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Venture Capital Contracts and Market Structure[*]

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Abstract

We examine the relation between optimal venture capital contracts and the supply and demand for venture capital. Both the composition and type of financial claims held by the venture capitalist and entrepreneur depend on the market structure. Beside, different market structures involve different optimal forms of transferring utility: sometimes it is optimal to transfer utility via equity stakes, sometimes it is optimal to use debt. Transferring utility via equity stakes affects incentives. Consequently, the net value created, the success probability, the market value, and the performance of venture-capital backed investments depend on the supply and demand for capital. Similarly, venture capitalists face different incentives to screen projects if the capital supply is low or high. We then endogenize the capital supply and study the relation between venture capital contracts and entry costs, public policy, investment profitability, and market transparency. Finally, we show that entry by inexperienced investors creates a negative externality for the value creation in ventures financed by (regular) venture capitalists.

1 Introduction

Do optimal financial contracts depend on the supply and demand for capital? And if so, how? In this paper, we address this question in the context of venture capital contracting. We examine whether, and how, optimal venture capital contracts vary with the supply and demand for capital, or more generally, the level of competition in the venture capital market. Pursuing this question within the traditional financial contracting framework is problematic, as it *assumes* that one side (typically the entrepreneur) has all the market power. We hence depart from the traditional paradigm and embed venture capital contracting in a search market where entrepreneurs and venture capitalists bargain over optimal contracts. Both the supply and demand for capital are competitive in the sense that there are many venture capitalists and many entrepreneurs. The market power of each side is determined by the ratio of supply to demand. A high ratio implies that the degree of competition among venture capitalists is high.

We find that both the composition and type of financial claims held by the venture capitalist and entrepreneur depend on the market structure, or degree of competition. If the degree of competition is either low or high, one side holds a mix of debt and equity and the other side holds straight equity. If the degree of competition is intermediate, both sides hold debt and equity. Generally, a change in the ratio of supply to demand affects market powers and therefore the distribution of surplus. The question is whether utility should be transferred via equity or debt. As a guiding principle, utility should be transferred at least cost, i.e., in a way that minimizes incentive distortions. We show that different competition levels entail different optimal forms of transferring utility. If the level of competition is either low or high, utility is transferred by changing the equity component of the optimal contract. If the level of competition is intermediate, utility is transferred by changing the debt component.

Work by Lerner (1995), Hellmann and Puri (2001), and Kaplan and Strömberg (2001b) indicates that venture capital contracting may be appropriately viewed as a double-sided moral-hazard problem.[1] Venture capitalists monitor the progress of the firm, give advice, provide entrepreneurs with access to consultants, investment bankers, and lawyers, negotiate with suppliers, and play an active role in the building up of human resources and recruiting of senior management. Under double-sided moral hazard, the second-best solution requires that both the venture capitalist and the entrepreneur hold sufficiently large equity stakes. Actual equity stakes, however, are determined by bargaining, where outside options depend on the level of competition in the capital market. For low and high levels of competition, outside options are strongly asymmetric, which implies that equity stakes , and hence incentives, deviate from the

[1] Theoretical papers modelling venture capital contracting as a double-sided moral-hazard problem include Casamatta (2000), Cestone (2001), Renucci (2000), Repullo and Suarez (2000), and Schmidt (2000).

second-best optimum. (While riskless debt can be used to transfer utility without affecting incentives, the total amount of riskless debt available is limited.)

This has the following additional implications: the net value created in ventures is a hump-shaped function of the degree of competition. If efforts are substitutes, the success probability, the market value, and the average performance of venture-capital backed investments are increasing in the degree of competition if the entrepreneur's contribution is more important, and decreasing if the venture capitalist's contribution is more important. If efforts are complements, the success probability, market value, and average performance are all hump-shaped functions of the degree of competition. Most of our results remain yet to be tested. The ones that have been tested, however, are consistent with the empirical evidence. We provide a discussion of the empirical evidence at the end of this paper.

In an extension of the model, we consider projects (or entrepreneurs) of low and high quality. Prior to making his investment, the venture capitalist can screen the project. We show that venture capitalists screen more if the degree of competition is low, and less if the degree of competition is high.

A caveat is in order here. The above implications are based on the notion that financial claims have a first-order effect on incentives. While it is difficult to say what motivates venture capitalists in practice, the following quote suggests that financial claims matter.[2]

> In an investment memo regarding a company in the financial information industry, a venture capitalist outlines a number of actions that he will undertake to assist the company. Among the risk factors which the venture capitalist worries about is whether he "can [...] get enough money at work, or ownership in the company, to warrant allocating these extra resources."

Similarly, Kaplan and Strömberg (2001b) conclude that equity incentives increase the likelihood that venture capitalists perform value-added support activities.

In the second part of the paper, we take the argument one step further and examine factors that potentially affect the capital supply. We consider entry costs, public policy, investment profitability, and market transparency. To investigate the role of these factors, we endogenize the entry decision of venture capitalists, thereby endogenizing the degree of competition in the capital market. An individual venture capitalist entering the market does not take into account the effect of his entry on the overall level of competition, and thus on the bargaining, contracting, and value creation in other ventures. Depending on the level of competition prevailing in the market, entry by an individual venture capitalist thus either creates a positive or negative *contracting externality*.

[2] We thank Per Strömberg for this quote.

It is frequently argued that the short-run supply of informed capital is fixed, for it takes time to develop the skills and experience which are necessary to be a good advisor (Gompers and Lerner 1999). In an extension of the model, we assume that informed capital is in short supply. There is, however, an abundant potential supply of *portfolio investors*, i.e., investors who have capital but no expertise.[3] We show that if the level of competition is strong, portfolio investors not only fail to add value in their own ventures, but also create a negative externality for the value added in ventures financed by (regular) venture capitalists.

We believe our model of bargaining and search captures important features of real-world venture capital contracting environments. In our model, deals are struck through bilateral negotiations, and not through auctions or a Walrasian tâtonnement mechanism. Both entrepreneurs and venture capitalists must actively look for deals. They can quit negotiations at any time and team up with somebody else. Finding a suitable partner takes time, however, and is easier the greater the supply of potential partners relative to the demand. Everyone is aware of his own and his partner's outside options, and this affects the outcome of the negotiations.[4] Finally, search models have appealing economic properties. For instance, in our model outside options, and hence contracts, adjust gradually to changes in the degree of competition. By contrast, in a Walrasian market the outcome is a bang-bang solution where the shorter side of the market has the entire market power. This can have extreme implications: if there are K venture capitalists and K " 1 entrepreneurs, entrepreneurs have the entire market power. If there are K + 1 entrepreneurs, however, venture capitalists have the entire market power.

By embedding venture capital contracting in a market environment, our paper goes beyond papers studying venture capital contracting in isolation. On the other hand, unlike many of these papers, our simple model falls short of explaining the richness of cash flow and control rights found in real-world venture capital contracts Michelacci and Suarez (2000) also have a search-based model of start-up financing. Unlike our model, Michelacci and Suarez do not consider incentive contracts or contracting inefficiencies. Instead, they consider search inefficiencies, using an insight from the search literature that entry creates externalities for the matching chances of other market participants. In Aghion, Bolton, and Tirole (2000), the limited partners

[3] There is evidence of inexperienced investors entering the venture capital industry at times when the market is "hot". For instance, the Economist notes: "[a] host of new entrants are now dabbling in venture capital, ranging from ad hoc groups of MBAs to blue-blooded investment banks such as J.P. Morgan, to sports stars and even the CIA." Money to Burn, the Economist, May 27, 2000.

[4] Entrepreneurs typically strike better deals when the supply of capital is abundant. The following is a statement comparing the boom period of the most recent venture capital cycle with the aftermath: "if you went into a [...] start-up three to six months ago, you almost certainly got a very bad deal. Companies could ask for anything they wanted [in terms of valuation]. Now entrepreneurs are much more realistic." Open Season for Europe's Turkeys, Financial Times, January 11, 2001.

in a venture partnership (i.e., the investors) must incentivize the venture capitalist to monitor an entrepreneur. Changes in the market environment that close the gap between the venture capitalist's private opportunity cost of capital and the investors' cost of capital improve the efficiency of the incentive contract. Inderst (2001) also examines the relationship between competition and contract design. Unlike this paper, he considers screening contracts in an adverse selection setting. To our knowledge, our paper is the first to investigate the relation between incentive contracts and market structure in a moral hazard setting. Finally, we are not the first to note that constraints on transfer payments may impair contract efficiency. Aghion and Bolton (1992) and Legros and Newman (2000) both discuss the implications of this for the optimal allocation of control rights.

2 Non-Technical Overview

Section 3 presents the model. The model consists of three building blocks: i) financial contracting, ii) bargaining, and iii) search. Section 3.1 studies a contracting problem between an entrepreneur and a venture capitalist. The optimal contract is a combination of equity and riskless debt.[5] Both the entrepreneur and the venture capitalist can exert effort. Equilibrium effort levels depend on the way in which the equity is allocated between the two. We first derive the utility possibility frontier generated by different equity allocations. The undominated segment of the frontier is called *equity frontier*. We subsequently add the riskless debt. The idea is to allocate the debt in a way that minimizes incentive distortions. The utility possibility frontier generated by different Pareto-optimal contracts is called *bargaining frontier*. In Section 3.2, the entrepreneur and venture capitalist bargain over an optimal contract. We apply the generalized Nash bargaining solution. Reservation values, i.e., outside options, are exogenous. In Section 3.3, we endogenize reservation values by embedding the bargaining problem in a search market with pairwise matching. Reservation values are determined by the market structure, or degree of competition, as expressed by the ratio of venture capitalists to entrepreneurs in the market. The degree of competition is assumed to be exogenous.

Section 4 summarizes the results. Via its effect on reservation values, the degree of competition determines i) the optimal financial contract, ii) the net value created, i.e., the expected project return minus aggregate effort costs, iii) the gross surplus, which corresponds to the market (or IPO) value of the venture if it was sold after effort and investment costs are sunk but before returns are realized, iv) the expected, or average, performance of the venture-backed

[5] An alternative interpretation is that the entrepreneur and venture capitalist hold combinations of common and straight preferred stock, or, with minor qualifications, participating convertible preferred stock. Such claims are widely used in real-world venture capital contracts (Kaplan and Strömberg 2001a).

investment, and v) the success probability of the venture, i.e., the likelihood that the investment pays off more than it costs. We also study the role of up-front payments.

Section 5 endogenizes the degree of competition by introducing free entry of capital. We examine the way in which the degree of competition, and thus the optimal contract, net value, etc., is affected by i) entry costs, ii) regulation and public policy, iii) investment profitability, and iv) market transparency.

