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**The Macroeconomic Role
of Unemployment Compensation**

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Abstract
The standard motivation for unemployment compensation is consumption smoothing and most papers in the literature have analyzed trade-offs involving consumption smoothing and moral hazard. This paper shows how such policy can increase output by enhancing the assignment of workers to jobs in the face of firm productivity heterogeneity and skill-biased technological change. It shows that in order to do so policy needs to be a function of the properties of the firm's productivity distribution. The paper undertakes an empirically-grounded, normative analysis of this issue. The analysis also bears upon the wage distribution, showing how optimal unemployment compensation policy is affected by wages and affects them in turn. A key insight emerging from the analysis is that the degree of firm productivity heterogeneity, in terms of skewness and variance, matters for the design of the time path of unemployment compensation.

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THE MACROECONOMIC ROLE OF UNEMPLOYMENT COMPENSATION [1]

1 Introduction

While there is growing evidence on the importance of productivity dispersion across firms,[2], there is little modelling of policy in the presence of such dispersion. In particular, unemployment compensation policy, which is a key macroeconomic and labor market policy tool, has, generally, not been linked with such productivity differences. Similarly, there has been no discussion of the possible response of unemployment compensation policy to technological change. This paper shows how such policy can increase output by enhancing the assignment of workers to jobs in the face of firm productivity dispersion and technological change. It shows that in order to do so policy needs to be a function of the properties of the firms productivity distribution. The paper undertakes an empirically-grounded, normative analysis of this issue.

More specifically, the analysis formulates policy as a function of two key properties of the productivity distribution: its variance, which quantifies the extent of dispersion, and its skewness, which quantifies its highly uneven nature. Noting that productivity and wage distributions are related and that wage dispersion and wage inequality are high on the research agenda,[3] the analysis bears upon the wage distribution as well. The paper shows how optimal unemployment compensation policy is affected by wages and affects them in turn.

The standard motivation for unemployment compensation is usually the provision of a tool for consumption smoothing over periods of employment and unemployment. Indeed, most of the literature has been concerned with trade-offs involving consumption smoothing and moral hazard. While it has been recognized that even when agents are risk-neutral unemployment compensation

[2] As found, for example, by Abowd, Kramarz and Margolis (1999), Davis, Haltiwanger and Schuh (1996), and Hatiwanger, Lane and Speltzer (1999, 2006). See the discussion in Section 5 below.

[3] Particularly with a growing sense of labor market polarization, see, for example, Autor, Katz and Kearney (2006).

can be used to induce them to search more intensively, thereby improving the quality of matches, not much attention has been devoted to the macroeconomic role of unemployment compensation policy.[4]

The paper derives an optimal time path for unemployment compensation policy. The intuition for this path may be summarized as follows. Consider a frictional environment where the assignment of unemployed workers to jobs with heterogenous productivity is uncoordinated and governed by a random matching process. All firms pay the reservation wage. At the beginning of their unemployment spell agents are relatively more choosy and tend to await better job offers. Later on, the reservation wage drops and agents are willing to take less attractive job offers. Heterogenous firms respond by introducing endogenous market segmentation. In equilibrium, more productive firms offer higher wages, anticipating a tighter 'sub-market', namely lower vacancy risk. Less productive firms offer lower wages and face higher vacancy risk as only long-term unemployed workers will accept the offer. A declining unemployment compensation profile results in voluntary unemployment by the short term unemployed but features improved matching, as it shifts the labor force towards the more productive firms. A random matching environment thus turns assortative via compensation policy. The mechanism described implies that heterogenous firms respond to the reservation wage time-profile induced by unemployment compensation policy. Wage setting takes this reservation profile into account so as to attract workers, while unemployment compensation policy takes wage setting into account to produce output gains. Optimal unemployment compensation policy is then shown to crucially depend on the nature of the productivity distribution, in particular on its variance and skewness. Note, too, that inducing agents to search more intensively is essentially a Pigouvian motive, i.e. internalizing positive matching externalities.

The paper makes two main contributions: first, it shows how the properties of the productivity distribution should affect unemployment compensation policy, providing a link between two

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[4]There is some work on productivity growth and policy [see for example Mortensen and Pissarides (1998)] and on the macroeconomic effects of unemployment compensation [see for example Pissarides (1998)] but these lines of research typically do not relate to the properties of the productivity distribution.

key issues that were unrelated. This link is mediated by the wage posting policies of firms that take both productivity and policy into account. A key point made is that the degree of productivity heterogeneity, in terms of skewness and variance, matters for the adoption of a flat unemployment compensation time profile or a declining path. Second, it demonstrates the assortative or assignment role of optimal unemployment compensation policy. It does so within a macroeconomic framework that is shown to be empirically relevant.

The paper is organized as follows. Section 2 presents the model. Section 3 discusses optimal unemployment compensation policy in the face of firm productivity dispersion and demonstrates the results with an illustrative numerical solution. Section 4 shows how optimal policy is affected by skill-biased technological change. Section 5 discusses the empirical relevance of key elements of the model, while Section 6 presents the relation of the paper with the relevant literature. Section 7 concludes. Technical matters are relegated to appendices. In what follows we shall use the terms assortative matching, worker assignment, and worker sorting interchangeably.

2   The model

In what follows we describe the general set up (2.1) and then look at two alternative time paths for unemployment compensation policy: constant (2.2) and declining (2.3).

2.1   The General Set-Up

There is a continuum of workers whose measure is given by $L > 0$, and a continuum of firms whose measure is given by $M > L$. . Firms differ in the technology they possess. Each firm can post a vacancy, incurring no costs, w.l.o.g. Once the vacancy is filled, the job produces x units of the single perishable consumption good, which price in normalized to unity. Production terminates with an exogenous Poisson parameter, $s > 0$. Otherwise the vacancy produces nothing.

The technological parameter, x, is assumed to be distributed according to the cumulative distribution function:

$$G(\cdot) \sim [\underline{x}, \overline{x}]$$

with strictly positive densities.

Workers are ex-ante identical in all respects and are assumed to be risk neutral. Let us note that this homogeneity assumption renders the model more tractable but is not crucial for the analysis. If workers were heterogenous, what would be needed is that their degree of heterogeneity be lower than the extent of technological dispersion. Firms are assumed to be expected-profit maximizers. To close the general equilibrium model we assume that workers possess an equal stake in each of the firms.

In every period (time is assumed to be discrete) firms post wage rates. Then, unemployed workers are randomly assigned to vacancies. Without loss of generality we assume that each active firm posts a single vacancy. Upon receiving a job offer, the worker decides whether to accept or reject the offer. This can be termed 'voluntary unemployment'. We assume that firms do not offer a menu of wages. This assumption is due to the idea that the individual's unemployment spell is unobserved by firms, so firms cannot condition the offered wage rate on the length of unemployment experienced by the individual applicant. Another reason is the prevalence of anti-discrimination laws that do now allow a firm to offer different wages for equally productive workers, due to differences in unemployment duration.

We assume that search intensity is fixed and normalized to one application per period. We further assume that all unassigned individuals are eligible for unemployment compensation, and that unemployment compensation is financed by neutral lump-sum taxes levied on all individuals.

We now turn to analyze the optimal behavior of the agents in steady-state equilibrium under different unemployment compensation time paths.

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2.2 A Constant Compensation Profile

Consider first an unemployment compensation scheme with a constant time profile. The sole purpose of this discussion is to serve as a benchmark for the analysis which follows. Note that throughout we assume that agents search only when unemployed.

Following Diamond's (1971) seminal contribution, by the homogeneity of workers, and since on-the-job-search is ruled out and firms are committed to the wages posted prior to the arrival of the job applicants, the wage distribution collapses to a singleton. Denote the equilibrium wage rate, which coincides with the uniform reservation wage, by w. It therefore follows that:[5]

$$w = a + h \tag{1}$$

where a and $h \leq \underline{x}$ denote the constant unemployment compensation and the imputed value of leisure, respectively.

The typical worker's optimal acceptance rule is trivial, namely accept the first offer received. Turning next to firms, it is easy to observe that firms, provided that they decide to operate, will choose to offer the uniform wage rate, w. This emerges from the fact that workers are assumed not to search on the job. Thus, applicants accept any job offer above the reservation instantaneously, and firms choose to offer precisely the reservation wage. Cutting this wage even slightly will increase vacancy risk to infinity, making profits drop to zero. Offering above it does not bring any gain, as it does not affect the vacancy risk because of the reservation strategy.

Firms differ in their productivity and correspondingly need to take a strategic decision, whether to enter the market or remain idle. Given w, the equilibrium wage rate, all firms possessing a technology $x \geq w$ will participate, for all expect the arrival of applicants with strictly positive probability.

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[5]We assume that unemployment compensation is financed by lump-sum taxation, and so the reservation wage rate only depends on the compensation level a and not on the tax rate, which thus has no distorting effect on workers' acceptance decisions.

Denote by U and V, the measure of unemployed workers and unfilled vacancies in steady state equilibrium, respectively. Let F denote the steady state measure of active firms, which is also the number of jobs, filled or vacant, in the economy. In equilibrium the following conditions hold true, in addition to the wage determination condition given by equation (1) above:

$$L - U = F - V \tag{2}$$

$$M\left(1 - G(w)\right) = F \tag{3}$$

$$m(U, V) = s\left(F - V\right) \tag{4}$$

The interpretation of the equations above is straightforward. Equation (2) defines the equilibrium condition according to which the measure of matched workers on the left-hand side is equal to the measure of filled vacancies on the right-hand side. Equation (3) defines the entry condition, given that the prevailing wage rate is w. Equation (4) states the familiar worker flow condition (the Beveridge curve). We assume a standard constant-returns to scale matching function, where $m(U, V)$ denotes the number of successful matches, $m(U, V)$ is strictly increasing with respect to its two arguments and $m\left(U, V\right) < \min\left(U, V\right)$. On the left-hand side we have, therefore, the flow into the pool of filled vacancies (successful matches). The right-hand side gives the flow out of the pool of filled vacancies.

With equations (1)-(4) we can solve for the equilibrium recursively, for any level of the constant level of unemployment compensation a. First we substitute for w (from (1)) into (3) and obtain an explicit solution for F. We then obtain a system of two equations ((2) and (4)) solved for two unknowns, U and V.[6]

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[6]Note that the solution is unique, as equation (2) is upward sloping in U-V space, whereas equation (4) is downward sloping, so that there exists a single crossing of the two curves.

By relaxing the fixed arrival rate paradigm of conventional wage-posting search equilibrium models, we obtain the inherent trade-off between sorting and employment using an extremely simple framework. To see this, let welfare be measured by per-capita utility, that is per-capita consumption plus the value imputed to per-capita leisure. By lowering a, the constant unemployment compensation , the wage rate, w, is lowered, thereby reducing unemployment. On the other hand, the fall in w, brings in less productive firms (the lower tail of the productivity distribution) that previously failed to break even.

