London School of Economics and Political Science

Essays in Information Economics and Political Economy

Weihan Ding

A thesis submitted to the Department of Economics of the London School of Economics and Political Science for the degree of Doctor of Philosophy, London, June 2019

Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

I declare that my thesis consists of approximately 35,000 words.

Statement of conjoint work

I confirm that Chapter 2 was jointly co-authored with Xingchen Zhu and I contributed 50% of this work.

To Leyi and Yilun

Acknowledgement

I am greatly indebted to my supervisors Philippe Aghion, Ricardo Alonso, and Gerard Padró i Miquel for their supervision, support, and continued guidance.

I am grateful for all the help from my friends Karun Adusumilli, Shan Aman-Rana, Andres Barrios Fernandez, Dita Eckardt, Friedrich Geiecke, Brandon Yueyang Han, Chao He, Eui Jung Lee, Clement Minaudier, Claudio Schilter, Xuezhu Shi, Chutiorn Tontivanichanon, and Tianle Zhang. I would like to especially thank Xingchen Zhu, who has been a great co-author and loyal friend. I would also like to thank Michel Azulai, Timothy Besley, Gilat Levy, Niclas Moneke, Inés Moreno De Barreda, Ronny Razin, Vincenzo Scrutinio, Stephane Wolton, seminar and workshop participants at Cheung Kong Graduate School of Business, London School of Economics, ShanghaiTech University, and Tulane University for their kind and helpful comments.

I am greatly indebted to my parents, who have been supporting me in my life with all they have. I would like to thank my parents-in-law for their great support in the last two years of my PhD study. I would also like to thank my son, Yilun, who brings me a lot of joy and happiness during my life. Last, I would like to especially thank my wife Leyi, without her unending love and accompany this thesis would never have been possible.

Abstract

The three essays of my PhD thesis study theoretical issues of Information Design and their applications in Political Economy and Finance. Those essays aim to understand that in various economic and political situations, how an informed party can design information structure that another party faces and thus shape that party's action for the informed party's own interests. By working on those issues, those essays aim to provide more insights on how information sender's incentives shape different levels of information quality, and how various variables of interest, including social welfare, are affected.

The first chapter studies in international disputes, how a government can get concessions from another government by encouraging its own people to protest against the foreign government. By choosing different levels of propaganda, a government not only affects the probability of protests happening but also the informativeness of protests. This chapter sheds more light on a government's optimal nation-building strategy, and also which kinds of countries can benefit most from stirring anti-foreign protests.

The second chapter studies the optimal information design problem in a financial market. A security issuer designs a signal to persuade an investment bank to underwrite, taking as given that there is a secondary market. This chapter shows that the existence of demand shock would lead to worsening of information quality in the primary market.

The third chapter studies that in a moral-hazard environment, how a principal can optimally design the information environment that the agent faces. This chapter shows how a principal can optimally combining providing information and pecuniary incentives to incentivise an agent to exert effort. It also generates some testable predictions about how the quality of equilibrium information structure is affected by factors such as cost of effort and noisiness of production technology.

Contents

List of Figures

List of Tables

Chapter 1

Propaganda, Patriotic Protest, and Diplomatic Persuasion

“ *During the first day and a half of the crisis, many of our colleagues, especially those in the Chancery and at some of the Consulates, were in significant danger. Though U.S. Marines protected the Chancery from direct assault, officers on the spot engaged in a full-scale destruction of classified materials that might fall into the hands of demonstrators should the Embassy be overrun. In hindsight, it appears the danger was never that close, but several Chinese did jump the compound wall and had to be confronted by Marines in full battle gear before they were persuaded to jump back over the wall. Except for Shanghai, with its own Marine guard contingent, the other Consulates were protected only by Chinese security guards. In Chengdu those guards were of virtually no help. Demonstrators climbed the compound wall, set fire to the Consul's residence, and smashed their way through the outer door of the Consulate. They were using a bike rack to try to crash into the interior - while screaming that they were going to exact vengeance - when city security forces finally arrived and routed them. Our colleagues were understandably terrified through this ordeal. They were frantically calling the Embassy and local contacts, and getting increasingly agitated by the slow, almost grudging response of the Chengdu authorities.*”

—Paul Blackburn, Foreign Service Officer, The Association for Diplomatic Studies and Training [1]

1.1 Introduction

When diplomatic disputes loom, governments often instil hostility in their own citizens to encourage protests against other countries. In May 1999, the US bombed the Chinese embassy in Yugoslavia, leading to the death of three journalists and twenty injuries.[2] After internal discussions following the attack, the Chinese leadership decided to take measures against the US, including encouraging protests: the Chinese leadership gave a speech on

[1] **Blackburn, Paul.** “Dealing with a PR Disaster-The U.S. Bombing of the Chinese Embassy in Belgrade”. The Association for Diplomatic Studies and Training: Foreign Affairs Oral History Project. Retrieved 8 May 2013. https://adst.org/2013/05/dealing-with-a-pr-disaster-the-u-s-bombing-of-the-chinese-embassy-in-belgrade/

[2] Whether this was the intentional behaviour of the US is subject to heavy debate. The US side claims this was not intentional and due to outdated information over the position of the Chinese embassy. However, this did not convince the Chinese side, and it is widely believed in China that there was some conspiracy or intentional provocation. **Mizokami, Kyle.** “In 1999, America Destroyed China's Embassy in Belgrade (And Many Chinese Think It Was on Purpose)”. The National Interest, January 21, 2017. https://nationalinterest.org/blog/the-buzz/1999-america-destroyed-chinas-embassy-belgrade-many-chinese-19124

national TV to condemn the US,[3] while the official Chinese media claimed that the Chinese people's anger was justified. This was seen as an effort to steer citizens into protesting, in marked contrast to the Chinese government's policy of quenching any public protest.

The US officials were perfectly aware of the Chinese government's efforts to incite protests. However, they also realized that the bombing did generate popular anger in the Chinese population,[4] and this may disrupt the US-China relationship. Eventually, US president Bill Clinton made a public apology and various commitments to investigate the event and improve the Sino-US relationship. After these public concessions, the Chinese government moved to subdue the protests successfully.[5]

This is far from an isolated example: in many other diplomatic disputes, there is intense hostility among the public of one or both sides[6] which leads to people protesting against the foreign opponent.[7] This chapter tries to understand the logic of governments making use of their citizens' activism in international disputes by answering the following questions: First, why does a government encourage protests against a foreign opponent, given that protests can be very costly?[8] Second, through which mechanisms can a government benefit from encouraging protests? Last, which kind of regimes can benefit most from these mechanisms?

To study these questions, I consider the following model: Two countries—Country 1 (Home country) and Country 2 (Foreign country)—seek to resolve an international dispute. Country 1 is the 'aggrieved' country, and its people may go to the streets to protest against Country 2. Country 1's government can choose the level of propaganda, which will affect Country 1's people's utility of participating in a protest. In Country 2 there is no internal politics, and its government can only choose to either concede or not.

In Country 1, its people is composed of two factions: the General Public (G) and the Nationalist (N). Both will be more likely to protest if there is a higher level of hostility. The opponent will only concede if the general public is protesting. However, the opponent cannot observe the identities of the protesters. Thus, the opponent is willing to concede as long as it is sufficiently likely that the general public will participate in that protest. This implies that a protest has to be sufficiently *informative* for the opponent to concede when

[3]Then Vice-President Hu said in a speech that "Chinese government firmly supports and protects any demonstration that is held according to law". "Vice-President Hu gave a speech over the bombing of China embassy on 9th May (Chinese)", Sina News, 25th May 2003. http://news.sina.com.cn/c/2003-05-25/14421097103.shtml

[4]In Weiss (2013), the author interviewed some senior level US diplomat. That diplomat claims "This thing got out of control. The government and the Foreign Ministry did not realize how determined and angry these people were ... at the United States, but also, as it went on, partially directed at the Chinese government".

[5]See Weiss (2013) and Weiss (2014) for more details.

[6]To name a few, the recent dispute over the name of Macedonia; Japan-South Korea's dispute over the issue of "comfort woman"; and various territorial disputes.

[7]Again, in the previous example of Macedonia naming dispute, large-scale protests broke out on both sides. In South-Korea, widespread protests over the issue of "comfort women" persist years after both governments reaching binding-agreement over this issue.

[8]Protests can lead to significant disruptions to society, reducing investors' confidence and sometimes creating threats to the regime.

seeing one. In the case of the US bombing the Chinese embassy, the US conceded over those protests because, although those protests were mobilised and manipulated, they still constituted strong signals of the general public's anger. After receiving concessions, it was much easier for the Chinese government to claim victory and calm down angry protesters.

Since the foreign opponent is willing to concede even with some doubt, there is space for the home government to manipulate protests for its own benefit. By choosing a higher level of hostility through propaganda and management of private media and activists, the home government can incite more protests, leading to more concessions if protests are sufficiently informative. However higher hostility means the general public is more likely to protest, which can be costly. I call the event of the general public protesting a *crisis*: placating the angry public is costly, even after some foreign concession. The home government, thus, chooses its optimal hostility balancing the benefit of more concessions versus the cost of more crises, with the additional constraint that protests are indeed sufficiently informative.

Two key parameters that shape the home government's choice of propaganda are the degrees of responsiveness to hostility of the different groups. I find the optimal level of propaganda for the home government can be non-monotonic over these *degrees of responsiveness*. I also find the optimal level of hostility can have a discontinuous jump over the relative benefit of hostility for the home government: if getting a concession is intermediately important, a small change in the relative benefit/cost ratio may generate a disproportionate increase in propaganda and thus protests. Therefore, big changes in propaganda are not necessarily associated with big changes in the value of receiving concessions or cost of facing protests. This means that occurrences of protest and strong hostility are most volatile over issues that are intermediately important for a government.

We then look at how the home government's equilibrium utility changes with different parameters of the model. We show that the home government would like to promote *restrained patriotism* among the general public: here *restrained patriotism* means an intermediate degree of responsiveness towards higher propaganda.[9] A more responsive general public means it is easier to convince the opponent with a low level of hostility. A more responsive nationalist, however, means it is harder to convince the opponent, so that the home government needs a higher level of hostility. Some responsiveness to hostility from the general public and not too much responsiveness from the nationalistare then necessary to avoid the need of generating high levels of costly hostility.

I then show that if the home government faces ex-post temptations to either promote fake protests or suppress protests, then countries with an intermediate level of media freedom and a high level of political fragility benefit the most from this mechanism.[10] If it is easy for the home government to generate some 'fake protests' *ex-post*, then to maintain the

[9]Under some parameter conditions, the home government would also prefer that the nationalist has an intermediate level of *degree of responsiveness*.

[10]This result holds under some parameter restrictions.

informativeness of protest, the home government has to choose extreme levels of hostility *ex-ante*. If *ex-post* a government can suppress a protest with some probability, then the foreign opponent will have less incentive to give concessions.[11] For the foreign opponent to concede after a protest, it requires the protest to be more informative. Therefore, both the ability to generate fake protests and the ability to suppress a protest without receiving concession could actually hurt the home government. This can explain why China often benefits from encouraging protests: China is a country with an intermediate level of media freedom and some political fragility, so a protest is a hard but still manipulable signal for China to seek a concession from a foreign government. Finally, I consider various further extensions to the baseline model.

The structure of this chapter is as follows: part 2 reviews the related literature; part 3 describes the model's setup and the equilibrium concept; part 4 solves the model and then characterizes comparative static analysis; part 5 explores which kinds of regimes can benefit most from encouraging protests; part 6 considers some further extensions to the original model; part 7 concludes. All formal proofs are in the Appendix.

1.2 Related Literature

1.2.1 Bargaining and Strategic Information Transmission

It has long been noticed that in bargaining situations, being irrational or 'crazy' could actually be an effective tactic to obtain better outcomes. Schelling (1960) argued that being inflexible or irrational can help one side commit to only accepting a good offer. Since the seminal work of Schelling (1960), there is a huge stream of literature discussing the particular ways that one side can benefit from commitment tactics (Crawford, 1982; Osborne and Rubinstein, 1990; Muthoo, 1996; Abreu and Gul, 2000; Kambe, 1999). That could be one reason why governments wants to promote hostility in diplomatic crises. By creating a demanding or even fanatical domestic audience behind the government, the government can credibly commit to only accepting a good offer. By promoting a high level of hostility among its people, the government will not accept a bad offer for fear of backlash from a domestic audience of zealots. That can be incorporated in a standard two-person zero-sum bargaining game with complete information.

However, this is far from the whole picture: far from playing a zero-sum game where one side's loss is always the other side's gain, in various crises both sides share a common interest to avoid costly conflicts breaking out. Either a hot war or an economic/diplomatic conflict could be detrimental to both sides and burn the potential surplus that comes from cooperation. Therefore, if one side is sure enough that the public from the other side is angry, it often has the incentive to make concessions and avoid conflict. The information

[11] This is consistent with the historical evidence that in the bombing crisis the American government believed the situation was under control, when they saw more signs of the Chinese government suppressing the level of protests.

that the public is angry is thus socially valuable.

Nevertheless, private incentives may hamper this socially-valuable information being transmitted. Receiving concessions is usually a good thing for a government, either due to actual gain or gain of prestige, while making concessions is usually costly. Hence the home government may have an incentive to bluff and exaggerate the probability of crises. This may lead to the foreign government being less sure whether the home public is angry. In the extreme case information transmission may completely break down, and thus the other side will always refuse to make a concession.

This is the now classical problem of *cheap talk* that socially valuable information is lost due to conflict of interests (Crawford and Sobel, 1982; Battaglini, 2002; Green and Stokey, 2007; Lipnowski and Ravid, 2017). It has been shown that a sender can achieve a better outcome for himself if he can persuade by committing to a pre-determined disclosure plan or an experiment (Kamenica and Gentzkow, 2011; Gentzkow and Kamenica, 2014; Alonso and Câmara, 2016, 2018; Bergemann and Morris, 2016). However, the level of commitment power required for the sender to persuade may be unrealistic here: there is no legally mandated commitment or ex-post verifiable independent experiment enabling the government to commit to a disclosure plan.

Given that persuasion is better for the sender, can he generate similar effects as if he is persuading the receiver? This model provides a specific way this can be done: increasing hostility through propaganda will increase the probabilities of protest for both the nationalist and the general public. However, both groups will respond at different speeds when hostility is higher. The foreign government is only concerned about the protest of the general public but cannot identify who is protesting. Therefore, when changing hostility, the home government changes the informativeness of protest as a signal of general public's action.

This chapter is thus related to the literature on signal-jamming and obfuscation (Holmström, 1999; Ellison and Ellison, 2009). This chapter is also related to the burgeoning literature about how the commitment assumption in Bayesian Persuasion can be relaxed or micro-founded (Best and Quigley, 2017; Margaria and Smolin, 2017).

1.2.2 War and Audience Cost

Since the pioneering work of Putnam (1988), there has been a vast strand of literature studying the interaction between domestic politics and international diplomacy. Putnam (1988) coined the concept of *two-level game* to describe the interconnection between international diplomacy and domestic politics. Domestic politics and diplomacy are usually closely entangled, and governments/leaders take both domestic and diplomatic decisions to maximise their own benefits.

Fearon(1994) popularized the concept of *audience cost* and ignited the long literature about *audience cost* and its various implications. Making credible statements about one country's resolution over crises is extremely difficult, and audience cost provides a particular way how this can be done. By making the threat to enter a war known to the public, the government increases its cost to back down, and this increases the credibility of the threat. Despite various studies that apply this concept (Eyerman and Hart, 1996; Mansfield, Milner, and Rosendorff, 2002; Schultz and Weingast, 2003), there are fewer studies discussing the micro-foundations of these concepts (Smith, 1998; Schultz, 1999; Slantchev, 2006). If leaders bluff for the national interests, it is not clear why domestic voters would like to punish leaders for backing down from their threats. This chapter provides a simple *preference channel* to explain why audience cost can work: by increasing hostility or salience of the current dispute through propaganda, the government makes the domestic audience more extreme and thus it would be harder for the government to back down without receiving some concessions, for fear of domestic backlash.

Another question related to this concept is which countries have high levels of audience cost and are thus more able to commit. Fearon (1994) argued that because leaders in democratic countries face stronger audience cost than leaders in nondemocratic countries, they will be at better positions to commit. Weiss (2013) argued that there can also be big audience costs in non-democratic countries. One important mechanism is through allowing anti-foreign protests. By allowing those risky protests, governments show their resolve and also domestic vulnerability. Weiss (2014) provided detailed discussion and case studies of how non-democratic countries can generate audience cost in the context of China. This chapter is also related to this stream of literature. In my model, protest is a noisy signal that conveys some information and may potentially prompt the foreign government to make concessions.

The differences between my chapter and previous conceptual and formal models are as follows.

First, this chapter focuses on *ex-ante* manipulation of protests instead of *ex-post*. In many cases, the government has to decide whether to encourage or hamper protests before knowing whether some particular dispute will become an uncontrollable diplomatic crisis.

Second, this chapter also provides predictions about the optimal strategy of governments' *nation-building* policy: the government would try to promote some form of *restrained patriotism*. It wants its people to be responsive, but not too responsive, over international controversies. That is consistent with governments' actions in real life, for example, the Chinese government's constant call for 'loving the country rationally'.

Third, this chapter provides predictions about which kind of countries can benefit from stirring domestic anti-foreign protests. This chapter emphasizes the importance of media freedom and political fragility in determining the profitability of encouraging protests. This provides additional testable predictions and could help us better understand why we see many anti-foreign protests in some particular countries.

Last but not least, this chapter emphasises the benefit rather than the cost of *obfuscation* for protests to persuade the foreign governments to concede. Previous studies such as Weiss (2014) have emphasised the value of credibility for protests to convince the opponents to concede. In this chapter I show the value of at least partial *obfuscation*: in many cases, there is no need to perfectly convince a foreign government to make it concede. Often it is willing to concede as long as a protest is informative enough about general population's widespread anger. A not perfectly precise but still informative enough signal can still guarantee concession and makes the probability of getting concession higher than the case of fully-revealing information.

1.3 Model Setup

1.3.1 Players, Actions, States and Payoff Functions

Player Set:

There are two countries, Country 1 and Country 2. This model includes the bargaining problem between Country 1 and Country 2 and the internal politics of Country 1.[12]

There are four players in the game:

$$\{H, F, N, G\}$$

H is the home government of Country 1; N is the nationalist group of Country 1 and G is the general public of Country 1; F is the foreign government (the government of Country 2).

Action Spaces:

The action spaces of all the players are as follows:

$A_H = \Psi = [0, \bar{\psi}], A_F = \{C, NC\}, A_N = A_G = \{P, NP\}$.

The home government chooses the level of hostility from a closed and bounded interval, $[0, \bar{\psi}]$; the foreign government chooses whether to concede or not; the nationalist and the general public each decides whether to protest or not.

$$\text{Country 1}: \begin{cases} H \rightarrow [0, \bar{\psi}] \\ N \rightarrow \{P, NP\} \\ G \rightarrow \{P, NP\} \end{cases}$$

$$\text{Country 2}: F \rightarrow \{C, NC\}$$

Home government chooses ψ from a compact interval, $[0, \bar{\psi}]$. ψ is the level of hostility the home government chooses. It captures a government's various methods of generating

[12] For simplicity, we do not consider the internal politics of Country 2, so foreign government represents a coherent Country 2.

higher hostility among its people towards a foreign opponent: 1) direct propaganda in government-controlled media; 2) censorship over independent media's coverage; 3) management over activities of the nationalist groups. This chapter will just call it propaganda; however, ψ basically models the various methods the home government can use to mobilize anti-foreign protests. In this chapter I will use *hostility* and *propaganda* interchangeably.

Here I model the foreign government's action as a discrete choice of whether to concede or not. This reflects the observation that in many cases, diplomatic responses are in discrete levels: instead of choosing from a continuous action space, the opponent usually has different 'steps' of potential responses. Also, many issues' indivisibility can further justify this assumption. [13]

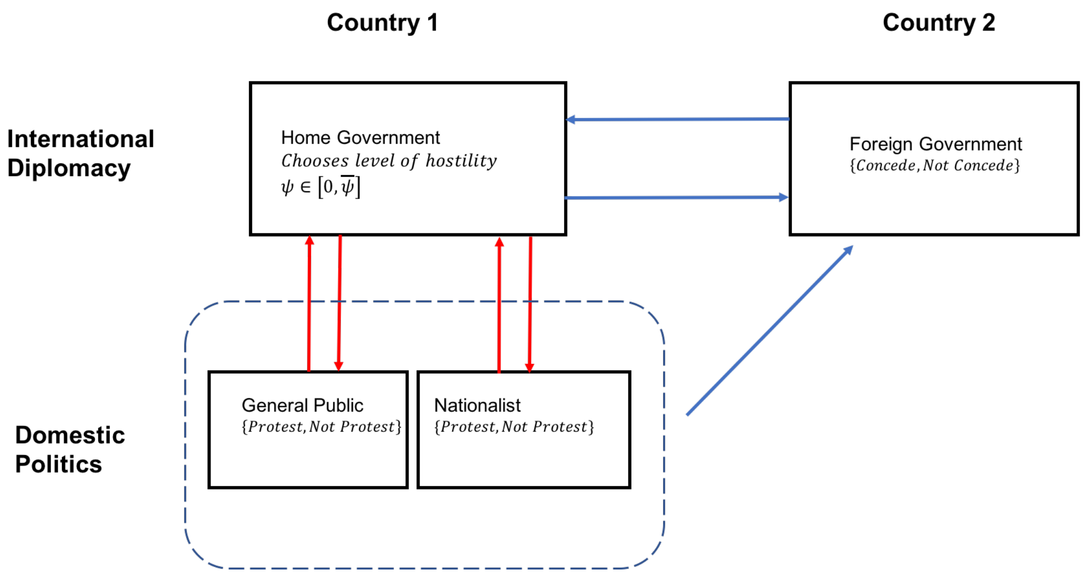


Figure 1.1: Players, Country Affiliations and Actions

State Space:

The state space, Θ, is a one-dimensional compact set. θ can be thought of as the latent common 'grievance' that country 1's people feel over some dispute. The higher is θ, the more annoying this event is to both the nationalist and the general public. I further assume θ is uniformly distributed among $[0, \bar{\theta}]$ and is statistically independent of any choice of ψ by the home government.

Pay-off Functions

In general, for any player $i, i = H, F, N, G$, its utility function $u_i(\theta, a_H, a_F, a_N, a_G)$ depends on the realization of the state, and the profile of all player's actions. I assume players' payoff functions are as follows:

Payoffs of Nationalist and General Public

[13] In theory, side payments can be possible. However, due to various reasons side payments or transfers cannot always solve this problem. For example, this can be due to the limited attention span or expertise in understanding diplomatic pacts from the public.

For $i = N, G$,

$$u_i(\theta, \psi, a_i) = \begin{cases} [w_i(\psi)\theta - c], & \text{if } a_i = P \\ 0, & \text{if } a_i = NP \end{cases}$$

I assume that $w_i(\psi) = w_0 + w_{1i}\psi$, and $w_{1N} > w_{1G} > 0$. Also I assume that $w_0\bar{\theta} > c$.

Payoffs of Home Government and Foreign Government

For H and F, their payoff functions only depend on the action of general public, a_G, and the action of foreign government, a_F.

Moreover, we assume the home government's payoff function has the following properties:

$$u_H(\cdot, C) > u_H(\cdot, NC)$$

$$u_H(NP, \cdot) > u_H(P, \cdot)$$

Those assumptions essentially mean that for the home government: 1) getting concessions is good; 2) general public protesting is costly.

We assume the foreign government's pay-off function have the following properties:

$$u_F(P, NC) < u_F(P, C)$$

$$u_F(NP, NC) > u_F(NP, C)$$

$$u_F(NP, \cdot) > u_F(P, \cdot)$$

Those assumptions essentially mean that: 1) home country's general public protesting is also costly to the foreign government; 2) giving concessions is costly, but the foreign government still prefers giving concessions if the general public in the home country is protesting.

Tie-breaking Rules

First, we assume that when the foreign government is indifferent between conceding or not, it will concede with probability 1. Following Kamenica and Gentzkow(2011), we are looking at the sender-optimal subgame perfect equilibrium.

Second, we assume that when either the nationalist or the general public is indifferent between protesting or not, it will protest with probability 1. That assumption is an innocuous one since for continuous distributions of θ, being at the indifference points are probability-zero events.

Last, without loss of generality, we assume that when the home government is indifferent between choosing a lower level of ψ or a higher level of ψ, he will choose the lower one with probability 1.[14]

[14] It can be shown that in this model the home government's optimal level of ψ is in general unique. The home government is only indifferent between more than one optimal level of hostility in rare cases when: 1) under the optimal level of hostility the informativeness constraint is binding; 2) the threshold of doubt equals the benefit/cost ratio.

Discussion

I will call the situation of general public protesting as a *crisis*, because general public protesting is always costly to both governments, and may get out of control if the foreign opponent does not concede.

There are several assumptions that are worth further discussion.

First, I have assumed that the two groups' payoff functions depend on whether they go to protest, but not on the future policy outcome. The event generates anger for both groups, and both groups benefit from expressing anger in the form of protesting in the street. Participating in a protest is still costly, so both groups are still rational in the sense that they will weigh the benefit and cost of going to the street.

So the two groups are not 'strategic' in the sense that they are not choosing whether to protest in a forward-looking way, i.e., protesting to induce the home government to take some particular actions. Instead, they are protesting for 'expressive' reasons. It has been long noticed in the sociology literature and the political science literature that protest can be 'instrumental' and/or 'expressive'. Passarelli and Tabellini(2017) consider a model in which protests are expressive. Protesters protest to express their emotions over the 'unfair' treatment they received. Here, similar to Passarelli and Tabellini(2017), protests are expressive. Two groups protest to react to the unfair treatment their country has received during the crisis, or the humiliation their country has received that needs proper compensation.

Second, I assume that both governments care about the action of the general public and the action of the foreign government. It is obvious that both governments are affected by the decision of the foreign government. It is also obvious that the home government cares about whether the public will go to the street. Here we assume the foreign government cares about the public's protest decision, not because we believe that the foreign government cares about the well-being of country 1's public, but as a reduced-form representation. One reason could be that foreign government dislikes the direct consequence of general public protesting, such as Country 1's general public boycotting Country 2's goods. Another reason is more aligned with the audience cost literature: the foreign government cares about the probability of escalation from the home government, and the home government's probability of escalation depends on whether the general public goes to the street. Intuitively, with angry public protesting on the street, the home government is usually facing mounting pressure to be 'tough' and escalate the crisis, if no concession is offered from the other side.

Third, I assume that both the home government and the foreign government do not care about the protest of nationalists intrinsically. Moreover, in the baseline model, θ and ψ do not enter directly into the utility functions of both governments. We show in later sessions that the conclusions are robust if we relax the first assumption. Intuitively, nationalists are people who care very strongly about national interest and may have very extreme views over diplomatic disputes. Suppose the great majority does not support this kind of extreme views, then nationalists are a thin minority which will not pose serious threats over the home government and will not force the home government into escalation. Also, the nationalists cannot bring huge direct costs to the foreign government due to its limited size. The opponent only cares about the protest behaviour of country 1's people either because they could generate huge direct costs, or they may pressure country 1's government into costly escalation. Therefore, given that only the general public can force the home government into a costly crisis or directly harm the foreign government, the foreign government will only care about the action of the general public but ignore any protest by the nationalists.

The assumption of θ and ψ not entering the payoff functions of both governments can be partially relaxed. Essentially, for θ to not affect the payoff functions for both governments, θ needs to only affect both governments indirectly: both governments will not be affected by the angry public as long as they stay at home and do not protest. For the ψ to not directly enter the payoff functions of both government, we are essentially assuming: 1) choosing ψ is costless;[15] 2) ψ will only affect the distribution of states, but not both governments' payoffs under each state.

1.3.2 Timeline and Information Structure

The timeline of the baseline model is as follows:

Period 1: The home government chooses the level of hostility, ψ; the level of ψ is common knowledge to all the players.

Period 2: nature chooses the level of θ. θ is private knowledge to the nationalist and the general public. The nationalist and the general public then decide simultaneously whether to protest or not. The actions of the nationalist and the general public are observable to themselves and the home government, but not to the foreign government. [16]

Period 3: The foreign government can only observe a binary signal, s, $s \in \{P, NP\}$, which is generated in such a way:

$$s = \begin{cases} P, & \text{if } a_N = P \vee a_G = P \\ NP, & \text{otherwise} \end{cases}$$

[15] We have assumed that for the home government, choosing any level of $\psi \in [0, \bar{\psi}]$ has zero cost. This is equivalent to choose $\psi \in [0, +\infty)$, and choosing any level $\psi \in [0, \bar{\psi}]$ has zero cost but choosing any $\psi > \bar{\psi}$ is infinitely costly:

$$C(\psi) = \begin{cases} 0, & \text{if} \quad \psi \in [0, \bar{\psi}] \\ +\infty, & \text{if} \quad \psi \in (\bar{\psi}, +\infty) \, . \end{cases}$$

[16] It is not essential to assume that θ is unobservable to the home government. What is essential here is that the foreign government cannot observe θ. However, it seems to be a reasonable assumption that even home government may have substantial difficulty to perfectly observe people's latent grievance.

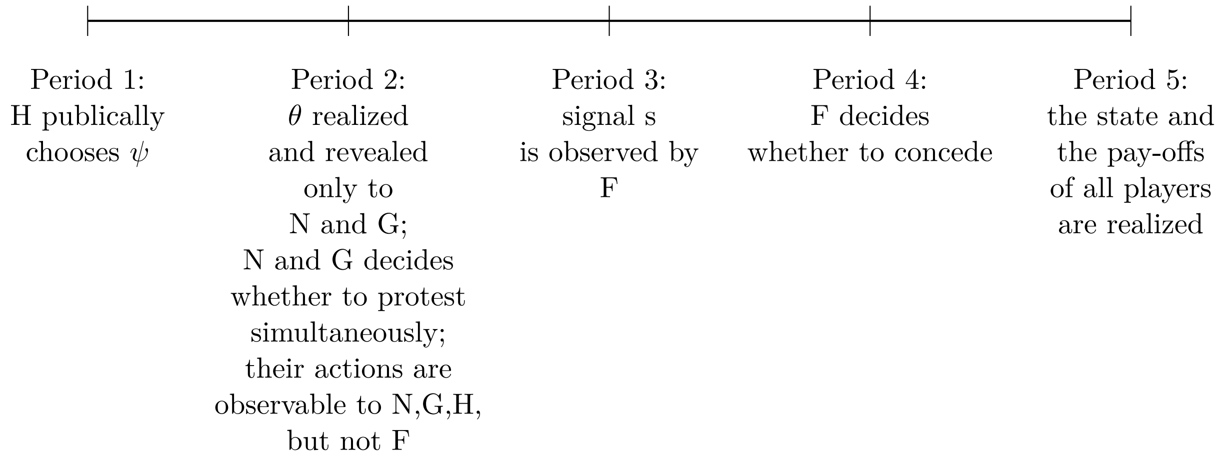


Figure 1.2: Timeline of the Baseline Model

This means the foreign government can only observe whether there is a protest but not who is protesting.

Period 4: The foreign government decides whether to concede or not after observing the realisation of signal s.

Period 5: the state and the pay-offs of all players are realised and publicly revealed.

Here we assume the latent grievance, θ, is unobservable to the home government. Given the timeline of the model, whether θ is observable to the home government is inconsequential because the home government have to choose ψ before θ is realized. However, in an alternative setting where the home government chooses ψ after θ is realized, whether θ is observable to him will make a great difference.

It seems to be a reasonable assumption that even the home government may have substantial difficulty to observe its people's latent grievance. Therefore, the home government may face a great level of uncertainty about θ and here a simplifying assumption is that θ is completely unobservable to the home government. An interesting future extension would be considering the case that the home government may have some noisy private information about θ.

Another assumption we have made here is that the home government chooses ψ before θ is realised. This assumption comes from the observation that the government cannot perfectly predict and control what will happen as the beginning of a potential crisis. When the home government are trying to affect the probability of protests, by either direct propaganda or act towards nationalist leniently, they do not know perfectly how its people will react towards its effort. It is precisely this property of protests being ex-ante uncontrollable and unpredictable that give the opponent incentive to concede and the home government room to manoeuvre the protests. Moreover, given that θ is unobservable to the home government, whether the home government chooses ψ before or after θ will not affect the equilibrium of this model.

Here ψ is essentially modelling the ways to *ex-ante* manage protests. Later we will discuss ways for the government to *ex-post* manage protests, such as suppressing a protest or generating some fake protests.

We also assume that θ is unobservable to the foreign government. For the government of Country 2, observing how the people in Country 1 feels about some diplomatic crisis before making the decision is extremely difficult. Observing or inferring what people have in mind is extremely difficult, and it would be even more difficult to observe a rival country's people's real feeling. There is only some limited amount of media information and some secret intelligence reports that the foreign country can rely on, and the home country has strong incentives to misrepresent the information the foreign government can receive. Therefore, it is possible that the foreign government to get some noisy signal over θ, but it is still safe to assume at least some unobservability of θ.

Another crucial assumption here is that the foreign government cannot distinguish between protests of only nationalist and protests of everyone (including both nationalist and the public). The only thing it can observe is whether there are some people on the street protesting. In reality, the foreign government probably can have some noisy signal over some characteristics of a protest, like the number of people appearing in the street. However, this is usually a very noisy signal, and the home government still has the incentive to misrepresent that signal. Therefore, this is a simplifying assumption that captures the fact that foreign government cannot perfectly observe who is protesting on the street. Later we will look at the case when the government can get some noisy and non-revealing signal over the identity of the protesters. That would also allow us to generate additional prediction over what level of media freedom would allow a regime to benefit most from manipulating protests.

1.3.3 Solution Concept

The equilibrium concept used in this chapter is standard weak Perfect Bayesian Equilibrium.

Definition 1.1. The profile of (a^*,μ^*) is *weak Perfect Bayesian Equilibrium* (PBE as follows) if:

1) μ^* is consistent: it is determined by Bayes' Rule according to a^* whenever possible;

1.1) In Period 4, $\mu(\psi, s)$ is foreign government's posterior belief that the general public is protesting, given the level of ψ and realization of s. $\mu(\psi, s)$ is determined by Bayes' Rule whenever possible, according to the signal-generating process and the strategies of the home government, the nationalist and the general public.

2) a^* is sequentially rational:

2.1) In period 4, given$(a_H^*, a_N^*, a_G^*, \mu^*(\psi, s))$, a_F^* is the optimal response of the foreign government;

2.2) In Period 2, given the choice of ψ by the home government in Period 0 and the realisation of θ in period 1, for $i = N, G$, a_i^* is the optimal response of group i given $a^*_{\{N,G\}/i}$ and a_F^*;

2.3) In Period 1, the home government chooses the optimal level of ψ, given $(a_N^*, a_G^*, a_F^*, \mu^*)$.

1.4 Equilibrium and Its Properties

In this section, we will solve the equilibrium of the baseline model and characterise its properties. The baseline model is solved by backwards induction. We will then discuss the comparative static analysis.

1.4.1 The Foreign Government's Optimal Strategy

The foreign government wants to concede if and only if the general public is protesting. Therefore, he will only concede if his posterior of general public protesting, $\mu(\psi, s)$, is higher than some threshold of doubt, $\bar{\mu}$. [17]

When he observes no protest, he knows that the general public is not protesting for sure. Therefore he will not concede in that case.

When he observes a protest, he understands that this can either be a protest where both groups (the Nationalist and the General Public) participate or a protest of only the nationalist. He only wants to concede if the general public protests, but he cannot observe who is protesting. Therefore, he will only concede if the posterior that the general public is protesting is high enough:

$$\begin{aligned}\mu(\psi, s = P) &\equiv Prob(a_G = P|\psi, s = P) \\ &= \frac{Prob(a_G = P|\psi)}{Prob(a_G = P|\psi) + Prob(a_G = NP, a_N = P|\psi)} \geqslant \bar{\mu}\end{aligned}$$

We define a protest to be *informative enough* if $\mu(\psi, s = P) \geqslant \bar{\mu}$. This is the case that protest is sufficiently informative about the general public's action so the foreign government will concede when a protest happens.

1.4.2 The Nationalist and the General Public's Optimal Strategy

For both the nationalist and the general public, their costs of protesting are constant but benefits of going to the street are higher if θ is higher: the benefit of going to streets,

[17] The *threshold of doubt* for the foreign government, $\bar{\mu} \in (0, 1)$, is defined as the level of μ such that the foreign government is indifferent between conceding or not conceding. That means $\bar{\mu}[u_F(P, C) - u_F(P, NC)] = (1 - \bar{\mu})[u_F(NP, NC) - u_F(NP, C)]$.

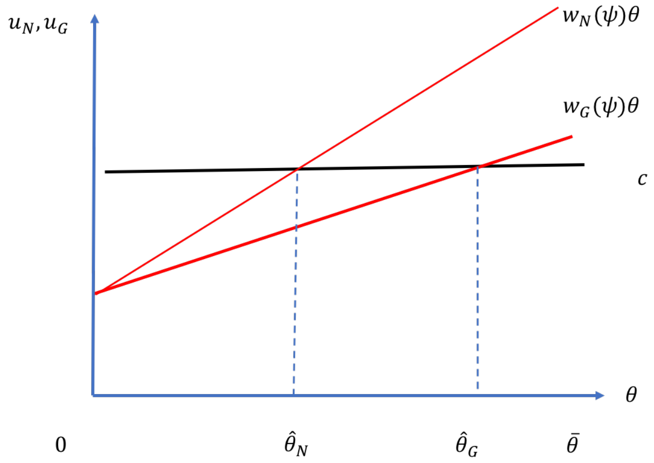


Figure 1.3: Nationalist and General Public's Best Responses

which is the grievance, $w_i(\psi)\theta$, is increasing in θ. Therefore, for both groups, their best responses are cut-off strategies.

For group $i, i = N, G$, it will protest if and only if $(w_0 + w_{1i}\psi)\theta - c \geqslant 0$. This means there exist a cut-off level, $\hat{\theta}_i(\psi)$, such that it will protest if and only if $\theta \geqslant \hat{\theta}_i(\psi)$.

Figure 3 describes the best response of both groups given the level of hostility, ψ. Because we have assume that the nationalist is more responsive to higher ψ, it is obvious that the nationalist has a lower cut-off level than the general public. That also means the we can divides the whole line of θ into three parts: 1) $\theta \in \left[\hat{\theta}_G, \bar{\theta}\right]$. When θ is high, both groups will protest. 2) $\theta \in \left[\hat{\theta}_N, \hat{\theta}_G\right)$. When θ is at an intermediate level, only the Nationalist will protest. 3)$\theta \in \left[0, \hat{\theta}_N\right)$. When θ is low, no one will protest.

Define the ex-ante probability of protesting [18] for group i as $H_i(\psi) \equiv Prob(Player\ i\ protests)$ $= Prob(\theta \geqslant \hat{\theta}_i)$. Because of the assumptions of our model, $H_N(\psi) \geqslant H_G(\psi)$: the nationalist will be more likely to protest. Given we have defined *crisis* as the general public protesting, $H_G(\psi)$ is also the probaility of *crises*. Moreover, given that the foreign government will observes a protest whenever the nationalist protests, $H_N(\psi)$ is also the probaility of the foreign government *observing* a protest. We know from the model's set up that higher hostility, ψ, will increase the benefit of protesting for both groups at any given θ. Figure 4 shows that this would reduce the cutoff levels of both groups and make them more likely to protest.

Now we can also characterise how the foreign government's posterior when observing a protest, $\mu(\psi, s = P)$, changes when ψ changes:

$$\mu(\psi, s = P) = \frac{Prob(a_G = P|\psi)}{Prob(a_G = P|\psi) + Prob(a_G = NP, a_N = P|\psi)} = \frac{H_G(\psi)}{H_N(\psi)}$$

Figure 5 shows that foreign government posterior is actually *U-shaped* over ψ. That

[18]Here ex-ante probability of protest means the *interim* ex-ante expected probability of protest. That means the probability of protesting after ψ is chosen but before θ is realized.

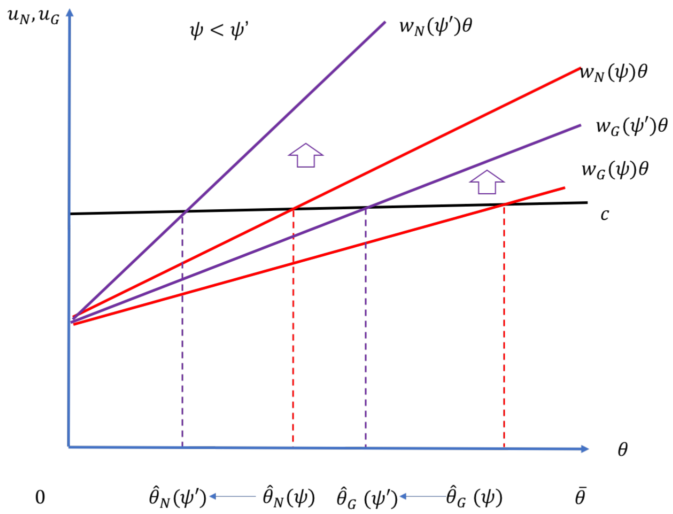


Figure 1.4: Nationalist and General Public's Best Responses

means for a high $\bar{\mu}$, a protest will only be informative enough if it is either very low or very high.

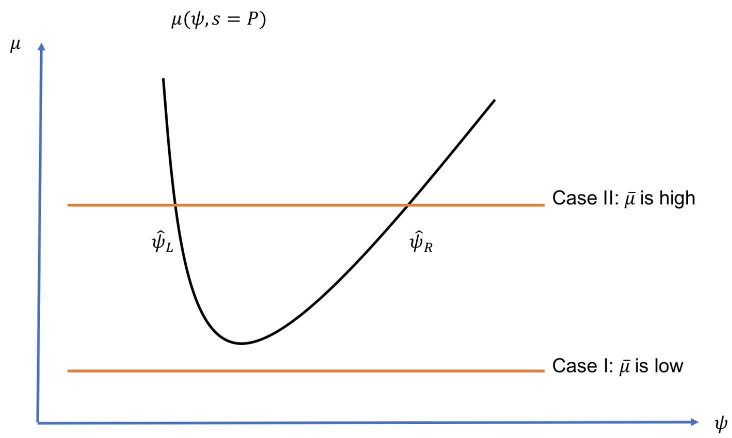


Figure 1.5: Foreign Government's posterior when observing protest, as function of ψ

The intuition of this U-shaped curve is as follows: when the ψ is low so there is not much manipulation of protests, the opponent knows that the nationalist and the public are both unlikely to be on the street, and their difference over probabilities of protesting is low. Thus when the opponent observes a protest, he knows with high probability both groups are protesting. When ψ is very high, the opponent knows the nationalist is almost always protesting on the street. However, since ψ is very high, the general public is also very likely to be protesting, so in this case, the difference over protest probabilities are also low. In the case that ψ is at intermediate level, $H_N(\psi)$ is much larger than $H_G(\psi)$. There would be many protests of only the nationalist but the general public is unlikely to protest, and this would make foreign government less sure that the general public supports a protest. [19]

1.4.3 The Home Government

The home government's choice of ψ will affect the home government's pay-off through: 1) the actions (to protest or not) of both groups (the nationalist and the general public); 2)the strategy of the foreign receiver.

First, it can be shown that the home government will never choose a level of ψ such that the $\mu(\psi, P) < \bar{\mu}$. The intuition is simple: the home government will only increase ψ to increase its probability of getting concessions, and the cost of doing so is a higher probability of crises. When ψ is at a level such that $\mu(\psi, P) < \bar{\mu}$, the opponent will not make a concession, and positive ψ will only generate cost but no benefit. Thus the home government can deviate and choose $\psi = 0$ instead, which secures concession when there is a protest and generates a lower probability of crises.

This simplifies the home government's problem substantially: we only need to consider the home government's optimal decision assuming that the foreign government will concede after observing a protest. Then to make sure it is indeed incentive compatible for the foreign government to concede, we just need to put a constraint that the level of ψ chosen by the home government makes it optimal for the opponent to concede. We will call it the *informativeness constraint.*

The home government's optimal propaganda problem is equivalent to: [20]

$$\max_{\psi \in [0,\bar{\psi}]} H_G(\psi) u_H(P, C) + [H_N(\psi) - H_G(\psi)] u_H(NP, C) + [1 - H_N(\psi)] u_H(NP, NC)$$

$$s.t.\ \mu(\psi, P) = \frac{H_G(\psi)}{H_N(\psi)} \geqslant \bar{\mu}$$

Now let us define two parameters of the home government's payoff function:

Definition 1.2.

- $t_H \equiv u_H(NP, C) - u_H(NP, NC)$
- $\tau_H \equiv u_H(P, C) - u_H(NP, C)$

[19]Mathematically, the reason of $\mu(\psi, s = P)$ being U-shaped comes from the elasticity of $H_N(\psi)$, $\frac{H'_N(\psi)}{H_N(\psi)}$, single-crossing the elasticity of $H_G(\psi)$, $\frac{H'_G(\psi)}{H_G(\psi)}$. Roughly speaking, when ψ is small, $H_N(\psi)$ is growing much faster than $H_G(\psi)$, and the levels of $H_N(\psi)$ and $H_G(\psi)$ are both small. Thus $H'_N(\psi)$ is substantially larger than $H'_G(\psi)$, while the difference between $H_N(\psi)$ and $H_G(\psi)$ is small. When ψ becomes large, $H_N(\psi)$ and $H_G(\psi)$ are both growing very slowly so $H'_N(\psi)$ is very near to $H'_G(\psi)$, while the difference between $H_N(\psi)$ and $H_G(\psi)$ are also small but still in larger magnitude.

[20]The objective function is continuous. The set of ψ that satisfies the constraint is non-empty and is a finite union of disjoint compact intervals. Therefore, the optimal solution and optimal value to this question both exist.

t_H can be thought of as the benefit of higher propaganda: a higher level of propaganda increases the probability of protest from only nationalist. When protests of only the nationalist happen, there is no crisis, but the foreign opponent will still concede. From our assumption, getting concession without a crisis happening is beneficial to the home government.

τ_H can be thought of as the cost of higher propaganda: higher propaganda also increases the probability of general public protesting. This is costly because having the general public protesting on the streets is costly. Therefore, the ratio of t_H over τ_H, $\frac{t_H}{\tau_H}$, can be thought of as a measure of the relative benefit of propaganda.

We will first characterise the objective function and the informativeness constraint:

Lemma 1.1.

- *The objective function is quasi-concave on the interval $[0, \bar{\psi}]$. Its maximal point on this region, ψ^I, is a weakly increasing function over the relative benefit/cost ratio, $\frac{t_H}{\tau_H}$.*
- *The posterior belief that the general public is protesting when there is a protest, $\mu(\psi, P)$, is a quasi-convex function over ψ. It is first decreasing and then increasing over ψ.*
- *define $\hat{\psi} = argmin_{\psi \in [0,+\infty)} \mu(\psi, P)$ and assume $\bar{\psi} > \hat{\psi}$. If $\bar{\mu} \in (\mu(\hat{\psi}, P), \mu(\bar{\psi}, P))$, there exist two levels of ψ, $\hat{\psi}_L(\bar{\mu})$ and $\hat{\psi}_R(\bar{\mu})$, $0 < \hat{\psi}_L(\bar{\mu}) < \hat{\psi}_R(\bar{\mu}) < \bar{\psi}$, such that $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. Moreover, for any $\psi \in [0, \bar{\psi}], \mu(\psi, P) \geqslant \bar{\mu}$ iff $\psi \in [0, \hat{\psi}_L(\bar{\mu})] \cup [\hat{\psi}_R(\bar{\mu}), \bar{\psi}]$*

The analytical forms of $\hat{\psi}_L(\bar{\mu})$ and $\hat{\psi}_R(\bar{\mu})$ can be found in the appendix. So there exists a unique ideal point of ψ , ψ^I, that maximises the objective function. The informativeness constraint is U-shaped over ψ. If the threshold of doubt $\bar{\mu}$ is high, then a protest is only informative if ψ is very high or very low. Therefore, the solution to the home government's optimisation problem, ψ^*, depends on whether the *informativeness constraint* is violated at ψ^I.

Now we are ready to characterise the optimal feasible level of ψ:

Proposition 1.1. *Assume $\bar{\psi} > \frac{w_0}{\sqrt{w_{1N} w_{1G}}} \sqrt{\frac{w_0 - \tilde{c}}{w_0}}$.*[21] *Then:*

1) If $\frac{t_H}{\tau_H} \leqslant \frac{w_{1G}}{w_{1N}}$, then $\psi^* = \psi^I = 0$ for any $\bar{\mu} \in (0, 1)$;

2) If $\frac{t_H}{\tau_H} \geqslant \frac{w_{1G}}{w_{1N}} (\frac{w_0 + w_{1N}\bar{\psi}}{w_0 + w_{1G}\bar{\psi}})^2$, then

- *If $\bar{\mu} \in \big(0, \mu(\bar{\psi}, P)\big]$, then $\psi^* = \psi^I = \bar{\psi}$*
- *If $\bar{\mu} \in \big(\mu(\bar{\psi}, P), 1\big)$, then there exists $\hat{\psi}_L(\bar{\mu})$, such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \hat{\psi}_L(\bar{\mu})$

[21] The other case leads to similar results. The complete results can be find in the Appendix.

3) If $\frac{t_H}{\tau_H} \in (\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)$*, then there exists a cut-off level* $\mu_1 \in (0,1)$*, such that:*

- *If* $\bar{\mu} \in (0, \mu_1]$ *, then* $\psi^* = \psi^I$
- *If* $\bar{\mu} \in \big(\mu_1, \mu(\bar{\psi}, P)\big]$*, then there exist* $\hat{\psi}_L(\bar{\mu})$ *and* $\hat{\psi}_R(\bar{\mu})$*, such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$
  - $$\psi^* = \begin{cases} \hat{\psi}_L, \text{ if } & \frac{t_H}{\tau_H} \leqslant \bar{\mu} \\ \hat{\psi}_R, \text{ if } & \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$
- *If* $\bar{\mu} \in \big(\mu(\bar{\psi}, P), 1\big)$*, then there exists* $\hat{\psi}_L(\bar{\mu})$*, such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \hat{\psi}_L(\bar{\mu})$

Figure 6 and 7 illustrate the intuition of Proposition 1.1 under the case that $\bar{\psi}$ is large and $\frac{t_H}{\tau_H}$ is at intermediate levels.[22] Proposition 1.1 says, when the threshold of doubt is so low enough such $\mu(\psi, P) \geqslant \bar{\mu}$ for any $\psi \in [0, \bar{\mu}]$, then the informativeness of the signal is not a concern for the home government. The optimal level of ψ equals to the home government's ideal point of ψ.

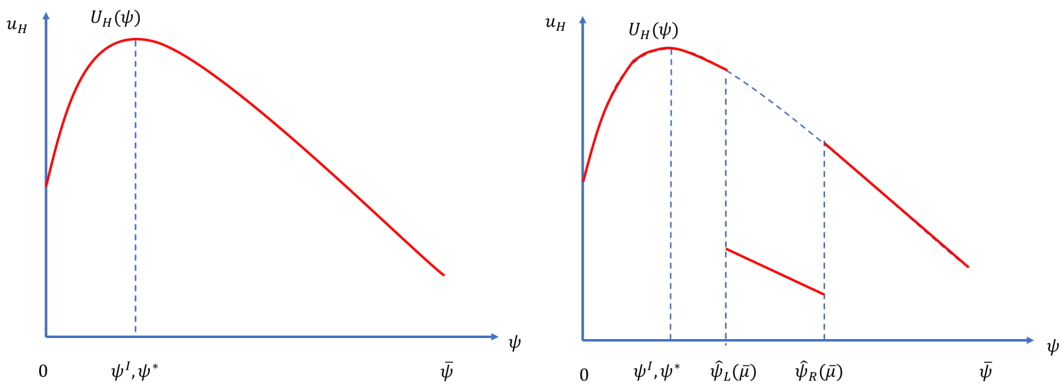


Figure 1.6: Optimal propaganda, when $\bar{\mu}$ is at low or middle levels

When the threshold of doubt is higher, then protest is only informative enough if $\psi \leqslant \hat{\psi}_L(\bar{\mu})$ or $\psi \geqslant \hat{\psi}_R(\bar{\mu})$. Now there are three cases: First, if $\bar{\mu}$ is in an intermediate range, then the interval that a protest is not informative enough, $\Big(\hat{\psi}_L(\bar{\mu}), \hat{\psi}_R(\bar{\mu})\Big)$, is small. The ideal point ψ^I lies out of this range and thus the optimal level ψ^* is the same as the previous case. Second, if $\bar{\mu}$ is larger, then the interval that a protest is not informative enough, $\Big(\hat{\psi}_L(\bar{\mu}), \hat{\psi}_R(\bar{\mu})\Big)$, is larger. Then the ideal point ψ^I lies out of this range and cannot be chosen. Since the objective function is single-peaked,the constrained optimal must be either $\hat{\psi}_L(\bar{\mu})$ or $\hat{\psi}_R(\bar{\mu})$. The relative benefit of $\hat{\psi}_L(\bar{\mu})$ over $\hat{\psi}_R(\bar{\mu})$ would be a lower probability of crises because of lower ψ. However, the relative cost would be a

[22] For other cases the intuition is the same.

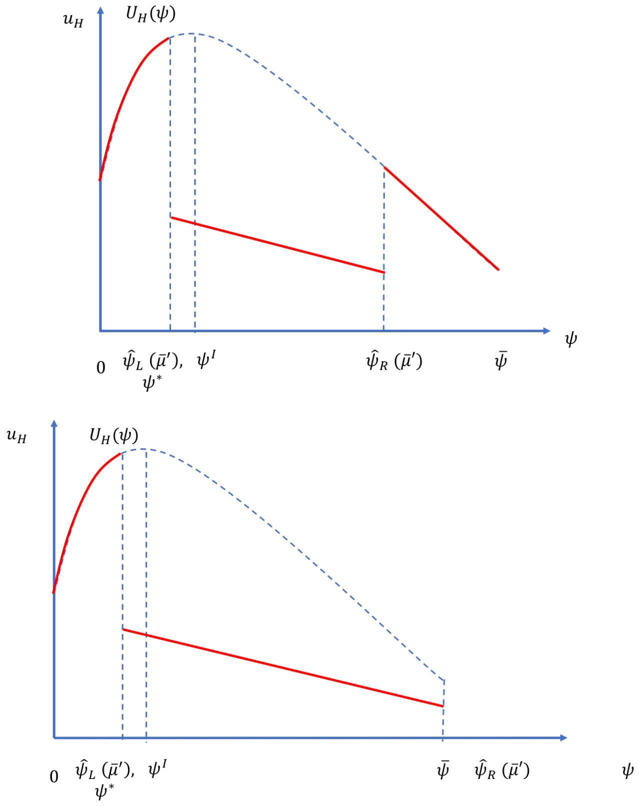


Figure 1.7: Optimal propaganda, when $\bar{\mu}$ becomes higher and higher

lower probability of a protest occurring and thus less concession. Which effect dominates depends on the comparison between $\frac{t_H}{\tau_H}$ and $\bar{\mu}$. Last, when ψ^I is even higher, $\hat{\psi}_R(\bar{\mu})$ is too high so the levels of propaganda that make a protest informative enough are in the range $\left[0, \hat{\psi}_L(\bar{\mu})\right]$.

1.4.4 Comparative Static Analysis

Naturally, the next step would be looking at the comparative static analysis of the baseline model. In this section, I aim to answer the following questions: (1) How does the optimal level of ψ, ψ^*, change when the parameters[23] of the model change? Moreover, how does ψ^* transit from unconstrained optimal point (home government's ideal point) to constrained optimal point ($\hat{\psi}_L(\bar{\mu})$ or $\hat{\psi}_L(\bar{\mu})$)? (2) How do the equilibrium probability of protest from the general public, $H_G(\psi^*)$, and the probability of protest from the nationalist, $H_N(\psi^*)$, change when the parameters change?

The complete comparative static analysis of ψ^*, $H_G(\psi^*)$, and $H_N(\psi^*)$ over various parameters can be found in the appendix. Some interest results can be found from the comparative static analysis.

First, ψ^* may have non-monotonic transitions over some parameters. In some cases,

[23] Parameters of the model: w_0; w_{1G} and w_{1N}; c; $\bar{\theta}$; $\bar{\mu}$; $u_H(P,C) - u_H(NP,C)$ and $u_H(NP,C) - u_H(NP,NC)$. We will look at last two parameters' effects jointly thorough $\frac{t_H}{\tau_H} \equiv \frac{u_H(NP,C)-u_H(NP,NC)}{u_H(P,C)-u_H(NP,C)}$, because that is the only way those two parameters affect ψ^*.

this is due to the non-monotonicity of the interior solution ψ^I itself. However, in many cases, this non-monotonic transition comes from the transitions between the interior solution ψ^I and the boundary solutions $\hat{\psi}_L(\bar{\mu}, P)$ and $\hat{\psi}_R(\bar{\mu}, P)$.

Second, ψ^* may have discontinuous jump over the relative benefit of concession, $\frac{t_H}{\tau_H}$, at the point $\frac{t_H}{\tau_H} = \bar{\mu}$. A very small change over $\frac{t_H}{\tau_H}$ would lead to discrete jump from $\hat{\psi}_L(\bar{\mu}, P)$ to $\hat{\psi}_R(\bar{\mu}, P)$ (or *vice versa*). This could possibly explain that when the fundamentals of some dispute remain the same, with some small change of benefit/cost of the home government, the home government will dramatically escalate or de-escalate the tension. Moreover, this only happens when the relative benefit of driving up tensions, $\frac{t_H}{t_H}$, is in an intermediate range. This would predict that we would find nationalism tensions to be most volatile when the dispute is intermediately important for the home government. In matters that are either too important or too trivial, we would not expect to see big shifts at the level of hostility unless there are some big changes on the fundamentals of the dispute.

The comparative static analysis of $H_G(\psi^*)$ and $H_N(\psi^*)$ also have similar patterns.

Here we will look at one example, the comparative static analysis of optimal hostility, ψ^*, over w_{1G}, the degree of responsiveness for the general public.

Corollary 1.1. *Assume $\frac{t_H}{\tau_H} < 1$ and $\bar{\mu}$ is large. There exists a level of w_{1G}, w_{1G}^*, such that*

- *ψ^* is increasing in $w_{1G} \in (0, w_{1G}^*)$*
- *ψ^* is decreasing in $w_{1G} \in [w_{1G}^*, w_{1N})$*

This corollary has an intuitive explanation: when w_{1G} is low compared to w_{1N}, as the home government increases ψ, the nationalist will increase its probability of protest in a much faster speed than the general public. In this case, the informativeness of the protest is problematic and to keep informativeness, the home government cannot choose a level of hostility that is too high.

However, when w_{1G} is very high, as ψ increases, the nationalist will not increase its probability of protest much faster than the general public. In this case, informativeness is not a concern. However, as w_{1G} becomes higher, costly crisis is more likely to occur and thus choosing a high level of ψ becomes more and more costly.

1.4.5 Home Government's Equilibrium Utility

In this part, we will look at how the home government's equilibrium utility changes as different parameters change. [24]

[24]The model is a stylized model and is not designed to describe the total social welfare. An interesting future research direction would be how social welfare, measured under different possible welfare functions, changes as various parameters change.

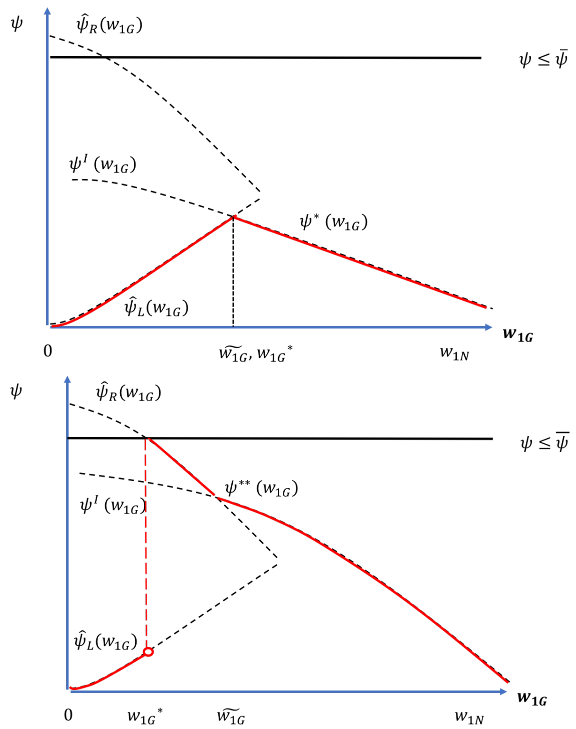


Figure 1.8: Comparative Static Analysis of ψ^* over w_{1G}: when $\frac{t_H}{\tau_H} < 1$ and $\bar{\mu}$ is high, two scenarios

I show that the home government's equilibrium utility can be non-monotonic over various parameters. Especially, the home government's equilibrium utility is single-peaked over the general public's *responsiveness to hostility.* Therefore, if a government can in the long-run cultivate its people's *responsiveness to hostility* through *nation-building* process, it would try to foster *restrained patriotism*—intermediate level of responsiveness to international controversies. The intuition is simple: a government would hope its people to be responsive enough to controversies, so it does not need to exert huge and very costly effort to convince the foreign opponent to concede. However, if the people are already responsive enough so informativeness of protest is not a concern for the foreign government, then a more responsive general public means more crises for the home government, which is also very costly. This prediction is consistent with what we observe in real life: for example, in Chinese textbooks and official media, the government often emphasise the importance of 'loving the country rationally'. This is consistent with the prediction of our model.

Baseline Analysis

In this section, we will look at how the home government's payoff depends on the various parameters of the model. We will focus on two parameters, the general public's *degree of responsiveness*, w_{1G}, and the nationalist's *degree of responsiveness*, w_{1N}. The complete analysis over all the parameters in the model can be found in the Appendix.

