The London School of Economics and Political Science

Essays in Financial Economics

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I declare that my thesis consists of **36456** words.

Abstract

This dissertation studies how financing shapes innovative entrepreneurship, and how promotions interact with the organization of knowledge in production.

The first chapter examines how venture capital affects entrepreneurial innovation in a multi-sector occupational choice model. Agents choose to work or to become entrepreneurs depending on labor market and venture capital financing conditions. A larger venture capital sector incentivizes entrepreneurship, but at the cost of fewer talented workers in the economy and higher wages that limit innovation. When a sector becomes more productive, demand for venture capital increases, raising monitoring costs across the economy. I finally show that distortions to R&D create spillovers through the market for venture capital. Competition for a shared pool of excludable ideas leads to excessive demand for venture capital, increasing monitoring costs for entrepreneurs and lowering innovation across all sectors.

The second chapter develops a model of early-stage financing in which entrepreneurs overcome uncertainty through experimentation and investors differ in scale, staging ability, and advising capability. The model shows that when investor advice and entrepreneurial effort reinforce each other by encouraging entrepreneurs to take on more risk, active investors back the most difficult ideas. When effort and advice substitute each other, investors shift to safer projects. The model helps explain the coexistence of diverse investors, the move of venture capital toward later stages, and the lack of active investors in "tough tech" sectors.

The third chapter studies how promotions interact with the organization of knowledge in production. Firms organize hierarchically when managers' expertise supports multiple workers, but prestige-driven promotion contests can distort production by diverting talent into inefficient lotteries, undermining knowledge complementarities between workers and managers across the economy. When promotion contests generate efficiency gains through, for instance, incentive effects, prestige can be socially valuable, especially in sectors with high communication costs.

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Chapter 1

Venture Capital, Entrepreneurship, and Innovation

How does venture capital affect innovative entrepreneurship? I present a multi-sector occupational choice model in which agents choose to work or to become entrepreneurs depending on labor market and venture capital financing conditions. My model shows how venture capital affects entrepreneurial innovation and the level of entrepreneurship in the economy. A larger venture capital sector lowers monitoring costs, leading to more entrepreneurs, each one less innovative. As more people choose entrepreneurship, labor costs rise, limiting entrepreneurial innovation. When a sector becomes more productive, demand for venture capital increases, raising monitoring costs across all sectors. Distortions to R&D create spillovers through the market for venture capital. Competition for a shared pool of excludable ideas leads to excessive demand for venture capital, increasing monitoring costs for entrepreneurs and lowering innovation. Reskilling workers across sectors lowers welfare, while R&D subsidies raise welfare by incentivizing some entrepreneurs to work.

1.1 Introduction

Venture capital financing in the U.S. grew over twentyfold during the last decades (Lerner and Nanda, 2020). Venture capital spurs technological progress by funding innovation (Lerner, 2000a), yet high-growth entrepreneurship and technological progress have been declining in every sector (Decker et al., 2016a). At the same time, venture capital has focused on software and consumer-business products, raising concerns about funding socially desirable sectors like renewable energy and materials (Lerner and Nanda, 2020).

The economic mechanisms underlying these trends remain unclear. The impact of venture capital on innovation depends on two effects. First, venture capitalists can add value to their firms – an intensive margin effect that is well documented.[1] Second, venture capital can change the decision of talented people to become entrepreneurs – an extensive margin effect that remains relatively unexplored (Da Rin et al., 2013), blurring our understanding of the effect of venture capital on innovation. How does venture capital affect the decision to become an entrepreneur and how innovative an entrepreneur becomes?

To answer this question, I present an occupational choice model with different sectors where agents choose to become workers or innovative entrepreneurs depending on labor market and venture capital financing conditions. Entrepreneurs hire workers to experiment and innovate. Venture capitalists finance and monitor entrepreneurs. My model endogenously determines entrepreneurship and the wage in each sector, the cost to monitor entrepreneurs, and the sectors financed in the economy.

The main mechanism of my model is the interplay between occupational choice, experimentation by entrepreneurs, and monitoring by venture capitalists. Agents choose to become workers or entrepreneurs. Entrepreneurs gather information about an idea's outcome through experimentation (Kerr et al., 2014), which reduces uncertainty and allows entrepreneurs to be more innovative. Experiments produce more information with more workers or higher research productivity. To operate, entrepreneurs obtain financing

[1] See for instance Lerner (2000a); Lerner and Nanda (2023); Nanda and Rhodes-Kropf (2017).

[1] See for instance Akcigit and Ates (2021, 2023), Akcigit and Goldschlag (2023), Hall and Woodward (2010), Ewens et al. (2020), Azoulay et al. (2020), Salgado (2020), and Bloom et al. (2020).

from venture capitalists, which requires costly monitoring. Entrepreneurs pay venture capitalists for monitoring, which influences entrepreneurial payoffs and thus agents' occupational choices. As the monitoring cost for entrepreneurs changes, the number of workers in each experiment shifts, affecting entrepreneurial innovation.

My model links venture capital to recent observations about innovative entrepreneurship. A larger venture capital sector increases entrepreneurship, but entrepreneurs are less innovative because labor costs rise. Therefore, if research productivity has declined, as Bloom et al. (2020) document, the simultaneous growth of the venture capital sector has mitigated the negative impact on entrepreneurship. Nonetheless, both trends imply less entrepreneurial innovation, which is consistent with more marginal patents, as Akcigit and Ates (2023) document. A new highly innovative sector – like artificial intelligence – increases the demand for venture capitalists' services in the economy, which raises the monitoring cost for entrepreneurs, crowding out financing and entrepreneurship from other sectors. The equilibrium is efficient but assumes that each entrepreneur has unique ideas, which is often not the case. When ideas are limited and excludable, the market has an inefficiently high demand for monitoring, raising the monitoring cost for entrepreneurs and creating a shortage of entrepreneurship and innovation in socially desirable sectors, which rationalizes the concerns of Lerner and Nanda (2023).

My main contribution is a framework to study how venture capital affects innovation. My model differs from the literature in two key ways.[2] First, I include the venture capital funding process in the decision to become an entrepreneur, while also considering labor market and technological factors as in standard models of occupational choice like Lucas Jr (1978). Second, while most venture capital research focuses on startup founders (Gompers et al., 2020; Kaplan and Strömberg, 2004), recent evidence by Choi et al. (2023) shows that early employees are crucial to a startup's success, creating a trade-off between allocating talent to entrepreneurship rather than labor. To the best of my knowledge, my model is the first to examine how venture capital and the trade-off in the allocation of talent interact.

[2]See for instance Lucas Jr (1978), Kihlstrom and Laffont (1979), Hall and Woodward (2010), Akcigit and Ates (2021), Hacamo and Kleiner (2022), and Akcigit and Ates (2023).

My model works as follows. The economy has different sectors, each characterized by a research productivity parameter and a cost to discover entrepreneurs. Each sector also has a competitive labor market and a mass of agents with heterogeneous entrepreneurial skill. Agents choose to become entrepreneurs or workers, which endogenously determines the supply and demand for workers. The wage clears the labor market. Agents in a sector represent highly-educated people with significant experience, but not top executives: they have the necessary human capital to become entrepreneurs, and their wages are low enough to consider entrepreneurship.

Workers supply a unit of labor time inelastically for a wage, while entrepreneurs obtain financing from venture capitalists to hire workers and pay the setup costs of an idea. Workers experiment to find the details of an idea. Experimentation reveals the details of the entrepreneur's idea and gives a signal about its outcome. Entrepreneurs decide to invest only after observing the signal. More innovative ideas have a higher upside and are costlier to set up, but the cost decreases with the skill of an entrepreneur.

An important feature of my model is that the innovativeness of an idea and experimentation complement each other. All ideas offer the same expected payoff without experimentation, but more innovative ideas have a wider payoff spread – a better upside and a worse downside. Experimentation reduces uncertainty and prevents investing in bad ideas, creating an option-like structure on payoffs. By reducing uncertainty, experimentation allows the entrepreneur to increase the payoff spread. More informative experiments result in more innovative entrepreneurs.

Venture capitalists provide two services: cash and monitoring. Cash is abundant, but monitoring is scarce. Venture capitalists pay a search cost to get randomly matched with an entrepreneur in a sector. Entrepreneurs only obtain funding if venture capitalists control the investment decision, which requires costly monitoring. The venture capitalist and the entrepreneur set up an efficient contract that maximizes the relationship's net surplus. Entrepreneurs give venture capitalists a share of net surplus and pay them the monitoring cost as a wage for their monitoring effort. In my model, the opportunity cost of financing an entrepreneur is not monitoring another entrepeneur.

Since the contract is efficient, the entrepreneur internalizes the monitoring cost. The monitoring cost for entrepreneurs is common across sectors, but entrepreneurs face different levels of research productivity and wages. Agents contemplating entrepreneurship compare their potential profits to two costs: the sector-specific opportunity cost, which is the wage, and the economy-wide monitoring cost.

In equilibrium, sectors with lower research productivity have fewer entrepreneurs, more conservative ideas, lower wages, and lower surplus. Lower research productivity makes experiments less informative, thereby lowering entrepreneurial profits. The monitoring cost entrepreneurs pay takes up a higher proportion of profits, lowering entrepreneurship. If monitoring were costless, entrepreneurship would be equal across sectors. Innovation declines when research productivity declines.

A larger pool of venture capitalists lowers monitoring costs for entrepreneurs in the economy, resulting in more entrepreneurs, each one less innovative. Wages increase with entrepreneurship, hence entrepreneurs run smaller firms. With fewer workers, experiments become less informative, reducing entrepreneurial innovation.

A shock that makes research in a single sector significantly more productive – a technological boom – lowers entrepreneurship in other sectors. Higher research productivity drives entrepreneurship up in the booming sector, increasing the demand for venture capitalists' monitoring services. As monitoring costs rise, entrepreneurship declines in other sectors, but those entrepreneurs who remain become more innovative, as they can run larger firms. Therefore, a booming sector implies venture capitalists finance fewer sectors. If, during a technological boom, potential venture capitalists become less skilled on average – for example, due to the sudden entry of inexperienced venture capitalists – negative spillovers to other sectors are amplified.

The equilibrium of my benchmark model is efficient, but several distortions can affect the development of new ideas. Often, ideas are not unique, with multiple entrepreneurs competing to bring the same idea to market. Innovation incentives can create races, leading to inefficiencies like the duplication of efforts (Reinganum, 1989). Moreover, Hill and Stein (2019, 2021) empirically show that priority races decrease the quality of ideas in science. Since entrepreneurs funded by venture capital have incentives to race to innovate or win the market, I study the policy implications of competition for a shared and excludable pool of resources – ideas, data, or infrastructure. Excessive entrepreneurship within a sector reduces each entrepreneur's chances of success.

When ideas are limited and excludable, the market generates an inefficiently high demand for venture capital monitoring services. The excessive demand for venture capital leads to reduced innovation and a shortage of entrepreneurs in socially desirable sectors. In the competitive market allocation, high research productivity sectors demand too much venture capital, inefficiently increasing the monitoring cost for all entrepreneurs. As a result, entrepreneurship in low research productivity sectors is below the planner's optimal level.

I evaluate three policy tools to address the inefficiency. First, a sector-specific profit tax implements the first best, but taxing innovation seems unrealistic. Second, incentivizing workers to switch sectors reduces overall welfare. Workers move to high-wage sectors, where there is also excess entrepreneurship. Lowering wages increases entrepreneurship, exacerbating the inefficiencies. Third, R&D subsidies can increase welfare by incentivizing excess entrepreneurs to work, but do not achieve the first best. R&D subsidies boost profits, but also raise wages. For high research productivity sectors, wages increase proportionally more than profits, which incentivizes entrepreneurs to work.

The literature on venture capital shows its positive impact on the economy and its firms – the intensive margin of innovation. Fewer than 0.5% of firms in the economy receive venture capital (Puri and Zarutskie, 2012), but these firms account for almost half of those that go public and 90% of the R&D spending of public firms (Lerner and Nanda, 2020). Kortum and Lerner (1998) find that venture capitalists increase the rate of patenting of firms and that they might be responsible for almost 8% of industrial innovations. Bernstein et al. (2016) find that venture capital actively creates value through monitoring. Different from these studies, I study how entrepreneurial innovation interacts with the level of entrepreneurship when there is a limited amount of talent in the economy.

The decision of talented individuals to become innovative entrepreneurs – the extensive margin of innovation – depends on technological, labor, and financing conditions. Ewens et al. (2020) show that for venture capital-backed entrepreneurs, wages, net wealth, and the speed to resolve uncertainty are important factors in selecting into entrepreneurship. Hall and Woodward (2010) show that the rewards to entrepreneurs backed by venture capital are small when considering idiosyncratic risk. Hacamo and Kleiner (2022) show there is untapped entrepreneurial potential across workers at the top of the income distribution. Samila and Sorenson (2011) shows that venture capital encourages entrepreneurship in local areas by lifting financing constraints. Finally, Choi et al. (2023) show the relevance of early workers for the outcomes of startups, suggesting the availability of talent is also an important determinant of entrepreneurship. My model incorporates the labor market and venture capital financing conditions into a common framework that determines selection into entrepreneurship.

1.2 A benchmark multi-sector innovative economy

Consider N sectors $i \in \{1 \dots N\}$. Each sector has a unit mass of agents with specialized skills who choose to work as researchers or become entrepreneurs and innovate. Entrepreneurs obtain financing from venture capitalists.

1.2.1 Timing

There are three periods and no discounting.

$t = 0$ **-** There is a unit mass of penniless risk-neutral agents in each sector i. Agents choose to become entrepreneurs or workers. Workers supply one unit of time inelastically for a wage w_i. Entrepreneurs seek financing from venture capitalists to experiment and potentially implement an idea of uncertain quality q at $t = 1$. There is a mass $V > N$ of venture capitalists that each finance an entrepreneur in exchange for a share of net surplus a_i and control of the investment decision at $t = 1$, which requires costly monitoring.

$t = 1$ **-** Experimentation yields the details to implement an idea and a signal $\mathcal{S}$ about the idea's quality, which is used to make an investment decision.

$t = 2$ - Payoffs realize. Figure 1.1 shows the timing of the model.

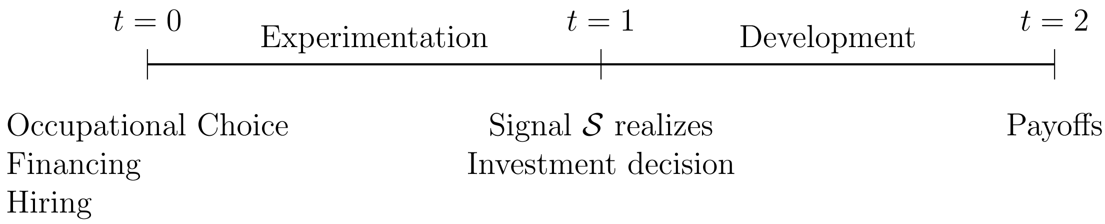


Figure 1.1: Timing of the model.

1.2.2 Ideas

When entrepreneurs seek financing from venture capitalists in $t = 0$, the details to implement an idea are unknown. For instance, an entrepreneur might want to create a new drug for weight-loss, or a better recommendation algorithm for online shopping, but the idea's implementation details and prospects are unknown.

Implementing any idea costs I at $t = 1$. An idea can be of good or bad quality $q \in \{G, B\}$ in $t = 2$ with equal probability. The quality of an idea and its payoff spread D characterize its revenue – a good idea has a revenue of $\mathrm{R}_G(D) = I + 2\sqrt{D}$ in $t = 2$, while a bad idea has a revenue of $\mathrm{R}_B(D) = I - 2\sqrt{D}$. The payoff spread D represents entrepreneurial innovation and is an intensive margin measure of innovation in my model. For instance, the payoff spread represents how much an entrepreneur deviates from the status-quo $D = 0$, increasing his upside while worsening his downside at the same time.

Each entrepreneur hires a mass of workers H at $t = 0$ to experiment with an idea. Experimentation yields a symmetric binary signal $\mathcal{S} \in \{\mathcal{S}_G, \mathcal{S}_B\}$ at $t = 1$ that informs the entrepreneur about the quality of the idea. Symmetry requires that the signal precision $\mathbb{P}(q|\mathcal{S}_q, H)$ is the same for both signal realizations, which I characterize by $p(H)$ such that $p(H) = \mathbb{P}(G|\mathcal{S}_G, H) = \mathbb{P}(B|\mathcal{S}_B, H)$. Without labor, signals are uninformative $p(0) = 0.5$. A more informative experiment implies a more precise signal $p(H)$, and signal precision is increasing with labor $p_H(H) > 0$. Given the signal is symmetric and the idea can be good or bad with equal probability, the probability of getting a good signal at $t = 1$ is 0.5.

The signal structure implies that an idea without experimentation produces expected revenue at $t = 1$ equivalent to the investment cost $\mathrm{E}_1[\mathrm{R}_q(D)|\mathcal{S}_q, H = 0] = I$. Moreover, a good signal implies $\mathrm{E}_1[\mathrm{R}_q(D)|\mathcal{S}_G, H] > I$ and vice versa. Figure 1.2 exemplifies the payoffs of an idea with a good signal.

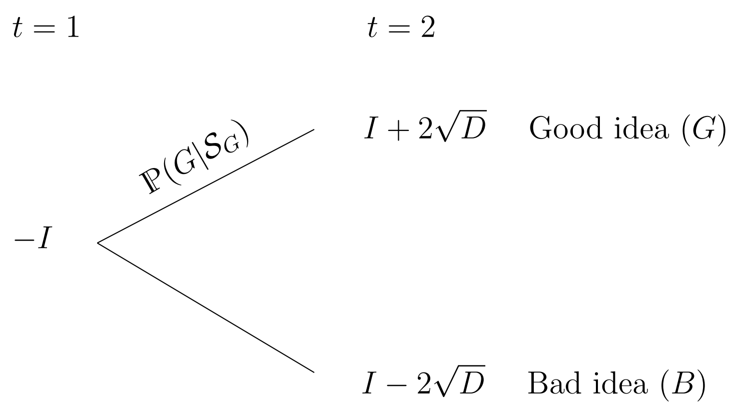


Figure 1.2: Payoffs of an idea given a good signal G.

Assuming that entrepreneurs lack a tangible idea when meeting venture capitalists is consistent with the evidence we have. Kerr et al. (2014) show evidence that for one large successful venture capitalist, the outcome of their startups is uncorrelated to their scores at the initial stage of investment, suggesting venture capitalists do not know who will succeed before experimenting, but retain the option to abandon ideas that yield negative signals. In selecting investments, venture capitalists attribute the success of a firm more to the team than to the business, as shown by Gompers et al. (2020). Moreover, Gompers et al. (2010) show that venture capitalists focus on the past success of an entrepreneur when evaluating investments. The current evidence suggests that venture capitalists select investments based mainly on the entrepreneurial skill of the team, consistent with my model.

1.2.3 Agents

Agents belong to a sector i, have an outside option within the same sector with income normalized to 0, and cannot switch sectors.[3] Agents have a unit labor endowment and entrepreneurial skill κ according to a smooth, continuous and bounded distribution with a cumulative distribution function $F(\kappa)$.

Agents choose to work or to become entrepreneurs within their sector. Workers are homogeneous and supply their labor inelastically for a wage w_i. Entrepreneurs choose how innovative to be, represented by the idea's payoff spread D, and how many workers to hire H to research the details of an idea.

An entrepreneur pays three costs to implement an idea. To choose an idea's payoff spread D, an entrepreneur pays a setup cost in $t = 0$ that is quadratic on D and decreasing on skill κ: $\frac{D^2}{\kappa}$.[4] To research the details of an idea, the entrepreneur pays a wage w_i to each worker $t = 0$. If the entrepreneur implements the idea at $t = 1$, he pays the investment cost I and gets the idea's revenue R_q at $t = 2$.

Then, an entrepreneur with skill κ and H workers has expected profits at $t = 0$:

$$\mathrm{E}_0[\Pi_i(\kappa, D, H)] = \mathrm{E}_0[\mathbb{1}_{\mathcal{S}}(\mathrm{R}_q(D) - I)|H] - w_i H - \frac{D^2}{\kappa} \tag{1.1}$$

where $\mathbb{1}_{\mathcal{S}} = 1$ if the signal $\mathcal{S}$ is such that the entrepreneur implements the idea in $t = 1$, and $\mathbb{1}_{\mathcal{S}} = 0$ if not. The entrepreneur's problem and the implementation decision depend on the financing contract from venture capitalists, which determines payoffs and control rights for each party.

1.2.4 Venture capitalists

There is a competitive sector of venture capitalists with free entry. Each venture capitalist has deep pockets and can monitor a single entrepreneur for a monitoring effort ϵ uniformly

[3]I assume agents cannot switch sectors for simplicity. Workers could switch sectors for a cost s, but my qualitative results remain the same. For simplicity, I consider $s = \infty$ in the benchmark scenario. I show how to allow workers to switch sectors in section 1.4.5.

[4]While it is tempting to interpret κ as risk aversion, it is not the right interpretation in my model. Higher entrepreneurial skill κ allows an entrepreneur to increase his upside by having a higher payoff spread D, but payoff spread is only one component of the risk of an idea. The adequate notion of risk for innovative entrepreneurs is idiosyncratic risk (Hall and Woodward, 2010). Idiosyncratic risk, measured as the variance of payoffs, has two components in a binomial outcome like my model: The probability of success, determined by research productivity θ_i and hiring H, and the payoff spread D. In canonical models of selection into entrepreneurship based on risk-aversion, such as Kihlstrom and Laffont (1979), positive technological shocks lead to more entrepreneurship because projects become safer. In my model, positive technological shocks also lead to more entrepreneurship, but the risk of projects can increase or decrease. Therefore, I interpret κ as entrepreneurial skill, such that agents select into entrepreneurship based on skill as in Lucas Jr (1978) and Hacamo and Kleiner (2022).

distributed in $[0, \bar{\epsilon}]$, $U \sim [0, \bar{\epsilon}]$.[5]

Venture capitalists pay a search cost $u(\#\ \text{Entrep}_i)$ to find a random entrepreneur in sector i. $u(\cdot)$ is decreasing in the amount of entrepreneurs in a sector ($\#\ \text{Entrep}_i$) – it is easier to find an entrepreneur when there are many of them. This is a reduced form search cost that accounts for the costs of generating deal flow. Gompers et al. (2020) show that venture capitalists report an average of 22 hours per 55 hour week spent sourcing deals, with most venture capitalists giving higher importance to deal flow and selection than to post-investment value added. Therefore, deal sourcing and selection amounts to a significant cost that is captured by $u(\cdot)$.

After finding an entrepreneur, the venture capitalist spends effort ϵ to monitor the entrepreneur's activities. The effort cost of the venture capitalist accounts for activities such as hiring a board member and processing information to evaluate an idea. Venture capitalists can offer their monitoring services to other venture capitalists for a wage m. The wage m then sets the outside opportunity for venture capitalists, and entrepreneurs have to pay at least m to venture capitalists to obtain funding and be monitored. Therefore, m is the monitoring cost for entrepreneurs and the wage that venture capitalists receive for their monitoring efforts.

The cost of monitoring m for entrepreneurs is equal across all sectors and set in equilibrium.[6] Since venture capitalists are heterogeneous in their cost of effort ϵ, they earn surplus $(m - \epsilon)$ from their monitoring activities. Monitoring is a key activity for venture capitalists. Lerner and Nanda (2020) show that venture capitalists sit on 6.1 boards on average, while Gompers et al. (2020) show that venture capitalists report an average of 18 hours per 55 hour week spent in monitoring activities. Bernstein et al. (2016) show that lower monitoring costs increase the value that a venture capitalist adds to their entrepreneurs, and Chemmanur et al. (2011) find that monitoring increases the efficiency of venture capital-backed firms.

[5] I use a uniform distribution for venture capitalists for simplicity, but it can be generalized to a continuous distribution $G(\cdot)$.

[6] I only consider a fixed monitoring cost for entrepreneurs in my main specification for simplicity. My model can accommodate a monitoring cost with a fixed and a variable component that depends on entrepreneurial characteristics. Section 1.B shows a specification in which more innovative ideas and less capable entrepreneurs are costlier to monitor. This specification allows for the venture capitalist

Financing contract

Gompers et al. (2020) and Kaplan and Strömberg (2003, 2004) empirically show that venture capitalists and entrepreneurs bargain over cash flow rights, however, venture capitalists are inflexible when it comes to control and liquidation rights.

Control The venture capitalist has control of the investment decision. Control can also be randomly allocated between the entrepreneur and the investor, but the results are qualitatively the same. Section 1.D shows the details of this setup.[7] Since a good signal implies $\mathrm{E}_1[\mathrm{R}_q(D)|\mathcal{S}_G, H] > I$ and vice versa, the venture capitalist implements the idea only when the signal is good: $\mathbb{1}_{\mathcal{S}} = 1$ when $\mathcal{S} = \mathcal{S}_G$.

Cash flows The venture capitalist gets share a_i of the expected net surplus $\mathrm{E}_0[S_i(\cdot)]$.[8] The share a_i is set in equilibrium. Net surplus is the revenue of an idea net of costs and outside opportunities for the entrepreneur and venture capitalist. Since the agent always has the option to work, his outside option is the wage w_i. Similarly, the venture capitalist gets a wage m for his monitoring effort, which is his outside opportunity. Consider an entrepreneur that hires H workers and has an idea D. The expected net surplus at $t = 0$ is

$$\mathrm{E}_0[S_i(\kappa, H, D)] = \underbrace{\mathrm{E}_0[\mathbb{1}_{\mathcal{S}}(\mathrm{R}_q(D) - I)|H] - w_i H - \frac{D^2}{\kappa}}_{\mathrm{E}_0[\Pi_i(\kappa, H, D)]} - w_i - m \tag{1.2}$$

The contract's payoffs are such that in expectation each party gets their share of surplus plus its outside opportunity and invested money. The venture capitalist covers all costs, so the contract ensures that the venture capitalist gets her invested money in full before splitting anything with the entrepreneur. The contract's cash flows at $t = 2$ to

to explicitly add value beyond just monitoring. Nonetheless, the qualitative conclusions of the model remain unchanged.

[7] Intuitively, suppose the entrepreneur controls the investment decision. Entrepreneurs have a downside of 0 but the upside is always positive, hence they invest regardless of the signal. Anticipating that entrepreneurs invest when they have control, venture capitalists expect lower revenue.

[8] I assume for simplicity that a_i is the same for every entrepreneur in the sector, but this is not necessary. Section 1.F shows some examples of bargaining powers that depend on entrepreneurial skill $a_i(\kappa)$ with the equilibrium condition that determines equilibrium bargaining power.

each party $\{E, VC\}$ depending on idea quality q, $\left(C^{E}_{2,q}, C^{VC}_{2,q}\right)$, are such that the expected cash flows from the contract at $t = 0$ are

$$\mathrm{E}_0\left[C^{E}_{2,q}(\kappa)\right] = (1 - a_i)\,\mathrm{E}_0[S_i(\kappa, H, D)] + w_i \tag{1.3}$$

$$\mathrm{E}_0\left[C^{VC}_{2,q}(\kappa)\right] = a_i\,\mathrm{E}_0[S_i(\kappa, H, D)] + m + w_i H + \frac{D^2}{\kappa} + \mathrm{E}_0[I\mathbb{1}_{\mathcal{S}}|H] \tag{1.4}$$

Notice the sum of the expected cash flows to each party are exactly equal to the expected revenue of the idea. Therefore, no matter what the cash flow structure is at $t = 2$, in expectation the entrepreneur and the venture capitalist share the total revenue such that the venture capitalist gets its invested money back ($w_i H + \frac{D^2}{\kappa} + \mathrm{E}_0[I\mathbb{1}_{\mathcal{S}}|H]$), and each party gets its outside opportunity before splitting anything. Figure 1.3 shows the timing of cash flows for the venture capitalist.

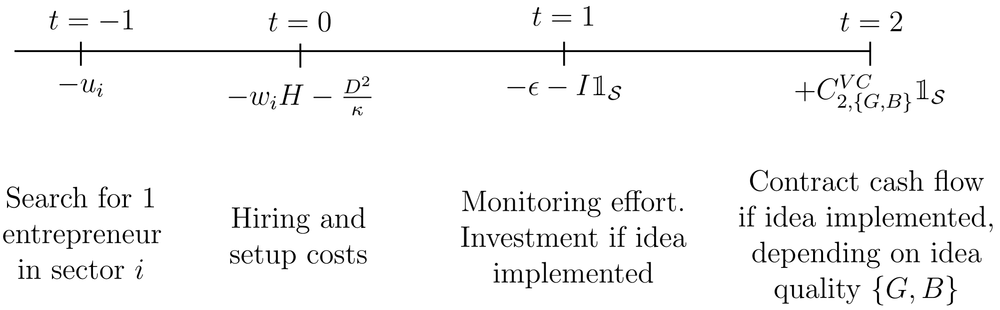


Figure 1.3: Timing of cash flows for venture capitalist

Since the venture capitalist incurs costs in every period, the payoffs of each party $\left(\mathrm{P}^{E}_{q}, \mathrm{P}^{VC}_{q}\right)$ are such that in $t = 0$:

$$\mathrm{E}_0[\mathrm{P}^{E}_{q}(\kappa)] = (1 - a_i)\,\mathrm{E}_0[S_i(\kappa, H, D)] + w_i \tag{1.5}$$

$$\mathrm{E}_0[\mathrm{P}^{VC}_{q}(\kappa, \epsilon)] = a_i\,\mathrm{E}_0[S_i(\kappa, H, D)] + m - \epsilon - u_i \tag{1.6}$$

The payoffs to the venture capitalist in equation (1.6) reflect that she recovers the entrepreneurs' operating costs, but also that she exerts effort ϵ to monitor and a cost u_i to find an entrepreneur in sector i. Summing both payoffs and considering that agents who choose to work get the wage shows that the deadweight costs in the economy are the setup cost of an idea $\frac{D^2}{\kappa}$, the investment cost I if the idea is implemented, the venture capitalist effort cost ϵ, and the search cost u_i. The wage w and the monitoring cost m are transfers between decision-makers. Importantly, the search cost u_i is not incorporated in the contract since u_i is sunk once the entrepreneur and the venture capitalist meet.

The financing contract is efficient, since the entrepreneur internalizes every cost of the relationship. The contract structure only determines payoffs in expectation, hence any cash flow structure in $t = 2$ that meets equations (1.3) and (1.4) is feasible. A variety of contracts can implement the desired control and cash flow structure given the absence of moral-hazard, asymmetric information, signal manipulation or any form of private costs.

The set of contracts that implement the expected payoffs includes the most common contracts used by venture capitalists – preferred equity, either convertible or with participation rights (Gornall and Strebulaev, 2023; Sahlman, 1990; Kaplan and Strömberg, 2003).[9] For instance, Kaplan and Strömberg (2003) show that venture capitalists use preferred equity in 189 of the 200 financing rounds in their sample, with participation rights in 72 of those. A natural implementation of the contract's expected payoff structure is a preferred equity contract. The venture capitalist gets her monitoring cost and invested money as a liquidation preference, and anything extra as an equity share depending on her bargaining weight a_i.

Entry condition

Venture capitalists enter a sector until the expected payoff to the marginal venture capitalist $\widehat{\epsilon}$ is 0, $\mathrm{E}_0[\mathrm{P}_q^{VC}(\kappa, \widehat{\epsilon})] = 0$:

$$\underbrace{(a_i \, \mathrm{E}_0[S_i(\kappa, H, D)] - u_i)}_{\substack{\text{Payoff from} \\ \text{search}}} + \underbrace{(m - \widehat{\epsilon})}_{\substack{\text{Payoff from} \\ \text{monitoring}}} = 0 \tag{1.7}$$

A venture capitalist's expected payoff can be understood as the payoff from searching an entrepreneur $(a_i \, \mathrm{E}_0[S_i(\kappa, H, D)] - u_i)$ plus the payoff from monitoring activities $(m - \epsilon)$. Finding an entrepreneur gives fraction a_i of the net surplus to the venture capitalist,

[9] Section 1.C shows the implementation of the expected payoffs using preferred equity with participation, preferred equity with convertibility, risky debt, and an equity contract.

while monitoring effort earns her the monitoring cost m. Since venture capitalists add value through monitoring and post-investment activities, I assume the payoff from search is 0.

Assumption 1.1 (Monitoring Profits). *Venture capitalists only profit from monitoring activities.*

Assumption 1.1 and the free entry condition in equation (1.7) determine the marginal venture capitalist that enters the market as $\widehat{\epsilon} = m$. Assumption 1.1 implies $(a_i \, \mathrm{E}_0[S_i(\kappa, H, D)] - u_i) = 0$, which sets the bargaining power a_i in equilibrium. Assumption 1.1 can be relaxed to allow for profits from search, which then implies the marginal venture capitalist from the free entry condition. Assumption 1.1 is without loss of generality to keep expressions simple – the marginal venture capitalist is solely determined by the monitoring cost.

1.2.5 An entrepreneurial sector

A sector i is characterized by its research productivity $\theta_i \in (0, 1)$, and how costly it is for venture capitalists to discover all entrepreneurs in that sector $\Delta_i > 0$. A sector is a tuple $(\theta_i, \Delta_i) \in (0, 1) \times R_+$ and a unit mass of risk-neutral agents.

Definition 1.1 (Sector). *A sector i is a tuple $(\theta_i, \Delta_i) \in (0, 1) \times R_+$ where*

- *θ_i is the productivity of research*
- *Δ_i is the cost to discover all entrepreneurs,*

and a unit mass of agents with heterogeneous entrepreneurial skill κ according to a bounded distribution with cdf $F(\kappa)$.

A sector operates on three key premises. First, occupational choice endogenously determines the wage of a sector via labor market clearing. In equilibrium, agents who decide to work are hired by agents who become entrepreneurs. Hence, the opportunity

[9] Assumption 1.1 is important as a second condition to pin down bargaining weights a_i and the marginal venture capitalist $\widehat{\epsilon}$. Other conditions suffice. For instance, one can assume that bargaining weights are set before nature reveals the venture capitalist types ϵ, hence, only the expected effort cost of the monitoring agent matters to set a_i. Then, given a_i, equation (1.7) determines $\widehat{\epsilon}$. Since my focus is on the interaction of the venture capital market with entrepreneurial sectors, I pick the simplest condition.

cost of entrepreneurship is the wage, consistent with the evidence shown by Ewens et al. (2020) and Hall and Woodward (2010) for highly educated workers.

Second, experimentation as a key feature of innovative entrepreneurs (Kerr et al., 2014). Entrepreneurs hire highly educated workers to research an idea, but entrepreneurs in sectors with lower research productivity face less informative experiments. Hence, these entrepreneurs need more workers to achieve a desired payoff. The need for research highlights the role of experimentation in R&D, since entrepreneurs' investments depend on the research workers generate.

Third, entrepreneurs pay the monitoring cost. Monitoring prevents the entrepreneur from making bad investments with venture capital money. Importantly, the entrepreneur bears the cost of monitoring, so entrepreneurs must produce at least enough surplus to pay m to venture capitalists.

Occupational choice, experimentation, and monitoring determine entrepreneurship and surplus in a sector. I focus on a sector's surplus as an overall measure of innovation, which can be decomposed in the share of innovative entrepreneurs as the extensive measure of innovation, and the payoff spread of ideas as an intensive measure of innovation.

Researching an idea

Workers increase the precision of the signal, reducing the uncertainty of an idea's outcome.

Assumption 1.2 (Signals are Informative)**.** *Signal precision with H workers and research productivity θ_i is such that:*

$$p(H) = \frac{1}{2}\left(1 + \theta_i\sqrt{H}\right) \tag{1.8}$$

Assumption 1.2 specifies a functional form that links signal informativeness to hiring, such that the signal is symmetric and informative. Whenever an entrepreneur hires workers to research ($H > 0$), the probability of the signal pointing to the true quality of the idea is more than 50%, which makes the signal informative of the outcome. As desired, a good signal implies an expected revenue that is larger than the investment

$\mathrm{E}_1[\mathrm{R}_q \,|\mathcal{S}_G, H > 0] > I$, and vice versa.