More formally, given the welfare measure defined above, the maximization problem solved by the social planner is given by:

$$\max_{a,\tau}\{(L-U)E[x \mid x > w] + Uh\} \tag{5}$$

s.t. equations (1)-(4), and the budget constraint, assuming no revenue needs for the government, given by:

$$Ua - L\tau = 0 \tag{6}$$

where, $E[\cdot]$ denotes the expectation operator, and τ denotes the uniform lump-sum tax.[7]

Using equation (3), the maximization may be re-formulated as a function of F:

$$\max_{F}\{(F-V)\left(E[x \mid x > G^{-1}(1 - \frac{F}{M})] - h\right) + Lh\} \tag{7}$$

subject to equations (2)-(4) and (6). Note that G^{-1} is well defined since by assumption densities are strictly positive. Equations (5) and (7) indicate that we can solve for the optimum by setting the number of posted vacancies F optimally, and implement it via setting the wage w through the determination of unemployment compensation a. The lump sum tax,τ, is then set to satisfy the budget constraint in (6)

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[7]Note that in the objective in (5) the fiscal instruments a and τ do not appear explicitly. This derives from the fact that individuals are assumed to be risk-neutral and the budget constraint is balanced.

By fully differentiating equation (2) and (4) with respect to F, it follows that $\frac{dV}{dF} < 1$, thus the term $F - V$ is rising with respect to F. At the same time the second term ($E[x \mid x > G^{-1}(1 - \frac{F}{M})]$) decreases, since firms possessing technologies of lower productivity enter the market, thereby reducing expected productivity. The optimal F balances these two opposing effects, namely employment versus enhanced matching. Note that without unemployment compensation (i.e., $a = 0$) wages are determined by the imputed value of leisure (h) and are, therefore, independent of the productivity distribution. Thus, unemployment compensation plays a corrective role by internalizing externalities associated with the matching frictions in the labor market.

2.3 A Declining Time Path

We now allow for a declining unemployment compensation profile. We confine attention to a two-tiered regime, in which agents are eligible for a short period of regular unemployment compensation, followed by an indefinite period of reduced compensation, which we refer to as income support. Let z denote the unemployment compensation and let the level of income support be denoted by a ($z > a$). We assume that z is paid during the first two periods of unemployment and that all agents who exhaust their eligibility for unemployment compensation are henceforth indefinitely eligible for income support. Later on, we relax the two period assumption. Unemployment compensation eligibility is assumed to be independent of work history, for simplicity.

First, consider the intuition. A declining profile implies a non-degenerate wage distribution, while, as we saw, with a constant profile, there exists a unique wage rate in equilibrium. This key feature derives from the wage-posting setting and the fact the agents search only when unemployed. While agents are ex-ante identical in all respects, the declining profile implies that short-term unemployed agents, faced with a non-degenerate distribution of wage offers, will have a higher reservation wage rate than long-term unemployed agents who have already exhausted their eligibility for unemployment compensation. The declining unemployment compensation profile yields ex-post heterogeneity among ex-ante identical agents. For a sufficiently dispersed set of technologies, a

two-wage equilibrium exists. Voluntary unemployment by short-term unemployed agents induces firms possessing more productive technologies to offer higher wages, thereby reducing their vacancy risk. Assuming two levels of unemployment compensation and two periods when the first level is in place, we can restrict attention to a two-wage equilibrium.

We turn now to a formal presentation of the model and the optimal policy.

Unemployed Workers Value Function. We start with the value functions for a typical agent in the economy. There are four states to consider: employment, two states of short-term unemployment for each period of unemployment compensation eligibility, and long term unemployment (income support recipients). We start with the three unemployment states, denoting by H_1, H_2, H, the continuation value functions for short-term unemployed agents during the first and second period of the unemployment spell, respectively, and income support recipients. In steady-state equilibrium the following asset-value conditions hold:

$$H_1 = z + h + \beta[\overline{n} \max(\overline{J}, H_2) + \underline{n} \max(\underline{J}, H_2) + (1 - \overline{n} - \underline{n})H_2] \tag{8}$$

$$H_2 = z + h + \beta[\overline{n} \max(\overline{J}, H) + \underline{n} \max(\underline{J}, H) + (1 - \overline{n} - \underline{n})H] \tag{9}$$

$$H = a + h + \beta[\overline{n} \max(\overline{J}, H) + \underline{n} \max(\underline{J}, H) + (1 - \overline{n} - \underline{n})H] \tag{10}$$

where $\overline{J}$, $\underline{J}$ denote the high and low wage jobs continuation values, respectively, $\overline{n}$, $\underline{n}$ denote re-employment chances in firms offering high and low wage rates, respectively, and $\beta \in (0,1)$ denotes the discount factor. In addition to the value of leisure h, short-term unemployed get unemployment compensation z and long-term unemployed get income support a. In the following period they move either to a job or to the next stage of unemployment.[8]

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[8]Note that the assignment of unemployed individuals to unfilled vacancies is assumed to occur at the end of each period. Thus, although there are three stages of unemployment, there will be only two reservation wages, and hence two wage rates, in equilibrium.

Employment Value Functions. The steady state value functions for the two types of jobs, those with high wage rate and low wage rate, respectively, are given by $\overline{J}$ and $\underline{J}$:

$$\overline{J} = \overline{w} + \beta[(1-s)\overline{J} + sH_1] \tag{11}$$

$$\underline{J} = \underline{w} + \beta[(1-s)\underline{J} + sH_1] \tag{12}$$

Workers get the relevant wage in each job and face the exogenous separation probability s.

Wages. By virtue of wage posting and the assumption that individuals search only while unemployed, in equilibrium firms offer such wages so as to satisfy the following reservation asset values:

$$\overline{J} = H_2 \tag{13}$$

$$\underline{J} = H \tag{14}$$

It is easy to verify that the reservation wage property is satisfied, by observing that $\overline{J} = H_2 > \underline{J} = H$. Thus, short-term unemployed agents will accept only high wage offers during their first period of unemployment compensation eligibility. All other unemployed agents will accept any offer.

Manipulating equations (8)-(14) yields the following two conditions which determine the equilibrium wage offers as a function of the policy parameters (z and a), the re-employment probabilities ($\overline{n}, \underline{n}$), the discount factor ($\beta$) and the separation rate (s):[9]

$$z - a = \frac{\overline{w} - \underline{w}}{1 - \beta(1-s)} \tag{15}$$

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[9]See appendix A for the full derivation.

$$\overline{w} = (z - a)[(1 - \beta) + \beta\overline{n} - \beta^2 s(1 - \overline{n})] + h + a \tag{16}$$

Matching. The re-employment probabilities in firms offering the high and low wage rate, respectively, are given by:

$$\overline{n} = \frac{m(U, V)}{U} \frac{\overline{V}}{V} \tag{17}$$

$$\underline{n} = \frac{m(U, V)}{U} \frac{\underline{V}}{V} \tag{18}$$

where $\overline{V}$ and $\underline{V}$ denote the measures of unfilled vacancies posted by firms offering the high wage rate and low wage rate, respectively, $V = \overline{V} + \underline{V}$ denotes the aggregate measure of unfilled vacancies and U denotes the measure of aggregate unemployment.

Steady State Flow Equations. The standard steady state flow conditions are given by:

$$U_2 = (1 - \overline{n})U_1 \tag{19}$$

$$(U - U_1 - U_2)(\overline{n} + \underline{n}) = (1 - \overline{n} - \underline{n})U_2 \tag{20}$$

$$m(U, V)\frac{\overline{V}}{V} = s\left(\overline{F} - \overline{V}\right) \tag{21}$$

$$m(U, V)\frac{\underline{V}}{V}\frac{(U - U_1)}{U} = s\left(F - \overline{F} - \underline{V}\right) \tag{22}$$

where , U_j, $j = 1, 2$ denotes the measure of unemployed agents during the jth period of unemployment compensation eligibility and $\overline{F}$ denotes the measure of vacancies posted by firms offering the high wage rate in equilibrium.[10]

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[10] Note that the last term on the left-hand-side of (22) captures the voluntary unemployment of individuals during

Steady State Equilibrium. To complete the characterization of steady state equilibrium we introduce the following conditions and interpret them:

$$L - U = F - \overline{V} - \underline{V} \tag{23}$$

$$F = M \left(1 - G(\underline{w}) \right) \tag{24}$$

$$\frac{m(U,V)}{V} \left(\widehat{x} - \overline{w} \right) = \frac{m(U,V)}{V} \frac{(U - U_1)}{U} \left(\widehat{x} - \underline{w} \right) \tag{25}$$

$$\overline{F} = M \left(1 - G(\widehat{x}) \right) \tag{26}$$

Equation (23) is the condition according to which the measure of filled vacancies is equal to the measure of employed agents. Equation (24) is a consistency condition, which requires that the total measure of active firms (hence posted vacancies) should be equal to the fraction of the firms possessing a technology above the lower bound wage rate times the measure of potential firms. Equation (25) determines the wage distribution by defining a cutoff technology, $\widehat{x}$, above which all firms maximize expected profits by offering the high wage rate, and below which all firms maximize expected profits by offering the low wage rate. To see this, note that the probability of filling a high-wage vacancy, given by $m(U,V)/V$, is higher than that of filling a low-wage vacancy, given correspondingly by $m(U,V)/V \cdot [(U - U_1)/U]$, by virtue of the voluntary unemployment by short-term unemployed. Thus, it follows from (25) that firms with productivity exceeding the threshold obtain higher expected profits by posting a high-wage vacancy, whereas firms with productivity below the threshold, prefer to post a low wage vacancy. Equation (26) is a consistency condition similar to (24) with regard to the firms offering the high wage rate in equilibrium.

the first period of unemployment. These individulas (of measure U_1), whose reservation wage is high, reject low-wage job offers. Thus, the number of successful matches is multiplied by an additional term, $\frac{U - U_1}{U} < 1$, to obtain the flow of newly formed low-wage jobs.

Optimality Problem. The optimality problem may be written as follows:

$$\max_{a,z,\tau}\{(F - \overline{F} - \underline{V})E[x \mid \underline{w} \le x \le \widehat{x}] \tag{27}$$

$$+ (\overline{F} - \overline{V})E[x \mid x > \widehat{x}]$$

$$+ Uh\}$$

subject to the above 12 equations [equations (15) – (26)] and the budget constraint, given by:

$$(U_1 + U_2)z + (U - U_1 - U_2)a - L\tau = 0 \tag{28}$$

Equilibrium is defined by this system of equations, where a, z and τ are solved according to (27).

