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Inflation Premium and Oil Price Volatility

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Abstract
This paper provides a fully micro-founded New Keynesian framework to study the interaction between oil price volatility, pricing behavior of firms and monetary policy. We show that when oil has low substitutability, firms find it optimal to charge higher relative prices as a premium in compensation for the risk that oil price volatility generates on their marginal costs. Overall, in general equilibrium, the interaction of the aforementioned mechanisms produces a positive relationship between oil price volatility and average inflation, which we denominate inflation premium. We characterize analytically this relationship by using the perturbation method to solve the rational expectations equilibrium of the model up to second order of accuracy. The solution implies that the inflation premium is higher when: a) oil has low substitutability, b) the Phillips Curve is convex, and c) the central bank puts higher weight on output fluctuations. We also provide some quantitative evidence showing that a calibrated model for the US with an estimated active Taylor rule produces a sizable inflation premium, similar to the levels observed in the US during the 70s.

JEL Classification: E52, E42, E12, C63
Keywords: Second Order Solution, Oil Price Shocks, Endogenous Trade-off

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1 Introduction

In an influential paper, Clarida, Gali and Gertler (2000, from now on CGG), advanced the idea that the high average levels of inflation observed in the USA during the 1970s could be explained mainly by the failure of monetary policy to properly react to higher expected inflation. In addition, they pointed out that oil price shocks played a minor role in generating those levels of inflation. CGG based their conclusions on the estimation of monetary policy reaction functions for two periods: pre- and post-Volcker[1]. Their estimates show that during the 1970s, as expected inflation rose the FED allowed the real short term interest rate to decline. In contrast, during the post-Volcker period the FED became more active, by raising the real interest rate in response to higher inflation expectations. Cogley and Sargent (2002) and Lubik and Schorfheide (2004) find similar evidence.

This evidence, however, is not conclusive. In a series of papers, Sims and Zha (2005), Canova, Gambetti and Pappa (2005), Primiceri (2004), Gordon (2005) and Leeper and Zha (2003) find weak evidence of a substantial change in the reaction function of monetary policy after Volcker [2]. In particular, they find evidence that the fall in both aggregate volatility and average inflation is related to a sizeable reduction of the volatility of the main business cycle driving forces[3]. Moreover, they highlight that in order to estimate the reaction function of the central bank, it is necessary to consider changes in the variance of structural shocks. Otherwise, these estimations may be biased towards finding significant shifts in coefficients in the monetary policy rule.

Motivated by this recent evidence, in this paper we provide an analytical and tractable framework that can be used to study the relationship between structural shocks volatility, in particular oil price shocks, and the average level of inflation. In doing so, we use a standard microfounded New Keynesian model with staggered Calvo pricing where the central bank implements its policy following a Taylor rule. We modify this simple framework considering oil as a production input for intermediate goods. A key assumption in our setup to generate this relationship is that oil is difficult to substitute in production, thus we use a constant elasticity of substitution (CES) production function with an elasticity lower than one as a prime of our model. Then, we solve the rational expectations equilibrium of this model up to second order

[1] It refers to the appointment of Paul Volcker as Chairman of the Federal Reserve System of the USA.

[2] Orphanides (2001) shows that when real time data are used to estimate policy reaction functions, the evidence of a change in policy after 1980 is weak.

[3] The literature has also associated oil prices to periods of recession. Bernanke, Gertler and Watson (1997) argue that monetary policy played a larger role during the 1970s in explaining the negative output dynamics. On the other hand, Hamilton (2001) and Hamilton and Herrada (2004) find out that the results of previous authors rely on a particular identification scheme and, on the contrary, they find that a contractionary monetary policy played only a minor role on the contractions in real output, oil prices being the main source of shock.

of accuracy using the perturbation method developed by Schmitt-Grohé and Uribe (2004). The second-order solution has the advantage of incorporating the effects of shocks volatility on the equilibrium, which are absent in the linear solution. We implement this method both analytically and numerically[4]. The former allows us to disentangle the key determinants of the relationship between volatility of oil price shocks and the average level of inflation, and the latter allows us to quantify the importance of each mechanism.

Using a similar model, CGG concluded that oil prices are not able to generate high average levels of inflation, unless monetary policy is passive. Instead our results give an important role to oil price volatility even with an active monetary policy. In our set up, oil prices play a central role on inflation determination and on the trade-off faced by the central bank. The key difference between CGG and our set up is that we use a second-order solution for the rational expectations equilibrium, instead of a log-linear one.

Thus, the second-order solution, by relaxing certainty equivalence, allows us to establish a link between the volatility of oil price shocks and the average level of inflation, absent in a log-linear model. We define this extra level of average inflation as the time varying *level of inflation premium*[5]. Moreover, the analytical solution allowed us to identify and to disentangle the sources of *inflation premium* in general equilibrium[6].

There are many novel results to highlight. First, the solution up to second order shows that oil price volatility produces an extra level of inflation by altering the way in which forward-looking firms set their prices. In particular, when oil has low substitutability, marginal costs are convex in oil prices, hence its price volatility increases the expected value of marginal costs.

Second, oil price volatility, by generating inflation volatility, induces price-setters to be more cautious to future expected marginal costs. In particular, their relative price becomes more sensitive to marginal costs, amplifying the previously mentioned channel.

Third, relative price dispersion, by increasing the amount of labour required to produce a given level of output, increases average wages. So, relative price dispersion amplifies the effect of expected marginal costs over average inflation.

Fourth, we find that, in general equilibrium, the weight that the central bank puts on output fluctuations is a key determinant for positive level of inflation premium. As a result, we show that the larger the endogenous responses of a central bank to output fluctuations,

[4] As part of our contribution, we use a novel strategy for the analytical solution. In contrast with other papers in which the perturbation method is applied directly to the non-linear system of equations, instead we first approximate the model up to second order and then we apply the perturbation method to this approximated model.

[5] The extra level of inflation generated by volatility is similar to the effect of consumption volatility on the level of average savings as in the literature of precautionary savings.

[6] We are not aware of any other paper in the literature that has obtained and developed the concept of inflation premium in general equilibrium.

the greater the level of inflation premium. This finding is consistent with the fact that, in the model, oil price shocks generate an endogenous trade-off between stabilizing inflation and output gap. Hence a benevolent central bank would choose to put a positive weight on output gap stabilization and would generate inflation premium.[7]

Finally, we also evaluate the implications of the model with numerical exercises calibrated for the US economy. For the calibration, we consider that oil price shocks have exhibited a change in their volatility across the pre- and post-Volcker periods. Our results are broadly consistent with predictions of the analytical solution. Remarkably, we are able to generate a level of inflation premium similar to the one observed during the 1970s in the USA even when an active monetary policy, as in CGG, is in place. Also, we show that the convexity of the Phillips curve accounts for 59 percent of the inflation premium in the pre-Volcker period, whereas the effects of oil price volatility on marginal costs accounts for another 45 percent. Overall, we find that the model can track quantitatively the average values of inflation fairly well. We check the robustness of our results with alternative estimated Taylor rules, yet the qualitative results do not change. Hence, our results provide support to the empirical findings of Sims and Zha (2005) that second moments of shocks might be important to understand the change in macroeconomic behavior observed in the US economy without relying on an accommodative monetary policy.

Closer to our work is the recent paper by Evans and Hnatkovska (2005), who evaluate the role of uncertainty in explaining differences in asset holdings in a two-country model. Also, Obstfeld and Rogoff (1998) develop an explicit stochastic New Open Economy model relaxing the assumption of certainty equivalence. Based on simplified assumptions, they obtain analytical solutions for the level exchange rate premium. Differently from Obstfeld and Rogoff (1998) and the aforementioned authors, in this paper we perform both a quantitative and analytical evaluation of the second-order approximation of the New Keynesian benchmark economy in order to account for the level of inflation premium generated by oil shocks volatility.

The plan of this paper is as follows. Section 2 presents some stylized facts for the US economy on the relationship between oil price volatility and the level of inflation. Also, this section presents an informal explanation of the link between oil price volatility and the inflation mean. In section 3 we outline a benchmark New Keynesian model augmented with oil as a non-produced input and we discuss its implications for monetary policy. Section 4 explains the mechanism at work in generating the level of inflation premium and we also find the analytical

[7]This trade-off emerges when we allow for a distorted steady-state along the CES production function. In constrast, Blanchard and Galí (2006) find that with a Cobb-Douglas production function, oil price shocks do not generate a trade-off between the stabilization of inflation and output gap. In order to generate the trade-off, they rely on a reduced form of real rigidities in the labor market. Montoro (2006) characterizes this trade-off from the quadratic approximation of the welfare of the representative agent

solution of inflation premium. In section 5 we report the numerical results. In the last section we draw conclusions.

2 Motivation

2.1 Average Inflation and Oil Price Volatility

Inspection of US inflation data seems to suggest that average inflation rate and the volatility of oil prices followed a similar pattern during the last 30 years. Thus, Figure 2.1 plots in the left hand axis, with a solid line the annual inflation rate of the US, measured by the non-farm business sector deflator (LXNFI), and in the right hand axis, with a dotted line, the real oil price in log [8]. As the figure shows, both the volatility of the real oil prices and the average quarterly annualized inflation rate has increased during the first half of the sample, 1970.1-1987.2, and has fallen in the second half, 1987.3-2005.2. In the first sub-sample, the standard deviation of real oil prices reached 0.57 and the average level of inflation 5.5 percent, whereas during the second sub-sample, the same statistics fall to 0.20 and 2.1 percent, respectively.

Interestingly, also the dynamics of inflation seems to closely mimic that of oil prices. Thus, in the first sub-sample we observe a persistent initial increase in inflation vis-a-vis the increase in oil prices following the oil price shock in 1974. Instead, from 1980 on we observe a steady decline in inflation accompanied by a persistent drop in oil prices. For the second sub-sample, we observe also a close co-movement between inflation and oil prices; from the early nineties until 1999 there is a downward trend in both oil prices and inflation, whereas from 2000 on we observe a markedly upward trend in oil and a moderate increase in inflation.

In a nutshell, the data seem to suggest that the change in oil price volatility may yield some information about the behavior of the mean of the inflation rate from the 1970s on. This evidence motivates the development of the model and the mechanism that we highlight in the coming sections in order to generate a link between average inflation and oil price volatility in a New Keynesian framework.

2.2 The Link between Average Inflation and Oil Price Volatility

Before moving to a general equilibrium analysis, in this section we provide the intuition of how the mechanism that links inflation and oil price volatility operates. For that purpose, we use a stylized two period price setting partial equilibrium model.

Suppose that some firms producing a differentiated good set prices one period in advance. They face a downward sloping demand function of the type, $Y_t(z) = \left(\frac{P_t^*(z)}{P_t}\right)^{-\varepsilon} Y$, where ε

[8] We obtain the data from the Haver USECON database (mnemonics are in parentheses).

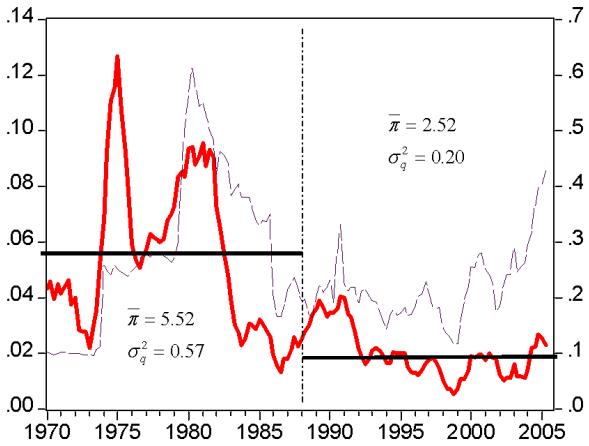

Figure 2.1: US inflation and oil prices

represents the elasticity of substitution across goods and Y aggregate output, which we assume is fixed[9]. Under these assumptions, the optimal pricing decision of a particular firm z for time t is given by mark-up over the expected next period marginal cost,

$$\frac{P_t^*(z)}{P_{t-1}} = \mu E_{t-1}\left[\Psi_t MC_t\right] \tag{2.1}$$

where μ, MC_t and $\Psi_t = \frac{\Pi_t^{\varepsilon+1}}{E_{t-1}\Pi_t^{\varepsilon}}$ denote the mark-up, firm´s marginal costs and a measure of the responsiveness of the optimal price to future marginal costs, respectively. A second order Taylor expansion of the expected responsiveness to marginal cost, Ψ_t,is given by:

$$E_{t-1}\left[\Psi_t\right] = E_{t-1}\left[\pi_t + \frac{1}{2}\left(2\varepsilon+1\right)\pi_t^2\right] \tag{2.2}$$

From the above expression it is worthnoting that $E_{t-1}\Psi_t$ is a convex function on expected inflation. This convexity implies that inflation volatility increases the weight that a firm put on expected marginal costs. In order to gain further insights, let's assume the following marginal cost function:

$$MC_t = \phi_1 q_t + \frac{\phi_2}{2} q_t^2 \tag{2.3}$$

where q_t represents the real price of oil, $\phi_1 > 0$ measures the linear effect of oil over the marginal

[9]This assumption helps to highlight the channels by which supply shocks as oil prices affect inflation. In section 3 we consider a fully general equilibrium model in which output is endogenous.

cost and ϕ_2 accounts for the impact of oil price volatility on marginal costs. When $\phi_2 > 0$, marginal costs are convex in oil prices, thus expected marginal costs become an increasing function of the volatility of oil prices.[10].