Assumption 1.2 also shows that lower research productivity implies less informative experiments and that there are decreasing returns to hiring. Figure 1.4 shows signal informativeness against hiring H for different levels of research productivity θ_i. At any level of hiring, marginal returns to research are lower for lower θ sectors.

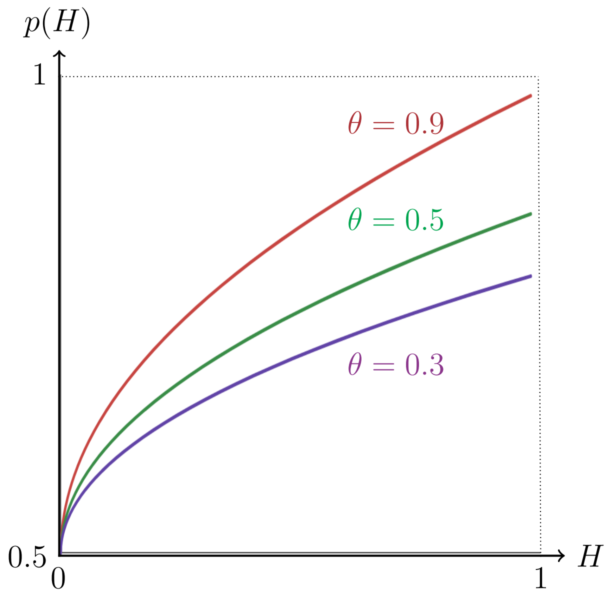


Figure 1.4: Signal precision for different levels of research productivity $\theta \in \{0.3, 0.5, 0.9\}$. Note the y-axis starts at 0.5.

Entrepreneur's problem

Managing the firm uses the entrepreneur's labor endowment.[10] Entrepreneurs are price takers and maximize their expected payoff in equation (1.5): fraction $(1 - a_i)$ of the expected net surplus $\mathrm{E}_0[S_i(\cdot)]$ plus the wage. Entrepreneurs choose hiring H and the payoff spread D of their idea. An entrepreneur with skill κ solves at $t = 0$:

$$\max_{H,D}(1 - a_i)\,\mathrm{E}_0[S_i(\kappa, H, D)] + w_i \tag{1.9}$$

Define the optimal solutions to the entrepreneur's problem in equation (1.9) as $H(\kappa)$

[10]This assumption can be relaxed as long as entrepreneurs cannot put their full unit of time into research. I keep the assumption in its extreme for simplicity.

and $D(\kappa)$, and the optimal expected net surplus as

$$\mathrm{E}_0[S_i^*(\kappa)] = \mathrm{E}_0[S_i^*(\kappa, H(\kappa), D(\kappa))] \tag{1.10}$$

.

All entrepreneurs in a sector have the same outside opportunity w_i even though they have different entrepreneurial skill and there is symmetric information. Two reasons justify this modeling choice. First, as Lazear and Oyer (2010) explain, finding the right compensation schemes for workers "appears to be largely about finding the right job for the person rather than finding the right pay for different people doing similar jobs". As there is only one job for workers in my model, it is reasonable to assume they earn the same wage. Second, section 1.E shows a version of the model similar to that of Hacamo and Kleiner (2022), in which agents can be high or low productivity workers, and entrepreneurial and working skills are correlated. This setup complicates the analysis, but the qualitative conclusions remain the same.

Agents as highly educated people

I interpret agents in each sector of my model as highly educated people in the mid-levels of their company, for instance, senior engineers or senior programmers. Agents have a decade of experience and the technical expertise to implement innovative ideas. For these agents, as shown by Ewens et al. (2020) and Hall and Woodward (2010), the main opportunity cost of entrepreneurship is their wage.

Innovative entrepreneurs are mainly financed by venture capitalists (Hurst and Pugsley, 2011; Ewens et al., 2020; Lee, 2014; Azoulay et al., 2020; Kaplan and Lerner, 2010). Ewens et al. (2020) show that the typical founder of startups backed by venture capital has 12 years of experience, is from the mid-levels of his company, and is highly educated, with 50% holding a master's degree and 20% an MBA.

Having the right amount of experience in a sector is important for agents to consider entrepreneurship. Agents with too much experience in a sector earn a wage that is too high to justify entrepreneurship (Hall and Woodward, 2010). On the other hand, young workers lack the necessary skills to become entrepreneurs. Young workers acquire the skills to start their own company through employment, which acts as a source of ideas and training for potential entrepreneurs (Babina et al., 2017; Gompers et al., 2005; Kim, 2018). After a decade of work, agents in a sector have a wage that is not too high and the technical know-how to start their own company. Thus, these agents face an occupational choice: to keep working, or to start their own business.

Occupational choice

By risk-neutrality, agents choose the occupation with the highest payoff. In case of indifference, agents can play any mixed strategy. Since the efficient contract pays the wage to the entrepreneur before sharing the net surplus, any agent who produces a positive expected optimal surplus $E_0[S^*(\kappa)]$ becomes an entrepreneur.

Lemma 1.1 (Occupational Choice). *An agent becomes an entrepreneur if $E_0[S_i^*(\kappa)] > 0$, works if $E_0[S_i^*(\kappa)] < 0$, and mixes with any probability if indifferent.*

Lemma 1.1 implies that an agent becomes an entrepreneur whenever his profits are higher than the wage plus the monitoring costs:

$$E_0[S_i^*(\kappa)] > 0 \Leftrightarrow \mathrm{E}_0[\Pi_i^*(\kappa)] > w_i + m \tag{1.11}$$

I interpret $(w_i + m)$ as the cost of entrepreneurship in my model. The wage is a sector-specific opportunity cost of entrepreneurship, while the monitoring cost is common to all sectors. As an entrepreneur, the agent must produce enough surplus to get the wage and pay the monitoring cost to the venture capitalist. Otherwise, the agent is better off as a worker.

A sector's labor market

Let $\mathcal{E}_i$ be the set of skills of agents who become entrepreneurs in sector i, such that $\mathcal{E}_i = \{\kappa | E_0[S_i^*(\kappa)] > 0\}$, and $\mathcal{E}_i^c$ its complement – the skills of agents who become workers.

Then, the wage w_i clears the labor market:

$$\int_{\mathcal{E}_i} H(\kappa)\mathrm{d}F(\kappa) = \int_{\mathcal{E}_i^c} \mathrm{d}F(\kappa) \tag{1.12}$$

Labor demand is the aggregate hiring of entrepreneurs. Labor supply is the amount of agents that choose to work.

A sector's search cost

Venture capitalists pay $u(\mathcal{E}_i)$ to find an entrepreneur in sector i. In equilibrium, the aggregate search cost paid by venture capitalists has to equal the aggregate cost to discover all entrepreneurs in sector i, Δ_i. Then,

$$u(\mathcal{E}_i) = \frac{\Delta_i}{\int_{\mathcal{E}_i} dF(\kappa)} \tag{1.13}$$

Equation (1.13) reflects that a thicker market makes it easier for venture capitalists to find an entrepreneur. Higher entrepreneurship $\int_{\mathcal{E}_i} dF(\kappa)$ in sector i implies a smaller search cost $u(\cdot)$, and vice versa.[11]

1.2.6 The monitoring market

The monitoring cost m clears the monitoring market:

$$\sum_{i}^{N} \int_{\mathcal{E}_i} dF(\kappa) = V\frac{m}{\bar{\epsilon}} \tag{1.14}$$

The demand for monitoring is the mass of entrepreneurs in the economy because each venture capitalist monitors one entrepreneur. The supply of monitoring is the mass of venture capitalists that have an effort cost smaller than the marginal venture capitalist $\widehat{\epsilon} = m$, given a uniform distribution of skill and a total mass of venture capitalists V.

A larger pool of potential venture capitalists V is equivalent to increasing the average

[11]The functional form of $u(\mathcal{E}_i)$ in equation (1.13) ensures market equilibrium efficiency. I adopt this form to focus on the impact of venture capital on entrepreneurship, rather than on the influence of search frictions on entrepreneurship.

skill of venture capitalists by lowering $\bar{\epsilon}$. Which force dominates – size vs average skill – in the venture capital sector ultimately determines the overall change in the supply of monitoring for a given monitoring cost m.

1.2.7 N sectors equilibrium

The equilibrium requires that the market for monitoring clears across sectors and that in each sector: agents behave optimally, venture capitalists earn zero expected profits, and the labor market clears. Figure 1.5 shows and summarizes the economy.

Definition 1.2 (Multi-Sector Equilibrium)**.** *Consider N sectors $(\theta_i, \Delta_i)_{i\in\{1...N\}}$, with a common distribution of entrepreneurial skill $F(\kappa)$. An equilibrium consists of a monitoring cost for entrepreneurs m, and for each sector: a set of entrepreneurs $\mathcal{E}_i$, a rule to hire workers $H(\kappa)$, a rule for the payoff spread $D(\kappa)$, a wage w_i, a bargaining weight a_i, and a search cost $u(\mathcal{E}_i)$. The equilibrium is such that:*

1. *Each financed sector is in equilibrium:*

   i) *$\{H(\kappa), D(\kappa)\}_i$ solve the entrepreneur's problem in equation (1.9).*

   ii) *Occupational choices are optimal: $\mathcal{E}_i = \{\kappa | E_0[S_i^*(\kappa)] > 0\}$.*

   iii) *The wage w_i clears the labor market:*

$$\int_{\mathcal{E}_i} H(\kappa) dF(\kappa) = \int_{\mathcal{E}_i^c} dF(\kappa) \tag{1.15}$$

   iv) *Free entry of venture capitalists:*

$$a_i \, \mathrm{E}_0[S_i^*(\kappa)] - u_i = 0 \tag{1.16}$$

   v) *The marginal venture capitalist has skill $\widehat{\epsilon} = m$.*

   vi) *Venture capitalists pay the aggregate cost to discover entrepreneurs:*

$$u(\mathcal{E}_i) \int_{\mathcal{E}_i} dF(\kappa) = \Delta_i \tag{1.17}$$

2. The monitoring market clears:

$$\sum_i^N \int_{\mathcal{E}_i} \mathrm{d}F(\kappa) = V\frac{m}{\epsilon} \tag{1.18}$$

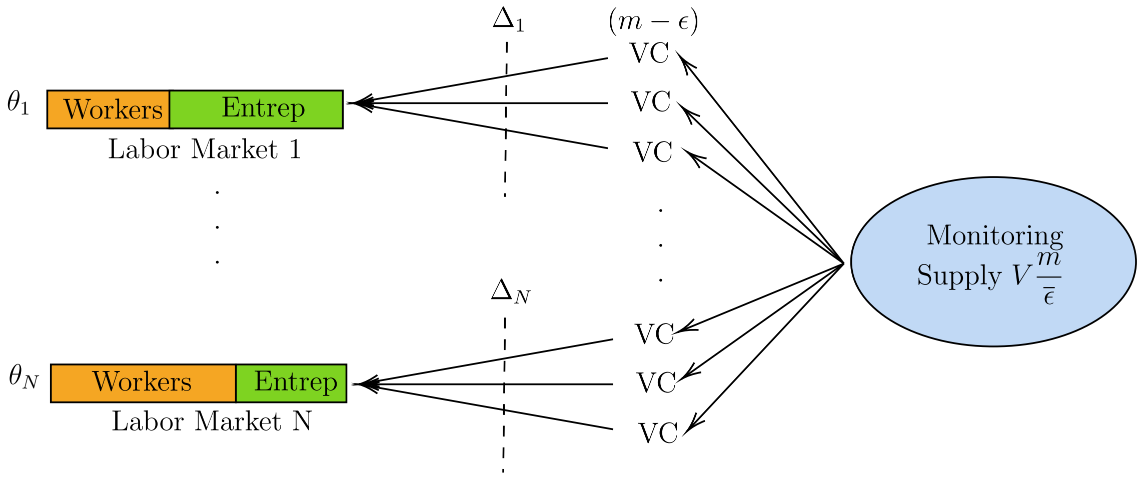


Figure 1.5: **A Multi-Sector Innovation Economy.** There are N sectors $i \in \{1 \dots N\}$ with a common distribution of entrepreneurial skill $F(\kappa)$. Each sector has a research productivity θ_i, a labor market, and a mass of agents that choose to become workers or entrepreneurs. Venture capitalists pay u_i to find a random entrepreneur in sector i. Once matched, the venture capitalist spends effort ϵ to monitor the entrepreneur until the firm produces. In expectation, venture capitalists get share a_i of net surplus plus the monitoring cost m, and entrepreneurs get $(1-a_i)$ of net surplus plus the wage w_i.

1.3 Benchmark model outcomes

Consider a sector i and an entrepreneur with skill κ. Hiring a mass of researchers H allows the entrepreneur to invest in good ideas and avoid bad ones, creating an option-like structure on payoffs.Moreover, since hiring increases the likelihood of success if the signal is good, research creates positive expected revenue net of investment costs. Figure 1.6 shows how hiring and the payoff spread impact the prospects of an idea in each period.

Lemma 1.2 (Expected Revenue)**.** *The expected revenue of an idea in sector i with H researchers and a payoff spread D is*

$$\mathrm{E}_0[\mathrm{R}_q(D)|H] = \frac{I}{2} + \theta_i\sqrt{HD} \tag{1.19}$$

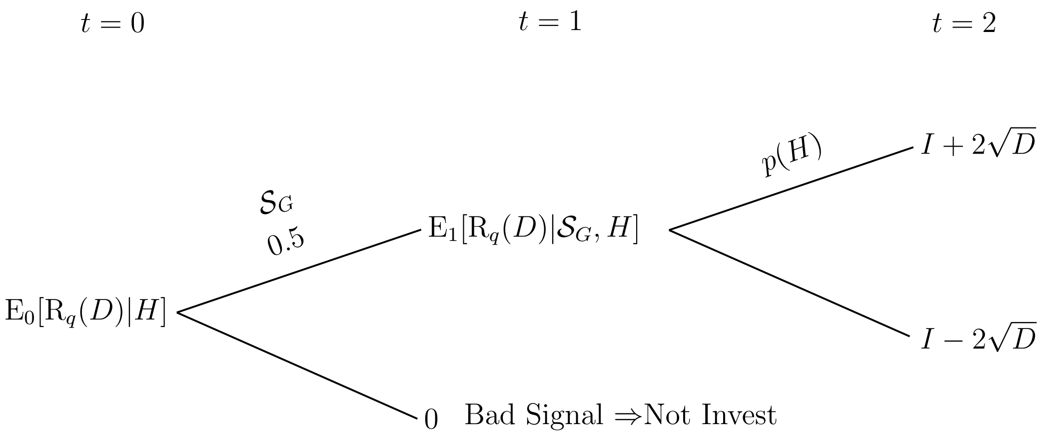


Figure 1.6: Effect of hiring workers H and the payoff spread D on an idea's prospects.

Lemma 1.2 shows that the hiring decision H and the payoff spread D are complements. A higher payoff spread increases the marginal payoff to hiring researchers. Research increases the likelihood of success, which allows the entrepreneur to increase the payoff spread. Sectors with low research productivity have less informative experiments and hence lower expected revenue.

The entrepreneur chooses hiring H and the payoff spread D given expected revenue and costs at $t = 0$, as shown in equation (1.9). First order conditions yield the optimal outcomes of an entrepreneur given his skill, his sector, and market prices.

Lemma 1.3 (Entrepreneur's Optimality)**.** *An entrepreneur's optimal choices and profits in sector i are*

$$H(\kappa) = \kappa \frac{\theta_i^4}{32 w_i^3} \qquad D(\kappa) = \kappa \frac{\theta_i^2}{8 w_i} \qquad E_0[\Pi^*(\kappa)] = \kappa \frac{\theta_i^4}{64 w_i^2} \tag{1.20}$$

Lemma 1.3 shows that entrepreneurs have constant returns to skill: hiring, payoff spreads, and profits are linear in κ. Labor market clearing determines the wage w_i, which together with research productivity in a sector θ_i controls the slope of the relationship between skill and entrepreneurial choices $(H(\kappa), D(\kappa))$.

Lemma 1.3 also shows that for a given sector $(\theta, \Delta)_i$, a change in the monitoring cost m only affects entrepreneurial choices through its effect on the wage. For entrepreneurs in a sector only a change in research productivity or in the wage are of first-order importance for their decisions.

The optimal expected profits of an entrepreneur in lemma 1.3 characterize the occupational choice of agents in a sector. By lemma 1.1 an agent becomes an entrepreneur if his optimal expected profits are larger than the wage plus the monitoring cost, $\mathrm{E}_0[\Pi^*(\kappa;\theta_i)] > w_i + m$. Since profits are increasing in entrepreneurial skill, a skill threshold determines occupational choice.

Lemma 1.4 (Occupational Choice)**.** *A threshold $\widehat{\kappa}(w_i, m; \theta_i)$ defines occupational choice. The most capable agents become entrepreneurs.*

Lemma 1.4 implies that the labor market clearing condition for sector i can be written as

$$F(\widehat{\kappa}(w_i, m; \theta_i)) = \int_{\widehat{\kappa}(w_i,m;\theta_i)} H(\kappa) dF(\kappa) \tag{1.21}$$

where $F(\widehat{\kappa}(w_i, m; \theta_i))$ is the supply of workers in the economy – agents with skill κ such that $\kappa < \widehat{\kappa}(w_i, m; \theta_i)$. Any agent with $\kappa > \widehat{\kappa}(w_i, m; \theta_i)$ is an entrepreneur that hires $H(\kappa)$ workers.

The labor market clearing condition determines entrepreneurial outcomes and surplus in a sector given a cost of monitoring m. Entrepreneurship then determines the search cost per entrepreneur $u(\#\ \text{Entrep}_i)$. The entry condition for venture capitalists in equation (1.16) and assumption 1.1 determine each party's bargaining power and the marginal venture capitalist in the economy.

Lemma 1.5 (Multi-Sector Equilibrium)**.** *Suppose the cost to discover entrepreneurs Δ_i is small relative to research productivity θ_i in some sectors, such that $\int_{\mathcal{E}} \mathrm{E}_0[S_i^*(\kappa)]\mathrm{d}F(\kappa) \geq \Delta_i$. Then, a unique multi-sector equilibrium with financed sectors exists. Moreover, the equilibrium is efficient.*

Lemma 1.5 shows that the research productivity of a sector relative to the cost to discover its entrepreneurs determines if the sector gets financed by venture capitalists. The set of financed sectors contains every sector that produces enough surplus to pay the discovery cost $\{(\theta_i, \Delta_i) | \int_{\mathcal{E}} \mathrm{E}_0[S_i^*(\kappa)]\mathrm{d}F(\kappa) \geq \Delta_i\}$. Since sectors with lower research productivity have lower surplus, they are less likely to be financed. Once a sector is financed, a unique wage w_i and bargaining weight a_i exist that clear the labor market and meet the venture capitalist entry condition.

A financing frontier characterizes the set of financed sectors, as shown in figure 1.7. The black line represents the individual rationality constraint for venture capitalists $\int_{\mathcal{E}} \mathrm{E}_0[S_i^*(\kappa)]\mathrm{d}F(\kappa) = \Delta_i$, where they get all the surplus of a sector $a_i = 1$. Any sector to the left does not produce enough surplus to pay for the cost to discover entrepreneurs Δ, while any sector to the right gets financed and has some bargaining weight $a_i \in [0, 1]$ to split surplus between entrepreneurs and venture capitalists. Finally, lemma 1.5 shows that the planner's allocation coincides with the market allocation – the competitive equilibrium is efficient.

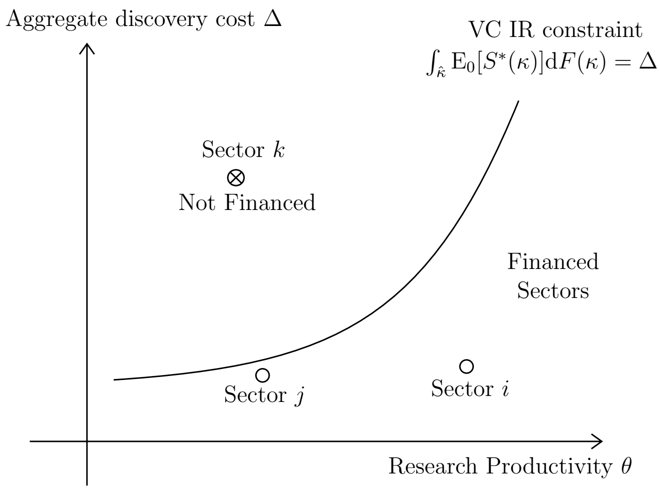


Figure 1.7: Three sectors and the venture capitalist individual rationality (IR) constraint. Sectors i and j get financed, sector k does not.

Suppose an equilibrium with financed sectors exists. In equilibrium, different research productivity across sectors imply each sector has different entrepreneurship levels and entrepreneurial innovation.

Lemma 1.6 (Equilibrium)**.** *In equilibrium, financed sectors with lower research productivity θ_i produce less surplus, with:*

i) Fewer entrepreneurs.

ii) Lower wages.

iii) More conservative ideas.

Lemma 1.6 shows that entrepreneurs in low research productivity sectors run larger firms and are less innovative. Intuitively, there are fewer entrepreneurs, more workers, and lower wages, so each entrepreneur hires more people. Lower research productivity makes signals less informative, but entrepreneurs hire more to offset the loss in informativeness. The decrease in research productivity dominates, making signals less informative, and hence entrepreneurs implement more conservative ideas – payoff spreads are lower.

The driver of different levels of entrepreneurship across sectors is the monitoring cost from venture capitalists. In a world where every agent has enough wealth to fund his projects and no monitoring is necessary, research productivity θ_i moves profits and wages in the same proportion across sectors, keeping occupational choice unchanged. When entrepreneurs need financing from venture capitalists, costly monitoring ($m > 0$) implies that entrepreneurs in vastly different technological conditions θ_i must clear the same bar m to obtain financing. Entrepreneurs in sectors with low research productivity have smaller profits. With smaller profits, the burden of the fixed monitoring cost is higher, reducing entrepreneurship.

One interpretation for the results in lemma 1.6 is that high research productivity sectors are more explorative, while low research productivity sectors are more exploitative. Explorative sectors are more entrepreneurial, with smaller firms, high wages, and highly innovative entrepreneurs. Given the same cost to find an entrepreneur in a sector $u(\cdot)$, competition from venture capitalists in explorative sectors lowers their bargaining power. On the other hand, exploitative sectors have low wages, with only a few entrepreneurs that run large firms and implement marginally innovative ideas.

More exploitative sectors – those with lower research productivity – might be sectors that are more mature, with access to different sources of financing. Any financier that monitors effectively can increase surplus in my model, however, it is unclear that financiers other than venture capitalists have developed such monitoring skills. Government grants to R&D like the SBIR programme (Howell, 2017; Lerner, 2000b) or angel investors (Da Rin et al., 2013) have gained relevance in financing the experimentation stage. Nonetheless, these financiers generally rely on venture capital to fund later stages and scale-ups (Da Rin et al., 2013), which in my model correspond to the investment period $t = 1$. I consider my model relevant for the venture capital environment given their specialized monitoring skill.

1.3.1 Venture capital and entrepreneurship

Over the last thirty years venture capital has grown from 5 to 130 billion invested in startups, with more and larger venture capital funds. The effect of venture capital on innovation depends on its effect on entrepreneurship and how innovative entrepreneurs are.

Proposition 1.1 (Venture Capital Sector Comparative Statics)**.** *A larger venture capital sector implies a smaller monitoring cost. Surplus is higher in every sector, with:*

i) More entrepreneurs.

ii) Higher wages.

iii) More conservative ideas.

Proposition 1.1 shows that while a larger venture capital sector leads to more entrepreneurship, entrepreneurs become less innovative. The effect of venture capital on innovation goes through the labor market. A larger supply of venture capitalists reduces the monitoring cost, which increases entrepreneurship. By increasing entrepreneurship, there are fewer workers, making labor more expensive for entrepreneurs. Hence, entrepreneurs run smaller firms with less informative experiments, resulting in less innovative ideas. Nonetheless, having more but less innovative entrepreneurs still results in a surplus gain.

The past decades have been characterized by the decline in business dynamism in the US (Akcigit and Ates, 2021). For instance, firm entry has declined steadily, and new firms are less likely to achieve high employment growth (Decker et al., 2016a,b) and to innovate (Akcigit and Ates, 2023). Decker et al. (2016a) show that the decline in firm entry is significant in every sector, including information technologies after the dot-com bubble. Salgado (2020) empirically shows that the decline is concentrated among highly educated individuals. My model rationalizes these trends as an efficient outcome of research productivity decreasing as sectors exploit their technologies and ideas. If research productivity is declining, as Bloom et al. (2020) document, entrepreneurship and innovation decline in every sector.

Corollary 1.1 (Entrepreneurship Decline)**.** *A larger venture capital sector dampens the effects of lower research productivity on entrepreneurship at the cost of even more marginal innovations.*

Lower research productivity in every sector implies lower entrepreneurship in the economy by lemma 1.6, hence a lower monitoring cost and lower surplus. Corollary 1.1 shows that if at the same time the venture capital sector is becoming larger, then the monitoring cost goes down, increasing surplus. An increase in the size of venture capital then dampens the decrease in entrepreneurship and surplus produced by lower research productivity. Nonetheless, by increasing entrepreneurship it makes labor more expensive, further reducing the innovativeness of entrepreneurs. Given entrepreneurship has declined despite the growth in venture capital Decker et al. (2016a), with patents that are more marginal Akcigit and Goldschlag (2023), the evidence suggests that the decline in research productivity effect has dominated the growth in venture capital.

1.3.2 Technological booms

Consider now a single sector in the economy where suddenly research becomes significantly more productive because of a new discovery. For instance, artificial intelligence with the advances in large language models, web services with the internet in the 2000's, or software in the last decade with the advent of cloud computing, which allowed for rapid prototyping in the software sector (Lerner and Nanda, 2023). These booming sectors have higher entrepreneurship, but also crowd out financing from other sectors in the economy.

Proposition 1.2 (Technological Booms)**.** *A shock that increases research productivity in a sector i increases the monitoring cost in the economy. Aggregate surplus increases, but in other sectors $j \neq i$ surplus decreases, with:*

i) Fewer entrepreneurs.

ii) Lower wages.

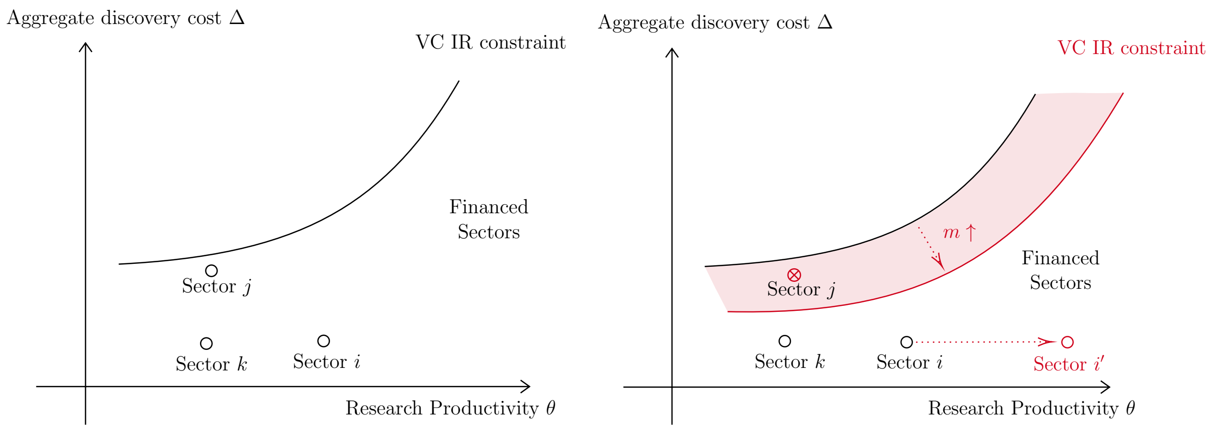
iii) More innovative entrepreneurs.

Proposition 1.2 shows that the sudden appearance of a new sector with high research productivity – a technological boom – has negative spillovers to the rest of the economy. All the spillovers act through the market for monitoring. Higher research productivity in a sector leads to an entrepreneurship boom. In equilibrium, more entrepreneurship in the booming sector leads to a higher cost of monitoring m, which lowers surplus, entrepreneurship, and wages in other sectors. With lower surplus, venture capitalists have higher bargaining power in non-booming sectors.

A positive spillover of a technological boom is that entrepreneurs that remain in non-booming sectors become more innovative. Since wages go down, entrepreneurs hire more and run larger firms. By hiring more, their probability of success increases which allows them to increase their payoff spread. However, surplus still declines. The extensive margin effect of having fewer entrepreneurs outweighs the intensive margin effect of having more innovative entrepreneurs. One interpretation of this result is that a technological boom lowers the parallel exploration of ideas in non-booming sectors in exchange for a deeper exploration of each idea, lowering innovation overall.

Figure 1.8 shows graphically that booming-sector i, for instance, artificial intelligence, increases the cost of monitoring for all sectors. A higher cost of monitoring makes the financing frontier move inwards, such that lower research productivity sectors like renewables end up with lower surplus and entrepreneurship, while some sectors like sector j are no longer financed.

Corollary 1.2 (Specialization)**.** *Venture capital finances fewer sectors in a technological boom.*



(a) Both sectors get financed. (b) New financing frontier.

Figure 1.8: An increase in productivity in sector i increases the monitoring cost m in the economy, which moves the financing frontier inwards, such that sector j is no longer financed.

Corollary 1.2 rationalizes recent trends in the venture capital and entrepreneurship environment. Lerner and Nanda (2020) show that venture capitalists have increasingly specialized in a handful of sectors, investing lower shares in socially desirable sectors like renewables and healthcare. My model rationalizes this specialization across sectors as an efficient outcome of the lifecycle of different sectors. Venture capitalists fund the rise in entrepreneurship in the booming sector, but with higher monitoring costs fewer sectors produce enough surplus to get financed. If we also consider a decline in research productivity, venture capitalists specialize even more.

Corollary 1.3 (Venture Capital Skill)**.** *If the average skill of potential venture capitalists decreases during a technological boom, the negative spillovers to non-booming sectors are exaggerated.*

Corollary 1.3 is a consequence of the supply curve for venture capitalists. A technological boom increases the demand for venture capialists' monitoring services, raising the monitoring cost m for entrepreneurs and lowering the skill of the marginal venture capitalist. Similarly, if the average skill of potential venture capitalists goes down because of, for instance, the sudden entry of new, inexperienced venture capitalists, the supply of monitoring at any given monitoring cost m is lower. To bring the monitoring market back to equilibrium, the monitoring cost increases even more to lower entrepreneurship and draw more venture capitalists to the market. By increasing the monitoring cost beyond what the technological boom does, lower venture capital skill exacerbates the negative spillovers to other sectors in the economy.

1.4 Innovation Races

The outcomes described so far are efficient, but rest on the assumption that ideas are unique and that each entrepreneur can appropriate the surplus of his idea. Unfortunately, several entrepreneurs might be competing to implement the same idea. Moreover, to incentivize innovation inventors must get some surplus from their innovations, which requires patents, prizes, or winner-takes-all markets (Reinganum, 1989). Nonetheless, such systems create innovation races in which only the first inventor is rewarded, while the effort of competitors is wasted. Too many inventors might inefficiently chase an idea. Recently, Hill and Stein (2019, 2021) empirically show that priority races in science have a significant positive effect in publication and career outcomes for the winner; however, the race makes the most important projects also the most rushed and lowest quality because of greater competition.

Entrepreneurs funded by venture capital have strong incentives to race to win the market. In a 2020 article in The New Yorker,[12] a managing partner at SoftBank explains that "Once Uber is founded, within a year you suddenly have three hundred copycats. The only way to protect your company is to get big fast by investing hundreds of millions." Given the strong incentives for venture capital-backed entrepreneurs to race to innovate and to win the market, I use my model to study the effects of priority races in my economy.

1.4.1 Setup

Consider a limited pool of excludable resources for which entrepreneurs compete independently. The resources could be ideas as in a regular duplication externality, private data in the case of artificial intelligence or academic researchers, or servers and labs in

[12] "How Venture Capitalists Are Deforming Capitalism", The New Yorker, 23rd of November 2020

the case of scientists. From now on, I call these resources "ideas".

The likelihood of success of an entrepreneur now is also characterized by a function $g(\#\text{ Entrep})$ that decreases as more entrepreneurs compete for a shared pool of ideas. Since setup costs are lower for more skilled entrepreneurs κ, profits increase with κ and a threshold $\widehat{\kappa}_i$ characterizes entrepreneurship.

Assumption 1.3 (Research in a Race)**.** *Signal precision with H workers and research productivity θ_i is such that:*

$$p(H) = \frac{1}{2}\left(1 + g(\widehat{\kappa}_i)\theta_i\sqrt{H}\right) \tag{1.22}$$

Formally, $g(\widehat{\kappa}_i)$ is increasing in the entrepreneurship threshold $g_{\widehat{\kappa}_i} > 0$ – fewer entrepreneurs imply higher chances to succeed. The results of the benchmark scenario are a special case in which $g(\widehat{\kappa}) = 1$. Analogous to lemma 1.2, the expected revenue of a project with research H and payoff spread D is $\mathrm{E}_0[\mathrm{R}_q(D)|H] = \frac{I}{2} + g(\widehat{\kappa}_i)\theta_i\sqrt{HD}$. As entrepreneurship increases entrepreneurs are less likely to succeed, lowering their expected revenue.

The externality captures in a one-shot setting the distortionary incentives of races to patent and to win the market. As in patent races, excludability of ideas is important. Every entrepreneur throws himself to search for an idea and hires researchers to increase his chances to succeed. If he finds an idea, he can exclude others from it. In the case of private data for artificial intelligence, contractual agreements allow the entrepreneur to exclude others from using it. As more artificial intelligence companies compete for private data, each one is less likely to get it and more effort is duplicated. This externality can also be thought of as the effect of crowding in the market, such as vaccines or artificial intelligence, where only some entrepreneurs can succeed in developing an idea.

1.4.2 The planner's problem

The planner picks a feasible allocation to maximize expected aggregate surplus net of deadweight costs: an idea's investment and setup costs, venture capitalists' effort, and the cost to discover entrepreneurs in a sector.

Definition 1.3 (Feasible Allocation)**.** *A planner's allocation is a set of financed sectors Θ, a set of entrepreneurs $\mathcal{E}_i$ for each sector, hiring $H(\kappa)$ and payoff spread $D(\kappa)$ policies for entrepreneurs in each sector, and a set of venture capitalists Ω.*

An allocation is feasible if each labor market clears, venture capitalists cover their entry costs in each financed sector, and the monitoring market clears across sectors.

The planner solves:[13]

$$\max_{\Theta,\{\mathcal{E},H,D\}_{i\in\Theta},\Omega} \quad \sum_{i\in\Theta}\int_{\mathcal{E}_i}\left[\mathrm{E}_0[\mathbb{1}_{\mathcal{S}}(\mathrm{R}_q(D)-I)|H]-\frac{D^2}{\kappa}\right]\mathrm{d}F(\kappa)-\Delta_i-\int_{\Omega}[V\epsilon]\mathrm{d}G(\epsilon) \tag{1.23}$$

$$\text{s.t.} \quad \int_{\mathcal{E}_i} H(\kappa)\mathrm{d}F(\kappa)=\int_{\mathcal{E}_i^c}\mathrm{d}F(\kappa) \qquad ,\forall i\in\Theta \qquad \text{(Labor Market)}$$

$$\int_{\mathcal{E}_i} E_0[S_i(\kappa)]\mathrm{d}F(\kappa)\geq\Delta_i \qquad ,\forall i\in\Theta \qquad \text{(Search)}$$

$$\sum_{i\in\Theta}\int_{\mathcal{E}_i}\mathrm{d}F(\kappa)=\int_{\Omega}V\mathrm{d}G(\epsilon) \qquad \text{(Monitoring Market)}$$

The planner's problem is simplified by having thresholds that determine which sectors get financed, entrepreneurship in each sector, and the set of venture capitalists.

