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Pascal Michaillat
Emmanuel Saez

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ABSTRACT

This paper analyzes optimal unemployment insurance (UI) over the business cycle. We consider a general matching model of the labor market. For a given UI, the economy is efficient if tightness satisfies a generalized Hosios condition, slack if tightness is too low, and tight if tightness is too high. The optimal UI formula is the sum of the standard Baily-Chetty term, which trades off search incentives and insurance, and an externality-correction term, which is positive if UI brings the economy closer to efficiency and negative otherwise. Our formula therefore deviates from the Baily-Chetty formula when the economy is inefficient and UI affects labor market tightness. In a model with rigid wages and concave production function, UI increases tightness; hence, UI should be more generous than in the Baily-Chetty formula when the economy is slack, and less generous otherwise. In contrast, in a model with linear production function and Nash bargaining, UI increases wages and reduces tightness; hence, UI should be less generous than in the Baily-Chetty formula when the economy is slack, and more generous otherwise. Deviations from the Baily-Chetty formula can be quantitatively large using realistic empirical parameters.

Camille Landais
Department of Economics
London School of Economics
Houghton Street
London WC2A 2AE
United Kingdom
c.landais@lse.ac.uk

Emmanuel Saez
Department of Economics
University of California, Berkeley
530 Evans Hall #3880
Berkeley, CA 94720
and NBER
saez@econ.berkeley.edu

Pascal Michaillat
Department of Economics
London School of Economics
Houghton Street
London, WC2A 2AE
United Kingdom
p.michaillat@lse.ac.uk

1 Introduction

Following Baily [1978], we analyze the optimal provision of unemployment insurance (UI) when individuals are risk averse, cannot insure themselves against unemployment, and job-search effort is not observable. UI helps workers smooth consumption when they become unemployed, but it also increases unemployment by discouraging job search. In Baily's model, optimal UI satisfies a simple and robust formula ensuring that the marginal benefit of smoothing consumption equal the marginal cost of increasing unemployment [Baily, 1978; Chetty, 2006a]. A limitation of Baily's model, however, is that the job-finding rate per unit of search effort does not depend on UI (it is a parameter of the model). In this paper, we allow the job-finding rate to depend on UI and generalize the Baily-Chetty formula accordingly. Our formula sheds light on the controversial debate about the optimal design of UI over the business cycle.

We begin in Section 2 by laying out a static labor market model with matching frictions and endogenous search effort. The production function and wage-determination mechanism are completely general. The job-finding rate depends on labor market tightness, which is determined in equilibrium as a function of UI. For a given UI, there is an optimal level of labor market tightness that maximizes expected welfare. We define the business cycle relative to this optimal tightness. The optimal tightness is given by a condition that generalizes the Hosios [1990] condition to a situation with risk aversion and imperfect insurance. We express this *generalized Hosios condition* in terms of estimable statistics so that the position in the cycle can be empirically evaluated. When the generalized Hosios condition holds, the marginal effect of UI on tightness has no impact on welfare and optimal UI is given by the conventional Baily-Chetty formula. Hence, the Baily-Chetty formula holds in an economy without business cycles.

In Section 3 we consider an economy with business cycles. The generalized Hosios condition may not hold and the economy may be inefficient. When the economy is inefficient, UI may also affect welfare by changing tightness. When the economy is slack, more UI is desirable if UI increases tightness and less UI is desirable if UI decreases tightness.

First, we show that the effect of UI on tightness is measured by the wedge between microelasticity and macroelasticity of unemployment with respect to UI. The microelasticity accounts only for the response of job search to UI while the macroelasticity also accounts for the response

of the job-finding rate to UI. Empirically, the microelasticity ϵ^m is the elasticity of the unemployment probability of a worker whose individual benefits change while the macroelasticity ϵ^M is the elasticity of aggregate unemployment when benefits change for all workers. When UI increases tightness, it increases the job-finding rate, which stimulates employment, dampening the negative microeffect of UI on unemployment. In that case, the macroelasticity is smaller than the microelasticity. Conversely, when UI decreases tightness, the macroelasticity is larger than the microelasticity. This property offers an empirical avenue to determine the effect of UI on tightness.

Next, we show that the optimal UI formula is a sum of the standard Baily-Chetty term and an externality-correction term. The externality-correction term is equal to the deviation from the generalized Hosios condition times the wedge between micro- and macroelasticity. The externality-correction term is positive if UI brings the economy closer to efficiency, and negative if it brings the economy further from efficiency. The externality-correction term is large (in absolute value) when the economy is far from efficiency, either very slack or very tight, and when UI has a large effect on tightness. Following the Baily-Chetty tradition, we express the optimal UI formula in terms of estimable sufficient statistics.

To illustrate the effect of UI on employment and labor market tightness, we represent the equilibrium as the intersection of a labor supply curve and a labor demand curve in an (employment, labor market tightness) plane. For a given tightness, the labor supply indicates the number of workers who find a job when they search optimally and the labor demand indicates the number of workers that firms desire to hire to maximize profits. The labor supply is upward sloping and the labor demand is horizontal or downward sloping. An increase in UI reduces search efforts and shifts the labor supply curve inward. The microelasticity measures the amplitude of the shift. The labor demand curve may also shift if wages respond to UI. The macroelasticity measures the horizontal distance between the old and the new equilibrium.

In Section 4 we derive the effect of UI on tightness in three specific matching models that illustrate the range of possibilities. First, we consider the model with rigid wage and concave production function of Michaillat [2012]. UI increases tightness so the microelasticity is larger than the macroelasticity. In our equilibrium diagram, the concavity of the production function generates a downward-sloping labor demand curve. Hence, the inward labor supply shift after an increase in UI leads to an increase in tightness along the labor demand curve. Since UI increases

tightness, optimal UI should be more generous than in the Baily-Chetty formula when the economy is slack and less generous when the economy is tight. Furthermore, the wedge between micro- and macroelasticity grows with slack. Hence, UI should be much more generous than in the Baily-Chetty formula in a very slack economy and only moderately less generous than Baily-Chetty in very tight economy. Second, we consider the model with rigid wage and linear production of Hall [2005]. Tightness is determined by the rigid wage independently of UI. As a result, UI has no impact on tightness, micro- and macroelasticities are identical, and the externality-correction term is zero. The Baily-Chetty formula therefore applies whether the economy is slack or tight. Third, we consider a conventional model with Nash bargaining over wages and linear production function. UI decreases tightness so the microelasticity is smaller than the macroelasticity. In our equilibrium diagram, the linearity of the production function generates a horizontal labor demand curve, which determines equilibrium tightness by itself. Higher UI increases the outside option of workers, which increases the Nash bargained wage and shifts the labor demand downward, reducing tightness. Since UI decreases tightness, optimal UI should be less generous than in the Baily-Chetty formula when the economy is slack, and more generous when the economy is tight.

In Section 5 we discuss existing empirical evidence on the sufficient statistics in our formula and particularly the wedge between micro- and macroelasticity, which is central to our analysis. We show how our optimal UI formula expressed in terms of estimable sufficient statistics can be implemented to provide quantitative guidance. To simplify exposition, Sections 2, 3, and 4 consider a static model. However, our formula carries over with minor modification to a dynamic model in which jobs are continuously created and destroyed. We use this extended formula for the numerical illustration. Because of lack of definitive evidence on the sign and magnitude of the wedge between micro- and macroelasticity, our illustration considers a range of scenarios. We show that the cyclical correction to the Baily-Chetty formula can be quantitatively large for plausible estimates of the sufficient statistics.

Section 6 concludes with two points. First, we emphasize that obtaining more empirical evidence on the wedge between micro- and macroelasticity is crucial to design optimal UI over the business cycle. Second, we conjecture that our model and methodology could be extended to analyse other policies over the business cycle, such as public-good spending or income taxation.

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2 Optimal UI with Perfect Stabilization

In this section we introduce the matching model on which we base our analysis. We characterize optimal UI when the government perfectly stabilizes the economy. By perfectly stabilizing the economy, we mean choosing the wage level that maximizes expected welfare. With perfect stabilization, optimal UI satisfies the conventional Baily [1978]-Chetty [2006a] formula. The optimal wage satisfies a generalization of the Hosios [1990] condition.

2.1 Matching Model

For simplicity of exposition, the model is static.[1] The economy is composed of a measure 1 of identical workers and a measure 1 of identical firms.

Labor Market. There are matching frictions on the labor market. Initially, all workers are unemployed and search for a job with effort e. Each firm posts o vacancies to recruit workers. The number l of workers who find a job is given by a matching function taking as argument aggregate search effort and vacancy: $l = m(e, o)$. The function m has constant returns to scale, is differentiable and increasing in both arguments, and satisfies the restriction that $m(e, o) \leq 1$ as the pool of potential workers has measure 1.

Labor market tightness is defined as the ratio of vacancies to aggregate search effort: $\theta \equiv o/e$. Since the matching function has constant returns to scale, labor market tightness determines the probabilities that a unit of search effort is successful and a vacancy is filled. A jobseeker finds a job at a rate $f(\theta) \equiv m(e, o)/e = m(1, \theta)$ per unit of search effort. Thus, a jobseeker searching with effort e finds a job with probability $e \cdot f(\theta)$. In the conventional Baily-Chetty model the job-finding rate per unit of effort, f, is a parameter. In our model, f varies with labor market tightness, which is determined in equilibrium as a function of UI. The property that f varies with UI is critical: it allows us to extend the Baily-Chetty analysis of optimal UI to an economy with business cycles.

A vacancy is filled with probability $q(\theta) \equiv m(e, o)/o = m(1/\theta, 1) = f(\theta)/\theta$. The function f is increasing in θ and the function q is decreasing in θ. That is, when the labor market is slacker, the

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[1]The continuous-time dynamic version of the model generates virtually the same results. It is described in the appendix and used for the quantitative exploration of Section 5.

job-finding rate per unit of effort is lower but the probability to fill a vacancy is higher. We denote by $1-\eta$ and $-\eta$ the elasticities of f and q: $1-\eta \equiv \theta \cdot f'(\theta)/f(\theta) > 0$ and $\eta \equiv -\theta \cdot q'(\theta)/q(\theta) > 0$.

Firms. The representative firm hires l workers, paid a real wage w, to produce a consumption good. As in Michaillat and Saez [2013], we assume that some workers are engaged in production while others are engaged in recruiting. A number $n < l$ of workers are producing an amount $y(n)$ of good, where the production function y is differentiable, increasing, and concave. Posting a vacancy requires a fraction $r > 0$ of a worker's time. Thus, $l - n = r \cdot o = r \cdot l/q(\theta)$ workers are recruiting a total of l workers so that $l \cdot [1 - r/q(\theta)] = n$. Hence, the numbers of workers and producers are related by

$$l = (1 + \tau(\theta)) \cdot n, \tag{1}$$

where $\tau(\theta) \equiv r/(q(\theta) - r)$ measures the number of recruiters for each producer. The function τ is positive and strictly increasing when $q(\theta) > r$, which holds in equilibrium. It is easy to show that the elasticity of τ is $\theta \cdot \tau'(\theta)/\tau(\theta) = \eta \cdot (1 + \tau(\theta))$.

The firm sells its output on a perfectly competitive market. Given θ and w, the firm chooses n to maximize profits $\pi = y(n) - (1 + \tau(\theta)) \cdot w \cdot n$. The optimal number of producers satisfies

$$y'(n) = (1 + \tau(\theta)) \cdot w. \tag{2}$$

At the optimum, the marginal revenue and marginal cost of hiring a producer are equal. The marginal revenue is the marginal product of labor, $y'(n)$. The marginal cost is the real wage, w, plus the marginal recruiting cost, $\tau(\theta) \cdot w$.

Government. The UI system provides employed workers with c^e consumption goods and unemployed workers with $c^u < c^e$ consumption goods. Job search effort is not observable, so the receipt of UI cannot be contingent on search. We introduce three measures of the generosity of UI. The UI system is more generous if the consumption gain from work $\Delta c \equiv c^e - c^u$ decreases, the utility gain from work $\Delta v \equiv v(c^e) - v(c^u)$ decreases, or the implicit tax rate on work $T \equiv 1 - (\Delta c/w)$ increases. When a jobseeker finds work, she keeps a fraction $1 - T$ of the wage and gives up a

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fraction T of the wage since UI benefits are lost. Hence, we interpret T as the replacement rate of the UI system and refer to it as such.[2] The government must satisfy the resource constraint

$$y(n) = (1 - l) \cdot c^u + l \cdot c^e. \tag{3}$$

If firms' profits π are equally distributed, the UI system can be implemented with a UI benefit b funded by a tax on wages so that $(1 - l) \cdot b = l \cdot t$ and $c^u = \pi + b$ and $c^e = \pi + w - t$. If profits are unequally distributed, a 100% tax on profits rebated lump sum implements the same allocation.

Workers. Workers cannot insure themselves against unemployment in any way, so they consume c^e and c^u. Utility from consumption is $v(c)$. The function v is twice differentiable, strictly increasing, and concave. We denote by ρ the coefficient of relative risk aversion at c^e: $\rho \equiv -c^e \cdot v''(c^e)/v'(c^e)$. Disutility from job search is $k(e)$. The function k is twice differentiable, strictly increasing, and convex.

