetilde{\theta}_2,\theta_1\right) > \widetilde{u}_A\left(\widetilde{\theta}_1,\theta_1\right)$ and $\widetilde{u}_A\left(\widetilde{\theta}_1,\theta_2\right) > \widetilde{u}_A\left(\widetilde{\theta}_2,\theta_2\right)$. It follows that

$$\widetilde{u}_A\left(\widetilde{\theta}_2,\theta_1\right) - \widetilde{u}_A\left(\widetilde{\theta}_1,\theta_1\right) > \widetilde{u}_A\left(\widetilde{\theta}_2,\theta_2\right) - \widetilde{u}_A\left(\widetilde{\theta}_1,\theta_2\right),$$

which contradicts the fact that $\frac{\partial^2}{\partial\widetilde{\theta}\partial\theta}\widetilde{u}_A\left(\widetilde{\theta},\theta\right) > 0$. Therefore it must be that, in equilibrium, the agent induces weakly higher posteriors (and thus decisions) as the state increases.

**S-Authority:** Under S-Authority the agent can influence the principal's choice of $y_1$ both through his message and his optimal response. However, sequential rationality requires the agent's response to be optimal given the principal's equilibrium choice. Now define $\widehat{u}_A\left(y_1,\theta\right) = \max_{y_2} u_A(y_1,y_2,\theta)$. It readily follows that $\frac{\partial^2}{\partial y_1\partial\theta}\widehat{u}_A\left(y_1,\theta\right) = 2(1+a)(1+\beta) > 0$ and $\frac{\partial^2}{\partial^2 y_1}\widehat{u}_A\left(y_1,\theta\right) = -2(1+\beta^2) < 0$, i.e. $\widehat{u}_A\left(y_1,\theta\right)$ is concave in its first argument and supermodular. To show that all equilibria are interval equilibria we need to establish that when the state increases the agent wants to induce a (weakly) higher posterior decision $y_1$. Suppose that this is not the case, i.e. there exist two

states $\bar{\theta} > \underline{\theta}$ and two decisions $\bar{y}_1 > \underline{y}_1$, such that the agent at state $\underline{\theta}$ prefers to induce decision $\bar{y}_1$ and at state $\bar{\theta}$ prefers to induce the decision $\underline{y}_1$. This implies that $\hat{u}_A (\bar{y}_1, \underline{\theta}) > \hat{u}_A (\underline{y}_1, \underline{\theta})$ and $\hat{u}_A (\bar{y}_1, \bar{\theta}) < \hat{u}_A (\underline{y}_1, \bar{\theta})$. But then we have that

$$\hat{u}_A (\bar{y}_1, \underline{\theta}) - \hat{u}_A (\underline{y}_1, \underline{\theta}) > \hat{u}_A (\bar{y}_1, \bar{\theta}) - \hat{u}_A (\underline{y}_1, \bar{\theta}),$$

which contradicts the fact that $\frac{\partial^2}{\partial y_1 \partial \theta} \hat{u}_A (y_1, \theta) > 0$. Therefore it must be that in equilibrium the agent induces a weakly higher decision $y_1$ as the state increases.

b-*Indifference condition:* We discuss the case of P-Authority and S-Authority separately.

**P-Authority**: Consider a partition $a^N$, $N \geq 2$, and two consecutive intervals $(a_{i-1}, a_i)$ and $(a_i, a_{i+1})$. After receiving message $m_{i-1} \in (a_{i-1}, a_i)$ the principal selects decisions $(y_{1i-1}, y_{2i-1})$ and after receiving message $m_i \in (a_i, a_{i+1})$ the principal selects decisions $(y_{1i}, y_{2i})$. It follows from (7) that after message $m_i$ the principal will select $y_{1i} = y_{2i} = \frac{1}{1-\beta} E_\theta [\theta | m_i] = \frac{1}{1-\beta} \left( \frac{a_i + a_{i+1}}{2} \right) \equiv \tilde{y}_i$. Then at state $a_i$ the agent is indifferent between sending message $m_{i-1}$ and $m_i$ iff $u_A (\tilde{y}_{i-1}, \tilde{y}_{i-1}, a_i) = u_A (\tilde{y}_i, \tilde{y}_i, a_i)$, which implies

$$
\begin{aligned}
K_A + \left( -2 (\tilde{y}_{i-1} - (1+a) a_i)^2 + 2\beta (\tilde{y}_{i-1})^2 \right) &= K_A + \left( -2(\tilde{y}_i - (1+a) a_i)^2 + 2\beta (\tilde{y}_i)^2 \right), \\
(1-\beta)\tilde{y}_i + (1-\beta)\tilde{y}_{i-1} &= 2 (1+a) a_i, \\
(a_i + a_{i+1}) + (a_{i-1} + a_i) &= 4 (1+a) a_i, \\
a_{i+1} - a_i &= a_i - a_{i-1} + 4a a_i,
\end{aligned}
$$

which is equivalent to (19) with $b_P = a$.

**S-Authority:** Consider a partition $a^N$, $N \geq 2$, and two consecutive interval $(a_{i-1}, a_i)$ and $(a_i, a_{i+1})$. After receiving message $m_{i-1} \in (a_{i-1}, a_i)$ the principal selects $y_{1i-1}$ according to (8). It follows from (8) that after message $m_i$ the principal will select $y_{1i} = \left( \frac{1}{1-\beta} + \frac{a\beta}{1-\beta^2} \right) E_\theta [\theta | m_i] = \left( \frac{1}{1-\beta} + \frac{a\beta}{1-\beta^2} \right) \left( \frac{a_i + a_{i+1}}{2} \right) \equiv \hat{y}_i$. Then the agent of type $a_i$ is indifferent between sending message $m_{i-1}$ and $m_i$ iff $\max_{y_2} u_A (\hat{y}_{i-1}, y_2, a_i) = \max_{y_2} u_A (\hat{y}_i, y_2, a_i)$, or, alternatively, $\hat{u}_A (\hat{y}_{i-1}, a_i) = \hat{u}_A (\hat{y}_i, a_i)$, which implies

$$K_A + \left( - (\hat{y}_{i-1} - (1+a) a_i)^2 - ((1+a)a_i + \beta\hat{y}_{i-1} - (1+a) a_i)^2 + 2\beta (\hat{y}_{i-1}) ((1+a)\theta + \beta\hat{y}_{i-1}) \right) =$$