It can be shown that the above formulation, which focuses on choosing the levels of unemployment compensation and income support, can be mapped into the following equivalent structure, focusing on the allocation of production:

$$\max_{\underline{x},\widehat{x}}\left\{ M \int_{\underline{x}}^{\overline{x}} x dG(x) - \underline{V}\frac{\displaystyle\int_{\underline{x}}^{\widehat{x}} x dG(x)}{G(\widehat{x}) - G(\underline{x})} - \overline{V}\frac{\displaystyle\int_{\widehat{x}}^{\overline{x}} x dG(x)}{1 - G(\widehat{x})} + Uh \right\} \tag{29}$$

subject to equations (21), (22),(23), (26) and:

$$U_1 = s\left(L - U\right). \tag{30}$$

$$F = M\left(1 - G(\underline{x})\right) \tag{31}$$

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In other words, the policymaker chooses $\underline{\underline{x}}$, the threshold productivity level above which firms enter the labor market, and $\widehat{x}$, the productivity level above which firms offer the high wage rate in equilibrium. The optimal allocation can be implemented by appropriate wage rates, $\underline{w}$ and $\overline{w}$ (employing equations (24) and (25)) through the choice of the two unemployment compensation policy instruments a and z (employing equations (15) and (16)).[11] The lump-sum tax, τ, is then set to satisfy the budget constraint in (28). Note that the firm productivity distribution $G(x)$ enters into the objective function (29) and into the constraints (26) and (31).

A declining unemployment compensation schedule creates enhanced sorting by inducing 'voluntary' unemployment by individuals at the initial phase of their unemployment spell in addition to that attained by a flat schedule via the crowding out of low-productivity firms from the labor market. Thus, it serves to "fine tune" the matching process.

3 Optimal Unemployment Compensation Policy and the Productivity Distribution

The question we would like to address is a normative one, namely under what circumstances is a declining unemployment compensation profile optimal. Moreover, we want to link this policy with the properties of the firm productivity distribution. Intuitively it seems that the answer should relate to the extent to which the set of technologies is dispersed. As the above problem has no closed form solution, it needs to be addressed by numerical methods. We opt for the simplest possible set-up, i.e., a discrete distribution of technologies, which comprises two elements in its support. We find that it is a rich enough formulation to demonstrate the linkages between the moments of the productivity distribution and optimal unemployment compensation policy. Hence we are able to undertake a comparative statics analysis, which shows policy responses to changes in the characteristics of the productivity distribution. First, we characterize the types of worker sorting

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[11]The flat UI profile is obtained as a special case of the allocation problem in (29) when $\underline{\underline{x}} = \widehat{x}$. In such a case, a would be set equal to z, so as to induce $\underline{w} = \overline{w}$.

which emerge under the different unemployment compensation paths analyzed above (3.1). We then study the relationship between the welfare levels induced by the different paths and the properties of the productivity distribution (3.2). Finally, we examine the case where policy determines the duration of the first tier of the unemployment compensation path (3.3).

3.1 Firm Technologies, Worker Sorting, and Optimal Unemployment Compensation Paths

Suppose there are two technologies, denoted by $\overline{x}$ and $\underline{x}$, where $\overline{x} > \underline{x} > h$. Denote by $0 < p < 1$ the fraction of firms, which measure is given by $M > 0$, possessing $\overline{x}$. We henceforth restrict attention to pure-strategy equilibria.

There are three equilibrium configurations to consider. A benchmark case is the one in which the unemployment compensation profile is constant and the compensation is set sufficiently low, so that all firms are active in equilibrium. We refer to this configuration as maximum employment or no-sorting, interchangeably. A second case, is the one in which the unemployment compensation time profile is constant, but the compensation is set high enough so as to crowd out the low-productivity firms. We refer to this configuration as high-sorting. In the third configuration both technologies are active, but due to a declining unemployment compensation time profile, voluntary unemployment by short-term unemployed agents yields partial-sorting.

Denote by N^{HS}, N^{PS} and N^{NS}, the steady-sate measures of employed workers in the high-sorting, partial-sorting and no-sorting configurations, respectively (where $N = L - U$).

We have already observed that $N^{NS} > N^{HS}$ (see the characterization of equilibrium in the constant profile unemployment compensation regime). Close inspection of (21)-(23) yields that $N^{NS} > N^{PS}$. This condition derives directly from the existence of 'voluntary' unemployment in the partial-sorting configuration.[12]

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[12] By aggregating (21) and (22) and comparing the expression to (4), it can be seen that for a given measure of active firms and for any level of unemployment, the market clears with a higher measure of aggregate unfilled vacancies relative to the constant UI regime.

However, one cannot relate N^{HS} and N^{PS} without making further assumptions. This ambiguity derives from the trade-off between unemployment in the high sorting case, when a lower measure of firms participates, and 'voluntary' unemployment in the partial sorting case, due to workers' ex-post heterogeneity emerging from the declining unemployment compensation profile.

Formulating the welfare measures for each one of the three configurations (denoted W, and maintaining our definition of welfare from the previous section), we obtain the following:

$$W^{NS} = N^{NS}[p\overline{x} + (1-p)\underline{x} - h] + Lh \tag{32}$$

$$W^{PS} = N^{PS}[q\overline{x} + (1-q)\underline{x} - h] + Lh \tag{33}$$

$$W^{HS} = N^{HS}[\overline{x} - h] + Lh \tag{34}$$

where $q = \frac{(pM - \overline{V})}{M - \overline{V} - \underline{V}}$ and it is easy to show, using (21) and (22), that $1 > q > p$.

Equation (32) is the "benchmark" case: a fraction p of workers go to the high technology firms and $1-p$ to the low technology ones and N^{NS} is determined solely through random matching. At the other extreme there is (34) with a constant unemployment compensation profile: here only the high technology is in operation. The two equations (32) and (34) express the trade-off between employment and sorting as $N^{NS} > N^{HS}$ and $[\overline{x} - h] > [p\overline{x} + (1-p)\underline{x} - h]$. The intermediate case is that of a declining unemployment compensation profile − equation (33). This policy balances the two considerations, employment and productivity. In order to increase the degree of sorting in the market, namely to shift workers away from low-productivity firms to high-productivity ones, the policymaker sacrifices a rise in unemployment. The optimal policy depends on the properties of the productivity distribution, as illustrated below.

It is easy to verify the existence of policy instruments that implement the social optimum. For the no sorting case set $\underline{x} \geq a$ and for the high sorting case set $\overline{x} \geq a \geq \underline{x}$. For the par-

tial sorting configuration, see appendix B. Note that if taxation were distortionary,[13] then when $N^{PS} > N^{HS}$ the case for a declining unemployment compensation profile is reinforced. This is so because expenditures on unemployment compensation are lower under the PS regime due to lower unemployment.

3.2 Unemployment Compensation Policy and the Variance and Skewness of the Productivity Distribution

We now study the relationship between the optimal unemployment compensation time path, welfare, and the productivity distribution. To do so we fix the average productivity in the economy and denote it by μ, where $\mu = p\overline{x} + (1-p)\underline{x} > h$. Consider a mean-preserving spread $\triangle \geq 0$ whereby $\overline{x} = \mu + \triangle$ and $\underline{x} = \mu - \frac{p\triangle}{1-p}$, with $\underline{x} \geq h$. Then the moments of this productivity distribution are given by:[14]

mean	μ
standard deviation σ	$\sqrt{\frac{p}{1-p}}\triangle$
skewness	$(1-2p)\sqrt{(1-p)p}$

Reformulation of equations (32) - (34) yields the following:

$$W^{NS} = N^{NS}[\mu - h] + Lh \tag{35}$$

$$W^{PS} = N^{PS}[\mu - h] + N^{PS}\frac{q-p}{1-p}\triangle + Lh \tag{36}$$

$$W^{HS} = N^{HS}[\mu - h] + N^{HS}\triangle + Lh \tag{37}$$

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[13]Recall that we have assumed throughout that benefits are financed by lump-sum taxation.

[14]Note that when $p = 0.5$ then $\sigma = \triangle$ and the skewness is zero; when $0.5 < p < 1$, the skewness is negative, and when $0 < p < 0.5$ the skewness is positive.

We turn now to characterize the general properties of optimal policy (in sub-section 3.2.1) and then present a numerical simulation that provides further illustration of this policy (in sub-section 3.2.2).

3.2.1 General Properties

The following proposition establishes some general properties of optimal unemployment compensation policy. It refers to the following illustrative figure, where we depict each of the three configurations in welfare-productivity spread space $(W - \triangle)$.[15] Note that social optimum is given by the welfare frontier (the upper envelope of the figure).

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[15]The mean and the skewness in the figure are fixed. Due to the fact that the skewness depends on the parameter p only, shifts in the (mean preserving) spread parameter $\triangle$ measure shifts in the standard deviation of the productivity distribution.
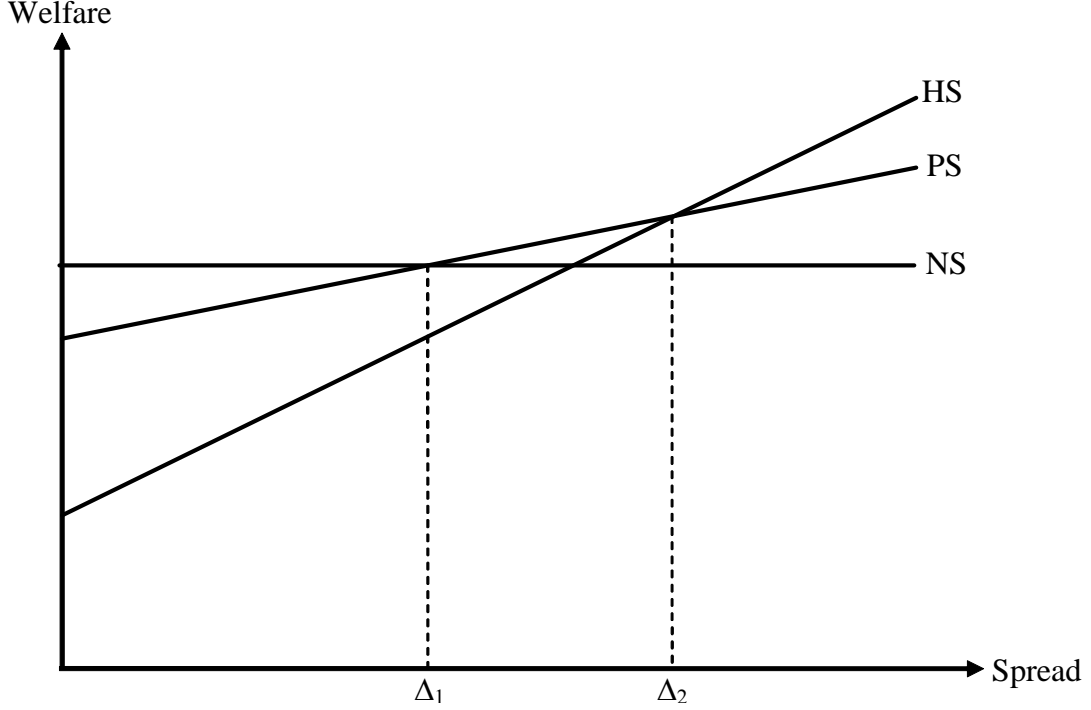
Welfare

Figure 1: Optimality and Productivity Variance (spread)

Proposition 1 *Holding fixed the mean (μ) and the skewness (p), there exist thresholds $\triangle_1$ and $\triangle_2$ where $0 < \triangle_1 \leq \triangle_2 \leq \frac{1-p}{p}\left[\mu - h\right]$, such that: for all $0 < \triangle \leq \triangle_1$ social welfare is maximized by the no sorting configuration, for all $\triangle_1 \leq \triangle \leq \triangle_2$ social welfare is maximized by the partial sorting configuration, and for all $\triangle_2 \leq \triangle \leq \frac{1-p}{p}\left[\mu - h\right]$ social welfare is maximized by the high sorting configuration.*

The proof is given in Appendix B.

