Intergenerational Risksharing
and Equilibrium Asset Prices

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Abstract

In the presence of overlapping generations, markets are incomplete because it is impossible to engage in risksharing trades with the unborn. In such an environment the government can use a social security system, with contingent taxes and benefits, to improve risksharing across generations. An interesting question is how the form of the social security system affects asset prices in equilibrium. In this paper we set up a simple model with two risky factors of production: human capital, owned by the young, and physical capital, owned by all older generations. We show that a social security system that optimally shares risks across generations exposes future generations to a share of the risk in physical capital returns. Such a system reduces precautionary saving and increases the risk-bearing capacity of the economy. Under plausible conditions it increases the riskless interest rate, lowers the price of physical capital, and reduces the risk premium on physical capital.

1   Introduction

The design of public pension systems is a subject of active discussion around the world. Important questions are how to combine pay-as-you-go with prefunded pension benefits, and how to adjust benefits and taxes to unanticipated shocks. Traditional public pension systems set fixed benefit rates to be financed by fixed rates of payroll taxation. Economic shocks may require adjustments in benefits, tax rates, or both, and adjustments have often been made (McHale 2001); but the nature of these adjustments is not always spelled out in advance. This lack of clarity is unfortunate, because pay-as-you-go pension systems with contingent taxes and benefits can be used to improve risksharing between generations. In effect, the government can use its powers of taxation to share capital and labor income risks across generations. Private markets cannot accomplish this because future generations are absent from the marketplace today.

Any analysis of a contingent public pension system, or contingent social security, should consider the effect of the system on private asset markets. The willingness of households to save, and to bear private investment risk, depends on their expectations about future social security payments and the correlation of these payments with risky asset returns. Thus the form of the social security system can influence the riskless interest rate and the pricing of risky assets.

To analyze such effects, we need a model with overlapping generations (OLG) in order to capture the special role of social security. Unfortunately, OLG models are hard to work with. The classic two-period OLG model of Samuelson (1958) and Diamond (1965) has inelastic supply of assets by the old (who have no reason to save), and inelastic demand for assets by the young (who have no reason to consume). A three-period extension of the model (Constantinides, Donaldson, and Mehra 2002, 2005) is more realistic, but analytically intractable. In this paper, we follow Blanchard (1985) and Gertler (1999) and use a model in which agents face a constant probability of death each period. Our model is most closely related to Farmer (2002). Like Farmer, we assume that agents have log utility and own both human and physical capital; unlike Farmer, however, we abstract from production and depreciation in order to concentrate on the asset pricing effects of social security.[3]

We simplify our analysis by assuming fixed supplies of two assets, human capital that is owned entirely by the youngest generation alive in each period, and physical capital that is used for savings. Our assumption that physical capital cannot be accumulated is often used in the finance literature on the pricing of the aggregate equity market, following Lucas (1978) and Mehra and Prescott (1985). Following Lucas, we can think of our economy as having two "fruit trees". The first tree produces fruit that is owned by each new generation, but no single generation owns the tree itself. The second tree is owned by older generations and sold to younger ones. The assumption of fixed asset supplies means that the social security system has large effects on asset prices but no effects on asset quantities; however our model suggests the direction of quantity effects that would arise in a model with factor accumulation.

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[3]Athanasoulis (2006) also uses the Blanchard model to study the determination of asset prices in an OLG model. Athanasoulis assumes constant absolute risk aversion and does not model a social security system.

We assume that both human capital and physical capital pay risky dividends. In a laissez-faire economy, these risks are shared across generations through their effects on the equilibrium price of physical capital. Although the old do not own human capital, they are affected by a low human capital dividend because this lowers the resources of the young and thus lowers the price of physical capital that the old sell to the young. Similarly, although the young do not initially own physical capital, the dividend on physical capital affects the price of the capital that they buy. It turns out that in our model, the laissez-faire equilibrium shares human capital risk, but not physical capital risk, optimally between the young and the old. Thus there is a role for a contingent social security system to improve risksharing.

Some previous authors, including Shiller (1999) and Ball and Mankiw (2001), have assumed that human capital is riskless and have concentrated on the need to share physical capital risk across generations. Bohn (2002, 2003) points out that in a standard production model, labor income and physical capital income are proportionately exposed to technology shocks.[4] Empirical estimates of human capital risk are often quite low (Campbell, Cocco, Gomes, and Maenhout 2001), but cointegration between human and capital income could increase long-run measures of human capital risk (Benzoni, Collin-Dufresne, and Goldstein 2005). Our model justifies the concentration on physical capital risk without relying on the assumption that human capital is riskless.

2   A Simple Model

We assume that each period $(1 - \chi)$ new agents are born and that each agent survives into the next period with probability χ. This implies that there is a unit measure of agents alive in every period. Agents stay young for one period and the young generation holds all the human capital in the economy. Labor supply is inelastic and the aggregate stream of wages earned by young workers is given by $\{h_t\}_{t=0}^{\infty}$, where $h_t > 0$ for all t. There are two tradable assets in the economy, riskfree one-period bonds and risky physical capital. There is a fixed supply of one unit of risky physical capital which pays a dividend stream $\{d_t\}_{t=0}^{\infty}$, where $d_t > 0$ for all t. The ex-dividend unit price of physical capital is denoted by p_t.

These assumptions could be justified by a production function of the following form:

$$Y_t = h_t H + d_t K, \quad \text{with } H = K = 1, \tag{1}$$

where H denotes aggregate human capital and K the total supply of physical capital.

2.1   Annuitization

We assume that individual investors purchase physical capital from annuity companies which operate in a competitive market. The contract investors sign with the annuity company

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[4]Bohn also assumes that physical capital can be accumulated or decumulated without adjustment costs. This implies a stable price of physical capital, so the owners of physical capital have a lower overall exposure to technology shocks than the owners of human capital.

specifies that their physical capital holdings are taken over by the company at the time of their death. In exchange for this contingent claim, the annuity company agrees to pay them an extra income stream $\{A_t\}$ per unit of physical capital during their lifetime.

At the beginning of every period, a fraction $(1-\chi)$ of the population dies and is replaced by an equal measure of young agents. Since the probability of death does not change with age in our simple model, the death rates are equal at all points of the wealth distribution. This implies that, at the beginning of every period, the annuity companies are left with a fraction $(1-\chi)$ of the aggregate claims on physical capital from those who die. Given that only a measure χ of agents survive from the previous period, the annuity companies must make total payments of χA_t at the beginning of period t. It follows that an annuity company can break even by offering individual investors an annuity payment each period, for each unit of physical capital, equal to:

$$A_t = \frac{1-\chi}{\chi}(d_t + p_t) \tag{2}$$

in exchange for a contingent claim to their physical capital holdings in the event of their death. This multiplies the gross return on physical capital by a factor $1/\chi$.