Section 6 considers projects of different quality. We investigate the impact of competition on the incentives of venture capitalists to screen projects ex ante.

Section 7 introduces a second class of investors: *portfolio investors*, i.e., investors who have money but no skills. We examine the effect of entry by portfolio investors on the performance of investments financed by (regular) venture capitalists.

Section 8 concludes. All proofs are provided in the Appendix.

3 The Basic Model

3.1 Financial Contracting

The Basic Setup

A penniless entrepreneur has a project which requires an investment outlay $I > 0$. Funding is provided by a venture capitalist. The project return is X_l # 0 with probability 1 " p and $X_h > I > X_l$ with probability p. The success probability p = $p(e,a)$ depends on both the entrepreneur's and the venture capitalist's (unobservable) efforts e $ [0,1] and a $ [0,1], respectively. The corresponding effort cost functions are $c_E(e)$:= $e^2/2\alpha_E$ and $c_F(a)$:= $a^2/2\alpha$. All agents are risk neutral.

The project return can be decomposed into a riskless return equal to X_l and a risky return paying 0 in the bad state and Δ_X := X_h " X_l in the good state. With the usual degree of caution, we refer to these as debt and equity. An optimal contract between the entrepreneur and the venture capitalist specifies i) the venture capitalist's investment I, ii) the debt 0 % S % X_l held by the venture capitalist, and iii) the fraction of equity s $ [0,1] held by the venture capitalist.[6] In Section 4, we additionally admit up-front payments by the venture capitalist. The entrepreneur keeps any return not paid to the venture capitalist. The entrepreneur's utility from the contract (s,S) is $U(s,S)$:= $u(s)$ + X_l " S, where $u(s)$:= $p(1$ " $s)\Delta_X$ " $e^2/2\alpha_E$ represents his utility from the equity allocation s. Likewise, the venture capitalist's utility from the contract (s,S) is $V(s,S)$:= $v(s)$ + S " I, where $v(s)$:= $ps\Delta_X$ " $a^2/2\alpha$ represents his utility from the equity allocation s.

[6] This rules out that one party receives a higher payment in the bad state than in the good state. It is easy to show that such contracts are never optimal.

To simplify the exposition, we proceed in two steps. We first derive the set of Pareto-optimal u " v combinations. The utility possibility frontier generated by Pareto-optimal equity allocations is called *equity frontier*. We then add the riskless debt and investment cost and derive the set of Pareto-optimal U " V combinations. The utility possibility frontier generated by Pareto-optimal contracts is called *bargaining frontier*.

The Equity Frontier

The equity frontier $u = \psi_E(v)$ depicts the utilities $u(s)$ and $v(s)$ for all Pareto-optimal equity allocations. In general, the precise shape of the equity frontier depends on the production technology $p(e,a)$. There are, however, some robust properties that any well-behaved equity frontier has. These are:

i) the equity frontier is strictly concave,

ii) the sum $u(s)$ + $v(s)$ attains its maximum in the interior of the domain, and

iii) along the equity frontier, $v(s)$ is strictly increasing in s.

The first two properties follow naturally from the fact that the incentive problem is two-sided and effort costs are strictly convex. Maximizing the sum of utilities then requires balancing the two incentive problems. Giving one side too big an equity stake is inefficient as it will then produce at a point where the marginal effort cost is relatively high. Accordingly, the equity allocation maximizing the total utility lies somewhere in the interior. Denote the point where the total utility is maximized by $(\hat{v},\hat{u})$, and the corresponding equity allocation by $\hat{s}$. We occasionally refer to this as the joint-surplus maximizing, or *second-best*, solution. Clearly, $\psi'_E(\hat{v}) = "1$. The third property states that the venture capitalist's utility increases with his equity stake. Note that this need not be true for dominated segments of the utility possibility frontier, i.e., segments that lie not on the equity frontier. (See the following examples.)

We illustrate these properties by means of two examples. The first is based on the linear technology $p(e,a) = da + (1 " d)e$ used in, e.g., Casamatta (2000). The second is based on the Cobb-Douglas technology $p(e,a) = a^d e^{1-d}$ used in, e.g., Repullo and Suarez (2000). Under the linear technology, the two efforts are substitutes while under the Cobb-Douglas technology, they are complements. While for the remainder of the paper we could work with either of these technologies (or any other well-behaved technology), we instead use the general notation $u = \psi_E(v)$ and assume that properties i) " iii) hold. To make the problem non-trivial, we assume that the second-best outcome is sufficiently good to allow the venture capitalist to break even, i.e., that $\hat{v} > I$.

Example 1: Linear Technology. To ensure that the equilibrium success probability has an interior solution, we assume that $\max\{\alpha_E(1 " d), \alpha_F d\} < 1/\Delta_X$. Given some equity allocation s, the corresponding equilibrium effort choices are $a^*(s) = \alpha_F d s \Delta_X$ and $e^*(s) =$

$\alpha_E (1 - d)(1 - s)\Delta_X$, respectively. The equilibrium success probability is

$$p^*(s) := p(e^*(s), a^*(s)) = \Delta_X \left[\alpha_F d^2 s + \alpha_E (1 - d)^2 (1 - s)\right]. \quad (1)$$

The venture capitalist's and entrepreneur's utility from the equity allocation s is

$$v(s) = \frac{1}{2}\alpha_F d^2 s^2 \Delta_X^2 + \alpha_E (1 - d)^2 s (1 - s) \Delta_X^2, \quad (2)$$

and

$$u(s) = \frac{1}{2}\alpha_E (1 - d)^2 (1 - s)^2 \Delta_X^2 + \alpha_F^2 d^2 s (1 - s) \Delta_X^2, \quad (3)$$

respectively.

Figure 1: Equity Frontier for Linear Technology.

Figure 1 depicts the utility possibility frontier generated by different equity allocations. By increasing the venture capitalist's equity share s from zero to one, we move along the curve clockwise. In the picture, we assume that $\alpha_E (1 - d)^2 > \alpha_F d^2$, i.e., the entrepreneur is more productive than the venture capitalist. If the reverse holds or if productivities are equal, the picture looks similar. Appendix A discusses all cases.

The equity frontier is the undominated segment of the utility possibility frontier. It is defined on the interval $[\underline{v}, \overline{v}]$, where in this example $\underline{v} = v(0) = 0$ and $\overline{v} = v(\overline{s})$, where $\overline{s} < 1$. If $s > \overline{s}$, the utility of *both* parties decreases with s, which is due to the fact that the entrepreneur is more productive. Hence, even if the venture capitalist had all the market power, he would want to leave the entrepreneur some rent. (If the venture capitalist were more productive, the reverse would hold.) A formal derivation of the equity frontier and a proof that properties i) - iii) hold is provided in Appendix A.

Example 2: Cobb-Douglas Technology. To ensure that the equilibrium success probability has an interior solution, we assume again that $\max\{\alpha_E (1 - d), \alpha_F d\} < 1/\Delta_X$. Equilibrium effort choices are then $e^*(s) = (\alpha_E (1 - d)(1 - s)\Delta_X [a^*(s)]^d)^{\frac{1}{1+d}}$ and $a^*(s) = (\alpha_F ds \Delta_X [e^*(s)]^{1-d})^{\frac{1}{2-d}}$, implying that

$$p^*(s) = \rho(s)\Delta_X, \quad (4)$$

where $\rho(s) := [\alpha_F ds]^d [\alpha_E (1 - d)(1 - s)]^{1-d}$. The venture capitalist's and entrepreneur's utility from the equity allocation s is

$$v(s) = \frac{1}{2}(2 - d) s \Delta_X^2 \rho(s), \quad (5)$$

and

$$u(s) = \frac{1}{2}(1 + d)(1 - s)\Delta_X^2 \rho(s). \tag{6}$$

respectively.

Figure 2: Equity Frontier for Cobb-Douglas Technology.

Figure 2 depicts the utility possibility and equity frontier for the Cobb-Douglas technology. Since efforts are complements, only a relatively small segment of the utility possibility frontier is undominated. In Appendix A we show that properties i)-iii) again hold.

The Bargaining Frontier

The bargaining frontier depicts the utility of the entrepreneur and venture capitalist, $U = u(s) + X_l - S$ and $V = v(s) + S - I$, respectively, for all Pareto-optimal contracts. We construct the bargaining frontier from the equity frontier by adding the riskless debt in a way that minimizes incentive distortions. (Adding the investment cost is trivial as it is borne by the venture capitalist.) The construction is simple. Suppose $s > \hat{s}$, in which case the venture capitalist holds too much and the entrepreneur too little equity relative to the second-best. Any Pareto-optimal contract where $s > \hat{s}$ must also have $S = X_l$, i.e., the venture capitalist must hold the entire debt. If not, a Pareto-improvement would be possible where the entrepreneur trades in debt for equity, thereby getting closer to the second-best solution. Similarly, if $s < \hat{s}$, the entrepreneur must hold the entire debt. If $s = \hat{s}$, any debt allocation is Pareto-optimal.

Figure 3: Construction of the Bargaining Frontier.

Figure 3 depicts the construction of the bargaining frontier. The figure is based on the equity frontier in Figure 1. We therefore have $\underline{v} = 0$, implying that $\max\{\underline{v} - I, 0\} = 0$. There are three regions. In the left interval, the entrepreneur holds the entire debt and an inefficiently large fraction of the equity. In the middle interval, both parties hold debt and equity. The equity allocation is second-best optimal. As we move along the frontier clockwise, debt is shifted to the venture capitalist until he holds all of it. As this is merely a wealth transfer, the slope of the bargaining frontier in the middle interval is minus one. Finally, in the right interval, the venture capitalist holds the entire debt and an inefficiently large fraction of the equity. This is summarized in the following lemma.

Lemma 1. *The bargaining frontier takes the following form*:

$$U = \psi_B(V) := \begin{cases} \psi_E(V + I) + X_l & \text{if } V \in [\max\{\underline{v} - I, 0\}, \hat{v} - I] \\ \psi_E(\hat{v}) + [X_l - I + \hat{v} - V] & \text{if } V \in [\hat{v} - I, \hat{v} + X_l - I] \\ \psi_E(V - X_l + I) & \text{if } V \in [\hat{v} + X_l - I, \overline{v} + X_l - I] \end{cases}. \tag{7}$$

3.2 Bargaining

It is reasonable to assume that when bargaining over a contract, the entrepreneur and venture capitalist choose a contract that is Pareto-efficient. Bargaining over a contract thus corresponds to choosing a utility pair (V,U) on the bargaining frontier. Let U^R and V^R denote the entrepreneur's and venture capitalist's reservation values, or outside options. For the moment we assume that reservation values are exogenous. In the following section, we derive reservation values endogenously as a function of the supply and demand in the capital market. We use the generalized Nash bargaining solution. Accordingly, the entrepreneur and venture capitalist select utilities $U = \psi_B(V)$ # U^R and V # V^R maximizing the $Nash\text{-}product$

$$\left[V \text{ " } V^R\right]^b \left[\psi_B(V) \text{ " } U^R\right]^{1-b}, \tag{8}$$

where b $ (0,1). For convenience, define $\beta := b/(1 " b)$.

