Mathematical modelling and its application to social care

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Methods Review 7

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ABSTRACT

This review summarises key mathematical modelling methods, with a focus upon those methods currently applied within healthcare modelling, and suggests previous and potential applications of these methods within the field of social care. A summary is offered of the model development process, including the use of problem structuring methods, literature reviewing, elicitation and uncertainty analysis. Specific modelling methods such as decision trees, state transition models and discrete event simulation are described, with an outline of their strengths and limitations. Current approaches for quantifying outcomes within health economic evaluation are also briefly discussed. The review highlights particular issues which may need to be considered when applying these modelling methods to social care.

RECOMMENDATIONS FOR RESEARCH ON ADULT SOCIAL CARE PRACTICE

- Development of methods for understanding the decision problem

- Methodological development of existing mathematical modelling techniques for use within social care

- Methods for quantifying outcomes associated with social care interventions

CONTENTS

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INTRODUCTION

This review presents an overview of mathematical modelling and its current and potential role in informing decisions surrounding the management and delivery of social care. The purposes of the review are threefold:

1. to briefly describe the model development process and the most common methodological forms of mathematical models used to inform health care policy decision-making;

2. to review the use of mathematical modelling in the evaluation of social care interventions, and;

3. to consider developing and applying existing modelling approaches within a social care context.

The use of mathematical models has gained much prominence through their use in facilitating economic evaluation (see accompanying review by Knapp 2011). However, models have also been used in a number of other decision-making contexts including risk assessment (Oscar 2004, Cook *et al*. 2006) and service planning and capacity modelling (Vlachos *et al*. 2007, Andersson and Varbrand 2007). The methods discussed within this review are applicable to any decision-making context, however the particular focus here is upon healthcare economic evaluation.

The role and value of mathematical modelling to inform policy decisions

A mathematical model is a representation of the real world, characterised by the use of mathematics to represent the parts of the real world that are of interest and the relationships between those parts (Eddy 1985). Such models can be used to inform policy decisions by synthesising a diverse range of evidence within a coherent and explicit framework. The use of mathematical models avoids intuition alone and, in some cases, the risk, time and cost associated with primary research. By definition, models involve assumption, abstraction and simplification. However, assumptions can be made explicit and the impact of uncertain assumptions upon the model results can be formally assessed. Mathematical models are developed with the explicit function of informing decision-making. Decision models generate information on the expected costs and consequences of alternative courses of action and the probability that a given decision alternative is optimal, given current information. Mathematical modelling rests within a Bayesian framework involving the synthesis of all relevant, often disparate, information in the development, implementation and interpretation of the model and its results (Spiegelhalter *et al*. 1999). Decision-analytic models can generally be characterised by three common features: (1) the definition of causal relationships between phenomena; (2) the use of some form of extrapolation, either the projection of short-term data to longer-term outcomes or the translation from intermediate to final outcomes through defined causal relationship, and (3) the synthesis of evidence from multiple sources.

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Whilst other applications exist, the two main areas in which mathematical modelling is used to inform policy decisions in health and well-being are economic decision modelling and service planning modelling. Within healthcare, the former usually involves the conceptual and mathematical representation of some underlying disease natural history and the impact of a given set of interventions upon that natural history to estimate expected costs and consequences. The costs and consequences are commonly described as the incremental cost per quality-adjusted life year (QALY) gained over a lifetime horizon, although disease specific outcomes may also be reported. The forthcoming SSCR review on cost-effectiveness analysis by Knapp (2011) provides further information about the economic framework in which such models are used. The latter application of modelling is less well-defined, often involving modelling systems across cross-sectional population groups within some organisation-based system (for example, a hospital ward) in order to identify some means of improvement such as cost savings, improved efficiency or shorter waiting times. The specific modelling methods described below are applicable to both areas.

THE MODEL DEVELOPMENT PROCESS

In a generic sense, the development of a mathematical model involves the following five non-distinct groups of activities:

1. understanding the decision problem;

2. conceptual modelling;

3. model implementation;

4. model checking; and

5. engaging with the decision (Chilcott *et al*. 2010).

Importantly, model development is not, nor should it be, a linear process; these five sets of activities may overlap; for example, model checking is advisable through the entire development process. This description of the model development process is largely generalisable across any modelling application.

Understanding the decision problem

The first stage in model development involves understanding what the problem is, why it is important, and what the decision-maker requires in order to reach a decision. This may involve a myriad of activities including immersing oneself in relevant research evidence, determining the broad scope of the model and establishing the purpose of the model i.e. what it needs to be able to output in order to be useful in informing the decision problem. This may involve formal problem structuring methods (Pidd 2009). With respect to decision modelling, this usually involves defining the population, interventions,

comparators, outcomes and perspective of the model. The *population* is the people that the intervention(s) will impact upon. Specific subgroups of people may also be considered relevant. Depending on the purpose of the model, *interventions* may include any use of resources which are expected to impact upon relevant outcomes for the population represented within the model. These may be broad ranging, and within healthcare may include pharmacological treatments, screening programmes, surgical techniques, public awareness campaigns, complex interventions or alternative configurations of an existing service. Model *outcomes* may be disease-specific, for example the incremental cost per cancer avoided, or generic, such as the incremental cost per life year gained (LYG) or the incremental cost per quality-adjusted life year (QALY) gained. In terms of service planning, outcomes are likely to be specific to the system being modelled, for example throughput, waiting times and test failures, but may also include more generic outcomes such as costs and cost savings.

Conceptual modelling

The development of the conceptual model involves the abstraction of the decision problem within a coherent framework; in effect it is the tangible expression of an intangible mental model of the system under consideration. The ultimate purpose of conceptual modelling is to determine what is relevant to the decision problem. This includes consideration of what should go into the model, what should stay out of the model, and the way in which those phenomena included in the model should be causally related. Each mathematical model is based on an explicit or implicit conceptual model developed either prior to, or in tandem with the quantitative mathematical model. Explicit methods for conceptual modelling include developing written documentation of the proposed model structure and population, drawing sketches of the proposed model, developing mock-ups and developing written interpretations of evidence. Conceptual modelling may involve consultation with individuals who impact upon or are impacted upon within the system to be modelled. It may also involve examination of research evidence, including previous models of similar systems. In theory, such conceptual models should reflect the shared understanding of the clinical community, the problem owners and the individual charged with model development. While generic frameworks for conceptual modelling have been suggested within the wider operational research literature (Robinson 2008), these have not been universally applied within healthcare and as such there remains considerable variability in approach between individual modellers (Chilcott *et al*. 2010).