While the discrete two-technology case is stylized, it provides us with some clear insights regarding the forces at play. When the set of technologies is almost degenerate, i.e., the spread $\triangle$

converges to zero, there is little to gain from introducing 'voluntary' unemployment and shifting the pool of workers away from low-productivity firms towards high-productivity ones. In this case unemployment compensation is redundant. When, however, technologies are sufficiently dispersed, the partial sorting configuration is preferred to the no-sorting one. In graphical terms, the partial sorting solution dominates in the interval $\triangle_1 - \triangle_2$. This interval is well-defined only when there is a substantial difference between the intercepts of the HS and PS schedules (on the vertical axis). This implies a sufficient degree of right-skewness of the productivity distribution. When dispersion is very large, it is desirable to increase unemployment by eliminating employment at low-productivity firms altogether and obtaining high (full) sorting. For intermediate values of the technological spread, social welfare is maximized by the partial sorting configuration, hence a declining unemployment compensation profile is optimal.[16] In the two other cases, a constant profile, either no sorting or high-sorting, suffices.

3.2.2 Numerical Solution

We turn now to a numerical solution of the model. We shall assume a standard constant-returns-to-scale Cobb-Douglas matching function of the form $m(U, V) = \gamma U^\alpha V^{1-\alpha}$, where $0 < \alpha, \gamma < 1$. In Figure 2 we plot four different curves in skewness (skew) – s.d. (σ) space.[17]

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[16] This discrete example extends to continuous cases with sufficient skewness. In cases that are not skewed (or not sufficiently skewed), such as the uniform distribution, the constant profile dominates.

[17] The figure is based on the numerical solution using the following parametric assumptions: $M = 100, L = 70, s = 0.01, h = 0, \mu = 10, \alpha = 0.5$ and $\gamma = 0.1$.
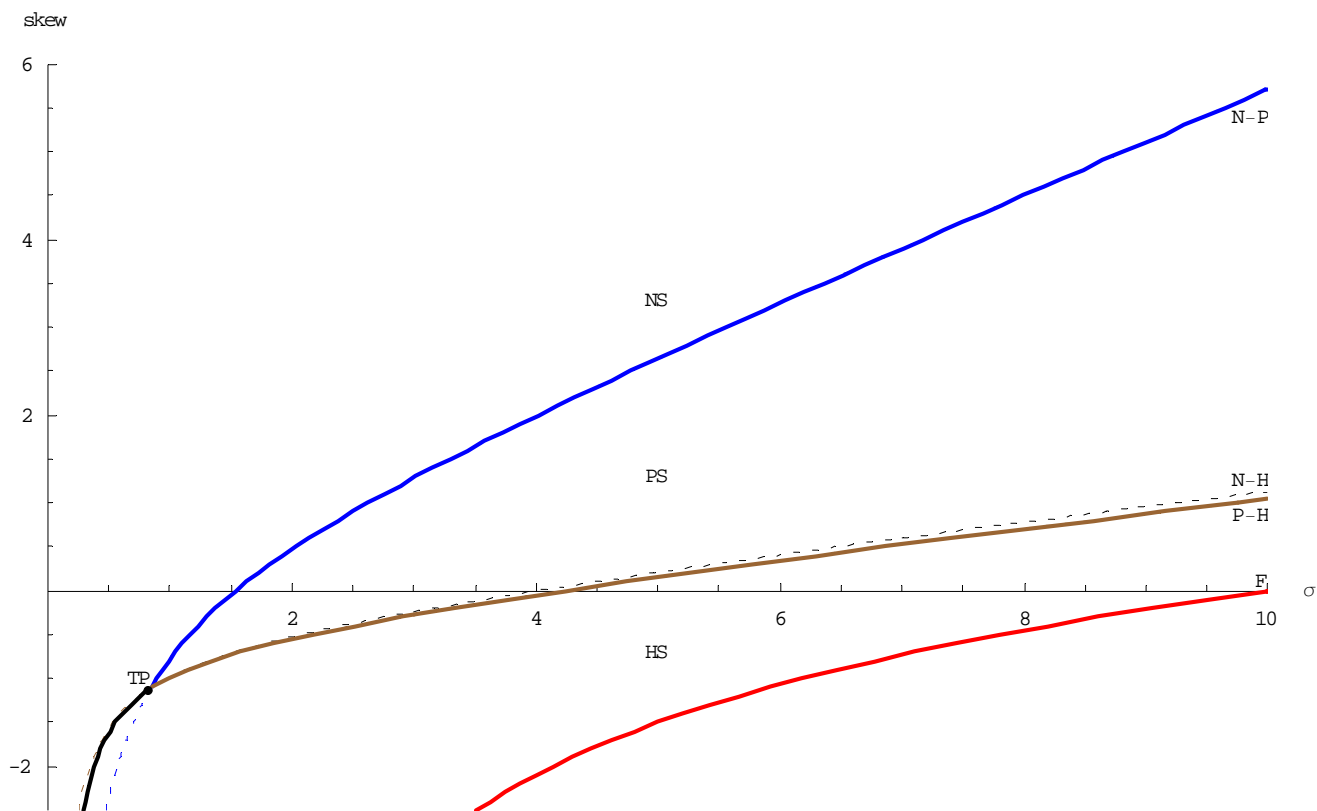
Figure 2: Skewness and Variance Relationships

The four curves depict the following:

The curve labeled F represents a feasibility constraint defined by the non-negativity of the lower-bound technology, $\underline{x}$. The set of feasible points lies above the curve.

Along the N-H curve the welfare attained by a no-sorting equilibrium configuration is equal to the welfare attained by the high-sorting configuration, where the lower-bound technology is crowded out of the market. Note that as the welfare in the no-sorting configuration case depends on the mean productivity of the distribution of technologies, and is completely insensitive to the other two moments, the curve N-H is essentially an indifference curve for the high-sorting configuration

along which the welfare attained by the high-sorting configuration is constant. The positive slope of the curve is due to the fact that the gains from sorting increase with respect to the standard deviation of the distribution of technologies and decrease with respect to its skewness. A larger value of skewness implies that a higher weight is shifted to the lower bound technology which is crowded out in the high-sorting configuration, which, in turn, implies a higher unemployment rate. Higher dispersion, captured by a higher value of the standard deviation, implies that the upper bound technology is farther away from mean productivity, which implies larger gains from sorting. As we move either downwards (decreasing skewness) or rightwards (increasing the s.d.) the welfare obtained by the high-sorting configuration rises. Thus, for any point which lies below the N-H curve, the high-sorting configuration dominates the no-sorting one and vice-versa.

The N-P curve represents points for which the welfare attained by the partial-sorting configuration is equal to the welfare attained by the no-sorting one. The positive slope of the curve derives from the same reasons given for the N-H curve. Similarly, any point lying above the curve implies that the no-sorting configuration dominates the partial-sorting one and vice-versa.

Along the P-H curve lie points for which the partial-sorting and the high-sorting configurations attain the same level of utility. The positive slope derives from the fact that the gain from sorting (shifting from partial-sorting to high-sorting) is increasing with respect to the standard deviation and is decreasing with respect to the skewness. It follows that for any point above the P-H curve, the partial-sorting configuration dominates the high-sorting configuration and vice versa.

Figure 3 depicts on a magnified scale the points in the neighborhood of the point T.P. in Figure 2, where all three curves (N-P, N-H and P-H) intersect.

skew
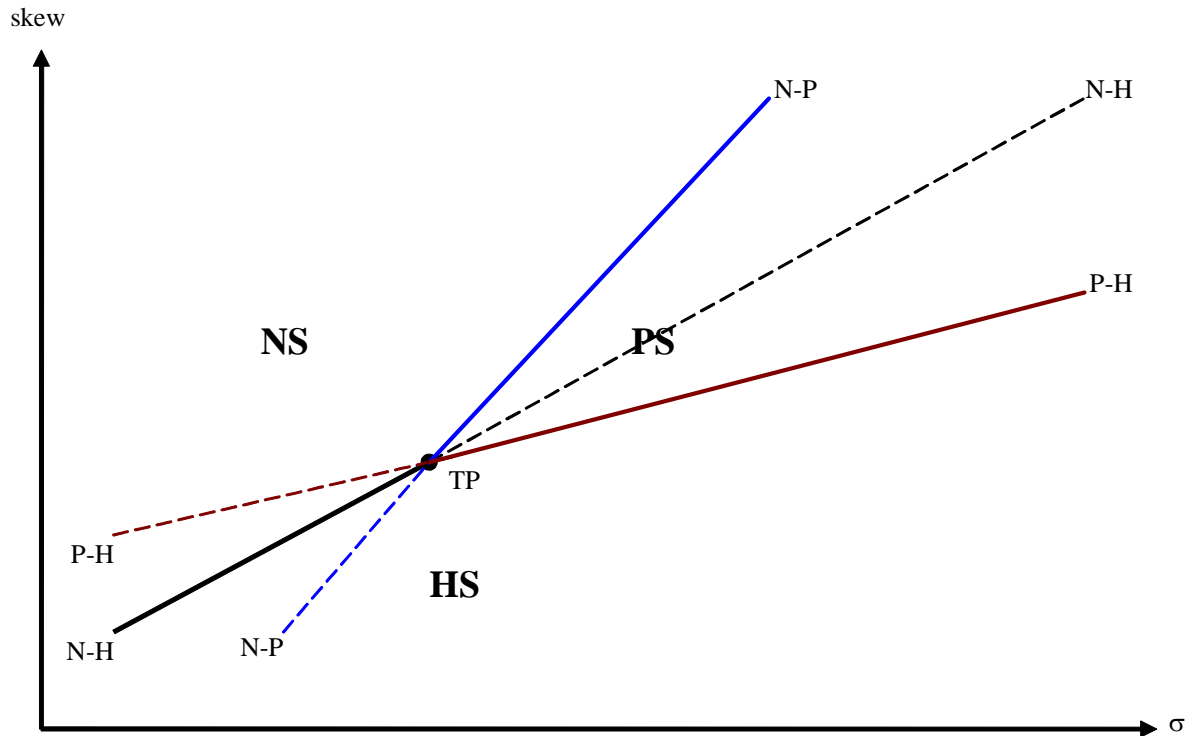
Figure 3: Optimal Unemployment Compensation Regions

Note that by construction, at the point T.P. all three different configurations yield the same level of utility. The solid parts of the curves define a fork-shape borderline which divides up the space into three regions, according to the dominating equilibrium configuration. For any point which lies above the upper envelope of the N-P and the N-H curves, the no-sorting configuration is socially desirable. By the same reasoning for any point which lies below the lower envelope of the P-H and the N-H curves, the high-sorting configuration is socially desirable. In the remaining region, the partial sorting is the socially desirable configuration

A number of conclusions emerge.