Different forms of aggregation of sticky prices in the literature show that the inflation rate is proportional to the optimal relative price of firms (equation (2.1)). Hence, when marginal costs are convex, both the optimal relative price and inflation are increasing in oil price volatility.

More importantly, other channels amplify the previous effect. To the extent that oil price volatility increases inflation volatility, price setters react by increasing the weight they put on marginal costs, Ψ_t, when setting prices. As equation (2.2) shows, up to second order, this weight depends not only on the level of expected inflation but also on its volatility. Yet, are those second order effects important? Two special features of oil prices, its high volatility and its low substitutability with other production factors, make those second order effects quantitative sizable. Hence, a linear approximation that omits the role of both oil price and inflation volatilities would not capture enough of the dynamics of inflation. In constrast, the second order solution of the rational expectations equilibrium of the model allows us to establish the link between average inflations and shocks volatility.

In the next section we formalize this link by obtaining a second order rational expectations solution of a New Keynesian general equilibrium model with oil prices. We use this model to show under which conditions the marginal cost of firms becomes a convex function of oil price shocks. Interestingly, we also show how relative price distortions and monetary policy might amplify the effect of uncertainty, inducing a meaningful level of inflation premium.

3 A New Keynesian Model with Oil Prices

The model economy corresponds to the standard New Keynesian Model in the line of CGG (2000). In order to capture oil shocks we follow Blanchard and Gali (2006) by introducing a non-produced input M, represented in this case by oil. Q denotes the real price of oil which is assumed to be exogenous.

3.1 Households

We assume the following period utility on consumption and labor

$$U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{L_t^{1+\nu}}{1+\nu}, \tag{3.1}$$

[10] In section 4 we show that when the production function is a CES with an elasticity of substitution between labor and oil lower than one, then marginal costs are convex on oil prices, that is $\phi_2 > 0$.

where σ and ν represent the coefficient of risk aversion and the inverse of the elasticity of labor supply, respectively. The optimizer consumer takes decisions subject to a standard budget constraint which is given by

$$C_t = \frac{W_t L_t}{P_t} + \frac{B_{t-1}}{P_t} - \frac{1}{R_t}\frac{B_t}{P_t} + \frac{\Gamma_t}{P_t} + \frac{T_t}{P_t} \tag{3.2}$$

where W_t is the nominal wage, P_t is the price of the consumption good, B_t is the end of period nominal bond holdings, R_t is the nominal gross interest rate , Γ_t is the share of the representative household on total nominal profits, and T_t are transfers from the government[11]. The first order conditions for the optimizing consumer´s problem are:

$$1 = \beta E_t \left[R_t \left(\frac{P_t}{P_{t+1}} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \right] \tag{3.3}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{\nu} = MRS_t \tag{3.4}$$

Equation (3.3) is the standard Euler equation that determines the optimal path of consumption. At the optimum the representative consumer is indifferent between consuming today or tomorrow, whereas equation (3.4) describes the optimal labor supply decision. MRS_t denotes the marginal rate of substitution between labor and consumption. We assume that labor markets are competitive and also that individuals work in each sector $z \in [0, 1]$. Therefore, L corresponds to the aggregate labor supply:

$$L = \int_0^1 L_t(z) dz \tag{3.5}$$

3.2 Firms

3.2.1 Final Good Producers

There is a continuum of final good producers of mass one, indexed by $f \in [0, 1]$ that operate in an environment of perfect competition. They use intermediate goods as inputs, indexed by $z \in [0, 1]$ to produce final consumption goods using the following technology:

$$Y_t^f = \left[\int_0^1 Y_t(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.6}$$

[11] In the model we assume that the government owns the oil´s endowment. Oil is produced in the economy at zero cost and sold to the firms at an exogenous price Q_t. The government transfers all the revenues generated by oil to consumers represented by T_t

where ε is the elasticity of substitution between intermediate goods. Then the demand function of each type of differentiated good is obtained by aggregating the input demand of final good producers

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} Y_t \tag{3.7}$$

where the price level is equal to the marginal cost of the final good producers and is given by:

$$P_t = \left[\int_0^1 P_t(z)^{1-\varepsilon}\, dz\right]^{\frac{1}{1-\varepsilon}} \tag{3.8}$$

and Y_t represents the aggregate level of output.

$$Y_t = \int_0^1 Y_t^f\, df \tag{3.9}$$

3.2.2 Intermediate Goods Producers

There is a continuum of intermediate good producers. All of them have the following CES production function

$$Y_t(z) = \left[(1-\alpha)\left(L_t(z)\right)^{\frac{\psi-1}{\psi}} + \alpha\left(M_t(z)\right)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}} \tag{3.10}$$

where M is oil which enters as a non-produced input, ψ represents the intratemporal elasticity of substitution between labor-input and oil and α denotes the share of oil in the production function. We use this generic production function in order to capture the fact that oil has few substitutes[12], in general we assume that.ψ is lower than one. The oil price shock, Q_t, is assumed to follow an $AR(1)$ process in logs,

$$\log Q_t = \log \overline{Q} + \rho \log Q_{t-1} + \varepsilon_t \tag{3.11}$$

[12]Since oil has few substitutes an appealing functional form to capture this feature is the CES production function. This function offers flexibility in the calibration of the degree of substitution between oil and labor. Some authors that have included oil in the analysis of RBC models and monetary policy, have omitted this feature. For example, Kim and Loungani (1992) assume for the U.S. a Cobb-Douglas production function between labor and a composite of capital and energy. Given that they calibrate their model considering that oil has a small share on output, they found that the impact of oil in the U.S. business cycle is small . Notice that when $\psi = 1$, the production function collapses to the standard Cobb-Douglas function as the one used by Blanchard and Gali (2006): $Y_t(z) = (L_t(z))^{1-\alpha} M_t^{\alpha}$.

where $\overline{Q}$ is the steady state level of oil price. From the cost minimization problem of the firm we obtain an expression for the real marginal cost given by:

$$MC_t(z) = \left[(1-\alpha)^{\psi} \left(\frac{W_t}{P_t} \right)^{1-\psi} + \alpha^{\psi} \left(Q_t \right)^{1-\psi} \right]^{\frac{1}{1-\psi}} \tag{3.12}$$

where $MC_t(z)$ represents the real marginal cost, W_t nominal wages and P_t the consumer price index. Note that since technology has constant returns to scale and factor markets are competitive, marginal costs are the same for all intermediate firms, i.e. $MC_t(z) = MC_t$. On the other hand, the individual firm´s labor demand is given by:

$$L_t^d(z) = \left(\frac{1}{1-\alpha} \frac{W_t/P_t}{MC_t} \right)^{-\psi} Y_t(z) \tag{3.13}$$

Intermediate producers set prices following a staggered pricing mechanism a la Calvo. Each firm faces an exogenous probability of changing prices given by $(1-\theta)$. The optimal price that solves the firm's problem is given by

$$\left(\frac{P_t^*(z)}{P_t} \right) = \frac{\mu E_t \left[\sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} MC_{t+k} F_{t,t+k}^{\varepsilon+1} Y_{t+k} \right]}{E_t \left[\sum_{k=0}^{\infty} \theta^k \zeta_{t,t+k} F_{t,t+k}^{\varepsilon} Y_{t+k} \right]} \tag{3.14}$$

where $\mu \equiv \frac{\varepsilon}{\varepsilon-1}$ is the price markup, $\zeta_{t,t+k} = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the stochastic discount factor, $P_t^*(z)$ is the optimal price level chosen by the firm, $F_{t,t+k} = \frac{P_{t+k}}{P_t}$ the cumulative level of inflation and Y_{t+k} is the aggregate level of output.

Since only a fraction $(1-\theta)$ of firms changes prices every period and the remaining one keeps its price fixed, the aggregate price level, the price of the final good that minimize the cost of the final goods producers, is given by the following equation:

$$P_t^{1-\varepsilon} = \theta P_{t-1}^{1-\varepsilon} + (1-\theta) \left(P_t^*(z) \right)^{1-\varepsilon} \tag{3.15}$$

Following Benigno and Woodford (2005), equations (3.14) and (3.15) can be written recursively introducing the auxiliary variables N_t and D_t (see appendix $B.1$ for details on the derivation):

$$\theta \left(\Pi_t \right)^{\varepsilon-1} = 1 - (1-\theta) \left(\frac{N_t}{D_t} \right)^{1-\epsilon} \tag{3.16}$$

$$D_t = Y_t \left(C_t\right)^{-\sigma} + \theta\beta E_t \left[\left(\Pi_{t+1}\right)^{\epsilon-1} D_{t+1}\right] \tag{3.17}$$

$$N_t = \mu Y_t \left(C_t\right)^{-\sigma} MC_t + \theta\beta E_t \left[\left(\Pi_{t+1}\right)^{\epsilon} N_{t+1}\right] \tag{3.18}$$

Equation (3.16) comes from the aggregation of individual firms prices. The ratio N_t/D_t represents the optimal relative price $P_t^*\left(z\right)/P_t$. Equations (3.16), (3.17) and (3.18) summarize the recursive representation of the non- linear Phillips curve. Writing the optimal price setting in a recursive way is necessary in order to implement both numerically and algebraically the perturbation method.

3.3 Monetary Policy

The central bank conducts monetary policy by targeting the nominal interest rate in the following way

$$R_t = R_{t-1}^{\phi_r}\left[\overline{R}\left(\frac{E_t\Pi_{t+1}}{\overline{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{\overline{Y}}\right)^{\phi_y}\right]^{1-\phi_r} \tag{3.19}$$

where, $\phi_\pi > 1$ and $\phi_y > 0$ measure the response of the nominal interest rate to expected future inflation and output, respectively. Also, the degree of interest rate smoothing is measured by $0 \leq \phi_r \leq 1$. The steady state values are expressed without time subscript and with and upper bar.

3.4 Market Clearing

In equilibrium labor, intermediate and final goods markets clear. Since there is neither capital accumulation nor government sector, the economywide resource constraint is given by

$$Y_t = C_t \tag{3.20}$$

The labor market clearing condition is given by:

$$L_t^s = L_t^d \tag{3.21}$$

Where the demand for labor comes from the aggregation of individual intermediate producers in the same way as the labor supply:

$$\begin{aligned} L^d &= \int_0^1 L_t^d(z)dz = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi}\int_0^1 Y_t(z)dz \\ L^d &= \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} Y_t\Delta_t \end{aligned} \tag{3.22}$$

where $\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} dz$ is a measure of price dispersion. Since relative prices differ across firms due to staggered price setting, input usage will differ as well. Implying that it is not possible to use the usual representative firm assumption. Therefore, the price dispersion factor, Δ_t appears in the aggregate labor demand equation. Also, from (3.22) we can see that higher price dispersion increases the labor amount necessary to produce a given level of output.

3.5 The Log Linear Economy

To illustrate the effects of oil in the dynamic equilibrium of the economy, we take a log linear approximation of equations (3.3), (3.4),(3.12), (3.16), (3.17), (3.18), (3.19) and (3.22) around the deterministic steady-state[13]. We denote variables in steady state with upper bar (i.e. $\overline{X}$) and their log deviations around the steady state with lower case letters (i.e. $x = \log(\frac{X_t}{X})$). After, imposing the goods and labor market clearing conditions to eliminate real wages and labor from the system, the dynamics of the economy are determined by the following equations:

$$mc_t = \chi\left(\nu + \sigma\right) y_t + \left(1-\chi\right) q_t \tag{3.23}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa mc_t \tag{3.24}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t\pi_{t+1}\right) \tag{3.25}$$

$$r_t = \phi_r r_{t-1} + \left(1-\phi_r\right)\left(\phi_\pi E_t\pi_{t+1} + \phi_y y_t\right) \tag{3.26}$$

$$q_t = \rho q_{t-1} + \eta\sigma_q e_t \tag{3.27}$$

where, $\chi \equiv \frac{1-\alpha^F}{1+v\psi\alpha^F}$, $\alpha^F \equiv \alpha^\psi\left(\frac{\overline{Q}}{\overline{MC}}\right)^{1-\psi}$, $\kappa \equiv \frac{1-\theta}{\theta}\left(1-\theta\beta\right)$; and $\overline{Q}$, and $\overline{MC}$, represent the steady-state value of oil prices and of marginal costs, respectively.