Proposition 1.2. *If* $\frac{t_H}{\tau_H} \leqslant \bar{\mu}$, $U_H(\psi^*)$ *is:*

- *increasing in* w_{1N}*;*

- *decreasing in* w_{1G}*;*

If $\frac{t_H}{\tau_H} > \bar{\mu}$, $U_H(\psi^*)$ *is:*

- *single-peaked over some interior level of* w_{1G}*;*
- *Increasing or single-peaked over* w_{1N}*;*

Therefore, when the relative benefit of hostility ($\frac{t_H}{\tau_H}$) is higher than the threshold of doubt ($\bar{\mu}$), the home government's utility is single-peaked over general public's *responsiveness to hostility*, w_{1G}. This result is robust even if we consider more general convex cost functions of propaganda. Essentially, this result comes from the two effects of w_{1G}: first, higher w_{1G} means the public is more likely to go to the street, and this would make it easier to convince the foreign government to concede. Second, higher w_{1G} and a higher probability of general public protesting also means that there will be more costly crises. Thus having a responsive or irritable general public can be a *double-edged sword*: it will bring costs to both the foreign government and home government.

Home Government's equilibrium utility can also be non-monotonic over other parameters. For example, $U_H(\psi^*)$ is non-monotonic over w_{1N}, nationalist's responsiveness to hostility, if $\bar{\psi}$ is larger than $\hat{\psi}_R$ when w_{1N} is small, but smaller than $\hat{\psi}_R$ when w_{1N} is large. This happens, for example, when w_{1G} is small. Having a very active nationalist fraction as noise-makers is not always a good thing; without an also active general public, the home government would find it hard to convince the opponent to concede.

Nation-building and *Restrained Patriotism*

It has been widely known that the coherent nation states that dominate contemporary international relations are very recent phenomena. According to Alesina and Reich (2013), "in 1860 French was still a foreign language to half of all French children". Also, "in 1860 at most 10% of the Italian population spoke what would become the Italian language". In the long run, a state can invest in systematic and gradual education and indoctrination that affects its people's language, beliefs, identity, and preference.

In the long run, a state has some considerable flexibility of choosing how its people will respond to hostility and diplomatic issues. By emphasising the importance of things like national pride and the importance of national interest, the government can affect how the public values the importance of going to the street. If in the long run, some parameters in the model can be changed by the home government, how would it change it?

Ideally, the home government will hope to make $C(\psi) = 0$ for any ψ. However this may not be feasible: there is a limited amount of hostility promotion the government can do, and any amount higher than that may be extremely costly or just impossible. Even with complete deregulation over the nationalist's activities, and putting all the available media resource on covering the issue in a provocative way, essentially the government needs the people to be responsive enough over propaganda and go to the street.

If it is always at least somehow costly to generate ψ, what would be the optimal level of w_{1G}? From the previous analysis, we know that some intermediate range of w_{1G} is optimal. Therefore, a government may hope its people to have 'restrained patriotism'. The government hope its people to care about national interest and go to the street if the event is serious, so there will be protests, and this can be used as hard evidence for the sake of getting concessions. However, the government does not hope its people to care too much over these issues. Otherwise, they are very sensitive and easy to be angry and protesting, which is very costly for the home government. This is consistent with the policies of governments in real life: for example, in Chinese textbooks and official media, there are often discussions about 'loving China rationally'. [25]

Similarly, the government may hope its nationalist to have intermediate responsiveness to international controversies. Systematic education and indoctrination can potentially also affect w_{1N}, although probably in a less degree: people usually self-select into being nationalists. However, other government policies and regulations can still affect w_{1N}. For example, the government can set various regulations over the activities of nationalist groups.

1.5 Which Regimes Benefit Most from Encouraging Protests? Role of Media and Political Fragility

In this section, we will try to answer the following questions: will every type of regime benefit at the same degree from the ability to generate propaganda and protests? If not, which kind of regimes would benefit most from this technology of generating hostility? More specifically, this chapter looks at how media freedom and political fragility affect regimes' ability to benefit from the technology of generating hostility and protests.

More precisely, in this chapter media freedom is treated as the level of media capture by the home government. With less media capture, the foreign government can get better information about protests through the home country's media platforms. This can be rationalized in classical models of media capture, such as Besley and Prat (2006). Political fragility here is modelled as the probability home government can successfully suppress a protest even without receiving a concession.

To explore these questions, I make a few extensions to the baseline model.

First, I consider the possibility that the foreign government can have better information than just observing whether there is a protest. The better the information the foreign government has, the harder it would be for the home government to manipulate protest-

[25] **Wang, Yankun and Ye Su.**"Rational Emotion of Loving China is needed in the path Of Rejuvenation of China (Chinese)", People.cn. http://theory.people.com.cn/n1/2016/0818/c40531-28646272.html

s to generate concessions. Related to the issue of media freedom, a freer media in the home country would make it easier for the foreign government to have better knowledge about a protest. This means stable democracies like the US or UK will be hard to obtain concessions, because the foreign government can easily judge whether a protest is important through the home country's media. However, this does not mean that completely totalitarian countries like North Korea would benefit most from encouraging anti-foreign protests. On the contrary, it may be very hard for totalitarian countries like North Korea to generate concessions.

Second, if the home government can probably suppress a protest even without receiving the concession of a foreign government, the foreign government would have smaller incentive to make concessions. Protests from the general public can be costly for various reasons, and governments usually have the incentive to suppress them or calm them done. Therefore, if the foreign government believes the home government can successfully suppress a protest with high probability, it would have less incentive to make concessions. This would increase the threshold of doubt of the foreign government and make it harder for the foreign government to make concessions. If this suppressing power becomes high enough, the foreign government will never concede even it knows for sure that the general public is protesting. If the probability of successfully suppressing a protest is positively related to political fragility, a high level of political fragility of the home government is needed for the foreign government to be willing to help.

Third, I consider the possibility of home government generating a 'fake' protest if no one is actually protesting. We show this ability to generate 'fake' protests is not always a blessing: if the foreign government always expects to see a protest, it would be very hard to convince her to concede. Therefore, countries like North Korea where generating 'fake' protests is easy, and no free media can be relied on to identify those protests, may have severe problems convincing their opponents to concede.

From those extensions, we can better answer the questions previously asked: neither stable democracies nor completely totalitarian regimes can benefit most from the technology of stirring anti-foreign protests. On the contrary, countries with intermediately level of media freedom and at least some political fragility may benefit most from this mechanism, especially those with strong nationalistic mood and historical grievance.

1.5.1 Additional Information for the Foreign Government

In the canonical model, we assumed a simple signal structure: the foreign government can observe whether there is a protest happening, but not who is protesting. Now we will relax this assumption.

We still assume the foreign government can observe whether there is a protest happening. Moreover, if there is a protest, it can now observe an additional signal, s, distributed in the following way:

We assume

$$f(s|a_G = P, a_N = P) = \begin{cases} \lambda_H \exp(-\lambda_H s), \text{ if} s \geqslant 0 \\ 0, \qquad \text{if} s < 0 \end{cases}$$

$$f(s|a_G = NP, a_N = P) = \begin{cases} \lambda_L \exp(-\lambda_L s), \text{ if} s \geqslant 0 \\ 0, \qquad \text{if} s < 0 \end{cases}$$

We assume $0 < \lambda_H < \lambda_L$, and thus $E(s|a_G = P, a_N = P) > E(s|a_G = NP, a_N = P)$. We can think of s as the number of people protesting in the street.

In expectation, if the general public is protesting, the number of people protesting on the street should be higher than the number of people protesting when there are only nationalists protesting. When no one is protesting, thus surely there are no people in the street. When at least one group is protesting, then the higher the number of people appearing in the street, the more likely the protest comes from the general public.

These assumptions essentially mean: when no one is protesting, the foreign government is perfectly aware. When there is at least one group protesting, the higher is the realisation of s, the surer the government will be that the general public is angry and protesting.

Then the posterior of the foreign government that the general public is protesting, when observing a protest with signal realization s, is:

$$\begin{aligned} Prob(a_N = P|\psi, P, s) =& \frac{H_G(\psi) f(s|a_N = P, a_G = P)}{H_G(\psi) f(s|a_N = P, a_G = P) + [H_N(\psi) - H_G(\psi)] f(s|a_N = P, a_G = NP)} \\ =& \frac{1}{1 + \frac{H_N(\psi) - H_G(\psi)}{H_G(\psi)} \frac{f(s|a_N = P, a_G = NP)}{f(s|a_N = P, a_G = P)}} \end{aligned}$$

Then the foreign government will only concede iff $Prob(a_N = P|\psi, P, s) \geqslant \bar{\mu}$ Therefore, there exists a cutoff level $\hat{s}(\psi)$ such that

$$a_F^*(s) = \begin{cases} C, \text{ if } & s \geqslant \hat{s}(\psi) \\ NC, \text{ if } & s < \hat{s}(\psi) \end{cases} \tag{1.1}$$

It can be shown easily that $\hat{s}(\psi)$ weakly decreases over $\frac{H_G(\psi)}{H_N(\psi)}$. That just means that the surer the foreign government thinks the general public is protesting, the more tolerate it will be over a low level of s.

We can now characterize the optimal solution with noisy signal.

Lemma 1.2. *1) For any ψ, there exist a cut-off level $\bar{s}$, such that F will only concede iff $s \geqslant \bar{s}$.*

$$\bar{s(\psi)} = \begin{cases} \log([(\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})(\frac{H_G(\psi)}{H_N(\psi)-H_G(\psi)})]^{\frac{-1}{\lambda_L - \lambda_H}}), & \textit{if} \quad (\frac{H_N(\psi)-H_G(\psi)}{H_G(\psi)}) \geqslant (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L}) \\ 0, & \textit{otherwise} \end{cases}$$

2) Home Government's utility function:

- *If* $(\frac{H_N(\psi)-H_G(\psi)}{H_G(\psi)}) > (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})$,

  $$U_H(\psi) = H_G(\psi)\{B[\frac{H_G(\psi)}{H_N(\psi)-H_G(\psi)}]^{\frac{\lambda_H}{\lambda_L-\lambda_H}} - A\} + u_H(NP, NC),$$

  in which
  $A \equiv u_H(NP, C) - u_H(NP, NC)$,
  and $B \equiv \{[(\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})]^{\frac{\lambda_H}{\lambda_L-\lambda_H}}[U_H(P,C) - U_H(P,NC)] + [(\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})]^{\frac{\lambda_L}{\lambda_L-\lambda_H}}[U_H(NP,C) - U_H(NP,NC)]\}$,
  $A, B > 0$

- *If* $(\frac{H_N(\psi)-H_G(\psi)}{H_G(\psi)}) \leqslant (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})$,

  $$U_H(\psi) = H_G(\psi)u_H(P,C) + [H_N(\psi) - H_G(\psi)]u_H(NP,C) + [1 - H_N(\psi)]u_H(NP,NC).$$

It can be shown that $U_H(\psi)$ under any ψ such that $(\frac{H_N(\psi)-H_G(\psi)}{H_G(\psi)}) > (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})$ will be strictly dominated by $U_H(\psi)$ under ψ that $(\frac{H_N(\psi)-H_G(\psi)}{H_G(\psi)}) = (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})$, which means it is optimal for the home government to choose a level of ψ such that $(\frac{H_G(\psi)}{H_N(\psi)-H_G(\psi)})$ is so high that the number of people, s, is obsolete as a signal.

Corollary 1.2. *1) The optimal solution to the home government's optimization problem with noisy signal is also the solution to the following optimization problem:*

$$\max_{\psi \in [0, \bar{\psi}]} H_G(\psi)u_H(P,C) + [H_N(\psi) - H_G(\psi)]u_H(NP,C) + [1 - H_N(\psi)]u_H(NP,NC)$$

$$s.t.\ \mu(\psi, P) = \frac{H_G(\psi)}{H_N(\psi)} \geqslant \frac{1}{1 + \frac{\lambda_H}{\lambda_L}\frac{1-\bar{\mu}}{\bar{\mu}}}$$

2) Denote The optimal value to this optimization problem as $u_H(\psi^{**})$. *Then* $u_H(\psi^{**})$ *is non-increasing over* $\frac{\lambda_L}{\lambda_H}$*;*

3) $\lim_{\frac{\lambda_L}{\lambda_H} \to \infty} \psi^{**} = 0$.

So it turns out that at least in the case that the number of people protesting on the strict is exponentially distributed, it is optimal for the home government to choose a level of ψ such that s becomes useless: home government would make the fact there is someone protesting so informative, so for any s, the opponent will always make concession.

Here the effect of a more precise signal is essentially increasing the threshold of doubt of the opponent. More specifically, the higher is $\frac{\lambda_L}{\lambda_H}$, the easier it would be to distinguish two types of protest from each other by looking at the number of people on the street. Therefore, the foreign government would be more demanding and has a higher level of threshold of doubt for it to concede whenever observing a protest.

A prediction from this section would be that free and informative media in the home country would harm the home government's ability to benefit from encouraging protests, because it limits the ability for the home government to affect the inference problem of the opponent. If we think that media freedom affects the quality of the signal, then countries with free media like the UK would be hard to benefit from stirring anti-foreign protests.

Another prediction would be that if the home government is in a country with very free and informative media, the optimal level of hostility it will choose will be very small. This also means the probability of protest and the probability of crisis would also be small. This implies that probabilities of disputes between stable democratic countries would be small and this could be thought as a natural extension to the concept of 'democratic peace': there are not only fewer wars and conflicts, but also fewer disputes and anti-foreign protests between democratic countries. However, the reason for this phenomenon comes from the fact that democratic countries usually have free media, instead of their representative governments.

1.5.2 Ex-post Incentive and Ability to Suppress

We have shown in the last session that free media could hurt the home government's ability to manipulate protests. Does this mean totalitarian regimes like North Korea would be the best country to use this approach?

The answer to this question is probably no. One reason could be North Korea can easily crush every protest even without receiving a concession from the opponent, and it will have every incentive to suppress it. The opponent will not be willing to giving a concession, knowing that North Korea will not be forced to escalate, even if there is a protest in North Korea and no concession is offered to calm protesters down. In this session, we will formalise this intuition.

Many protests, even massive ones, are suppressed by governments. In our environment, one fundamental reason for the opponent to concede is that if the protesters don't see concessions then the home government may be replaced or toppled, which is also bad for the opponent. However, given that protests are threats to the regime's survival, the regime would have every incentive to suppress it if possible.

So assume that if the opponent chooses not to concede, the protest by only nationalist is suppressed with probability 1[26], while the protest attended by the general public is suppressed with probability $q \in [0,1]$. We assume that when the general public's protest is suppressed, it is equivalent to the general public not protesting for both governments.

[26]Any probability between $[0,1]$ will lead to exactly the same result.

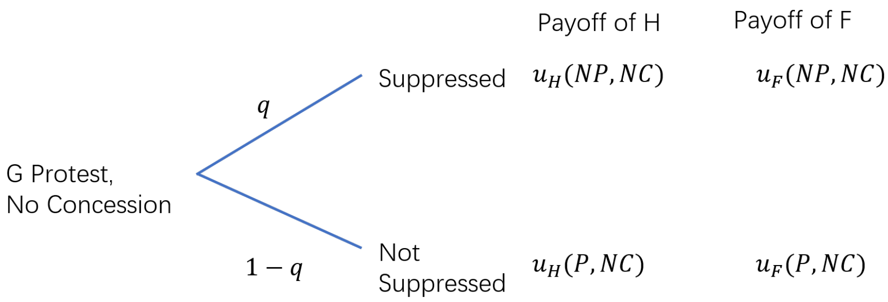


Figure 1.9: General Public Protests but No Concession is Made

We'll look at the threshold of doubt, $\bar{\mu}$, as a function of q, and for notation simplicity we defined $\bar{\mu}$ in the following way: $\bar{\mu}$ is defined as the number such that $\bar{\mu}u_F(P,C) + (1-\bar{\mu})u_F(NP,C) = \bar{\mu}(1-q)u_F(P,NC) + [\bar{\mu}q + (1-\bar{\mu})]\, u_F(NP,NC)$, and now $\bar{\mu}$ can be greater than 1. $\bar{\mu}$ being greater than 1 just means the foreign government will not concede when observing a protest.

So we can show that:

Proposition 1.3. *1)* $\bar{\mu} = \frac{u_F(NP,NC)-u_F(P,NC)}{[u_F(P,C)-(1-q)u_F(P,NC)-qu_F(NP,NC)]+[u_F(NP,NC)-u_F(P,NC)]}$*;*

2) the threshold of doubt $\bar{\mu}$ *is weakly increasing over* q*;*

3) there exist a cut-level $\bar{q}$ *such that* $\bar{\mu} > 1$ *iff* $q > \bar{q}$*.*

The logic is straightforward: if the home government is more likely to suppress the protest anyway even without receiving a concession, then the foreign government's benefit of making concession is smaller. So the foreign government would have to be more certain that the general public is protesting, for her to be willing to concede. In the extreme case that the home government can suppress the protest by himself very easily even without the opponent's concession, the opponent will have no incentive to give concessions, even if he is perfectly sure that the general public is protesting. Another prediction from this result would be that if one government or regime has stabilised its control over the country, the opponent should be more demanding regarding the level of evidence to grant a concession.

So if a regime is more totalitarian and more immune to street protesters, the threshold of doubt for the foreign government would increase. This would reduce the utility of the home government. Moreover, if the threshold of doubt is high enough, the optimal level of hostility it will choose will be minimal.

1.5.3 Ex-post Ability to Bias the Signal

Another reason why North Korea would not be able to make use of this mechanism could be that ex-post, it can always generate some 'fake' protests, and the opponent cannot distinguish between 'fake' protests and real protests. But if North Korea is willing to generate fake protests ex-post, this will harm the informativeness of protest. In the extreme case that it can always successfully generate fake protests and the opponent can never distinguish the difference between the two, then the opponent will not make any concession after observing a protest.

When facing nationalist protests, foreign governments are often suspicious whether the home governments have designed fake protests by themselves just to get more concessions. Ex-post, the home government does have the incentive to generate fake protests if there has been no protest, because protests often bring concessions. For example, the government can send their own people or hire people to go to the street to pretend they are genuinely angry and protesting.

The new timeline is as follows:

Period 1: the home government chooses the level of hostility, ψ; the level of ψ is common knowledge to all the players.

Period 2: the nature chooses the level of θ. θ is private knowledge to the nationalist and the general public. The nationalist and the general public then decide simultaneously whether to protest or not. The actions of the nationalist and the general public are observable to themselves and the home government, but not to the foreign government. [27]

Period 3: a binary signal, s, $s \in \{P, NP\}$ is generated and observed to only players in country 1. s is generated in such a way:

$$s = \begin{cases} P, & \text{if } a_N = P \vee a_G = P \\ NP, & \text{otherwise} \end{cases}$$

Home government can then take a decision $b_H \in \{T, NT\}$, whether to tamper with the signal s.

Period 4: The signal $\tilde{s}$ is generated according to s and b_H in the following way:

$$\tilde{s} = \begin{cases} P, & \text{if} s = P \\ P \quad \text{with prob} \quad p, & \text{if} s = NP \wedge b_H = T \\ NP \quad \text{with prob} \quad 1-p, & \text{if } s = NP \wedge b_H = T \\ NP, & \text{if} s = NP \wedge b_H = NT \end{cases}$$

$\tilde{s}$ is revealed to every player and the foreign government then decides whether to concede or not after observing the realization of signal $\tilde{s}$.

Period 5: the state and the pay-offs of all players are realised and publicly revealed.

Here we consider a case that after the nationalist and the public make the decision of protesting or not, and before the foreign government observes the protest outcome, the

[27] It is not essential to assume that θ is observable to the home government. What is essential here is that the home government can observe *who* is protesting.

government can choose whether to arrange a fake protest. Suppose the foreign government can distinguish between a real protest and a fake protest with probability $1-p$. Then a higher p would reduce the informativeness of protest as a signal.

Proposition 1.4 shows that sometimes, the home government's payoff could be maximised at an intermediate level of p:

Proposition 1.4. *:*

1) If $\frac{t_H}{\tau_H} \geqslant \bar{\mu}$ & $\bar{\psi} \geqslant \frac{\frac{\tilde{c}}{1-\bar{\mu}}-w_0}{w_{1G}}$, $U_H(\psi^*)$ *is increasing over p*

2) If $\frac{t_H}{\tau_H} \geqslant \bar{\mu}$ & $\bar{\psi} < \frac{\frac{\tilde{c}}{1-\bar{\mu}}-w_0}{w_{1G}}$, $U_H(\psi^*)$ *is single-peaked at some interior level of p,* p^*

3) If $\frac{t_H}{\tau_H} < \bar{\mu}$, $U_H(\psi^*)$ *can be either (weakly) increasing over p, or single-peaked at some interior level of p,* p^**.*

A higher p would allow the home government to generate more protests and thus more concessions. However, a lower p would make protest less informative and requires the home government to choose a higher level of hostility to make protests convincing to the opponent. If this is higher than the maximal possible hostility, the home government would be forced to choose a very low level of hostility.

Therefore, if p is very high, i.e., the ability to generate fake protest without being identified is too high, the home government could actually benefit from decreasing p. This happens when $\bar{\psi} < \frac{\frac{\tilde{c}}{1-\bar{\mu}}-w_0}{w_{1G}}$, which is likely to be true if: 1) the threshold of doubt, $\bar{\mu}$, is high; 2) the public's responsiveness to hostility, w_{1G}, is low; 3) People's cost of protesting, c, is high; 4) people's initial weight on the dispute, w_0, is low; the maximal possible hostility, $\bar{\psi}$, is low.

In totalitarian countries like North Korea, as we discussed in the last subsection, $\bar{\mu}$ could be very high due to its ability to suppress protests easily.

Moreover, for those protests to convince the foreign opponent to concede, they have to be *independent* instead of *state-sponsored.* Participating in *state-sponsored* protests usually will not bring any risk to individuals, but only potential benefits. However those orchestrated protests will not put any pressure over the home government, and the home government will not have the incentive to give concessions. *Independent* protests can potentially force the home government to escalate, and thus could prompt the foreign government to concede. However, those *Independent* protests can easily become anti-regime and would bear very high risks for individuals in totalitarian countries. Therefore we would expect people's cost to participate in those independent protests, c, is high, and their initial weight on this issue,w_0 is low. In summary, people's potential low willingness to participate in those independent protests, and the inability to distinguish real versus fake protests make totalitarian countries very hard to convince the opponent to give in and concede.

In later sections, this chapter will further discuss the difference between *independent* and *state-sponsored* protests, especially the different roles they serve for governments.

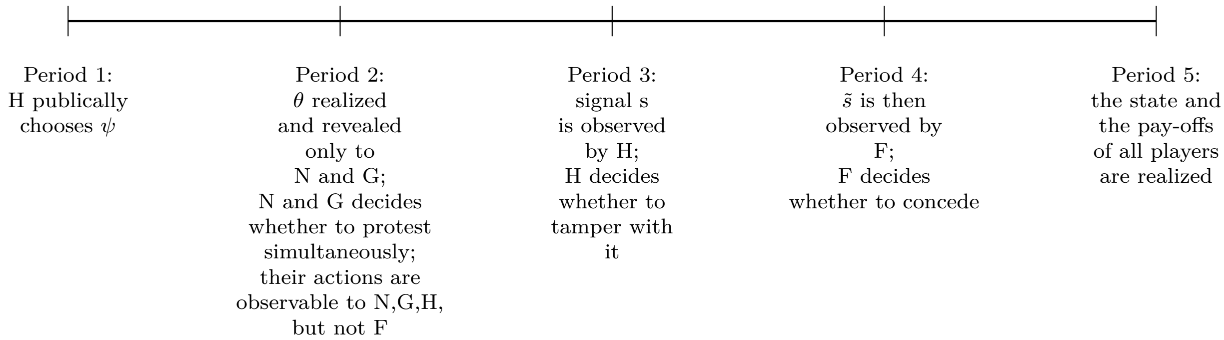


Figure 1.10: Timeline: when Home Government can generate fake protests

High media freedom may reduce p, as real rather than orchestrated protests may get more coverage from independent media. Also, free media may want to reveal the fake protest to get more readership for its services. This can be rationalized again by classical media capture models such as Besley and Prat (2006)

So to make the signal to be informative enough and not completely useless, at least some level of media freedom is needed to retain credibility.

1.5.4 Comparative Politics: Who Would Benefit Most from Encouraging Protests?

After considering the previous extensions, now we can answer the previous question, i.e., which regimes benefit most from the technology of manipulating protests.

Stable democratic countries, such as the UK and the US, have high levels of media freedom and the rule of law and thus may not be able to benefit much from this technology of encouraging protests. Free media will leave not much space for the home government to affect the opponent's inference, and the cost of generating hostility may be too high.

Completely totalitarian regimes like North Korea may also not be able to benefit much from this technology of encouraging protests. State control over the society is too strong in totalitarian countries, so totalitarian regimes are unlikely to be forced into escalation. Also, there is no free media at all, so the government can easily produce fake protests ex-post and the opponent cannot distinguish between fake protests and real protests.

Countries with intermediate levels of media freedom and regimes that are not too strong may be the best candidate to benefit from this technology. They have the necessary levels of media freedom to keep the signal informative enough, and also they are to some extent fragile to angry protesters. Among those countries, those with strong nationalism mood and colonial history may be in even better positions to encourage protests to receive concessions. People will be responsive to the government's promotion of hostility, so it is not too costly to generate protests that are informative enough. Good candidates for this set of countries include China, Turkey and so on. That can possibly explain why the Chinese government can often generate concessions from foreign governments by encouraging anti-foreign protests.

1.6 Further Extensions and Discussions

In this section, we will consider various further extensions to the canonical model. We show that many conclusions of the baseline model are robust to changes to some assumptions of the canonical model.

We also considered whether it is possible to apply the model we have developed in the context of International Disputes to other situations. Our model can easily apply to two scenarios: 1) foreign aid and domestic reform; 2) anti-terrorism aid and anti-terrorism efforts. We explained in the incoming sessions why these two cases may fit well with our model, and we show in both cases, the incentive to help actually generates adverse incentive that makes the socially inefficient states happening more often.

1.6.1 Protest of Nationalists also Costly

In the baseline model, I made the assumption that both the home government and foreign government do not care intrinsically about the protest of nationalists. That assumption can be substantially relaxed. We show in this section that if the nationalist's protest is also costly, it is either the case that the protest of a nationalist is very costly that the results are trivial, or the protest of the nationalist is not so costly, and hence all the previous results remain qualitatively the same.

We first looked at the optimal response of the foreign government. We still assume that $u_F(a_G = P, a_N, \psi, a_F = C) \geqslant u_F(a_G = P, a_N, \psi, a_F = NC)$, so regardless of the action of the nationalist, the foreign government would prefer conceding if the general public is protesting.

We can also safely assume that $u_F(a_G = NP, a_N = NP, \psi, a_F = C) < u_F(a_G = NP, a_N = NP, \psi, a_F = NC)$. This assumption means that if no one is protesting, the foreign government won't concede for sure.

Now consider two cases:

Case1 $u_F(a_G = NP, a_N = P, \psi, a_F = C) \geqslant u_F(a_G = P, a_N = P, \psi, a_F = NC)$

Case2 $u_F(a_G = NP, a_N = P, \psi, a_F = C) < u_F(a_G = P, a_N = P, \psi, a_F = NC)$

In Case 1, the cost of a nationalist protest to the foreign government is also high, hence regardless of who is protesting, the foreign government will always concede when there is a protest. Then informativeness is never a problem, and the home government will always choose its ideal level of propaganda, i.e., $\psi^* = \psi^I$.

In Case 2, the cost of nationalist protest is low, then the foreign government will not concede if she is sure only nationalists are protesting. In this case, when the foreign government observes a protest, it will concede if and only if it thinks the general public participates in this protest with high probability. Otherwise, it will not concede. Therefore, there still exists a threshold of doubt, $\bar{\mu}$, similar to the one in the canonical model.

Assuming Case 2, we can now write down the optimal decision of the home government. We can safely assume that a protest of nationalist is also costly for the home government: for any a_G, a_H, a_F, if $a_N = P$, $u_H(a_G, a_N = P, a_H = \psi, a_F) < u_H(a_G, a_N = NP, a_H = \psi, a_F)$.

Now the home government's optimization problem:

$$\max_{\psi \in [0,\bar{\psi}]} H_G(\psi)u_H(a_G = P, a_N = P, C) + [H_N(\psi) - H_G(\psi)]u_H(a_G = NP, a_N = P, C)$$
$$+ [1 - H_N(\psi)]u_H(a_G = NP, a_N = NP, NC)$$

$$s.t.\ \mu(\psi, P) = \frac{H_G(\psi)}{H_N(\psi)} \geqslant \bar{\mu}$$

The objective function is equivalent to:

$$\max_{\psi \in [0,\bar{\psi}]} H_G(\psi)[u_H(a_G = P, a_N = P, C) - u_H(a_G = NP, a_N = P, C)]$$
$$+ H_N(\psi)[u_H(a_G = NP, a_N = P, C) - u_H(a_G = NP, a_N = NP, NC)]$$
$$+ u_H(a_G = NP, a_N = NP, NC)$$

Define $t_H \equiv u_H(a_G = NP, a_N = P, C) - u_H(a_G = NP, a_N = NP, NC)$ and $\tau_H \equiv u_H(a_G = NP, a_N = P, C) - u_H(a_G = P, a_N = P, C)$. Again, we can safely assume τ_H is positive. t_H can be either positive or negative. The difference, $u_H(a_G = NP, a_N = P, C) - u_H(a_G = NP, a_N = NP, NC)$, now depends on two factors: 1) the benefit of concession 2) the cost of having nationalists protesting. As long as the cost of nationalist protesting is smaller compared to the benefit of getting a concession, then t_H is also positive. Then all the previous results will carry through qualitatively.

If t_H is negative, then the result is trivial. There is no benefit but only cost of increasing hostility, and thus the optimal level of ψ would be zero. $\psi^* = \psi^I = 0$.

So in summary, as long as the protest of nationalists is not so costly, our main results carry through, even when a protest of nationalists does affect the payoff functions of home government and foreign government intrinsically.

1.6.2 Alternative Distributions of Crisis

In the canonical model, I assumed that the seriousness of the event, θ, is uniformly distributed. One possible concern could be that some important conclusions of the model are specific to the assumption of uniform distribution.

Here we show that two important patterns that I showed before are to some extent robust to the distribution of θ: first, I show here that the fact the informativeness of a protest, $\frac{H_G(\psi)}{H_N(\psi)}$, is first decreasing and then increasing is not specific to the assumption of uniform distribution. Second, I also show that the objective function has a single optimal solution under general assumptions.

Assume θ is distributed with a distribution, $F(\theta)$, with the following properties:

- $F(\theta)$ is independent of ψ
- $F(\theta)$ is non-degenerate continuous distribution. Denote $f(\theta)$ as the pdf of the distribution
- $F(\theta)$ is second-order continuously differentiable

Therefore, we have

$$\frac{H_G(\psi)}{H_N(\psi)} = \frac{Prob(\theta \geqslant \hat{\theta}_G)}{Prob(\theta \geqslant \hat{\theta}_N)} = \frac{Prob(\theta \geqslant \frac{c}{w_0+w_{1G}\psi})}{Prob(\theta \geqslant \frac{c}{w_0+w_{1N}\psi})} = \frac{1-F(\frac{c}{w_0+w_{1G}\psi})}{1-F(\frac{c}{w_0+w_{1N}\psi})}$$

U-shaped Informativeness Curve

$$\begin{aligned}\frac{\partial \frac{H_G(\psi)}{H_N(\psi)}}{\partial \psi} \geqslant 0 \quad & \textit{iff} \quad \frac{f(\frac{c}{w_0+w_{1G}\psi})\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}}{1-F(\frac{c}{w_0+w_{1G}\psi})} \Big/ \frac{f(\frac{c}{w_0+w_{1N}\psi})\frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}}{1-F(\frac{c}{w_0+w_{1N}\psi})} \geqslant 0 \\ & \textit{iff} \quad \left[h(\frac{c}{w_0+w_{1G}\psi})\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}\right] \Big/ \left[h(\frac{c}{w_0+w_{1N}\psi})\frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}\right] \geqslant 0\end{aligned}$$

, in which $h(\theta) \equiv \frac{f(\theta)}{1-F(\theta)}$ is the hazard rate function.

$\frac{H_G(\psi)}{H_N(\psi)}$ to be first decreasing and then increasing if and only if:

(Quasi-convexity condition) there exist a $\hat{\psi}$ such that:

$$sgn(\frac{\partial \frac{H_G(\psi)}{H_N(\psi)}}{\partial \psi}) = \begin{cases} 1, & \text{if } \psi > \hat{\psi} \\ 0, & \text{if } \psi = \hat{\psi} \\ -1, & \text{if } \psi < \hat{\psi} \end{cases}$$

Two examples that satisfy this condition are Exponential Distribution and Weibull Distribution (with some constraints on parameters).

More generally, here are some sufficient conditions for quasi-convexity condition to be true:

- $\frac{f(\frac{c}{w_0+w_{1G}\psi})\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}}{1-F(\frac{c}{w_0+w_{1G}\psi})}/\frac{f(\frac{c}{w_0+w_{1N}\psi})\frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}}{1-F(\frac{c}{w_0+w_{1N}\psi})}$ is increasing over ψ
- $\frac{f(\frac{c}{w_0+w_{1G}\psi})\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}}{1-F(\frac{c}{w_0+w_{1G}\psi})}/\frac{f(\frac{c}{w_0+w_{1N}\psi})\frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}}{1-F(\frac{c}{w_0+w_{1N}\psi})}|_{\psi=0} < 1$
- $\frac{f(\frac{c}{w_0+w_{1G}\psi})\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}}{1-F(\frac{c}{w_0+w_{1G}\psi})}/\frac{f(\frac{c}{w_0+w_{1N}\psi})\frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}}{1-F(\frac{c}{w_0+w_{1N}\psi})}|_{\psi=\bar{\psi}} > 1$

The conditions above essentially require the hazard rate to be 'flat' enough and bounded.

Single-peaked Objective Function

For the objective function to be single-peaked, we need the propensity density function to be flat enough and bounded. Intuitively, this requires the distribution to be not too volatile, so the $\frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}/\frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}$ is dominating in determining $\frac{H_G(\psi)}{H_N(\psi)}$. Again, exponential distribution can work as well, although it requires the parameter λ to be not too small. With some constraints on parameters, Weibull distribution also satisfies this condition.

1.6.3 Other Roles of Anti-Foreign Protests

We have shown that in countries where regimes' control over information and society are too strong, it would be hard for anti-foreign protests to generate concessions. However, we still often observe those protests in autocracies, for example, anti-US demonstrations in North Korea and Syria. Moreover, even in countries where anti-foreign or patriotic protests can potentially generate concessions from foreign governments, those protests can serve more roles than just bargaining chips for international disputes. In this subsection, we will look at the other roles of anti-foreign protests.

One potential role of anti-foreign protests would be providing domestic legitimacy and shifting domestic political focus. Many regimes' political legitimacies are based on acting as the defender of national interests. Therefore they may have the incentive to use those protests to emphasise that they care about national interests. Also, those protests can shift people's focus from some domestic problems. Domestic legitimacy and shifting people's focus can be very important reasons for the home government to encourage protests, but as long as those protests will generate pressure for the home government to escalate in international disputes, most predictions of our model will still carry through.

Another role of anti-foreign protests would be for domestic mobilisation. Those foreign rivals provide useful 'straw man' that allow regimes to mobilise its domestic audience for various reasons. Again, our model points out the international concerns a regime needs to take into account when deciding whether to mobilise protests.

Moreover, as discussed before, anti-foreign protests can be *independent*, *state-organized*, or a mixture of two with various weights. *State-organized* protests are usually pro-government and under government controls, thus less threatening to the government. They could potentially boost government legitimacy or other domestic agendas but are unlikely to convince foreign opponents to concede. On the contrary, *independent* protests can be dangerous to regimes but are useful signals that could convince foreign governments the necessity to concede to avoid costly escalation. Our model mainly applies to cases where protests are *independent*. In cases where protests are mixtures of both, our model would predict that the higher the level of regime participation, the harder it would be for those protests to generate concessions from foreign governments.

1.6.4 Other Applications

Although this chapter considers a model of how the sender tries to affect the receiver's inference problem of the receiver under the setting of crisis diplomacy, some conclusion of the model can be extended to other settings.

One possible environment this model may apply to is foreign aid and institutional reform. There is vast literature on the effects of foreign aids on the receiving countries. Especially, various studies have discussed how foreign aid can either promote or hamper necessary domestic reforms.

This chapter can easily fit into this discussion: suppose in some underdeveloped country, some serious disasters may have significant impacts over itself and other countries, so some developed country may have the incentive to provide foreign aid to help relieve the disaster. However, providing foreign aid is costly. There may be some other disasters that are also severe but not destabilising enough, so the donor would not want to provide foreign aid for that issue. The problem is that the donor may find it hard to distinguish different kinds of disasters perfectly. If the receiving country takes necessary institutional reforms that reduce the possibilities of real disasters, then it will be harder for it to credibly convince the donor to provide foreign aids. Then one implication from the previous model would be that the foreign aid would actually reduce the incentive to implement crucial institutional reforms and may actually make the humanitarian crisis perpetual.

Another environment this model applies to would be anti-terrorism. Some country fighting remote terrorist groups may have the incentive to support its local partner (can be country or local groups) with aids, especially if the terrorism level has been very high and destabilising the local community.

By providing aids, that country may stop the area from being further radicalized. However, providing aid is very costly, and the country would not provide aid to some non-destabilizing events. Again, the donor country may not perfectly observe the type of terrorism the local group is facing. If the local group takes anti-terrorism efforts that reduce the possibilities of destabilizing events, then it will be harder for it to credibly convince the donor to provide anti-terrorism aids.Then one implication from the previous model would be that the ex-post socially beneficial action of providing aid in the local area may reduce the local group's incentive to fight terrorism and thus actually destabilising the area.[28]

More generally, this chapter can be applied to a sender-receiver environment, where the state can be endogenously affected by the sender. The receiver may want to take some action that is preferred by the sender if the receiver is sure enough about some states happening. However, the receiver cannot perfectly observe the states of the world, and this gives the sender an incentive to change the state to affect the receiver's inference problem. Sometimes this could create an incentive for the sender to spend effort on increasing the frequency of socially inefficient states.

1.7 Conclusion

We have considered a model exploring how a government can manipulate anti-foreign protests of its people to get more concessions from foreign countries. By promoting a high level of hostility through propaganda, the home government generates more protests and thus more concessions. However, this comes with more crises, which are costly for the home government. Moreover, the government has to choose a level of hostility that keeps a protest informative enough about a real crisis. These factors jointly determine the optimal and feasible level of hostility the home government would choose and also the equilibrium probabilities of protests and crises.

I characterise the comparative static analysis and find some interesting patterns that are consistent with real-life examples. I also show that due to the cost of propaganda, governments may hope their people to be responsive but not too responsive over international disputes. Moreover, government's *ex-post* ability to tamper with protest can sometimes harm its *ex-ante* welfare. Therefore, regimes that are in some sense 'weak' can be those that can benefit more from encouraging anti-foreign protests: in our context, 'weak' regimes means regimes with intermediate levels of media freedom and a high level of fragility to political protests.

This chapter suggests some possible directions for future research: first, it would be interesting to see what will happen in a repeated environment where reputations and dynamic interactions kick in. Second, combining this chapter with models of political competitions in a democratic environment may generate additional insights over how protests will have effects over democratic countries' domestic politics and international affairs. Third, a crisis-bargaining model with domestic politics and protests on both sides

[28] One newspaper report described Pakistan's incentive to keep terrorists active: "Over the years successive governments - both military and civilian - have developed the fine art of using the begging bowl as an extortion racket. Zia played the Soviets off against the US after the invasion of Afghanistan. The US wrote the bigger cheque. Musharraf adroitly squeezed money out of Bush for his war on terror, capturing a few al Qaeda operatives, whilst at the same time letting the Taliban off the hook (proving that the Pakistani Army is the largest mercenary force in the world). Nor have civilian administrations been any better. We have become particularly good of late of warning the international community of dire consequences of militants getting hold of our nukes if they don,t pay up. Kerry-Lugar Zindabad!". **Fulton, George.** "Cry Wolf". The Express Tribune, August 18 2010. https://tribune.com.pk/story/40220/cry-wolf/

may generate additional insights over whether promoting hostility can act as ’defence’ over the opponent’s investment in hostility. That model may also help us understand better why inefficient deadlocks like World War I may occur in equilibrium. Last, this chapter focuses on protest as ’street diplomacy’ instead of formal diplomacy. A richer model may be needed to explore the interaction between ‘street diplomacy’ and ‘formal diplomacy’.

Chapter 2

Strategic Disclosure, Primary Market Uncertainty, and Informed Trading

"In today's aggressive marketplace, listed companies can no longer rely on their numbers to do the talking. If companies can't communicate their achievements and strategy, mounting research evidence suggests, they will be overlooked, their cost of capital will increase and stock price will suffer."

–Westbrook (2014)

2.1 Introduction

The design and transmission of information play a vital role in security offering in that it shapes issuers', intermediaries' and investors' expectations of the future, and thus profoundly influences the resulting supply-demand equilibrium. One overarching friction which plagues the well-being of the market participants is information asymmetry: usually, one party holds a payoff-relevant informational advantage over another. Issuers have considerable discretion in the disclosure of information to advance their own interests. Intermediaries, by underwriting and investing in the deals, acquire proprietary information which helps them predict future performance but cannot be credibly communicated to other investors. Moreover, they may gain from trading on their private information. Accordingly, understanding such friction and evaluating feasible options for alleviating it is of great importance.

The goal of this chapter is to provide a comprehensive theoretical framework to address the following questions. First, how does information disclosure by the issuer potentially affect a financial intermediary's decision to retain and trade the issued securities? Second, can strategic information disclosure help the issuer maximize proceeds from security offering, mitigate adverse selection, and induce the investment bank to underwrite even if some unfavourable market friction (e.g. weak demand) may initially deter the bank from doing so? Third, what are the effects of the issuer's fundamentals, the underwriter's cost of capital, the primary market condition, and the secondary market liquidity on the informativeness of the optimal disclosure policy?

In this chapter, we develop a tractable yet comprehensive model that links the issuer's information disclosure in the capital raising process to various primary and secondary market activities by the underwriter and other investors. We model the optimal design of disclosure policy by the issuer as a Bayesian persuasion game *à la* Kamenica and Gentzkow (2011). In their paper, Kamenica and Gentzkow (2011) present a model where a sender chooses a signal to reveal to a receiver, who then takes an action that affects the welfare of both players. They solve for the sender-optimal signal by reframing the problem as maximizing the sender's payoff over distributions of posterior beliefs subject to the Bayesian plausibility condition that the average posteriors should be consistent with the prior. The effectiveness of Bayesian persuasion is that it improves the sender's expected payoff by inducing the receiver to choose a better action. The maximal value is obtained by finding the concave closure of the sender's payoff function for any posterior held by the receiver.

In general, the Bayesian persuasion approach fits the process of security issuance very well. The issuing party (sender) has to first draft a proposal which will be sent to a potential underwriting bank (receiver). Routinely, the issuer possesses marked flexibility in selecting what to disclose and how precise the disclosure is. In effect, issuers usually exercise discretion in reporting forward-looking information which contributes to the valuation of the proposed security. Such information includes but is not limited to forecasts of future sales, earnings, and growth opportunities, which can be either purely qualitative, or quantitative with varying precision – a range or a point estimate. Moreover, issuers often choose to release unique marketing information about business models, corporate strategy, and prospects of the industry to attract potential investors. In sum, the proposal-drafting stage resembles the sender's communication about the optimally designed signal system to the receiver. After seeing the proposal, the investment bank further investigates the realization of the signal through due diligence if it still cannot decide whether it should underwrite. If the bank agrees to underwrite, it engages in information production with the issuer to prepare the information memorandum (for debt) or prospectus (for equity), which is then circulated to potential investors (other receivers). In this sense, the information memorandum or prospectus reflects the informativeness of the issuer's disclosure. The underwriter then prices the security based on the collected information. This stage corresponds to the mapping from the signal realization to the pricing of the security.

Specifically, we consider an issuer who designs an information disclosure system and reveals it to an investment bank to invite it to underwrite the deal. The issuer may represent a borrower in a debt issue, an originator in securitization, or an entrepreneur in an equity issue. The investment bank may serve as a lead bank in loan syndication, an arranger in the sale of asset-backed security (ABS), or an underwriter in equity and bond offering.[1] If the investment bank decides to underwrite, it further helps communicate the signal to potential investors, chooses its stake, and allocates the remaining securities to the participant investors. We assume that the underwriter obtains proprietary information from its underwriting activity and retention. Similar assumptions regarding the generation of private information are commonly used in the literature on banking and blockholders (e.g. Parlour and Plantin, 2008; Edmans and Manso, 2011), and well documented empirically (e.g. Lummer and McConnell, 1989; Edmans, Fang and Zur, 2013). Nevertheless, the acquisition of material information in our model is an inevitable but adverse consequence of the underwriter's involvement in the issue. As a result, the underwriting bank can profit from insider trading when the secondary market opens. Following Maug (1998), Hennessy and Zechner (2011), and Chemla and Hennessy (2014), who model the secondary markets of equity, bond, and ABS respectively, the market structure is in the spirit of Kyle (1985) where investors submit their market orders to a continuum of deep-pocketed risk-neutral market makers who price the security competitively after observing aggregate demand. If the participant investors anticipate that there is an adverse selection in the secondary market, they will demand a discount in the issue price to offset their future losses, a fact widely used in the literature (e.g. Holmström and Tirole, 1993; Maug, 1998; Edmans and Manso, 2011).

In our baseline model, we consider a secondary market where the underwriter is banned from selling the security short (alternatively, short sale is prohibitively costly for him). In reality, it is almost impossible to sell certain assets such as loans short. Furthermore, short sale of securities by underwriters has long been contended as highly controversial and is viewed unfavourably by regulators as well as market participants. Moreover, the SEC has made an effort to restrict the short sale of the ABS by securitization participants. For instance, in a proposed rule of "Prohibition against Conflicts of Interests in Certain Securitizations" in September 2011, they prohibit a large group of interested parties including underwriters from engaging in certain transactions, among which a particular one is short sale. Moreover, investors are fiercely opposed to short-selling securities by underwriters, and petitions from institutional investors to urge constraint on short sale in the City of London in recent years are common occurrences. In other financial markets such as the ones in China, short sale of any securities is strictly forbidden. This is why we primarily focus on the case in which there is a short sale constraint for the underwriters.

Like in Aghion, Bolton and Tirole (2004), we assume that the underwriter's capital is scarce and he incurs an opportunity cost (i.e., cost of capital) proportional to his investment in the security. Consequently, even though the underwriter can free ride on the adverse selection discount, in equilibrium the additional cost due to the retention depress-

[1] The investment bank can also be viewed as an extant blockholder in the firm who makes the decision on whether to support and participate in a seasoned security offering. See more discussion on the corporate governance implication of the blockholder on Page 55.

es his stake to the level that is just enough for him to camouflage as liquidity traders and gain from informed trading. Interestingly, a unique equilibrium of informed-sales arises naturally in which the underwriter liquidates his holdings if his private information indicates that the security will subsequently underperform, and he refrains from trading otherwise. Our results speak to the issues associated with the rise of the originate-to-distribute (OTD) lending model in debt markets (Bord and Santos, 2012). Because of the development of active secondary markets, banks' incentives to screen and monitor loans have diminished (Keys, Mukherjee, Seru and Vig, 2010). Moreover, they tend to sell loans that are of excessively poor quality (Purnanandam, 2010), and underperform their peers by about 9% per year subsequent to the initial sales (Berndt and Gupta, 2009). To this end, our model fully captures the resultant adverse selection problem from OTD.

Working backwards, we consider the optimal design of disclosure by the issuer. If she does not disclose additional information, the underwriter will choose to retain a stake only when the *ex ante* uncertainty about the security's payoff is relatively high. Because otherwise his private information has low value and his trading profits are not enough to compensate for his opportunity cost of investment. As a result, underpricing occurs only if the security is riskier. This is consistent with the evidence in Cai, Helwege and Warga (2007) that find significant underpricing on speculative-grade debt offerings but no significant underpricing on investment-grade bond IPOs. Since the underpricing undermines the issuer's proceeds, she can do better by inducing posteriors beliefs which reduce the uncertainty to the degree that the investment bank is just indifferent between no retention and a positive stake. In this case, the optimal disclosure is partially informative. A sender-preferred equilibrium prescribes that the underwriter should not retain any share, thus no discount will occur in equilibrium.

Next, we extend our model by introducing *demand uncertainty* (i.e., that demand may fall short of supply) in the primary market. With a positive probability, the shares net of the underwriter's planned retention cannot be fully subscribed by the participant investors. In order to complete the deal, the underwriter has to acquire all the remaining shares. Unlike before where the investment bank's decision to underwrite is trivial, the bank will shy away from the deal if his expected payoff is negative. This creates a hold-up problem arising from the possibility of a demand shock. Intuitively, the bank will choose to underwrite and hold a stake only if uncertainty about the cash flows from the security is sufficiently high. Then, the underwriter is able to exploit his private information, and his expected trading gain is enough to offset his expected loss from excessive retention. Therefore, the issuer's optimal information design will be as follows. If the *ex ante* uncertainty about the security is so high that the investment bank is always willing to underwrite, the issuer will design a signal system inducing posteriors beliefs which reduce the uncertainty to the level that makes the investment bank just indifferent between whether or not to underwrite. This, in turn, reduces adverse selection and increases the issuer's expected revenue. However, if the *ex ante* uncertainty about the security is relatively low, the investment bank will not underwrite unless the signal changes his prior. The issuer's overriding interest in this scenario is to be able to sell the security and maximize her expected payoff with strategic disclosure. Thanks to the Bayesian plausibility constraint which requires that the average posteriors to be equal to the prior, Bayesian persuasion by the issuer can induce the investment bank to underwrite with positive probability and balances this with a worse belief that leaves the bank's underwriting decision unchanged, which improves the issuer's expected payoff. The optimal disclosure is such that on the one hand, it may induce the worst belief which leads to the investment bank's withdrawal from underwriting, but on the other hand, it may generate a signal that makes the investment bank just willing to underwrite at the relevant beliefs. At the latter belief, the security's uncertainty is in fact increased, and the underwriter's private information thus becomes sufficiently valuable again, although on average the disclosure system still reduces the uncertainty relative to that at the prior belief. Our model features an interesting mechanism where increased payoff uncertainty can mitigate the hold-up problem brought about by demand uncertainty. We contribute to the literature by demonstrating a possible way to avoid security issuance failure due to weak demand, and by offering alternative insight into the "pipeline risk" in Bruche, Malherbe and Meisenzahl (2018), where they document the successful issuance of leveraged syndicated loans along with the costly excessive retention by the underwriting banks. We argue that it may stem from the fact that the banks are successfully persuaded by the borrowers albeit the presence of high demand uncertainty.

Our model yields novel empirical predictions that relate the informativeness of the optimal disclosure to various aspects of the primary and secondary markets. We show that the effects are not simply monotonic and depend on the *ex ante* uncertainty of the security's payoff. Specifically, when the *ex ante* payoff uncertainty is relatively high, both better growth option of the firm/borrower and more secondary market liquidity lead to more transparent disclosure. Conversely, greater issue size, larger cost of underwriting bank's capital, a higher probability of demand shock, and weaker demand are associated with less informative disclosure. Better growth option and more liquidity allow the underwriter to enjoy more profits by trading on his private information. Hence the optimal system only needs to induce less uncertainty at posteriors that make the underwriter just break-even. In contrast, larger issue size and cost of underwriting bank's capital make it more costly for the underwriter to hold a stake in order to gain from informed trading. Thus more uncertainty should be introduced to make the underwriter's private information more valuable. Likewise, a higher probability of demand shock and weaker demand make it more costly for the bank to underwrite, thus the optimal system should induce beliefs with higher uncertainty so that his stake carries more trading value in the secondary market. Our result is similar to the model of Pagano and Volpin (2012) which shows that coarse information enhances primary market liquidity at the cost of reducing secondary market liquidity. In contrast, the motivation for the revelation of coarse information in our model is to solve the hold-up problem and promote an active primary market with the underwriter's participation. Moreover, the issuer cannot control over the realizations of the signal, thus the coarse information does not come with certainty.

The results for the security with *ex ante* relatively low payoff uncertainty in the presence of demand uncertainty is just the opposite: better growth option and more liquidity dampen the informativeness of the disclosure, while greater issue size, larger cost of capital, a higher probability of demand shock, and weaker improve the informativeness. Especially noteworthy is that in the latter cases although the overall uncertainty is reduced by the optimal disclosure, to attract the bank to underwrite, the inherent uncertainty at the posterior beliefs that make the bank just indifferent actually becomes larger than that at the prior. The uncertainty at these posteriors should vary according to the intuition discussed in the previous paragraph. But the informativeness hinges on how *dispersed* the distribution of the posteriors is.

Finally, we extend our model by relaxing the assumption on the short sale constraint in the secondary market. Without short-sale constraint, it is optimal for the underwriter not to acquire any security in the primary market, but to exploit his private information by selling the asset short in the secondary market. If there is no demand uncertainty, only a fully informative disclosure can deter the underwriter from engaging in informed trading. Nevertheless, when demand is uncertain, all of the results on optimal disclosure we have obtained with short-selling constraint extends to the case without it. Compared with the case where short sale is prohibited, the issuer only needs less transparent disclosure to persuade the investment bank to underwrite when the uncertainty about the security's payoff is relatively low. But she has to design more transparent disclosure to alleviate adverse selection when the payoff uncertainty is relatively high.

This chapter is related to several strands of the literature. First, our work contributes to the theoretical literature that attempts to address the question of how the rapidly evolving debt markets can go awry (e.g. Chemla and Hennessy, 2014; Pagano and Volpin, 2012; Parlour and Plantin, 2008). We model the adverse selection problem in the OTD lending model, and show that strategic disclosure not only benefits the issuer but also reduces this informational friction.

Second, our theoretical framework enriches the large literature on blockholders' governance by exit (e.g. Aghion, Bolton and Tirole, 2004; Edmans and Manso, 2011; Faure-Grimaud and Gromb, 2004). Importantly, the applicability of our model naturally goes beyond debt markets and extends to equity markets if we view the underwriter as an extant blockholder in a firm. Under this interpretation, we model the blockholder's decision to support and participate in a security offering (e.g. seasoned equity offering). As long as he participates, the blockholder has an informational advantage over other dispersed investors from holding and learning. As we have explained, he can exert governance by exiting to push the firm to *ex ante* disclose more transparent information when the payoff uncertainty of the security is relatively high.

Third, this chapter adds to a growing body of literature on information design theory (e.g. Kamenica and Gentzkow, 2011; Alonso and Câmara, 2016; Bergemann and Morris, 2018; Rayo and Segal, 2010) as well as its application in corporate finance (e.g. Azarmsa, 2017; Azarmsa and Cong, 2018; Boleslavsky, Carlin and Cotton, 2017; Goldstein and Leitner, 2018; Huang, 2016; Szydlowski, 2016). We extend the basic Bayesian persuasion framework by including a second receiver (the participant investors) who indirectly affects the welfare of both the sender and the first receiver.

Fourth, our theoretical analysis offers new insight into the empirical literature on the effect of disclosure on liquidity (e.g. Balakrishnan, Billings, Kelly and Ljungqvist, 2014). In contrast with the extant literature, we focus on how firms will design their disclosure in security issuance when faced with varying market liquidity. Our model provides a rationale for whether a liquid secondary market contributes to a better information environment of the issuing firm. To our best knowledge, we are the first to consider the security issuer's optimal design of information disclosure in the presence of both the financing and the trading frictions. We thus call for empirical investigations of the relationship between the informativeness of disclosure (through the lens of the information memoranda and the prospectuses) and the subsequent market activities as predicted in our model.

This chapter is organized as follows. Section 2 introduces the setup of the model. Section 3 solves for the secondary market trading equilibrium and the primary market issue price given an active secondary market. Section 4 presents the core results of the model with a secondary market that has short sale constraint. The equilibrium disclosure policies are analyzed both with and without demand uncertainty in the primary market. Section 5 changes the secondary market structure by removing the short sale ban and solve for the optimal disclosure policies. Section 6 conducts welfare analysis for the investment bank and the issuer under different primary market conditions and secondary market structures. Section 7 concludes. All proofs not in the main body of the chapter are deferred to the Appendix.

2.2 The Model

The model has four dates and no discounting. There are three types of players: an issuer, an investment bank, and a group of investors, all of whom are risk-neutral.

2.2.1 The Issuer

The issuer (also called "she" or "firm") wants to sell claims to cash flows from a productive asset. Examples of such claims include bonds, (syndicated/securitized) loans, or equity stocks. For brevity, we shall simply call them securities. We normalize the number of securities to be issued to 1. The state ω is binary: it can be Good (G) or Bad (B) with prior probability distribution $\mathbb{P}[\omega = G] = \mu_0$ and $\mathbb{P}[\omega = B] = 1 - \mu_0$ respectively. Cash flows $\tilde{v}$ from state B and state G are $V_H \equiv V_L + \Delta V$ and V_L respectively.

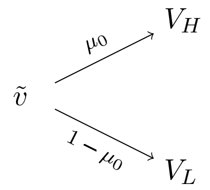


Figure 2.1: Cash Flows Distribution under the Prior

The issuer designs an experiment which we refer to as a disclosure system π with binary signal $s \in \{h, \ell\}$. The signal realization follows the conditional distribution: $\pi_G \equiv \mathbb{P}[s = h|\omega = G] \geqslant \pi_B \equiv \mathbb{P}[s = h|\omega = B]$, which also represents the precision of the system. Figure 2.2 illustrates how the disclosure system maps each state to a signal. Using Bayes' Rule, the posteriors μ_s upon observing $s \in \{h, \ell\}$ are

$$\mu_h \equiv \mathbb{P}[\omega = G|s = h] = \frac{\pi_G \mu_0}{\pi_G \mu_0 + \pi_B(1-\mu_0)},$$

$$\mu_\ell \equiv \mathbb{P}[\omega = G|s = \ell] = \frac{(1-\pi_G)\mu_0}{(1-\pi_G)\mu_0 + (1-\pi_B)(1-\mu_0)}.$$

Moreover, Bayesian updating requires that the average posterior is consistent with the prior, which gives the Bayesian plausibility condition:

$$\mathbb{P}[s = h] \cdot \mu_h + \mathbb{P}[s = \ell] \cdot \mu_\ell = \mu_0.$$

Therefore, the information design problem for the issuer is equivalent to choosing a pair of posteriors $\{\mu_h, \mu_\ell\}$ whose distribution must satisfy the above constraint.

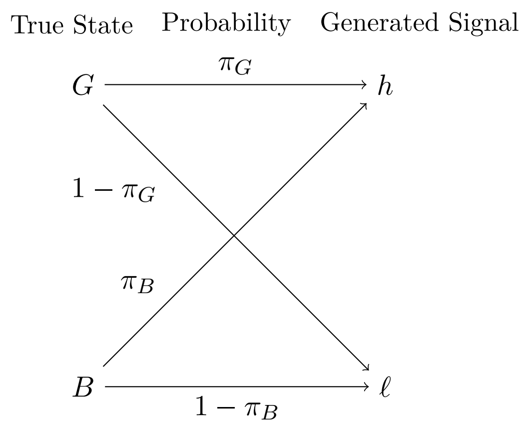


Figure 2.2: The disclosure system π

One important assumption in the Bayesian Persuasion framework *à la* Kamenica and Gentzkow (2011) is the *commitment assumption.* Commitment assumption means that whatever the outcome of the signal system, the sender has to truthfully report the outcome without the ability to tamper with or hide the outcome. This is a reasonable modelling assumption in our paper, as when the investment bank underwrites, due diligence requires independent third parties such as auditing firms to assess and scrutinize the firm's financial situation and other details. The firm owner/issuer will not be able to change the result of the report. But by providing more or less details, he can affect the precision of signal.

Another potential concern of using Bayesian Persuasion framework here is that the issuer may have private information about the state before even designing the experiment. Then the choice of experiment itself may already reveal the state of the world. It is possible that the issuer could have some private information about the state. However, it is safe to argue that the issuer still faces substantial uncertainty about the state. For example, a new firm that plans to go through Initial Public Offering may face substantial uncertainty about its future growth prospectives and profitability. Even as a firm owner, who is an insider of the firm, he may still have very limited information about the firm's future prospectives. A interesting future research direction would be considering a more realistic model where the issuer has some imperfect private information about the state ω before designing the signal. [2]

2.2.2 Informativeness of the Disclosure System

Following Gentzkow and Kamenica (2014), we use the entropy measure to gauge the uncertainty associated with a given belief. In our binary-state economy, if the belief that the state is G conditional on observing s is μ_s, its entropy is $H(\mu_s) = -\mu_s \ln \mu_s - (1 - \mu_s) \ln (1 - \mu_s)$. Hence the belief achieves the highest uncertainty when $\mu_s = 1/2$, and the closer it is to the endpoints of its support (i.e., 0 or 1), the less uncertain the belief is. Moreover, the informativeness of a disclosure system π is measured as the reduction in entropy $L(\pi) = H(\mu_r) - \mathbb{E}_{\langle \pi | \mu_r \rangle}[H(\mu_s)]$, where μ_r is a fixed reference belief independent of the system π, and the subscript $\langle \pi | \mu_r \rangle$ indicates that the expectation is taken under the distribution of posteriors (i.e., the probabilities of $s = h$ and $s = \ell$) given the reference prior μ_r.[3]

The fact that the above $L(\pi)$ function is convex in μ_s implies a simpler yet more intuitive interpretation of the informativeness: the more dispersed the distribution of posteriors, the more informative the disclosure system. Formally, consider two systems π and π' with possible signal realizations $\{h, \ell\}$ and $\{h', \ell'\}$, and induced posteriors $\{\mu_h, \mu_\ell\}$ and $\{\mu_{h'}, \mu_{\ell'}\}$. Suppose that

$$0 \leqslant \mu_\ell \leqslant \mu_{\ell'} \leqslant \mu_{h'} \leqslant \mu_h \leqslant 1$$

with either the second or fourth inequality (or both) holding strictly, then we claim that system π is more informative than system π' in the spirit of Blackwell (1951). Furthermore, from the Bayesian updating formulas of the two posteriors, both a higher π_G and a lower

[2]For example, Alonso and Câmara (2018) study the question that when a sender can benefit from becoming privately informed before choosing an experiment.