We also assume that individuals can use annuity companies to borrow or lend at a deterministic rate. Again, the idea is that the annuity companies take over the debt or assets of individuals when they die. With a large enough population of agents, an annuity company can break even by offering a higher rate on savings and asking for a higher rate on loans than the going riskless rate in the bond market. With a competitive market for annuities and a death rate that does not vary with age, the gross interest rate in the annuities market $R_{a,t+1}$ is $1/\chi$ times the gross riskless rate in the bond market $R_{f,t+1}$:

$$1 + R_{a,t+1} = \frac{1}{\chi}(1 + R_{f,t+1}). \tag{3}$$

2.2 Laissez Faire Equilibrium

We now consider how consumption and wealth are determined in laissez faire equilibrium. We adopt the following notational convention. We denote by $\widehat{C}_t^r$ the per capita consumption in period t of an individual born in period $r \leqslant t$ who survives to period t. We denote by C_t^r the aggregate consumption of all the agents who were born in period r and who survived to period t. We also refer to this as the aggregate consumption of generation r in period t. Finally, C_t denotes aggregate consumption in period t of all agents alive in period t.

For tractability, we assume that all agents have log utility of consumption. Thus an agent born at date $r \leqslant t$ (a member of generation r) who survives to date t, maximizes the following objective function at date t:

$$E_t\left\{\sum_{s=0}^{\infty}(\beta\chi)^s \log(\widehat{C}_{t+s}^r)\right\}.$$

3

The assumption of log utility implies that in period t, agents of any generation r consume a constant fraction of their wealth at date t:

$$\widehat{C}_t^r = (1 - \beta\chi)\widehat{W}_t^r, \tag{4}$$

where $\widehat{W}_t^r$ denotes per capita wealth in period t of an agent born in period $r \leqslant t$.

Summing equation (4) for all consumers alive in period t gives the following relationship between aggregate consumption and aggregate wealth:

$$C_t = (1 - \beta\chi)W_t. \tag{5}$$

We assume that only net output can be consumed. Then aggregate consumption needs to equal net output in equilibrium:

$$C_t = Y_t = (h_t + d_t). \tag{6}$$

Finally, aggregate wealth of the economy is given by current output plus the ex dividend value of physical capital:

$$W_t = (h_t + d_t) + p_t. \tag{7}$$

These equilibrium conditions pin down the current price of physical capital in terms of current output and parameters of the model:

$$p_t = \frac{\beta\chi(h_t + d_t)}{1 - \beta\chi}. \tag{8}$$

The one period return on physical capital is then given by:

$$1 + R_{t+1} = \frac{d_{t+1} + p_{t+1}}{p_t} = \frac{\beta\chi h_{t+1} + d_{t+1}}{\beta\chi(h_t + d_t)}. \tag{9}$$

These results can also be derived from equilibrium in the physical capital market. Since all consumers invest the same fraction α_t of their savings in the risky asset, the value of physical capital and aggregate wealth need to satisfy the following relationship:

$$p_t = \alpha_t(\beta\chi W_t). \tag{10}$$

But because physical capital is the only asset in positive net supply, we must have $\alpha_t = 1$, which implies the solution for the physical capital price given in equation (8).

The ex dividend value of physical capital in (8) is increasing in both labor income h_t and the physical capital dividend d_t. Labor income increases the value of physical capital by increasing the desired saving of the young, while the physical capital dividend increases it by reducing the desire of the old to sell physical capital to finance their current consumption. That is, labor income increases the demand for physical capital, while dividend income reduces the supply.

The consumption of the young and the old in the laissez faire equilibrium are given by

$$C_t^t = (1 - \beta\chi)h_t \tag{11}$$

$$C_t^{r<t} = d_t + \beta\chi h_t = (1 - \beta\chi)(d_t + p_t), \tag{12}$$

where C_t^t denotes the aggregate consumption of the young generation and $C_t^{r<t}$ denotes the aggregate consumption of older generations. The young consume a fraction $(1 - \beta\chi)$ of their wealth h_t, and use the rest of their wealth to buy physical capital. The old consume the dividend on physical capital and the proceeds from their capital sales to the young. Equivalently, they consume a fraction $(1 - \beta\chi)$ of their wealth $(d_t + p_t)$.

2.3 Asset Pricing Under Laissez Faire

Aggregating the marginal utilities of all agents alive in *both periods t and $t + 1$* leads to a valid stochastic discount factor (SDF). This group's consumption in period t is equal to a fraction χ of aggregate consumption C_t since only a fraction χ of all agents alive in period t survive into period $t + 1$. In period $t + 1$ the group's consumption is given by $C_{t+1}^{r<t+1}$. With log utility, marginal utility is the reciprocal of consumption so a valid SDF is given by:

$$M_{t+1} = \frac{\beta\chi C_t}{C_{t+1}^{r<t+1}}. \tag{13}$$

Using the equilibrium conditions from the previous section, the SDF simplifies to:

$$M_{t+1} = \frac{\beta\chi(h_t + d_t)}{\beta\chi h_{t+1} + d_{t+1}} = \frac{1}{1 + R_{t+1}}. \tag{14}$$

This condition, that the SDF is the reciprocal of the gross return on wealth, is standard in a model with log utility. It is straightforward to check that the SDF in equation (14) is consistent with the equilibrium price for physical capital derived in equation (8):

$$
\begin{aligned}
p_t &= E_t[M_{t+1}(d_{t+1} + p_{t+1})] \\
&= E_t\left[\frac{\beta\chi(h_t + d_t)}{\beta\chi h_{t+1} + d_{t+1}}\left(d_{t+1} + \frac{\beta\chi(h_{t+1} + d_{t+1})}{1 - \beta\chi}\right)\right] \\
&= \frac{\beta\chi(h_t + d_t)}{1 - \beta\chi}.
\end{aligned}
\tag{15}
$$

The riskless rate in the laissez faire economy is given by:

$$\frac{1}{1 + R_{f,t+1}} = E_t[M_{t+1}] = E_t\left(\frac{1}{1 + R_{t+1}}\right). \tag{16}$$

2.4 Risk Exposures Under Laissez Faire

Expected lifetime utility of an agent of generation t is given by:

$$\widehat{U}_t^t \equiv E_t\left\{\sum_{s=0}^{\infty}(\beta\chi)^s \log(\widehat{C}_{t+s}^t)\right\}. \tag{17}$$

As before, hats denote per capita variables and time superscripts index an agent's generation. In period t, there is a measure $(1-\chi)$ of agents of generation t, a measure $\chi(1-\chi)$ of agents of generation $t-1$, a measure $\chi^2(1-\chi)$ of agents of generation $t-2$, and so on.

The first thing to note is that all generations have some exposure to both human capital risk and physical capital dividend risk in the laissez faire equilibrium. In particular, young agents are exposed to dividend risk indirectly through the price of physical capital, which determines the share of physical capital they can acquire with their savings. More formally, agents of generation t have a per capita labor income of $\frac{h_t}{1-\chi}$ in the first period of their life but no initial claims to physical capital. With log utility, they consume a fraction $(1-\beta\chi)$ of their wealth:

$$\widehat{C}_t^t = (1-\beta\chi)\widehat{W}_t^t = (1-\beta\chi)\frac{h_t}{1-\chi}. \tag{18}$$

The remainder of their wealth is invested in physical capital. The fraction of aggregate physical capital $\widehat{\theta}_t^t$ an agent of generation t acquires depends on the price of physical capital in period t:

$$\widehat{\theta}_t^t = \frac{\beta\chi\widehat{W}_t^t}{p_t} = \frac{1-\beta\chi}{1-\chi}\frac{h_t}{h_t + d_t}. \tag{19}$$

A high dividend in period t has a negative effect on the expected lifetime utility of agents of generation t because it results in a higher physical capital price, which reduces the amount of physical capital that these agents can acquire with their initial savings. High labor income has an intuitive positive effect on the expected lifetime utility of the agents who earn it.

