# Denis Sargan : some perspectives


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# DENIS SARGAN: SOME PERSPECTIVES

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Abstract

We attempt to present Denis Sargan's work in some kind of historical perspective, in two ways. First, we discuss some previous members of the Tooke Chair of Economic Science and Statistics, which was founded in 1859 and which Sargan held. Second, we discuss one of his articles "Asymptotic Theory and Large Models" in relation to modern preoccupations with semiparametric econometrics.

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# 1 INTRODUCTION

Unlike other contributors to this special issue, I was not a research student or colleague of Denis Sargan's, and little of my own research has been very close to his. I am not quali…ed, therefore, to o¤er a detailed appraisal of his contributions, or a fund of personal anecdotes. However, I am very much aware not only of the striking originality of his research, but also of his role, along with T.W. Anderson, E.J. Hannan, and others, in creating from the 1950's and 1960's, today's rigorous discipline of econometric theory. I would like to try to place his work in some sort of historical context, but I am not a historian of econometrics and so I will try to do so in a rather "individual" way.

From 1982 until his retirement in 1984, Denis Sargan held the Tooke Chair of Economic Science and Statistics at the London School of Economics (LSE). This Chair dates from 1859 - by way of comparison, the …rst university department of statistics, at University College London, was founded in 1911, while the Econometric Society was founded in 1930. I hope that a brief survey, in Section 2, of the previous incumbents of the Tooke Chair, may give a (very casual) picture of how the interface between statistics and econometrics developed prior to Sargan, whose own treatment of econometric problems was in the highest traditions of mathematical statistics. Such a review seems especially apt in that one of the previous incumbents of the Chair was F.Y. Edgeworth, who gave his name to Edgeworth expansions, one of Sargan's major contributions to econometrics being his development of higher-order asymptotic theory. Nowadays, the boundary between the sort of theoretical and empirical research pursued by econometricians on the one hand, and many statisticians on the other, has become very blurred. The in‡uence has not been all in one direction, from statistics to econometrics. In particular, "identi…ability" is a general issue with statistical models, but it was studied early on in the context of linear simultaneous equations systems in econometrics, and Sargan's own contributions (e.g., Sargan, 1983) to this subject were of particular depth. Also, increasingly many statisticians have become interested in problems arising in economics or …nance. On the other hand, in recent years econometricians have drawn on an ever wider range of statistical models and techniques. A particular topic which has attracted great econometric attention, at both the theoretical and empirical levels, and has largely developed since Sargan's retirement, is semiparametric inference, involving smoothed nonparametric estimation. (Indeed it has been much studied at the LSE since the mid-1980's.) The interest in semiparametric methods re‡ects the incomplete information typically available in econometric model building,

as well as increasing availability of large data sets, and improved computing power. Sargan's own work was predominantly of a "parametric" character, but one of his papers "Asymptotic theory and large models" (Sargan, 1975), can be related to subsequent research on semiparametric estimation, and I will discuss this paper in section 3, providing also some, minor, development of it. Section 4 contains some brief …nal comments.

# 2 THE TOOKE CHAIR

Denis Sargan was the ninth Tooke Professor of Economic Science and Statistics. The Tooke Chair was founded in 1859 at Kings College London, by public subscription and in memory of Thomas Tooke, who had died the previous year. Born in Cronstadt, Russia, the son of a chaplain of a British factory there, Tooke established himself …rst as a merchant, and as an early supporter of the free trade movement, then founding, with Ricardo, Malthus, James Mill and others, the Political Economy Club. As an economist, he devoted himself to systematically collecting and analyzing statistical and historical information concerning price changes from 1793 onwards, concluding that the ‡uctuations over the following 45 years were due to circumstances a¤ecting the supply of commodities rather than to changes in the system of currency, the popular view then being that the suspension of convertibility from 1793, and its resumption in 1819, led to a depreciation, and then an increase, in the value of currency. Tooke's investigations also led him to oppose the quantity theory of money. The data he collected were subsequently used by other researchers, including Jevons.

Both the …rst, and the third, holders of the Tooke Chair, at Kings College, had been ordained as Church of England clergymen, and were pioneers of economic history. James Thorold Rogers held the Chair from 1859 to 1890. He espoused controversial political opinions, and was brie‡y a Member of Parliament, but also conducted minute historical and statistical investigations leading to many publications, the best known being a history of agriculture and prices from 1759 to 1793. William Cunningham held the Chair from 1891 to 1897 and wrote the …rst textbook on economic history, which went to seven editions; Cunningham's e¤orts had much to do with the establishment of economic history as a separate discipline.