[3]The introduction of this reference belief μ_r ensures that the disclosure informativeness does not vary with the prior μ_0.

π_B imply a more informative signal system. It is because such changes in the precision parameters lead to a higher μ_h and a lower μ_ℓ, which are consistent with our definition of the informativeness above. In this chapter we use "informativeness" and "transparency" interchangeably to describe the quality of a disclosure system.

2.2.3 The Investment Bank and the Participant Investors

In addition to the issuer, there are two other types of players: an investment bank and a group of participant investors. To issue the securities, the issuer has to find an investment bank (also called "underwriter" or "he") to help her underwrite the deal in the primary market. The investment bank can be an underwriting bank in a public offering of bond or equity, a lead bank in loan syndication, or an arranger in securitization. The issuer reveals the disclosure system π to the investment bank. The investment bank then engages in due diligence to find out the realization of the signal s. After observing s the investment bank makes a decision on whether to underwrite. If he agrees to underwrite, he further chooses the fraction of securities β to retain. Instead, he can also withdraw from underwriting if he finds it unfavourable, and thus the issue fails.[4] We denote the action set of the investment bank as follow

$$a_{IB} \in \{(\text{Underwrite \& Retain } \beta), (\text{Not Underwrite})\}.$$

Following Aghion, Bolton and Tirole (2004), we assume that capital is scarce for the investment bank and he incurs an opportunity cost (i.e., cost of capital) $r > 0$ per unit of investment.[5] Moreover, there are a unit mass of participant investors who can also invest in a risk-free asset with zero return. They will invest in the remaining $(1 - \beta)$ shares as long as they are break-even.

2.2.4 Time Line

At $T = 0$, nature determines the prior distribution of the states. The issuer designs a signal system π which will generate a signal s at $T = 1$. She finds an investment bank and reveals this experiment π to him.

At $T = 1$, signal s realizes. The investment bank first engages in due diligence to discover s and then decides if he will underwrite the issuance. If the investment bank chooses to underwrite, he materializes and communicates the signal s to participant investors. He

[4]In practice when primary market demand for the security is weak and the underwriter is not willing to retain additional shares, he may choose to delay (suspend) the issuance indefinitely, and only to close the deal when the securities can be fully subscribed. For simplicity, we also regard this scenario as failure.

[5]We assume throughout this chapter that the investment bank will always incur this opportunity cost of his capital expenditure in both the primary and the secondary markets. This helps to eliminate multiple equilibria in the secondary market. Removal of such assumption in the secondary market does not affect the equilibrium we will characterize. Moreover, r cannot be too large as otherwise the investment bank will always find it unfavorable to underwrite. We characterize the exact requirements that r should satisfy in order to ensure the existence of interior solutions of the model in the appendix.

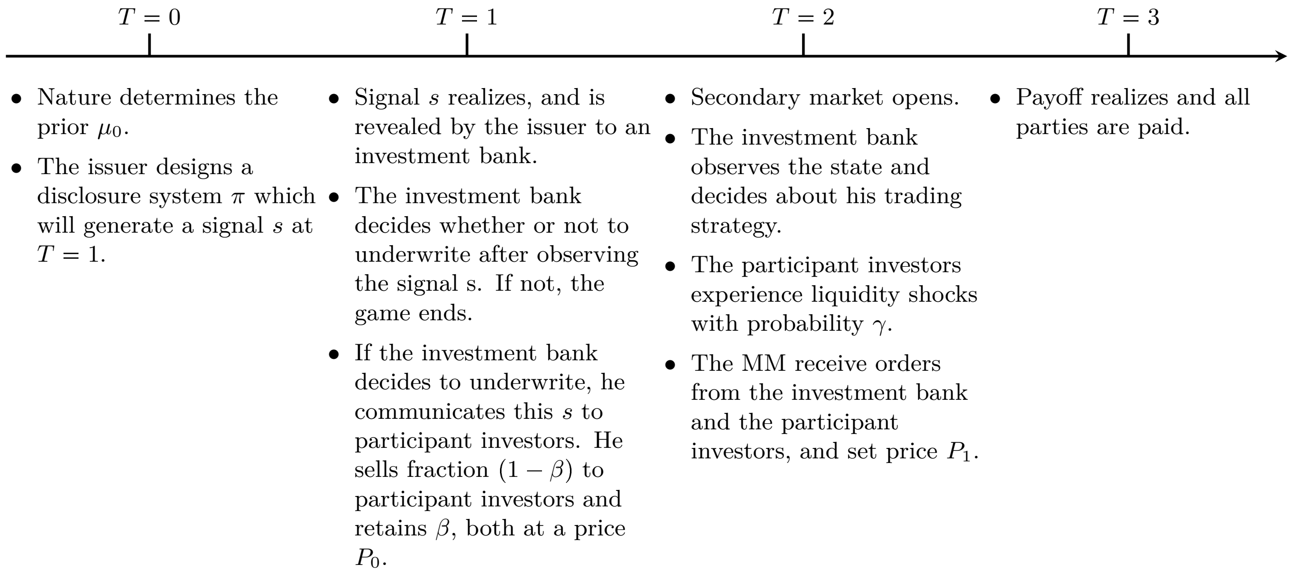


Figure 2.3: Time line

sells $(1-\beta)$ to the participant investors and acquires the remaining β, both at price P_0.

At $T = 2$, a secondary market opens. The market structure is like Kyle (1985). The investment bank and the participant investors submit their market orders to a continuum of deep-pocketed risk-neutral market makers (MM) who price the security competitively after observing the total net order flow y. The market maker sets price $P_1 = \mathbb{E}_s[\tilde{v}|y]$. The trading episode proceeds with three sub-stages:

1. The investment bank observes the true state ω and determines his trading strategy, i.e., the amount of securities $\{x_{IB}\}$ to trade.

2. Liquidity shocks happen with probability $\gamma \in (0,1)$. The participant investors submit their aggregate market order $\{x_{PI}\}$, whereby

   a. with probability γ a fraction $\phi \in (0, \frac{1}{2})$ of the participant investors experience liquidity shocks and have to liquidate their holdings;

   b. with probability $(1-\gamma)$, there is no liquidity shock and these participant investors don't sell.

3. The MM receive the net order flow from the investment bank and the participant investors $y \equiv x_{IB} + x_{PI}$, and set P_1.

At $T = 3$, payoffs of the underlying securities are realized, and all parties get paid.

The time line is summarized in Figure 2.3.

2.2.5 Payoff Functions

We next define the expected payoff functions of the issuer, the investment bank, and the participant investors at $T = 1$ in the primary market. Consider the situation after the signal s has realized. The issuer's expected payoff is

$$U_E(a_{IB}, P_0) = \mathbb{1}_{a_{IB}=\{\text{Underwrite, } \beta\}}\, P_0.$$

$\mathbb{1}_{a_{IB}=\{\text{Underwrite, } \beta\}}$ is an indicator function which takes value 1 if $a_{IB} = \{\text{Underwrite, } \beta\}$ (the investment bank underwrites and acquires β), and 0 otherwise. Since the investment bank will make his underwriting and retention decisions after observing s, it follows that a_{IB} will be a function of posterior belief μ_s. P_0 is the price of the securities and the money she will obtain in the primary market conditional on the investment bank choosing to underwrite. We follow the Bayesian persuasion literature (e.g. Kamenica and Gentzkow, 2011; Huang, 2016; Szydlowski, 2016) by assuming that information design incurs no cost, and when the issuer is indifferent between two disclosure systems, she always selects the one that is less informative.[6]

Back to $T = 0$ when the issuer designs the disclosure system π, she rationally anticipates the best response by the investment bank conditional on induced posterior belief. Her payoff given posterior μ_s is therefore

$$U_E(\mu_s) \equiv a^*_{IB}(\mu_s) P_0(a^*_{IB}(\mu_s), \mu_s),$$ [7]

and her expected payoff under the disclosure system is

$$\mathbb{E}_\pi \left[U_E(\mu_s)\right] = \mathbb{E}_\pi \left[a^*_{IB}(\mu_s) P_0(a^*_{IB}(\mu_s), \mu_s)\right],$$

$a^*_{IB}(\mu_s)$ is investment bank's optimal decision in the primary market (whether to underwrite and fraction to retain if he agrees to underwrite), given that posterior is μ_s, and in the secondary market all the players will make optimal decisions. $P_0(a^*_{IB}(\mu_s), \mu_s)$ is the price that makes participant investor breaks even in expectation in the primary market, given that posterior is μ_s, investment bank optimally chooses $a^*_{IB}(\mu_s)$, and in the secondary market all the players will make optimal decisions. Here the subscript π implies that the expectation is taken under the distribution of signal realizations (posteriors).

The investment bank's expected payoff after observing s depends on whether he be-

[6]This assumption ensures the tractability of our model as well as the uniqueness of the equilibrium. Alternatively, we can define the issuer's expected payoff as

$$U_E(a_{IB}, P_0, \pi) = \mathbb{1}_{a_{IB}=\{\text{Underwrite, } \beta\}}\, P_0 - C(\pi),$$

where C represents a sunk cost of disclosure which varies with the informativeness of the disclosure system π as in Gentzkow and Kamenica (2014):

$$C(\pi) \equiv kL(\pi) = k\{H(\mu_r) - \mathbb{E}_{\langle \pi|\mu_r \rangle}[H(\mu_s)]\}.$$

Note that $k > 0$ is the cost of a one-unit reduction in entropy. Therefore, at $T = 0$ when two disclosure systems deliver the issuer the same expected proceeds, she prefers the one that is less informative and thus less costly. When the unit cost $k \to 0^+$, the optimal disclosure policies converge to the ones in this chapter. Also, for small k our main intuitions still go through and thus our results are robust to costly information disclosure.

[7]Based on our model setting and tie-breaking assumptions,$a^*_{IB}(\mu_s)$ and $P_0(a^*_{IB}(\mu_s), \mu_s)$'s distrbutions are degenerate and thus we do not need to take expectation here.

comes an underwriter as well as his retention β if he chooses to underwrite:

$$\mathbb{E}U_{IB}(a_{IB}, \mu_s, P_0, \Pi) = \mathbb{1}_{a_{IB}=\{\text{Underwrite, } \beta\}} \times \{\beta[(\mu_s \Delta V + V_L) - (1+r)P_0] + \mathbb{E}_s[\Pi]\},$$

where $\beta[(\mu_s \Delta V + V_L) - (1+r)P_0]$ is his net payoff from retaining β shares in the primary market, and $\mathbb{E}_s[\Pi]$ is his expected trading profits in the secondary if there is any at $T = 2$. Here the subscript s implies that we take the expectation under the distribution of underlying states induced by signal s.

Finally, for the participant investors to acquire the remaining $(1-\beta)$ shares, they will demand a price P_0 which makes them at least break even. Therefore the issuer will offer a price such that their expected payoff is $\mathbb{E}U_{PI}(\beta, \mu_s) = 0$.

2.3 Secondary Market Trading and Primary Market Discount

In this section, we solve the *weak Perfect Bayesian Equilibrium* (weak PBE)[8]of the game by backward induction. Suppose that the investment bank chooses to underwrite at $T = 1$. Then at $T = 2$, the disclosure system π, the signal realization s, the share price P_0 in the primary market, and the investment bank's retention β are all taken as given.

Now that the investment bank has observed the true underlying state at $T = 2$, he decides about the optimal market order x_{IB} he should submit. We characterize the weak PBE that generates *highest expected profit for the investment bank*, where the investment bank does not trade in state G and sells $(1-\beta)\phi$ in state B as follows.

In state G, the true value of the security is V_H. The investment bank has no incentive to sell simply because the secondary market price cannot exceed the security's intrinsic value, i.e., $P_1 \leqslant V_H$. It will only stay put or purchase positive amounts. We will discuss those two cases separately.

Case I: the investment bank under state G chooses neither to buy or sell. In state B, since the price is always at least as much as the security's intrinsic value (i.e., $P_1 \geqslant V_L$), the investment bank can potentially benefit from sale. The maximal amount that can be sold in order to at least partially conceal his private information is u. In this case the aggregate order flow will be $y = -2u$ if participant investors are hit by liquidity shocks, and $y = -u$ otherwise. Therefore, the MM cannot tell which state the economy is in when the net order flow is $-u$, and the investment bank enjoys informed trading profits if the true state happens to be bad. It is straightforward to check that under appropriate out-of-equilibrium beliefs, the investment bank under state G has no incentive to deviate and the investment bank do not trade in state G and sell $(1-\beta)\phi$ in state B is part of an weak PBE.

[8]Our results in this section are robust to the equilibrium definition used in this game. In the appendix, we show the results using the weaker solution concept of maximizing investment bank's ex-ante pay-off.

Table 2.1: Secondary Market Trading and Pricing

State	Liquidity Shocks	$\tilde{v}$	Probability	x_{PI}	x_{IB}	y	P_1
G	Yes	V_H	$\mu_s\gamma$	$-u$	0	$-u$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} + V_L$
G	No	V_H	$\mu_s(1-\gamma)$	0	0	0	V_H
B	Yes	V_L	$(1-\mu_s)\gamma$	$-u$	$-u$	$-2u$	V_L
B	No	V_L	$(1-\mu_s)(1-\gamma)$	0	$-u$	$-u$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} + V_L$

Note: $u \equiv (1-\beta)\phi$.

Case II: the investment bank under state G chooses to buy some positive amount. For this to be a part of an equilibrium profile, the only possibility is that the investment bank under state B sells some positive amount in equilibrium. It can be shown that this profile will lead to same amount of expected income for the investment bank, but the expected net profit would be lower because holding more stocks is costly for the investment bank. Therefore, we can just focus on Case I if we are looking at the weak PBE that generates the highest expected profit for the investment bank.

In sum, to best exploit his private information, the investment bank refrains from trading in good state and liquidates $(1-\beta)\phi$ in bad state to maximize his expected informed trading profits while not fully reveal his identity.

We tabulate the equilibrium in the secondary market in Table 2.1, and summarize in the following proposition.

Proposition 2.1. *(Secondary market equilibrium):*

1. *The investment bank's optimal trading strategy is to submit an order $x_{IB} = 0$ in state G, and an order $x_{IB} = -(1-\beta)\phi$ in state B.*
2. *The MM's posterior belief about the probability of state G is*

$$\mu_{MM} = \begin{cases} 1 & \text{if } y = 0, \\ \frac{\mu_s\gamma}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} & \text{if } y = -(1-\beta)\phi, \\ 0 & \text{if } y = -2(1-\beta)\phi. \end{cases}$$

3. *The MM set price*

$$P_1 = \begin{cases} V_H & \text{if } y = 0, \\ \frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} + V_L & \text{if } y = -(1-\beta)\phi, \\ V_L & \text{if } y = -2(1-\beta)\phi. \end{cases}$$

Having obtained the trading equilibrium, we now derive the primary market issue price taking into account the adverse selection in the secondary market. Recall from Table 2.1 that the investment bank's trading strategy mixes case {State G, Liquidity Shocks} with case {State B, No Liquidity Shocks}, and he only makes profits in the second case where he manages to camouflage as liquidity traders. His informed-sale profits per share are

$$G \equiv P_1 - V_L = \frac{\mu_s \gamma \Delta V}{\mu_s \gamma + (1-\mu_s)(1-\gamma)}.$$

The next proposition derives the investment bank's total expected trading profits and the primary market issue price when he observes signal s at $T = 1$.

Proposition 2.2. *(Expected trading profits, and Primary market underpricing):*

1. *The investment bank's total expected trading profits are*

$$\mathbb{E}_s[\Pi] = (1-\beta)\phi \ \mathbb{E}_s[G] = \frac{(1-\beta)\phi(1-\mu_s)(1-\gamma)\mu_s\gamma\Delta V}{\mu_s \gamma + (1-\mu_s)(1-\gamma)}.$$

2. *Since the investment bank's gain per share is just the participant investors' loss per share, in order for these investors to purchase at $T = 1$,*

$$\begin{aligned} P_0 &\equiv \mathbb{E}_s[\tilde{v}] - \Delta P \\ &= (\mu_s \Delta V + V_L) - \frac{\mathbb{E}_s[\Pi]}{1-\beta} \\ &= (\mu_s \Delta V + V_L) - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s \gamma + (1-\mu_s)(1-\gamma)}. \end{aligned}$$

The fact that securities are issued with a discount due to adverse selection in the secondary market has been commonly mentioned in the literature (e.g. Holmström and Tirole, 1993; Maug, 1998; Edmans and Manso, 2011).

2.4 With Short Sale Constraint (SS)

As we will see, whether short sale by the underwriter is allowed in the secondary market has somewhat different implications for the equilibrium in the primary market at $T = 1$ as well as the issuer's choice of optimal disclosure policy at $T = 0$. Note that whether there is short sale constraint in the secondary market does not affect the equilibrium strategies we have characterized in the previous section. We first consider the baseline model where the investment bank cannot sell the security short. Then we proceed with the model in which there is no short sale constraint.

The next lemma establishes the condition under which strategic trading by the investment bank is feasible when there is short sale constraint in the secondary market.

Lemma 2.1. *(Minimal stake): When selling the security short is not allowed in the secondary market, the investment bank can engage in strategic informed trading iff* $\frac{\phi}{1+\phi} \leqslant$

$\beta < 1$.

Suppose that part of the participant investors are hit by liquidity shocks. They will liquidate a fraction of $u \equiv (1-\beta)\phi$ shares in total. To gain informed trading profits, the investment bank has to camouflage as liquidity traders. Because he cannot short sell, to achieve this goal his holdings β should not be too samll, i.e., no less than $(1-\beta)\phi$. Also note that β should be strictly less than 1 because otherwise the market is completely illiquid and there will be no liquidity traders.

2.4.1 No Demand Uncertainty (NDU)

In this section we first consider the benchmark model where there is no demand uncertainty in the primary market, i.e., all the shares can be fully subscribed by the participant investors even if the investment bank does not acquire any.

At $T = 1$, from Lemma 2.1 we have already established that when $\beta \in [0, \frac{\phi}{1+\phi})$ or $\beta = 1$, the investment bank cannot gain from trading on his private information, because either his stake is not enough or the secondary market is completely illiquid. Thus the issue price will not include the adverse selection discount. The following proposition characterizes the price in the primary market for different levels of retention by the investment bank.

Proposition 2.3. *(Primary market issue price): The issue price in the primary market is*

$$P_0(\beta, \mu_s) = \begin{cases} \mu_s \Delta V + V_L - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} & \text{if } \beta \in [\frac{\phi}{1+\phi}, 1), \\ \mu_s \Delta + V_L & \text{if } \beta \in [0, \frac{\phi}{1+\phi}) \text{ or } \beta = 1. \end{cases}$$

Investment Bank's Optimal Decision I

Absent any demand uncertainty, the investment bank can always stay break-even by choosing to underwrite yet retaining no shares. Therefore, the investment bank's decision to underwrite is trivial in our benchmark model here.

At $T = 1$ after signal s has realized and posterior belief μ_s has been formed, the investment bank decides on his stake β to maximize his expected payoff, denoted $U_{IB}^1(\beta, \mu_s)$:

$$\max_{\beta \in [0,1]} \left\{ \begin{array}{c} \beta \cdot [(\mu_s \Delta V + V_L) - (1+r)P_0(\beta, \mu_s)] \\ + \mathbb{1}_{\{\beta \geqslant (1-\beta)\phi\}} \cdot (1-\beta)\phi \cdot \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} \end{array} \right\}.$$

The first term above represents the investment bank's expected payoff in the primary market which is the intrinsic value of the β shares net of his capital expenditure and opportunity cost. The second term is his expected trading profits as we have shown in Proposition 2.2 if he has acquired adequate stake in the primary market. Observe that the above expected utility function $U_{IB}^1(\beta, \mu_s)$ is in fact piece-wise linear in β. Hence its maximum must be attained at $\beta^* = 0$, or 1, or $\frac{\phi}{1+\phi}$, or $\beta^* \uparrow 1$ (i.e., $\beta^* = 1^-$). The investment bank's optimal retention problem thus becomes

$$\beta^* = \underset{\beta \in \{0,1,\frac{\phi}{1+\phi},1^-\}}{\arg\max} \left\{ U_{IB}^1(0,\mu_s), U_{IB}^1(1,\mu_s), U_{IB}^1(\frac{\phi}{1+\phi},\mu_s), U_{IB}^1(1^-,\mu_s) \right\}.$$

The investment bank's expected payoff $U_{IB}^1(\beta,\mu_s)$ is calculated as follows:

(i). If $\beta^* = 0$, there will be no informed trading in the secondary market and no price discount in the primary market, $U_{IB}^1(0,\mu_s) = 0$.

(ii). If $\beta^* = 1$, the secondary market is completely illiquid and the issue price has no discount,

$$U_{IB}^1(1,\mu_s) = (\mu_s \Delta V + V_L) - (1+r)P_0(1,\mu_s) = -r(\mu_s \Delta V + V_L).$$

(iii). If $\beta^* = \frac{\phi}{1+\phi}$, informed trading is feasible and thus issue price must be discounted,

$$U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) = \frac{\phi}{1+\phi}\left[(\mu_s \Delta V + V_L) - (1+r)P_0(\frac{\phi}{1+\phi},\mu_s)\right] + \frac{1}{1+\phi} \cdot \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

(iv). Finally, if $\beta^* = 1^-$, there is (infinitesimal) informed trading profit yet still a relatively sizable adverse selection discount,

$$\begin{aligned} U_{IB}^1(1^-,\mu_s) &= 1^- \cdot [(\mu_s \Delta V + V_L) - (1+r)P_0(1^-,\mu_s)] + 0^+ \cdot \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \\ &\doteq (\mu_s \Delta V + V_L) - (1+r)P_0(1^-,\mu_s) \\ &= (\mu_s \Delta V + V_L) - (1+r)\left[(\mu_s \Delta V + V_L) - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}\right]. \end{aligned}$$

To pin down the optimal retention by the investment bank in response to the observed signal s, it suffices to show for different μ_s which of the above U_{IB}^1's achieve the largest value. The lemma below provides some important properties of the investment bank's expected payoff function if he chooses to retain $\beta = \frac{\phi}{1+\phi}$.

Lemma 2.2. *(Indifference cut-off posteriors I):*

1. *There exists a pair* $\{\underline{\mu},\overline{\mu}\}$ *with* $0 < \underline{\mu} < \frac{1}{2} < \overline{\mu} < 1$ *such that* $U_{IB}^1(\frac{\phi}{1+\phi},\underline{\mu}) = U_{IB}^1(\frac{\phi}{1+\phi},\overline{\mu}) = 0$.

2. $U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) > 0$ *if* $\mu_s \in (\underline{\mu},\overline{\mu})$, *and* $U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) < 0$ *if* $\mu_s \in [0,\underline{\mu})$ *or* $\mu_s \in (\overline{\mu},1]$.

Therefore, at posteriors $\mu_s = \underline{\mu}$ and $\overline{\mu}$, the investment bank is indifferent between holding $\beta = \frac{\phi}{1+\phi}$ and $\beta = 0$. Furthermore, the investment bank will only consider purchasing

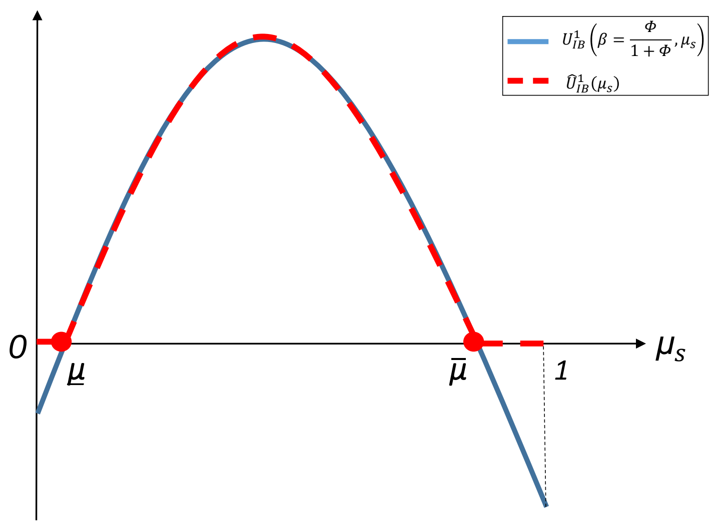


Figure 2.4: The investment bank's payoff (i)

a fraction of the shares when uncertainty about the security is large (i.e., the posterior belief μ_s lies in an intermediate range).

The following proposition characterizes the investment bank's optimal strategy and the relevant equilibrium payoffs under different posterior beliefs.

Proposition 2.4. *(Investment bank's optimal strategy and relevant payoffs I): The investment bank's optimal stake is*

$$\beta^* = \begin{cases} \frac{\phi}{1+\phi} & \text{if } \mu_s \in (\underline{\mu}, \overline{\mu}), \\ 0 & \text{if } \mu_s \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]. \end{cases}$$

His equilibrium payoff is

$$\hat{U}_{IB}^1(\mu_s) = \begin{cases} U_{IB}^1(\frac{\phi}{1+\phi}, \mu_s) & \text{if } \mu_s \in (\underline{\mu}, \overline{\mu}), \\ 0 & \text{if } \mu_s \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]. \end{cases}$$

In Figure 2.4 the blue line shows the payoff of the investment bank if he chooses to retain $\beta = \frac{\phi}{1+\phi}$, i.e., $U_{IB}^1(\frac{\phi}{1+\phi}, \mu_s)$. The red dashed line depicts his equilibrium payoff under his optimal retention strategy, denoted by $\hat{U}_{IB}^1(\mu_s)$. In equilibrium when $\mu_s \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]$, the investment bank does not retain any share, and his payoff is zero. Yet when $\mu_s \in (\underline{\mu}, \overline{\mu})$, he chooses his retention $\beta = \frac{\phi}{1+\phi}$ and his payoff is $U_{IB}^1(\frac{\phi}{1+\phi}, \mu_s)$, which corresponds to the hump-shaped part of the red dashed line. So in equilibrium both the investment bank's optimal stake and his expected payoff depend only on his belief μ_s.

The intuition of Proposition 2.4 is straightforward: when uncertainty about the security's payoff is relatively small, the investment bank's informed trading profits in the secondary market is not enough to cover his cost of capital in the primary market, even though he free rides on the discounted issue price. This results in zero retention by the bank. When the uncertainty about the security is relatively large, it is profitable for the investment bank to acquire some shares in order to later trade on his private information strategically. Yet such gain in the secondary market trades off against the opportunity cost incurred from his primary market capital expenditure. In equilibrium the investment bank optimally chooses his retention such that it is just enough for him to camouflage as liquidity traders in the secondary market. This minimizes his total cost of capital while maximizes his expected trading profits. Our result contrasts with the retention equilibrium in Leland and Pyle (1977) where a firm holds a large fraction of its shares to have some skin in the game and signal to the market its quality when information asymmetry problem is severe. In our model, the investment bank acquires a stake to later gain from informed sales in the secondary market when the security's cash flows are relatively more uncertain. In this regard, such retention exacerbates the adverse selection problem.

Optimal Disclosure System I

Given the optimal retention scheme by the investment bank described in Proposition 2.4, it follows naturally that the issuer's expected revenue conditional on signal s at $T = 1$ will be either the intrinsic value of the security if the bank does not acquire any share, or the expected cash flows from the security net of an adverse selection discount if the bank holds a positive stake $\frac{\phi}{1+\phi}$. This gives the following proposition.

Proposition 2.5. *(Issuer's payoff after information design I): At $T = 1$ the issuer's expected payoff conditional on signal s is:*

$$U_E^1(\mu_s) = \begin{cases} \mu_s \Delta V + V_L - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} & \text{if } \mu_s \in (\underline{\mu}, \overline{\mu}), \\ \mu_s \Delta V + V_L & \text{if } \mu_s \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]. \end{cases}$$

Note that at the two posteriors $\underline{\mu}$ and $\overline{\mu}$, the investment bank is actually indifferent between retaining 0 and a positive stake $\frac{\phi}{1+\phi}$. Following the convention of information disclosure literature, we select the sender-preferred equilibrium in which the investment bank does not acquire any share in the primary market when he is indifferent, and thus there will be no discount. In reality, given the high cost of bank capital, we have reason to believe that if the issuer is not opposed to it, investment banks are more prone to no retention although a positive stake gives him the same expected payoff.

At $T = 0$ the issuer designs the optimal disclosure policy to maximize her expected proceeds from issuing the security. She has to choose the precision of her signal π_G and π_B for the disclosure system π. By Bayes' rule, essentially her problem is equivalent to the optimal choice of two posteriors μ_h and μ_ℓ.

Because we have assumed that demand never falls short of the supply in the primary market, the investment bank does not have to worry about the risk of retaining more shares than his privately optimal level. Thus he will always underwrite, and his decision problem is reduced to the choice of stake β. We can write the issuer's payoff at $T = 1$ as

$$U_E^1(\beta, \mu_s) = \mathbb{1}_{a_{IB}=\{\text{Underwrite, } \beta\}} P_0(\beta, \mu_s).$$

Since we already know from Proposition 2.4 that the investment bank's optimal retention β^* depends on μ_s, the issuer's expected proceeds will only depend on μ_s in equilibrium, which we denote by $U_E^1(\mu_s) \equiv U_E^1(\beta^*, \mu_s) = P_0(\beta^*, \mu_s)$. So the issuer solves the following maximization problem:

$$\hat{U}_E^1(\mu_0) \equiv \max_{\{\mu_\ell, \mu_h\}} \mathbb{E}_\pi[U_E^1(\mu_s)] = \max_{\{\mu_\ell, \mu_h\}} \mathbb{E}_\pi[a_{IB}^*(\mu_s) P_0(a_{IB}^*(\mu_s), \mu_s)]$$

$$\begin{aligned} \text{s.t.} \qquad & \mathbb{P}[s = h] \cdot \mu_h + \mathbb{P}[s = \ell] \cdot \mu_\ell = \mu_0, \\ & \mathbb{P}[s = h] + \mathbb{P}[s = \ell] = 1. \end{aligned}$$

The issuer chooses the optimal disclosure system taking into account that the subsequent players will choose their optimal behaviours. Especially, she is aware that in this scenario, the investment bank will always underwrite and it will retain $\beta^*(\mu_s) = \arg\max_{\beta \in [0,1]} U_{IB}^1(\beta, \mu_s)$ amount of shares.

The first constraint is the Bayesian plausibility condition in which the expectation of posteriors must equal the prior. The second constraint requires that the probabilities of signal realizations should sum to one.

To solve this problem, we use the concavification technique in Kamenica and Gentzkow (2011). In particular, the issuer's *ex ante* optimal design of disclosure system can be derived by finding the concave closure of $U_E^1(\mu_s)$, which we define as $\hat{U}_E^1(\mu_s)$. A graphic representation is given in Figure 2.5. The black line depicts the issuer's expected payoff conditional on different posteriors. When the uncertainty is relatively large, the investment bank retains a stake and there is underpricing. Thus we observe a dent from the graph when $\mu_s \in (\underline{\mu}, \overline{\mu})$. The blue dashed line illustrates $\hat{U}_E^1(\mu_s)$ – the issuer's maximized expected payoff from the optimal disclosure system.

Intuitively, for any given prior μ_0, it must be equal to some convex combination of two posteriors μ_ℓ and μ_h induced by the optimal system due to the Bayesian plausibility condition (i.e., $\mu_0 = \lambda\mu_\ell + (1 - \lambda)\mu_h$ for some $\lambda \in [0, 1]$). So the issuer's *ex ante* expected payoff under the distribution of posteriors must be a convex combination of two expected payoffs conditional on relevant signal realizations too (i.e., $\mathbb{E}_\pi[U_E^1(\mu_s)] = \lambda U_E^1(\mu_\ell) + (1 - \lambda)U_E^1(\mu_h)$). Obviously, the optimal $\mathbb{E}_\pi[U_E^1(\mu_s)]$ is attained on the concave closure of $U_E^1(\mu_s)$. The optimal μ_ℓ and μ_h are obtained at the intersections of $U_E^1(\mu_s)$ and its concave closure, which are to the left and right of μ_0 respectively.[9] λ and $(1 - \lambda)$ are the probabilities of posteriors μ_ℓ and μ_h. The proposition below characterizes the optimal

[9] In a completely uninformative system, $\mu_\ell = \mu_h = \mu_0$.

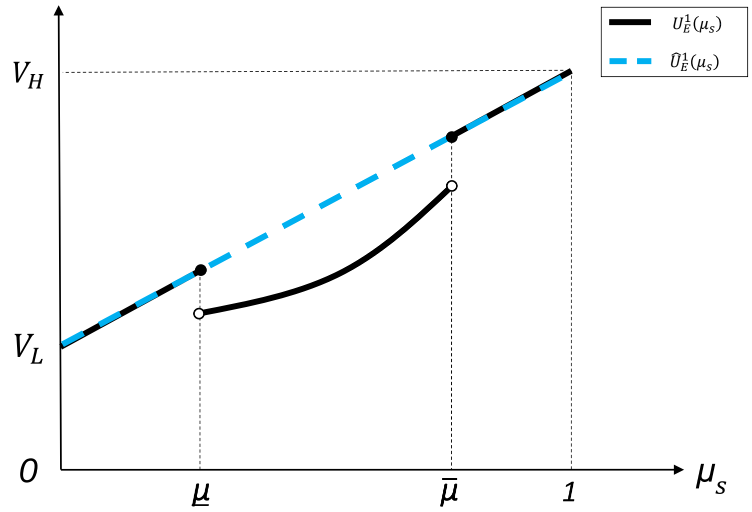


Figure 2.5: The issuer's payoff (i)

disclosure policy employed by the issuer at $T = 0$.

Proposition 2.6. *(Optimal information design I): At $T = 0$ the issuer's optimal disclosure policy is:*

1. *If $\mu_0 \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]$, the optimal disclosure system has $\pi_G = \pi_B \in (0, 1)$, and is therefore completely uninformative, yielding posteriors $\mu_\ell = \mu_h = \mu_0$.*
2. *If $\mu_0 \in (\underline{\mu}, \overline{\mu})$, the optimal disclosure system has $\pi_G = \frac{\overline{\mu}(\mu_0 - \underline{\mu})}{\mu_0(\overline{\mu} - \underline{\mu})}$ and $\pi_B = \frac{(1-\overline{\mu})(\mu_0 - \underline{\mu})}{(1-\mu_0)(\overline{\mu} - \underline{\mu})}$, yielding posteriors $\mu_\ell = \underline{\mu}$ and $\mu_h = \overline{\mu}$.*

One caveat is worth some discussion here. When $\mu_0 \in (\underline{\mu}, \overline{\mu})$, there are multiple disclosure systems which gives the issuer the same expected payoff. In fact she can set any arbitrary π_G and π_B, as long as they induce posteriors $\mu_\ell \in [0, \underline{\mu}]$ and $\mu_h \in [\overline{\mu}, 1]$ subject to $\mathbb{P}[s = h] \cdot \mu_h + \mathbb{P}[s = \ell] \cdot \mu_\ell = \mu_0$. But since we have assumed before that if multiple disclosure policies give the issuer the same expected payoff, she selects the one that is the least informative (and thus the least costly if we assume an infinitesimal cost of reduction in entropy due to the disclosure that varies with the informativeness of the system). Accordingly, Proposition 2.6 characterizes the least informative optimal disclosure system at $T = 0$.

From Figure 2.5 it is clear that if the issuer does not release information, underpricing happens when uncertainty about the firm is relatively large. This is consistent with Cai, Helwege and Warga (2007) that find significant underpricing on speculative-grade debt IPOs but no significant underpricing on investment-grade bond IPOs. We take a further step by showing that in fact issuer can strategically design her disclosure policy to curb underpricing even if *ex ante* the uncertainty about the security is relatively large. This is achieved by designing a system which decreases the uncertainty associated with the security to the degree that the investment bank is just indifferent between holding either zero or a positive stake. Also, a security with its payoff uncertainty below some thresholds will in turn have no discount. In practice, because of other possible frictions such as issuer's limited capability in reducing uncertainty, we will still observe some underpricing. Later we will show that when there is demand uncertainty in the primary market, underpricing always arises in equilibrium, but strategic disclosure can reduce it on average.

Since we have derived the optimal disclosure policy, it is natural to ask what factors may potentially affect the informativeness of the optimal system. Moreover, how do firms with different levels of uncertainty alter their optimal strategies in response to changes in those factors? We address these important questions in Proposition 2.7.

Proposition 2.7. *(Comparative statics I):*

(1) $\frac{\partial \underline{\mu}}{\partial V_L} > 0$ *and* $\frac{\partial \overline{\mu}}{\partial V_L} < 0$.

(2) Define $\eta \equiv \frac{\Delta V}{V_L}$, *then* $\frac{\partial \underline{\mu}}{\partial \eta} < 0$ *and* $\frac{\partial \overline{\mu}}{\partial \eta} > 0$.

(3) $\frac{\partial \underline{\mu}}{\partial r} > 0$ *and* $\frac{\partial \overline{\mu}}{\partial r} < 0$.

(4) $\frac{\partial \underline{\mu}}{\partial \phi} < 0$ *and* $\frac{\partial \overline{\mu}}{\partial \phi} > 0$.

Result (1) states that as V_L increases, the lower-bound cut-off posterior $\underline{\mu}$, at which the investment bank is indifferent between holding 0 and $\frac{\phi}{1+\phi}$, becomes larger and the similar upper-bound cut-off posterior $\overline{\mu}$ becomes smaller. This implies that the range $(\underline{\mu}, \overline{\mu})$ shrinks inward. V_L is the reservation value of the security, and can be viewed as a proxy for the issue size. We first discuss the implications of the comparative statics if the system is completely uninformative. In this case the posterior belief is simply the prior. A larger V_L makes it more costly for the underwriter to retain a stake. So at the cut-off posterior beliefs, only marginally higher uncertainty will induce the underwriter to have a positive retention and stay break-even. The enhanced uncertainty makes the bank's private information more valuable in the secondary market trading, hence offsetting the additional cost brought about by the larger V_L.

Turning to the optimal disclosure, a larger V_L means that only firms that are relatively more uncertain (i.e., $\mu_0 \in (\underline{\mu}, \overline{\mu})$) will employ a system which induces a pair of posteriors $\{\underline{\mu}, \overline{\mu}\}$. Yet as V_L becomes larger, the resulting optimal system will be less transparent because of the inward-shrunken $(\underline{\mu}, \overline{\mu})$, (i.e., less dispersed distribution of posteriors).[10] Therefore, for firms whose security payoffs are *ex ante* highly uncertain, larger issue size allows them to use less transparent disclosure to curb underpricing in the primary market.

Result (2) concerns the effect of the firm's growth option η on the optimal disclosure policy used by the issuer. Better growth option is potentially beneficial to the underwriting

[10]Recall from our definition of informativeness in Section 2.2.2, an inward (outward) shrunken range of posteriors $(\underline{\mu}, \overline{\mu})$ indicates less (more) informativeness of the system.

bank because it makes his informed trading more profitable. Consequently, at the cut-off beliefs, even marginally lower uncertainty still ensures a non-negative payoff from his retention and subsequent informed trading. As a result, the range $(\underline{\mu}, \overline{\mu})$ expands, and the issuer will use more transparent system as the growth option improves if the security's *ex ante* payoff uncertainty is high.

Result (3) shows that the greater cost of capital of the investment bank will push the two cut-off posteriors inward. Similar to Result (1), at the cut-off beliefs, only marginally higher uncertainty will compensate the underwriter's increased cost of capital by making his private information more valuable in the secondary market trading. Therefore, greater cost of capital of the investment bank results in less transparent disclosure by the issuer with high *ex ante* payoff uncertainty.

Finally, result (4) relates disclosure to market liquidity. A more liquid secondary market pushes the two threshold posteriors outward. In effect, higher liquidity is beneficial to the underwriter as it improves his trading profits. Hence at the margins, cut-off beliefs with relatively lower uncertainty are sufficient to make the underwriter just break-even by holding a stake. Also, the optimal disclosure reduces more uncertainty, rendering it more transparent if the prior is associated with high uncertainty. Result (4) implies a benefit of the market liquidity in that potentially a more liquid secondary market can push the issuer to design a more transparent disclosure system when issuing securities although this is not the complete story as we will see in the next section.

2.4.2 Demand Uncertainty (DU)

In this section, we extend the model by introducing the possibility of negative demand shock in the primary market. When demand shock happens, the securities are under-subscribed and the underwriting bank has to acquire additional shares to close the deal if he chooses to underwrite the issue. Note that the demand shock does not affect our secondary market equilibrium as well as the discounted issue price due to informed trading discussed in Section 3. We thus proceed with our analysis from $T = 1$ and then work backward to determine the optimal disclosure policy at $T = 0$.

Formally, we assume that if demand shock happens in the primary market, the demand for the issuer's security is only ψ which satisfies the following inequality:

$$0 < \psi < 1 - \frac{\phi}{1 + \phi}.$$

Therefore, if the investment bank plans to retain a fraction $\beta \leqslant \frac{\phi}{1+\phi}$, the aggregate demand for the security will fall short of the supply (i.e., $\beta + \psi < 1$). We further assume that if initially the investment bank has entered into an agreement to underwrite the issue, he has to acquire all of the remaining $(1 - \psi)$ shares. Also, recall from Lemma 2.1 that with short sale constraint informed trading is feasible for the investment bank if and only if the fraction of his retention is at least $\frac{\phi}{1+\phi}$ yet strictly less than 1, and the pricing of shares in the primary still follows Proposition 2.3.

More specifically, suppose that at $T = 1$ after the investment bank has agreed to underwrite and makes his initial retention plan $\hat{\beta}$,

a. with probability $\epsilon \in (0,1)$, the total demand of shares by the participant investors is only ψ. So the investment bank has to acquire $\beta = 1-\psi$. The issue price is $P_0(1-\psi, \mu_s)$;

b. with probability $(1-\epsilon)$, there is no demand shock. The investment bank's ultimate retention is $\beta = \hat{\beta}$ and the issue price is $P_0(\hat{\beta}, \mu_s)$.[11]

Investment Bank's Optimal Decision II

In this scenarior, even if the investment bank initially decides to retain only $\hat{\beta} = 0$, the possible demand shock may force him to acquire more than he plans and depress his expected payoff below zero. Nevertheless, the investment bank has an exit option "Not Underwrite" to stay break-even. So the decision to underwrite is no longer trivial, and it depends crucially on the posteriors induced by the issuer's disclosure. We denote the investment bank's payoff by $U_{IB}^2(\hat{\beta}, \mu_s)$ if he enters into the underwriting contract and makes his initial retention plan $\hat{\beta}$.

Consider the situation in which the investment bank chooses to underwrite. He needs to determine his initial retention plan $\hat{\beta}$ to maximize his expected payoff before the demand uncertainty is resolved. With probability ϵ, the demand shock happens and the investment bank has to buy $(1-\psi)$. His expected payoff is:

$$A(1-\psi,\mu_s) \equiv (1-\psi)[(\mu_s\Delta V + V_L) - (1+r)P_0(1-\psi,\mu_s)] + \frac{\psi(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

With probability $(1-\epsilon)$, the demand shock does not occur, and the underwriter's payoff is the same as in the no demand uncertainty case:

$$\begin{aligned} B(\hat{\beta},\mu_s) \equiv\ & \hat{\beta}\cdot[(\mu_s\Delta V + V_L) - (1+r)P_0(\hat{\beta},\mu_s)] \\ & + \mathbb{1}_{\{\hat{\beta}\geqslant(1-\hat{\beta})\phi\}}\cdot\frac{(1-\hat{\beta})(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}. \end{aligned}$$

Therefore, after observing signal s, the investment bank has to first decide whether he will underwrite. If he underwrites, he further chooses a planned retention $\hat{\beta}$ to maximize his expected payoff. Formally, he chooses his optimal action a_{IB}^* to solve the following maximization problem

$$\max_{a_{IB}\in\{\{\text{NU}\},\{\text{U},\hat{\beta}\}\}} \mathbb{1}_{a_{IB}=\{\text{U},\hat{\beta}\}}\cdot[\epsilon A(1-\psi,\mu_s) + (1-\epsilon)B(\hat{\beta},\mu_s)].$$

[11] To avoid confusion, we use β and $\hat{\beta}$ respectively to distinguish between the issuers planned and ultimate retention.

To derive the investment bank's optimal action, we first characterize the investment bank's optimal planned retention $\hat{\beta}$ if he chooses to underwrite based on the observed signal in the proposition below.

Proposition 2.8. *(Investment bank's optimal planned retention): If the investment bank decides to underwrite, it is a dominant strategy for him to choose an initial retention* $\hat{\beta} = \frac{\phi}{1+\phi}$ *before demand uncertainty is unraveled.*

Proposition 2.8 implies that the investment bank's planned retention is independent of the issuer's disclosure. Such planned purchase serves as an insurance scheme against the demand uncertainty. The result can be understood in the following way. If demand shock happens, the investment bank is forced to complete the deal by acquiring all the remaining $(1-\psi)$ shares. In this case any *ex ante* planned retention $\hat{\beta} \leqslant 1-\psi$ will not affect his expected payoff. Meanwhile, any initial stake that is larger than $(1-\psi)$ is never optimal. As we have seen in Proposition 2.4, any stake β that is larger than $\frac{\phi}{1+\phi}$ for the range of more uncertain beliefs $(\underline{\mu}, \overline{\mu})$ is sub-optimal in that it incurs more cost of capital while the informed trading profits become less owing to lower liquidity $\phi(1-\beta)$. Therefore, acquiring a stake that is larger than $(1-\psi)$ is even less desirable. When there is no demand shock, a retention which is just enough for the investment bank to camouflage as liquidity traders, i.e., $\frac{\phi}{1+\phi}$, is optimal as we have shown before. Consequently, it is optimal for the investment bank to choose an initial retention $\hat{\beta} = \frac{\phi}{1+\phi}$. In order for the investment bank to underwrite, his expected payoff should be at least zero. Compared with the cut-off posteriors $\underline{\mu}$ and $\overline{\mu}$ before, it is obvious that the new thresholds satisfy $\underline{\mu}^* > \underline{\mu}$ and $\overline{\mu}^* < \overline{\mu}$. It is because at the old posteriors the investment bank's expected payoff when demand shock happens, i.e., $A(\mu_s)$, will be strictly negative as a result of the higher-than-optimum retention $(1-\psi)$. Thus only a larger lower bound $\underline{\mu}^*$ and a smaller upper bound $\overline{\mu}^*$ will suffice to make the investment bank just break-even by accepting to underwrite.

Recall that $U^2_{IB}(\hat{\beta}, \mu_s) \equiv \epsilon A(\mu_s) + (1-\epsilon)B(\hat{\beta}, \mu_s)$ is the investment bank's expected payoff conditional on posterior μ_s if he accepts to underwrite. Also, $\hat{\beta}$ represents his planned retention before demand uncertainty is resolved. We summarize our discussion above in Lemma 2.3.

Lemma 2.3. *(Indifference cut-off posteriors II):*

1. *There exists a pair* $\{\underline{\mu}^*, \overline{\mu}^*\}$ *with* $0 < \underline{\mu} < \underline{\mu}^* < \frac{1}{2} < \overline{\mu}^* < \overline{\mu} < 1$ *such that* $U^2_{IB}(\frac{\phi}{1+\phi}, \underline{\mu}^*) = U^2_{IB}(\frac{\phi}{1+\phi}, \overline{\mu}^*) = 0$.
2. $U^2_{IB}(\frac{\phi}{1+\phi}, \mu_s) > 0$ *if* $\mu_s \in (\underline{\mu}^*, \overline{\mu}^*)$*, and* $U^2_{IB}(\frac{\phi}{1+\phi}, \mu_s) < 0$ *if* $\mu_s \in [0, \underline{\mu}^*)$ *or* $\mu_s \in (\overline{\mu}^*, 1]$.

Unlike before, if the investment bank's expected payoff is negative conditional on the observed signal s, he will choose not to underwrite. This happens when the induced posterior μ_s lies in either $[0, \underline{\mu}^*)$ or $(\overline{\mu}^*, 1]$. In general, the bank will not always underwrite, and he withdraws from underwriting when $\mu_s \in [0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1]$. Proposition 2.9 summarizes the investment bank's best response to different posteriors induced by the issuer's disclosure system and his equilibrium payoff given his optimal action.

Proposition 2.9. *(Investment bank's optimal strategy and relevant payoffs II): The investment bank's optimal action is*

$$a^*_{IB}(\mu_s) = \begin{cases} \textit{Underwrite and } \hat{\beta}^* = \frac{\phi}{1+\phi} & \textit{if } \mu_s \in [\underline{\mu}^*, \overline{\mu}^*], \\ \textit{Not Underwrite} & \textit{if } \mu_s \in [0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1]. \end{cases}$$

His equilibrium payoff is

$$\hat{U}^2_{IB}(\mu_s) = \begin{cases} U^2_{IB}(\frac{\phi}{1+\phi}, \mu_s) & \textit{if } \mu_s \in [\underline{\mu}^*, \overline{\mu}^*], \\ 0 & \textit{if } \mu_s \in [0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1]. \end{cases}$$

Since $\hat{\beta}^*$ in equilibrium depends on the posterior μ_s only, we can simply write the investment bank's expected payoff as $\hat{U}^2_{IB}(\mu_s)$, a function of μ_s too. In Figure 2.6, the red dashed line depicts the investment bank's expected payoff given his optimal action a^*_{IB}, while the yellow solid line is his expected payoff if he sticks to a planned retention $\hat{\beta} = \frac{\phi}{1+\phi}$ regardless of his posterior. For comparison, we also draw the investment bank's expected payoff if he always retains $\frac{\phi}{1+\phi}$ shares when there is no demand uncertainty (i.e., the blue dashed line, which corresponds to the blue solid line in Figure 2.4). The yellow line is beneath the blue dashed one in that the presence of possible demand shock extracts a rent from the investment bank thus decreases its expected payoff in general. In this case the two cut-off posteriors are less dispersed. Indeed, to induce the investment bank to underwrite, higher uncertainty in the primary market is needed. Then the losses due to unfortunate retention can be offset by larger trading profits from the underwriter's private information in the secondary market.

Accordingly, when the uncertainty in the primary market is relatively small (i.e., $\mu_s \in [0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1]$), the investment bank's private information is less valuable and on average he expects to suffer a loss from accepting to underwrite. His optimal strategy is to withdraw from underwriting the issue. Only when the uncertainty is relatively large (i.e., $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$) can the investment bank's expected loss from unfortunate retention be compensated by his informed trading profits owing to more valuable private information. In this case, he will agree to underwrite even though he may end up with more retention than he originally plans.

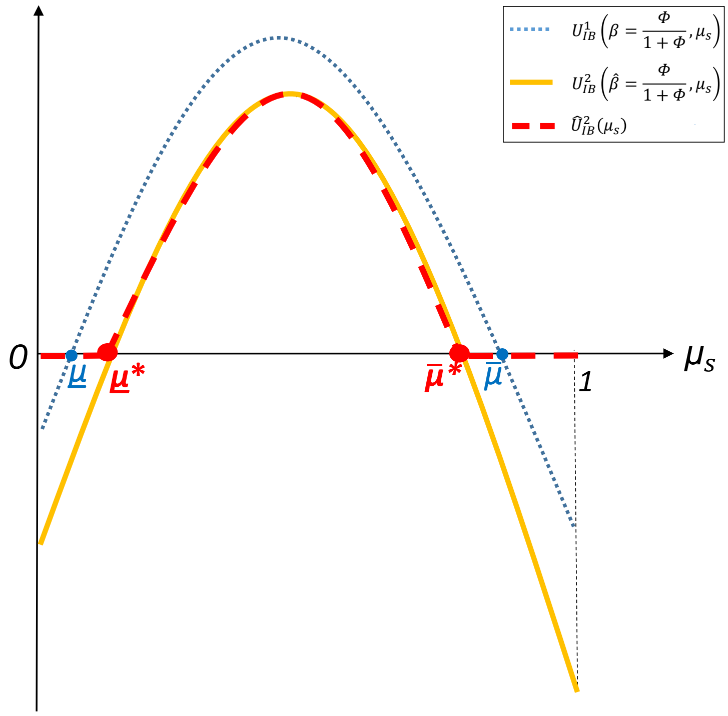


Figure 2.6: The investment bank's payoff (ii)

Optimal Disclosure System II

Since we have solved for the optimal strategy of the investment bank, it is easy to derive the issuer's expected proceeds from security issuance conditional on different signal realizations at $T = 1$.

Proposition 2.10. *(Issuer's payoff after information design II):*

1. *When* $\mu_s \in [0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1]$, *the investment bank does not underwrite, and* $U_E^2(\mu_s) = 0$.
2. *When* $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$, $U_E^2(\mu_s) \equiv U_E^2(\frac{\phi}{1+\phi}, \mu_s) = \epsilon P_0(1-\psi, \mu_s) + (1-\epsilon)P_0(\frac{\phi}{1+\phi}, \mu_s)$

$$= \mu_s \Delta V + V_L - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

The second part of Proposition 2.10 implies that the issue prices are the same under two different levels of retention by the investment bank, $(1-\psi)$ and $\frac{\phi}{1+\phi}$. This is because as long as the bank acquires a stake of at least $\frac{\phi}{1+\phi}$, the issue price will always have an adverse selection discount. Yet such discount does not vary with the investment bank's retention in that each participant investor's expected loss per share from trading in the secondary market is independent of the investment bank's ultimate stake β, a result that has already been shown in Proposition 2.2. From Proposition 2.10 it is easy to see that conditional on signal s, the issuer's expected revenue $U_E^2(\hat{\beta}^*, \mu_s)$ depends on posterior μ_s only, thus we denote it by $U_E^2(\mu_s)$.

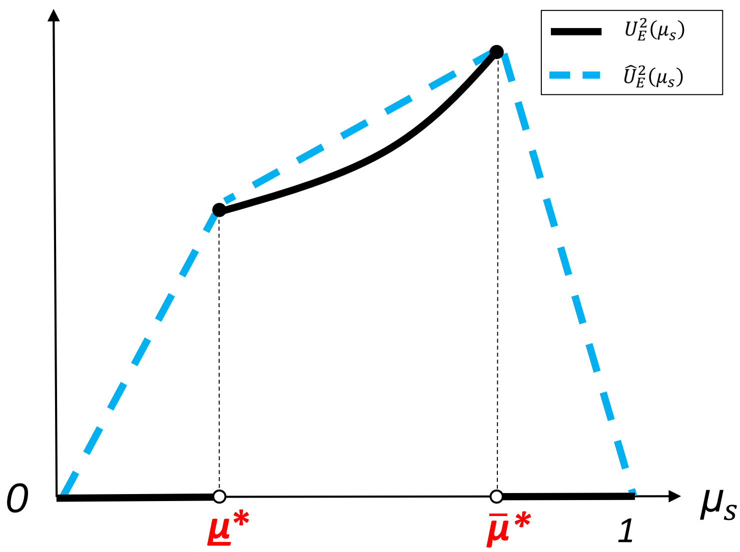


Figure 2.7: The issuer's payoff (ii)

At $T = 0$, taking into account the optimal action that will be taken by the investment bank at different posteriors, the issuer designs the disclosure system to maximize her expected payoff. In particular, she chooses a distribution of posteriors to solve

$$\hat{U}_E^2(\mu_0) \equiv \max_{\{\mu_\ell,\mu_h\}} \mathbb{E}_\pi[U_E^2(\mu_s)] = \max_{\{\mu_\ell,\mu_h\}} \mathbb{E}_\pi[a_{IB}^*(\mu_s)P_0(a_{IB}^*(\mu_s),\mu_s)]$$

$$\text{s.t.} \qquad \mathbb{P}[s=h]\cdot\mu_h + \mathbb{P}[s=\ell]\cdot\mu_\ell = \mu_0,$$

$$\mathbb{P}[s=h] + \mathbb{P}[s=\ell] = 1.$$

Again, The issuer chooses the optimal disclosure system taking into account that the subsequent players will choose their optimal behaviours. Especially, she is aware that in this scenario, the investment bank will only underwrite under some posteriors and its optimal strategy is $a_{IB}^*(\mu_s) = \arg\max_{a_{IB}\in\{\{\text{U},\hat{\beta}\},\{\text{NU}\}\}} \mathbb{1}_{\{a_{IB}=\{\text{U},\hat{\beta}\}\}} \cdot U_{IB}^2(\hat{\beta},\mu_s)$.

The first constraint is the Bayesian plausibility condition. The second constraint ensures that the sum of probabilities of high signal h and low signal ℓ equals 1. We solve this constrained maximization problem by finding the concave closure of $U_E^2(\mu_s)$. In Figure 2.7 the black solid line depicts the issuer's expected payoff $U_E^2(\mu_s)$ as characterized in Proposition 2.10. The blue dashed line is the concave closure of $U_E^2(\mu_s)$, which is denoted by $\hat{U}_E^2(\mu_s)$. Hence we can read off the optimal disclosure system directly from the graph.

Proposition 2.11. *(Optimal information design II): At $T = 0$, the issuer's optimal disclosure policy is:*

1. *If $\mu_s \in [0,\underline{\mu}^*)$, the optimal disclosure system has $\pi_B = \frac{\mu_0(1-\underline{\mu}^*)}{\underline{\mu}^*(1-\mu_0)}$ and $\pi_G = 1$, yielding posteriors $\mu_\ell = 0$ and $\mu_h = \underline{\mu}^*$.*
2. *If $\mu_s \in (\underline{\mu}^*,\overline{\mu}^*)$, the optimal disclosure system has $\pi_G = \frac{\overline{\mu}^*(\mu_0-\underline{\mu}^*)}{\mu_0(\overline{\mu}^*-\underline{\mu}^*)}$ and $\pi_B =$*

$\frac{(1-\overline{\mu}^*)(\mu_0-\underline{\mu}^*)}{(1-\mu_0)(\overline{\mu}^*-\underline{\mu}^*)}$, *yielding posteriors* $\mu_\ell = \underline{\mu}^*$ *and* $\mu_h = \overline{\mu}^*$.

3. *If* $\mu_s \in (\overline{\mu}^*, 1]$, *the optimal disclosure system has* $\pi_B = \frac{\mu_0-\overline{\mu}^*}{\mu_0(1-\overline{\mu}^*)}$ *and* $\pi_G = 0$, *yielding posteriors* $\mu_\ell = \overline{\mu}^*$ *and* $\mu_h = 1$.

4. *If* $\mu_0 = \underline{\mu}^*$ *or* $\overline{\mu}^*$, *the optimal disclosure system has* $\pi_G = \pi_B \in (0, 1)$, *and is therefore completely uninformative, yielding posteriors* $\mu_\ell = \mu_h = \mu_0$.

Again, we have characterized the sender-preferred equilibrium. At the two cut-off posteriors $\underline{\mu}^*$ and $\overline{\mu}^*$, the investment bank is indifferent between declining and underwriting with a planned retention $\frac{\phi}{1+\phi}$. Yet the latter is strictly preferred by the issuer in that she would otherwise fail to issue the security. So we assume that for the sake of the issuer's interest, the investment bank will underwrite when he is indifferent. Here the merit of strategic disclosure lies in that even though an *ex ante* prior $\mu_0 \in (0, \underline{\mu}^*) \cup (\overline{\mu}^*, 1)$ implies failure of issuance owing to the investment bank's unwillingness to underwrite, the optimal disclosure policy is still able to induce the investment bank to underwrite with strictly positive probability. In this sense, strategic disclosure may solve the hold-up problem introduced by the demand shock in the primary market. The other advantage of this disclosure policy manifests in that when uncertainty is higher $\mu_0 \in (\underline{\mu}^*, \overline{\mu}^*)$, the expected issue-price discount is reduced compared with that under no informative disclosure, as is clear from the wedge between the blue dashed line and the black line in Figure 2.7.

Moreover, although the optimal disclosure reduces payoff uncertainty on average, with some particular signal realization, the uncertainty is actually enhanced. For instance, if the prior $\mu_0 \in (0, \underline{\mu}^*)$, an h signal leads to a posterior belief of $\underline{\mu}^*$. Also, $\underline{\mu}^*$ is more uncertain than μ_0 as it has higher entropy. When the signal realization is ℓ, the disclosure is fully revealing and the underlying state is B. The same logic applies to posterior $\overline{\mu}^*$ induced by signal ℓ as it has higher entropy than μ_0 when $\mu_0 \in (\overline{\mu}^*, 1)$. Also, an h signal indicates that the state is G. Thanks to the Bayesian plausibility constraint, the strategic disclosure by the issuer can induce the investment bank to underwrite with positive probability and balances this with a worse belief that leaves the bank's underwriting decision unchanged, which generally improves the issuer's expected payoff. The optimal disclosure is such that, on the one hand, it induces the worst beliefs which lead to the investment bank's withdrawal from underwriting, and on the other hand, it generates signals that make the investment bank just willing to underwrite at the other beliefs. At these beliefs that the underwrite chooses to underwrite, the security's uncertainty is in fact enhanced, and the underwriter's private information becomes sufficiently valuable, although on average the disclosure system reduces the uncertainty compared with the situation at the prior belief. In the meantime, the issuer's expected proceeds from the issue is maximized. In this regard, the optimal disclosure features a mechanism in which the increased payoff uncertainty can offset the loss brought about by the demand uncertainty so that the investment bank will change to the better action that is favored by the issuer.

Nevertheless, a posterior of either $\underline{\mu}^*$ or $\overline{\mu}^*$ does not necessarily mean that the demand risk is alleviated. In fact it is entirely possible that the investment bank will acquire more than his planned retention eventually. Our result sheds some light on the empirically documented "Pipeline Risk" (or "Unfortunate Retention") in leveraged loan syndication by Bruche, Malherbe and Meisenzahl (2018). We have shown that because of the issuer's disclosure policy, even in the presence of demand uncertainty a fully rational investment bank will still agree to underwrite. But when demand shock happens, the investment bank will suffer large losses as a result of excessive retention.

The next proposition provides some empirical predictions that relate the optimal disclosure to various aspects of the primary and secondary markets.

Proposition 2.12. *(Comparative statics II):*

(1) $\frac{\partial \underline{\mu}^*}{\partial \epsilon} > 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial \epsilon} < 0$.

(2) $\frac{\partial \underline{\mu}^*}{\partial \psi} < 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial \psi} > 0$.

(3) $\frac{\partial \underline{\mu}^*}{\partial V_L} > 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial V_L} < 0$.