Interestingly, the effects of oil prices on marginal costs, equation (3.23), depends crucially on both the share of oil in the production function, α, and the elasticity of substitution between oil and labor,ψ. Thus, when α is large, χ is small making marginal costs more responsive to oil

[13] See appendix A for the derivation of the steady-state of the economy.

prices. Also, the smaller the ψ, the greater the impact of oil on marginal costs. It is important to note that even though the share of oil in the production function, α, can be small, its impact on marginal cost, α^F, can be magnified when oil has few substitutes (that is when ψ is low) [14]. Note also that a permanent increase in oil prices, that is an increase in $\overline{Q}$, makes marginal cost of firms more sensitive to oil price shocks given its effect on α^F. Finally, when $\alpha = 0$, the model collapses to a standard closed economy New Keynesian model without oil.

The model also has a key implication for monetary policy. Notably, it delivers an endogenous trade-off for the central bank when stabilizing inflation and output gap. We denote output gap by x_t and it is defined as the difference between the sticky price level of output and its corresponding efficient level, $x_t = y_t - y_t^E$, where y_t^E denotes the log deviations of the efficient level of output. In this economy, the efficient allocation is achieved when $\overline{MC} = 1$, since this equilibrium corresponds to one where intermediate firms are perfectly competitive. Therefore, when the equilibrium is efficient we have that $\alpha^F \neq \alpha^E$, where, $\alpha^E = \alpha^{\psi} \left(\overline{Q}\right)^{1-\psi}$. Using the previous definition of output gap, the economy can be represented by two equations in terms of the efficient output gap, x_t and inflation, π_t (see appendix C for details),

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^E\right) \tag{3.28}$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y x_t + \mu_t \tag{3.29}$$

where $\mu_t = \frac{\kappa_q}{\kappa_y}\left(\frac{1}{(1-\alpha^E)\alpha^F}\right)\left(\alpha^F - \alpha^E\right) q_t$, $\kappa_q = (1-\chi)$ and $\kappa_y = \chi\left(\nu + \sigma\right)\kappa$. In our model the endogenous trade off emerges from the combination of a distorted steady state and a CES production function[15]. When the elasticity of substitution between oil and labor is equal to one, the Cobb-Douglas case as in Blanchard and Gali (2006), the trade off disappears. Hence, in that case, the flexible and efficient level of output only differ by a constant term, which in turn implies that $\alpha^E = \alpha^F$. In addition, when monopolistic competition distortion is eliminated, using a proportional subsidy tax, as in Woodford (2003), the trade-off is inhibited, since again $\alpha^E = \alpha^F$. The existence of this endogenous trade off implies that it is optimal for the central bank to allow higher levels of inflation in response to supply shocks.

[14] For example, considering an oil share in the order of 1%. and an elasticity of substitution of 0.6, and assuming $\overline{Q} = \overline{W/P} = \overline{MC}$, gives $\alpha^F = (0.01)^{0.56} = 6\%$. This share would be even higher if we consider a higher steady state value of the oil price,$\overline{Q}$.

[15] Benigno and Woodford (2005), in a similar model but without oil price shocks, have found an endogenous trade-off by combining a distorted steady state with a government expenditure shock. In their framework, the combination of a distorted steady state along with a non-linear aggregate budget constraint due to government expenditure is crucial for the existence of this endogenous trade-off. Analogously to Benigno and Woodford´s finding, in our model the combination of a distorted steady state and the non-linearity of the CES production function delivers a trade-off when considering an efficient level of output such that eliminate monopolistic distortions. Montoro (2005) demonstrates that this trade-off remains even when monopolistic distortions are eliminated

The special features of oil, such as high price volatility and low substitutability in production, induce the volatility of oil prices to have a non trivial second order effects that the log-linear representation described by equations (3.23) to (3.27) does not take into account[16]. These second order effects are crucial elements in establishing the link between oil price volatility and inflation premium. The next section provides a log-quadratic approximation of the economy around its steady-state to study the link between oil price volatility and inflation.

4 Inflation Premium in General Equilibrium

4.1 The second order representation of the model

In this sub-section we present a log-quadratic (Taylor series) approximation of the fundamental equations of the model around the steady state. A detailed derivation is provided in Appendix $B.2$. The second-order Taylor-series expansion serves to compute the equilibrium fluctuations of the endogenous variables of the model up to a residual of order $O\left(\|q_t, \sigma_q\|^2\right)$, where $\|q_t, \sigma_q\|$ is a bound on the deviation and volatility of the oil price generating process around its steady state[17]. Up to second order, equations (3.23) - (3.26) are replaced by the following set of log-quadratic equations:

Aggregate Supply	
Marginal Costs	
$mc_t = \kappa_y y_t + \kappa_q q_t + \frac{1}{2}(1-\chi)\chi^2 \frac{1-\psi}{1-\alpha^F}\left((\nu+\sigma)y_t - q_t\right)^2 + \chi v \widehat{\Delta}_t + O\left(\|q_t, \sigma_q\|^3\right)$	(i)
Price dispersion	
$\widehat{\Delta}_t = \theta \widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon \frac{\theta}{1-\theta}\pi_t^2 + O\left(\|q_t, \sigma_q\|^3\right)$	(ii)
Phillips Curve	
$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\left(2(1-\sigma)y_t + mc_t\right) + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\|q_t, \sigma_q\|^3\right)$	(iii)
where we have defined the auxiliary variables:	
$v_t \equiv \pi_t + \frac{1}{2}\left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}(1-\theta\beta)\pi_t z_t$	(iv)
$z_t \equiv 2(1-\sigma)y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\|q_t, \sigma_q\|^2\right)$	(v)
Aggregate Demand	
$y_t = E_t y_{t+1} - \frac{1}{\sigma}(r_t - E_t\pi_{t+1}) - \frac{1}{2}\sigma E_t\left[(y_t - y_{t+1}) - \frac{1}{\sigma}(r_t - \pi_{t+1})\right]^2 + \left(\|q_t, \sigma_q\|^3\right)$	(vi)

Table 4.1: Second order Taylor expansion of the equations of the model

Equation (i) is obtained taking a second-order Taylor-series expansion of the real marginal cost equation, and using the labor market equilibrium condition to eliminate real wages. $\widehat{\Delta}_t$

[16] In a log-linear representation certainty equivalence holds, thus uncertainty does not play any role.

[17] Since we want to make explicit the effects of changes in the volatility of oil prices in the equilibrium of the endogenous variables, we solve the policy functions as in Schmitt-Grohe and Uribe (2004) in terms of q_t and σ_q. This is different to the approach taken by other authors, for example Woodford (2003), who consider the policy function in terms of the shocks (e_t).

is the log-deviation of the price dispersion measure Δ_t, which is a second order function of inflation and its dynamics is represented by equation (ii). Importantly, the second order approximation adds two new ingredients in the determination of marginal costs. The first one is related to the convexity of marginal costs with respect to oil prices. From this expression, when, $\psi < 1$, marginal costs become a convex function of oil prices, hence, the volatility of oil prices increases expected marginal costs. This is an important channel through which oil price volatility generates higher inflation rates. Notice, however that when the production function is a Cobb-Douglas, $\psi = 1$, this second order effect disappears, and the marginal cost equation does not depends directly on the volatility of oil prices, but only indirectly through its effects on relative price dispersion, $\widehat{\Delta}_t$,. In this particular case, marginal costs are given by,

$$mc_t = \kappa_y y_t + \kappa_q q_t + \chi v \widehat{\Delta}_t$$

the second new ingredient is associated to the indirect effect of oil price volatility through $\widehat{\Delta}_t$. From equation (3.22), it is clear that as price dispersion increases, the required number of hours to produce a given level of output also rises. Thus, this higher labor demand increases real wages, and consequently marginal costs. This effect is higher when the elasticity of labor supply, $\frac{1}{v}$ is lower and when the participation of oil in production is higher.

Equations (iii), (iv) and (v) in turn represent the second order version of the Phillips curve, and equation (vi) is the quadratic representation of the aggregate demand which includes the negative effect of the real interest rate on consumption and the precautionary savings effect. The second order representation of the aggregate demand considers, additional to the linear approximation, the effect of the volatility of the growth rate of consumption on savings. Thus, when the volatility of consumption increases, consumption falls, since households increase their savings for precautionary reasons. Next we further simplify the model economy by writing it as a second order two equation system of output and inflation. This canonical second order representation of the economy with oil allows us to disentangle the determinants of the inflation premium.

4.2 Determinants of Inflation Premium

Since the second order terms of the equations (i) - (vi) depend on the first order solution of the model, we can use the latter to express the second order terms as quadratic functions of the oil process as in Sutherland (2002). Then, we replace equations (i),(ii),(iv) and (v) in (iii), and the policy rule of the central bank in equation (vi), to write the model as second order

system of two equations on inflation, output and the oil price[18]:

$$\pi_t = \kappa_y y_t + \kappa_q q_t + \beta E_t \pi_{t+1} + \frac{1}{2}\omega_v \sigma_q^2 + \frac{1}{2}\left(\Omega_{mc} + \Omega_\pi + \Omega_v\right) q_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{4.1}$$

$$y_t = E_t\left(y_{t+1}\right) - \frac{1}{\sigma}\left(\left(\phi_\pi - 1\right) E_t \pi_{t+1} + \phi_y y_t\right) + \frac{1}{2}\omega_y \sigma_q^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{4.2}$$

where κ_y and κ_q were defined in the previous section.

We represent the second order terms as function of σ_q^2, q_t^2 and the *"omega"* coefficients $\{\Omega_{mc}, \Omega_\pi, \Omega_v, \omega_v, \omega_y\}$, which are defined in appendix B. Each of these "omega" coefficients represent the second order term in the equations for the marginal costs (subscript mc), the Phillips Curve (subscript π) , the auxiliary variable v_t (subscript v) and the aggregate demand (subscript y). Given $\{q_t\}$, the rational expectations equilibrium for $\{\pi_t\}$ and $\{y_t\}$ is obtained from, equations (4.1) and (4.2).

The *"omega"* coefficients are the sources of inflation premium in general equilibrium and capture the interaction between the nonlinearities of the model and the volatility of oil price shocks. Coefficients denoted by capital omega (Ω) represent the time variant components of the inflation premium, whereas coefficients denoted by small omega (ω) are time invariant and depend on the unconditional variance of oil prices. Note that if the aforementioned coefficients were equal to zero the model would collapse to a standard version of a New Keynesian model in log linear form. In what follows we provide economic interpretation to the determinants of the inflation premium.

The **coefficient** Ω_{mc} captures the direct effect of oil price volatility on marginal costs and its indirect effect through the labor market. Let's consider first its direct effect. When oil has few substitutes, $\psi < 1$, marginal costs are convex in oil prices, hence expected marginal costs become an increasing function of oil price volatility. To compensate the increase in expected marginal costs generated by oil price volatility, forward looking firms react by optimally charging higher prices. This response of firms, in turn, leads to higher aggregate inflation when prices are sticky [19]. Interestingly, the increase on marginal costs and inflation in response to oil price volatility is larger when the elasticity of substitution between oil and labor is small.

Additionally, oil price volatility affects marginal cost indirectly, through its effects on the labor market. Since oil price volatility generates inflation volatility, which is costly because it

[18] To make the analysis analytically tractable , we have eliminated state variables such us the lagged nominal interest rate by setting the smoothing parameter in the Taylor rule equal to zero. Similarly, we assume an small initial price dispersion, that is $\widehat{\Delta}_{t_o-1} \approx 0$ up to second order. However, in the next section, the numerical exercises consider the more general specification of the model.

[19] This mechanism can be understood by observing equation (i), where $\frac{\partial^2 mc_t}{\partial q_t^2} = (1-\chi)\chi^2 \frac{1-\psi}{1-\alpha^F}$. When $\psi < 1 (\psi > 1)$, $\frac{\partial^2 mc_t}{\partial q_t^2} > 0 (< 0)$

increases relative price distortions, efficiency in production falls as the volatility of oil prices increases. In particular, firms require, at the aggregate level, more hours of work to produce the same amount of output. Hence, the demand for labor rises, making labor more expensive and increasing marginal cost even further. Then, the increase in marginal costs through both effects, the direct and indirect, lead to an increases on aggregate inflation.

We illustrate these mechanisms in figure 4.1. In panel (a) we plot the relation between Ω_{mc} and the parameter ψ. We see that Ω_{mc} increases exponentially as ψ decreases. Also, the steady state oil price affects the impact of oil prices in marginal costs: the higher the oil price in steady-state, *ceteris paribus*, also the higher the effect of oil price volatility on marginal costs. According to this, in economies where oil is more difficult to substitute in production, or when the oil price level is relatively high, oil price volatility would be more important in the determination of the dynamics of inflation. Similarly, in panel (b) we plot the relation between Ω_{mc} and the elasticity of labor supply $(1/v)$. We see that a more elastic labor supply increase the effects of oil price volatility. This latter effect works through the indirect impact of oil price volatility on the labor market.