Lemma 1.7 (Thresholds)**.** *The planner finances sectors within a financing frontier, anyone more skilled than threshold $\widehat{\kappa}_i^P$ becomes an entrepreneur, and venture capitalists with an effort cost lower than threshold $\widehat{\epsilon}$ enter the market.*

Lemma 1.7 shows the planner's solution shares the threshold properties of the market outcome. The planner finances sectors within a financing frontier, the most skilled agents in a sector are entrepreneurs, and the most skilled venture capitalists finance entrepreneurs. Intuitively, if an allocation does not meet the threshold properties of lemma 1.7, surplus increases by reshuffling agents across occupations or financing across sectors.

[13] The planner gives the same weight to each group of agents, hence he does not care about the wage or monitoring cost as they represent transfers among decision-makers.

1.4.3 Optimal policy

The planner's first order conditions for payoff spreads and hiring coincide with the market outcome, but he allocates entrepreneurs differently in each sector. The planner sets the marginal entrepreneur in sector i $(\widehat{\kappa}_i^P)$ to balance the marginal loss of some surplus producing entrepreneurs against the benefit of increasing the likelihood of success of every other entrepreneur. By rearranging, the first order condition results in:[14]

$$\mathrm{E}_0[\Pi(\widehat{\kappa}_i^P)]\left(1-\frac{\phi(\widehat{\kappa}_i^P)}{\widehat{\kappa}_i^P}\right)=(\lambda+m^P) \tag{1.24}$$

where $\phi(\widehat{\kappa}_i^P)$ is a factor that determines how much the planner moves entrepreneurship because of innovation races, λ is the lagrange multiplier of the labor market clearing condition, and m^P is the lagrange multiplier on the monitoring market condition.

The cost of entrepreurship in the planner's problem is the marginal social value of labor plus the social value of monitoring $(\lambda + m^P)$ adjusted for crowding effects through factor $\phi(\widehat{\kappa}_i^P)$, which represents the optimal policy that the planner can implement. The planner's optimal policy $\phi(\widehat{\kappa}_i^P)$ has three elements:

$$\phi(\widehat{\kappa}_i^P)=4\underbrace{\varepsilon_{\widehat{\kappa}}^g}_{\substack{\text{Elasticity of}\\ \text{Inefficiency}}}\times\underbrace{\frac{\mathrm{E}[\kappa|\kappa\geq\widehat{\kappa}_i^P]}{\widehat{\kappa}_i^P}}_{\substack{\text{Relative}\\ \text{Capability}}}\times\underbrace{\frac{[1-F(\widehat{\kappa}_i^P)]}{f(\widehat{\kappa}_i^P)}}_{\substack{\text{Relative}\\ \text{Mass}}} \tag{1.25}$$

where $\varepsilon_{\widehat{\kappa}}^g$ is the elasticity of the inefficiency function $g(\widehat{\kappa}_i^P)$ with respect to the entrepreneurial threshold $\widehat{\kappa}_i^P$.

The three elements of the optimal policy measure the benefit of reducing inefficiencies in the market by changing entrepreneurship. $\varepsilon_{\widehat{\kappa}}^g \times \frac{\mathrm{E}[\kappa|\kappa\geq\widehat{\kappa}_i^P]}{\widehat{\kappa}_i^P}$ measures the intensity of the benefit – how much the likelihood of success of an entrepreneur increases times how much more talented the average entrepreneur is relative to the marginal entrepreneur. $\frac{1-F(\widehat{\kappa}_i^P)}{f(\widehat{\kappa}_i^P)}$ measures the extensive margin of the benefit – how many entrepreneurs benefit relative

[14]The proof to proposition 1.3 shows the steps to reach equation (1.24) in equation (1.A.32) to equation (1.A.34).

to how many are sacrificed $f(\widehat{\kappa}_i^P)$.[15]

Assumption 1.4 (Decreasing Policy)**.** *The optimal policy $\phi(\widehat{\kappa}_i^P)$ is decreasing in the entrepreneurial threshold $\widehat{\kappa}_i^P$.*

The optimal policy $\phi(\widehat{\kappa}_i^P)$ can increase or decrease with the skill threshold $\widehat{\kappa}_i^P$ depending on the distribution and the inefficiency functional forms. Assumption 1.4 implies that the overall effect of the inefficiency decreases when there are fewer entrepreneurs competing for ideas. Assumption 1.4 is met under a variety of circumstances. For instance, any CES function for $g(\widehat{\kappa}_i^P)$ with a uniform or pareto distribution for skill $F(\kappa)$; or a decreasing elasticity $\varepsilon_{\widehat{\kappa}}^g$ with an exponential distribution for skill $F(\kappa)$.

1.4.4 Innovation races make monitoring too costly

Crowding in a sector creates spillovers through the market for monitoring. Suppose a single sector is crowded. Then the crowded sector has excess entrepreneurship, pushing the monitoring cost inefficiently high, which results in less surplus and entrepreneurship in other sectors. For generality, suppose the sectors financed in any equilibrium have a sufficiently wide range of research productivity θ.[16]

The planner allocates resources similarly to the market to sectors with lower research productivity. Lower research productivity sectors have fewer entrepreneurs, lower wages, and less innovative entrepreneurs. Different to the market, the planner allocates more entrepreneurs to low research productivity sectors.

Proposition 1.3 (Planner vs Market)**.** *Innovation races imply an inefficiently high monitoring cost in the market equilibrium, which results in:*

i) Excess entrepreneurs in high research productivity sectors.

ii) A shortage of entrepreneurs in low research productivity sectors.

[15] The planner might implement a large correction even if the inefficiency reacts little (small $\varepsilon_{\widehat{\kappa}}^g$) if there is a group of highly capable "superstar" entrepreneurs who make $\mathrm{E}[\kappa|\kappa \geq \widehat{\kappa}_i^P]/\widehat{\kappa}_i^P$ large. Intuitively, entrepreneurial skill and the likelihood of success are complements, such that the planner wants a higher probability of success when entrepreneurs are very skilled.

[16] If the financed sectors have a narrow range of research productivity, only the results to a specific range of θ apply.

iii) Less innovative projects in each sector.

Proposition 1.3 shows that the market has an inefficiently high demand for the monitoring of venture capitalists, raising the monitoring cost too much when there are innovation races. Entrepreneurship in any sector has two forces acting in opposite directions. First, given a monitoring cost, innovation races imply excess entrepreneurship. Second, inefficiently expensive monitoring pushes entrepreneurship down. The excess entrepreneurship force dominates in high research productivity sectors, and vice versa.[17]

Proposition 1.3 justifies the concerns of Lerner and Nanda (2020) about venture capital not providing enough funding to socially desirable sectors like renewables and new materials. Booming sectors like artificial intelligence or the internet in the 2000's have excess entrepreneurship that creates an inefficiently high monitoring cost, crowding out entrepreneurship and venture capital from socially desirable, low research productivity sectors, and making every entrepreneur in the economy less innovative.

1.4.5 Policy implications

I explore three policy tools to improve efficiency by changing entrepreneurship in each sector. A first natural candidate is a sector-specific profit tax, however, profit taxes on innovation do not seem realistic. Therefore, I then explore worker reskilling across sectors, and finally R&D subsidies.

Proposition 1.4 (Policy Implications)**.** *To control the inefficiencies from innovation races:*

i) Sector-specific taxes achieve the first best.

ii) Worker reskilling across sectors lowers welfare.

iii) R&D subsidies improve welfare by incentivizing excess entrepreneurs to work.

[17]Wages could increase or decrease depending on how strongly the inefficiency reacts to changes in entreprenurship $\varepsilon^g_{\widehat{\kappa}}$. If $\varepsilon^g_{\widehat{\kappa}}$ is small, wages decrease in high research productivity sectors, and vice versa.

Sector-specific taxes Suppose the planner implements a sector-specific profit tax T_i. The net surplus of an entrepreneur in sector i is

$$\mathrm{E}_0[S_i^*(\kappa)] = \mathrm{E}_0[\Pi_i^*(\kappa)](1 - T_i) - w_i - m \tag{1.26}$$

Proposition 1.4 shows that a different tax for each sector achieves the first best. With taxes, the marginal entrepreneur $\widehat{\kappa}$ meets $\mathrm{E}_0[\Pi^*(\widehat{\kappa})](1 - T_i) = w_i + m$. Higher taxes reduce entrepreneurship. The optimal sector-specific tax equals the optimal policy $T_i^* = \frac{\phi(\widehat{\kappa}_i^P)}{\widehat{\kappa}_i^P}$. The optimal tax makes the market and the planner's allocation of entrepreneurs in equation (1.24) coincide. By matching entrepreneurship in every sector, prices adjust and the market achieves the first best. The optimal tax grows with research productivity since the optimal policy $\phi(\widehat{\kappa}_i^P)$ decreases with the skill threshold $\widehat{\kappa}_i^P$, and vice versa.[18]

While sector-specific taxes achieve the first-best, taxing innovation seems unrealistic. Most innovative firms fail or do not produce profits in many years, making the collection of taxes highly uncertain. Moreover, a large literature in accounting and law[19] shows that startups backed by venture capital are organized in a tax-inefficient form. Poterba (1989) argues that capital gains taxes are "a relatively blunt device for encouraging venture investment", and Allen et al. (2023) estimates that the inefficiencies are as much as 4.9% of the equity invested in firms in foregone tax savings. Therefore, I turn to worker reskilling across sectors and R&D subsidies as alternative policy tools.

Worker reskilling The development of artificial intelligence in the next 5 years is threatened by a shortage of engineers in the chipmaking sector.[20] One of the short-term solutions proposed for the lack of engineers is the sourcing of talent from adjacent sectors.[21] Suppose the planner can lower the switching cost s of workers for some cost. Agents can only become entrepreneurs in the sector they are born in (i), but workers can

[18] An alternative is to keep a balanced budget, using tax income from some sectors to subsidize other sectors. A balanced budget tax increases welfare but cannot achieve the first best.

[19] See for instance Fleischer (2003), Goldberg (2001), Morse and Allen (2015), and Allen et al. (2023).

[20] "Chipmakers face a labour crisis", Financial Times, 13th of August 2024

pay s to move to any other sector $j \neq i$. An agent becomes an entrepreneur if

$$\mathrm{E}_0[S_i^*(\kappa)] > 0 \Leftrightarrow \mathrm{E}_0[\Pi_i^*(\kappa)] > \max\{w_i, w_{j\neq i} - s\} + m \tag{1.27}$$

Proposition 1.4 shows that worker reskilling across sectors reduces welfare by worsening the inefficiencies in high research productivity sectors. Consider a sector with high research productivity – artificial intelligence – and a sector with low research productivity – renewables – and assume we start from the market equilibrium. Wages are higher in artificial intelligence. As switching costs go down, workers move to artificial intelligence, lowering the wage. With lower wages, entrepreneurship increases, which crowds the market even more. On the other hand, renewables suffer from fewer workers, higher wages, and hence fewer entrepreneurs. Incentivizing worker reallocation is good for workers in low research productivity sectors, but it lowers welfare by increasing the inefficiencies.

Sector-specific R&D subsidies R&D subsidies are a common way for governments to incentivize innovative entrepreneurship. For instance, consider the SBIR programme in the U.S. (Lerner, 2000b; Howell, 2017) which gives R&D subsidies to highly innovative firms, that was later adopted in the UK as the SBRI programme (Tredgett and Coad, 2013). Suppose the planner subsidizes experimentation by covering a fraction of the setup and labor costs of entrepreneurs. The planner's subsidy is sector-specific and has an opportunity cost of \$1 dollar per \$1 of subsidy, equivalent to investing at the baseline rate of zero. The expected profits of an entrepreneur in sector i with subsidy U_i are

$$\mathrm{E}_0[\Pi_i(\kappa, H, D)] = \mathrm{E}_0[\mathrm{R}_i(D) - I|H] - \frac{(wH + \frac{D^2}{\kappa})}{1 + U_i} \tag{1.28}$$

Proposition 1.4 shows that an R&D subsidy can improve welfare by decreasing entrepreneurship in some sectors. Subsidizing the costs of entrepreneurs leads to higher profits and higher wages. Since entrepreneurs face lower costs, they achieve higher profits, but also demand more workers, increasing wages. The effect of a subsidy in en-

[21] "Reimagining labor to close the expanding US semiconductor talent gap", McKinsey & Company, 2nd of August 2024

trepreneurship depends on the relative strength of the effect on profits versus wages. In low research productivity sectors, the effect on profits dominates, increasing entrepreneurship. In high research productivity sectors, the effect on wages dominates, incentivizing excess entrepreneurs to work.

R&D subsidies do not achieve the first best. The sector in which the subsidy shifts from lowering to increasing entrepreneurship need not be the same sector in which the economy moves from an excess to a shortage of entrepreneurs. For some sectors the planner wants to decrease entrepreneurship but the subsidy increases it, or vice versa. These sectors don't get subsidized.[22]

1.5 Conclusion

We do not fully understand the effect of venture capital on the decision to become an entrepreneur, blurring our understanding of the effect of venture capital on innovation. I contribute to the literature by presenting a multi-sector occupational choice model in which agents choose to become workers or innovative entrepreneurs. I deviate from the literature by considering that talented agents are both the suppliers of labor for innovation and the entrepreneurs that innovate, and by incorporating venture capital financing conditions into the occupational choice of agents. The mechanism of my model is such that the cost of monitoring by venture capitalists alters occupational choice, which in turn alters the availability of talented workers for entrepreneurs to innovate.

My model shows that a larger venture capital sector lowers monitoring costs, which incentivizes entrepreneurship but lowers entrepreneurial innovation because labor becomes more expensive. If research productivity declines over time, entrepreneurship declines and ideas become more marginal. If at the same time the venture capital sector grows, it dampens the declines in entrepreneurship at the cost of even more marginal innovations. If a sector suddenly becomes more innovative, it increases the monitoring cost, which crowds out financing from other sectors.

[22] An alternative is an economy-wide R&D subsidy ($U_i = U$). An economy-wide subsidy exaggerates the inefficiencies in intermediate research productivity sectors. This welfare loss against the benefit of correcting high and low research productivity sectors determines if aggregate welfare increases.

The outcome of the competitive market is unlikely to be efficient because of the distortionary effects of races to innovate and to win the market. When entrepreneurs compete for a shared pool of ideas, sectors are crowded and the monitoring cost of the market is inefficiently high, which crowds out entrepreneurship and financing from socially desirable sectors like renewables.

Appendix to Chapter 1

Appendix 1.A Proofs

Lemma 1.1. An agent becomes an entrepreneur if his optimal payoff as an entrepreneur exceeds the wage: $(1-a_i)\,\mathrm{E}_0[S_i^*(\kappa)] + w_i > w_i$, which gives the result. □

Lemma 1.2. The result follows directly by applying conditional expectations at $t = 0$, the law of total probability using assumption 1.2, and that the project is dropped after a bad signal given $E[Rev|B, H, D] < 0$. Figure 1.6 graphically shows the steps. □

Lemma 1.3. Taking the first order conditions of the entrepreneur's problem in equation (1.9), solving for the optimal choices, and replacing for the optimal profits yields the results. □

Lemma 1.4. Replacing the entrepreneur's optimal profits of lemma 1.3 into the expression for surplus yields

$$S_i^*(\kappa) = \kappa\frac{\theta_i^4}{64w_i^2} - (w_i + m) \tag{1.A.1}$$

Anyone with positive surplus becomes an entrepreneur by lemma 1.1. Since surplus is increasing in capabilities κ, a unique κ determines who is indifferent between work or entrepreneurship $S_i^*(\kappa) = 0$ – the marginal entrepreneur. Define $\widehat{\kappa}$ as the marginal entrepreneur. Then $\widehat{\kappa} = \frac{64w_i^2}{\theta_i^4}(w_i + m)$. Anyone with capabilities higher than $\widehat{\kappa}$ becomes an entrepreneur, and vice versa. □

Lemma 1.5. A sector is financed if venture capitalists cover their search costs in expectation $\int_{\widehat{\kappa}}^{\infty} a(\kappa)S(\kappa;\theta)\mathrm{d}F(\kappa) = u$. Since the bargaining weight a does not alter entrepreneurial choices nor sector outcomes, a sector produces the same aggregate surplus for any a. Then, a sector gets financed if the expected surplus of an entrepreneur exceeds search costs $\int_{\widehat{\kappa}}^{\infty} S(\kappa;\theta)\mathrm{d}F(\kappa) \geq u$ which implies the sector is financed if expected aggregate surplus is greater than aggregate search costs $(1-F(\widehat{\kappa}))\int_{\widehat{\kappa}}^{\infty} S(\kappa;\theta)\mathrm{d}F(\kappa) \geq \Delta$. Then, since expected surplus increases in research productivity θ, a sector is financed is Δ is sufficiently small relative to θ.

Consider a sector that gets financed.

a) Given a monitoring cost m, a marginal entrepreneur $\widehat{\kappa} = \frac{64w_i^2}{\theta_i^4}(w_i+m)$ and the hiring policy $H(\kappa)$ in lemma 1.3m the excess supply of labor is:

$$\text{Excess Labor Supply} = F(\widehat{\kappa}) - \int_{\widehat{\kappa}}^{\bar{\kappa}} H(\kappa, w; \theta)\mathrm{d}F(\kappa) \tag{1.A.2}$$

The excess labor supply is continuous, increasing in the wage, negative when $w=0$, and positive for a wage w high enough such that $\widehat{\kappa} = \bar{\kappa}$. By the intermediate value theorem, a unique wage that clears the market exists.

b) If a sector (θ, Δ) gets financed, venture capitalist cover their entry cost u in expectation. By assumption 1.1 $a = \frac{u}{\int_{\mathcal{E}} S(\kappa)\mathrm{d}F(\kappa)} \leq 1$.

Then, if a sector is financed, a unique wage w that clears the market and a unique bargaining weight a exist.

The supply of monitoring is increasing, continuous, and 0 for a monitoring cost $m=0$, so a unique $m>0$ exists for any level of entreprenurship in the economy.

Finally, the model has no externalities, and agents internalize the effects on both the labor and monitoring markets through prices, so the equilibrium is efficient. For a formal treatment of the planner's problem, refer to the proof to proposition 1.3. □

Lemma 1.6. Let ε_b^a be the elasticity of variable a with respect to variable b, $\varepsilon_b^a = \frac{\mathrm{d}\ln(a)}{\mathrm{d}\ln(b)} = \frac{\mathrm{d}a}{\mathrm{d}b}\frac{b}{a}$. Since capabilities κ, productivities θ, wages w, and the monitoring cost m are all positive, the elasticities have the same sign as the derivatives.

Given a monitoring cost m in equilibrium, consider the change in the marginal entrepreneur $\widehat{\kappa} = \frac{64w_i^2}{\theta_i^4}(w_i + m)$ as a sector varies in research productivity $\frac{\mathrm{d}\widehat{\kappa}}{\mathrm{d}\theta}$. Then:

$$\varepsilon_\theta^{\widehat{\kappa}} = -4 + 3\frac{(w + \frac{2}{3}m)}{(w+m)}\varepsilon_\theta^w \tag{1.A.3}$$

From the optimal hiring policy $H(\kappa)$

$$\varepsilon_\theta^H = [4 - 3\varepsilon_\theta^w] = -\left[\varepsilon_\theta^{\widehat{\kappa}} + \frac{m}{w+m}\varepsilon_\theta^w\right] \tag{1.A.4}$$

Differentiating the market clearing condition with respect to research productivity $\frac{\mathrm{d}}{\mathrm{d}\theta}$, noting that $\frac{\mathrm{d}H}{\mathrm{d}\theta} \propto H$, and using that aggregate hiring is the labor supply $F(\widehat{\kappa}) = \int_{\widehat{\kappa}}^{\bar{c}} H(\kappa)\mathrm{d}F(c)$, yields

$$\left[\frac{f(\widehat{\kappa}) + H(\widehat{\kappa})}{F(\widehat{\kappa})}\right]\varepsilon_\theta^{\widehat{\kappa}}\widehat{\kappa} = \varepsilon_\theta^H \tag{1.A.5}$$

Then using equations (1.A.18) to (1.A.20):

$$\varepsilon_\theta^{\widehat{\kappa}}\left[\left(\frac{f(\widehat{\kappa}) + H(\widehat{\kappa})}{F(\widehat{\kappa})}\right)\widehat{\kappa} + 1\right] = -\frac{m}{w+m}\varepsilon_\theta^w \tag{1.A.6}$$

Then:

$$\frac{\varepsilon_\theta^w}{\varepsilon_\theta^{\widehat{\kappa}}} < 0 \tag{1.A.7}$$

Rewriting equation (1.A.18):

$$\varepsilon_\theta^{\widehat{\kappa}}\underbrace{\left(1 - 3\frac{(w+\frac{2}{3}m)}{(w+m)}\frac{\varepsilon_\theta^w}{\varepsilon_\theta^{\widehat{\kappa}}}\right)}_{>0} = -4 \tag{1.A.8}$$

Then $\varepsilon_\theta^{\widehat{\kappa}} < 0$ and $\varepsilon_\theta^w > 0$. $2\,\mathrm{E}[\Pi(\kappa)] = wH$ by the first order conditions of the entrepreneur's problem, then for profits:

$$\varepsilon_\theta^\Pi = \varepsilon_\theta^H + \varepsilon_\theta^w \tag{1.A.9}$$

$$\varepsilon_\theta^\Pi = \varepsilon_\theta^{\widehat{\kappa}}\left\{-\left(\frac{m}{w}\cdot\frac{f(\widehat{\kappa}) + H(\widehat{\kappa})}{F(\widehat{\kappa})}\right)\widehat{\kappa} - \frac{(w+m)}{w}\right\} \tag{1.A.10}$$

Therefore $\varepsilon_{\theta}^{\Pi} > 0$. $\frac{D^2}{\kappa} = \Pi(\kappa)$ by the first order conditions of the entrepreneur's problem, then $\varepsilon_{\theta}^{D} > 0$.

For surplus of an entrepreneur, we can rewrite surplus as:

$$S(\kappa) = (w+m)\left(\frac{\kappa}{\widehat{\kappa}} - 1\right) = \Pi(\kappa)\left(\frac{\kappa - \widehat{\kappa}}{\kappa}\right) \tag{1.A.11}$$

$$\varepsilon_{\theta}^{S} = \varepsilon_{\theta}^{\Pi} - \frac{\widehat{\kappa}}{k - \widehat{\kappa}}\varepsilon_{\theta}^{\widehat{\kappa}} \tag{1.A.12}$$

Since only those with $\kappa > \widehat{\kappa}$ produce surplus and $\varepsilon_{\theta}^{\widehat{\kappa}} < 0$, $\varepsilon_{\theta}^{S} > 0$. Then for the total surplus of a sector $\int_{\widehat{\kappa}}^{\infty} S(\kappa)\mathrm{d}F(\kappa)$ it's enough to note that when research productivity goes up θ, there are more entrepreneurs $\left(\frac{\mathrm{d}\widehat{\kappa}}{\mathrm{d}\theta} < 0\right)$, and each entrepreneur produces more surplus $\frac{\mathrm{d}S(\kappa)}{\mathrm{d}\theta} > 0$, so the surplus of a sector goes up.

□

Proposition 1.1. Consider the same steps for the proof of lemma 1.6 as the economy varies in its monitoring cost m. Let ε_{b}^{a} be the elasticity of variable a with respect to variable b, $\varepsilon_{b}^{a} = \frac{\mathrm{d}\ln(a)}{\mathrm{d}\ln(b)} = \frac{\mathrm{d}a}{\mathrm{d}b}\frac{b}{a}$. Then for a given sector θ the capabilities threshold $\widehat{\kappa}$ implies:

$$\varepsilon_{m}^{\widehat{\kappa}} = \frac{m}{(w+m)} + 3\frac{(w + \frac{2}{3}m)}{(w+m)}\varepsilon_{m}^{w} \tag{1.A.13}$$

From the optimal hiring policy $H(\kappa)$

$$\varepsilon_{m}^{H} = -H3\varepsilon_{m}^{w} \tag{1.A.14}$$

Differentiating the market clearing condition with respect to the monitoring cost $\frac{\mathrm{d}}{\mathrm{d}m}$, noting that $\frac{\mathrm{d}H}{\mathrm{d}m} \propto H$, and using that aggregate hiring is the labor supply $F(\widehat{\kappa}) = \int_{\widehat{\kappa}}^{\bar{c}} H(\kappa)\mathrm{d}F(c)$, yields

$$\left[\frac{f(\widehat{\kappa}) + H(\widehat{\kappa})}{F(\widehat{\kappa})}\right]\varepsilon_{m}^{\widehat{\kappa}}\widehat{\kappa} = \varepsilon_{m}^{H} \tag{1.A.15}$$

Then using the three conditions

$$\varepsilon_{m}^{\widehat{\kappa}}\left[1 + \left(\frac{f(\widehat{\kappa}) + H(\widehat{\kappa})}{F(\widehat{\kappa})}\right)\widehat{\kappa}\frac{(w + \frac{2}{3}m)}{(w+m)}\right] = \frac{m}{(w+m)} \tag{1.A.16}$$

Therefore $\varepsilon_m^{\widehat{\kappa}} > 0$, which implies $\varepsilon_m^w < 0$ and $\varepsilon_m^H < 0$. Since the wage goes down and productivity stays the same, the payoff spread D and the profits of entrepreneurs $\mathrm{E}_0[\Pi(\kappa)]$ increase by lemma 1.3.

Consider now an increase in the size of the venture capital sector N. Differentiation the monitoring market clearing condition $\frac{\mathrm{d}}{\mathrm{d}N}$:

$$\frac{\mathrm{d}m}{\mathrm{d}N} = -\frac{\frac{m}{\bar{\epsilon}}}{\left(\sum f(\widehat{\kappa}_i)\frac{\mathrm{d}\widehat{\kappa}_i}{\mathrm{d}m} + \frac{N}{\bar{\epsilon}}\right)} \tag{1.A.17}$$

Hence $\frac{\mathrm{d}m}{\mathrm{d}N} < 0$ since $\frac{\mathrm{d}\widehat{\kappa}}{\mathrm{d}m} > 0$ for every sector. Therefore, an increase in the size of the venture capital sector, or equivalently, an increase in the skill (lower $\bar{\epsilon}$), imply a lower monitoring cost. A lower monitoring cost implies more entrepreneurship, higher wages, and less innovative ideas in every sector. □

Corollary 1.1. By lemma 1.6, for a given monitoring cost, a sector has lower entrepreneurship if research productivity decreases. Suppose research productivity decreases for every sector in the economy. Entrepreneurship declines in each sector, so the demand for monitoring decreases. For the monitoring market to clear, the monitoring cost decreases. The new equilibrium has sectors with fewer entrepreneurs, less surplus, and less innovative ideas.

Consider a larger venture capital sector. Then, the monitoring cost decreases, and it increases surplus in each sector, with more entrepreneurs, but less innovative ideas, by proposition 1.1. □

Proposition 1.2. Consider the same steps for the proof of lemma 1.6 as as a single sector i varies in research productivity θ_i and the economy-wide monitoring cost m adjusts. Let ε_b^a be the elasticity of variable a with respect to variable b, $\varepsilon_b^a = \frac{\mathrm{d}\ln(a)}{\mathrm{d}\ln(b)} = \frac{\mathrm{d}a}{\mathrm{d}b}\frac{b}{a}$. Then for a given sector θ the capabilities threshold $\widehat{\kappa}$ implies:

$$\varepsilon_\theta^{\widehat{\kappa}} = -4 + 3\frac{(w+\frac{2}{3}m)}{(w+m)}\varepsilon_\theta^w + \frac{m}{w+m}\varepsilon_\theta^m \tag{1.A.18}$$

From the optimal hiring policy $H(\kappa)$

$$\varepsilon_\theta^H = [4 - 3\varepsilon_\theta^w] = -\left[\varepsilon_\theta^{\widehat{\kappa}} + \frac{m}{w+m}\varepsilon_\theta^w - \frac{m}{w+m}\varepsilon_\theta^m\right] \tag{1.A.19}$$

Differentiating the market clearing condition with respect to research productivity $\frac{\mathrm{d}}{\mathrm{d}\theta}$, noting that $\frac{\mathrm{d}H}{\mathrm{d}\theta} \propto H$, and using that aggregate hiring is the labor supply $F(\widehat{\kappa}) = \int_{\widehat{\kappa}}^{\bar{c}} H(\kappa)\mathrm{d}F(c)$, yields

$$\left[\frac{f(\widehat{\kappa}) + H(\widehat{\kappa})}{F(\widehat{\kappa})}\right]\varepsilon_\theta^{\widehat{\kappa}}\widehat{\kappa} = \varepsilon_\theta^H \tag{1.A.20}$$

By the Monitoring Market Clearing:

$$\varepsilon_{\theta_i}^m = -\varepsilon_{\theta_i}^{\widehat{\kappa}_i}\underbrace{\left(\frac{\widehat{\kappa}_i}{m} \cdot \frac{f(\widehat{\kappa}_i)}{\frac{N}{\bar{\epsilon}} + 2\sum_{j\neq i} f(\widehat{\kappa}_j)\frac{\partial \widehat{\kappa}_j}{\partial m}}\right)}_{>0} \tag{1.A.21}$$

Then using the four previous conditions:

$$\varepsilon_\theta^{\widehat{\kappa}}\underbrace{\left[\left(\frac{f(\widehat{\kappa}) + H(\widehat{\kappa})}{F(\widehat{\kappa})}\right)\widehat{\kappa} + 1 + \frac{m}{w+m}\left(\frac{\widehat{\kappa}_i}{m} \cdot \frac{f(\widehat{\kappa}_i)}{\frac{N}{\bar{\epsilon}} + 2\sum_{j\neq i} f(\widehat{\kappa}_j)\frac{\partial \widehat{\kappa}_j}{\partial m}}\right)\right]}_{>0} = -\frac{m}{w+m}\varepsilon_\theta^w \tag{1.A.22}$$

Therefore from equations (1.A.19), (1.A.21) and (1.A.39):

$$\frac{\varepsilon_\theta^H}{\varepsilon_\theta^{\widehat{\kappa}}} > 0 \quad \text{and} \quad \frac{\varepsilon_\theta^m}{\varepsilon_\theta^{\widehat{\kappa}}} < 0 \quad \text{and} \quad \frac{\varepsilon_\theta^w}{\varepsilon_\theta^{\widehat{\kappa}}} < 0 \tag{1.A.23}$$

Rewriting equation (1.A.18):

$$\varepsilon_\theta^{\widehat{\kappa}}\underbrace{\left(1 - 3\frac{(w + \frac{2}{3}m)}{(w+m)}\frac{\varepsilon_\theta^w}{\varepsilon_\theta^{\widehat{\kappa}}} - \frac{m}{w+m}\frac{\varepsilon_\theta^m}{\varepsilon_\theta^{\widehat{\kappa}}}\right)}_{>0} = -4 \tag{1.A.24}$$

Then $\varepsilon_\theta^{\widehat{\kappa}} < 0$, which implies by equation (1.A.41) that $\varepsilon_\theta^H < 0$, $\varepsilon_\theta^w > 0$, and $\varepsilon_\theta^m > 0$. For profits:

$$\varepsilon_\theta^\Pi = \varepsilon_\theta^H + \varepsilon_\theta^w \tag{1.A.25}$$

$$\varepsilon_\theta^\Pi = \varepsilon_\theta^{\widehat{\kappa}} \left\{ -\left(\frac{m}{w} \cdot \frac{f(\widehat{\kappa}) + H(\widehat{\kappa})}{F(\widehat{\kappa})}\right) \widehat{\kappa} - \frac{(w+m)}{w} \cdot \left[1 + \frac{m}{w+m} \left(\frac{\widehat{\kappa}_i}{m} \cdot \frac{f(\widehat{\kappa}_i)}{\frac{N}{\bar{\epsilon}} + 2\sum_{j\neq i} f(\widehat{\kappa}_j)\frac{\partial \widehat{\kappa}_j}{\partial m}}\right)\right]\right\} \tag{1.A.26}$$

Therefore $\varepsilon_\theta^\Pi > 0$. Since $\frac{D^2}{\kappa} = \Pi(\kappa)$, $\varepsilon_\theta^D > 0$.

For surplus, similar to the proof of lemma 1.6:

$$\varepsilon_\theta^S = \varepsilon_\theta^\Pi - \frac{\widehat{\kappa}}{k - \widehat{\kappa}} \varepsilon_\theta^{\widehat{\kappa}} \tag{1.A.27}$$

Since only those with $\kappa > \widehat{\kappa}$ produce surplus and $\varepsilon_\theta^{\widehat{\kappa}} < 0$, $\varepsilon_\theta^S > 0$. Then for the total surplus of a sector $\int_{\widehat{\kappa}}^\infty S(\kappa)\mathrm{d}F(\kappa)$ it's enough to note that when research productivity goes up θ, there are more entrepreneurs $\left(\frac{\mathrm{d}\widehat{\kappa}}{\mathrm{d}\theta} < 0\right)$, and each entrepreneur produces more surplus $\frac{\mathrm{d}S(\kappa)}{\mathrm{d}\theta} > 0$, so the surplus of a sector goes up.

□

Corollary 1.2. Since entrepreneurship increases in the booming sector and declines in other sectors, entrepreneurship and venture capital concentrate in the booming sectors relative to before the boom. Since surplus declines in other sectors, fewer sectors get financed. □

Corollary 1.3. A shock that increases the research productivity of sector i increases the monitoring cost of the economy by proposition 1.2. The average skill of venture capitalists decreases when $\bar{\epsilon}$ goes up, which is equivalent to a decrease in the venture capital sector size N by the proof to proposition 1.1. Then, a decrease in the average skill of venture capitalists also increases the monitoring cost m, increasing the negative spillovers to other sectors. □

Lemma 1.7. The planner's problem is

$$\max_{\Theta,\{\mathcal{E},H,D\}_{i\in\Theta},\widehat{\epsilon}} \quad \sum_{i\in\Theta}\int_{\mathcal{E}_i}\left[\mathrm{E}_0[Rev(D)-I|H]-\frac{D^2}{\kappa}\right]\mathrm{d}F(\kappa)-\int_0^{\widehat{\epsilon}}[Ve]\mathrm{d}G(e)-\Delta_i \tag{1.A.28}$$

$$\text{s.t.} \quad \int_{\mathcal{E}_i}H(\kappa)\mathrm{d}F(\kappa)=\int_{\mathcal{E}_i^c}\mathrm{d}F(\kappa) \qquad ,\forall i\in\Theta \tag{Labor Market}$$

$$\int_{\mathcal{E}_i}S(\kappa)\mathrm{d}F(\kappa)\geq\Delta_i \qquad ,\forall i\in\Theta \tag{Search}$$

$$\sum_{i\in\Theta}\int_{\mathcal{E}_i}\mathrm{d}F(\kappa)=\int_0^{\widehat{\epsilon}}V\mathrm{d}G(e) \tag{Monitoring Market}$$

Any optimal allocation is characterized by a financing frontier $\int_{\mathcal{E}}S(\kappa;\theta)\mathrm{d}F(\kappa)=\Delta$. Any sector inside is financed, any sector outside is not. Suppose not for some feasible allocation $\{\Theta,\{\mathcal{E},H,D\}_{i\in\Theta},\widehat{\epsilon}\}$. Then, there is a sector i outside the financing frontier that gets financed, but it produces less surplus than its entry cost $\int_{\mathcal{E}_i}S(\kappa;\theta_i)\mathrm{d}F(\kappa)<\Delta_i$. Aggregate surplus increases by simply not financing sector i, hence the original allocation cannot be optimal. The logic is analogous for a sector inside the frontier that does not get financed.

The financing frontier is characterized by a threshold level of research productivity $\widehat{\theta}(\Delta)$ such that for each level of search costs Δ: $\int_{\mathcal{E}}S(\kappa;\widehat{\theta}(\Delta))\mathrm{d}F(\kappa)=\Delta$. $\widehat{\theta}(\Delta)$ gives the minimum research productivity the sector needs to cover it's entry costs. Consider the entry condition for a sector with a given Δ. Surplus is decreasing in research productivity, so a unique $\widehat{\theta}(\Delta)$ determines indifference for the entry condition. Any sector with lower research productivity is not financed. Considering $\widehat{\theta}(\Delta)$ for any possible Δ determines the financing frontier.