Neither Rogers nor Cunningham were theoretically inclined, and in fact held strongly critical attitudes to economic theorizing, but their tenures of the Tooke Chair were separated by that of Francis Ysidro Edgeworth, between 1890 and 1891, who made contributions of lasting value to both math-

ematical economics and statistics. Born of an Irish landowning father and Spanish refugee mother, Edgeworth had no early formal training in advanced mathematics, and started working in the moral sciences, but he went on to father the mathematical approach to economic theory. Edgeworth wrote extensively on such topics as taxation, monopoly and duopoly, international trade, and index numbers. He was a pioneer in using the indi¤erence curve, and his invention of the core was not appreciated by the economics profession for many years. At the same time, he o¤ered numerous important statistical insights. One, which we take for granted today, is the representation of the multivariate normal distribution in terms of pairwise correlation coe¢cients. Edgeworth's work on correlation greatly in‡uenced Karl Pearson. He derived the t-distribution as the posterior distribution for the sample mean. Some of his papers contain some element of a proof of asymptotic e¢ciency of maximum likelihood estimates, prior to Fisher. To many statisticians and economists he is most associated with the "Edgeworth expansion", however. Recognizing that much real data are not normally distributed, he introduced a series expansion which, as a rival to the Pearson family, is able to ‡exibly describe skewness, kurtosis and other phenomena. The "Edgeworth expansion" we refer to nowadays is employed to improve on the central limit theorem in approximating sampling distributions, and has led to re…nements in statistical inference in many areas of statistics and econometrics; more recently, strong theoretical support for the bootstrap has been provided by its ability to achieve an Edgeworth correction. Edgeworth was the …rst editor of the Economic Journal, serving in that role for twenty-one years.

Edgeworth was the epitome of the absent-minded professor, and an incoherent lecturer. The political economist and historian William Albert Samuel Hewins, Tooke Professor between 1897 and 1904, was more successful as a lecturer, in his early career speaking to working class audiences in the north of England on trade unionism and factory legislation. Hewins became the …rst director (that is, chief executive) of the LSE. He was succeeded as Tooke Professor by Charles Stewart Loch, later knighted, and then in 1908 by Edmund John Urwick; both Loch and Urwick wrote on social questions.

In 1919 the Tooke Chair was transferred from Kings College to the LSE. By then, both were colleges of London University, and the aim was to avoid overlapping by concentrating economic study in one of the two colleges - Kings College has focussed on the sciences and humanities, LSE on social science. However, the Tooke Chair was then in abeyance between 1920 and 1931, until it was taken up by Friedrich August von Hayek, arriving from Vienna. Hayek's scholarly output spreads over six decades and beyond economics. He is widely known for his in‡uence on the monetarist properties later pursued by western governments, for his critique of Keynesianism, and

his integration of price theory, capital theory and monetary theory, providing a base for reinterpreting 19th and 20th century economic history and promoting institutional reform. Hayek was awarded the Nobel Prize in Economics in 1974.

Hayek left the LSE in 1950, for Chicago, and the Tooke Chair was then vacant until 1958, when it was …lled by the promotion of Albert William Housego ("Bill") Phillips. Phillips had a background in electrical engineering, which provided him with a distinctive perspective on economic problems. He pioneered the application of optimal control theory to economics, and even built a hydraulic perspex model of a dynamic Keynesian-type economy, the "Phillips machine". Phillips' single most widely-known contribution is the "Phillips curve", relating wage in‡ation and unemployment. Phillips' research on continuous time dynamic models may also be mentioned, particularly in‡uencing work of fellow-New Zealanders at the LSE and the University of Essex.

Phillips left the LSE in 1967 and the Tooke Chair was vacant until, in 1982, it was transferred to Denis Sargan, then Professor of Econometrics. None of the previous incumbents of the Tooke Chaire comes close to …tting the modern description of an econometrician (though one could imagine a synthesis of Edgeworth's ideas leading to an earlier development of econometrics). But in many of his contributions - his early work in economic theory (Sargan, 1955), his in‡uential study of wages and prices in the UK (Sargan, 1964), his deep theoretical development of Edgeworth expansions (Sargan, 1976), and even his brief interest in continuous time models (Sargan, 1974), Denis Sargan echoed concerns and achievements of a number of his predecessors.

The preparation of this section has relied very heavily on the references Blaug (1986), Blyth (1987), Dahrendorf (1995), Davis and Weaver (1927), Garrison and Kirzner (1987), Lee (1897, 1899), Mai (1975), Morgan (1990), Newman (1987), Pavetti (1987), Stigler (1987), Weaver (1937), Wickham Legg (1949), and Who was Who 1929-1940 and 1941-1950.