$$= K_A + \left( -(\hat{y}_i - (1+a) a_i)^2 - ((1+a)a_i + \beta\hat{y}_i - (1+a) a_i)^2 + 2\beta (\hat{y}_i) ((1+a)\theta + \beta\hat{y}_i) \right),$$

$$
\begin{aligned}
2 (1+a) a_i &= (1-\beta) \hat{y}_i + (1-\beta) \hat{y}_{i-1}, \\
\frac{4 (1+\beta) (1+a)}{(1 + (1+a)\beta)} a_i &= a_i + a_{i+1} + a_i + a_{i-1}, \\
a_{i+1} - a_i &= a_i - a_{i-1} + 4 \frac{a}{(1 + (1+a)\beta)} a_i,
\end{aligned}
$$

which is equivalent to (19) with $b_S = \frac{a}{(1+(1+a)\beta)}$. ∎

The proofs of propositions 3, 5 and 7 will make use of the following lemma that provides conditions under which the expected utility of the principal and the agent increase with the informativeness of equilibrium.

**LEMMA 3** (Expected Utility of Principal and Agent under Communication Equilibria). *The expected utility of principal and agent increase with the informativeness of communication whenever*
  i-*P-Authority*    a-*Principal's expected utility increases if* $\left(\frac{1}{1-\beta}\right) > 0$.
                b-*Agent's expected utility increases if* $\left(\frac{1}{1-\beta}\right) > 0$ *and* $b_P > -\frac{1}{2}$.
  ii-*S-Authority*    a-*Principal's expected utility increases if* $\beta b_S < \frac{1}{2}$.
                b-*Agent's expected utility increases if* $b_S > -\frac{1}{2}$.

**Proof**: Let $a^N$ be a communication equilibrium, and for any message $m \in (a_{i-1}, a_i)$ let $\overline{m} = \mathrm{E}_\theta [\theta | m]$. We analyze first P-Authority.

**P-Authority:** The expected utility of the principal $\mathrm{E}_\theta [u_P (y_1, y_2, \theta) | m]$ satisfies

$$\mathrm{E}_\theta [u_P (y_1, y_2, \theta) | m] = \mathrm{E}_\theta \left[ -(\frac{\overline{m}}{1 - \beta} - \theta)^2 - (\frac{\overline{m}}{1 - \beta} - \theta)^2 + 2\beta \left(\frac{\overline{m}}{1 - \beta}\right)^2 \right] =$$
$$= 2 \left(\frac{1}{1 - \beta}\right) \left( \beta \mathrm{E}_\theta [\theta^2] - \mathrm{E}_\theta [\theta^2 - (\overline{m})^2] \right). \tag{20}$$

Thus the expected utility of the principal increases with the informativeness of communication if $1/(1 - \beta) > 0$. Since this condition is always satisfied, under P-Authority the principal's is ex-ante better off with improved communication.

The expected utility of the agent $\mathrm{E}_\theta [u_A (y_1, y_2, \theta) | m]$ satisfies

$$\mathrm{E}_\theta [u_A (y_1, y_2, \theta) | \overline{m}] = \mathrm{E}_\theta \left[ -(\frac{\overline{m}}{1 - \beta} - (1 + a) \theta)^2 - (\frac{\overline{m}}{1 - \beta} - (1 + a) \theta)^2 + 2\beta \left(\frac{\overline{m}}{1 - \beta}\right)^2 \right] =$$
$$= 2 \left(\frac{1}{1 - \beta}\right) \mathrm{E}_\theta \left[ (1 + 2a) (\overline{m})^2 - (1 + a)^2 (1 - \beta) \theta^2 \right].$$

Therefore the agent benefits from improved communication if $(1/1 - \beta)(1 + 2a) > 0$, which is satisfied whenever $b_P > -\frac{1}{2}$.

**S-Authority:** Given equilibrium decisions (8), the expected utility of the principal $\mathrm{E}_\theta [u_P (y_1, y_2, \theta) | m]$ under S-Authority satisfies

$$\mathrm{E}_\theta [u_P (y_1, y_2, \theta) | m] = \mathrm{E}_\theta [ -(\frac{1 + (a + 1) \beta}{1 - \beta^2} \overline{m} - \theta)^2 - (\frac{(a + 1) + \beta}{1 - \beta^2} \overline{m} +$$
$$(a + 1) (\theta - \overline{m}) - \theta)^2 + 2\beta \frac{1 + (a + 1) \beta}{1 - \beta^2} \overline{m} \left( \frac{(a + 1) + \beta}{1 - \beta^2} \overline{m} + (a + 1) (\theta - \overline{m}) \right)] \tag{21}$$
$$= \frac{(\beta (a + 1) + 1) (1 + \beta - \beta a)}{1 - \beta^2} \mathrm{E}_\theta [\overline{m}^2] - (1 + a^2) \mathrm{E}_\theta [\theta^2], \tag{22}$$

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where we have made use of the fact that $\mathrm{E}_\theta\left[\theta \mathrm{E}_\theta\left[\theta\,|m\right]\right] = \mathrm{E}_\theta\left[\left(\mathrm{E}_\theta\left[\theta\,|m\right]\right)^2\right]$. Thus $\mathrm{E}_\theta\left[u_P\left(y_1, y_2, \theta\right)|m\right]$ increases with $\mathrm{E}_\theta\left[\overline{m}^2\right]$ if $\left(\beta\left(a+1\right)+1\right)\left(1+\beta-\beta a\right) > 0$ which, by (13), can be simplified to $1 - 2\beta b_S > 0$.