As the bargaining frontier is strictly concave, the bargaining problem has a unique solution. Denote this solution by $\left(V^B, U^B\right)$, where $U^B = \psi_B(V^B)$. We can restrict attention to the case where V^B $ (max $\{\underline{v} " I, 0\}, \overline{v} + X_l " I)$, i.e., where the solution lies in the interior (see Appendix B, Proof of Proposition 1). It then follows from the first-order condition that

$$\beta = " \frac{V^B " V^R}{U^B " U^R} \psi_B'(V^B), \tag{9}$$

implying that V^B is continuous and strictly increasing (decreasing) in V^R (in U^R).

As is well known, the axiomatic Nash bargaining solution can be derived as the limit of a non-cooperative bargaining game where the two parties bargain with an open time horizon under the risk of breakdown (Binmore, Rubinstein, and Wolinsky 1986). If β = 0.5, the two players alternate in making proposals or are chosen with equal probability each period. If $\beta \neq$ 0.5, the players are chosen with unequal probabilities. Finally, it is worth noting that our results do not depend on the specifics of the Nash bargaining solution. All we need for our results is that an agent's bargaining utility is positively related to his own and negatively related to his counterparty's outside option.

3.3 Search

We finally embed the bargaining problem in a market environment. We consider a stationary search market populated by entrepreneurs and venture capitalists. The measure of entrepreneurs and venture capitalists in the market is M_E and M_F, respectively. A key variable is the ratio of venture capitalists to entrepreneurs, or $degree\ of\ competition$, $M_F/M_E =: \theta$. A high value of θ implies that the capital supply is highly competitive.

Time is continuous. Both sides discount future utilities at interest rate $r > 0$. From the perspective of a venture capitalist, the arrival rate of a deal is given by a decreasing function

$q(\theta)$, where $\lim_{\theta\to 0} q(\theta) = \&$ and $\lim_{\theta\to\infty} q(\theta) = 0$. Hence a venture capitalist is more likely to meet an entrepreneur in a given time interval if the ratio of venture capitalists to entrepreneurs is low. It is convenient to assume that $q(\theta)$ is continuously differentiable. Since the mass of deals per unit of time is $M_E q(\theta)$, the arrival rate of a deal from the perspective of an entrepreneur equals $\theta q(\theta)$, which is increasing in θ. Define $q_F(\theta) := q(\theta)$ and $q_E(\theta) := \theta q(\theta)$.

Example 3: Search Efficiency. Suppose the mass of deals per unit of time is given by $\xi[M_E M_F]^{0.5}$, where $\xi > 0$ represents an efficiency measure. For instance, ξ could be a measure of market transparency: matching is easier if the market is more transparent, which, holding the market size fixed, results in more deals per unit of time. Given this specification, arrival rates are $q_E(\theta) = \xi\theta^{0.5}$ and $q_F(\theta) = \xi\theta^{-0.5}$, respectively. We will return to this matching technology in Section 5.4 below.

If the search is successful, the venture capitalist and entrepreneur bargain over a contract.[7] Reservation values derive from the standard $asset\ value\ equations$[8]

$$rU^R = q_E(\theta)(U^B - U^R), \tag{10}$$

and

$$rV^R = q_F(\theta)(V^B - V^R). \tag{11}$$

If the venture capitalist and entrepreneur reach an agreement, they leave the market.

Stationarity requires that the inflow of venture capitalists and entrepreneurs matches the respective outflow. Denote the measure of entrepreneurs and venture capitalists arriving in the market over one unit of time by m_E and m_F, respectively. The market is stationary if

$$q_F(\theta)M_F = m_F, \tag{12}$$

[7] The model can be extended (e.g., by introducing heterogeneity or match complementarities) such that on average, a suitable partner is found only after several unsuccessful visits.

[8] Outside options can be valued as assets. Consider the entrepreneur's reservation value U^R. The (Poisson) arrival rate of a deal from the perspective of an entrepreneur is $q_E(\theta)$. The probability that a deal occurs in the next small time interval Δ is thus $q_E(\theta)\Delta$. With probability $1 - q_E(\theta)\Delta$ no deal occurs, and the entrepreneur continues searching. The expected discounted utility from searching is therefore

$$U^R = q_E(\theta)\Delta \exp(-r\Delta)\, U^B + (1 - q_E(\theta)\Delta)\exp(-r\Delta)\, U^R.$$

Solving for U^R and letting $\Delta \to 0$ using L'Hôpital's rule, we have

$$U^R = \frac{q_E(\theta)U^B}{q_E(\theta) + r}.$$

Rearranging terms yields (10). The intuition for (11) is analogous.

and

$$q_E(\theta)M_E = m_E. \tag{13}$$

As stationarity can always be ensured by scaling flows and stock accordingly, the market is fully characterized by the ratio of venture capitalists to entrepreneurs θ.

We conclude by summarizing the equilibrium conditions.

Definition: Market Equilibrium. *An equilibrium in the market with contracting, bargaining, and search is defined by the following conditions.*

i) *Bargaining utilities* $\left(V^B, U^B\right)$ *maximize the Nash product* (8).

ii) *Reservation values* $\left(V^R, U^R\right)$ *satisfy the asset value equations* (10)-(11).

iii) *Flows* (m_E, m_F) *and stocks* $(M,$

Region III: $S = 0$ *and* $s < \hat{s}$, *i.e., the venture capitalist holds no debt and an inefficiently small fraction of the equity compared to the second-best. The entrepreneur holds the entire debt and an inefficiently large fraction of the equity.*

3) Changes in Financial Contracts.

Region I: *An increase in* θ *leads to a decrease in the venture capitalist's equity stake and an increase in the entrepreneur's equity stake. Debt holdings remain constant.*

Region II: *An increase in* θ *leads to a decrease in the venture capitalist's debt and an increase in the entrepreneur's debt. Equity stakes remain constant.*

Region III: *see Region I.*

4) Net Value Creation. *The net value created* $U^B + V^B$ *is increasing in* θ *in Region I, constant in Region II, and decreasing in* θ *in Region III.*

The proof is in Appendix B. We proceed with a discussion.

Utilities. Bargaining and search utilities both move in the same direction. Consider, for instance, the entrepreneur. An increase in θ makes it easier for him to find a counterparty, which reduces his cost of delay. Hence U^R increases (and V^R decreases), which implies the bargaining outcome shifts in favor of the entrepreneur. The increase in U^B, in turn, feeds back into the search market dynamics. As the utility from doing a deal has gone up, searching for a deal becomes more valuable. U^R therefore increases again, which shifts the bargaining outcome further, and so on. All together, an increase in θ implies that we move along the bargaining frontier counterclockwise.

Financial Contracts. Regions I-III correspond to the three intervals in Figure 3 (in reverse order). The idea is to transfer utility in a way that minimizes incentive distortions. In Region I, the degree of competition and hence the entrepreneur's bargaining position are weak, implying that the bulk of the project return goes to the venture capitalist. At the margin, it is better to pay the venture capitalist with debt than with equity, for his equity stake is already too high compared to the second-best. Consequently, as θ increases, efficiency dictates that utility be transferred by reducing the venture capitalist's equity stake. If $s = \hat{s}$, we enter Region II. In this region, the equity allocation is second-best optimal. As θ increases, efficiency dictates that the venture capitalist's debt be reduced and the equity allocation remains constant. If $S = 0$, this possibility is exhausted, and we enter Region III. The only way to transfer utility to the entrepreneur is now to reduce s, which worsens efficiency as the venture capitalist's equity stake in this region is already inefficiently low.

Net Value Creation. Consider again Figure 3. The result follows immediately from the fact that in Region I the bargaining frontier has slope $\psi'_B < -1$, in Region II it has slope $\psi'_B = -1$, and in Region III it has slope $\psi'_B > -1$.

Perhaps the main insight to be taken away is that equity is used to transfer utility if the level of competition is either low or high, while riskless debt is used to transfer utility if the level of competition is intermediate. Accordingly, the equity component of venture capital contracts should be relatively stable for intermediate competition levels, but vary substantially with changes in competition if competition is either weak or strong. The opposite holds for the debt component. In relative terms, Proposition 1 implies that the $fraction$ of the entrepreneur's compensation consisting of debt is a U-shaped function of the competition level. Conversely, the fraction of the venture capitalist's compensation consisting of debt is a hump-shaped function of the competition level.

4.2 Market Value, Performance, and Success Probability

The gross value $p(e,a)\Delta_X + X_l$ represents the expected project return after effort and investment costs are sunk, but before returns are realized. It measures the average performance of venture-backed investments, assuming that effort costs cannot be observed. In monetary terms, it represents the fair value realized by the entrepreneur and venture capitalist if the firm was sold at an interim date. We shall refer to $p(e,a)\Delta_X + X_l$ as the market (or IPO) value of the venture.

In our model, the market value and the average performance depend on s or θ in the same way as the success probability $p(e,a)$. Unlike the net value $U^B + V^B$, the success probability is not necessarily a hump-shaped function of s and θ. Its functional behavior depends on whether the two efforts are complements or substitutes. For illustrative purposes, consider the CES family $p(e,a) = (da^\rho + (1 - d)e^\rho)^{\frac{1}{\rho}}$, where $\rho \neq 1$ and $d \in (0,1)$. The parameter ρ measures the degree of complementarity between the two efforts. If $\rho = 1$ the CES technology coincides with the linear technology, whereas if $\rho \to 0$ the isoquants of the CES technology approach those of the Cobb-Douglas technology. As ρ decreases, the degree of complementarity between e and a increases.