(4) Recall that $\eta = \frac{\Delta V}{V_L}$, *then* $\frac{\partial \underline{\mu}^*}{\partial \eta} < 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial \eta} > 0$.

(5) $\frac{\partial \underline{\mu}^*}{\partial r} > 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial r} < 0$.

(6) $\frac{\partial \underline{\mu}^*}{\partial \phi} < 0$ *and* $\frac{\partial \overline{\mu}^*}{\partial \phi} > 0$.

From result (1), it is easy to see that as the probability of demand shock in the primary market becomes higher, the two cut-off posteriors shrink inward. So when the prior belief about the security's cash flow is relatively more uncertain (i.e., $\mu_0 \in (\underline{\mu}^*, \overline{\mu}^*)$), higher likelihood of under-subscription results in less transparent disclosure designed by the issuer. Indeed, since the demand shock is more likely to occur in the primary market, in order for the investment bank to at least stay break-even from underwriting the issue, the disclosure should bring in more uncertainty so that his stake carries more trading value with his private information in the secondary market. Also, the additional informed trading profits can offset his expected loss from "unfortunate retention" due to demand shock. Anticipating this, the issuer will employ a relatively more opaque disclosure *ex ante*.

However, when the uncertainty about the security's payoff is relatively low (i.e., $\mu_0 \in (0, \underline{\mu}^*)$ or $\mu_0 \in (\overline{\mu}^*, 1)$), larger ϵ leads to more transparent disclosure. In this case, π_B is smaller, suggesting that the h signal is more indicative of the good state and the ℓ signal is more indicative of the bad state. As in this case, only a marginally higher payoff uncertainty will be enough to compensate for the additional expected loss due to higher probability of demand shock and make the unwilling bank to accept the deal again at the high-uncertainty posterior belief.

Result (2) contrasts with result (1) above: if demand shock happens, a stronger demand (larger ψ), or equivalently, a smaller unfortunate retention (smaller $(1-\psi)$) by the underwriter, expands the two cut-off posteriors outward. Therefore, if demand shock happens, this in turn reduces the additional cost of capital incurred from the investment bank's unfortunate retention and increases his future trading profits thanks to more liquidity traders. As a result, when the *ex ante* payoff uncertainty is relatively large, a more transparent disclosure will be employed in equilibrium as in this case marginally less uncertain cut-off posteriors are enough to make the investment bank indifferent between whether or not to underwrite. Nevertheless, when *ex ante* uncertainty is relatively small, a higher ψ result in less transparent disclosure. Both lower $\underline{\mu}$ and higher $\overline{\mu}$ bring about less transparent disclosure systems for $\mu_0 \in (0, \underline{\mu}^*)$ and $\mu_0 \in (\overline{\mu}, 1)$ respectively. In both cases, due to Bayesian plausibility condition, the probabilities of full revelation will be smaller, and the probabilities of the more uncertain posteriors will be higher, making the systems less informative.

The dichotomy remains valid regarding result (3). Higher V_L (issue size or reservation value of the firm) expands the range of posterior beliefs $(\underline{\mu}, \overline{\mu})$. As V_L grows, it is more costly for the investment bank to underwrite and retain a positive stake. So when prior belief about the uncertainty of the security's payoff is relatively large, marginally more uncertain cut-off posteriors (i.e., higher $\underline{\mu}$ and lower $\overline{\mu}$) should be generated for the system so that the investment bank will be just willing to underwrite. Yet when the *ex ante* payoff uncertainty is relatively small, both higher $\underline{\mu}$ and lower $\overline{\mu}$ result in more transparent disclosure systems for $\mu_0 \in (0, \underline{\mu}^*)$ and $\mu_0 \in (\overline{\mu}, 1)$ respectively. So the probabilities of fully revealing states will be higher, and the probabilities of the more uncertain posteriors will be lower, rendering the systems more informative.

Result (4) asserts that higher growth option (η) gives rises to the expansion of $(\underline{\mu}, \overline{\mu})$. When prior belief about uncertainty is relatively large, as growth option improves, the investment bank will benefit more from his informed sales in the secondary market. Hence the optimal disclosure will be more informative as now marginally less uncertain cut-off posteriors are still able to induce the investment bank to underwrite. When the *ex ante* payoff uncertainty is relatively small, better growth option leads to less informative disclosure. The reasoning is similar to what we have discussed in result (3): less uncertain cut-off posteriors give rise to higher probabilities of high-uncertainty posteriors and lower probability of fully revealing signals. In addition, a less informative information disclosure arises naturally in equilibrium.

With the same token, result (5) states that higher r makes the disclosure system less informative when *ex ante* uncertainty is relatively high, but it leads to less informative disclosure when the uncertainty is relatively low. Higher opportunity cost per unit of investment by the bank makes him less willing to retain a positive stake at the old cut-off posteriors. To induce him to underwrite and compensate his additional cost of capital, posteriors with higher uncertainty must be generated from the optimal system.

Likewise, the implications of result (6) depend on the prior μ_0. When the *ex ante* payoff uncertainty is relatively high, a more liquid secondary market leads to more transparent disclosure. This is because better liquidity in the secondary market allows the underwriter to gain more from trading on his private information. Therefore, a more transparent system, although decreases the value of the investment bank's private information, is still able to make the investment bank just break-even by underwriting the deal. Yet when uncertainty about the firm is relatively low, the disclosure becomes less transparent as the secondary market liquidity increases. Recall that in order to change the investment bank's decision of not underwriting, the system should produce one particular signal which increases the payoff uncertainty to the extent that the investment bank is just willing to serve as an underwriter. As liquidity pumps up, the optimal disclosure only needs to generate a marginally less uncertain high-uncertainty posterior (higher $\underline{\mu}^*$ or lower $\overline{\mu}^*$) such that the bank still wants to underwrite. As a result the disclosure becomes less informative than before.

2.5 Without Short Sale Constraint (NSS)

In this section, we briefly layout the equilibria by relaxing the previous assumption that the underwriter is not allowed to sell the security short in the secondary market. We also assume that short sale does not incur any other cost to the underwriter. As before, we divide into two scenarios: 1. the security can always be fully subscribed by the participant investors even in the absence of underwriter retention; and 2. there is demand uncertainty in the primary market. In face, in case 2, the results on the optimal disclosure we have obtained with short-selling constraint extend to the scenario without the ban on short sale.

2.5.1 No Demand Uncertainty (NDU)

We first consider the case in which there is neither demand uncertainty in the primary market nor ban on short sale in the secondary market. Since the demand for the security will never fall short of the supply, the investment bank is always willing to underwrite.

Proposition 2.13. *(Investment bank's optimal retention): It is optimal for the investment bank to retain zero stake in the primary market regardless of the signal realization (i.e., $\beta^*(\mu_s) = 0$).*

The intuition is fairly straightforward: recall from Part 1 of Proposition 2.2, the underwriting bank's informed trading profits are proportional to the fraction of liquidity traders $(1-\beta)\phi$. Hence such profits are maximized at $\beta = 0$ when the liquidity in the secondary

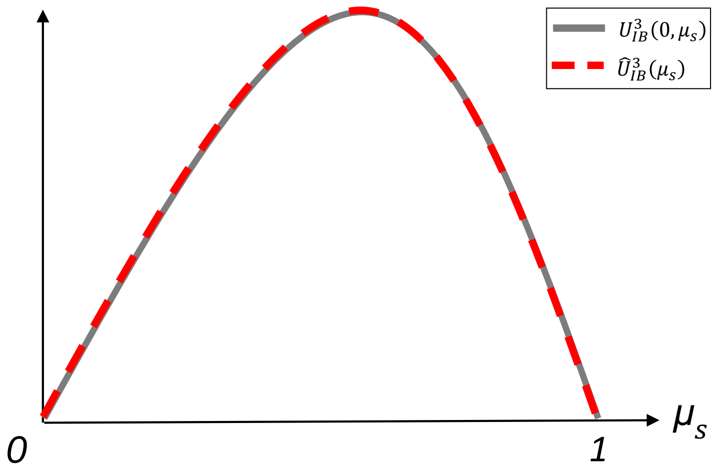


Figure 2.8: The investment bank's payoff (iii)

market is maximized. Since now the underwriter can sell the security short, he no longer has to hold a stake, but is still able to camouflage as liquidity traders. Meanwhile, zero retention is optimal in the primary market in that any positive retention in the primary market would incur an opportunity cost for the investment bank while his gain per share from primary market underpricing is the same as his informed trading profit per share in the secondary market. Hence the investment bank's expected payoff is just his expected trading profits from the secondary market:

$$\hat{U}_{IB}^3(\mu_s) = U_{IB}^3(0, \mu_s) \equiv \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

Figure 2.8 depicts the investment bank's expected payoff as a function of the posterior belief μ_s. Also, given the investment bank's zero retention and short-sale trading strategy, from Part 2 of Proposition 2.2 the issuer's expected proceeds conditional on signal s at $T = 1$ is

$$U_E^3(\mu_s) \equiv (\mu_s\Delta V + V_L) - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

To solve the optimal information design problem faced by the issuer at $T = 0$, it suffices to find the concave closure of $U_E^3(\mu_s)$, which we denote by $\hat{U}_E^3(\mu_s)$. In Figure 2.9, the black line represents $U_E^3(\mu_s)$ and the blue dashed line is its concave closure $\hat{U}_E^3(\mu_s)$. Since $U_E^3(\mu_s)$ is concave on the support of μ_s, the optimal disclosure system is fully revealing.

Proposition 2.14. *(Optimal information design III): At $T = 0$, the issuer's optimal disclosure policy is completely informative, i.e., $\pi_G = 1$ and $\pi_B = 0$, yielding posteriors $\mu_\ell = 0$ and $\mu_h = 1$.*

2.5.2 Demand Uncertainty (DU)

We next explore the scenario where there is demand uncertainty in the primary market.

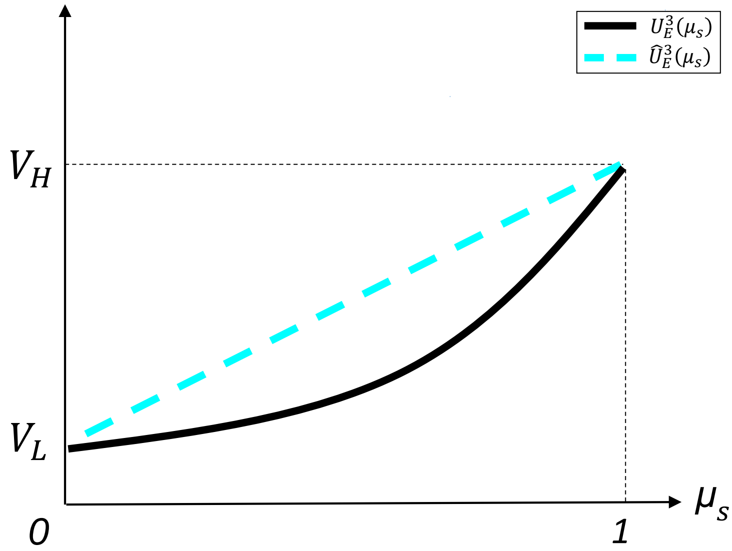


Figure 2.9: The issuer's payoff (iii)

First suppose that the investment bank chooses to underwrite. Then if demand shock does not happen, the investment bank's optimal underwriting, retention and short selling strategy coincides with what we have obtained in the previous subsection. Yet if demand shock happens, the investment bank is forced to acquire a stake of $(1-\psi)$. As he is able to short sell in the secondary market, his planned retention should still be zero before the demand uncertainty is unraveled. His expected payoff from underwriting with zero planned retention is

$$U_{IB}^4(0,\mu_s) \equiv \epsilon \cdot \left\{ (1-\psi)\cdot[(\mu_s \Delta V + V_L) - (1+r)P_0(1-\psi,\mu_s)] + \psi \cdot \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \right\}$$

$$+(1-\epsilon)\cdot\frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)},$$

where $P_0(1-\psi,\mu_s)$ is the issue price defined in Part 2 of Proposition 2.2. The first term above represents the investment bank's expected payoff if demand shock happens while the second is his expected payoff if the demand shock does not occur, both at posterior belief μ_s. The second term is always strictly positive while the first one can be negative for some set of beliefs which are associated with low uncertainty.

Consequently, choosing to underwrite regardless of his posterior belief is not a best response for the investment bank. This is because when the *ex ante* uncertainty about the security's payoff is relatively small, the expected profits from trading on his private information are far from enough to cover the investment bank's opportunity cost of unfortunate retention. Although the bank can always enjoy a strictly positive payoff from short selling when the demand shock does not occur, the investment bank's expected payoff before the resolution of the demand uncertainty under these low-uncertainty beliefs will still be negative. As a result, the investment bank will shy away from underwriting the deal.

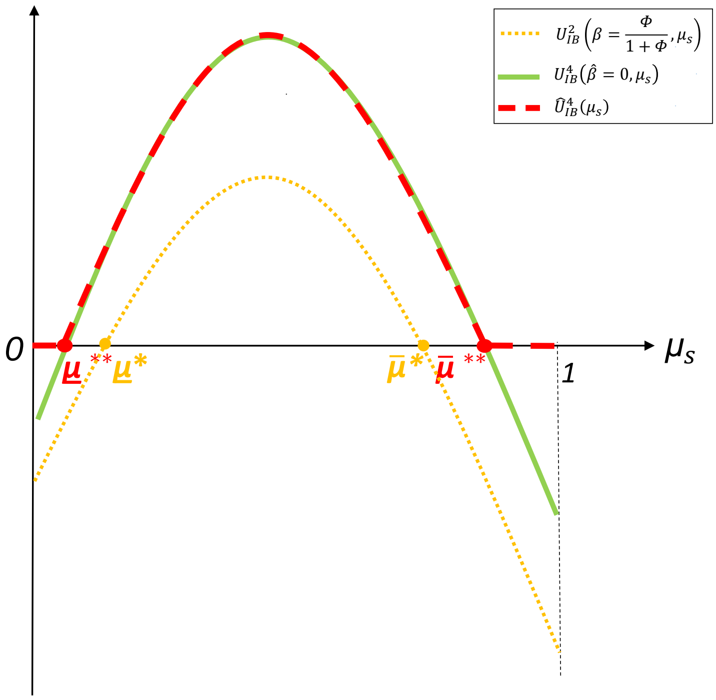


Figure 2.10: The investment bank's payoff (iv)

Lemma 2.4. *(Indifference cut-off posteriors III):*

1. *There exists a pair* $\{\underline{\mu}^{**}, \overline{\mu}^{**}\}$ *with* $0 < \underline{\mu}^{**} < \underline{\mu}^{*} < \frac{1}{2} < \overline{\mu}^{*} < \overline{\mu}^{**} < 1$ *such that* $U_{IB}^4(0, \underline{\mu}^*) = U_{IB}^4(0, \overline{\mu}^{**}) = 0$.

2. $U_{IB}^4(0, \mu_s) > 0$ *if* $\mu_s \in (\underline{\mu}^{**}, \overline{\mu}^{**})$*, and* $\tilde{U}_{IB}(0, \mu_s) < 0$ *if* $\mu_s \in [0, \underline{\mu}^{**})$ *or* $\mu_s \in (\overline{\mu}^{**}, 1]$.

Proposition 2.15. *(Investment bank's optimal strategy and relevant payoffs III): The investment bank's optimal action is*

$$a_{IB}^*(\mu_s) = \begin{cases} \textit{Underwrite and } \hat{\beta}^* = 0 & \textit{if } \mu_s \in [\underline{\mu}^{**}, \overline{\mu}^{**}], \\ \textit{Not Underwrite} & \textit{if } \mu_s \in [0, \underline{\mu}^{**}) \cup (\overline{\mu}^{**}, 1]. \end{cases}$$

His equilibrium payoff is

$$\hat{U}_{IB}^4(\mu_s) = \begin{cases} U_{IB}^4(0, \mu_s) & \textit{if } \mu_s \in [\underline{\mu}^{**}, \overline{\mu}^{**}], \\ 0 & \textit{if } \mu_s \in [0, \underline{\mu}^{**}) \cup (\overline{\mu}^{**}, 1]. \end{cases}$$

In Figure 2.10, the green line depicts $U_{IB}^4(0, \mu_s)$ (i.e., the investment bank's expected payoff from underwriting with zero planned retention) while the red dashed line depicts the investment bank's expected payoff under his optimal underwriting and retention s-trategy. For comparison, the yellow dashed line is the investment bank's expected payoff by underwriting and retaining $\frac{\phi}{1+\phi}$ when there is demand uncertainty yet short sale is not allowed, the scenario that we have discussed in Section 4.2. An interesting observation is that compared with before, even if the issuer does not disclosure additional information,

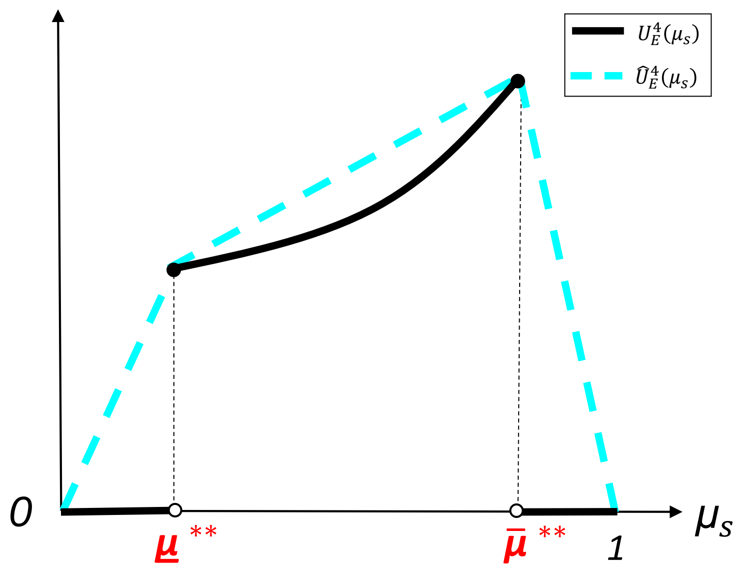


Figure 2.11: The issuer's payoff (iv)

there is a wider range of beliefs under which the investment bank is willing to underwrite. This is because the feasibility of short sale by underwriter enables the investment bank to enjoy positive expected payoffs under two sets of relatively less uncertainty beliefs $(\underline{\mu}^{**}, \underline{\mu}^{*})$ and $(\overline{\mu}^{*}, \overline{\mu}^{**})$. The removal of short sale constraint reduces the total cost of capital due to primary market retention to zero, yet allows the underwriter to trade more intensively on his private information. In turn the indifference cut-off posteriors only need to involve less uncertainty.

Given the optimal strategy of the investment bank, the next proposition follows naturally.

Proposition 2.16. *(Issuer's payoff after information design III):*

1. *When $\mu_s \in [0, \underline{\mu}^{**}) \cup (\overline{\mu}^{**}, 1]$, the investment bank does not underwrite, and $U_E^4(\mu_s) = 0$.*

2. *When $\mu_s \in [\underline{\mu}^{**}, \overline{\mu}^{**}]$, $U_E^4(\mu_s) \equiv U_E^4(\hat{\beta}^* = 0, \mu_s) = (\mu_s \Delta V + V_L) - \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}$.*

Concavification of $U_E^4(\mu_s)$ gives us the optimal disclosure system designed by the issuer at $T = 0$, as illustrated in Figure 2.11.

Proposition 2.17. *(Optimal information design III): At $T = 0$, the issuer's optimal disclosure policy is:*

1. *If $\mu_s \in [0, \underline{\mu}^{**})$, the optimal disclosure system has $\pi_B = \frac{\mu_0(1-\underline{\mu}^{**})}{\underline{\mu}^{**}(1-\mu_0)}$ and $\pi_G = 1$, yielding posteriors $\mu_\ell = 0$ and $\mu_h = \underline{\mu}^{**}$.*

2. *If $\mu_s \in (\underline{\mu}^{**}, \overline{\mu}^{**})$, the optimal disclosure system has $\pi_G = \frac{\overline{\mu}^{**}(\mu_0 - \underline{\mu}^{**})}{\mu_0(\overline{\mu}^{**} - \underline{\mu}^{**})}$ and $\pi_B = \frac{(1-\overline{\mu}^{**})(\mu_0 - \underline{\mu}^{**})}{(1-\mu_0)(\overline{\mu}^{**} - \underline{\mu}^{**})}$, yielding posteriors $\mu_\ell = \underline{\mu}^{**}$ and $\mu_h = \overline{\mu}^{**}$.*

3. *If* $\mu_s \in (\overline{\mu}^{**}, 1]$, *the optimal disclosure system has* $\pi_B = \frac{\mu_0 - \overline{\mu}^{**}}{\mu_0(1-\overline{\mu}^{**})}$ *and* $\pi_G = 0$, *yielding posteriors* $\mu_\ell = \overline{\mu}^{**}$ *and* $\mu_h = 1$.

4. *If* $\mu_0 = \underline{\mu}^{**}$ *or* $\overline{\mu}^{**}$, *the optimal disclosure system has* $\pi_G = \pi_B \in (0,1)$, *and is therefore completely uninformative, yielding posteriors* $\mu_\ell = \mu_h = \mu_0$.

Proposition 2.18. *(Comparative statics III):*

(1) $\frac{\partial \underline{\mu}^{**}}{\partial \epsilon} > 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial \epsilon} < 0$.

(2) $\frac{\partial \underline{\mu}^{**}}{\partial \psi} < 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial \psi} > 0$.

(3) $\frac{\partial \underline{\mu}^{**}}{\partial V_L} > 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial V_L} < 0$.

(4) Recall that $\eta = \frac{\Delta V}{V_L}$, *then* $\frac{\partial \underline{\mu}^{**}}{\partial \eta} < 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial \eta} > 0$.

(5) $\frac{\partial \underline{\mu}^{**}}{\partial r} > 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial r} < 0$.

(6) $\frac{\partial \underline{\mu}^{**}}{\partial \phi} < 0$ *and* $\frac{\partial \overline{\mu}^{**}}{\partial \phi} > 0$.

Note that Proposition 2.17 and 2.18 are identical to what we have obtained in Proposition 2.11 and 2.12. Therefore, all the intuitions go through.

2.6 Welfare Analysis

We have explored the four possible scenarios: 1. (No Short Sale, No Demand Uncertainty), 2. (No Short Sale, Demand Uncertainty), 3. (Short Sale, No Demand Uncertainty), and 4. (Short Sale, Demand Uncertainty). Now suppose that the economy is populated with a continuum of mass 1 issuers with their types μ_0 drawn from a uniform distribution $U[0,1]$, and each issuer invites an investment bank to underwrite.[12]

Let $i \in \{1,2,3,4\}$ denote one of the above four scenarios. Recall that $U_E^i(\mu_0)$ is a type-μ_0 issuer's expected payoff and $\hat{U}_{IB}^i(\mu_0)$ is the relevant investment bank's expected payoff conditional on his prior (or equivalently if the issuer does not disclose additional information). Moreover, $\hat{U}_E^i(\mu_0)$ is the type-μ_0 issuer's maximized expected payoff under optimal disclosure system in scenario i.[13] Since the optimal disclosure always makes the investment bank just break-even at any of the posteriors induced by the signal generated from the optimal system, the investment bank's expected utility will be zero given the issuer's optimal disclosure strategy. Also, because the primary mark price P_0 will always

[12] Alternatively, assume that a generic issuer has type $\mu_0 \sim U[0,1]$. Hence the welfare is just the issuer's expected payoff.

[13] Note that we have already characterized $U_E^i(\mu_0)$, $U_{IB}^i(\mu_0)$, and $\hat{U}_E^i(\mu_0)$, each corresponds to the issuer's expected payoff at $T=1$ given the investment bank's best response (the black solid line in Figure 2.5, 2.7, 2.9, and 2.11), the investment bank's expected payoff at $T=1$ with his optimal underwriting and retention decision (the red dashed line in Figure 2.4, 2.6, 2.8, and 2.10), and the issuer's expected payoff at $T=0$ under the optimal disclosure system (the blue dashed line in Figure 2.5, 2.7, 2.9, and 2.11).

be set to the level such that the general investors only break even, the general investor's equilibrium utility is always zero.

Therefore, if the issuers do not disclose additional information at $T = 0$, their welfare in scenario i is

$$W_E(i) \equiv \int_0^1 U_E^i(\mu_0)\, d\mu_0,$$

and the investment banks' welfare in scenario i is

$$W_{IB}(i) \equiv \int_0^1 \hat{U}_{IB}^i(\mu_0)\, d\mu_0.$$

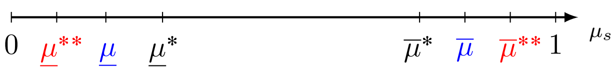
The issuers' welfare with their optimal disclosure policies in scenario i is

$$\hat{W}_E(i) \equiv \int_0^1 \hat{U}_E^i(\mu_0)\, d\mu_0.$$

We first look at the investment banks' welfare if the issuers do not disclose any informative signal. The ranking of their welfare in the four scenarios depends on the probability of demand shocks ϵ.

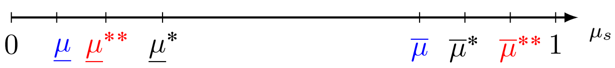
Proposition 2.19. *(Investment banks' welfare): For any posterior* $\mu \in (0, 1)$*,*

(1) When $0 < \epsilon < \frac{\phi}{(1-\psi)(1+\phi)}$,



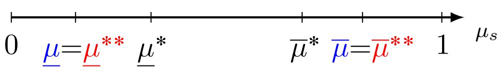
$$W_{IB}(SS, NDU) \geqslant W_{IB}(SS, DU) \geqslant W_{IB}(NSS, NDU) \geqslant W_{IB}(NSS, DU).$$

(2) When $\frac{\phi}{(1-\psi)(1+\phi)} < \epsilon < 1$,



$$W_{IB}(SS, NDU) \geqslant W_{IB}(NSS, NDU) \geqslant W_{IB}(SS, DU) \geqslant W_{IB}(NSS, DU).$$

(3) When $\epsilon = \frac{\phi}{(1-\psi)(1+\phi)}$,



$$W_{IB}(SS, NDU) \geqslant W_{IB}(NSS, NDU) = W_{IB}(SS, DU) \geqslant W_{IB}(NSS, DU).$$

The red, blue, and black cut-offs posteriors represent the threshold beliefs that make the investment bank just break-even as an underwriter in scenarios (SS, DU), (NSS,NDU), and (NSS, DU) respectively. Also, $\{0, 1\}$ are relevant beliefs in scenario (SS,NDU). In general, the more dispersed the cut-off posteriors, the better off the investment banks as a whole. (NSS,DU) is the least desirable. This is because demand uncertainty gives rise to possible unfortunate retention by the investment banks. Furthermore, the ban on short

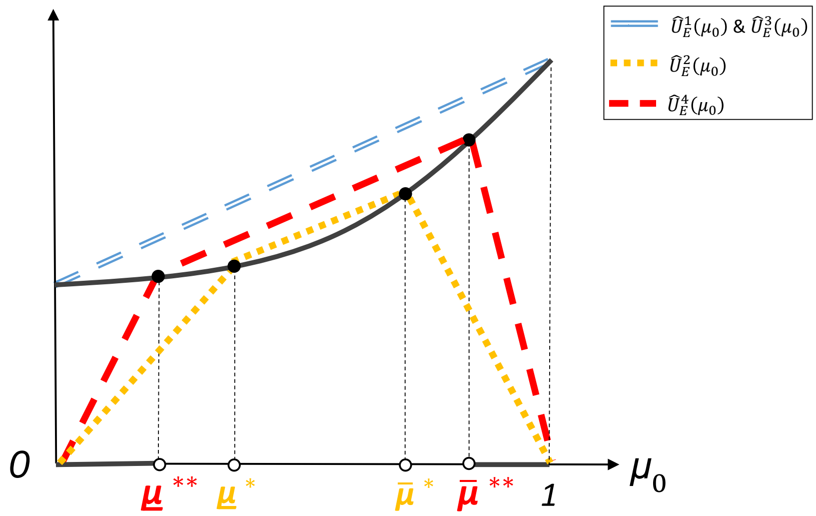


Figure 2.12: The Entrepreneur's Expected Payoffs

sale forces the investment bank to retain a stake so that he can trade strategically. Yet his stake incurs additional cost of bank capital. In contrast, (SS,NDU) renders the investment banks the highest welfare in that they can always sell the security short to gain informed trading profits in the secondary market while they do not have to acquire any stake in the primary market. The comparison between the welfare of the remaining two scenarios is more involved. When ϵ is small (Case (1)), the investment banks' welfare is still higher if short sale is allowed compared to the scenario where there is no demand uncertainty but short selling is banned. Yet when ϵ is large (Case (2)), the investment banks are strictly better off without demand uncertainty even if short sale is prohibited. The trade-off hinges on whether the gain brought about by short sale is able to compensate for the loss due to the demand shock.

Finally, we summarize the rankings of the issuers' welfare in the next proposition.

Proposition 2.20. *(Issuers' Welfare): For any prior $\mu_0 \in \{0,1\}$, if the issuers do not disclosure additional information, their welfare have the following ranking:*

$$W_E(NSS, NDU) > W_E(SS, NDU) > W_E(SS, DU) > W_E(NSS, DU).$$

Yet if they use Bayesian persuasion to maximize their expected proceeds,

$$\hat{W}_E(NSS, NDU) = \hat{W}_E(SS, NDU) > \hat{W}_E(SS, DU) > \hat{W}_E(NSS, DU).$$

A graphical illustration of Proposition 2.20 is given in Figure 2.12. The proposition asserts that if issuers do not reveal informative signals, they achieve the highest welfare when there is no demand uncertainty in the primary market and short sale is not allowed in the secondary market. A primary market without demand uncertainty along with a short selling ban in the secondary market delivers the issuers the second highest welfare. They are worse off if demand shocks may happen in the primary market and underwriters are allowed to sell the security short. Their welfare is the lowest if it is probable that the security will be under-subscribed by participant investors in the primary market and there is short sale constraint in the secondary market. From the perspective of the issuers, they strictly prefer a primary market that has no demand uncertainty. Then the investment banks are always willing to underwrite, and the issuers can sell off their securities with certainty. Absent any possibility of demand shocks, they prefer a secondary market where underwriters are prohibited from short selling the securities. However, if demand is uncertain, the option of short sale allows the investment banks to reduce the opportunity cost associated with primary market retention and gain more from informed trading when demand shocks do not happen. This induces more banks to underwrite and thus enables more issuers to successfully issue their securities.

Under the issuers' optimal persuasion mechanisms, most parts of the ranking remain the same. They still dislike demand uncertainty in the primary market. However, with strategic disclosure the issuers will be indifferent between whether or not there is short sale constraint if there is no demand uncertainty. In both scenarios, the aim of the optimal disclosure is to discourage the investment bank from trading on his private information in the secondary market. To achieve this goal the optimal disclosure needs to be fully informative if short sale is allowed in the secondary market while a partially informative disclosure suffices to do the job if there is the short-sale ban.

2.7 Conclusion

This chapter presents a Bayesian Persuasion model of security offering and trading taking into account issuer's strategic disclosure problem. We show that disclosure can be used to boost the issuer's expected revenue, mitigate underpricing resulting from underwriter's informed trading, and increase the likelihood of security issue even when demand is weak and underwriters may shy away. On average, the optimal disclosure reduces the uncertainty of the security's payoff. Nevertheless, full transparency is not always optimal. Signal realizations that introduce more uncertainty can potentially solve the hold-up problem brought about by demand uncertainty. In general, the optimal information design depends crucially on the *ex ante* level of payoff uncertainty. We provide new empirical predictions which relate the informativeness of the optimal disclosure to the issue size and the issuer's growth option, the underwriter's cost of capital, the uncertainty about demand, and the secondary market liquidity. Moreover, the underwriter in our model can be viewed as an existing blockholder in the firm who makes a decision on whether to support and participate in a security issue (e.g. seasoned debt/equity offering). We show that the blockholder, by participating, may exert governance by exit to push the firm to disclose more transparent information. In sum, corporate finance application of information design theory appears to be a promising topic to work on. Future work can be done by extending our model with issuer's moral hazard and signal manipulation as well as investors' information acquisition. Empirical side, textually analysis of the information memoranda and the prospectuses in both debt and equity issuance can be performed to test the new empirical predictions generated from our model.

Chapter 3

Information Design for Incentive Contracting

3.1 Introduction

It is widely known that in a standard principal-agent environment with hidden action, a principal can often provide pecuniary incentives to encourage an agent to exert effort and implement particular effort levels.

In many cases, a principal can also affect another factor that changes the incentive of an agent exerting effort: information. Information about the usefulness or effectiveness of exerting effort can have important implications on the agent's choice of effort. For example, in a firm, the perspective of that firm, fundamental conditions of the economy, or the ability of a worker can all change that worker's incentive to work hard.

What if a principal can change the information environment that its agent faces, besides providing pecuniary incentives? For example, a firm owner (the principal) can hire an employee (the agent) to work in the firm and generate some output. The output generated is a random variable and the agent's hidden effort could increase the probability of high output. Besides higher effort, another factor that can affect the distribution of outputs is the level of fit between the worker and this firm.[1] This information can be either complementary with, substitute to, or independent of the agent's effort. The firm owner can first arrange an interview with the agent, which is a public experiment that generates and reveals a noisy signal about the level of fit to both players. After the outcome of the experiment, the owner could then offer the employee a contract that specifies his payment based on the output generated.

In this chapter, I aim to explore the following questions: 1) under what conditions can the principal benefit from providing information to the agent? 2) What are the principal's optimal information environment, and the effort level that is implemented

[1] For example, Alonso (2018) considers a model where the level of fit between the worker and the firm determines the value of a match. This level of fit is unobservable to both players. Both sides enter an interview that provides a noisy signal about this information.

given this information environment? 3) Would this lead to any social welfare loss? By answering those questions, we can have better understandings about how a principal could optimally combine providing information and providing pecuniary incentives to incentivize an agent and reduce its costs.

I build a model where the principal first designs a public direct experiment [2] about some state of the world that could affect the effectiveness of the agent's effort. After the result of this experiment is revealed to both players, the principal can then provide a contract to the agent that pays conditioning on the realization of output. Output can be low or high, and the probability of output being high is a function of both the agent's effort and the state. The probability of high output is marginally increasing in higher effort or the probability of state being good, and effort and state could be complements or substitutes. [3] Moreover, I assume both players are risk-neutral but there is limited-liability on the agent's side.

This game can be solved by backward induction: given any experiment and its outcome, we can solve the level of effort the principal prefers to implement and the corresponding optimal contract. Then, given the continuation value for the principal under each possible experiment and its outcome, we can solve the optimal experiment.

I first show that the value of implementing high effort for the principal can be either concave or convex over the common posterior belief about the state. Moreover, it is concave if and only if output is *log-supermodular* in state and effort. This is a stronger condition that state and effort being merely complementary and requires state and effort to be *complementary enough*: the relative ratio of probabilities of high output from a high effort versus low effort has to be higher when the state is good, compared to the case when the state is bad.

I then look at the optimal experiments. I show that if the principal's value of implementing high effort is convex, i.e., state and effort being not highly complementary,[4] a full-revealing experiment is weakly optimal. The optimal experiment would be no information being revealed instead, if the principal prefers to implement high effort regardless of the state and the principal's value of implementing high effort is concave. The more interesting case occurs when it is profitable for the principal to implement high effort when the state is good but not when the state is bad, and principal's value of implementing high effort is concave: the optimal experiment may be *partial obfuscating*, in the sense that bad news will be fully-revealing but not good news. I characterised conditions for the principal's experiment to be fully-revealing, providing no information, or partial obfuscating. I then look at how the principal's optimal experiment will change when various parameters change.

[2] Direct experiment here means the signal space equals to the state space.

[3] In the baseline model, I assumed state and effort are complements. Later I also considered the scenario that state and effort are substitutes.

[4] Here 'not highly complementary' means that output is *log-submodular* in state and effort.

I also consider various extensions to the baseline model. First, restricting the signal space to the state space, i.e., focusing on direct experiments, has no loss of generality. Second, I show that making the principal offer a contract before the experiment outcome is realized will make no difference. Third, contrary to what I previously assumed, I consider the optimal experiment and contract when state and effort are actually substitutes. I show that partial obfuscating is still possible, but in a different way: good news is now fully-revealing, but not bad news. Last, I look at whether potential transparency policies that require some kind of minimum informativeness of the experiment could improve or harm social welfare. I show that more transparency can be welfare-decreasing and we should be cautious before implementing policies that force more transparent or informative experiments.

This chapter contributes to both the moral hazard literature (Mirrlees, 1976, 1999; Holmström, 1979; Grossman and Hart, 1983; Rogerson, 1985) and the information design literature (Aumann and Maschler, 1995; Rayo and Segal, 2010; Kamenica and Gentzkow, 2011; Alonso and Câmara, 2016; Bergemann and Morris, 2016). This chapter aims to combine both streams of literature by considering a framework where a principal can induce an agent to exert unobservable effort by providing both pecuniary incentives and information about the effectiveness of effort at the same time.

Crémer and Khalil (1992) study a principal-agent adverse-selection model where the agent can gather information with cost about the state before deciding whether to accept an offer. They show the principal will offer contracts that leave the agent no incentive to gather its own information.

Boleslavsky and Kim (2018) consider a model where there are three players, a sender who decides on the public experiment, an agent who chooses the distribution of the state through her effort, and a receiver of the experiment. They provide a method to solve the optimal signal under the environment where an experiment will also affect the effort provision of the agent. Rodina (2017) studies how different information structures affect the agent's incentive to exert efforts due to career-concerns. Rodina and Farragut (2016) build a model where a principal can design a signal about an agent's productivity to encourage the agent to make useful investments. Boleslavsky and Cotton (2015) builds a model where schools may choose less informative grading schemes for better placement of their students.

This chapter is different from those papers above in the sense that it allows the principal to provide not only information but also formal contracts that offer pecuniary incentives. In many environments where principal-agent relationships are formal and institutionalized, such as firms and public organisations, it is feasible for the principal to provide both formal contracts relating agent's payment to its performance, and also design a public experiment such as some internal information disclosure rule, auditing system, or interview process. Also, this chapter highlights how factors including the level of complementarity between information and effort affect the experiment chosen by the principal.

Prévet (2018) considers a model with a similar setting to this chapter, where a principal chooses to design an informational disclosure system before offering the agent a contract, and the agent faces limited liability constraints. [5] However, different from Prévet (2018), this chapter focuses on how different *degrees of complementarity/substitutability* determines the optimal experiment and optimal contract. This feature allows us to generate predictions on how the level of complementarity between effort and state can be important in determining both experiments and contract forms, and also the correlation between them. Due to its specific assumption about the productivity of the agent under different effort levels, the output generating process in Prévet (2018) can be thought as a specific case of this chapter where state and effort are log-supermodular. [6] Also, in our model, the constrained social optimal experiment can either require more informative or less informative than the equilibrium experiment, while in Prévet (2018) the social optimal signal is generally more 'vague' than the equilibrium signal.

The structure of this chapter is as follows: section 2 describes the basic settings and assumptions of the baseline model. Section 3 briefly discusses how we will solve the model and introduce some simplifying notations. Section 4 solves the optimal contract given any realization of the experiment. Section 5 solves the optimal experiment. Section 6 conducts comparative static analysis. Section 7 considers some extensions of our baseline model. All the formal proofs are in the Appendix.

3.2 Model Setting

3.2.1 Players, Payoffs, and Actions

There are two players, a principal (P) and an agent (A). The principal can generate some output, y, with the help of the agent. The agent can exert some unobservable effort, e, $e \in \{0, 1\}$. The principal can provide a contract $\{w(y)\}$, $w \in [0, +\infty)$, that pays the agent depending on the realization of output. The outside options of both players should the agent reject the contract are normalized to 0.

The output y has two levels, y_L and y_H. Assume that $0 < y_L < y_H$. The distribution

[5] One important difference is that in Prévet (2018), the author consider a model with continuous state space. Due to the continuous state space, the author introduces an upper bound on the amount ('the Budget') that the principal can award to the agent. This is irrelevant to this chapter. This chapter works on a binary state space but allows a more general output generating process $p(e, \theta)$. The main results of this chapter look into how properties of $p(e, \theta)$ will determine the optimal experiment and the optimal contract.

[6] In Prévet (2018), the author assumes the productivity of the agent if he choose not to exert effort to be p_0, which is constant over the state of world. The agent's productivity of exerting high effort, p, is a random variable with support $\left[\underline{p}, \bar{p}\right]$. In our model, this can be roughly translated as log-supermodularity between effort and state: for $\theta > \theta'$, $p(1, \theta')p(0, \theta) - p(1, \theta)p(0, \theta') = p(1, \theta')p_0 - p(1, \theta)p_0 > 0$. Therefore, in this sense Prevet (2018)'s conclusion about the optimal experiment is a specific case of this paper when state and effort are complementary enough. However, to make a rigorous comparison, a possible future extension would be to look at cases with compact state spaces.

of output is affected by the effort of the agent, e, and a state variable, θ, $\theta \in \{G, B\}$: $p(e,\theta) \equiv Prob(y = y_H|e,\theta)$. The prior probability that $\theta = G$, $\mu_0 \equiv Prob(\theta = G)$, is common knowledge and $\mu_0 \in (0,1)$.

We assume that $p(e,\theta)$ has the following properties:

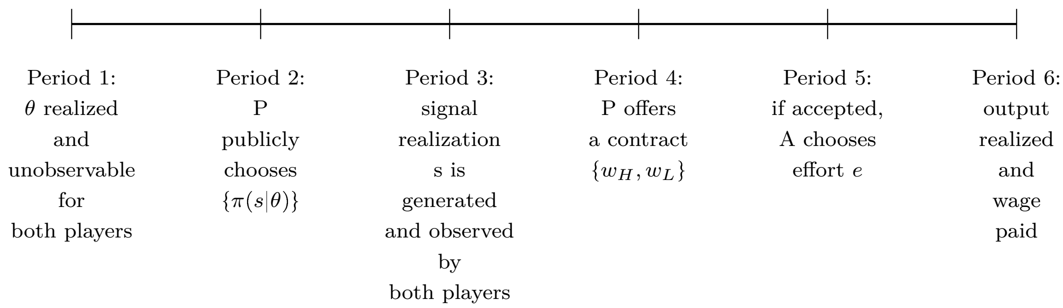
- $p(1,\theta) > p(0,\theta)$
- $p(e,G) > p(e,B)$
- $p(1,G) - p(0,G) > p(1,B) - p(0,B)$

Those assumptions just mean that: 1) higher effort increases the probability of high outcome; 2) better state increases the probability of high outcome; 3) higher effort and better state are complementary.

Denote the payoff function of the principal as $V(e,\theta,y,w)$ and the payoff function of the agent as $U(e,\theta,y,w)$. Assume both principal and agent are risk-neutral: $V(e,\theta,y,w) = y - w$ and $U(e,\theta,y,w) = w - c(e)$. $c(0) = 0$ and $c(1) = c > 0$. Assume that there is limited liability for the agent: $w_H, w_L \geqslant \underline{w}$, and without loss of generality we will just assume $\underline{w} = 0$.

Besides offering a contract, the principal can also costlessly design a direct public experiment, $\{\pi(s|\theta)\}_{s\in\{G,B\}}$. [7] The only restriction on $\{\pi(s|\theta)\}$ is *Bayesian Plausibility*: expected posterior generated by any experiment must equal to its prior.

3.2.2 Timeline



The timeline of the baseline model is as follows: [8]

Period 1: state θ is realized but unobservable to both players.

Period 2: the principal publicly chooses a direct experiment $\{\pi(s|\theta)\}, s \in \{G, B\}$.

Period 3: the realization of the experiment, s, is generated according to $\{\pi(s|\theta)\}$ and the realization of θ. s is publicly observable to both players.

[7] Direct experiment here means the signal space, S, equals to the state space, Θ. Any other signal space S that $|S| = |\Theta|$ would not change anything. Also, in later part of this chapter, I show that it is without loss of generality to focus on signal space S such that $|S| = |\Theta|$.

[8] Whether θ is determined before the principal designs the experiment is inconsequential, as long as θ is not revealed to the Principal and the Agent.

Period 4: the principal offers a contract $\{w_H, w_L\}$ to the agent. If the agent rejects the contract, the game ends and both players get an outside payoff of 0.

Period 5: if the contract is accepted, the agent chooses effort e that is unobservable to the Principal.

Period 6: output is realized and wage is paid to the agent according to the previously agreed contract.

3.2.3 Equilibrium Concept

The equilibrium concept used in this chapter is *Perfect Bayesian Equilibrium* (PBE) similar to Tirole and Fudenberg (1991). Intuitively, Perfect Bayesian Equilibrium is needed here for its "no-signalling-what-you-don't-know" property as in Tirole and Fudenberg (1991). In this chapter, in equilibrium, the optimal contract is generally unique, while the potential contract space is rich (R^{+2}) and off-equilibrium contract space is thus also rich. For weak PBE, There is no restriction on off-equilibrium beliefs. Therefore, the posterior belief of the agent about the state being good when observing an off-equilibrium contract being offer can be any number between 0 and 1.

However, this does not seem to be reasonable: in this model, the principal does not have any private information, and his action should not bring any additional information to the agent about the state being good or bad. Therefore, Perfect Bayesian Equilibrium would be a more appropriate equilibrium concept here: because the principal does not have additional information about the state, the agent's belief should be the same when facing any contract offered by the principal, no matter whether this contract is in equilibrium path or off equilibrium path.

To formally define Perfect Bayesian Equilibrium in this game, we need to first formally define the Game. The formal definitions of the Game can be founded in the Appendix.

Following the traditions of the game theory literature, we will define a fictional player called 'Nature' (N). Nature is the player who chooses the realization of θ, realization of the experiment, s, and the realization of output level. Nature has no payoff function. θ will be nature's private information and the principal's action will not signal any information about nature.

The Bayesian Game is roughly defined as follows:

Period 0: Nature chooses the realization of θ from the prior distribution $\Delta(\Theta)$. Nature privately observes θ. $\Delta(\Theta)$ is common knowledge.

Period 1: the principal publicly chooses a direct experiment $\{\pi(s|\theta)\}, s \in \{G, B\}$.

Period 2: Nature randomly reports the state, according to the design of the experiment, $Prob(s = G|\theta) = \pi(s|\theta)$. s is publicly observable to both players.

Period 3: the principal offers a contract $\{w_H, w_L\}$ to the agent. If the agent rejects the contract, the game ends and both players get an outside payoff of 0.

Period 4: if the contract is accepted, the agents choose effort e that is unobservable to the Principal.

Period 5: Nature chooses the output level, y, according to the output generating process $p(e, \theta)$, the state θ, and the effort e. Wage is paid to the agent according to the previously agreed contract and output level y.

Denote player i's strategy as σ_i, his action at period t as a_{it}, and his belief as μ_i. Denote the continuation strategy of player i after history h_t as $\sigma_i(h_t)$, and the continuation strategies of all the other players after history h_t as $\sigma_{-i}(h_t)$. Denote the tuple of all players' strategies and beliefs as (σ, μ).

Definition 3.1. For any (σ, μ), μ is *reasonable* if: [9]

- μ is decided by σ according to Bayes' Rule whenever possible;
- For any player j, $j \neq i$, its belief about player i at history h_t, $\mu_j(\theta_i|\theta_j, h_t)$, only depends on previous history and player i's action at period t, a_{it}.

Definition 3.2. Any (σ, μ) is a *Perfect Bayesian Equilibrium* if:

- μ is reasonable given that σ;
- Given any history of the game h_t, any non-nature player's continuation strategy $\sigma_i(h_t)$ is optimal given μ and all the other players' continuation strategies, $\sigma_{-i}(h_t)$.

Given this equilibrium concept, our previous concern is solved: given any contract the principal offers, the agent's belief about the state being good is the same. Because the state is Nature's private information, the principal should not signal what he doesn't know. Therefore, given any contract the principal may offer, the agent's belief about the state remains the same.

3.2.4 Tie-breaking Rules

First, I assume that when the principal is indifferent between implementing high effort or not, it will choose to implement high effort with probability 1.

Second, I assume that when the agent is indifferent between exerting high effort or low effort, it will exert high effort with probability 1. I also assume that when the agent is indifferent between accepting the principal's contract or not, it will choose to accept the principal's contract with probability 1. Similar to Kamenica and Gentzkow(2011), I am looking at the *sender-optimal* Perfect Bayesian Equilibrium.

[9] There is only private information for Nature and thus the requirement that posterior beliefs being independent in Tirole and Fudenberg (1991) is not needed here.

3.3 Solving the Model

The baseline model can be solved in two steps by backward induction: first, given the experiment and realization of signal, we can solve the optimal contract the principal will set; then, we can solve the optimal experiment.

Because here an experiment only affects both sides' pay-offs through the posteriors induced, we can just look at the distribution of posteriors instead of the experiment itself. [10] From now on, we will just look at the distribution of posteriors $\{\tau_s, \mu_s\}_{s\in\{G,B\}}$, in which $\tau_s \equiv Prob(\mu = \mu_s)$.

For notation simplicity, define the following notations:

- $\mu_s \equiv Prob(\theta = G|s)$
- $\Delta p(\theta) \equiv p(1,\theta) - p(0,\theta)$
- $\Delta\Delta p \equiv (p(1,G) - p(0,G)) - (p(1,B) - p(0,B))$
- $E_\mu p(e) \equiv (\mu)p(e,G) + (1-\mu)p(e,B)$
- $E_\mu \Delta p \equiv (\mu)\Delta p(G) + (1-\mu)\Delta p(B)$

3.4 Optimal Contract

Given any posterior μ_s, the general optimal contract problem is as follows:

$$\begin{aligned}
&\max_{\{w_H,w_L,e\}} E_{\mu_s}p(e)(y_H - w_H) + (1 - E_{\mu_s}p(e))(y_L - w_L)\\
&s.t.\\
&E_{\mu_s}p(e)w_H + (1 - E_{\mu_s}p(e))w_L - c(e) \geqslant 0 \text{ (IR)}\\
&e \in \operatorname*{argmax}_{e'} E_{\mu_s}p(e')w_H + (1 - E_{\mu_s}p(e'))w_L - c(e') \text{ (IC)}\\
&w_H, w_L \geqslant 0 \text{ (LLC)}
\end{aligned}$$

We will use the standard approach of Grossman and Hart (1983) to solve this optimal contract problem. Given any effort the principal would like to implement, we will first solve the optimal contract that implements that effort level. Then we will compare the values the principal can get under different effort levels, $V(e=1,\mu)$ and $V(e=0,\mu)$. Afterwards, we can solve the value function, $V(\mu) = \max\{V(e=1,\mu), V(e=0,\mu)\}$, and also the optimal contract that implements it.

Given any posterior about the state, μ, if the Principal wants to implement high effort,

[10] See Kamenica and Gentzkow (2011) for more details.

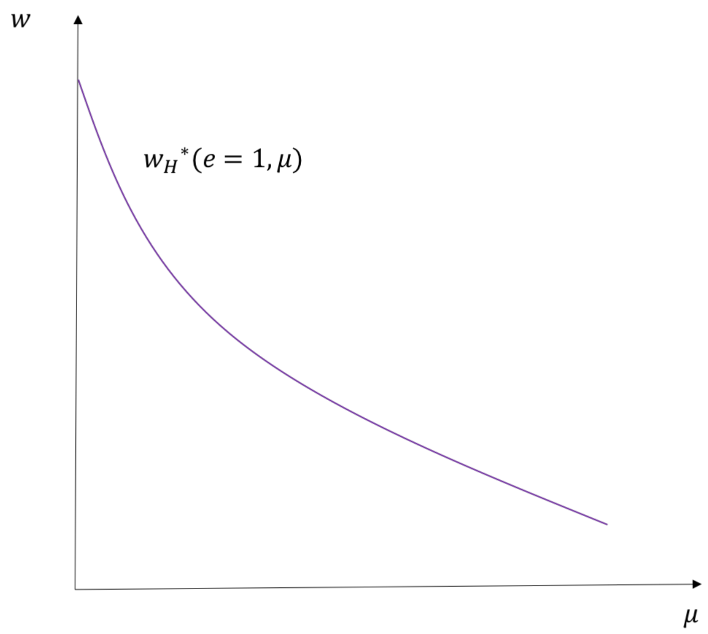


Figure 3.1: High Performance Compensation decreases if state is more likely to be good

then its optimal contract problem is:

$$\max_{w_H, w_L} E_\mu p(e=1)(y_H - w_H) + (1 - E_\mu p(e=1))(y_L - w_L)$$
$$s.t.$$
$$E_\mu p(e=1) w_H + (1 - E_\mu p(e=1)) w_L - c \geqslant 0 \text{ (IR)}$$
$$E_\mu p(e=1) w_H + (1 - E_\mu p(e=1)) w_L - c \geqslant E_\mu p(e=0) w_H + (1 - E_\mu p(e=0)) w_L \text{ (IC)}$$
$$w_H, w_L \geqslant 0 \text{ (LLC)}$$

Under the optimal contract, only the Incentive Compatibility Constraint and the Limited liability Constraint of the low output case are binding. [11] Therefore, his optimal contract under this case would be $w_H^*(e=1) = \frac{c}{E_\mu \Delta p}, w_L^*(e=1) = 0$.

What is important to our further analysis is that $w_H^*(e=1)$ is decreasing over μ. If the state is more likely to be good, $E_\mu \Delta p$ is higher, which means that effort is more likely to be effective. This will reduce the level of compensation needed for the agent to be willing to exert effort.

Given any posterior about the state, μ, if the Principal wants to implement low effort,

[11]The Incentive Compatibility Constraint (IC) with the Limited Liability Constraints (LLC) together imply that the Individual Rationality Constraint is slack. Then if either IC constraint or LLC constraint of the low type is slack, the contract would never be optimal.

then its optimal contract problem is:

$$\max_{w_H, w_L} E_\mu p(e=0)(y_H - w_H) + (1 - E_\mu p(e=0))(y_L - w_L)$$

$$s.t.$$

$$E_\mu p(e=0) w_H + (1 - E_\mu p(e=0)) w_L \geqslant 0 \text{ (IR)}$$

$$E_\mu p(e=0) w_H + (1 - E_\mu p(e=0)) w_L \geqslant E_\mu p(e=1) w_H + (1 - E_\mu p(e=1)) w_L - c \text{ (IC)}$$

$$w_H, w_L \geqslant 0 \text{ (LLC)}$$

The optimal contract if the Principal hopes to implement low effort is $w_H^*(e=0) = w_L^*(e=0) = 0$. By providing a constant wage of zero, the Principal implements low effort in the least costly way.[12]

Lemma 3.1.

The optimal contract if the Principal would like to implement low effort is $w_H^*(e=0) = w_L^*(e=0) = 0$. *The value of him implementing low effort is* $V(e=0,\mu) = E_\mu p(e=0)\,(y_H - y_L) + y_L$.

The optimal contract if the Principal would like to implement high effort is $w_H^*(e=1) = \frac{c}{E_\mu \Delta p}, w_L^*(e=1) = 0$. *The value of him implementing high effort is* $V(e=1,\mu) = E_\mu p(e=1)\left(y_H - y_L - \frac{c}{E_\mu \Delta p}\right) + y_L$.

The optimal value of the principal is $V(\mu) = \max\{V(e=0,\mu), V(e=1,\mu)\}$.

Before we solve the optimal experiment, we need to first establish some properties of the value functions mentioned above.

We will first look at how the values of implementing low and high effort levels change when μ changes.

Lemma 3.2.

- $V(e=0,\mu)$ *is linearly increasing in* μ.
- $V(e=1,\mu)$ *is:*
  - *If* $p(1,G)p(0,B) = p(1,B)p(0,G)$, *then* $V(e=1,\mu)$ *is linearly increasing in* μ.
  - *If* $p(1,G)p(0,B) > p(1,B)p(0,G)$, *then* $V(e=1,\mu)$ *is increasing and strictly concave in* μ.
  - *If If* $p(1,G)p(0,B) < p(1,B)p(0,G)$, *then* $V(e=1,\mu)$ *is strictly convex in* μ. *Moreover,*
    - *If* $y_H - y_L \geqslant -\frac{c[p(1,G)p(0,B) - p(1,B)p(0,G)]}{(p(1,G)-p(1,B))(E_\mu \Delta p)^2}_{|\mu=0}$, *then* $V(e=1,\mu)$ *is increasing in* μ;

[12] Because of the LLC constraints, IR constraint is slack. IC constraint is violated only when the difference between w_H and w_L are too big. Last, it is optimal to set both LLC constraints to be binding, and this would not violate IC constraint.

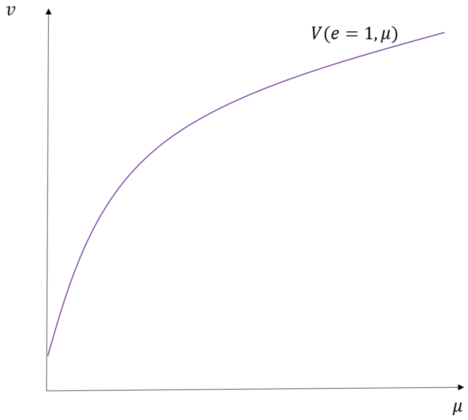


Figure 3.2: $V(e=1,\mu)$ concave: $p(1,G)p(0,B) > p(1,B)p(0,G)$

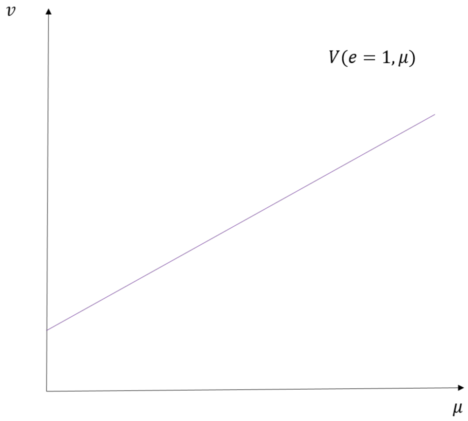


Figure 3.3: $V(e=1,\mu)$ linear: $p(1,G)p(0,B) = p(1,B)p(0,G)$

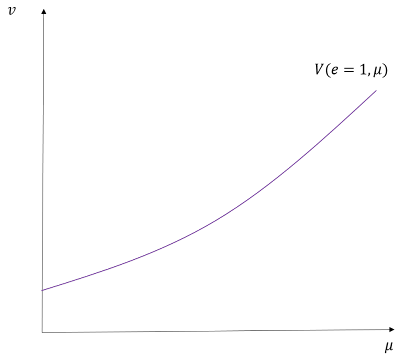
  * *If* $y_H - y_L \leqslant -\frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]}{(p(1,G)-p(1,B))(E_\mu \Delta p)^2}_{|\mu=1}$, *then* $V(e=1,\mu)$ *is decreasing in* μ*;*
  * *If* $-\frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]}{(p(1,G)-p(1,B))(E_\mu \Delta p)^2}_{|\mu=1} < y_H - y_L < -\frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]}{(p(1,G)-p(1,B))(E_\mu \Delta p)^2}_{|\mu=0}$, *then* $V(e=1,\mu)$ *is first decreasing and then increasing in* μ*.*

- $V(e=1,\mu) \geqslant V(e=0,\mu)$ *iff* $y_H - y_L \geqslant \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}$.
- $V(\mu)$ *is increasing in* μ*.*

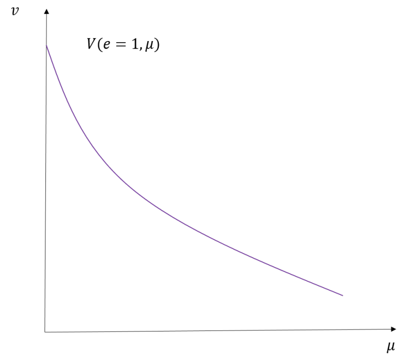
Lemma 3.2 shows that the *log-supermodularity* of $p(e,\theta)$ will have important implications on the monotonicity and concavity/convexity of the $V(e=1,\mu)$, the value for the Principal to implement high effort. By the definition of log-supermodularity, $p(e,\theta)$ is log-supermodular if and only if $p(1,G)p(0,B) > p(1,B)p(0,G)$.

As long as $p(0,B) > 0$, $p(1,G)p(0,B) - p(1,B)p(0,G) > 0$ is equivalent to $\frac{p(1,G)}{p(0,G)} > \frac{p(1,B)}{p(0,B)}$, which means that being in a good state would make high effort *exponentially* more effective. Notice that this condition is stronger than our assumption that higher state being complementary to higher effort: $\frac{p(1,G)}{p(0,G)} > \frac{p(1,B)}{p(0,B)}$ is equivalent to $\frac{p(1,G)-p(0,G)}{p(1,B)-p(0,B)} > \frac{p(0,G)}{p(0,B)}$, which is stricter than $\frac{p(1,G)-p(0,G)}{p(1,B)-p(0,B)} > 1$.

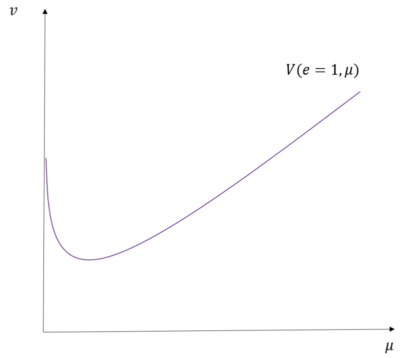
I find that the value of implementing high effort, $V(e=1,\mu)$ can be decreasing over μ: better news can sometimes reduce the value of implementing high effort. The intuition



(a) $V(e=1,\mu)$ increasing



(b) $V(e=1,\mu)$ decreasing



(c) $V(e=1,\mu)$ first decreasing and then increasing

Figure 3.4: $V(e=1,\mu)$ convex: $p(1,G)p(0,B) < p(1,B)p(0,G)$,different scenarios

is as follows: when μ increases, $E_\mu p(e=1)$, the effect of higher effort increases, but $E_\mu p(e=0)$ also increases. $E_\mu \Delta p = E_\mu p(e=1) - E_\mu p(e=0)$ still increases over μ, but the growth rate can be low if $E_\mu p(e=0)$ grows very fast. $E_\mu \Delta p$ is important in determining the agency cost, and when it increases slowly over μ, this means the agency cost is not very responsive to the change of information. All else equal, $E_\mu \Delta p$ increases faster if there is stronger complementary between state and effort.