Any optimal allocation is characterized by a capabilities threshold $\widehat{\kappa}$. Anyone above is an entrepreneur, anyone below is a worker. Suppose not for some feasible allocation $\{\Theta,\{\mathcal{E},H,D\}_{i\in\Theta},\widehat{\epsilon}\}$. Then, there is some worker i and some entrepreneur j such that $\kappa_i>\kappa_j$. Switch the roles between these two agents, and assign the hiring and payoff spread of agent j $\{H(\kappa_j),D(\kappa_j)\}$ to the new entrepreneur i. Then, the allocation is still feasible but the setup cost of agent i is smaller than that for agent j $\frac{D_j^2}{\kappa_i}<\frac{D_j^2}{\kappa_j}$, and hence surplus increased by just reshuffling entrepreneurs. Therefore, the original allocation $\{\Theta, \{\mathcal{E}, H, D\}_{i\in\Theta}, \widehat{\epsilon}\}$ cannot be optimal.

A threshold determines venture capital entry. The effort cost of venture capitalists is a deadweight loss in the economy. To minimize the deadweight loss, the planner allocates the more skilled venture capitalists to finance and monitor entrepreneurs, according to a threshold $\widehat{\epsilon}$. □

Proposition 1.3. Since the search constraint is active only for those sectors in the frontier, omit it from the problem for now. The planner's Lagrangian is:

$$\mathcal{L} = \sum_{i\in\Theta} \int_{\mathcal{E}_i} \left[\mathrm{E}_0[Rev(D) - I|H] - \frac{D^2}{\kappa} \right] \mathrm{d}F(\kappa) - \int_0^{\widehat{\epsilon}} [Ve] \mathrm{d}G(e) - \Delta_i \tag{1.A.29}$$

$$- \sum_{i\in\Theta} \lambda_i \left(\int_{\mathcal{E}_i} H(\kappa)\mathrm{d}F(\kappa) - \int_{\mathcal{E}_i^c} \mathrm{d}F(\kappa) \right) - m^P \left(\sum_{i\in\Theta} \int_{\mathcal{E}_i} \mathrm{d}F(\kappa) - \int_0^{\widehat{\epsilon}} V\mathrm{d}G(e) \right) \tag{1.A.30}$$

Rewriting $\mathcal{L}$, and considering the thresholds of lemma 1.7, maximizing $\mathcal{L}$ is equivalent to maximizing $\mathcal{L}^*$

$$\mathcal{L}^* = \sum_{i\in\Theta} \int^{\widehat{\kappa}} \left[\mathrm{E}_0[Rev(D) - I|H] - \lambda H - \frac{D^2}{\kappa} - \lambda - m^P \right] \mathrm{d}F(\kappa) = \sum_{i\in\Theta} \int^{\widehat{\kappa}} S(\kappa)\mathrm{d}F(\kappa) \tag{1.A.31}$$

which is to maximize the net surplus each entrepreneur creates considering λ as the marginal social value of labor and m^P as the marginal social value of monitoring.

The planner's first order conditions for an entrepreneurs hiring and payoff spread are then equivalent to the market ones, under prices λ and m^P. The planner's first order condition for entrepreneurship in sector i, $\widehat{\kappa}_i$, balances the marginal cost of losing entrepreneurs against the marginal benefit of increasing the likelihood of other entrepreneurs to succeed, which gives:

$$f(\widehat{\kappa}_i)S(\widehat{\kappa}_i) = \int_{\widehat{\kappa}_i} \frac{\partial S(\kappa)}{\partial \widehat{\kappa}_i} \mathrm{d}F(\kappa) \tag{1.A.32}$$

The partial derivative of surplus is equivalent to the partial derivative of revenues $\mathrm{E}_0[\max\{Rev(D)-$

$I\}|H] = g(\widehat{\kappa})\theta\sqrt{HD}$, and using the optimal hiring and spread equation (1.A.32) can be rewritten as

$$f(\widehat{\kappa}_i)S(\widehat{\kappa}_i) = \varepsilon^{g(\widehat{\kappa})}_{\widehat{\kappa}_i}\frac{1}{\widehat{\kappa}}g(\widehat{\kappa})^4\frac{\theta^4}{16\lambda^2}\int_{\widehat{\kappa}_i}^{\bar{\kappa}}\kappa \mathrm{d}F(\kappa) = \varepsilon^{g(\widehat{\kappa})}_{\widehat{\kappa}_i}4\frac{\Pi(\widehat{\kappa})}{\widehat{\kappa}}\frac{\mathrm{E}[\kappa|\kappa \geq \widehat{\kappa}]}{\widehat{\kappa}}[1-F(\widehat{\kappa})] \quad (1.A.33)$$

reordering terms, replacing the expression for surplus as $\mathrm{E}_0[S^*(\widehat{\kappa})] = \mathrm{E}_0[\Pi^*(\widehat{\kappa})] - \lambda - m^P$, and defining $\phi(\widehat{\kappa}) = 4\varepsilon^{g(\widehat{\kappa})}_{\widehat{\kappa}_i}\frac{\mathrm{E}[\kappa|\kappa\geq\widehat{\kappa}]}{\widehat{\kappa}}\frac{[1-F(\widehat{\kappa})]}{f(\widehat{\kappa})}$ gives

$$\Pi(\widehat{\kappa})\left(1 - \frac{\phi(\widehat{\kappa})}{\widehat{\kappa}}\right) = (\lambda + m^P) \quad (1.A.34)$$

To see how the planner's threshold moves with research productivity, following the same steps as in the proof to lemma 1.6, taking $(\widehat{\kappa} - \phi(\widehat{\kappa}))$ as the object of interest, and rewriting equation (1.A.34) in terms of $(\widehat{\kappa} - \phi(\widehat{\kappa}))$ gives:

$$\varepsilon^{(\widehat{\kappa}-\phi(\widehat{\kappa}))}_{\theta} = -4 + 3\frac{(w + \frac{2}{3}m)}{(w+m)}\varepsilon^w_\theta + \frac{m}{w+m}\varepsilon^m_\theta - 4\varepsilon^g_{\widehat{\kappa}}\varepsilon^{\widehat{\kappa}}_\theta \quad (1.A.35)$$

From the optimal hiring policy $H(\kappa)$

$$\varepsilon^H_\theta = \left[4 - 3\varepsilon^w_\theta + 4\varepsilon^g_{\widehat{\kappa}}\varepsilon^{\widehat{\kappa}}_\theta\right] = -\left[\varepsilon^{(\widehat{\kappa}-\phi(\widehat{\kappa}))}_\theta + \frac{m}{w+m}\varepsilon^w_\theta - \frac{m}{w+m}\varepsilon^m_\theta\right] \quad (1.A.36)$$

Differentiating the labor market clearing condition with respect to research productivity $\frac{\mathrm{d}}{\mathrm{d}\theta}$, noting that $\frac{\mathrm{d}H}{\mathrm{d}\theta} \propto H$, and using that aggregate hiring is the labor supply $F(\widehat{\kappa}) = \int_{\widehat{\kappa}}^{\bar{c}} H(\kappa)\mathrm{d}F(c)$, yields

$$\left[\frac{f(\widehat{\kappa}) + H(\widehat{\kappa})}{F(\widehat{\kappa})}\right]\varepsilon^{\widehat{\kappa}}_\theta\widehat{\kappa} = \varepsilon^H_\theta \quad (1.A.37)$$

Differentiating the monitoring market clearing condition with respect to research productivity $\frac{\mathrm{d}}{\mathrm{d}\theta}$ yields

$$\varepsilon^m_{\theta_i} = -\varepsilon^{\widehat{\kappa}_i}_{\theta_i}\underbrace{\left(\frac{\widehat{\kappa}_i}{m}\cdot\frac{f(\widehat{\kappa}_i)}{\frac{N}{\bar{\epsilon}} + 2\sum_{j\neq i}f(\widehat{\kappa}_j)\frac{\partial\widehat{\kappa}_j}{\partial m}}\right)}_{>0} \quad (1.A.38)$$

Then using equations (1.A.35) to (1.A.38) gives:

$$\underbrace{\varepsilon_\theta^{\widehat{\kappa}}\left[\left(\frac{f(\widehat{\kappa})+H(\widehat{\kappa})}{F(\widehat{\kappa})}\right)\widehat{\kappa}+\frac{\varepsilon_\theta^{(\widehat{\kappa}-\phi(\widehat{\kappa}))}}{\varepsilon_\theta^{\widehat{\kappa}}}+\frac{m}{w+m}\left(\frac{\widehat{\kappa}_i}{m}\cdot\frac{f(\widehat{\kappa}_i)}{\frac{N}{\epsilon}+2\sum_{j\neq i}f(\widehat{\kappa}_j)\frac{\partial\widehat{\kappa}_j}{\partial m}}\right)\right]}_{>0}=-\frac{m}{w+m}\varepsilon_\theta^w \tag{1.A.39}$$

which implies the change in entrepreneurship and the wage have opposite signs as long as $\frac{\varepsilon_\theta^{(\widehat{\kappa}-\phi(\widehat{\kappa}))}}{\varepsilon_\theta^{\widehat{\kappa}}}>0$:

$$\frac{\varepsilon_\theta^{(\widehat{\kappa}-\phi(\widehat{\kappa}))}}{\varepsilon_\theta^{\widehat{\kappa}}}=\frac{\widehat{\kappa}}{(\widehat{\kappa}-\phi(\widehat{\kappa}))}\left(1-\frac{\partial\phi(\widehat{\kappa})}{\partial\widehat{\kappa}}\right)>0 \tag{1.A.40}$$

since $(\widehat{\kappa}-\phi(\widehat{\kappa}))>0$ and $\frac{\partial\phi(\widehat{\kappa})}{\partial\widehat{\kappa}}<0$ by assumption 1.4. Therefore from equations (1.A.36), (1.A.38) and (1.A.39):

$$\frac{\varepsilon_\theta^H}{\varepsilon_\theta^{\widehat{\kappa}}}>0 \quad \text{and} \quad \frac{\varepsilon_\theta^m}{\varepsilon_\theta^{\widehat{\kappa}}}<0 \quad \text{and} \quad \frac{\varepsilon_\theta^w}{\varepsilon_\theta^{\widehat{\kappa}}}<0 \tag{1.A.41}$$

Rewriting equation (1.A.35):

$$\underbrace{\varepsilon_\theta^{\widehat{\kappa}}\left(\frac{\widehat{\kappa}}{(\widehat{\kappa}-\phi(\widehat{\kappa}))}\left(1-\frac{\partial\phi(\widehat{\kappa})}{\partial\widehat{\kappa}}\right)+4\varepsilon_{\widehat{\kappa}}^g\widehat{\kappa}-3\frac{(w+\frac{2}{3}m)}{(w+m)}\frac{\varepsilon_\theta^w}{\varepsilon_\theta^{\widehat{\kappa}}}-\frac{m}{w+m}\frac{\varepsilon_\theta^m}{\varepsilon_\theta^{\widehat{\kappa}}}\right)}_{>0}=-4 \tag{1.A.42}$$

Therefore $\varepsilon_\theta^{\widehat{\kappa}}>0$, which implies $\varepsilon_\theta^w>0$, $\varepsilon_\theta^m>0$, $\varepsilon_\theta^H<0$. For profits $\varepsilon_\theta^\Pi=\varepsilon_\theta^H+\varepsilon_\theta^w+4\varepsilon_\theta^g$, which implies:

$$\varepsilon_\theta^\Pi=\varepsilon_\theta^{\widehat{\kappa}}\Bigg\{-\left(\frac{m}{w}\cdot\frac{f(\widehat{\kappa})+H(\widehat{\kappa})}{F(\widehat{\kappa})}\right)\widehat{\kappa}-\frac{(w+m)}{w}\cdot\Bigg[\frac{\widehat{\kappa}}{(\widehat{\kappa}-\phi(\widehat{\kappa}))}\left(1-\frac{\partial\phi(\widehat{\kappa})}{\partial\widehat{\kappa}}\right) + \frac{m}{w+m}\left(\frac{\widehat{\kappa}_i}{m}\cdot\frac{f(\widehat{\kappa}_i)}{\frac{N}{\epsilon}+2\sum_{j\neq i}f(\widehat{\kappa}_j)\frac{\partial\widehat{\kappa}_j}{\partial m}}\right)\Bigg]\Bigg\}+4\varepsilon_\theta^g \tag{1.A.43}$$

Then $\varepsilon_\theta^\Pi>0$. $\frac{D^2}{\kappa}=\Pi$ hence $2\varepsilon_\theta^D=\varepsilon_\theta^\Pi$ and $\varepsilon_\theta^D>0$. Finally, for surplus consider

$\mathrm{E}_0[S^*(\widehat{\kappa})] = \mathrm{E}_0[\Pi^*(\widehat{\kappa})] - \lambda - m^P$ which can be rewritten as:

$$S(\kappa) = \frac{\Pi(\widehat{\kappa})}{\widehat{\kappa}}[\kappa - \widehat{\kappa} + \phi(\widehat{\kappa})] \tag{1.A.44}$$

and implies

$$\varepsilon_\theta^S = \varepsilon_\theta^\Pi - \varepsilon_\theta^{\widehat{\kappa}} - \varepsilon_\theta^{\widehat{\kappa}} \frac{\widehat{\kappa}}{(\kappa - \widehat{\kappa} + \phi(\widehat{\kappa}))} \left[1 - \frac{\partial \phi(\widehat{\kappa})}{\partial \widehat{\kappa}}\right] > 0 \tag{1.A.45}$$

Therefore, $\varepsilon_\theta^S > 0$, which completes the proof for the comparative statics of the planner's allocations.

To consider the change in each sector's allocation for a given equilibrium with a monitoring cost m, consider $\varepsilon_m^a = 0$ for any measure a. Similar to the market, lower research productivity implies less surplus, fewer entrepreneurs, less innovative ideas.

□

Proposition 1.4. **Sector-specific taxes achieve the first best** The proof to proposition 1.3 shows the planner's rule to allocate sectors, venture capitalists, hiring, and payoff spreads is equivalent to the market one. Then, if the planner allocates entrepreneurship equally to the market, prices coincide and allocations coincide. From equation (1.A.34) a tax such that $T_i^* = \frac{\phi(\widehat{\kappa})}{\widehat{\kappa}}$ makes the market allocation of entrepreneurs coincide with the planner's allocation.

Worker reskilling across sectors lowers welfare Consider any switching cost s low enough such that some workers move sectors. Workers move to the highest research productivity sectors given higher wages by proposition 1.3. With a higher worker supply, wages decrease and entrepreneurship increases, decreasing the likelihood of success of entrepreneurs in the high research productivity sector, which lowers welfare. In the low research productivity sector, wages increase and entrepreneurship decreases even more, lowering welfare.

R&D subsidies improve welfare by incentivizing excess entrepreneurs to work With a subsidy U_i, the marginal entrepreneur in a sector is $\widehat{\kappa} = \frac{64w_i^2}{\theta_i^4}\frac{(w_i+m)}{(1+U_i)^3}$. Consider the effect in entrepreneurship of a change in the subsidy $\frac{\mathrm{d}\widehat{\kappa}}{\mathrm{d}U_i}$:

$$\frac{\mathrm{d}\widehat{\kappa}}{\mathrm{d}U_i} = \frac{\partial\widehat{\kappa}}{\partial U_i} + \frac{\partial\widehat{\kappa}}{\partial w}\frac{\mathrm{d}w}{\mathrm{d}U_i} + \frac{\partial\widehat{\kappa}}{\partial m}\frac{\mathrm{d}m}{\mathrm{d}U_i} \tag{1.A.46}$$

Taking into account the hiring policy, the labor market clearing condition, and the monitoring market clearing condition, analogously to the proof of proposition 1.2, shows that

$$\frac{\mathrm{d}\widehat{\kappa}}{\mathrm{d}U_i} \propto \frac{\left(\frac{4}{3}w - 3\widehat{\kappa}\right)}{(1+U_i)} \tag{1.A.47}$$

and hence the change in entrepreneurship depends on the difference between the wage and the entrepreneurship threshold. A sector with a sufficiently low research productivity has a low wage w and high entrepreneurship $\widehat{\kappa}$ such that $\frac{\mathrm{d}\widehat{\kappa}}{\mathrm{d}\theta} < 0$, and the opposite is true for a sector with a sufficiently high research productivity. Qualitatively similar results hold for a subsidy that only goes to hiring H or spreads D, given the complementarity between both. $\square$

Appendix 1.B Variable monitoring cost

Consider monitoring $\mathcal{M}_i$ with a fixed and variable part:

$$\mathcal{M}_i(\kappa) = m_f + m_v \frac{D^2}{\kappa} \tag{1.B.1}$$

Equation (1.B.1) considers that it is costlier to monitor more innovative (larger D) and less skilled (lower κ) entrepreneurs. For instance, asymmetries of information between the investor and the entrepreneur might increase with how innovative the idea is, making monitoring more difficult. At the same time, more skilled entrepreneurs might communicate better with the venture capitalist or not need such close monitoring, lowering the monitoring cost for venture capitalists.

Equation (1.B.1) can accommodate for the venture capitalist directly adding value to the entrepreneur $m_V < 0$. The joint surplus is

$$\mathrm{E}_0[S_i(\kappa, H, D)] = \mathrm{E}_0[\mathbb{1}_{\mathcal{S}}(\mathrm{R}_q(D) - I)|H] - w_i H - \frac{D^2}{\kappa} - w_i - \mathcal{M}_i(\kappa) \tag{1.B.2}$$

$$\mathrm{E}_0[S_i(\kappa, H, D)] = \mathrm{E}_0[\mathbb{1}_{\mathcal{S}}(\mathrm{R}_q(D) - I)|H] - w_i H - (1 + m_v)\frac{D^2}{\kappa} - w_i - m_f \tag{1.B.3}$$

Equation (1.B.3) is the same condition as in the benchmark scenario if we define an auxiliary skill level for entrepreneurs $\tilde{\kappa} = \frac{\kappa}{1+m_v}$. The variable cost of monitoring acts as a discount on the entrepreneurial skill of agents. Then, the overall entrepreneurial skill distribution depends both on the agent's entrepreneurial skill κ and on the value added of venture capitalists m_v. $m_v < 0$ implies venture capitalists add value as if entrepreneurs where more skilled.

Appendix 1.C Contract implementation

The most common contract used by venture capitalists is a preferred equity contract (Kaplan and Strömberg, 2003; Gompers et al., 2020), which can take two forms: preferred equity with participation or preferred equity with convertibility.

The venture capitalist has full control so the idea is only implemented when there is a good signal. Since revenue is enough to cover the costs and wages in expectation, a contract is feasible in equilibrium as long as cash flows to the venture capitalist in the bad state are high enough.

Let $p = P(G|\mathcal{S}_G, H; \theta_i)$ be the signal informativeness in sector i for a given hiring H. Let $CF_{2,q}^{VC}$ be the contract's payoffs to the entrepreneur at $t = 2$ depending on idea quality $q \in \{G, B\}$.

Preferred equity with participation The cash flow to the venture capitalist for an idea of quality q is:

$$CF_{2,q}^{VC} = \underbrace{\min\{CF_{2,q}^{VC}, R\}}_{\text{Liquidity Preference}} + \underbrace{\gamma_p \max\left\{0, CF_{2,q}^{VC} - R\right\}}_{\text{Participation}} \tag{1.C.1}$$

The venture capitalist always gets his liquidity preference, and a share γ_p of any cash flows beyond R.

Consider the liquidity preference such that the venture capitalist gets everything in case of failure, $R > I - 2\sqrt{D}$, and that the liquidity preference returns all his invested money in expectation.[23] The venture capitalist wage m and surplus share a come from the variable component γ_p. Then the liquidity preference and variable component γ_p are such that:

$$pR + (1-p)(I - 2\sqrt{D}) = I + \frac{D^2}{\kappa} + w_i H \tag{1.C.2}$$

$$\gamma_p p(I + 2\sqrt{D} - R) = a\,\mathrm{E}_0[S^*(\kappa)] + m, \tag{1.C.3}$$

which yields

$$R = \frac{I - (1-p)(I - 2\sqrt{D}) + \frac{D^2}{\kappa} + w_i H + m}{p} \tag{1.C.4}$$

$$\gamma_p = \frac{a\,\mathrm{E}_0[S^*(\kappa)] + m}{p(I + 2\sqrt{D} - R)} \tag{1.C.5}$$

Preferred equity with convertibility The cash flow to the venture capitalist for an idea of quality q is:

$$CF_{2,q}^{VC} = \max\{\underbrace{\min\{CF_{2,q}^{VC}, R\}}_{\text{Liquidity Preference}}, \underbrace{\gamma_c CF_{2,q}^{VC}}_{\text{Convertibility}}\} \tag{1.C.6}$$

The venture capitalist always gets his liquidity preference or a share γ_c of the cash flows if he decides to convert. Assume R is such that the venture capitalist gets all the cash flows in the low state and converts in the high state. $R = CF_{2,B} = I - 2\sqrt{D}$. Then the variable component γ_c is such that:

$$p\gamma_c(I + 2\sqrt{D}) + (1-p)(I - 2\sqrt{D}) = a\,\mathrm{E}_0[S^*(\kappa)] + m + I + \frac{D^2}{\kappa} + w_i H, \tag{1.C.7}$$

which yields

$$\gamma_c = \frac{a\,\mathrm{E}_0[S^*(\kappa)] + m + I + wH + \frac{D^2}{\kappa}}{p(I + 2\sqrt{D})} - \frac{(1-p)(I - 2\sqrt{D})}{p(I + 2\sqrt{D})} \tag{1.C.8}$$

Debt The cash flow to the venture capitalist for an idea of quality q is:

$$CF_{2,q}^{VC} = R_q \tag{1.C.9}$$

Debt is always risky in my setup. Assume $R_B = I - 2\sqrt{D}$. Then the repayment in the good state R_G just needs to be equivalent to the expected payoff from converting in the

[23] Other assumptions yield different liquidity preferences R and participation rights γ_p. The cash flows that implement the contract are not unique.

case of a preferred equity with convertibility contract, such that:

$$R_G = \gamma_c(I + 2\sqrt{D}) = \frac{a\,\mathrm{E}_0[S^*(\kappa)] + m + I + wH + \frac{D^2}{\kappa}}{p} - \frac{(1-p)(I - 2\sqrt{D})}{p} \tag{1.C.10}$$

Equity The cash flow to the venture capitalist for an idea of quality q is:

$$CF^{VC}_{2,q} = \gamma_e CF^{VC}_{2,q} \tag{1.C.11}$$

Then the equity share γ_e is:

$$p\gamma_c(I + 2\sqrt{D}) + (1-p)\gamma_c(I - 2\sqrt{D}) = a\,\mathrm{E}_0[S^*(\kappa)] + m + I + \frac{D^2}{\kappa} + w_i H \tag{1.C.12}$$

which yields

$$\gamma_c = \frac{a\,\mathrm{E}_0[S^*(\kappa)] + m + I + \frac{D^2}{\kappa} + w_i H}{I + 2\sqrt{D}(2p-1)} \tag{1.C.13}$$

Appendix 1.D Entrepreneur control

Let $CF^E_{2,q}$ be the contract's payoffs to the entrepreneur at $t = 2$ depending on idea quality $q \in \{G, B\}$. Since the entrepreneur does not invest any money, he implements the idea in $t = 1$ regardless of the signal:

$$\begin{aligned}
\mathrm{E}_1[CF^E_{2,q}|\mathcal{S}_G] =& P(G|\mathcal{S}_G)(>0) + P(B|\mathcal{S}_G)(0) \\
\mathrm{E}_1[CF^E_{2,q}|\mathcal{S}_B] =& P(G|\mathcal{S}_B)(>0) + P(B|\mathcal{S}_B)(0)
\end{aligned}$$

Suppose the venture capitalist has control with probability z. The venture capitalist expects to lose money whenever there is a negative signal and the entrepreneur has control, which happens with probability $\frac{(1-z)}{2}$. The expected revenue for an idea net of investment is:

$$\mathrm{E}_0[\mathbb{1}_{\mathcal{S}}(Rev(D) - I)|H;\theta_i] = \theta_i\sqrt{HD}z \tag{1.D.1}$$

where $\mathbb{1}_{inv} = 1$ if the idea is implemented at $t = 1$, which depends on the signal realization and who has control over the investment decision.

Equation (1.D.1) is the same expression as for the benchmark scenario, multiplied by the venture capitalist's probability of control z. The venture capitalist probability of control z acts as a discount on research productivity θ_i. With higher venture capitalist control z revenues are higher in expectation, which allows for more surplus, more entrepreneurs, smaller firms, and more innovative ideas.

If the entrepreneur has full control $z = 0$, venture capitalists expect to break even in any scenario at $t = 1$, so they do not finance any research or entrepreneurial innovation ($H = 0$, $D = 0$) once they enter. Moreover, since no surplus will be created, venture capitalists do not expect to recover their search cost u_i, and hence do not enter the sector. With full control of the entrepreneur $z = 0$, the sector is non-innovative, with no venture capital financing and every agent taking the outside option with income normalized to 0.

Appendix 1.E Heterogeneous worker skill

Consider a setup similar to that of Hacamo and Kleiner (2022). Agents can be of two types $\gamma \in \{h, l\}$ in addition to their entrepreneurial skill κ. With probability z they are of type h. h agents have an entrepreneurial skill of $\kappa_h = \xi\kappa$ where $\xi > 1$, while l agents have an entrepreneurial skill of $\kappa_l = \kappa$. As workers, h type agents have higher productivity, such that their unit of labor endowment produces $\delta > 1$ units of research, while l type agents produce a unit of research. Then, signal informativeness for an entrepreneur that hires H_h high type workers and H_l low type workers is:

$$P(\mathcal{S}_G|G,H) = P(\mathcal{S}_B|B,H) = \frac{1}{2}\left(1 + \theta_i\left(\delta\sqrt{H_h} + \sqrt{H_l}\right)\right) \tag{1.E.1}$$

δ and ξ allow for h type agents to be both better entrepreneurs and better workers on average. For a given hiring and payoff spread D, expected revenue of an idea is:

$$\mathrm{E}_0[\mathrm{R}_q(H_h, H_l, D)] = \frac{I}{2} + \theta_i(\delta\sqrt{H_h} + \sqrt{H_l})\sqrt{D} \tag{1.E.2}$$

Consider a type γ entrepreneur with skill κ_γ and wages $w_{i,h}$ and $w_{i,l}$ for workers $\{h, l\}$ respectively. The entrepreneur solves:

$$\max_{H,D}(1 - a_i)\left(\mathrm{E}_0[\max\{\mathrm{R}_q(D) - I, 0\}|H] - w_{i,h}H_h - w_{i,l}H_l - \frac{D^2}{\kappa_\gamma}\right) + w_i \tag{1.E.3}$$

Then optimal profits are still proportional to skill $\kappa_{\{h,l\}}$

$$\mathrm{E}_0[\Pi(\kappa_{\{h,l\}})] = \kappa_\gamma\frac{\theta_i^2}{64}\left(\frac{\delta^2}{w_{i,h}} + \frac{1}{w_{i,l}}\right) \tag{1.E.4}$$

and occupational choice in each labor market is determined by a threshold $\widehat{\kappa}_\gamma$. Hence, market clearing in each market implies:

$$z\int_{\widehat{\kappa}_h} H_h(\kappa;w_i)\mathrm{d}F(\kappa)+(1-z)\int_{\widehat{\kappa}_l} H_h(\kappa;w_i)\mathrm{d}F(\kappa)=z\int^{\widehat{\kappa}_h}\mathrm{d}F(\kappa) \tag{1.E.5}$$

$$z\int_{\widehat{\kappa}_h} H_l(\kappa;w_i)\mathrm{d}F(\kappa)+(1-z)\int_{\widehat{\kappa}_l} H_l(\kappa;w_i)\mathrm{d}F(\kappa)=(1-z)\int^{\widehat{\kappa}_l}\mathrm{d}F(\kappa) \tag{1.E.6}$$

Then, the entrepreneur's problem and sector equilibrium remain equivalent, with two wages and two levels of entrepreneurship for each sector.

Appendix 1.F Equilibrium bargaining power

The surplus an entrepreneur creates in a sector in terms of the entrepreneurial threshold $\widehat{\kappa}$ is:

$$\mathrm{E}_0[S^*(\kappa;\theta_i)] = (w+m)\Big(\frac{\kappa}{\widehat{\kappa}} - 1\Big) \tag{1.F.1}$$

- **Example 1:** Bargaining power decreasing and convex in skill κ, $a(\kappa) = \frac{\delta}{\kappa}$, unknown δ. Then

$$\int_{\widehat{\kappa}} a(\kappa)S(\kappa)\mathrm{d}F(\kappa) = u_i(\widehat{\kappa}) \tag{1.F.2}$$

$$a(\widehat{\kappa}) \equiv \frac{\delta}{\widehat{\kappa}} = \frac{u_i(\widehat{\kappa})}{(w+m)\int_{\widehat{\kappa}}\big(1-\frac{\widehat{\kappa}}{\kappa}\big)\mathrm{d}F(\kappa)} \tag{1.F.3}$$

- **Example 2:** Bargaining power decreasing and linear in distance from most skilled entrepreneur $(\bar{\kappa}-\kappa)$, $a(\kappa) = \delta(\bar{\kappa}-\kappa)$. Analogously to example 1,

$$\int_{\widehat{\kappa}} a(\kappa)S(\kappa)\mathrm{d}F(\kappa) = u_i(\widehat{\kappa}) \tag{1.F.4}$$

$$\delta = \frac{u_i(\widehat{\kappa})}{(w+m)\int_{\widehat{\kappa}}(\bar{\kappa}-\kappa)\left(\frac{\kappa}{\widehat{\kappa}}-1\right)\mathrm{d}F(\kappa)} \tag{1.F.5}$$

Chapter 2

Who Funds Which Ideas?

I develop a model of early-stage financing in which entrepreneurs overcome uncertainty through experimentation and investors differ in scale, staging ability, and advice. The model shows how investor-entrepreneur matches depend on project difficulty, experimental scale, and the complementarity between entrepreneurial effort and investor advice. When advice and effort reinforce each other by encouraging entrepreneurs to take on more risk, active investors back the most uncertain and difficult ideas. When entrepreneurial effort and investor advice substitute each other, investors shift to safer projects. The model helps explain the coexistence of diverse investors, the move of venture capital toward later and larger stages, and the lack of active investors in "tough tech" sectors. It also yields predictions on how changes in experimentation costs or investor scale affect the allocation of ideas and capital in the economy.

2.1 Introduction

Entrepreneurial finance today is marked by an unprecedented diversity of investors – yet this diversity hides sharp limits in who and what gets funded. Venture capitalists pioneered risky early-stage financing, but deal sizes have ballooned, shifting their focus toward later-stage, more certain investments (Lerner and Nanda, 2020). At the same time, alternative financing sources emerged, including accelerators, angels, institutional investors, sovereign funds, and syndicates, to fill funding gaps along the entrepreneurial process (Kerr and Nanda, 2015). The result is a segmented ecosystem where entrepreneurs begin with smaller, hands-on investors and later transition to larger, more distant investors as their ventures mature and scale.

Despite this expansion in funding sources, financing remains uneven across sectors. Socially valuable sectors such as renewables and energy have not been able to attract enough private funding, earning the label "tough tech sectors" (Kerr and Nanda, 2015). Even in sectors with rich entrepreneurial ecosystems, concerns grow that innovation is becoming less radical and impactful (Akcigit and Goldschlag, 2023), reflecting both idea scarcity (Bloom et al., 2020) and funding constraints (Nanda and Rhodes-Kropf, 2017). In this context, a central question in entrepreneurial finance is how the diversity of investors interacts with the uncertainty of ideas: who finances which projects, and how does the financing landscape shape the innovation landscape?

I present a theoretical model to explain when and why different types of investors back different kinds of ideas. An entrepreneur with an uncertain idea faces different investors. Investors have access to different financing technologies: stage financing, advise, or managing other investor's money. By turning to stage financing investors, the entrepreneur can transform negative-NPV ideas into positive ones through experimentation, to then approach other investors with a new, improved project. Depending on their idea and the quality of the advice technology, entrepreneurs turn to different investors in the economy.

My main result proposes that the complementarity between the investor's advice and the entrepreneur's effort is a key determinant of which investor finances which ideas. If the investor's advice encourages the entrepreneur to exert significantly more effort, such that he is willing to undertake a riskier project in equilibrium, then investors who offer advice will finance the most uncertain ideas in the economy. However, if the entrepreneur does not increase his effort enough, or even decreases it, such that his project becomes safer in response to the help from an investor, advice-giving investors retreat to safer ideas.

The model works as follows. Entrepreneurs differ in the difficulty of their ideas: some are highly novel and uncertain, while others are more incremental and easier to implement. Initially, all ideas are negative-NPV. The idea's payoffs have the structure of a prize – the entrepreneur gets a fixed amount if successful, and 0 otherwise. Importantly, entrepreneurial effort only impacts the probability to succeed, not the prize amount. Therefore, the entrepreneur's effort choice is tailored to maximize his chance to succeed.

The entrepreneur faces different groups of investors. Each investor is endowed with the same amount of cash. Investors can have up to three financing technologies, each with its own cost of running it. First, stage financing allows them to break an idea in two stages – experimentation and implementation – which increases the odds of success of the project and allows for early abandonment. Second, advising allows them to actively offer advice to the entrepreneur, reducing the project's risk. Third, managing other investor's money allows them to achieve a larger scale.

Stage financing creates value through experimentation, which is a key element of my model, as in Kerr et al. (2014). With stage financing, the entrepreneur runs an experiment that, if successful, improves the idea's probability of success in the implementation stage. Experiments are costly to run and stochastic. Importantly, the experiment has a mean outcome and some risk, which can be altered by the entrepreneur and advice-giving investors. The entrepreneur can exert effort to increase the mean outcome of the experiment in exchange for increased risk, while the investor's advice lowers the experiment's risk. In equilibrium, the effort of the entrepreneur and the advice of the investor jointly determine the chance that the experiment succeeds.

The market for financing is competitive, with a large mass of investors ready to enter.

Since only stage financiers can turn the entrepreneurs' idea into a positive NPV opportunity, the entrepreneur first seeks stage financiers to go through the experimentation stage. If successful, the project improves by having an increased chance to succeed, being derisked enough (Nanda and Rhodes-Kropf, 2017) so larger, less specialized investors can fund the implementation stage. An entrepreneur seeks investors from different groups depending on the difficulty of his idea, the experimental scale that it requires, and the quality of advice the investor offers.

I first show that investor advice is not always valuable. If the experiment is unlikely to succeed even when the entrepreneur exerts his maximum effort, investor advice is undesirable. Since the expected outcome of the experiment is to fail, the entrepreneur can maximize his chances to succeed by maximizing the risk of the experiment – he is out-of-the-money, so risk is valuable. Since investor advice lowers variance, it is undesirable from the perspective of the entrepreneur.

I then show that advice giving investors are less prevalent at mid-experimental scales. In low and large experimental scales, advice is more likely to be valuable. Hence, angel investors and venture capitalists seem to be efficient in the extremes of the experimental scale distribution, while in the middle either syndicates or more arm's length investors dominate unless the advice technology creates significant value.

Finally, I show that in my model the complementarity between the entrepreneur's effort and investor advice determines wether active investors focus on difficult or easy ideas. This mechanism helps explain why venture capitalists have moved to later stages, with larger scales and already tested, safer ideas. First, lower experimentation costs in sectors like software (Kerr et al., 2014) allowed small investors to emerge, pushing venture capitalists toward larger scales. Second, talent flows driven by an abundance of venture capital may have lowered the average quality of entrepreneurs, weakening complementarities and reducing the appeal of early uncertain projects. Third, in "tough-tech" sectors Nanda and Rhodes-Kropf (2017), the deep expertise required might limit the returns to advice, pushing venture capitalists toward other sectors.

I contribute to the literature on early-stage financing of innovative ideas in three ways. First, I develop a model where investor heterogeneity is central, yet investors coexist in equilibrium, financing different ideas at different scales. Second, the model incorporates entrepreneurial experimentation (Kerr et al., 2014) as part of the idea's life cycle, offering insights into which projects are funded by which investors depending on the experimentation process. Third, the model delivers empirical predictions on how shifts in the financing environment affect the allocation of ideas and capital. For instance, while a decline in experimentation costs increases the number of early projects funded (Nanda and Rhodes-Kropf, 2017), my model shows it also strengthens sorting: more ideas flow to small specialized investors, while venture capitalists focus even more on scaling proven ventures.