Given θ, c^e, and c^u, a representative worker chooses effort e to maximize expected utility

$$l \cdot v(c^e) + (1 - l) \cdot v(c^u) - k(e) \tag{4}$$

subject to the matching constraint

$$l = e \cdot f(\theta), \tag{5}$$

where l is the probability to be employed and $1 - l$ is the probability to remain unemployed. The optimal search effort satisfies

$$k'(e) = f(\theta) \cdot [v(c^e) - v(c^u)]. \tag{6}$$

At the optimum, the marginal utility cost and marginal utility gain of search are equal. The marginal utility cost is $k'(e)$. The marginal utility gain from search is the probability $f(\theta)$ that

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[2]In reality, the UI system provides a benefit b funded by a tax t on work so that $\Delta c = w - t - b$ and $T = (t + b)/w$. The conventional replacement rate in the literature is b/w; it ignores the tax t and is not exactly the same as T. However, unemployment is small relative to employment so $t \ll b$ and $T \approx b/w$.

a unit of effort leads to a job times the utility gain $v(c^e) - v(c^u)$ from having a job.

Equilibrium. An equilibrium is a collection of variables $\{e, l, n, \theta, w, c^e, \Delta c\}$ such that workers maximize utility given tightness and UI, firms maximize profits given tightness and wage, and the government satisfies a resource constraint. In equilibrium, the seven variables satisfy five equations: (1), (2), (3), (5), and (6).

Since there are two more variables than equations, two variables are indeterminate. One variable is the consumption gain from work, Δc. In the rest of the paper, Δc is determined by an optimal UI formula when the government chooses the generosity of UI to maximize welfare. The other variable is the wage, w. As is well-known, the indeterminacy of w arises because of the matching frictions on the labor market.[3] We consider three different resolutions to the indeterminacy. In Section 2, the government chooses w to bring the economy to efficiency, and w is determined by an optimality condition that generalizes the Hosios condition to an environment with risk aversion and imperfect insurance.[4] In Section 3, w cannot be controlled by the government; instead, w follows a completely general wage schedule. In Section 4, we consider various models in which w follows specific wage schedules, arising for instance from bargaining.

We solve for the equilibrium taking as given the value w of the real wage and the values Δc and c^e of the UI system. Search effort is given by $e = e^s(f(\theta), \Delta c, c^e)$, where e^s is the function implicitly defined by (6). The function e^s increases with θ and Δc. The number of producers, n, is given by (1) once l and θ are known.

We determine employment, l, and labor market tightness, θ, as the intersection of a labor supply and a labor demand curve, as depicted on Figure 1(a). We define the labor supply function l^s by

$$l^s(\theta, \Delta c, c^e) = e^s(f(\theta), \Delta c, c^e) \cdot f(\theta). \tag{7}$$

The labor supply gives the number of workers who find a job when workers search optimally for a given labor market tightness and UI system. The labor supply increases with θ and with Δc. The labor supply is higher when UI is less generous because search efforts are higher. The labor supply

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[3]Hall [2005] exploits this indeterminacy to introduce rigid wages in a matching model. Michaillat and Saez [2013] discuss this indeterminacy in detail.

[4]The resulting equilibrium is only constrained efficient because the government cannot remediate the matching frictions or the moral hazard problem.

is higher when the labor market is tighter because the job-finding rate per unit of effort is higher and search efforts are higher. We define the labor demand function l^d implicitly by

$$y'\left(\frac{l^d(\theta, w)}{1 + \tau(\theta)}\right) = w \cdot (1 + \tau(\theta)). \tag{8}$$

The labor demand gives the number of workers hired by firms when firms maximize profits for a given labor market tightness and real wage.

In equilibrium, labor market tightness equalizes labor supply to labor demand:

$$l^s(\theta, \Delta c, c^e) = l^d(\theta, w). \tag{9}$$

The equilibrium is represented in Figure 1(a) in a (l, θ) plane. The labor supply curve is necessarily upward sloping, and it shifts outwards when UI falls. The labor demand curve may be horizontal or downward sloping, and it may or may not respond to UI. The intersection of the supply and demand curves gives equilibrium labor market tightness and employment. The figure also shows the composition of employment between producers and recruiters. Producers are only a fraction $1/(1 + \tau(\theta))$ of all workers, and recruiters are workers who are not producers. The share of recruiters grows with θ. Finally, unemployment is the number of labor force participants who did not find employment.[5] The representation in the (l, θ) plane will be useful to understand the key economic mechanisms behind our analysis.[6]

2.2 Microelasticity of Unemployment and Discouraged-Worker Elasticity

We measure how workers' search effort responds to a change in UI using an elasticity concept:

DEFINITION 1. The *microelasticity of unemployment with respect to UI* is

$$\epsilon^m \equiv \frac{\Delta c}{1 - l} \cdot \left.\frac{\partial l^s}{\partial \Delta c}\right|_{\theta, c^e}.$$

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[5]Michaillat and Saez [2013] provide more details about the equilibrium concept and offer a complete characterization of the equilibrium and its properties, in a model with inelastic search effort.

[6]More generally, Michaillat [2014] and Michaillat and Saez [2013] show that this equilibrium representation is useful to study the cyclical properties of a number of macroeconomic policies, including public employment or the provision of public good.

(a) Labor market equilibrium   (b) Microelasticity of unemployment
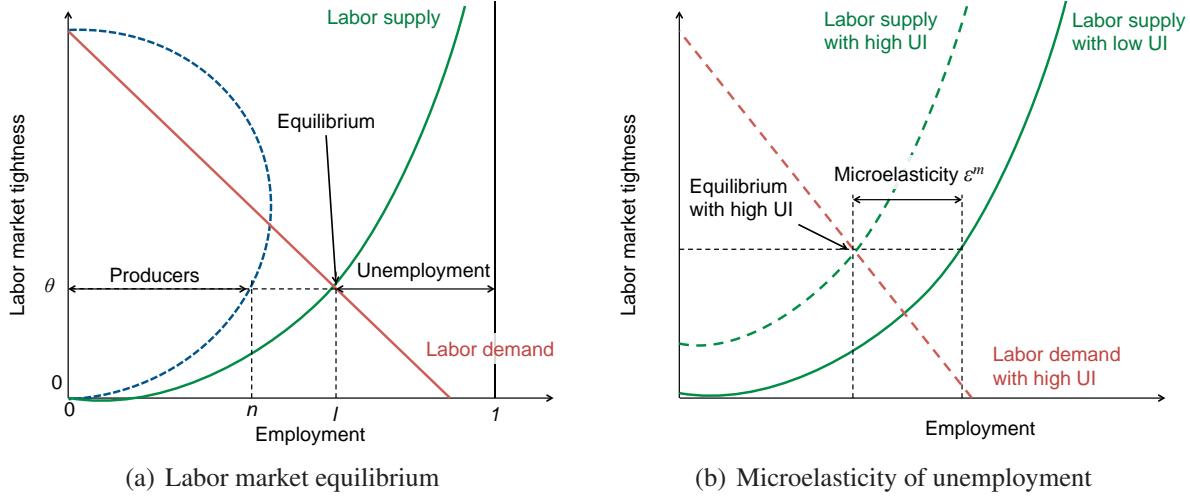
Figure 1: Equilibrium in an (employment, labor market tightness) plane

The microelasticity measures the percentage increase in unemployment when the consumption gain from work decreases by 1%, taking into account jobseekers' reduction in search effort but ignoring the equilibrium adjustment of labor market tightness. The microelasticity is computed keeping c^e constant, which means that c^e does not adjust to meet the government's budget constraint when Δc varies. The microelasticity can be estimated by measuring the reduction in the job-finding probability of an unemployed worker whose unemployment benefits are increased, keeping the benefits of all other workers constant. In Figure 1(b), the microelasticity measures the shift of the labor supply curve after a change in UI. This shift is caused by a change in search efforts following the change in UI.

In equilibrium a change in UI may also affect tightness, which affects workers' search efforts. We measure how search efforts respond to a change in labor market conditions with an elasticity:

DEFINITION 2. The *discouraged-worker elasticity* is

$$\epsilon^d \equiv \frac{f(\theta)}{e} \cdot \left. \frac{\partial e^s}{\partial f} \right|_{\Delta c, c^e}.$$

The discouraged-worker elasticity measures the percentage increase in search effort when the job-finding rate per unit of effort increases by 1%, keeping UI constant. If $\epsilon^d > 0$, workers search less when the job-finding rate decreases; hence, $\epsilon^d > 0$ captures the discouragement of jobseekers when labor market conditions deteriorate.

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The discouraged-worker elasticity is closely related to two labor supply elasticities:

LEMMA 1. *The microelasticity of unemployment with respect to UI satisfies*

$$\epsilon^m = \frac{l}{1-l} \cdot \frac{\Delta c \cdot v'(c^u)}{\Delta v} \cdot \epsilon^d.$$

Proof. Consider a infinitesimal change $d\Delta c$, keeping c^e and θ constant. This change leads to changes $d\Delta v$, dl and de. As $\Delta v = v(c^e) - v(c^u)$ and c^e is constant, we have $d\Delta v = -v'(c^u)dc^u = v'(c^u)d\Delta c$. Since the optimal search effort satisfies, $k'(e) = f \cdot \Delta v$, the elasticity of e with respect to Δv is the same as the elasticity of e with respect to f. Hence, $de = \epsilon^d \cdot (e/\Delta v) \cdot d\Delta v = \epsilon^d \cdot (e/\Delta v) \cdot v'(c^u) \cdot d\Delta c$. As $l = e \cdot f(\theta)$ and θ is constant, we have $dl = (l/e) \cdot de$. By definition, $\epsilon^m = [\Delta c/(1-l)] \cdot dl/d\Delta c$, which yields the desired result. $\square$

LEMMA 2. *The elasticity of labor supply with respect to labor market tightness satisfies*

$$\frac{\theta}{l} \cdot \frac{\partial l^s}{\partial \theta}\bigg|_{\Delta c, c^e} = (1 + \epsilon^d) \cdot (1 - \eta)$$

Proof. Obvious because $l^s(\theta, \Delta c, c^e) = e \cdot f(\theta)$, ϵ^d is the elasticity of e with respect to f, and $1 - \eta$ is the elasticity of f with respect to θ. $\square$

2.3 Formulas for Jointly Optimal UI and Labor Market Tightness

We determine the optimal policy when the government can simultaneously stabilize the economy and provide UI. By stabilizing the economy, we mean choosing the wage paid by firms to determine the level of labor demand and thus the level of employment and labor market tightness prevailing in equilibrium. As equation (9) shows that for a given UI system there is a one-to-one relationship between wage and equilibrium labor market tightness, we consider that the government can choose directly equilibrium labor market tightness. Taking this perspective greatly simplifies the analysis. The wage level that implements a level of tightness is found with (9). Assuming that the government can stabilize the economy may be unrealistic, but the results of this analysis are a useful building block for the subsequent analysis where the government cannot stabilize the economy.

The government chooses c^e, Δc, and θ to maximize social welfare (4) subject to the budget

constraint, given by (3), the matching frictions on the labor market, given by (1) and (5), and workers' optimal search choice, given by (6). Proposition 1 characterizes the optimal policy:

PROPOSITION 1. *The optimal UI policy (at constant θ) satisfies the Baily-Chetty formula*

$$\frac{w - \Delta c}{\Delta c} = \frac{l}{\epsilon^m} \cdot \left(\frac{v'(c^u)}{v'(c^e)} - 1 \right). \tag{10}$$

The optimal labor market tightness (at constant Δv) satisfies the generalized Hosios condition

$$\frac{\Delta v}{\phi \cdot w} + \left(1 - \frac{\Delta c}{w} \right) \cdot \left(1 + \epsilon^d \right) - \frac{\eta}{1 - \eta} \cdot \tau(\theta) = 0, \tag{11}$$

where ϕ satisfies

$$\frac{1}{\phi} = \left(\frac{l}{v'(c^e)} + \frac{1 - l}{v'(c^u)} \right). \tag{12}$$

Proof. We write the Lagrangian of the government's problem as

$$\mathcal{L} = \underbrace{l \cdot v(c^e) + (1 - l) \cdot v(c^u) - k(e)}_{SW} + \phi \cdot \underbrace{[y(n) - l \cdot c^e - (1 - l) \cdot c^u]}_{X},$$

where ϕ is the Lagrange multiplier on the resource constraint, SW is social welfare, and X is the net budget of the government. We first determine ϕ. Consider infinitesimal changes $dc^e = dc/v'(c^e)$ and $dc^u = dc/v'(c^u)$ keeping θ constant. The changes have no first-order impact on $\Delta v = v(c^e) - v(c^u)$ and hence no impact on e and l. The effect on welfare is $dSW = l \cdot v'(c^e) \cdot dc^e + (1 - l) \cdot v'(c^u) \cdot dc^u = dc$. The effect on the budget is $dX = -l \cdot dc^e - (1 - l) \cdot dc^u = -dc \cdot \{[l/v'(c^e)] + [(1 - l)/v'(c^u)]\}$. At the optimum $d\mathcal{L} = dSW + \phi \cdot dX = 0$, which establishes equation (12).

Next, consider an infinitesimal change $d\Delta c$ keeping c^e and θ constant. This change leads to changes dl and de. Given that $l = e \cdot f(\theta)$, the effect on welfare is $dSW = [f(\theta) \cdot \Delta v - k'(e)] \cdot de - (1 - l) \cdot v'(c^u) \cdot d\Delta c = -(1 - l) \cdot v'(c^u) \cdot d\Delta c$ because workers choose e optimally. The effect on the government's budget is $dX = \{[y'(n)/(1 + \tau(\theta))] - \Delta c\} \cdot dl + (1 - l) \cdot d\Delta c$. By definition of the microelasticity, $dl = [(1 - l)/\Delta c] \cdot \epsilon^m \cdot d\Delta c$. Using (2), we can write $w = y'(n)/(1 + \tau(\theta))$.