# 3 SEMIPARAMETRIC ESTIMATION

So much of Sargan's work, on instrumental variables, simultaneous equations, speci…cation testing, and Edgeworth expansions, can be seen as fundamental to, and in‡uential in, the development of econometrics. "Asymptotic Theory and Large Models" (Sargan, 1975) can be seen as something of a one-o¤ contribution, which turns out to contain ideas related to a more modern econo-

metric preoccupation, semiparametric estimation; I referred to Sargan (1975) in my review article on semiparametric econometrics (Robinson, 1988), but did not discuss it in any depth there. The paper is not included among the two volumes of selections of Sargan's papers (Sargan, 1988) and has indeed rather escaped attention, perhaps because interest in large econometric models was already on the wane by the time the paper was published, in an issue of International Economic Review devoted to the proceedings of a symposium in econometric model performance. Though connected to some of Sargan's other work, including instrumental variables estimation of linear simultaneous equations systems (see Sargan, 1958), the main innovation in Sargan (1975) is the allowance for an increase, with sample size, of the number of variables, and equations, in the model, and the justification of parametric convergence rates with respect to the part being estimated, and asymptotic efficiency, relative to an underlying model involving infinitely many variables and equations. It thus deserves to be associated with the statistical and econometric literature on "adaptive" estimation in the context of a semiparametric model: the finite-dimensional parameter vector is estimated asymptotically as efficiently (in either the Cramer-Rao or Gauss Markov senses) as if the infinite-dimensional nuisance function were of finite dimension. Hannan (1963) seems to have obtained the first results of this kind, showing adaptivity with respect to disturbance autocorrelation of unknown, nonparametric form, in time series regression models; Hannan followed this up with work on related semiparametric models, including distributed lag and linear simultaneous equations models. (Sargan's own work on frequency domain semiparametric estimation - Espasa and Sargan (1977) - and his early paper on the periodogram - Sargan (1953) - also warrant mention here.) The model in Sargan (1975) is perhaps not really semiparametric in the sense that, even though he was concerned with estimation of only a finite subset of equations, whose number remains constant as sample size increases, all equations in his underlying infinite system are assumed to be identified, each involving variables whose number also remains fixed as sample size increases. I will point out later that the same statistical achievement is possible without this kind of "full-information" requirement, presenting a version of the problem more in keeping with the modern semiparametric literature. First, however, I would like to describe in more detail the basic setup and accomplishments of Sargan (1975).

Define arrays $\{y_{st};\ s;t = 1, \ldots\}$, $\{z_{st};\ s;t = 1, \ldots\}$, $\{u_{st};\ s;t = 1, \ldots\}$, $\{b_{st};\ s;t = 1, \ldots\}$, $\{c_{st};\ s;t = 1, \ldots\}$; and consider the model

$$\sum_{j=1}^{p} \{b_{ij} y_{jt} + c_{ij} z_{jt}\} = u_{it}; \qquad i;t = 1. \tag{3.1}$$

In (3.1), i indexes structural equations and t indexes observations (I will mostly try to follow Sargan's notation, though he employed instead an in…nite-dimensional matrix presentation here). We will refer to the $y_{jt}$ as "endogenous" variables and the $z_{jt}$ as "exogenous" variables. The $u_{it}$ are unobservable disturbances, satisfying at least

$$E(u_{it}) = 0; \tag{3.2}$$

$$E(u_{it}u_{jt}) = \omega_{ij} \tag{3.3}$$

$$E(u_{is}u_{jt}) = 0; \ s \neq t; \tag{3.4}$$

for all i, j, s, t, and the $z_{jt}$ are nonstochastic and uniformly bounded. The i-th equation in (3.1) is well-de…ned, for all t, by the requirements that for some $p < 1$, $q < 1$,

$$b_{ij} \neq 0 \text{ for at most } p \text{ values of } j; \tag{3.5}$$

$$c_{ij} \neq 0 \text{ for at most } q \text{ values of } j, \tag{3.6}$$

and is normalized by the requirement

$$b_{ii} = 1. \tag{3.7}$$

It is supposed that (3.5)-(3.7) hold for all $i \geq 1$. Further, Sargan assumed that for all i

$$b_{ij} = 0; \quad j > i + P; \quad \text{for some } P > 0; \tag{3.8}$$

$$c_{ij} = 0; \quad j > qi \tag{3.9}$$

$$\sum_{j:j\neq i} |b_{ij}| < 1; \tag{3.10}$$

and

$$\max_{i,j} |c_{ij}| < 1. \tag{3.11}$$

The $b_{ij}$, $c_{ij}$ are otherwise unknown. Condition (3.8) is a kind of near-recursivity requirement, and along with (3.10), which is clearly very strong, and (3.11), ensures the existence of a stable solution of (3.1), while, given (3.6), condition (3.9) is always satis…ed by a suitable re-ordering.