Older cohorts are exposed indirectly to human capital risk through its effects on the price of physical capital and thereby on the returns on their savings. The purpose of the remainder of this section is to derive precise expressions for the exposures of the different cohorts to the two types of risk in this economy. In the Appendix to this paper (Campbell and Nosbusch 2007), we show that, for an agent of generation t, expected lifetime utility at the beginning of period t is given by:

$$
\begin{aligned}
\widehat{U}_t^t &= \frac{1}{1-\beta\chi}\log(\widehat{C}_t^t) - \frac{\beta\chi}{1-\beta\chi}\log(h_t + d_t) + \varphi_t \\
&= \frac{1}{1-\beta\chi}\log\frac{1-\beta\chi}{1-\chi} + \frac{1}{1-\beta\chi}\log(h_t) - \frac{\beta\chi}{1-\beta\chi}\log(h_t + d_t) + \varphi_t, \tag{20}
\end{aligned}
$$

where

$$\varphi_t = E_t\sum_{s=1}^{\infty}(\beta\chi)^s\log\left\{\frac{1}{\chi^s}\left[\prod_{r=1}^{s-1}\frac{\beta\chi h_{t+r} + d_{t+r}}{h_{t+r} + d_{t+r}}\right](\beta\chi h_{t+s} + d_{t+s})\right\}. \tag{21}$$

If there is persistence in the dividend process, the term φ_t is time-varying. It becomes a constant in the special case of i.i.d. dividends.

Agents of any generation $r < t$ also consume a constant fraction of their wealth in period t:

$$\widehat{C}_t^r = (1-\beta\chi)\widehat{W}_t^r = (1-\beta\chi)\widehat{\theta}_{t-1}^r\frac{1}{\chi}(d_t + p_t) = \widehat{\theta}_{t-1}^r\frac{1}{\chi}(\beta\chi h_t + d_t), \tag{22}$$

where $\widehat{\theta}^r_{t-1}$ denotes the share of physical capital brought forward from period $t-1$ by an agent born at date r. The factor $\frac{1}{\chi}$ is due to the fact that consumers purchase the physical capital from annuity companies in a competitive market.

The remainder of wealth is invested in physical capital. The share of physical capital acquired in period t by an agent of generation r is:

$$\widehat{\theta}^r_t = \frac{\beta \chi \widehat{W}^r_t}{p_t} = \frac{1}{\chi} \frac{\beta \chi h_t + d_t}{h_t + d_t} \widehat{\theta}^r_{t-1}. \tag{23}$$

Expected lifetime utility at the beginning of period t for an agent of generation $r < t$ is given by:

$$\begin{aligned} \widehat{U}^r_t &= \frac{1}{1-\beta\chi} \log(\widehat{C}^r_t) - \frac{\beta\chi}{1-\beta\chi} \log(h_t + d_t) + \varphi_t \\ &= \frac{1}{1-\beta\chi} \log\widehat{\theta}^r_{t-1} + \frac{1}{1-\beta\chi} \log\frac{1}{\chi}(\beta\chi h_t + d_t) - \frac{\beta\chi}{1-\beta\chi} \log(h_t + d_t) + \varphi_t. \tag{24} \end{aligned}$$

The four terms in this expression may be interpreted as follows. The first term is a function of the state variable $\widehat{\theta}^r_{t-1}$ which gives the share of physical capital acquired in the previous period by an agent of generation $r < t$. It incorporates the effects of all past shocks since the birth of generation r and up to period $t-1$ on the expected lifetime utility of the agent. The second term gives the effect of the current shocks h_t and d_t on the unit value of physical capital holdings (inclusive of annuity payments) brought forward from last period. Agents of generation r consume a constant fraction of their wealth during period t and reinvest the rest in physical capital. The amount of physical capital they can buy to carry forward into period $t+1$ depends negatively on the current price of physical capital and thus on current output. This effect is captured by the third term and could be described as reinvestment risk. Finally, the last term measures the expected effect of future output realizations on lifetime utility. This term is identical for all generations. The $\frac{1}{1-\beta\chi}$ factors multiplying these terms arise because the effects on consumption are permanent.

Equation (24) shows that agents of all generations born before the current period have exactly the same exposure of expected lifetime utility to current shocks. This allows us to aggregate these generations into a single group of old agents. Furthermore, comparing equations (20) and (24), one can see that both young and old agents are exposed to the same reinvestment risk and the same effects of future output on lifetime utility. Hence the only difference in risk exposure across the generations arises from the second term in equations (20) and (24), capturing different exposures of period t wealth to contemporaneous realizations of h_t and d_t.

From this argument it also follows that, in order to share the exposure to h_t and d_t equally among all agents, it is sufficient to have a tax-transfer system that equalizes the sensitivity of current wealth, and hence current consumption, to h_t and d_t across agents.

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3 Social Security

3.1 First Best Intergenerational Consumption Allocation

Suppose that a social planner designs a social security system behind a Rawlsian veil of ignorance. The purpose of the social security system is to allocate net output optimally between the different cohorts alive in any given period. We assume that the planner places equal weight on the welfare of all agents. The first-order conditions of the planner's problem imply that the expected consumption path of any individual should decline at rate β, conditional on survival of the agent. Given that only a fraction χ of any cohort survives into the next period, this implies the following optimal consumption allocations for the different cohorts:

$$C_t^t = (1 - \beta\chi)(h_t + d_t) \tag{25}$$

$$C_t^r = (1 - \beta\chi)(\beta\chi)^{t-r}(h_t + d_t), \text{ for } r < t, \tag{26}$$

where C_t^r denotes aggregate consumption in period t of the cohort born in period r. Aggregating over all the old cohorts, the solution to the social planner's problem allocates a fraction $(1-\beta\chi)$ of net output to the consumption of the young generation and the remaining fraction $\beta\chi$ to the consumption of old generations.

Comparing this to the consumption allocations in the decentralized equilibrium, (11) and (12), which divide human capital income in this way but allocate all physical capital income to the consumption of the old, we see that the optimal policy in this economy requires a net consumption transfer of $(1 - \beta\chi)d_t$ from the old to the young. This can be accomplished by a wealth transfer of d_t from the old to the young each period, but such a transfer is contrary to what we usually think of as a social security system. This analysis also highlights the fact that human capital risk is shared optimally in the laissez faire equilibrium. The old have an optimal indirect exposure to human capital risk through the price of physical capital.

