

Sponsoring Company Finance, Investment and Pension Plan Funding.

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Abstract

This paper presents a model of the interaction of a company's financial and real investment decisions with the financing of its defined benefit pension plan. The pension plan deficit is a debt of the company, with explicit funding requirements and priority in the event of company insolvency. Pension plan deficits and options on future deficits and surpluses affect investment incentives as does the size and composition of company debt. We illustrate the incentives for the firm to pay dividends rather than fund the pension plan and the general incentives to overfund the pension plan. We also illustrate the impact of pension benefit insurance and minimum funding requirements.

1 Introduction

The asset-liability management problem for a company's pension plan can impact directly on its own capital structure and investment decisions. However, the nature of this relationship can be affected to a considerable extent by the institutional environment in which the relationship is couched. In particular it will be affected by pension funding requirements such as the MFR in the UK, pension plan insurance such as the PBGC in the USA and rules regarding the treatment of pension plan liabilities in the event of the sponsoring firm being both solvent and insolvent.¹

A number of papers have examined the interaction of company pension plans and financial structure. Sharpe (1976) argued that if the risk of underfunded pension liabilities can be fully insured, as would be the case in a complete market, pension-funding policy is irrelevant. Arnott and Gesovitz (1980) argue that (small) risk averse firms may have unfunded pension liabilities as a way to share risk with risk averse workers. They also examined the risk-shifting problem that exists when pension liabilities are not fully funded and corporate bonds are senior to pensions in the event of company insolvency. They show that in an incomplete market firms may have an incentive to increase leverage so as to shift risk in an uncompensated way to the pension plan. Ippolito (1985) argued that underfunded pension liabilities are a way for firms to improve their bargaining position with trades unions that can make excessive wage demands that could jeopardise the financial position of the company. Cooper and Ross (2002) show that pension underfunding can result if the sponsoring company encounters imperfections in the capital market

¹ In the USA since 1974, in the event of the sponsoring company declaring bankruptcy, underfunded pension benefits have a senior claim on company assets (See Martin and Henderson (1983)). In the UK, the Pensions Act (1995) imposed a statutory debt on employers when a pension plan winds up. Before that, when a final salary plan was in deficit on winding-up and the employer was solvent, benefits of pensioners were required to be bought out in full and the deficit in respect of other members had to be met on the Minimum Funding Requirement (MFR) basis. However, as this basis is less than the full cost of buying out the benefits with insurance annuities, members could receive only a percentage of their benefits under the plan unless the employer decides to fund a full buy-out of the benefits. There was thus the potential for companies to default on a significant portion of pension liabilities, with damage being limited to the impact on company reputation and employee relations. On 11 June 2003, however, the UK Government's Winding up and Deficiency on Winding up (Amendment) Regulations 2003 were issued. These require the calculation of the liability for an employer not in liquidation to be based on benefits under the plan being secured on a full buy-out basis. Employers winding up a plan may therefore need to make a substantial additional contribution to enable them to wind up the plan and discharge their liabilities. But in the event of company insolvency, unfunded pension liabilities are junior to the company's debt.

when raising external funds to finance investments and resort to borrowing from the pension fund, which then effectively purchases the firm's risky debt.²

Other studies such as Tepper and Ack (1974), Black (1980), Tepper (1980) and Thomas (1988) all discuss the implications of taxes for pension plan overfunding. This analysis argues that firms should fund pension liabilities to the maximum to take advantage of the tax exemption of payments into the plan and also the tax advantage that the plan has in holding taxable bonds. This literature poses a puzzle as to why firms do not uniformly do this, for which a number of explanations have been given in the literature.³ Petersen (1992) is an interesting empirical study of the links between the closure of overfunded pension plans (seen as wealth transfers from workers to shareholders) and the current financial position of the sponsoring firm, that explicitly takes into account tax considerations.

Relatively little published work has attempted to examine the interaction of the sponsoring company's financial and real investment decisions with the financing of its defined benefit pension plan. A primary feature of the model developed in the paper is its treatment of the company pension deficit as a debt of the company, with explicit funding requirements and priority in the event of company insolvency. The funding of current pension plan deficits and options on future deficits and surpluses play a central role in the analysis. This model is used to analyse the impact of both the size and composition of company debt and pension plan funding position on real investment decisions. The model is also used to illustrate the incentives for the firm to pay dividends rather than fund the pension plan and the general incentives to overfund the pension plan. We also illustrate how the incentives of both the pension plan trustees and the sponsoring firm are affected by pension plan insurance and minimum funding requirements.

² See also Tepper and Ack (1974).

³ See Feldstein and Seligman (1981).

2 Basic Model

We present a simple stylised model of a firm with a defined-benefit pension plan. For simplicity, the firm has a two-period horizon. It is owned by shareholders but borrows from banks on a short-term basis and can also issue bonds. It also has a commitment to the company pension plan. In what follows we treat the holders of all the firm's liabilities as risk neutral. But the analysis is readily adapted to incorporate risk. Assuming no discounting all income streams are valued as expected cash flows.

2.1 The Firm's Final Salary Defined-Benefit Pension Plan

The pension plan is overseen by trustees who choose a fund manager to allocate the assets of the plan between equities and fixed income investments available in the economy. The trustees can be assumed to be risk averse utility maximisers.⁴ Consulting actuaries assess the ability of the plan's contribution rate and asset allocation policy to meet the plan's liabilities. Given the liability profile, in choosing its investment policy the pension plan trustees are assumed to have optimised over risk and return, subject to solvency constraints and the sponsoring firm's commitments to the plan. Having said this, throughout the rest of the present paper the outcome of this optimisation problem will be taken as given.

The investment policy of the pension plan can be led by either an equity or bond strategy. In the absence of default on pension liabilities, the choice of a debt versus an equity led investment strategy may be based on tax considerations. Black (1980) and Tepper (1981) both argued that given their tax advantage in holding debt, pension plans will specialise in debt led strategies. However, Harrison and Sharpe (1982) show, extending the analysis in Sharpe (1976), that in a complete market, with both the possibility of default on pension liabilities and pension benefit insurance, there is a trade-off between the tax advantage of debt and the increased risk of an equity led strategy that is subsidised by a pension benefit insurance scheme.

⁴ For current purposes broader investment opportunities are ignored.

The corporate pension plan has total liabilities of \star to be paid over two periods. The breakdown of \star between the two periods can be thought of as respectively the obligations to current pensions, \star_0 , and prospective pensions, \star_1 . The pension claims are long-term and their value can be affected by the firm's financial structure and investment strategy. At the same time, the pension liabilities of the firm affect its financial position and its real investment incentives. The framework developed in this paper allows the examination of these interactions. The basic model is an extension of the simple models of the interaction of firm's investment policy and financial structure developed by Bulow and Shoven (1978) and extended by Gertner and Scharfstein (1992). For simplicity the approach assumes that the sponsoring firm acts in the interests of shareholders to maximise their wealth.

2.2 The Balance Sheets of the Firm and Pension Fund

\star_0 and \star_0^* are respectively the date $\star = 0$ returns on the firm's liquid and fixed investments in place at the end of date $\star = 1$. \star_1^* is the random return on the firm's fixed assets at date $\star = 1$ and \star_1 is the random return on the firm's investment made at date $\star = 0$. At date $\star = 1$ the firm is financed by equity with realised value at date $\star = 0$ of \star_0 and bank debt that matures at date $\star = 0$ with a face value of \star_0 . For the moment we assume that the firm issues no bonds.⁵

\star_0^* is the realisation of the return on the pension plan's investment policy, determined at date $\star = 1$. Contributions to the plan at this date are made by the sponsoring firm and are denoted by \star_1 . The pension plan invests in either riskless bonds or in risky assets. Let the return at date $\star = 0$ on the former be given by \star^s and the random return on the latter by \star and suppose that a fraction $\star_1 \in [0, 1]$ of plan assets is invested in the risky asset. Throughout the rest of the paper we assume that the risky asset has a stationary distribution with cumulative distribution function $\star(\star)$. Moreover, we assume that the first two moments of the distribution exist and the expected return on the risky asset exceeds that on the safe asset.

⁵ The presentation of the model abstracts from a theory determining the ex ante choice of the level of debt finance.

\star_1^* is the return at date $\star = 1$ on assets in the plan at date $\star = 0$. These assets are equal to assets carried forward from the previous period, \star_0^* , less withdrawals of any pension payments, \star_0 , and possibly some part of any surplus in the plan, denoted by \star_0^* , plus possibly contributions to cover some part of any current deficit denoted by \star_0^* and a further contribution from the firm, \star_0 . Of course the pension plan is in deficit or surplus, but not both.