On the other hand, the **coefficient** Ω_π accounts for the effects of oil price volatility on the way price setters weight future marginal costs. When prices are sticky and firms face a positive probability of not being able to change prices, as in the Calvo price-setting model, the weight that firms assign to future marginal cost depends on both future expected inflation and future expected inflation's volatility. Oil price volatility by raising inflation volatility induces prices setters to put a higher weight on future marginal costs. Hence, oil price volatility not only increases expected marginal costs but also make relative price of firms more responsive to those future marginal costs.

Panel (c) shows that when the elasticity of substitution of goods ε increases, it increases the effect of inflation volatility on the price of individual firms and Ω_π increases. Similarly, panel (d) shows that lower price stickiness θ makes the Phillips curve steepper and also more convex, then the effects of inflation volatility on Ω_π increases.

The **coefficients** Ω_v **and** ω_v accounts for the time variant and constant effects of inflation volatility on the composite of inflation v_t. This mechanism is similar to that of Ω_π, however both coefficients are quantitatively small. Finally, the **coefficient** ω_y is negative and accounts for the standard precautionary savings effect, by which the uncertainty that oil price volatility generates induces households to increase savings to buffer future states of the nature where income can be low.

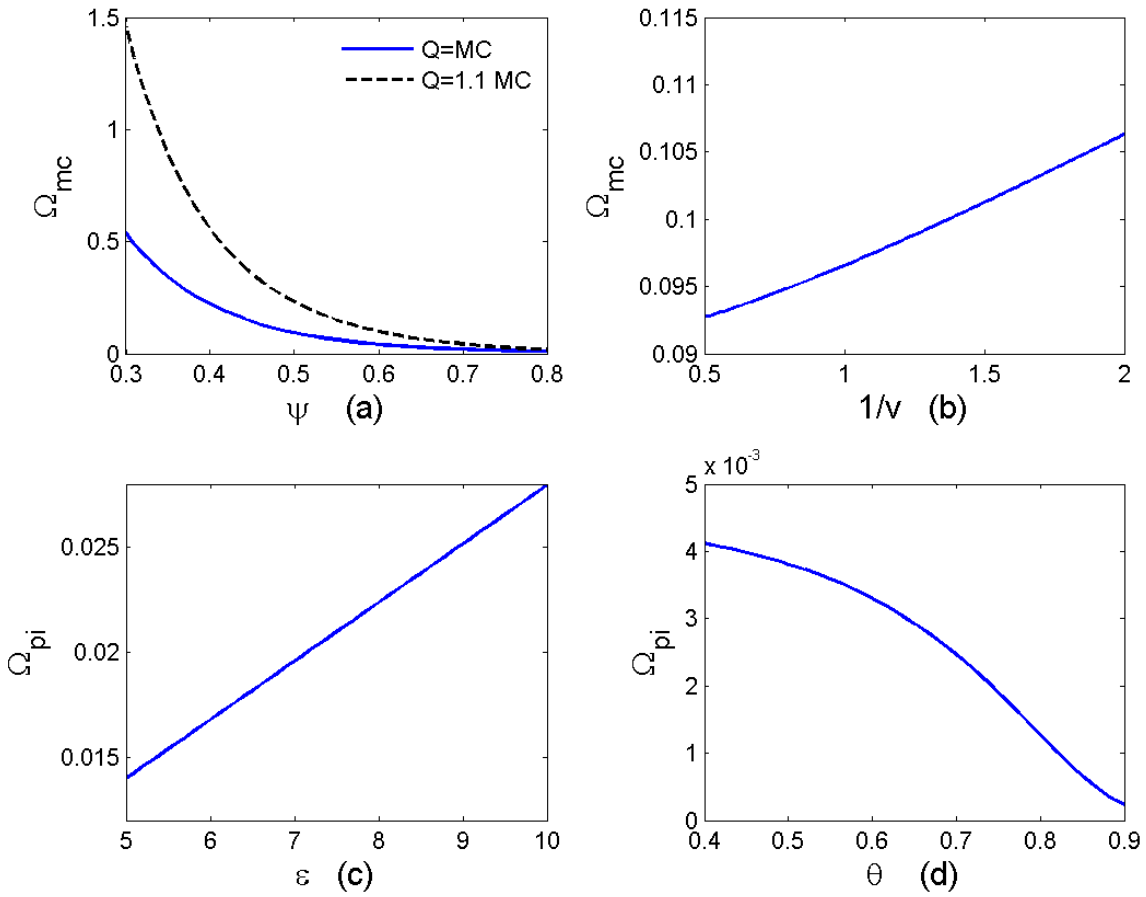

Figure 4.1: Inflation premium components (uses benchmark calibration presented in section 5). a) Effects of elasticity of substitution (ψ) on Ω_{mc}. b) Effects of labor supply elasticity ($1/v$) on Ω_{mc}. c) Effects of elasticity of substitution of goods (ε) on Ω_{π}. d) Effects of price stickyness (θ) on Ω_{π}.

4.3 The analytical solution for inflation premium

We use the perturbation method, implemented by Schmitt-Grohe and Uribe (2004)[20], to obtain the second order rational expectations solution of the model. The second order solution makes explicitly the potential effects of oil price´s volatility and the dynamics of endogenous variables. As we mentioned before, we define inflation premium as the extra level of inflation that arises in equilibrium once the second order solution is considered[21]. Also, different from other papers which apply perturbation methods directly to the non-linear system of equations, we first approximate the model up to second order and then apply the perturbation method[22]. Our proposed approach has the advantage that makes it easier to obtain clear analytical results for the sources of the level of inflation premium.

The rational expectations second order solution of output and inflation, in log-deviations from the steady state, can be written as quadratic polynomials in both the level and the standard deviation of oil prices:

$$y_t = \frac{1}{2}a_o\sigma_q^2 + a_1 q_t + \frac{1}{2}a_2\left(q_t\right)^2 + O\left(\left\|q_t, \sigma_q\right\|^3\right) \tag{4.3}$$

$$\pi_t = \frac{1}{2}b_o\sigma_q^2 + b_1 q_t + \frac{1}{2}b_2\left(q_t\right)^2 + O\left(\left\|q_t, \sigma_q\right\|^3\right) \tag{4.4}$$

where the *á*s and *b́*s are the unknown coefficients that we need to solve for and $O\left(\left\|q_t, \sigma_q\right\|^3\right)$ denotes terms on q and σ_q of order equal or higher than three[23]. Notice that the linear terms ($a_1 q_t$ and $b_1 q_t$) correspond to the policy functions that we would obtain using any standard method for linear models (i.e. undetermined coefficients), whereas the additional elements account for the effects of uncertainty (premium) on the equilibrium variables.

The quadratic terms in the policy function of inflation have two components: $\frac{1}{2}b_o\sigma_q^2$, which is constant and $\frac{1}{2}b_2\left(q_t\right)^2$, which is time varying. The analytical solution obtained with the

[20] The perturbation method was originally developed by Judd (1998) and Collard and Julliard (2001). The fixed point algorithm proposed by Collard and Julliard introduces a dependence of the coefficients of the linear and quadratic terms of the solution with the volatility of the shocks. In contrast, the advantage of the algorithm proposed by Schmitt-Grohe and Uribe is that the coefficients of the policy are invariant to the volatility of the shocks and the corresponding ones to the linear part of the solution are the same as those obtained solving a log linear approximated model, which makes both techniques comparable.

[21] It is important to remark that this extra level of average inflation is part of the dynamic rational expectations equilibrium up to second order, and it can not be interpreted as a part of the steady state equilibrium. This second order effect on the level inflation is similar to the effect of the volatility of consumption on savings that is known in the literature as precautionary savings.

[22] Since a second order Taylor expansion is an exact approximation up to second order of any non-linear equation, having the system expressed in that way would give the same solution as the system in its non-linear form.

[23] Schmitt-Grohe and Uribe (2004) show that the quadratic solution does not depend neither on σ_q nor on $q_t\sigma_q$. That is, they show that the coefficients in the solution for those terms are zero.

perturbation method implies the following expression for the overall expected level of inflation premium

$$E(\pi) = \frac{1}{2}(b_o + b_2)\sigma_q^2$$

which can be expressed as:

$$E(\pi) = \frac{1}{2}\frac{1}{\Lambda_0}\left[\phi_y(\Omega_{mc} + \Omega_\pi + \Omega_v)(1 + \Psi) + \phi_y\omega_v + \sigma\kappa_y\omega_y\right]\sigma_q^2 \tag{4.5}$$

for $\Lambda_0 = (\phi_\pi - 1)\kappa_y + (1 - \beta)\phi_y > 0$ and $\Psi > 0$ defined in the appendix B.4. According to this closed form, the inflation premium is proportional to the oil price volatility and depends on a linear combination of the *"omega's"* coefficients. Moreover, these sources of inflation premium interact with monetary policy to determine the sign and size of the premium. Under a Taylor rule, inflation premium will be positive if monetary policy reacts also to fluctuations in output due to oil shocks. From equation (4.5), the inflation premium will be positive when :

$$\phi_y > -\omega_y\sigma\kappa_y / \left[\omega_v + (\Omega_{mc} + \Omega_\pi + \Omega_v)(1 + \Psi)\right] > 0 \tag{4.6}$$

since ω_y is negative, the right hand side is positive. When the coefficient of output fluctuations in the Taylor rule, ϕ_y, is positive and above this threshold, then the inflation premium is always positive. The higher ϕ_y, the higher the inflation premium. Therefore, when the central bank reacts also to output fluctuations it also generates, in equilibrium, an inflation premium. Yet, if the central bank cares only about inflation and does not react to output fluctuations, that is $\phi_y = 0$, then the inflation premium would be negative and small. Although oil price volatility is an important determinant of inflation, the previous result shows that in general equilibrium, the reaction of the central bank turns out to be crucial. A central bank that reacts only to inflation can fully eliminate the effects of oil price volatility on inflation raising output volatility. However, this type of reaction would come at a considerable cost, since output fluctuations are inefficient when they are generated by oil price shocks.

In figure 4.2 we depict the relation between the level of inflation premium an the parameter ϕ_y. There is a small positive threshold for ϕ_y such that the premium becomes positive. Also, the higher the reaction to output fluctuations, the higher the premium. Remarkably, the existence of the inflation premium depends crucially on the existence of a trade-off between inflation and output. When the central bank does not face this trade-off, it is always possible to find a policy rule where the inflation premium is zero. The previous implication steams from the fact that the second order solution depends upon the log-linear one[24]. Therefore, in order to

[24] In a log-linear solution, when the central bank does not face a meaningful trade-off between stabilizing inflation and output, the optimal policy implies both zero inflation and output gap.

observe a positive inflation premium a necessary condition is the existence of an endogenous trade-off for the central bank. Moreover, as shown in the previous section, such trade-off exists when the elasticity of substitution between oil and labor is lower than one.

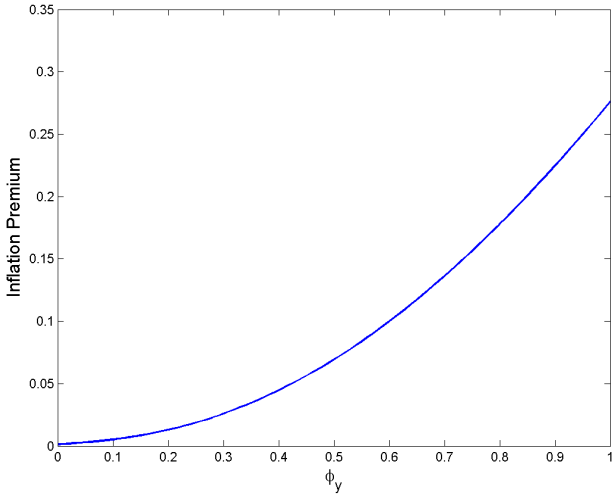

Figure 4.2: Inflation premium and the output parameterϕ_y in the policy rule.

5 Some Numerical Experiments

In this section we explore the ability of the model to explain high average levels of inflation in periods of high volatility of oil prices. To obtain the numerical results we use the method developed by Schmitt-Grohe and Uribe (2004), which provides second order numerical solutions to non-linear rational expectations models .