3.2 Social Security with Full Risksharing

In order to distinguish between redistribution of the average physical capital dividend and reallocation of physical capital risk, we now write the physical capital dividend as

$$d_t = \mu_{dt} + \varepsilon_t, \tag{27}$$

where $\mu_{dt} = E_{t-1}(d_t)$ and ε_t is the pure risk component of the dividend on physical capital. We assume that ε_t is independent and identically distributed over time.

Suppose that, instead of implementing the first best consumption allocation, the social planner only partially reallocates the mean consumption level between generations but still achieves the optimal allocation of consumption risk:

$$C_t^t = (1 - \beta\chi)(h_t + \varepsilon_t) + (1 - \theta)\mu_{dt} \tag{28}$$

$$C_t^{r<t} = \beta\chi(h_t + \varepsilon_t) + \theta\mu_{dt}, \tag{29}$$

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where $\theta > \beta\chi$, with $\theta = \beta\chi$ corresponding to the first best consumption allocation. This setup allows for deterministic transfers from old to young ($\theta < 1$) or from young to old ($\theta > 1$).

In the absence of any restrictions on the dividend processes and the social security transfers, the previous expressions can imply negative values for the consumption of the old or the young. In order to avoid such a situation, we assume that the human dividend process has a lower bound $\underline{h} > \mu_{dt} > 0$, and we restrict the range of the deterministic part of the transfer to $\beta\chi < \theta < \beta\chi + (1 - \beta\chi)\underline{h}/\mu_{dt}$.

The knife-edge case of $\theta = 1$ corresponds to "pure risksharing". In the pure risksharing equilibrium, the desired consumption allocations are given by:

$$C_t^t = (1 - \beta\chi)(h_t + \varepsilon_t) \tag{30}$$

$$C_t^{r<t} = \beta\chi(h_t + \varepsilon_t) + \mu_{dt}. \tag{31}$$

We now consider how the social planner can redistribute income between generations to achieve full risksharing. We assume that the social planner can levy a payroll tax on the young generation and use it to make social security payments to the old. We require aggregate payroll tax revenue T_t to equal the aggregate social security payouts to all old cohorts S_t, so that the system is balanced each period. We allow payroll taxes and social security payouts to be negative, corresponding to a tax on physical capital income and subsidy on labor income.

In the presence of social security, the expected lifetime wealth of agents includes expected future social security transfers. The anticipated transfer stream accruing to the cohort that is young in period t is given by:

$$\{-T_t, (1 - \beta\chi)S_{t+1}, (1 - \beta\chi)\beta\chi S_{t+2}, (1 - \beta\chi)(\beta\chi)^2 S_{t+3}, ...\}.$$

The value of this expected income stream in period t may be written as:

$$-T_t + (1 - \beta\chi)z_t,$$

where z_t is the value of the following stream of payments:

$$\{S_{t+1}, \beta\chi S_{t+2}, (\beta\chi)^2 S_{t+3}, ...\}.$$

Similarly, the value of present and future social security transfers to those who are old in period t may be written as:

$$S_t + \beta\chi z_t.$$

z_t can be interpreted as the present value of future social security payouts accruing to all generations currently alive.

The correct measure of wealth in the presence of social security incorporates anticipated future payouts:

$$W_t^t = [h_t - T_t + (1 - \beta\chi)z_t] \tag{32}$$

$$W_t^{r<t} = [d_t + p_t + S_t + \beta\chi z_t]. \tag{33}$$

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With log utility, individual optimization implies that consumption is a constant fraction of wealth:

$$C_t^t = (1 - \beta\chi)[h_t - T_t + (1 - \beta\chi)z_t] \tag{34}$$

$$C_t^{r<t} = (1 - \beta\chi)[d_t + p_t + S_t + \beta\chi z_t]. \tag{35}$$

A useful expression for the optimal transfer policy is:

$$T_t = S_t = \mu_{dt}\frac{\theta - \beta\chi}{1 - \beta\chi} + \beta\chi h_t - (1 - \beta\chi)(d_t + p_t). \tag{36}$$

It is easy to verify that this policy implements the social planner's consumption allocations in equation (28).

The interpretation of (36) is subtle. Two points are particularly important to keep in mind. First, the price of physical capital on the right hand side of this expression is endogenous, so (36) is not a closed-form solution for the optimal transfer. In particular, (36) does not show that the optimal transfer decreases with the physical capital dividend. While this is normally the case, for certain extreme parameter values it is possible that a high dividend justifies a high payroll tax to reduce the demand for physical capital and drive down its price.

Second, the current-period transfer given in (36) is not the total change in wealth for either the young or the old generation, because these generations also anticipate receiving future transfers with current market value z_t. Full risksharing requires that the total change in the wealth of the young generation caused by the social security system is increasing in the physical capital dividend, even though we cannot unambiguously sign the response of the current-period transfer to that dividend. The Appendix (Campbell and Nosbusch 2007) gives further details on these points.

3.3   Social Security and the Stochastic Discount Factor

In the presence of a social security system, the SDF is given by:

$$M_{t+1}^s = \frac{\beta\chi C_t}{C_{t+1}^{r<t+1}} = \frac{\beta\chi(h_t + d_t)}{(1 - \beta\chi)[d_{t+1} + p_{t+1} + S_{t+1} + \beta\chi z_{t+1}]} \tag{37}$$

When social security achieves full risksharing, as in the previous section, the expression for the SDF simplifies to:

$$M_{t+1}^s = \frac{\beta\chi(h_t + d_t)}{\beta\chi(h_{t+1} + \varepsilon_{t+1}) + \theta\mu_{dt+1}} = \frac{h_t + d_t}{h_{t+1} + \frac{\theta\mu_{dt+1}}{\beta\chi} + \varepsilon_{t+1}} \tag{38}$$

In the presence of social security, aggregate consumption in period t is given by:

$$\begin{aligned} C_t &= C_t^t + C_t^{r<t} \\ &= (1 - \beta\chi)[h_t - T_t + (1 - \beta\chi)z_t] + (1 - \beta\chi)[d_t + p_t + S_t + \beta\chi z_t] \\ &= (1 - \beta\chi)[h_t + d_t + p_t + z_t]. \end{aligned} \tag{39}$$

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Aggregate consumption needs to equal net output in equilibrium: $C_t = Y_t = h_t + d_t$. This equilibrium condition pins down the sum of the aggregate ex dividend value of physical capital and expected future social security payouts accruing to those currently alive:

$$p_t + z_t = \frac{\beta \chi (h_t + d_t)}{1 - \beta \chi}. \tag{40}$$

Thus a higher value for future social security payouts implies a lower physical capital price, and vice versa.

The stochastic discount factor for general social security derived in equation (37) can be used to price physical capital and the social security payout stream $\{S_{t+1}, \beta \chi S_{t+2}, (\beta \chi)^2 S_{t+3}, ...\}$:

$$p_t = \mathrm{E}_t[M_{t+1}^s (d_{t+1} + p_{t+1})] \tag{41}$$

$$z_t = \mathrm{E}_t[M_{t+1}^s (S_{t+1} + \beta \chi z_{t+1})] \tag{42}$$

It is easy to verify that these equations are consistent with the equilibrium condition (40).