The value of the pension deficit that must be funded if the firm is to continue at date $\star = 0$ is $\star_0^* = \star_0^* (\star_i \star_0^*)^+$, where $\star_0^* \in [0, 1]$ is the fraction of the deficit that the solvent firm must finance. We assume that $\star_0^* = 1$, so the solvent firm pays \star_0 in full and $\star_0^* = (\star_0 \star_i \star_0^*)^+$. In the event of a pension plan surplus at date $\star = 0$, $\star_0^* = \star_0^* (\star_0^* \star_i \star)^+$ is recovered by the sponsoring firm, where $\star_0^* \in [0, 1]$. The firm's liabilities include a value for the put option, \star_0 , issued by the firm to the pension plan to cover the future pension deficits, $\star_1^* = \star_1^* [\star_1 \star_i \star_1^*]^+$, where $\star_1^* \in [0, 1]$ is the fraction of the deficit covered by the solvent firm. The investment policy of the plan at date $\star = 0$ is given by $\star_0 \in [0, 1]$. Thus

$$\star_1^* = [(\star_0^* \star_i \star_0)^+ \star_i \star_0^* + \star_0] [\star^a + \star_0 (\star_i \star^a)] \quad (1)$$

where \star_0 is determined by the optimisation problem described above.

The firm's assets also include a call option on any surpluses of the pension plan at date $\star = 1$, $\star_1^* = \star_1^* [\star_1 \star_i \star_1]^+$ where for simplicity $\star_1^* = 1$ is the fraction of the surplus recovered by the firm.⁶ The value of this option is given by \star_0 ⁷

We assume that all assets and liabilities are valued at market prices. The firm's consolidated balance sheet at the current date, $\star = 0$, before any decisions are made is as follows:

⁶ In a more general framework \star_1^* will be determined by bargaining between the firm and workers.

⁷ For a discussion of pension liabilities as options see Sharpe (1976) and Blake (1998).

Assets	Liabilities
Liquid Assets: \star_0 .	Bank Debt: \star_0 due at date $t=0$.
Fixed Assets: \star_0^* .	Contribution to the pension plan at date $t=0$: \star_0
Share of pension plan surplus: \star_0^*	Current pension plan deficit that must be funded: \star_0^* .
Call option on share of future pension plan surplus: \star_0	Put option on share of future pension plan deficit: \star_0 .
	Equity: \star_0 .

The pension plan balance sheet at date $t=0$, before new decisions are made is:

Assets	Liabilities
Securities: \star_0^* .	Pensions: \star_0 due at date $t=0$.
Claim on sponsor at date $t=0$: $\star_0 + \star_0^* + \star_0$.	Pensions: \star_1 due at date $t=1$.
	Share of pension plan surplus: \star_0^* .
	Call option on future pension plan surplus: \star_0

2.3 The Firm's Investment Decision

At date $\star = 0$ the firm can invest \star_0 to generate a random return of \star_1 at date $\star = 1$. \star_1 is distributed continuously on $[0, \bar{\star}_1]$ with distribution function $\star_1(\star_1 | \star_0)$, where \star is a risk index of the distribution. The risk index of the distribution is chosen by the firm and for simplicity a higher \star corresponds to a mean-preserving spread in the sense of Rothschild and Stiglitz (1971). Given no discounting and risk neutrality for valuation purposes, the date $\star = 0$ value of the firm's investment is $\star_0 = \star_0[\star_1 | \star_0]$, where expectations are taken with respect to $\star_1(\star_1 | \star_0)$.

At date $\star = 0$, the values of \star_0 , \star_0^* and \star_0^* are realised. Once \star_0^* is known it is determined whether the pension plan is in deficit or surplus. If $\star_0^* > 0$ then $\star_0^* = 0$ and vice versa. To continue the firm must pay \star_0 and $\star_0^* = (\star_0 + \star_0^*)^+$. The firm cannot finance continuation from internal resources if $\star_0 + \star_0^* < \star_0 + \star_0 + \star_0^* + \star_0^*$. If the firm discontinues at date $\star = 0$ it is insolvent if $\star_0 + \star_0^* + \star_0^* < \star_0 + \star_0^*$. On the other hand, it is solvent if $\star_0 + \star_0^* + \star_0^* > \star_0 + \star_0^*$.

When the firm continues the pension plan has assets of $\star_0^* i \star_0 + \star_0^* + \star_0 + \star_0$. The gain or loss to the pension plan if the firm continues is given by $\$*(\star)$, which is derived in the next sub-section. However, continuation also offers the prospect of transfers from the pension plan to the firm equal to the value of the call option that the firm has on future pension surpluses, \star_0 .

2.4 Modelling the Effect of the Firm's Investment Decision on the Pension Plan

We will now be more explicit about the treatment of the pension deficit in the event of liquidation. Suppose that if the firm is liquidated at date $\star = 0$, then a fraction \star_0 of the pension deficit is as senior as bank debt, the remainder being a junior (subordinated) claim. So, for example, we may have that

$$\star_0(\star_i \star_0^*)^+ = \star_0^* = (\star_0 i \star_0^*)^+ \quad (2)$$

and then only current pensions are given priority.⁸ The total value of senior claims at date $\star = 0$ is

$$\star_0 = \star_0 + \star_0(\star_i \star_0^*)^+ \quad (3)$$

In the event of liquidation the pension plan receives:

$$\star_0^* = \left(\frac{\star_0(\star_i \star_0^*)^+}{\star_0} \right) (\star_0^* + \star_0^* + \star_0) \text{ if } \star_0^* + \star_0^* + \star_0 > \star_0 \quad (4)$$

and $\star_0(\star_i \star_0^*)^+ + \min[\star_0^* + \star_0^* + \star_0 i \star_0, (1 i \star_0)(\star_i \star_0^*)^+]$ otherwise.

Note that if $\star_0 = 0$, pension-deficit liabilities are a junior claim and simply receive the residual value of the firm after other higher priority claims have been paid, $\star_0^* + \star_0 i \star_0$.

For the moment assume that if the firm continues at date $\star = 0$, \star_0 is invested in the firm. To finance continuation, if $\star_0 i \star_0 + \star_0 + \star_0 i \star_0^* + \star_0^* > 0$ more bank debt, \star_1 , is issued

$$\star_1 = \star_0 i \star_0 + \star_0 + \star_0 i \star_0^* + \star_0^* \quad (5)$$

⁸ This was the case in the UK until the June 6th Winding-Up Amendment to the Pensions Act.

Given continuation at date $\star = 0$, at date $\star = 1$ \star_1^* and \star_1^* are realised. After the realisation of \star_1^* , total debt at is

$$\star_1 = \star_1 + \star_1^* \quad (6)$$

Through this term, the realisation of \star_1 is a function of \star_1^* .

At date $\star = 1$, a fraction \star_1 of any pension debt is assumed to be as senior as new bank debt. The face value of senior debt and the fraction, \star_1 , of the pension debt is

$$\star_1 = \star_1 + \star_1 [\star_1 \ i \ \star_1^*]^+ \quad (7)$$

If \star_0 has been paid in full at date $\star = 0$, then at date $\star = 1$ there are the following possible payment to pension holders:

$$\star_1 \text{ if either } \star_1^* = 0 \text{ or } \star_1^* > 0 \text{ and } \star_1^* + \star_1 < \star_1, \quad (8)$$

$$\text{and } \star_1^* \text{ if } \star_1^* > 0 \text{ and } \star_1^* + \star_1 > \star_1.$$

where

$$\star_1^* = \left(\frac{\star_1 [\star_1 \ i \ \star_1^*]^+}{\star_1} \right) (\star_0^* + \star_0^* + \star_0) \text{ if } \star_0^* + \star_0^* + \star_0 < \star_1, \quad (9)$$

$$\text{and } \star_1 [\star_1 \ i \ \star_1^*]^+ + \min[\star_0^* + \star_0^* + \star_0 \ i \ \star_1^* (1 \ i \ \star_1^*) \star_1^*] \text{ otherwise.}$$

In (9), if $\star_1^* > \star_1$, then $\star_1^* > 0$ and $\star_1^* = 0$.