Thus, the first-order condition $\partial \mathcal{L}/\partial \Delta c = 0$ implies that

$$0 = -v'(c^u) \cdot (1 - l) + \phi \cdot (1 - l) + \phi \cdot (1 - l) \cdot \frac{w - \Delta c}{\Delta c} \cdot \epsilon^m.$$

The sum of the first two terms reflects the welfare effect of transferring resources from the unemployed to the employed by changing Δc. The third term captures the budgetary effect of changing employment through the response of job search to a change in Δc. Rearranging the terms and dividing by $\phi \cdot (1 - l) \cdot \epsilon^m$ yields

$$\frac{w - \Delta c}{\Delta c} = \frac{1}{\epsilon^m} \cdot \left[\frac{v'(c^u)}{\phi} - 1 \right].$$

Equation (12) implies that

$$\frac{v'(c^u)}{\phi} - 1 = l \cdot \left[\frac{v'(c^u)}{v'(c^e)} - 1 \right]. \tag{13}$$

Combining the last two equations yields the Baily-Chetty formula.

Last, consider an infinitesimal change $d\theta$ keeping c^e and Δc constant. This change leads to changes dl, df, and de. The effect on welfare is $dSW = \Delta v \cdot (l/f) \cdot df$ because workers choose e optimally. By definition, $df = (1 - \eta) \cdot (f/\theta) \cdot d\theta$ and $dSW = \Delta v \cdot (1 - \eta) \cdot (l/\theta) \cdot d\theta$. The effect on the budget is $dX = \{[y'(n)/(1 + \tau(\theta))] - \Delta c\} \cdot dl - [y'(n)/(1 + \tau(\theta))] \cdot n \cdot d\tau$. By definition, $d\tau = \eta \cdot (1 + \tau(\theta)) \cdot (\tau(\theta)/\theta) \cdot d\theta$. Given $l = l^s(\theta, \Delta c, c^e)$, Lemma 2 implies that $dl = (1 + \epsilon^d) \cdot (1 - \eta) \cdot (l/\theta) \cdot d\theta$. We rewrite $dX = (w - \Delta c)(1 + \epsilon^d) \cdot (1 - \eta) \cdot (l/\theta) \cdot d\theta - w \cdot l \cdot \eta \cdot (\tau(\theta)/\theta) \cdot d\theta$. Thus, the first-order condition $\partial \mathcal{L}/\partial \Delta c = 0$ implies

$$0 = \frac{l}{\theta} \cdot (1 - \eta) \cdot \Delta v + \phi \cdot \frac{l}{\theta} \cdot (1 - \eta) \cdot (1 + \epsilon^d) \cdot (w - \Delta c) - \phi \cdot \frac{l}{\theta} \cdot w \cdot \eta \cdot \tau(\theta).$$

The first term, proportional to Δv, is the welfare gain from increasing employment by increasing tightness. The welfare effect includes solely the job-finding rate channel and ignores the employment effect due to a change in effort as e maximizes individual expected utility (a standard envelope theorem argument). The second term, proportional to $w - \Delta c$, captures the budgetary gain from increasing employment by increasing tightness. Each new job created increases government revenue

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by $w - \Delta c$. For the second term, the increase in employment results both from a higher job-finding rate and from a higher search effort as no envelope theorem applies in that case. The term $(1 + \epsilon^d)$ captures the combination of the two effects. The third term is the loss in resources $y(n)$ due to a higher tightness. Increasing tightness requires to devote a larger share of the workforce to recruiting and a smaller share to producing. Dividing the expression by $\phi \cdot w \cdot (l/\theta) \cdot (1 - \eta)$ yields the generalized Hosios condition. $\qquad\square$

Three points are worth making about Proposition 1. First, the Baily-Chetty formula gives the optimal budget-balanced UI system when labor market tightness is fixed, regardless of whether tightness is optimal or not. It captures the trade-off between the need for insurance, measured by $v'(c^u)/v'(c^e) - 1$, and the need for incentives to search, captured by ϵ^m, exactly as in the analysis of Baily [1978] and Chetty [2006a] with fixed job-finding rate per unit of search effort.

Second, the generalized Hosios condition determines the optimal labor market tightness, θ, for a given UI system, c^e and Δc. More precisely, it provides the value of θ that maximizes welfare assuming that c^e and Δc adjust to maintain budget balance while keeping Δv constant.[7] If workers are risk neutral, the Baily-Chetty formula implies that it is optimal to set $w = \Delta c$ and provide no UI. In that case, $\Delta v / (\phi \cdot w) = 1$ and the generalized Hosios condition simplifies to

$$\frac{\eta}{1 - \eta} \cdot \tau(\theta) = 1.$$

This condition is another formulation of the standard condition of Hosios [1990] for efficiency in a matching model with risk-neutral workers. With risk aversion and UI, formula (11) generalizes the Hosios condition. Assuming $1 - l \ll l \approx 1$ and using a second-order expansion for v yields

$$\frac{\eta}{1 - \eta} \cdot \tau(\theta) = \frac{1}{\phi} \cdot \frac{\Delta v}{w} + \left(1 - \frac{\Delta c}{w}\right) \cdot (1 + \epsilon^d) \approx 1 + \epsilon^d \cdot \left(1 - \frac{\Delta c}{w}\right) + \frac{\rho}{2} \cdot \frac{\Delta c}{w} \cdot \frac{\Delta c}{c^e},$$

As $\tau(\theta)$ is increasing in θ and $\epsilon^d \geq 0$, the presence of insurance and risk aversion raises the optimal θ relative to the Hosios condition.

Third and most important, when the government controls perfectly both UI and labor market

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[7]To see this, suppose the government changes θ by $d\theta$ while changing c^e and c^u by $dc/v'(c^e)$ and $dc/v'(c^u)$ to keep budget balance and keep Δv constant. Computations paralleling the proof of Proposition 1 show that $dX = 0$ requires $dc = \phi \cdot (w - \Delta c) \cdot (1 + \epsilon^d) \cdot e \cdot df - \phi \cdot w \cdot \tau(\theta) \cdot [\eta/(1 - \eta)] \cdot e \cdot df$. Furthermore, $dSW = dc + e \cdot \Delta v \cdot df$. Imposing $dSW = 0$ yields (11).

tightness, the conventional UI trade-off between insurance and search incentives is disconnected from the efficiency concerns of stabilization. Hence, the Baily-Chetty formula holds in a matching model only when the government can perfectly stabilize the economy by optimally choosing tightness. This assumption provides a useful benchmark but may not be realistic as economies seem to be subject to inefficient cycles that cannot be perfectly stabilized.

3 Optimal UI with Business Cycles

In this section, we characterize optimal UI under the assumption that the government cannot stabilize the economy. The wage is not chosen by the government but is determined by a completely general wage schedule. As a result, labor market tightness is determined endogenously to equalize labor supply and labor demand. The generalized Hosios condition does not necessarily hold, and departures from the condition have important implications for optimal UI.

Let H be the departure from the generalized Hosios condition: $H \equiv (1/\phi) \cdot (\Delta v/w) + (1 - (\Delta c/w)) \cdot (1 + \epsilon^d) - [\eta/(1 - \eta)] \cdot \tau(\theta)$, where ϕ satisfies (12). H has the same sign as the derivative of the Lagrangian of the government's problem with respect to tightness, adjusting c^e and Δc to maintain budget balance and keep Δv constant. The economy can be in three regimes:

DEFINITION 3. The economy is *slack* if a marginal increase in tightness increases welfare, *tight* if it decreases welfare, and *efficient* if it has no effect on welfare. Equivalently, slackness and tightness can be measured by the departure from the generalized Hosios condition. The economy is slack if $H > 0$, efficient if $H = 0$, and tight if $H < 0$.

We interpret business cycles as a succession of slack and tight episodes. According to our definition, business cycles are necessarily inefficient. This definition of business cycles is unconventional but is very useful: optimal UI is systematically different in the three regimes.

3.1 Macroelasticity of Unemployment and Elasticity Wedge

We assume that the wage is determined by a completely general wage schedule. The wage schedule may depend on UI, and the response of the wage to UI has important implications for optimal UI because it determines how the labor demand respond to UI. However, we do not need to give

an explicit expression for the wage schedule. The only information we need is the response of equilibrium employment to UI, which is measured by the following elasticity:

DEFINITION 4. The *macroelasticity of unemployment with respect to UI* is

$$\epsilon^M \equiv \frac{\Delta c}{1 - l} \cdot \left.\frac{\partial l}{\partial \Delta c}\right|_{c^e}.$$

The macroelasticity measures the percentage increase in unemployment when the consumption gain from work decreases by 1%, taking into account jobseekers' reduction in search effort and the equilibrium adjustment of labor market tightness. The macroelasticity is computed keeping c^e constant, which means that c^e does not adjust to meet the budget constraint of the government when Δc varies. The macroelasticity can be estimated by measuring the increase in aggregate unemployment following a general increase in unemployment benefits financed by deficit spending.

Microelasticity and macroelasticity are not necessarily the same. A wedge between microelasticity and macroelasticity appears when labor market tightness responds to UI:

DEFINITION 5. The *elasticity wedge* is $1 - (\epsilon^M/\epsilon^m)$.

PROPOSITION 2. *The elasticity wedge depends on the response of labor market tightness to UI:*

$$1 - \frac{\epsilon^M}{\epsilon^m} = -(1 - \eta) \cdot \frac{1 + \epsilon^d}{\epsilon^d} \cdot \frac{\Delta v}{v'(c^u) \cdot \Delta c} \cdot \frac{\Delta c}{\theta} \cdot \left.\frac{\partial \theta}{\partial \Delta c}\right|_{c^e}.$$

The elasticity wedge is positive if and only if labor market tightness increases with the generosity of UI. If labor market tightness does not depend on UI, the elasticity wedge is zero.

Proof. Consider an infinitesimal change $d\Delta c$, keeping c^e constant. This change leads to variations dl and $d\theta$. We write dl in two different ways. First, the definition of the macroelasticity implies that $dl = \epsilon^M \cdot [(1 - l)/\Delta c] \cdot d\Delta c$. This variation corresponds to a movement from A to B in Figure 2. Second, the equilibrium condition $l = l^s(\theta, \Delta c, c^e)$ implies that we can decompose the variation in employment as $dl = dl_\theta + dl_{\Delta c}$. The variation $dl_{\Delta c} = (\partial l^s/\partial \Delta c) \cdot d\Delta c$ is the variation keeping θ constant. It corresponds to a movement from A to C in Figure 2. The definition of the microelasticity implies that $dl_{\Delta c} = [(1 - l)/\Delta c] \cdot \epsilon^m \cdot d\Delta c$. The variation $dl_\theta = (\partial l^s/\partial \theta) \cdot d\Delta c$ is the additional variation through the change in θ. It corresponds to a movement from C to B in

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Figure 2. Lemma 2 implies that $dl_\theta = (1 + \epsilon^d) \cdot (1 - \eta) \cdot (l/\theta) \cdot d\theta$. Multiplying the two expressions for dl by $\Delta c / [(1 - l) \cdot d\Delta c]$, we obtain

$$\epsilon^M = \epsilon^m + \frac{l}{1 - l} \cdot \left(1 + \epsilon^d\right) \cdot (1 - \eta) \cdot \frac{\Delta c}{\theta} \cdot \left. \frac{\partial \theta}{\partial \Delta c} \right|_{c^e}. \tag{14}$$

Combining this equation with the expression for ϵ^m in Lemma 1 yields the desired result. $\qquad\square$

Optimal UI depends on the response of θ to UI when the government cannot control θ. This proposition is therefore important for our analysis because it shows that the response of θ to UI can be captured by the wedge between micro- and macroelasticity. The result of Proposition 2 is illustrated in Figure 2. In Figures 2(a) and 2(b), the horizontal distance A–B measures the macroelasticity and the horizontal distance A–C measures the microelasticity. In Figure 2(a), the labor demand curve is downward sloping, and it does not shift with a change in UI because w does not respond to UI. Labor market tightness falls after a reduction in UI along the demand curve. We call this decrease in tightness a *labor-demand externality*. Because of the labor-demand externality, the macroelasticity is smaller than the microelasticity. In Figure 2(b), the labor demand shifts with a change in UI because w increases with UI. Labor market tightness first falls along the old demand curve and then increases along the new supply curve. We call a *wage externality* this increase in tightness due to the shift in labor demand. In net, tightness can either increase or decrease. In Figure 2(b), the wage externality dominates and labor market tightness increases. Thus, the macroelasticity is larger than the microelasticity.