Likewise, given equilibrium decisions (8), the expected utility of the agent $\mathrm{E}_\theta\left[u_A\left(y_1, y_2, \theta\right)|m\right]$ satisfies

$$\mathrm{E}_\theta\left[u_A\left(y_1, y_2, \theta\right)|m\right] = \mathrm{E}_\theta[-(\frac{1+\left(a+1\right)\beta}{1-\beta^2}\overline{m} - \left(a+1\right)\theta)^2 - (\frac{\left(a+1\right)+\beta}{1-\beta^2}\overline{m} + \left(a+1\right)\left(\theta-\overline{m}\right) - \left(a+1\right)\theta)^2 +$$

$$+ \left(a+1\right)\left(\theta-\overline{m}\right) - \left(a+1\right)\theta)^2 + 2\beta\frac{1+\left(a+1\right)\beta}{1-\beta^2}\overline{m}\left(\frac{\left(a+1\right)+\beta}{1-\beta^2}\overline{m} + \left(a+1\right)\left(\theta-\overline{m}\right)\right),$$

$$\mathrm{E}_\theta\left[u_A\left(y_1, y_2, \theta\right)|m\right] = \frac{\left(2a+1+\beta(1+a)\right)\left(1+\beta(1+a)\right)}{1-\beta^2}\mathrm{E}_\theta\left[\overline{m}^2\right] - \left(\left(a+1\right)+1\right)^2\mathrm{E}_\theta\left[\theta^2\right].$$

Thus $\mathrm{E}_\theta\left[u_A\left(y_1, y_2, \theta\right)|m\right]$ increases with $\mathrm{E}_\theta\left[\overline{m}^2\right]$ if $\left(2a+1+\beta(1+a)\right)\left(1+\beta(1+a)\right) > 0$ which, by (13), can be simplified to $1 + 2b_S > 0$. ∎

**Proof of Proposition 2** For a given communication equilibrium, parts (i.-iv.) follow from Lemma 1-ii, Lemma 1-iii, and Lemma 2. To show that an equilibrium $a^N$ exists for each $N$, we construct for each $N$ an equilibrium with $N$ intervals

Using the boundary conditions $a_0 = 0$ and $a_N = s$ to solve the difference equation (19) gives

$$a_i = \frac{x_H^i - y_H^i}{x_H^N - y_H^N}s \text{ for } 0 \leq i \leq N, \tag{23}$$

where $x_H$ and $y_H$ are

$$x_H = \left(2b_l + 1\right) + 2\sqrt{b_l(1+b_l)} \text{ and}$$
$$y_H = \left(2b_l + 1\right) - 2\sqrt{b_l(1+b_l)},$$

and satisfy $x_H\,y_H = 1$. Therefore $a_i$ is well defined and $a_i \geq a_{i-1}, 1 \leq i \leq N$, thus $a^N$ is a communication equilibrium. ∎

**Proof of Proposition 3:** We first compute the variance of communication for each equilibrium with $N$ intervals, and show that this variance is increasing in $N$. We then compute the limit value as $N \to \infty$. The proof that the limit of strategy profiles and beliefs constitutes a communication equilibria can be found in Alonso, Dessein and Matouschek (2006).

Consider a communication equilibrium $a^N$. To compute the residual variance of communication

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$\sigma^2 - \sigma_l^2$ we first compute $\mathrm{E}_\theta\left[\overline{m}^2\right]$ where $\overline{m} = \mathrm{E}_\theta\left[\theta\,|m\right]$.

$$
\begin{aligned}
\mathrm{E}_\theta\left[\overline{m}^2\right] &= \sum_{i=1}^{N}\int_{a_{i-1}}^{a_i}\left(\frac{a_i + a_{i-1}}{2}\right)^2\frac{1}{s}d\theta = \frac{1}{4s}\sum_{i=1}^{N}(a_i - a_{i-1})(a_i + a_{i-1})^2 = \\
&= \frac{1}{4s}\sum_{i=1}^{N}\left(\left(a_i^3 - a_{i-1}^3\right) + a_i a_{i-1}\left(a_i - a_{i-1}\right)\right) = \\
&= \frac{1}{4s}\left(s^3 + \frac{s^3}{\left(x_H^N - y_H^N\right)^3}\sum_{i=1}^{N}\left(x_H^i - y_H^i\right)\left(x_H^{i-1} - y_H^{i-1}\right)\left(x_H^i - y_H^i - x_H^{i-1} + y_H^{i-1}\right)\right) = \\
&= \frac{s^2}{4}(1 + \frac{1}{\left(x_H^N - y_H^N\right)^3}\sum_{i=1}^{N}[x_H^{3i}\left(x_H^{-1} - x_H^{-2}\right) + x_H^i\left(2 - x_H + x_H^{-2} - 2x_H^{-1}\right) \\
&\qquad\qquad + y_H^{3i}\left(y_H^{-2} - y_H^{-1}\right) + y_H^i\left(-2 + y_H - y_H^{-2} + 2y_H^{-1}\right)]) \\
&= \frac{s^2}{4}(1 + \frac{1}{\left(x_H^N - y_H^N\right)^3}[\frac{\left(x_H^{3N} - 1\right)x_H}{\left(x_H^2 + x_H + 1\right)} - \frac{\left(x_H^N - 1\right)\left(x_H^2 - x_H + 1\right)}{x_H} \\
&\qquad\qquad - \frac{\left(y_H^{3N} - 1\right)y_H}{\left(y_H^2 + y_H + 1\right)} + \frac{\left(y_H^N - 1\right)\left(y_H^2 - y_H + 1\right)}{y_H}]) = \\
&= \frac{s^2}{4}\left(1 + \frac{x_H^{3N} - y_H^{3N}}{\left(x_H^N - y_H^N\right)^3}\frac{x_H}{\left(x_H + x_H^2 + 1\right)} - \frac{1}{\left(x_H^N - y_H^N\right)^2}\frac{\left(x_H^2 - x_H + 1\right)}{x_H}\right)
\end{aligned}
$$

where the last equality follows from

$$
\begin{aligned}
\frac{y_H}{\left(y_H + y_H^2 + 1\right)} &= \frac{x_H}{\left(x_H + x_H^2 + 1\right)}, \\
\frac{\left(y_H^2 - y_H + 1\right)}{y_H} &= \frac{\left(x_H^2 - x_H + 1\right)}{x_H}.
\end{aligned}
$$

Since $x_H\,y_H = 1$ and letting $p = x_H^{2N}$ we obtain

$$
\begin{aligned}
\mathrm{E}_\theta\left[\overline{m}^2\right] &= \frac{s^2}{4}\left(1 + \frac{\left(p - x_H^2\right)\left(px_H^2 - 1\right)}{\left(x_H + x_H^2 + 1\right)(p - 1)^2\,x_H}\right), \\
\sigma_l^2 &= \mathrm{E}_\theta\left[\overline{m}^2\right] - \frac{s^2}{4} = \frac{s^2}{4}\left(\frac{\left(p - x_H^2\right)\left(px_H^2 - 1\right)}{\left(x_H + x_H^2 + 1\right)(p - 1)^2\,x_H}\right).
\end{aligned}
$$