The total cost of implement high effort,$\frac{cE_\mu p(e=1)}{E_\mu \Delta p}$ is composed of two parts, $E_\mu p(e=1)$, the frequency of good outcome, and $\frac{c}{E_\mu \Delta p}$, the cost to incentive worker or the agency cost. When μ increases, a good outcome is more likely to happen and therefore compensation is more likely to happen. At the same time, $\frac{c}{E_\mu \Delta p}$ will decrease as μ increases. Therefore, whether the total cost increases or decreases will depends on which force is dominating. If $\frac{c}{E_\mu \Delta p}$ decreases very slowly, which happens when the complementary between state and effort is low, the total cost will go up if μ increases. And this could lead to the value of implementing the high effort to decrease when μ decreases, if the value of high effort is relatively small compared to the cost of implementing it.

Although $V(e=1,\mu)$ can be non-monotonic over μ, $V(\mu)$ is always increasing over μ. This is because that first of all, $V(e=0,\mu)$ is increasing over μ; second, whenever

$V(e=1,\mu)$ is decreasing over μ, it is strictly smaller than $V(e=0,\mu)$. [13] Therefore, we can focus on the ranges of $V(e=1,\mu)$ that is increasing in μ.

I also find that $V(e=1,\mu)$ is concave if $p(e,\theta)$ is log-supermodular and convex if $p(e,\theta)$ is log-submodular. The total expected revenue from implementing high effort, $E_\mu(y_H - y_L)$, is linear in μ. The total expected cost $\frac{cE_\mu p(e=1)}{E_\mu \Delta p}$ equals to $E_\mu p(e=1)$ times $\frac{c}{E_\mu \Delta p}$. $E_\mu p(e=1)$ is also linear in μ while $\frac{c}{E_\mu \Delta p}$ is convex in μ. For the total cost to be convex and thus the value function to be concave, $\frac{c}{E_\mu \Delta p}$'s second-order derivative has to be large enough compared to its first-order derivative, which is equivalent to high enough complemenrtarity in our model.

Now we can characterize how many intersections points $V(e=0,\mu)$ and $V(e=1,\mu)$ can possibly have.

Lemma 3.3.
- *If $p(1,G)p(0,B) = p(1,B)p(0,G)$, then $V(e=1,\mu)$ either has no intersection with $V(e=0,\mu)$, or cross $V(e=0,\mu)$ from below for once.*
- *If $p(1,G)p(0,B) > p(1,B)p(0,G)$, then $V(e=1,\mu)$ either has no intersection with $V(e=0,\mu)$, or cross $V(e=0,\mu)$ from below for once.*
- *If $p(1,G)p(0,B) < p(1,B)p(0,G)$, then $V(e=1,\mu)$ can either has no intersection with $V(e=0,\mu)$, cross $V(e=0,\mu)$ from below or above for once, or cross $V(e=0,\mu)$ first from below and then from above.*

There are multiple cases that could happen when $V(e=1,\mu)$ is convex. However, under those different cases the implied optimal experiment turns out to be the same. We will show in the next section why it is the case. Formal results about some properties of $V(e=1,\mu)$ when it is convex can be found in the appendix.

What really matters is that when $V(e=1,\mu)$ is concave, it can only cross $V(e=0,\mu)$ for at most once and it must cross $V(e=0,\mu)$ from below. $V(e=1,\mu)$ is higher than $V(e=0,\mu)$ if the expected benefit of switching from low effort to high effort, $E_\mu \Delta p(y_H - y_L)$, is higher than the expected cost $E_\mu p(e=1)\frac{c}{E_\mu \Delta p}$. When state and effort are highly complementary, if μ increases, $E_\mu \Delta p$ would grow in fast speed and hence the ratio of benefit/cost would also be increasing.

Now we can characterize the value function of the Principal when the $V(e=1,\mu)$ is weakly concave, which happens when $p(1,G)p(0,B) \geqslant p(1,B)p(0,G)$:

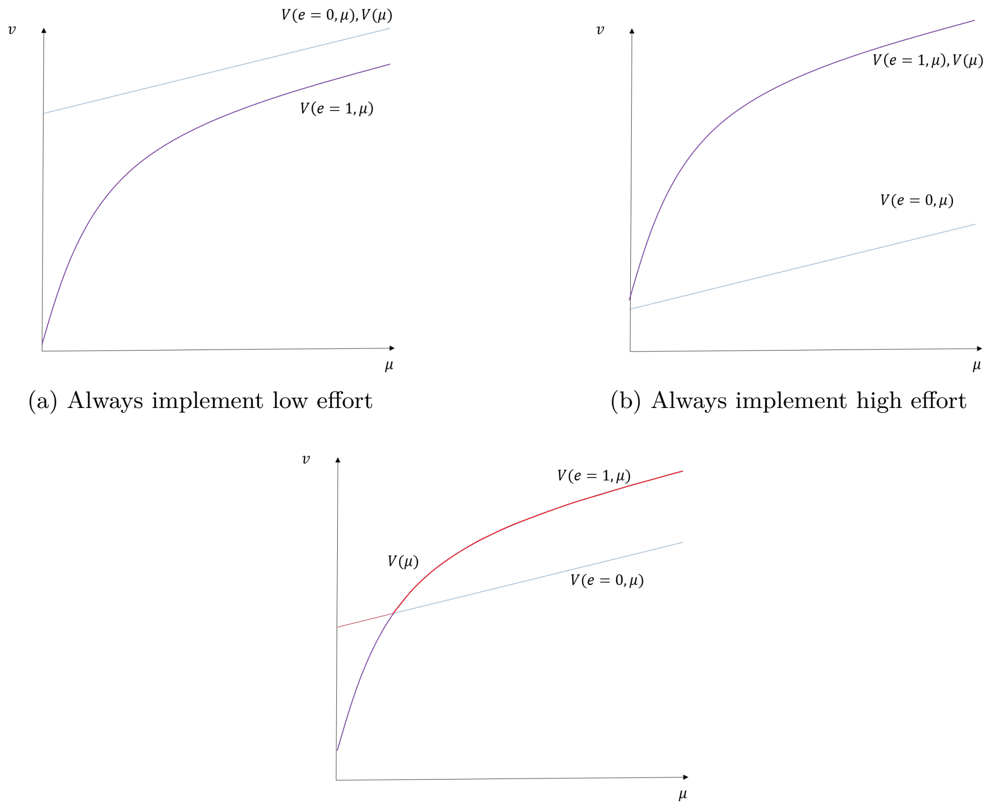
Proposition 3.1. *If $p(1,G)p(0,B) \geqslant p(1,B)p(0,G)$, then $V(e=1,\mu)$ is strictly increasing and weakly concave. Moreover,*

- *if $y_H - y_L \leqslant \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}_{|\mu=1}$, $V(\mu) = V(e=0,\mu)$.*

[13] This is because that $V(e=1,\mu)$ is decreasing over μ only if $y_H - y_L - \frac{c}{E_\mu \Delta p} < 0$. However, then $V(e=1,\mu) = E_\mu p(e=1)\left(y_H - y_L - \frac{c}{E_\mu \Delta p}\right) + y_L < E_\mu p(e=0)\left(y_H - y_L - \frac{c}{E_\mu \Delta p}\right) + y_L < y_L < E_\mu p(e=0)(y_H - y_L) + y_L = V(e=0,\mu)$.

- *if* $y_H - y_L \geqslant \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}\Big|_{\mu=0}$, $V(\mu) = V(e=1,\mu)$.

- *if* $\frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}\Big|_{\mu=1} < y_H - y_L < \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}\Big|_{\mu=0}$, *then there exist a* $\bar{\mu} \in (0,1)$ *that* $y_H - y_L = \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}\Big|_{\mu=\bar{\mu}}$, *and* $V(\mu) = V(e=0,\mu)$ *if* $\mu \leqslant \bar{\mu}$ *and* $V(\mu) = V(e=1,\mu)$ *if* $\mu > \bar{\mu}$.



(a) Always implement low effort

(b) Always implement high effort

(c) High effort if and only if good state is likely

Figure 3.5: $V(\mu) = \max\{V(e=1,\mu), V(e=0,\mu)\}$, when $V(e=1,\mu)$ is concave: different scenarios

3.5 Optimal Experiment

We have solved the optimal contract that the principal would provide given any posterior μ, and his value from offering this optimal contract. Now we can solve the optimal experiment by solving the optimal distribution of posteriors that can be generated by some experiment.

The optimal experiment problem:

$$\max_{\mu_G,\mu_B,\tau} \tau V(\mu_G) + (1-\tau)V(\mu_B)$$

$$\text{s.t.}$$

$$0 \leqslant \mu_G, \mu_B, \tau \leqslant 1$$

$$\tau\mu_G + (1-\tau)\mu_B = \mu_0$$

First, some definitions about the informativeness/precision of an experiment:

Definition 3.3.

- An experiment is *partial obfuscating* if there exist posteriors μ_s, μ_t, and $Prob(\mu_s) > 0$, $Prob(\mu_t) > 0$, such that
  - $\mu_s \in (0,1)$
  - $\mu_t \neq \mu_s$
- An experiment *provides no information* if for any posteriors μ_s, μ_t, such that $Prob(\mu_s) > 0$, $Prob(\mu_s) > 0$, $\mu_s = \mu_t = \mu_0$
- An experiment is *fully revealing* if for any posterior μ_s such that $Prob(\mu_s) > 0$, $Prob(\mu_s)$ must be either 0 or 1.

Kamenica and Gentzkow (2011) showed that the optimal experiment is fully revealing if the value function of the sender is globally strictly convex and the optimal experiment is providing no information if sender's value function is globally strictly concave.

Lemma 3.4.
- *If* $y_H - y_L \leqslant \min_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}$, $V(\mu) = V(e=0,\mu)$, *then any experiment would be weakly optimal.*
- *If* $p(1,G)p(0,B) \leqslant p(1,B)p(0,G)$ *and* $y_H - y_L > \min_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}$, *then any optimal experiment must be fully revealing.*
- *If* $p(1,G)p(0,B) > p(1,B)p(0,G)$ *and* $y_H - y_L \geqslant \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}_{\mu=0}$, *then any optimal experiment must provide no information.*

Because the maximum of two convex functions must be a convex function, when $V(e = 1,\mu)$ is convex then fully revealing must be weakly optimal for the Principal.

This lemma leaves only one case: $p(1,G)p(0,B) > p(1,B)p(0,G)$, and $\frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}_{|\mu=1} < y_H - y_L < \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}_{\mu=0}$. From previous lemmas, this means that $V(e = 1,\mu)$ crosses $V(e=0,\mu)$ at some $\bar{\mu} \in (0,1)$ from below.

Again, following Kamenica and Gentzkow (2011), to solve the optimal experiment, we just need to find the convex hull of the graph $(\mu, V(\mu))$.

Proposition 3.2. *Assume that* $y_H - y_L > \min_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}$, *then:*

1. If $p(1,G)p(0,B) \leqslant p(1,B)p(0,G)$, *then the optimal distribution of posteriors is* $\{\tau(\mu_G) = \mu_0, \tau(\mu_B) = 1-\mu_0, \mu_B = 0, \mu_G = 1\}$, *and the probability high effort is implemented is*

$$
\begin{aligned}
Prob(e^* = 1) = {} & \mathbb{1}(V(e=1,\mu=1) \geqslant V(e=0,\mu=1))\mu_0 \\
& + \mathbb{1}(V(e=1,\mu=0) \geqslant V(e=0,\mu=0))(1-\mu_0)
\end{aligned}
$$

The optimal value of the principal is $\tilde{V}(\mu_0) = \mu_0 V(\mu=1) + (1-\mu_0)V(\mu=0)$.

2. If $p(1,G)p(0,B) > p(1,B)p(0,G)$*, and* $y_H - y_L \geqslant \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}_{\mu=0}$*, then the optimal distribution of posteriors is* $\{\tau(\mu_G) \in [0,1], \tau(\mu_B) = 1-\tau(\mu_G), \mu_B = \mu_G = \mu_0\}$*.*
$Prob(e^* = 1) = 1$
The optimal value of the principal is $\tilde{V}(\mu_0) = V(\mu_0)$*.*

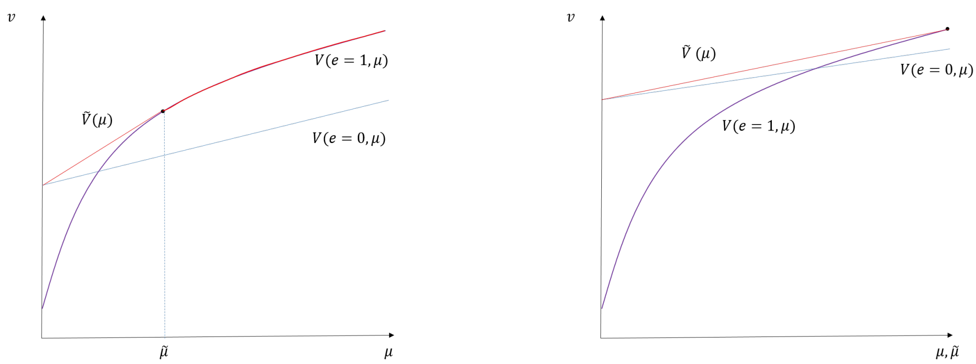
3. If $p(1,G)p(0,B) > p(1,B)p(0,G)$ *and* $\frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}_{|\mu=1} < y_H - y_L < \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}_{\mu=0}$*, then there exist* $\tilde{\mu} \in [0,1]$*, such that* $\tilde{\mu} \in \operatorname{argmax}_\mu \frac{V(e=1,\mu)-V(e=0,0)}{\mu-0}$*, and the optimal distribution of posterior is as follows:*

$$\{\tau*, \mu*\} = \begin{cases} \left\{\tau(\mu_G) = \frac{\mu_0}{\tilde{\mu}}, \tau(\mu_B) = 1-\tau(\mu_G), \mu_B = 0, \mu_G = \tilde{\mu}\right\}, \text{if } \mu_0 \leqslant \tilde{\mu} \\ \{\tau(\mu_G) \in [0,1], \tau(\mu_B) = 1-\tau(\mu_G), \mu_B = \mu_G = \mu_0\}, \text{if } \mu_0 > \tilde{\mu} \end{cases}$$

$\tilde{\mu} = \min\{\tilde{\mu}^I, 1\}$*, in which* $\tilde{\mu}^I$ *is solved by*
$$-(p(1,B)-p(0,B))(y_H - y_L) + \left(\frac{cE_\mu p(e=1)}{E_\mu \Delta p} + \frac{c(p(1,G)p(0,B)-p(1,B)p(0,G))}{(E_\mu \Delta p)^2}\right)_{|\mu=\tilde{\mu}^I} = 0.$$

$Prob(e^* = 1) = \min\left\{\frac{\mu_0}{\tilde{\mu}}, 1\right\}$*.*

The optimal value of the principal,
$\tilde{V}(\mu_0) = \mathbb{1}(\mu_0 \leqslant \tilde{\mu})\left\{V(e=0,\mu=0) + \frac{V(e=1,\tilde{\mu})-V(e=0,\mu=0)}{\mu-0}(\mu_0 - 0)\right\}$
$+ \mathbb{1}(\mu_0 > \tilde{\mu})V(e=1,\mu_0)$*.*



(a) Partial Obfuscation

(b) Fully Revealing

Figure 3.6: Optimal Experiment when $V(e=1,\mu)$ is concave

When $V(e=1,\mu)$ is concave, if the principal would like to, or is forced to implement high effort, then his optimal experiment will reveal no information. Similarly, because $V(e=0,\mu)$ is weakly concave, revealing no information is also weakly optimal if the principal chooses to always implement low effort. However, if the effort he would like to prefer depends on the state, an optimal experiment could actually be fully revealing or partial obfuscation, even when both $V(e=1,\mu)$ and $V(e=0,\mu)$ are concave.

3.6 Comparative Static Analysis

We can now look at how various variables of interest are affected by parameters in the model.

Variables that may be of interest:

- μ_G and μ_B, posteriors under different signal realization and 'quality of information' [14]

- The probability of agent exerting high effort, $Prob(e^* = 1)$

And parameters in the model: y_L, $y_H - y_L$, c, μ_0, $p(e, \theta)$.

There are multiple ways we can look at how the change of $p(e, \theta)$ affects variable affect variables of interest. One way could be looking at keeping other cases the same, how a change of $p(e, \theta)$ at a particular (e, θ) can affect the variables of interest.

A more interesting way could be looking at what happens if the production process becomes 'noisier'. Here we will look at an incomplete order that one production technology is 'noisier than' another production technology, with the following definition:

Definition 3.4. A production function $\{\tilde{p}(e,\theta)\}_{(e,\theta)\in\{(0,0),(1,0),(0,1),(1,1)\}}$ is *noisier than* another $\{p(e,\theta)\}_{(e,\theta)\in\{(0,0),(1,0),(0,1),(1,1)\}}$ if $\tilde{p}(e,\theta) = ap(e,\theta) + (1-b)(1-p(e,\theta))$, $\frac{1}{2} < a, b < 1$.

This incomplete order, noisier than, is defined similarly as the classical concept of *Blackwell informativeness*: the likelihood function of the more noisy production function can be written as a linear transformation of the less noisy production function.

Given our assumptions, the order of $\{p(e,\theta)\}_{(e,\theta)\in\{(0,0),(1,0),(0,1),(1,1)\}}$ is preserved by $\{\tilde{p}(e,\theta)\}_{(e,\theta)\in\{(0,0),(1,0),(0,1),(1,1)\}}$: $\tilde{p}(e,\theta) < \tilde{p}(e',\theta')$ *iff* $p(e,\theta) < p(e',\theta')$. However, being noisier means the production function is less response to the change of (e,θ) and reflects more about randomness: for any (e,θ) and (e',θ'), $|\tilde{p}(e,\theta) - \tilde{p}(e',\theta')| = (a+b-1)|p(e,\theta) - p(e',\theta')| < |p(e,\theta) - p(e',\theta')|$.

Proposition 3.3. *If* $p(1,G)p(0,B) > p(1,B)p(0,G)$ *and* $\frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}_{|\mu=1} < y_H - y_L < \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}_{\mu=0}$, *then under the optimal experiment,*

- μ_G *is weakly decreasing in* $y_H - y_L$

- μ_G *is weakly increasing in* c

- μ_G *is weakly higher if the production process becomes noisier.*

- $Prob(e = 1)$ *is weakly increasing in* μ_0

[14]Formally, we can define the quality of information as reduction of uncertainty using measures like information entropy. However, notice that in our model, besides the case that any experiment is weakly optimal, in general under optimal experiment $\mu_B = 0$. Therefore, we can just focus on the changes of μ_G instead.

- *$Prob(e=1)$ is weakly increasing in $y_H - y_L$*
- *$Prob(e=1)$ is weakly decreasing in c*
- *$Prob(e=1)$ is weakly lower if the production process becomes noisier.*

From previous results, when would information quality be the highest? Information quality would be the highest first when complementarity between information and effort is weak, i.e., when $\frac{p(1,G)}{p(0,G)} \leqslant \frac{p(1,B)}{p(0,B)}$. When complementarity between information and effort is strong, i.e., when $\frac{p(1,G)}{p(0,G)} > \frac{p(1,B)}{p(0,B)}$, information quality would be higher when the benefit/cost ratio of effort is small, and the production process is noisier.

Especially, *partial obfuscation* happens when information is important, the benefit/cost ratio of effort is intermediately and the production process is intermediately noisier, and the prior probability of a state being good is not very high.

3.7 Extensions

3.7.1 More General Signal Space

I assumed before that the signal space S equals to the state space Θ. This assumption turns out to be innocuous. In this part, I will always assume that $|S| \geqslant |\Theta|$.

First, it is obvious that as long as $|S| = |\Theta|$, it does not matter whether $S = \Theta$ or not. We can always find a one-to-one relationship between elements of S and elements of Θ and changing the signal space from S to Θ is merely relabelling.

Kamenica and Gentzkow (2011) show in their online appendix that in their model, they can restrict the state space to $|S| = \min\{|\Theta|, |A|\}$ without any loss of generality. Different from Kamenica and Gentzkow (2011), in our model, it is not enough to require $|S| \geqslant |A|$. This is due to the fact that the principal would first offer a contract before the agent chooses its action, and can always implement high or low effort if it wants to. The value of offering a contract for the Principal would thus depend on the probability distribution of outcomes and the compensation needed, which are both functions of the distribution of the state. Therefore, the value of offering a contract for the Principal is a function of the distribution of Θ but not the action. However, there proof on why $|S| = |\Theta|$ is sufficient [15] can still be directly applied to our case so we will not provide a proof here.

3.7.2 Timing of Offering Contract

I assumed that the principal first designs the experiment and then offers a contract after observing the realization of the experiment. Would it be different if the principal can already offer a contract before the result of the experiment is realized ?[16]

[15] See Proposition 4 of their online appendix, "Bayesian Persuasion Web Appendix".

[16] I assume here that there is no contract incompleteness and when the principal writes down a contract, the terms of that contract can depend on all observable variables.

The answer is no. The intuition is: regardless of the timing of offering contract in this model, there is no information asymmetry between the principal and the agent. Therefore, offering a contract after the experiment has realized will not give the principal any additional information advantage over the agent. After the experiment outcome is realized, the principal just chooses the optimal contract only based on the public information generated by the experiment.

If the principal chooses a contract before an experiment is realized, he could make the terms of the contract completely depending on the realization of the signal. Under each realization, he will always choose the ex-post optimal contract, based on all available information conditional on that realization of the signal. Given that he already chooses the ex-post optimal contract for each scenario ex-ante, even if we allow renegotiation, the result would be completely the same.

If the principal chooses the contract before the realization of signal, the contract he can offer is in general in the form of $\{w_H(s), w_L(s)\}_{s\in\{G,B\}}$. Then given any experiment $\{\mu_G, \mu_B, \tau\}$, the optimal contract problem is thus:

$$
\begin{aligned}
\max_{\{w_H(s),w_L(s),e(s)\}_{s\in\{G,B\}}} \quad & \tau\left[E_{\mu_G}p(e(G))(y_H - w_H(G)) + (1 - E_{\mu_G}p(e(G)))(y_L - w_L(G))\right] \\
& + (1-\tau)\left[E_{\mu_B}p(e(B))(y_H - w_H(B)) + (1 - E_{\mu_B}p(e(B)))(y_L - w_L(B))\right] \\
& s.t. \\
& E_{\mu_s}p(e(s))w_H(s) + (1 - E_{\mu_s}p(e(s)))w_L(s) - c(e(s)) \geqslant 0, s = G, B \\
& e(s) \in \operatorname*{argmax}_e E_{\mu_s}p(e)w_H(s) + (1 - E_{\mu_s}p(e))w_L(s) - c(e), s = G, B \\
& w_H(s), w_L(s) \geqslant 0, s = G, B
\end{aligned}
$$

Given any distribution of posterior generated by the experiment, μ_G, μ_B, τ, the terms of the contract if the experiment outcome is 'Good', $\{w_H(G), w_L(G), e(G)\}$, will have no effect on the principal's payoff function and the constraints when $s = B$ (and vice versa). Therefore, we could solve two cases separately. However, the results would then be exactly the same as the case that the principal chooses the optimal contract only after s is realized.

Another interesting question is whether the order between writing a contract and designing an experiment matters.[17] The answer is no. That is simply because for a function $f(x, y)$, if $\max_{(x,y)} f(x, y)$,$\max_y f(x, y)$ for any x, and $\max_x f(x, y)$ for any y all exists, then $\max_{(x,y)} f(x, y) = \max_x \max_y f(x, y) = \max_y \max_x f(x, y)$.

In summary, the results of this chapter are robust to the timing of the Principal offering contract and designing experiment.

[17] Again, this is based on the assumption that experiment outcome is contractible.

3.7.3 Good State Being Substitute to High Effort

I have assumed that good state is complementary with high effort, i.e., that $p(1,G) - p(0,G) > p(1,B) - p(0,B)$. What if good state is actually substitute to high effort, i.e., that $p(1,G) - p(0,G) < p(1,B) - p(0,B)$? [18]

In this session, we will assume that a good state is actually a substitute for a high effort, while everything else is the same. [19]

First, given any distribution, the optimal way to implement low effort is still $w_L = w_H = 0$. The principal's value of implementing low effort is $V(e=0,\mu) = E_\mu p(e=0)\,(y_H - y_L) + y_L$.

To implement high effort, the optimal contract is still $w_H^*(e=1) = \frac{c}{E_\mu \Delta p}$, $\quad w_L^*(e=1) = 0$. What is different here is that now $w_H^*(e=1)$ will increase with μ. The principal's value of implementing high effort is $V(e=1,\mu) = E_\mu p(e=1)\left(y_H - y_L - \frac{c}{E_\mu \Delta p}\right) + y_L$.

We can then show that $V(e=0,\mu)$ is increasing and linear in μ.

$V(e=1,\mu)$ can be increasing or decreasing in μ and is always concave in μ. Also, $\frac{y_H - y_L}{c} - \frac{E_\mu p(e=1)}{(E_\mu \Delta p)^2}$ is decreasing over μ, so $V(e=1,\mu)$ either has no intersection with $V(e=0)$, or crosses $V(e=0,\mu)$ from above for at most once.

We can now establish the optimal experiment when good state and high effort are substitutes:

Corollary 3.1.

- *If* $\frac{y_H - y_L}{c} \leqslant \frac{E_\mu p(e=1)}{(E_\mu \Delta p)^2}_{|\mu=0}$, *then* $V(\mu) = V(e=0,\mu)$ *for any* μ, *and any experiment would be weakly optimal.*
- *If* $\frac{y_H - y_L}{c} \geqslant \frac{E_\mu p(e=1)}{(E_\mu \Delta p)^2}_{|\mu=1}$*, then* $V(\mu) = V(e=1,\mu)$ *for any* μ, *and any optimal experiment must provide no additional information compared to the prior distribution.*
- *If* $\frac{E_\mu p(e=1)}{(E_\mu \Delta p)^2}_{|\mu=0} < \frac{y_H - y_L}{c} < \frac{E_\mu p(e=1)}{(E_\mu \Delta p)^2}_{|\mu=1}$, *then there exist* $\tilde{\tilde{\mu}} \in [0,1]$, *such that* $\tilde{\tilde{\mu}} \in \operatorname{argmax}_\mu \frac{V(e=1,\mu) - V(e=0,\mu=1)}{1-\mu}$, *and the optimal distribution of posterior is as follows:*

$$\{\tau*, \mu*\} = \begin{cases} \left\{\tau(\mu_G) = 1 - \tau(\mu_B), \tau(\mu_B) = \frac{1-\mu_0}{1-\tilde{\tilde{\mu}}}, \mu_B = \tilde{\tilde{\mu}}, \mu_G = 1\right\}, \text{if } \mu_0 \geqslant \tilde{\tilde{\mu}} \\ \{\tau(\mu_G) = 1 - \tau(\mu_B), \tau(\mu_B) \in [0,1], \mu_B = \mu_G = \mu_0\}, \text{if } \mu_0 < \tilde{\tilde{\mu}} \end{cases}$$

$Prob(e^* = 1) = \min\left\{\frac{1-\mu_0}{1-\tilde{\tilde{\mu}}}, 1\right\}$.

The optimal value of the principal,

$\tilde{V}(\mu_0) = \mathbb{1}(\mu_0 \geqslant \tilde{\tilde{\mu}})\left\{V(e=0,\mu=1) + \frac{V(e=1,\tilde{\tilde{\mu}}) - V(e=0,\mu=1)}{1-\mu}(1-\mu_0)\right\}$
$+ \mathbb{1}(\mu_0 < \tilde{\tilde{\mu}})V(e=1,\mu_0)$.

[18] If $p(1,G) - p(0,G) = p(1,B) - p(0,B)$, i.e., that state and effort are independent, then the result is trivial: either any experiment is weakly optimal, or the optimal experiment is fully revealing.

[19] For example, I still assume that: 1) higher effort increases the probability of high outcome; 2) better state increases the probability of a high outcome.

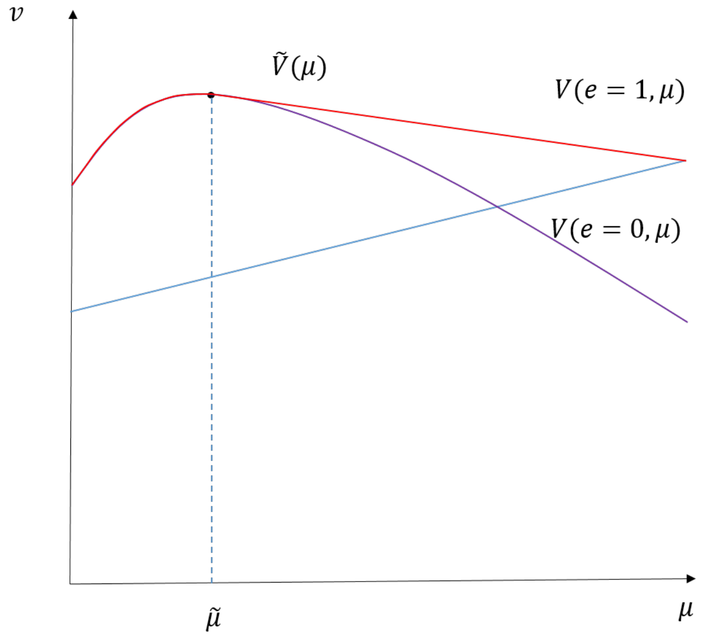


Figure 3.7: Optimal Experiment When State and Effort are Substitutes: an Example

Figure 3.7 provides an example that if state and effort are substitutes, the optimal experiment can also lead to partial obfuscation. The difference is that now a good signal is fully revealing, while when effort and state are complementary, a bad signal is fully revealing. This corollary provides us with a *necessary* condition to judge the complementarity between effort and state in this environment: when we find the principal designs a partially obfuscating experiment, then we can determine the level of complementarity by looking which signal is fully revealing. If the good signal is fully revealing then the effort and state must be substitutes; if the bad signal is fully revealing, then effort and state must be complementary (actually highly complementary).

3.7.4 Welfare Analysis

In this section, we will look at how individual-rational information disclosure and effort provision are different from social optimal levels. We will consider utilitarian social welfare function. The social welfare function, W is defined as the expected sum of the Principal and agent's welfare:

$$W \equiv E_\theta E_e E_{y|\theta,e}\{V(e,\theta,y,w)+U(e,\theta,y,w)\} = E_\theta E_e E_{y|\theta,e}\{y-w+w-c(e)\}$$
$$= E_\theta E_e E_{y|\theta,e}\{y-c(e)\} = E_\theta E_e\{y_L + Prob(y=y_H|\theta,e)(y_H-y_L)-c(e)\}$$
$$= y_L + E_\theta E_e\{p(\theta,e)(y_H-y_L)-c(e)\}$$

The social welfare maximizing level of effort is simple:

$$e^O(\theta) \in \max_{\{e(\theta)\}_{\theta\in\{G,B\}}} \{y_L + E_\theta E_e\{p(\theta,e)(y_H-y_L)-c(e)\}\}$$

$$e^{O}(\theta) = \begin{cases} 1, & \text{if } \Delta p(\theta)(y_H - y_L) \geqslant c \\ 0, & \text{if } \Delta p(\theta)(y_H - y_L) < c \end{cases}$$

It is socially optimal to implement effort as long as the expected social gain from high effort is higher than its cost, i.e., that $\Delta p(\theta)(y_H - y_L) > c$. Also, because a wage is purely a transfer between principal and agent, the only thing that matters for social welfare is the probability of high effort being implemented in each state.

The Equilibrium social welfare:

$$\begin{aligned}
&W^{Eq} \\
&\quad \equiv E_\theta E_{\mu^*|\theta} E_{y|\theta,e^*(\mu^*,\{w^*(y,\mu^*)\})} V(e^*(\mu^*,\{w^*(y,\mu^*)\}),\theta,y,w^*(y,\mu^*)) \\
&\quad + U(e^*(\mu^*,\{w^*(y,\mu^*)\}),\theta,y,w^*(y,\mu^*)) \\
&\quad = E_\theta E_{\mu^*|\theta} E_{y|\theta,e^*(\mu^*,\{w^*(y,\mu^*)\})} \{y - w^*(y,\mu^*) + w^*(y,\mu^*) - c(e^*(\mu^*,\{w^*(y,\mu^*)\}))\} \\
&\quad = E_\theta E_{\mu^*|\theta} E_{y|\theta,e^*(\mu^*,\{w^*(y,\mu^*)\})} \{y - c(e^*(\mu^*,\{w^*(y,\mu^*)\}))\} \\
&\quad = y_L + E_\theta E_{\mu^*|\theta} \{p(\theta,(e^*(\mu^*,\{w^*(y,\mu^*)\}))(y_H - y_L) - c((e^*(\mu^*,\{w^*(y,\mu^*)\}))\}
\end{aligned}$$

There is efficiency loss from social optimal as long as the equilibrium probability of high effort being implemented under each state is different from the social optimal probability of high effort being implemented under each state, and the efficiency losses under different states do not cancel out. There are three different cases:

Case I: $\Delta p(\theta)(y_H - y_L) \geqslant c$ for both $\theta \in \{G, B\}$. Then it is socially efficient to always implement high effort. Therefore, unless $Prob(e^* = 1) = Prob(e^{SO} = 1) = 1$, there is under-provision of effort in equilibrium. From previous analysis, we know that $Prob(e^* = 1) = 1$ if and only if $V(e = 1, \mu) > V(e = 0, \mu)$ for both $\mu = 0, 1$, which is a stricter condition than $\Delta p(\theta)(y_H - y_L) \geqslant c$ due to agency friction.

Case II: $\Delta p(\theta = G)(y_H - y_L) \geqslant c \geqslant \Delta p(\theta = B)(y_H - y_L)$. It is thus socially optimal to implement effort when the state is good but not when the state is bad. In this case, in equilibrium, there could be either under-provision or over-provision of effort. There is under-provision of effort if and only if $V(e = 1, \mu) < V(e = 0, \mu)$ for both $\mu = 0, 1$, and over-provision of effort if and only if $V(e = 1, \mu)$ is concave, $V(e = 1, \mu) > V(e = 0, \mu)$ for $\mu = 1$, and $\tilde{\mu} < 1$. Therefore, there can be over-provision when: 1) effort and state are complementary enough; 2)implementing high effort is privately beneficial when the state is good but not when the state is bad; 3) the principal will choose partial obfuscation.

Case III: $\Delta p(\theta)(y_H - y_L) < c$ for both $\theta \in \{G, B\}$. It is never efficient to implement high effort. In this case high effort will never be implemented anyway, so there is no efficiency loss.

A more interesting comparison would be between equilibrium experiments and *con-*

strained social optimal experiments. Here constrained social optimal experiment means the experiment(s) that maximizes the social welfare, taking as given that effort is unobservable, and the principal and the agent optimally choose what contract to offer and whether to accept the contract.

Hidden effort could be an inherent feature in this environment and it may be hard to change this feature. However, information quality is different: the precision of the public experiment is chosen by the principal, and regulations on the precision of the experiment are feasible: for example, transparency requirements in government public reports, disclosure rule that requires enough details in a financial report, or rules that state the minimum precision. Comparing the constrained social optimal experiment and the equilibrium experiment may thus provide us with policy implications over transparency issues.

Definition 3.5. An experiment, $\left\{\pi^{CSO}(s|\theta)\right\}_{s\in\{G,B\}}$, is called *constrained socially optimal experiment* if
$\left\{\pi^{CSO}(s|\theta)\right\}_{s\in\{G,B\}} \in \text{argmax}_{\{\pi(s|\theta)\}_{s\in\{G,B\}}} \sum_{\theta'\in\{G,B\}} Prob(\theta=\theta') \sum_{s\in\{G,B\}} \pi^{SO}(s|\theta')\hat{W}(s)$.

$\hat{W}(s) \equiv y_L + E_{\theta|s}E_e\left\{p(\theta,e^*(s))(y_H-y_L)-c(e)\right\}$, in which $w_H^*(s), w_L^*(s), e^*(s)$ are defined as:

$$\begin{aligned}
&\{w_H^*(s), w_L^*(s), e^*(s)\} \in \\
&\max_{\{w_H(s),w_L(s),e(s)\}} E_{\mu_s}p(e(s))(y_H-w_H)+(1-E_{\mu_s}p(e(s)))(y_L-w_L) \\
&s.t. \\
&E_{\mu_s}p(e(s))w_H(s)+(1-E_{\mu_s}p(e(s)))w_L(s)-c(e(s)) \geqslant 0 \\
&e(s) \in \underset{e}{\text{argmax}}\, E_{\mu_s}p(e)w_H(s)+(1-E_{\mu_s}p(e))w_L(s)-c(e) \\
&w_H(s), w_L(s) \geqslant 0
\end{aligned}$$

$\{w_H^*(s), w_L^*(s), e^*(s)\}$ is exactly the optimal contract we solved in the previous section of 'Optimal Contract'(Section 4).

Now we will compare the effort levels under equilibrium and the constrained social optimal. Denote the effort level of e under constrained social optimal as e^{CSO}.

Proposition 3.4. *If $\Delta p(\theta)(y_H-y_L) \geqslant c$ for both $\theta \in \{G,B\}$, then:*

If $y_H - y_L \leqslant \min_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}$, then $V(e=1,\mu) \leqslant V(e=0,\mu)$ for any $\mu \in [0,1]$, and any experiment will be both constrained social optimal and individually optimal for the principal, and in general $Prob(e^{CSO}=1) = Prob(e^*=1) = 0$. [20]

[20]Due to our tie-breaking rule, in the case that $y_H - y_L = \min_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}$, define $\mu_{\min} = \text{argmin}_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}$, then $Prob(e^{CSO}=1)$ can be greater than 0 if the experiment put positive weights on $\mu_{\min}$. Otherwise $Prob(e^{CSO}=1)=0$. Moreover, there is no *strict* social welfare gain to implement high effort anyway.

If $y_H - y_L \geqslant \max_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}$, *then* $V(e=1,\mu) \geqslant V(e=0,\mu)$ *for any* $\mu \in [0,1]$, *and any experiment will be constrained social optimal, and* $Prob(e^{CSO} = 1) = Prob(e^* = 1) = 1$.

If $y_H - y_L \in \left(\min_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}, \max_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}\right)$, *and* $p(1,G)p(0,B) \leqslant p(1,B)p(0,G)$, *then* $Prob(e^{CSO} = 1) \geqslant Prob(e^* = 1)$.

If $y_H - y_L \in \left(\min_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}, \max_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}\right)$, *and* $p(1,G)p(0,B) > p(1,B)p(0,G)$, *then* $Prob(e^{CSO} = 1) = \min\left\{\frac{\mu_0}{\bar{\mu}}, 1\right\} \geqslant Prob(e^* = 1) = \min\left\{\frac{\mu_0}{\tilde{\mu}}, 1\right\}$.

The constrained socially optimal experiment is:

$$\left\{\tau^{CSO}, \mu^{CSO}\right\} = \begin{cases} \left\{\tau(\mu_B) = 1 - \tau(\mu_G), \tau(\mu_G) = \frac{\mu_0}{\bar{\mu}}, \mu_B = 0, \mu_G = \bar{\mu}\right\}, \textit{if } \mu_0 \leqslant \bar{\mu} \\ \left\{\tau(\mu_B) = 1 - \tau(\mu_G), \tau(\mu_G) \in [0,1], \mu_B = \mu_G = \mu_0\right\}, \textit{if } \mu_0 > \bar{\mu} \end{cases}$$

, in which $\bar{\mu}$ *is the level of* μ *such that* $V(e=1,\bar{\mu}) = V(e=0,\bar{\mu})$.

If $\Delta p(\theta = G)(y_H - y_L) \geqslant c \geqslant \Delta p(\theta = B)(y_H - y_L)$, *then:*

If $y_H - y_L \leqslant \min_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}$, *then* $V(e=1,\mu) \leqslant V(e=0,\mu)$, *and any experiment will be both constrained social optimal and individually optimal for the principal, and in general* $Prob(e^{CSO} = 1) = Prob(e^* = 1) = 0$.

If $y_H - y_L \in \left(\min_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}, \max_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}\right)$, *and* $p(1,G)p(0,B) > p(1,B)p(0,G)$, *then the constrained socially optimal experiment is fully revealing.* $Prob(e^{CSO} = 1) = \mu_0 \leqslant Prob(e^* = 1) = \min\left\{\frac{\mu_0}{\tilde{\mu}}, 1\right\}$.

If $y_H - y_L \in \left(\min_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}, \max_{\mu\in[0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}\right)$, *and* $p(1,G)p(0,B) \leqslant p(1,B)p(0,G)$, *then both the constrained socially optimal experiment and the principal's optimal experiment are fully revealing.* $Prob(e^{CSO} = 1) = Prob(e^* = 1) = \mu_0$.

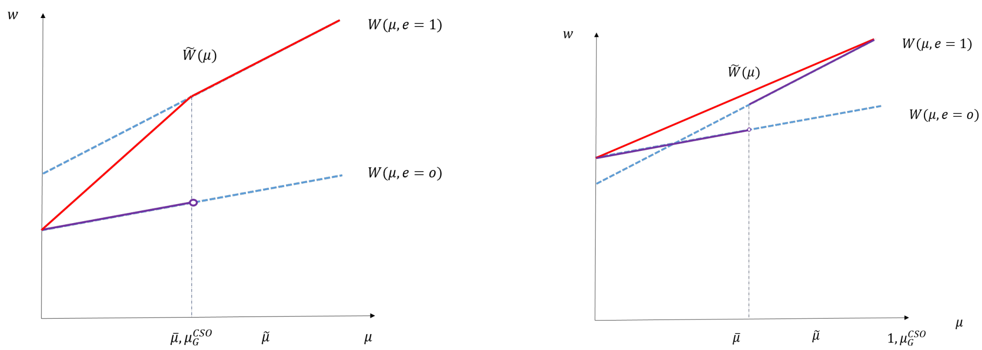
If $\Delta p(\theta)(y_H - y_L) < c$, *then* $V(e=1,\mu) < V(e=0,\mu)$ *for any* $\mu \in [0,1]$, *and any experiment will be both constrained socially optimal and individually optimal for the principal, and in general* $Prob(e^{CSO} = 1) = Prob(e^* = 1) = 0$.

Proposition 3.4 shows that when implementing high effort is constrained social optimal regardless of the state, in equilibrium high effort could be under-provided. If implementing high effort is constrained social optimal when the state is good but not when the state is bad, in equilibrium high effort could be over-provided. Those under-provision and over-provision happen only if the principal's preferred effort level depends on the posterior μ: if the principal would like to implement either high or low effort regardless of the state, then any experiment would be weakly optimal because it cannot affect the probability of high effort being implemented.

Moreover, Proposition 3.4 shows that constrained social optimal experiment can either reveal more or less information than the principal's optimal experiment. This means more transparency is not always social welfare improving, and we should be cautious before promoting regulations on transparency. Figure 3.8 provides two examples. We know from Proposition 3.4 that $\mu_B^{CSO} = 0$ in both cases, while μ_G^{CSO} can be different in those two cases. Case (a) shows an example that constrained social optimal experiment reveals less information than the equilibrium experiment. When effort and state are highly complementary, and it is socially optimal to implement high effort in both states, but the principal would only like to implement high effort when the state is good due to agency problem, less transparency could actually improve social welfare. In this scenario, agency problem makes it costly for the principal to implement high effort under bad state, and less transparent information eases the friction of agency problem by providing good news more frequently.

Case (b) shows an example that constrained social optimal experiment reveals more information than the equilibrium experiment. Again, effort and state are highly complementary. Now it is both constrained social optimal and individual rational for the principal to implement high effort in good state but not in bad state. To minimize the distortions, the constrained social optimal experiment will be fully revealing, while we know from previous results that equilibrium experiment could be partial obfuscation, which provides less information than a fully-revealing experiment.

Therefore, mandatory transparency regulation will only be social welfare improving under some scenarios and can sometimes reduce social welfare.



(a) Implementing High Effort Social Optimal Under Both States

(b) Implementing High Effort Social Optimal Under Good State But Not Bad State

Figure 3.8: Constrained Social Optimal Experiment: two Examples

Another interesting comparison could be between constrained social optimal experiment and the situation when there is no information design. When there is no information design, the only information available to both players is the prior μ_0, so the principal will choose to implement high effort *iff* $V(e = 1, \mu_0) \geqslant V(e = 0, \mu_0)$. Again, if it is always optimal for the principle to implement high or low effort regardless of the state, then any experiment would lead to exactly same level of social welfare.

Now let us look at the case that $y_H - y_L \in \left(\min_{\mu \in [0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}, \max_{\mu \in [0,1]} \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}\right)$, and $p(1, G)p(0, B) > p(1, B)p(0, G)$. Under this case, if there is no information design, the principal will exert low effort with probability 1 if $\mu_0 \in (0, \bar{\mu})$ and exert high effort with probability 1 if $\mu_0 \in [\bar{\mu}, 1)$. Now consider two sub-cases: (1) $\Delta p(\theta)(y_H - y_L) \geqslant c$ for both $\theta \in \{G, B\}$; (2) $\Delta p(\theta = G)(y_H - y_L) \geqslant c \geqslant \Delta p(\theta = B)(y_H - y_L)$.

In the first case, under the constrained social optimal experiment, $Prob(e^{CSO} = 1) = \min\left\{\frac{\mu_0}{\bar{\mu}}, 1\right\}$, which is greater than the probability of high effort being implemented when $\mu_0 \in (0, \bar{\mu})$. In the second case, $Prob(e^{CSO} = 1) = \mu_0$ which can be higher or lower than the probability of high effort being implemented under no information, depending on whether μ_0 is greater than $\bar{\mu}$. So in summary, compared to constrained social optimal, under no information design we can also have under-provision or over-provision of high effort.

3.8 Conclusion

This chapter considers a principal-agent hidden-effort model, where the principal can also affect the information environment the agent faces by designing a public experiment. Information is relevant in the sense that good state increases the effectiveness of the agent's effort in generating high outcomes. More optimistic news about the future will thus reduce the compensation that the agent requires to exert effort and make implementing high effort less costly.

I show that the concavity of the principal's value function of implementing high effort depends on the level of complementarity between state and effort. The value function is concave when the probability of high output is log-supermodular in state and effort, which means state and effort are complementary enough, and convex otherwise. The concavity of that value function will affect the optimal experiment: if the value function is convex, the optimal experiment is generally full revealing; while if the value function is concave, then the optimal experiment can also be no information revealed or partial obfuscation. I then explore the situations for the optimal experiment to be fully revealing, revealing no information, or partial obfuscation.

I then show that relaxing some assumptions I made in the baseline model such as focusing on direct experiment or the particular timing of the model I have assumed will not lead to losses of generality. I then consider what happens if state and effort are actually substitutes. Last, I show that in the welfare analysis part that forcing the principal to disclosure high-quality information can be sometimes welfare-reducing. This requires us to be cautious when considering transparency policies.

This chapter suggests some possible directions for future research: first, I assume in this chapter that the principal designs a public experiment. It would be natural to consider the alternative scenario of the principal designing a private experiment and compare the differences. Second, I show that whether the principal offers a contract before or after the experiment outcome is realized has no impact. This is based on the assumption that experiment outcomes are contractible ex-ante. If the experiment outcome is not contractible, the results could be quite different. Third, both the principal and the agent can have some private information besides the public experiment. Last, the principal could face multiple agents at the same time. A model similar to the framework in this chapter but with multiple agents could help us better understand how public information affect agents' incentive to exert effort and collaborate.

Appendix A

Proofs and Further Results of Chapter 1

The foreign government's optimal strategy

The foreign government's optimal choice is simple:

based on the information available to the foreign government (level of ψ, the public signal s) in Period 3, denote its posterior belief that the general public of home country is protesting as $\mu(\psi, s)$. Then by Bayes' Rule,

$$\mu(\psi, s = S) \equiv Prob(a_G = P|\psi, s = S) = \frac{Prob(a_G = P, s = S|\psi)}{Prob(s = S|\psi)}$$
$$= \begin{cases} \frac{Prob(a_G=P,s=NP|\psi)}{Prob(s=NP|\psi)} = 0, \text{if} \quad S = NP \\ \frac{Prob(a_G=P|\psi)}{Prob(a_G=P|\psi)+Prob(a_G=NP,a_N=P|\psi)} \quad ,\text{if} \quad S = P \end{cases}$$

The previous result comes from the signal structure of s:

$$Prob(s = P|a_G, a_N, \psi, \theta) = \begin{cases} 1, \text{if } a_G = P \text{ or } a_N = P \\ 0, \qquad \text{otherwise} \end{cases}$$

The optimization problem of the foreign government:

$$\max_{a_F \in \{C,NC\}} I(a_F = C)\{\mu(\psi, s)u_F(P, C) + (1 - \mu(\psi, s))u_F(NP, C)\}+$$
$$I(a_F = NC)\{\mu(\psi, s)u_F(P, NC) + (1 - \mu(\psi, s))u_F(NP, NC)\}$$

The optimal solution is simple:

$a_F^* = NC, \text{if}$

$\{\mu(\psi, s)u_F(P, NC) + (1 - \mu(\psi, s))u_F(NP, NC)\} > \{\mu(\psi, s)u_F(P, C) + (1 - \mu(\psi, s))u_F(NP, C)\};$

$a_F^* = C, \text{otherwise.}$

This just means that:

$$a_F^* = \begin{cases} NC, \text{ if } \mu(\psi, s) < \bar{\mu} \\ C, \text{ if } \mu(\psi, s) \geqslant \bar{\mu} \end{cases}$$

in which $\bar{\mu} \in (0,1)$, *the threshold of doubt* for foreign government, is defined as the level of μ such that the foreign government is indifferent between conceding or not: $\bar{\mu}[u_F(P,C) - u_F(P,NC)] = (1-\bar{\mu})[u_F(NP,NC) - u_F(NP,C)]$.

From the signal structure, $s = NP$ means neither group is protesting so the general public is not protesting for sure.

Therefore, when $s = NP$, $a_F^* = NC$, since $\mu(\psi, NP) = 0 < \bar{\mu}$.

When $s = P$, $a_F^* = C$ if and only if $\mu(\psi, P) \geqslant \bar{\mu}$.

The foreign government will never concede if there is no protest. Moreover, it will concede after seeing a protest, if and only if it thinks protest is an informative enough signal.

the Nationalist and the Public's Best Response

In the baseline model, the decision problems for the nationalist and the public are straightforward. Their actions depend only on ψ and θ, and there is no strategic interaction between them.

For i=N,G, the optimization problem is:

$$\max_{a_i \in \{P,NP\}} I(a_i = P)\{w_i(\psi)\theta - c\},$$

in which $w_i(\psi) = w_0 + w_{1i}\psi$.

The optimal strategy of player i, $i = N, G$ is: $a_i^* = P$, iff $\theta \geqslant \hat{\theta}_i(\psi)$, in which $\hat{\theta}_i(\psi) \equiv \frac{c}{w_0 + w_{1i}\psi}$.

$\hat{\theta}_i$ decreases with ψ, and the tuple of both types' cut-off levels, $(\hat{\theta_N}(\psi), \hat{\theta_G}(\psi))$ are function of ψ in the following way:

$$(\hat{\theta_N}(\psi), \hat{\theta_G}(\psi)) = (\frac{c}{w_0 + w_1 N\psi}, \frac{c}{w_0 + w_1 G\psi})$$

Because w_0 is the same among the two groups and $w_{1N} > w_{1G}$, $\hat{\theta_N} \leqslant \hat{\theta_G}$ for any ψ.

The interim ex-ante expected probability of protest (after ψ is chosen but before θ is realized) for group i is $H_i(\psi) \equiv Prob(\text{Player i protests}) = Prob(\theta \geqslant \hat{\theta}_i)$, and $H_N(\psi) \geqslant H_G(\psi)$.

$H_i(\psi)$ decreases with ψ, and the tuple of both types' ex-ante probabilities of protesting,

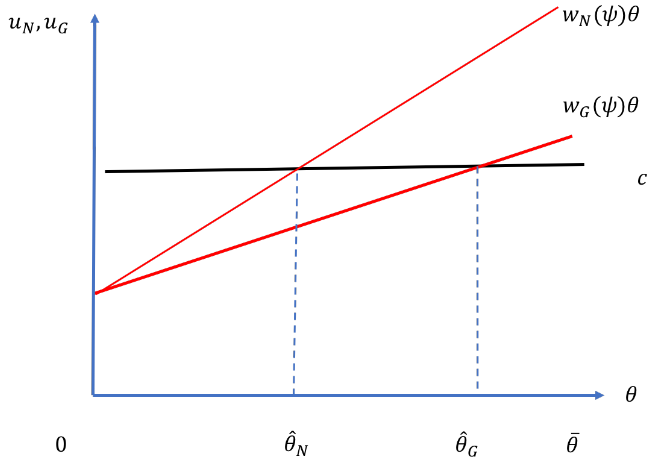


Figure A.1: Nationalist and General Public's Best Responses

$(H_N(\psi), H_G(\psi))$ are increasing functions of ψ:

$$(H_N(\psi), H_G(\psi)) = (\frac{\bar{\theta} - \frac{c}{w_0 + w_1 N\psi}}{\bar{\theta}}, \frac{\bar{\theta} - \frac{c}{w_0 + w_1 G\psi}}{\bar{\theta}})$$

From last section we know: $\mu(\psi, P) = \frac{H_G(\psi)}{H_N(\psi)}$.

And $\mu(\psi, P)$ is a function of ψ:

$$\mu(\psi, P) = \frac{\bar{\theta} - \frac{c}{w_0 + w_1 G\psi}}{\bar{\theta} - \frac{c}{w_0 + w_1 N\psi}}$$

We show that there is an inverse-U shaped relationship between the level of hostility, ψ, and the posterior of the foreign government when observing a protest, $\mu(\psi, P)$. This means that for $\bar{\mu}$ large enough, there exists two levels of ψ, $\hat{\psi}_L$ and $\hat{\psi}_R$, such that $\mu(\hat{\psi}_L, P) = \mu(\hat{\psi}_R, P) = \bar{\mu}$. Moreover, $\mu(\psi, P) \geqslant \bar{\mu}$ if and only if $\psi \in \left[0, \hat{\psi}_L\right] \cup \left[\hat{\psi}_R, \bar{\psi}\right]$

Mathematically, the reason of inverse-U shaped we observe here comes from the elasticity of $H_N(\psi)$, $\frac{H_N'(\psi)}{H_N(\psi)}$, single-crosses the elasticity of $H_G(\psi)$, $\frac{H_G'(\psi)}{H_G(\psi)}$. Roughly speaking, when ψ is small, $H_N(\psi)$ is growing much faster than $H_G(\psi)$, and the levels of $H_N(\psi)$ and $H_G(\psi)$ are both small. Thus $H_N'(\psi)$ is substantially larger than $H_G'(\psi)$, while the difference between $H_N(\psi)$ and $H_G(\psi)$ is small. When ψ becomes large, $H_N(\psi)$ and $H_G(\psi)$ are both growing very slowly so $H_N'(\psi)$ is very near to $H_G'(\psi)$, while the difference between $H_N(\psi)$ and $H_G(\psi)$ are also small but still in larger magnitude.

Intuitively, when the ψ is low so there is not much manipulation of protests, the opponent knows that the nationalist and the public are both unlikely to be on the street, and their differences over protest probabilities are low. Thus when the opponent observes a protest, he knows that with high probability both groups are protesting. When ψ is very high, the opponent knows the nationalist is almost always protesting on the street. However, since ψ is very high, the general public is also very likely to be protesting, so in this case the difference over protest probabilities are also low. In the case that ψ is at intermediate level, $H_N(\psi)$ is much larger than $H_G(\psi)$. There would be many protests of only nationalist, and this would make foreign government less sure that general public supports a protest when it observe one.

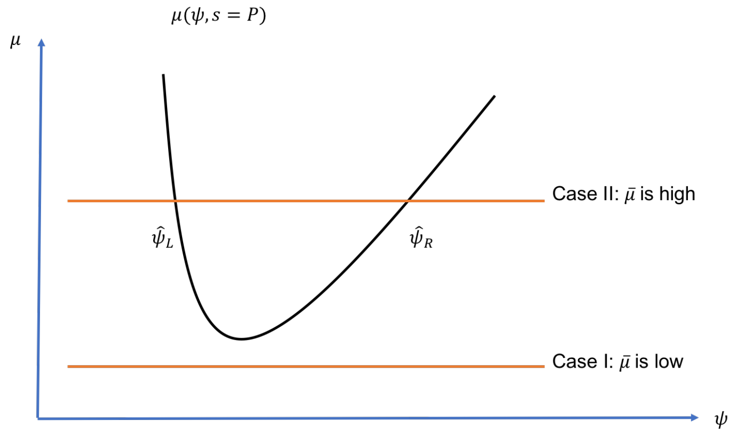


Figure A.2: Foreign Government's posterior when observing Protest, as function of ψ

The home government's general optimization problem

The home government's optimization problem, in the most general form, is:

Definition A.1 (**General Optimization Problem**).

$$\max_{\psi \in [0,\bar{\psi}]} \sum_{a_G} \sum_{a_N} \sum_{S} \sum_{a_F} [Prob(a_F^* = a_F|\psi, S) Prob(s = S|\psi, a_G, a_N) \\ Prob(a_G^* = a_G, a_N^* = a_N|\psi) u_H(a_G^*, a_F^*)]$$

$a_i^*, i = N, G, F$ is the optimal strategy of player i.

However, this problem can be substantially simplified: In the canonical model, $Prob(s = S|\psi, a_G, a_N)$, $Prob(a_F^* = a_F|\psi, S)$, and $Prob(a_G^* = a_G, a_N^* = a_N|\psi)$ are all degenerate due to our model setting and tie-breaking rules.

In the baseline model the general optimization problem becomes:

$$\max_{\psi \in [0,\bar{\psi}]} H_G(\psi) u_H(P, a_F^*(\psi, P)) + [H_N(\psi) - H_G(\psi)] u_H(NP, a_F^*(\psi, P)) \\ + [1 - H_N(\psi)] u_H(NP, a_F^*(\psi, NP))$$

, in which $a_F^*(\psi, NP) = NC$, and $a_F^*(\psi, P) = C$ iff $\mu(\psi, P) \geqslant \bar{\mu}$.

It can be show that it is never optimal to choose some level of ψ such that $a_F^*(\psi, P) = NC$ (We know already that $a_F^*(\psi, NP) = NC$).

Definition A.2 (Simplified Optimization Problem).

$$\max_{\psi\in[0,\bar{\psi}]} H_G(\psi)u_H(P,C) + [H_N(\psi) - H_G(\psi)]u_H(NP,C) + [1 - H_N(\psi)]u_H(NP,NC)$$

$$s.t.\ \mu(\psi,P) = \frac{H_G(\psi)}{H_N(\psi)} \geqslant \bar{\mu}$$

Lemma A.1. *1. Denote ψ^S as the optimal solution to the Simplified Optimization Problem. ψ^S exists.*

2. Denote ψ^* as the optimal solution to the General Optimization Problem. ψ^* exists. $\psi^S = \psi^*$.

3. $a_F^*(\psi^*, s = P) = C$

Proof. Define $Eu_H(\psi) \equiv H_G(\psi)u_H(P,C) + [H_N(\psi) - H_G(\psi)]u_H(NP,C)$ $+[1-H_N(\psi)]u_H(NP,NC)$, the expected utility the home government can get from choose ψ, assuming the other players will make sequentially rational decisions.

1)For the simplified Optimization Problem:
The objective function is continuous. The set of ψ that satisfies the constraint is non-empty and is a finite union of disjoint compact intervals. Therefore, the optimal solution and optimal value to this question both exist.

2) and 3): We prove 2) and 3) by showing that ψ^S is the unique solution to the general optimization problem.

Suppose the contrary is true. Then there exists a level $\psi > 0$ such that $a_F^*(\psi, s = P) = NC$ and $Eu_H(\psi) \geqslant Eu_H(\psi^S)$.
Then H's payoff is
$Eu_H(\psi) = H_G(\psi)u_H(P,NC)+[H_N(\psi)-H_G(\psi)]u_H(NP,NC)+[1-H_N(\psi)]u_H(NP,NC)$.
If deviating to $\psi' = 0$, $\mu(\psi = 0, s = P) = 1 \geqslant \bar{\mu}$ for any $\bar{\mu}$,
and H's payoff is
$Eu_H(\psi') = H_G(0)u_H(P,C) + [H_N(0) - H_G(0)]u_H(NP,C) + [1 - H_N(0)]u_H(NP,NC) >$
$H_G(0)u_H(P,NC) + [H_N(0) - H_G(0)]u_H(NP,NC) + [1 - H_N(0)]u_H(NP,NC) >$
$H_G(\psi)u_H(P,NC)+[H_N(\psi)-H_G(\psi)]u_H(NP,NC)+[1-H_N(\psi)]u_H(NP,NC) = Eu_H(\psi)$

However, by definition, $Eu_H(\psi^S) \geqslant Eu_H(\psi')$. Therefore, $Eu_H(\psi^S) > Eu_H(\psi)$.

Contradiction. Therefore, the opposite statement is correct.

□

The intuition of this proof is as follows: the potential benefits of a higher ψ are: 1) more concessions; 2) keeping the signal (Protest) informative enough. At the same time, the cost of higher ψ is a higher probability of bad states.

When $a_F^*(\psi, P) = NC$, a positive ψ has no benefit but only cost. The home government can always improve by reducing ψ which reduces the cost. Therefore a level of ψ such that $a_F^*(\psi, P) = NC$ can never be optimal.

Therefore we can solve for ψ^* just by solving the simplified optimization problem.

Definition A.3. Define $\tilde{c} \equiv c/\bar{\theta}$.

Now let us look at the posterior of foreign government when observing a protest, $\mu(\psi, P)$:

Lemma A.2. *1 $\mu(\psi, P)$ is quasi-convex; it is decreasing if $0 \leqslant \psi \leqslant \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}$ and increasing if $\psi > \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}$.*

2 Define $\hat{\psi} \equiv \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{\bar{\theta}w_0-c}{\bar{\theta}w_0}}$.

Then $\mu(\hat{\psi}, P) = \min_{\psi\in[0,+\infty)} \mu(\psi, P) = \frac{w_{1G}}{w_{1N}}\left(\frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}}\right)^2$.