If $\rho < 1$ efforts are complements. It is straightforward to show that the equilibrium success probability $p^*(s)$ is then a hump-shaped function of s with interior maximum

$$s_p := \frac{1}{1+\phi}, \text{ where } \phi := \left[\left(\frac{\alpha_E}{\alpha_F}\right)^{\frac{\rho}{2}} \left(\frac{1-d}{d}\right)\right]^{\frac{1}{1-\rho}}.$$

The parameter $\phi > 0$ measures the relative productivity of the entrepreneur compared to the venture capitalist. If $\phi > 1$ the entrepreneur is more productive, which means his effort is less costly and/or the success probability is more responsive to his effort, implying that his contribution is more important. If $\phi < 1$ the venture capitalist is more productive, and if $\phi = 1$ productivities are equal. By inspection, the peak of $p^*(s)$ lies further to the left the greater the

entrepreneur's productivity relative to that of the venture capitalist. If productivities are equal, the peak is at s_p = 1/2.

If ρ = 1 efforts are substitutes. We showed in Example 1 that the success probability $p^*(s)$ is then a monotonic function of s. If $\phi > 1$ the function is strictly decreasing, while if $\phi < 1$ it is strictly increasing. If the entrepreneur is more productive, the success probability is thus maximized at s_p = 0. Conversely, if the venture capitalist is more productive, the success probability is maximized at s_p = 1. (This result can be obtained directly by maximizing (1) with respect to s.)

To establish the relation between market value, average performance, and success probability on the one side and competition on the other, recall that the venture capitalist's equity stake is a weakly decreasing function of θ, with a flat segment in Region II. Accordingly, if efforts are complements the market value, average performance, and success probability are all hump-shaped functions of the competition level (with a flat segment). On the other hand, if efforts are substitutes they are either weakly increasing (if $\phi > 1$) or weakly decreasing (if $\phi < 1$) functions of the competition level.

4.3 Up-front Payments

Suppose in addition to financing the investment cost I, the venture capitalist can make an up-front payment $P \in \left(0, \overline{P}\right]$. (It is reasonable to assume that the entrepreneur cannot make a noteworthy up-front payment). Like riskless debt, up-front payments can be used to transfer utility without affecting incentives. Consequently, the middle, i.e., linear, segment of the bargaining frontier will extend to the left. There are two cases: if $\overline{P} \geq \hat{v} - I$ the left segment vanishes completely. By contrast, if $0 < \overline{P} < \hat{v} - I$ all three segments of the bargaining frontier remain. The following result is then immediate.

Proposition 2. *Suppose up-front payments can take values* $P \in \left(0, \overline{P}\right]$.

i) If $\overline{P} < \hat{v} - I$ *Proposition 1 holds as is. The middle segment of the bargaining frontier is defined on* $V \in [\hat{v} - I - \overline{P}, \hat{v} + X_l - I]$, *while the leftmost segment is defined on* $V \in [\max\left\{\underline{v} - I - \overline{P}, 0\right\}, \hat{v} - I - \overline{P}]$.

ii) If $\overline{P} \geq \hat{v} - I$ *Region II extends to all* $\theta \geq \underline{\theta}$, *implying that Region III disappears. The middle segment of the bargaining frontier is defined on* $V \in [0, \hat{v} + X_l - I]$.

Introducing up-front payments has either no effect ($\overline{P}$ is small), or it causes Regions II and III to collapse into a single region ($\overline{P}$ is large), which has the same properties as Region II. From our perspective, what is important is that up-front payments have no effect on Region I. As the venture capitalist's equity stake in this region is inefficiently high, efficiency dictates that utility be transferred by reducing his equity stake. Hence even if up-front payments are

possible, they will not be used. Introducing (potentially large) up-front payments therefore does not affect our main conclusions: it is still true that for some competition levels equity is used to transfer utility, while for others riskless debt (or up-front payments) is used. A change in the level of competition continues to affect incentives, and thus the net value, success probability, market value, and performance of venture-backed investments. Likewise, it still holds that for some competition levels one side holds a mix of debt and equity and the other holds straight equity, while for other competition levels both sides hold a mix. The literature appears to favor the view that up-front payments should be small or even zero, however:

1) The promise to make up-front payments may attract a large pool of fraudulent entrepreneurs, or "fly-by-night operators" (Rajan), who take the money and run (Rajan 1992, von Thadden 1995. In the context of venture capital: Hellmann 2001). According to this argument, up-front payments should be zero.

2) Up-front payments may be limited due to incentive problems between the venture capitalist and *his* providers of capital (Holmström and Tirole 1997, Michelacci and Suarez 2000). To mitigate the problem, the venture capitalist must put up a fraction of his own wealth, which is arguably limited. Indeed, covenants in venture partnerships frequently require the "venture capitalist to invest a set dollar amount or percentage in every investment made by the fund" (Gompers and Lerner 1999, p. 40).

3) As the claims of limited partners in venture partnerships are senior, the claim of the general partner (i.e., the venture capitalist) has features of a call option. To counteract risk-taking by the venture capitalist, partnership agreements frequently include restrictions imposing "a maximum percentage of capital invested in the fund [...] that can be invested in any one firm" (Gompers and Lerner 1999, p. 38).

5 Endogenous Capital Supply

5.1 Closing the Model: Endogenous Entry

So far we have assumed that the degree of competition between venture capitalists is exogenous. While this may be an adequate description of the market in the short run, the long-run supply of venture capital is likely to be elastic. In what follows, we endogenize the supply side of venture capital, thereby endogenizing the degree of competition in the market. This allows us to study the relationship between venture capital contracts, the performance and value creation of ventures, and various exogenous factors. The factors are entry costs, government subsidies and regulation, investment profitability, and market transparency.

We take as given the flow of new ideas that are continuously created in the economy. Each idea is associated with a single entrepreneur and cannot be traded. We normalize the flow of

new ideas such that the mass m_E = 1 of ideas is created in the market over one unit of time. The inflow of venture capital is determined by a zero-profit constraint. We assume that venture capitalists entering the market incur entry costs $k > 0$, which may be conveniently thought of as the cost of raising capital or acquiring information. Zero profit then implies that $V^R = k$: to recoup entry costs, venture capitalists must realize a positive utility in the market. While we make the simplifying assumption that each venture capitalist can finance at most one project, all our results hold if venture capitalists can finance a finite number of projects. The equilibrium conditions are the same as before, except that m_E = 1 and $V^R = k$.

While it seems reasonable to assume that the supply of venture capital adjusts more quickly to changes in exogenous parameters than the supply of new ideas, our model can be easily extended to incorporate endogenous entry by both venture capitalists and entrepreneurs. Suppose, for instance, that entrepreneurs face entry costs c $ [0, \overline{c}]. These costs may, e.g., reflect initial R&D or the cost of setting up a business plan. Otherwise all projects are identical. If $\overline{c}$ is sufficiently large, there exists a threshold $\hat{c} < \overline{c}$ such that only entrepreneurs with cost c % $\hat{c}$ will enter the market. This setup, in particular, generates the same results as the one considered here.

5.2 Financing Costs

In our model, venture capitalists face two costs of providing finance: entry costs k and investment costs I. In what follows, we examine how changes in each of these costs affect the degree of competition in the capital market.

Entry Costs

The venture capital industry is highly cyclical. In the past decades, there have been two cycles: one starting in the late 1970s and ending in the late 1980s, the other starting in the early 1990s. Each cycle witnessed entry by new venture capital funds: between 1991 and 1999 alone, the number of independent venture capital funds in the United States increased from 34 to 204. A factor potentially related to this massive entry of funds are changes in entry costs. As more information about a technology or product becomes available, both the cost of information acquisition and the uncertainty premium required by investors fall, implying that entry costs decrease.

Consider the implications of a decrease in entry cost for the entry decision of venture capitalists.[9] From the equality $V^R = k$ we have that V^R must decrease by the same amount. By Proposition 1, this implies that the degree of competition θ must rise.

Proposition 3. *For each level of entry cost k, there exists a unique equilibrium. A decrease*

[9] The market will only open up if venture capitalists can potentially break even, i.e., if $\overline{v} + X_l - I > k$. We assume that this holds.

in k corresponds to an increase in the level of competition θ and vice versa. The effects on utilities and financial contracts are the same as in Proposition 1.

The proof is in Appendix B. Analogous to Proposition 1, there exist three regions: $k > \overline{k}$ (Region I), k \$ $\left[\underline{k}, \overline{k}\right]$ (Region II), and $k < \underline{k}$ (Region III). In each region, the respective statements from Proposition 1 hold. Among other things, this implies that the net value created in ventures is a hump-shaped function of entry costs: if entry costs are high, there are too few venture capitalists relative to the second-best. If entry costs are low, there are too many venture capitalists. An individual venture capitalist entering the market does not take into account the effect of his entry on the overall level of competition, and hence on the bargaining, contracting, and value creation in other ventures. Entry by an individual venture capitalist therefore either creates a positive (if there are too few venture capitalists) or negative (if there are too many) *contracting externality*.

Regulation and Public Policy

In 1978 the United States abandoned the "prudent man rule", a rule that prohibited pension fund investments in securities issued by venture capital funds. The regulatory change was accompanied by a cut in the capital gains tax from 49.5 to 28 percent. In 1981 taxes were cut further to 20 percent.[10] These changes triggered a surge in the supply of venture capital: commitments to private equity partnerships during 1980-82 totaled more than $3.5 billion, two and a half times the commitments to private equity during the entire decade of the 1970s.

Around the world, public programs have been designed to stimulate venture capital investment. In the United States, the bulk of these activities takes place on the state and local level (Florida and Smith 1992). In Europe, the I-TEC project sponsored by the European Union subsidizes initial (i.e., appraisal and management) costs, and the European Investment Fund provides subsidized investment loans. The intention of these programs is not to replace, but to stimulate the private provision of venture capital.

To examine the effect of investment subsidies, suppose they reduce the investment cost by γI. As the venture capitalist's investment outlay is now only (1 " γ) I, fewer financial claims are needed to make him break even. An increase in γ therefore either reduces the venture capitalist's equity stake s or his debt holdings S, whichever is more efficient.[11] (If $s > \hat{s}$ a decrease in s is more efficient, while if s % $\hat{s}$ a decrease in S is more efficient).

[10] Fenn, Liang, and Prowse (1995) provide a detailed account of the events.

[11] Technically, an increase in γ shifts the bargaining frontier to the right. There are two effects at work. First, V^B increases, implying that θ must increase since $V^R = k$. Second, U^B increases, implying that either s or S must decrease. Both effects counteract the increase in γ, hence ensuring that $V^R = k$.

Proposition 4. *An investment subsidy γI has the same effect on financial contracts as an (exogenous) increase in the degree of competition θ or a decrease in entry cost k.*

The proof is in Appendix B. Of course, government can also directly subsidize or tax entry, thereby changing entry costs. The effect is then as described in Proposition 3.