It is straightforward to show that $\mathrm{E}_\theta\left[\overline{m}^2\right]$ increases in $p$. Since $x_H > 1$ it follows that $\mathrm{E}_\theta\left[\overline{m}^2\right]$ (and thus $\sigma_l^2$) increases in $N$. From Lemma 3, if $b_l > 0$ (which requires $a > 0$) then the agent always benefits from improved communication. The principal always benefits from improved communication with a reactive agent under P-Authority, and benefits from improved communication

with a reactive agent under S-Authority if, and only if, $\beta b_S < \frac{1}{2}$. From Assumption 2 we have that $\beta b_S < \frac{1}{2}$. Thus the most informative equilibrium Pareto dominates all other equilibria. We next compute the residual variance of the agent's message for this case.

From $\lim_{N \to \infty} \frac{\left(p - x_H^2\right)\left(p x_H^2 - 1\right)}{\left(x_H + x_H^2 + 1\right)(p - 1)^2 x_H} = \frac{x_H}{x_H^2 + x_H + 1}$ we have

$$\lim_{N \to \infty} \mathrm{E}_\theta\left[\overline{m}^2\right] = \frac{s^2}{4}\left(1 + \frac{x_H}{x_H^2 + x_H + 1}\right) = \frac{s^2}{4} \frac{(x_H + 1)^2}{x_H^2 + x_H + 1} = \frac{b_l + 1}{4b_l + 3}s^2.$$

Therefore for the most informative equilibrium we have

$$\sigma^2 - \sigma_l^2 = \mathrm{E}_\theta\left[\theta^2\right] - \mathrm{E}_\theta\left[\overline{m}^2\right] = \frac{s^2}{3} - \frac{b_l + 1}{4b_l + 3}s^2 = \frac{4b_l}{4b_l + 3}\sigma^2.$$

∎

**Proof of Proposition 4:** For a given communication equilibrium, parts (i.-iv.) follow from Lemma 1-ii, Lemma 1-iii, and Lemma 2. We now show that an equilibrium $a^N$ exists for each $N \le N^*$ and thus the number of equilibria is necessarily finite.

Consider a partition $a^N$. To solve the second order difference equation (19) we note that the roots of the characteristic equation are given by (with $j = \sqrt{-1}$ being the imaginary unit)

$$x_H = (2b_l + 1) + 2j\sqrt{|b_l|(1 + b_l)} = \cos\varphi + j\sin\varphi, \text{ and}$$
$$y_H = (2b_l + 1) - 2j\sqrt{|b_l|(1 + b_l)} = \cos\varphi - j\sin\varphi,$$

where the angle $\varphi$ is defined implicitly by $\tan\varphi = 2\frac{\sqrt{|b_l|(1+b_l)}}{(2b_l+1)}, -\frac{\pi}{2} \le \varphi \le \frac{\pi}{2}$. Therefore, the general solution to (19) is given by $a_i = 2\rho\cos(i\varphi + \psi)$. To determine $\rho$ and $\psi$ we use the boundary conditions $a_0 = 0$ and $a_N = s$ to obtain

$$2\rho\cos\psi = 0$$
$$2\rho\cos(N\varphi + \psi) = s,$$

which yields $2\rho = \frac{s}{\sin N\varphi}$ and $\psi = -\frac{\pi}{2}$. It follows that each element of the partition is given by

$$a_i = s\frac{\cos\left(i\varphi - \frac{\pi}{2}\right)}{\sin N\varphi} = s\frac{\sin i\varphi}{\sin N\varphi}. \tag{24}$$

*Monotonicity condition:* The sequence of end-points $\{a_i\}_{i=0}^{i=N}$ will qualify as an equilibrium of the communication game only if the monotonicity condition $a_i \ge a_{i-1}, 1 \le i \le N$, holds. From (24) this requires that

$$\frac{\sin i\varphi - \sin(i-1)\varphi}{\sin N\varphi} = \frac{2\cos\left(i - \frac{1}{2}\right)\varphi\sin\frac{\varphi}{2}}{\sin N\varphi} \ge 0,$$

which implies $\cos\left(i - \frac{1}{2}\right)\varphi \geq 0$, $1 \leq i \leq N$, or, alternatively, $-\frac{\pi}{2} \leq \left(i - \frac{1}{2}\right)\varphi \leq \frac{\pi}{2}$. Since for $b_l < 0$ we have $\varphi \neq 0$ then there exists an integer $N(b_l)$ such that $\left|\left(N(b_l) - \frac{1}{2}\right)\varphi\right| \leq \frac{\pi}{2}$ and $\left|\left(N(b_l) + \frac{1}{2}\right)\varphi\right| > \frac{\pi}{2}$, i.e. for a given $b_l < 0$ the number of communication equilibria is necessarily finite. In particular, $N(b_l)$ satisfies

$$\frac{\pi}{2\,|\varphi|} - \frac{1}{2} < N(b_l) \leq \frac{\pi}{2\,|\varphi|} + \frac{1}{2}.$$

Also, if $b_l < -\frac{1}{2}$ then $N(b_l) = 1$, i.e. the only equilibrium is the babbling equilibrium in which no information is transmitted.∎

**Proof of Proposition 5:** To compute the residual variance $\sigma^2 - \sigma_l^2$ we first compute $\mathrm{E}_\theta\left[\overline{m}^2\right]$ where $\overline{m} = \mathrm{E}_\theta\left[\theta\,|m\right]$.