The simultaneous equations system (3.1) involves in…nitely-many equations and in…nitely-many endogenous variables $y_{jt}$, $j \geq 1$, and exogenous variables $z_{jt}$, $j \geq 1$, observed at time points t. In practice we have observations at $t = 1; ...; T$, for T …nite, and construct a working model of n(T) equations in $y_{jt}$, $1 \leq j \leq n(T)$ and $z_{jt}$, $1 \leq j \leq m(T)$, namely

$$\sum_{j=1}^{n(T)} b_{ij}y_{jt} + \sum_{j=1}^{m(T)} c_{ij}z_{jt} = u_{it}; \quad 1 \leq i \leq n(T); \quad 1 \leq t \leq T; \tag{3.12}$$

with $m(T)+n(T) < T$. In discussing asymptotic statistical theory for estimation of the unknown parameters explaining the $b_{ij}; c_{ij}$ in (3.12), traditionally (see e.g. Johnston, 1984) $m(T)$ and $n(T)$ are …xed as $T \to 1$, so $m(T) = m$, $n(T) = n$, say. In (3.12), however, $m(T)$ and $n(T)$ are regarded as increasing slowly with $T$, so that (3.12) "tends to" (3.1), to re‡ect an attitude often adopted by practical modellers, that the more data we have, the more unknown parameters one can hope to estimate with reasonable precision. Even with the restrictions (3.5) and (3.6), when $n(T) \to 1$ the number of unrestricted, unknown $b_{ij}; c_{ij}$ in (3.12) also tends to in…nity, so that (3.12) has something of a nonparametric, in…nite-dimensional character. However, Sargan (1975) was concerned with estimating only the …rst $N$ equations of (3.12), where $N$ stays …xed as $T \to 1$, so that a …nite number of no more than $N(p+q-1)$ unknowns are being estimated. The situation is then more like that in various semiparametric models, where it is desired to estimate a …nite-dimensional vector as well as possible, or at least with convergence rate $T^{\frac{1}{2}}$, in the presence of an unknown nuisance function. In fact, Sargan (1975) was not concerned with developing new estimates to deal with this situation, but rather with justifying established ones, developed for the traditional case of $m(T); n(T)$ …xed, in the context of the "increasing" model (3.12) nested in the in…nite-dimensional system (3.1). Speci…cally, he showed that estimates which are known to achieve a certain asymptotic e¢ciency bound with respect to (3.12) with $m(T); n(T)$ …xed, achieve the same bound with respect to (3.1).

To signi…cantly simplify the presentation I will take $N = 1$ throughout; since $N$ is in any case …xed, there is no essential loss of generality, the only substantive di¤erence arising in the case $N > 1$ being that an estimate of the covariance matrix of $(u_{1t}; ...; u_N)$ is involved in order to achieve e¢ciency when those $u_{it}$ are contemporaneously correlated; in the asymptotic theory Sargan developed, and that which I will later develop, this is handled relatively straightforwardly, given the other techniques employed to address the problem, and the allowance for this kind of extension to the theory I will subsequently give is likewise straightforward. We write the …rst equation of (3.1) or (3.12) as

$$y_{1t} + \sum_{j=2}^{p} b_{1j} y_{jt} + \sum_{j=1}^{q} c_{1j} z_{jt} = u_{1t}; \tag{3.13}$$

for $p_1 \cdot p$, $q_1 \cdot q$, where $b_{12}; ...; b_{1p_1}; c_{11}; ...; c_{1q_1}$ are unknown and we have employed a convenient ordering of the $y_{jt}; z_{jt}$. Write

$$y_j = (y_{j1}; ...; y_{jT})'; \quad Y_1 = (y_1; ...; y_{p_1}) \tag{3.14}$$

$$z_j = (z_{j1}; ...; z_{jT})'; \quad Z_1 = (z_1; ...; z_{q_1}): \tag{3.15}$$

The estimates considered by Sargan are all of instrumental variables type. For a $T \times (p_1 - 1)$ matrix $W$ we consider the instrumental variables estimate

$$\hat{\alpha}_{IV} = \begin{pmatrix} W'Y_1 & W'Z_1 \\ Z_1'Y_1 & Z_1'Z_1 \end{pmatrix}^{-1} \begin{pmatrix} W'y_1 \\ Z_1'y_1 \end{pmatrix} \tag{3.16}$$

of $\alpha = (\beta_{12}, \ldots, \beta_{1p_1}, \gamma_{11}, \ldots, \gamma_{1q_1})'$, assuming the inverse exists. (Our notation here differs somewhat from Sargan's.)