Model implementation

Model implementation involves the quantitative programming of a conceptual model within some software platform. This stage may involve considerable iteration in achieving a trade-off between the perceived 'ideal' model and the 'best' model possible given the limitations in the evidence base. The most appropriate modelling methodology is dependent on the decision problem under consideration and the information

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requirements of the decision-maker; the most common modelling methods are described on pages 5 to 9. Structural assumptions such as those around the extrapolation of trial outcomes are required within the quantitative model.

While evidence review is appropriate across the whole model development process, it is particularly important here in terms of the identification, selection and use of relevant evidence to inform the model parameters. In healthcare economic evaluation 'relevant evidence' may include data relating to the effectiveness of interventions, costs, resource use, utility values, epidemiology and aetiology, probabilities of events occurring and longer term outcomes (Glanville and Paisley 2010). Evidence sources may include randomised controlled trials, observational data and routine data sources (for example, hospital episode statistics and unit cost sources). Ideally, many of these parameters would be identified through systematic, comprehensive and reproducible literature reviews. In practice however, time and resource constraints often make such processes feasible only for key model parameters. Formal or informal methods of elicitation of clinical experts may be required for specific assumptions or model parameters where relevant literature is unavailable (O'Hagan 2006).

Model checking

A key element of model development involves the verification and validation of models. The former asks 'is the model correct?' while the latter asks 'is it the correct model?' Eddy (1985) splits model validation into four levels: (1) Would experts believe the model?; (2) Is the model doing what it is expected to be doing?; (3) Do model predictions agree with non-source data which has not been used to populate the model; and (4) Can the model predict the outcomes of trials? There are a wealth of methods for identifying and avoiding errors including engaging with clinical experts to check face validity, testing extreme values, checking logic, and checking data sources. A detailed taxonomy of model checking approaches is provided in Chilcott *et al*. (2010). Model checking should be undertaken throughout the model development process.

Engaging with the decision

The final stage in model development involves using the model to inform the decision problem in question. This involves the generation, interpretation and communication of model results to the decision-maker(s). With reference to economic decision modelling, interventions should be compared incrementally, which means that the most effective option is compared with the next most effective option, and this in turn is compared with the next most effective option, and so on. Uncertainty may be assessed using one-way sensitivity analysis and probabilistic sensitivity analysis. One-way sensitivity analysis indicates which parameters are the key drivers of the model results by varying individual parameters in turn within plausible ranges. Probabilistic sensitivity analysis involves the use of Monte Carlo sampling across distributions of all uncertain parameters to estimate the probability that each intervention is optimal. An extension of this approach involves

the use of *value of information analysis* to estimate the monetary value of reducing or eliminating uncertainty in current information, thus providing a rational basis for planning and prioritising future research. Expected value of perfect information (EVPI) involves calculating the maximum investment a decision-maker would be willing to pay to eliminate all parameter uncertainty from the decision problem (Claxton and Posnett 1996). More advanced methods of value of information analysis are described within *Decision Modelling for Health Economic Evaluation* by Briggs *et al*. (2006).
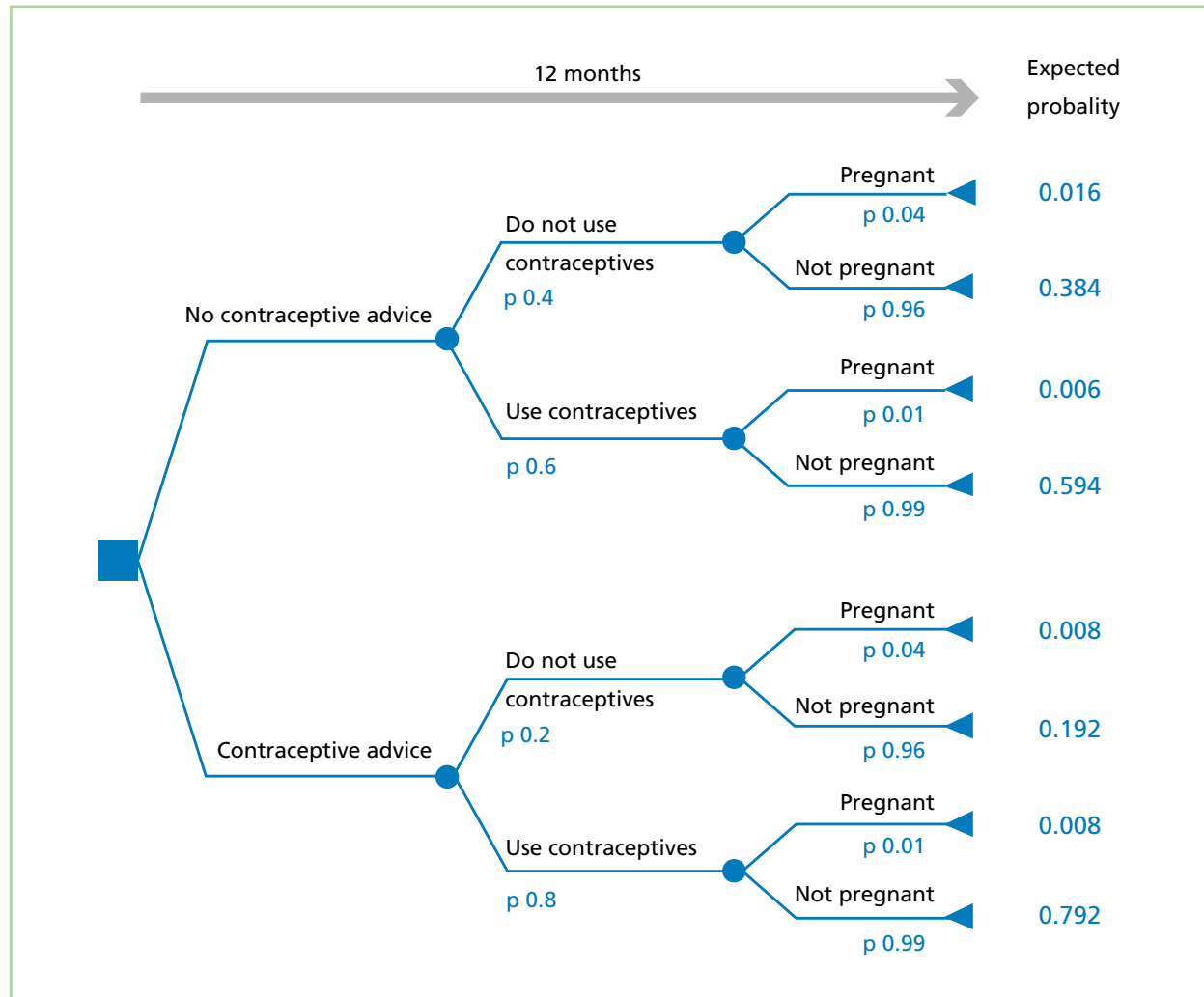
MODELLING METHODS