In what follows \star_1^* and \star_1 appear as a sum with distribution function $\star(\star_1^* + \star_1 \star)$. Then, if the firm invests at date $\star = 0$, continuation yields a transfer to the pension plan of

$$\mathbb{C}\star(\star) = \star_0^* + \star_0 + \sum_0^{\infty} (\star_1^* \ i \ \star_1^*) \star(\star_1^* + \star_1 \star) [\star_1^* + \star_1 \star] \star_1 \ i \ \star_1^*. \quad (10)$$

The decomposition of the right-hand-side of this expression illustrates the main factors determining the impact of the firm's investment decision on the pension plan. \star_0^* is the firm's contribution to the pension plan at date $\star = 0$. The second term, \star_0 , equals $\star_0(\star_1^*)$, where expectations are taken with respect to $\star(\star_1)$. This is the date $\star = 0$ value of the put option that the pension plan

has against the ...rm at date $\star = 1$, which remains alive if the ...rm continues at date $\star = 0$. Its value will be affected by the size of any contributions that the ...rm makes to the pension plan at date $\star = 0$, including \star_0 . The third term is the net transfer of value from the ...rm to the pension plan if the ...rm is liquidated at date $\star = 1$. It is the expectation of the difference between the value of the liquidation proceeds to the pension plan, \star_1^* , and the put option, \star_1^* , conditional upon the event of liquidation at date $\star = 1$. The ...nal term is the contribution of the ...rm to the pension plan in the event of liquidation at date $\star = 0$, which is not paid in the event of continuation.

The ...rm has a call option on future pension surpluses, \star_1^* , with a total value of $\star_0 = \star_0(\star_1^*)$. The value of the call depends upon the ...rms contribution to the pension plan and the pension plan's investment policy as well as the face value of the plans date $\star = 1$ liabilities. In the event that the ...rm defaults these surpluses are used to make settlement upon creditors. The impact of the default on the value of this option to the ...rm is given by

$$\star_0 = \frac{\mathbf{Z} \mathbf{Z}^{\star_1}}{0} \star_1^* \star_1^* (\star_1^* + \star_1^*) \star_1^* (\star_1^*) \quad (11)$$

so the current value of this option to the ...rm is \star_0 if \star_0 .

2.5 Impact of the Pension Plan on the Firm's Decisions

The ...rm's investment decision at date $\star = 0$ is to invest if

$$\star_0(\star) \geq \mathbb{E}[\star_1^* (\star_1^* \star_0) + \star_1^* (\star_1^* \star_0)] + \star_0 \quad (12)$$

The cost of undertaking the investment includes the transfer to the pension fund, $\mathbb{E}[\star_1^* (\star_1^*)]$. Continuation will also have an effect on the value of the call option on future surpluses held by the ...rm, denoted by $\mathbb{E}[\star_0 | \star_0]$. If it continues the ...rm must contribute \star_0 to the pension plan. The investment choice also involves choosing the risk of the project.

De...nition 1 Assume \star can take one of two values two values $\star_2 > \star_1$, with \star_2 being riskier in the Rothschild Stiglitz (1971) sense than \star_1 . The value of the call option $\star_0(\star_1^* \star_0)$ is independent of \star but does depend upon the investment policy of the pension plan and the contributions that the ...rm makes if it continues, \star_0 . However, the default term $\star_0 = \star_0(\star_1^* \star_0)$ depends upon both \star and \star_0 . Both $\mathbb{E}[\star_1^* (\star_1^*)]$ and $\mathbb{E}[\star_1^* (\star_1^* \star_0)]$ are decreasing in risk \star . Then we have the possibility of both under and over-investment.

(a). If the risk of the investment is given by \star_1 and $\star_0(\star_1) > 0$ and $\mathbb{C}(\star_1\star_0) < \mathbb{C}(\star_0)$, then the shareholders and the bank will gain if $\star_0(\star_1) > \mathbb{C}(\star_1\star_0)$ and $\mathbb{C}(\star_1\star_0) < \mathbb{C}(\star_0)$. If $0 < \star_0(\star_1) < \mathbb{C}(\star_1\star_0)$ and $\mathbb{C}(\star_1\star_0) < \mathbb{C}(\star_0)$, there will be under-investment.

(b). With the alternative riskier investment \star_2 , $\star_0(\star_2) > \star_0(\star_1)$ but $\mathbb{C}(\star_2) < \mathbb{C}(\star_1)$. Then if $\star_0(\star_2) > \mathbb{C}(\star_2) + \mathbb{C}(\star_1)$, the riskier investment will be adopted. Suppose $\star_0(\star_2) > 0$, then if $\mathbb{C}(\star_2) < \mathbb{C}(\star_1)$, we have over-investment.

The former problem is analogous to the Myers (1977) under-investment problem and the latter to the Jensen and Meckling (1976) over-investment problem.

Proposition 2 Under Definition 1, in the absence of effects on the value of the pension plan the firm will undertake the low risk investment. However, if $\mathbb{C}_*(\star\star\star_1) + \mathbb{C}_*(\star\star\star_0\star\star_2) > \mathbb{C}_*(\star\star\star_2) + \mathbb{C}_*(\star\star\star_0\star\star_2) \star \star_0(\star_1) + \star_0(\star_2)$ the firm will switch to the riskier investment.

3 Dividends and Pension Plan Overfunding

3.1 Dividends

In the above we made a working assumption that the firm will retain liquid resources. We now test this assumption by allowing the firm to use these resources to pay dividends or overfund the pension plan. Consider the firm's liquid assets \star_0 and first examine the possibility of paying dividends.

Proposition 3 If free to do so, at date $\star = 0$ the firm may pay all liquid assets as a dividend to shareholders.

Proof. Paying \star_0 out as a dividend at date $\star = 0$ means that bank borrowing is increased so that by (6), \star_1 is increased. The new borrowing is at a fair market price. But this means that the absolute value of the third term in (10) declines. In particular, differentiating the third term in (10) with respect to \star_0 and using (5) and (6), we find that $\frac{\frac{\partial \star_1}{\partial \star_0}}{\star_0} = \frac{R}{\star_1} [(\star_1 \star_1) \frac{\frac{\partial \star_1}{\partial \star_0}}{\star_0} \star_1 \star_1] \star_1 \star_0 \star_0$, so for a reduction in \star_0 there will be a loss to the pension plan and a gain to the sponsoring firm. However, offsetting this is an increased chance that the firm forfeits the full value of any pension plan surplus. From (11), $\frac{\frac{\partial \star_0}{\partial \star_0}}{\star_0} = \frac{R}{\star_1} [\star_1 \frac{\frac{\partial \star_1}{\partial \star_0}}{\star_0} \star_1 \star_1] \star_1 \star_0 \star_0$

so for a fall in \star_0 , \star_0 will increase. If this last effect is relatively small, by reducing \star_0 the firm will gain from paying \star_0 as a dividend. ■

The incentive illustrated here will be reduced if the seniority of pensions in the event of liquidation at date $\star = 1$ is increased, thereby raising \star_1^* . The result though illustrates the conflict of interest between the firm and the pension plan and the need for the pension plan to monitor the firm's behaviour and protect itself from discretionary dividends through covenants.⁹

However, the firm has an alternative to paying money out to shareholders now. It can consider overfunding the pension plan. This will reduce the value of the put option on future pension deficits held by the pension plan and at the same time raise the value of the call option on pension plan surpluses held by the firm. These two effects are easy to demonstrate and are denoted by $\frac{\star_1^* - \star_0}{\star_0} > 0$ and $\frac{\star_1^* - \star_0}{\star_0} > 0$. The gain to the firm from this strategy is limited because in some states of the world it defaults on funding the pension deficit, so that the put option is substituted by a less valuable liquidation claim. This follows even if we assume, as we do in the sequel, that the firm captures the plan surplus even if insolvent, because in the event of insolvency creditors have first claim on this amount.

Proposition 4 If free to do so, at date $\star = 0$ the firm will pay all liquid assets out as a dividend to shareholders rather than overfund the pension plan.