2.1 If $\bar{\psi} > \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}$,

2.1.1 If $\bar{\mu} \in \left(0, \mu(\hat{\psi}, P)\right]$, then for any $\psi \in [0, \bar{\psi}]$, $\mu(\psi, P) \geqslant \bar{\mu}$;

2.1.2 If $\bar{\mu} \in (\mu(\hat{\psi}, P), \mu(\bar{\psi}, P))$, there exist two levels of ψ, $\hat{\psi}_L(\bar{\mu})$ and $\hat{\psi}_R(\bar{\mu})$, $0 < \hat{\psi}_L(\bar{\mu}) < \hat{\psi}_R(\bar{\mu}) < \bar{\psi}$, such that $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. Moreover, for any $\psi[0, \bar{\psi}], \mu(\psi, P) \geqslant \bar{\mu}$ iff $\psi \in [0, \hat{\psi}_L(\bar{\mu})] \cup [\hat{\psi}_R(\bar{\mu}), \bar{\psi}]$

2.1.3 For any $\bar{\mu} \in \left(\mu(\bar{\psi}, P), 1\right)$, there exists a level of ψ, $\hat{\psi}_L(\bar{\mu})$, $0 < \hat{\psi}_L(\bar{\mu}) < \bar{\psi}$, such that $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$. Moreover, for any $\psi[0, \bar{\psi}], \mu(\psi, P) \geqslant \bar{\mu}$ iff $\psi \in [0, \hat{\psi}_L(\bar{\mu})]$

2.1.4

$$\hat{\psi}_L(\bar{\mu}) = \frac{-\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]-\sqrt{(\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4w_{1G}w_{1N}w_0(w_0-\tilde{c})}}{2w_{1G}w_{1N}}$$

$$\hat{\psi}_R(\bar{\mu}) = \frac{-\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]+\sqrt{(\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4w_{1G}w_{1N}w_0(w_0-\tilde{c})}}{2w_{1G}w_{1N}}$$

2.2 If $\bar{\psi} \leqslant \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}$,

2.2.1 For any $\bar{\mu} \in \left(0, \mu(\bar{\psi}, P)\right]$, then for any $\psi \in [0, \bar{\psi}]$, $\mu(\psi, P) \geqslant \bar{\mu}$;

2.2.2 For any $\bar{\mu} \in \left(\mu(\bar{\psi}, P), 1\right)$, there exists a level of ψ, $\hat{\psi}_L(\bar{\mu})$, $0 < \hat{\psi}_L(\bar{\mu}) < \bar{\psi}$, such that $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$. Moreover, for any $\psi[0, \bar{\psi}], \mu(\psi, P) \geqslant \bar{\mu}$ iff $\psi \in [0, \hat{\psi}_L(\bar{\mu})]$

2.2.3

$$\hat{\psi}_L(\bar{\mu}) = \frac{-\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]-\sqrt{(\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4w_{1G}w_{1N}w_0(w_0-\tilde{c})}}{2w_{1G}w_{1N}}$$

Proof. 1) $\mu(\psi, P) = \frac{H_G(\psi)}{H_N(\psi)}$

then $\frac{\partial\mu(\psi,P)}{\partial\psi} = \frac{H_G(\psi)}{H_N(\psi)}\left[\frac{\frac{\partial H_G(\psi)}{\partial\psi}}{H_G(\psi)} - \frac{\frac{\partial H_N(\psi)}{\partial\psi}}{H_N(\psi)}\right]$

$= \frac{H_G(\psi)}{H_N(\psi)}\left[\frac{\tilde{c}w_{1G}}{(w_0+w_{1G}\psi)^2}\frac{w_0+w_{1G}\psi}{(w_0-\tilde{c})+w_{1G}\psi} - \frac{\tilde{c}w_{1N}}{(w_0+w_{1N}\psi)^2}\frac{w_0+w_{1N}\psi}{(w_0-\tilde{c})+w_{1N}\psi}\right]$
$= \frac{H_G(\psi)}{H_N(\psi)}\left[\frac{\tilde{c}w_{1G}}{(w_0+w_{1G}\psi)[(w_0-\tilde{c})+w_{1G}\psi]} - \frac{\tilde{c}w_{1N}}{(w_0+w_{1N}\psi)[(w_0-\tilde{c})+w_{1N}\psi]}\right]$

Therefore, $\frac{\partial\mu(\psi,P)}{\partial\psi} \geqslant 0$ *iff* $\psi \geqslant \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{\theta w_0 - c}{\theta w_0}}$

2) $\hat{\psi}_L(\bar{\mu})$ and $\hat{\psi}_R(\bar{\mu})$ are two solutions (if those solutions exist) to $\mu(\psi, P) = \bar{\mu}$. This is equivalent to: $w_{1G}w_{1N}\psi^2 + \left[w_{1N}(w_0 - \frac{\tilde{c}}{1-\bar{\mu}}) + w_{1G}(w_0 + \frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]\psi + w_0(w_0 - \tilde{c}) = 0$
Therefore,

$$\hat{\psi}_L(\bar{\mu}) = \frac{-\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]-\sqrt{(\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4w_{1G}w_{1N}w_0(w_0-\tilde{c})}}{2w_{1G}w_{1N}}$$

and

$$\hat{\psi}_R(\bar{\mu}) = \frac{-\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right]+\sqrt{(\left[w_{1N}(w_0-\frac{\tilde{c}}{1-\bar{\mu}})+w_{1G}(w_0+\frac{\tilde{c}\bar{\mu}}{1-\bar{\mu}})\right])^2-4w_{1G}w_{1N}w_0(w_0-\tilde{c})}}{2w_{1G}w_{1N}}.$$

The other conclusions of this lemma are obvious implications of 1) and 2).

□

Now let's look at the objective function:

$$OB(\psi) \equiv H_G(\psi)[u_H(P,C) - u_H(NP,C)] + H_N(\psi)[u_H(NP,C) - u_H(NP,NC)] + u_H(NP,NC)$$

It is also the expected utility of the home government for choosing hostility ψ, if the foreign government's best response is $a_F(\psi, P) = C$.

Now we can provide some basic characterizations of the objective function:

Definition A.4. $t_H \equiv u_H(NP,C) - u_H(NP,NC)$

$\tau_H \equiv u_H(NP,C) - u_H(P,C)$

$\frac{t_H}{\tau_H} \equiv \frac{u_H(NP,C)-u_H(NP,NC)}{u_H(NP,C)-u_H(P,C)}$

Lemma A.3. *1 The objective function $OB(\psi)$ is quasi-concave on the interval $[0, \bar{\psi}]$. There exists a unique maximum point of the objective function on that interval. Denote ψ^I as the maximum point of the objective function on $[0, \bar{\psi}]$: $\psi^I = argmax_{[0,\psi]}OB(\psi)$. Then:*

1.1 If $\frac{t_H}{\tau_H} \leqslant \frac{w_{1G}}{w_{1N}}$, the objective function is decreasing on $[0, \bar{\psi}]$ and $\psi^I = 0$.

1.2 If $\frac{t_H}{\tau_H} \geqslant \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2$, the objective function is increasing on $[0, \bar{\psi}]$ and $\psi^I = \bar{\psi}$.

1.3 If $\frac{t_H}{\tau_H} \in (\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)$, $\psi^I = \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}$. The objection function is increasing over ψ on $[0, \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}]$ and decreasing over

ψ *on* $\left(\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}, \bar{\psi}\right]$. *Moreover,* $\psi^I = \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}$ *is strictly increasing over* $\frac{t_H}{\tau_H}$.

2 the utility of the home government at $\hat{\psi}_L(\bar{\mu})$, $Eu_H(\hat{\psi}_L(\bar{\mu}))$, *is greater or equal to the its utility at* $\hat{\psi}_R(\bar{\mu})$,$Eu_H(\hat{\psi}_R(\bar{\mu}))$,*if and only if* $\frac{t_H}{\tau_H} \leqslant \bar{\mu}$.

Proof. 1) $\frac{\partial OB(\psi)}{\partial \psi} = \frac{\partial H_N(\psi)}{\partial \psi}t_H - \frac{\partial H_G(\psi)}{\partial \psi}\tau_H = \frac{cw_{1N}}{(w_0+w_{1N}\psi)^2}t_H - \frac{cw_{1G}}{(w_0+w_{1G}\psi)^2}\tau_H$
$= \frac{cw_{1G}}{(w_0+w_{1N}\psi)^2}\left[\frac{t_H}{\tau_H}\frac{w_{1N}}{w_{1G}} - \frac{(w_0+w_{1N}\psi)^2}{(w_0+w_{1G}\psi)^2}\right]$.
$\left[\frac{t_H}{\tau_H}\frac{w_{1N}}{w_{1G}} - \frac{(w_0+w_{1N}\psi)^2}{(w_0+w_{1G}\psi)^2}\right]$ is decreasing over ψ.
Therefore, either 1) $\frac{\partial OB(\psi)}{\partial \psi} > 0$ for any $\psi \in [0, \bar{\psi}]$; 2) $\frac{\partial OB(\psi)}{\partial \psi} < 0$ for any $\psi \in [0, \bar{\psi}]$; or 3) $\frac{\partial OB(\psi)}{\partial \psi}$ is first positive and then negative.

2)

$$\begin{aligned}
&Eu_H(\hat{\psi}_R(\bar{\mu})) \\
&= H_G(\hat{\psi}_R(\bar{\mu}))[u_H(P,C) - u_H(NP,C)] + H_N(\hat{\psi}_R(\bar{\mu}))[u_H(NP,C) - u_H(NP,NC)] \\
&+ u_H(NP,NC) \\
&= H_N(\hat{\psi}_R(\bar{\mu}))\tau_H[\frac{t_H}{\tau_H} - \frac{H_G(\hat{\psi}_R(\bar{\mu}))}{H_N(\hat{\psi}_R(\bar{\mu}))}] + u_H(NP,NC) \\
&= H_N(\hat{\psi}_R(\bar{\mu}))\tau_H[\frac{t_H}{\tau_H} - \bar{\mu}] + u_H(NP,NC)
\end{aligned}$$

, because $\mu(\hat{\psi}_R(\bar{\mu}), P) = \frac{H_G(\hat{\psi}_R(\bar{\mu}))}{H_N(\hat{\psi}_R(\bar{\mu}))} = \bar{\mu}$.
Similarly,

$$Eu_H(\hat{\psi}_L(\bar{\mu})) = H_N(\hat{\psi}_L(\bar{\mu}))\tau_H[\frac{t_H}{\tau_H} - \bar{\mu}] + u_H(NP,NC)$$

Because $H_N(\hat{\psi}_R(\bar{\mu})) \geqslant H_N(\hat{\psi}_L(\bar{\mu}))$,
$Eu_H(\hat{\psi}_R(\bar{\mu})) \geqslant Eu_H(\hat{\psi}_L(\bar{\mu}))$ iff $\frac{t_H}{\tau_H} - \bar{\mu} \geqslant 0$.

□

So if $\frac{t_H}{\tau_H}$ is small enough, ψ^I will be constrained to the left boundary 0; when $\frac{t_H}{\tau_H}$ becomes large enough, ψ^I starts to strictly increase over $\frac{t_H}{\tau_H}$, until $\frac{t_H}{\tau_H}$ becomes so high that ψ^I hits the right boundary $\bar{\psi}$.

The second part of the lemma says that if the home government is forced to choose between $\hat{\psi}_L(\bar{\mu})$ and $\hat{\psi}_R(\bar{\mu})$, it will strictly prefer $\hat{\psi}_L(\bar{\mu})$ iff $\frac{t_H}{\tau_H} < \bar{\mu}$, i.e., that the relative benefit/cost ratio is smaller than the threshold of doubt.

Proof of Lemma 1.1. Lemma A.2 and A.3 immediately imply Lemma 1.1. □

Now we are ready to characterize the optimal and feasible level of ψ.

Proposition A.1. *1. Suppose* $\bar{\psi} \geqslant \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}$. *Then:*

1.1 If $\frac{t_H}{\tau_H} \leqslant \frac{w_{1G}}{w_{1N}}$, *then* $\psi^* = \psi^I = 0$ *for any* $\bar{\mu} \in (0,1)$;

1.2 If $\frac{t_H}{\tau_H} \geqslant \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2$, *then*

- *If* $\bar{\mu} \in \big(0, \mu(\bar{\psi}, P)\big]$, *then* $\psi^* = \psi^I = \bar{\psi}$
- *If* $\bar{\mu} \in \big(\mu(\bar{\psi}, P), 1\big)$, *then there exists* $\hat{\psi}_L(\bar{\mu})$, *such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \hat{\psi}_L(\bar{\mu})$

1.3 If $\frac{t_H}{\tau_H} \in (\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)$ *There exist a cut-off level* $\mu_1 \in (0,1)$, *such that:*

- *If* $\bar{\mu} \in (0, \mu_1]$, *then* $\psi^* = \psi^I$
- *If* $\bar{\mu} \in \big(\mu_1, \mu(\bar{\psi}, P)\big]$, *then there exist* $\hat{\psi}_L(\bar{\mu})$ *and* $\hat{\psi}_R(\bar{\mu})$, *such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$
  - $$\psi^* = \begin{cases} \hat{\psi}_L, \text{if } & \frac{t_H}{\tau_H} \leqslant \bar{\mu} \\ \hat{\psi}_R, \text{if } & \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$
- *If* $\bar{\mu} \in \big(\mu(\bar{\psi}, P), 1\big)$, *then there exists* $\hat{\psi}_L(\bar{\mu})$, *such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \hat{\psi}_L(\bar{\mu})$

2. Suppose $\bar{\psi} < \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}$. *Then there exist a cut-off level* $\mu_2 \in (0,1)$, *such that:*

2.1 If $\frac{t_H}{\tau_H} \leqslant \frac{w_{1G}}{w_{1N}}$, *then* $\psi^* = \psi^I = 0$ *for any* $\bar{\mu} \in (0,1)$;

2.2 If $\frac{t_H}{\tau_H} \geqslant \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2$, *then*

- *If* $\bar{\mu} \in \big(0, \mu(\bar{\psi}, P)\big]$, *then* $\psi^* = \psi^I = \bar{\mu}$
- *If* $\bar{\mu} \in \big(\mu(\bar{\psi}, P), 1\big)$, *then there exists* $\hat{\psi}_L(\bar{\mu})$, *such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \hat{\psi}_L(\bar{\mu})$

2.3 If $\frac{t_H}{\tau_H} \in (\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)$, *then there exists a cut-off level* $\mu_2 \in (0,1)$, *such that:*

- *If* $\bar{\mu} \in (0, \mu_2]$, *then* $\psi^* = \psi^I$
- *If* $\bar{\mu} \in (\mu_2, 1)$, *then there exists* $\hat{\psi}_L(\bar{\mu})$, *such that:*
  - $\mu(\hat{\psi}_L(\bar{\mu}), P) = \bar{\mu}$
  - $\psi^* = \hat{\psi}_L(\bar{\mu})$

Proof. Proposition A.1 is obvious given Lemma A.1, A.2, A.3 and the fact that the set $\big(\hat{\psi}_L(\bar{\mu}), \hat{\psi}_L(\bar{\mu})\big)$ is larger when $\bar{\mu}$ is larger. When $\bar{\mu}$ is small this set is null set, and when $\bar{\mu}$ converges to 1 this set will converge to $(0,1)$. □

Proof of Proposition 1.1. Proposition 1.1 is an immediate implication of Proposition A.1. □

Characterisation of optimal level of ψ

We will first look at the effects of marginal changes of the parameters on the ideal point. Then, we will look at how marginal changes of parameters affect $\hat{\psi}_L(\bar{\mu})$ and $\hat{\psi}_L(\bar{\mu})$ respectively.

Lemma A.4.

- $\hat{\psi}_L$ *is marginally:*
  - *increasing in* $\bar{\theta}$ *;*
  - *decreasing in* w_{1N}*;*
  - *increasing in* w_{1G}*;*
  - *increasing in* w_0
  - *decreasing of* c*;*
  - *independent of the utility function of the sender;*
  - *decreasing in* $\bar{\mu}$*, the threshold of doubt;*
- *the signs of* $\hat{\psi}_R$ *over the respective parameters are exactly the opposite to the signs of* $\hat{\psi}_L$*.*

Proof. For any fixed ψ, it is obvious that $\mu(\psi, P)$ is:

- decreasing in c
- increasing in $\bar{\theta}$
- increasing in w_0
- decreasing in w_{1N}
- increasing in w_{1G}

Also, $\frac{\partial \mu(\psi,P)}{\partial \psi}|_{\psi=\hat{\psi}_L} < 0$ and $\frac{\partial \mu(\psi,P)}{\partial \psi}|_{\psi=\hat{\psi}_R} > 0$ because of the quasi-convexity of $\mu(\psi, P)$ over ψ.

By Implicit Function Theorem the results follow through. □

Lemma A.5.

ψ^I*, is marginally:*

- *independent of* $\bar{\theta}$ *;*

– *if* $\frac{t_H}{\tau_H} \geqslant 1$, ψ^I *is weakly decreasing in* w_{1N}*; if* $\frac{t_H}{\tau_H} < 1$, ψ^I *is weakly decreasing in* w_{1N} *if* $\sqrt{\frac{w_{1N}}{w_{1G}}} > \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}$ *and weakly increasing in* w_{1N} *if* $1 < \sqrt{\frac{w_{1N}}{w_{1G}}} < \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}$

– *if* $\frac{t_H}{\tau_H} \leqslant 1$, ψ^I *is weakly decreasing in* w_{1G}*; if* $\frac{t_H}{\tau_H} > 1$, ψ^I *is weakly decreasing in* w_{1G} *if* $\sqrt{\frac{w_{1N}}{w_{1G}}} > \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H - \tau_H}{\tau_H}}$ *and weakly increasing in* w_{1G} *if* $\sqrt{\frac{w_{1N}}{w_{1G}}} < \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H - \tau_H}{\tau_H}}$

– *increasing in* w_0*;*

– *independent of* c*;*

– *increasing in* $\frac{t_H}{\tau_H}$*, the benefit/cost ratio;*

– *independent of the pay-off function of the foreign government.*

Proof. It is obvious that ψ^I is:

- independent over $\bar{\theta}$ and c
- (weakly) increasing over w_0
- (weakly) increasing in $\frac{t_H}{\tau_H}$
- independent of the pay-off function of the foreign government

Moreover, If $1 < \sqrt{\frac{w_{1N}}{w_{1G}}} < \max\left\{\sqrt{\frac{t_H}{\tau_H}}, \sqrt{\frac{\tau_H}{t_H}}\right\}$, $\psi^I = 0$. For the interior cases,

$$\frac{\partial \psi^I}{\partial \sqrt{w_{1N}}} = -\frac{w_0\sqrt{\frac{t_H}{\tau_H}}}{w_{1N}\sqrt{w_{1G}}(\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}})^2}\left[\left(\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{\tau_H}{t_H}}\right)^2 + \left(1 - \frac{\tau_H}{t_H}\right)\right]$$

$$\frac{\partial \psi^I}{\partial \sqrt{w_{1G}}} = -\frac{w_0\sqrt{\frac{t_H}{\tau_H}}}{w_{1G}\sqrt{w_{1N}}(\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}})^2}\left[\left(\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}}\right)^2 + \left(1 - \frac{t_H}{\tau_H}\right)\right]$$

□

Now given that we know how ψ^I, $\hat{\psi}_L$ and $\hat{\psi}_R$ change as different parameters change, and how ψ^* depends on ψ^I, $\hat{\psi}_L$ and $\hat{\psi}_R$, we'll look at how ψ^* transits between ψ^I, $\hat{\psi}_L$ and $\hat{\psi}_R$ when parameters change. After then, we can give a complete characterization about how ψ^* changes as different parameters change.

Lemma A.6. $\mu(\psi^I, P)$ *is:*

- *continuously weakly decreasing over* $w_{1N} \in (w_{1G}, +\infty)$*, and* $\mu(\psi^I, P) \in (\frac{\theta w_0 - c}{\theta w_0}, 1)$
- *continuously weakly increasing over* $w_{1G} \in (0, w_{1N})$*, and* $\mu(\psi^I, P) \in (\frac{\theta w_0 - c}{\theta w_0}, 1)$
- *continuously weakly increasing over* $w_0 \in \left(c/\bar{\theta}, +\infty\right)$*, and*

$$\mu(\psi^I, P) \in \left(\frac{w_{1G}}{w_{1N}}\frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, 1\right);$$

- *continuously weakly decreasing over* $c \in (0, \bar{\theta} w_0)$*, and*
$$\mu(\psi^I, P) \in \left(\frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, 1 \right);$$
- *continuously weakly increasing over* $\bar{\theta} \in \left(\frac{c}{w_0}, +\infty\right)$*, and*
$$\mu(\psi^I, P) \in \left(\frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, 1 \right);$$
- *continuously weakly decreasing over* $\frac{t_H}{\tau_H}$ *when* $0 \leqslant \frac{t_H}{\tau_H} < \mu(\hat{\psi}, P)$ *and increasing over* $\frac{t_H}{\tau_H}$ *when* $\frac{t_H}{\tau_H} > \mu(\hat{\psi}, P)$*,* $\mu(\psi^I, P) \in \left[\mu(\hat{\psi}, P), 1\right]$

Proof. We will show the case when ψ^I is interior. The cases when ψ^I is zero or $\bar{\psi}$ are trivial.

$\psi^I = \frac{w_0}{\sqrt{w_{1N}w_{1G}}} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}$,

Therefore $H_G(\psi^I) = 1 - \frac{\tilde{c}}{w_0+w_{1G}\psi^I} = 1 - \frac{\tilde{c}}{w_0+w_{1G}\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}} =$
$1 - \frac{\tilde{c}}{w_0} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{w_{1G}}{w_{1N}}}} = 1 - \frac{\tilde{c}}{w_0} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{w_{1G}}{w_{1N}}}(\frac{w_{1N}}{w_{1G}}-1)}$

and $H_N(\psi^I) = 1 - \frac{\tilde{c}}{w_0+w_{1N}\psi^I} = 1 - \frac{\tilde{c}}{w_0+w_{1N}\frac{w_0}{\sqrt{w_{1N}w_{1G}}}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}}$
$= 1 - \frac{\tilde{c}}{w_0} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}$

Therefore,
$$\mu(\psi^I) = \frac{H_G(\psi^I)}{H_N(\psi^I)} = \frac{1 - \frac{\tilde{c}}{w_0} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{w_{1G}}{w_{1N}}}(\frac{w_{1N}}{w_{1G}}-1)}}{1 - \frac{\tilde{c}}{w_0} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}}$$

, which is obviously decreasing over $\frac{\tilde{c}}{w_0}$. $\frac{\tilde{c}}{w_0}$ is increasing over c, and decreasing over w_0 and $\bar{\theta}$.

Therefore $\mu(\psi^I, P)$ is decreasing over c, and increasing over w_0 and $\bar{\theta}$.

For $\frac{t_H}{\tau_H}$:$\mu(\psi, P)$ is independent over $\frac{t_H}{\tau_H}$. ψ_I is weakly increasing over $\frac{t_H}{\tau_H}$. Also, $\mu(\psi, P)$ is decreasing over ψ when $\psi < \hat{\psi}$ and increasing over ψ when $\psi > \hat{\psi}$.

For the interior case, $\psi^I < \hat{\psi}$ if and only if
$\frac{w_0}{\sqrt{w_{1N}w_{1G}}} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{t_H}{\tau_H}}-1}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}} < \frac{w_0}{\sqrt{w_{1N}w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}$,
if and only if $\frac{t_H}{\tau_H} < \frac{w_{1G}}{w_{1N}} \left(\frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}}\right)^2 = \mu(\hat{\psi}, P)$. Also, from previous analy-

sis, ψ_I is interior solution if and only if $\frac{w_{1G}}{w_{1N}} < \frac{t_H}{\tau_H} < \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2$, and $\frac{w_{1G}}{w_{1N}} < \frac{w_{1G}}{w_{1N}}\left(\frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\sqrt{\frac{w_0-\tilde{c}}{w_0}}}\right)^2 = \mu(\hat{\psi}, P)$, and $\mu(\hat{\psi}, P) \leqslant \mu(\bar{\psi}, P) = \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})(\frac{\frac{w_0-c}{w_{1G}}+\bar{\psi}}{\frac{w_0-c}{w_{1N}}+\bar{\psi}}) < \frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2$ when $\bar{\psi} > \hat{\psi}$.

For w_{1N} and w_{1G}: when $\frac{t_H}{\tau_H} < 1$,

$$\mu(\psi^I, P) = \frac{H_G(\psi^I)}{H_N(\psi^I)} = \frac{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{w_{1G}}{w_{1N}}}(\frac{w_{1N}}{w_{1G}}-1)}}{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}} = \frac{1-\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}}{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}} + \sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}$$

$$\begin{aligned}\frac{d\mu(\psi^I, P)}{d\sqrt{\frac{w_{1N}}{w_{1G}}}} = &\left[\frac{-\sqrt{\frac{t_H}{\tau_H}}}{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}} + \sqrt{\frac{t_H}{\tau_H}}\right] - \\ &\frac{1-\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}}{\left(1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}\right)^2}\frac{\partial\left(1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}\right)}{\partial\sqrt{\frac{w_{1N}}{w_{1G}}}} = \\ &\frac{-\sqrt{\frac{t_H}{\tau_H}}}{H_N(\psi^I)}\left[\frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{\tau_H}{t_H}}}{\frac{w_{1N}}{w_{1G}}-1}\right] \\ &\left[[\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}}] - \frac{\frac{\frac{w_{1N}}{w_{1G}}-2\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}+1}{\frac{w_{1N}}{w_{1G}}-1}}{1 - \frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}}\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}}\right]\end{aligned}$$

$\left[[\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}}] - \frac{\frac{\frac{w_{1N}}{w_{1G}}-2\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}+1}{\frac{w_{1N}}{w_{1G}}-1}}{1-\frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}}\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}}\right]$ is decreasing over $\frac{\tilde{c}}{w_0}$, and $\left[[\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}}] - \frac{\frac{\frac{w_{1N}}{w_{1G}}-2\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}+1}{\frac{w_{1N}}{w_{1G}}-1}}{1-\frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}}\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}}\right]|_{\frac{\tilde{c}}{w_0}=1} = \sqrt{\frac{t_H}{\tau_H}} - \sqrt{\frac{w_{1G}}{w_{1N}}} > 0$ at interior cases.

Therefore, $\left[[\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}}] - \frac{\frac{\frac{w_{1N}}{w_{1G}}-2\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}+1}{\frac{w_{1N}}{w_{1G}}-1}}{1-\frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}}-1)}}\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}}\right]$ is always positive and $\frac{d\mu(\psi^I)}{d\sqrt{\frac{w_{1N}}{w_{1G}}}}$ is always negative.

When $\frac{t_H}{\tau_H} \geqslant 1$,

$$\left[[\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}}] - \frac{\frac{\frac{w_{1N}}{w_{1G}} - 2\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}} + 1}{\frac{w_{1N}}{w_{1G}} - 1}}{1 - \frac{\tilde{c}}{w_0} \frac{\sqrt{\frac{w_{1N}}{w_{1G}}} - \sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}(\frac{w_{1N}}{w_{1G}} - 1)}} \frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}} - 1}{\sqrt{\frac{t_H}{\tau_H}}} \right]$$

is always positive using the same argument and $\frac{d\mu(\psi^I, P)}{d\sqrt{\frac{w_{1N}}{w_{1G}}}} < 0$.

When ψ^I is either 0 or $\bar{\psi}$, $\mu(\psi^I, P)$ is increasing in w_{1G} and decreasing in w_{1N}. Moreover,there is no discontinuous jump of $\mu(\psi^I, P)$ with respect to w_{1G} or w_{1N}.

□

From this lemma we know that if we are interested in the transition of ψ^* between unconstrained case (ψ^I, information constraint not binding) and constrained case ($\hat{\psi}_L$ or $\hat{\psi}_R$, information constraint binding), the transition would happen *at most once* with respect to various parameters.

Therefore if we draw a graph of ψ^I, $\hat{\psi}_L$ and $\hat{\psi}_R$ with respect to various parameters, there is at most one intersection between 1) ψ^I and 2) $\hat{\psi}_L$ or $\hat{\psi}_R$.

Proof of Corollary 1.1. From Lemma A.4, we know that $\hat{\psi}_L$ is increasing in w_{1G} and $\hat{\psi}_R$ is decreasing in w_{1G}. Also, Lemma A.5 shows that ψ^I is decreasing over w_{1G} if $\frac{t_H}{\tau_H} < 1$.

By lemma A.6, $\mu(\psi^I, P)$ is increasing over w_{1G}, and there exists at most one intersection point between $\mu(\psi^I, P)$ (as a function of w_{1G}) and $\bar{\mu}$. For appropriate levels of $\bar{\mu}$, therefore, $\mu(\psi^I, P) < \bar{\mu}$ for w_{1G} being small, $\mu(\psi^I, P) > \bar{\mu}$ for w_{1G} being larger.

If ψ_I intersects with $\hat{\psi}_L$, then it is obvious that $\psi^* = \min\left\{\psi^I, \hat{\psi}_L\right\}$ will first increase over w_{1G} when it's small, and then decrease over w_{1G} when it's larger (due to the fact that ψ_I decreases over w_{1G} when $\frac{t_H}{\tau_H} < 1$).

If ψ_I intersects with $\hat{\psi}_R$, then when w_{1G} is very small, $\hat{\psi}_L < \psi^I < \bar{\psi} < \hat{\psi}_R$, $\psi^* = \hat{\psi}_L$ and increases over w_{1G}. When w_{1G} is larger, ψ^* equals to ψ^I or $\hat{\psi}_R$ and will decrease over w_{1G}.

□

Now we can give a complete characterization of ψ^*:

Proposition A.2.

- $\bar{\theta}$:
  - *if* $\bar{\mu} \in \left(0, \frac{w_{1G}}{w_{1N}}\right]$, *then for any level of* $\bar{\theta} \in (\frac{c}{w_0}, +\infty)$, $\mu(\psi, P) \geqslant \bar{\mu}$ *for any* $\psi \in [0, \bar{\psi}]$. $\psi^* = \psi^I$ *is independent of* $\bar{\theta}$.
  - *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}} \frac{1 + \sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}} - 1}{\sqrt{\frac{t_H}{\tau_H}} - \sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1 + \sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}} - 1}{\sqrt{\frac{t_H}{\tau_H}} - \sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}\right]$, *then there exists a cutoff level* $\hat{\bar{\theta}}$, *such that for* $\bar{\theta} \leqslant \hat{\bar{\theta}}$, *there exist two levels of* ψ, $\hat{\psi}_L(\bar{\mu}, \bar{\theta})$ *and* $\hat{\psi}_R(\bar{\mu}, \bar{\theta})$, $0 <$

$\hat{\psi}_L(\bar{\mu},\bar{\theta}) < \hat{\psi}_R(\bar{\mu},\bar{\theta})$, such that $\mu(\hat{\psi}_L(\bar{\mu}),P) = \mu(\hat{\psi}_R(\bar{\mu}),P) = \bar{\mu}$. $\psi^* = \psi^I$ is independent of $\bar{\theta}$.

– *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, \mu(\bar{\psi},P) \right]$*, then there exists a cutoff level $\hat{\hat{\theta}}$, such that for $\bar{\theta} \leqslant \hat{\hat{\theta}}$, there exist two levels of ψ, $\hat{\psi}_L(\bar{\mu},\bar{\theta})$ and $\hat{\psi}_R(\bar{\mu},\bar{\theta})$, $0 < \hat{\psi}_L(\bar{\mu},\bar{\theta}) < \hat{\psi}_R(\bar{\mu},\bar{\theta})$, such that $\mu(\hat{\psi}_L(\bar{\mu}),P) = \mu(\hat{\psi}_R(\bar{\mu}),P) = \bar{\mu}$. Then there exists a level of $\tilde{\tilde{\theta}}$, such that*

$$\psi^* = \begin{cases} \psi^I, & \text{if } \bar{\theta} \geqslant \tilde{\tilde{\theta}} \\ \hat{\psi}_L(\bar{\mu},P), & \text{if } \bar{\theta} < \tilde{\tilde{\theta}} \text{ and} \frac{t_H}{\tau_H} \leqslant \bar{\mu} \\ \hat{\psi}_R(\bar{\mu},P), & \text{if } \bar{\theta} < \tilde{\tilde{\theta}} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

If $\frac{t_H}{\tau_H} \leqslant \bar{\mu}$, ψ^* is increasing over $\bar{\theta}$ if $\bar{\theta} < \tilde{\tilde{\theta}}$ and flat over $\bar{\theta} \geqslant \tilde{\tilde{\theta}}$; If $\frac{t_H}{\tau_H} > \bar{\mu}$, ψ^* is decreasing over $\bar{\theta}$ if $\bar{\theta} < \tilde{\tilde{\theta}}$ and flat over $\bar{\theta} \geqslant \tilde{\tilde{\theta}}$.

- *c:*

  – *if $\bar{\mu} \in \left(0, \frac{w_{1G}}{w_{1N}}\right]$, then for any level of $c \in (0, \bar{\theta}w_0)$, $\mu(\psi,P) \geqslant \bar{\mu}$ for any $\psi \in [0,\bar{\psi}]$. $\psi^* = \psi^I$ is independent of c.*

  – *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)} \right]$*, then there exists a cut-off level $\hat{c}$, such that for $c \geqslant \hat{c}$, there exist two levels of ψ, $\hat{\psi}_L(\bar{\mu},c)$ and $\hat{\psi}_R(\bar{\mu},c)$, $0 < \hat{\psi}_L(\bar{\mu},c) < \hat{\psi}_R(\bar{\mu},c)$, such that $\mu(\hat{\psi}_L(\bar{\mu}),P) = \mu(\hat{\psi}_R(\bar{\mu}),P) = \bar{\mu}$. $\psi^* = \psi^I$ is independent of c.*

  – *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, \mu(\bar{\psi},P) \right]$*, then there exists a cutoff level $\hat{c}$, such that for $c \geqslant \hat{c}$, there exist two levels of ψ, $\hat{\psi}_L(\bar{\mu},c)$ and $\hat{\psi}_R(\bar{\mu},c)$, $0 < \hat{\psi}_L(\bar{\mu},c) < \hat{\psi}_R(\bar{\mu},c)$, such that $\mu(\hat{\psi}_L(\bar{\mu}),P) = \mu(\hat{\psi}_R(\bar{\mu}),P) = \bar{\mu}$. Then there exists a level of $\tilde{\tilde{\theta}}$, such that*

$$\psi^* = \begin{cases} \psi^I, & \text{if } c \leqslant \hat{c} \\ \hat{\psi}_L(\bar{\mu},P), & \text{if } c > \hat{c} \text{ and} \frac{t_H}{\tau_H} \leqslant \bar{\mu} \\ \hat{\psi}_R(\bar{\mu},P), & \text{if } c > \hat{c} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

If $\frac{t_H}{\tau_H} \leqslant \bar{\mu}$, ψ^* is decreasing over c if $c > \hat{c}$ and flat over c if $c \leqslant \hat{c}$; If $\frac{t_H}{\tau_H} > \bar{\mu}$, ψ^* is increasing over c if $c > \hat{c}$ and flat over $c \leqslant \hat{c}$.

- *w_0:*

- *if* $\bar{\mu} \in \left(0, \frac{w_{1G}}{w_{1N}}\right]$, *then for any level of* $w_0 \in (\frac{c}{\bar{\theta}}, +\infty)$, $\mu(\psi, P) \geqslant \bar{\mu}$ *for any* $\psi \in [0, \bar{\psi}]$. $\psi^* = \psi^I$ *is increasing over* w_0.

- *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}}, \frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}\right]$, *then there exists a cutoff level* $\hat{w}_0$, *such that for* $w_0 \leqslant \hat{w}_0$, *there exist two levels of* ψ, $\hat{\psi}_L(\bar{\mu}, w_0)$ *and* $\hat{\psi}_R(\bar{\mu}, w_0)$, $0 < \hat{\psi}_L(\bar{\mu}, w_0) < \hat{\psi}_R(\bar{\mu}, w_0)$, *such that* $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. $\psi^* = \psi^I$ *is increasing over* w_0.

- *if* $\bar{\mu} \in \left(\frac{w_{1G}}{w_{1N}} \frac{1+\sqrt{\frac{w_{1N}}{w_{1G}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}{1+\sqrt{\frac{w_{1G}}{w_{1N}}}\left(\frac{\sqrt{\frac{t_H}{\tau_H}}\sqrt{\frac{w_{1N}}{w_{1G}}}-1}{\sqrt{\frac{t_H}{\tau_H}}-\sqrt{\frac{w_{1N}}{w_{1G}}}}\right)}, \mu(\bar{\psi}, P)\right]$, *then there exists a cutoff level* $\hat{w}_0$, *such that for* $w_0 \leqslant \hat{w}_0$, *there exist two levels of* ψ, $\hat{\psi}_L(\bar{\mu}, w_0)$ *and* $\hat{\psi}_R(\bar{\mu}, w_0)$, $0 < \hat{\psi}_L(\bar{\mu}, w_0) < \hat{\psi}_R(\bar{\mu}, w_0)$, *such that* $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. *Then there exists a level of* $\tilde{w_0}$, *such that*

$$\psi^* = \begin{cases} \psi^I, & \text{if } w_0 \geqslant \tilde{w_0} \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } w_0 < \tilde{w_0} \text{ and} \frac{t_H}{\tau_H} \leqslant \bar{\mu} \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } w_0 < \tilde{w_0} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

  If $\frac{t_H}{\tau_H} \leqslant \bar{\mu}$, ψ^* *is increasing over* w_0; *If* $\frac{t_H}{\tau_H} > \bar{\mu}$, ψ^* *is decreasing over* w_0 *if* $w_0 < \tilde{w_0}$ *and increasing* w_0 *if* $w_0 \geqslant \tilde{w_0}$.

- w_{1N}:

  - *if* $\bar{\mu} \in \left(0, \frac{\bar{\theta} w_0 - c}{\bar{\theta} w_0}\right]$, *then for any level of* $w_{1N} \in (w_{1G}, +\infty)$,$\mu(\psi, P) \geqslant \bar{\mu}$ *for any* $\psi \in [0, \bar{\psi}]$. $\psi^* = \psi^I$.

    * *if* $\frac{t_H}{\tau_H} \geqslant 1$, ψ^* *is decreasing in* w_{1N};
    * *if* $\frac{t_H}{\tau_H} < 1$, ψ^* *is decreasing in* w_{1N} *if* $\sqrt{\frac{w_{1N}}{w_{1G}}} > \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}$ *and increasing in* w_{1N} *if* $1 < \sqrt{\frac{w_{1N}}{w_{1G}}} < \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}$

  - *if* $\bar{\mu} \in \left(\frac{\bar{\theta} w_0 - c}{\bar{\theta} w_0}, \mu(\bar{\psi}, P)\right)$, *then exist a level of* $\hat{w_{1N}}$ *such that for any* $w_{1N} \geqslant \hat{w_{1N}}$, *there exist two levels of* ψ, $\hat{\psi}_L(\bar{\mu}, w_{1N})$ *and* $\hat{\psi}_R(\bar{\mu}, w_{1N})$, $0 < \hat{\psi}_L(\bar{\mu}, w_{1N}) < \hat{\psi}_R(\bar{\mu}, w_{1N})$, *such that* $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. *Then there exists a level of* $\tilde{w_{1N}} \in [\hat{w_{1N}}, +\infty)$, *such that*

$$\psi^* = \begin{cases} \psi^I, & \text{if } w_{1N} \leqslant \tilde{w_{1N}} \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } w_{1N} > \tilde{w_{1N}} \text{ and} \frac{t_H}{\tau_H} \leqslant \bar{\mu} \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } w_{1N} > \tilde{w_{1N}} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

    * *if* $\frac{t_H}{\tau_H} \geqslant 1$, *then* ψ^* *is decreasing in* w_{1N} *if* $w_{1N} \in (w_{1G}, \tilde{w_{1N}})$ *and increasing in* w_{1N} *if* $w_{1N} \in (\tilde{w_{1N}}, +\infty)$

* if $\frac{t_H}{\tau_H} < 1$, then there exist a cut-off level $\tilde{\bar{\mu}} \in (\frac{\bar{\theta}w_0-c}{\theta w_0}, 1)$, and for any $\bar{\mu}$ a level of $\frac{\bar{t_H}}{\tau_H}(\bar{\mu}) \in (0,1)$ such that:
  · if $\bar{\mu} \in \left(\frac{\bar{\theta}w_0-c}{\theta w_0}, \tilde{\bar{\mu}}\right]$, then $\frac{\bar{t_H}}{\tau_H}(\bar{\mu}) \leqslant \bar{\mu}$. Moreover:
  (1) if $\frac{t_H}{\tau_H} \in \left(0, \frac{\bar{t_H}}{\tau_H}\right]$, ψ^* is increasing over w_{1N} if $w_{1N} \in (w_{1G}, \tilde{w_{1N}})$ and decreasing over w_{1N} if $w_{1N} \in (\tilde{w_{1N}}, +\infty)$
  (2) if $\frac{t_H}{\tau_H} \in \left(\frac{\bar{t_H}}{\tau_H}, \bar{\mu}\right]$, then ψ is increasing over w_{1N} if $w_{1N} \in (w_{1G}, \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}})$ and decreasing over w_{1N} if $w_{1N} \in (\sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}, +\infty)$
  (3) if $\frac{t_H}{\tau_H} \in (\bar{\mu}, 1)$, then ψ^* is increasing over w_{1N} if $w_{1N} \in (w_{1G}, \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}})$ and decreasing over w_{1N} if $w_{1N} \in (\sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}, \tilde{w_{1N}})$ and then increasing over w_{1N} if $w_{1N} \in (\tilde{w_{1N}}, +\infty)$
  · if $\bar{\mu} \in \left(\frac{\bar{\theta}w_0-c}{\theta w_0}, \tilde{\bar{\mu}}\right]$, then $\frac{\bar{t_H}}{\tau_H}(\bar{\mu}) > \bar{\mu}$. Moreover:
  (1) if $\frac{t_H}{\tau_H} \in (0, \bar{\mu}]$, ψ^* is increasing over w_{1N} for $w_{1N} \in (w_{1G}, \tilde{w_{1N}})$ and decreasing over w_{1N} if $w_{1N} \in (\tilde{w_{1N}}, +\infty)$
  (2) if $\frac{t_H}{\tau_H} \in \left(\bar{\mu}, \frac{\bar{t_H}}{\tau_H}\right]$, then ψ is increasing over w_{1N} for $w_{1N} \in (w_{1G}, +\infty)$
  (3) if $\frac{t_H}{\tau_H} \in (\bar{\mu}, 1)$, then ψ^* is increasing over w_{1N} if $w_{1N} \in (w_{1G}, \sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}})$ and decreasing over w_{1N} if $w_{1N} \in (\sqrt{\frac{\tau_H}{t_H}} + \sqrt{\frac{\tau_H - t_H}{t_H}}, \tilde{w_{1N}})$ and then increasing over w_{1N} if $w_{1N} \in (\tilde{w_{1N}}, +\infty)$

- w_{1G}:
  - if $\bar{\mu} \in \left(0, \frac{\bar{\theta}w_0-c}{\theta w_0}\right]$, then for any level of $w_{1G} \in (0, w_{1N})$,$\mu(\psi, P) \geqslant \bar{\mu}$ for any $\psi \in [0, \bar{\psi}]$. $\psi^* = \psi^I$.
    * if $\frac{t_H}{\tau_H} \leqslant 1$, ψ^* is decreasing in w_{1G};
    * if $\frac{t_H}{\tau_H} > 1$, ψ^* is decreasing in w_{1G} if $\sqrt{\frac{w_{1N}}{w_{1G}}} > \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H - \tau_H}{\tau_H}}$ and increasing in w_{1G} if $\sqrt{\frac{w_{1N}}{w_{1G}}} < \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H - \tau_H}{\tau_H}}$
  - if $\bar{\mu} \in \left(\frac{\bar{\theta}w_0-c}{\theta w_0}, \mu(\bar{\psi}, P)\right)$, then exist a level of $\hat{w_{1G}}$ such that for any $w_{1G} \leqslant \hat{w_{1G}}$, there exist two levels of ψ, $\hat{\psi}_L(\bar{\mu}, w_{1G})$ and $\hat{\psi}_R(\bar{\mu}, w_{1G})$, $0 < \hat{\psi}_L(\bar{\mu}, w_{1G}) < \hat{\psi}_R(\bar{\mu}, w_{1G})$, such that $\mu(\hat{\psi}_L(\bar{\mu}), P) = \mu(\hat{\psi}_R(\bar{\mu}), P) = \bar{\mu}$. Then there exists a level of $\tilde{w_{1G}} \in (0, \hat{w_{1G}}]$, such that

$$\psi^* = \begin{cases} \psi^I, & \text{if } w_{1G} \geqslant \tilde{w_{1G}} \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } w_{1G} < \tilde{w_{1G}} \text{ and} \frac{t_H}{\tau_H} \leqslant \bar{\mu} \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } w_{1G} < \tilde{w_{1G}} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

    * if $\frac{t_H}{\tau_H} \in (0, \bar{\mu}]$, then ψ^* is increasing in w_{1G} if $w_{1G} \in (0, \tilde{w_{1G}})$ and decreasing in w_{1G} if $w_{1G} \in (\tilde{w_{1G}}, w_{1N})$
    * if $\frac{t_H}{\tau_H} \in (\bar{\mu}, 1]$, then ψ^* is decreasing in w_{1G} for $w_{1G} \in (0, w_{1N})$
    * if $\frac{t_H}{\tau_H} \in (1, +\infty)$, then there exist a cut-off point $\frac{\bar{t_H}}{\tau_H}$ such that:
      · if $\frac{t_H}{\tau_H} \geqslant \frac{\bar{t_H}}{\tau_H}$, then ψ^* is decreasing over w_{1G} if $w_{1G} \in (0, \tilde{w_{1G}})$ and increasing in w_{1G} if $w_{1G} \in (\tilde{w_{1G}}, w_{1N})$

$\cdot$ *if* $\frac{t_H}{\tau_H} < \frac{\bar{t_H}}{\tau_H}$, *then* ψ^* *is decreasing over* w_{1G} *if* $w_{1G} \in (0, \sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H}{\tau_H} - 1})$ *and increasing in* w_{1G} *if* $w_{1G} \in (\sqrt{\frac{t_H}{\tau_H}} + \sqrt{\frac{t_H}{\tau_H} - 1}, w_{1N})$

- $\bar{\mu}$:
  - *if* $\bar{\mu} \in \left(0, \mu(\hat{\psi}, P)\right]$, $\psi^* = \psi^I$ *is independent of* $\bar{\mu}$.
  - *if* $\bar{\mu} \in (\mu(\hat{\psi}, P), \mu(\bar{\psi}, P))$, *there exist a level of cutoff point* $\tilde{\bar{\mu}}$ *such that:*

$$\psi^* = \begin{cases} \psi^I, & \text{if } \bar{\mu} \leqslant \tilde{\bar{\mu}} \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } \bar{\mu} > \tilde{\bar{\mu}} \text{ and} \frac{t_H}{\tau_H} \leqslant \bar{\mu} \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } \bar{\mu} > \tilde{\bar{\mu}} \text{ and} \frac{t_H}{\tau_H} > \bar{\mu} \end{cases}$$

    - *if* $\frac{t_H}{\tau_H} \in (0, \bar{\mu}]$, ψ^* *is flat over* $\bar{\mu}$ *if* $\bar{\mu} \leqslant \tilde{\bar{\mu}}$ *and decreasing over* $\bar{\mu}$ *if* $\bar{\mu} > \tilde{\bar{\mu}}$
    - *if* $\frac{t_H}{\tau_H} \in (\bar{\mu}, +\infty)$, ψ^* *is flat over* $\bar{\mu}$ *if* $\bar{\mu} \leqslant \tilde{\bar{\mu}}$ *and increasing over* $\bar{\mu}$ *if* $\bar{\mu} > \tilde{\bar{\mu}}$

- $\frac{t_H}{\tau_H}$
  - *if* $\bar{\mu} \in (0, \mu(\hat{\mu})]$, $\psi^* = \psi^I$. ψ^* *is weakly increasing over* $\frac{t_H}{\tau_H}$
  - *if* $\bar{\mu} \in \left(\mu(\hat{\mu}), \mu(\hat{\psi}, P)\right]$, *then there exists two cut-off points,* $\hat{\frac{t_H}{\tau_H}}_1$ *and* $\hat{\frac{t_H}{\tau_H}}_2$ *such that*

$$\psi^* = \begin{cases} \psi^I, & \text{if } \frac{t_H}{\tau_H} \in \left(0, \hat{\frac{t_H}{\tau_H}}_1\right] \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } \frac{t_H}{\tau_H} \in \left(\hat{\frac{t_H}{\tau_H}}_1, \bar{\mu}\right] \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if } \frac{t_H}{\tau_H} \in \left(\bar{\mu}, \hat{\frac{t_H}{\tau_H}}_2\right) \\ \psi^I, & \text{if } \frac{t_H}{\tau_H} \in \left(\hat{\frac{t_H}{\tau_H}}_2, +\infty\right] \end{cases}$$

  ψ^* *is increasing over* $\frac{t_H}{\tau_H}$ *if* $\frac{t_H}{\tau_H} \in \left(0, \hat{\frac{t_H}{\tau_H}}_1\right] \cup \left(\hat{\frac{t_H}{\tau_H}}_2, +\infty\right]$, *flat over* $\frac{t_H}{\tau_H}$ *if* $\left(\hat{\frac{t_H}{\tau_H}}_1, \bar{\mu}\right]$ *and* $\left(\bar{\mu}, \hat{\frac{t_H}{\tau_H}}_2\right)$. *There is discrete jump at the point* $\frac{t_H}{\tau_H} = \bar{\mu}$

Proof. Proposition A.2 is a natural implication of previous lemmas and proposition A.1. □

Comparative Static Analysis for $H_N(\psi^*)$ and $H_G(\psi^*)$

We have looked at the comparative static analysis of ψ^*, i.e., the home government's optimal effort of driving up tension. However, the government has no obvious reason to reveal its effort and it could be hard or impossible to measure\observe its effort. A variable that are easier to observe could be $H_N(\psi)$, the probability of a protest.

Also $H_N(\psi)$ and $H_G(\psi)$ would be interesting for their own sake: $H_N(\psi)$ is the probability of nationalist protest and also the probability of home government receiving a concession. $H_G(\psi)$ is the probability of general public protest and also the probability of a real crisis.

We can similarly look at how $H_N(\psi^*)$ and $H_G(\psi^*)$ change according to the parameters:

First, some intermediate results:

Lemma A.7. • $H_N(\psi^I)$, $H_G(\psi^I)$

$$H_N(\psi^I)=\begin{cases}1-\frac{\tilde{c}}{w_0}, & if \frac{t_H}{\tau_H}\leqslant\frac{w_{1G}}{w_{1N}}\\ 1-\frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{t_H}{\tau_H}}[\frac{w_{1N}}{w_{1G}}-1]}, & if \frac{t_H}{\tau_H}\in(\frac{w_{1G}}{w_{1N}},\frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)\\ 1-\frac{\tilde{c}}{w_0+w_{1N}\bar{\psi}}, & if \frac{t_H}{\tau_H}\geqslant\frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2\end{cases}$$

$$H_G(\psi^I)=\begin{cases}1-\frac{\tilde{c}}{w_0}, & if \frac{t_H}{\tau_H}\leqslant\frac{w_{1G}}{w_{1N}}\\ 1-\frac{\tilde{c}}{w_0}\frac{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}}}{\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{w_{1G}}{w_{1N}}}}, & if \frac{t_H}{\tau_H}\in(\frac{w_{1G}}{w_{1N}},\frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)\\ 1-\frac{\tilde{c}}{w_0+w_{1G}\hat{\psi}}, & if \frac{t_H}{\tau_H}\geqslant\frac{w_{1G}}{w_{1N}}(\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2\end{cases}$$

- *$H_N(\psi^I)$ and $H_G(\psi^I)$ are increasing over w_0 and $\bar{\theta}$, and decreasing over c*
- *If $\frac{t_H}{\tau_H}<1$, then $H_N(\psi^I)$ is increasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$; If $\frac{t_H}{\tau_H}>1$ $H_N(\psi^I)$ is decreasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$ if $\sqrt{\frac{w_{1N}}{w_{1G}}}<\sqrt{\frac{t_H}{\tau_H}}+\sqrt{\frac{t_H-\tau_H}{\tau_H}}$ and increasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$ if $\sqrt{\frac{w_{1N}}{w_{1G}}}>\sqrt{\frac{t_H}{\tau_H}}+\sqrt{\frac{t_H-\tau_H}{\tau_H}}$;*
- *If $\frac{t_H}{\tau_H}>1$, $H_G(\psi^I)$ is decreasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$; If $\frac{t_H}{\tau_H}\leqslant 1$, $H_G(\psi^I)$ is decreasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$ if $\sqrt{\frac{w_{1N}}{w_{1G}}}>\frac{1}{\sqrt{\frac{\tau_H}{t_H}}-\sqrt{\frac{\tau_H-t_H}{t_H}}}$, and increasing over $\sqrt{\frac{w_{1N}}{w_{1G}}}$ if $\sqrt{\frac{w_{1N}}{w_{1G}}}<\frac{1}{\sqrt{\frac{\tau_H}{t_H}}-\sqrt{\frac{\tau_H-t_H}{t_H}}}$,*
- *$H_N(\psi^I)$ and $H_G(\psi^I)$ are weakly increasing over $\frac{t_H}{\tau_H}$*
- *$H_N(\psi^I)$ and $H_G(\psi^I)$ are independent over $\bar{\mu}$*

• $H_N(\hat{\psi}_L(\bar{\mu}))$,$H_G(\hat{\psi}_L(\bar{\mu}))$

- *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ decrease over $\frac{w_{1N}}{w_{1G}}$*
- *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ increase over $\bar{\theta}$*
- *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ increase over w_0*
- *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ decrease over c*
- *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ are independent of $\frac{t_H}{\tau_H}$*
- *$H_N(\hat{\psi}_L(\bar{\mu}))$ and $H_G(\hat{\psi}_L(\bar{\mu}))$ are decreasing over $\bar{\mu}$*

• $H_N(\hat{\psi}_R(\bar{\mu}))$,$H_G(\hat{\psi}_R(\bar{\mu}))$

- *$H_N(\hat{\psi}_R(\bar{\mu}))$ and $H_G(\hat{\psi}_R(\bar{\mu}))$ increase over $\frac{w_{1N}}{w_{1G}}$*
- *$H_N(\hat{\psi}_R(\bar{\mu}))$ and $H_G(\hat{\psi}_R(\bar{\mu}))$ decrease over $\bar{\theta}$*
- *$H_N(\hat{\psi}_R(\bar{\mu}))$ and $H_G(\hat{\psi}_R(\bar{\mu}))$ decrease over w_0*

- $H_N(\hat{\psi}_R(\bar{\mu}))$ *and* $H_G(\hat{\psi}_R(\bar{\mu}))$ *increase over* c
- $H_N(\hat{\psi}_R(\bar{\mu}))$ *and* $H_G(\hat{\psi}_R(\bar{\mu}))$ *are independent of* $\frac{t_H}{\tau_H}$
- $H_N(\hat{\psi}_R(\bar{\mu}))$ *and* $H_G(\hat{\psi}_R(\bar{\mu}))$ *are increasing over* $\bar{\mu}$

Proof. If ψ^I is interior solution, then $\frac{\partial H_G(\psi^I)}{\partial\sqrt{\frac{w_{1N}}{w_{1G}}}} = \frac{-\tilde{c}\sqrt{\frac{t_H}{\tau_H}}}{w_0(\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{w_{1G}}{w_{1N}}})^2}\left[(\sqrt{\frac{w_{1G}}{w_{1N}}}-\sqrt{\frac{\tau_H}{t_H}})^2+1-\frac{\tau_H}{t_H}\right]$

$\frac{\partial H_N(\psi^I)}{\partial\sqrt{\frac{w_{1N}}{w_{1G}}}} = \frac{\tilde{c}}{w_0(\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{w_{1G}}{w_{1N}}})^2}\left[(\sqrt{\frac{w_{1N}}{w_{1G}}}-\sqrt{\frac{t_H}{\tau_H}})^2+1-\frac{t_H}{\tau_H}\right]$

Results about $H_N(\hat{\psi}_L(\mu))$,$H_N(\hat{\psi}_R(\mu))$,$H_G(\hat{\psi}_L(\mu))$,$H_G(\hat{\psi}_R(\mu))$ are straightforward due to the Implicit Function Theorem and $\mu(\psi, P) = \frac{H_G(\psi)}{H_N(\psi)}$. We can express $\mu(\psi, P)$ as function of $H_N(\psi)$ or function of $H_G(\psi)$, and then because $\mu(H_N(\hat{\psi}_L(\mu)), P) = \bar{\mu}$, we can get the comparative statics of $H_N(\hat{\psi}_L(\mu))$ by the Implicit Function Theorem (and similarly for $H_G(\hat{\psi}_L(\mu)), H_N(\hat{\psi}_R(\mu)), H_G(\hat{\psi}_R(\mu))$). □

Corollary A.1.

- *If* $\frac{t_H}{\tau_H} \leqslant \bar{\mu}$, $H_N(\psi^*)$ *and* $H_G(\psi^*)$ *are*
  - *increasing in* w_0 *and* $\bar{\theta}$ *and decreasing in* c
  - *first increasing and then decreasing over* $\frac{w_{1N}}{W_{1G}}$
- *If* $\frac{t_H}{\tau_H} > \bar{\mu}$,
  - $H_N(\psi^*)$ *and* $H_G(\psi^*)$ *are:*
    - * *first decreasing and then increasing over* $\bar{\theta}$
    - * *first decreasing and then increasing over* w_0
    - * *first decreasing and then increasing over* c
  - *If* $\frac{t_H}{\tau_H} \in (\bar{\mu}, 1]$, $H_N(\psi^*)$ *is increasing over* $\frac{w_{1N}}{w_{1G}}$, *and* $H_G(\psi^*)$ *is increasing and then decreasing and then increasing again over* $\frac{w_{1N}}{w_{1G}}$
  - *if* $\frac{t_H}{\tau_H} > 1$, $H_N(\psi^*)$ *and* $H_G(\psi^*)$ *are first decreasing and then increasing over* $\frac{w_{1N}}{w_{1G}}$
- $H_N(\psi^*)$ *and* $H_G(\psi^*)$ *are weakly increasing over* $\frac{t_H}{\tau_H}$. $H_N(\psi^*)$ *and* $H_G(\psi^*)$ *both jump discontinuously over* $\frac{t_H}{\tau_H}$ *at point* $\frac{t_H}{\tau_H} = \bar{\mu}$
- $\bar{\mu}$*:*
  - *if* $\frac{t_H}{\tau_H}$ *is small,* $H_N(\psi^*)$ *and* $H_G(\psi^*)$ *are first constant over* $\bar{\mu}$ *and then decreasing over* $\bar{\mu}$*;*
  - *if* $\frac{t_H}{\tau_H}$ *is in intermediate range,* $H_N(\psi^*)$ *and* $H_G(\psi^*)$ *are first constant over* $\bar{\mu}$ *,then increasing over* $\bar{\mu}$*, and then decreasing over* $\bar{\mu}$

- *if $\frac{t_H}{\tau_H}$ is large enough, $H_N(\psi^*)$ and $H_G(\psi^*)$ are first constant over $\bar{\mu}$ and then increasing over $\bar{\mu}$*

Proof. Corollary A.1 is a direct implication from Lemma A.7 and Proposition A.2. □

So the probability of protest, $H_N(\psi^*)$, and the probability of crisis, $H_G(\psi^*)$, have similar patterns as the equilibrium level of hostility, ψ^*.

Home Government's Equilibrium Utility

Lemma A.8.

$U_H(\psi^I)$ is:

- *increasing in w_{1N};*
- *decreasing in w_{1G};*
- *increasing in c if $\frac{t_H}{\tau_H} \in (0, \frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})$ and decreasing in c if $\frac{t_H}{\tau_H} \in (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}}, +\infty)$*
- *decreasing in w_0 if $\frac{t_H}{\tau_H} \in (0, (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)$ and increasing in w_0 if $\frac{t_H}{\tau_H} \in ((\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2, +\infty)$*
- *decreasing in $\bar{\theta}$ if $\frac{t_H}{\tau_H} \in (0, \frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})$ and increasing in $\bar{\theta}$ if $\frac{t_H}{\tau_H} \in (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}}, +\infty)$;*

Proof.

$$\begin{aligned} U_H(\psi^I) = \max_{\psi \in [0,\bar{\psi}]} & H_G(\psi)u_H(P,C) + \\ & [H_N(\psi) - H_G(\psi)]u_H(NP,C) + [1 - H_N(\psi)]u_H(NP,NC) \end{aligned}$$

By Envelope Theorem, for parameter θ, $\frac{dU_H(\psi^I)}{d\theta} = \frac{\partial U_H(\psi^I)}{\partial \theta}$. The results are then proved by taking partial derivative over each parameter. □

Lemma A.9. *If $\frac{t_H}{\tau_H} < \bar{\mu}$, $U_H(\hat{\psi}_L)$ is:*

- *increasing in w_{1N};*
- *decreasing in w_{1G};*
- *increasing in c;*
- *decreasing in w_0;*
- *decrease in $\bar{\theta}$*

If $\frac{t_H}{\tau_H} \geqslant \bar{\mu}$, $U_H(\hat{\psi}_L)$ is:

- *(weakly) decreasing in w_{1N};*
- *(weakly) increasing in w_{1G};*

– *(weakly) decreasing in c;*

– *(weakly) increasing in w_0;*

– *(weakly) increasing in $\bar{\theta}$*

The sign of $U_H(\hat{\psi}_L)$ over those parameters are reversed.

Proof. For $\psi = \hat{\psi}_L$, $\frac{H_G(\psi)}{H_N(\psi)} = \bar{\mu}$, and $u_H(\psi) = H_N(\psi)t_H - H_G(\psi)\tau_H + u_H(NP, NC) = H_N(\psi)t_H - \bar{\mu}H_N(\psi)\tau_H + u_H(NP, NC) = H_N(\psi)\tau_H \left[\frac{t_H}{\tau_H} - \bar{\mu}\right] + u_H(NP, NC) = \frac{1}{\bar{\mu}}H_G(\psi)\tau_H \left[\frac{t_H}{\tau_H} - \bar{\mu}\right] + u_H(NP, NC)$. By Lemma A.7, the results are proved.