In health economic evaluation and service planning, mathematical models may reflect the experience of individuals or cohorts of patients and may be evaluated in terms of events or discrete increments of time. A detailed taxonomy of alternative model structures is available from Brennan *et al.* (2006). In practice, the most common manifestations of mathematical models are decision-trees, state transition models and patient-level models. These three broad classes of models are outlined below.

Decision trees

Decision trees are one of the simplest forms of decision model. These are typically cohort models which means that they follow a cohort of people over a period of time but do not distinguish between individual characteristics. This means that the probabilities of events occurring within the model are associated with an 'average' person. Decision trees are made up of a decision node which describes all possible decision alternatives, chance nodes which describe the probability of events occurring dependent upon the decision node, and outcome nodes which describe the probability of all outcomes. Outcome nodes are calculated using an approach known as 'folding back' by multiplying the chance nodes leading to that outcome (Drummond *et al.* 2005). Figure 1 shows an example of a decision tree, estimating the probability of becoming pregnant in a group of sexually active 18-year olds who are either given, or not given, contraceptive advice. Probabilities are illustrative only. Based on this decision tree, the probability of becoming pregnant given no contraceptive advice would therefore be 0.016+0.006 which equates to 2.2%, and the probability of becoming pregnant given contraceptive advice would be 0.008+0.008 which equates to 1.6%. Costs and outcomes for each branch of the tree would also be calculated to estimate the difference in costs and difference in outcomes between the two options, and hence the incremental cost-effectiveness ratio associated with the two options.

Decision trees are useful when a decision process can be easily broken down into a tree-like structure, and events of interest occur once and over a short period of time. The advantage of decision trees is that they are relatively simple to develop and understand. However, decision trees are not appropriate when a problem is more complex. For example, when the risk of events is continuous or the number of events occurring is high since the number of branches would become unmanageable. It is also preferable to use

Figure 1: Decision tree model of providing contraceptive advice to 18-year olds

12 months

Expected probality

No contraceptive advice

Do not use contraceptives
p 0.4

Pregnant
p 0.04 — 0.016

Not pregnant
p 0.96 — 0.384

Use contraceptives
p 0.6

Pregnant
p 0.01 — 0.006

Not pregnant
p 0.99 — 0.594

Contraceptive advice

Do not use contraceptives
p 0.2

Pregnant
p 0.04 — 0.008

Not pregnant
p 0.96 — 0.192

Use contraceptives
p 0.8

Pregnant
p 0.01 — 0.008

Not pregnant
p 0.99 — 0.792

other modelling methods when the timing of events or variability between individuals within the model is important.

State transition models

Within state transition models, events of interest are modelled as transitions from one state to another. The time period of the model is divided into cycles of time, for example a year, and at each cycle there is a probability of remaining in the same state or progressing to a different state within the model. The key assumption is that all people must exist within one of a finite number of states at any one time. State transition models are usually cohort models, although they can also follow individual patients.

For example, chronic myeloid leukaemia (CML) can be described by three clinical phases as the patient progresses: chronic phase, accelerated phase and blastic phase. Within a state

transition model of CML, all patients would exist within one of these health states or in the dead state. Probabilities of transitioning between the states are assigned so that patients can progress from chronic phase to accelerated phase, from accelerated phase to blastic phase, and from blastic phase to dead. Reverse transitions are not possible. There would also be a probability of dying from other causes from any of the CML states. Once a person has reached the dead state, there would be a probability of 1 of remaining in that state. This is known as an absorbing state, since eventually everyone within the model will progress to this state. This example is shown in Figure 2. Arrows are used to represent possible transitions.

Within a state transition model, the behaviour of the process subsequent to any cycle depends only on its description in that cycle (Sonnenberg and Beck 1993). In other words, the probability of transitioning between the health states is independent of which states the patient has been in previously. However, what happens to a person in the future is often dependent on what happens to them in the past. This can be accounted for by building in additional states which are explicit about the history of the people within the model. For instance, within the example above, people may receive a transplant in the accelerated phase which would lead to the patient clinically being in the chronic phase; however the probability of subsequent events is different for these patients. The chronic