3.2   Formula for Optimal UI

The government chooses c^e and Δc to maximize social welfare, given by (4), subject to the budget constraint, given by (3), the matching frictions on the labor market, given by (5), and workers' optimal choice of search effort, given by (6), and the equilibrium condition on the labor market, given by (9). The following proposition characterizes the optimal policy:

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(a) No response of labor demand   (b) Large response of labor demand
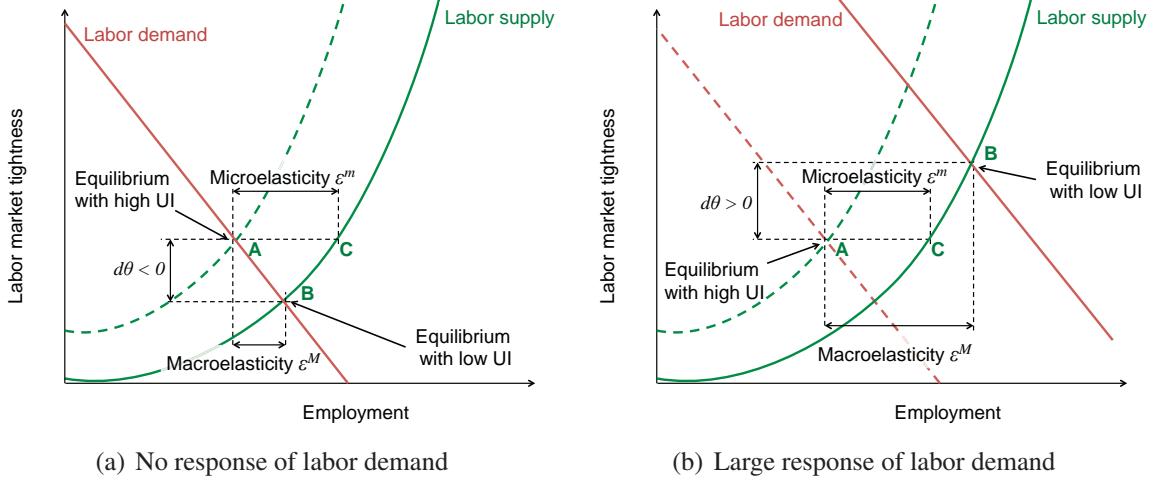
Figure 2: Microelasticity and macroelasticity for different responses of labor demand to UI

PROPOSITION 3. *The optimal UI policy satisfies the formula*

$$
\frac{w - \Delta c}{\Delta c} = \frac{l}{\epsilon^m} \cdot \left(\frac{v'(c^u)}{v'(c^e)} - 1 \right)
$$

$$
+ \frac{1}{1 + \epsilon^d} \cdot \frac{w}{\Delta c} \cdot \left(1 - \frac{\epsilon^M}{\epsilon^m} \right) \cdot \left[\frac{\Delta v}{w \cdot \phi} + \left(1 - \frac{\Delta c}{w} \right) \cdot \left(1 + \epsilon^d \right) - \frac{\eta}{1 - \eta} \cdot \tau(\theta) \right], \quad (15)
$$

where ϕ satisfies (12)*. The first term in the right-hand side is the* Baily-Chetty term*, and the second term is the* externality-correction term*.*

Proof. Let $\mathcal{L}$ be the Lagrangian of the government's problem and ϕ the Lagrange multiplier on the budget constraint. Following the same reasoning as in the proof of Proposition 1, we can show that ϕ satisfies equation (12).

Next, we consider a infinitesimal change $d\Delta c$, keeping c^e constant. We decompose the variation in the Lagrangian as $\partial \mathcal{L}/\partial \Delta c\big|_{c^e} = \partial \mathcal{L}/\partial \Delta c\big|_{\theta,c^e} + \partial \mathcal{L}/\partial \theta\big|_{\Delta c,c^e} \cdot \partial \theta/\partial \Delta c\big|_{c^e}$. Therefore, the first-order condition $d\mathcal{L} = 0$ in the current problem is a linear combination of the first-order conditions $\partial \mathcal{L}/\partial \Delta c\big|_{\theta,c^e} = 0$ and $\partial \mathcal{L}/\partial \theta\big|_{\Delta c,c^e} = 0$ in the joint optimization problem of Proposition 1. Hence, the optimal formula is also a linear combination of the Baily-Chetty formula and the generalized Hosios condition. Moreover, the generalized Hosios condition is multiplied by the wedge $1 - (\epsilon^M/\epsilon^m)$ because the factor $\partial \theta/\partial \Delta c\big|_{c^e}$ is proportional to that wedge.

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Table 1: Prediction of the optimal UI formula compared to the Baily-Chetty formula

	(1)	(2)	(3)
Elasticity wedge, $1 - (\epsilon^M/\epsilon^m)$	positive	zero	negative
Effect of UI on tightness	positive	zero	negative
Regime	Optimal UI replacement rate compared to Baily-Chetty		
Slack	higher	same	lower
Efficient	same	same	same
Tight	lower	same	higher
Cyclicality of optimal UI	countercyclical	acyclical	procyclical

Notes: The UI replacement rate is $T = 1 - (\Delta c/w)$. This table summarizes the results of Proposition 3. The table reports whether optimal UI should be above, at, or below the Baily-Chetty formula (10) depending on the position of the economy in the cycle (slack, efficient, or tight) and the sign of the elasticity wedge. Optimal UI is defined as being higher than Baily-Chetty if the externality-correction term in formula (15) is positive. Cyclicality of optimal UI refers to the cyclicality of optimal UI relative to the Baily-Chetty formula.

More precisely, we showed in the proof of Proposition 1 that

$$\left.\frac{\partial \mathcal{L}}{\partial \Delta c}\right|_{\theta,c^e} = \epsilon^m \cdot \phi \cdot (1-l) \cdot \left[\frac{w - \Delta c}{\Delta c} - \frac{l}{\epsilon^m} \cdot \left(\frac{v'(c^u)}{v'(c^e)} - 1 \right) \right]$$

$$\left.\frac{\partial \mathcal{L}}{\partial \theta}\right|_{\Delta c,c^e} = \phi \cdot l \cdot (1-\eta) \cdot \frac{w}{\theta} \cdot \left[\frac{\Delta v}{\phi \cdot w} + \left(1 - \frac{\Delta c}{w} \right) \cdot (1 + \epsilon^d) - \frac{\eta}{1-\eta} \cdot \tau(\theta) \right].$$

In addition, equation (14) shows that the labor market tightness variation is given by

$$\left.\frac{\partial \theta}{\partial \Delta c}\right|_{c^e} = \frac{1-l}{l} \cdot \frac{1}{1+\epsilon^d} \cdot \frac{1}{1-\eta} \cdot \frac{\theta}{\Delta c} \cdot \left(\epsilon^M - \epsilon^m \right).$$

Combining these equations to write $\left.\partial \mathcal{L}/\partial \Delta c\right|_{c^e} = 0$ and dividing the result by $\epsilon^m \cdot \phi \cdot (1-l)$ yields the desired formula. □

Formula (15) applies to a broad range of matching models (see Section 4). The formula is expressed with estimable sufficient statistics so that it can be combined with empirical estimates to evaluate optimal UI (see Section 5). As is standard in optimal taxation, the right-hand-side is endogenous to UI. Hence, one needs to make assumptions on how each term varies with UI when implementing the formula. Alternatively, the formula can be used to assess the desirability of a

small reform around the current system : if the left-hand-side term is lower than the right-hand-side evaluated in the current system then increasing UI increases social welfare, and conversely.

Formula (15) shows that optimal UI, measured by $(w-\Delta c)/\Delta c = T/(1-T)$ where T is the UI replacement rate, is the sum of an insurance term, which is the term in the Baily-Chetty formula, and an externality-correction term, which is proportional to the deviation from the generalized Hosios condition. This form of additive decomposition between a standard term and a corrective term is well-known in the literature on optimal taxation in the presence of externalities.[8] When the generalized Hosios condition holds, the externality-correction term vanishes and the optimal replacement rate is given by the Baily-Chetty formula.

Formula (15) generates an optimal UI higher, equal, or lower than the Baily-Chetty formula if the product of the deviation from the generalized Hosios condition and the elasticity wedge, $1 - (\epsilon^M/\epsilon^m)$, is positive, zero, or negative. Intuitively, increasing UI above Baily-Chetty is desirable if UI brings the economy closer to efficiency.

Table 1 shows the nine possibilities depending on the sign of the deviation from the generalized Hosios condition and the sign of the elasticity wedge. Section 4 shows that different models predict different signs for the elasticity wedge and hence different cyclicality for optimal UI. Optimal UI is countercyclical relative to Baily-Chetty if the elasticity wedge is positive and procyclical if the elasticity wedge is negative.

It is the combination of a low ϵ^M relative to ϵ^m and a low tightness relative to efficiency that makes more UI desirable. A low ϵ^M alone is not sufficient. Consider a model in which the number of jobs is fixed: $\epsilon^M = 0$. Increasing UI redistributes from the employed to the unemployed without destroying jobs, but, unlike what intuition suggests, full insurance is not desirable. This can be seen by plugging $\epsilon^M = 0$ and $\epsilon^m > 0$ in (15). The reasons is that increasing UI increases tightness and forces firms to allocate more workers to recruiting instead of producing, thus reducing output available to consumption. In fact if the generalized Hosios condition holds, UI is given by the standard Baily formula and the magnitude of ϵ^M is irrelevant.

Formula (15) also justifies the public provision of UI even in the presence of private provision of UI. Small private insurers would solely take into account the microelasticity of unemployment.

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[8]See for instance the general framework laid out in Farhi and Werning [2013] to design optimal macroeconomic policies in the presence of price rigidities.

Table 2: Optimal UI in various matching models

Model	(1) Section 4.1	(2) Section 4.2	(3) Section 4.3
	Assumptions		
Wage schedule	rigid	rigid	Nash bargaining
Production function	concave	linear	linear
	Macroeconomic effect of UI		
Elasticity wedge, $1 - (\epsilon^M/\epsilon^m)$	positive	zero	negative
Effect of UI on tightness	positive	zero	negative
Cyclicality of optimal UI	countercyclical	acyclical	procyclical
Cyclicality of elasticity wedge	countercyclical	acyclical	acyclical

Notes: The table summarizes the results for the three models of Section 4 (across columns). The top panel presents the assumptions of each model on wage determination and production function. The bottom panel summarizes the effects of UI in each model. Cyclicality of the elasticity wedge refers to how it changes with tightness. Cyclicality of optimal UI refers to how optimal UI deviates from Baily-Chetty when the economy is slack or tight. Countercyclical optimal UI means that optimal UI is above the Baily-Chetty formula in a slack economy and below in a tight economy.

They would offer insurance according to the Baily-Chetty formula and would not internalize tightness externalities. Hence, it would be optimal for the government to provide the cyclical component of UI equal to the second term in (15). Related, even in the absence of any concern for insurance (if workers are risk neutral or already perfectly insured), positive UI should be offered if UI brings the economy closer to the generalized Hosios condition.

4   Predictions of Specific Matching Models

Section 3 shows that the Baily-Chetty formula, conventionally used to calibrate optimal UI, may underestimate or overestimate the optimal level of UI depending on the state of the business cycle and the sign of the elasticity wedge, $1 - (\epsilon^M/\epsilon^m)$. The sign of the wedge matters because it determines the effect of UI on tightness. This section determines the sign of the wedge in specific matching models. The wedge can be positive, zero, or negative depending on the model.

4.1 A Model With Rigid Wage and Concave Production Function

We analyze a matching model that shares the main features of the model of Michaillat [2012]. The model makes the following assumptions on the production function and the wage:

ASSUMPTION 1. The production function is concave: $y(n) = n^\alpha$ with $\alpha \in (0, 1)$.

ASSUMPTION 2. The wage is a parameter of the model: $w \in (0, +\infty)$.

Figures 3(a) and 3(b) represent the equilibrium in this model. Under Assumption 1, the labor demand function l^d is strictly decreasing with θ and with w. The labor demand decreases with θ because when the labor market is tighter, hiring workers is less profitable as it requires more recruiters. As a consequence, the labor demand curve is downward sloping in the (l, θ) plane, and the labor demand curve shifts inward when the wage increases. Figure 3(a) plots the labor demand curve for a low wage and Figure 3(b) plots it for a high wage, thus illustrating the effect of the wage on the labor demand. We interpret a change in real wage as an aggregate activity shock that could arise from a technology shock or an aggregate demand shock.[9] Under Assumption 2, the wage does not respond to UI so the labor demand curve does not shift after a change in UI. The labor supply curve represents $l^s(\theta, \Delta c, c^e)$, which is defined by (7). Unlike the labor demand curve, the labor supply curve does shift after a change in UI. Figure 3(a) and Figure 3(b) plot one labor supply curve for high UI (dotted line) and one labor supply curve for low UI (solid line), thus illustrating the effect of UI on labor supply.

Proposition 4 characterizes the response of tightness to a change in UI and the elasticity wedge:

PROPOSITION 4. *Under Assumptions 1 and 2, $\partial\theta/\partial\Delta c\big|_{c^e} < 0$ and $1 - (\epsilon^M/\epsilon^m) > 0$. More precisely,*

$$1 - \frac{\epsilon^M}{\epsilon^m} = \left[1 + \frac{\eta}{1-\eta} \cdot \frac{\alpha}{1-\alpha} \cdot \frac{1}{1+\epsilon^d} \cdot \tau(\theta)\right]^{-1}.$$

Proof. Consider an infinitesimal change $d\Delta c$, keeping c^e constant. This change leads to variations dl and $d\theta$. We can express $d\theta$ by moving along the labor demand since $l = l^d(\theta, w)$ in equilibrium.

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The labor demand is implicitly defined by (8). Under Assumptions 1 and 2, the labor demand is

$$l^d(\theta, w) = w^{\frac{-1}{1-\alpha}} \cdot (1 + \tau(\theta))^{\frac{-\alpha}{1-\alpha}}.$$

Since the elasticity of $1 + \tau(\theta)$ with respect to θ is $\eta \cdot \tau(\theta)$, the elasticity of l^d with respect to θ is $[-\alpha/(1-\alpha)] \cdot \eta \cdot \tau(\theta)$. Using the definition of the macroelasticity, $dl = \epsilon^M \cdot [(1-l)/\Delta c] \cdot d\Delta c$, we therefore obtain

$$d\theta = -\frac{1-\alpha}{\alpha} \cdot \frac{1}{\eta} \cdot \frac{1}{\tau(\theta)} \cdot \frac{1-l}{l} \cdot \frac{\theta}{\Delta c} \cdot \epsilon^M \cdot d\Delta c.$$

This expression reflects a movement from A to C in Figure 3(a).