5.3 Investment Profitability

Another factor that might potentially trigger a positive influx of venture capital is an increase in investment profitability. In our model, an increase in investment profitability can mean a number of things: an increase in Δ_X or X_l, or a decrease in I. In either case, the bargaining frontier shifts outward. In what follows, we show that if an increase in profitability brings about a change in the *type* of financial contract, i.e., if we move across the three regions in Proposition 1, the only possible direction is from Region I to Region II to Region III. This is the same direction as in the case of an exogenous increase in the degree of competition θ or a decrease in entry cost k. The result does not depend on the particular source of profitability increase.

The argument proceeds in two steps. First, it is reasonable to assume that the relative importance of the entrepreneur's and venture capitalist's contribution does not change as the investment becomes more profitable. Formally, this implies that the ratio of second-best utilities $\hat{v}/\hat{u}$ remains unchanged. Second, as the investment becomes more profitable, a smaller share of the proceeds is needed to make the venture capitalist break even. As an illustration, suppose we are in Region III where $s < \hat{s}$ and $S = 0$. The only way to reduce the venture capitalist's share of the proceeds is to reduce his equity stake s. As $\hat{v}/\hat{u}$ is unchanged, this implies we must still be in Region III, for moving to Region II or I would mean giving the venture capitalist *more* equity. Similar arguments hold if we are in Regions I or II. The proof of the following result is in Appendix B.

Proposition 5. *Holding the ratio of second-best utilities $\hat{v}/\hat{u}$ fixed, an increase in investment profitability changes the type of financial contract in the same way as an (exogenous) increase in the degree of competition θ or a decrease in entry cost k.*

5.4 Market Transparency

So far we have studied changes affecting the costs and returns to financing. The market for venture financing itself may undergo changes, however. In this section, we study how a change in market transparency affects optimal financial contracts. We use the Cobb-Douglas matching technology introduced in Example 3. According to this specification, the mass of new deals per unit of time is $\xi[M_E M_F]^{0.5}$, where $\xi > 0$ is a measure of market transparency, or more generally, market efficiency.

Consider first the case where the degree of competition θ is exogenous. Inserting the reservation values (10)-(11) in the first-order condition for the Nash product (9), we obtain

$$\beta \frac{r + \xi\theta^{-\frac{1}{2}}}{r + \xi\theta^{\frac{1}{2}}} = (1-\beta)\psi_B(V^B)\frac{V^B}{\psi(V^B)}. \tag{14}$$

Differentiating V^B with respect to ξ while holding θ fixed shows that if $\theta < 1$, i.e., if entrepreneurs outnumber venture capitalists, an increase in transparency increases the bargaining utility of venture capitalists. Conversely, if $\theta > 1$ an increase in transparency reduces V^B. Intuitively, the greater the market transparency, the stronger is the impact of competition, or relative supply and demand, on the bargaining outcome. As this benefits the short side of the market, its bargaining utility increases. Second, implicitly differentiating (9) reveals that the cross-derivative of V^B with respect to θ and ξ is negative. In words, the greater the market transparency, the faster we move along the bargaining frontier as the level of competition changes. Again, a greater market transparency amplifies the impact of market structure on the bargaining outcome. To summarize, the effect of an exogenous change in θ on contracts and utilities described in Proposition 1 is more pronounced the greater the market transparency.

In the case where entry is endogenous, the effect of transparency on the bargaining utilities is ambiguous if $\theta < 1$, but unambiguous if $\theta > 1$. Consider, for instance, the effect on V^B. There are two effects. First, we know from the above discussion that an increase in transparency has a positive effect on V^B if $\theta < 1$ and a negative effect if $\theta > 1$. Second, an increase in transparency reduces frictions and therefore the cost of delay. In principle, this implies that the venture capitalist's overall utility from being the market, V^R, must rise. But V^R cannot rise as it is determined by the equality $V^R = k$. To ensure that V^R remains constant, the bargaining utility V^B must therefore decrease. Accordingly, there are two forces driving V^B. If $\theta > 1$ they move in the same direction, whereas if $\theta < 1$ they move in the opposite direction. This is summarized in the following proposition. The proof is in Appendix B.

Proposition 6. *If entry costs k are low (implying that $\theta > 1$), an increase in market transparency has the same effects on financial contracts and bargaining utilities as an (exogenous) increase in the degree of competition θ or a decrease in entry cost k. If entry costs are high, the effect is ambiguous.*

6 Ex-Ante Project Screening

Thus far we have focused on two functions ascribed to venture capitalists: providing capital and coaching projects. We now turn to a third, equally important function: separating good ideas from bad ones. While entrepreneurs are always optimistic about their ideas, venture capitalists,

due to their experience and industry knowledge, are likely to have a better sense of whether a given idea is workable or not. Suppose projects come in two qualities, or types: $t \$ T := \{L, H\}$. The probability that a project (or entrepreneur) is of a high quality is $\pi \$ (0, 1]$. Only high-quality projects are profitable. For simplicity, suppose a low-quality project yields a zero return for sure. A priori, neither the venture capitalist nor the entrepreneur knows the project's quality. The venture capitalist can, however, learn it by screening the project. Screening comes at a cost $C > 0$.

In what follows, we will argue that venture capitalists are less likely to screen projects if the level of competition is strong. This is consistent with casual evidence that tapping venture capital is easier in times when the venture capital market is booming.[12] Before stating the result, let us briefly lay out the argument. Each venture capitalist can finance and coach only a certain number of projects. For simplicity, we assume in our model that this number is one. Before sinking time and capital into a particular project, a venture capitalist will weigh the benefit from doing so against the opportunity cost from (not) searching for a better candidate. If the opportunity cost is high, the venture capitalist will carefully screen the project before making the investment. If the opportunity cost if low, the benefits from screening are also low. Recall that the utility from searching is V^R. Since $V^R = k$, we have that the venture capitalist is more likely to screen if entry costs are high, or alternatively, if the degree of competition is low.

The sequence of moves is as follows. The venture capitalist and the entrepreneur bargain over a contract which grants the venture capitalist the right to withdraw if he finds out that the project is of low quality. Subsequently, the venture capitalist decides whether to screen or not. If he screens, he learns the project quality for sure. If he does not screen, he holds prior beliefs that he faces a high-quality project with probability π. If the venture capitalist does not withdraw, he sinks the investment cost I. After the investment cost is sunk, the project quality is fully revealed. The last assumption simplifies the analysis as it implies that effort choices are made under complete information.

In a slight abuse of notation, denote the venture capitalist's expected utility from investing in a high-quality project by V^B. If the venture capitalist screens and finds out that the quality is low, it is optimal for him not to invest and to search anew. Moreover, as a low-quality project generates a zero return, it does not pay an entrepreneur who has been rejected to stay in the market.[13] The expected utility from screening is then πV^B + (1 " π)k " C. By contrast, the expected utility from not screening is πV^B " (1 " π)I. Consequently, screening is optimal if and

[12] See, e.g., Finance is a Siren Song, Financial Times, March 13, 2000.

[13] This rules out the existence of (negative) $pool\ externalities$. See Broecker (1990) for details.

only if

$$C \leq (1 - \pi)(k + I). \tag{15}$$

From Proposition 3, we know that for each entry cost k there exists a unique corresponding competition level θ. We thus have the following result.

Proposition 7. *Ceteris paribus, screening is more likely if i) the cost of screening is low, ii) the fraction of low-type projects is large, iii) the investment outlay (which is lost if a low-quality project is financed) is large, and iv) entry cost k are high, or alternatively, the degree of competition θ is low.*

7 Portfolio Investors vs. Advisors

Gompers and Lerner (1999, p.4) point out that "[t]he skills needed for successful venture capital investing are difficult and time-consuming to acquire. During periods when the supply or demand for venture capital has shifted, adjustments in the number of venture capitalists and venture capital organizations appear to take place very slowly." The consequences of this are particular apparent in boom periods such as the 1990s, where the demand for venture capital increased sharply. In an attempt to fill the gap, an increasing number of new, inexperienced investors entered the market.[14] In this section, we model both the short-run stickiness of informed capital and the entry by inexperienced investors who have money but no skills. We call the latter *portfolio investors*.

Portfolio investors may be viewed as agents whose cost of providing effort is infinitely high. To distinguish between regular venture capitalists and portfolio investors, we use the subscripts $f \in F := \{A, P\}$, where A stands for *advisor*. There are two main differences between the bargaining frontiers $\psi_{B,A}$ and $\psi_{B,P}$. First, the fact that advisors add value while portfolio investors do not shifts $\psi_{B,A}$ outward relative to $\psi_{B,P}$. Second, unlike $\psi_{B,A}$, the bargaining frontier with portfolio investors is strictly concave with slope $\psi'_{B,P} < -1$ everywhere. As the moral hazard problem is only one-sided, efficiency dictates that portfolio investors hold as little equity as possible. Accordingly, utility should be transferred by reducing the portfolio investor's equity stake. The bargaining frontier does not have a linear segment since the second-best allocation $\hat{s}_P = 0$ is not feasible. ($\hat{s}_P = 0$ implies $\hat{v}_P = 0$, which in turn implies $\hat{v}_P + X_l - I < 0$.)

As before, we assume that the mass $m_E = 1$ of new ideas is created in the market over one unit of time. We now additionally assume that over the same time period, the mass $\overline{m} < 1$

[14] See, e.g., Banks Seek to Mine a Rich Seam: Private Equity, Financial Times, June 30, 2000, as well as the reference in footnote 3.

of advisors arrives at the market fringe, where they must decide whether to enter or not.[15] The inflow of portfolio capital is endogenous and determined by a zero-profit constraint. (The formal equilibrium conditions, which extend the definitions in Sections 3.3 and 5.1, are stated in Appendix C.) To ensure that portfolio investors can potentially break even, we assume that $\overline{v}_P + X_l - I > k$. That is, the bargaining utility of portfolio investors under the most favorable bargaining outcome must be sufficiently large to cover the entry cost k. The following result shows that Proposition 3 extends to multiple investor types. The proof is in Appendix C.

Proposition 8. *For each entry cost k, there exists a unique equilibrium where both types of investors enter. Analogous to Proposition 3, a decrease in k raises the level of competition θ and reduces the bargaining utility of both advisors and portfolio investors. As for ventures between entrepreneurs and advisors, the effect on financial contracts* (*and thus on incentives and value creation*) *is the same as in Proposition 1. As for ventures between entrepreneurs and portfolio investors, a decrease in k leads to a decrease in the portfolio investor's equity stake and an increase in the entrepreneur's equity stake. The portfolio investor holds the entire debt, and his debt holdings remain constant throughout.*

Entry by an individual portfolio investor raises the overall level of competition. The extent to which this entails a positive or negative (contracting) externality depends on whether we consider ventures with advisors or portfolio investors. As for ventures between entrepreneurs and portfolio investors, the externality is unambiguously positive. As competition increases, utility is transferred to the entrepreneur by reducing the portfolio investor's equity stake. Since the moral hazard problem is one-sided, this improves efficiency. Also, both the market value and the success probability increase. With regard to ventures between entrepreneurs and advisors, the result is ambiguous. We showed in Section 5.2 that the externality is positive if competition is weak but negative if competition is strong. In the latter case, entry by portfolio investors reduces the net value in ventures financed by advisors and, provided efforts are complements, also the market value and success probability in these ventures.