$$
\begin{aligned}
\mathrm{E}_\theta\left[\overline{m}^2\right] &= \sum_{i=1}^{N}\int_{a_{i-1}}^{a_i}\left(\frac{a_i + a_{i-1}}{2}\right)^2\frac{1}{s}d\theta = \frac{1}{4s}\sum_{i=1}^{N}(a_i - a_{i-1})(a_i + a_{i-1})^2 \\
&= \frac{1}{4s}\sum_{i=1}^{N}\left(\left(a_i^3 - a_{i-1}^3\right) + a_i a_{i-1}\left(a_i - a_{i-1}\right)\right) \\
&= \frac{1}{4s}\left(\left(a_N^3 - a_0^3\right) + \sum_{i=1}^{N}a_i a_{i-1}\left(a_i - a_{i-1}\right)\right) \\
&= \frac{1}{4s}\left(s^3 + \frac{s^3}{(\sin N\varphi)^3}\sum_{i=1}^{N}\left(\sin i\varphi\sin(i-1)\varphi\left(\sin i\varphi - \sin(i-1)\varphi\right)\right)\right).
\end{aligned}
$$

From $\sin(i-1)\varphi = \sin i\varphi\cos\varphi - \cos i\varphi\sin\varphi$ we obtain, after some calculations,

$$
\begin{aligned}
\mathrm{E}_\theta\left[\overline{m}^2\right] &= \frac{s^2}{4}\left(1 + \cos\varphi - \frac{\sin\varphi}{(\sin N\varphi)^3}\sum_{i=1}^{N}\left(\sin^2 i\varphi\cos i\varphi + \sin^2(i-1)\varphi\cos(i-1)\varphi\right)\right) \\
&= \frac{s^2}{4}\left(1 + \cos\varphi - \frac{\sin\varphi}{(\sin N\varphi)^3}\sum_{i=1}^{N}\left(\cos i\varphi - \cos^3 i\varphi + \cos(i-1)\varphi - \cos^3(i-1)\varphi\right)\right).
\end{aligned}
$$

In order to compute the summation terms we note that (with $\mathrm{Re}\left[\eta\right]$ denoting the real part of the complex number $\eta$)

$$
\sum_{i=1}^{N}\left(\cos i\varphi - \cos^3 i\varphi + \cos(i-1)\varphi - \cos^3(i-1)\varphi\right) = \mathrm{Re}\left[\sum_{i=1}^{N}\left(e^{ji\varphi} - e^{j3i\varphi} + e^{j(i-1)\varphi} - e^{j3(i-1)\varphi}\right)\right] =
$$

$$
\mathrm{Re}\left[\left(\frac{1 + e^{j\varphi}}{1 - e^{j\varphi}}\left(1 - e^{jN\varphi}\right) - \frac{1 + e^{j3\varphi}}{1 - e^{j3\varphi}}\left(1 - e^{j3N\varphi}\right)\right)\right] = \mathrm{Re}\left[\left(\frac{j}{\tan\frac{\varphi}{2}}\left(1 - e^{jN\varphi}\right) - \frac{j}{\tan\frac{3\varphi}{2}}\left(1 - e^{j3N\varphi}\right)\right)\right] =
$$

$$
\frac{\sin N\varphi}{\tan\frac{\varphi}{2}} - \frac{\sin 3N\varphi}{\tan\frac{3\varphi}{2}} = \frac{\sin N\varphi}{\tan\frac{\varphi}{2}} - \frac{3\sin N\varphi - 4\left(\sin N\varphi\right)^3}{\tan\frac{3\varphi}{2}}.
$$

where the last inequality follows from $\sin 3N\varphi = 3\sin N\varphi - 4(\sin N\varphi)^3$. Thus

$$E_\theta\left[\overline{m}^2\right] = \frac{s^2}{4}\left(1 + \cos\varphi - \frac{1}{4}\frac{\sin\varphi}{(\sin N\varphi)^2}\left(\frac{\tan\frac{3\varphi}{2} - 3\tan\frac{\varphi}{2}}{\tan\frac{3\varphi}{2}\tan\frac{\varphi}{2}}\right) - \frac{\sin\varphi}{\tan\frac{3\varphi}{2}}\right). \qquad (25)$$

From $\cos\varphi = 2b_l + 1$, $\sin\varphi = 2\sqrt{(b_l+1)|b_l|}$, $\tan\frac{\varphi}{2} = \frac{\sin\varphi}{1+\cos\varphi} = \sqrt{\frac{|b_l|}{1+b_l}}$ and $\tan\frac{3\varphi}{2} = \frac{3\tan\frac{\varphi}{2} - \tan^3\frac{\varphi}{2}}{1 - 3\tan^2\frac{\varphi}{2}}$ we have

$$\sin\varphi\left(\frac{\tan\frac{3\varphi}{2} - 3\tan\frac{\varphi}{2}}{\tan\frac{3\varphi}{2}\tan\frac{\varphi}{2}}\right) = (1+\cos\varphi)\left(1 - 3\frac{\tan\frac{\varphi}{2}}{\tan\frac{3\varphi}{2}}\right) = 2(b_l+1)\left(1 - 3\frac{1 - 3\tan^2\frac{\varphi}{2}}{3 - \tan^2\frac{\varphi}{2}}\right) = 16\frac{(b_l+1)|b_l|}{4b_l+3},$$

$$\frac{\sin\varphi}{\tan\frac{3\varphi}{2}} = (1+\cos\varphi)\frac{\tan\frac{\varphi}{2}}{\tan\frac{3\varphi}{2}} = 2(b_l+1)\frac{(4b_l+1)}{(4b_l+3)},$$

which applied to (25) implies

$$\begin{aligned}
E_\theta\left[\overline{m}^2\right] &= \frac{s^2}{4}\left(2(b_l+1) - \frac{4}{(\sin N\varphi)^2}\frac{(b_l+1)|b_l|}{4b_l+3} - 2(b_l+1)\frac{4b_l+1}{4b_l+3}\right) = \\
&= \frac{b_l+1}{4b_l+3}\left(1 - \frac{|b_l|}{(\sin N\varphi)^2}\right)s^2. \qquad (26)
\end{aligned}$$

We also note that as $N$ increases the variance of the message $\sigma_l^2$ also increases, i.e. the informativeness of equilibrium increases with the number of messages sent by the agent. For the equilibrium with the maximum number of partitions $N(b_l)$ we have that the quality of communication is given by

$$\sigma^2 - \sigma_l^2 = \frac{s^2}{3} - \frac{b_l+1}{4b_l+3}\left(1 - \frac{|b_l|}{(\sin N(b_l)\varphi)^2}\right)s^2 = \frac{4|b_l|}{4b_l+3}\left[\frac{3(1+b_l)}{\sin^2 N(b_l)\varphi} - 1\right]\sigma^2.$$