(i) For a wide range of parameters, the partial sorting equilibrium configuration constitutes the social optimum, implying a declining unemployment compensation time path.

(ii) For this partial sorting to prevail, skewness has to be sufficiently large, exceeding the skewness associated with the T.P. intersection point. As already demonstrated in Figure 1, for a given skewness of such magnitude, partial sorting dominates the other two configurations for intermediate levels of dispersion (standard deviation).

(iii) Similarly, for intermediate levels of skewness, partial sorting dominates when dispersion is large enough, exceeding the standard deviation associated with the T.P. intersection point.

To see the rationale behind these results, note that when skewness is sufficiently small, the costs associated with crowding out the low-productivity firms are relatively small. In such a case, the high-sorting configuration prevails, as it attains enhanced matching in exchange for a moderate loss in employment. Similarly, when skewness is large enough, the costs associated with foregoing the low-productivity firms are significant, rendering the no-sorting configuration the socially desirable equilibrium, as the gains from sorting (partial or high) are outweighed by the increase in unemployment. In the intermediate range, partial sorting attains the optimal balance between employment and matching considerations. As productivity dispersion rises, the ranges in which the high-sorting and the partial sorting configurations prevail expand, at the expense of the no-sorting configuration. This is due to the increase in the gains from sorting.

3.3 Duration Policy

In the previous sub-sections we have confined attention to two kinds of unemployment compensation schemes. We compared a flat time profile, where a constant compensation is paid indefinitely, to a two-tiered scheme, where individuals are eligible for high compensation during a limited period, and, thereafter, are eligible for reduced compensation indefinitely. In what follows we generalize our duration analysis. Denoting the duration of the first tier of the unemployment compensation scheme by $\bar{t}$, we now allow for $0 \leq \bar{t} \leq \infty$, while we have thus far considered only the two schemes $\bar{t} = 0$, i.e., the flat profile, and $\bar{t} = 2$, i.e., the declining profile with two periods for the higher compensation. We retain the structure of the two-tiered unemployment compensation regime, i.e.,

regular unemployment compensation and income support, and allow the number of periods in the first tier to be optimally determined by policy. We examine optimal policy, analyzing the forces that affect optimal duration in the general setup. In an appendix we re-examine the relationship between optimal duration and the properties of the productivity distribution, in particular its standard deviation and skewness in the two-technology case.

In order to maintain the simple property of a two-wage equilibrium, but gain more flexibility with respect to duration policy, we confine attention to the case of myopic agents.[18] Thus we assume that the discount factor is given by $\beta = 0$. This simplifying assumption is made in order to focus the analysis on the essential duration implications. The insights gained carry over to the more general case $(0 \leq \beta \leq 1)$.[19] Myopia implies that along each of the two tiers, the reservation wages, and hence the equilibrium wages, are constant. Regardless of the length of the first tier of the unemployment compensation regime, these will be given respectively by:

$$\underline{w} = a + h$$

$$\overline{w} = z + h$$

---

[18] A different approach to simplify the wage structure in equilibrium is to assume a two tiered system (like the one we have) but allow for a stochastic shift between the two phases of the unemployment compensation policy. Under such a regime, the expected duration of the unemployment compensation phase can be determined optimally, by changing the switching probability across the two phases.

[19] When agents are non-myopic, namely, $\beta > 0$, individuals' reservation wages decline over the first phase of the unemployment benefits schedule. That is, as individuals approach the period in which they exhaust their eligibility for unemployment benefits, they are willing to accept lower wage offers. Hence without myopia there will be more than two wage rates. In particular, if the duration is set to $\widetilde{t}$ periods there may potentially be $\widetilde{t}$ wage rates in equilibrium, making the model intractable. In order to keep the model tractable with only two wage rates, but allow for the flexibility with respect to the level of search-induced unemployment, we assume myopia. In the case of two technologies this is a harmless assumption since there will be at most two wage rates under any duration length, even without myopia

This result can be verified by substituting into the wage determination equations (15) and (16).

Under this set-up the social planner has one additional degree of freedom in choosing optimal unemployment compensation policy, namely, setting the duration of the first tier of the regime, $\bar{t}$. Let U_t denote the measure of unemployed agents during period t of unemployment compensation (first-tier) eligibility. During the first $\bar{t}-1$ periods of unemployment compensation eligibility agents will only accept high-wage offers. Maintaining our notation from previous sections, it follows that:

$$U_t = U_{t-1}(1 - \overline{n}) \quad 2 \leq t \leq \bar{t}$$

Denote by $\overline{U}$ the aggregate measure of unemployed agents whose reservation wage is high. By construction:

$$\overline{U} = \sum_{t=1}^{\bar{t}-1} U_t = U_1 \frac{(1 - (1 - \overline{n})^{\bar{t}-1})}{\overline{n}} \tag{38}$$

Reformulating the unemployment compensation optimization focusing on the production allocation problem (see equation (29)) yields:

$$\max_{\underline{x}, \widehat{x}, \overline{U}} \left\{ M \int_{\underline{x}}^{\overline{x}} x dG(x) - \underline{V} \frac{\int_{\underline{x}}^{\widehat{x}} x dG(x)}{G(\widehat{x}) - G(\underline{x})} - \overline{V} \frac{\int_{\widehat{x}}^{\overline{x}} x dG(x)}{1 - G(\widehat{x})} + Uh \right\} \tag{39}$$

subject to the same constraints as above, or re-formulated wherever relevant, reproduced here:

$$m(U,V)\frac{\overline{V}}{V} = s(\overline{F} - \overline{V}) \tag{40}$$

$$m(U,V)\frac{V}{V}\frac{(U - \overline{U})}{U} = s\left[F - \overline{F} - \underline{V}\right] \tag{41}$$

$$L - U = F - \overline{V} - \underline{V} \tag{42}$$

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$$F = M[1 - G(\underline{\underline{x}})]]$$ (43)

$$\overline{F} = M[1 - G(\widehat{x})]]$$ (44)

$$\overline{U} \geq U_1$$ (45)

$$\overline{U} \leq \frac{U_1}{\overline{n}}$$ (46)

$$U_1 = s[L - U]$$ (47)

$$\overline{n} = \frac{m(U, V)\frac{\overline{V}}{V}}{U}$$ (48)

With (45) and (46) denoting constraints derived from (38) for the two limiting cases of $\overline{t} = 2$ and $\overline{t} \to \infty$.

Any change in duration policy, namely in $\overline{t}$, translates into a change in $\overline{U}$ (by virtue of (38)). To examine the implication of a change in duration policy, suppose that $\overline{U}$ is increased, fixing $\underline{\underline{x}}$ and $\widehat{x}$. Fully differentiating equations (40)-(42) with respect to $\overline{U}$ yields:

$$0 < \frac{dU}{d\overline{U}} < 1$$ (49)

It follows that the matching probability for a firm posting a vacancy offering the high wage rate, given by the term in brackets on the left-hand-side of (40), rises, whereas the corresponding matching probability for a low wage vacancy, given by the term in brackets on the left-hand-side of (41), declines. Thus the rise in $\overline{U}$ implies a higher aggregate level of unemployment (U) accompanied

by enhanced sorting, that is a shift from low-wage vacancies towards high-wage vacancies, which in equilibrium results in a shift from low-productivity firms towards high-productivity ones.

Balancing the two opposing forces will determine the optimum. To illustrate the point consider the following figure derived for a numerical solution of the model based on the two-point discrete example. The figure shows welfare as a function of duration, with the maximum of the function determining optimal duration:[20]
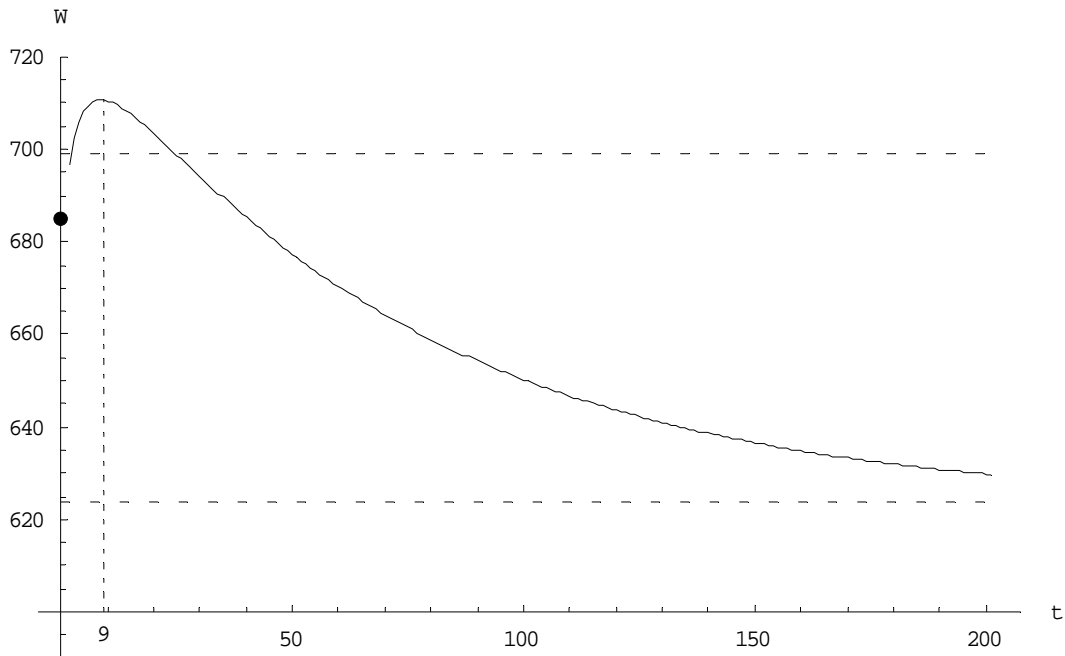
Figure 4: Optimal Duration

The black dot (W=685 and $\bar{t} = 0$) represents the welfare level attained by the no-sorting configuration (where duration is set to zero, both technologies operate and matching is purely

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[20] There is no particular importance to the numerical values here, but rather to the shape of the function. The following parameter values are used: $\bar{x} = 14.3, \underline{x} = 5.7, p = 0.5, M = 100, L = 70, s = 0.01, h = 0, \alpha = 0.1$ and $\gamma = 0.5$.