5.1 Calibration

To calibrate the model we choose standard parameter values in the literature. We set a quarterly discount factor, β, equal to 0.99 which implies an annualized rate of interest of 4%. For the coefficient of risk aversion parameter, σ, we choose a value of 1 and the inverse of the elasticity of labor supply, v, is calibrated to be equal to 0.5, similar to those used in the *RBC* literature and consistent with the micro evidence. We choose a degree of monopolistic competition, ε, equal to 11, which implies a firm mark-up of 10% over the marginal cost. The steady state level of oil price, $\overline{Q}$ is set equal to the inverse of the mark-up in order to isolate the effect of the share of oil in the production function. The elasticity of substitution between

oil and labor, ψ, is set equal to 0.6 and we use modest value for $\alpha = 0.01$, so that the share of oil prices in the marginal cost is around 6%[25]. The probability of the Calvo lottery is set equal to 0.66 which implies that firms adjust prices, on average, every three quarters. Finally, the log of real oil price follows an $AR(1)$ stochastic process with $\rho_q = 0.95$ and standard deviation, $\sigma_\varepsilon = 0.14$ for the first sample and $\rho_q = 0.82$ and standard deviation, $\sigma_\varepsilon = 0.13$ for the second one. These processes imply standard deviations for real oil prices of 0.46 and 0.22 in each sample, respectively. Our benchmark monetary policy rule is the estimated by CGG for the post-Volcker period. We also perform robustness exercises by comparing the results of this benchmark rule with those obtained with the estimated rules by Orphanides (2001) and Judd and Rudebush (1998)[26]. The coefficients of the alternative policy rules analysed are presented in the following table:

	CGG	Taylor	Orphanides	Judd-Rudebush
ϕ_r	0.79	0.00	0.79	0.72
ϕ_π	2.15	1.53	1.80	1.54
ϕ_y	0.93	0.77	0.27	0.99

Table 5.1: Alternative Policy Rule Coefficients

5.2 Explaining the U.S. Level of Inflation Premium with Oil Price Shocks

In this section we evaluate how the model does at capturing the conditional mean of the key macro variables, in particular of inflation. In Table 5.2 we report the means of inflation, output gap and nominal interest rates compared with the values observed in the data based on our benchmark parameterization[27]. Notice that by comparing the sub-samples we observe an important change in means and volatilities in inflation, GDP gap, and interest rates across sub-samples (columns 3 and 5 of table 5.2). Thus, quarterly inflation standard deviation has

[25] We consider a conservative calibration for the share of oil in production. Other authors have considered a larger share of oil in production or costs. For example, Atkeson and Kehoe (1999) use a share of energy in production of 0.043 and Rotemberg and Woodford (1996) a share of energy equal to 5.5% of the labour costs..

[26] Importantly, we have used the same Taylor type rule for the overall sample. Values $\phi_\pi > 1$ and $\phi_y > 0$ are consistent with recent estimation using bayesian methods by Rabanal and Rubio-Ramirez (2005). Although the previous authors find that from 1982 on, both parameters are estimated to be higher with respect to the overall sample.

[27] We use the data from the Haver USECON database (mnemonics are in parentheses). Our measure of the price level is the non-farm business sector deflator (LXNFI), the measure of GDP corresponds to the non-farm business sector output (LXNFO), we use the quarterly average daily of the 3-month T-bill (FTB3) as the nominal interest rate, and our measure of oil prices is the Spot Oil Prices West Texas Intermediate (PZTEXP). We express output in per-capita terms by dividing LXNFO by a measure of civilian non-institutional population aged above 16 (LNN) and oil prices are deflated by the non-farm business sector deflator.

decreased from 0.8% to 0.3% and the mean has moved from 1.4% to 0.5%, between the pre-Volcker and post-Volcker periods, respectively. Similarly, the three-month T-bill has decreased in both means and volatilities. Finally, GDP gap has decreased in volatility (from a standard deviation of 2.8% to 1.3%) and has experimented and increase in its mean (from -0.20% to 0.26%).

To clarify, the simulations that follow are a first step at exploring whether the mechanisms we have just have emphasized have potential for explaining the inflation-premium. In the model, we interpret oil price shocks as the main driven force of the inflation premium, although we are aware that in order to closely match the moments of other macro variables, additional shocks might be necessary. Thus, by performing these numerical exercises we intend to confront the data to the mechanism previously described. We do so by generating the unconditional mean of inflation, output and interest rates implied by the calibrated model for the pre and post Volcker periods. The only difference in the calibration between these two periods is the assumption on the data generating process of oil prices. We fit an AR (1) process for oil prices in each period and find that both the persistence and the variance of oil price shocks have fallen from the first to the second period.

	Pre-Volcker		Post-Volcker	
	Simulated	Observed	Simulated	Observed
Mean Inflation	1.09	1.38	0.19	0.53
Mean Output Gap (HP)	-1.35	-0.20	-0.23	0.26
Mean Nominal Interest Rate	4.32	7.65	0.72	5.36
Standard Deviation Inflation	1.91	0.80	0.75	0.29
Standard Deviation Output Gap (HP)	2.02	2.79	0.56	1.33
Standard Deviation Nominal Interest Rate	1.64	2.84	0.45	1.44
Standard Deviation Real Oil Price	0.46	0.57	0.22	0.21

Al variables are quarterly, except the nominal interest rate which is annualized.

Table 5.2: Unconditional Moments Generated by the Benchmark Model

The key result to highlight from table 5.2 is that we are able to generate a positive level of inflation premium that allows the model to mimic the average inflation level in the US in the pre-Volcker and post Volcker periods without relying on different monetary policy regimes across periods. Remarkably, the model can match very closely the mean of inflation for the two sub-periods. Thus, inflation mean during the first period is 1.38% while the model delivers a value of 1.09%. Similarly, for the second period we observe a mean inflation of 0.53% and the model predicts a value of 0.19%. The model is much less successful at matching the moments of the nominal interest rate and to a less extent those of output. Yet, the model does a fairly good job at matching qualitatively changes in average levels of inflation, output and interest

rates across sub-samples.

5.3 Decomposition of the Determinants of Inflation Premium

As described in the previous section, in general equilibrium, the determinants of inflation premium can be de-composed in four components: those coming from the non-linearity (convexity) of the Phillips curve (Ω_π), the non-linearity of the marginal costs (Ω_π), the auxiliary variable v_t (ω_v and Ω_v) and the precautionary savings effect (ω_y). In table 5.3 we show the decomposition of inflation premium across samples by these determinants for our calibrated economy. It is worthnoting that the convexity of the Phillips curve with respect to oil prices, accounts for roughly 59 and 55 percent of the inflation premium in the pre and post Volcker periods, respectively. The second determinant in importance is the convexity of the marginal cost with respect to oil that accounts for 45 and 48 percent, respectively. For instance, out of this effect, the level of inflation premium attributed to price distortions represents about 50 percent in each sample. Finally, the precautionary savings effect is negative and almost negligible.

	CGG	
	Pre-Volcker	Post-Volcker
Convexity Phillips curve (Ω_π)	58.9	55.4
Marginal costs (Ω_{mc})	45.2	48.2
Indirect effect: price dispersion	27.4	24.8
Direct effect: convexity respect to oil prices	17.9	23.4
Auxiliary variable v_t (ω_v and Ω_v)	-3.9	-2.9
Precautionary Savings (ω_y)	-0.3	-0.6
Total	100.0	100.0

Table 5.3: Inflation Premium - Effects Decomposition

5.4 Comparing Different Monetary Policy Rules

We now evaluate how monetary policy can affect the level of inflation premium. We do so by comparing the benchmark specification (CGG) with the estimated Taylor rules suggested by Orphanides (2001) and Judd and Rudebush (1998). Table 5.4 shows that Orphanides´s generate a smaller average inflation in both sub-samples. This finding is explained by the smaller weight assigned on output in the Orphanides´s rule with respect to the CGG´s rule. This result is consistent with threshold for the parameter ϕ_y from our analytical results, equation (4.6)

Notice also, that the smaller average level of inflation is consistent with a smaller mean level of the nominal interest rate. Hence, the aggressiveness of the central bank towards inflation determines how the premium is distributed between inflation and output means. The more aggressive at fighting inflation the central bank is, the smaller the level of inflation premium and the larger the reduction of average output. Note also that Rudebush´s rule delivers an excessive inflation premium during the pre-Volcker period (6.38%). This result is basically explained by the higher weight over output fluctuations that this rule implies.

	CGG		Orphanides		Judd-Rudebush	
	Pre-Volcker	Post-Volcker	Pre-Volcker	Post-Volcker	Pre-Volcker	Post-Volcker
Mean Inflation	1.09	0.19	0.19	0.05	6.38	0.64
Mean Output Gap (HP)	-1.35	-0.23	-0.57	-0.15	-3.49	-0.35
Mean Nominal Interest Rate	4.32	0.72	0.76	0.20	25.48	2.52
S.D Deviation Inflation	1.91	0.75	1.01	0.54	3.34	1.00
S.D Output Gap (HP)	2.02	0.56	2.22	0.68	1.73	0.43
S.D Nominal Interest Rate	1.64	0.45	0.82	0.28	2.91	0.62
S.D Real Oil Price	0.46	0.22	0.46	0.22	0.46	0.22

Table 5.4: Alternative Policy Rules

6 Conclusions

Traditionally New Keynesian log-linear models have been used to match second order moments. However, they have the limitation that their solution implies certainty equivalence, neglecting any role of uncertainty and volatility over the level of inflation. To the extent that uncertainty is important in real economies, a second order solution of the New Keynesian model is required to improve their fit to the data. In particular, this type of solution provides a link between volatility of shocks and the average values of endogenous variables offering a non-conventional way to analyze business cycles. In this paper we have taken this approach and we show how the interaction between volatility and the convexity of both the marginal costs and the Phillips curve improves the ability of a standard New Keynesian model to explain the history of inflation in the USA.

The second order solution allows us to provide an additional element to the explanation suggested by CGG for the high inflation episode during the 70s. Our hypothesis puts at the centre of the discussion the volatility of supply shocks, in particular oil price shocks. Contrary to what a linear solution implies, a second order solution establishes the link between volatility of oil prices and expected inflation, what we called inflation premium. In this paper we show that a calibrated version of our model can match very closely the inflation behavior observed in

the USA during both the pre-Volcker and post-Volcker periods. In particular we show that the high volatility of oil price shocks during the 70s implied an endogenous high level of inflation premium that can account for the high average inflation levels observed in US during that period. The analytical solution obtained by implementing the perturbation method shows that the existence of the inflation premium depends crucially on, first, the convexity of both the marginal costs and the Phillips curve and second, the response of the monetary authority. We find that when oil has low substitutability, marginal costs are convex in oil prices, hence its price volatility increases the expected value of marginal costs. In addition, the reaction of the central bank determines in equilibrium how higher volatility generated by oil price shocks is distributed between a higher average inflation and lower growth rate. Moreover, in order to observe a positive inflation premium it is required that the central bank partially reacts to supply shocks. Another interesting and novel finding is that relative price dispersion, by increasing the amount of labour required to produce a given level of output, increases average wages. So, relative price dispersion amplifies the effect of expected marginal costs over average inflation.

Our results can be extended in many directions. First, it will be worth to explore the effect of openness in inflation premium. Second, the analytical perturbation method strategy proposed in this paper can be used to capture the effects of change in a monetary policy regime over inflation. Third, it will be worth also to explore the implications of other source of shocks in the determination in the level of inflation premium. Finally, the estimation of a non-linear Phillips curve considering the effects of oil price volatility on inflation will be an issue to further explore.

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A Appendix: Equations of the Model

A.1 The system of equations

Using the the market clearing conditions that close the model, the dynamic equilibrium of the model described in section 3 is given by the following set of 10 equations:

AGGREGATE SUPPLY	
Marginal Costs	
$MC_t = \left[(1-\alpha)^{\psi}\left(W_t/P_t\right)^{1-\psi} + \alpha^{\psi}\left(Q_t\right)^{1-\psi}\right]^{\frac{1}{1-\psi}}$	A-i
Labor market	
$\frac{W_t}{P_t} = Y_t^{\sigma} L_t^{v}$	A-ii
$L_t = \left(\frac{1}{1-\alpha}\frac{W_t/P_t}{MC_t}\right)^{-\psi} Y_t \Delta_t$	A-iii
Price dispersion	
$\Delta_t = (1-\theta)\left(\frac{1-\theta(\Pi_t)^{\varepsilon-1}}{1-\theta}\right)^{\varepsilon/(\varepsilon-1)} + \theta\Delta_{t-1}\left(\Pi_t\right)^{\varepsilon}$	A1-iv
Phillips Curve	
$\theta\left(\Pi_t\right)^{\varepsilon-1} = 1-(1-\theta)\left(\frac{N_t}{D_t}\right)^{1-\varepsilon}$	A-v
$N_t = \mu Y_t^{1-\sigma} MC_t + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\varepsilon} N_{t+1}\right]$	A-vi
$D_t = Y_t^{1-\sigma} + \theta\beta E_t\left[\left(\Pi_{t+1}\right)^{\varepsilon-1} D_{t+1}\right]$	A-vii
AGGREGATE DEMAND	
$1 = \beta E_t\left[\left(\frac{Y_{t+1}}{Y_t}\right)^{-\sigma}\frac{R_t}{\Pi_{t+1}}\right]$	A-vii
MONETARY POLICY	
$R_t = \overline{R}\left(\frac{E_t\Pi_{t+1}}{\overline{\Pi}}\right)^{\phi_{\pi}}\left(\frac{Y_t}{\overline{Y}}\right)^{\phi_y}$	A-ix
OIL PRICES	
$Q_t = \overline{Q}Q_{t-1}^{\rho}\exp\left(\eta\sigma_q e_t\right)$	A-x

Table A.1: Equations of the model

The first block represents the aggregate supply, which consists on the marginal costs, the labor market equilibrium and the Phillips curve, which has been written recursively using the auxiliary variables N_t and D_t. The aggregate demand block is represented with the Euler equation and Monetary Policy block is given by the Taylor rule. The last equation describes the dynamics of oil prices. We use this set of ten non-linear equations to obtain numerically the second order solution of the model.