As in Proposition 2, we can also express $d\theta$ as a shift of labor supply followed by a movement along the labor supply curve. This expression reflects a movement from A to B and from B to C in Figure 3(a). Equation (14) implies that

$$d\theta = \frac{\theta}{\Delta c} \cdot \frac{1}{1 + \epsilon^d} \cdot \frac{1}{1 - \eta} \cdot \frac{1-l}{l} \cdot \left(\epsilon^M - \epsilon^m\right) \cdot d\Delta c.$$

Combining these results yields

$$\epsilon^M = \frac{\alpha}{1-\alpha} \cdot \frac{1}{1 + \epsilon^d} \cdot \frac{\eta}{1 - \eta} \cdot \tau(\theta) \cdot \left(\epsilon^m - \epsilon^M\right).$$

Dividing this expression by ϵ^m and re-arranging yields the desired result. □

In this model the macroelasticity is smaller than the microelasticity because cutting UI reduces labor market tightness. As discussed above, this reduction in tightness is a labor-demand externality. Figure 3(a) illustrates the externality. After a reduction in UI, jobseekers search more, shifting the labor supply curve outward by a distance A–C, which represents the microelasticity. But the job-finding rate per unit of search effort, $f(\theta)$, does not remain constant. If it did, labor market tightness, θ, and marginal recruiting cost, $\tau(\theta)$, would remain constant. As the wage remains constant, the marginal cost of labor would remain constant. Simultaneously, firms need to absorb the additional workers who find a job and who have lower productivity because the production function has diminishing marginal returns to labor. Firms would face the same marginal cost of labor but a

(a) Low rigid wage and concave production function (b) High rigid wage and concave production function

(c) Rigid wage and linear production function (d) Nash bargaining and linear production function
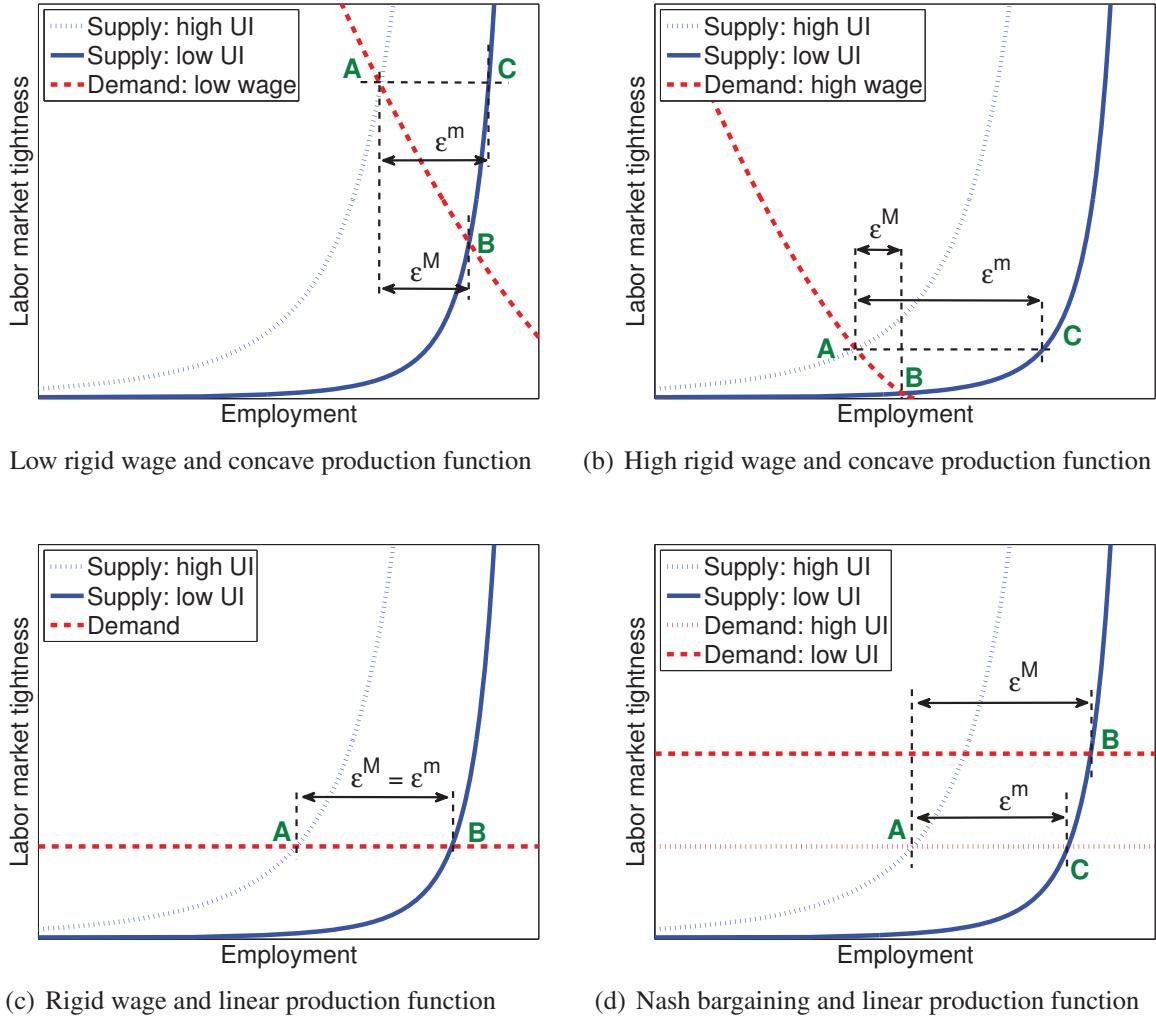
Figure 3: Equilibrium of the three matching models of Section 4

lower marginal product of labor, which would not be optimal. Thus, the new equilibrium has lower labor market tightness and lower job-finding rate. A lower job-finding rate mechanically reduces the number of new hires and also leads jobseekers to search less. The corresponding reduction in employment is the distance C–B. The increase in equilibrium employment is the distance A–B, which represents the macroelasticity. Since A–B is smaller than A–C, macroelasticity is smaller than microelasticity.

As depicted in Table 2, column (1), Proposition 4 implies that optimal UI should be counter-cyclical relative to Baily-Chetty because the elasticity wedge $1 - (\epsilon^M / \epsilon^m)$ is positive. Optimal UI is higher than Baily-Chetty in a slack economy and lower than Baily-Chetty in a tight economy.

To explore further how the externality-correction term varies over the cycle, we establish the cyclicality of the elasticity wedge under isoelasticity assumptions:

ASSUMPTION 3. The matching function and search cost function are isoelastic: $h(e, v) = \omega_h \cdot e^\eta \cdot v^{1-\eta}$ and $k(e) = \omega_k \cdot e^{\kappa+1}$ for $\eta \in (0, 1)$, $\kappa > 0$, $\omega_h > 0$, and $\omega_k > 0$.

PROPOSITION 5. *Under Assumptions 1, 2, and 3, the elasticity wedge is countercyclical. An increase in wage decreases tightness and increases the wedge:* $\partial \left[1 - (\epsilon^M/\epsilon^m) \right] /\partial w \big|_{\Delta v} > 0$ *and* $\partial \theta/\partial w \big|_{\Delta v} < 0.$

Proof. Under Assumption 3, the optimal search choice implies that $\omega_k \cdot (1 + \kappa) \cdot e^\kappa = f \cdot \Delta v$ so $\epsilon^d = 1/\kappa$. The cyclicality of ϵ^M/ϵ^m follows from the expression in Proposition 4 and the fact that the function τ is increasing with θ. □

Proposition 5 describes comparative statics with respect to the real wage, w, keeping the UI system, Δv, constant. An increase in real wage represents a negative aggregate activity shock. The proposition shows that the elasticity wedge is countercyclical so that the externality-correction term is higher in a slack economy than in a tight economy for a deviation from the generalized Hosios condition of a given amplitude. Hence, we expect the cyclical correction for optimal UI to be larger in a slack economy than in a tight economy.

This result is illustrated by comparing the low-wage equilibrium in Figure 3(a) to the high-wage equilibrium in Figure 3(b). Clearly, the wedge between ϵ^M and ϵ^m is driven by the slope of the labor supply curve relative to the slope of the labor demand curve. In the low-wage equilibrium, the labor supply is steep at the equilibrium point because the matching process is congested by the large number of vacancies. Hence, ϵ^M is close to ϵ^m. Conversely, in the high-wage equilibrium, the labor supply is flat at the equilibrium point because the matching process is congested by search efforts. Hence, ϵ^M is much lower than ϵ^m. Formally, let $\epsilon^{ls} \equiv (\theta/l) \cdot (\partial l^s/\partial \theta)$ be the elasticity of labor supply with respect to tightness and $\epsilon^{ld} \equiv -(\theta/l) \cdot (\partial l^d/\partial \theta)$ be the elasticity of labor demand with respect to tightness (normalized to be positive). We could rewrite the elasticity wedge as $1 - (\epsilon^M/\epsilon^m) = 1/ \left[1 + (\epsilon^{ld}/\epsilon^{ls}) \right]$. The elasticity wedge is countercyclical because $\epsilon^{ld}/\epsilon^{ls}$ is procyclical.[10]

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[10]The result of Proposition 5 is closely connected to the result on the cyclicality of the public-employment multiplier in Michaillat [2014] as both results rely on the cyclicality of the ratio $\epsilon^{ld}/\epsilon^{ls}$. In a matching model with rigid wage

4.2 A Model With Rigid Wage and Linear Production Function

We analyze a model that shares the main features of the model of Hall [2005]. The model assumes a rigid wage (Assumption 2) and makes the following assumption on the production function:

ASSUMPTION 4. The production function is linear: $y(n) = n$.

In this model, UI has no effect on tightness and micro- and macroelasticity are equal:

PROPOSITION 6. *Under Assumptions 4 and 2, $\partial\theta/\partial\Delta c\big|_{c^e} = 0$ and $\epsilon^M = \epsilon^m$.*

This result is illustrated in Figure 3(c). It arises because labor demand is perfectly elastic and independent of UI, such that equilibrium tightness is independent of UI. There is no labor-demand externality because the labor demand is perfectly elastic. There is no wage externality either because the wage does not respond to UI. Table 2, column (2), shows that optimal UI is always given by the Baily-Chetty formula. Tightness may be inefficient in this model but this inefficiency does not affect optimal UI because UI has no effect on tightness. In other words, the generalized Hosios condition may not hold but the externality-correction term is always zero because UI cannot affect tightness.

4.3 A Model With Nash Bargaining and Linear Production Function

We analyze a matching model that shares the main features of the model of Pissarides [2000, Chapter 1]. The model assumes a linear production function (Assumption 4) and makes the following assumption on the wage-setting mechanism:

ASSUMPTION 5. The wage is determined using the generalized Nash solution to the bargaining problem faced by firm-worker pairs. The bargaining power of workers is $\beta \in (0, 1)$.

We begin by determining the equilibrium wage. The worker's surplus from a match with a firm is $\mathcal{W} = \Delta v$. We assume that the government uses a lump-sum tax to finance the UI system such that an increase dw in the bargained wage raises the worker's post-tax income by dw and

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and concave production function, Michaillat [2014] shows that the public-employment multiplier λ, defined as the additional number of workers employed when one more worker is employed in the public sector, is countercyclical. The public-employment multiplier satisfies $\lambda = 1/\left[1 + (\epsilon^{ld}/\epsilon^{ls})\right]$ and it is countercyclical because $\epsilon^{ld}/\epsilon^{ls}$ is procyclical.

leads to a utility gain $\mathcal{W}'(w)dw = v'(c^e)dw.$[11] The firm's surplus from a match with a worker is $\mathcal{F} = 1 - w$ because once a worker is recruited, she produces 1 unit of good and receives a real wage w. Accordingly, $\mathcal{F}'(w) = -1$. The generalized Nash solution to the bargaining problem is the wage w that maximizes $\mathcal{W}(w)^\beta \cdot \mathcal{F}(w)^{1-\beta}$. The first-order condition of the maximization problem implies that the worker's surplus each period is related to the firm's surplus by

$$\beta \cdot \frac{\mathcal{W}'(w)}{\mathcal{W}(w)} + (1 - \beta) \cdot \frac{\mathcal{F}'(w)}{\mathcal{F}(w)} = 0.$$

Using $\mathcal{W}'(w)/\mathcal{W}(w) = v'(c^e)/\Delta v$ and $\mathcal{F}'(w)/\mathcal{F}(w) = -1/(1-w)$, we obtain

$$w = 1 - \frac{1-\beta}{\beta} \cdot \frac{\Delta v}{v'(c^e)}. \tag{16}$$

After a cut in UI benefits, which increases Δv but does not affect c^e, the bargained wage decreases. The reason is that the outside option of jobseekers decreases after the cut in benefits, so they are forced to settle for a lower wage during bargaining.

With a linear production function, equation (2) imposes that $w \cdot (1 + \tau(\theta)) = 1$, which can be interpreted as a free-entry condition. Combining this condition with the expression for the wage yields

$$\frac{\tau(\theta)}{1 + \tau(\theta)} = \frac{1-\beta}{\beta} \cdot \frac{\Delta v}{v'(c^e)}. \tag{17}$$

Figure 3(d) depicts the equilibrium of this model in a (l, θ) plane. Equation (17) defines a perfectly elastic labor demand, which determines equilibrium tightness. Equilibrium employment is read off the labor supply curve. The following proposition characterizes the response of labor market tightness to a change in UI and the elasticity wedge:

PROPOSITION 7. *Under Assumptions 4 and 5, $\partial\theta/\partial\Delta c\big|_{c^e} > 0$ and $1 - (\epsilon^M/\epsilon^m) < 0$. More precisely,*

$$1 - \frac{\epsilon^M}{\epsilon^m} = -\frac{1-\eta}{\eta} \cdot \frac{1+\epsilon^d}{\epsilon^d}.$$

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[11]If the government used a linear income tax to finance UI, $\mathcal{W}'(w)$ would also depend on the tax rate.