Figure 2: State transition model of chronic myeloid leukaemia
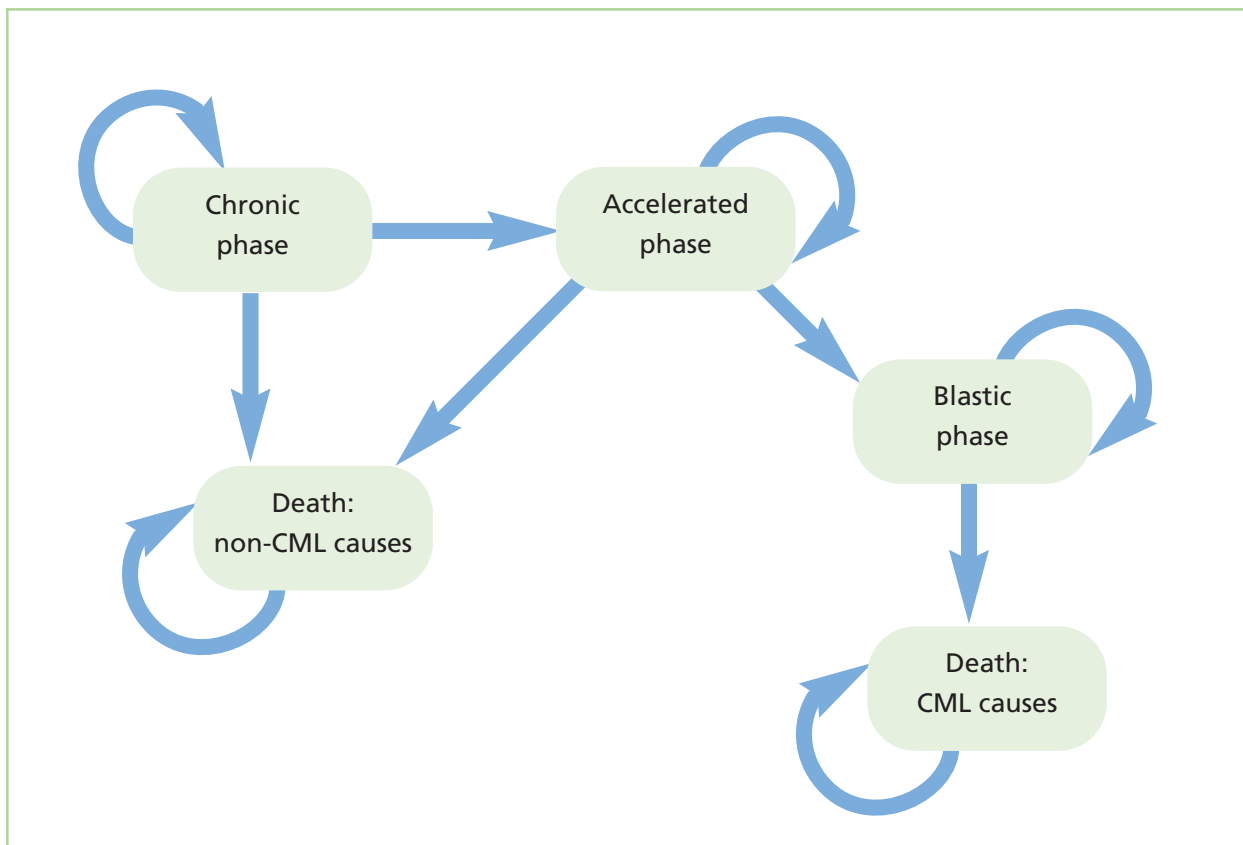
phase may therefore be split into two states, 'chronic phase' and 'chronic phase following transplant'.

State transition models are useful when the risk of an event is ongoing over time, when the timing of events is important and when events may occur more than once (Sonnenberg and Beck 1993). Other modelling methods are preferable when there are many events which are dependent upon the history of persons within the model and when variability between individuals within the model is important.

Discrete event simulation

Discrete event simulations (DES) are individual-level models where events can be based upon the previous experiences of the entities (for example, people) within the model and their characteristics. Within DES, timing is explicitly modelled by calculating when the next event will occur following the occurrence of each event (Pidd 2004). Resources, such as staff or hospital beds, can be explicitly modelled using shift patterns.

Figure 3 shows an example of a discrete event simulation of the cervical screening process which assesses the impact of a range of options for change to the process upon the turn-

Figure 3: Discrete event simulation of cervical screening process
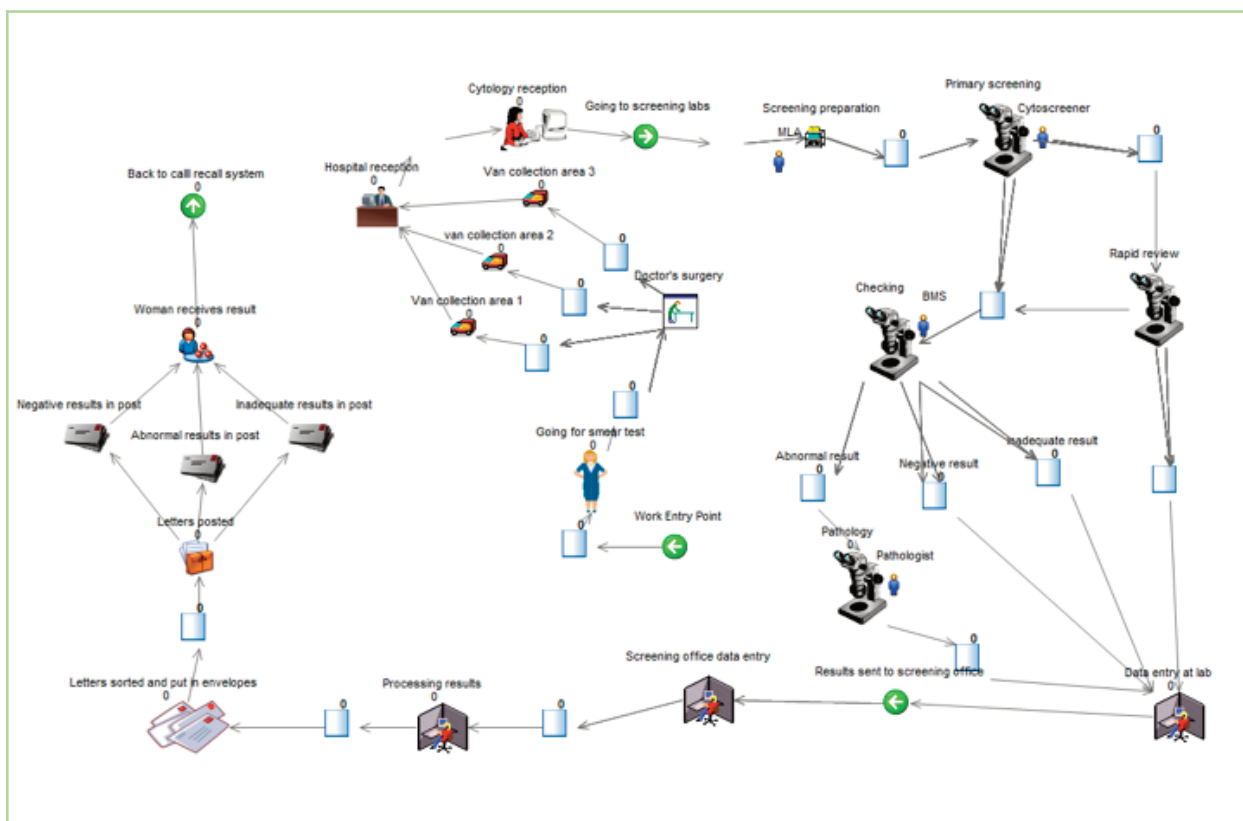
Source: Pilgrim H and Chilcott J (2008). Reproduced with permission of Palgrave Macmillan.