Being unskilled, portfolio investors have no advantage vis-a-vis advisors in our model. In practice, this need not be the case. For instance, portfolio investors might have access to cheaper funding, e.g., from in-house sources. On the other hand, advisors might have both better and cheaper access to industry-specific information. In a young industry where the information acquisition problem is important, advisors may thus face lower entry cost. By contrast, in a mature industry portfolio investors may face lower entry costs. Given these assumptions, our model has the following implication: as entry costs decrease and the relative cost advantage of portfolio investors increases, advisors are driven out of the market. The reason for this is that i)

[15] If $\overline{m} > 1$, we are back to the previous setting. If $\overline{m} = 1$, there exist multiple equilibria.

portfolio investors eventually face lower entry costs, and, more interestingly, ii) the increase in competition and the associated negative contracting externality decrease the surplus produced by advisors.

8 Concluding Remarks

Our paper yields numerous empirical predictions relating venture capital contracts, the intensity of ex-ante project screening, and the value, success probability, and performance of venture-backed investments to the relative supply and demand for venture capital (or degree of capital market competition), entry costs, market transparency, investment profitability, and public policy. Most of these predictions remain yet to be tested. The ones that have been tested, however, appear to be consistent with the empirical evidence. Perhaps the most crucial aspect of our model is that the entrepreneur's equity share increases with the level of competition (except for intermediate levels, where it remains constant). This implies that incentives, the market value, and the success probability of ventures all vary with the supply and demand for venture capital. Indeed, Gompers and Lerner (2000) find that valuations are higher in times of strong competition, which implies that the equity share of entrepreneurs increases as capital inflows and competition among venture capitalists become more intense.[16] The authors also examine the relation between competition and the success of ventures. They find no statistically significant difference for investments made during the late 1980s, a period when competition was strong, and the early 1990s, a period of relatively low inflows. This is consistent with the hump-shaped relation predicted by our model (if efforts are complements), which implies that the success rate is low if competition is either strong or weak. It is, however, also consistent with the hypothesis that there is no relation between competition and success. To discriminate between these two hypotheses, more than two periods will be needed. Finally, Kaplan and Strömberg (2001a) find that the entrepreneur's equity share increases with investment performance, which is consistent with the results stated in Proposition 5.

Our model can be extended in several directions. As it stands, the focus is on cash-flow rights. Real-world venture capital contracts, however, include cash-flow rights, voting rights, liquidation rights, board rights, and other instruments (Kaplan and Strömberg 2001a). One conceivable extension is to examine how changes in competition affect the mix of different contractual provisions. Again, the underlying principle must be that utility is transferred in a way that minimizes incentive distortions. Second, we treat venture capitalists as a single entity. In practice, venture capital partnerships consist of general and limited partners, which are tied together by a contract. It would be interesting to analyze how competition simultaneously

[16] We thank Paul Gompers for bringing this to our attention.

affects both types of contracts, i.e., contracts between venture capitalists and entrepreneurs and contracts between the general and limited partners. The question is whether the two contracts move in the same or opposite direction. Third, we do not allow for project choice. Suppose there is a choice between projects that rely heavily on the effort of venture capitalists and projects that do not. If competition is strong and venture capitalists' equity stakes are inefficiently small, it will become optimal for venture capitalists to move away from effort-intensive activities such as early-stage seed financing toward less effort-intensive activities.

9 Appendix

9.1 Appendix A: Specific Production Technologies

Example 1: Linear Technology. The equity frontier derives from the following program: the entrepreneur choose s to maximize $u(s)$ subject to the constraint that $v(s) \# v$. The solution is characterized for all feasible reservation values $v \# 0$. (If v is too large, the solution is not feasible). In a slight abuse of notation, we denote the solution by $s^*(v)$. Both $v(s)$ and $u(s)$ are strictly quasiconcave. Accordingly, s^* is a solution to the entrepreneur's problem if and only if $v(s)$ is nondecreasing and $u(s)$ is nonincreasing at s^*. Define

$$\underline{s} := \frac{\alpha_F d^2 \text{ " } \alpha_E (1 \text{ " } d)^2}{2\alpha_F d^2 \text{ " } \alpha_E (1 \text{ " } d)^2},$$

and

$$\overline{s} := \frac{\alpha_E (1 \text{ " } d)^2}{2\alpha_E (1 \text{ " } d)^2 \text{ " } \alpha_F d^2},$$

where $0 < \underline{s} < \overline{s} < 1$. We obtain the following result:

i) if $\alpha_E (1 \text{ " } d)^2 > \alpha_F d^2$, the set of Pareto-optimal equity allocations is $[0, \overline{s}]$,

ii) if $\alpha_E (1 \text{ " } d)^2 = \alpha_F d^2$, the set of Pareto-optimal equity allocations is $[0, 1]$, and

iii) if $\alpha_E (1 \text{ " } d)^2 < \alpha_F d^2$, the set of Pareto-optimal equity allocations is $[\underline{s}, 1]$.

Case i) is the case discussed in the text where the entrepreneur is more productive. In case i), define $\underline{v} := 0$ and $\overline{v} =: v(\overline{s})$. In case ii), define $\underline{v} := 0$ and $\overline{v} := v(1)$. Finally, in case iii), define $\underline{v} := v(\underline{s})$ and $\overline{v} := v(1)$. For any $v \$ [\underline{v}, \overline{v}]$, the solution $s^*(v)$ satisfies

$$v = \frac{1}{2}\alpha_F d^2 (s^*(v))^2 \Delta_X^2 + \alpha_E (1 \text{ " } d)^2 s^*(v) (1 \text{ " } s^*(v)) \Delta_X^2. \tag{16}$$

Solving (16) for $s^*(v)$, we obtain

$$s^*(v) = \frac{\alpha_E (1 \text{ " } d)^2 \Delta_X \text{ " } \Phi}{\left(2\alpha_E (1 \text{ " } d)^2 \text{ " } \alpha_F d^2\right) \Delta_X}, \tag{17}$$

where $\Phi := \sqrt{\alpha_E^2 (1 - d)^4 \Delta_X^2 - 2v(2\alpha_E (1 - d)^2 - \alpha_F d^2)}$. Clearly, $s^*(v)$ is strictly increasing. Inserting (17) in (3) generates the equity frontier $\psi_E(v)$. Differentiating $\psi_E(v)$ twice with respect to v, we have

$$\frac{d^2\psi_E(v)}{dv^2} = -\Phi^{-3}\left[\left(\alpha_E (1 - d)^2 - \alpha_F d^2\right)^2 + \alpha_E (1 - d)^2 \alpha_F d^2\right] < 0.$$

To show that the sum of utilities $v + \psi_E(v)$ attains its maximum in the interior of $[\underline{v}, \overline{v}]$, we compute the derivative of $\psi_E(v)$ at the boundaries. In case i), we have $\psi'_E(\underline{v}) = [\alpha_F d^2 - \alpha_E (1 - d)^2]/[\alpha_E (1 - d)^2] > -1$ and $\lim_{v \to \overline{v}} \psi'_E(v) = -\infty$. In case ii), we have $\psi'_E(\underline{v}) = 0$ and $\lim_{v \to \overline{v}} \psi'_E(v) = -\infty$. Finally, in case iii), we have $\psi'_E(\underline{v}) = 0$ and $\psi'_E(\overline{v}) = -\alpha_F d^2/[\alpha_F d^2 - \alpha_E (1 - d)^2] < -1$. Since $\psi_E(v)$ is strictly concave, this implies that in each case there exists a unique $\hat{v} \in (\underline{v}, \overline{v})$ such that $\psi'_E(\hat{v}) = -1$. Q.E.D.

Example 2: Cobb-Douglas Technology. Again, $v(s)$ and $u(s)$ are both quasiconcave, implying that s^* solves the entrepreneur's problem for some v if and only if $v(s)$ is nondecreasing and $u(s)$ is nonincreasing at s^*. Differentiating (5) with respect to s, we have that $v(s)$ is nondecreasing if and only if $s \geq [1 + d]/2$. Similarly, differentiating (6) with respect to s, we have that $u(s)$ is nonincreasing if and only if $s \leq d/2$. Accordingly, the set of Pareto-optimal equity allocations is $[d/2, [1 + d]/2]$, implying that $\underline{v} := v(d/2)$ and $\overline{v} := v([1 + d]/2)$. Moreover, $v(s)$ is strictly increasing for all $s < [1 + d]/2$, implying that $s^*(v)$ is strictly increasing for all $v < \overline{v}$. We next show that ψ_E is strictly concave. Differentiating $u(s)$ and $v(s)$ twice, we obtain

$$\begin{aligned}\frac{d^2\psi_E(v)}{dv^2} = & -\frac{1}{2}(1 + d)\Delta_X^2 \rho(s^*(v))\left(\frac{1}{v'(s^*(v))}\right)^2 \\ & \cdot\left(\frac{d}{(s^*(v))^2} + \frac{(1 - d)(2s^*(v) - d)}{s^*(v)(1 - s^*(v))(1 + d - 2s^*(v))}\right),\end{aligned}$$

which is strictly negative for all $s^* \in [d/2, (1 + d)/2]$. To show that the sum of utilities $v + \psi_E(v)$ attains its maximum in the interior of $[\underline{v}, \overline{v}]$, we compute the derivative of $\psi_E(v)$ at the boundaries. The derivative of $\psi_E(v)$ is

$$\frac{d\psi_E(v)}{dv} = -\frac{(1 + d)(2s^*(v) - d)(1 - s^*(v))}{s^*(v)(2 - d)(1 + d - 2s^*(v))}. \tag{18}$$

Evaluating (18) at $\underline{v}$ and $\overline{v}$, we obtain $\psi'_E(\underline{v}) = 0$ and $\lim_{v \to \overline{v}} \psi'_E(v) = -\infty$. Since $\psi_E(v)$ is strictly concave, this implies there exists a unique $\hat{v} \in (\underline{v}, \overline{v})$ such that $\psi'_E(\hat{v}) = -1$. Q.E.D.