3.4 Social Security and the Riskless Interest Rate

The simple expressions for the SDF under laissez faire and full risksharing, presented in equations (14) and (38), allow us to assess the effect of social security on the riskless rate of interest. In what follows, ρ_{hd} denotes the correlation between the human and physical capital dividends. The variances of the innovations to the human and physical dividend processes are denoted by σ_h^2 and σ_d^2.

Proposition 1 *In the case of deterministic dividends to human capital, a pure risksharing social security policy unambiguously increases the riskless rate of interest. For $\theta = 1$ and $\sigma_h^2 = 0$, $R_{f,t+1}^s > R_{f,t+1}^{lf}$.*

In the case of stochastic dividends to human capital, a pure risksharing social security policy increases the riskless rate of interest if and only if $\sigma_d / \sigma_h > -\rho_{hd}(2\beta\chi)/(1 + \beta\chi)$. A sufficient condition for this to hold is that the dividends to human and physical capital are positively correlated. For $\theta = 1$ and $\sigma_h^2 > 0$, $R_{f,t+1}^s > R_{f,t+1}^{lf}$ if $\rho_{hd} \geq 0$.

Proof. See Appendix (Campbell and Nosbusch 2007).

The intuition for this effect is that pure risksharing social security is a form of insurance whereby all future generations effectively hedge some of the rate of return risk on the savings of those cohorts that are currently alive. As a result, those currently alive have a reduced need for precautionary savings. Given that the riskless asset is in zero net supply in this economy, the equilibrium riskless rate needs to rise in order to clear the bond market.

To understand the necessary and sufficient condition for this result to hold, consider what happens if it fails. If the two dividends are negatively correlated ($\rho_{hd} < 0$) and if the human capital dividend is sufficiently riskier than the physical capital dividend ($\sigma_h > \sigma_d$), the old bear too little consumption risk under laissez faire. In this case physical capital is a valuable hedge against human capital risk and the first best policy *increases* the consumption risk of the old by giving future young generations the benefit of this hedge. But this perverse case

is unlikely to be empirically relevant, since estimates of the correlation between human and physical capital risk tend to be positive (Benzoni, Collin-Dufresne, and Goldstein 2005) and physical capital dividends appear to be riskier than human capital dividends.

Proposition 2 *The effect of a purely deterministic transfer stream from the young to the old is to increase the riskless rate of interest. The opposite is true of a deterministic transfer stream from the old to the young.*

Proof. See Appendix.

Under a purely deterministic transfer τ, the consumption of the old and the young are given by $C_t^t = (1 - \beta\chi)h_t - \tau$ and $C_t^{r<t} = \beta\chi h_t + d_t + \tau$, where $\tau > 0$ implies a net transfer from young to old and $\tau < 0$ a net transfer from old to young. The allocation of risk is identical to the decentralized equilibrium. In the absence of restrictions on the processes for dividends and social security transfers, these expressions can imply negative values for the consumption of the old or the young, a problem already pointed out in Section 3.3. In order to rule out the possibility of negative consumption in some states of the world for either age group, we assume that the human capital dividend has a strictly positive lower bound $\underline{h} > 0$, and that the social security transfer τ is in the range $-\beta\chi\underline{h} < \tau < (1 - \beta\chi)\underline{h}$. These two assumptions, together with the assumption that the physical capital dividend d_t is positive, ensure that both the old and the young have strictly positive consumption in all states of the world.

A deterministic transfer scheme from the young to the old in all future periods reduces the need to save for those who are currently alive. In order for the bond market to clear, the equilibrium riskless interest rate needs to rise. The opposite holds for a deterministic transfer from the old to the young.

3.5   Social Security and Physical Capital

The SDF derived in equation (38) can be used to solve explicitly for the price of physical capital in the equilibrium with social security. Details are given in the Appendix. In the case of i.i.d. dividends ($\mu_{dt} = \mu_d$ for all t), the price of physical capital in the presence of social security is equal to a constant multiple of the price under laissez faire:

$$p_t^s = F p_t^{lf}, \tag{43}$$

where an explicit expression for the constant F is given in the Appendix. Depending on the form of the social security system, this factor F can be smaller or larger than one. The next two propositions give more precise conditions.

Proposition 3 *In the case of deterministic dividends to human capital and i.i.d. dividends to physical capital, a pure risksharing social security policy ($\theta = 1$) unambiguously leads to a fall in the price of physical capital (F<1).*

In the case of i.i.d. dividends to human capital and i.i.d. dividends to physical capital, a pure risksharing social security policy leads to a fall in the price of physical capital provided that $Cov_t \left[d_{t+1}, \frac{1}{h_{t+1}+d_{t+1}+\frac{1-\beta\chi}{\beta\chi}\mu_d} \right] < 0.$

Proof. See Appendix.

As noted in the previous section, a pure risksharing social security system reduces the need for precautionary savings by those currently alive. The only savings vehicle available in this economy is the risky physical capital asset. Since physical capital is in fixed supply, this reduction in demand results in a lower equilibrium price.

The covariance condition for Proposition 2a holds unambiguously when human capital dividends are deterministic. It also holds for the particular stochastic processes we consider for human capital dividends in our calibration exercise.

Proposition 4 *The effect of a deterministic transfer stream from the young to the old is to decrease the price of physical capital. The opposite is true of a deterministic transfer stream from the old to the young.*

Proof. See Appendix.

As in the previous section, these effects may be interpreted as the result of changes in savings needs of those currently alive.

Equation (40) provides an alternative way of stating the intuition for these effects. The sum $(p_t + z_t)$ takes the same value irrespective of the particular form of the social security system. A pure risksharing policy means that social security is valuable, implying a positive value for z_t and therefore a lower physical capital price p_t compared to laissez faire. The same is true of a social security system that consists of deterministic transfers from the young to the old.

Given the price of physical capital with social security and i.i.d. dividends, we can solve for the implied return on physical capital:

$$1 + R_{t+1}^s = \frac{h_{t+1} + \frac{1+F\beta\chi-\beta\chi}{F}\frac{d_{t+1}}{\beta\chi}}{h_t + d_t}. \tag{44}$$

Proposition 5 *When dividends are i.i.d., $E_t(1 + R_{t+1}^s) > E_t(1 + R_{t+1}^{lf})$ iff $F < 1$ and $Var_t(1 + R_{t+1}^s) > Var_t(1 + R_{t+1}^{lf})$ iff $F < 1$.*

Proof. See Appendix.

If the price of physical capital falls, its average return increases but its volatility also increases because the volatile current dividend has a larger proportional impact on the return. This result implies that a pure risksharing social security system leads to an increase in the expected rate of return on physical capital as well as an increase in its return volatility compared to the laissez faire equilibrium. A deterministic transfer from the young to the old has the same qualitative effects, while a deterministic transfer from the old to the young reduces expected returns and return volatilities on the risky asset.