around time of screening results. The model begins at the General Practice surgery and follows the patients' samples to the laboratory where they are examined and graded. These grades are then sent to the 'call and recall' offices for processing the results and posting them out to the women. Each entity is assigned patient characteristics, such as underlying disease status, which determine the subsequent pathway of the entities. Each process (or 'work centre') within the model is programmed to require time to be completed and staff to be available where appropriate. The white boxes within Figure 3 allow queues to develop prior to each work centre if there are insufficient resources available at that time. Within this example, the DES was developed such that the result turn-around times closely matched what happened in practice. Options for change were then tested within the model in terms of result turn-around times (Pilgrim and Chilcott 2008).

DES is useful when the decision problem is complex and requires explicit modelling of time and when variability between people within the model is important. However, DES is associated with greater data requirements and usually requires greater expertise and development time; hence there is a trade-off between these advantages and disadvantages (Karnon 2003).

Other modelling methods

There are a wide range of other modelling methods which have not been applied as widely within healthcare. It is not feasible to cover all of these methods here. However two key methods will be briefly described: system dynamics and agent-based modelling. System dynamics aims to capture feedback loops within systems by firstly mapping these relationships diagrammatically, and secondly using differential equations to represent them mathematically. Within healthcare, system dynamics has mainly been applied to infectious disease modelling, where the complex nature of an infectious disease needs to be captured (Atun *et al*. 2007). Agent-based modelling is another individual-level approach which simulates agents, for example patients, with potentially adaptive behaviour according to their environment. Within the model, agents are able to adapt their behaviour according to interactions between each other and their environment. Agent-based modelling has also been applied to infectious diseases (Meng *et al*. 2010).

It is important to note that while there are advantages and disadvantages of using each of the modelling methods outlined here, the choice of appropriate modelling approach is not always clear. For example, there is a trade-off between the increased flexibility associated with individual-level modelling such as DES and the increased programming expertise, data and time requirements. Additionally, the choice of modelling methodology is not always distinct; in some cases it may be appropriate to combine two or more modelling methods. Further discussion of which type of mathematical model would be appropriate for decision problems with certain characteristics is provided by Brennan *et al*. (2006).

APPLICATIONS OF MATHEMATICAL MODELS IN SOCIAL CARE

A literature search was undertaken to identify previous applications of mathematical models in social care. This review was not intended to be exhaustive. The aim was to capture the types of mathematical models currently being developed within social care and the key issues associated with social care modelling.

Social Care Online was searched for economic evaluations using the terms 'economic evaluation', 'cost effectiveness' or 'cost utility' within the title. In addition, the Health Technology Assessment (HTA) website was searched for any assessments around social care using the search term 'social care'. Studies were included if they assessed the cost-effectiveness of UK social care interventions, which are not aimed at children and which are not provided within a primary care or a hospital-based setting, using a mathematical model. Within this review, the use of a 'mathematical model' is characterised by: (1) the definition of causal relationships between phenomena; (2) the use of some form of extrapolation, either the projection of short-term data to longer-term outcomes or the translation from intermediate to final outcomes through defined causal relationship; and (3) the synthesis of evidence from multiple sources.

Three hundred and seventy six potentially relevant studies were identified by the searches. Following title and abstract sifting and after obtaining potentially relevant full papers, no studies were identified which used a mathematical model as defined above to assess the cost-effectiveness of social care interventions. Five economic evaluations alongside clinical trials were identified from the review. These are described in Table 1.

The use of evidence from only one trial means that all relevant comparators may not be included within the analysis. This is important because it may be that the most economically attractive comparator is not included within the analysis. In addition, it may not be possible to adequately capture the uncertainty around the population mean. Moreover, extrapolation of trial outcomes is important to capture all differences in costs and outcomes between the intervention and the comparator. Using only the trial data is likely to underestimate the differences in both costs and outcomes, and hence will lead to different results and potentially different conclusions.

Extrapolation within a mathematical model may involve extending outcomes reported within the trials over the long-term or it may involve the estimation of final outcomes which have not been collected within the trial by modelling a relationship between these and the intermediate outcomes collected within the trials. For example, suppose a model of the impact of contraceptive advice upon costs and outcomes is required, but trial data are only available reporting contraceptive use. The model would need to capture the relationship between contraceptive use and pregnancy rates, between pregnancy rates and birth rates, and the way in which this impacts upon health-related quality of life.