After laying out the model, I analyze how investor's advice and entrepreneurial effort interact in experimentation, when does advice create enough value, and finally show that active investors finance more difficult ideas when their advice induces the entrepreneur to undertake riskier projects.

2.2 Model setup

2.2.1 Ideas

An idea ι is a triplet (z, π, I), where $z \in [-\infty, +\infty]$ represents the difficulty of the idea, π its payoff if successful, and I its implementation cost. A function $p(z)$ determines the probability of success of any idea. $p(z)$ is strictly decreasing, such that a higher difficulty implies a lower probability to succeed ex-ante $(z < z') \Rightarrow p(z) > p(z')$.

From now on, an idea is summarized by its base difficulty z, and let $p_\iota = p(z)$ be the base probability of success of idea z_0. Hence, implementing the idea creates an expected value of $p_\iota \pi - I$.

2.2.2 Experimentation

Suppose a penniless risk-neutral entrepreneur has gathered financing to run an experiment on his idea before implementing it. If successful, the experiment reduces the difficulty of implementing the idea from z to z_1, increasing the probability of success.

An experiment $\mathcal{E}$ is a tuple $(z_0, \mathcal{X}_0, z_1, I_0)$. z_0 is the difficulty of the experiment, $\mathcal{X}_0$ is a normally distributed random variable with mean $\mu(\cdot)$ and standard deviation $\sigma(\cdot)$, $z_1 < z$ is the difficulty of implementing the idea if the experiment succeeds, and I_0 is the cost to run the experiment. I assume that running an experiment is cheaper than implementing the idea $I_0 < I$, otherwise, it is always better to implement the idea directly since it resolves all uncertainty.

The entrepreneur can exert effort $e \in [0, \overline{e}]$ to alter the experiment's mean outcome and risk, while the investor can advise the entrepreneur, represented by $v \in \{0, \overline{v}\}$, which lowers the experiment's risk – $\mu(e)$ and $\sigma(e, v)$. The investor offers advice before the entrepreneur exerts effort. If the investor offers advice ($v = \overline{v}$), she reduces the experiment's risk $\sigma_v < 0$. Entrepreneurial effort improves the mean outcome of the experiment in exchange for increased risk,[1] $\mu_e > 0$ and $\sigma_e > 0$.

One way to interpret the setup is that the entrepreneur faces a multitasking problem, where effort is split between improving the technology (the mean outcome) and managing business risk. These tasks have opposing effects on the probability to succeed, and therefore on the expected payoff. The venture capitalist cannot directly enhance the technology but can offer advice that reduces business risk and lowers the entrepreneur's managerial load. By easing these burdens, the entrepreneur can focus more effort on the technology, potentially leading to better outcomes.

Assumption 2.1. *Harder ideas have harder experiments:z_0 increases with z.*

Then, running the experiment means the entrepreneur pays I_0 to get a draw from $\mathcal{X}_0 \sim \mathcal{N}(\mu(e), \sigma(e, v))$. If the draw exceeds the experiment's difficulty $x_0 > z_0$ the experiment succeeds, lowering the idea's difficulty of implementation from z to z_1. Therefore, the expected surplus of an experiment with entrepreneurial effort e, and investor advice v is

$$\mathrm{E}[S|e, v, z] = \mathbb{P}(x_0 > z_0)[p(z_1)\pi - I] - I_0 \tag{2.1}$$

[1] Functions with subscripts represent partial derivatives $f_x(a) = \frac{\partial f}{\partial x}(a)$

By standardizing the experiment's outcome and using the symmetry of the normal distribution[2], rewrite the NPV as

$$\mathrm{E}[S|e,v,z] = \Phi\left(\frac{\mu(e) - z_0}{\sigma(e,v)}\right)[p(z_1)\pi - I] - I_0 \tag{2.2}$$

where Φ is the standard normal CDF. Define the *Success Ratio* of an experiment as $SR(e,v,z) = \frac{\mu(e)-z}{\sigma(e,v)}$, which captures how far the expected outcome exceeds the success threshold, scaled by the experiment's risk.

The Success Ratio of the experiment is the key object of my model that encapsulates the tradeoffs faced by the entrepreneur and the investor. It controls the probability to succeed since $\mathbb{P}(x_0 > z_0) = \Phi(SR(e,v,z_0))$ and is affected by both the entrepreneur's effort and the investor's advice. The success of the experiment depends on how the entrepreneur and the investor interact through the success ratio $SR(e,v,z_0)$.

2.2.3 Financing environment

Now consider the more interesting case where the entrepreneur has not yet secured financing, and the idea is negative NPV ex ante $p_\iota\pi - I < 0$. The financing sector is competitive and there is a large enough mass of financiers with no entry cost, so the entrepreneur captures all surplus. However, different groups of investors have access to different financing technologies and pay an ongoing cost to run those technologies. Each investor initially has cash c to invest.

Investor technologies

There are three financing technologies that investors might have access to: stage financing, advising, and managing other investor's money.

Stage financing For an ongoing cost of ι_S, stage financing breaks the idea in two stages and gives the entrepreneur an experiment $\mathcal{E}$ to run. The first stage (0) is for experimentation. The second stage (1) is for implementation. If the experiment $\mathcal{E}$ succeeds, the idea

[2] Roy (1952, 1956) shows that on any distribution by Chebychev's inequality.

has a lower difficulty $z_1 < z$,[3] while if $\mathcal{E}$ fails, the idea retains its base difficulty z. The idea succeeds if and only if both stages succeed, which happens with probability $p_0 p_1$.

Both stages have uncertainty, but they differ in scale and uncertainty resolution. The experimentation stage is cheaper and does not fully resolve uncertainty: it provides information such that the odds of success at implementation increase. The implementation stage has a larger scale and fully resolves uncertainty – all the investment is laid out and the idea either works or fails. Therefore, the experimentation stage is a learning device that represents the hallmark of stage financing that distinguishing specialized from non-specialized investors (Kerr et al., 2014).

For simplicity, the probability that the experiment succeeds is notated as $p_0(e, v) = \Phi(SR(e, v, z_0))$, and the implementation as $p_1 = p(z_1)$. Since the idea is negative NPV ex-ante under non-specialized financiers, specialized financiers are of interest only if stage financing can create enough value.

Assumption 2.2. *The idea is positive NPV under stage-financing*

$$p_0(e, v)[p_1\pi - I] - I_0 > 0$$

Assumption 2.2 makes stage financing efficient, which has several implications. First, it implies that the experiment creates significant value relative to the base idea. Assumption 2.2 can be written as

$$[p_0(e, v)p_1 - p_\iota]\pi - I_0 + I(1 - p_0(e, v)) > I - p_\iota\pi \tag{2.3}$$

where the left hand side is the gain in NPV from stage financing. Equation (2.3) says that stage financing has to create enough value to at least overcome the value loss of the base idea. The gain in NPV from stage financing can be decomposed in two terms. $([p_0(e, v)p_1 - p_\iota]\pi - I_0)$ measures the gains from experimentation: the change in the overall success probability minus the cost to experiment. $I(1 - p_0(e, v))$ measures the savings

[3] Δz can be interpreted as a measure of how informative experiments are. Moreover, $z_i = \frac{z_{i-1}}{1+\beta^i}$ can give β an interpretation of experiment informativeness, which could be a parameter of interest across sectors.

from abandoning the project early without sinking the full implementation cost I. Even if the increase in the chance of success is small, stage financing can be valuable if it allows the entrepreneur to abandon early and cheaply.

Second, it enables financing from non-specialized investors, as the idea's NPV becomes positive at stage 1 if the experiment succeeds. If the experiment at 0 fails, the idea is abandoned. Therefore, assumption 2.2 implies that the experiment has to be sufficiently informative, resulting in a low enough z_1 that guarantees a probability of success such that $p_1\pi - I \geq 0$.

Third, the experimentation stage cost I_0 has to be small enough relative to its success probability $p_0(e, v)$. Combining assumption 2.2 with the negative NPV condition of the base idea implies

$$\left(\frac{I_0}{p_0(e,v)}\right)\frac{1}{p_1} + \frac{I}{p_1} \leq \pi \leq \frac{I}{p_\iota}, \tag{2.4}$$

which is always true if $I_0/p_0(e, v)$ is small enough. The space of prizes in which stage financing is efficient grows with decreases in the likelihood of success p_ι. Moonshot ideas – those with a very low p_ι – may offer large payoffs and still only be suitable for stage financiers, who have less stringent requirements on the prize if experimentation is sufficiently cheap, as, for instance, could be the case for software companies Kerr and Nanda (2015).

From now on, consider $I_0/p_0(0, 0)$ to be small enough so that stage financing is efficient and the experiment meets assumption 2.2 for any values of e and v.

Advising For an ongoing cost ι_A, some stage financiers can offer advice $v \in \{0, \overline{v}\}$ which decreases the riskiness $\sigma(e, v)$ of experiments. I assume stage financing is a requirement to have the advice technology, reflecting the active involvement needed from investors and that both technologies complement each other. Hence, investors that offer advice are a strict subset of investors with the stage financing technology.

Managing funds For an ongoing cost ι_M, investors can manage the cash of K investors, themselves included, which increases their investing scale from c to Kc.

Interpretation of ongoing costs of financing technologies ι_M represents the costs to manage other people's money. For instance, any frictions induced by principal-agent problems. Both ι_S and ι_A represent the cost for the investor to have an active involvement in the company, which could be physical resources, time, or expertise. However, they represent different dimensions of the investor's involvement.

ι_S considers the expertise needed from the investor to recognize under what conditions a project should be split in stages and what the experimentation stage entails. However, it does not require active involvement. For instance, it could represent the investor putting the experiment as a condition for his next round of founding, but not getting involved with the entrepreneur until he shows up with the experiment's result. The active involvement and help during the experimentation process is represented by ι_A.

For instance, consider Genentech, considered the one of the first biotech companies and a pioneer in producing synthetic human insulin. In their beginnings, Genentech approached Kleiner Perkins for half a million dollars to cover expenses until they manufactured their first product. While Kleiner Perkins recognized the potential, they suggested cheaper experimentation by subcontracting research laboratories to lower the risks of the company before going to market. If that experimentation was successful, following financing rounds would come, as narrated by Mallaby (2022). In this case, Kleiner Perkines only set up a stage financing scheme for Genentech, suggesting a route for de-risking the company with cheaper experimentation, while not getting involved with the technology and science behind it directly.

Differently to Genentech, Mallaby (2022) also narrates the case of Cisco technologies, which represents stage financing with active involvement ($\iota_S + \iota_A$) from the main investor, Don Valentine. Given the eccentric personality of Cisco's founders, Don Valentine actively participated in the development of the company, monitoring the founders, looking for appropriate management, serving as interim chief executive if necessary, and even firing the Cisco's founders for the sake of Cisco.

Groups of investors

The available financing technologies give a total of 6 investor groups. Each group is represented by a triplet (S, A, M) with binary elements that represent their available technologies – S for stage financing, A for advising, and M for managing money. For instance, $(0, 0, 1)$ represents an investor without stage financing nor advising, that can invest at scale Kc given he can manage other investor's money $M = 1$. $(1, 1, 1)$ represents an investor that can implement stage financing, offer advice, and manage other investor's money. Notice 2 types are removed because stage financing is a pre-requisite for advice.

The efficiency of stage financing (assumption 2.2) separates investors by the stage in which they can participate. Initially, only investors with stage financing are willing to finance the idea. Through experimentation, these investors can turn the idea into a profitable opportunity for other investors. Conditional on experimental success, investors without stage financing can now implement the idea given the higher chance to succeed.

I assume that investors only fund a single stage and a single idea, staying until the idea succeeds or fails. One way to interpret this assumption is that while funds might have a limited amount of time to produce returns or exit the idea, for instance, 10 years for venture capitalists Lerner and Nanda (2020), there is always a secondary market of outside investors where they can resell their share. This assumption clearly separates investors by stage: stage financing-investors belong exclusively to the experimentation stage while investors without access to stage financing belong exclusively to the implementation stage.

Also notice that advice is only valuable for the experimentation stage. Even though a venture capitalist might be on board by the implementation stage, the advice is useless at this stage as the probability of success is solely determined by the new difficulty p_1. One way to interpret the lack if advice for the implementation stage is that all the active involvement from the investor comes from the experimentation stage. The experimentation stage reduced uncertainty as much as possible, such that the implementation stage represent uncertainty that cannot be controlled nor reduced anymore – it is the last leap of faith before success or failure.

Therefore, each group of investors has a participation constraint and a scale constraint given by their financing technologies.

Group	Participation constraint	Scale constraint
$SAM \equiv (1,1,1)$	$p_0(e,v)p_1R_{SAM} - I_0 \geq \iota_S + \iota_A + \iota_M$	$I_0 \leq Kc - (\iota_S + \iota_A + \iota_M)$
$SA \equiv (1,1,0)$	$p_0(e,v)p_1R_{SA} - I_0 \geq \iota_S + \iota_A$	$I_0 \leq c - (\iota_S + \iota_A)$
$SM \equiv (1,0,1)$	$p_0(e,0)p_1R_{SM} - I_0 \geq \iota_S + \iota_M$	$I_0 \leq Kc - (\iota_S + \iota_M)$
$S \equiv (1,0,0)$	$p_0(e,0)p_1R_S - I_0 \geq \iota_S$	$I_0 \leq c - \iota_S$
$M \equiv (0,0,1)$	$p_1R_M - I \geq \iota_M$	$I \leq Kc - \iota_M$
$n \equiv (0,0,0)$	$p_1R_n - I \geq 0$	$I \leq c$

Table 2.1: Participation and scale constraints of each investor group.

Assumption 2.3. *All stage financiers can coexist and those who manage funds $M = 1$ have a larger scale:*

$$(\iota_S + \iota_A) < c < Kc - (\iota_S + \iota_A + \iota_M) \tag{2.5}$$

The condition in assumption 2.3 first establishes that all stage financiers have some money leftover after investing. Even the smallest scale investor SA can invest in some entrepreneur with a small enough scale $0 < I_0 \leq c - (\iota_S + \iota_A)$. Second, it establishes that accounting for all the ongoing costs of their financing technologies, managing funds gives a larger scale than any single investor. A large enough initial cash c from investors and pooling scale K are enough to meet assumption 2.3 for any set of costs $\{\iota_S, \iota_A, \iota_M\}$.

Assumption 2.4. *Implementation requires pooling of resources $I > c$.*

Finally, I assume that a single investor does not have enough cash to implement the idea, so managing funds $M = 1$ is necessary for the implementation stage. Assumption 2.4 can be interpreted as a minimum scale requirement for the idea, determined by investors. From equation (2.4), a higher investment I requires a larger prize given the same probabilities of success and experiment. This assumptions simplifies the investor space by removing group $n = (0,0,0)$ – they do not have the necessary scale for implementation, nor the technologies for experimentation.

2.2.4 Entrepreneur's problem

The entrepreneur approaches a stage financing investor to fund his idea. Let R_0 be the repayment to the stage financing investor, and R_1 the repayment to whoever finances the implementation stage if the experiment is successful. The entrepreneur chooses his effort in the experimentation stage after getting advice v (if any) from the financier to maximize his expected payoff:

$$\max_e p_0(e,v)p_1(\pi - R_0 - R_1) \quad \text{s.t.} \quad 0 \leq e \leq \overline{e} \tag{2.6}$$

where $p_0(e,v) = \Phi(SR(e,v,z_0))$.

2.2.5 Equilibrium

I concentrate on an allocation such that every agent maximizes and no one wants to deviate.

Definition 2.1. *An equilibrium for a given idea with difficulty z is an entrepreneurial effort e^*, investor advice v^*, and investor $\mathcal{I}$ such that*

i) e^* solves the entrepreneur's problem,

ii) v^* maximizes the idea's surplus,

iii) the investor $\mathcal{I}$ breaks even in expectation.

2.3 Entrepreneur's outcomes and investor advice

Since the financing sector is competitive, the entrepreneur captures all the surplus from the idea. Irrespective of repayments R_0 and R_1, surplus only depends on $p_0(e,v)$, and the entrepreneur's problem is equivalent to maximizing $p_0(e,v) = \Phi(SR(e,v,z_0))$.

Moreover, since the normal CDF is strictly increasing, the entrepreneur chooses effort to maximize the Success Ratio of the experiment:

$$SR^*(v) = \max_{0 \le e \le \bar{e}} \frac{\mu(e) - z_0}{\sigma(e, v)} \tag{2.1}$$

where $\mu(e)$ is log-concave and $\sigma(e, v)$ is log-convex in entrepreneurial effort to ensure an interior solution. Let $SR^*(v)$ be the optimal success ratio.

The entrepreneur's problem shows that in my setup ideas have the structure of fixed prizes – success grants π irrespective of your effort. For instance, the prize represents the market that the idea can capture if successful. While the entrepreneur always chooses ideas with a larger prize given the same difficulty, the effort decision remains unchanged, so effort is independent of the prize π. Entrepreneurial effort only changes in response to factors that alter the experiment, like the distribution of random variable $\mathcal{X}_0$ or the experiment's difficulty z_0. As long as the idea is profitable, the entrepreneur is driven by a desire to succeed, and not the prize amount.

2.3.1 Entrepreneur's optimal effort

Let $e^*(v)$ be the optimal effort of the entrepreneur from his problem in equation (2.1) and $\overline{\mu} = \mu(\overline{e})$ the maximum mean outcome achievable by the entrepreneur.

Lemma 2.1. *If $\overline{\mu} \le z_0$, optimal entrepreneurial effort is $e^*(v) = \bar{e}$. If $\overline{\mu} > z$, optimal entrepreneurial effort solves:*

$$\frac{\mu(e^*(v)) - z_0}{\sigma(e^*(v), v)} = \frac{\mu_e(e^*(v))}{\sigma_e(e^*(v), v)} \tag{2.2}$$

To understand lemma 2.1, first note that $\overline{\mu} = z_0$ implies $SR(\bar{e}, v, z_0) = 0$ – the experiment is a coin toss, $\Phi(0) = 0.5$. Hence, if $\overline{\mu} < z_0$ the experiment is worse than a coin toss since $SR(\cdot) < 0$ for any effort, and vice versa if $\overline{\mu} > z$. Then, if $\overline{\mu} = z$, any $e < \bar{e}$ implies $SR(\cdot) < 0$, so the best outcome for the entrepreneur is $e^*(v) = \bar{e}$ with $SR^*(v) = 0$.

Lemma 2.1 first shows that whenever the best experiment achievable is worse than a coin toss, the entrepreneur exerts maximum effort. Since $\overline{\mu} < z_0$ implies a negative Success Ratio, risk increases the chances for the experiment to succeed. Mechanically, increased risk makes the success ratio closer to zero. Intuitively, increased risk increases the spread of the distribution, putting more mass in the success region, beyond z_0. When success is unlikely, the entrepreneur resorts to risk-shifting, exerting maximum effort.

When the experiment can be better than a coin toss $\overline{\mu} > z_0$, lemma 2.1 shows that there is an interior solution for effort. First, since the Success Ratio can be positive, the entrepreneur works as much as needed to make it positive. Once the Success Ratio is positive, risk becomes an endogenous cost of effort which leads to an interior solution.

Lemma 2.1 also implies that the optimal success ratio when $\overline{\mu} > z_0$ is the marginal rate of substitution of effort between mean outcomes and risk

$$SR^*(v) = \frac{\mu_e(e^*(v))}{\sigma_e(e^*(v), v)}, \tag{2.3}$$

which implies the optimal Success Ratio represents how much the entrepreneur can improve the mean outcome of the experiment for each unit of increased risk on the margin.

Since lemma 2.1 implies the optimal success ratio, one can understand how the probability to succeed changes with the idea's difficulty ex-post and with investor advice.

Lemma 2.2. *The optimal Success Ratio of the experiment* $SR^*(v)$

i) decreases in experimental difficulty, $\frac{\mathrm{d}SR^*(v)}{\mathrm{d}z_0} > 0$.

ii) increases with investor's advice $\frac{\mathrm{d}SR^*(v)}{\mathrm{d}v} > 0$ *if and only if* $\overline{\mu} > z$.

The first part of lemma 2.2 supports the interpretation of z_0 as experimental difficulty. Lemma 2.2 first shows that higher z_0 also implies a lower chance to succeed ex-post, once the entrepreneur optimally chooses effort.

The second part of lemma 2.2 shows that an investor's advice technology does not create value for the entrepreneur when he is more likely to fail. When the experiment is worse than a coin toss ($\overline{\mu} < z$), risk is desirable. As the potential advice of an investor decreases risk, it lowers the chance of success for the entrepreneur. Conversely, when

$\overline{\mu} > z$, advice creates value since risk is undesirable. Hence, entrepreneurs with ideas so difficult that the experiment has a small chance to succeed $z_0 > \overline{\mu}$ do not value investor advice.

One way to intuitively understand lemma 2.2 and the value of advice is to consider the idea as a digital option that pays $p_1(\pi - I)$ if $x_0 > z_0$ and 0 otherwise. When the entrepreneur is in-the-money, reductions in risk are valuable as they decrease the likelihood of not being paid ($x_0 < z_0$). When the entrepreneur is out-of-the-money for any effort ($\overline{\mu} \leq z$), risk is valuable as it increases the probability of getting paid. Figure 2.1 considers ideas with an easy ($z_{low} < \overline{\mu}$) and a more difficult ($z_{high} > \overline{\mu}$) experiment to graphically show the intuition behind the desirability of risk and the value of advice from lemmas 2.1 and 2.2.

Finally, the key of my model rests on how entrepreneurial effort reacts to difficulty and to investor's advice.

Proposition 2.1. *If advice is valuable ($\overline{\mu} > z_0$), optimal entrepreneurial effort $e^*(v)$*

i) increases with idea difficulty z, $\frac{\mathrm{d}e^*(v)}{\mathrm{d}z} > 0$.

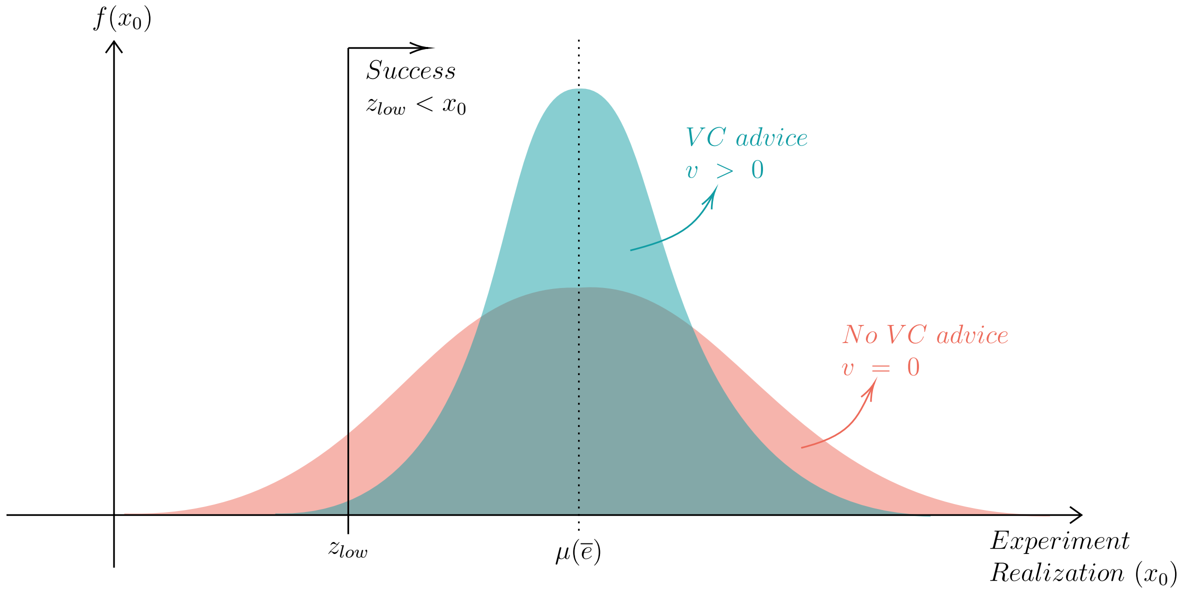
ii) reacts to investor advice v depending on σ_{ev}:

(a) if $\sigma_{ev} < \frac{\sigma_e \sigma_v}{\sigma(e,v)} < 0$, then $e^*(v)$ and v are complements, $\frac{\mathrm{d}e^*(v)}{\mathrm{d}v} > 0$.

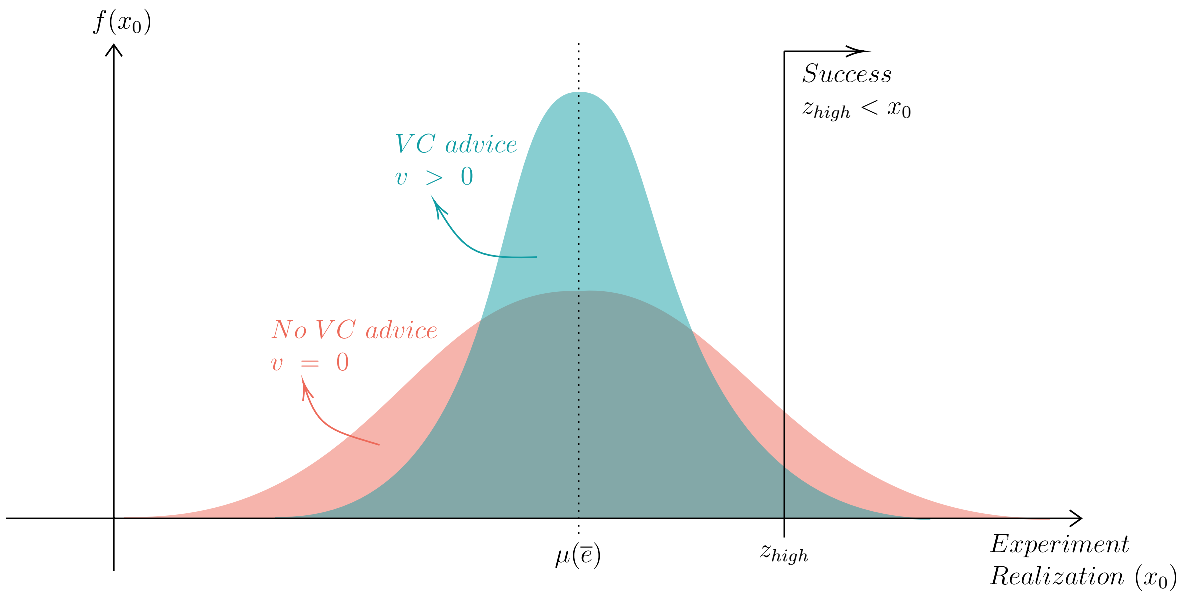
(b) if $\sigma_{ev} >> \frac{\sigma_e \sigma_v}{\sigma(e,v)}$, then $e^*(v)$ and v are substitutes, $\frac{\mathrm{d}e^*(v)}{\mathrm{d}v} \leq 0$.

The first part of proposition 2.1 shows that more difficult problems demand more effort from the entrepreneur, yet still deliver a lower chance to succeed (lemma 2.2). Since more difficult ideas have more difficult experiments by assumption 2.1, proposition 2.1 implies effort increases with experimental difficulty z_0. Once experimental difficulty is so high that $z_0 \geq \overline{\mu}$, effort is maximum $\overline{e}$. Therefore, effort is increasing in idea difficulty z.

The second part of proposition 2.1 establishes the complementarity of effort and advice in risk σ_{ev} as the key determinant of the entrepreneur-investor relationship. If effort and advice are strong complements in the experimental risk, then the entrepreneur works harder in response to investor advice.[4] Conversely, if effort and advice are mild comple-



(a) Easier idea $z_{low} < \overline{\mu}$ – Investor advice is valuable.



(b) Harder idea $\overline{\mu} < z_{high}$ – Investor advice is not valuable.

Figure 2.1: Value of risk and investor advice across ideas of different difficulty.

ments or substitutes in any way, the entrepreneur will lower his efforts in response to investor advice.

Therefore, proposition 2.1 shows that in my model the entrepreneur and investor actions have sinergies if and only if the complementarities in risk are strong enough. Even under mild complementarities, the entrepreneur free-rides on the investor's advice, reducing his own effort. Proposition 2.1 establishes the degree of complementarities in the experiment's risk σ_{ev} between the entrepreneur and the investor as a core determinant of the model's outcomes.

2.4 Investor's financing conditions

Investors in any group are in abundance relative to ideas and behave competitively. Therefore, their participation constraint binds and the entrepreneur captures all the surplus. Investors give advice before the entrepreneur works and hence anticipate the entrepreneur's effort decision. Similarly, the entrepreneur recognizes which investor brings the most value to his idea, either through advice or cheaper financing.

2.4.1 Implementation stage

Suppose a stage financing investor has already participated and the experiment $\mathcal{E}$ was successful. Because $I > c$ from assumption 2.4, only investors SAM, SM, and M have the necessary scale to fund the implementation of the idea. Let R_1 and C_1 represent the repayment and the cost of running the financing technologies of whoever finances the implementation stage. From the investor's participation constraint, the repayment is

$$R_1 = \frac{C_1 + I}{p_1} \tag{2.1}$$

The costs to run their technologies are $\iota_A + \iota_S + \iota_M$ for SAM, $\iota_A + \iota_M$ for SM, and ι_M for M. Investor M has the smallest cost to run his financing technologies. Moreover,

[4] Since risk is an endogenous cost of effort, $\sigma_{ev} < 0$ implies complementarity – advice makes the marginal cost of effort lower.

stage financing and advice provide no value once the idea is in the implementation stage. Therefore, the entrepreneur picks investor M given his cheaper financing.

2.4.2 Experimentation stage

The experimentation stage is the key to my model. Financing for the implementation of the idea is guaranteed from investors M if the experiment succeeds. Since stage financing makes the idea profitable, the entrepreneur can always find an investor that offers stage financing. Let R_0 and C_0 represent the repayment and the cost of running the financing technologies of whoever does the stage financing, and E_0 the expected payoff to the entrepreneur. From the investor's participation constraint

$$R_0 = \frac{C_0 + I_0}{p_0(e,v)p_1} \quad \text{and} \quad R_1 = \frac{\iota_M + I}{p_1} \tag{2.2}$$

and since the entrepreneur captures all the surplus of the idea

$$E_0 = p_0(e,v)(p_1\pi - I) - I_0 - (C_0 + p_0(e,v)\iota_M) \tag{2.3}$$

To be feasible, the idea needs to produce enough surplus under stage financing to cover ongoing costs of the investors $C_0 + p_0(e,v)\iota_M$. Therefore, investors who run cheaper technologies have a relative advantage over more sophisticated investors.[5] To keep cases simple and focus on the relationship of the entrepreneur and the investor without other considerations, I assume from now on not only that stage financing is efficient, but so efficient that

$$p_0(e,v)(p_1\pi - I) - I_0 > \iota_S + \iota_A + 2\iota_M \tag{2.4}$$

which implies that any idea under stage financing can cover any potential financing costs. Hence, the presence of a financier in an idea is not driven by the cost of their technologies, but by the relative advantages they can offer the entrepreneur.

[5]For instance, any idea with expected surplus $E[S] < \iota_S + \iota_A$ simply cannot be financed by SA, but might be financed by S given his cheaper ongoing costs.

Advice is not valuable

Suppose $z_0 \geq \overline{\mu}$, such that advice is not valuable for the entrepreneur. Then, investors without the advice technology have higher scale and offer cheaper financing since their ongoing costs are lower. Therefore, investors who cannot advise dominate ($\succ$) their advising counterpart, $SM \succ SAM$ and $S \succ SA$.

Lemma 2.3. *Suppose advice is not valuable ($z_0 \geq \overline{\mu}$) and $I_0 \leq Kc - \iota_S - \iota_M$. The entrepreneur funds experimentation through*

i) investor S if $I_0 \leq c - \iota_S$,

ii) investor SM if $c - \iota_S < I_0$.

Lemma 2.3 shows that when ideas are so difficult that advice does not add value, only investors SM and S finance entrepreneurs. The choice between them depends solely on the experiment's scale I_0, since S offers cheaper financing whenever her scale is sufficient $I_0 \leq c - \iota_S$. When experimentation requires pooling of resources, $c - \iota_S \leq I_0$ investor SM finances. When the experimentation cost is larger than investor's SM, the entrepreneur does not get funded.

Advice is valuable

Suppose advice is valuable for the entrepreneur $z_0 < \overline{\mu}$. Then, the entrepreneur picks an investor depending on the scale of experimentation I_0 and the quality of the advice technology. Figure 2.2 shows the set of investors the entrepreneur may approach depending on the experimentation scale. If the experimental scale is too large, no investor has the necessary funds.[6]

The entrepreneur approaches a specific investor group at each experimentation scale depending on the value created by the advice technology. The entrepreneur always picks

[6] In terms of algebra, from smallest to largest experimental scale: 1. If $I_0 \leq c - \iota_A - \iota_S$, every investor can offer funding $\{SAM, SA, SM, S\}$, however, pooling money is unnecessary so $SA \succ SAM$ and $S \succ SM$. Hence, only $\{SA, S\}$ may finance the idea. 2. If $c - \iota_A - \iota_S \leq I_0 \leq c - \iota_S$, investors $\{SAM, SM, S\}$ can offer funding, but $S \succ SM$. Hence, only $\{SAM, S\}$ may finance the idea. 3. If $c - \iota_S \leq I_0 \leq Kc - \iota_S - \iota_A - \iota_M$, $\{SAM, SM\}$ may finance the idea. 4. If $Kc - \iota_S - \iota_A - \iota_M \leq I_0 \leq Kc - \iota_A - \iota_M$, $\{SM\}$ finances the idea. 5. $Kc - \iota_A - \iota_M < I_0$, no one has the necessary scale.

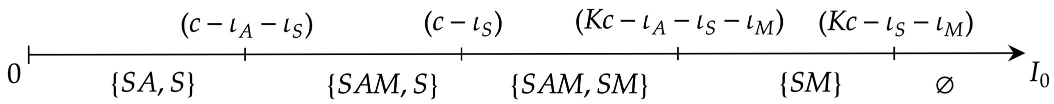


Figure 2.2: Investment scale and feasible investors

the cheaper investor accounting for the value they create. As financing markets are competitive, the entrepreneur captures any value created by the investor beyond her ongoing costs. While the advice giving investor needs a larger repayment, her advice might create enough value to increase the payoffs to the entrepreneur.

For instance, consider the smallest experimental scale $I_0 < c - \iota_A - \iota_S$. Let $p_0^*(v) = p_0(e^*(v), v)$ be the optimal probability of success under advice v, which accounts for the entrepreneur's effort choice. SA dominates S if it offers more value for the entrepreneur, which happens when

$$(p_0^*(\overline{v}) - p_0^*(0)) \geq \frac{\iota_A}{(p_1\pi - I - \iota_M)} \tag{2.5}$$

which states that the entrepreneur prefers the advice giving investor if advice creates positive net value. The expected benefit $(p_0^*(\overline{v}) - p_0^*(0))(p_1\pi - I - \iota_M)$ has to exceed the cost ι_A.

The same procedure yields the rules for which investor dominates at each experimental scale interval, creating two thresholds

$$\underline{\Delta} = \frac{\iota_A}{(p_1\pi - I - \iota_M)} \quad \text{and} \quad \overline{\Delta} = \frac{\iota_A + \iota_M}{(p_1\pi - I - \iota_M)} \tag{2.6}$$

where $\underline{\Delta} < \overline{\Delta}$. Then, the gains from advice relative to $\underline{\Delta}$ and $\overline{\Delta}$, and the experimental scale determine the chosen investor.

Define $\Delta : [0, Kc - \iota_A - \iota_S - \iota_M] \rightarrow R^+$ as the function that from an investment scale I_0 determines the threshold where advice giving investors dominate, such that if $(p_0^*(\overline{v}) - p_0^*(0)) > \Delta(I_0)$, the entrepreneur picks the advice giving investor. Advice giving investors are not feasible for $I_0 > Kc - \iota_A - \iota_S - \iota_M$.