A.2 The deterministic steady state

The non-stochastic steady state of the endogenous variables is given by: where

Inflation	$\overline{\Pi} = 1$
Auxiliary variables	$\overline{N} = \overline{D} = \overline{Y}/\left(1-\theta\beta\right)$
Interest rate	$\overline{R} = \beta^{-1}$
Marginal costs	$\overline{MC} = 1/\mu$
Real wages	$\overline{W/P} = \tau_y \frac{1}{\mu}\left(1-\alpha^F\right)^{\frac{1}{1-\psi}}$
Output	$\overline{Y} = \tau_y \left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}} \left(1-\alpha^F\right)^{\frac{1+\psi\nu}{\sigma+\nu}\frac{1}{1-\psi}}$
Labor	$\overline{L} = \tau_l \left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}} \left(1-\alpha^F\right)^{\frac{1-\sigma\psi}{\sigma+\nu}\frac{1}{1-\psi}}$

Table A.2: The steady state

$$\alpha^F = \alpha^{\psi}\left(\frac{\overline{Q}}{\overline{MC}}\right)^{1-\psi} = \alpha^{\psi}\left(\mu\overline{Q}\right)^{1-\psi}$$

α^F is the share of oil in the marginal costs, τ_y and τ_l are constants[28]. Notice that the steady state values of real wages, output and labor depend on the steady state ratio of oil prices with respect to the marginal cost. This implies that permanent changes in oil prices would generate changes in the steady state of this variables. Also, as the standard New-Keynesian models, the marginal cost in steady state is equal to the inverse of the mark-up ($\overline{MC} = 1/\mu = (\varepsilon-1)/\varepsilon$). Since monopolistic competition affects the steady state of the model, output in steady state is below the efficient level. We call to this feature a distorted steady state.

A.3 The flexible price equilibrium

The flexible price equilibrium of the endogenous variables is consistent with zero inflation in every period (i.e. $.\Pi_t^F = 1$). In this case marginal costs are constant, equal to its steady state value, and the other variables are affected by the oil shock.

Notice that the flexible price equilibrium is not efficient, since there are distortions from monopolistic competition in the intermediate goods market (i.e. $MC_t^F > 1$).

[28] More precisely: $\tau_y = \left(\frac{1}{1-\alpha}\right)^{\frac{\psi}{1-\psi}\frac{1+v}{\sigma+v}}$ and $\tau_l = \left(\frac{1}{1-\alpha}\right)^{\frac{\psi}{1-\psi}\frac{1-\sigma}{\sigma+v}}$.

Inflation	$\Pi_t^F = 1$
Interest rate	$1/R_t^F = E_t \left(\frac{1-\alpha^F \left(Q_{t+1}/\overline{Q}\right)^{1-\psi}}{1-\alpha^F \left(Q_t/\overline{Q}\right)^{1-\psi}} \right)^{-\sigma\gamma}$
Marginal costs	$MC_t^F = 1/\mu$
Real wages	$W_t^F/P_t^F = \tau_y \frac{1}{\mu} \left(1 - \alpha^F \left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1}{1-\psi}}$
Output	$Y_t^F = \tau_y \left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}} \left(1 - \alpha^F \left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1+\psi\nu}{\sigma+\nu}\frac{1}{1-\psi}}$
Labor	$L_t^F = \tau_l \left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\nu}} \left(1 - \alpha^F \left(Q_t/\overline{Q}\right)^{1-\psi}\right)^{\frac{1-\sigma\psi}{\sigma+\nu}\frac{1}{1-\psi}}$

Table A.3: The flexible price equilibrium

B Appendix: The second order solution of the model

B.1 The recursive AS equation

We divide the equation for the aggregate price level (3.15) by $P_t^{1-\varepsilon}$ and make $P_t/P_{t-1} = \Pi_t$

$$1 = \theta \left(\Pi_t\right)^{-(1-\varepsilon)} + (1-\theta) \left(\frac{P_t^*(z)}{P_t}\right)^{1-\varepsilon} \tag{B.1}$$

Aggregate inflation is function of the optimal price level of firm z. Also, from equation (3.14) the optimal price of a typical firm can be written as:

$$\frac{P_t^*(z)}{P_t} = \frac{N_t}{D_t}$$

where, after using the definition for the stochastic discount factor: $\zeta_{t,t+k} = \beta C_{t+k}^{-\sigma}/C_t^{-\sigma} P_t/P_{t+k}$, we define N_t and D_t as follows:

$$N_t = E_t \left[\sum_{k=0}^{\infty} \mu(\theta\beta)^{k+1} F_{t,t+k}^{\varepsilon} Y_{t+k} C_{t+k}^{-\sigma} MC_{t+k} \right] \tag{B.2}$$

$$D_t = E_t \left[\sum_{k=0}^{\infty} (\theta\beta)^{k+1} F_{t,t+k}^{\varepsilon-1} Y_{t+k} C_{t+k}^{-\sigma} \right] \tag{B.3}$$

N_t and D_t can be expanded as:

$$N_t = \mu Y_t C_t^{-\sigma} MC_t + E_t \left[\Pi_{t+1}^{\varepsilon} \sum_{k=0}^{\infty} \mu(\theta\beta)^k F_{t+1,t+1+k}^{\varepsilon} Y_{t+1+k} C_{t+1+k}^{-\sigma} MC_{t+1+k} \right] \quad \text{(B.4)}$$

$$D_t = Y_t C_t^{-\sigma} + E_t \left[\Pi_{t+1}^{\varepsilon-1} \sum_{k=0}^{\infty} (\theta\beta)^k F_{t+1,t+1+k}^{\varepsilon-1} C_{t+1+k}^{-\sigma} Y_{t+1+k} \right] \quad \text{(B.5)}$$

where we have used the definition for $F_{t,t+k} = P_{t+k}/P_t$.

The Phillips curve with oil prices is given by the following three equations:

$$\theta \left(\Pi_t\right)^{\varepsilon-1} = 1 - (1-\theta) \left(\frac{P_t^*(z)}{P_t} \right)^{1-\varepsilon} \quad \text{(B.6)}$$

$$N_t = \mu Y_t^{1-\sigma} MC_t + \theta\beta E_t \left(\Pi_{t+1}\right)^{\varepsilon} N_{t+1} \quad \text{(B.7)}$$

$$D_t = Y_t^{1-\sigma} + \theta\beta E_t \left(\Pi_{t+1}\right)^{\varepsilon-1} D_{t+1} \quad \text{(B.8)}$$

where we have reordered equation (B.1) and we have used equations (B.2) and (B.3) evaluated one period forward to replace N_{t+1} and D_{t+1} in equations (B.4) and (B.5).

B.2 The second order approximation of the system

B.2.1 The second order approximation of the Phillips Curve

The second order expansion for equations $(B.6)$, $(B.7)$ and $(B.8)$ are:

$$\pi_t = \frac{(1-\theta)}{\theta} \left(n_t - d_t\right) - \frac{1}{2} \frac{(\varepsilon-1)}{1-\theta} \left(\pi_t\right)^2 + O\left(\left\| q_t, \sigma_q \right\|^3 \right) \quad \text{(B.9)}$$

$$n_t = (1-\theta\beta) \left(a_t + \frac{1}{2} a_t^2 \right) + \theta\beta \left(E_t b_{t+1} + \frac{1}{2} E_t b_{t+1}^2 \right) - \frac{1}{2} n_t^2 + O\left(\left\| q_t, \sigma_q \right\|^3 \right) \quad \text{(B.10)}$$

$$d_t = (1-\theta\beta) \left(c_t + \frac{1}{2} c_t^2 \right) + \theta\beta \left(E_t e_{t+1} + \frac{1}{2} E_t e_{t+1}^2 \right) - \frac{1}{2} d_t^2 + O\left(\left\| q_t, \sigma_q \right\|^3 \right) \quad \text{(B.11)}$$

Where we have defined the auxiliary variables a_t,b_{t+1},c_t and e_{t+1} as:

$$a_t \equiv (1-\sigma)\, y_t + mc_t \qquad b_{t+1} \equiv \varepsilon \pi_{t+1} + n_{t+1}$$
$$c_t \equiv (1-\sigma)\, y_t \qquad e_{t+1} \equiv (\varepsilon - 1)\, \pi_{t+1} + d_{t+1}$$

Subtract equations $(B.10)$ and $(B.11)$, and using the fact that $X^2 - Y^2 = (X - Y)(X + Y)$, for any two variables X and Y :

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)(a_t - c_t) + \frac{1}{2}(1-\theta\beta)(a_t - c_t)(a_t + c_t) \\ &\quad +\theta\beta E_t(b_{t+1} - e_{t+1}) + \frac{1}{2}\theta\beta E_t(b_{t+1} - e_{t+1})(b_{t+1} + e_{t+1}) \\ &\quad -\frac{1}{2}(n_t - d_t)(n_t + d_t) + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{B.12}$$

Plugging in the values of a_t, b_{t+1}, c_t and e_{t+1} into equation $(B.12)$, we obtain $(B.13)$

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)mc_t + \frac{1}{2}(1-\theta\beta)mc_t(2(1-\sigma)y_t + mc_t) \\ &\quad +\theta\beta E_t(\pi_{t+1} + n_{t+1} - d_{t+1}) + \frac{1}{2}\theta\beta E_t(\pi_{t+1} + n_{t+1} - d_{t+1})((2\varepsilon - 1)\pi_{t+1} + n_{t+1} + d_{t+1}) \\ &\quad -\frac{1}{2}(n_t - d_t)(n_t + d_t) + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{B.13}$$

Taking forward one period equation $(B.9)$, we can solve for $n_{t+1} - d_{t+1}$:

$$n_{t+1} - d_{t+1} = \frac{\theta}{1-\theta}\pi_{t+1} + \frac{1}{2}\frac{\theta}{1-\theta}\frac{(\varepsilon - 1)}{1-\theta}(\pi_{t+1})^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{B.14}$$

replace equation $(B.14)$ in $(B.13)$ and make use of the auxiliary variable $z_t = (n_t + d_t) / (1 - \theta\beta)$

$$\begin{aligned} n_t - d_t &= (1-\theta\beta)mc_t + \frac{1}{2}(1-\theta\beta)mc_t(2(1-\sigma)y_t + mc_t) \\ &\quad +\frac{\theta}{1-\theta}\beta\left[E_t\pi_{t+1} + \left(\frac{\varepsilon - 1}{1-\theta} + \varepsilon\right)E_t\pi_{t+1}^2 + (1-\theta\beta)E_t\pi_{t+1}z_{t+1}\right] \\ &\quad -\frac{1}{2}\frac{\theta}{1-\theta}(1-\theta\beta)\pi_t z_t + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{B.15}$$

Notice that we use only the linear part of equation $(B.14)$ when we replace $n_{t+1} - d_{t+1}$ in the quadratic terms because we are interested in capture terms only up to second order of accuracy. Similarly, we make use of the linear part of equation $(B.9)$ to replace $(n_t - d_t) = \frac{\theta}{1-\theta}\pi_t$ in the right hand side of equation $(B.15)$.