Finally, we can combine our solutions for the riskless interest rate and the return on physical capital to calculate the risk premium on physical capital. Social security has two offsetting effects on the risk premium. First, a pure risksharing social security system improves the allocation of risk in the economy, resulting in a higher overall riskbearing capacity. A way to see this is to look at the portfolios of investors in the two economies. In

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the presence of social security, investors effectively hold an implicit second asset (their claim to future social security benefits) in their portfolio. By design, this second asset hedges the returns on the original asset (physical capital) held in the portfolio. This makes investors less averse to the risk on the physical capital asset, thereby reducing the risk premium they demand in equilibrium. There is however a second offsetting effect. Proposition 5 shows that a pure risksharing social security system increases the return volatility of the risky asset, which, by itself, would tend to increase the risk premium.

In our numerical analysis we find that, for all empirically plausible parameter values, the first effect dominates and a pure risksharing social security system reduces the risk premium on physical capital.[5]

4   Calibrating the Model

We interpret one period to last for twenty years. We set the survival probability χ equal to 2/3. This implies an expected economic lifetime of sixty years. The idea is that economic life starts at around age twenty, there is an initial period of twenty years where agents earn labor income, followed on average by two twenty year periods of financing consumption through savings. The discount factor β is set to 0.96 on an annual basis.

For the purpose of the simulations, we assume i.i.d. lognormal processes. The mean of the human capital dividend μ_h is normalized to one. We set the mean of the physical capital dividend μ_d equal to 1/2 in order to match the relative magnitudes of capital and labor shares in national income for the United States. The standard deviation of the human capital dividend σ_h is set to 0.2 and we assume a correlation of 0.5 between human and physical capital dividends at the 20 year horizon[6]. We report results for several values of the standard deviation of the physical capital dividend; σ_d ranges from 0 to 0.5. These parameter values imply that the endogenous standard deviation of the return on physical capital is in a range between 0 and 35 per cent on an annual basis. In our figures, all variables are plotted against the standard deviation of returns on physical capital.

Figure 1 plots the riskless interest rate in the laissez faire and pure risksharing equilibria. It confirms the result in Proposition 1 that a shift to a pure risksharing social security system increases the riskless interest rate. The corresponding change in the risk premium on physical capital is plotted in Figure 2. For our range of parameter values, the risk premium always falls as a result of pure risksharing. The effect of increased riskbearing capacity thus dominates the effect of the increased return volatility. Indeed, Figure 3 shows

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[5] We can find counterexamples but they are somewhat artificial. In particular, if the physical capital dividend is lognormally distributed with low mean and high volatility (e.g. $\mu_d = 0.1$, $\sigma_d = 1.5$), it is possible for the risk premium to increase. This counterexample works with a deterministic human capital dividend ($\mu_h = 1$, $\sigma_h = 0$). However, the implied return volatilities on the risky asset are implausibly high, on the order of 80% on an annual basis.

[6] In order to rule out the possibility of negative values of consumption for the young under pure risksharing, we assume that h_t follows the following process: $h_t = \mu_d + z$, where z is distributed lognormally with mean ($\mu_h - \mu_d$) and standard deviation σ_h. This specification implies a lower bound on the human capital dividend of $\underline{h} = \mu_d$.

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that the increase in the volatility of returns on physical capital is relatively modest. We should also note that the risk premia predicted by our model are generally much lower than those observed empirically. This is a manifestation of the familiar equity premium puzzle. It arises here in part because of our assumption of log utility and the low associated coefficient of relative risk aversion. Figure 4 illustrates the fact that the price of physical capital is lower under pure risksharing, as shown in Proposition 3.

5   Conclusion

In this paper we have studied the effects of government intergenerational transfers on asset prices. Real-world social security systems can be interpreted as combinations of deterministic transfers from young to old, and contingent transfers that enable young and old to share their income risks. We have shown that both elements of social security systems have similar effects on asset prices. They reduce life-cycle and precautionary motives to save, and thus increase the riskless interest rate and lower the price of physical capital. The lower price for physical capital increases the expected return on capital, but also increases the volatility of that return because volatile dividends have a larger proportional impact. The effect on the risk premium is theoretically ambiguous; on the one hand the riskbearing capacity of the economy increases when risks are better shared across generations, but on the other hand the return risk of physical capital is greater. In realistic examples the former effect dominates, and social security reduces the risk premium for physical capital.

We have derived these results using a stylized model in which physical capital cannot be accumulated. A natural extension of our approach would allow capital accumulation. Social security would then have smaller effects on asset prices, but would lower the capital stock in long-run equilibrium[7]. If risky and riskless capital could be separately accumulated, our results suggest that social security systems would have a milder negative effect on the accumulation of risky capital because the increased riskbearing capacity of the economy partially offsets the effect of reduced saving.

All these results apply in reverse if we consider recent proposals to reduce intergenerational transfers and encourage private retirement saving. These proposals have the potential to increase overall capital accumulation and drive down interest rates, but if they reduce intergenerational risksharing the increased saving may be disproportionately directed towards safe assets in which case the equity premium may increase.

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[7]Krueger and Kuebler (2006) show that if this crowding-out effect of physical capital is sufficiently strong, a social security system that reduces the consumption risk of the old may not be Pareto improving. The focus of their paper is on the welfare properties of risksharing social security systems, while we concentrate instead on their effects on asset prices in equilibrium. In the same context, there is an earlier literature that argues that it is subtle to determine when welfare effects in OLG models are due to market incompleteness. Baxter (1989), generalizing results in Marshall, Sonstelie and Gilles (1987), shows that while the introduction of money in an OLG economy can be welfare improving, money does not complete markets.

References

Abel, Andrew, 2003, "The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security," *Econometrica*, 71, 551–578.

Athanasoulis, Stefano G., 2006, "Asset Prices and Consumption in a Model of Perpetual Youth", *Journal of Business* 79, 2835–2868.

Ball, Laurence and N. Gregory Mankiw, 2001, "Intergenerational Risk Sharing in the Spirit of Arrow, Debreu and Rawls, with Applications to Social Security Design," NBER Working Paper No. 8270.

Baxter, Marianne, 1989, "Money and Market Incompleteness in Overlapping Generations Models", *Journal of Monetary Economics* 24, 69-91.

Benzoni, Luca, Pierre Collin-Dufresne, and Robert S. Goldstein, 2005, "Portfolio Choice over the Life Cycle in the Presence of 'Trickle-Down' Labor Income", NBER Working Paper No. 11247.

Blanchard, Olivier J., 1985, "Debt, Deficits, and Finite Horizons", *Journal of Political Economy* 93, 223–247.

Bohn, Henning, 2002, "Retirement Savings in an Aging Society: A Case for Innovative Government Debt Management", in Alan Auerbach and Heinz Herrman eds., *Ageing, Financial Markets and Monetary Policy*, Springer, Berlin.

Bohn, Henning, 2003, "Intergenerational Risksharing and Fiscal Policy", unpublished paper, University of California Santa Barbara.

Campbell, John Y., Joao Cocco, Francisco Gomes, and Pascal Maenhout, 2001, "Investing Retirement Wealth: A Life-Cycle Model", in John Y. Campbell and Martin Feldstein eds., *Risk Aspects of Investment-Based Social Security Reform*, University of Chicago Press, Chicago, IL.

Campbell, John Y., and Yves Nosbusch, 2007, "Intergenerational Risksharing and Equilibrium Asset Prices: Appendix", available online at

http://post.economics.harvard.edu/faculty/campbell/campbell.html.