Proposition 2.2. *Let* $(p_0^*(\overline{v}) - p_0^*(0)) \in [\underline{\Delta}, \overline{\Delta}]$. *Then* $\Delta(I_0)$ *is a step wise function that peaks at mid-level experimentation scales:*

$$\Delta(I_0) = \begin{cases} \overline{\Delta} & I_0 \in (c - \iota_A - \iota_S, c - \iota_S) \\ \underline{\Delta} & otherwise \end{cases} \tag{2.7}$$

Proposition 2.2 shows that the presence of advice giving investors follows an inverted U-pattern across experimental scales. Conditional on advice giving investors being feasible, they are most likely to be present at small or large experimental scales ($I_0 \leq c - \iota_S - \iota_A - \iota_M$ and $c - \iota_S \leq I_0$), and to be the least present in mid-scale levels.

Corollary 2.1. *Suppose* $I_0 \leq Kc - \iota_A - \iota_S - \iota_M$,

i) if $(p_0^*(\overline{v}) - p_0^*(0)) \geq \overline{\Delta}$ *then advice giving investors dominate.*

ii) if $(p_0^*(\overline{v}) - p_0^*(0)) \leq \underline{\Delta}$ *then non-advice giving investors dominate.*

Corollary 2.1 shows that if the advice technology is so good that the gains from advice are larger than $\overline{\Delta}$, then the entrepreneur will always pick the advice giving investor. In this situation, the gains from advice exceed $\iota_A + \iota_M$, justifying the use of every financing technology. Conversely, when the gains from advice are lower than $\underline{\Delta}$, the gains from advice are not enough to cover ι_A. Hence, the entrepreneur always picks the non-advice giving financier.

Figure 2.3 graphically depicts proposition 2.2 and Corollary 2.1. It graphs $\Delta(I_0)$ and which investor dominates under each advice-quality×investment-scale combination. The red line depicts the threshold function $\Delta(I_0)$. The shaded area is the area where the entrepreneurs prefers advice giving investors. When scale is too large, only SM can finance. When advice is too good ($(p_0^*(\overline{v}) - p_0^*(0)) > \overline{\Delta}$), advice giving investors A dominate in every experiemntal scale, and vice versa when advice creates too little value $(p_0^*(\overline{v}) - p_0^*(0)) < \underline{\Delta}$.

Figure 2.3 allows to interpret the groups of investors in the model. Investor SAM looks like a venture capitalist and SA like an angel investor – both offer stage financing

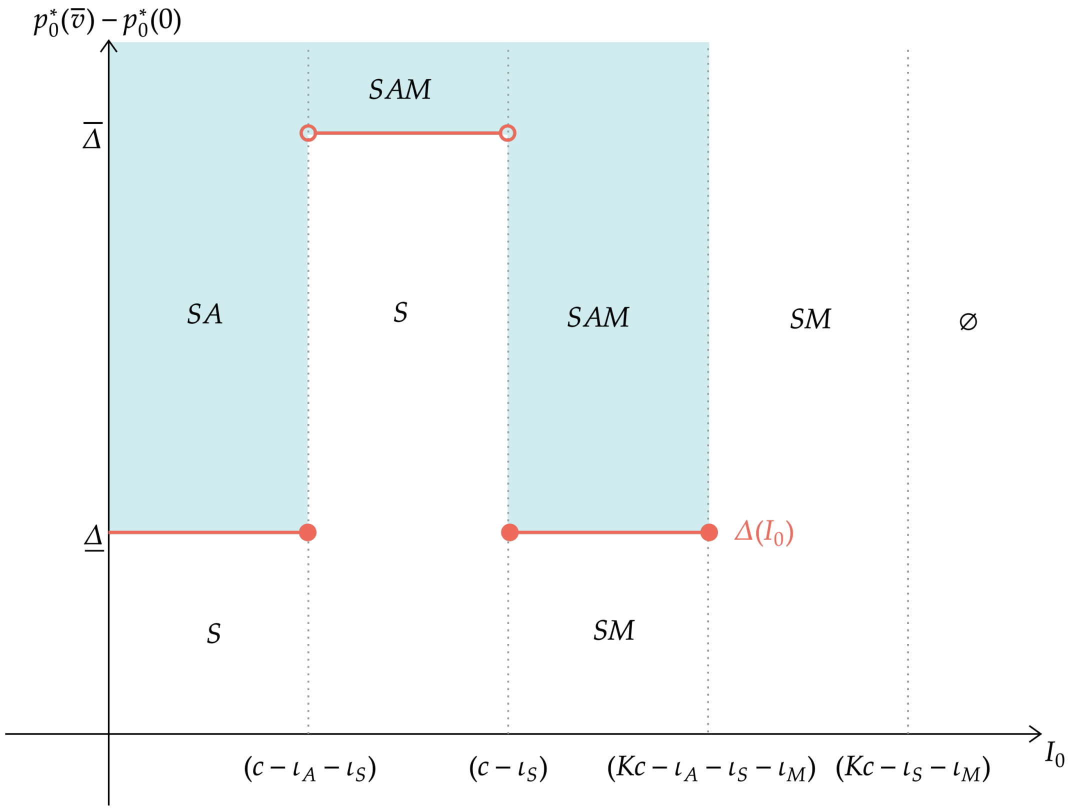


Figure 2.3: Best investor depending on gains from advice and investment scale.

and active advice but differ in their scales. Comparatively, SM and S have a more arm's length relationship with the entrepreneur by not advising, which allows them to achieve higher scales. Therefore, SM and S could represent the syndication of venture capitalists and angels, which removes their advising because of, for instance, coordination costs, in favor of larger scale investments. Finally, at the implementation stage, M represents large equity markets, with the largest scale and agency costs ι_M. Moreover, stage financiers are a requisite for M to participate.

Proposition 2.2 helps explain the recent retreat of venture capitalists toward later stages in the funding process, as recently documented Lerner and Nanda (2020), which coincides with the greater prevalence of SAM investors for larger scales. While during the 70's and 80's venture capitalist where the only stage financing investors available, the appearance of smaller, more specialized investors like angel investors SA have resulted in venture capital retreating to larger experimental scales.

Finally, notice that any factor that makes the expected benefit of the implementation stage $(p_1\pi - I - \iota_M)$ smaller makes advice less prevalent at any scale. With smaller benefits, the increase in the probability of success of the experiment has to be higher to justify the cost of advice, which increases $\underline{\Delta}$ and $\overline{\Delta}$ in figure 2.3. However, it also increases the distance between thresholds $(\overline{\Delta} - \underline{\Delta})$, making advice proportionally less prevalent in mid levels of experimental scale. Therefore, less informative experiments – those of lower p_1 – smaller prizes π, larger implementation costs, or larger costs of pooling money ι_M, for instance, agency costs, all make advice less prevalent, specially at mid experimental scales.

Lerner and Nanda (2020) discuss the worrying absence of venture capital in sectors with high social value, experimental scales, and high uncertainty – "tough tech" sectors. There are several reasons this could happen in my model. First, experimental scale might be outside of the scope of advice giving investors. Second, the required advice might be so specialized, given the nature of these "tough tech" sectors like renewables and nuclear energy, that acquiring the technology is too costly for financiers relative to it's benefit – the gains from advice are lower than ι_A. Third, as mentioned by Lerner and Nanda (2020), it could be that the resolution of uncertainty is not enough – in terms of my model, the experiment does not produce enough information and p_1 is too low despite a high prize π. Finally, the base difficulty of ideas z might be too large in these sectors, with a very small chance to succeed, rendering advice not valuable at all for entrepreneurs since they need to maximize the variability of their outcomes.

2.5 Returns to advice across problem difficulty

The mapping between investors and ideas depends on how the value of advice changes with idea difficulty z. If $(p_0^*(\overline{v}) - p_0^*(0))$ goes up with z, advice giving financiers tackle more difficult ideas. If $(p_0^*(\overline{v}) - p_0^*(0))$ goes down with difficulty z, advice giving financiers take a step-back and move towards more certain ideas.

Proposition 2.3. *If entrepreneurial effort and investor advice are strong complements in risk* $\sigma_{ev} << 0$*, such that* $\frac{\mathrm{d}e^*(v)}{\mathrm{d}v} \geq \frac{-\sigma_v}{\sigma_e}$*, then the gains from advice go up with difficulty* $\frac{\mathrm{d}(p_0^*(\overline{v}) - p_0^*(0))}{\mathrm{d}z} > 0$.

Proposition 2.3 shows that when the investor's advice reduces the marginal cost of effort enough, such that the entrepreneur greatly increases his effort, advice giving investors will finance more uncertain, less-likely to succeed problems. On the other hand, non-advice giving investors will finance safer ideas. Importantly, the requirement for the complementarity of effort and advice represents a sufficient condition with an intuitive interpretation.

Advice-giving investors will concentrate on the most difficult ideas available if

$$\sigma(e^*(\overline{v}), \overline{v}) > \sigma(e^*(0), 0). \tag{2.1}$$

Hence, if the advice technology lowers risk and the marginal cost of effort so much that the entrepreneur ends up taking more risk than without advice, the advice giving investors will concentrate on the hardest ideas available. The requirement from investors to tackle valuable-difficult ideas is that both, as a team, end up with a bolder, more uncertain experiment.

Similarly, if the entrepreneur slacks-off and free rides on the advice of the investor, such that he ends with significantly less risk, advice giving investors will retreat to safer ideas.

Lemma 2.4. *If effort and advice are strong substitutes, such that* $\frac{\sigma(e^*(\overline{v}),\overline{v})}{\sigma(e^*(0),0)} < e^{-\frac{(SR^*(\overline{v})^2 - SR^*(0)^2)}{2}}$ *for any difficulty level* z*, then the gains from advice go down with difficulty* z*.*

Lemma 2.4 shows that when efforts are strong substitutes, such that the experiment's risk goes down by more than the upper bound given by $e^{-\frac{(SR^*(\overline{v})^2 - SR^*(0)^2)}{2}}$, returns to advice go down in difficulty, inducing active investors to concentrate in easy problems – those with a high unconditional probability to succeed. Lemma 2.4 presents a similar condition to proposition 2.3, however, it is a necessary condition. Notably, the more impactful the advice technology is on the success probability of the experiment, reflected in $(SR^*(\overline{v})^2 - SR^*(0)^2)$, the more difficult it is to meet the equatio in lemma 2.4, all else constant. Hence, investors with better advice technologies will concentrate on harder problems.

2.6 Conclusion

This paper develops a model of early-stage financing that highlights how investor heterogeneity and entrepreneurial experimentation jointly shape which ideas get funded. By focusing on the complementarity between entrepreneurial effort and investor advice, the model explains why active investors sometimes back the riskiest projects and at other times retreat to safer ones. The framework sheds light on the coexistence of diverse investors, the shift of venture capital toward later and larger stages, and the persistent absence of active investors in "tough tech" sectors.

Appendix to Chapter 2

Appendix 2.A Proofs

Proof of lemma 2.1. Consider different cases depending on the value of $\overline{\mu}$.

Case 1 – $\overline{\mu} \leq z_0$. Then $SR < 0$ always.

$$\max_{e \leq \overline{e}} SR(e, v, z_0) \equiv \max_{e \leq \overline{e}} \frac{\mu(e) - z_0}{\sigma(e, v)} \tag{2.A.1}$$

which is minimized when $e^*(v) = \overline{e}$ – the numerator is as close to 0 as possible, and the denominator is as large as possible.

Case 2 – $\overline{\mu} > z_0$. Then $SR > 0$ always and the problem in equation (2.A.1) has an interior solution given the log concavity of $\mu(e)$ and the log convexity of $\sigma(e, v)$. The first-order condition gives:

$$\frac{\mu(e)(e^*(v)) - z_0}{\mu_e(e)(e^*(v))} = \frac{\sigma(e^*(v), v)}{\sigma_e(e^*(v), v)} \tag{2.A.2}$$

Implying that the optimal Success Ratio is:

$$SR(e^*(v); v, z_0) = \frac{\mu(e)(e^*(v)) - z_0}{\sigma(e^*(v), v)} = \frac{\mu(e)_e(e^*(v))}{\sigma_e(e^*(v), v)} > 0 \tag{2.A.3}$$

Case 3 – $\mu(0) < z_0$ **and** $\overline{\mu} > z_0$. Then there is some effort level $e_m$such that $\mu(e_m) = z_0$. Since e_m gives $SR = 0$ it dominates any effort $e < e_m$. Then, for any $e > e_m$, $SR > 0$ and the problem is well defined. Therefore, $e^*(v) \in (e_m, \overline{e}]$ and solves the same first order condition shown in equation (2.A.2). □

Proof of lemma 2.2. Assume advice is valuable ($\overline{\mu} > z_0$). Given $SR(e, v, z_0) = \frac{(\mu(e)-z_0)}{\sigma(e,v)}$, the envelope theorem implies:

$$\frac{\mathrm{d}SR^*(v)}{\mathrm{d}z_0} = -\frac{1}{\sigma(e^*(v), v)} < 0 \tag{2.A.4}$$

$$\frac{\mathrm{d}SR^*(v)}{\mathrm{d}v} = -\frac{SR^*(v)}{\sigma(e^*(v), v)^2}\sigma_v(e^*(v), v) > 0 \tag{2.A.5}$$

given $\sigma_v(e^*(v), v) < 0$. □

Proof of proposition 2.1. For the effort of the entrepreneur, since $SR_{ee} < 0$, using samuelson's comparative statics principle implies:

$$\text{sign}\left(\frac{\mathrm{d}e^*(v)}{\mathrm{d}z_0}\right) = \text{sign}\left(SR_{ez}\right) \quad , SR_{ez} = \frac{\sigma_e}{\sigma(e^*(v), v)^2} > 0 \tag{2.A.6}$$

and for the investor's advice

$$\text{sign}\left(\frac{\mathrm{d}e^*(v)}{\mathrm{d}v}\right) = \text{sign}\left(SR_{ev}\right) \tag{2.A.7}$$

$$SR_{ev} = \underbrace{-\frac{\mu_e(e^*(v))}{\sigma(e^*(v), v)^2}\sigma_v}_{>0} - (\mu(e^*(v)) - z_0)\left(\frac{\sigma_{ev}\sigma(e^*(v), v)^2 - \sigma(e^*(v), v)\sigma_e\sigma_v}{\sigma(e^*(v), v)^4}\right) \tag{2.A.8}$$

Since in the region of interest $\mu(e) > z_0$ the sign of SR_{ev} depends on σ_{ev}. A sufficient condition for $SR_{ev} > 0$ is:

$$\sigma_{ev} < \frac{\sigma_e\sigma_v}{\sigma(e^*(v), v)} < 0 \tag{2.A.9}$$

given $\sigma_v < 0$. Then, $\sigma_{ev} < \frac{\sigma_e\sigma_v}{\sigma(e^*(v),v)} \Rightarrow SR_{ev} > 0 \Rightarrow \frac{\mathrm{d}e^*(v)}{\mathrm{d}v} > 0$.

□

Proof of lemma 2.3. Suppose there are two investors A and B at stage 0 with total cost C_0^A and C_0^B of running their technologies. Then, since the entrepreneur captures the whole surplus, he prefers the investor that maximizes the surplus, either by being cheaper, or by creating value through advice.

Then, the entrepreneur chooses A if

$$p_0^{A*}(v^A)(p_1\pi - I - \iota_M) - I_0 - C_0^A \geq p_0^{B*}(v^B)(p_1\pi - I - \iota_M) - I_0 - C_0^B \tag{2.A.10}$$

not if advice is not valuable, then $v^A = v^B = 0$ and both financiers achieve the same probability of success, and equation (2.A.10) reduces to

$$C_0^A \leq C_0^B \tag{2.A.11}$$

so the entrepreneur picks the cheapest financier at every investment scale. If $Kc - \iota_S - \iota_M < I_0$, then no investor has enough scale to finance the project. □

Proof of proposition 2.2. Suppose advice is valuable. At any investment scale such that $I_0 \leq Kc - \iota_S - \iota_M - \iota_A$ there are at least two potential investors: one that gives advice, and who does not. Following the same steps as in the proof of lemma 2.3, the entrepreneur picks an investor A over an investor B if equation (2.A.10) holds.

First, if two feasible investors can offer the same level of advice, the cheaper one dominates. Second, suppose A can offer advice and B cannot. Then, rearranging equation (2.A.10), the entrepreneur picks A if

$$p_0^{A*}(\overline{v}^A) - p_0^{B*}(0) \geq \frac{C_0^A - C_0^B}{(p_1\pi - I - \iota_M)}. \tag{2.A.12}$$

If the only difference between investors is the advice technology, then the value of advice has to be such that

$$p_0^{A*}(\overline{v}^A) - p_0^{B*}(0) \geq \frac{\iota_A}{(p_1\pi - I - \iota_M)} \tag{2.A.13}$$

if the difference between investors is both the advice and the scale technologies, then advice has to create enough value such that

$$p_0^{A*}(\overline{v}^A) - p_0^{B*}(0) \geq \frac{\iota_M + \iota_A}{(p_1\pi - I - \iota_M)} \tag{2.A.14}$$

Suppose $(p_0^*(\overline{v}) - p_0^*(0)) \in [\underline{\Delta}, \overline{\Delta}]$, such that which investor dominates depends on the experimental scale I_0, which is represented by function $\Delta(I_0)$. Then, when $I_0 \leq c-\iota_A-\iota_S$ or $c-\iota_S \leq I_0$, the only difference between the feasible investors is the advice technology, so $\underline{\Delta}$ determines if advice is valuable or not.

When $I_0 \in (c-\iota_A-\iota_S, c-\iota_S)$, the difference between investors is both the advice technology and the scale technology, so the relevant threshold that determines if advice is valuable is $\overline{\Delta}$. □

Proof of corollary 2.1. Let $\underline{\Delta} = \frac{\iota_A}{(p_1\pi - I - \iota_M)}$ and $\overline{\Delta} = \frac{\iota_M+\iota_A}{(p_1\pi-I-\iota_M)}$. Then, $\underline{\Delta} < \overline{\Delta}$, and if $p_0^{A*}(\overline{v}^A) - p_0^{B*}(0) > \overline{\Delta}$, the returns to advice are so high that they cover for the cost of all the technologies of the financier, even if a higher scale is not needed. In such case, the entrepreneur always picks the advice-giving investor. If $p_0^{A*}(\overline{v}^A) - p_0^{B*}(0) < \underline{\Delta}$, advice does not create enough value to even justify its own cost, so the entrepreneur never picks advice giving investors. □

Proof of proposition 2.3. The value of advice across the difficulty distribution z is $(p_0^*(\overline{v}) - p_0^*(0)) = F(SR^*(\overline{v})) - F(SR^*(0))$, hence

$$\frac{\mathrm{d}(p_0^*(\overline{v}) - p_0^*(0))}{\mathrm{d}z} = f(SR^*(\overline{v}))\frac{\mathrm{d}SR^*(\overline{v})}{\mathrm{d}z_0} - f(SR^*(0))\frac{\mathrm{d}SR^*(0)}{\mathrm{d}z_0}. \tag{2.A.15}$$

For $\frac{\mathrm{d}(p_0^*(\overline{v}) - p_0^*(0))}{\mathrm{d}z} > 0$,

$$\frac{f(SR^*(\overline{v}))}{f(SR^*(0))} < \frac{\frac{\mathrm{d}SR^*(0)}{\mathrm{d}z_0}}{\frac{\mathrm{d}SR^*(\overline{v})}{\mathrm{d}z_0}} \equiv \frac{\sigma(e^*(\overline{v}),\overline{v})}{\sigma(e^*(0),0)} \tag{2.A.16}$$

since $\frac{\mathrm{d}SR}{\mathrm{d}z_0} < 0$. Now advice is always better than a coin toss, and since the experiment is normally distributed, whenever advice is valuable $f(SR^*(\overline{v})) < f(SR^*(0))$. Hence,

$$\frac{\mathrm{d}SR^*(\overline{v})}{\mathrm{d}z_0} \geq \frac{\mathrm{d}SR^*(0)}{\mathrm{d}z_0} \tag{2.A.17}$$

is a sufficient condition for $\frac{\mathrm{d}(p_0^*(\overline{v}) - p_0^*(0))}{\mathrm{d}z} > 0$, which always holds when $\frac{\partial^2 SR}{\partial v \partial z_0} > 0$. $\frac{\partial^2 SR}{\partial v \partial z_0} > 0$

requires

$$\frac{\partial(-1/\sigma(e^*(v),v))}{\partial v} \geq 0 \tag{2.A.18}$$

$$\frac{\partial\sigma(e^*(v),v)}{\partial v} + \frac{\partial\sigma(e^*(v),v)}{\partial e}\frac{\partial e^*(v)}{\partial v} \geq 0 \tag{2.A.19}$$

$$\frac{\mathrm{d}e^*(v)}{\mathrm{d}v} \geq \frac{-\sigma_v}{\sigma_e} \tag{2.A.20}$$

where $\sigma_{vv} < 0$ $\sigma_{ee} > 0$. Since $\sigma_v < 0$, e and v must be strong enough complements to guarantee that $\frac{\mathrm{d}(p_0^*(\overline{v})-p_0^*(0))}{\mathrm{d}z} > 0$.

□

Proof of lemma 2.4. Following the proof of proposition 2.3, $\frac{\mathrm{d}(p_0^*(\overline{v})-p_0^*(0))}{\mathrm{d}z} < 0$ when

$$\frac{f(SR^*(\overline{v}))}{f(SR^*(0))} > \frac{\frac{\mathrm{d}SR^*(0)}{\mathrm{d}z_0}}{\frac{\mathrm{d}SR^*(\overline{v})}{\mathrm{d}z_0}} \equiv \frac{\sigma(e^*(\overline{v}),\overline{v})}{\sigma(e^*(0),0)} \tag{2.A.21}$$

which given $f()$ is a standard normal distribution, $\frac{f(SR^*(\overline{v}))}{f(SR^*(0))} = e^{-\frac{(SR^*(\overline{v})^2-SR^*(0)^2)}{2}}$. A tighter bound is given by $\frac{f(SR^*(\overline{v}))}{f(SR^*(0))} = e^{-\frac{SR^*(\overline{v})^2}{2}}$ □

Chapter 3

Promotions Within Knowledge Hierarchies

I study how promotions interact with the organization of knowledge in production. In a model in which solving problems requires scarce knowledge and communication is costly, firms organize hierarchically to exploit complementarities that arise when managers' expertise supports multiple workers. Firms can use spot contracts, directly hiring managers and workers, or career-path contracts, in which young workers accept low wages for a chance at promotion to a prestigious managerial role. I show that promotion contests based purely on prestige are inefficient relative to spot contracts: they divert knowledgeable individuals into lotteries instead of employing them as managers, undermining knowledge complementarities in the economy. When promotion contests generate efficiency gains through, for instance, incentive effects, prestige can be socially valuable, especially in sectors with high communication costs. In these sectors, knowledge complementarities in production are low, but incentive effects can be large if the managerial position is highly prestigious.

3.1 Introduction

Two central features of knowledge-intensive sectors like finance, law, and consulting are the organization of production in knowledge hierarchies and the use of promotions, yet these features have largely been studied separately. The literature on knowledge hierarchies pioneered by Garicano (2000) emphasizes the efficiency gains from assigning routine problems to workers and escalating complex problems to managers. In parallel, the literature on promotions emphasizes their role for incentives (Lazear and Rosen, 1981), talent discovery (Waldman, 1984), or rent extraction (Ferreira and Nikolowa, 2024). Hierarchies and promotions are mutually reinforcing – hierarchies make promotions attractive, and promotions imply a hierarchy of jobs. Too much talent allocated to promotion contests may undermine the efficiency of knowledge hierarchies, while the absence of promotions may forgo incentives, talent discovery, or rent extraction. How do promotions and the organization of knowledge in production interact?

This paper combines the knowledge-based economy of Fuchs et al. (2015) with the prestige-based promotions framework of Ferreira and Nikolowa (2024) to study how promotions interact with the use and organization of knowledge in production. Firms produce by solving problems and have two layers: production workers and a manager who helps when workers lack the knowledge to solve a problem. As in Ferreira and Nikolowa (2024), managerial positions are valuable because they pay more and carry prestige, which complements pay. Firms can offer spot contracts, hiring workers and managers for one period, or career-path contracts, in which young workers accept low wages in exchange for a chance at future promotion to a prestigious managerial role.

The central insight of my model is that prestige-based promotions are undesirable unless promotion contests generate efficiency gains – such as incentives – and there are high communication costs. The mechanism is that promotion contests based solely on prestige waste talent in two ways as prestige increases. First, the number of participants in the lottery rises, diverting young talent into contests instead of management and undermining the exploitation of knowledge complementarities across the economy.

Second, at any knowledge level, fewer agents become managers and more exit to their outside option, wasting talent. However, when contests generate efficiency gains and communication costs are high, prestige becomes desirable. In these sectors, knowledge complementarities are low, limiting the impact of wasting talent, while prestige allows the firm to get more benefits from promotions by widening the scope of the contest.

My model contributes to the literature by showing that promotion-based contracts serve a dual role – allocating scarce managerial knowledge and extracting benefits from promotions – and by pointing to communication costs as the key determinant of which role dominates. If communication costs are low, the allocation of knowledge is more relevant in the economy, best achieved through spot contracts. If communication costs are high, incentives from promotions dominate, making prestigious managerial positions desirable because they help capture efficiency gains from promotion contests. Hence, my model points to sectors with intangible and abstract problems that are difficult to communicate – such as finance, law, and consulting – as the ones benefiting from having prestigious roles, offering an explanation for the prestige hierarchy among sectors first explored in Hayes (1971) that motivates Ferreira and Nikolowa (2024).

I build on the prestige-based promotions framework of Ferreira and Nikolowa (2024) rather than tournament theory (Lazear and Rosen, 1981) or promotions as signaling (Waldman, 1984) for three reasons. First, Ferreira and Nikolowa (2024)'s framework is tailored to knowledge-intensive sectors such as finance, consulting, and law, which are also the focus of the knowledge-hierarchies literature. Second, tournament theory is less applicable because performance in these sectors is difficult to measure. Third, as Ferreira and Nikolowa (2024) emphasize, in these sectors the managerial role itself seems to be the primary reward, which is tied to prestige and pay, more than signaling talent to outside firms. Building on Fuchs et al. (2015) and Ferreira and Nikolowa (2024), the model is as follows.

The model features overlapping generations of young and old agents across infinite periods. Each agent has one unit of time and a knowledge level, which they use to solve problems of unknown difficulty. There is an abundance of problems relative to the resources available to solve them – knowledge and time. An agent produces one unit of output if her knowledge exceeds the problem's difficulty.

Agents can either produce independently or join firms that organize production hierarchically. Independent producers use only their own time and knowledge to solve problems. Firms hire agents and have two layers: production workers at the bottom and a manager at the top who provides help when workers cannot solve a problem. Providing help is costly because it consumes some of the manager's time, but it allows to spread the manager's specialized knowledge across many workers. Within a firm and across the economy, production workers demand time from managers to help them solve problems that are beyond their knowledge, while managers supply help time to production workers. Workers are matched to managers so that the market for time clears.

I consider two contractual regimes. Under spot contracts, firms hire workers for one period to be either production workers or managers at a given wage. Under career-path contracts, young agents work as production workers at low wages and enter a lottery for promotion to a managerial role in the next period. Managerial positions are prestigious, which is a non-pecuniary trait that is valued by agents and complements pay. Spot contracts replicate the setup of Fuchs et al. (2015), while career-path contracts introduce the interaction between knowledge-based production and prestige-driven promotions.

The knowledge of production workers and their manager within the firm are complements in both contractual regimes. Hence, workers are matched with managers in a positively assortative way – more knowledgeable workers have more knowledgeable managers. Positive assortative matching in turn implies that the equilibrium with spot contracts agents is stratified: the least knowledgeable agents are production workers, while the most knowledgeable ones are managers. For career-path contracts, only young agents are stratified, and the most knowledgeable agents are the ones up for promotion. For old agents, the lottery of the promotion naturally breaks stratification: from any level of knowledge of promotion workers, some become managers, and some take their outside option and leave since they lost the lottery.

Given a contractual regime, I concentrate on the efficient matching function and allocation that results from solving the planner's problem. The planner maximizes aggregate output by allocating agents to occupations, making sure that the demand for help-time by production workers equals the supply of help-time by managers, and respecting the stratification of occupations across the economy.

I first show that the equilibria under both contractual regimes share common features. As communication costs rise, the number of workers and managers falls, pushing more agents into independent production. Intuitively, higher communication costs limit managers' ability to help workers, reducing knowledge complementarities. Each regime is characterized by a unique threshold of communication costs: below it, communication is so cheap that all production is hierarchical; above it, knowledge complementarities are sufficiently low such that some agents prefer independent production. Relative to spot contracts, career-path contracts yield fewer workers for any given communication cost.

Career-path contracts based on prestige are inefficient relative to spot contracts. In my model, prestige adds nothing to output but creates two opportunity costs. First, before the lottery, talented young agents who could help as managers are diverted into promotion contests, undermining knowledge complementarities that firms could exploit with spot contracts. Second, after the lottery, some knowledgeable agents are not promoted and become independent producers, leaving the old cohort no longer fully stratified – some independent producers are more knowledgeable than some managers. In both cases, talent is wasted in favor of running the promotion lottery.

Finally, I show that when promotion contests generate efficiency gains – incentives or talent discovery – then the sectors with the highest communication costs are the ones that benefit from increased prestige. This setup, while deviating in spirit from Ferreira and Nikolowa (2024), realigns the model with the empirical observations that inspire their work. Sectors with high communication costs, like finance, law, and consulting, where capital is intangible and problems are abstract, stand to gain the most from running promotion contests and being more prestigious. Higher prestige enables firms to put more participants to compete for the same position, increasing incentive effects. Higher communication costs reduce the opportunity cost of running such contests by limiting the knowledge complementarities in the economy.

I build on two complementary strands of organizational economics. The first strand is the knowledge-hierarchy framework, which emphasizes the scarcity of expert time in organizing production. In these models (Garicano, 2000; Garicano and Rossi-Hansberg, 2006, 2015; Fuchs et al., 2015), production requires solving problems of heterogeneous difficulty. Workers with limited knowledge attempt routine tasks, escalating unsolved problems to more knowledgeable "solvers." Hierarchies economize on expert time by structuring organizations so that scarce high-knowledge agents focus on exceptional problems. The bottleneck is thus the limited capacity of experts to solve complex problems, which determines spans of control, organizational design, and wages. More recently, Ide and Talamas (2024) incorporate modern artificial intelligence into this framework, showing how AI substitutes for or complements human knowledge in hierarchical structures. My model extends this literature by incorporating promotions and long-term contracts into their structure.

The second strand is the prestige-based promotions framework developed by Ferreira and Nikolowa (2024), which emphasizes how scarce prestigious positions shape firm and worker outcomes. Its central insight is that firms extract rents from workers by offering scarce prestigious positions, such as partner or managing director roles, when pay and prestige are complements in utility. Entry-level jobs function as a form of "currency" that workers must endure in order to compete for scarce top jobs. Firms optimally create more entry-level positions than strictly necessary, using internal career paths to sell access to prestigious positions. Promotions thus ration access to top positions: only a subset of workers advances, and wages rise steeply upon promotion because pay and prestige jointly raise utility. I contribute to the literature on promotions (Lazear and Rosen, 1981; Waldman, 1984; Ferreira and Nikolowa, 2024) by incorporating how promotions alter the organization of knowledge in production.

After presenting the model, I first analyze spot contracts, then career-path contracts, and finally show how efficiency gains and high communication costs can make prestige in managerial positions socially desirable.

3.2 Model Setup

Consider an overlapping generations setup. Time is discrete and there are infinite periods denoted by τ. There is no discounting. The following describes the setup on a single period, and hence the subscript τ is removed for notational convenience.

3.2.1 Agents

In each period, a unit mass of agents enters the economy. Agents live for two periods: they are born young, denoted y, and in the second period they get old, denoted o. Each agent is endowed with knowledge $z \in [0, 1]$ according to a distribution with cdf $F(z)$, and with a unit of time.

Agents produce output by solving problems. At any point in time, the supply of problems exceeds the economy's available resources – knowledge and time – for solving them. Each problem has an unknown difficulty x which is independently and uniformly distributed in $[0, 1]$, $X \sim U[0, 1]$. An agent solves a problem in one unit of time if the problem's difficulty is lower than the agent's knowledge, $x \leq z$.

Agents have time-separable preferences and they value both the prestige of a job, represented by θ, and the pay it offers, such that the utility they receive in a period from job j with knowledge z is $U(z, \theta_j) = \theta_j u(w_t(z))$, with $u'(\cdot) > 0$. Prestige is a non-pecuniary attribute of the job that complements pay, which is the crucial assumption of Ferreira and Nikolowa (2024).

3.2.2 Production

Production can be organized with agents acting as independent producers or working within firms. As an independent producer, the agent's knowledge equals the probability of solving a problem because problems are uniformly distributed in $[0, 1]$. Hence, the expected output of an independent producer equals their knowledge.

$$\text{E}[\text{Output}|\text{Independent}, z] = P(x \leq z) \cdot 1 = z \tag{3.1}$$

Firms hire workers and produce one unit of output for every problem solved by its workers.[1] There is a large mass of firms ready to enter the market. Production in a firm is organized hierarchically in two layers. The bottom layer is job position b, with as many vacancies as the firm needs, which are filled by production workers. The top layer is job position t, with a single vacancy and filled by a manager that helps production workers if they cannot solve a problem. Importantly, only the managerial position t carries prestige $\theta_t > 1$. Independent production, and producing in the bottom layer both have $\theta_b = 1$. Therefore, from now on I consider $\theta = \theta_t$ as the only relevant variable for prestige in the model.

Production within a firm follows a two step process. If production workers have knowledge z, they cannot solve a problem with probability $(1-z)$, in which case they can ask the manager for help for a communication cost of h time units. The communication cost is paid in full by the manager. Following Fuchs et al. (2015), I assume managerial time is used efficiently, such that there are as many production workers asking for help as managerial time available in expectation. Hence, let $n(z)$ be the amount of production workers asking the manager for help. To ensure that in expectation no time is wasted

$$n(z)(1-z)h = 1 \tag{3.2}$$

which says that within the firm, the expected demand of managerial time has to equal the unit of managerial time available. The demand for managerial time is the expected amount of workers that ask for help – $n(z)$ workers ask for help with probability $(1-z)$ – multiplied by the time cost of helping for the manager h. When a manager helps, the total amount of time used attempting to solve the problem is $1+h$ – the production worker's time plus the communication cost for the manager. A single firm has one manager and $n(z) = 1/(h(1-z))$ production workers.

[1]While independent producers are technically also firms, I reserve the term "firm" for two-layered organizations since, unlike independent producers, they hire workers.

3.2.3 Matching

Following Fuchs et al. (2015), I describe firms as point-wise matches between a number of producers with knowledge $z \in W$ and a manager with knowledge $m \in \mathcal{M}$ – all producers that require help from a given manager have the same knowledge z.[2] Let $\mathcal{M} : W \to M$ represent the function describing the pointwise matching arrangement generated by the hiring decisions of firms. $\mathcal{M}(z)$ represents the knowledge of the manager that advises workers of knowledge z. Similarly, define $\mathcal{Z} : M \to W$ as the inverse of $\mathcal{M}(z)$. Therefore, $\mathcal{Z}(m)$ gives the knowledge z of the worker matched with manager m.

3.2.4 Contracts

I assume there are no frictions, such that the firm can condition contracts on the knowledge of agents, there is full commitment, and the worker can leave the firm and start as an independent producer in any period. The problems faced by the firm are of unknown difficulty when hiring workers.

I consider two types of contracts that could be offered to an agent of knowledge z. First, spot contracts, in which the firm offers a position in $\{b, t\}$ and the corresponding wage in $\{w_b(z), w_t(z)\}$ for the current period.[3]

Second, long-term contracts that last for two periods in which the firm offers job b and wage $w_b(z_b)$ to the agent today and a chance to be promoted to job t tomorrow for wage $w_t(z)$. The initial wage $w_b(z_b)$ is set by the knowledge of production workers that are under spot contracts, allowing the firm to potentially extract rents from more knowledgeable workers as in Ferreira and Nikolowa (2024) – the worker is offered a lower pay relative to his knowledge, in exchange for the possibility to become a prestigious,

[2]This is without loss of generality. Section 3.2.5 shows that z and m are complements in the firm's output function, in which case Eeckhout and Kircher (2018) show that the matching function that induces the efficient allocation must exhibit positive assortative matching – so each knowledge level z is matched to a unique m.