Proof. We differentiate equation (17) with respect to Δc, keeping c^e constant. Since the elasticity of τ with respect to θ is $\eta \cdot (1 + \tau(\theta))$, we obtain

$$\eta \cdot \frac{\Delta c}{\theta} \cdot \left.\frac{\partial \theta}{\partial \Delta c}\right|_{c^e} = v'(c^u) \cdot \frac{\Delta c}{\Delta v}.$$

It follows that $\left.\partial\theta/\partial\Delta c\right|_{c^e} > 0$. Using (14), we infer that

$$\epsilon^M - \epsilon^m = \frac{l}{1-l} \cdot \left(1 + \epsilon^d\right) \cdot \frac{1-\eta}{\eta} \cdot \frac{v'(c^u) \cdot \Delta c}{\Delta v}.$$

Lemma 1 tells us that $\epsilon^m/\epsilon^d = [l/(1-l)] \cdot (v'(c^u) \cdot \Delta c/\Delta v)$. Combining the expressions for $\epsilon^M - \epsilon^M$ and ϵ^m yields the desired expression for $1 - (\epsilon^M/\epsilon^m)$. As $\epsilon^d > 0$ and $0 < \eta < 1$, the expression establishes that $1 - (\epsilon^M/\epsilon^m) < 0$. $\qquad\square$

Figure 3(d) provides an illustration for the results of the proposition. After a reduction in UI, jobseekers search more, shifting the labor supply curve outward by a distance A–C, which measures the microelasticity. In addition when UI falls, jobseekers face a worse outside option, the bargained wage falls, which shifts the labor demand upward and raises equilibrium labor market tightness. This increase in tightness is a wage externality. The corresponding increase in employment is represented by distance C–B. The increase in equilibrium employment is given by the distance A–B, which measures the macroelasticity. Because of the wage externality, A–B is larger than A–C and the macroelasticity is larger than the microelasticity.

As depicted in Table 2, column (3), Proposition 7 implies that optimal UI should be procyclical relative to Baily-Chetty: optimal UI is higher than Baily-Chetty in a tight economy and lower in a slack economy. Intuitively, the economy is slack when the bargaining power of workers and wages are too high. In that situation, lowering UI is a way to reduce wages and increase tightness, which improves welfare. The converse holds in a tight economy.

In this model, fluctuations in workers' bargaining power are a simple to obtain fluctuations in tightness and unemployment. Proposition 8 establishes that the elasticity wedge is acyclical; therefore, the externality-correction term is expected to be symmetric in slack and tight economies:

PROPOSITION 8. *Under Assumptions 4, 5, and 3, the elasticity wedge is acyclical. An increase in workers' bargaining power decreases tightness and has no effect on the elasticity wedge:*

$\partial \theta / \partial \beta \big|_{\Delta v} < 0$ *and* $\partial \left[1 - (\epsilon^M / \epsilon^m) \right] / \partial \beta \big|_{\Delta v} = 0.$

Proof. Equation (6) implies that $\epsilon^d = 1/\kappa$. Under Assumption 3, both κ and η are constant parameters. The cyclicality of $1 - (\epsilon^M / \epsilon^m)$ follows from the expression in Proposition 7. □

It is conceivable that both the labor-demand externality of Section 4.1 and the wage externality of Section 4.3 are present in reality.[12] In that general case, our formula (15) would remain true and the cyclicality of UI would still be given by the sign of the elasticity wedge. If the elasticity wedge is negative in tight economies because the wage externality dominates and positive in slack economies because the labor-demand externality dominates, then optimal UI could be more generous than Baily-Chetty both in slack and tight economies. In the end, empirical analysis is required to evaluate the sign and cyclicality of the elasticity wedge.

5 A Quantitative Exploration

In this section we show how our optimal UI formula can be combined with available empirical evidence to explore quantitatively how optimal UI varies over the business cycle. We first approximate our formula to express it in terms of estimable sufficient statistics. To obtain realistic numbers, we introduce a minor modification to the formula such that it applies to a dynamic environment. Next, we discuss the estimates of the statistics found in the empirical literature. As there is no definitive empirical evidence on the sign, let alone the magnitude, of the elasticity wedge, we explore a range of scenarios corresponding to the three models of Section 4.

5.1 Approximation of the Formula in Sufficient Statistics

The appendix shows that formula (15) remains valid in the steady state of a continuous-time dynamic model with no time discounting by dividing $\tau(\theta)$ by u, where $u = s/(s + e \cdot f(\theta))$ is steady-state unemployment with job-destruction rate s. We use this modified formula below.

To implement the formula, we assume $v(c) = \ln(c)$, corresponding to a coefficient of relative

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[12]Cahuc and Wasmer [2001] and Elsby and Michaels [2013] develop matching models in which firms have a concave production function and wages are determined with the bargaining protocol of Stole and Zwiebel [1996]. In these models, UI would generate both a labor-demand externality and a wage externality.

risk aversion $\rho = 1$.[13] We normalize the disutility from job search, k, so that $k(e) = 0$ at the optimum. One simple way to interpret this assumption is that the costs of search while unemployed are of same magnitude as the costs of work while employed (which are not modeled) so that the welfare gain from employment is exactly $\Delta v = v(c^e) - v(c^u)$. We also assume that unemployment is small relative to employment so that $l \approx 1$ and hence $1/\phi \approx v'(c^e) = 1/c^e$. Under these assumptions, optimal UI approximately satisfies

$$
\frac{w}{\Delta c} - 1 \approx \frac{1}{\epsilon^m} \left(\frac{c^e}{c^u} - 1 \right)
$$
$$
+ \frac{1}{1 + \epsilon^d} \cdot \left(1 - \frac{\epsilon^M}{\epsilon^m} \right) \left[\frac{\ln(c^e/c^u)}{1 - c^u/c^e} + \left(1 + \epsilon^d \right) \left(\frac{w}{\Delta c} - 1 \right) - \frac{\eta}{1 - \eta} \cdot \frac{w}{\Delta c} \cdot \frac{\tau(\theta)}{u} \right]. \quad (18)
$$

The first term is the Baily-Chetty term. The second term is the externality-correction term, proportional to the elasticity wedge times the deviation from the generalized Hosios condition.

Naturally, the replacement rate depends on the generosity of UI. In our model, $c^u/c^e \approx 1 - \alpha \cdot (\Delta c/w)$ when $l \approx 1$. When $\Delta c = 0$, there is perfect insurance and $c^u/c^e = 1$. In reality, the unemployed receive less profits than average but they are also partially self-insured with savings, spousal income, or home production. Formula (18) carries over unchanged with partial self-insurance in the form of home production when unemployed by replacing c^u/c^e by the actual consumption smoothing benefits generated by UI.[14] We ignore this aspect in our numerical illustration because relatively little is known on the cyclicality of the consumption smoothing benefits of UI.[15]

5.2   Empirical Evidence

Formula (18) combined with $c^u/c^e \approx 1 - \alpha \cdot (\Delta c/w)$ relates the replacement rate to six estimable statistics: $\eta, \alpha, \tau(\theta)/u, \epsilon^d, \epsilon^m$, and ϵ^M/ϵ^m. We survey the empirical literature providing empirical estimates for these statistics. We use these estimates in the illustration of Table 3.

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[13]Many studies estimate the coefficient of relative risk aversion. A value of 1 is on the low side of available estimates but is consistent with labor supply behavior [Chetty, 2004, 2006b]. Naturally the higher risk aversion, the more generous optimal UI.

[14]Gruber [1997] estimates the consumption smoothing benefits of UI and uses these empirical results to simulate optimal UI using the conventional Baily-Chetty formula.

[15]See Chetty and Finkelstein [2012] for a survey.

Estimates of $\eta, \alpha, \tau(\theta)/u, \epsilon^d, \epsilon^m$. We set $\eta = 0.7$, in line with empirical evidence [Petrongolo and Pissarides, 2001; Shimer, 2005]. We set $\alpha = 2/3$ which is a standard estimate for the labor share. There is relatively little empirical evidence on $\tau(\theta)$ defined as the share of employees devoted to recruiting. Villena Roldan [2010] provides an estimate of $\tau(\theta) = 2.5\%$ when the unemployment rate is 5%, based on the National Employer Survey conducted by the Bureau of Labor Statistics in 1997. More evidence on the amplitude and cyclicality of $\tau(\theta)$ would be very valuable. The statistic $\tau(\theta)/u$ captures the slack or tightness of the economy, and we will present optimal UI for various values of $\tau(\theta)/u$ corresponding to different points in the business cycle.

There is little empirical work estimating the elasticity ϵ^d of job search effort with respect to the job-finding rate. Empirically, ϵ^d seems to be close to zero because labor market participation and other measures of search intensity are, if anything, slightly countercyclical even after controlling for changing characteristics of unemployed workers over the business cycle [Shimer, 2004]. Hence, we set $\epsilon^d = 0$.

Many studies estimate the microelasticity ϵ^m.[16] The ideal experiment to estimate ϵ^m is to offer higher unemployment benefits to a randomly selected and small subset of individuals within a labor market and compare unemployment durations between these treated individuals and the other jobseekers. In practice, ϵ^m is estimated by comparing individuals with different benefits in the same labor market at a given time, while controlling for individual characteristics. Most studies evaluate the elasticity of the job-finding probability with respect to benefits, which approximately equals ϵ^m in normal circumstances with a replacement rate around 50%. In US administrative data from the 1980s, the classic study of Meyer [1990] finds an elasticity of 0.9 with few individual controls and 0.6 with more individual controls. In a larger US administrative dataset from the early 1980s, and using a regression kink design to better identify the elasticity, Landais [2012] finds an elasticity around 0.3. Based on this evidence, we set $\epsilon^m = 0.5$.

Estimates of the Elasticity Wedge, $1 - (\epsilon^M/\epsilon^m)$. The elasticity wedge can be estimated either directly or indirectly by obtaining an estimate for ϵ^M and comparing it with an estimate for ϵ^m. The wedge is the most important sufficient statistic in our theory and yet the hardest to estimate.

The indirect approach is complicated. Estimating ϵ^M is inherently more difficult than estimat-

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[16]See Krueger and Meyer [2002] for a survey.

ing ϵ^m because it necessitates exogenous variation in UI benefits across comparable labor markets, instead of exogenous variations across comparable individuals within a single labor market. The ideal experiment to estimate ϵ^M is to offer higher unemployment benefits to all individuals in a randomly selected subset of labor markets and compare unemployment rates between these treated labor markets and the other labor markets. Having obtained an estimate of ϵ^M, we would need to compare it with estimates of ϵ^m to sign the elasticity wedge. This is an additional difficulty as estimates of ϵ^m vary significantly across samples, settings, and identification strategies.

A more promising approach is to estimate directly the elasticity wedge. We show in Section 4 that the labor-demand externality (increased search reduces tightness and the job-finding rate) leads to a positive elasticity wedge while the wage externality (higher UI increases bargained wages) leads to a negative elasticity wedge.

Several papers have tried to estimate directly the sign and magnitude of the labor-demand externality. Early studies find that an increase in the search effort of some jobseekers, induced by a reduction in UI or by job training programs, has a negative effect on the job-finding probability of other jobseekers. Absent a wage externality, this result implies $1 - (\epsilon^M/\epsilon^m) > 0$ [Burgess and Profit, 2001; Ferracci, Jolivet and van den Berg, 2010; Gautier et al., 2012; Levine, 1993]. The estimates range from $1 - (\epsilon^M/\epsilon^m) \approx 0.3$ in Denmark [Gautier et al., 2012] to $1 - (\epsilon^M/\epsilon^m) \approx 0.5$ in the US [Levine, 1993].[17] The literature however has not reached a complete consensus: for instance, Blundell et al. [2004] do not find any significant spillover effects of a job training program in the UK.

More recently, using a large change in UI duration for a subset of workers in a subset of geographical areas in Austria, Lalive, Landais and Zweimüller [2013] find a significant labor-demand externality that translates into a wedge $1 - (\epsilon^M/\epsilon^m) \approx 0.25 > 0$. Crepon et al. [2013] analyze a large randomized field experiment in France in which some young educated jobseekers are treated by receiving job placement assistance. The experiment has a double-randomization design: (1) some areas are treated and some are not, (2) within treated areas some jobseekers are treated and some are not. Interpreting the treatment as an increase in search effort from e^C for control jobseekers to e^T for treated jobseekers, their empirical results for long-term employment

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[17]See Table 3, column (1) in Gautier et al. [2012] and Table 5, column (1) in the working-paper version of Levine [1993], available at http://dataspace.princeton.edu/jspui/handle/88435/dsp01wh246s14w.

translate into an elasticity wedge $1 - (\epsilon^M/\epsilon^m) = 0.37$.[18] Crepon et al. [2013] also find suggestive evidence that the elasticity wedge $1 - (\epsilon^M/\epsilon^m)$ is countercyclical as in the model of Section 4.1.[19]

The presence of a labor-demand externality does not necessarily rule out a wage externality that also affects the elasticity wedge. The studies discussed above cannot typically capture a wage externality.[20] The best way to measure the wage externality might be to look directly at whether a more generous UI increases wages. A number of studies have investigated whether longer unemployment durations due to more generous UI benefits affect the re-employment wage. Most studies find no effect on wages or even slightly negative effects [for example, Card, Chetty and Weber, 2007]. However, longer durations induced by more generous UI benefits may affect the wage offers received (for example, if the duration of unemployment spells affects the productivity of the unemployed or is interpreted by employers as a negative signal of productivity). Schmieder, von Wachter and Bender [2012] find a negative effect on wages of longer durations due to more generous UI in Germany. Interestingly, they further decompose the effect on wages into a reservation-wage effect and an effect on wage offers and find zero reservation-wage effect. The reservation wage effect is obtained by controlling for the duration of the unemployment spell. Lalive, Landais and Zweimüller [2013], using the same approach, do find a positive reservation-wage effect for the population of Austrian unemployed eligible for a very large increase in the potential duration of their UI benefits but this effect is very small in magnitude. These results suggest that the wage externality might be small although more empirical work would be very valuable.