*Pareto dominance.* From Lemma 3 the principal's expected utility increases in $\sigma_l^2$ under P-Authority and S-Authority (the latter given Assumption 2). Thus the principal prefers the equilibrium $a^{N(b_l)}$ to any other equilibrium. Also from Lemma 3 the agent benefits from more communication if and only if, $b_l > -\frac{1}{2}$, $l \in \{P, S\}$. As discussed above, however, when $b_l \leq -\frac{1}{2}$ the only communication equilibrium is the babbling equilibrium. Thus the agent always prefers the most informative equilibrium to any other equilibria and as a result the most informative equilibrium Pareto dominates all other equilibria. ∎

**Proof of Proposition 6:** We first show that the quality of communication is decreasing whenever the communication bias $|b_l|$, $l \in \{P, S\}$, increases. Then we establish that if activities interact as complements the communication bias under S-Authority $|b_S|$ is smaller than the communication bias under P-Authority $|b_P|$ where the reverse is true in activities interact as substitutes.

In the case of a reactive agent $\partial \left(\sigma^2 - \sigma_l^2\right)/\partial b_l = \frac{12}{(4b_l+3)^2} > 0$ which indicates that increasing $b_l$ worsens communication. For a passive agent we first note that $\sigma^2 - \sigma_l^2$ is a continuous function of $b_l$ and admits a derivative except when $\varphi\left(b_l\right) = \frac{\pi}{2N+1}$ for each $N \geq 1$ (this corresponds to the values of the communication bias where the number of intervals of the most informative equilibrium vary). If $\sigma^2 - \sigma_l^2$ is differentiable we have that

$$\frac{\partial \left(\sigma^2 - \sigma_l^2\right)}{\partial b_l} = \frac{-12}{(4b_l+3)^2}\left[\left(\frac{4b_l^2 + 6b_l + 3}{(\sin N\left(b_l\right)\varphi)^2} - 1\right) + \frac{|b_l|\left(b_l+1\right)(4b_l+3)}{(\sin N\left(b_l\right)\varphi)^4}2N\left(b_l\right)\left(\sin 2N\left(b_l\right)\right)\frac{\partial \varphi}{\partial b_l}\right]$$

It then follows from $\sin 2N\left(b_l\right) < 0$, $\frac{\partial \varphi}{\partial b_l} < 0$, and $4b_l^2 + 6b_l + 3 > 1$ if $b_l > -1/2$, that $\partial \left(\sigma^2 - \sigma_l^2\right)/\partial b_l < 0$. Thus reducing $|b_l|$, $l \in \{P, S\}$, improves the quality of communication with a passive agent.

Comparing the communication bias under P-Authority and S-Authority we have that $b_P - b_S = a - a/\left(1 + \beta\left(a+1\right)\right) = a\beta\left(a+1\right)/\left(\beta + a\beta + 1\right)$. From our assumptions it follows that $\left(a+1\right)/\left(\beta + a\beta + 1\right) > 0$. Therefore for a reactive agent $a > 0$ and thus $b_P > b_S$ iff $\beta > 0$. Likewise for a passive agent $a < 0$ and thus $b_P < b_S$ iff $\beta > 0$. In summary, $|b_P| > |b_S|$ if activities interact as complements, and $|b_P| < |b_S|$ if activities interact as substitutes.

Finally, since $\partial |b_S|/\partial \beta = -\left(a+1\right)|a|/\left(\beta + a\beta + 1\right)^2 < 0$ and $b_P$ does not vary with $\beta$, we conclude that increasing $\beta$ improves communication under S-Authority while it does not affect the quality of communication under P-Authority.∎

**Proof of Proposition 7**: The principal's expected utility under the first-best decision making (2) is given by

$$\mathrm{E}_\theta\left[u_P\left(y_1^P\left(\theta\right), y_1^P\left(\theta\right), \theta\right)\right] = 2\left(\frac{\beta}{1-\beta}\right)\mathrm{E}_\theta\left[\theta^2\right]. \tag{27}$$

*P-Authority*: When the principal relies on the information provided by the agent then her expected utility is given in (20). Comparing now (27) and (20) we have that the loss of information under P-Authority is

$$\begin{aligned} \mathrm{L}_I^P &= \mathrm{E}_\theta\left[u_P\left(y_1^P\left(\theta\right), y_1^P\left(\theta\right), \theta\right)\right] - \mathrm{E}_\theta\left[u_P\left(y_1^P(m), y_2^P(m), \theta\right)|m\right] = \\ &= 2\left(\frac{1}{1-\beta}\right)\left(\sigma^2 - \sigma_P^2\right). \end{aligned} \tag{28}$$

*A-Authority*: The principal's expected utility in this case is

$$\mathrm{E}_\theta\left[u_P\left(y_1^A(\theta), y_2^A(\theta), \theta\right)\right] = 2\frac{a^2 - \beta}{1 - \beta}\mathrm{E}_\theta\left[\theta^2\right], \tag{29}$$

and thus the loss of control under A-Authority is given by comparing (27) and (29)

$$\begin{aligned} \mathrm{L}_C^A &= \mathrm{E}_\theta\left[u_P\left(y_1^P\left(\theta\right), y_1^P\left(\theta\right), \theta\right)\right] - \mathrm{E}_\theta\left[u_P\left(y_1^P(m), y_2^P(m), \theta\right)|m\right] = \\ &= 2\frac{a^2}{1-\beta}\mathrm{E}_\theta\left[\theta^2\right] = 4\frac{2a^2}{1-\beta}\sigma^2. \end{aligned} \tag{30}$$