Replace equation $(B.15)$ in $(B.9)$:

$$\begin{aligned} \pi_t &= \kappa mc_t + \frac{1}{2}\kappa mc_t(2(1-\sigma)y_t + mc_t) \\ &\quad +\beta\left[E_t\pi_{t+1} + \left(\frac{\varepsilon - 1}{1-\theta} + \varepsilon\right)E_t\pi_{t+1}^2 + (1-\theta\beta)E_t\pi_{t+1}z_{t+1}\right] \\ &\quad -\frac{1}{2}(1-\theta\beta)\pi_t z_t - \frac{1}{2}\frac{(\varepsilon - 1)}{1-\theta}(\pi_t)^2 + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{B.16}$$

for

$$\kappa \equiv \frac{(1-\theta)}{\theta}(1-\theta\beta)$$

where z_t has the following linear expansion:

$$z_t = 2(1-\sigma)y_t + mc_t + \theta\beta E_t\left(\frac{2\varepsilon-1}{1-\theta\beta}\pi_{t+1} + z_{t+1}\right) + O\left(\|q_t, \sigma_q\|^2\right) \tag{B.17}$$

Define the following auxiliary variable:

$$v_t = \pi_t + \frac{1}{2}\left(\frac{\varepsilon-1}{1-\theta} + \varepsilon\right)\pi_t^2 + \frac{1}{2}(1-\theta\beta)\pi_t z_t \tag{B.18}$$

Using the definition for v_t, equation $(B.16)$ can be expressed as:

$$v_t = \kappa mc_t + \frac{1}{2}\kappa mc_t\left(2(1-\sigma)y_t + mc_t\right) + \frac{1}{2}\varepsilon\pi_t^2 + \beta E_t v_{t+1} + O\left(\|q_t, \sigma_q\|^2\right) \tag{B.19}$$

which is equation (4.3) in the main text.

Moreover, the linear part of equation (B.19) is:

$$\pi_t = \kappa mc_t + \beta E_t(\pi_{t+1}) + O\left(\|q_t, \sigma_q\|^2\right)$$

which is the standard New Keynesian Phillips curve, inflation depends linearly on the real marginal costs and expected inflation.

B.2.2 The MC equation and the labor market equilibrium

The real marginal cost (3.12) and the labor market equations (3.4 and 3.22) have the following second order expansion:

$$mc_t = \left(1-\alpha^F\right)w_t + \alpha^F q_t + \frac{1}{2}\alpha^F\left(1-\alpha^F\right)(1-\psi)(w_t-q_t)^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{B.20}$$

$$w_t = \nu l_t + \sigma y_t \tag{B.21}$$

$$l_t = y_t - \psi(w_t - mc_t) + \widehat{\Delta}_t \tag{B.22}$$

Where w_t and $\widehat{\Delta}_t$ are, respectively, the log of the deviation of the real wage and the price dispersion measure from their respective steady state. Notice that equations $(B.21)$and $(B.22)$ are not approximations, but exact expressions.

Solving equations $(B.21)$ and $(B.22)$ for the equilibrium real wage:

$$w_t = \frac{1}{1+\nu\psi}\left[(\nu+\sigma)\, y_t + \nu\psi mc_t + v\widehat{\Delta}_t\right] \tag{B.23}$$

Plugging the real wage in equation $(B.20)$ and simplifying:

$$\begin{aligned} mc_t &= \chi(\sigma+v)\, y_t + (1-\chi)(q_t) + \chi v\widehat{\Delta}_t \\ &\quad + \frac{1}{2}\frac{1-\psi}{1-\alpha^F}\chi^2(1-\chi)\left[(\sigma+v)\, y_t - q_t\right]^2 + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{B.24}$$

where $\chi \equiv \left(1-\alpha^F\right)/\left(1+v\psi\alpha^F\right)$ .This is the equation (i) in the main text. This expression is the second order expansion of the real marginal cost as a function of output and the oil prices.

B.2.3 The price dispersion measure

The price dispersion measure is given by

$$\Delta_t = \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon} dz$$

Since a proportion $1-\theta$ of intermediate firms set prices optimally, whereas the other θ set the price last period, this price dispersion measure can be written as:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{-\varepsilon} + \theta\int_0^1 \left(\frac{P_{t-1}(z)}{P_t}\right)^{-\varepsilon} dz$$

Dividing and multiplying by $(P_{t-1})^{-\varepsilon}$ the last term of the RHS:

$$\Delta_t = (1-\theta)\left(\frac{P_t^*(z)}{P_t}\right)^{-\varepsilon} + \theta\int_0^1 \left(\frac{P_{t-1}(z)}{P_{t-1}}\right)^{-\varepsilon}\left(\frac{P_{t-1}}{P_t}\right)^{-\varepsilon} dz$$

Since $P_t^*(z)/P_t = N_t/D_t$ and $P_t/P_{t-1} = \Pi_t$, using equation (3.8) in the text and the definition for the dispersion measure lagged on period, this can be expressed as

$$\Delta_t = (1-\theta)\left(\frac{1-\theta(\Pi_t)^{\varepsilon-1}}{1-\theta}\right)^{\varepsilon/(\varepsilon-1)} + \theta\Delta_{t-1}(\Pi_t)^{\varepsilon} \tag{B.25}$$

which is a recursive representation of Δ_t as a function of Δ_{t-1} and Π_t.

Benigno and Woodford (2005) show that a second order approximation of the price dispersion depends solely on second order terms on inflation. Then, the second order approximation

of equation (B.25) is:

$$\widehat{\Delta}_t = \theta\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\pi_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{B.26}$$

which is equation $(2-ii)$ in the main text. Moreover, we can use equation $(B.26)$ to write the infinite sum:

$$\begin{aligned}
\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_t &= \theta\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_{t-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|q_t, \sigma_q\|^3\right) \\
(1-\beta\theta)\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_t &= \theta\widehat{\Delta}_{t_o-1} + \frac{1}{2}\varepsilon\frac{\theta}{1-\theta}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|q_t, \sigma_q\|^3\right)
\end{aligned}$$

Dividing by $(1-\beta\theta)$ and using the definition of κ :

$$\sum_{t=t_o}^{\infty}\beta^{t-t_o}\widehat{\Delta}_t = \frac{\theta}{1-\beta\theta}\widehat{\Delta}_{t_o-1} + \frac{1}{2}\frac{\varepsilon}{\kappa}\sum_{t=t_o}^{\infty}\beta^{t-t_o}\frac{\pi_t^2}{2} + O\left(\|q_t, \sigma_q\|^3\right) \tag{B.27}$$

The discounted infinite sum of $\widehat{\Delta}_t$ is equal to the sum of two terms, on the initial price dispersion and the discounted infinite sum of π_t^2.

B.2.4 The IS

Similarly, the second order expansion of the IS is:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t\pi_{t+1}\right) - \frac{1}{2}\sigma E_t\left[\left(y_t - y_{t+1}\right) - \frac{1}{\sigma}\left(r_t - \pi_{t+1}\right)\right]^2 + \left(\|q_t, \sigma_q\|^3\right) \tag{B.28}$$

Replacing the linear solution of y_t inside the quadratic part of equation $(B.28)$:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(r_t - E_t\pi_{t+1}\right) - \frac{1}{2}\sigma E_t\left[y_{t+1} + \frac{1}{\sigma}\pi_{t+1} - E_t\left(y_{t+1} + \frac{1}{\sigma}\pi_{t+1}\right)\right]^2 + \left(\|q_t, \sigma_q\|^3\right) \tag{B.29}$$

where $E_t\left[y_{t+1} + \frac{1}{\sigma}\pi_{t+1} - E_t\left(y_{t+1} + \frac{1}{\sigma}\pi_{t+1}\right)\right]^2$ is the variance of $\left(y_{t+1} + \frac{1}{\sigma}\pi_{t+1}\right)$.

B.3 The system in two equations

Since the quadratic terms of the second order Taylor expansions of the equations depend on the linear solution, we can use the latter to solve for the formers. Let´s assume the linear solution for output, inflation and the auxiliary variable z_t[29]

$$\begin{aligned} y_t &= a_1 q_t + O\left(\|q_t, \sigma_q\|^2\right) \\ \pi_t &= b_1 q_t + O\left(\|q_t, \sigma_q\|^2\right) \\ z_t &= c_1 q_t + O\left(\|q_t, \sigma_q\|^2\right) \end{aligned}$$

Additionally, we have the transition process for the oil price:

$$q_t = \rho q_{t-1} + \eta \sigma_q e_t$$

where $e \tilde{} iid\,(0,1)$ and $\eta = \sqrt{1-\rho^2}$.

B.3.1 The AS

Replacing the equation for the price dispersion in the equation for the marginal costs, the latter can be expressed as:

$$mc_t = \chi\,(v+\sigma)\,y_t + (1-\chi)\,q_t + \chi v \widehat{\Delta}_t + \frac{1}{2}\widetilde{\Omega}_{mc} q_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{B.30}$$

where $\widetilde{\Omega}_{mc} = (1-\chi)\,\chi^2 \frac{1-\psi}{1-\alpha^F}\left((\nu+\sigma)\,a_1 - 1\right)^2 + \varepsilon \frac{\theta}{1-\theta}\left(b_1\right)^2$.

Similarly, the Phillips curve equation can be expressed as:

$$v_t = \kappa mc_t + \beta E_t v_{t+1} + \frac{1}{2}\Omega_\pi q_t^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{B.31}$$

where $\Omega_\pi = \varepsilon\left(b_1\right)^2 + \kappa\left[\chi\,(v+\sigma)\,a_1 + (1-\chi)\right]\left[2\,(1-\sigma)\,y_t + \chi\,(v+\sigma)\,a_1 + (1-\chi)\right]$. We have used the linear solution of output and inflation to express Ω_π in terms of a_1 and b_1.

Replace the equation for the marginal costs in the second order expansion of the Phillips Curve and iterate forward, the Phillips curve can be expressed as the discounted infinite sum:

$$v_t = \sum_{t=t_o}^{\infty} \beta^{t-t_o}\left\{\kappa_y y_t + \kappa_q q_t + \kappa\chi v\widehat{\Delta}_t + \frac{1}{2}\kappa\widetilde{\Omega}_{mc} q_t^2 + \frac{1}{2}\Omega_\pi q_t^2\right\} + \left(\|q_t, \sigma_q\|^3\right) \tag{B.32}$$

[29] From the linear expansion of the definition of z_t we can solve for c_1, where $c_1 = \frac{1}{1-\theta\beta\rho}\left\{\left[2\,(1-\sigma)+\chi\,(\sigma+v)\right]a_1 + (1-\chi) + \theta\beta\frac{2\varepsilon-1}{1-\theta\beta}\rho b_1\right\}$

where $\kappa_y = \kappa\chi(\sigma+\nu)$ and $\kappa_q = \kappa(1-\chi)$. Make use of equation $(B.27)$, the discounted infinite sum of $\widehat{\Delta}_t$, v_t becomes:

$$\begin{aligned} v_t &= \sum_{t=t_o}^{\infty} \beta^{t-t_o} \left\{ \kappa_y y_t + \kappa_q q_t + \frac{1}{2}\varepsilon\chi v\pi_t^2 + \frac{1}{2}\kappa\widetilde{\Omega}_{mc} q_t^2 + \frac{1}{2}\Omega_\pi q_t^2 \right\} \\ &+ \frac{\kappa\chi v\theta}{1-\beta\theta}\widehat{\Delta}_{t-1} + \left(\|q_t, \sigma_q\|^3 \right) \end{aligned} \tag{B.33}$$

Assuming that we depart from an initial state where the price dispersion is small, that is $\widehat{\Delta}_{t-1} \simeq 0$ up to second order, then equation (B.33) can be expressed recursively as[30]:

$$v_t = \kappa_y y_t + \kappa_q q_t + \frac{1}{2}\varepsilon\chi v\pi_t^2 + \frac{1}{2}\kappa\widetilde{\Omega}_{mc} q_t^2 + \frac{1}{2}\Omega_\pi q_t^2 + \beta E_t v_{t+1} + \left(\|q_t, \sigma_q\|^3 \right) \tag{B.34}$$

Let´s consider the total second order terms coming from the marginal costs:

$$\Omega_{mc} q_t^2 = \varepsilon\chi v\pi_t^2 + \kappa\widetilde{\Omega}_{mc} q_t^2 \tag{B.35}$$

then, $\Omega_{mc} = \varepsilon\chi v\left(b_1\right)^2 + \kappa\widetilde{\Omega}_{mc}$.

The auxiliary variable v_t is also affected by second order terms:

$$v_t = \pi_t + \frac{1}{2}\widetilde{\Omega}_v q_t^2 \tag{B.36}$$

where $\widetilde{\Omega}_v = \left[\left(\frac{\varepsilon-1}{1-\theta} + \varepsilon \right) b_1^2 + (1-\theta\beta)\, b_1 c_1 \right]$. $E_t v_{t+1}$ becomes:

$$\begin{aligned} E_t v_{t+1} &= E_t\pi_{t+1} + \frac{1}{2}\widetilde{\Omega}_v E_t q_{t+1}^2 \\ &= E_t\pi_{t+1} + \frac{1}{2}\widetilde{\Omega}_v \left(\rho^2 q_t^2 + \eta^2\sigma_q^2 \right) \end{aligned} \tag{B.37}$$

Replacing equations (B.35), (B.36) and (B.37) in (B.34), we obtain the equation (4.1) in the text:

$$\pi_t = \kappa_y y_t + \kappa_q q_t + \beta E_t\pi_{t+1} + \frac{1}{2}\left(\Omega_{mc} + \Omega_\pi + \Omega_v \right) q_t^2 + \frac{1}{2}\omega_v\sigma_q^2 + \left(\|q_t, \sigma_q\|^3 \right) \tag{B.38}$$

where $\Omega_v = -\widetilde{\Omega}_v\left(1-\beta\rho^2\right)$ and $\omega_v = \widetilde{\omega}_v\beta\eta^2$. $\Omega_{mc}, \Omega_\pi, \Omega_v$ and ω_v are respectively the second order terms coming from the marginal costs, the Phillips Curve and the auxiliary variable v_t.