An efficient choice of $W$ involves the reduced form for $y_t = (y_{2t}, \ldots, y_{p_1 t})'$. Sargan deduced that

$$E(y_t) = \sum_{j=1}^{\infty} \pi_j z_{jt}, \tag{3.17}$$

where the $\pi_j$ are $(p_1 - 1) \times 1$ vectors, the $\pi_j$ indeed converging exponentially to zero as $j \to \infty$ under the above conditions (due especially to (3.10)), so that (3.17) is bounded. Introduce the $T \times (p_1 - 1)$ matrix

$$Y_1^* = \sum_{j=1}^{\infty} z_j \pi_j'. \tag{3.18}$$

Then take $W = Y^*$ in (3.16), and thus consider the estimate

$$\hat{\alpha}_{INF} = \begin{pmatrix} Y_1^{*\prime}Y_1 & Y_1^{*\prime}Z_1 \\ Z_1'Y_1 & Z_1'Z_1 \end{pmatrix}^{-1} \begin{pmatrix} Y_1^{*\prime}y_1 \\ Z_1'y_1 \end{pmatrix}; \tag{3.19}$$

where "INF" stands for "infeasible", both because the $\pi_j$ are unknown and because the $z_{jt}$ are observable only for $j \le n(T)$ (see (3.12)). Sargan argued that $W = Y^*$ is an efficient choice of instruments, in the sense that under suitable additional conditions the limiting variance matrix of $T^{\frac{1}{2}}(\hat{\alpha}_{IV} - \alpha)$ exceeds that of $T^{\frac{1}{2}}(\hat{\alpha}_{INF} - \alpha)$ by a non-negative definite matrix; for this and his other results he assumed that the limits

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} z_{it} z_{jt} = m_{ij} \tag{3.20}$$

exist for all $i$ and $j$, and the $m(T) \times m(T)$ matrix $(m_{ij})$ has eigenvalues that are uniformly bounded away from zero and infinity as $T \to \infty$ (implying $m(T) \to \infty$).

Sargan then considered feasible efficient estimates, taking $W = \hat{Y}$, where

$$\hat{Y}_1 = \sum_{j=1}^{m(T)} z_j \hat{\pi}_p' = Z \hat{\pi}_1', \tag{3.21}$$

writing

$$\hat{\Pi}_1 = \begin{pmatrix} \hat{\pi}_1, \ldots, \hat{\pi}_{m(T)} \end{pmatrix}; \quad Z = \begin{pmatrix} z_1, \ldots, z_{m(T)} \end{pmatrix}; \tag{3.22}$$

for $(p_1 - 1) \times 1$ vector estimates $\hat{\pi}_j$ of the $\pi_j$. We then have the feasible estimate

$$\hat{\alpha}_{FEA} = \begin{bmatrix} \hat{Y}_1'Y_1 & \hat{Y}_1'Z_1 \\ Z_1'Y_1 & Z_1'Z_1 \end{bmatrix}^{-1} \begin{bmatrix} \hat{Y}_1'y_1 \\ Z_1'y_1 \end{bmatrix}. \tag{3.23}$$

For $\hat{\Pi}_1$ Sargan used preliminary instrumental variables estimates computed from all equations of (3.12). Given estimates $\hat{B}$ and $\hat{C}$ of the $n(T) \times n(T)$ and $n(T) \times m(T)$ matrices $(b_{ij})$ and $(c_{ij})$, take

$$\hat{\Pi}_1 = [I_{p_1 - 1}; 0]\hat{\Pi}; \tag{3.24}$$

where $\hat{\Pi} = -\hat{B}^{-1}\hat{C}$, $I_r$ is the $r \times r$ identity matrix and 0 is here a $(p_1 - 1) \times (m(T) - p_1 + 1)$ matrix of zeros. In Sargan (1975) $\hat{B}$ and $\hat{C}$ are formed from preliminary single-equation instrumental variables estimates, as indicated by Brundy and Jorgensen (1971). These authors argued that using the least squares estimate $Y_1'Z(Z'Z)^{-1}$ in place of $\hat{\Pi}$ in (3.24) is computationally unattractive when $m(T)$ is large, due to the need to invert $Z'Z$; with $p_j$ unexcluded $y_{jt}$ and $q_j$ unexcluded $z_{jt}$ in the j-th equation, $p_j \leq p$, $q_j' \leq q$, single equation instrumental variables estimation of the j-th equation involves inversion of only a $(p_j - 1 + q_j)$-dimensional square matrix, and Brundy and Jorgensen (1971) preferred the prospect of $n(T)$ such inversions over least squares. Imposing some unprimitive conditions on $\hat{B}$ and $\hat{C}$ (though indicating circumstances under which they would be met), Sargan showed that $\hat{\alpha}_{FEA} - \hat{\alpha}_{INF} = o_p(T^{-\frac{1}{2}})$, so that $T^{\frac{1}{2}}(\hat{\alpha}_{FEA} - a)$ has the same limit distribution as $T^{\frac{1}{2}}(\hat{\alpha}_{INF} - a)$, and is thus asymptotically as efficient.