9.2 Appendix B: Proofs of Propositions 1 and 3-6

Proof of Proposition 1. To apply the first-order condition (9), we must first show that the bargaining outcome lies in the interior of the domain of ψ_B. Given that $V^R \geq 0$ and $U^R \geq 0$,

there are only two possible cases where this might not hold: if $\underline{v}$ " $I > 0$ and $\psi'_B(\underline{v}$ " $I) < 0$, and if $\psi_B(\overline{v} + X_l$ " $I) > 0$ and $\lim_{V \to \overline{v}+X_l-I} \psi'_B(V) >$ " &. To show that these cases cannot arise, we argue to a contradiction. Suppose that $\underline{v}$ " $I > 0$ and $\psi'_B(\underline{v}$ " $I) < 0$, implying that $s^*(\underline{v}) > 0$. But $\psi'_E(v) = u'(s^*)/v'(s^*)$, $\psi'_E(\underline{v}) < 0$, and Pareto optimality imply that $u'(s^*(\underline{v})) < 0$. Hence there exists some equity allocation $s < s^*(\underline{v})$ which makes the entrepreneur better off, contradicting the construction of ψ_E, which requires that $u(s)$ is maximized at $s^*(\underline{v})$. An analogous argument holds for the second case.

We next prove uniqueness. Inserting (10)-(11) into (9) yields

$$\beta \frac{r + q_F(\theta)}{r + q_E(\theta)} = \text{"} \psi'_B(V^B) \frac{V^B}{\psi_B(V^B)}. \tag{19}$$

From the uniqueness of the bargaining solution, it follows that (19) has a unique solution (U^B, V^B), where V^B $ (0, \overline{v} + X_l$ " $I)$. As for claim 1), implicitly differentiating (19) with respect to θ gives

$$\frac{dV^B}{d\theta} = \beta \frac{[r + q_F]\, q'_E \text{"} [r + q_E]\, q'_F}{[r + q_E]^2} \frac{\psi_B^2}{\psi_B \left[\psi'_B + V^B \psi''_B\right] \text{"} V^B \left(\psi'_B\right)^2} < 0,$$

implying that $dU^B/d\theta > 0$. This, in conjunction with the monotonicity of q_E and q_F, implies that V^R is decreasing and U^R increasing in θ.

Claims 2)-3) follow now from the construction of the bargaining frontier and the monotonicity of U^B and V^B. In particular, by (19), the threshold $\underline{\theta}$ is uniquely determined by $V^B = \hat{v} + X_l$ " I , while $\overline{\theta}$ is uniquely determined by $V^B = \widehat{v}$ " I. Q.E.D.

Proof of Proposition 3. In equilibrium, θ, U^B, and V^B are determined by (19) and the zero-profit constraint $V^R = k$. The total derivatives from these two equations generate the following equation system:

$$\begin{pmatrix} \text{"} \beta \frac{[r+q_F]q'_E - [r+q_E]q'_F}{[r+q_E]^2} & \frac{\psi_B\left[\psi'_B V^B \psi''_B\right] - V^B\left(\psi'_B\right)^2}{\psi_B^2} \\ V^B \frac{rq'_F}{[q_F+r]^2} & \frac{q_F}{q_F+r} \end{pmatrix} \begin{pmatrix} d\theta \\ dV^B \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} dk. \tag{20}$$

The determinant of this system, D, is negative. In conjunction with the limit properties of q_F and q_E for θ ' 0 and θ ' &, this establishes the existence and uniqueness of a solution to the equation system given by (19) and $V^R = k$. We can now apply Cramer's rule and obtain

$$\frac{d\theta}{dk} = \text{"} \frac{1}{D} \left[\text{"} V^B \frac{rq'_F}{[q_F + r]^2} \frac{\psi_B \left[\psi'_B + V^B \psi''_B\right] \text{"} V^B \left(\psi'_B\right)^2}{\psi_B^2} \right] < 0.$$

The rest follows from Proposition 1. Q.E.D.

Proof of Proposition 4. An increase in γ shifts the bargaining frontier outward. (The bargaining frontier is the same as in the basic model, except that I is replaced by $\Delta_I :=$

(1 " γ)I.) Define $\zeta := \psi'_B(V)/\psi_B(V)$, and observe that $d\zeta/d\Delta_I < 0$. (More precisely, $d\zeta/d\Delta_I = [\psi_B\psi''_B$ " $(\psi'_B)^2]/\psi_B^2 < 0$ if V % $\hat{v}$ " Δ_I or V # $\hat{v}$ + X_l " Δ_I, and $d\zeta/d\Delta_I = \psi'_B/\psi_B^2 < 0$ if $\hat{v}$ " Δ_I % v % $\hat{v}$ + X_l " Δ_I.) In what follows, we show that $dU^B/d\Delta_I < 0$, which proves the claim in the proposition.

We argue to a contradiction, assuming that U^B is nondecreasing in Δ_I for some Δ_I. Inspection of the equilibrium conditions shows that this can only happen if θ is nondecreasing in Δ_I. However, from the equation system generated by the total derivatives of (19) and $V^R = k$, we have that

$$\frac{d\theta}{d\Delta_I} = \text{"}\frac{1}{D}V^B\frac{d\zeta}{d\Delta_I}\frac{q_F}{q_F + r} < 0,$$

contradiction. Q.E.D.

Proof of Proposition 5. Proposition 4 considers a decrease in I. It therefore remains to prove the result for an increase in Δ_X and/or X_l. Take any two profitability levels L (low) and H (high). We argue to a contradiction, assuming that an increase in profitability from L to H induces a clockwise move across regions. The proof proceeds in two steps. We first show that this must imply that

$$\frac{V_H^B}{U_H^B} < \frac{V_L^B}{U_L^B}. \tag{21}$$

Recall that $V_H^R = V_L^R = k$. Suppose first that θ_H % θ_L. From the zero-profit constraint $V_H^R = k$, it follows that V_H^B % V_L^B, implying $U_H^B > U_L^B$ and therefore (21). Suppose next that $\theta_H > \theta_L$. Moving clockwise across regions implies that

$$\text{"}\psi'_{B,H}(V_H^B) > \text{"}\psi'_{B,L}(V_L^B). \tag{22}$$

Moreover, equations (9)-(11) imply that

$$\text{"}\psi'_{B,H}(V_H^B)\frac{V_H^B}{\psi_{B,H}(V_H^B)}\frac{r + q_E(\theta_H)}{r + q_F(\theta_H)} = \text{"}\psi'_{B,L}(V_L^B)\frac{V_L^B}{\psi_{B,L}(V_L^B)}\frac{r + q_E(\theta_L)}{r + q_F(\theta_L)}. \tag{23}$$

Together, (22)-(23), $\theta_H > \theta_L$, and monotonicity of q_E and q_F, imply (21).

Having established that (21) must hold, we can now consider all possible moves across regions. Suppose first we move from Region II or III to Region I. By the definition of Regions I-III, this implies that

$$\frac{V_H^B}{U_H^B} > \frac{\hat{v}_H + X_l \text{ " } I}{\hat{u}_H}, \tag{24}$$

and

$$\frac{V_L^B}{U_L^B} \text{ \% } \frac{\hat{v}_L + X_l \text{ " } I}{\hat{u}_L}. \tag{25}$$

Suppose first that Δ_X increases while X_l remains constant, implying that $\hat{u}_H > \hat{u}_L$. In conjunction with (24)-(25) and the assumption that $\hat{v}_H/\hat{u}_H = \hat{v}_L/\hat{u}_L$, this contradicts (21). Suppose next that X_l increases while Δ_X remains constant. As the equity frontier remains unchanged, we have $\hat{u}_H = \hat{u}_L$. Again, by (24)-(25), this contradicts (21). Given these arguments, the argument if both Δ_X and X_l increase is obvious. The argument for why a move from Region III to Region II can be ruled out is analogous. Q.E.D.

Proof of Proposition 6. Define $\omega_1 := (r + \xi\theta^{-\frac{1}{2}})/(r + \xi\theta^{\frac{1}{2}})$ and $\omega_2 := \xi\theta^{-\frac{1}{2}}/(r + \xi\theta^{-\frac{1}{2}})$. Note that $d\omega_2/d\xi > 0$, $d\omega_1/d\xi < 0$ if $\theta > 1$, and $d\omega_1/d\xi > 0$ if $\theta < 1$. From the equation system generated by the total derivatives of (19) and $V^R = k$, we have that

$$\frac{dV^B}{d\xi} = \frac{1}{D}\left[\omega_2' V^B \beta \frac{[r + q_F]\, q_E' - [r + q_E]\, q_F'}{[r + q_E]^2} + \omega_1' \beta V^B \frac{r q_F'}{[q_F + r]^2}\right],$$

which is strictly negative if $\theta > 1$. It remains to prove that θ becomes arbitrarily large if k is sufficiently small. Suppose not. There then exists a sequence of equilibria indexed by n such that $\theta_n \to \overline{\theta}$ as $k_n \to 0$, where $\overline{\theta}$ is finite. By (19), this implies that V_n^B converges to some strictly positive value $\overline{V}^B$. Since $\theta_n \to \overline{\theta}$, and since r remains constant, this contradicts the zero-profit constraint for large n. Q.E.D.

9.3 Appendix C: Proof of Proposition 8

We first state the conditions for a market equilibrium with different investor types $f \in F := \{A, P\}$. The distribution of investors in the market is $\mu_f := M_f / \sum_{f \in F} M_f$, while the degree of competition is $\theta := \sum_{f \in F} M_f / M_E$. Investor utilities are denoted by V_f^R and V_f^B, respectively, while the utility of an entrepreneur bargaining with a type-f investor is denoted by U_f^B. Equilibrium utilities derive from the equations

$$V_f^R = \frac{q_F(\theta)}{r + q_F(\theta)} V_f^B \text{ for all } f \in F, \tag{26}$$

and

$$U^R = \frac{q_E(\theta)}{r + q_E(\theta)} \sum_{f \in F} \mu_f U_f^B. \tag{27}$$

Stationarity requires that

$$q_f(\theta) M_f = m_f \text{ for any } f \in F \text{ with } \mu_f > 0, \tag{28}$$

and

$$q_E(\theta) M_E = m_E. \tag{29}$$

We can now state the equilibrium conditions as follows:

Definition: Market Equilibrium with Two Types of Investors. *A market equilibrium in the market with contracting, bargaining, and search with two types of investors is defined by the following conditions, which must hold for all* $f \in F$.

i) *Bargaining utilities* (V_f^B, U_f^B) *maximize the Nash product* (8).

ii) *Reservation values* (V_f^R, U^R) *and* V_f^R *satisfy the asset value equations* (26)-(27).

iii) *Flows* (m_E, m_f) *and stocks* $(M_E,\ M_f)$ *satisfy the stationarity conditions* (28)-(29).

iv) *Entry must be optimal for each type of investor*:

- *Type* $f = A$ (*in short supply*): $m_A = \overline{m}$ *if* $V_A^R > k$, $m_A \in [0, \overline{m}]$ *if* $V_A^R = k$, *and* $m_A = 0$ *if* $V_A^R < k$.
- *Type* $f = P$ (*not in short supply*): $V_P^R = k$ *if* $m_P > 0$ *and* $V_P^R \leq k$ *if* $m_P = 0$.