random across all firms). The upper dashed curve represents the welfare level (W=699) attained by the high-sorting configuration, where all low productivity firms are crowded out of the market and, again, duration is set to zero, so that workers are randomly assigned across all high productivity firms. The solid curve represents the welfare level associated with the partial sorting configuration (as a function of the duration of the first tier). The curve starts at $\bar{t} = 2$ (minimal duration of two periods). The solid curve is asymptotic to the lower dashed flat curve, which represents the welfare level (W=624) associated with the limiting case where duration of the first tier is set at infinity. As can be observed from the figure, the solid curve is initially rising (for $\bar{t} < 9$) and then monotonically falling (for $\bar{t} > 9$). This reflects the tradeoff between two opposing forces – matching versus unemployment. For sufficiently short durations, raising the duration yields matching gains that outweigh the costs associated with the rise in unemployment (thus overall welfare is rising with respect to $\bar{t}$). For long enough duration the balance of forces reverse, hence, the welfare is decreasing with respect to the length of duration. The optimum is obtained for the partial sorting where duration is set at $\bar{t} = 9$ This attains a higher welfare level than those associated with the no-sorting and the high sorting configurations, respectively.[21]

A general pattern that emerges from the figure is that the optimal duration will always be finite. That is, setting the duration to infinity (corresponding to the lower dashed flat curve in the figure) will never be the optimal solution. As can be observed from the figure (and will be the case under any parametric assumptions) implementing a partial sorting equilibrium and setting the duration to infinity will always be dominated by the high-sorting configuration (the upper dashed flat curve in the figure). The reason for this is simple. By setting the duration of the first tier of high benefits to be extremely long, nearly all unassigned workers will reject any offer received from a low productivity firm. In such a case, the gain from having only a small number of individuals

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[21]Note that when $\bar{t}^*$ is set then the control variable $\overline{U}$ is set according to (38). Note, also, that the other controls $\underline{x}, \widehat{x}$ in (39) are set as a function of the properties of the productivity distribution. In the numerical example, as we consider a two type discrete distribution, the two threshold levels are correspondingly set at the low productivity and the high productivity levels.

willing to accept job offers from low productivity firms is outweighed by the cost associated with the significant increase in voluntary unemployment, due to the massive job rejection. The economy can do better by crowding out low-productivity firms altogether, i.e., implementing the high-sorting configuration.

In appendix D we re-examine the relationship between the optimal duration and the properties of the productivity distribution (in the case of flexible duration). We demonstrate that our key insights in section 3.2.2 carry over to the more general case. Notably, the optimal degree of sorting (captured by the length of the duration of the first tier of benefits) is rising with respect to the dispersion of the firm productivity distribution and decreasing with respect to its skewness

4   Consequences of Technological Change for Policy

There is evidence according to which recent changes in technology have generated increases in the variance and skewness of the productivity distribution. Our analysis suggests that such changes should engender a response of optimal unemployment compensation policy. In this brief section we present the evidence and discuss the implications for policy.

Dunne, Foster, Haltiwanger, and Troske (2004) report that the between-plant measures of wage and productivity dispersion have increased substantially over recent decades. They build on the following logic proposed by Caselli (1999): technical change occurs through differential technology adoption by plants. If plants adopt new technologies at different rates, and new technology is skill- biased, this should lead to cross-plant changes in the dispersion of wages and productivity. The authors measure productivity as the log of output per hour worked and use U.S. LRD plant level data with 12,904 plants. The main findings are that a significant percentage of the observed changes in the dispersion of wages and productivity is accounted for by changes in the distributions of computer investment as well as changes in the wage and productivity differentials associated with computer investment. More specifically, rising wage and productivity dispersion is accounted for by rising wage and productivity differentials across plants with different computer intensities.

Productivity dispersion increased steadily after the early 1980s recession. It now also appears to be the case that it is not only variance which had increased. Dew-Becker and Gordon (2005) report, using U.S. data, a huge increase in the skewness of the income distribution. Between 1966 and 2001, skewness rose from 11 to 319 (in 2000 dollars, see their Table 9).

Our analysis caters for such changes. They represent a movement in a North Easterly direction in the terms of Figures 2 and 3. Earlier we had noted that as we move in a South-Easterly direction, either towards higher standard deviation and/or towards lower skewness, the optimal degree of sorting rises. Optimal policy goes from no sorting to partial sorting to high sorting. The empirical developments described here imply that the optimal policy response is ambiguous: with higher variance of the productivity distribution, unemployment compensation policy needs to generate greater sorting, while with higher skewness the opposite policy is required. This suggests that policy will depend on the relative magnitudes of the changes in the moments of the distribution, i.e., optimal policy will be a function of the relative increase in variance vs. increase in skewness. If the former is dominant, sorting will need to increase; if the latter is dominant sorting will need to decrease. Inspection of the above figures shows that as we move in the North Easterly direction the region we are more likely to get into is the partial sorting zone, i.e., a declining benefit schedule being the optimal policy. Except for the knife edge case of an exact offset, there needs to be an unemployment compensation policy response to skill-biased technological change. This lesson is absent from policy discussions, both in the domain of skill-biased technological change and in the domain of unemployment compensation policy.

5   Empirical Consistency

The model has examined optimal unemployment compensation policy from a normative perspective, focusing on the response of policy to the productivity distribution. It has built on a number of key elements: heterogeneity in firm productivity, a positive relationship between productivity and wages, random matching, and a declining unemployment compensation time profile. How relevant

empirically are these elements?

The assumption of firm heterogeneity is well supported by empirical studies. In U.S. data, Davis, Haltiwanger and Schuh (1996) using LRD establishment level data, Haltiwanger, Lane and Spletzer (1999), and Haltiwanger, Lane and Spletzer (2007) using matched employer-employee data of the Bureau of Census, find that: (i) firms locate along a productivity/earnings/skill locus with some firms being high productivity, high wage, and high skill while others are low productivity, low wage, and low skill; (ii) firm performance and behavior, even within quite narrowly defined industries, is quite heterogeneous and this is a substantial and persistent phenomenon.

Abowd, Kramraz and Margolis (1999), using a longitudinal sample from France of over one million workers from more than five hundred thousand firms, find that firms that hire high-wage workers are more productive per worker. Mortensen (2003) argues that dispersion in wages paid for observably equivalent workers is to be explained by differences in firm productivity and that more productive firms offer higher wages. He provides evidence from a matched employer-employee data set for the Danish economy.

Though the current analysis has focused on a simple two-wage case, it is conducive to the analysis of economies that are characterized by 'polarization' of the labor market. This term refers to the phenomenon whereby employment polarizes into high-wage and low-wage jobs at the expense of middle-wage work. Recent work by Autor, Katz, and Kearney (2006) shows that this process is manifest in U.S. data.

For empirical evidence on random matching see the survey by Petrongolo and Pissarides (2001) and the structural estimates in Yashiv (2000).[22]Note too, that the current paper is fully consistent with w including non pecuniary elements as well as wages, so workers may be compensated not only by wage payments but also by other job attributes, such as job risk, work environment, promotion chances, amenities etc. In this setting it seems reasonable to assume imperfections in worker information about vacant jobs, and this is captured by the random matching assumption.

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[22]For issues of 'mismatch' which underlie the analysis here see Manaconda and Petrongolo (1999).

A declining unemployment compensation time profile is a very prevalent phenomenon. Unemployment compensation duration is typically limited and is then replaced by social or income assistance which is lower (and often means-tested, thereby leading to lower take up rates). Thus, for example, data from OECD (2004) indicates that unemployment compensation duration in OECD countries ranges between 6 and 60 months across 28 member countries, with only Belgium having unlimited duration in some cases. In particular, 13 countries have a compensation duration of 12 months and less.

6 Relation to the Literature

We turn to discuss the place of the current paper in two literatures: the search literature and the literature on optimal unemployment compensation policy. Starting with the former, the recent survey by Rogerson, Shimer and Wright (2005) characterizes three main classes of search models: random matching and bargaining (see Pissarides (2000)), directed search and wage posting (see Moen (1997)), and random matching and wage posting (two key models and many references are discussed in Section 6 of Rogerson, Shimer and Wright (2005)). The current paper belongs in the last class. The first class, that includes bargaining, does not allow for the wage posting behavior of firms, which is crucial for the effects of unemployment compensation policy in the current set up. Indeed that class of models is not geared to explain wage dispersion. The second class, sometimes referred to as "competitive search theory," does share a key feature with the current approach: firms set wages optimally, knowing that the probability of filling a job rises with the wage offer. Additionally, as in Moen (2003), labor market segmentation arises due to the fact that firms cannot condition wage offers on the worker type and workers' productivities differ across matches. But the segmentation in the current paper does not take the form of sub-markets and the operation of market makers; rather, it is due to exogenous productivity dispersion and the effects of unemployment compensation policy.

The main line of research on the optimal design of unemployment compensation policy

has focused on issues of moral hazard and consumption smoothing (see Karni (1999) for a survey). This literature examined the impact of work disincentives on the design of optimal schemes (the seminal papers are by Baily (1978), Flemming (1978) and Shavell and Weiss (1979)). The main insight provided by the early models was the desirability of a declining schedule, i.e. compensation should decline over the spell of unemployment so as to mitigate the moral hazard effect. The early models have been recently extended in several directions, some of them into general equilibrium frameworks. Hopenhayn and Nicolini (1997), as a notable example, enlarge the set of instruments by allowing for a wage tax after re-employment. This model preserves the sequencing structure of Shavel and Weiss (1979) and attains enhanced consumption smoothing.

The current paper does not belong in the above strand, as it does not consider issues of risk aversion, consumption smoothing, moral hazard, or adverse selection. Rather it focuses on the role unemployment compensation can play in attaining a better match between jobs and workers, deriving optimal policy in the face of productivity dispersion. A seminal contribution in this context has been made by Diamond (1981), who discussed the role of unemployment compensation in enhancing efficiency in the context of a steady state search model. In his model unemployment compensation makes job-taking use more stringent standards, thereby raising the vacancy rate and improving the distribution of job offers. Another paper in this spirit is Albrecht and Axell (1984). The paper obtains a non-degenerate wage offer distribution in a simple general equilibrium setup. It demonstrates the role of unemployment benefits as a search subsidy that brings about re-allocation of workers to more productive firms at the cost of increased unemployment. Our framework builds on Albrecht and Axell (1984) but modifies their setup in a substantial way. Most importantly we seek to make benefits a function of the properties of the productivity distribution. Thus we relax the assumption of fixed arrival rates so firms can respond through wage posting and we allow benefits to vary over time. These changes allow us to compare a declining time path (of different duration lengths) to a flat scheme, as a function of the variance and skewness of the productivity distribution.

There are a number of more recent contributions that have dealt with related issues and

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it is worthwhile delineating their relation to the current paper: in Marimon and Zilibotti (1999) unemployment compensation improves matching between ex-ante heterogenous workers and ex-ante heterogenous firms under random matching. Unemployment compensation serves to reduce worker-job mismatch, as without unemployment compensation workers would tend to accept unsuitable jobs. This paper however does not deal with optimal unemployment compensation policy, as does the current one, and does not make any connection between unemployment compensation policy and heterogeneous firm productivity. The model of Acemoglu and Shimer (1999) shares with the current paper the idea that unemployment compensation generates an increase in output, whereby more productive firms choose to offer higher wages and more workers are assigned to those firms. However Aceomglu and Shimer have risk aversion at the heart of their analysis and unemployment compensation has an insurance role. By offering unemployment compensation, apart from the consumption smoothing argument, the policymaker induces risk-averse workers to take on a higher degree of unemployment risk, boosting investment by firms. Their set-up is one with directed search, so externality issues do not arise. In the model here a key point is unemployment compensation policy turning random matching into assortative matching against the backdrop of heterogeneity in productivity. Thus the mechanism studied is entirely different; it does not relate to risk aversion (agents are risk-neutral) but rather explores the role of unemployment compensation policy in affecting firm and worker behavior to obtain enhanced matching. Finally, Albrecht and Vroman (2005) present a model of wage posting, matching, declining unemployment compensation, and a two-tier wage system in equilibrium, as is the case here. They show how time-varying unemployment compensation can generate wage dispersion even though firms and workers are homogenous . However, their paper does not contain two key ingredients of the current paper: firm productivity dispersion (in their model firms are identical) and a normative analysis of optimal policy.