From a technical stand-point, Sargan (1975) solved highly challenging problems by both deriving the reduced form (3.17) and justifying its estimation by the preliminary structural form estimates of B and C. However, computing power has developed considerably over the past thirty years, so that least squares estimation of the reduced form in the presence of a very large number of exogenous variables seems a less daunting prospect nowadays than it did to Brundy and Jorgenson (1971). Thus, the two-stage least squares estimate of a,

$$\hat{\alpha}_{2SLS} = \begin{bmatrix} Y_1'Z(Z'Z)^{-1}Z'Y_1 & Y_1'Z_1 \\ Z_1'Y_1 & Z_1'Z_1 \end{bmatrix}^{-1} \begin{bmatrix} Y_1'Z(Z'Z)^{-1}Z'y_1 \\ Z_1'y_1 \end{bmatrix}; \tag{3.25}$$

seems worth considering. This is (3.16) with $W = Y_1'Z(Z'Z)^{-1}$. $\hat{\alpha}_{2SLS}$ has a further, minor, advantage over $\hat{\alpha}_{FEA}$ in that the first factor on the right of

(3.25) is proportional to a consistent estimate of the limiting covariance matrix of the estimate, whereas the …rst factor on the right of (3.23), with Sargan's choice of $\hat{Y}_1$, is not symmetric so will not be satisfactory for covariance matrix estimation. The main distinction from Sargan's work is theoretical, however. Though exploitation of overidentifying restrictions on all structural equations of (3.12) may produce desirable …nite-sample statistical properties, as well as avoiding inversion of a very large matrix, there is a cost: some misspeci…cation of the restrictions could lead to inconsistent estimation of the reduced form, and thence at least to a large-sample e¢ciency loss in the estimates of the …rst equation. In fact, the large-sample e¢ciency achievement of Sargan is possible without identifying other structural equations, as two-stage least squares allows in the traditional …xed-m(T) context. We demonstrate this by a representation of the problem that is more in keeping with the modern semiparametric literature than Sargan's. A further, minor, observation is that Sargan did not actually present a central limit theorem for his $\hat{a}_{FEA}$, whereas we will do so, requiring some additional conditions, though in other respects our conditions are weaker.

We assume (3.13) with

$$y_{1t} = \sum_{j=1}^{\infty} \gamma_j z_{jt} + v_t; \tag{3.26}$$

with

$$\sum_{j=1}^{\infty} j^{\frac{1}{2}} |\gamma_j| < \infty; \tag{3.27}$$

where $u_{1t}$ and the $(p_1 - 1) \times 1$ unobservable vector $v_t$ of reduced form disturbances are such that the $(u_{1t}, v_t')$ are independent and identically distributed with zero mean and …nite fourth moment. Like Sargan (1975), we assume the $z_{jt}$, $j \geq 1$, $t \geq 1$, are nonstochastic and uniformly bounded, and also impose the condition in the sentence surrounding (3.20). Denoting by $e_j$ the $(p_1 - 1) \times 1$ vector whose j-th element is 1 and whose other elements are zero, by 0 the $q_1 \times 1$ vector of zeros, and $\gamma_j = (\gamma_j', e_j')'$, $1 \leq j \leq q_1$, $\gamma_j = (\gamma_j', 0')'$, $j > q_1$, we assume that

$$(\gamma_1, \gamma_2, \ldots) \text{ has full row rank;} \tag{3.28}$$

this is just an identi…ability condition. Then with also

$$m(T)^{-1} + m(T)^2/T \to 0, \qquad \text{as } T \to \infty; \tag{3.29}$$

we have that

$$T^{\frac{1}{2}}(\hat{a}_{LNF} - a) \to_d N\left(0, \sigma^2 \cdot \left(\cdot\right)^{-1}\right); \tag{3.30}$$

$$T^{\frac{1}{2}}(\hat{a}_{2SLS} - a) \to_d N\left(0, \sigma^2 \cdot \left(\cdot\right)^{-1}\right); \tag{3.31}$$

where

$$\sigma_{11} = V(u_{11}); \qquad \Omega = \sum_{j=1}^{\infty}\sum_{k=1}^{\infty} \psi_j m_{jk} \psi_k'; \tag{3.32}$$

and $\sigma_{11}\Omega^{-1}$ can be estimated consistently by $\hat{\sigma}_{11}\hat{\Omega}^{-1}$; where

$$\hat{\sigma}_{11} = \frac{1}{T} y_1' y_1 - \hat{\alpha}_{2SLS}' \hat{\Omega} \hat{\alpha}_{2SLS} \tag{3.33}$$

$$\hat{\Omega} = \frac{1}{T} \begin{bmatrix} Y_1' Z (Z'Z)^{-1} Z' Y_1 & Y_1' Z_1 \\ Z_1' Y_1 & Z_1' Z_1 \end{bmatrix}. \tag{3.34}$$

The proof is described in the Appendix.