□

Proposition A.3. *If $\frac{t_H}{\tau_H} \leqslant \bar{\mu}$, $U_H(\psi^*)$ is:*

* *increasing in w_{1N};*
* *decreasing in w_{1G};*
* *increasing in c ;*
* *decreasing in w_0*
* *decreasing in $\bar{\theta}$*
* *weakly decreasing in $\bar{\mu}$*

If $\frac{t_H}{\tau_H} > \bar{\mu}$, $U_H(\psi^*)$ is:

* *single-peaked over some interior level of w_{1G};*
* *Increasing or single-peaked over w_{1N};*
* *If $\frac{t_H}{\tau_H} \in (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}}, +\infty)$,decreasing over c; If $\frac{t_H}{\tau_H} \in (\bar{\mu}, \frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})$, increasing or single-peaked over c;*
* *If $\frac{t_H}{\tau_H} \in (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}}, +\infty)$,increasing over $\bar{\theta}$; If $\frac{t_H}{\tau_H} \in (\bar{\mu}, \frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})$, decreasing or single-peaked over $\bar{\theta}$;*
* *If $\frac{t_H}{\tau_H} \in ((\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2, +\infty)$, increasing over w_0, if $\frac{t_H}{\tau_H} \in (\bar{\mu}, (\frac{w_0+w_{1N}\bar{\psi}}{w_0+w_{1G}\bar{\psi}})^2)$, decreasing or single-peaked over w_0*

Proof of Proposition A.3. When ψ^* is interior solution, which means $\psi^* = \psi^I$, $u_H(\psi^I)$ is increasing in w_{1N} and decreasing in w_{1G} by Lemma A.8.

By Lemma A.3, $Eu_H(\hat{\psi}_L(\bar{\mu})) \leqslant Eu_H(\hat{\psi}_R(\bar{\mu}))$,if and only if $\frac{t_H}{\tau_H} \leqslant \bar{\mu}$.

The results follow naturally from Lemma A.8,Lemma A.9 and proposition A.2.

□

Proof of Proposition 1.2. Proposition A.3 immediately implies Proposition 1.2. □

Proof of Corollary 1.2. 1) First result follows directly from Lemma 1.2. Because $u_H(\psi)$ under any ψ such that $(\frac{H_N(\psi)-H_G(\psi)}{H_G(\psi)}) > (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})$ will be strictly dominated by $u_H(\psi)$

under ψ that $(\frac{H_N(\psi)-H_G(\psi)}{H_G(\psi)}) = (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})$. Therefore, for a level of ψ to be optimal, it must be the case that $(\frac{H_N(\psi)-H_G(\psi)}{H_G(\psi)}) \leqslant (\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})$. This is equivalent to: $\frac{H_G(\psi)}{H_N(\psi)} \geqslant \frac{1}{1+(\frac{1-\bar{\mu}}{\bar{\mu}})(\frac{\lambda_H}{\lambda_L})}$.

2) $\frac{1}{1+\frac{\lambda_H}{\lambda_L}\frac{1-\bar{\mu}}{\bar{\mu}}}$, the effective threshold of doubt, is increasing over $\frac{\lambda_L}{\lambda_H}$. The space of ψ that makes protest informative will thus shrink and this would (weakly) decrease the home government's equilibrium utility.

3) when $\frac{\lambda_L}{\lambda_H} \to \infty$, $\frac{1}{1+\frac{\lambda_H}{\lambda_L}\frac{1-\bar{\mu}}{\bar{\mu}}}$ converges to 1. For any finite $\bar{\psi}$, $\frac{H_G(\bar{\psi})}{H_N(\bar{\psi})} \leqslant \frac{1}{1+\frac{\lambda_H}{\lambda_L}\frac{1-\bar{\mu}}{\bar{\mu}}}$ for $\frac{\lambda_L}{\lambda_H}$ large enough. Therefore, the only set of ψ where protest is informative enough is $[0, \hat{\psi}_L]$. $\hat{\psi}_L$ decreases over $\frac{\lambda_L}{\lambda_H}$ and will converge to zero as $\frac{\lambda_L}{\lambda_H} \to \infty$.

□

Proof of Proposition 1.3. Given the posterior μ, the foreign government's payoff if conceding: $\mu u_F(P, C) + (1-\mu)u_F(NP, C)$

his payoff if not conceding: $\mu(1-q)u_F(P, NC) + [\mu q + (1-\mu)]\, u_F(NP, NC)$

The level of μ that the foreign government is indifferent between conceding or not, $\bar{\mu}$, is: $\bar{\mu}u_F(P, C) + (1-\bar{\mu})u_F(NP, C) = \bar{\mu}(1-q)u_F(P, NC) + [\bar{\mu}q + (1-\bar{\mu})]\, u_F(NP, NC)$

$\bar{\mu}\,[u_F(P, C) - qu_F(P, NC) - (1-q)u_F(NP, NC)]$

$= (1-\bar{\mu})\,[u_F(NP, NC) - u_F(NP, C)]$

$\bar{\mu} = \frac{u_F(NP,NC)-u_F(P,NC)}{[u_F(P,C)-(1-q)u_F(P,NC)-qu_F(NP,NC)]+[u_F(NP,NC)-u_F(P,NC)]}$

$\bar{\mu} > 0$ because $u_F(NP, NC) - u_F(P, NC) > 0$ and

$[u_F(P, C) - (1-q)u_F(P, NC) - qu_F(NP, NC)] + [u_F(NP, NC) - u_F(P, NC)]$

$\geqslant [u_F(P, C) - u_F(NP, NC)] + [u_F(NP, NC) - u_F(P, NC)] = u_F(P, C) - u_F(P, NC) > 0.$

$\bar{\mu}$ increases over q because $[u_F(P, C) - (1-q)u_F(P, NC) - qu_F(NP, NC)] + [u_F(NP, NC) - u_F(P, NC)]$ decreases over q, since $u_F(NP, NC) > u_F(P, NC)$

$\bar{\mu}_{|q=0} \in (0, 1)$ and $\bar{\mu}_{|q=1} > 1$. Because $\bar{\mu}$ is continuous over q, when q is near enough to μ, $\bar{\mu}$ must be greater than 1. □

Proof of Proposition 1.4.

Part I: the home government's ideal optimal propaganda:

Define $OB(\psi) \equiv H_G(\psi)u_H(P, C) + (H_N(\psi) - H_G(\psi))u_H(NP, C) + p(1 - H_N(\psi))u_H(NP, C) + (1-p)(1 - H_N(\psi))u_H(NP, NC)$.

$$\begin{aligned}
&\max_{\psi\in[0,\bar{\psi}]} H_G(\psi)u_H(P,C) + (H_N(\psi) - H_G(\psi))u_H(NP,C) \\
&+ p(1-H_N(\psi))u_H(NP,C) + (1-p)(1-H_N(\psi))u_H(NP,NC) \\
=&\max_{\psi\in[0,\bar{\psi}]} H_G(\psi)\left[u_H(P,C) - u_H(NP,C)\right] \\
&+ (p+(1-p)H_N(\psi))\left[u_H(NP,C) - u_H(NP,NC)\right] + u_H(NP,NC) \\
=&\max_{\psi\in[0,\bar{\psi}]} (p+(1-p)H_N(\psi))\left[u_H(NP,C) - u_H(NP,NC)\right] \\
&- H_G(\psi)\left[u_H(NP,C) - u_H(P,C)\right] + u_H(NP,NC) \\
=&\max_{\psi\in[0,\bar{\psi}]} (p+(1-p)H_N(\psi))t_H - H_G(\psi)\tau_H + u_H(NP,NC)
\end{aligned}$$

Define $\tilde{\psi}^I$ as the level of ψ that maximizes the new objective function (without the informativeness constraint). Define $\tilde{t}_H = (1-p)t_H$.

Taking F.O.C.:

$$\begin{aligned}
&(1-p)t_H H_N'(\psi) - \tau_H H_G'(\psi) = 0 \\
\iff &\tilde{t}_H H_N'(\psi) - \tau_H H_G'(\psi) = 0
\end{aligned}$$

$\tilde{\psi}^I$ is just the ψ_I in the baseline model with t_H now replaced by $\tilde{t}_H$.

By Envelope theorem, $OB(\tilde{\psi}^I)$ is increasing over p because $u_H(NP,C) > u_H(NP,NC)$.[1]

Part II: We now look at the informativeness constraint.

When the foreign government observes a protest, his posterior belief that this protest comes from the general public:
$\mu(\psi,\tilde{s}=P) \equiv Prob(a_G = P|\psi,\tilde{s}=P) = \frac{H_G(\psi)}{H_G(\psi)+(H_N(\psi)-H_G(\psi))+p(1-H_N(\psi))}$, where the $p(1-H_N(\psi))$ part comes from the fake protests.

The foreign government will only concede iff:
$\frac{H_G(\psi)}{H_G(\psi)+(H_N(\psi)-H_G(\psi))+p(1-H_N(\psi))} \geqslant \bar{\mu}$

The home government's objective function: $H_G(\psi)u_H(P,C)+(H_N(\psi)-H_G(\psi))u_H(NP,C)+p(1-H_N(\psi))u_H(NP,C)+(1-p)(1-H_N(\psi))u_H(NP,NC)$

$$\begin{aligned}
&\frac{H_G(\psi)}{H_G(\psi)+(H_N(\psi)-H_G(\psi))+p(1-H_N(\psi))} \geqslant \bar{\mu} \\
\iff &\frac{H_G(\psi)}{p+(1-p)H_N(\psi)} \geqslant \bar{\mu}
\end{aligned}$$

$\mu(\psi,P) = \frac{H_G(\psi)}{p+(1-p)H_N(\psi)}$

[1]In the boundary cases, $u_H(\tilde{\psi}^I)$ is also increasing over p because $u_H(NP,C) > u_H(NP,NC)$.

then $\frac{\partial\mu(\psi,P)}{\partial\psi} = \frac{H_G(\psi)}{p+(1-p)H_N(\psi)}\left[\frac{\frac{\partial H_G(\psi)}{\partial\psi}}{H_G(\psi)} - \frac{\frac{\partial(p+(1-p)H_N(\psi))}{\partial\psi}}{(p+(1-p)H_N(\psi))}\right]$

$= \frac{H_G(\psi)}{p+(1-p)H_N(\psi)}\left[\frac{\tilde{c}w_{1G}}{(w_0+w_{1G}\psi)^2}\frac{w_0+w_{1G}\psi}{(w_0-\tilde{c})+w_{1G}\psi} - \frac{\tilde{c}w_{1N}}{(w_0+w_{1N}\psi)^2}\frac{w_0+w_{1N}\psi}{(\frac{1}{1-p}w_0-\tilde{c})+\frac{1}{1-p}w_{1N}\psi}\right]$

$= \frac{H_G(\psi)}{p+(1-p)H_N(\psi)}\left[\frac{\tilde{c}w_{1G}}{(w_0+w_{1G}\psi)[(w_0-\tilde{c})+w_{1G}\psi]} - \frac{\tilde{c}w_{1N}}{(w_0+w_{1N}\psi)\left[(\frac{1}{1-p}w_0-\tilde{c})+\frac{1}{1-p}w_{1N}\psi\right]}\right]$

$= \frac{H_G(\psi)}{p+(1-p)H_N(\psi)}\left[\frac{\tilde{c}}{(\frac{w_0}{w_{1G}}+\psi)[(w_0-\tilde{c})+w_{1G}\psi]} - \frac{\tilde{c}}{(\frac{w_0}{w_{1N}}+\psi)\left[(\frac{1}{1-p}w_0-\tilde{c})+\frac{1}{1-p}w_{1N}\psi\right]}\right]$

□

$$\begin{aligned}
&\frac{\partial\mu(\psi,P)}{\partial\psi} \geqslant 0\\
&\iff (\frac{w_0}{w_{1G}}+\psi)\left[(w_0-\tilde{c})+w_{1G}\psi\right] \leqslant (\frac{w_0}{w_{1N}}+\psi)\left[(\frac{1}{1-p}w_0-\tilde{c})+\frac{1}{1-p}w_{1N}\psi\right]\\
&\iff (\frac{1}{1-p}w_{1N}-w_{1G})\psi^2 + 2w_0\frac{p}{1-p}\psi + \left[\frac{w_0(\frac{w_0}{1-p}-\tilde{c})}{w_{1N}} - \frac{w_0(w_0-c)}{w_{1G}}\right] \geqslant 0
\end{aligned}$$

$\frac{1}{1-p}w_{1N} - w_{1G} \geqslant w_{1N} - w_{1G} > 0$,

function $(\frac{1}{1-p}w_{1N} - w_{1G})\psi^2 + 2w_0\frac{p}{1-p}\psi + \left[\frac{w_0(\frac{w_0}{1-p}-\tilde{c})}{w_{1N}} - \frac{w_0(w_0-c)}{w_{1G}}\right]$ is convex and reaches its minimum at $\psi = -2\frac{(\frac{1}{1-p}w_{1N}-w_{1G})}{2w_0\frac{p}{1-p}\psi}$. Therefore, it is growing on the interval $[0, +\infty)$.

Thus $(\frac{1}{1-p}w_{1N} - w_{1G})\psi^2 + 2w_0\frac{p}{1-p}\psi + \left[\frac{w_0(\frac{w_0}{1-p}-\tilde{c})}{w_{1N}} - \frac{w_0(w_0-c)}{w_{1G}}\right] = 0$ can have at most one solution on the interval $[0, +\infty)$.

If that solution exist, then $\mu(\psi, P)$ is first decreasing and then increasing over ψ. For appropriate levels of $\bar{\mu}$, again there exist levels of ψ, $\hat{\psi}_L$ and $\hat{\psi}_R$, such that $0 < \hat{\psi}_L < \hat{\psi}_R$, and $\mu(\hat{\psi}_L, P) = \mu(\hat{\psi}_R, P) = \bar{\mu}$

Similar as before, we can easily show that:

1) $\frac{\partial\hat{\psi}_R}{\partial p} > 0$ and $\frac{\partial\hat{\psi}_L}{\partial p} < 0$

2) $u_H(\hat{\psi}_R)$ increases over p if $\frac{t_H}{\tau_H} > \bar{\mu}$ and decreases over p if $\frac{t_H}{\tau_H} > \bar{\mu}$

3) $u_H(\hat{\psi}_L)$ decreases over p if $\frac{t_H}{\tau_H} > \bar{\mu}$ and increases over p if $\frac{t_H}{\tau_H} > \bar{\mu}$

4) $u_H(\hat{\psi}_R) \geqslant u_H(\hat{\psi}_L)$ *iff* $\frac{t_H}{\tau_H} \geqslant \bar{\mu}$

Moreover, as p goes to 1, eventually only $\hat{\psi}_R$ can exist but not $\hat{\psi}_L$.

This is because $\left\{(\frac{1}{1-p}w_{1N} - w_{1G})\psi^2 + 2w_0\frac{p}{1-p}\psi + \left[\frac{w_0(\frac{w_0}{1-p}-\tilde{c})}{w_{1N}} - \frac{w_0(w_0-c)}{w_{1G}}\right]\right\}_{\psi=0} > 0$ when p is close enough to 1.

We can define the optimal solution to the home government's problem,ψ^*, in the revised game same as before:

$$\psi^* \in \underset{\psi\in[0,\bar{\psi}]}{\operatorname{argmax}} H_G(\psi)u_H(P,C) + (H_N(\psi) - H_G(\psi))u_H(NP,C)$$
$$+ p(1 - H_N(\psi))u_H(NP,C) + (1-p)(1 - H_N(\psi))u_H(NP,NC)$$

$$s.t. \ \frac{H_G(\psi)}{H_G(\psi) + (H_N(\psi) - H_G(\psi)) + p(1 - H_N(\psi))} \geqslant \bar{\mu}$$

Similar as before, we know that:

$$\psi^* = \begin{cases} \psi^I, & \text{if } \psi_I \notin \left(\hat{\psi}_L, \hat{\psi}_R\right) \\ \hat{\psi}_L(\bar{\mu}, P), & \text{if } \psi_I \in \left(\hat{\psi}_L, \hat{\psi}_R\right) \text{and} \frac{t_H}{\tau_H} < \bar{\mu} \\ \hat{\psi}_R(\bar{\mu}, P), & \text{if if } \psi_I \in \left(\hat{\psi}_L, \hat{\psi}_R\right) \text{and} \frac{t_H}{\tau_H} \geqslant \bar{\mu} \end{cases}$$

Part III: Now we can show that the graph of $\psi^I(p)$ cannot intersect with both the graph of $\hat{\psi}_L(p)$ and the graph of $\hat{\psi}_R(p)$: When $\frac{t_H}{\tau_H} > \bar{\mu}$, $\psi^I(p)$ cannot intersect with $\hat{\psi}_L(p)$; When $\frac{t_H}{\tau_H} < \bar{\mu}$, $\psi^I(p)$ cannot intersect with $\hat{\psi}_R(p)$.

Suppose not.

First, it is obvious that $OB(\psi, p)$ is continuous over ψ and p. Also, it is obvious that $\psi^I(p)$, $\hat{\psi}_L(p)$, and $\hat{\psi}_R(p)$ are continuous over p.

Then when $\frac{t_H}{\tau_H} > \bar{\mu}$, there exist a level of $\hat{p}$ such that $\psi^I(p)$ and $\hat{\psi}_L(p)$ intersects, so $\psi^I(\hat{p}) = \hat{\psi}_L(\hat{p})$. Hence $u_H(\psi^I(p)) = u_H(\hat{\psi}_L(p))$. Because it is a intersection point, moving from $\hat{p}$ a little bit either to the left or right, we must have $\hat{\psi}_R(p) > \psi^I(p) > \hat{\psi}_L(p)$. Without any loss of generality, let's assume it is the right side.

Then pick $p = \hat{p} + \epsilon$, $\epsilon > 0$, for any $\delta > 0$, for ϵ small enough we have $|\hat{\psi}_L(p) - \psi^I(\hat{p})| = |\hat{\psi}_L(p) - \hat{\psi}_L(\hat{p})| < \delta$. Then for any $h > 0$, for small enough δ and small enough ϵ we have $|u_H(\hat{\psi}_L(p), p) - u_H(\psi^I(\hat{p}), \hat{p})| < h$, which means, $u_H(\hat{\psi}_L(p), p) > u_H(\psi^I(\hat{p}), \hat{p}) - h$.

However, $u_H(\hat{\psi}_L(p), p) = u_H(\hat{\psi}_R(p), p) - (\frac{t_H}{\tau_H} - \bar{\mu})(H_N(\hat{\psi}_R(p)) - H_N(\hat{\psi}_L(p)))$. Therefore, $u_H(\hat{\psi}_R(p), p) > u_H(\psi^I(\hat{p}), \hat{p}) - h + (\frac{t_H}{\tau_H} - \bar{\mu})(H_N(\hat{\psi}_R(p)) - H_N(\hat{\psi}_L(p)))$

Also, for any $j > 0$, we can pick ϵ to be very small so $|u_H(\hat{\psi}_R(p), p) - u_H(\hat{\psi}_R(\hat{p}), \hat{p})| < j$, and hence $u_H(\hat{\psi}_R(\hat{p}), \hat{p}) > u_H(\hat{\psi}_R(p), p) - j$.

However, then $u_H(\hat{\psi}_R(\hat{p}), \hat{p}) > u_H(\psi^I(\hat{p}), \hat{p}) - h - j + (\frac{t_H}{\tau_H} - \bar{\mu})(H_N(\hat{\psi}_R(p)) - H_N(\hat{\psi}_L(p))) > u_H(\psi^I(\hat{p}), \hat{p})$ if we pick h and j small enough. This leads to contradiction because $u_H(\psi^I(\hat{p}), \hat{p}) \geqslant u_H(\psi, \hat{p})$ for any other ψ by definition.

Therefore, $\psi^I(\hat{p})$ and $\hat{\psi}_L(p)$ cannot have intersection points if $\frac{t_H}{\tau_H} > \bar{\mu}$. The other part of the assertion is proved using exactly the same method.

Part IV: we know that $\hat{\psi}_R(p)$ is increasing over p and when p goes to 1, $\hat{\psi}_R(p)$ converges to $\frac{\frac{c}{1-\bar{\mu}} - w_0}{w_{1G}}$. Therefore, if $\bar{\psi} \geqslant \frac{\frac{c}{1-\bar{\mu}} - w_0}{w_{1G}}$, the upper-bound of ψ has no effort.

If $\bar{\psi} < \frac{\frac{c}{1-\bar{\mu}} - w_0}{w_{1G}}$, From part III, we know that if $\frac{t_H}{\tau_H} \geqslant \bar{\mu}$, for $\psi \leqslant \bar{\psi}$, $\psi^*(p) = \max\left\{\psi_I(p), \hat{\psi}_R(p)\right\}$. The maximal of two continuous functions must be continuous so $\psi^*(p)$ is continuous over p. Because $u_H(\psi^I(p), p)$ and $u_H(\hat{\psi}_R(p), p)$ are both increasing over p, inside each region of ψ^* where there is no switching between ψ^* and $\hat{\psi}_L$, $u_H(\psi^*(p), p)$ is increasing. Also, because both $u_H(\psi, p)$ and $\psi^*(p)$ are continuous,$u_H(\psi^*(p), p)$ is continuous. Therefore, there is no discrete jump and $u_H(\psi^*(p), p)$ is increasing. At the point of p that $\bar{\psi} = \hat{\psi}_R(p)$, for any $p' > p$, $\bar{\psi} < \hat{\psi}_R(p')$. Therefore, the home government has to either choose $\psi^*(p') = \hat{\psi}_L(p')$ (if it exists) or $\psi^*(p') = 0$ (if $\hat{\psi}_L(p')$ doesn't exist). There is a discontinuous downwards jump of utility in either case, and $u_H(\psi^*(p''), p'') \leqslant u_H(\psi^*(p'), p')$ for $p'' > p'$. Therefore, $u_H(p)$ is maximized at the point of that $\bar{\psi} = \hat{\psi}_R(p)$.

If $\frac{t_H}{\tau_H} < \bar{\mu}$, from part III we know that for $\psi \leqslant \bar{\psi}$, $\psi^* = \min\left\{\psi_I, \hat{\psi}_L\right\}$. Without loss of generality, assume that $\hat{\psi}_L(p) < \bar{\mu}$ for any p. Both ψ_I and $\hat{\psi}_L$ are continuously decreasing over p, so $\psi^* = \min\left\{\psi_I, \hat{\psi}_L\right\}$ is also continuously decreasing over p. Because $u_H(\psi^I(p), p)$ and $u_H(\hat{\psi}_L(p), p)$ are both increasing over p (if $\frac{t_H}{\tau_H} < \bar{\mu}$), therefore, inside each region of ψ^* where there is no switching, $u_H(\psi^*(p), p)$ is increasing. Also, because both $u_H(\psi, p)$ and $\psi^*(p)$ are continuous,$u_H(\psi^*(p), p)$ is continuous. Therefore, there is no discrete jump and $u_H(\psi^*(p), p)$ is increasing. At the point of p such that $\hat{\psi}_L(p) = 0$, for $p' = p + \epsilon$, either $\hat{\psi}_R(p') \geqslant \bar{\psi}$ or $\hat{\psi}_R(p') < \bar{\psi}$. In the first scenario, $\psi^* = 0$ and $u_H(0, p)$ for $p \geqslant p'$ is constant over p. In the second scenario, $u_H(\psi^*(p), p) = \max\left\{u_H(0, p), u_H(\hat{\psi}_R(p), p)\right\}$ which is weakly decreasing over p for $p \geqslant p'$, because $u_H(0, p)$ for $p \geqslant p'$ is constant over p, and $u_H(\hat{\psi}_R(p), p)$ is decreasing over p when $\frac{t_H}{\tau_H} < \bar{\mu}$.

Appendix B

Proofs and Further Results of Chapter 2

Proof of Proposition 2.1. Suppose that the investment bank trades x when the state is G, and z when the state is B. He incurs additional cost of capital if he further acquires shares in the secondary market (i.e., either $x > 0$ or $z > 0$). Also, recall that $u \equiv (1-\beta)\phi$.

	State	Liquidity Shocks	$\tilde{v}$	Probability	x_{PI}	x_{IB}	y
(I).	G	Yes	V_H	$\mu_s\gamma$	$-u$	x	$y_I \equiv -u + x$
(II).	G	No	V_H	$\mu_s(1-\gamma)$	0	x	$y_{II} \equiv x$
(III).	B	Yes	V_L	$(1-\mu_s)\gamma$	$-u$	z	$y_{III} \equiv -u + z$
(IV).	B	No	V_L	$(1-\mu_s)(1-\gamma)$	0	z	$y_{IV} \equiv z$

To camouflage as liquidity traders, the investment bank has to design his trading strategy such that two of the above four scenarios have the same aggregate order flows. This gives four possibilities: $y_I = y_{III}$ (i.e. $-u + x = -u + z$), $y_I = y_{IV}$ (i.e. $-u + x = z$), $y_{II} = y_{III}$ (i.e. $x = -u + z$) or $y_{II} = y_{IV}$ (i.e. $x = z$). Note that the first and the last coincide. Hence we investigate the following three cases: 1. $x = z$, 2. $z = -u + x$, and 3. $z = u + x$.

Case 1. $x = z$:

	State	Liquidity Shocks	Probability	x_{PI}	x_{IB}	y	P_1
(I).	G	Yes	$\mu_s\gamma$	$-u$	x	$-u + x$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)\gamma} + V_L$
(II).	G	No	$\mu_s(1-\gamma)$	0	x	x	$\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)(1-\gamma)} + V_L$
(III).	B	Yes	$(1-\mu_s)\gamma$	$-u$	x	$-u + x$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)\gamma} + V_L$
(IV).	B	No	$(1-\mu_s)(1-\gamma)$	0	x	x	$\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)(1-\gamma)} + V_L$

It is easy to see that $P_1 = \mu_s\Delta V + V_L$ since the net order flows are only indicative of whether or not there is liquidity shock, but reveals no information concerning the underlying state due to the investment bank's consistent trading strategy regardless of his private information. So the market maker will set a price to the intrinsic value of the security conditional on the posterior belief μ_s. The investment bank's expected payoff from this trading strategy is

$$
\begin{aligned}
\mathbb{E}_s[\Pi_1] = & \ [V_H - (\mu_s \Delta V + V_L)][\mu_s \gamma + \mu_s(1-\gamma)]x \\
& + [V_L - (\mu_s \Delta V + V_L)][(1-\mu_s)\gamma + (1-\mu_s)(1-\gamma)]x - \mathbb{1}_{\{x>0\}}\, r(\mu_s \Delta V + V_L)x \\
= & \ -\mathbb{1}_{\{x>0\}}\, r(\mu_s \Delta V + V_L)x \\
\leqslant & \ 0.
\end{aligned}
$$

Case 2. $z = -u + x$:

	State	Liquidity Shocks	Probability	x_{PI}	x_{IB}	y	P_1
(I).	G	Yes	$\mu_s\gamma$	$-u$	x	$-u+x$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} + V_L$
(II).	G	No	$\mu_s(1-\gamma)$	0	x	x	V_H
(III).	B	Yes	$(1-\mu_s)\gamma$	$-u$	$-u+x$	$-2u+x$	V_L
(IV).	B	No	$(1-\mu_s)(1-\gamma)$	0	$-u+x$	$-u+x$	$\frac{\mu_s\gamma\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} + V_L$

The investment bank's expected trading profits from this trading strategy are

$$
\begin{aligned}
\mathbb{E}_s[\Pi_2] = & \left(V_H - \frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} - V_L\right)\mu_s\gamma x \\
& + \left(V_L - \frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} - V_L\right)(1-\mu_s)(1-\gamma)(-u+x) \\
& - \mathbb{1}_{\{x>0\}}\, rx\left[\mu_s\gamma\frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} + V_L + \mu_s(1-\gamma)V_H\right] \\
& - \mathbb{1}_{\{-u+x>0\}}\, r(-u+x)\left[(1-\mu_s)\gamma V_L + (1-\mu_s)(1-\gamma)\left(\frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} + V_L\right)\right] \\
= & \ \frac{\mu_s\gamma(1-\mu_s)(1-\gamma)\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}\cdot u - \mathbb{1}_{\{x>0\}}\, rx\left[\mu_s\gamma\frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} + V_L + \mu_s(1-\gamma)V_H\right] \\
& - \mathbb{1}_{\{-u+x>0\}}\, r(-u+x)\left[(1-\mu_s)\gamma V_L + (1-\mu_s)(1-\gamma)\left(\frac{\mu_s\gamma\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} + V_L\right)\right] \\
\leqslant & \ \frac{\mu_s\gamma(1-\mu_s)(1-\gamma)\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}\cdot u.
\end{aligned}
$$

In this case, it is optimal to set $x = 0$ and $z = -u$ such that the investment bank can achieve the maximal expected trading profits $\frac{\mu_s\gamma(1-\mu_s)(1-\gamma)\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)} \cdot u$ while do not incur additional cost of capital from acquiring shares in the secondary market. It is an informed sales equilibrium where the investment bank only sell his stake when his private information is unfavorable. Moreover, such trading strategy is sequentially rational as well.

Finally, we consider Case 3. ($z = u + x$):

	State	Liquidity Shocks	Probability	x_{PI}	x_{IB}	y	P_1
(I).	G	Yes	$\mu_s\gamma$	$-u$	x	$-u+x$	V_H
(II).	G	No	$\mu_s(1-\gamma)$	0	x	x	$\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma}+V_L$
(III).	B	Yes	$(1-\mu_s)\gamma$	$-u$	$u+x$	x	$\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma}+V_L$
(IV).	B	No	$(1-\mu_s)(1-\gamma)$	0	$u+x$	$u+x$	V_L

His relevant expected trading profits are

$$\begin{aligned}
\mathbb{E}_s[\Pi_3] = & \left[V_H - \frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma} - V_L\right]\mu_s(1-\gamma)x \\
& + \left[V_L - \frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma} - V_L\right](1-\mu_s)\gamma x \\
& - \mathbb{1}_{\{x>0\}}\, rx\left[\mu_s\gamma V_H + \mu_s(1-\gamma)\left(\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma} + V_L\right)\right] \\
& - \mathbb{1}_{\{x+u>0\}}\, r(x+u)\left[(1-\mu_s)\gamma\left(\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)(1-\gamma)} + V_L\right) + (1-\mu_s)\gamma V_L\right] \\
\leqslant & -\mathbb{1}_{\{x>0\}}\, rx\left[\mu_s\gamma V_H + \mu_s(1-\gamma)\left(\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)\gamma} + V_L\right)\right] \\
& - \mathbb{1}_{\{x+u>0\}}\, r(x+u)\left[(1-\mu_s)\gamma\left(\frac{\mu_s(1-\gamma)\Delta V}{\mu_s(1-\gamma)+(1-\mu_s)(1-\gamma)} + V_L\right) + (1-\mu_s)\gamma V_L\right] \\
\leqslant & \; 0.
\end{aligned}$$

This strategy is obviously suboptimal.

In sum, the investment bank's optimal trading strategy is $x_{IB} = 0$ in state G and $x_{IB} = -u$ in state B. This gives the equilibrium characterized in Proposition 2.1. ■

Proof of Lemma 2.2.

$$\begin{aligned}
U_{IB}^1\left(\frac{\phi}{1+\phi}, \mu_s\right) &= \frac{\phi}{1+\phi}\left[(\mu_s\Delta V + V_L) - (1+r)P_0\left(\frac{\phi}{1+\phi}, \mu_s\right)\right] + \frac{1}{1+\phi}\cdot\frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \\
&= \frac{\phi}{1+\phi}\cdot\left[-r(\mu_s\Delta V + V_L) + (1+r)\cdot\frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}\right] \\
&\quad + \frac{1}{1+\phi}\cdot\frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} \\
&= \frac{\phi}{1+\phi}\cdot\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} + \frac{1}{1+\phi}\cdot\Delta P \\
&= -\frac{r\phi}{1+\phi}\cdot\mathbb{E}_s[\tilde{v}] + \left(1 + \frac{r\phi}{1+\phi}\right)\cdot\Delta P.
\end{aligned}$$

Note that

$$\frac{\partial\,\mathbb{E}_s[\tilde{v}]}{\partial\mu_s} = \frac{\partial(\mu_s\Delta V + V_L)}{\partial\mu_s} = \Delta V, \qquad \frac{\partial^2\,\mathbb{E}_s[\tilde{v}]}{\partial\mu_s^2} = 0,$$

and

$$\frac{\partial\,\Delta P}{\partial \mu_s} = \frac{\partial}{\partial \mu_s}\left(\frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)}\right)$$
$$= \gamma(1-\gamma)\phi\Delta V\cdot\frac{(1-2\gamma)\mu_s^2-2(1-\gamma)\mu_s+(1-\gamma)}{[\mu_s\gamma+(1-\mu_s)(1-\gamma)]^2}.$$

Moreover,

$$\frac{\partial^2\,\Delta P}{\partial \mu_s^2} = \gamma(1-\gamma)\phi\Delta V\cdot\frac{-2\gamma(1-\gamma)}{[\mu_s\gamma+(1-\mu_s)(1-\gamma)]^3} < 0.$$

Therefore

$$\frac{\partial\,U_{IB}^1}{\partial \mu_s} = -\frac{r\phi\Delta V}{1+\phi} + \left(1+\frac{r\phi}{1+\phi}\right)\cdot\frac{\gamma(1-\gamma)\phi\Delta V[(1-2\gamma)\mu_s^2-2(1-\gamma)\mu_s+(1-\gamma)]}{[\mu_s\gamma+(1-\mu_s)(1-\gamma)]^2},$$

and

$$\frac{\partial^2\,U_{IB}^1}{\partial \mu_s^2} = \left(1+\frac{r\phi}{1+\phi}\right)\cdot\left(\frac{\partial^2\,\Delta P}{\partial \mu_s^2}\right) < 0,$$

i.e. U_{IB}^1 is concave and $\partial U_{IB}^1/\partial\mu_s$ is decreasing in $\mu_s \in (0,1)$.

To ensure that the interior optimum is attained at some $\mu^* \in (0,1)$, the following must be satisfied:

$$\left.\frac{\partial\,U_{IB}^1}{\partial \mu_s}\right|_{\mu_s=0} = -\frac{r\phi\Delta V}{1+\phi} + \left(1+\frac{r\phi}{1+\phi}\right)\gamma\phi\Delta V > 0;$$
$$\left.\frac{\partial\,U_{IB}^1}{\partial \mu_s}\right|_{\mu_s=1} = -\frac{r\phi\Delta V}{1+\phi} - \left(1+\frac{r\phi}{1+\phi}\right)(1-\gamma)\phi\Delta V < 0.$$

The first implies that $r < \frac{\gamma(1+\phi)}{1-\gamma\phi}$ while the second is always satisfied. Then $\partial U_{IB}^1/\partial\mu_s = 0$ when $\mu_s = \mu^*$. Also, for $\mu_s \in [0,\mu^*)$, $\partial U_{IB}^1/\partial\mu_s > 0$ yet $\partial U_{IB}^1/\partial\mu_s < 0$ for $\mu_s \in (\mu^*,1]$. Therefore, U_{IB}^1 is single-peaked and has a hump shape on $[0,1]$.

Since from above we know that

$$U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) = -\frac{r\phi}{1+\phi}\cdot(\mu_s\Delta V+V_L) + \left(1+\frac{r\phi}{1+\phi}\right)\cdot\frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma+(1-\mu_s)(1-\gamma)},$$

it is obvious that there always exists a set of $\mu_s \in (0,1)$ such that $U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) > 0$ as long as r is not too large. In particular, we impose that for $\mu_s = \frac{1}{2}$, $U_{IB}^1(\frac{\phi}{1+\phi},\frac{1}{2}) > 0$. This implies $r < \frac{\gamma(1-\gamma)(1+\phi)\Delta V}{\Delta V-\gamma(1-\gamma)\phi\Delta V+2V_L}$. Therefore, $0 < r < \min\{\frac{\gamma(1+\phi)}{1-\gamma\phi}, \frac{\gamma(1-\gamma)(1+\phi)\Delta V}{\Delta V-\gamma(1-\gamma)\phi\Delta V+2V_L}\}$, i.e. $r \in (0, \frac{\gamma(1-\gamma)(1+\phi)\Delta V}{\Delta V-\gamma(1-\gamma)\phi\Delta V+2V_L})$.

In the meantime, $U_{IB}^1(\frac{\phi}{1+\phi},0) = -\frac{\phi r V_L}{1+\phi} < 0$ and $U_{IB}^1(\frac{\phi}{1+\phi},1) = -\frac{\phi r(\Delta V+V_L)}{1+\phi} < 0$. Hence there must be a pair of $\{\underline{\mu},\overline{\mu}\}$ with $0 < \underline{\mu} < \frac{1}{2} < \overline{\mu} < 1$ such that $U_{IB}^1(\frac{\phi}{1+\phi},\underline{\mu}) = U_{IB}^1(\frac{\phi}{1+\phi},\overline{\mu}) = 0$. In addition, $U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) > 0$ if $\mu_s \in (\underline{\mu},\overline{\mu})$, and $U_{IB}^1(\frac{\phi}{1+\phi},\mu_s) < 0$ if $\mu_s \in [0,\underline{\mu}) \cup (\overline{\mu},1]$.

Last but not least, it follows naturally that $\partial U_{IB}^1/\partial\mu_s > 0$ at $\mu_s = \underline{\mu}$ but $\partial U_{IB}^1/\partial\mu_s < 0$ at $\mu_s = \overline{\mu}$, an important observation that will be useful to calculate the comparative static analysis of the optimal disclosure later. ■

Proof of Proposition 2.4. When $\beta \in [\frac{\phi}{1+\phi}, 1)$, there will be discount in the issue price. Also,

$$U_{IB}^1(\beta, \mu_s) = \beta\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} + (1-\beta)\Delta P.$$

Note that

$$\begin{aligned}\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} - \Delta P &= -r(\mathbb{E}_s[\tilde{v}] - \Delta P) \\ &= -r\left[V_L + \mu_s\Delta V \cdot \frac{\mu_s\gamma + (1-\mu_s)(1-\gamma)(1-\gamma\phi)}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}\right] \\ &< 0.\end{aligned}$$

Hence to maximize $U_{IB}^1(\beta, \mu_s)$, we want $(1-\beta)$ to be as large as possible. This is achieved by choosing the smallest $\beta = \frac{\phi}{1+\phi}$ such that informed trading is still feasible. Also, it is easy to see that $U_{IB}^1(\frac{\phi}{1+\phi}, \mu_s) > U_{IB}^1(1^-, \mu_s)$ for all $\mu_s \in (0,1)$. So in equilibrium, stake $\frac{\phi}{1+\phi}$ strictly dominates stake 1^-. Moreover, we know that for $\beta = 0$, $U_{IB}^1(0, \mu_s) = 0$, and for $\beta = 1$, $U_{IB}^1(1, \mu_s) < 0$. So $\beta = 0$ strictly dominates $\beta = 1$. To characterize the investment bank's optimal retention at posterior belief μ_s, it suffices to compare $U_{IB}^1(0, \mu_s)$ with $U_{IB}^1(\frac{\phi}{1+\phi}, \mu_s)$. From Lemma 2.2, it follows that the investment bank's optimal stake is

$$\beta^* = \begin{cases} \frac{\phi}{1+\phi} & \text{if } \mu_s \in (\underline{\mu}, \overline{\mu}), \\ 0 & \text{if } \mu_s \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]. \end{cases}$$

His equilibrium payoff is

$$\hat{U}_{IB}^1(\mu_s) = \begin{cases} U_{IB}^1(\frac{\phi}{1+\phi}, \mu_s) & \text{if } \mu_s \in (\underline{\mu}, \overline{\mu}), \\ 0 & \text{if } \mu_s \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]. \end{cases}$$

■

Proof of Proposition 2.5. From Proposition 2.4 we know that the investment bank will hold a positive stake $\frac{\phi}{1+\phi}$ only when $\mu_s \in (\underline{\mu}, \overline{\mu})$. So for this set of posterior beliefs, there will be informed trading by the bank and thus an adverse selection discount in the issue price. The issuer's expected proceeds are

$$U_E^1(\mu_s) = \mu_s\Delta V + V_L - \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

At any other posterior belief, the investment bank retains zero stake and cannot engage in informed trading. The issue price will just be the intrinsic value of the security, i.e.

$$U_E^1(\mu_s) = \mu_s\Delta V + V_L.$$

■

Proof of Proposition 2.6. At any prior belief $\mu_0 \in [0, \underline{\mu}] \cup [\overline{\mu}, 1]$, a sender-preferred equilibrium prescribes that the investment bank should not retain any shares. In this case, the issue price will be the expected value of the cash flows from the security with no discount. Thus the issuer does not benefit from persuasion and the optimal disclosure system should be completely uninformative, i.e $\pi_G = \pi_B \in (0,1)$, yielding posteriors $\mu_\ell = \mu_h = \mu_0$.

At prior belief $\mu_0 \in (\underline{\mu}, \overline{\mu})$, the investment bank holds a strictly positive stake, and there will be a discounted associated with the issue price. The issuer's expected payoff under any Bayesian plausible posteriors μ_h and μ_ℓ is

$$\begin{aligned}
\mathbb{E}_\pi[U_E^1(\mu_s)] &= \mathbb{E}_\pi[\mathbb{1}_{\{\mu_0 \in [0,\underline{\mu}] \cup [\overline{\mu},1]\}} \cdot (\mu_s \Delta V + V_L) + \mathbb{1}_{\{\mu_0 \in (\underline{\mu}, \overline{\mu})\}} (\mu_s \Delta V + V_L - \Delta P)] \\
&= \mathbb{P}[\mu_h] \cdot [\mathbb{1}_{\{\mu_h \in [0,\underline{\mu}] \cup [\overline{\mu},1]\}} \cdot (\mu_h \Delta V + V_L) + \mathbb{1}_{\{\mu_h \in (\underline{\mu}, \overline{\mu})\}} (\mu_h \Delta V + V_L - \Delta P)] \\
&\quad + \mathbb{P}[\mu_\ell] \cdot [\mathbb{1}_{\{\mu_\ell \in [0,\underline{\mu}] \cup [\overline{\mu},1]\}} \cdot (\mu_\ell \Delta V + V_L) + \mathbb{1}_{\{\mu_\ell \in (\underline{\mu}, \overline{\mu})\}} (\mu_\ell \Delta V + V_L - \Delta P)] \\
&\leqslant \mathbb{P}(\mu_h)(\mu_h \Delta V + V_L) + \mathbb{P}(\mu_\ell)(\mu_\ell \Delta V + V_L),
\end{aligned}$$

where the last inequality is satisfied with if $\mu_\ell \in [0, \underline{\mu}]$, $\mu_h \in [\overline{\mu}, 1]$ and $\mathbb{P}(\mu_h)\mu_h + \mathbb{P}(\mu_\ell) = \mu_0$. Hence the least informative optimal disclosure yields posteriors $\mu_\ell = \underline{\mu}$ and $\mu_h = \overline{\mu}$. In this case $\hat{U}_E^1(\mu_0) = \max \mathbb{E}_\pi[U_E^1(\mu_s)] = \mu_0 \Delta V + V_L$. Using Bayes' theorem, simple algebra gives $\pi_B = \frac{(1-\overline{\mu})(\mu_0 - \underline{\mu})}{(1-\mu_0)(\overline{\mu} - \underline{\mu})}$ and $\pi_G = \frac{\overline{\mu}(\mu_0 - \underline{\mu})}{\mu_0(\overline{\mu} - \underline{\mu})}$. ■

Proof of Proposition 2.7. Recall that $\overline{\mu}$ and $\underline{\mu}$ are two roots to the equation $U_{IB}^1(\frac{\phi}{1+\phi}, \mu_s) = 0$. Write explicitly,

$$\begin{aligned}
U_{IB}^1(\frac{\phi}{1+\phi}, \mu_s) &= \frac{\phi}{1+\phi} \cdot \left[-r(\mu_s \Delta V + V_L) + (1+r) \cdot \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi \Delta V}{\mu_s \gamma + (1-\mu_s)(1-\gamma)} \right] \\
&\quad + \frac{1}{1+\phi} \cdot \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi \Delta V}{\mu_s \gamma + (1-\mu_s)(1-\gamma)} \\
&= 0.
\end{aligned}$$

Multiply both sides by $\frac{1+\phi}{\phi}$, and define

$$\begin{aligned}
F(\mu_s, \theta) &\equiv \frac{1+\phi}{\phi} \cdot U_{IB}^1(\frac{\phi}{1+\phi}, \mu_s) \\
&= -r(\mu_s \Delta V + V_L) + [1 + (1+r)\phi] \cdot \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\Delta V}{\mu_s \gamma + (1-\mu_s)(1-\gamma)},
\end{aligned}$$

where $\theta \in \{V_L, \frac{\Delta}{V_L}, r, \phi\}$. By the implicit function theorem, at $\mu_s = \underline{\mu}$ or $\overline{\mu}$,

$$\frac{\partial F}{\partial \mu_s} \cdot \frac{\partial \mu_s}{\partial \theta} + \frac{\partial F}{\partial \theta} = 0.$$

This gives

$$\operatorname{sign}\left(\frac{\partial \mu_s}{\partial \theta} \right) = -\operatorname{sign}\left(\frac{\partial F}{\partial \mu_s} \cdot \frac{\partial F}{\partial \theta} \right).$$

Next we calculate $F(\mu_s, \theta)$'s partial derivatives with respect to different $\theta \in \{V_L, \eta, r, \phi\}$:

$$\frac{\partial F}{\partial V_L} = -r < 0;$$
$$\frac{\partial F}{\partial r} = -V_L - \mu_s \Delta V \left[1 - \frac{(1-\mu_s)(1-\gamma)\gamma\phi}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}\right] < 0;$$
$$\frac{\partial F}{\partial \phi} = (1+r) \cdot \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} > 0.$$

Define $\eta \equiv \frac{\Delta V}{V_L}$, and

$$f \equiv \frac{F}{\Delta V} = -r(\mu_s \Delta V + \frac{1}{\eta}) + [1 + (1+r)\phi] \cdot \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

So

$$\frac{\partial f}{\partial \eta} = \frac{r}{\eta^2} > 0 \Rightarrow \frac{\partial F}{\partial \eta} = \frac{r\Delta V}{\eta^2} > 0.$$

Moreover, in the proof of Lemma 2.2 we have shown that $\frac{\partial F}{\partial \mu_s} > 0$ at $\mu_s = \underline{\mu}$ but $\frac{\partial F}{\partial \mu_s} < 0$ at $\mu_s = \overline{\mu}$. Consequently, we have (1) $\frac{\partial \underline{\mu}}{\partial V_L} > 0$ and $\frac{\partial \overline{\mu}}{\partial V_L} < 0$; (2) $\frac{\partial \underline{\mu}}{\partial \eta} < 0$ and $\frac{\partial \overline{\mu}}{\partial \eta} > 0$; (3) $\frac{\partial \underline{\mu}}{\partial r} > 0$ and $\frac{\partial \overline{\mu}}{\partial r} < 0$; (4) $\frac{\partial \underline{\mu}}{\partial \phi} < 0$ and $\frac{\partial \overline{\mu}}{\partial \phi} > 0$. ■

Proof of Proposition 2.8. If the investment bank chooses to underwrite and his planned retention is $\hat{\beta}$, we can write his expected payoff as

$$U_{IB}^2(\hat{\beta}, \mu_s) = \epsilon A(1-\psi, \mu_s) + (1-\epsilon)B(\hat{\beta}, \mu_s) = [\epsilon(1-\psi) + (1-\epsilon)\hat{\beta}] \cdot [-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P] + [\epsilon\psi + (1-\epsilon)(1-\hat{\beta})] \cdot \Delta P.$$

Recall from the proof of Proposition 2.4 that $-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P < \Delta P$, thus we want $\hat{\beta}$ to be as small as possible yet such stake still allows the underwriter to engage in informed trading if demand shock does not happen. The optimal planned retention is $\hat{\beta} = \frac{\phi}{1+\phi}$, the stake that is just enough for the bank to camouflage as liquidity traders. ■

Proof of Lemma 2.3. The proof resembles that of Lemma 2.2. Specifically, the equation now becomes

$$U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) = [\epsilon(1-\psi) + (1-\epsilon)\hat{\beta}][-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P] + [\epsilon\psi + (1-\epsilon)(1-\hat{\beta})]\Delta P = 0.$$

$\frac{\partial^2 U_{IB}^2}{\partial \mu_s^2} < 0$ because $\frac{\partial^2 \Delta P}{\partial \mu_s^2} < 0$. So U_{IB}^2 is concave in μ_s. To ensure that the interior optimum is attained at some $\mu^* \in (0,1)$, the following must be satisfied:

$$\left.\frac{\partial U_{IB}^2}{\partial \mu_s}\right|_{\mu_s=0} = -Kr\Delta V + [(1+r)K + (1-K)]\gamma\phi\Delta V > 0;$$

$$\left.\frac{\partial U_{IB}^2}{\partial \mu_s}\right|_{\mu_s=1} = -Kr\Delta V - [(1+r)K + (1-K)](1-\gamma)\phi\Delta V < 0,$$

where $K \equiv \epsilon(1-\psi) + (1-\epsilon)(\frac{\phi}{1+\phi})$ and $1 - K = \epsilon\psi + (1-\epsilon)(\frac{1}{1+\phi})$. The first inequality implies

$$r < \frac{\gamma\phi(1+\phi)}{[(1+\phi)(1-\psi)\epsilon + \phi(1-\epsilon)](1-\gamma\phi)},$$

while the second is always satisfied.

Some simple algebra reveals that $U_{IB}^2(\frac{\phi}{1+\phi}, 0) < 0$ and $U_{IB}^2(\frac{\phi}{1+\phi}, 1) < 0$. Moreover, we need $U_{IB}^2(\frac{\phi}{1+\phi}, \frac{1}{2}) > 0$. This implies

$$r < \frac{\gamma\phi(1+\phi)(1-\gamma)\Delta V}{[(1+\phi)(1-\psi)\epsilon + \phi(1-\epsilon)][\Delta V - \gamma\phi(1-\gamma)\Delta V + 2V_L]}.$$

Therefore, $r < \min\left\{\frac{\gamma\phi(1+\phi)}{[(1+\phi)(1-\psi)\epsilon+\phi(1-\epsilon)](1-\gamma\phi)}, \frac{\gamma\phi(1+\phi)(1-\gamma)\Delta V}{[(1+\phi)(1-\psi)\epsilon+\phi(1-\epsilon)][\Delta V-\gamma\phi(1-\gamma)\Delta V+2V_L]}\right\}$, i.e. $r < \frac{\gamma\phi(1+\phi)(1-\gamma)\Delta V}{[(1+\phi)(1-\psi)\epsilon+\phi(1-\epsilon)][\Delta V-\gamma\phi(1-\gamma)\Delta V+2V_L]}$. Note that both U_{IB}^1 and U_{IB}^2 are convex combinations of two ingredients $-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P$ and ΔP with the latter strictly larger than the former. It is easy to see that U_{IB}^1 puts more weight on ΔP and thus less weight on $-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P$ than U_{IB}^2. Hence $U_{IB}^1 > U_{IB}^2$, $\forall \mu_s \in [0,1]$.

With the same logic used in the proof of Lemma 2.2, it follows naturally:

1. There exists a pair $\{\underline{\mu}^*, \overline{\mu}^*\}$ with $0 < \underline{\mu} < \underline{\mu}^* < \frac{1}{2} < \overline{\mu}^* < \overline{\mu} < 1$ such that $U_{IB}^2(\frac{\phi}{1+\phi}, \underline{\mu}^*) = U_{IB}^2(\frac{\phi}{1+\phi}, \overline{\mu}^*) = 0$.
2. $U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) > 0$ if $\mu_s \in (\underline{\mu}^*, \overline{\mu}^*)$, and $\tilde{U}_{IB}(\frac{\phi}{1+\phi}, \mu_s) < 0$ if $\mu_s \in [0, \underline{\mu}^*)$ or $\mu_s \in (\overline{\mu}^*, 1]$.

■

Proof of Proposition 2.9 and Proposition 2.10. It follows naturally from Proposition 2.8 and Lemma 2.3 that at $T = 1$, the investment bank will agree to underwrite if his planned retention $\frac{\phi}{1+\phi}$ gives him a non-negative expected payoff. So he chooses to underwrite if $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$, and not underwrite otherwise. The issuer is only able to issue the security when $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$, and get an expected payoff of $\mathbb{E}_s[\tilde{v}] - \Delta P$. ■

Proof of Proposition 2.11. The optimal information design depends on the prior μ_0.

1. First we investigate the optimal system when prior $\mu_0 \in [0, \underline{\mu}^*)$. In this case the investment bank does not underwrite if no additional information is disclosed. Consider any two arbitrary posteriors μ_ℓ and μ_h with $0 \leqslant \mu_\ell \leqslant \mu_0 \leqslant \mu_h \leqslant 1$ and $\mathbb{P}[s=\ell]\mu_\ell + \mathbb{P}[s=h]\mu_h = \mu_0$. To maximize her expected proceeds, the issuer will set $\mu_\ell = 0$ to have the maximal $\mathbb{P}[s=h]\mu_h$ which is μ_0. Also, the issuer will set a $\mu_h \in [\underline{\mu}^*, \overline{\mu}^*]$ so that the investment bank is willing to underwrite. Her expected payoff is therefore $\mathbb{P}[s=h]P_0(\mu_h) = \frac{\mu_0 P_0(\mu_h)}{\mu_h}$. Recall that

$$U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) = K[-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P] + (1-K)\Delta P = -rKP_0(\mu_s) + \Delta P(\mu_s),$$

where $K = \epsilon(1-\psi) + (1-\epsilon)(\frac{\phi}{1+\phi})$, and $\Delta P(\mu_s)$ means ΔP is a function of μ_s.

At $\mu_s = \underline{\mu}^*$ or $\overline{\mu}^*$, $U^2_{IB}(\frac{\phi}{1+\phi}, \mu_s) = 0$. This implies

$$-rKP_0(\mu_s) + \Delta P(\mu_s) = 0$$
$$\Rightarrow \frac{P_0(\mu_s)}{\mu_s} = \frac{\Delta P(\mu_s)}{rK\mu_s} = \frac{(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{rK[\mu_s\gamma + (1-\mu_s)(1-\gamma)]}.$$

The last term is decreasing in $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$. Since $\underline{\mu}^* < \overline{\mu}^*$, we have $\frac{P_0(\underline{\mu}^*)}{\underline{\mu}^*} > \frac{P_0(\overline{\mu}^*)}{\overline{\mu}^*}$. Moreover, at $\mu_s \in [\underline{\mu}^*, \overline{\mu}^*]$, $U^2_{IB}(\frac{\phi}{1+\phi}, \mu_s) \geqslant 0$. This implies

$$-rKP_0(\mu_s) + \Delta P(\mu_s) \geqslant 0$$
$$\Rightarrow \frac{P_0(\mu_s)}{\mu_s} \leqslant \frac{\Delta P(\mu_s)}{rK\mu_s} \leqslant \frac{\Delta P(\underline{\mu}^*)}{rK\underline{\mu}^*} = \frac{P_0(\underline{\mu}^*)}{\underline{\mu}^*}.$$

Therefore, the optimal system will induce two posteriors $\mu_\ell = 0$ and $\mu_h = \underline{\mu}^*$. The relevant precision parameters are $\pi_B = \frac{\mu_0(1-\underline{\mu}^*)}{\underline{\mu}^*(1-\mu_0)}$ and $\pi_G = 1$.

2. Second, we derive the optimal system when $\mu_0 \in (\overline{\mu}^*, 1]$. Consider any two arbitrary posteriors μ_ℓ and μ_h with $0 \leqslant \mu_\ell \leqslant \mu_0 \leqslant \mu_h \leqslant 1$ and $\mathbb{P}[s = \ell]\mu_\ell + \mathbb{P}[s = h]\mu_h = \mu_0$. To maximize her expected proceeds, the issuer will set $\mu_h = 1$. This ensures that for any fixed μ_ℓ, the probability of achieving this posterior $\mathbb{P}[s = \ell] = \frac{\mu_h - \mu_0}{\mu_h - \mu_l}$ will be maximized, i.e. the probability of underwriting will be the highest. Her expected payoff is therefore $\mathbb{P}[s = \ell]P_0(\mu_\ell) = \frac{1-\mu_0}{1-\mu_\ell} \cdot P_0(\mu_\ell)$. Since both $\frac{1-\mu_0}{1-\mu_\ell}$ and $P_0(\mu_\ell)$ are increasing in μ_ℓ, it is optimal to set $\mu_\ell = \overline{\mu}^*$. Hence the optimal system yields two posteriors $\mu_\ell = \underline{\mu}^*$ and $\mu_h = 1$. This gives $\pi_B = \frac{\mu_0 - \overline{\mu}^*}{\mu_0(1-\overline{\mu}^*)}$ and $\pi_G = 0$.

3. Third, when $\mu_0 = \underline{\mu}^*$ or $\overline{\mu}^*$, the investment bank is break-even by underwriting the deal. In this case, a completely uninformative disclosure system is optimal. It has $\pi_G = \pi_B \in (0, 1)$, yielding posteriors $\mu_\ell = \mu_h = \mu_0$.

4. Finally, we find the optimal system when $\mu_0 \in (\underline{\mu}^*, \overline{\mu}^*)$. Since $\Delta P(\mu_s)$ is concave in μ_s, $P_0(\mu_s) = \mathbb{E}_s[\tilde{v}] - \Delta P$ is convex and increases in μ_s. First consider any arbitrary posteriors μ_ℓ and μ_h such that $\underline{\mu}^* \leqslant \mu_\ell \leqslant \mu_0 \leqslant \mu_h \leqslant \overline{\mu}^*$.

   In order for the two pairs of posteriors $\{\underline{\mu}^*, \overline{\mu}^*\}$ and $\{\mu_\ell, \mu_h\}$ to be Bayesian plausible, they should satisfy

$$\mu_0 = \lambda\underline{\mu}^* + (1-\lambda)\overline{\mu}^*,$$
$$\mu_0 = \bar{\lambda}\mu_\ell + (1-\bar{\lambda})\mu_h.$$

   Moreover, we can write

$$\mu_\ell = \lambda_\ell\underline{\mu}^* + (1-\lambda_\ell)\overline{\mu}^*,$$
$$\mu_h = \lambda_h\underline{\mu}^* + (1-\lambda_h)\overline{\mu}^*.$$

   Here λ, λ_ℓ, λ_h, and $\bar{\lambda}$ all lie in $[0, 1]$.

So we have

$$\begin{aligned}\mu_0 &= \bar{\lambda}[\lambda_\ell \underline{\mu}^* + (1-\lambda_\ell)\overline{\mu}^*] + (1-\bar{\lambda})[\lambda_h \underline{\mu}^* + (1-\lambda_h)\overline{\mu}^*] \\ &= [\bar{\lambda}\lambda_\ell + (1-\bar{\lambda})\lambda_h]\underline{\mu}^* + [\bar{\lambda}(1-\lambda_\ell) + (1-\bar{\lambda})(1-\lambda_h)]\overline{\mu}^* \\ &= \lambda\underline{\mu}^* + (1-\lambda)\overline{\mu}^*.\end{aligned}$$

By Jensen's inequality,

$$\begin{aligned}U_E^2(\frac{\phi}{1+\phi}, \mu_0) &= P_0(\mu_0) \\ &\leqslant \bar{\lambda}P_0(\mu_\ell) + (1-\bar{\lambda})P_0(\mu_h) \\ &\leqslant \bar{\lambda}[\lambda_\ell P_0(\underline{\mu}^*) + (1-\lambda_\ell)P_0(\overline{\mu}^*)] + (1-\bar{\lambda})[\lambda_h P_0(\underline{\mu}^*) + (1-\lambda_h)P_0(\overline{\mu}^*)] \\ &= [\bar{\lambda}\lambda_\ell + (1-\bar{\lambda})\lambda_h]P_0(\underline{\mu}^*) + [\bar{\lambda}(1-\lambda_\ell) + (1-\bar{\lambda})(1-\lambda_h)]P_0(\overline{\mu}^*) \\ &= \lambda P_0(\underline{\mu}^*) + (1-\lambda)P_0(\overline{\mu}^*) \\ &= \hat{U}_E^2(\mu_0),\end{aligned}$$

where $\lambda = \frac{\overline{\mu}^* - \mu_0}{\overline{\mu}^* - \underline{\mu}^*} = \mathbb{P}[s = \ell]$. The issuer achieves expected payoff $\hat{U}_E^2(\mu_0)$ by setting $\mu_\ell = \underline{\mu}^*$ and $\mu_h = \overline{\mu}^*$.

We further consider two other possibilities. If we set $\mu_\ell = 0$, then the issuer's expected payoff upon observing $s = \ell$ is zero. Her expected payoff is thus $\frac{\mu_0}{\mu_h} \cdot P_0(\mu_h) < P_0(\mu_h)$. Since $P_0(\mu_s)$ is convex in μ_s, we have $P_0(\mu_h) \leqslant \lambda P_0(\underline{\mu}^*) + (1-\lambda)P_0(\overline{\mu}^*) = \hat{U}_E^2(\mu_0)$. Hence $\frac{\mu_0}{\mu_h} \cdot P_0(\mu_h) < P_0(\mu_h) \leqslant \hat{U}_E^2(\mu_0)$, rendering this strategy suboptimal. If we set $\mu_h = 1$, under this system, the issuer's expected payoff is $\frac{1-\mu_0}{1-\mu_\ell} \cdot P_0(\mu_\ell) < P_0(\mu_0) < \hat{U}_E^2(\mu_0)$ because $P_0(\mu_s)$ is convex and increasing in μ_s. Again, such system is not optimal too.