7　Conclusions

This paper has studied optimal unemployment compensation policy from a macroeconomic perspective. A key insight is that the degree of productivity heterogeneity (in terms of skewness and variance) matters for the design of the time path of unemployment compensation. Workers react to unemployment compensation policy through job acceptance decisions; firms react to unemployment compensation policy through wage posting, with the associated market segmentation. In a world of random matching, output gains are induced by a declining unemployment compensation profile that induces worker sorting. The longer the duration of the first phase, the higher the degree of induced heterogeneity and sorting. The main elements of the model – productivity dispersion, positive association of productivity and wages, random matching, and a declining unemployment compensation profile – were shown to be empirically relevant. Thus, while the model is essentially normative, it is consistent with known empirical regularities. Note that although we have not allowed for capital investment and endogenous formation of heterogeneous firms, the distribution of posted jobs is endogenously determined, as some low productivity jobs are crowded out, and the distribution of filled vacancies is endogenously determined, as high productivity firms face a lower vacancy risk. Allowing for capital investment would maintain the sorting-unemployment tradeoff but would complicate the analysis.

A key lesson is that even in the absence of moral hazard arguments there is a role for a declining time profile of unemployment compensation. By enhancing matching it operates to increase output and efficiency (in a constrained setting). Particularly important is the dependence of optimal policy on the properties of the productivity distribution. This kind of connection has received little, if any, attention thus far. In future work we hope to provide a mapping from empirically-relevant dispersion of firms' productivities to a larger set of policy instruments.

References

[1] Abowd, J.M., F. Kramarz, and D.N.Margolis, 1999. "High Wage Workers and High Wage Firms," **Econometrica** 67,2, 251-333.

[2] Acemoglu, D. and R. Shimer, 1999. "Efficient Unemployment Insurance", **Journal of Political Economy** 107, 5, 893-928.

[3] Albrecht, J. and B.Axell, 1984. "An Equilibrium Model of Search Unemployment, " **Journal of Political Economy** 92, 824-840.

[4] Albrecht, J. and. S. Vroman, 2005. "Equilibrium Search with Time Varying Unemployment Benefits," **Economic Journal** 115,631-648.

[5] Autor, D.H., L.F. Katz, and M.S. Kearney, 2006. "The Polarization of the U.S. Labor Market," NBER Working Paper No. 11986.

[6] Baily, M., 1978."Some Aspects of Optimal Unemployment Insurance", **Journal of Public Economics**, 10, 379-402.

[7] Caselli, F., 1999. "Technological Revolutions, "**American Economic Review** 89, 78-102.

[8] Davis, S.J, J.C. Haltiwanger, and S.Schuh, 1996. **Job Creation and Destruction.** MIT Press, Cambridge.

[9] Dew-Becker, I. and R. J. Gordon, 2005. "Where did the Productivity Growth Go? Inflation Dynamics and the Distribution of Income," **Brookings Papers on Economic Activity,** 2, 67-127.

[10] Diamond, P. 1971. "A Model of Price Adjustment", **Journal of Economic Theory**, 3, 156-168.

[11] Diamond, P. 1981. "Mobility Costs, Frictional Unemployment and Efficiency", **Journal of Political Economy**, 89, 798-812.

[12] Dunne, T., L.Foster, J. Haltiwanger, and K.R. Troske, 2004. " Wage and Productivity Dispersion in United States Manufacturing: The Role of Computer Investment," **Journal of Labor Economics**, 2004, 22, 2, 397-430.

[13] Flemming, J., 1978. "Aspects of Optimal Unemployment Insurance: Search, Leisure, Savings and Capital Market Imperfections", **Journal of Public Economics**, 10, 403-425.

[14] Haltiwanger, J.C., J.I. Lane, and J.R. Spletzer, 1999. Productivity differences across employers: the role of employer size, age, and human capital, **American Economic Review Papers and Proceedings**, 94–98.

[15] Haltiwanger, J.C., J.I. Lane, and J.R. Spletzer, 2007. "Wages, Productivity, and the Dynamic Interaction of Businesses and Workers," **Labour Economics,**14, 3, 575-602.

[16] Hopenhayn, H. and J. Nicolini, 1997. "Optimal Unemployment Insurance", **Journal of Political Economy,** 105,412-438.

[17] Karni, E., 1999. "Optimal Unemployment Insurance – A Survey", **Southern Economic Journal,** 66, 442-465

[18] Manaconda. M. and B. Petrongolo, 1999. "Skill mismatch and unemployment in OECD countries," **Economica** 66,181-207.

[19] Marimon, R. and F.Zilibotti, 1999. "Unemployment vs. Mismatch of Talents: Reconsidering Unemployment Benefits," **The Economic Journal** 109, 266-291.

[20] Moen, E., 1997. "Competitive Search Equilibrium", **Journal of Political Economy**, 105, 385-411.

[21] Moen, E., 2003. "Do Good Workers Hurt Bad Workers or is it the Other Way Round?" **International Economic Review,** 44,779-799.

[22] Mortensen, D.T., 2003. **Wage Dispersion**. MIT Press, Cambridge.

[23] Mortensen, D.T. and C.A. Pissarides, 1998. "Technological Progress, Job Creation, and Job Destruction, "**Review of Economic Dynamics** 1 (1998) 733-753

[24] OECD, 2004. **Benefits and Wages.** OECD, Paris.

[25] Petrongolo B. and C.A. Pissarides, 2001. "Looking into the Black Box: A Survey of the Matching Function," **Journal of Economic Literature** 39,2**.**

[26] Pissarides, C.A., 1998. "The Impact of Employment Tax Cuts on Unemployment and Wages: The Role of Unemployment Benefits and Tax Structure," **European Economic Review** 42, 155-183

[27] Pissarides, C.A., 2000. **Equilibrium Unemployment Theory**. 2nd edition, MIT Press, Cambridge.

[28] Rogerson, R., R. Shimer and R. Wright, 2005. "Search Theoretic Models of the Labor Market: A Survey," **Journal of Economic Literature,** 43,4,959-988.

[29] Shavell, S. and L. Weiss, 1979. "The Optimal Payment of Unemployment Insurance Over Time", **Journal of Political Economy**, 87, 1347-1362.

[30] Yashiv, E., 2000. "The Determinants of Equilibrium Unemployment," **American Economic Review** 90, 1297-1322.

8 Appendix A

Derivation of the Wage Equations

We reproduce the relevant equations for convenience:

$$H_1 = z + h + \beta[\overline{n}\max(\overline{J}, H_2) + \underline{n}\max(\underline{J}, H_2) + (1 - \overline{n} - \underline{n})H_2] \tag{50}$$

$$H_2 = z + h + \beta[\overline{n}\max(\overline{J}, H) + \underline{n}\max(\underline{J}, H) + (1 - \overline{n} - \underline{n})H] \tag{51}$$

$$H = a + h + \beta[\overline{n}\max(\overline{J}, H) + \underline{n}\max(\underline{J}, H) + (1 - \overline{n} - \underline{n})H] \tag{52}$$

$$\overline{J} = \overline{w} + \beta[(1 - s)\overline{J} + sH_1] \tag{53}$$

$$\underline{J} = \underline{w} + \beta[(1 - s)\underline{J} + sH_1] \tag{54}$$

$$\overline{J} = H_2 \tag{55}$$

$$\underline{J} = H \tag{56}$$

Subtracting (54) from (53) yields:

$$\overline{J} - \underline{J} = \frac{\overline{w} - \underline{w}}{1 - \beta(1 - s)} \tag{57}$$

Subtracting (52) from (51) yields:

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$$H_2 - H = z - a \tag{58}$$

Substituting (55) and (56) into (57), and then substituting (57) into (58) yields:

$$z - a = \frac{\overline{w} - \underline{w}}{1 - \beta(1 - s)}$$

This is equation (15) in the main text.

Substituting (55) into (50) and noting that $\overline{J} = H_2 > \underline{J}$ yields:

$$H_1 = z + h + \beta H_2 \tag{59}$$

Substituting (55) into (53) yields:

$$[1 - \beta(1 - s)]H_2 = \overline{w} + \beta s H_1 \tag{60}$$

Solving (59) and (60) for H_2 and simplifying yields:

$$[1 - \beta(1 - s) - \beta^2 s]H_2 = \overline{w} + \beta s(z + h) \tag{61}$$

Substituting (55) and (56) into (51) and re-formulating yields:

$$(1 - \beta)H_2 = z + h - \beta(1 - \overline{n})[H_2 - H] \tag{62}$$

Substituting (58) into (62), then substituting from (61) into (62) for H_2 and simplifying, yields:

$$\overline{w} = (z - a)[(1 - \beta) + \beta\overline{n} - \beta^2 s(1 - \overline{n})] + h + a$$

This is equation (16) in the main text.

9 Appendix B

Proof of Proposition in Section 3

Proof. First observe that by virtue of linearity with respect to $\triangle$ for each one of the schedules (35)-(36), each configuration will appear at most once on the welfare frontier (single crossing property).

Next, note that the finiteness of set of configurations ensures non-emptiness.

Note further that for $\triangle = 0$, the no-sorting configuration is welfare maximizing, since $\mu > h$ and $N^{NS} > \max[N^{PS}, N^{HS}]$. Then, by continuity, for sufficiently small $\triangle$, the no-sorting configuration is socially desirable.

Let NS denote the set of all $\triangle$, for which the no-sorting configuration is welfare maximizing. The set NS is non-empty (as just shown) and bounded from above (by construction of the spread $\triangle$). Thus, it has a least upper bound. We denote it by $\triangle_1$.

Next, note that if the high-sorting configuration is welfare maximizing for some $\triangle'$, then it remains the maximizing configuration for all $\triangle' \leq \triangle$. To see that, suppose, by way of contradiction, that the opposite holds true. Since the schedule of the no-sorting configuration is flat, whereas the high-sorting configuration is rising with respect to $\triangle$, the only case we need to examine is the possibility where partial-sorting attains a higher level of welfare than high sorting for some $\triangle$, $\triangle \geq \triangle'$ This necessarily implies that the slope of the partial sorting schedule with respect to $\triangle$ is steeper than the corresponding slope of the high sorting schedule. Formally:

$$N^{PS} \frac{q - p}{1 - p} > N^{HS}$$

which implies

$$N^{PS} > N^{HS}$$

Thus, we obtain a contradiction, since it follows that for all $\triangle$, partial sorting is preferred

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to high sorting.