[30] We make the assumption that the initial price dispersion is small to make the analysis analytically tractable. However, in the numerical exercise we work with the general case and the results are quantitatively similar.

B.3.2 The aggregate demand

Replace the policy rule (3.26) in the second order expansion of the IS (B.29), assuming there is not interest rate smoothing (that is $\phi_r = 0$) :

$$\begin{aligned} y_t &= E_t y_{t+1} - \frac{1}{\sigma}\left[(\phi_\pi - 1) E_t \pi_{t+1} + \phi_y y_t\right] + \\ &\quad -\frac{1}{2}\sigma E_t\left[y_{t+1} + \frac{1}{\sigma}\pi_{t+1} - E_t\left(y_{t+1} + \frac{1}{\sigma}\pi_{t+1}\right)\right]^2 + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{B.39}$$

This can be expressed as:

$$y_t = E_t\left(y_{t+1}\right) - \frac{1}{\sigma}\left[(\phi_\pi - 1) E_t \pi_{t+1} + \phi_y y_t\right] + \frac{1}{2}\omega_y \sigma_q^2 + O\left(\|q_t, \sigma_q\|^3\right) \tag{B.40}$$

where:

$$\omega_y \sigma_q^2 = -\sigma E_t\left[a_1 q_{t+1} + \frac{1}{\sigma} b_1 q_{t+1} - E_t\left(a_1 q_{t+1} + \frac{1}{\sigma} b_1 q_{t+1}\right)\right]^2 \tag{B.41}$$

Similar to the previous sub-section, the IS risk premium can be written as a function of the linear solution of inflation and output:

$$\omega_y = -\sigma\left(a_1 + \frac{1}{\sigma} b_1\right)^2 < 0 \tag{B.42}$$

Note that the risk premium component of the IS is negative, capturing precautionary savings due to output and inflation volatility.

B.4 The perturbation method

The policy functions of the second order solution for output and inflation can be written in the following form:

$$\begin{aligned} y_t &= \frac{1}{2} a_o \sigma_q^2 + a_1 q_t + \frac{1}{2} a_2 \left(q_t\right)^2 + O\left(\|q_t, \sigma_q\|^3\right) \\ \pi_t &= \frac{1}{2} b_o \sigma_q^2 + b_1 q_t + \frac{1}{2} b_2 \left(q_t\right)^2 + O\left(\|q_t, \sigma_q\|^3\right) \end{aligned} \tag{B.43}$$

where the *á*s and *b́*s are the unknown coefficients that we need to solve for and $O\left(\|q_t, \sigma_q\|^3\right)$ denotes terms on q and σ_q of order equal or higher than 3. We express the dynamics of the oil price as:

$$q_t = \rho q_{t-1} + \eta \sigma_q e_t \tag{B.44}$$

where the oil shock has been normalized to have mean zero and standard deviation of one, i.e. $e \tilde{} iid\,(0,1)$ .Also, we set $\eta = \sqrt{1-\rho^2}$ in order to express $V\,(q_t) = \sigma_q^2$.

In order to solve for the 6 unknown coefficients, we use the following algorithm that consist in solving recursively for three systems of two equations. This allow us to obtain algebraic solutions for the unknown coefficients. We follow the following steps:

1. We replace the closed forms of the policy functions (B.43) and the transition equation for the shock (B.44) in the equations for the AS (B.38) and the AD (B.40).

2. Solve for a_1 and b_1: we take the partial derivatives with respect to q_t to the two equations of step 1, then we proceed to evaluate them in the non-stochastic steady state (i.e. when $q_t = 0$ and $\sigma_q = 0$). Then, the only unknowns left are a_1 and b_1 for two equations. We proceed to solve for a_1 and b_1 as function of the deep parameters of the model.

$$\begin{aligned} a_1 &= -\left[\left(\phi_\pi - 1\right)\rho\right]\kappa_q \frac{1}{\Lambda_1} < 0 \\ b_1 &= \left[\sigma\left(1-\rho\right) + \phi_y\right]\kappa_q \frac{1}{\Lambda_1} > 0 \end{aligned}$$

3. Solve for a_2 and b_2: similar to step 2, we take successive partial derivatives with respect to q_t and q_t to the two equations of step 1 and we evaluate them at the non-stochastic steady state. Then, we solve for the unknowns a_2 and b_2.

$$\begin{aligned} a_2 &= -\left[\left(\phi_\pi - 1\right)\rho^2\right]\left(\Omega_\pi + \Omega_{mc}\right)\frac{1}{\Lambda_2} < 0 \\ b_2 &= \left[\sigma\left(1-\rho^2\right) + \phi_y\right]\left(\Omega_\pi + \Omega_{mc}\right)\frac{1}{\Lambda_2} > 0 \end{aligned}$$

4. Solve for a_0 and b_0: similar to steps 2 and 3, we take successive partial derivatives with respect to σ_q and σ_q to the two equations of step 1 and we evaluate them at the non-stochastic steady state. Then, we solve for the unknowns a_0 and b_0. The solution for the coefficients is given by:

$$\begin{aligned} a_o &= -\left(\phi_\pi - 1\right)\left[\left(b_2\eta^2 + \omega_\pi\right) - \sigma\left(1-\beta\right)\left(a_2\eta^2 + \omega_y\right)\right]\frac{1}{\Lambda_0} \\ b_o &= -b_2\eta^2 + \left[\phi_y\left(b_2\eta^2 + \omega_\pi\right) + \sigma\kappa_y\left(a_2\eta^2 + \omega_y\right)\right]\frac{1}{\Lambda_0} \end{aligned}$$

where we have defined the following auxiliary variables:

$$\begin{aligned}\Lambda_0 &= (\phi_\pi - 1)\,\kappa_1 + (1-\beta)\,\phi_y \\ \Lambda_1 &= (\phi_\pi - 1)\,\rho\kappa_y + (1-\beta\rho)\left[\sigma\,(1-\rho) + \phi_y\right] \\ \Lambda_2 &= (\phi_\pi - 1)\,\rho^2\kappa_y + \left(1-\beta\rho^2\right)\left[\sigma\,(1-\rho)^2 + \phi_y\right]\end{aligned}$$

where Λ_0, Λ_1, and Λ_2 are all positive.

B.4.1 The Inflation premium

The inflation premium is given by:

$$E\,(\pi) = \frac{1}{2}\,(b_o + b_2)\,\sigma_q^2$$

replace the solution for b_o:

$$b_o + b_2 = b_2\rho^2 + \frac{\phi_y\left(b_2\eta^2 + \omega_\pi\right) + \sigma\kappa_y\left(a_2\eta^2 + \omega_y\right)}{\Lambda_0} \tag{B.45}$$

Replace the solution of a_2 and the definition of η, and collect for b_2 :

$$b_o + b_2 = \frac{1}{\Lambda_0}\left\{b_2\left[\rho^2\Lambda_o + \left(\phi_y - \sigma\kappa_y\frac{\phi_\pi - 1}{\sigma\,(1-\rho^2) + \phi_y}\right)\left(1-\rho^2\right)\right] + \phi_y\omega_\pi + \sigma\kappa_y\omega_y\right\} \tag{B.46}$$

After some algebra, it can be expressed as:

$$b_o + b_2 = \frac{1}{\Lambda_0}\left\{\frac{b_2\phi_y}{\sigma\,(1-\rho^2) + \phi_y}\left[\Lambda_2 + 2\left(1-\beta\rho^2\right)\sigma\,(1-\rho)^2\right] + \phi_y\omega_\pi + \sigma\kappa_y\omega_y\right\} \tag{B.47}$$

Replace the definition for b_2 :

$$b_o + b_2 = \frac{1}{\Lambda_0}\left\{\phi_y\,(\Omega_\pi + \Omega_{mc} + \Omega_v)\,(1+\Psi) + \phi_y\omega_v + \sigma\kappa_y\omega_y\right\} \tag{B.48}$$

where $\Psi = 2\left(1-\beta\rho^2\right)\sigma\,(1-\rho)^2/\Lambda_2$. Ψ is positive and very small for ρ close to 1

C Appendix: Endogenous Trade-off

From equation (B.24), we can derive linearly the marginal cost as function of output and oil price shocks, as follows:

$$mc_t = \frac{\left(1-\alpha^F\right)(\sigma+v)}{1+v\psi\alpha^F} y_t + \alpha^F \frac{(1+v\psi)}{1+v\psi\alpha^F} q_t + O\left(\|q_t, \sigma_q\|^2\right) \tag{C.1}$$

This equation can be also written in terms of parameters κ_y and κ_q, defined previously in the main text, as follows:

$$mc_t = \frac{\kappa_y}{\kappa} y_t + \frac{\kappa_q}{\kappa} q_t + O\left(\|q_t, \sigma_q\|^2\right) \tag{C.2}$$

Under flexible prices, $mc_t = 0$. Condition that defines the natural level of output in terms of the oil price shock :

$$y_t^F = -\frac{\kappa_q}{\kappa_y} q_t + O\left(\|q_t, \sigma_q\|^2\right) \tag{C.3}$$

Notice that in this economy the flexible price level of output does not coincide with the efficient one since the steady state is distorted by monopolistic competition The efficient level of output is defined as the level of output with flexible prices under perfect competition, we use equation $(C.2)$ to calculate this efficient level of output under the condition that $\mu = 1$ as follows:

$$y_t^E = -\frac{\alpha^E}{(1-\alpha^E)} \frac{\left(1-\alpha^F\right)}{\alpha^F} \frac{\kappa_q}{\kappa_y} q_t + O\left(\|q_t, \sigma_q\|^2\right) \tag{C.4}$$

Where $\alpha^E = \alpha^\psi \left(\overline{Q}\right)^{1-\psi}$. This parameter can be also expressed in terms of the participation of oil under flexible prices as follows:

$$\alpha^E = \alpha^F \mu^{\psi-1}$$

Notice that when there is no monopolistic distortion or when $\psi = 1$ we have that $\alpha^E = \alpha^F$ and $y_t^E = y_t^F$.

Using the definition of efficient level of output, we can write the marginal costs equation in terms an efficient output gap, x_t. Where $x_t = \left(y_t - y_t^E\right)$ in the following way

$$mc_t = \frac{\kappa_y}{\kappa}\left(y_t - y_t^E\right) + \frac{1}{\kappa}\mu_t + O\left(\|q_t, \sigma_q\|^2\right) \tag{C.5}$$

Where

$$\mu_t = \kappa_y \left(1 - \frac{\alpha^F}{(1-\alpha^F)} \frac{\left(1-\alpha^E\right)}{\alpha^E}\right) y_t^E$$

Using equations (C.5) and (3.24), the Phillips curve can be written as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_y x_t + \mu_t + O\left(\|q_t, \sigma_q\|^2\right) \tag{C.6}$$

This equation corresponds to equation (3.28) in the main text. We can further write μ_t in terms of the oil price shocks using the definition of the efficient level of output:

$$\mu_t = \frac{\kappa_q}{\kappa_y}\left(\frac{\alpha^F - \alpha^E}{(1-\alpha^E)\,\alpha^F}\right) q_t$$

The dynamic IS equation can also be written in terms of the efficient output gap.

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^E\right) + O\left(\|q_t, \sigma_q\|^2\right) \tag{C.7}$$

where r_t^E is the natural interest rate, the real interest rate consistent with y_t^E:

$$r_t^E = \sigma\,(1-\rho)\, y_t^E + O\left(\|q_t, \sigma_q\|^2\right)$$

which in turn can be written as follows:

$$r_t^E = -\sigma\,(1-\rho)\,\frac{\alpha^E}{(1-\alpha^E)}\,\frac{\left(1-\alpha^F\right)}{\alpha^F}\,\frac{\kappa_q}{\kappa_y} q_t + O\left(\|q_t, \sigma_q\|^2\right)$$

Notice that when there is no monopolistic distortion or when $\psi = 1$ we have that $\alpha^E = \alpha^F$, which implies that there is no an endogenous trade off.

$$\mu_t = 0 \;\forall t$$

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