[3]The terminology of "spot contracts" follows the one of Ferreira and Nikolowa (2024) and is different to the one used in Fuchs et al. (2015). Fuchs et al. (2015) use the term "spot contracts" for contracts offered once it is known that the worker was not able to solve the problem, and "ex-ante" contracts for contracts offered before knowing the problem. Here, spot contracts is used to signify contracts that last for the current period and nothing more. In that sense, my solution with spot contracts is equivalent to the benchmark of Fuchs et al. (2015) which works under their "ex-ante" contracts.

well-paid manager tomorrow. If the agent loses the lottery, he can remain a production worker with a wage $w_b(z_b)$ unless he quits. By construction, long-term contracts can only be offered to young agents.

For notational convenience, from now on I assume that a firm hires workers of knowledge z to be production workers under spot contracts, and workers of knowledge m to either be the manager in the case of spot contracts, or to participate in the promotion lottery in the case of long-term contracts.

Assumption 3.1. *Prestige is valuable if and only if there is a single helper within the firm.*

Assumption 3.1 prevents the firm from potentially using long-term contracts to hire knowledgeable workers for promotion lotteries and in practice using them as cheap managers – offering help to other workers. One interpretation of this assumption is that promotion workers do not offer any help time and only use their time to produce, since helping is "beyond their pay grade".[4] If any other worker is offering help, prestige θ is meaningless and anticipating this behavior, no one accepts a long-term contract.

In a long-term contract, the agent is promoted with probability $1/\eta(\theta, m)$, similar to Ferreira and Nikolowa (2024) and Ferreira et al. (2025). I take function $\eta(\theta, m)$ as an exogenous function and impose that $\eta(\theta, m)$ is increasing in prestige, $\frac{\partial \eta(\theta,m)}{\partial \theta} > 0$, following Ferreira and Nikolowa (2024) since in equilibrium they find the span of control of the manager to be increasing in prestige, which translates in a lower probability to succeed.[5] Moreover, I assume that the function $\eta(\theta, z)$ is such that the agent is always willing to participate as long as there is a single helper, in line with assumption 3.1:

$$u(w_b(z_b)) + \frac{\theta}{\eta(\theta, m)} u(w_t(m)) \geq 2u(m) \tag{3.3}$$

which states that the expected utility of accepting the long-term contract is at least as

[4] A simple way to microfound this assumption is to say that their chances of success depend on their production. Hence, since promotion workers would not perceive any benefits from helping, they use all their time in production.

[5] In Ferreira and Nikolowa (2024), the lower probability to succeed is compensated by the higher prestige and pay of the managerial position.

high as the outside option – to produce independently for both periods.[6] Finally, since I take $\eta(\theta, m)$ as an exogenous function, I consider $u(w)$ as a linear function such that $u(w) = w$, to not deviate from the setup of Fuchs et al. (2015).[7]

Contractual regimes I analyze two contractual regimes. In the first, firms use only spot contracts, which is equivalent to the setup of Fuchs et al. (2015). In the second, firms may use both spot and long-term contracts, but managers are hired only through long-term contracts. This assumption is without loss of generality: it is a device to focus on equilibria with long-term contracts, even if they are not always efficient. Finally, I compare the two regimes to determine which is efficient and under what conditions.

Therefore, define the contractual regime as s for spot contracts and c for long-term contracts. Each contractual regime $i \in \{s, c\}$ defines a matching function $\mathcal{M}_i$ with its inverse $\mathcal{Z}_i$, a set of workers W_i, a set of independent producers I_i, and a set of managers M_i in the economy. For long-term contracts, the regime also defines a set of promotion workers P.

3.2.5 Firm output and profits

With spot contracts a firm has profits

$$\pi_s(z, m) = n(z)(m - w_b(z)) - w_t(m) \tag{3.4}$$

which corresponds to the cost of hiring $n(z)$ workers for $w_b(z)$ and a single manager for $w_t(m)$. The output produced by the firm is $n(z)m$, which is equivalent to the expected output of workers plus the expected output of the manager:

$$n(z)\underbrace{P(x < z)}_{z} + n(z)(1 - z)\underbrace{P(x < m | x > z)}_{\frac{m-z}{1-z}} = n(z)m \tag{3.5}$$

[6]Since wages are endogenous and m a parameter of the agent, equilibrium conditions deliver a boundary that $\eta(\theta, m)$ has to meet at each knowledge level m.

[7]However, this is an important deviation from Ferreira and Nikolowa (2024), where the concavity of the utility is key to their results. A microfoundation of $\eta(\theta, m)$ based on Ferreira and Nikolowa (2024) needs to consider a concave $u(\cdot)$.

Importantly, equation (3.5) shows the knowledge of production workers and the manager are complements($\pi_{s,zm} > 0$): better workers benefit from better managers and vice versa. One way to look at the output of the firm is to think of the manager as "augmenting" the knowledge of the $n(z)$ workers from z to m by using his unit of time.[8] The marginal value of managerial knowledge increases for harder problems because the manager can now augment the knowledge of more workers as $n(z)$ increases with z, and vice versa.

For career-path contracts, managers are promoted from a subset of production workers – the set of promotion workers. Promotion workers produce according to their knowledge m and do not offer nor require help, since the manager can solve the same type of problems that they can. With career-path contracts, a firm has profits

$$\pi_c(z, m) = n(z)(m - w_b(z_b)) - w_t(m) + \eta(\theta, m)(m - w_b(z_b)) \tag{3.6}$$

which amounts to the same expression as that for spot contracts in equation (3.5), plus the amount of workers participating in the lottery $\eta(\theta, m)$ times their benefit: output minus their cost $(m - w_b(z_b))$. Note that under long term contracts the knowledge of production workers z and the manager m are also complements ($\pi_{c,zm} > 0$).

3.2.6 Equilibrium

I am interested in a stationary equilibrium for each contractual regime in which the allocation of occupations across agents does not vary over time. Since the problem can be analyzed as an infinitely repeated two-period problem where every period behaves as described, I also drop the subscript τ from the analysis and problem outcomes.

Moreover, I center my analysis on the solutions achieved by a social planner that maximizes aggregate output in the economy by choosing the occupations of agents.

Definition 3.1. *Given a contractual regime $i \in \{s, c\}$, a stationary equilibrium is an allocation of occupations to agents $\{W_i, I_i, M_i, P\}$ and a matching function $\mathcal{M}_i$ such that*

[8] Then, hiring the manager is like buying a knowledge augmenting technology for $w(m)$.

i) $\{W_i, I_i, M_i, P\}$ *and* $\mathcal{M}_i$ *are time independent,*

ii) the allocation maximizes social surplus,

iii) the market for time clears every period.

3.3 Equilibrium with spot and long-term contracts

3.3.1 Matching is positively assortative

Let $\mathcal{M}_s(z)$ be the matching function for spot contracts, and $\mathcal{M}_c(z)$ the matching function for long-term contracts. The knowledge complementarities between workers and managers in solving problems define the efficient matching function.

Lemma 3.1. *The matching functions* $\mathcal{M}_s(z)$ *and* $\mathcal{M}_c(z)$ *are positively assortative.*

Lemma 3.1 shows that the interest in a pointwise matching arrangement is without loss of generality, and that better workers are matched with better managers. Therefore, under any contractual regime the matching function is strictly increasing because the knowledge of workers and managers complement each other in solving problems.

Both matching functions are related through their derivatives as

$$\frac{\mathcal{M}_s'(z)}{\mathcal{M}_c'(z)} = \frac{\frac{f(\mathcal{M}_c(z))}{2\eta(\theta,\mathcal{M}_c(z))}}{f(\mathcal{M}_s(z))}. \tag{3.1}$$

The difference in steepness between the matching functions depends on prestige θ and the chance to be promoted as summarized by $\eta(\theta, \mathcal{M}_c(z))$. The more prestigious the managerial position, the less likely it is to succeed, and fewer of the potential candidates managers for workers z, $f(\mathcal{M}_c(z))$, get promoted – only $\frac{f(\mathcal{M}_c(z))}{2\eta(\theta,\mathcal{M}_c(z))}$ become managers. For sufficiently high prestige, the long-term contracts regime matching function is much steeper than the matching function of spot contracts, with $\mathcal{M}_c'(z) > \mathcal{M}_s'(z)$. Intuitively, as prestige grows, there are fewer managers at each level, so the matching function has to move faster across the knowledge distribution to match the same amount of workers with knowledge z.

Lemma 3.1 greatly simplifies the planner's problem of choosing a matching function and occupations for every agent. Since the matching function is positively assortative, it must be that if there are two workers such that $z < z'$, then their managers follow the same relationship $\mathcal{M}_i(z) < \mathcal{M}_i(z')$. Therefore, if the worst worker ($z = 0$) gets matched with manager $\mathcal{M}_i(0)$, it must be that every other worker gets matched with better managers until the best worker is matched with the best manager.

3.3.2 Occupations are stratified

Lemma 3.1 implies that for outcomes that are not random, the sets of agents allocated to each occupation are disjoint and ordered.

Lemma 3.2. *i) The spot-contracts equilibrium is stratified:* $W_s \preceq I_s \preceq M_s$.

ii) The long-term contracts equilibrium is stratified only for young agents: $W_c \preceq I_c \preceq P$.

Lemma 3.2 shows that a consequence of lemma 3.1 is that under any contractual regime there are knowledge thresholds that determine who becomes a worker, a manager, and an independent producer or promotion worker. If this were not the case, and the most knowledgeable individual were an independent producer or a production worker, there are immediate gains from switching occupations.

The spot contracts equilibrium partitions agents into occupations according to a threshold z_s. Production workers are $W_s = [0, z_s]$. By positive assortative matching, everyone more knowledgeable than the manager of the worst worker is also a manager, so that $M_s = [\mathcal{M}_s(0; z_s), 1]$. Everyone in-between is an independent producer $I_s = (z_s, \mathcal{M}_s(0; z_s))$.

The long-term contracts equilibrium partitions agents into occupations according to a threshold z_c, such that $W_c = [0, z_c]$, $I_c = (z_c, \mathcal{M}_c(0; z_c))$, and $P = [\mathcal{M}_c(0; z_c), 1]$. The set of managers and the set of agents who failed promotion and became independents are equivalent to the set of promotion workers $M_c \cup I_P \equiv P$.

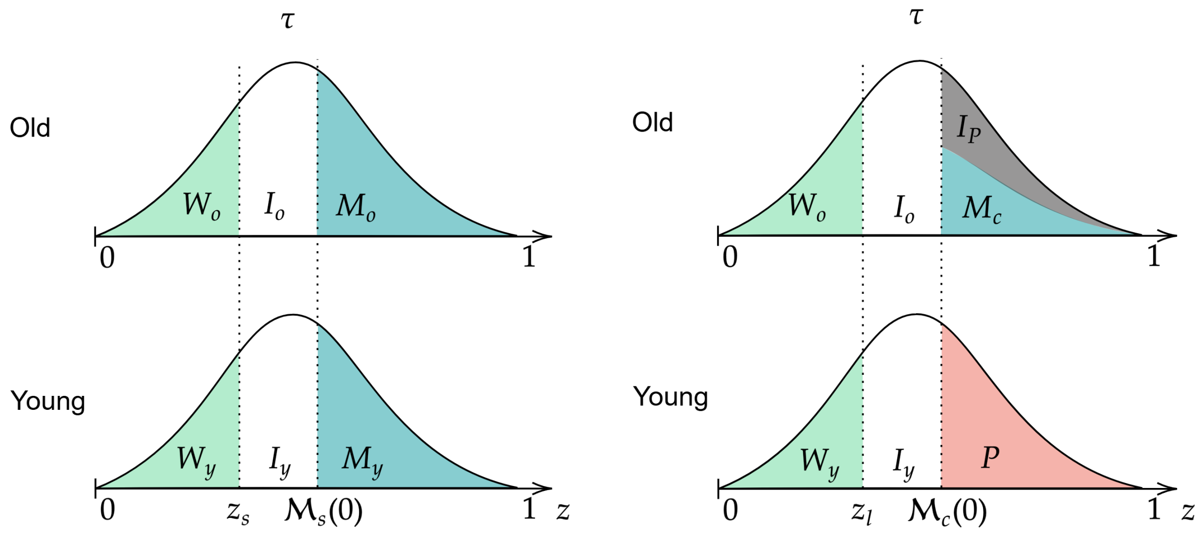
Figure 3.1 shows how agents are stratified in different occupation under each contractual regime. Figure 3.1a shows that for spot contracts the age cohort of workers is irrelevant and there are three broad groups of agents. Workers $W_s = W_y \cup W_o$, independent producers $I_s = I_y \cup I_o$, and managers $M_s = M_y \cup M_o$. The long-term contracts regime is shown in figure 3.1b and has 4 broad groups of agents. Workers $W_c = W_y \cup W_o$, independent producers $I_c = I_y \cup I_o \cup I_P$, managers M_c, and workers up for promotion P. Managers M_c are those workers in P who win the lottery for promotion.

Importantly, those who fail promotion quit and become independent producers. One alternative for firms would be to hire them under spot contracts as managers to help other workers, however, this behavior is ruled out by assumption 3.1. In such scenario, workers would anticipate the firm's behavior and would never accept a long-term contract since the managerial position would lose it's prestige. Moreover, the firm can fully commit to not hiring failed promotion workers by assumption. Then, promotion workers who fail the lottery can stay as production workers for a wage $w_b(z_b)$ or leave the firm and become independent producers. Since the firm only hires promotion workers from which it can extract rents, the firm's profit function in equation (3.6) implies that $m > w_b(z_b)$ so the agent prefers to become an independent producer and leaves the firm.

In a long term contract a fraction $1/\eta(\theta, m)$ of the agents in the promotion track become managers at each knowledge level m, so the resulting distribution of managers is a scaled down version of the original distribution. Figure 3.2 shows how long-term contracts allocate workers over two periods. Importantly, since spot contracts determine a definite threshold to select managers, spot contracts cannot replicate the outcome of long term contracts. Therefore, and considering long-term contracts offer a career for the life of the agent, I follow Ferreira and Nikolowa (2024) and call long-term contracts *career-path* contracts from now on.

3.3.3 Spot contracts equilibrium

The stratification of occupations greatly simplifies the market clearing conditions for both contractual regimes. For the spot contracts regime the set of workers is everyone in $[0, z_s]$,



(a) Groups of agents under spot contracts for a given threshold z_s and matching function $\mathcal{M}_s(z)$.

(b) Groups of agents under long-term contracts for a given threshold z_c and matching function $\mathcal{M}_c(z)$.

Figure 3.1: Distribution of agents by contractual regime.

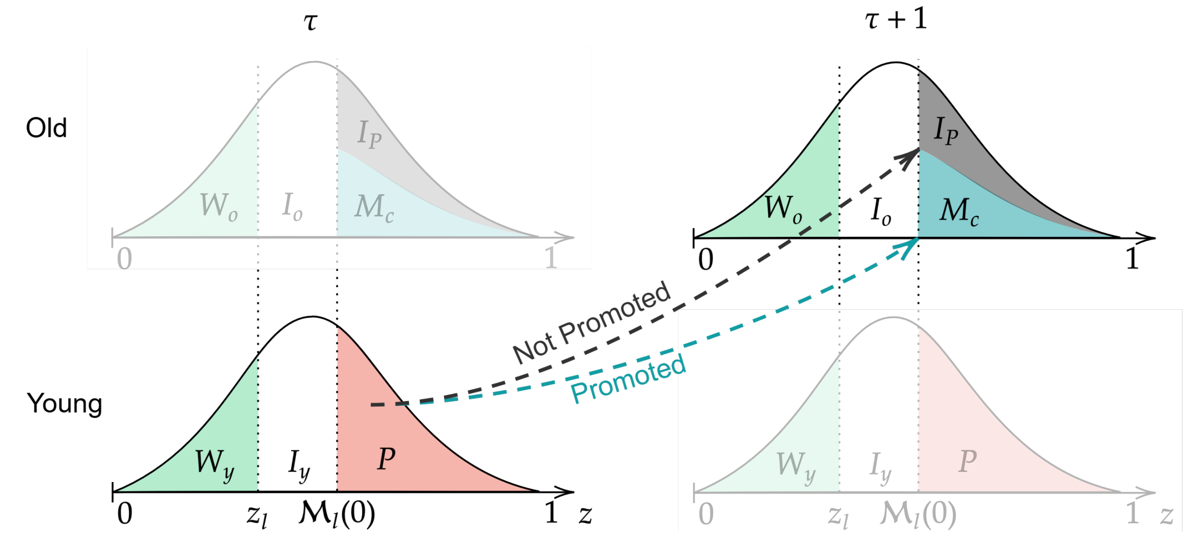


Figure 3.2: Allocation of agents across time for career-path contracts.

and the set of managers is everyone in $[\mathcal{M}_s(0; z_s), 1]$. Equilibrium requires that for any allocation the time demand of workers equals the time supply of managers, hence the time market clearing conditions simplifies to

$$2 \int_{z}^{z_s} h(1-t)\mathrm{d}F(t) = 2 \int_{\mathcal{M}_s(z;z_s)}^{1} \mathrm{d}F(m) \tag{3.2}$$

Differentiating the time market clearing condition with respect to any knowledge level $\frac{\mathrm{d}}{\mathrm{d}z}$ gives a differential equation that characterizes the matching function. $\mathcal{M}_s(z)$ is the matching function that meets:

$$\mathcal{M}_s'(z) = h(1-z)\frac{f(z)}{f(\mathcal{M}_s(z))} \tag{3.3}$$

and the border condition $\mathcal{M}_s(z_s; z_s) = 1$ – the manager of the best worker is the most knowledgeable agent in the economy.

Taking the matching function that ensures market clearing $\mathcal{M}_s(z)$ as given, the planner maximizes surplus by choosing the threshold z_s that determines the distribution of workers and managers in the economy. For spot contracts, the planner's problem is equivalent to the one in Fuchs et al. (2015). The planner solves:

$$\max_{z_s} \quad 2\left[\int_0^{z_s} \mathcal{M}_s(z; z_s)\mathrm{d}F(z) + \int_{z_s}^{\mathcal{M}_s(0;z_s)} z\mathrm{d}F(z)\right] \tag{3.4}$$

$$\text{s.t.} \quad z_s \leq \mathcal{M}_s(0; z_s) \tag{3.5}$$

where the restriction imposes that the most knowledgeable production worker cannot be better than the worst manager, such that the equilibrium respects stratification. When the restriction is active, there are no independent producers in the economy. The total output in the economy is the sum of the output of each production worker, which produces according to the knowledge of its matched manager $\mathcal{M}_s(z; z_s)$, plus the output of every independent worker in the economy who produces their knowledge z.

Lemma 3.3. *The spot-contracts equilibrium is unique and characterized by:*

i) Fewer workers and managers as communication costs increase $\frac{\mathrm{d}z_s^*}{\mathrm{d}h} \leq 0$.

ii) $W_s \neq \emptyset$ *and* $M_s \neq \emptyset$*. However,* $I_s \neq \emptyset$ *if and only if* $h > h_s \in (0, 1)$.

Lemma 3.3 restates the result of Fuchs et al. (2015) in their proposition 1, given the spot contracts case is equivalent to their benchmark scenario. First, the amount of workers and managers in the economy, summarized by z_s^*, decreases with communication costs. Intuitively, as communication costs increase, the knowledge complementarities and efficiency gains from organizing production in firms go down, pushing the marginal workers and managers into independent production.

Lemma 3.3 then shows that for any level of communication costs, some workers and managers always exist. However, independent producers do not exist if communication is cheaper than h_s. When $h \leq h_s$, communication is so cheap that each manager can oversee a lot of workers, increasing knowledge complementarities and making two-layered organizations the only efficient form of organization in the economy. When communication costs are high, $h > h_s$, knowledge complementarities are not high enough, and some agents become independent producers $I \neq \emptyset$.

3.3.4 Career-path contracts equilibrium

For the career-path contracts regime, the set of workers is $[0, z_c]$ and the set of promotion workers, from which some become managers, is $[\mathcal{M}_c(0; z_c), 1]$. Then, the time market clearing is

$$2 \int_{z}^{z_c} h(1-t)\mathrm{d}F(t) = \int_{\mathcal{M}_c(z;z_c)}^{1} \frac{1}{\eta(\theta, m)} \mathrm{d}F(m) \tag{3.6}$$

Similar to spot contracts, differentiating the time market clearing condition with respect to any knowledge level $\frac{\mathrm{d}}{\mathrm{d}z}$ characterizes the matching function. $\mathcal{M}_c(z)$ is the matching function that meets the following equation

$$\mathcal{M}_c'(z) = h(1-z)\frac{f(z)}{f(\mathcal{M}_c(z))} \cdot 2\eta(\theta, \mathcal{M}_c(z)) \tag{3.7}$$

and the border condition $\mathcal{M}_c(z_c; z_c) = 1$.

The planner maximizes social surplus by choosing the threshold that determines occupations in the economy z_c taking as given the matching function that ensures market clearing $\mathcal{M}_c(z)$. The planner solves

$$\max_{z_c} \quad 2\left[\int_0^{z_c} \mathcal{M}_c(z; z_c)\mathrm{d}F(z) + \int_{z_c}^{\mathcal{M}_c(0;z_c)} z\mathrm{d}F(z)\right] + \int_{\mathcal{M}_c(0;z_c)}^{1} \left(2 - \frac{1}{\eta(\theta, m)}\right) m\mathrm{d}F(m) \tag{3.8}$$

$$\text{s.t.} \quad z_c \leq \mathcal{M}_c(0; z_c) \tag{3.9}$$

where the restriction is equivalent to the one in spot contracts – it imposes stratification among young agents. Total output has two main components. First, as before and in square brackets, the output of each production worker which produces according to the knowledge of its matched manager, plus the output of independents, both young and old. Second, and different to spot contracts, the production of promotion workers P – all young produce their knowledge m, while from the old ones, a fraction $1/\eta(\theta, m)$ become managers, and the rest, $(1 - 1/\eta(\theta, m))$ are not promoted and become independents, producing their knowledge m. Hence, at any point m there are two agents – a young and an old one – and with probability $1/\eta(\theta, m)$ one of them becomes a manager and does not produce independently.

Proposition 3.1. *The equilibrium with career-path contracts is unique and characterized by:*

i) Fewer workers and managers as communication costs increase $\frac{\mathrm{d}z_c^*}{\mathrm{d}h} \leq 0$.

ii) $W_c \neq \emptyset$, $M_c \neq \emptyset$, $P \neq \emptyset$, $I_c \neq \emptyset$. *However,* $I_o \cup I_y \neq \emptyset$ *if and only if* $h > h_c \in (0, 1)$.

Proposition 3.1 is equivalent to lemma 3.3 but for the career-path contracts regime. The first part shows that the economy needs fewer promotion workers as communication costs increase, since there are fewer production workers and the demand for help time decreases.

The second part shows that regardless of communication costs, some production workers, promotion workers, and managers always exist. However, different to spot contracts, some independent producers always exist as they are a natural outcome of the promotion contest. Only the set of agents that are directly allocated to independent production

$I_y \cup I_o$ is empty whenever communication is cheaper than h_c, following the same logic as that for spot contracts – cheap communication makes firm production so efficient that everyone who did not lose the promotion lottery works in a firm.

Notably, the first order conditions under both contracting scenarios are equivalent and share similar properties. The differences in both equilibria, for instance, the different thresholds for the existence of independent producers $h_s \neq h_c$ come from the matching function that each type of contracts induces in the market. For an interior solution, the first order condition in both cases $i \in \{s, c\}$ is

$$\frac{1}{h} - \mathcal{M}_i(0; z_i) = \int_{\mathcal{M}_i(0;z_i)}^{1} n(\mathcal{Z}_i(m))\mathrm{d}m \tag{3.10}$$

and the intuition is as explained by Fuchs et al. (2015). Consider a marginal increase in z_i such that because of the time market clearing condition it transforms one independent producer into a manager. Through the matching function, the marginal agent who becomes a manager is $\mathcal{M}_i(0; z_i)$. The left hand side of the first order condition shows the extensive margin gain of such change: The new manager stops producing its independent output $\mathcal{M}_i(0; z_i)$ and takes on the worst workers, so that the most skilled manager is now free to solve $1/h$ problems. The right hand side is the intensive margin cost of the extra manager and workers. Every worker is matched with a marginally worse manager m_i, which lowers team output since it is less likely that the manager can help workers.

3.4 Prestige in career-path contracts

The differences between both regimes are driven by the career-path contracts' matching function, which is directly affected by prestige through the lottery $\eta(\theta, m)$.

Proposition 3.2. *In the career-path contracts equilibrium, higher prestige θ:*

i) increases the amount of promotion workers,

ii) lowers the amount of workers and managers $\frac{\mathrm{d}z_c^*}{\mathrm{d}\theta} < 0$,

iii) lowers the match type of every worker $\frac{\mathrm{d}\mathcal{M}_c(z;z_c^*)}{\mathrm{d}\theta} < 0$.

Proposition 3.2 shows that as prestige increases, the equilibrium adjusts across three margins. Since fewer agents get promoted, the supply of managerial time decreases, and the first margin of adjustment is to increase the amount of agents in the promotion contest to compensate for the lost supply of help time. This first adjustment lowers the average quality of managers, hurting firm output.

Since the supply of managerial time is lower, the second margin of adjustment is to allocate fewer agents to be production workers. Demand for help time decreases, helping the market for time clear. Therefore, even though there are more promotion workers, there are also fewer workers and hence in equilibrium there are fewer managers in the economy.

Finally, to achieve all of these adjustments, proposition 3.2 shows that the matching function has to become steeper. Since the distribution of managerial talent becomes wider and there are fewer managers, the matching function allocates a worse manager to the same worker. Moreover, the matching function has to move faster across the knowledge distribution to compensate for the lower supply of managers at each level.

3.4.1 Career-path contracts are inefficient

The previous sections characterize the equilibria under both contractual regimes but do not establish which one is the efficient outcome.

Corollary 3.1. *Career-path contracts are inefficient relative to spot contracts.*

Corollary 3.1 shows that spot contracts dominate career-path contracts, creating more surplus. The intuition comes from the stratification of agents, shown in lemma 3.2. Stratification improves efficiency. Career-path contracts break stratification across two margins. The first margin is within old agents. The career-path contract wastes some managerial talent – a fraction of the most talented agents become independent producers instead of managers when old. Therefore, there are immediate efficiency gains from switching their positions, which spot contracts achieve. The second margin is across young and old agents. Promotion workers are the most talented agents of the population, such that the most talented are also managers in the future. However, they produce like independent producers while participating in the promotion lottery. Diverting young talent towards promotion lotteries undermines knowledge complementarities in the current period. Output would increase if these promotion workers were assigned managerial roles instead.

Note that corollary 3.1 is in stark contrast to the empirical and anecdotal evidence that inspires Ferreira and Nikolowa (2024). As Ferreira and Nikolowa (2024) explain, prestige drives the desire of workers to work in certain sectors like finance, law, and consulting, having a strict ranking of which sectors, companies, and positions are the most prestigious ones. In my framework, prestige based promotions cannot be the outcome of any efficient equilibrium.

What would it take to make career-path contracts efficient The difference in the optimal surplus achieved under both regimes shows why long-term contracts are inefficient. A sufficient condition such that career-path contracts are more efficient than spot contracts $\mathcal{S}_s^*(h) < \mathcal{S}_c^*(h,\theta)$ is that

$$\mathrm{E}\left[\mathcal{M}_s(z;z_s^*)|0<z<z_s^*\right] - \mathrm{E}\left[\mathcal{M}_c(z;z_c^*)|0<z<z_s^*\right] < 0 \tag{3.1}$$

Hence, within the common interval of workers, the average manager assigned by the career-path matching function has to be more skilled than the average manager assigned by the spot contracts regime. But meeting equation (3.1) is impossible. The career-path contract can only worsen the average quality of managers, as it wastes scarce and valuable talent in favor of running promotion contests and replaces it with marginally worse agents. However, any efficiency gain resulting from promotion contests would soften the condition in equation (3.1), allowing career-path contracts to allocate worse managers on average in exchange for the efficiency gains brought by promotion contests.

3.5 Efficiency gains in career-path contracts

Deviating from Ferreira and Nikolowa (2024), equation (3.1) shows that one way to make career-path contracts efficient is to include some efficiency gain related to promotion workers. For instance, increased effort as in tournament theory Lazear and Rosen (1981), or talent discovery and signaling as in Waldman (1984). For now, I ignore any microfoundation of these efficiency gains to explore how they interact with promotions and the organization of knowledge in production.

Let δ be a small average efficiency gain per worker that is subject to the promotion lottery and call this regime with efficiency gains δ. The efficiency gains are unrelated to the time demand or supply of agents, such that they produce more output with their same endowment of time. Therefore, aggregate surplus now increases by $(1-F(\mathcal{M}_e(0; z_e)))\delta$ for a given occupational threshold δ and matching function $\mathcal{M}_\delta(\cdot)$ – the mass of promotion workers times the average benefit from each one. For instance, promotion workers could have incentives to search for more productive problems, that have an output greater than 1 while being equally difficult.

Lemma 3.4. *Let $\eta_z(\theta, z) > 0$. Higher efficiency gains δ increase surplus and the amount of workers in the economy: $\frac{\mathrm{d}\mathcal{S}_e^*}{\mathrm{d}\delta} > 0$ and $\frac{\mathrm{d}z_e^*}{\mathrm{d}\delta}$.*

Lemma 3.4 shows that career-path contracts can be efficient, and that the efficiency gains of career-path contracts increase the amount of workers in equilibrium. As the efficiency gains from promotions increase, having more promotion workers is beneficial. More promotion workers, for the same lottery $\eta(\theta, m)$, will result in more managers, and hence the planner allocates more production workers in the economy. This increase in workers is the cost to take advantage of the efficiency gains derived from promotion contests – it is not to take advantage of knowledge complementarities.

Moreover, prestige θ can increase or decrease the amount of workers in equilibrium depending on the effect on complementarities versus the efficiency gains of having more promotion workers. The sign of the marginal contribution of z_c to social surplus, when in an interior solution, is determined by

$$1 - h\left[\mathcal{M}_e(0; z_e) + \int_{\mathcal{M}_e(0;z_e)}^{1} n(\mathcal{Z}_e(v))\mathrm{d}v - 2\eta(\theta, \mathcal{M}_e(0; z_e))\delta\right] \tag{3.1}$$

where $\mathcal{M}_e(0; z_e) + \int_{\mathcal{M}_e(0;z_e)}^{1} n(\mathcal{Z}_e(v))\mathrm{d}v$ measures the cost of increases in prestige θ, which lower knowledge complementarities by worsening the average manager in the economy. However, $-2\eta(\theta, \mathcal{M}_e(0; z_e))\delta$ measures the efficiency gains of higher prestige, which allows a firm to have more promotion workers. Therefore, prestige lowers z_e when knowledge complementarities dominate, and it increases z_e when efficiencies from promotion contests dominate, such that there are workers demanding time from the newly, artificially created managerial positions.

Corollary 3.2. *In equilibrium, prestige θ and efficiency gains δ are complements, $\frac{\partial^2 \mathcal{S}_s}{\partial\theta\partial\delta} > 0$.*

Corollary 3.2 shows that higher prestige θ increases the marginal gains from the efficiencies derived from lotteries δ. Intuitively, as positions are more prestigious, the firm can have more promotion workers, taking advantage of the efficiencies of promotion contests. When efficiencies are higher, prestige also becomes more valuable as each individual put into promotion contests creates more value. However, increases in prestige might still lower surplus, since as more agents are moved into promotion contests, the average quality of managers worsens, undermining knowledge complementarities in production.

Lemma 3.5. *If the efficiency gains δ and prestige θ are such that $\delta\eta(\theta, \mathcal{M}_e(0; z_e^*)) \geq F(z_e^*)$, higher prestige increases social surplus.*

Lemma 3.5 shows that higher prestige is socially valuable in a sector in which the marginal gains of running a promotion contest exceed the marginal losses in knowledge complementarities. $\delta\eta(\theta, \mathcal{M}_e(0; z_e^*))$ represents the aggregate marginal gains of running a promotion contest – $\eta(\theta, \mathcal{M}_e(0; z_e^*))$ promotion workers will each increase their output by δ. The right hand side is a strict upper bound on the loss in knowledge complementarities from lowering the average quality of managers in the economy. Notice that $F(z_e^*)$ is the maximum possible production of workers, if they were all matched with a manager of maximum knowledge $m = 1$: $\int_0^{z_e^*} 1 \mathrm{d}F(z)$. Then, $F(z)$ is an upper bound on the losses in knowledge complementarities if the quality of managers decreases.

Proposition 3.3. *Consider a sector θ^* such that*

$$\lim_{h \to y} [\delta\eta(\theta^*, \mathcal{M}_e(0; z_e^*)) - F(z_e^*)] \begin{cases} < 0 & , y = 0, \\ > 0 & , y = 1. \end{cases} \tag{3.2}$$

Then, there is a unique threshold h_θ such that for $h \geq h_\theta$, increasing prestige is valuable.

Proposition 3.3 show that communication costs are key to the relative advantage of career-path versus spot contracts. Communication costs are relevant on two different margins. First, high communication costs lower knowledge complementarities, making career-path contracts cheaper to implement – diverting and wasting talent is not as costly. Second, high communication costs increase the mass of independent workers under both regimes. Moving an agent from independent production into a promotion contest increases output, and if he is unlikely to become a manager – as it happens when prestige is high – the distortion to knowledge complementarities from switching the agent's occupation is small. Nonetheless, higher communication costs and lower knowledge complementarities also imply that there are fewer reasons to run promotion contests, because there is less demand for help time in the economy.

Proposition 3.3 shows that provided that a sector can increase its surplus through higher prestige, higher communication costs will make prestige socially valuable at some point. From lemma 3.5, higher communication costs make $\delta\eta(\theta, \mathcal{M}_e(0; z_e^*))$ weakly larger by increasing $\mathcal{M}_e(0; z_e^*)$, and lower $F(z_e^*)$ by reducing the amount of workers in the economy – both effects are a result of smaller knowledge complementarities. Therefore, as communication costs increase, it is easier to meet the condition that makes prestige valuable (lemma 3.5). Intuitively, while communication costs ensure that only a few managers are needed in the economy because knowledge complementarities are low, higher prestige ensures that promotion contests can still be large, reaping any incentive or talent discovery benefits that career-path contracts offer.

Overall, the set of results in this section show those knowledge sectors with high communication costs and high prestige will use career-path contracts in equilibrium, while other sectors with low communication costs and low potential for prestige use spot contracts. Moreover, it shows that in a sector were communication costs are high, there are incentives to increase prestige if this were an endogenous decision. In my model, in Fuchs et al. (2015), and in Ferreira and Nikolowa (2024), the interest is in knowledge industries like finance, law and consulting, in which the problems are naturally more abstract and difficult to communicate leading to a high h relative to other sectors of the economy. Taking h as a technological parameter, proposition 3.3 suggests that it is these sectors that would see the greater benefit of creating prestigious managerial positions. The use of career-path contracts in these sectors aligns with the evidence described by Ferreira and Nikolowa (2024), although in stark difference to the spirit of their paper, in my model promotion contests increase output.

3.6 Conclusion

This paper incorporates prestige-based promotions into a knowledge-based economy. I show that career-path contracts based only on prestige are inefficient because they pull talent away from management and undermine knowledge complementarities in the economy. Prestige becomes valuable only when promotion contests generate benefits – for instance, incentives or talent discovery – and when the sector has high communication costs. My model helps explain why prestige plays a larger role in sectors like finance, law, and consulting, where problems are abstract and hence communication costs are particularly high.

Appendix to Chapter 3

Appendix 3.A Proofs

Proof of lemma 3.1. Given that each firms' output is supermodular in the production workers' and manager's knowledge ($\pi_{zm} > 0$), Eeckhout and Kircher (2018) prove in a more general setting that the efficient matching function is positively assortative (see their section 3.1, proposition 1).