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[18]Compared to control jobseekers in the same area, treated jobseekers face a higher job-finding probability: $\left[e^T - e^C\right] \cdot f^T = 5.7\%$. But compared to control jobseekers in control areas, control jobseekers in treated areas face a lower job-finding probability: $e^C \cdot \left[f^T - f^C\right] = -2.1\%$ (Table 10, column 1, Panel B). Therefore the increase in the job-finding probability of treated jobseekers in treated areas compared to control jobseekers in control areas is only $\left[e^T \cdot f^T\right] - \left[e^C \cdot f^C\right] = 5.7 - 2.1 = 3.6\%$. By definition, the microelasticity is proportional to $\left[e^T - e^C\right] \cdot f^T$ and the macroelasticity is proportional to $\left[e^T \cdot f^T\right] - \left[e^C \cdot f^C\right]$, implying that $\epsilon^M/\epsilon^m = 3.6/5.7 = 0.63$.

[19]They estimate that the wedge is larger in geographical areas and time periods with higher unemployment. For example, $\epsilon^M/\epsilon^m = (14.5 - 7.6)/14.5 = 0.48$ during the 2008–2009 recession in areas with high unemployment, compared with $\epsilon^M/\epsilon^m = (3.5 - 0.9)/3.5 = 0.74$ otherwise (Table 11, Panel A, column (2)). The numbers vary somewhat across groups and specifications so this evidence is only suggestive.

[20]To see this, suppose group T receives more generous UI benefits while group C does not, and group T bargains for higher wages while group C does not. The ability of each group to bargain separately for a wage seems the most plausible assumption as differential UI benefits are typically based on observable characteristics such as age, geographical location, or industry. With linear production, the higher wage of group T does not affect group C so the studies focusing on group C cannot capture the wage externality. In the case of job-placement treatment, as in Crepon et al. [2013], no wage externality is expected in treatment or control group so the analysis captures a pure labor demand externality.

Table 3: Solutions of the optimal UI formula across the business cycle

	(1)	(2)	(3)
	positive	zero	negative
Elasticity wedge, $1 - (\epsilon^M/\epsilon^m)$	positive	zero	negative
Effect of UI on tightness	positive	zero	negative
Panel A: Very slack		$\tau(\theta)/u = 0.03$	
Notional unemployment: 11%	$\epsilon^M/\epsilon^m = 0.3$	$\epsilon^M/\epsilon^m = 1$	$\epsilon^M/\epsilon^m = 1.5$
Optimal UI replacement rate	0.76	0.50	0.36
Panel B: Slack		$\tau(\theta)/u = 0.15$	
Notional unemployment: 7.3%	$\epsilon^M/\epsilon^m = 0.5$	$\epsilon^M/\epsilon^m = 1$	$\epsilon^M/\epsilon^m = 1.5$
Optimal UI replacement rate	0.62	0.50	0.40
Panel C: Efficient		$\tau(\theta)/u = 0.47$	
Notional unemployment: 5.2%	$\epsilon^M/\epsilon^m = 0.7$	$\epsilon^M/\epsilon^m = 1$	$\epsilon^M/\epsilon^m = 1.5$
Optimal UI replacement rate	0.50	0.50	0.50
Panel D: Tight		$\tau(\theta)/u = 0.8$	
Notional unemployment: 4.5%	$\epsilon^M/\epsilon^m = 0.9$	$\epsilon^M/\epsilon^m = 1$	$\epsilon^M/\epsilon^m = 1.5$
Optimal UI replacement rate	0.48	0.50	0.64

Notes: The table presents the replacement rate $T = 1 - (\Delta c/w)$ of the optimal UI system for a range of values for the elasticity wedge and $\tau(\theta)/u$. The replacement rate is obtained by solving formula (18) given that $c^u/c^e = 1 - \alpha \cdot (\Delta c/w)$. The formula provides optimal UI in a dynamic model with partial self-insurance when $v(c) = \ln(c)$ (corresponding to a coefficient of relative risk aversion $\rho = 1$) and $l \approx 1$. We set the labor share at $\alpha = 0.66$, the elasticity of the matching function with respect to aggregate effort at $\eta = 0.7$, the elasticity of search effort with respect to the job-finding rate at $\epsilon^d = 0$, and the microelasticity of unemployment with respect to the consumption gain from work at $\epsilon^m = 0.5$.

5.3 Illustrative Results

As the empirical evidence is suggestive of a labor-demand externality but cannot definitively rule out a wage externality, we consider various scenarios for the elasticity wedge, corresponding to the range of models discussed in Section 4. Table 3 displays the optimal UI obtained with our approximated formula (18) across the business cycle for different values of the elasticity wedge, $1 - (\epsilon^M/\epsilon^m)$. The goal of the table is to quantify the deviations from Baily-Chetty caused by the business cycle in several numerical examples. The levels of UI in the table are only illustrative as they depend on the magnitudes of a number of statistics that are not well known.

Column (1) considers the case $\epsilon^M/\epsilon^m < 1$ as in the model with rigid wage and concave production presented in Section 4.1. In this model, ϵ^M/ϵ^m is expected to be procyclical so we set it from a low value of 0.3 is a very slack economy up to 0.9 in a tight economy. Column (2) considers the case $\epsilon^M/\epsilon^m = 1$ as in the model with rigid wage and linear production presented in Section 4.2. Column (3) considers the case $\epsilon^M/\epsilon^m > 1$ as in the model with Nash bargaining and linear production presented in Section 4.3. In this model, ϵ^M/ϵ^m is not expected to be cyclical so we set $\epsilon^M/\epsilon^m = 1.5$ uniformly across rows.

Each panel corresponds to a certain stage of the business cycle proxied by the value of $\tau(\theta)/u$. To help interpretation, we construct the notional labor market tightness and unemployment rate that correspond to the value of $\tau(\theta)/u$ of each panel when search effort is constant at $e = 1$. The notional values of labor market tightness and unemployment rate are not the equilibrium values because they do not account for the adjustment of search effort to labor market conditions and UI. In the steady-state of a continuous-time matching model with effort $e = 1$ and job-destruction rate s, the unemployment rate is related to labor market tightness by $u(\theta) = f(\theta)/(s + f(\theta))$ and the ratio of recruiters to producers is related to labor market tightness by $\tau(\theta) = (s \cdot r)/[q(\theta) - (s \cdot r)]$. We set $s = 0.009$, $f(\theta) = 0.17 \cdot \theta^{0.3}$, and $q(\theta) = 0.17 \cdot \theta^{-0.7}$ using the calibration of Michaillat [2014].[21] In addition, we set $r = 0.5$ such that the fraction of workers who are recruiters is 2.5% when the unemployment rate is 5%, in line with the empirical evidence in Villena Roldan [2010] discussed above. Panel A report results for $\tau(\theta)/u = 0.03$, corresponding to a notional tightness of 0.06 and a notional unemployment rate of 11%. Panel B report results for $\tau(\theta)/u = 0.15$,

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[21]This calibration relies mostly on data collected by the Bureau of Labor Statistics with the Job Opening and Labor Turnover Survey over the 2001–2011 period.

corresponding to a notional tightness of 0.27 and a notional unemployment rate of 7.3%. Panel C report results for $\tau(\theta)/u = 0.47$, corresponding to a notional tightness of 0.87 and a notional unemployment rate of 5.2%. Panel D report results for $\tau(\theta)/u = 0.80$, corresponding to a notional tightness of 1.48 and a notional unemployment rate of 4.5%. The economy is slack in Panel A and Panel B because the optimal replacement rate is higher in column (1) than in column (2) and higher in column (2) than in column (3), indicating that the deviation from the generalized Hosios condition is necessarily positive. The economy is efficient in Panel C because the optimal replacement rate is the same in the three columns, indicating that the generalized Hosios condition holds. The economy is tight in Panel D because the optimal replacement rate is higher in column (3) than in column (2) and higher in column (2) than in column (1), indicating that the deviation from the generalized Hosios condition is necessarily negative.

Three points are worth noting from Table 3. First, the optimal replacement rate given by the Baily-Chetty formula (0.50 in column (2)) is constant across the cycle because we assume that ϵ^m is constant.[22] This replacement rate is slightly higher than in the optimal UI simulations of Gruber [1997] because we abstract from self-insurance. Second, in the scenario $1 - (\epsilon^M/\epsilon^m) > 0$ of column (1), optimal UI is strongly countercyclical, going from 0.76 in a very slack economy to 0.48 in a tight economy. The fluctuations in optimal UI are asymmetric: the increases are very large in slack economies and the decrease is only modest in a tight economy. The asymmetry is due to the fact that the elasticity wedge is very positive in slack economies ($1 - (\epsilon^M/\epsilon^m) = 0.7$ in Panel A) and close to zero in a tight economy ($1 - (\epsilon^M/\epsilon^m) = 0.1$ in Panel D). Third, in the scenario $1 - (\epsilon^M/\epsilon^m) = -0.5$ of column (3), the optimal UI replacement rate is strongly procyclical, going from 0.36 in a very slack economy to 0.64 in a tight economy. Table 3 shows that the discrepancy from the Baily-Chetty formula can be quantitatively large over the business cycle for realistic estimates of the sufficient statistics. The striking difference between the scenarios in columns (1) and (3) underscores how crucial it is to obtain accurate estimates of the elasticity wedge.

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[22]Obviously, if ϵ^m or the ability to self-insure change with the business cycle, the optimal UI replacement rate given by the Baily-Chetty formula also changes with the business cycle. Our theory focuses on the cyclical deviation from Baily-Chetty.

6 Conclusion

Our paper analyzes optimal UI over the business cycle in a generic matching model. For a given UI system, the level of labor market tightness that maximizes welfare is given by the generalized Hosios condition. We say that the economy is efficient if tightness is at this optimal level, slack if tightness is too low, and tight if tightness is too high. Our formula for optimal UI is the sum of the standard Baily-Chetty term, which trades off search incentives and insurance, and an externality-correction term, which is positive if UI brings the economy closer to efficiency, negative if UI drives the economy farther away from efficiency, and zero if UI has no effect on aggregate efficiency. Formally, the externality-correction term is equal to the deviation from the generalized Hosios condition times the marginal effect of a change in UI on tightness. When the economy is slack, UI should be more generous than the Baily-Chetty formula if an increase in UI raises tightness and less generous if an increase in UI reduces tightness.

The effect of UI on tightness is measured by the wedge between micro- and macroelasticity of unemployment with respect to UI. Hence, this wedge is the crucial statistic that governs optimal UI relative to the Baily-Chetty benchmark over the business cycle. The wedge can be negative or positive depending on the model. Hence, empirical evidence on the sign and magnitude of the wedge is required to implement our formula fruitfully and deliver quantitative guidance for policy. A number of recent studies have found evidence of labor-demand externality whereby a decrease in search effort for some group of workers has a positive impact on the job-finding rate of other workers in the same labor market. This result suggests that the macroelasticity is smaller than the microelasticity, and that an increase in UI raises tightness. However, these findings are not definitive as little is known empirically about the effect of UI on wages. More empirical work comparing the two elasticities would be very valuable.

In principle, our methodology could be applied to the optimal design of other public policies over the business cycle. We conjecture that a policy that maximizes welfare in an economy with inefficient fluctuations obeys the same general rule as the one derived in this paper for UI. The optimal policy is the sum of (a) the optimal policy absent any inefficient cycle and (b) an externality-correction term when the economy is slack or tight. If a marginal increase of the policy increases tightness, the corrective term is positive in a slack economy and negative in a tight

economy. For example, using the same methodology in a model in which public good provision increases tightness, Michaillat and Saez [2013] show that it is desirable to provide more public good than in the Samuelson rule when the economy is slack, and less when the economy is tight. We conjecture that the rule could also be applied to income taxation. In the model of Michaillat and Saez [2013], if high-income earners have a lower propensity to consume than low-income earners, then transfers from high incomes to low incomes stimulate aggregate demand and increase tightness. As a result, the top income tax rate should be higher than in the Mirrleesian optimal top income tax formula in a slack economy, and lower in a tight economy. The rule could also apply to monetary policy, which can be seen as a special case where the traditional term is zero as monetary policy is only useful to stabilize the economy. This broad agenda in normative analysis could help bridge the gap between optimal policy analysis in public economics and business cycle analysis in macroeconomics.

This agenda is closely related in spirit to the general conceptual framework laid out by Farhi and Werning [2013] to study optimal macroeconomic policies in environments with price rigidities. They obtain the same decomposition of optimal policy into standard formulas plus a corrective term. Their corrective term arises not because of matching frictions but because of the price rigidities. In related work, they apply their framework to a standard New Keynesian model to analyze macroeconomic policies in fiscal unions [Farhi and Werning, 2012b] and to analyze optimal capital controls [Farhi and Werning, 2012a].

References

Baily, Martin N. 1978. "Some Aspects of Optimal Unemployment Insurance." *Journal of Public Economics*, 10(3): 379–402.

Blundell, Richard, Monica Costa Dias, Costas Meghir, and John van Reenen. 2004. "Evaluating the Employment Impact of a Mandatory Job Search Program." *Journal of the European Economic Association*, 2(4): 569—606.

Burgess, Simon, and Stefan Profit. 2001. "Externalities in the Matching of Workers and Firms in Britain." *Labour Economics*, 8(3): 313–333.

Cahuc, Pierre, and Etienne Wasmer. 2001. "Does Intrafirm Bargaining Matter in the Large Firm's Matching Model?" *Macroeconomic Dynamics*, 5(5): 742–747.

Card, David, Raj Chetty, and Andrea Weber. 2007. "Cash-On-Hand and Competing Models of Intertemporal Behavior: New Evidence from the Labor Market." *Quarterly Journal of Economics*, 122(4): 1511–1560.