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Table 1: Summary of identified studies

Population	Intervention	Model outcomes	Perspective	Key methods	Results	Key limitations
Charlesworth et al. 2008						
People with dementia & their carers	Befriending of carers by trained lay workers	Cost per QALY gained	Societal	Economic evaluation alongside RCT	Befriending is unlikely to be economically attractive	No extrapolation beyond trial follow up; Evidence from only 1 trial
Graff et al. 2008						
People with dementia & their carers	Occupational therapy	Cost per number with 'successful treatment' gained	Societal	Economic evaluation alongside RCT	Occupational therapy is likely to be cost saving	No extrapolation beyond trial follow up; Evidence from only 1 trial
Mason et al. 2007						
Frail older people & their carers	Respite care services	Wide range of outcomes assessed within the studies identified	Societal or NHS	Systematic review of economic evaluations	5 studies identified (1 UK). All economic evaluations alongside RCTs. Authors concluded it was not feasible to undertake a *de novo* economic evaluation due to paucity of effectiveness data	No economic evaluation undertaken
Kalra et al. 2005						
Stroke patients & their carers	(i) 24 hour stroke care; (ii) general ward care + specialist team support; (iii) home care + specialist team + community support	Cost per avoidance of death or institutionalisation; cost per Barthel Index; cost per QALY gained	Societal	Economic evaluation alongside RCT	24 hour stroke care is the most effective option; however home care is the most cost-effective option	No extrapolation beyond trial follow up; Evidence from only 1 trial
Patel et al. 2004						
Stroke patients & their carers	Care giver training	Cost per QALY gained	Societal and NHS	Economic evaluation alongside RCT	Care giver training is likely to be cost saving	No extrapolation beyond trial follow up; Evidence from only 1 trial

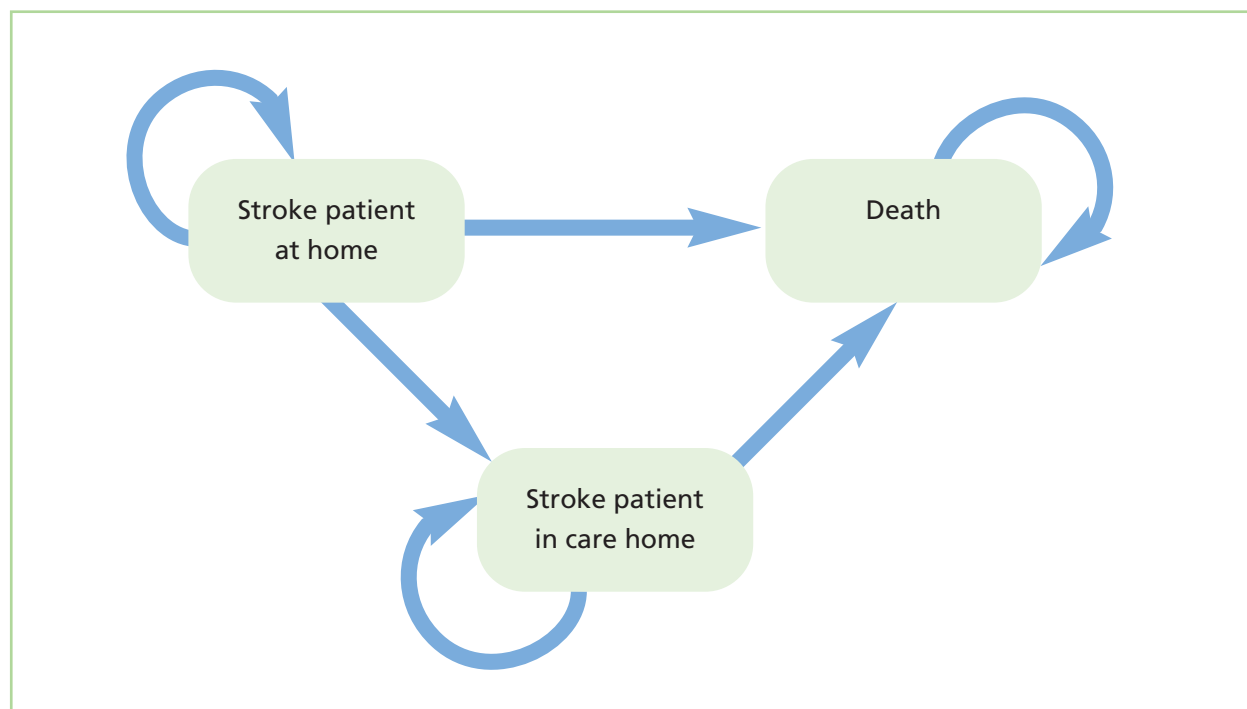## POTENTIAL APPLICATION OF MATHEMATICAL MODELS IN SOCIAL CARE

There is no fundamental reason why the modelling methods outlined on pages 2 to 9 of this review cannot be applied to social care economic evaluation. An illustrative example of this is described for assessing the cost-effectiveness of carer training.

Model assessing the cost-effectiveness of carer training

To assess the cost-effectiveness of carer training upon stroke patients, a state transition model could be developed with three states; stroke patient at home with carer, stroke patient in care home and death as shown in Figure 4.

The model could follow a cohort of patients over their lifetime. Patients would begin in the 'stroke patient at home' state and eventually progress to the 'death' state. The model would include the disease natural history of stroke, and the impact of the intervention upon that disease natural history. In practice, it may be that more states would be required to incorporate more states associated with the illness or other comorbidities associated with stroke. The model would also include death from other causes. Costs could be incorporated for the intervention (cost of training carers), direct healthcare costs (such as outpatient and stroke unit costs), social service costs (such as the cost of the social care worker and other assistance), community-based care costs (such as general practitioner and nurse costs) and carer costs (the cost of caring by relatives/friends). Different costs

Figure 4: State transition model of carer training

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could be associated with each health state according to the care received. Health-related quality of life could also be associated with each state dependent upon the intervention. This model could be developed using the model development process outlined within pages 2 to 4 of this review.