Campbell, John Y. and Luis M. Viceira, 2002, *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*, Oxford University Press, New York, NY.

Constantinides, George M., John B. Donaldson, and Rajnish Mehra, 2002, "Junior Can't Borrow: A New Perspective on the Equity Premium Puzzle", *Quarterly Journal of Economics* 117, 269–296.

Constantinides, George M., John B. Donaldson, and Rajnish Mehra, 2005, "Junior Must Pay: Pricing the Implicit Put in Privatizing Social Security", *Annals of Finance* 1, 1–34.

Diamond, Peter A., 1965, "National Debt in a Neoclassical Growth Model", *American Economic Review* 55, 1126–1150.

Farmer, Roger E. A., 2002, "Fiscal Policy, Equity Premia and Heterogeneous Agents", unpublished paper, UCLA.

Gertler, Mark, 1999, "Government Debt and Social Security in a Life-Cycle Economy", *Carnegie-Rochester Conference Series on Public Policy* 50, 61–110.

Krueger, Dirk and Felix Kubler, 2006, "Pareto Improving Social Security Reform when Financial Markets are Incomplete!?", *American Economic Review* 96, 737-755.

Lucas, Robert E., Jr., 1978, "Asset Prices in an Exchange Economy", *Econometrica* 46, 1429–1446.

Marshall, John M., Jon Sonstelie, and Christian Gilles, 1987, "Money and Redistribution: Revisionist Notes on a Problem by Samuelson", *Journal of Monetary Economics* 20, 3-23.

McHale, John, 2001, "The Risk of Social Security Benefit-Rule Changes: Some International Evidence", in John Y. Campbell and Martin Feldstein eds., *Risk Aspects of Investment-Based Social Security Reform*, University of Chicago Press, Chicago, IL.

Mehra, Rajnish and Edward C. Prescott, 1985, "The Equity Premium: A Puzzle", *Journal of Monetary Economics* 15, 145–161.

Samuelson, Paul A., 1958, "An Exact Consumption-Loan Model of Interest With and Without the Social Contrivance of Money", *Journal of Political Economy* 66, 467–482.

Shiller, Robert J., 1999, "Social Security and Institutions for Intergenerational, Intragenerational, and International Risk Sharing", *Carnegie-Rochester Conference Series on Public Policy* 50, 165–204.