Chetty, Raj. 2004. "Optimal Unemployment Insurance when Income Effects are Large." National Bureau of Economic Research Working Paper 10500.

Chetty, Raj. 2006a. "A General Formula for the Optimal Level of Social Insurance." *Journal of Public Economics*, 90(10-11): 1879–1901.

Chetty, Raj. 2006b. "A New Method of Estimating Risk Aversion." *American Economic Review*, 96(5): 1821–1834.

Chetty, Raj, and Amy Finkelstein. 2012. "Social Insurance: Connecting Theory to Data." National Bureau of Economic Research Working Paper 18433.

Crepon, Bruno, Esther Duflo, Marc Gurgand, Roland Rathelot, and Philippe Zamora. 2013. "Do Labor Market Policies Have Displacement Effect? Evidence from a Clustered Randomized Experiment." *Quarterly Journal of Economics*, 128(2): 531–580.

Elsby, Michael W. L., and Ryan Michaels. 2013. "Marginal Jobs, Heterogeneous Firms, and Unemployment Flows." *American Economic Journal: Macroeconomics*, 5(1): 1–48.

Farhi, Emmanuel, and Iván Werning. 2012*a*. "Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates." National Bureau of Economic Research Working Paper 18199.

Farhi, Emmanuel, and Iván Werning. 2012*b*. "Fiscal Unions." National Bureau of Economic Research Working Paper 18280.

Farhi, Emmanuel, and Iván Werning. 2013. "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities." National Bureau of Economic Research Working Paper 19313.

Ferracci, Marc, Grégory Jolivet, and Gérard J. van den Berg. 2010. "Treatment Evaluation in the Case of Interactions within Markets." Institute for the Study of Labor (IZA) Discussion Paper 4700.

Gautier, Pieter, Paul Muller, Bas van der Klaauw, Michael Rosholm, and Michael Svarer. 2012. "Estimating Equilibrium Effects of Job Search Assistance." Institute for the Study of Labor (IZA) Discussion Paper 6748.

Gruber, Jonathan. 1997. "The Consumption Smoothing Benefits of Unemployment Insurance."

American Economic Review, 87(1): 192–205.

Hall, Robert E. 2005. " Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review*, 95(1): 50–65.

Hosios, Arthur J. 1990. "On the Efficiency of Matching and Related Models of Search and Unemployment." *Review of Economic Studies*, 57(2): 279–298.

Krueger, Alan B., and Bruce Meyer. 2002. "Labor Supply Effects of Social Insurance." In *Handbook of Public Economics*. Vol. 4, , ed. Alan J. Auerbach and Martin Feldstein, 2327–2392. Elsevier.

Lalive, Rafael, Camille Landais, and Josef Zweimüller. 2013. "Market Externalities of Large Unemployment Insurance Extension Programs." http://econ.lse.ac.uk/staff/clandais/cgi-bin/Articles/Austria.pdf.

Landais, Camille. 2012. "Assessing the Welfare Effects of Unemployment Benefits Using the Regression Kink Design." http://econ.lse.ac.uk/staff/clandais/cgi-bin/Articles/rkd.pdf.

Levine, Phillip B. 1993. "Spillover Effects Between the Insured and Uninsured Unemployed." *Industrial and Labor Relations Review*, 47(1): 73–86.

Meyer, Bruce. 1990. "Unemployment Insurance and Unemployment Spells." *Econometrica*, 58(4): 757–782.

Michaillat, Pascal. 2012. "Do Matching Frictions Explain Unemployment? Not in Bad Times." *American Economic Review*, 102(4): 1721–1750.

Michaillat, Pascal. 2014. "A Theory of Countercyclical Government Multiplier." *American Economic Journal: Macroeconomics*, 6(1).

Michaillat, Pascal, and Emmanuel Saez. 2013. "A Model of Aggregate Demand and Unemployment." National Bureau of Economic Research Working Paper 18826.

Petrongolo, Barbara, and Christopher A. Pissarides. 2001. "Looking into the Black Box: A Survey of the Matching Function." *Journal of Economic Literature*, 39(2): 390–431.

Pissarides, Christopher A. 2000. *Equilibrium Unemployment Theory.* . 2nd ed., Cambridge, MA:MIT Press.

Schmieder, Johannes F., Till M. von Wachter, and Stefan Bender. 2012. "The Effect of Unemployment Insurance Extensions on Reemployment Wages."

Shimer, Robert. 2004. "Search Intensity." https://sites.google.com/site/robertshimer/intensity.pdf.

Shimer, Robert. 2005. "The Cyclical Behavior of Equilibrium Unemployment and Vacancies." *American Economic Review*, 95(1): 25–49.

Stole, Lars A., and Jeffrey Zwiebel. 1996. "Intra-Firm Bargaining under Non-Binding Contracts." *Review of Economic Studies*, 63(3): 375–410.

Villena Roldan, Benjamin. 2010. "Aggregate Implications of Employer Search and Recruiting Selection." Center for Applied Economics, University of Chile Working Paper 271.

Appendix: Dynamic Model

We extend our static model to a dynamic environment. We work in continuous time.

Labor market. At time t, the number of employed workers is $l(t)$ and the number of unemployed workers is $u(t) = 1 - l(t)$. Labor market tightness is $\theta_t = v_t/(e_t \cdot u_t)$. Employed workers become unemployed at rate $s > 0$. Unemployed workers find a job at rate $e(t) \cdot f(\theta(t))$. Thus, the law of motion of employment is

$$\dot{l}(t) = e(t) \cdot f(\theta(t)) \cdot (1 - l(t)) - s \cdot l(t). \tag{A1}$$

In steady state $\dot{l}(t) = 0$. Hence, employment, effort, and tightness are related by

$$l = \frac{e \cdot f(\theta)}{s + e \cdot f(\theta)}. \tag{A2}$$

It is convenient to define the function L by $L(x) = x/(s + x)$. The elasticity of L with respect to x is $1 - L$. It is because $l = L(e \cdot f(\theta))$ in the dynamic model instead of $l = e \cdot f(\theta)$ in the static model that the factor $1 - l$ appears in many formulas of the dynamic model.

Firms employ $n(t)$ producers and $l(t) - n(t)$ recruiters. Each recruiter handles $1/r$ vacancy so the law of motion of the number of employees is

$$\dot{l}(t) = -s \cdot l(t) + \frac{l(t) - n(t)}{r} \cdot q(\theta(t)). \tag{A3}$$

In steady state $\dot{l}(t) = 0$. Hence, number of employees, number of producers, and tightness are related by

$$l = (1 + \tau(\theta)) \cdot n \tag{A4}$$

where

$$\tau(\theta) = \frac{s \cdot r}{q(\theta) - (s \cdot r)}.$$

Optimization Problems and Equilibrium. Let $\sigma > 0$ be the time discount factor. Given $\{w(t)\}$ and $\{\theta(t)\}$, the firm's problem is to choose the number of employees $\{l(t)\}$ and the number of producers $\{n(t)\}$ to maximize the discounted stream of profits

$$\int_{t \geq 0} e^{-\sigma \cdot t} \cdot [y(n(t)) - w(t) \cdot l(t)] \, dt$$

subject to the law of motion (A3). Given $\{c^e(t)\}$, $\{c^u(t)\}$, and $\{\theta(t)\}$, the worker's problem is to choose effort $\{e(t)\}$ and employment probability $\{l(t)\}$ to maximize the discounted utility stream

$$\int_{t \geq 0} e^{-\sigma \cdot t} \cdot \{l(t) \cdot v(c^e(t)) + [1 - l(t)] \cdot v(c^u(t)) - [1 - l(t)] \cdot k(e(t))\} \, dt \tag{A5}$$

subject to the law of motion (A1). In equilibrium, $\{\theta(t)\}$ ensures that that the time paths $\{l(t)\}$ given by the laws of motion (A1) and (A3) coincide. The government's problem is to choose $\{c^e(t)\}$ and $\{c^u(t)\}$ to maximize the discounted welfare stream (A5) subject to firms and workers behaving optimally, the labor market being in equilibrium, and the resource constraint (3).

Steady State with No Time Discounting. We focus on the steady state of the model with no time discounting ($\sigma = 0$). Firms, workers, and government maximize the flow value of profits, utility, and welfare subject to steady-state constraints. Given w and θ, the firm chooses n to maximize $y(n) - w \cdot l$ subject to (A4). The optimal employment level satisfies

$$y'(n) = (1 + \tau(\theta)) \cdot w. \tag{A6}$$

Given θ, c^e, and c^u, the representative worker chooses e to maximize

$$l \cdot v(c^e) + (1 - l) \cdot v(c^u) - (1 - l) \cdot k(e) \tag{A7}$$

subject to (A2). The optimal search effort satisfies

$$k'(e) = \frac{l}{e} \cdot (\Delta v + k(e)) \,. \tag{A8}$$

Finally, the government chooses c^e and c^u to maximize (A7) subject to (A2), (A4), (A6), (A8), and to the resource constraint (3).

The resource constraint links c^u/c^e to $w/\Delta c$ when the production function is $y(n) = n^\alpha$. First, we can rewrite the firm's optimality condition as $\alpha \cdot y(n)/n = (1 + \tau(\theta)) \cdot w$, which imposes

$$y(n) = \frac{1}{\alpha} \cdot w \cdot l.$$

The labor share is α in the model. Then, we rewrite the resource constraint as follows:

$$c^u + l \cdot (c^e - c^u) = \frac{1}{\alpha} \cdot w \cdot l$$
$$\frac{c^u/c^e}{1 - c^u/c^e} = \frac{l}{\alpha} \cdot \left(\frac{w}{\Delta c} - \alpha\right)$$
$$\frac{c^u}{c^e} = \frac{(w/\Delta c) - \alpha}{(w/\Delta c) + \alpha \cdot (1 - l)/l}.$$

If $l \approx 1$, we obtain $c^u/c^e \approx 1 - \alpha \cdot (\Delta c/w)$.

Modification of the Static Results. We now describe how the static results are modified in the dynamic model with no time discounting. These results are obtained by following exactly the same steps as in the static model. To obtain most of the results, we assume that $k(e) = 0$ in steady state.

Lemma 1 remains valid. The formula of Lemma 2 becomes

$$\frac{\theta}{l} \cdot \left.\frac{\partial l^s}{\partial \theta}\right|_{\Delta c, c^e} = (1 - l) \cdot (1 + \epsilon^d) \cdot (1 - \eta)$$

Equation (14) becomes

$$\epsilon^M = \epsilon^m + l \cdot \left(1 + \epsilon^d\right) \cdot (1 - \eta) \cdot \frac{\Delta c}{\theta} \cdot \frac{\partial \theta}{\partial \Delta c}\Big|_{c^e} \tag{A9}$$

and the formula of Proposition 2 becomes

$$1 - \frac{\epsilon^M}{\epsilon^m} = -(1 - l) \cdot (1 - \eta) \cdot \frac{1 + \epsilon^d}{\epsilon^d} \cdot \frac{\Delta v}{v'(c^u) \cdot \Delta c} \cdot \frac{\Delta c}{\theta} \cdot \frac{\partial \theta}{\partial \Delta c}\Big|_{c^e}.$$

The formula of Proposition 3 becomes

$$\frac{w - \Delta c}{\Delta c} = \frac{l}{\epsilon^m} \cdot \left(\frac{v'(c^u)}{v'(c^e)} - 1\right)$$
$$+ \frac{1}{1 + \epsilon^d} \cdot \frac{w}{\Delta c} \cdot \left(1 - \frac{\epsilon^M}{\epsilon^m}\right) \cdot \left[\frac{\Delta v}{w \cdot \phi} + \left(1 - \frac{\Delta c}{w}\right) \cdot \left(1 + \epsilon^d\right) - \frac{\eta}{1 - \eta} \cdot \frac{\tau(\theta)}{1 - l}\right].$$

where ϕ satisfies equation (12). The formula of Proposition 4 becomes

$$1 - \frac{\epsilon^M}{\epsilon^m} = \left[1 + \frac{\eta}{1 - \eta} \cdot \frac{\alpha}{1 - \alpha} \cdot \frac{1}{1 + \epsilon^d} \cdot \frac{\tau(\theta)}{1 - l}\right]^{-1}.$$

Proposition 5 remains valid: $1 - (\epsilon^M/\epsilon^m)$ is countercyclical. Proposition 6 remains the same. With no time discounting, the wage obtained with Nash bargaining is

$$w = 1 - (1 - l) \cdot \frac{1 - \beta}{\beta} \cdot \frac{\Delta v}{v'(c^e)}$$

Using (A9), we infer that the gap between ϵ^M and ϵ^m in the model of Section 4.3 satisfies

$$\epsilon^M - \epsilon^m = l \cdot (1 + \epsilon^d) \cdot \frac{1 - \eta}{\eta} \cdot \left[\frac{\Delta c \cdot v'(c^u)}{\Delta v} - \epsilon^M\right]$$

The discouraged-worker elasticity satisfies $\epsilon^d = (1 - l)/(\kappa + l)$ (in the static model, $\epsilon^d = 1/\kappa$). Thus, Lemma 1 implies that

$$\epsilon^m = \frac{l}{\kappa + l} \cdot \frac{\Delta c \cdot v'(c^u)}{\Delta v}$$

Under Assumption 3, the formula of Proposition 7 becomes

$$1 - \frac{\epsilon^M}{\epsilon^m} = -\kappa \cdot \left(\frac{l + \kappa}{1 + \kappa} \cdot \frac{\eta}{1 - \eta} + l\right)^{-1}.$$

Unlike in Proposition 8, $1 - (\epsilon^M/\epsilon^m)$ is not acyclical but slightly procyclical; however, the fluctuations of $1 - (\epsilon^M/\epsilon^m)$ are small because l does not fluctuate much over the business cycle.