Proof. Absent corporate default, the pension payment \star_1 is a guaranteed promise so that with fair pricing $\frac{\star_1^* - \star_0}{\star_0} = i \frac{\star_1^* - \star_0}{\star_0} + \frac{\star_1^* - \star_0}{\star_0}$. With corporate default, the effect of \star_0 on the cost of put options to the firm is given by $\frac{\star_1^* - \star_0}{\star_0} i \frac{R}{[0^{\star-1}(\frac{\star_1^* - \star_0}{\star_0})] \star (\star_1^* + \star_1^* \star_0)} \star (\star_1) i \frac{R}{[\star_1^* \star (\star_1^* \star_0) \frac{\star_1^* - \star_0}{\star_0}]} \star (\star_1)$, where the second two terms measure the effect of firm default on the net cost of the put. The increase in the value of the call option is $\frac{\star_1^* - \star_0}{\star_0}$ but this is only captured by the solvent firm. In the event of insolvency, an additional amount with maximum value equal to $\frac{R}{[0^{\star-1}(\frac{\star_1^* - \star_0}{\star_0})] \star (\star_1^* + \star_1^* \star_0)} \star (\star_1) + [\star_1^* \star (\star_1^* \star_0) \frac{\star_1^* - \star_0}{\star_0}]} \star (\star_1)$ is lost. Thus the firm loses some of both the saving in the cost of the put option and the increase in the value of the call. Hence the firm prefers to

⁹ See the classic analysis in Smith and Warner (1979).

pay \star_0 as a dividend. ■

This result is modified if there are taxes.¹⁰ Suppose that payments to shareholders at date $\star = 0$ are taxed at the total rate of \star_* . We can think of this as the current composite dividend and corporate tax rate. Assets retained in the pension plan avoid these taxes, so that \star_* is saved. However when assets are withdrawn by the firm at date $\star = 1$, an effective tax rate of \star_* is incurred.¹¹ Tax rates will depend upon the overall tax status of the firm that reflects loss carry forwards and other available tax deductions. Higher tax status firms will have a relatively higher value of \star_* .¹² The call option held by the firm is a claim on the after tax value of pension plan surpluses. Also assume that future funding of pension plan deficits is tax deductible. It then follows that at the margin, the firm will overfund the pension plan if $(1 + \star_*)^{\frac{1}{\star_0}} \star_0 \geq (1 + \star_*)^{\frac{1}{\star_0}} + (1 + \star_*)^{\frac{1}{\star_0}}$. This will be satisfied if \star_* is sufficiently high relative to \star_0 . The firm will overfund the pension plan when it faces high taxes and take the money out when taxes are lower. Notice that if we remove the tax deductibility of funding future pension plan deficits the incentive to overfund the plan is increased.

If $(1 + \star_0) = (1 + \star_*)$ and without corporate default firms will only generally be indifferent between paying money out and overfunding the pension plan if funding deficits are tax deductible, so $(1 + \star_0)^{\frac{1}{\star_0}} = (1 + \star_*)^{\frac{1}{\star_0}} + (1 + \star_*)^{\frac{1}{\star_0}}$. But this condition again breaks down once company default is introduced. Then the bias is in favour of underfunding. This may be a partial explanation of the “pension funding puzzle” described by Feldstein and Seligman (1981), as to why given favourable tax treatment pension plan deficits are not immediately funded and that in general plans will be overfunded.¹³

¹⁰ Tepper and Ack (1974), Black (1980) and Tepper (1980) all discuss the implications of taxes for pension plan overfunding. Thomas (1988) provides a thorough investigation of the empirical issues.

¹¹ Because of its tax status the pension plan has an advantage in holding taxable bonds. To reflect this a further adjustment in the effective tax rate applied to assets held within the fund should be made.

¹² Petersen (1992) describes how in the USA the IRS restricts the ability of firms to undertake tax arbitrage through the pension plan.

¹³ The investment policy of the pension plan can be led by either an equity or bond strategy. In the absence of default on pension liabilities, the choice of a debt versus an equity led investment strategy may be based on tax considerations. Black (1980) and Tepper (1981) both argued that given their tax advantage in holding debt,

We can now go back and ask what the firm will do at date $\star = 1$. Given optimal decisions at date $\star = 0$, as with raising \star_0 , raising \star_{i1} will lower $\mathbb{C}(\star\star)$ by $\frac{\star\star_0}{\star\star_{i1}} \in \mathbb{R}^+ \left[\frac{\star\star_1}{\star\star_0} \right] \star\star (\star_1 + \star_{1\star\star}) \star\star (\star_1) \in \mathbb{R}^+ [\star_{i1} \star\star_0] \star\star (\star_1)$ and raise $\mathbb{C}[\star_{0i} \star_0]$. Then in condition (12) $\mathbb{C}(\star\star) \in \mathbb{C}[\star_{0i} \star_0] + \star_0$ will be higher but by the same argument as in Proposition 4 by less than the cost, \star_{i1} . Another way to see this point is to think of overfunding the pension plan as trapping equity that will only be realised as a return to shareholders if the firm continues at date $\star = 0$. Hence the incentive to invest at date $\star = 0$ is increased, but since the efficiency gain is only partially captured by shareholders they will not increase \star_{i1} . This argument will naturally be modified by the introduction of tax considerations and other long-term claims. The tax argument is the same as that already given. The point about long-term claims will be taken up later.¹⁴

4 Pension Plan Deficits and Firm Investment

We now examine the impact of pension plan deficits on the sponsoring company's investment decisions.

Definition 5 Given \star_0 , a low realisation of \star_0^* , implies a high current pension plan deficit (or overhang) of $\star_0^* \star 0$.

Before presenting the next result, note that if the firm continues and funds the current pension plan deficit, this will not affect the distribution of future pension plan surpluses, \star_0^* , so $\mathbb{C}\star_0$ will be unchanged. But the default effect $\mathbb{C}\star_0$ will be affected as higher values of funded current deficits, \star_0^* , will mean that \star_1 is higher. However, both $\mathbb{C}\star_0$ and $\mathbb{C}\star_0^*$ will be affected by the contributions made to the plan at date $\star = 0$, \star_0 , which are contingent upon firm continuation.

pension plans will specialise in debt led strategies. However, Harrison and Sharpe (1982) show, extending the analysis in Sharpe (1976), that in a complete market, with both the possibility of default on pension liabilities and pension benefit insurance, that there is a trade-off between the tax advantage of debt and the increased risk of an equity led strategy that is subsidised by a pension benefit insurance scheme.

¹⁴ The present analysis ignores informational problems. Suppose that these were present along lines discussed in Myers and Majluf (1984). Assume date $\star = 1$, if the firm is not financially constrained, in the sense that it does not need to raise external finance to meet its investment and pension commitments. It can consider overfunding the pension plan as a tax efficient way of accumulating financial slack to mitigate the lemons problem in the market for external finance at date $\star = 0$, when it has planned investment expenditures. The tax advantages of this strategy may be limited by the tax authority.

Consider expression (10) and take as given the risk index of the firm's investment and that $\star_0^* > 0$. At date $\star = 1$ pensions cannot receive more in liquidation than the value of the deficit foregone, $\star_1^* i \star_1^* > 0$, so it is clear that although $\star_0^* + \star_0 + \frac{R}{Z} [\star_0^* (\star_1^* i \star_1^*) \star_1^* (\star_1^* + \star_1 \star_1)] \star_1^* > 0$ it will be smaller the higher the value of \star_1 . Therefore, as $\star_0^* > \star_0^*$, $\frac{R}{Z} (\star_1^* + \star_1 \star_1) < 0$. However, given Definition 1, this value will be smaller under project \star_2 than project \star_1 . We now establish how, given project risk, this term is affected by the size of the pension deficit.

Proposition 6 A high pension deficit, \star_0^* , gives an incentive for the firm to under-invest at date $\star = 0$.

Proof. The effect of \star_0^* on $\star(\star)$ is given by differentiating (10)

$$\begin{aligned} \frac{\star \frac{d\star}{d\star_0^*}}{\star_0^*} &= 1 + \frac{Z}{Z} \left[\star_0^* \left[\frac{\star_1^*}{\star_0^*} i \frac{\star_1^*}{\star_0^*} \right] \star_1^* (\star_1^* + \star_1 \star_1) \right] \star_1^* (\star_1^*) \\ &+ \left[(\star_1^* i \star_1^*) \star_1^* (\star_1 \star_1) \frac{\star_1^*}{\star_0^*} \right] \star_1^* (\star_1^*) i \frac{\star_1^*}{\star_0^*} \end{aligned} \quad (13)$$

The direct effect of funding the pension deficit in continuation is given as unity. The second two terms show the impact of a higher \star_0^* on \star_1 and hence on the value of future pensions. $\frac{\star_1^*}{\star_0^*} > 0$ but the assumption that the firm must fund the current pension deficit to continue means that the distribution of the future pension deficit is independent of \star_0^* , $\frac{\star_1^*}{\star_0^*} = 0$. Therefore the second term is negative. The positive impact of \star_0^* on \star_1 raises \star_1 so $\frac{\star_1^*}{\star_0^*} > 0$ and as $(\star_1^* i \star_1^*) > 0$ so the third term is also negative. However, the direct effect of funding the pensions deficit will dominate and this will be particularly so for \star_1 not too high. From (4) $0 < \frac{\star_1^*}{\star_0^*} < 1$. Then if \star_1 is not too high $\frac{\star_1^*}{\star_0^*} < 0$. Finally, note the effect of higher deficits on the value of the firm's option on pension plan surpluses,

$$\frac{\star_0^*}{\star_0^*} = \left[\star_1^* \star_1^* (\star_1 \star_1) \frac{\star_1^*}{\star_0^*} \right] \star_1^* (\star_1^*) \quad (14)$$

which is unambiguously positive. Hence, a higher deficit increases the incentive to under-invest.