Methodological issues for modelling within social care

Social care modelling both accentuates some of the methodological issues within health technology assessment and highlights additional issues which do not require consideration within other contexts. Within health technology assessment, there is often only one relevant disease to be modelled, and the disease natural history and the impact of health technologies upon the disease natural history are relatively well understood, which facilitates extrapolation of data beyond the trial follow-up. Costs associated with health technologies are usually either incurred by the NHS or personal social services or are considered to be indirect costs; there are generally no costs incurred by other sectors within health technology assessment models.

In contrast, social care modelling appears to face similar issues to public health modelling; namely the attribution of effects (in other words quantifying the effectiveness of the interventions), measuring and valuing outcomes, identifying intersectoral costs and consequences, and incorporating equity considerations (Weatherley *et al*. 2009).

Attribution of effects

This relates to the limited evidence from the UK setting, particularly in the form of randomised controlled trials (RCT), and the short length of follow-up within these studies. Statistical methods for adjusting effectiveness estimates from other settings and for extrapolating these outcomes should be developed. In addition, social care models may need to incorporate human behaviour and there are many factors influencing this which increases the complexity of modelling. For instance, mental health disorders present additional challenges in modelling patient behaviour, and these impacts are often not collected within the trials of effectiveness. Econometric techniques can be used to analyse the outcomes of non-RCT evidence by aiming to control for confounding factors.

Measuring and valuing outcomes

Within health technology assessment the QALY measure is used to value outcomes. NICE recommend that the EQ-5D questionnaire is used where possible to measure these outcomes (NICE 2008). This consists of five dimensions including mobility, self care, usual activities, pain/ discomfort and anxiety/ depression. However, these dimensions may not be sufficiently broad within social care to capture all of the effects of the interventions (for example, non health-related effects). Research is currently underway in this area, including research assessing the use of multicriteria decision analysis for valuing stakeholders' preferences. There may also be issues with valuing these health dimensions from people with mental health needs (see accompanying review by Knapp 2011).

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Identifying intersectoral costs and consequences

Social care interventions may affect costs and outcomes across numerous sectors in addition to health; for example, education and employment. Expenditure within one sector may reduce expenditure within another sector and the value of this should be considered within the model. The perspective chosen will affect which costs and consequences are accounted for within the model.

Incorporating equity considerations

Equity relates to the fair allocation of resources. One aim of social care interventions is to reduce inequalities by providing care to those who are unable to adequately care for themselves. It may therefore be appropriate to incorporate equity concerns within economic models developed within social care and this should be explored further.

The characterisation of uncertainty

There may be considerable uncertainty regarding the disease natural history and some of the benefits of social care interventions. A coherent conceptual model should be developed with input from social care experts to minimise the uncertainties within the model. Structural uncertainties as well as parameter uncertainties should be incorporated into the probabilistic sensitivity analysis (Bojke *et al*. 2009).

In 2005, NICE began undertaking assessments of the effectiveness and cost-effectiveness of public health interventions. As a result, much research is currently underway around public health modelling of many of these issues and the outcomes of much of this research will be useful to social care modelling (Weatherley *et al*. 2009). However, there are three key differences between social care modelling and public health modelling. Firstly, within social care the majority of the population which the intervention is aimed at will benefit from the intervention, whereas within public health modelling generally only a small subset of the population provided with the intervention will benefit from it since a substantial proportion will often not engage with the intervention. Therefore, while within public health uptake may need to be explicitly modelled, within social care it may be less important. This also means that the budget impact of social care interventions can be assessed with more certainty than public health interventions. Secondly, the benefits of social care interventions are generally apparent within the short to medium-term, whereas due to the preventative nature of public health interventions, the benefits are often unlikely to be observed until several years after the intervention is provided. This means that extrapolating outcomes based upon clinical trials within social care is less likely to require the mapping of intermediate outcomes onto final outcomes than within public health modelling.

Finally, it is suggested that all interventions within an economic evaluation should be compared with a 'do nothing' option (Drummond *et al*. 2005). However, in practice this is usually the minimum treatment which is considered to be ethically reasonable. Within

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social care, this 'do nothing' option – for example providing a carer for a person with intellectual disability – is often associated with high costs in comparison to 'do nothing' options within public health or health technology assessment. It may not always be clear which comparator should be modelled within the economic evaluation and this should be considered within the 'understanding the decision problem' phase of the model development process (see pages 2 to 3). It is important that the choice of comparator(s) within an economic model is appropriate since the cost-effectiveness of an intervention is defined by reference to its comparator(s).

Strengths and limitations associated with mathematical modelling within social care

Decisions around resource allocation are essential where there are insufficient resources for the demands upon a service. Mathematical models provide a framework for synthesising all relevant evidence associated with the decision in an explicit way so that assumptions can be questioned and explored. Although there are currently methodological limitations associated with undertaking economic evaluations within social care, the use of the outlined modelling methods provide a more appropriate framework for resource allocation decisions than either an informal decision-making process – which is likely to be subject to implicit assumptions and biased evidence – or economic evaluations alongside clinical trials. The current limitations within the modelling methodology can be explored and developed based upon the methodological research currently being undertaken within public health modelling and alongside its application within social care.

Resources required for undertaking economic evaluation within social care

Mathematical modelling expertise would be required for developing a model within social care. The cost and time requirements of developing a mathematical model are wide-ranging depending upon the aim and scope of the work. However as a guide, a time period of between three to six months would usually be required to develop a model and an accompanying report. The aim of economic evaluation is to improve the efficacy of the system, and hence the cost of the model development should be relatively small compared with the potential cost savings associated with changing current practice within social care throughout England. The cost of the development of mathematical models is also relatively low compared with primary research.

In instances when a mathematical model suggests that an intervention is economically attractive over the long-term, it should be noted that the intervention may require substantial capital within the short-term. The short-term costs associated with the interventions may also be estimated within a model, as has been undertaken previously within social care economic evaluations.

CONCLUSION

This review has described the generic model development process and the most common application of mathematical models within healthcare. A literature review was undertaken which suggests that, while some economic evaluations are undertaken alongside clinical trials within social care, modelling which synthesises relevant evidence and extrapolates outcomes beyond trial follow-up is not generally employed within social care. The modelling methods currently used within healthcare could be applied to social care and there are clear benefits to doing this. However, further methodological development would be required as part of the application to social care. Research around public health modelling may be useful for applying modelling methods to social care since there are important similarities between the two applications.