One can verify that the matching function is indeed positively assortative by differentiating the time-market clearing conditions in equations (3.2) and (3.6) with respect to z, $\frac{\mathrm{d}}{\mathrm{d}z}$. In the case of spot contracts, differentiation of equation (3.2) gives

$$\mathcal{M}_s'(z) = h(1-z)\frac{f(z)}{f(\mathcal{M}_s(z))} > 0 \tag{3.A.1}$$

which implies $\mathcal{M}_s(z)$ is strictly increasing. For career-path contracts, differentiation of equation (3.6) gives

$$\mathcal{M}_c'(z) = h(1-z)\frac{f(z)}{f(\mathcal{M}_c(z))} \cdot 2\eta(\theta, \mathcal{M}_c(z)) > 0 \tag{3.A.2}$$

which implies $\mathcal{M}_c(z)$ is strictly increasing. Since both matching functions are strictly increasing, they match better workers with better managers – they are positively assortative. □

Proof of lemma 3.2. The proof follows the proof of lemma 2 in Fuchs et al. (2015). Consider any contractual regime and proceed by contradiction. Consider first workers and managers. Suppose there is a set of managers $\mathcal{M}_0$ below a set of workers W_0, both with the same measure $\mathcal{Q}$ that fit on an interval of length ε. Let $\mathcal{P}_0$ be the set of problems allocated to managers in $\mathcal{M}_0$, and let $\mathcal{M}_s$ be the managers of workers W_0. First, note that $\mathcal{M}_0 \cap \mathcal{M}_s = \emptyset$ – workers in W_0 do not benefit from any help from the managers in $\mathcal{M}_0$ since workers are more knowledgeable.

First, transform every agent in W_0 into managers and allocate them the problems $\mathcal{P}_0$. This change increases surplus by at least $\frac{(\inf W_0 - \sup \mathcal{M}_0)}{h}\mathcal{Q}$, which represents how many more problems are solved in the economy in expectation. $(\inf W_0 - \sup \mathcal{M}_0)$ is the change in the probability of solving a problem, the knowledge difference between the worst new manager $(\inf W_0)$ and the greatest old manager, which is scaled by $1/h$ since managers can utilize their knowledge across $1/h$ problems.

Second, transform every agent in $\mathcal{M}_0$ into workers and match them with the managers in $\mathcal{M}_s$ – the ones that helped the original workers in W_0. Surplus went down by at most $(\sup W_0 - \inf \mathcal{M}_0)\mathcal{Q}$ – the knowledge of the best initial worker minus the knowledge of the worst new worker.

Finally, Since $1/h > 1$, the gains from reshuffling occupations exceed the losses, so it cannot be that there is a set of managers $\mathcal{M}_0$ below a set of workers W_0.

Now consider production workers and independent producers. Suppose there is a set of independent producers I_0 below a mass of production workers W_0, such that both leave the same amount of problems unsolved for managers P_0. If we swap the roles of I_0 with W_0, two things happen. First, the new independent producers have higher knowledge, so their output is larger. Second, the mass of problems left behind by the new workers W_0 is easier to solve for managers – because the workers who supply the problems are now lower knowledge. Hence, managers are more likely to solve the problems, without spending more time, increasing social surplus. Therefore, production workers have to be below independent producers.

Now consider a set of independent producers I_0 that lies above some managers $\mathcal{M}_0$. Transform I_0 into managers and $\mathcal{M}_0$ into independents, giving all the unsolved problems of $\mathcal{M}_0$ to the new managers in I_0. Then, the proof is identical to the one for production workers and managers. Independent production decreases, but the new managers are more likely to solve problems and use their knowledge across $1/h$ problems, so the gains outweigh the losses.

Therefore, workers lie below independent producers, and independents lie below managers, which implies the equilibrium is stratified.

For long term contracts, the stratification only applies to young agents – the lottery process does not allow full stratification of the old cohort. Some independents that are more skilled than some managers must exist given the random nature of the contract. However, to ensure that managers are as skilled as possible, the young agents have to be stratified accordingly. □

Proof of lemma 3.3. The proof follows the same steps and algebra as in Fuchs et al. (2015). Given the matching function $\mathcal{M}_s(z)$ that meets the market clearing condition in equation (3.2), the planner maximizes social surplus:

$$\max_{z_s} \quad 2\left[\int_0^{z_s} \mathcal{M}_s(z; z_s) z_s \mathrm{d}F(z) + \int_{z_s}^{\mathcal{M}_s(0;z_s)z_s} z\mathrm{d}F(z)\right] \tag{3.A.3}$$

$$\text{s.t.} \quad z_s \leq \mathcal{M}_s(0; z_s) z_s \tag{3.A.4}$$

The constraint can be replaced by $z_s \leq \hat{z}_s$ where $\hat{z}_s = \sup\{z_s | z_s \leq \mathcal{M}_s(0; z_s) z_s\}$. $\hat{z}_s$ is the largest possible z_s such that the constraint is met, and it is a fixed value.

Setting λ_s as the lagrange multiplier of the stratification constraint sets the complementary slackness condition as complementary slackness $\lambda_s(z_s - \hat{z}_s)$. Taking the first order condition of the lagrangian with respect to z_s and noting that $\mathcal{M}_s(z_s; z_s) = 1$ gives:

$$\int_0^{z_s} \frac{\partial \mathcal{M}_s(z; z_s)}{\partial z_s} \mathrm{d}F(z) + (1 - z_s) f(z_s) + \mathcal{M}_s(0; z_s) f(\mathcal{M}_s(0; z_s)) \frac{\partial \mathcal{M}_s(0; z_s)}{\partial z_s} = \lambda_s \tag{3.A.5}$$

The first step is to get an expression for $\frac{\partial \mathcal{M}_s(z;z_s)z_s}{\partial z_s}$ and $\frac{\partial \mathcal{M}_s(0;z_s)z_s}{\partial z_s}$. Taking the partial derivative of the market clearing condition in equation (3.2) with respect to the threshold z_s, $\frac{\partial}{\partial z_s}$, gives:

$$\frac{\partial \mathcal{M}_s(z; z_s)}{\partial z_s} = -\frac{h(1 - z_s) f(z_s)}{f(\mathcal{M}_s(z; z_s))} \tag{3.A.6}$$

so at $z = 0$

$$\frac{\partial \mathcal{M}_s(0; z_s)}{\partial z_s} = -\frac{h(1 - z_s)f(z_s)}{f(\mathcal{M}_s(0; z_s))} \tag{3.A.7}$$

Replacing equations (3.A.6) and (3.A.7) into the first order condition in equation (3.A.5) gives:

$$-\int_0^{z_s} \frac{f(z)}{f(\mathcal{M}_s(z; z_s)z_s)} \mathrm{d}z + \frac{1}{h} - \mathcal{M}_s(0; z_s)z_s = \frac{\lambda}{h(1 - z_s)f(z_s)} \tag{3.A.8}$$

The second step is to replace $\int_0^{z_s} \frac{f(z)}{f(\mathcal{M}_s(z;z_s)z_s)}\mathrm{d}z$ to make the first order condition more interpretable. Notice that $f(z)/f(\mathcal{M}_s(z; z_s)z_s)$ is the ratio of production workers to managers at any point in the distribution. From the derivative of the time market clearing condition with respect to z, $\frac{\mathrm{d}}{\mathrm{d}z}$, shown in equation (3.A.1), replacing $(1 - z)h$ by $1/n(z)$, and integrating from 0 to an arbitrary value y with respect to knowledge z one gets

$$\int_0^y \mathcal{M}'_s(z)n(z)\mathrm{d}z = \int_0^y \frac{f(z)}{f(\mathcal{M}_s(z))}\mathrm{d}z. \tag{3.A.9}$$

Doing a change of variables $v = \mathcal{M}_s(z)$, $\mathrm{d}v = \mathcal{M}'_s(z)\mathrm{d}z$, then setting $y = z_s$ and noting that $\mathcal{M}_s(z_s; z_s) = 1$ gives

$$\int_{\mathcal{M}_s(0;z_s)}^1 n(\mathcal{Z}_s(v))\mathrm{d}v = \int_0^{z_s} \frac{f(z)}{f(\mathcal{M}_s(z))}\mathrm{d}z \tag{3.A.10}$$

which now gives an economic interpretation to $\int_0^{z_s} \frac{f(z)}{f(\mathcal{M}_s(z;z_s)z_s)}\mathrm{d}z$: it is the aggregate number of workers matched with managers, whose knowledge goes from $\mathcal{M}_s(0; z_s)$ to 1.

Replacing equation (3.A.10) in the first order condition of equation (3.A.8):

$$-\int_{\mathcal{M}_s(0;z_s)}^1 n(\mathcal{Z}_s(v))\mathrm{d}v + \frac{1}{h} - \mathcal{M}_s(0; z_s) = \frac{\lambda_s}{h(1 - z_s)f(z_s)} \tag{3.A.11}$$

so when the constraint is slack ($\lambda_s = 0$), the interior solution is defined by

$$\mathcal{M}_s(0; z_s) + \int_{\mathcal{M}_s(0;z_s)}^1 n(\mathcal{Z}_s(v))\mathrm{d}v = \frac{1}{h} \tag{3.A.12}$$

when the constraint is active ($\lambda_s > 0$), the corner solution is such that

$$\mathcal{M}_s(0; z_s) + \int_{\mathcal{M}_s(0;z_s)}^{1} n(\mathcal{Z}_s(v))\mathrm{d}v \geq \frac{1}{h} \tag{3.A.13}$$

With these equations the optimal solution is characterized. Now, it remains to be shown that i) the solution is unique and a maximum, ii) there is a unique h that characterizes the optimal solution, iii) z_s^* decreases with h, iv) and there is an h_s that characterizes slackness,

Solution is unique and a maximum. Consider the left hand side of the first order conditions that characterize the solution in equations (3.A.12) and (3.A.13), and differentiate it with respect to z_s:

$$\frac{\mathrm{d}\left[\mathcal{M}_s(0; z_s) + \int_{\mathcal{M}_s(0;z_s)}^{1} n(\mathcal{Z}_s(v))\mathrm{d}v\right]}{\mathrm{d}z_s} = \frac{\mathrm{d}\mathcal{M}_s(0; z_s)}{\mathrm{d}z_s}[1 - n(\mathcal{Z}_s(\mathcal{M}_s(0; z_s)))] + \int_{\mathcal{M}_s(0;z_s)}^{1} \frac{\mathrm{d}n(\mathcal{Z}_s(v))}{\mathrm{d}z_s}\mathrm{d}v \tag{3.A.14}$$

The derivative of the LHS in in equations (3.A.12) and (3.A.13) is positive. First, $\frac{\mathrm{d}\mathcal{M}_s(0;z_s)}{\mathrm{d}z_s} < 0$. Second, $[1 - n(\mathcal{Z}_s(\mathcal{M}_s(0; z_s)))] < 0$. $n(\mathcal{Z}_s(\mathcal{M}_s(0; z_s)))$ is the worker matched with manager $\mathcal{M}_s(0; z_s)$, which in turn is the manager of worker 0, so $\mathcal{Z}_s(\mathcal{M}_s(0; z_s)) = 0$, and $n(0) = 1/h$, so $[1 - n(\mathcal{Z}_s(\mathcal{M}_s(0; z_s)))] = \left(1 - \frac{1}{h}\right) < 0$. Third, $\int_{\mathcal{M}_s(0;z_s)}^{1} \frac{\mathrm{d}n(\mathcal{Z}_s(v))}{\mathrm{d}z_s}\mathrm{d}v > 0$, as increasing z_s implies consultants are matched with producers of a higher type, which all else constant, leaves them with available time, so they can be matched with more producers. Mathematically, $n(\mathcal{Z}(v))$ increases with $\mathcal{Z}_s(v)$, and for a given manager v, the matched worker's type increases $\mathcal{Z}_s(v)$ as z_s increases. The signs of these three conditions imply that the LHS is increasing. Moreover, at $z_s = 0$ the LHS in equations (3.A.12) and (3.A.13) is

$$\mathcal{M}_s(0; 0) + \int_{\mathcal{M}_s(0;0)}^{1} n(\mathcal{Z}_s(v))\mathrm{d}v = 1 < \frac{1}{h},$$

so there is a unique solution.

To see that it is a maximum, evaluate the first order condition taken in equation (3.A.5) at $z_s = 0$, noting that $\lambda_s = 0$ since $z_s = 0 < \mathcal{M}_s(0;0) = 1$ – everyone is an independent producer. The evaluation gives

$$f(0)(1-h) > 0 \tag{3.A.15}$$

and since the objective function grows as z_s increases from 0, the unique z_s^* achieves a maximum.

Unique h that characterizes z_s. The marginal contribution of z_s to social surplus is the first order condition represented in equation (3.A.11). Rewriting gives

$$(1-z_s)f(z_s)\left\{1 - h\left[\mathcal{M}_s(0;z_s) + \int_{\mathcal{M}_s(0;z_s)}^{1} n(\mathcal{Z}_s(v))\mathrm{d}v\right]\right\} \tag{3.A.16}$$

Rewriting the expression in square brackets as $\left[\int_0^{\mathcal{M}_s(0;z_s)} \mathrm{d}v + \int_{\mathcal{M}_s(0;z_s)}^{1} n(\mathcal{Z}_s(v))\mathrm{d}v\right]$ and noting that $n(\mathcal{Z}_s(v)) > 1$ for any z shows that $\left[\mathcal{M}_s(0;z_s) + \int_{\mathcal{M}_s(0;z_s)}^{1} n(\mathcal{Z}_s(v))\mathrm{d}v\right] > 1$. Then, for any z_s, the marginal contribution of z_s to social surplus can be positive or negative depending on the value of h, and exactly 0 for some specific $h^* \in (0,1)$.

z_s decreasing with h. When the constraint is slack, $\lambda_s = 0$, z_s decreases with h because the marginal contribution of z_s becomes negative at the original point as h increases.

When the constraint binds, z_s eventually decreases with h because as h increases each manager can help fewer workers, so z_s goes down, increasing the ratio of managers to workers.

h_s that determines slack or active. When slack, z_s is strictly increasing as communication costs decrease, so for sufficiently small h, z_s will keep increasing until the constraint is active, where h_s determines the cutoff.

□

Proof of proposition 3.1. The proof follows the exact same steps as those for spot contracts, taking into consideration that the set of managers is the set of promotion workers scaled down by their probability of being promoted $1/\eta(\theta,m)$. The planner solves:

$$\max_{z_c} \quad 2\left[\int_0^{z_c} \mathcal{M}_c(z;z_c)\mathrm{d}F(z) + \int_{z_c}^{\mathcal{M}_c(0;z_c)} z\mathrm{d}F(z)\right] \tag{3.A.17}$$

$$+ \int_{\mathcal{M}_c(0;z_c)}^{1} \left(2 - \frac{1}{\eta(\theta,m)}\right) m\mathrm{d}F(m) \tag{3.A.18}$$

$$\text{s.t.} \quad z_c \leq \mathcal{M}_c(0;z_c) \tag{3.A.19}$$

The constraint can be replaced by $z_c \leq \hat{z_c}$ where $\hat{z_c} = \sup\{z_c | z_c \leq \mathcal{M}_c(0;z_c)\}$. $\hat{z_c}$ is the largest possible z_c such that the constraint is met, and it is a fixed value.

Setting λ_c as the lagrange multiplier of the stratification constraint sets the complementary slackness condition as complementary slackness $\lambda_c(z_c - \hat{z_c})$. Taking the first order condition of the lagrangian with respect to z_c and noting that $\mathcal{M}_c(z_c;z_c) = 1$ gives:

$$\int_0^{z_c} \frac{\partial \mathcal{M}_c(z;z_c)}{\partial z_c} \mathrm{d}F(z) + (1-z_c)f(z_c)$$
$$+ \mathcal{M}_c(0;z_c) \frac{f(\mathcal{M}_c(0;z_c))}{2\eta(\theta,\mathcal{M}_c(0;z_c))} \frac{\partial \mathcal{M}_c(0;z_c)}{\partial z_c} = \lambda_c \tag{3.A.20}$$

The partial derivative of the market clearing condition with respect to z_c gives:

$$\frac{\partial \mathcal{M}_c(z;z_c)}{\partial z_c} = -\frac{h(1-z_c)f(z_c)}{f(\mathcal{M}_c(z;z_c))} \cdot 2\eta(\theta,\mathcal{M}_c(z;z_c)) \tag{3.A.21}$$

Consider the derivative of the time market clearing condition with respect to z, $\frac{\mathrm{d}}{\mathrm{d}z}$, which is shown in equation (3.A.2). As in the section 3.A, i) replace $(1-z)h$ by $1/n(z)$, ii) integrate from 0 to an arbitrary value y with respect to knowledge z, iii) do a change of variables $v = \mathcal{M}_c(z)$, $\mathrm{d}v = \mathcal{M}_c'(z)\mathrm{d}z$, iv) set $y = z_c$ and note that $\mathcal{M}_c(z_c;z_c) = 1$. These steps give

$$\int_{\mathcal{M}_c(0;z_c)}^{1} n(\mathcal{Z}_c(v))\mathrm{d}v = \int_0^{z_c} \frac{f(z)}{f(\mathcal{M}_c(z))} 2\eta(\theta,\mathcal{M}_c(z;z_c))\mathrm{d}z \tag{3.A.22}$$

Replacing $\frac{\partial \mathcal{M}_c(z;z_c)}{\partial z_c}$, $\frac{\partial \mathcal{M}_c(0;z_c)}{\partial z_c}$ and using the expression for $\int_0^{z_c} \frac{f(z)}{f(\mathcal{M}_c(z))} 2\eta(\theta,\mathcal{M}_c(z;z_c))\mathrm{d}z$

in the first order condition one gets:

$$-\int_{\mathcal{M}_c(0;z_c)}^{1} n(\mathcal{Z}(v))\mathrm{d}v + \frac{1}{h} - \mathcal{M}_c(0;z_c) = \frac{\lambda_c}{h(1-z_c)f(z_c)} \tag{3.A.23}$$

so when the constraint is slack, $\lambda_c = 0$ and

$$\mathcal{M}_c(0;z_c) + \int_{\mathcal{M}_c(0;z_c)}^{1} n(\mathcal{Z}_c(v))\mathrm{d}v = \frac{1}{h} \tag{3.A.24}$$

when the constraint is active, $\lambda_c > 0$ and

$$\mathcal{M}_c(0;z_c) + \int_{\mathcal{M}_c(0;z_c)}^{1} n(\mathcal{Z}_c(v))\mathrm{d}v \geq \frac{1}{h} \tag{3.A.25}$$

These conditions are the exact same algebraically than those for spot contracts shown in equation (3.A.11). Any difference comes from the different matching function summarized in $\mathcal{M}_c(0;z_c)$ and $\mathcal{Z}_c(v)$ induced by long term contracts. Moreover, the matching functions share the same properties and signs.

It remains to show that i) the solution is unique and a maximum, ii) there is a unique h that characterizes the optimal solution, iii) z_c^* decreases with h, iv) and there is an h_c that characterizes slackness. These proofs follow the exact same algebra and steps outlined in the section 3.A, since the lottery $\eta(\theta, z)$ induced by long-term contracts do not directly enter the first order condition and the properties of the matching function are the same $-\frac{\partial \mathcal{M}_c(z;z_c)}{\partial z_c} < 0$ and $\frac{\mathrm{d}\mathcal{M}_c(z)}{\mathrm{d}z} > 0$). Hence, these steps are omitted from the proof. □

Proof of proposition 3.2. To see that the amount of promotion workers increases, taking the partial derivative of the market clearing condition in equation (3.6) at the optimum z_c^* with respect to θ gives

$$\frac{\partial \mathcal{M}_c(0;z_c^*)}{\partial \theta} = -\frac{\eta(\theta, \mathcal{M}_c(0;z_c^*))}{f(\mathcal{M}_c(0;z_c^*))} \int_{\mathcal{M}_c(0;z_c^*)}^{1} \frac{\eta_\theta(\theta,m)}{\eta(\theta,m)^2}\mathrm{d}F(m) \tag{3.A.26}$$

To see that prestige lowers the amount of workers and managers, the marginal contribution of z_c to social surplus is the first order condition represented in equation (3.A.23). Rewriting gives

$$(1 - z_c)f(z_c)\left\{1 - h\left[\mathcal{M}_c(0; z_c) + \int_{\mathcal{M}_c(0;z_c)}^{1} n(\mathcal{Z}_c(v))\mathrm{d}v\right]\right\}. \tag{3.A.27}$$

Then, when θ increases $\mathcal{M}_c(0; z_c^*)$ goes down, which makes

$$\left[\mathcal{M}_c(0; z_c) + \int_{\mathcal{M}_c(0;z_c)}^{1} n(\mathcal{Z}_c(v))\mathrm{d}v\right]$$

larger and hence the marginal contribution of z_c^* becomes negative. Therefore, z_c^* has to decrease with prestige θ.

To see that prestige θ lowers the match type of every worker, taking the total derivative of the market clearing condition in equation (3.6) at the optimum z_c^* with respect to θ gives

$$\frac{\mathrm{d}\mathcal{M}_c(z; z_c^*)}{\mathrm{d}\theta} = \frac{\eta(\theta, \mathcal{M}_c(z; z_c^*))}{f(\mathcal{M}_c(z; z_c^*))}\left[2f(z_c^*)h(1 - z_c^*)\frac{\partial z_c^*}{\partial \theta} - \int_{\mathcal{M}_c(z;z_c)}^{1} \frac{1}{\eta(\theta, m)^2}\frac{\mathrm{d}\eta(\theta, m)}{\mathrm{d}\theta}\mathrm{d}F(s)\right]. \tag{3.A.28}$$

Given that the amount of workers decreases with prestige $\frac{\partial z_c^*}{\partial \theta}$, $\frac{\mathrm{d}\mathcal{M}_c(z;z_c^*)}{\mathrm{d}\theta}$ is negative and the match decreases for every production worker z.

□

Proof of corollary 3.1. Consider the equilibrium with career-path contracts and the set of promotion workers P. First, consider only the cohort of old workers. Consider a subset of independent producers from the set of promotion workers that failed promotion $I_0 \subset I_P$ and a set of managers $\mathcal{M}_0 \subset \mathcal{M}_P$ of the same measure $\mathcal{Q}$ such that they both fit in an interval of length $\varepsilon > 0$ and every agent in I_0 is more knowledgeable than $\mathcal{M}_0$. I_0 and $\mathcal{M}_0$ exist given that at any knowledge level z, only a fraction $1/\eta(\theta, m)$ become managers.

Transform I_0 into managers and $\mathcal{M}_0$ into independents, giving all the unsolved problems of $\mathcal{M}_0$ to the new more knowledgeable managers in I_0. Independent production decreases by at most $(\sup I_0 - \inf \mathcal{M}_0)\mathcal{Q}$. However, the probability to solve the unsolved problems passed on to the managers increases by at least $(\inf I_0 - \sup \mathcal{M}_0)$, which is spread across $1/h$ problems, for a total expected gain of $\frac{\mathcal{Q}}{h}(\inf I_0 - \sup \mathcal{M}_0)$. Since $1/h > 1$ and both sets fit in an interval of length ε, the gains exceed the losses and social surplus increases.

To increase efficiency by reallocating occupations across cohorts, consider a set of managers $\mathcal{M}_0 \subset \mathcal{M}_P$ and the a set of most talented young promotion workers $P_0 \subset P$ such that $\mathcal{M}_0$ lies below P_0 in the knowledge distribution. Then, transform $\mathcal{M}_0$ into independents, and transform P_0 into managers and assign to them the unsolved problems of the old managers. Social surplus increases by at least $\frac{\mathcal{Q}}{h}(\inf P_0 - \sup \mathcal{M}_0)$ and it decreases by at least $\mathcal{Q}(\sup P_0 - \inf \mathcal{M}_0)$. As before, since $1/h > 1$ and both sets fit in an interval of length ε, the gains exceed the losses and social surplus increases.

The consequence of repeating the first reallocation within the old cohort is a completely stratified old cohort – within any knowledge level, there are only agents of a single occupation, and managers are the most knowledgeable of the distribution. Once the old cohort is stratified, we can repeatedly reallocate agents across cohorts – the best promotion workers become managers, and the worst (old) managers become promotion workers. Finally, transform any promotion worker into an independent producer, which has no impact on output since they both produce according to their knowledge. The result of these two repeated reallocations is an allocation with the same threshold for production workers across cohorts, which has not been changed, and the same threshold for managers across cohorts. The time market still clears since it has been respected in every reallocation of occupations.

Therefore, we have progressively moved the career-path equilibrium to an allocation that has increased social surplus and is perfectly stratified. Moreover, since the thresholds are the same across cohorts and there are no lotteries nor promotion workers, this allocation is feasible under spot contracts. Therefore, we have found an allocation that improves upon the career-path equilibrium and is implementable under spot-contracts. Hence, the spot contracts equilibrium is always more efficient than the career-path equilibrium. □

Proof of lemma 3.4. Suppose there are efficiency gains to promotion lotteries, such that in expectation each worker that participates in promotion contests produces δ more with their unit of time. Call this regime by the subscript z_e. The planner now solves

$$\max_{z_e} \quad 2\left[\int_0^{z_e} \mathcal{M}_e(z;z_e)\mathrm{d}F(z) + \int_{z_e}^{\mathcal{M}_e(0;z_e)} z\mathrm{d}F(z)\right] \tag{3.A.29}$$

$$+\int_{\mathcal{M}_e(0;z_e)}^{1}\left(2-\frac{1}{\eta(\theta,m)}\right)m\mathrm{d}F(m) + (1-F(\mathcal{M}_e(0;z_e)))\delta \tag{3.A.30}$$

$$\text{s.t.} \quad z_e \leq \mathcal{M}_e(0;z_e) \tag{3.A.31}$$

Setting λ_e as the lagrange multiplier of the stratification constraint and defining $\hat{z}_e = \sup\{z_e | z_e \leq \mathcal{M}_e(0;z_e)\}$ sets the complementary slackness condition as $\lambda_e(z_e - \hat{z}_e)$. Taking the first order condition of the lagrangian with respect to z_e and noting that $\mathcal{M}_e(z_e;z_e) = 1$ gives:

$$\int_0^{z_e} \frac{\partial \mathcal{M}_e(z;z_e)}{\partial z_e}\mathrm{d}F(z) + (1-z_e)f(z_e) + \left[\mathcal{M}_e(0;z_e)\frac{f(\mathcal{M}_e(0;z_e))}{2\eta(\theta,\mathcal{M}_e(0;z_e))} - f(\mathcal{M}_e(0;z_e))\delta\right]\frac{\partial \mathcal{M}_e(0;z_e)}{\partial z_e} = \lambda_e \tag{3.A.32}$$

The partial derivative of the market clearing condition with respect to z_e gives:

$$\frac{\partial \mathcal{M}_e(z;z_e)}{\partial z_e} = -\frac{h(1-z_e)f(z_e)}{f(\mathcal{M}_e(z;z_e))} \cdot 2\eta(\theta,\mathcal{M}_e(z;z_e)) \tag{3.A.33}$$

Consider the derivative of the time market clearing condition with respect to z, $\frac{\mathrm{d}}{\mathrm{d}z}$, which is shown in equation (3.A.2). As in the section 3.A, i) replace $(1-z)h$ by $1/n(z)$, ii) integrate from 0 to an arbitrary value y with respect to knowledge z, iii) do a change of variables $v = \mathcal{M}_e(z)$, $\mathrm{d}v = \mathcal{M}_e'(z)\mathrm{d}z$, iv) set $y = z_c$ and note that $\mathcal{M}_e(z_e;z_e) = 1$. These steps give

$$\int_{\mathcal{M}_e(0;z_e)}^{1} n(\mathcal{Z}_e(v))\mathrm{d}v = \int_0^{z_e} \frac{f(z)}{f(\mathcal{M}_e(z))} 2\eta(\theta,\mathcal{M}_e(z;z_e))\mathrm{d}z \tag{3.A.34}$$

Replacing $\frac{\partial \mathcal{M}_e(z;z_e)}{\partial z_e}$, $\frac{\partial \mathcal{M}_c(0;z_c)}{\partial z_c}$ and using the expression for $\int_0^{z_e} \frac{f(z)}{f(\mathcal{M}_e(z))} 2\eta(\theta,\mathcal{M}_e(z;z_e))\mathrm{d}z$

in the first order condition one gets:

$$-\int_{\mathcal{M}_e(0;z_e)}^{1} n(\mathcal{Z}_e(v))\mathrm{d}v + \frac{1}{h} - \mathcal{M}_e(0;z_e) + 2\eta(\theta,\mathcal{M}_e(0;z_e))\delta = \frac{\lambda_e}{h(1-z_e)f(z_e)} \quad (3.A.35)$$

so when the constraint is slack, $\lambda_e = 0$ and

$$\mathcal{M}_e(0;z_e) + \int_{\mathcal{M}_e(0;z_e)}^{1} n(\mathcal{Z}_e(v))\mathrm{d}v - 2\eta(\theta,\mathcal{M}_e(0;z_e))\delta = \frac{1}{h} \quad (3.A.36)$$

when the constraint is active, $\lambda_e > 0$ and

$$\mathcal{M}_e(0;z_e) + \int_{\mathcal{M}_e(0;z_e)}^{1} n(\mathcal{Z}_e(v))\mathrm{d}v - 2\eta(\theta,\mathcal{M}_e(0;z_e))\delta \geq \frac{1}{h} \quad (3.A.37)$$

These conditions are the equivalent for the regimes of spot contracts and career-path contracts.

It remains to show that i) the solution is unique and a maximum, ii) there is a unique h that characterizes the optimal solution, iii) z_e^* decreases with h, iv) and there is an h_e that characterizes slackness. These proofs follow the exact same algebra and steps outlined in the section 3.A, considering that the properties of the matching function are the same $-\frac{\partial \mathcal{M}_e(z)}{\partial z_e} < 0$ and $\frac{\mathrm{d}\mathcal{M}_e(z)z}{\mathrm{d}z} > 0$). Importantly, notice that given $\eta_z > 0$, the left hand side of the first order condition in equation (3.A.36) $\mathcal{M}_e(0;z_e) + \int_{\mathcal{M}_e(0;z_e)}^{1} n(\mathcal{Z}_e(v))\mathrm{d}v - 2\eta(\theta,\mathcal{M}_e(0;z_e))\delta$ increases with z_c, so the solution is unique and all the properties are the same.

Moreover, the marginal value of z_e is

$$(1-z_e)f(z_e)\left\{1 - h\left[\mathcal{M}_e(0;z_e) + \int_{\mathcal{M}_e(0;z_e)}^{1} n(\mathcal{Z}_e(v))\mathrm{d}v - 2\eta(\theta,\mathcal{M}_e(0;z_e))\delta\right]\right\}, \quad (3.A.38)$$

becomes positive when efficiency gains δ are higher, hence z_e increases with δ. □

Proof of corollary 3.2. Define $\mathcal{S}_e^*(h,\theta,\delta)$ as the optimal surplus with efficiency gains from promotion contests

$$\mathcal{S}_e^*(h,\theta,\delta) = 2\left[\int_0^{z_e^*} \mathcal{M}_e(z;z_e^*)\mathrm{d}F(z) + \int_{z_e^*}^{\mathcal{M}_e(0;z_e^*)} z\mathrm{d}F(z)\right] + \int_{\mathcal{M}_e(0;z_e^*)}^1 \left(2 - \frac{1}{\eta(\theta,m)}\right)m\mathrm{d}F(m) + (1 - F(\mathcal{M}_e(0;z_e^*)))\delta \quad (3.A.39)$$

then $\frac{\partial \mathcal{S}_e^*(h,\theta,\delta)}{\partial \delta} = 1 - F(\mathcal{M}_e(0;z_e^*))$ since the efficiency gains, by assumption, do not go through the time endowment of agents, and hence do not alter the matching function. Then

$$\frac{\partial^2 \mathcal{S}_e^*(h,\theta,\delta)}{\partial\theta\partial\delta} = -f(\mathcal{M}_e(0;z_e^*))\frac{\partial \mathcal{M}_e(0;z_e^*)}{\partial\theta} \quad (3.A.40)$$

which is positive since $\frac{\partial \mathcal{M}_e(0;z_e^*)}{\partial\theta} < 0$. Hence, θ and δ are complements in surplus. □

Proof of lemma 3.5. Similar to the proof of proposition 3.2, the derivative of surplus with respect to prestige θ gives

$$\frac{\mathrm{d}\mathcal{S}_e^*(h,\theta,\delta)}{\mathrm{d}\theta} = \int_{\mathcal{M}_e(0;z_e^*)}^1 \frac{\eta_\theta(\theta,m)}{\eta(\theta,m)^2}[m - \mathcal{M}_e(0;z_e^*)]\mathrm{d}F(m) - \int_0^{z_e^*}\int_{\mathcal{M}_e(z;z_e^*)}^1 \frac{\eta_\theta(\theta,m)}{\eta(\theta,m)^2}\mathrm{d}F(m)\mathrm{d}F(z) - \delta f(\mathcal{M}_e(0;z_e^*))\frac{\partial \mathcal{M}_e(0;z_e^*)}{\partial\theta} \quad (3.A.41)$$

then a sufficient condition for $\frac{\mathrm{d}\mathcal{S}_e^*(h,\theta,\delta)}{\mathrm{d}\theta} > 0$ is that

$$-\delta f(\mathcal{M}_e(0;z_e^*))\frac{\partial \mathcal{M}_e(0;z_e^*)}{\partial\theta} > \int_0^{z_e^*}\int_{\mathcal{M}_e(z;z_e^*)}^1 \frac{\eta_\theta(\theta,m)}{\eta(\theta,m)^2}\mathrm{d}F(m)\mathrm{d}F(z) \quad (3.A.42)$$

where

$$\frac{\partial \mathcal{M}_e(0;z_e^*)}{\partial\theta} = -\frac{\eta(\theta,\mathcal{M}_e(0;z_e^*))}{f(\mathcal{M}_e(0;z_e^*))}\int_{\mathcal{M}_e(0;z_e^*)}^1 \frac{\eta_\theta(\theta,m)}{\eta(\theta,m)^2}\mathrm{d}F(m) \quad (3.A.43)$$

but since the matching function is positively assortative and it increases with z

$$\int_0^{z_e^*}\mathrm{d}F(z) > \int_0^{z_e^*}\frac{\int_{\mathcal{M}_e(z;z_e^*)}^1 \frac{\eta_\theta(\theta,m)}{\eta(\theta,m)^2}}{\int_{\mathcal{M}_c(0;z_c^*)}^1 \frac{\eta_\theta(\theta,m)}{\eta(\theta,m)^2}}\mathrm{d}F(m)\mathrm{d}F(z) \quad (3.A.44)$$

and hence $\delta\eta(\theta,\mathcal{M}_e(0;z_e^*)) > F(z_e^*)$ is a sufficient condition for prestige to increase social surplus $\frac{\mathrm{d}\mathcal{S}_e^*(h,\theta,\delta)}{\mathrm{d}\theta} > 0$. □

Proof of proposition 3.3. Consider the derivative of the condition in lemma 3.5 with respect to h:

$$\frac{\mathrm{d}[\delta\eta(\theta, \mathcal{M}_e(0; z_e^*)) - F(z_e^*)]}{\mathrm{d}h} = \delta\eta_z(\theta, \mathcal{M}_e(0; z_e^*))\frac{\mathrm{d}\mathcal{M}_e(0; z_e^*)}{\mathrm{d}h} - f(z_e^*)\frac{\mathrm{d}z_e^*}{\mathrm{d}h} > 0 \quad (3.A.45)$$

The condition is increasing since $\frac{\mathrm{d}\mathcal{M}_e(0;z_e^*)}{\mathrm{d}h} > 0$ and $\frac{\mathrm{d}z_e^*}{\mathrm{d}h} < 0$, both due to lower knowledge complementarities. If a sector θ^* is such that

$$\lim_{h \to y} [\delta\eta(\theta^*, \mathcal{M}_e(0; z_e^*)) - F(z_e^*)] \begin{cases} < 0 & , y = 0, \\ > 0 & , y = 1, \end{cases} \quad (3.A.46)$$

a threshold h_θ exists such that for $h \geq h_\theta$, $[\delta\eta(\theta^*, \mathcal{M}_e(0; z_e^*(h))) - F(z_e^*(h))] \geq 0$.

□

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