*S-Authority*: The principal's expected utility is given by (22). Therefore the loss with respect to the first best (27) is

$$
\begin{aligned}
\mathrm{L}^{S} &= 2\left(\frac{\beta}{1-\beta}\right)\mathrm{E}_{\theta}\left[\theta^{2}\right] + \left(1+a^{2}\right)\mathrm{E}_{\theta}\left[\theta^{2}\right] - \frac{(1+\beta)^{2}-a^{2}\beta^{2}}{1-\beta^{2}}\mathrm{E}_{\theta}\left[\left(\mathrm{E}_{\theta}\left[\theta\,|m\right]\right)^{2}\right] = \\
&= \frac{a^{2}}{1-\beta^{2}}\mathrm{E}_{\theta}\left[\theta^{2}\right] + \frac{(1+\beta)^{2}-a^{2}\beta^{2}}{1-\beta^{2}}\mathrm{E}_{\theta}\left[\left(\mathrm{E}_{\theta}\left[\theta\,|m\right]-\theta\right)^{2}\right] = \\
&= \frac{a^{2}}{1-\beta^{2}}4\sigma^{2} + \left(2\left(\frac{1}{1-\beta}\right) - \frac{a^{2}}{1-\beta^{2}} + \left(a^{2}-1\right)\right)\left(\sigma^{2}-\sigma_{S}^{2}\right).
\end{aligned}
\tag{31}
$$

∎

**Proof of Proposition 8:** If communication is exogenous the loss under each organizational form is given by

$$
\begin{aligned}
\mathrm{L}^{P} &= 2\left(\frac{1}{1-\beta}\right)\left(\sigma^{2}-\sigma_{E}^{2}\right), \\
\mathrm{L}^{A} &= 4\frac{2a^{2}}{1-\beta}\sigma^{2}, \\
\mathrm{L}^{S} &= 4\frac{a^{2}}{1-\beta^{2}}\sigma^{2} + \frac{(1+\beta)^{2}-a^{2}\beta^{2}}{1-\beta^{2}}\left(\sigma^{2}-\sigma_{E}^{2}\right).
\end{aligned}
$$

We proceed now to make pairwise comparisons between organizational forms. In particular, A-Authority dominates P-Authority whenever

$$
2\left(\frac{1}{1-\beta}\right)\left(\sigma^{2}-\sigma_{E}^{2}\right) > 4\frac{2a^{2}}{1-\beta}\sigma^{2},
$$

or $\left(\sigma^{2}-\sigma_{E}^{2}\right)/\sigma^{2} \geq 4a^{2}$. For shared control to dominate centralization it must be that

$$
4\frac{a^{2}}{1-\beta^{2}}\sigma^{2} + \frac{(1+\beta)^{2}-a^{2}\beta^{2}}{1-\beta^{2}}\left(\sigma^{2}-\sigma_{E}^{2}\right) < 2\left(\frac{1}{1-\beta}\right)\left(\sigma^{2}-\sigma_{E}^{2}\right),
$$

which reduces to the condition

$$
\frac{\left(\sigma^{2}-\sigma_{E}^{2}\right)}{\sigma^{2}} > \frac{4a^{2}}{1+\beta^{2}\left(a^{2}-1\right)} \equiv m(a,\beta).
\tag{32}
$$

Alternatively, for shared control to dominate delegation we must have that

$$
4\frac{a^{2}}{1-\beta^{2}}\sigma^{2} + \frac{(1+\beta)^{2}-a^{2}\beta^{2}}{1-\beta^{2}}\left(\sigma^{2}-\sigma_{E}^{2}\right) < 4\frac{2a^{2}}{1-\beta}\sigma^{2},
$$

which reduces to

$$
\left(\sigma^{2}-\sigma_{E}^{2}\right)/\sigma^{2} < \frac{4a^{2}\left(1+2\beta\right)}{(1+\beta)^{2}-\beta^{2}a^{2}} \equiv n(a,\beta).
\tag{33}
$$

with $n(a, \beta) > 0$ from Assumptions 2 and 3. Comparing (32) and (33) and the fact that $\left(\sigma^2 - \sigma_E^2\right)/\sigma^2 < 1$, shared control is optimal only if

$$n(a, \beta) - m(a, \beta) > 0, \tag{34}$$

$$n(a, \beta) < 1. \tag{35}$$

Condition (34) can be written using (32) and (33) as

$$\frac{8\left(\beta + 1\right)\left(a^2 - 1\right)a^2\beta^2}{\left(a^2\beta^2 + 1 - \beta^2\right)\left((1 + \beta)^2 - \beta^2 a^2\right)} > 0.$$

It is readily seen that this expression is only satisfied if $a > 1$. However noting that $n(1, \beta) = 4$ and that $\frac{\partial n(a, \beta)}{\partial a} > 0$ we can see that conditions (34) and (35) are mutually exclusive. Therefore shared control cannot simultaneously dominate P-Authority and A-Authority when the quality of communication is exogenously given and does not depend on the allocation of decision rights.■

**Proof of Proposition 9**: We first proof the case where activities interact as substitutes and then proceed to the proof for complementary activities.

*i- Substitutes.* We first note that if activities interact as substitutes, shared control can never be strictly optimal. This follows immediately from the fact that under Assumption 3 the principal always increases the loss under S-Authority as the total amount of information worsens and, from Proposition 8, if the quality of communication under P-Authority and S-Authority is the same then S-Authority can never be optimal. We now investigate under what conditions the principal will centralize all decision making or delegate both activities to the agent.

If the agent has an incentive to overstate his private information (thus $a > 0$) by (28) and (30) P-Authority dominates A-Authority whenever

$$\frac{\left(\sigma^2 - \sigma_P^2\right)}{\sigma^2} < 4a^2. \tag{36}$$

*Reactive agent*: By using (14) with $b_P = a$ it can readily be seen that (36) is satisfied if and only if $4a - 1 < 0$. Thus whenever the conflict of interest $a$ is sufficiently large (i.e. $a > c_R = 1/4$) the principal centralizes decision making and for $0 < a < 1/4$ delegation takes place.

*Passive agent*: If the agent has an incentive to understate his private information (thus $a < 0$) the quality of communication under P-Authority is given by (15) with $b_P = a$. By noting that

$$\frac{\left(\sigma^2 - \sigma_P^2\right)}{\sigma^2} \geq \frac{4\left|b_l\right|}{4b_l + 3}\left[3\left(1 + b_l\right) - 1\right] = \frac{-4a\left(3a + 2\right)}{4a + 3}, \tag{37}$$

and $\left[-4a\left(3a + 2\right)\right]/\left(4a + 3\right) > 4a^2$ for $-1/2 < a < 0$, we have that A-Authority dominates P-Authority if the adaptation bias is larger than $-1/2$. Since for $a \leq -1/2$ the agent's message is

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completely uninformative (i.e. $\sigma^2 - \sigma_P^2 = \sigma^2$), condition (36) is always satisfied and P-Authority dominates A-Authority if $a < -1/2$.

ii- Complements: If activities interact as complements shared control has a communication advantage over centralization. Since the quality of communication is qualitatively different for a passive and reactive agent we analyze each case separately.