It follows that $\hat{\alpha}_{2SLS}$ has the same asymptotic efficiency as the infeasible optimal estimate $\hat{\alpha}_{INF}$, and inference can be carried out in the manner presented in elementary textbooks (e.g. Johnston, 1984) for two-stage least squares with a fixed number of explanatory variables. Note that condition (3.27) is much milder than the exponential decay of the $\psi_j$ derived by Sargan (1975) from his set-up. The prescription (3.26) is rather appealing because it may be felt that whereas we may be prepared to identify a single structural equation of interest, the included endogenous variables $y_{jt}$ are affected by a potentially unlimited number of exogenous variables $z_{jt}$, only some of which we observe. Moreover, the $z_{jt}$ could be nonlinear functions of a possibly finite number of observables, so that $\sum_{j=1}^{m(T)} \psi_j z_{jt}$ constitutes a series approximation to a nonlinear $E(y_t)$ in (3.17), and $\sum_{j=1}^{m(T)} \psi_j z_{jt}$ a corresponding series estimate, $\psi_j$ being the j-th column of $Y_1'Z(Z'Z)^{-1}$. Our set-up can then be compared with Newey's (1990), who considered semiparametric estimation using nearest neighbour (instead of our series) nonparametric estimation, and assumed stochastic exogenous variables, unlike our nonstochastic ones. Indeed, Andrews (1991) and Newey (1994) have provided extensive theoretical treatments of the use of series estimation in semiparametric problems, in a wide context, thought they did not explicitly consider the present problem, and their regularity conditions were motivated by more general and elaborate situations; a closer paper to ours is Koenker and Machado (1999), who considered, under different kinds of conditions, generalized method of moments estimation with an increasing number of moment conditions. Our discussion here aims to draw parallels between Sargan (1975) paper and later econometric developments, rather than offer something distinctively novel relative to the current literature. Extended versions of the problem discussed here might involve the structural equation or equations of interest being semiparametric or nonparametric, or the number of equations of interest being thought of as increasing slowly with T.

# 4 FINAL COMMENTS

The London School of Economics was a sympathetic environment for research in quantitative economics and statistics before Sargan's arrival, and in a more global sense these areas were already quite developed. However, Denis Sargan deserves to be regarded as a founder of high-level econometric theory at the LSE, and in the UK, as well as, alongside a few others, internationally. Econometrics has developed considerably since, and interests have changed, but nevertheless much of Sargan's work was ahead of its time and remains relevant and inspiring today.

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# 5 APPENDIX

We describe the main details of the proof of (3.31); the proofs of consistency of $\hat{\gamma}_4^2$ and $\hat{\Sigma}$ for $\gamma_4^2$ and $\Sigma$ use similar techniques but are easier, while the proof of (3.30) involves straightforward use of standard central limit theorem for independent non-identically distributed variables, noting the conditions on the $z_{jt}$ and $(u_{1t}; v_t')$; it is perhaps only necessary here to observe that $\Sigma$ is non-singular owing to the condition on the $m_{jk}$ and (3.28).

Given (3.30), we immediately deduce (3.31) on proving

$$\hat{\alpha}_{INF} - \hat{\alpha}_{2SLS} = o_p\left(T^{-\frac{1}{2}}\right): \tag{A.1}$$

We may write the left side of (A.1) as

$$D_1^{-1}d_1 - D_2^{-1}d_2 = \left(D_1^{-1} - D_2^{-1}\right)d_1 + D_2^{-1}(d_1 - d_2);$$

where

$$D_1 = \frac{1}{T}\begin{bmatrix} Y_1'Y_1 & Y_1'Z_1 \\ Z_1'Y_1 & Z_1'Z_1 \end{bmatrix}; \qquad d_1 = \frac{1}{T}\begin{bmatrix} Y_1'u_1 \\ Z_1'u_1 \end{bmatrix};$$

$$D_2 = \frac{1}{T}\begin{bmatrix} Y_1'Z(Z'Z)^{-1}Z'Y_1 & Y_1'Z_1 \\ Z_1'Y_1 & Z_1'Z_1 \end{bmatrix}; \quad d_2 = \frac{1}{T}\begin{bmatrix} Y_1'Z(Z'Z)^{-1}Z_1'u_1 \\ Z_1'u_1 \end{bmatrix};$$

with $u_1 = (u_{11}; \ldots; u_{1T})'$. Since $D_1 \to_p \Sigma$ is entailed in the proof of (3.30), it suffices to show that $D_1 - D_2 \to_p 0$ and $d_1 - d_2 = o_p(T^{-\frac{1}{2}})$. We give the proof only for the latter, the proof of the former being easier due to the slower convergence that suffices.