■

In presenting this result we assumed that any current pension deficit is paid in full before the firm continues, $\star_0^* = \star_0^*[\star_0^* + \star_0]^+$ with $\star_0^* = 1$. Any reduction in \star_0^* will weaken the effect demonstrated.

Corollary 7 Suppose that ...rm the ...rm currently will choose the project with risk index \star_1 . Then as $\mathbb{P}(\star\star_1) \downarrow \mathbb{P}(\star_0(\star_1\star_0) \downarrow \star(\star_1\star_0\star_1))$ increases, a point is reached where the ...rm switches from the project with risk index \star_1 to that with risk index \star_1

This follows from condition (13) and Definition 1. Indeed, throughout the remainder of the paper we will present local properties such as that in condition (13) but note that these results hold when project type is given but will be reversed at the switching point between the two types of project.

Firm continuation means that the pension plan is paid \star_0 for sure and holds a risky prospect on \star_1 . This package must be more attractive to the pension plan under the safer distribution. For $\star_1 = 0$, $\mathbb{C}(\star\star_1) = \mathbb{C}(\star\star_2)$ but for $\star_1 > 0$ $\mathbb{C}(\star\star_1) > \mathbb{C}(\star\star_2)$ as the pension fund has a strict preference for the safer technology. Moreover, other things being equal $\mathbb{C}(\star\star_1) > \mathbb{C}(\star\star_2)$ is continuous and increasing in \star_1 . Although the value of the call option that the firm has on pension plan surpluses is affected by the value of \star_1 , the value of this option is not affected by the risk of the firm's investments. Then we have the following:

Corollary 8 As \star_1 increases, the firm will have an incentive to invest in risky investments that have a negative net present value.

Finally, consider the implications for the firm's investment policy of the priority of pensions in the event of firm liquidation:

Proposition 9 Raising \star_0 , raises \star_0^* , so that the pension plan does well in the case of liquidation at date $\star = 0$, $\Phi(\star)$ falls, so increasing the incentive for the firm to invest at date $\star = 0$. On the other hand if the priority of pensions at date $\star = 1$, \star_1 , is increased, $\Phi(\star)$ rises and the gain to the firm from investing is reduced, so there is less incentive for the firm to take risk.

This proposition follows directly from differentiating conditions (4) and (9) with respect to \star_0 and \star_1 . In the UK, in June 2003, the priority of pensions in the event of the firm going into liquidation and the pension plan being closed has increased. This proposition shows that the effect of this on investment is ambiguous, only if the current effect of a higher value of \star_1 dominates will the effect on investment be positive. In this case the firm's shareholders will gain more from the investment at this date than before and since the most valuable action is taken more often welfare will be increased.

5 Introducing Long-Term Debt

Long-term debt is a significant mode of finance for many companies with large defined-benefit pension plans. If debt is risky it distorts the firm's investment incentives. However, the effect will depend upon the debt's seniority relative to pension obligations in the event of the firm being insolvent.

The principal modification we make to the preceding model is the introduction of long-term debt issued at date $\star = 1$ with face value at date $\star = 1$ of \mathbf{b}_1 . \mathbf{b}_1 is set at date $\star = 1$, given rational expectations over future events including firm liquidation, to fairly price the debt. We assume that this debt simply takes the form of a claim against the firm's date $\star = 1$ assets.

5.1 The Balance Sheets of the Firm and Pension Fund

The firm's date $t=0$ consolidated balance sheet before new decisions is as follows:

Assets	Liabilities
Liquid Assets: \star_0 .	Bank debt: \star_0 due at date $t=0$.
Fixed Assets: \star_0^* .	Long term debt: \star_1 due at date $t=1$.
Share of pension plan surplus: \star_0^*	Contribution to the pension plan at date $t=0$: \star_0
Call option on future pension plan surplus: \star_0 .	Current pension plan deficit that must be funded: \star_0^* .
	Put option for future pensions plan deficit: \star_0 .
	Equity: \star_0 .

The firm's pension fund date $t=0$ balance sheet is as before:

Assets	Liabilities
Securities: \star_0^* .	Pensions: \star_0 due at date $t=0$.
Claim on sponsor at date $t=0$: $\star_0 + \star_0^* + \star_0$.	Pensions: \star_1 due at date $t=1$.
	Share of pension plan surplus: \star_0^* .
	Call option on future pension plan surplus: \star_0 .

5.2 The Effect of the Firm's Investment Decision on the Value of the Pension Plan and Long-Term Debt

Long-term debt has to be distinguished from other corporate liabilities in terms of where it ranks in priority if the firm is liquidated. A fraction \star_0 of the long-term debt is as senior as bank debt.

Note that if $\star_0^* + \star_0 + \star_0^* \geq \star_0 + \star_0 \star_1 + \star_0 (\star_1 \star_0^*)^+$, the long-term bondholders and pension plan either receive full payment from the firm or respectively \star_1 and $(\star_1 \star_0^*)^+$; or respectively \star_0 and $(1 - \star_0)$ of the residual. At date $t=0$ the total value of senior claims is

$$\star_0 = \star_0 + \star_0 \star_1 + \star_0 (\star_1 \star_0^*)^+ \quad (15)$$

In the event that the firm is liquidated at date $t=0$, the long-term debt is paid:

$$\star_0^b = \left(\frac{\star_0 \star_0^*}{\star_0} \right) (\star_0^* + \star_0^* + \star_0) \text{ if } \star_0^* + \star_0^* + \star_0 > \star_0, \quad (16)$$

and $\star_0 \star_0^* + \min[\star_0 (\star_0^* + \star_0^* + \star_0) i \star_0] (1 i \star_0) \star_0^b$ otherwise.

and pensions are paid

$$\star_0^* = \left(\frac{\star_0 (\star_i \star_0^*)^+}{\star_0} \right) (\star_0^* + \star_0^* + \star_0) \text{ if } \star_0^* + \star_0^* + \star_0 > \star_0, \quad (17)$$

and $\star_0 (\star_i \star_0^*) + \min[(1 i \star_0) (\star_0^* + \star_0^* + \star_0) i \star_0]$,

$(1 i \star_0) (\star_i \star_0^*)^+$ otherwise.

In continuation more bank debt is issued:

$$\star_1 = \star_0 i \star_0 + \star_0 + \star_0 i \star_0^* + \star_0^*. \quad (18)$$

Let a fraction \star_1 of any pension debt at date $t=1$ rank as senior as new bank debt, as does a fraction \star_1 of long-term debt. Total debt plus pension debt at $\star = 1$ is

$$\star_1 = \star_1 + \star_1^b + \star_1^*. \quad (19)$$

The face value of the sum of bank debt, senior long-term debt and priority pensions is

$$\star_1 = \star_1 + \star_1 [\star_1 i \star_1^*]^+ + \star_1 \star_1^b. \quad (20)$$

Then at date $t=1$, long-term debt receives

$$\star_1^b \text{ if } \star_1^* + \star_1^* + \star_1 > \star_1, \quad (21)$$

and \star_1^b if $\star_1^* + \star_1^* + \star_1 > \star_1$,

where

$$\star_1^b = \left(\frac{\star_1 \star_1^b}{\star_1} \right) (\star_1^* + \star_1^* + \star_1) \text{ if } \star_1^* + \star_1^* + \star_1 > \star_1, \quad (22)$$

and $\star_1 \star_1^b + \min[\star_1 (\star_1^* + \star_1^* + \star_1) i \star_1] (1 i \star_1) \star_1^b$ otherwise.

Here, \star_1 is the sharing rule used in the event of liquidation at date $t=1$, that divides residual value between the pension plan and the long-term debt.