Reactive agent: Comparing the loss under A-Authority and S-Authority, with $\left(\sigma^2 - \sigma_S^2\right)$ given by (14) and $b_S$ given by (13), we have that S-Authority dominates A-Authority if

$$8\frac{a^2}{1-\beta}\sigma^2 - 4\frac{\left(\beta - a^2\beta + 4a^2 + 3a + 1\right)a}{\left(4a + 3\beta + 3a\beta + 3\right)\left(1 - \beta\right)}\sigma^2 > 0,$$
$$4\frac{\left(4a - \beta + 7a\beta - 1\right)\left(a + 1\right)a}{\left(4a + 3\beta + 3a\beta + 3\right)\left(1 - \beta\right)}\sigma^2 > 0.$$

Thus S-Authority dominates A-Authority whenever $4a - \beta + 7a\beta - 1 > 0$. Therefore the principal is indifferent between sharing control and delegating to the agent whenever $a = \underline{a}_R\left(\beta\right) = \left(\beta + 1\right)/\left(7\beta + 4\right).$

Comparing now the loss under P-Authority and S-Authority, where $\left(\sigma^2 - \sigma_P^2\right)$ is given by (15) and $b_S$ given by (13), we have that S-Authority dominates P-Authority if

$$8\frac{a}{\left(4a + 3\right)\left(1 - \beta\right)}\sigma^2 - 4\frac{\left(\beta - a^2\beta + 4a^2 + 3a + 1\right)a}{\left(4a + 3\beta + 3a\beta + 3\right)\left(1 - \beta\right)}\sigma^2 > 0,$$
$$\frac{\left(8a + a\beta + 16a^2 - 3\beta - 4a^2\beta - 3\right)\left(a + 1\right)a}{\left(4a + 3\beta + 3a\beta + 3\right)\left(4a + 3\right)\left(1 - \beta\right)}\sigma^2 > 0.$$

Thus S-Authority dominates P-Authority whenever $\left(8a + a\beta + 16a^2 - 3\beta - 4a^2\beta - 3\right) > 0$. Therefore the principal is indifferent between sharing control and delegating to the agent whenever $a = \overline{a}_R\left(\beta\right) = \left[\sqrt{160\beta - 47\beta^2 + 256} - \beta - 8\right]/\left[8\left(4 - \beta\right)\right]$. For $\overline{a}_R\left(0\right) = \underline{a}_R\left(0\right) = 1/4$ and for $\beta > 0$ $\overline{a}_R\left(\beta\right) > \underline{a}_R\left(\beta\right)$. Furthermore, it can be shown that $\frac{d}{d\beta}\left[\overline{a}_R\left(\beta\right) - \underline{a}_R\left(\beta\right)\right] > 0$. In summary, whenever $a \in \left(\underline{a}_R\left(\beta\right), \overline{a}_R\left(\beta\right)\right)$ sharing control is strictly optimal, while if $a < \underline{a}_R\left(\beta\right)$ A-Authority is optimal and for $a > \overline{a}_R\left(\beta\right)$ the principal finds it optimal to centralize all decision making.

Passive agent: Comparing the loss under A-Authority and S-Authority as given by (30) and (31), with $\left(\sigma^2 - \sigma_S^2\right)$ given by (14) and $b_S$ as in (13), we have that S-Authority dominates A-Authority if

$$8\frac{a^2}{1-\beta}\sigma^2 - \frac{a^2}{1-\beta^2}4\sigma^2 > \frac{\left(1 + \beta\right)^2 - a^2\beta^2}{1 - \beta^2}\left(\sigma^2 - \sigma_S^2\right),$$
$$\frac{\left(\sigma^2 - \sigma_S^2\right)}{\sigma^2} < 4\frac{\left(2\beta + 1\right)a^2}{\left(1 + \beta\right)^2 - a^2\beta^2} \equiv p(a, \beta).$$

However the quality of communication under S-Authority in this case is bounded from below by

$$\frac{\left(\sigma^2 - \sigma_S^2\right)}{\sigma^2} > \frac{4\,|b_S|}{4b_S + 3}\left[3\left(1 + b_S\right) - 1\right] = \frac{-4\left(3a + 2\beta + 2a\beta + 2\right)a}{\left(4a + 3\beta + 3a\beta + 3\right)\left(\beta + a\beta + 1\right)} \equiv q(a, \beta). \qquad (38)$$

It can be verified that $q(a, \beta) > p(a, \beta)$ for $-1/2 < a < 0$ implying that A-Authority dominates S-Authority if the adaptation bias is larger than $-1/2$. Comparing now the loss under P-Authority and S-Authority as given by (28) and (31) (with $\left(\sigma^2 - \sigma_S^2\right)$ given by (15), $b_S$ as in (13), $\left(\sigma^2 - \sigma_P^2\right)$ given by (15) and $b_P$ as in (11)) we have that P-Authority dominates S-Authority if

$$2\left(\frac{1}{1 - \beta}\right)\left(\sigma^2 - \sigma_P^2\right) < 4\frac{a^2}{1 - \beta^2}\sigma^2 + \frac{(1 + \beta)^2 - a^2\beta^2}{1 - \beta^2}\left(\sigma^2 - \sigma_S^2\right). \qquad (39)$$

We need only verify this inequality for $a < -1/2$ since for $a > -1/2$ P-Authority and S-Authority are both dominated by A-Authority. Noting that for $a < -1/2$ we have $\sigma^2 - \sigma_P^2 = \sigma^2$, (39) simplifies to

$$\frac{\left(\sigma^2 - \sigma_S^2\right)}{\sigma^2} > 2\frac{1 + \beta - 2a^2}{(1 + \beta)^2 - a^2\beta^2}.$$

Next we have that for $a < -1/2$, $2\left(1 + \beta - 2a^2\right) < 4\left(2\beta + 1\right)a^2$ which implies that.

$$2\frac{1 + \beta - 2a^2}{(1 + \beta)^2 - a^2\beta^2} < p(a, \beta).$$

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