Now

$$\begin{aligned} Y_1'u_1 &= \Pi' Z'u_1 + c_1; \\ Y_1'Z(Z'Z)^{-1}Z'u_1 &= \Pi' Z'u_1 + c_2 + c_3; \end{aligned}$$

where

$$\begin{aligned} c_1 &= \sum_{j=m(T)+1}^{\infty} \gamma_j \sum_{t=1}^{T} z_{jt}u_{1t}; \\ c_2 &= \sum_{t=1}^{T} v_t z_t'(Z'Z)^{-1} \sum_{t=1}^{T} z_t u_{1t}; \\ c_3 &= \sum_{j=m(T)+1}^{\infty} \gamma_j \sum_{t=1}^{T} z_{jt} z_t'(Z'Z)^{-1} \sum_{t=1}^{T} z_t u_{1t}; \end{aligned}$$

writing $z_t = \left(z_{1t}; \ldots; z_{m(T);t}\right)'$. It thus remains to show that

$$c_i = o_p\left(T^{\frac{1}{2}}\right), \quad i = 1; 2; 3: \tag{A.2}$$

First, $c_1$ has mean zero and variance matrix

$$\sigma^2 \sum_{j=m(T)+1}^{p} \sum_{k=m(T)+1}^{p} \gamma_j \sum_{t=1}^{T} z_{jt} z_{kt} \gamma_k';$$

whose largest eigenvalue is

$$O\left(T \sum_{j=m(T)+1}^{p} \|\gamma_j\|^2\right) = o(T), \quad \text{as } T \to \infty;$$

where $\|A\|$ denotes the square root of the largest eigenvalue of $A'A$. This proves (A.2) for $i = 1$.

Next, $c_2$ has mean

$$\begin{aligned} E(v_1 u_{11}) \sum_{t=1}^{T} \bar{z}_t' (Z'Z)^{-1} \bar{z}_t &= E(v_1 u_{11}) \operatorname{tr}\left[(Z'Z)^{-1} Z'Z\right] \\ &= O(m(T)) = o(T^{\frac{1}{2}}); \end{aligned}$$

and variance matrix

$$\begin{aligned} & E\{v_1 u_{11} - E v_1 u_{11}\}\{v_1 u_{11} - E v_1 u_{11}\}' \sum_{t=1}^{T} \left[\bar{z}_t (Z'Z)^{-1} \bar{z}_t\right]^2 \\ & + \left[E(v_1 v_1') E(u_{11}^2) + E(v_1 u_{11}) E(v_1' u_{11})\right] \sum_{t=1}^{T} \sum_{\substack{s=1 \\ s \neq t}}^{T} \left[\bar{z}_s' (Z'Z)^{-1} \bar{z}_t\right]^2 \\ = \; & O\left(\max_{1 \le t \le T} \|\bar{z}_t\|^2 \left\|(Z'Z)^{-1}\right\|\right) + O(m(T)) \\ = \; & O\left(\frac{m(T)^2}{T} + m(T)\right) = o(T); \end{aligned}$$

so (A.2) is proved for $i = 2$.

Finally, $c_3$ has mean zero and variance matrix

$$\sigma^2 \sum_{j=m(T)+1}^{p} \sum_{k=m(T)+1}^{p} \gamma_j \sum_{t=1}^{T} z_{jt} \bar{z}_t (Z'Z)^{-1} \sum_{t=1}^{T} \bar{z}_t z_{kt} \gamma_k'. \tag{A.3}$$

Now

$$\left\| \sum_{t=1}^{T} z_{jt} \bar{z}_t \right\| \le \left(\sum_{t=1}^{T} z_{jt}^2\right)^{\frac{1}{2}} \left(\sum_{t=1}^{T} \|\bar{z}_t\|^2\right)^{\frac{1}{2}} = O\left(T m(T)^{\frac{1}{2}}\right);$$

so (A.3) is

$$O\left(T m(T) \left(\sum_{j=m(T)+1}^{p} \|\gamma_j\|\right)^2\right) = O\left(T \sum_{j=m(T)+1}^{p} j^{\frac{1}{2}} \|\gamma_j\|\right) = o(T);$$

to prove (A.3) for $i = 3$.