Granted that \star_0 was paid in full at date $\star=0$, at date $\star=1$ pensions are paid

$$\star_1 \text{ if } \star_1^* = 0 \text{ or } \star_1^* > 0 \text{ and } \star_1^* + \star_1 > \star_1, \quad (23)$$

$$\text{and } \star_1^* \text{ if } \star_1^* < 0 \text{ and } \star_1^* + \star_1 < \star_1,$$

where

$$\star_1^* = \left(\frac{\star_1[\star_1 i \star_1^*]^+}{\star_1} \right) (\star_1^* + \star_1) \text{ if } \star_1^* + \star_1 > \star_1, \quad (24)$$

$$\text{and } \star_1[\star_1 i \star_1^*]^+ + \min[(1 i \star_1)(\star_1^* + \star_1 i \star_1) \star (1 i \star_1) \star_1^*] \text{ otherwise.}$$

We can now write down expressions for the impact of the firm's investment policy at date $t=0$ on the value of the pension plan and the long-term debt. Continuation again yields a transfer to pensioners of

$$\mathbb{C}\star(\star) = \star_0^* + \star_0 + \left[\sum_0^{\star_1} (\star_1^* i \star_1^*) \star (\star_1^* + \star_1 \star) \right] \star (\star_1) i \star_0^*, \quad (25)$$

which as we can see is affected by the existence of long-term debt in three ways, through the impact of b on \star_0^* and \star_1^* and through \star_1 .

The value of the company's long-term debt at date $t=0$ if the firm continues is

$$\begin{aligned} & b_1 + \left[\sum_0^{\star_1} (\star_1^* i b_1) \star (\star_1^* + \star_1 \star) \right] \star (\star_1) = \\ & b_1 [1 i \sum_0^{\star_1} \star (\star_1^* + \star_1 \star) \star (\star_1)] + \left[\sum_0^{\star_1} \star_1^* b_1 \star (\star_1^* + \star_1 \star) \right] \star (\star_1). \end{aligned} \quad (26)$$

The first term on the right-hand-side of the equality is the present value of the face value of the bonds less the shareholders default option. This option is increasing in the risk of $\star_1^* + \star_1$, as defined in Rothschild Stiglitz (1971).¹⁵ The second term is approximately equal to the probability of default times the amount recovered by bonds in the event of default. If the firm is liquidated

¹⁵ See Merton (1973).

at date $t=0$ the bondholders obtain \star_0^b . In the event of continuation, the transfer to long-term bondholders is given by (26) less this amount,

$$\mathbb{C}(\mathbf{b}_1) = \mathbf{b}_1 + \left[\frac{\mathbf{Z}^*}{0} (\star_1^b - \mathbf{b}_1) \star (\star_1^* + \star_1 \star) \right] \star (\star_1) - \star_0^b. \quad (27)$$

The interaction with the value of pension liabilities is through the terms \star_0^b and \star_1^b , and also \star_1 .

5.3 The Firm's Investment Policy

The firm is faced with the following investment decision at date $t=0$, it invests and continues if

$$\star_0 = \star_1 \star_0 + \mathbb{C}(\mathbf{b}_1) + \mathbb{C}(\star) \star_0 \star (\star_0 \star_1) + \star_0. \quad (28)$$

Where $\mathbb{C}(\mathbf{b}_1)$, $\mathbb{C}(\star)$ and $\star (\star_0 \star_1)$ are given in (27), (25) and (11) respectively. In the absence of renegotiation new bank debt will be issued to finance continuation when $\mathbb{C}(\mathbf{b}_1)$ and $\mathbb{C}(\star)$ are relatively small or negative. This is most likely when new bank debt ranks senior to long-term debt and pension liabilities. With fully funded or insured pensions, $\mathbb{C}(\star) = 0$. Only when long-term debt is riskless will $\mathbb{C}(\mathbf{b}_1) = 0$. The pension plan pension plan has not been immunised from the firm's decision, hence, wealth transfers to and from the pension plan will take place if the firm continues. We also assume that long-term debt is risky. Thus in what follows, neither $\mathbb{C}(\star)$ nor $\mathbb{C}(\mathbf{b}_1)$ can be assumed to be zero.

The impact of the size of \mathbf{b}_1 and \star_0^* on $\mathbb{C}(\mathbf{b}_1)$ and $\mathbb{C}(\star)$ are illustrated in the following Claims. The properties illustrated in the Claims are all local and are derived taking the firm's risk index as given. The issue of what happens in the neighbourhood of a switch in project risk will be discussed subsequently.

We first note the effect of the amount of long-term debt on $\mathbb{C}(\mathbf{b}_1)$, where the sign of $\mathbb{C}(\mathbf{b}_1)$ depends upon how long-term debt is treated in liquidation at dates $t=0$ and $t=1$. Given \mathbf{b}_1 set at date $\star = 1$ and the risk index \star , the value of bonds at date $\star = 0$ is given by $\mathbf{b}_1 + \frac{\mathbf{R}}{0} \left[\star_1^* (\star_1^b - \mathbf{b}_1) \star (\star_1^* + \star_1 \star) \right] \star (\star_1) \star 0$. Since $\star_1^b - \mathbf{b}_1 \star 0$, as \star_1 is increased the value of bonds declines. But certainly for lower values of \star_1 , the bonds will be worth more under continuation

than liquidation, \star_0^b , so that $\mathbb{C}(\star_0^b) < 0$.

Claim 10 For a given treatment of long-term debt in liquidation, the effect of higher \star_1^b on $\mathbb{C}(\star_1^b)$ will initially be positive but at higher values of \star_1^b this can turn negative, as the marginal effect on default at date $\star = 1$ becomes large.

Proof. To see the effect of higher values of \star_1^b on $\mathbb{C}(\star_1^b)$, differentiate (27) with respect to \star_1^b

$$\frac{\star \mathbb{C}(\star_1^b)}{\star \star_1^b} = 1 + \frac{Z}{Z} \left[\frac{\star_1^b}{\star \star_1^b} \left(\frac{\star_1^b}{\star \star_1^b} - 1 \right) \star (\star_1^b + \star_1^b) \right] \star (\star_1^b) + \frac{[(\star_1^b - \star_1^b) \star (\star_1^b)] \star (\star_1^b)}{\star \star_1^b} \star (\star_1^b) \geq 0 \quad (29)$$

Since $0 < \frac{\star_1^b}{\star \star_1^b} < 1$, the sum of the first two terms is positive. As $\star_1^b > \star_1^b$ and $\frac{\star_1^b}{\star \star_1^b} < 0$, the third term is negative. Finally, $0 < \frac{\star_1^b}{\star \star_1^b} < 1$. As \star_1^b rises, \star_1^b also rises and so the relative importance of the third term increases and can come to dominate the positive terms and $\frac{\star \mathbb{C}(\star_1^b)}{\star \star_1^b}$ will change from being initially positive to negative. ■

Claim 11 When there are unfunded pension liabilities, a higher value of \star_1^b implies a decrease in $\mathbb{C}(\star) < 0$.

Proof. This result follows directly from expression (25):

$$\frac{\star \mathbb{C}(\star)}{\star \star_1^b} = \frac{Z}{Z} \left[\frac{\star_1^b}{\star \star_1^b} \star (\star_1^b + \star_1^b) \right] \star (\star_1^b) + \frac{[(\star_1^b - \star_1^b) \star (\star_1^b)] \star (\star_1^b)}{\star \star_1^b} \star (\star_1^b) \geq 0. \quad (30)$$

The first total-term measures the effect of higher \star_1^b on the value of pensions under firm continuation. As $\frac{\star_1^b}{\star \star_1^b} < 0$, $\star_1^b - \star_1^b < 0$, and $\frac{\star_1^b}{\star \star_1^b} < 0$, this term is certainly negative. Hence, unless $\frac{\star_1^b}{\star \star_1^b} < 0$ is large in absolute terms, higher \star_1^b will be associated with lower $\mathbb{C}(\star)$. ■

The above effects will be zero if the firm's debt is all short-term or if long-term debt is riskless and pension plan liabilities are matched by equivalent duration assets. If this is not possible, collective action may be achieved through institutional arrangements that overcome free-rider problems by dispersed bondholders. For example, in the event of financial distress at date $\star = 1$, bondholders could be offered increased seniority, \star_1^b , of their claim in exchange for a cut in its face value to \star cents in the dollar. They are then faced with a trade-off of more in the event of

insolvency against less in the event of solvency at date $\star = 1$. Failing this, formal mechanisms may be introduced to reduce the free-rider problem. However, since by Claim 11, the forgiveness of debt has a positive spillover effect on the value of the pension plan, $\mathbb{C}_\star(\star)$ increases, so this effect is limited.¹⁶

Next, consider the impact of a pension deficit on the value of $\mathbb{C}_\star(\star_1)$.

Claim 12 Given suitable values of \star_0 and \star_0 , a higher pension plan deficit, \star_0^* , has a negative impact upon $\mathbb{C}_\star(\star_1)$.