Let HS denote the set of all $\triangle$ for which the high-sorting configuration is welfare maximizing. The set is bounded from below (by construction of the spread $\triangle$). If it is non-empty, it has a highest lower bound. Let $\triangle_2$ denote the highest lower bound (if it exists) and set $\triangle_2 = \frac{1-p}{p(\mu-h)}$, otherwise. This completes the proof. ■

10   Appendix C

Implementation of the Partial Sorting Equilibrium

We set $F = M$ and $\overline{F} = pM$.

We substitute (17) and (18) in (19) and (20) correspondingly, and solve the system (19)-(23) for five unknowns: $\overline{V}, \underline{V}, U_1, U_2$, and U.

To insure existence of partial-sorting equilibrium, we need to verify that our solution satisfies (15), (16) and (25). Modified to the discrete case, eq. (25) turns into two inequality conditions:

$$\frac{m(U,V)}{V} \frac{(U-U_1)}{U} (\underline{x} - \underline{w}) \geq \frac{m(U,V)}{V} (\underline{x} - \overline{w}) \tag{63}$$

$$\frac{m(U,V)}{V} (\overline{x} - \overline{w}) \geq \frac{m(U,V)}{V} \frac{(U-U_1)}{U} (\overline{x} - \underline{w}) \tag{64}$$

This is easy to observe for we have two instruments at our disposal – a and z. Using (15) we fix some $\varepsilon > 0$ arbitrarily small, and set $z - a$ small enough such that $\overline{w} - \underline{w} = \varepsilon$. Using (16), we adjust a, such that $\overline{w} = \underline{x} + \frac{\varepsilon}{2}$. For $\varepsilon > 0$ sufficiently small, (25) is satisfied i.e. all high-productivity firms choose to offer $\overline{w}$, whereas all low-productivity firms choose to offer $\underline{w}$.

11   Appendix D

Duration Policy Simulation

We provide further analysis of the duration simulation in terms of the moments of the productivity distribution. We re-do the analysis of section 3.2.2, setting the duration at one period longer than in the benchmark model, i.e., using $\bar{t} = 3$ as opposed to $\bar{t} = 2$. Crucially note, that as we focus on the two-point distribution, there would be at most two wage rates in equilibrium regardless of the duration. We allow for an additional equilibrium configuration, to be denoted P_2S, which we shall refer to as enhanced partial sorting. Note that it is different from the partial sorting configuration, PS, discussed above, where duration was set at $\bar{t} = 2$. Figure D-1 depicts seven different curves in the skewness - standard deviation space. Three new curves, labeled $N - P_2, P - P_2, P_2 - H$, are added to the four curves that appear in Figure 2 above. We maintain the notation used above. Thus, for instance, along the $P - P_2$ curve the partial sorting and the enhanced partial sorting attain the same level of welfare. The figure is based on a numerical solution, using the same parametric assumptions used to derive Figure 2.
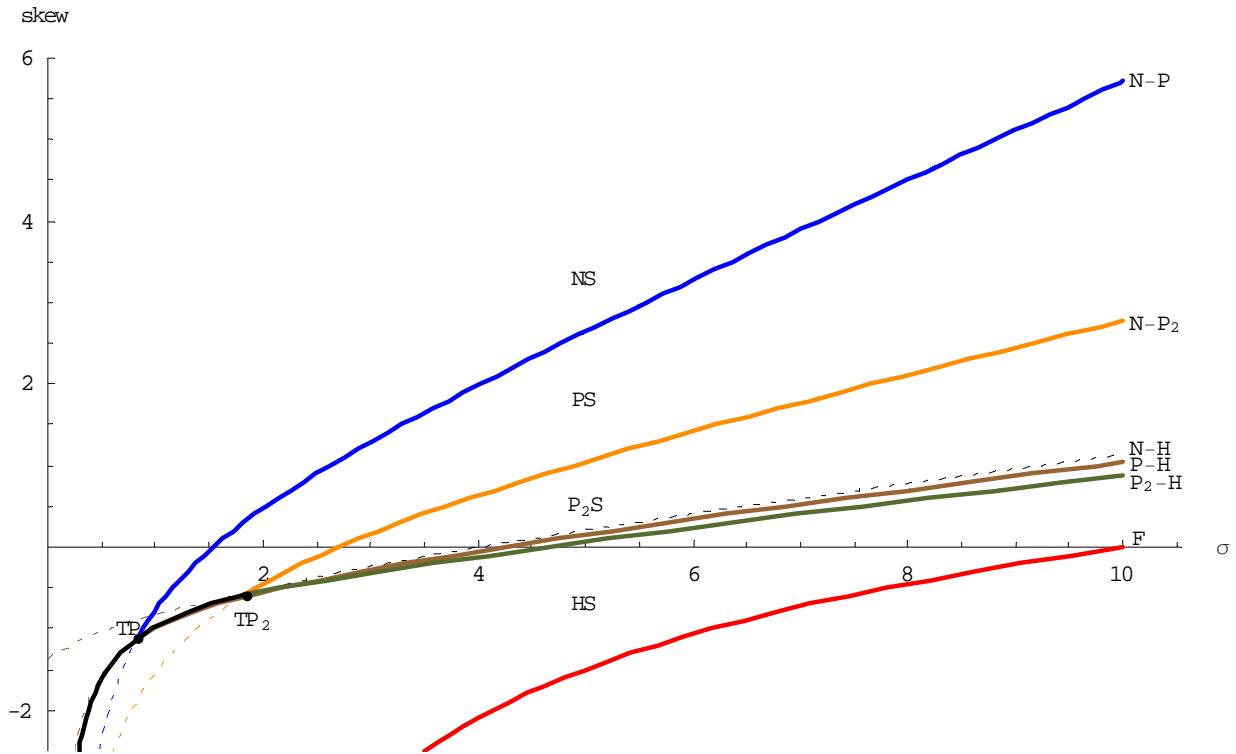
Figure D-1: Skewness and Variance Relationships

We provide again an illustrative figure on a magnified scale (Figure D-2). The solid parts divide the space into four disjoint regions. For any point which lies in the region labeled NS, the no-sorting configuration attains the highest level of welfare and is therefore socially desirable. Similarly, for points that lie in the regions labeled, respectively, PS, P_2S and HS, the partial sorting, enhanced partial sorting, and high-sorting configurations prevail, correspondingly.
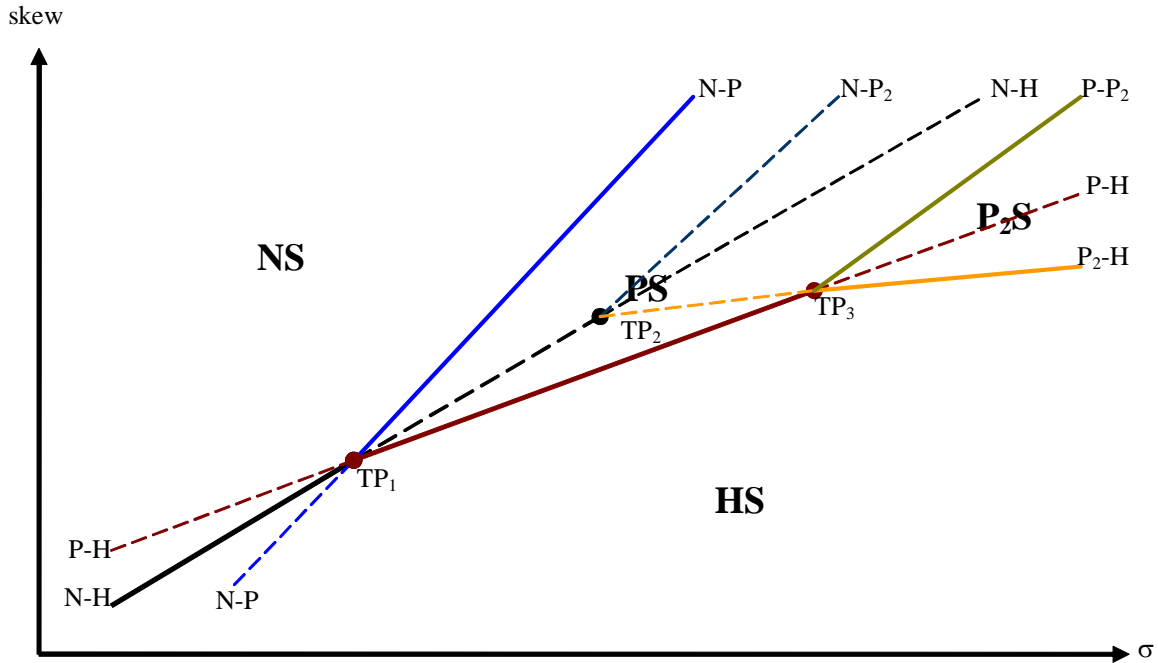
Figure D-2: Optimal Unemployment Compensation Regions

Several insights emerge from inspection of the figure:

First, compare Figure D-2 to Figure 3. It can be seen that allowing for the setting of an extended duration of the first tier of the unemployment compensation scheme expands the range in which a declining unemployment compensation time path prevails, i.e., either partial sorting (PS) or enhanced partial sorting (P_2S). Notably, the added flexibility of setting the duration of the first tier implies that the enhanced partial sorting configuration crowds out both the partial sorting configuration and the high-sorting one, as the social planner can now achieve better balance between the opposing matching and employment considerations. In other words, there is some "fine tuning" of the high sorting and the partial sorting configurations.

Second, one can see that the optimal degree of sorting, which is determined by the length of the duration of the first tier, is rising with respect to the dispersion (s.d.) of the firm technological distribution, and is decreasing with respect to the skewness. Thus, as we move either rightwards, increasing the s.d, or downwards, reducing skewness, we gradually increase the optimal degree of sorting by extending the duration. This reflects the fundamental trade-off between matching and employment considerations.

While the figure demonstrates only three different durations $(\bar{t} = 0, 2, 3)$,[23] noting that the no-sorting configuration is essentially the limiting case, whereby the duration is set to zero $(\bar{t} = 0)$, it is straightforward to see how the analysis can be extended to any range of durations. The more flexibility we add, the more refined will be the partition of the variance-skewness space. Thus, for example, allowing for a fifth configuration, where the duration would be set at $\bar{t} = 4$, we would obtain another intersection point along the $P_2 - H$ curve to the right of TP_3, which would yield a fork-shaped region stemming from the intersection point. In this region the new configuration would prevail, crowding out the high-sorting and the enhanced partial sorting configurations. As we refine duration policy, we would obtain additional fork-shaped regions like those stemming from intersection points TP_1 and TP_3.

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[23] The fourth equilibrium configuration depicted in the figure is the high-sorting one, for which duration policy is irrelevant, as we crowd out all low-productivity firms and hence can gain nothing from 'voluntary' unemployment.

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