Viceira, Luis M., 2001, "Optimal Portfolio Choice for Long-Horizon Investors with Non-tradable Labor Income", *Journal of Finance* 56, 433–470.
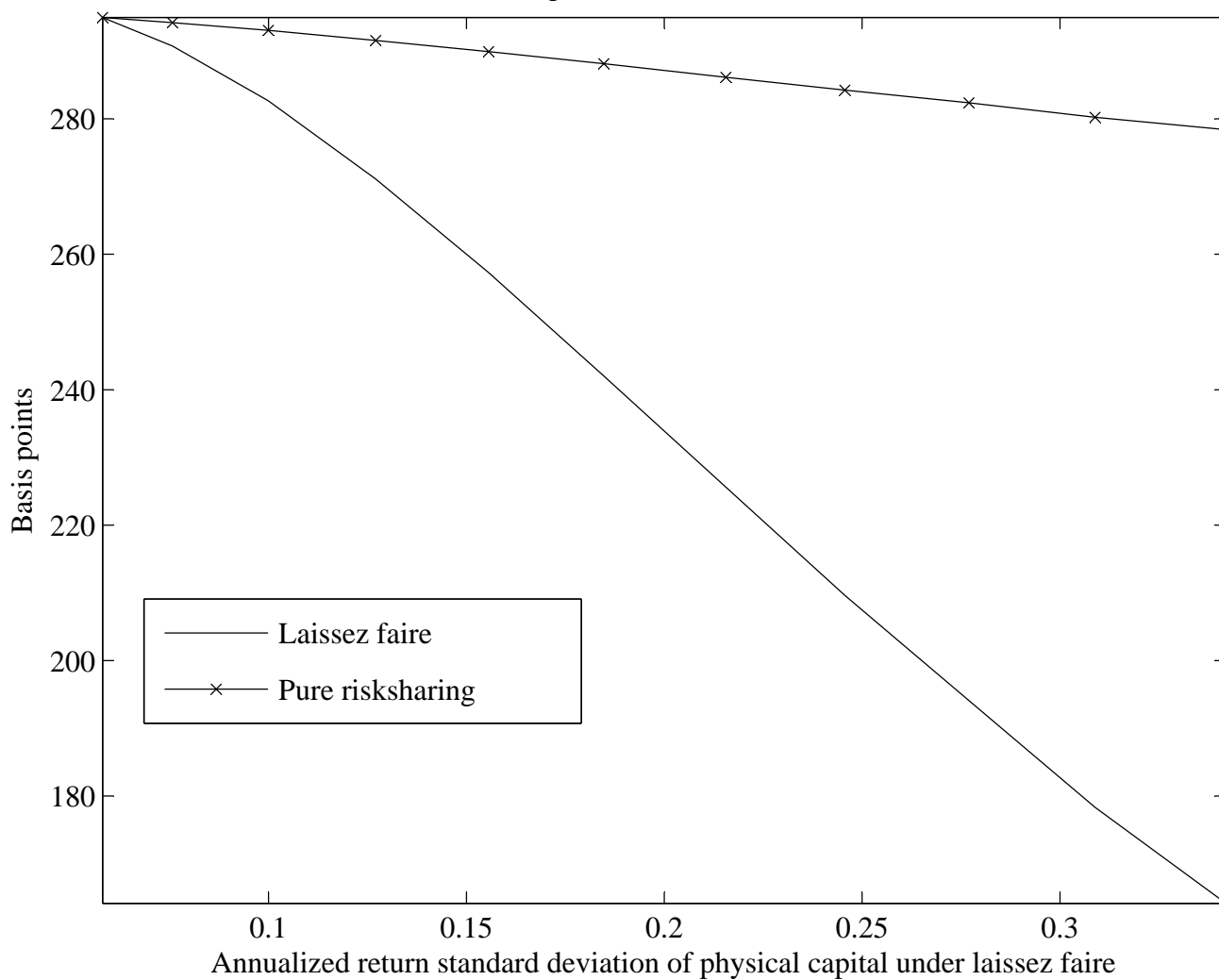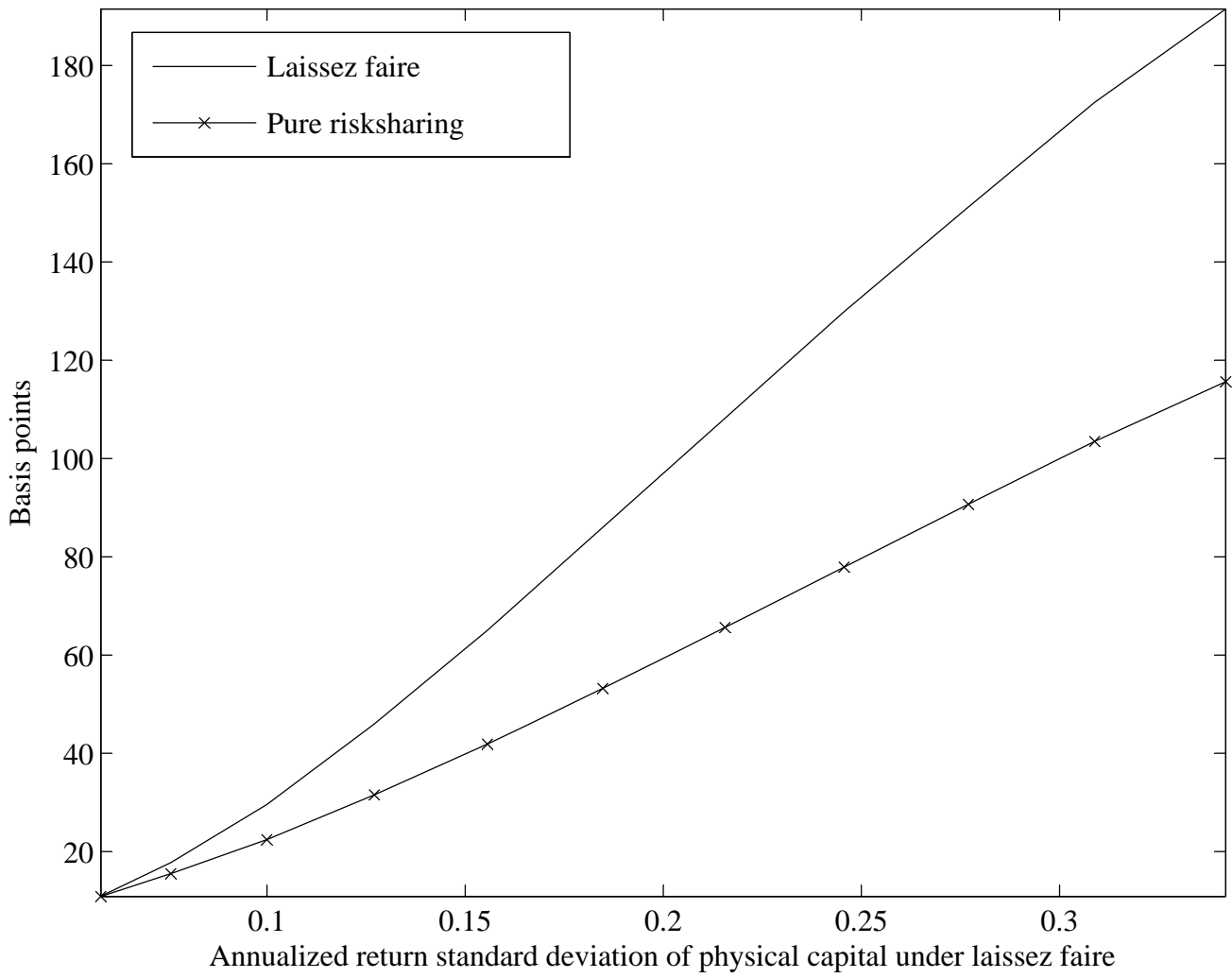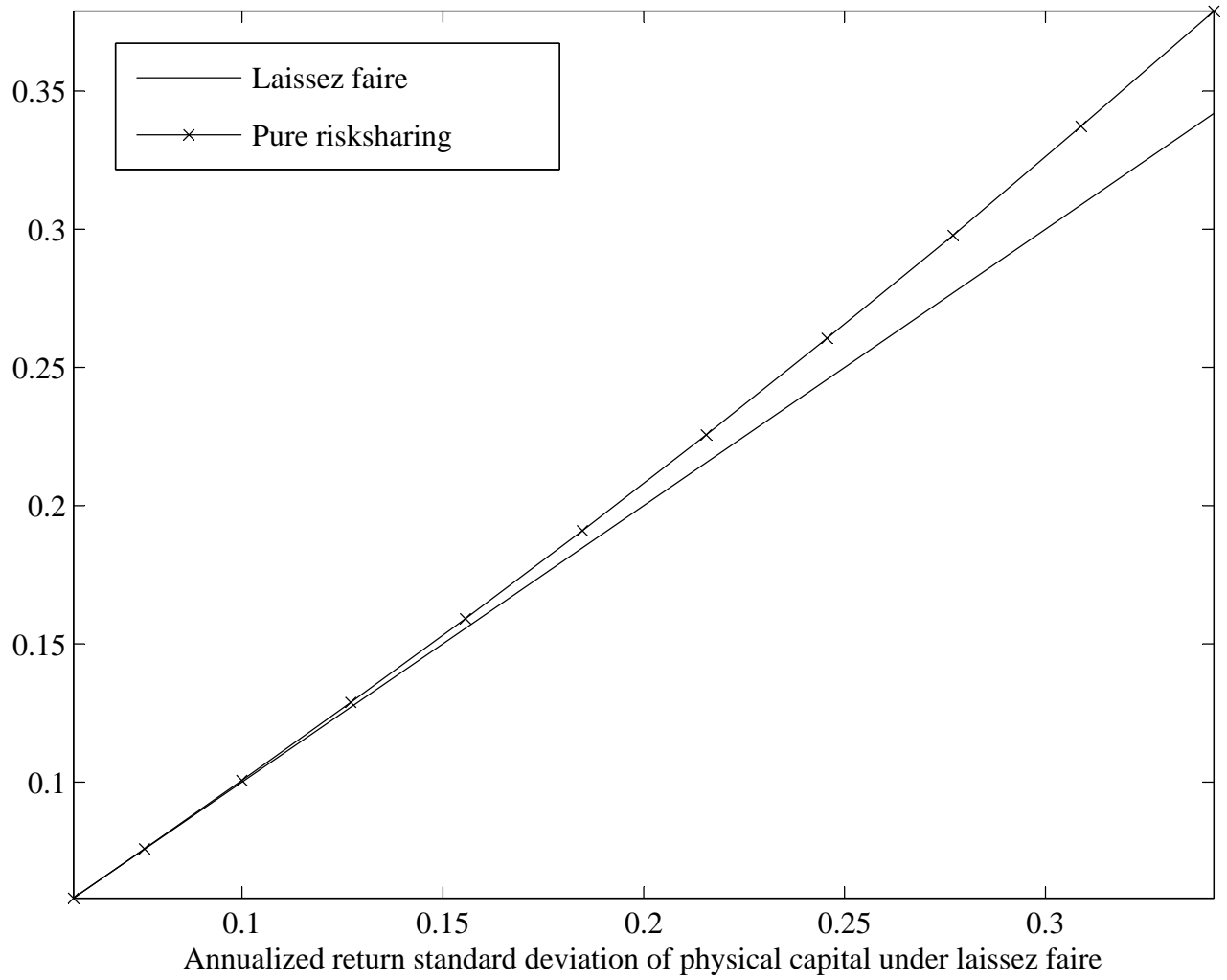
Figure 1: Riskless Rate

Figure 2: Risk Premium

Figure 3: Return Standard Deviation

Figure 4: Physical Capital Price Ratio