Proof. Given the risk index \star , \star_1 and $\star_0^* \neq 0$, differentiate (27) with respect to \star_0^*

$$\frac{\partial \mathbb{C}_\star(\star_1)}{\partial \star_0^*} = \frac{\mathbb{Z}}{\star_0^*} \left[\left(\frac{\star_0^* \star_1^b}{\star_0^* \star_0^*} \right) \star_1^* (\star_1^* + \star_1 \star_1^b) \right] \star_1^* (\star_1) + \frac{\mathbb{Z}}{\star_0^*} \left[(\star_1^b \star_1 \star_1^b) \star_1^* (\star_1) \frac{\star_0^* \star_1^b}{\star_0^*} \right] \star_1^* (\star_1) + \frac{\star_0^* \star_1^b}{\star_0^* \star_0^*} \quad (31)$$

The impact of \star_0^* on the value of long-term debt if the firm continues is given by $\frac{\mathbb{R}}{\star_0^*} \left[\left(\frac{\star_0^* \star_1^b}{\star_0^* \star_0^*} \right) \star_1^* (\star_1^* + \star_1 \star_1^b) \right] \star_1^* (\star_1) + \frac{\mathbb{R}}{\star_0^*} \left[(\star_1^b \star_1 \star_1^b) \star_1^* (\star_1) \frac{\star_0^* \star_1^b}{\star_0^*} \right] \star_1^* (\star_1)$. Since $\frac{\star_0^* \star_1^b}{\star_0^* \star_0^*} \neq 0$, $\star_1^b \star_1 \star_1^b$ and from (19) $\frac{\star_0^* \star_1^b}{\star_0^* \star_0^*} \star 0$, this total effect is seen to be negative. Moreover, if the negative impact on the current value of the bonds in the event of liquidation at date $\star = 0$, $\frac{\star_0^* \star_1^b}{\star_0^* \star_0^*} \star 0$, is relatively small, because either \star_0 is small or \star_0 is large, then the former total effect dominates. Thus a high realisation of \star_0^* will have a negative impact on bondholders. ■

We also note at this stage that this negative effect will be greater the higher the priority of pension obligations at date $\star = 1$, and hence the lower the value of \star_1^b .

This result is consistent with the findings of both Maher (1987) and Carroll and Niehaus (1998), that underfunding of pension liabilities has a negative impact upon the sponsoring companies bond rating. Moreover, as pension deficits increase bond ratings decline. The relationship between pension plan overfunding and bond ratings is, however, weak.

Claim 13 A higher pension deficit, \star_0^* , has a positive impact on $\mathbb{C}_\star(\star)$.

Proof. Consider condition (24), note that $\star_1^* \star_1^* \star 0$ so that for \star_1 not too high, $\star_0^* + \frac{\mathbb{R}}{\star_0^*} \left[\left(\star_1^* \star_1^* \star_1^* \right) \star_1^* (\star_1^* + \star_1 \star_1^b) \right] \star_1^* (\star_1) \star \star_0^*$ and $\mathbb{C}_\star(\star) \star 0$. To see the effect of higher

¹⁶ There is an argument here for also including the pension plan in any formal financial restructuring of the insolvent firm

pension deficits, differentiate (25) with respect to \star_0^*

$$\begin{aligned} \frac{\partial \mathbb{C}_*(\star)}{\partial \star_0^*} &= 1 + \frac{Z}{z} \left[\frac{\star_1^*}{\star_0^*} \left(\frac{\star_1^*}{\star_0^*} + \frac{\star_1^*}{\star_0^*} \right) \right] \star_1^* \\ &+ \left[(\star_1^* + \star_1^*) \star_1^* \right] \frac{\star_1^*}{\star_0^*} \end{aligned} \quad (32)$$

This expression is analogous to (13), except that the liquidation terms are given in (17) and (24) and will depend upon the seniority of pensions in liquidation at dates $\star = 0$ and $\star = 1$ and the value of \star_1 . Otherwise the argument is the same as in Proposition 6. Hence, at lower values of \star_1 , $\mathbb{C}_*(\star) \star_0^*$ and is increasing in the size of the pension deficit, as indeed is the incentive for the firm to underinvest. ■

Claims 12 and 13 together imply:

Proposition 14 Other things being equal, a high pension deficit will redistribute value from long-term debt to the pension plan. This effect will be greater, the greater the amount of long-term debt and the more senior pension liabilities are at date $\star = 1$.

Given project risk, an increase in the pension deficit raises $\mathbb{C}_*(\star)$ and lowers $\mathbb{C}_*(\mathbf{b}_1)$ but by less than the increase in $\mathbb{C}_*(\star)$ and because \star_1 increases \star_0^* increases. Hence $\mathbb{C}_*(\mathbf{b}_1) + \mathbb{C}_*(\star) \mathbb{C}_*(\star_0^* + \star_1^*)$ increases so that using Definition 1, a point is reached at which the firm switches to the riskier project. Then as in Corollary 6, there will be a local reversal of the above result.

The presence of long-term debt in the company's financial structure also has implications for pension plan overfunding.

Proposition 15 Long-term debt reduces the incentive to overfund the pension plan at date $\star = 0$ but increases the incentive to overfund the pension plan at date $\star = 1$.

We have seen in Proposition 4, that in the absence of long-term debt the firm will not fund it profitable to overfund the pension plan at date $\star = 0$. Since \mathbf{b}_1 is set at date $\star = 1$, raising \star_0^* will raise \star_1 but can raise the value of this debt, through the impact of increased expected pension plan surpluses on the amount recovered by the long-term debt holders if the firm defaults at date $\star = 1$, without compensation for the shareholders. Things are more favourable for the shareholders if \star_1 is increased, as there will be a compensating reduction in the face value of the

debt, b_1 , when it is set. However, in the absence of tax incentives to overfund the pension plan, the firm will still not fund it in the interests of shareholders to do so.

6 Pension Benefit Insurance

Pension benefit insurance can, at least theoretically, be provided either publicly or privately.¹⁷ In the United States, the PBGC is a government agency that guarantees the shortfall between the level of insured benefit and the assets securing those benefits. Bodie (1996) argued that the PBGC's exposure to shortfall risk depends on three factors: the financial strength of the sponsoring firm; the degree of underfunding of insured benefits; and the mismatch between the market risk exposure of insured benefits and the market-risk exposure of the assets securing that benefit. It is important to recognise that the degree of underfunding will be a function of the contribution and investment policy of the pension plan. Moreover, potential mismatches between assets and liabilities and hence potential shortfalls, will be greater under an equity-led than a bond led investment strategy. But of course there will be the offsetting benefit to the pension plan of greater potential surpluses.

Under any pension benefit insurance plan, the firm makes a payment into an insurance fund and in the event of a pension deficit, which cannot be met from company contributions; the pension plan draws on the insurance. There is scope for variation in the setting of premia and in the benefits guaranteed. In the case of the PBGC, the insurance fund only pays-out if the company is insolvent. Then, in the event that the sponsoring firm is insolvent, the pension insurance fund has a claim on the sponsoring firm that is determined by the bankruptcy law. In the United States the PBGC has a priority claim of 30 per cent of a firm's net-worth excluding the pension plan. After this the PBGC has a claim equal to that of an unsecured creditor on the balance of the

¹⁷ The United States administers pension guarantees through the Pension Benefit Guaranty Corporation. The United Kingdom Government proposes to introduce a similar scheme, the Pension Protection Fund (PPF). Of the companies in the Standard & Poor's 500 index, 360 have defined-benefit pension plans. Since 2000, stock market declines in the last three years have caused the pension funds of companies in the S&P 500 to lose more than \$200 billion in value, according to recent studies. The PBGC reported a \$11.4-billion loss for the fiscal year ended Sept. 30, 2003, the biggest in the agency's history after it assumed liability for pensions at steel and airline companies that failed for bankruptcy protection.

...rm's net-worth.

The pension benefit insurance can be thought of as a put option paid for by the sponsoring ...rm. Sharpe (1976), Treynor (1977) and Langetieg, Findlay and da Motta (1982) all examined the basic option pricing approach to valuing pension benefit insurance. Sharpe for example showed the problems that arise when the pension insurance fund does not receive full compensation for the risk taken by the sponsoring ...rm. However, Marcus (1987) was the ...rst author to use option pricing to value the PBGC put. Lewis and Pennachi (1994,1999) extended the Marcus analysis in a number of interesting directions.¹⁸

In our simple risk-neutral, two-period setting the value of pension benefit insurance is approximately equal to the probability of ...rm insolvency multiplied by the expected insured de...cit of the pension plan in the event of sponsor default.¹⁹ A full pension insurance means that in the event of the sponsoring ...rm being insolvent, the pension insurance fund pays for the loss of the put option that the pension plan has against the sponsoring ...rm.