References

Andersson T, Varbrand P (2007) Decision support tools for ambulance dispatch and relocation, *Journal of the Operational Research Society*, 58, 195–201.

Atun R, Lebcirb R, McKeec M, Habichtd J, Cokerc R (2007) Impact of joined-up HIV harm reduction and mulitdrug resistant tuberculosis control programmes in Estonia: system dynamics simulation model, *Health Policy*, 81, 2, 207–217.

Bojke L, Claxton K, Sculpher M, Palmer S (2009) Characterizing structural uncertainty in decision-analytic models: a review and application of methods, *Value in Health*, 12, 739–749.

Brennan A, Chick S, Davies R (2006) A taxonomy of model structures for economic evaluation, *Health Economics*, 15, 12, 1295–1310.

Briggs A, Sculpher M, Claxton K (2006) *Decision Modelling for Health Economic Evaluation*, Oxford University Press Inc, New York.

Charlesworth G, Shepstone L, Wilson E, Thalanany M, Mugford M, Poland F (2008) Does befriending by trained lay workers improve psychological well-being and quality of life for carers of people with dementia, and at what cost? A randomised controlled trial, *Health Technology Assessment*, 12, 4,

Chilcott J, Tappenden P, Rawdin A, Johnson M, Kaltenthaler E, Paisley S, Papaioannou D, Shippam A (2010) Avoiding and identifying errors in health technology assessment models: qualitative study and methodological review, *Health Technology Assessment*, 14, 25, 1–136.

Claxton K, Posnett J (1996) An economic approach to clinical trial design and research priority setting, *Health Economics*, 5, 6, 513–524.

Cook N, Buring J, Ridker P (2006) The effect of including C-reactive protein in cardiovascular risk prediction models for women, *Annals of Internal Medicine*, 145, 1, 21–29.

Drummond MF, Sculpher MJ, Torrance GW, O'Brien BJ, Stoddart GL (2005) *Methods for the Economic Evaluation of Health Care Programmes*, Third edition, Oxford University Press, Oxford.

Eddy D (1985) Technology assessment: the role of mathematical modelling, in Mosteller F (ed.), *Assessing Medical Technologies*, National Academy Press, Washington, DC, pp. 144–160.

Glanville J, Paisley S (2010) Chapter 7: Searching for evidence for cost-effectiveness decisions, in Shemilt I, Mugford M, Vale L, Marsh K, Donaldson C (eds.) *Evidence-based Decisions and Economics: Health care, Social Welfare, Education and Criminal Justice*, 2nd Edition, Wiley-Blackwell, pp. 79–92.

Graff M, Adang E, Vernooij-Dassen M, Dekker J, Jönsson L, Thijssen M, Hoefnagels W, Olde Rikkert M (2008) Community occupational therapy for older patients with dementia and their care givers: cost effectiveness study, *British Medical Journal*, 336, 7636, 134–138.

Kalra L, Evans A, Perez I, Knapp M, Swift C, Donaldson N (2005) A randomised controlled comparison of alternative strategies in stroke care, *Health Technology Assessment*, 9, 18.

Karnon J (2003) Alternative decision modelling techniques for the evaluation of health care technologies: Markov processes versus discrete event simulation, *Health Economics*, 12, 837–848.

Knapp M (2011) *Cost-effectiveness analysis and social care*, SSCR Methods Review 16, NIHR School for Social Care Research, London.

Mason A, Weatherly H, Spilsbury K, Arksey H, Golder S, Adamson J, Drummond M, Glendinning C (2007) A systematic review of the effectiveness and cost-effectiveness of different models of community-based respite care for frail older people and their carers, He*alth Technology Assessment*, 11, 15.

Meng Y, Davies R, Hardy K, Hawkey P (2010) An application of agent-based simulation to the management of hospital-acquired infection, *Journal of Simulation*, 4, 60–67.

NICE (2008) *Guide to the Methods of Technology Appraisal*, National Institute for Health and Clinical Excellence, London.

O'Hagan A (2006) Research in elicitation, in Upadhyay SK, Singh U, Dey DK (eds) *Bayesian Statistics and its Applications*, Anamaya, New Delhi, pp. 375–382.

Oscar T (2004) A quantitative risk assessment model for Salmonella and whole chickens. *International Journal of Food Microbiology*, 93, 2: 231–247.

Patel A, Knapp M, Evans A, Perez I, Kalra L (2004) Training care givers of stroke patients: economic evaluation, *British Medical Journal*, 328, 1102–1104.

Pidd M (2004) *Computer Simulation in Management Science*, John Wiley and Sons Ltd, Chichester.

Pidd M (2009) *Tools for Thinking; Modelling in Management Science*, John Wiley and Sons Ltd, Chichester.

Pilgrim H, Chilcott J (2008) Assessment of a 7-day turn-around for the reporting of cervical smear results using discrete event simulation, *Journal of the Operational Research Society*, 59, 7, 902–910.

Robinson S (2008) Conceptual modelling for simulation part I: definition and requirements, *Journal of the Operational Research Society*, 59, 278–290.

Sonnenberg F, Beck J (1993) Markov models in medical decision making, *Medical Decision Making*, 13, 322–338.

Speigelhalter D, Myles J, Jones D, Abrams K (1999) An introduction to Bayesian methods in health technology assessment, *British Medical Journal*, 319, 508–512.

Vlachos D, Georgiadis P, Iakovou E (2007) A system dynamics model for dynamic capacity planning of remanufacturing in closed-loop supply chains, *Computers & Operations Research*, 34, 2, 367–394.

Weatherly H, Drummond M, Claxton K, Cookson R, Ferguson B, Godfrey C, Rice N, Sculpher M, Sowden A (2009) Methods for assessing the cost-effectiveness of public health interventions: key challenges and recommendations, *Health Policy*, 93, 2, 85–92.