Proof of Proposition 8. Stationarity in conjunction with $V_A^R > V_P^R$ implies that $m_A = \overline{m}$ and $m_E = 1 - \overline{m}$. We now show that each θ is associated with a unique set of bargaining utilities V_f^B, which are continuous and decreasing in θ. One implication of this is that the reservation values V_f^R are also decreasing in θ. In conjunction with the constraint $V_P^R = k$, this implies that the equilibrium is unique and that the bargaining utilities of both investor types are increasing in k, as claimed in the proposition. Following a decrease in k, we therefore move counterclockwise along each of the two bargaining frontiers. The rest follows from the construction of $\psi_{B,A}$ and $\psi_{B,P}$.

To simplify the notation, define $\rho_1(\theta) := r/[r + q_F(\theta)]$ and $\rho_2(\theta) := q_E(\theta)/[r + q_F(\theta)]$, where $\rho_1'(\theta) > 0$ and $\rho_2'(\theta) < 0$. From (26), we have that $V_f^B - V_f^R = \rho_1(\theta) V_f^B$. Substituting this and (27) into (9) (with appropriately added subscripts $f \in F$) and computing total derivatives, we obtain the following equation system:

$$\begin{pmatrix} \beta\psi'_{B,A}[1 - \rho_2\mu_A] + \rho_1\left[\psi'_{B,A} + V_A^B\psi''_{B,A}\right] & -\beta\psi'_{B,P}\rho_2\mu_P \\ -\beta\psi'_{B,A}\rho_2\mu_A & \beta\psi'_{B,P}[1 - \rho_2\mu_P] + \rho_1\left[\psi'_{B,P} + V_P^B\psi''_{B,P}\right] \end{pmatrix} \cdot \begin{pmatrix} dV_A^B \\ dV_P^B \end{pmatrix} = \begin{pmatrix} \beta\psi'_{B,A}\rho_2'\mu_A - \rho_1' V_A^B\psi'_{B,A} \\ \beta\psi'_{B,P}\rho_2'\mu_P - \rho_1' V_P^B\psi'_{B,P} \end{pmatrix} d\theta.$$

The determinant of this system, $\widetilde{D}$, is

$$\begin{aligned} \widetilde{D} \;=\; & \left[\beta\psi'_{B,A} + \rho_1\left(\psi'_{B,A} + V_A^B\psi''_{B,A}\right)\right]\left[\beta\psi'_{B,P} + \rho_1\left(\psi'_{B,P} + V_P^B\psi''_{B,P}\right)\right] \\ & - \beta\psi'_{B,A}\rho_2\mu_A\left[\beta\psi'_{B,P} + \rho_1\left(\psi'_{B,P} + V_P^B\psi''_{B,P}\right)\right] \\ & - \beta\psi'_{B,P}\rho_2\mu_P\left[\beta\psi'_{B,A} + \rho_1\left(\psi'_{B,A} + V_A^B\psi''_{B,A}\right)\right]. \end{aligned}$$

Since μ_A = 1 " μ_P, $\widetilde{D}$ is a linear function of μ_A and thus minimized at either μ_A = 0 or μ_A = 1. If the minimum is attained at μ_A = 1, $\widetilde{D}$ is bounded from below by

$$\left[\beta\psi_{B}{}'_{A}(1 \text{ " } \rho_2) + \rho_1\left(\psi_{B}{}'_{A} + V_A^B\psi_{B}{}''_{A}\right)\right]\left[\beta\psi_{B}{}'_{P} + \rho_1\left(\psi_{B}{}'_{P} + V_P^B\psi_{B}{}''_{P}\right)\right],$$

which is positive since $\rho_2 < 1$. Similarly, if the minimum is attained at μ_A = 0, $\widetilde{D}$ is also positive. Applying Cramer's rule, we have that $dV_A^B/d\theta = D_A/\widetilde{D}$, where

$$\begin{aligned} D_A \;:\; &= \left[\beta\psi'_{B,A}\rho'_2\mu_A \text{ " } \rho'_1V_A^B\psi'_{B,A}\right]\left[\beta\psi'_{B,P}(1 \text{ " } \rho_2\mu_P) + \rho\left(_1\psi'_{B,P} + V_P^B\psi''_{B,P}\right)\right] \\ &\text{ " } \left[\beta\psi'_{B,P}\rho'_2\mu_P \text{ " } \rho'_1V_P^B\psi'_{B,P}\right]\left[\text{ " }\beta\psi'_{B,P}\rho_2\mu_P\right]. \end{aligned}$$

Given that $\rho'_1 > 0$ and $\rho'_2 < 0$, we have that $D_A < 0$ and thus $dV_A^B/d\theta < 0$. Analogous calculations show that $dV_P^B/d\theta < 0$. By (26), this implies that $dV_f^R/d\theta < 0$. Q.E.D.

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Figure - 1

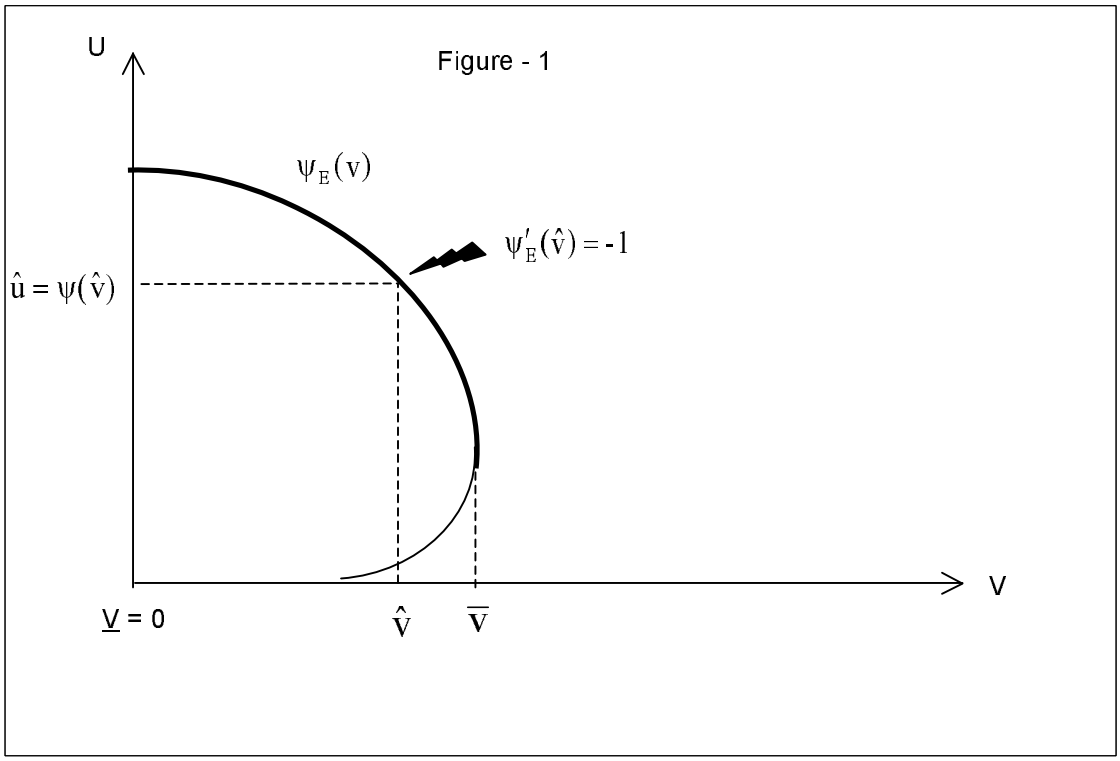

Figure-2

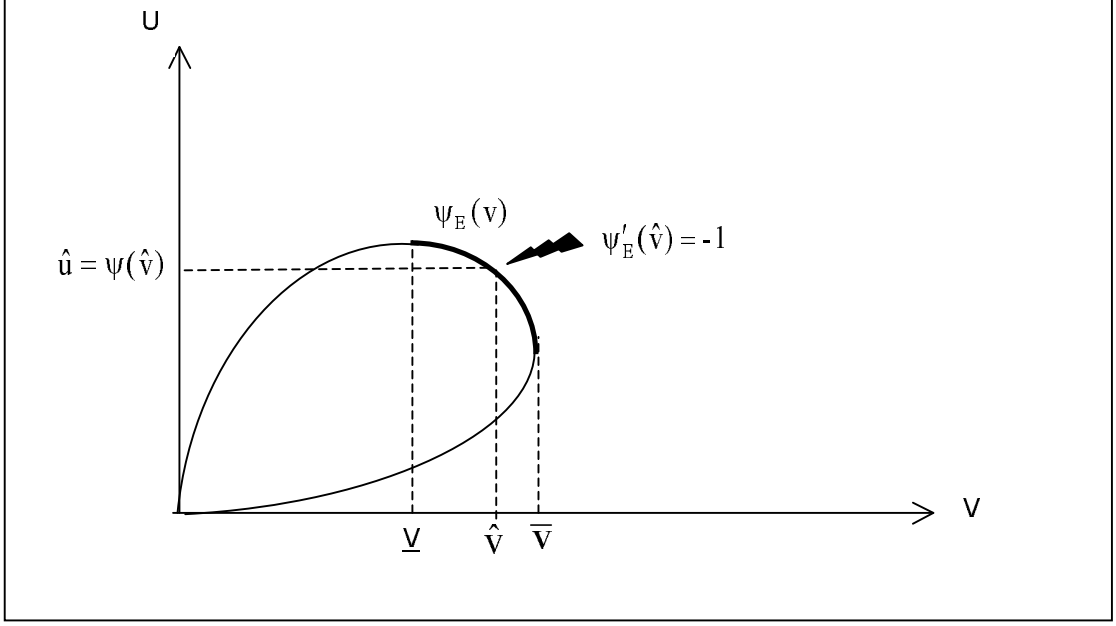

Figure - 3