As shown by Sharpe (1976), in a complete ...nancial market, if the level of pension benefit is guaranteed by fairly priced insurance, the cost to shareholders of pension liabilities is independent of the investment policy of the ...rm. In this context it is easy to see that an equity-led investment

¹⁸ Marcus (1987) assumed that the pension asset, the pension liabilities, the debt of the ...rm, and the value of the ...rm all follow Brownian motions. If the ...rm is insolvent he assumed that the PBGC took over the pension scheme de...cit. He also assumed that contributions are linear, so they fall as the ...rm approaches insolvency. He found that the value of the PBGC put option was very sensitive to assumptions about funding behaviour. Pennachi and Lewis (1994, 1999) extended the analysis of Marcus (1987) in several ways. They recognised that: a ...rm only becomes insolvent when its value sinks below some fraction of ...rm value; and also recognised that when a scheme terminates, its liabilities increase due to changes in retirement assumptions. They also allowed for courts setting recovery of assets by the PBGC at less than the 30 per cent ...gure. Finally they allowed the sponsoring ...rm to pay dividends prior to insolvency, so reducing ...rm value. In a further paper, Lewis and Pennachi (1999) extend their earlier results to include the present value of PBGC premiums, which they modelled as a series of put options on PBGC assets. Applying this model to a sample of ...rms, they found that actual premiums only offset a small amount of the insurance provided. Moreover, they found that the net liability assumed by the PBGC is affected by the riskiness of the sponsoring ...rm, the funding status of the pension scheme and the asset portfolio of the pension scheme, with premia rising for riskier equity-led strategies.

Vanderhei (1990) estimated the correct premium as the product of the probability of a loss in each year and the severity of a loss given that it occurred. He used PBGC data to estimate these variables. He then estimated the appropriate premia for each plan on existing PBGC lines, a ...xed fee per participant and a variable premium per \$1,000 of underfunding. He found signi...cant cross-subsidies in the PBGC premium structure and that the PBGC was undercharging for its insurance on average. The most striking feature of this literature is the wide variation in the values of PBGC insurance.

¹⁹ This is more complicated in a multi-period environment, in which the exercise time of the option is endogenous, and with an unde...ned horizon.

strategy will benefit shareholders to the extent that it does not provide full cover through the purchase of these put options.

There are well-known moral hazard problems linked to the above, with the incentives of both the sponsoring firm and the pension plan trustees being affected by the existence and extent of the pension benefit insurance.²⁰ The value of pension benefit insurance is increasing in the value of the pension deficit and in the probability of insolvency of the sponsoring firm. If the firm increases the risk of its investments so its probability of insolvency increases and the insurance increases in value, so there is an increase in the liability of the insurance fund and an offsetting reduction in the value of the firm's liability.²¹

At the same time, pension benefit insurance, in providing the pension plan with a put option, exercisable in the event of the sponsoring firm's failure, also creates an incentive for the pension plan to take greater investment risk through a high-risk, equity-led investment strategy.

An important problem to consider is how best to design and price insurance for some level of benefit in a way that eliminates, or at least minimises the potential to "game" the insurance fund. Suppose that the pension benefit insurance, viewed as a put option, is only exercised if the sponsoring firm is insolvent and then it pays the deficit of the fund. Consider the incentives facing the firm. As noted, there is an incentive for the firm to take greater risks and shift wealth from the insurance fund to the shareholders of the firm. However, there is also be an impact on the pension plan. In expression (25), in the event of the sponsoring firm defaulting, the liquidation value \star_0^* is recovered, at least in part, by the pension insurance fund and the default on the put option, \star_1^* , is in turn covered by the insurance fund. This has the following implication:

Proposition 16 Pension benefit insurance reduces the impact of the sponsoring firm's investment policy on the pension plan, so $\mathbb{P}(\star)$ is reduced. At the same time there is a transfer of value away from the insurance plan. The reduction in $\mathbb{P}(\star)$ reduces the underinvestment effect.

²⁰ The above literature has not accounted for these factors.

²¹ It should be noted here that Gersovitz (1982) found that with PBGC insured pension plans there was little evidence that the size of pension funding deficits affects the value of sponsoring firm's equity. This contrasts with earlier work by Feldstein and Seligman (1981) that found that funding deficits have an offsetting negative effect on company capitalisation. This latter finding suggests that, at least for the sample period, at the margin the impact of funding deficits is similar to that which we would expect in an environment without insurance such as the UK.

If the firm has risky long-term debt outstanding the above result is modified, as the existence of pension benefit insurance will increase the recovery rate on these bonds.

One way to reduce the incentive for the firm to take greater risk is to give the losing claim-holder the right to convert into an equity claimant.²² In this case the insurance fund could take a significant holding of the sponsoring company's equity. In a multi-period setting, linking risk-based insurance premia to the level of pension plan underfunding may also constrain this incentive. A regulatory solution could be a risk-based minimum funding requirement.²³ This forces pension plan sponsors to ensure that the pension plan assets can fund some minimum level of pension plan liabilities and thereby limits the incentive for the firm to take risk as this will raise the funding requirement. This will tend to bias investment incentives in favour of underinvestment.

7 Conclusion

A defined benefit pension is not a fully guaranteed promise but is closely tied to the company's fortunes and is affected by its opportunities and incentives. In the model presented, pension plan surpluses and unfunded deficits are a source of equity to the firm but deficits that must be funded are a debt burden. But as shown, absent tax arbitrage, firms have no incentive to retain funds on their own balance sheet or as equity in an overfunded pension plan. We showed that increases in the maturity of a company's pension plan as measured by a high value of α_0 will, other things being equal, induce under-investment by the firm. On the other hand, relatively high future pension commitments create an incentive to undertake risky investments. We considered the implications of the priority of pensions in the event of firm liquidation and showed that if the pension plan does well when the firm is liquidated at the current date, the incentive for the firm to invest is high. On the other hand, if the priority of pensions in the future is high, the gain to

²² See Green (1984).

²³ In the UK the Minimum Funding Requirement (MFR) was introduced by the Pensions Act 1995 as a measure designed to promote security for pension plan members. It requires funded defined benefit pension plans to hold a minimum level of assets to meet their liabilities, and sets out time limits within which any underfunding must be made good. The terms of the MFR were significantly relaxed in 2002. Shortfalls must be funded over a five-year period. The MFR will soon be replaced by scheme specific funding standards.

the firm from investing is reduced and there is more incentive to take risk.

When a firm has both long-term debt and outstanding pension liabilities, the value of new investment to shareholders is lowered by the presence of both. Continuation versus liquidation of the firm depends upon the costs of debt and pensions. The solvent firm must fund the pension deficit. But these transfers to the pension plan reduce shareholder equity and at the same time increase the risk of the company's long-term debt. This has the implication that a firm in these circumstances may have an incentive to close its defined benefit pension plan to new members. This threat may force the pension plan to renegotiate the benefit level or the future level of contributions.

In our model, a pension benefit insurance scheme reduces the under-investment problem facing the firm but increases the over-investment problem. Pension insurance creates an incentive for the sponsoring firm to take greater risk. In providing the pension plan with a put option that can be exercised in the event of the sponsoring firm's failure, the insurance also creates an incentive for the pension plan to take greater investment risk through a high-risk, equity-led investment strategy. We argued that the incentive for the sponsoring firm to take greater risk could be constrained in a number of ways, including risk based premia. A minimum funding requirement can also reduce risk taking by the firm but this is limited if the pension plan is induced to take greater risk in its investment policy.²⁴

The paper stepped back from drawing any significant policy conclusions. However, it makes it clear that the pension plan must have an interest in ensuring that the firm's financial policy and investment behaviour does not jeopardise the pension promise through paying out excessive dividends and taking too much risk. Moreover, the sponsoring firm has an interest in controlling the risks transferred back to it through the pension plan's own investment policy. We considered the dual role of pension fund insurance and minimum funding requirements in limiting the impact

²⁴ Arnott and Gersovitz (1980) show that with fixed factor supplies, imposing full funding on pension plans will affect risk sharing and can raise the welfare of shareholders at the expense of workers.

of sponsoring company risk on the pension plan. However, this raises a whole series of further issues of how best to control the multitude of moral hazard problems. The impact of the company's own financial and investment decisions on the pension plan will be less if the maturity of the company's debt is short-term and structured so as not to distort subsequent investment opportunities. At the same time the pension plan will have less effect on the firm's investment decisions if plan assets have the same duration as the plan's liabilities. There would appear to be no perfect solution to these problems leading some authors to advocate that company pension plans should be defined contribution, with the employee bearing the investment risk of the plan; but with plan assets being independent of the firm's own financial position.²⁵

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²⁵ See for example the discussion in Blake (2000).

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