In sum, the optimal system will induce two posteriors $\mu_\ell = \underline{\mu}^*$ and $\mu_h = \overline{\mu}^*$. By Bayes' theorem, $\pi_G = \frac{\overline{\mu}^*(\mu_0 - \underline{\mu}^*)}{\mu_0(\overline{\mu}^* - \underline{\mu}^*)}$ and $\pi_B = \frac{(1-\overline{\mu}^*)(\mu_0 - \underline{\mu}^*)}{(1-\mu_0)(\overline{\mu}^* - \underline{\mu}^*)}$. ■

Proof of Proposition 2.12. Recall that if the investment bank chooses to underwrite and his planned retention is $\frac{\phi}{1+\phi}$, then

$$\begin{aligned}U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) = &-r\left[\epsilon(1-\psi) + (1-\epsilon)\frac{\phi}{1+\phi}\right](\mu_s \Delta V + V_L) \\ &+ \left\{1 + r\left[\epsilon(1-\psi) + (1-\epsilon)\frac{\phi}{1+\phi}\right]\right\} \cdot \frac{(1-\mu_s)\mu_s(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}\end{aligned}$$

Define $G(\mu_s, \theta_1) \equiv U_{IB}^2(\frac{\phi}{1+\phi}, \mu_s) = 0$ where $\theta_1 \in \{\epsilon, \psi, V_L, \frac{\Delta V}{V_L}, r, \phi\}$. By the implicit function theorem, at $\mu_s = \underline{\mu}^*$ or $\overline{\mu}^*$,

$$\frac{\partial G}{\partial \mu_s} \cdot \frac{\partial \mu_s}{\partial \theta_1} + \frac{\partial G}{\partial \theta_1} = 0.$$

Like before,

$$\text{sign}\left(\frac{\partial \mu_s}{\partial \theta_1}\right) = -\text{sign}\left(\frac{\partial G}{\partial \mu_s}\cdot\frac{\partial G}{\partial \theta_1}\right).$$

Moreover,

$$\begin{aligned}
\frac{\partial G}{\partial \epsilon} &= -r\left[(1-\psi) - \frac{\phi}{1+\phi}\right](\mathbb{E}_s[\tilde{v}] - \Delta P) < 0;\\
\frac{\partial G}{\partial \psi} &= r\epsilon(\mathbb{E}_s[\tilde{v}] - \Delta P) > 0;\\
\frac{\partial G}{\partial V_L} &= -r\left[\epsilon(1-\psi) + (1-\epsilon)\frac{\phi}{1+\phi}\right] < 0;\\
\frac{\partial G}{\partial r} &= -\left[\epsilon(1-\psi) + (1-\epsilon)\frac{\phi}{1+\phi}\right](\mathbb{E}_s[\tilde{v}] - \Delta P) < 0.
\end{aligned}$$

Multiply $G(\mu_s, \phi)$ by $(1+\phi)$ we obtain

$$g_1(\mu_s, \phi) \equiv (1+\phi)G(\mu_s,\phi) = -rK_1\,\mathbb{E}_s[\tilde{v}] + \{1 + rK_1\}\Delta P = 0,$$

where $K_1 \equiv \epsilon(1-\psi)(1+\phi) + (1-\epsilon)\phi$. This implies

$$\mathbb{E}_s[\tilde{v}] = \frac{(1+rK_1)\Delta P}{rK_1}.$$

Note that

$$\frac{\partial K_1}{\partial \phi} = 1 - \psi\epsilon = \frac{K_1 - \epsilon(1-\psi)}{\phi}.$$

Therefore,

$$\begin{aligned}
\frac{\partial g_1}{\partial \phi} &= -r\cdot\frac{K_1 - \epsilon(1-\psi)}{\phi}\cdot\frac{(1+rK_1)\Delta P}{rK_1}\\
&\quad + r\cdot\frac{K_1 - \epsilon(1-\psi)}{\phi}\cdot\Delta P + (1+rK_1)\cdot\frac{\Delta P}{\phi}\\
&= \frac{\Delta P}{\phi}\cdot\left\{-r[K_1 - \epsilon(1-\psi)]\cdot\frac{1}{rK_1} + (1+rK_1)\right\}\\
&= \frac{\Delta P}{\phi}\cdot\left[\frac{\epsilon(1-\psi)}{K_1} + rK_1\right]\\
&> 0.
\end{aligned}$$

We then divide $g_1(\mu_s, \phi)$ by ΔV, and obtain

$$g_2 \equiv -rK_1(\mu_s + \frac{1}{\eta}) + (1+rK_1)\cdot\frac{\mu_s(1-\mu_s)\gamma(1-\gamma)}{\mu_s\gamma + (1-\mu_s)(1-\gamma)},$$

where $\eta = \frac{\Delta V}{V_L}$. So

$$\frac{\partial g_2}{\partial \eta} = \frac{rK_1}{\eta^2} > 0.$$

Recall from the proof of Lemma 2.3, we know that $\partial U_{IB}^2/\partial \mu_s > 0$ at $\mu_s = \underline{\mu}^*$ yet $\partial U_{IB}^2/\partial \mu_s < 0$ at $\mu_s = \overline{\mu}^*$. Hence at $\mu_s = \underline{\mu}^*$, $\partial G/\partial \mu_s > 0$, $\partial g_1/\partial \mu_s > 0$, and $\partial g_2/\partial \mu_s > 0$. Meanwhile at $\mu_s = \overline{\mu}^*$, $\partial G/\partial \mu_s < 0$, $\partial g_1/\partial \mu_s < 0$, and $\partial g_2/\partial \mu_s < 0$.

Accordingly, by the implicit function theorem, (1) $\frac{\partial \underline{\mu}^*}{\partial \epsilon} > 0$ and $\frac{\partial \overline{\mu}^*}{\partial \epsilon} < 0$; (2) $\frac{\partial \underline{\mu}^*}{\partial \psi} < 0$ and $\frac{\partial \overline{\mu}^*}{\partial \psi} > 0$; (3) $\frac{\partial \underline{\mu}^*}{\partial V_L} > 0$ and $\frac{\partial \overline{\mu}^*}{\partial V_L} < 0$; (4) Recall that $\eta = \frac{\Delta V}{V_L}$, then $\frac{\partial \underline{\mu}^*}{\partial \eta} < 0$ and $\frac{\partial \overline{\mu}^*}{\partial \eta} > 0$; (5) $\frac{\partial \underline{\mu}^*}{\partial r} > 0$ and $\frac{\partial \overline{\mu}^*}{\partial r} < 0$; (6) $\frac{\partial \underline{\mu}^*}{\partial \phi} < 0$ and $\frac{\partial \overline{\mu}^*}{\partial \phi} > 0$. ■

Proof of Proposition 2.13. If there is no demand uncertainty, the investment bank chooses his optimal retention β to maximize his expected payoff:

$$U_{IB}^3(\beta, \mu_s) = \beta\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} + (1-\beta)\Delta P.$$

Because we know that $\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} < \Delta P$, it is optimal to choose the largest possible $(1-\beta)$. Since the underwrite can sell the security short in the secondary market, he no longer has to retain any share in the primary market. Thus he chooses the optimal $\beta^*(\mu_s) = 0$, and his maximal expected payoff is just

$$\hat{U}_{IB}^3(\mu_s) = U_{IB}^3(0, \mu_s) = \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}.$$

■

Proof of Proposition 2.14. Given the investment bank's best response in the primary market, the issuer's expected payoff conditional on posterior belief is

$$U_E^3(\mu_s) = (\mu_s\Delta V) - \frac{\mu_s(1-\mu_s)\gamma(1-\gamma)\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)} = P_0(\mu_s).$$

As we have shown before, this function is convex in $\mu_s \in [0,1]$. For any posteriors μ_ℓ and μ_h that are Bayesian plausible,

$$\begin{aligned} U_E^3(\mu_0) &\leqslant \mathbb{P}[s=\ell]P_0(\mu_\ell) + \mathbb{P}[s=h]P_0(\mu_h) \\ &\leqslant \mathbb{P}[s=\ell]P_0(0) + \mathbb{P}[s=h]P_0(1). \end{aligned}$$

The last inequality follows form the convexity of the function, and it holds with strict inequality if $\mu_0 \in (0,1)$. Therefore, the optimal system generates a low posterior $\mu_\ell = 0$ and a high posterior $\mu_h = 1$. The system is fully informative in that $\pi_G = 1$ and $\pi_B = 0$. ■

Proof of Lemma 2.4. If the investment bank agrees to underwrite and chooses a planed retention $\hat{\beta} = 0$, his expected payoff is

$$\begin{aligned} U_{IB}^4(\hat{\beta} = 0, \mu_s) &= \epsilon\{(1-\psi)[\mathbb{E}_s[\tilde{v}] - (1+r)(\mathbb{E}_s[\tilde{v}] - \Delta P)] + \psi\Delta P\} + (1-\epsilon)\Delta P \\ &= \epsilon(1-\psi)\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} + [\epsilon\psi + (1-\epsilon)]\Delta P \\ &= -\epsilon(1-\psi)r\,\mathbb{E}_s[\tilde{v}] + [(1+r)\epsilon(1-\psi) + \epsilon\psi + (1-\epsilon)]\Delta P \\ &> U_{IB}^2(\hat{\beta} = \frac{\phi}{1+\phi}, \mu_s). \end{aligned}$$

The last inequality holds because when demand shock does not happen and there is short sale constraint, the underwriter has to retain a positive stake to engage in informed trading, which incurs cost of capital and undermines the informed trading profits.

It is easy to see that $U_{IB}^4(\hat{\beta}=0,\mu_s)$ is concave in μ_s because of the concavity of ΔP. Like in the proofs of Lemma 2.2 and Lemma 2.3, to ensure its optimum appears at some $\mu^{**}\in(0,1)$, we require

$$\left.\frac{\partial U_{IB}^4}{\partial \mu_s}\right|_{\mu_s=0} = -\epsilon(1-\psi)r\Delta V + [(1+r)\epsilon(1-\psi)+\epsilon\psi+(1-\epsilon)]\gamma\phi\Delta V > 0;$$

$$\left.\frac{\partial U_{IB}^4}{\partial \mu_s}\right|_{\mu_s=1} = -\epsilon(1-\psi)r\Delta V - [(1+r)\epsilon(1-\psi)+\epsilon\psi+(1-\epsilon)](1-\gamma)\phi\Delta V < 0.$$

The first requires that $r<\frac{\gamma\phi}{\epsilon(1-\psi)(1-\gamma\phi)}$, while the second always holds.

It's easy to see that $U_{IB}^4(\hat{\beta}=0,0)<0$ and $U_{IB}^4(\hat{\beta}=0,1)<0$. We further require that $U_{IB}^4(\hat{\beta}=0,\frac{1}{2})>0$. This implies that $r<\frac{\gamma(1-\gamma)\phi\Delta V}{\epsilon(1-\psi)[\Delta V-\gamma(1-\gamma)\phi\Delta V+2V_L]}$. So $r<\min\{\frac{\gamma\phi}{\epsilon(1-\psi)(1-\gamma\phi)},\frac{\gamma(1-\gamma)\phi\Delta V}{\epsilon(1-\psi)[\Delta V-\gamma(1-\gamma)\phi\Delta V+2V_L]}\}$, i.e. $r<\frac{\gamma(1-\gamma)\phi\Delta V}{\epsilon(1-\psi)[\Delta V-\gamma(1-\gamma)\phi\Delta V+2V_L]}$.

As long as all of the above are satisfied, it follows naturally that:

1. There exists a pair $\{\underline{\mu}^{**},\overline{\mu}^{**}\}$ with $0<\underline{\mu}^{**}<\underline{\mu}^{*}<\frac{1}{2}<\overline{\mu}^{*}<\overline{\mu}^{**}<1$ such that $U_{IB}^4(0,\underline{\mu}^{*})=U_{IB}^4(0,\overline{\mu}^{**})=0$.
2. $U_{IB}^4(0,\mu_s)>0$ if $\mu_s\in(\underline{\mu}^{**},\overline{\mu}^{**})$, and $\tilde{U}_{IB}(0,\mu_s)<0$ if $\mu_s\in[0,\underline{\mu}^{**})$ or $\mu_s\in(\overline{\mu}^{**},1]$.

■

Proof of Proposition 2.15. From Proposition 2.13 we know that if demand shock does not happen, it is optimal for the investment bank not to retain any share in the primary market. If demand shock happens, he is forced to retain $(1-\psi)$. Therefore, his optimal planned retention should always be zero if the bank decides to underwrite. From Lemma 2.4 we know that the investment bank will choose to underwrite only at posteriors $\mu_s\in[\underline{\mu}^{**},\overline{\mu}^{**}]$, otherwise he will withdraw from underwriting. This gives his expected payoff

$$\hat{U}_{IB}^4(\mu_s)=\begin{cases}U_{IB}^4(\hat{\beta}=0,\mu_s) & \text{if } \mu_s\in[\underline{\mu}^{**},\overline{\mu}^{**}],\\ 0 & \text{if } \mu_s\in[0,\underline{\mu}^{**})\cup(\overline{\mu}^{**},1].\end{cases}$$

■

Proof of Proposition 2.16. Proposition 2.16 follows naturally from Proposition 2.15. ■

Proof of Proposition 2.17. Much of the proof resembles that of Proposition 2.11. Likewise, we consider four cases respectively.

1. If $\mu_0\in[0,\underline{\mu}^{**})$, like part 1 of Proposition 2.11's proof, it is optimal to set $\mu_\ell=0$

and the issuer's expected payoff is $\frac{\mu_0 P_0(\mu_h)}{\mu_h}$. Define $K_2 = \epsilon(1-\psi)$, so

$$U_{IB}^4(0,\mu_s) = -rK_2\,\mathbb{E}_s[\tilde{v}] + (1+rK_2)\Delta P \geqslant 0$$

$$\Rightarrow rK_2(\mathbb{E}_s[\tilde{v}] - \Delta P) \leqslant \Delta P$$
$$\Rightarrow \frac{\mu_0 P_0(\mu_s)}{\mu_s} \leqslant \frac{\mu_0 \Delta P}{rK_2\mu_s}.$$

The last holds with equality when $\mu_s = \underline{\mu}^{**}$ or $\overline{\mu}^{**}$. Since

$$\frac{\Delta P}{\mu_s} = \frac{(1-\mu_s)(1-\gamma)\gamma\phi\Delta V}{\mu_s\gamma + (1-\mu_s)(1-\gamma)}$$

which is decreasing in μ_s and achieves the maximum at $\mu_s = \underline{\mu}^{**}$. Therefore it is optimal for the issuer to set $\mu_h = \underline{\mu}^{**}$ so that she gets the highest expected payoff $\frac{\mu_0 P_0(\underline{\mu}^{**})}{\underline{\mu}^{**}}$. In sum, the optimal system will induce two posteriors $\mu_\ell = 0$ and $\mu_h = \underline{\mu}^{**}$. The relevant precision parameters are $\pi_B = \frac{\mu_0(1-\underline{\mu}^{**})}{\underline{\mu}^{**}(1-\mu_0)}$ and $\pi_G = 1$.

2. If $\mu_0 \in (\overline{\mu}^{**}, 1]$, with the same reasoning as part 2 of Proposition 2.11's proof, it is optimal to set $\mu_h = 1$. Her expected payoff is therefore $\mathbb{P}[s = \ell]P_0(\mu_\ell) = \frac{1-\mu_0}{1-\mu_\ell} \cdot P_0(\mu_\ell)$. Since both $\frac{1-\mu_0}{1-\mu_\ell}$ and $P_0(\mu_\ell)$ are increasing in μ_ℓ, it is optimal to set $\mu_\ell = \overline{\mu}^{**}$. Hence the optimal system yields two posteriors $\mu_\ell = \underline{\mu}^{**}$ and $\mu_h = 1$. This gives $\pi_B = \frac{\mu_0 - \overline{\mu}^{**}}{\mu_0(1-\overline{\mu}^{**})}$ and $\pi_G = 0$.

3. Third, when $\mu_0 = \underline{\mu}^{**}$ or $\overline{\mu}^{**}$, the investment bank is break-even by underwriting the deal. In this case, a completely uninformative disclosure system is optimal. It has $\pi_G = \pi_B \in (0,1)$, yielding posteriors $\mu_\ell = \mu_h = \mu_0$.

4. Finally, we explore the cae when $\mu_0 \in (\underline{\mu}^{**}, \overline{\mu}^{**})$. Using a similar argument as in part 3 of Proposition 2.11's proof, we have $\mu_\ell = \underline{\mu}^{**}$ and $\mu_h = \overline{\mu}^{**}$ due to the convexity of $U_{IB}^4(0,\mu_s)$ in μ_s on $[\underline{\mu}^{**}, \overline{\mu}^{**}]$. Again, setting either $\mu_\ell = 0$ or $\mu_h = 1$ is suboptimal. Hence the optimal system has $\pi_G = \frac{\overline{\mu}^{**}(\mu_0 - \underline{\mu}^{**})}{\mu_0(\overline{\mu}^{**} - \underline{\mu}^{**})}$ and $\pi_B = \frac{(1-\overline{\mu}^{**})(\mu_0 - \underline{\mu}^{**})}{(1-\mu_0)(\overline{\mu}^{**} - \underline{\mu}^{**})}$. ∎

Proof of Proposition 2.18. Note that $\overline{\mu}^{**}$ and $\underline{\mu}^{**}$ are two roots of the following equation:

$$U_{IB}^4(0,\mu_s) = -rK_2\,\mathbb{E}_s[\tilde{v}] + (1+rK_2)\Delta P = 0.$$

Define

$$J(\mu_s, \theta_1) = -rK_2\,\mathbb{E}_s[\tilde{v}] + (1+rK_2)\Delta P,$$

where $K_2 = \epsilon(1-\psi)$ and $\theta_1 \in \{\epsilon, \psi, V_L, \frac{\Delta V}{V_L}, r, \phi\}$. Some simple algebra gives

$$\frac{\partial J}{\partial \epsilon} = -r(1-\psi)(\mathbb{E}_s[\tilde{v}] - \Delta P) < 0;$$
$$\frac{\partial J}{\partial \psi} = r\epsilon(\mathbb{E}_s[\tilde{v}] - \Delta P) > 0;$$

$$\frac{\partial J}{\partial V_L} = -r\epsilon(1-\psi) < 0;$$
$$\frac{\partial J}{\partial r} = -\epsilon(1-\psi)(\mathbb{E}_s[\tilde{v}] - \Delta P) < 0;$$
$$\frac{\partial J}{\partial \phi} = [1 + r\epsilon(1-\psi)] \cdot \frac{\Delta P}{\phi} > 0$$

Let $j = J/\Delta V$, we obtain
$$\frac{\partial j}{\partial \eta} = \frac{r\epsilon(1-\psi)}{\eta^2} > 0.$$

Moreover, from the proof of Lemma 2.4, at $\mu_s = \underline{\mu}^{**}$, $\frac{\partial J}{\partial \mu_s} > 0$, while at $\mu_s = \overline{\mu}^{**}$, $\frac{\partial J}{\partial \mu_s} < 0$.

So by the implicit function theorem, we have (1) $\frac{\partial \underline{\mu}^{**}}{\partial \epsilon} > 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial \epsilon} < 0$; (2) $\frac{\partial \underline{\mu}^{**}}{\partial \psi} < 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial \psi} > 0$; (3) $\frac{\partial \underline{\mu}^{**}}{\partial V_L} > 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial V_L} < 0$; (4) Recall that $\eta = \frac{\Delta V}{V_L}$, then $\frac{\partial \underline{\mu}^{**}}{\partial \eta} < 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial \eta} > 0$; (5) $\frac{\partial \underline{\mu}^{**}}{\partial r} > 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial r} < 0$; (6) $\frac{\partial \underline{\mu}^{**}}{\partial \phi} < 0$ and $\frac{\partial \overline{\mu}^{**}}{\partial \phi} > 0$. ■

Proof of Proposition 2.19. Recall that $i \in \{1, 2, 3, 4\}$ represents one of the following four scenarios: 1. (No Short Sale, No Demand Uncertainty), 2. (No Short Sale, Demand Uncertainty), 3. (Short Sale, No Demand Uncertainty), and 4. (Short Sale, Demand Uncertainty).

We have already shown that $U^1_{IB}(\beta = \frac{\phi}{1+\phi}, \mu_s) > U^2_{IB}(\hat{\beta} = \frac{\phi}{1+\phi}, \mu_s)$ and $0 < \underline{\mu} < \underline{\mu}^* < \frac{1}{2} < \overline{\mu}^* < \overline{\mu} < 1$, as well as $U^4_{IB}(\hat{\beta} = 0, \mu_s) > U^2_{IB}(\hat{\beta} = \frac{\phi}{1+\phi}, \mu_s)$ and $0 < \underline{\mu}^{**} < \underline{\mu}^* < \frac{1}{2} < \overline{\mu}^* < \overline{\mu}^{**} < 1$. Thus it remains to compare $U^1_{IB}(\beta = \frac{\phi}{1+\phi}, \mu_s)$ and $U^4_{IB}(\hat{\beta} = 0, \mu_s)$ to rank the welfare of the investment banks. Recall that

$$\begin{aligned} U^4_{IB}(\hat{\beta} = 0, \mu_s) &= \epsilon\{(1-\psi)\{\mathbb{E}_s[\tilde{v}] - (1+r)(\mathbb{E}_s[\tilde{v}] - \Delta P)\} + \psi\Delta P\} + (1-\epsilon)\Delta P, \\ &= (\epsilon - \epsilon\psi)\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} + (\psi\epsilon + 1 - \epsilon)\Delta P, \end{aligned}$$

and
$$U^1_{IB}(\beta = \frac{\phi}{1+\phi}, \mu_s) = \frac{\phi}{1+\phi} \cdot \{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} + \frac{1}{1+\phi} \cdot \Delta P.$$

Since we have shown that $\{-r\,\mathbb{E}_s[\tilde{v}] + (1+r)\Delta P\} < \Delta P$, it is easy to see:

(1) If $\epsilon - \epsilon\psi < \frac{\phi}{1+\phi}$, i.e. $\epsilon < \frac{\phi}{(1-\psi)(1+\phi)}$, then $U^1_{IB}(\beta = \frac{\phi}{1+\phi}, \mu_s) < U^4_{IB}(\hat{\beta} = 0, \mu_s)$, and $0 < \underline{\mu}^{**} < \underline{\mu} < \underline{\mu}^* < \frac{1}{2} < \overline{\mu}^* < \overline{\mu} < \overline{\mu}^{**} < 1$. Note that the investment banks' welfare is
$$W_{IB}(i) = \int_0^1 \hat{U}^i_{IB}(\mu_0)\, d\mu_0 = \int_{\underline{\mu}_{(i)}}^{\overline{\mu}_{(i)}} U^i_{IB}(\,\cdot\,, \mu_0)\, d\mu_0.$$
where $\underline{\mu}_{(i)}$ and $\overline{\mu}_{(i)}$ denote the relevant cut-offs in scenario i, and "$\cdot$" denotes the investment banks' relevant retention in $U^i_{IB}(\,\cdot\,, \mu_s)$. Hence we obtain the following ranking:

$$W_{IB}(SS, NDU) > W_{IB}(SS, DU) > W_{IB}(NSS, NDU) > W_{IB}(NSS, DU).$$

(2) Similarly, if $\epsilon > \frac{\phi}{(1-\psi)(1+\phi)}$, then $0 < \underline{\mu} < \underline{\mu}^{**} < \underline{\mu}^{*} < \frac{1}{2} < \overline{\mu}^{*} < \overline{\mu}^{**} < \overline{\mu} < 1$ and

$$W_{IB}(SS, NDU) > W_{IB}(NSS, NDU) > W_{IB}(SS, DU) > W_{IB}(NSS, DU).$$

(3) Finally, if $\epsilon = \frac{\phi}{(1-\psi)(1+\phi)}$, then $0 < \underline{\mu} = \underline{\mu}^{**} < \underline{\mu}^{*} < \frac{1}{2} < \overline{\mu}^{*} < \overline{\mu}^{**} = \overline{\mu} < 1$ and

$$W_{IB}(SS, NDU) > W_{IB}(NSS, NDU) = W_{IB}(SS, DU) > W_{IB}(NSS, DU).$$

■

Proof of Proposition 2.20. If the issuers do not disclose additional information, the investment banks' decisions to underwrite and the issuers' expected payoffs will depend directly on μ_0. Also,

$$W_E(1) = \int_0^{\underline{\mu}} (\mu_0 \Delta V + V_L)\, d\mu_0 + \int_{\underline{\mu}}^{\overline{\mu}} \left[(\mu_0 \Delta V + V_L) - \frac{(1-\mu_0)\mu_0(1-\gamma)\gamma\phi\Delta V}{\mu_0\gamma + (1-\mu_0)(1-\gamma)} \right] d\mu_0 + \int_{\overline{\mu}}^{1} (\mu_0 \Delta V + V_L)\, d\mu_0,$$

$$W_E(2) = \int_0^{\underline{\mu}^{*}} 0\, d\mu_0 + \int_{\underline{\mu}^{*}}^{\overline{\mu}^{*}} \left[(\mu_0 \Delta V + V_L) - \frac{(1-\mu_0)\mu_0(1-\gamma)\gamma\phi\Delta V}{\mu_0\gamma + (1-\mu_0)(1-\gamma)} \right] d\mu_0 + \int_{\overline{\mu}^{*}}^{1} 0\, d\mu_0,$$

$$W_E(3) = \int_0^{1} \left[(\mu_0 \Delta V + V_L) - \frac{(1-\mu_0)\mu_0(1-\gamma)\gamma\phi\Delta V}{\mu_0\gamma + (1-\mu_0)(1-\gamma)} \right] d\mu_0,$$

$$W_E(4) = \int_0^{\underline{\mu}^{**}} 0\, d\mu_0 + \int_{\underline{\mu}^{**}}^{\overline{\mu}^{**}} \left[(\mu_0 \Delta V + V_L) - \frac{(1-\mu_0)\mu_0(1-\gamma)\gamma\phi\Delta V}{\mu_0\gamma + (1-\mu_0)(1-\gamma)} \right] d\mu_0 + \int_{\overline{\mu}^{**}}^{1} 0\, d\mu_0.$$

Therefore, the ranking is as follow,

$$W_E(NSS, NDU) > W_E(SS, NDU) > W_E(SS, DU) > W_E(NSS, DU).$$

We can write

$$P_0(\mu) = (\mu \Delta V + V_L) - \frac{(1-\mu)\mu(1-\gamma)\gamma\phi\Delta V}{\mu\gamma + (1-\mu)(1-\gamma)},$$

which is increasing in μ and does not exceed $(\mu \Delta V + V_L)$. Then if all of the issuers design

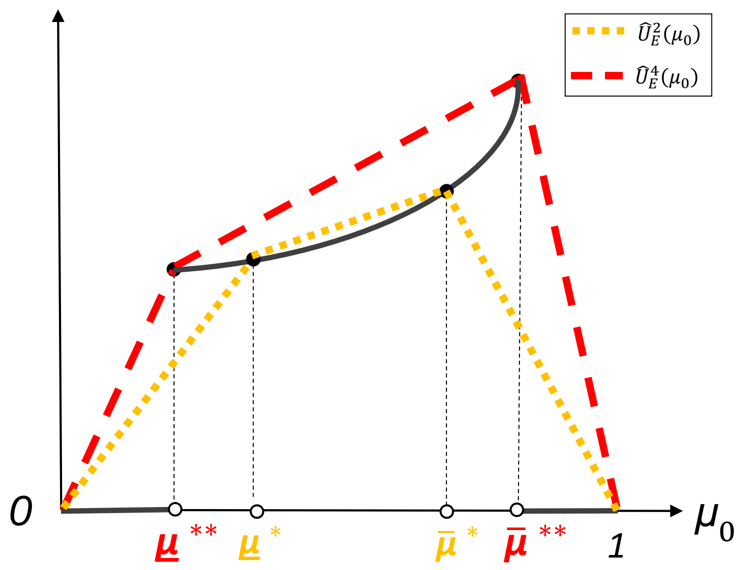


Figure B.1: Welfare Comparison

their disclosure policies optimally, their welfare under four different scenarios are

$$\hat{W}_E(1) = \int_0^1 (\mu_0 \Delta V + V_L)\, d\mu_0,$$

$$\hat{W}_E(2) = \int_0^{\underline{\mu}^*} P_0(\underline{\mu}^*) \cdot \frac{\mu_0}{\underline{\mu}^*}\, d\mu_0 + \int_{\underline{\mu}^*}^{\overline{\mu}^*} \left[P_0(\underline{\mu}^*) + \frac{P_0(\overline{\mu}^*) - P_0(\underline{\mu}^*)}{\overline{\mu}^* - \underline{\mu}^*} \cdot (\mu_0 - \underline{\mu}^*) \right] d\mu_0$$
$$+ \int_{\overline{\mu}^*}^1 \left[P_0(\overline{\mu}^*) - \frac{P_0(\overline{\mu}^*)}{1 - \overline{\mu}^*} \cdot (\mu_0 - \overline{\mu}^*) \right] d\mu_0,$$

$$\hat{W}_E(3) = \int_0^1 (\mu_0 \Delta V + V_L)\, d\mu_0,$$

$$\hat{W}_E(4) = \int_0^{\underline{\mu}^{**}} P_0(\underline{\mu}^{**}) \cdot \frac{\mu_0}{\underline{\mu}^{**}}\, d\mu_0 + \int_{\underline{\mu}^{**}}^{\overline{\mu}^{**}} \left[P_0(\underline{\mu}^{**}) + \frac{P_0(\overline{\mu}^{**}) - P_0(\underline{\mu}^{**})}{\overline{\mu}^{**} - \underline{\mu}^{**}} \cdot (\mu_0 - \underline{\mu}^{**}) \right] d\mu_0$$
$$+ \int_{\overline{\mu}^{**}}^1 \left[P_0(\overline{\mu}^{**}) - \frac{P_0(\overline{\mu}^{**})}{1 - \overline{\mu}^{**}} \cdot (\mu_0 - \overline{\mu}^{**}) \right] d\mu_0.$$

It is easy to see that $\hat{W}_E(1) = \hat{W}_E(3)$, and both achieve the highest possible welfare. It suffices to show that $\hat{W}_E(4) > \hat{W}_E(2)$. Intuitively, this is because the graph of $\hat{U}_E^2(\mu)$ is beneath that of $\hat{U}_E^4(\mu)$ for $\forall \mu \in (0,1)$ due to the convexity of $P_0(\mu)$.

Next we formally show that indeed $\hat{U}_E^4(\mu_0)$ is piece-wise larger than $\hat{U}_E^2(\mu_0)$ for any prior belief $\mu_0 \in (0,1)$. A graphical illustration is given in Figure B.1.

1. When $\mu_0 \in (0, \underline{\mu}^{**}]$, we have shown in the proofs of Proposition 2.11 and 2.17 that because $\underline{\mu}^{**} < \underline{\mu}^*$, we have $\frac{P_0(\underline{\mu}^{**})}{\underline{\mu}^{**}} > \frac{P_0(\underline{\mu}^*)}{\underline{\mu}^*}$. Hence $\frac{P_0(\underline{\mu}^{**})\mu_0}{\underline{\mu}^{**}} > \frac{P_0(\underline{\mu}^*)\mu_0}{\underline{\mu}^*}$, i.e. $\hat{U}_E^4(\mu_0) > \hat{U}_E^2(\mu_0)$.

2. When $\mu_0 \in (\underline{\mu}^{**}, \underline{\mu}^*)$, $\hat{U}_E^4(\mu_0)$ is a convex combination of $P_0(\underline{\mu}^{**})$ and $P_0(\overline{\mu}^{**})$, which is strictly larger than $P_0(\underline{\mu}^*)$ due to convexity of $P_0(\mu)$. Since $\hat{U}_E^2(\mu_0) = \frac{P_0(\underline{\mu}^*)\mu_0}{\underline{\mu}^*} <$

$P_0(\underline{\mu}^*)$, we have $\hat{U}_E^4(\mu_0) > \hat{U}_E^2(\mu_0)$.

3. When $\mu_0 \in [\underline{\mu}^*, \overline{\mu}^*]$, the convexity of $P_0(\mu)$ implies that the convex combination of $P_0(\underline{\mu}^{**})$ and $P_0(\overline{\mu}^{**})$ strictly dominates the convex combination of $P_0(\underline{\mu}^*)$ and $P_0(\overline{\mu}^*)$. This implies $\hat{U}_E^4(\mu_0) > \hat{U}_E^2(\mu_0)$.

4. When $\mu_0 \in (\overline{\mu}^*, \overline{\mu}^{**})$, $\hat{U}_E^2(\mu_0) = P_0(\overline{\mu}^*) - \frac{P_0(\overline{\mu}^*)}{1-\overline{\mu}^*} \cdot (\mu_0 - \overline{\mu}^*) < P_0(\overline{\mu}^*)$. Also, $P_0(\overline{\mu}^*)$ is strictly smaller than the convex combination of $P_0(\underline{\mu}^{**})$ and $P_0(\overline{\mu}^{**})$. Hence $\hat{U}_E^4(\mu_0) > \hat{U}_E^2(\mu_0)$.

5. When $\mu_0 \in [\overline{\mu}^{**}, 1)$, we define

$$\Delta U(\mu_0) \equiv \hat{U}_E^2(\mu_0) - \hat{U}_E^4 = [(\mu_0) = P_0(\overline{\mu}^*) - \frac{P_0(\overline{\mu}^*)}{1-\overline{\mu}^*} \cdot (\mu_0 - \overline{\mu}^*)] - [P_0(\overline{\mu}^{**}) - \frac{P_0(\overline{\mu}^{**})}{1-\overline{\mu}^{**}} \cdot (\mu_0 - \overline{\mu}^{**})].$$

It is easy to see that $\frac{\partial \Delta U}{\partial \mu_0} = \frac{P_0(\overline{\mu}^{**})}{1-\overline{\mu}^{**}} - \frac{P_0(\overline{\mu}^*)}{1-\overline{\mu}^*} > 0$ and $\Delta U(\mu_0) = 0$ if $\mu_0 = 1$. Hence at $\mu_0 \in [\overline{\mu}^{**}, 1)$, $\Delta U(\mu_0) < 0$, i.e. $\hat{U}_E^4(\mu_0) > \hat{U}_E^2(\mu_0)$.

Therefore, it follows naturally that

$$\hat{W}_E(NSS, NDU) = \hat{W}_E(SS, NDU) > \hat{W}_E(SS, DU) > \hat{W}_E(NSS, DU).$$

■

Appendix C

Proofs and Further Results of Chapter 3

Definitions and Notations

Here is a list of some definitions and notations used in Chapter 3:

- μ_0: common prior probability that state is good
- $\bar{\mu}$: the level of posterior such that the principal is indifferent between implementing high or low effort, i.e., $V(e=1,\bar{\mu}) = V(e=0,\bar{\mu})$ (when $V(e=1,\bar{\mu})$ and $V(e=0,\bar{\mu})$ only have one intersection point)
- $\bar{\mu}_1$ and $\bar{\mu}_2$: the levels of posterior such that the principal is indifferent between implementing high or low effort, i.e., $0 < \bar{\mu}_1 < \bar{\mu}_2 < 1$ such that $V(e=1,\bar{\mu}_1) - V(e=0,\bar{\mu}_1) = V(e=1,\bar{\mu}_2) - V(e=0,\bar{\mu}_2) = 0$ (when $V(e=1,\bar{\mu})$ and $V(e=0,\bar{\mu})$ have two intersection points)
- $\mu_s \equiv Prob(\theta = G|s)$: (the principal's and the agent's) common posterior belief[1] about the state, given the realization of experiment s
- $\tilde{\mu}$: the posterior generated by the optimal experiment and the realization of the experiment is "Good", when:1) state and effort are compliments; 2) $p(1,G)p(0,B) > p(1,B)p(0,G)$, 3) $V(\mu) = V(e=0,\mu)$ for some μ and $V(\mu) = V(e=1,\mu)$ for some other μ
- $\tilde{\mu}^I$ is solution to the equation that $\frac{\partial \frac{(V(e=1,\mu)-V(e=0,0))}{(\mu-0)}}{\partial\mu} = 0$.
- $\tilde{\tilde{\mu}}$: the posterior generated by the optimal experiment and the realization of the experiment is "Bad", when: 1)state and effort are substitutes; 2) $V(\mu) = V(e=1,\mu)$ for some μ, and $V(\mu) = V(e=0,\mu)$ for some other μ.

[1] Due to common knowledge and information symmetry

- $\hat{\mu}$: the a level of μ such that $\frac{\partial log\left(\frac{E_\mu p(e=1)}{(E_\mu \Delta p)^2}\right)}{\partial \mu}_{|\mu=\hat{\mu}} = 0$ (if existing)
- $V(\mu)$: principal's continuation value when the experiment has realized and lead to a posterior μ
- $\tilde{V}(\mu)$: principal's value of the whole game
- $\Delta p(\theta) \equiv p(1, \theta) - p(0, \theta)$
- $\Delta\Delta p \equiv (p(1, G) - p(0, G)) - (p(1, B) - p(0, B))$
- $E_\mu p(e) \equiv (\mu)p(e, G) + (1 - \mu)p(e, B)$
- $E_\mu \Delta p \equiv (\mu)\Delta p(G) + (1 - \mu)\Delta p(B)$

Formal Definition of the Game

Here I present a (slightly) more formal definition of the baseline model, mainly to formally define Perfect Bayesian Equilibrium in this chapter.

Player Set:

$N = \{P, A, N\}$

As mentioned earlier, besides the Principal (P), the Agent (A), there is a pseudo-player called Nature (N).

Action and Strategy Spaces:

Denote player i's action space as A_i and strategy space as Σ_i.

$A_A = \{0, 1\}$

$A_P = [0, 1]^2 \times R^{+2}$

$A_N = \{G, B\} \times \{G, B\}^2 \times \{y_H, y_L\}$

$\Sigma_A = \Delta(A_A)$, $\Sigma_P = \Delta(A_P)$

$\Sigma_N = \Sigma_{N1} \times \Sigma_{N2} \times \Sigma_{N3}$

$\Sigma_{N1} = \{(\sigma_{N1}(G), \sigma_{N1}(B))|\sigma_{N1}(G) = \mu_0, \sigma_{N1}(B) = 1 - \sigma_{N1}(G)\}$

$\Sigma_{N2} = \times_{\theta\in G,B} \left\{(\sigma_{N2|\theta}(G), \sigma_{N2|\theta}(B))|\sigma_{N2|\theta}(G) = \pi(G|\theta), \sigma_{N2|\theta}(B) = 1 - \sigma_{N2|\theta}(G)\right\}$

$\Sigma_{N3} =$

$\times_{\theta\in G,B} \times_{e\in 0,1} \left\{(\sigma_{N3|\theta,e}(y_H), \sigma_{N3|\theta,e}(y_L))|\sigma_{N3|\theta,e}(y_H) = p(\theta, e), \sigma_{N3|\theta,e}(y_L) = 1 - \sigma_{N3|\theta,e}(y_H)\right\}$

The agent chooses an effort of 0 or 1; the principal designs the experiment and the performance contract. Individual randomization is allowed in this game. Nature chooses the realization of the state and also later chooses the realization of the signal, based on the experiment that the principal designed. It also chooses the realization of output, based on the effort of the agent and the state.

Pay-off functions:

$u_P(a_A, a_P, a_N) = y - w$, $u_A(a_A, a_P, a_N) = w - c(e)$, $c(0) = 0, c(1) = c > 0$

The payoff of nature is not defined.

Expected utility functions for P and A, $Eu_P(\sigma_A, \sigma_P, \sigma_N)$, $Eu_A(\sigma_A, \sigma_P, \sigma_N)$, are defined accordingly in the standard way.

Beliefs

Denote the public history at the beginning of period t as h_t.

In this game, the only private information that matters for P's and A's decisions is the realization of state, θ. This is the private information of Nature. Therefore, the only beliefs that matters are $\mu_j(\theta|h_t)$, $j = P, A$. [2]

Timeline

Period 1:	Period 2:	Period 3:	Period 4:	Period 5:	Period 6:	Period 7:
N choose θ and unobservable for P and A	P publicly chooses $\{\pi(s\|\theta)\}$	N chooses s according to θ and $\{\pi(s\|\theta)\}$; s is observed by both players	P offers a contract $\{w_H, w_L\}$	if accepted, A chooses effort e	N chooses output according to e and θ	output realized and wage paid

The timeline of the baseline model under the formal definition is as follows:

Period 1: Nature chooses the state θ according to the prior probability, μ_0.

Period 2: the principal publicly chooses a direct experiment $\{\pi(s|\theta)\}, s \in \{G, B\}$.

Period 3: Nature chooses s according to $\{\pi(s|\theta)\}$ and its choice of θ. s is publicly observable to both the principal and the agent.

Period 4: the principal offers a contract $\{w_H, w_L\}$ to the agent. If the agent rejects the contract, the game ends and both players get an outside payoff of 0.

Period 5: if the contract is accepted, the agent chooses effort e that is unobservable to the Principal.

Period 6: Nature chooses the output according to the state θ and effort e.

Period 7: output is realized and wage is paid to the agent according to the previously agreed contract.

Restriction on beliefs: We have defined in Definition 3.1 that for any (σ, μ), μ is *reasonable* if:

- μ is decided by σ according to Bayes' Rule whenever possible;
- For any player j, $j \neq i$, its belief about player i at history h_t, $\mu_j(\theta_i|\theta_j, h_t)$, only depends on previous history and player i's action at period t, a_{it}.

Given those restrictions, for $j = P, A$, $\mu_j(h_1) = \mu_j(h_2) = \mu_j(h_3) = \mu_0$, $\mu_j(h_4) = \mu_j(h_5) = \mu_s$.

[2] Also, neither the principal nor the agent has any private information here.

Especially, at the beginning of period 5 (after the principal offers a contract in period 4), since the principal does not know more about the state than the agent, his action should not change the agent's belief given any contract the principal offers.

Proofs and Additional Results

Proof of Lemma 3.1. 1) Implementing high effort: Because $w_h, w_L \geqslant 0$, $E_\mu p(e = 1)w_H + (1 - E_\mu p(e = 1))w_L - c \geqslant E_\mu p(e = 0)w_H + (1 - E_\mu p(e = 0))w_L \geqslant 0$. Therefore, the IR constraint must be slack.

Because of the IC constraint, $w_H - w_L \geqslant \frac{c}{E_\mu \Delta p}$. Then it is trivial to show that IC constraint and the Limited liability constraint of the low output must be binding at the optimal.

2) Implementing low effort: again, it is easy to show that the IR constraint is slack. By choosing $w_H = w_L = 0$, the IC constraint is not violated and it minimizes the cost of the Principal. □

Proof of Lemma 3.2. 1) $V(e = 0, \mu) = (E_\mu p(e = 0))(y_H - w_H^*(e = 0)) + (1 - E_\mu p(e = 0))(y_L - w_L^*(e = 0)) = (E_\mu p(e = 0))(y_H - y_L) + y_L$

$\frac{\partial V(e=0,\mu)}{\partial \mu} = (p(0, G) - p(0, B))(y_H - y_L) > 0$

$\frac{\partial^2 V(e=0,\mu)}{\partial \mu^2} = 0$

2) $V(e = 1, \mu) = (E_\mu p(e = 1))(y_H - w_H^*(e = 1)) + (1 - E_\mu p(e = 1))(y_L - w_L^*(e = 1)) = (E_\mu p(e = 1))(y_H - y_L - \frac{c}{E_\mu \Delta p}) + y_L$

$\frac{\partial V(e=1,\mu)}{\partial \mu} = (p(1, G) - p(1, B))(y_H - y_L) + \frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]}{(E_\mu \Delta p)^2}$

$\frac{\partial^2 V(e=1,\mu)}{\partial \mu^2} = (-2)\frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]\Delta\Delta p}{(E_\mu \Delta p)^3}$

If $p(1, G)p(0, B) - p(1, B)p(0, G) = 0$, $\frac{\partial V(e=1,\mu)}{\partial \mu} > 0$ and $\frac{\partial^2 V(e=1,\mu)}{\partial \mu^2} = 0$; $V(e = 1, \mu)$ is increasing and linear in μ.

If $p(1, G)p(0, B) - p(1, B)p(0, G) > 0$, $\frac{\partial V(e=1,\mu)}{\partial \mu} > 0$ and $\frac{\partial^2 V(e=1,\mu)}{\partial \mu^2} < 0$; $V(e = 1, \mu)$ is increasing and strictly concave in μ.

If $p(1, G)p(0, B) - p(1, B)p(0, G) < 0$, $\frac{\partial^2 V(e=1,\mu)}{\partial \mu^2} > 0$, $V(e = 1, \mu)$ is strictly convex in μ.

$\frac{\partial V(e=1,\mu)}{\partial \mu} = (p(1, G) - p(1, B))(y_H - y_L) + \frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]}{(E_\mu \Delta p)^2}$.

- If $(p(1, G) - p(1, B))(y_H - y_L) + \frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]}{(E_\mu \Delta p)^2}_{|\mu=0} \geqslant 0$, $V(e = 1, \mu)$ is increasing in μ
- If $(p(1, G) - p(1, B))(y_H - y_L) + \frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]}{(E_\mu \Delta p)^2}_{|\mu=1} \leqslant 0$, $V(e = 1, \mu)$ is decreasing in μ
- If $(p(1, G) - p(1, B))(y_H - y_L) + \frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]}{(E_\mu \Delta p)^2}_{|\mu=0} < 0 < (p(1, G) - p(1, B))(y_H - y_L) + \frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]}{(E_\mu \Delta p)^2}_{|\mu=1}$, then there exists a $\hat{\mu} \in (0, 1)$, such that $V(e = 1, \mu)$ is decreasing in μ for $\mu \in [0, \hat{\mu})$ and increasing in μ for $\mu \in (\hat{\mu}, 1]$.

□

Proof of Lemma 3.3. $V(e=0,\mu)=(E_\mu p(e=0))(y_H-y_L)+y_L$. $V(e=1,\mu)=E_\mu p(e=1)\left(y_H-y_L-\frac{c}{E_\mu \Delta p}\right)+y_L$. $\Delta V(\mu)\equiv V(e=1,\mu)-V(e=0,\mu)=E_\mu\Delta p(y_H-y_L)-\frac{cE_\mu p(e=1)}{E_\mu \Delta p}$
$=E_\mu p(e=1)\left[(y_H-y_L)-\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\right]$

$V(e=1,\mu)\geqslant V(e=0,\mu)$ iff $(y_H-y_L)-\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\geqslant 0$

Case I: If $p(1,G)p(0,B)=p(1,B)p(0,G)$,
$V(e=1,\mu)=(E_\mu p(e=1))(y_H-y_L-\frac{c}{E_\mu\Delta p})+y_L=(E_\mu p(e=1))(y_H-y_L)-\frac{cp(1,B)}{p(1,B)-p(0,B)}+y_L$

$\Delta V(\mu)\equiv V(e=1,\mu)-V(e=0,\mu)$,
$\frac{\partial \Delta V(\mu)}{\partial\mu}=(p(1,G)-p(1,B))(y_H-y_L)-(p(0,G)-p(0,B))(y_H-y_L)$
$=\Delta\Delta p(y_H-y_L)>0$
$\frac{\partial^2\Delta V(\mu)}{\partial\mu^2}=0$

Case II: If $p(1,G)p(0,B)>p(1,B)p(0,G)$, $(y_H-y_L)-\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}$ is increasing over μ.

$\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\in\left[\frac{cp(1,G)}{(p(1,G)-p(0,G))^2},\frac{cp(1,B)}{(p(1,B)-p(0,B))^2}\right]$

- If $y_H-y_L\geqslant\frac{cp(1,B)}{(p(1,B)-p(0,B))^2}$, $V(e=1)\geqslant V(e=0)$ for any $\mu\in[0,1]$
- If $y_H-y_L<\frac{cp(1,G)}{(p(1,G)-p(0,G))^2}$, $V(e=1)\leqslant V(e=0)$ for any $\mu\in[0,1]$
- If $y_H-y_L\in\left(\frac{cp(1,G)}{(p(1,G)-p(0,G))^2},\frac{cp(1,B)}{(p(1,B)-p(0,B))^2}\right)$,
  there exist a cut-off point $\bar\mu\in(0,1)$, $[y_H-y_L]=\frac{cE_{\bar\mu}p(e=1)}{(E_{\bar\mu}\Delta p)^2}$,
  such that $V(e=1)\leqslant V(e=0)$ for any $\mu\in[0,\bar\mu]$ and $V(e=1)>V(e=0)$ for any $\mu\in(\bar\mu,1]$

Case III: If $p(1,G)p(0,B)<p(1,B)p(0,G)$, then $log\left(\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\right)=log(c)+log(E_\mu p(e=1))-2log(E_\mu\Delta p)$
$\frac{\partial log\left(\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\right)}{\partial\mu}=\frac{p(1,G)-p(1,B)}{E_\mu p(e=1)}-2\frac{\Delta\Delta p}{E_\mu\Delta p}=\frac{1}{\mu+\frac{p(1,B)}{p(1,G)-p(1,B)}}-2\frac{1}{\mu+\frac{p(1,B)-p(0,B)}{\Delta\Delta p}}$
$=\frac{1}{\left(\mu+\frac{p(1,B)}{p(1,G)-p(1,B)}\right)\left(\mu+\frac{p(1,B)-p(0,B)}{\Delta\Delta p}\right)}\left\{\left(\mu+\frac{p(1,B)-p(0,B)}{\Delta\Delta p}\right)-2\left(\mu+\frac{p(1,B)}{p(1,G)-p(1,B)}\right)\right\}$
$=\frac{1}{\left(\mu+\frac{p(1,B)}{p(1,G)-p(1,B)}\right)\left(\mu+\frac{p(1,B)-p(0,B)}{\Delta\Delta p}\right)}\left\{-\mu+\left(\frac{p(1,B)-p(0,B)}{\Delta\Delta p}-\frac{p(1,B)}{p(1,G)-p(1,B)}\right)-\frac{p(1,B)}{p(1,G)-p(1,B)}\right\}$

Therefore, if $-1+\left(\frac{p(1,B)-p(0,B)}{\Delta\Delta p}-\frac{p(1,B)}{p(1,G)-p(1,B)}\right)-\frac{p(1,B)}{p(1,G)-p(1,B)}\geqslant 0$, $log\left(\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\right)$ is increasing for any level of $\mu\in[0,1]$;

if $\left(\frac{p(1,B)-p(0,B)}{\Delta\Delta p}-\frac{p(1,B)}{p(1,G)-p(1,B)}\right)-\frac{p(1,B)}{p(1,G)-p(1,B)}\leqslant 0$, $log\left(\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\right)$ is decreasing for any level of $\mu\in[0,1]$;

if $-1+\left(\frac{p(1,B)-p(0,B)}{\Delta\Delta p}-\frac{p(1,B)}{p(1,G)-p(1,B)}\right)-\frac{p(1,B)}{p(1,G)-p(1,B)}<0<\left(\frac{p(1,B)-p(0,B)}{\Delta\Delta p}-\frac{p(1,B)}{p(1,G)-p(1,B)}\right)-\frac{p(1,B)}{p(1,G)-p(1,B)}<0$, $log\left(\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\right)$ is first increasing and then decreasing

Case III.1:

- If $-1+\frac{p(1,B)-p(0,B)}{p(1,G)-p(0,G)-p(1,B)+p(0,B)}-2\frac{p(1,B)}{p(1,G)-p(1,B)}>0$, $\frac{\partial log\left(\frac{E_\mu p(e=1)}{(E_\mu\Delta p)^2}\right)}{\partial\mu}>0$ for any $\mu\in[0,1]$, $\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\in\left[\frac{cp(1,B)}{(p(1,B)-p(0,B))^2},\frac{cp(1,G)}{(p(1,G)-p(0,G))^2}\right]$
- Then $\left[(y_H-y_L)-\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\right]$ is decreasing
  - If $y_H-y_L\geqslant\frac{cp(1,G)}{(p(1,G)-p(0,G))^2}$, $V(e=1)\geqslant V(e=0)$ for any $\mu\in[0,1]$
  - If $y_H-y_L\leqslant\frac{cp(1,B)}{(p(1,B)-p(0,B))^2}$, $V(e=1)\leqslant V(e=0)$ for any $\mu\in[0,1]$
  - If $y_H-y_L\in\left(\frac{cp(1,B)}{(p(1,B)-p(0,B))^2},\frac{cp(1,G)}{(p(1,G)-p(0,G))^2}\right)$,
    there exist a cut-off point $\bar\mu\in(0,1)$, $[y_H-y_L]=\frac{cE_{\bar\mu}p(e=1)}{(E_{\bar\mu}\Delta p)^2}$,
    such that $V(e=1)\geqslant V(e=0)$ for any $\mu\in[0,\bar\mu]$ and $V(e=1)<V(e=0)$ for any $\mu\in(\bar\mu,1]$

- **Case III.2:** If $\frac{p(1,B)-p(0,B)}{p(1,G)-p(0,G)-p(1,B)+p(0,B)}-2\frac{p(1,B)}{p(1,G)-p(1,B)}<0$, $\frac{\partial log\left(\frac{E_\mu p(e=1)}{(E_\mu\Delta p)^2}\right)}{\partial\mu}<0$ for any $\mu\in[0,1]$, then $\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\in\left[\frac{cp(1,G)}{(p(1,G)-p(0,G))^2},\frac{cp(1,B)}{(p(1,B)-p(0,B))^2}\right]$
- Then $\left[(y_H-y_L)-\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\right]$ is increasing
  - If $y_H-y_L\geqslant\frac{cp(1,B)}{(p(1,B)-p(0,B))^2}$, $V(e=1)\geqslant V(e=0)$ for any $\mu\in[0,1]$
  - If $y_H-y_L\leqslant\frac{cp(1,G)}{(p(1,G)-p(0,G))^2}$, $V(e=1)\leqslant V(e=0)$ for any $\mu\in[0,1]$
  - If $y_H-y_L\in\left(\frac{cp(1,G)}{(p(1,G)-p(0,G))^2},\frac{cp(1,B)}{(p(1,B)-p(0,B))^2}\right)$,
    there exist a cut-off point $\bar\mu\in(0,1)$, $[y_H-y_L]=\frac{cE_{\bar\mu}p(e=1)}{(E_{\bar\mu}\Delta p)^2}$,
    such that $V(e=1)\leqslant V(e=0)$ for any $\mu\in[0,\bar\mu]$ and $V(e=1)>V(e=0)$ for any $\mu\in(\bar\mu,1]$

- **Case III.3:** If
  $-1+\frac{p(1,B)-p(0,B)}{p(1,G)-p(0,G)-p(1,B)+p(0,B)}-2\frac{p(1,B)}{p(1,G)-p(1,B)}<0<\frac{p(1,B)-p(0,B)}{p(1,G)-p(0,G)-p(1,B)+p(0,B)}-2\frac{p(1,B)}{p(1,G)-p(1,B)}$,
  then there exist a level of μ, $\hat\mu$, such that $\frac{\partial log\left(\frac{E_\mu p(e=1)}{(E_\mu\Delta p)^2}\right)}{\partial\mu}_{|\mu=\hat\mu}=0$, and $\frac{\partial log\left(\frac{E_\mu p(e=1)}{(E_\mu\Delta p)^2}\right)}{\partial\mu}\geqslant 0$ if $\mu\leqslant\hat\mu$ and $\frac{\partial log\left(\frac{E_\mu p(e=1)}{(E_\mu\Delta p)^2}\right)}{\partial\mu}<0$ if $\mu>\hat\mu$,
  $\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\in\left[\min\left\{\frac{cp(1,G)}{(p(1,G)-p(0,G))^2},\frac{cp(1,B)}{(p(1,B)-p(0,B))^2}\right\},\frac{cE_{\hat\mu}p(e=1)}{(E_{\hat\mu}\Delta p)^2}\right]$
- Then $\left[(y_H-y_L)-\frac{cE_\mu p(e=1)}{(E_\mu\Delta p)^2}\right]$ is first decreasing and then increasing
  - If $y_H-y_L\geqslant\frac{cE_{\hat\mu}p(e=1)}{(E_{\hat\mu}\Delta p)^2}$, $V(e=1)\geqslant V(e=0)$ for any $\mu\in[0,1]$

- If $y_H - y_L \leqslant \min\left\{\frac{cp(1,G)}{(p(1,G)-p(0,G))^2}, \frac{cp(1,B)}{(p(1,B)-p(0,B))^2}\right\}$, $V(e=1) \leqslant V(e=0)$ for any $\mu \in [0,1]$
- If $y_H - y_L \in \left(\min\left\{\frac{cp(1,G)}{(p(1,G)-p(0,G))^2}, \frac{cp(1,B)}{(p(1,B)-p(0,B))^2}\right\}, \max\left\{\frac{cp(1,G)}{(p(1,G)-p(0,G))^2}, \frac{cp(1,B)}{(p(1,B)-p(0,B))^2}\right\}\right)$,
there exist a cut-off point $\bar{\mu} \in (0,1)$, $[y_H - y_L] = \frac{cE_{\bar{\mu}}p(e=1)}{(E_{\bar{\mu}}\Delta p)^2}$,
such that $V(e=1) \leqslant V(e=0)$ for any $\mu \in [0, \bar{\mu}]$ and $V(e=1) > V(e=0)$ for any $\mu \in (\bar{\mu}, 1]$
- If $y_H - y_L \in \left(\max\left\{\frac{cp(1,G)}{(p(1,G)-p(0,G))^2}, \frac{cp(1,B)}{(p(1,B)-p(0,B))^2}\right\}, \frac{cE_{\hat{\mu}}p(e=1)}{(E_{\hat{\mu}}\Delta p)^2}\right)$,
there exist two cut-off points $\bar{\mu}_1, \bar{\mu}_2 \in (0,1)$, $\bar{\mu}_1 < \bar{\mu}_2$, $[y_H - y_L] = \frac{cE_{\bar{\mu}_1}p(e=1)}{(E_{\bar{\mu}_1}\Delta p)^2} = \frac{cE_{\bar{\mu}_2}p(e=1)}{(E_{\bar{\mu}_2}\Delta p)^2}$,
such that $V(e=1) \geqslant V(e=0)$ for any $\mu \in [0, \bar{\mu}_1] \cup [\bar{\mu}_2, 1]$ and $V(e=1) < V(e=0)$ for any $\mu \in (\bar{\mu}_1, \bar{\mu}_2)$

□

Proof of Proposition 3.1. Proposition 3.1 is obvious given the proof of lemma 3.3. □

Corollary C.1. *If $p(1,G)p(0,B) < p(1,B)p(0,G)$, then*

- *If $-1 + \frac{p(1,B)-p(0,B)}{p(1,G)-p(0,G)-p(1,B)+p(0,B)} - 2\frac{p(1,B)}{p(1,G)-p(1,B)} > 0$, then $V(e=1,\mu)$ either has no intersection with $V(e=0,\mu)$, or crosses $V(e=0,\mu)$ for once from above;*
- *If $\frac{p(1,B)-p(0,B)}{p(1,G)-p(0,G)-p(1,B)+p(0,B)} - 2\frac{p(1,B)}{p(1,G)-p(1,B)} < 0$, then $V(e=1,\mu)$ either has no intersection with $V(e=0,\mu)$, or crosses $V(e=0,\mu)$ for once from below;*
- *If $-1 + \frac{p(1,B)-p(0,B)}{p(1,G)-p(0,G)-p(1,B)+p(0,B)} - 2\frac{p(1,B)}{p(1,G)-p(1,B)} < 0 < \frac{p(1,B)-p(0,B)}{p(1,G)-p(0,G)-p(1,B)+p(0,B)} - 2\frac{p(1,B)}{p(1,G)-p(1,B)}$, then $V(e=1,\mu)$ can either has no intersection with $V(e=0,\mu)$, cross $V(e=0,\mu)$ from below or above for once, or cross $V(e=0,\mu)$ first from below and then from above.*

Proof. The proof comes directly from the proof of Lemma 3.3. □

Proof of Lemma 3.4. If $V(\mu) = V(e=0,\mu)$, it is obvious that $E_\tau V(\mu) = V(E_\tau \mu) = V(\mu_0)$. Therefore, any experiment would lead to exactly same pay-off for the principle.

If $p(1,G)p(0,B) \leqslant p(1,B)p(0,G)$ and for $V(\mu) = V(e=1,\mu)$ for some levels of μ, then $V(\mu) = \max\{V(e=0,\mu), V(e=1,\mu)\}$ is (weakly) convex because both $V(e=0,\mu)$ and $V(e=1,\mu)$ are (weakly) convex. Kamenica and Gentzkow (2011) showed that when the value function is convex, the optimal distribution of posterior is fully revealing.

If $p(1,G)p(0,B) > p(1,B)p(0,G)$ and for $V(\mu) = V(e=1,\mu)$ for any μ, then $V(\mu)$ is strictly concave. Kamenica and Gentzkow (2011) showed that when the value function is concave, the optimal distribution of posterior reveals no information. □

Proof of Proposition 3.2. Part 1 and Part 2 of Proposition 3.2 has been proved in Lemma 3.4.

Part 3:

$$\tilde{\mu} \in \underset{\mu}{\text{argmax}} \frac{V(e=1,\mu) - V(e=0,0)}{\mu - 0}$$
$$\in \underset{\mu}{\text{argmax}}\, log\left(\frac{V(e=1,\mu) - V(e=0,0)}{\mu - 0}\right)$$

F.O.C.: (neglecting the constraint $\mu \in [0,1]$)

$$\frac{\partial log\left(\frac{V(e=1,\mu)-V(e=0,0)}{\mu-0}\right)}{\partial \mu} = 0$$
$$\frac{V'(e=1,\mu) - V'(e=0,0)}{V(e=1,\mu) - V(e=0,0)} - \frac{1}{\mu} = 0$$
$$\left[V'(e=1,\mu) - V'(e=0,0)\right]\mu = V(e=1,\mu) - V(e=0,0)$$
$$\left[(p(1,G) - p(1,B))(y_H - y_L) + \frac{c\left[p(1,G)p(0,B) - p(1,B)p(0,G)\right]}{(E_\mu \Delta p)^2}\right]\mu$$
$$= \mu(p(1,G) - p(1,B))(y_H - y_L) - \frac{cE_\mu p(e=1)}{E_\mu \Delta p} + (p(1,B) - p(0,B))(y_H - y_L)$$
$$-(p(1,B) - p(0,B))(y_H - y_L) + \left[\frac{c\left[p(1,G)p(0,B) - p(1,B)p(0,G)\right]}{(E_\mu \Delta p)^2}\mu + \frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right]_{|\mu=\tilde{\mu}^I} = 0$$
$$-(p(1,B) - p(0,B))(y_H - y_L) + \left[\left(-\frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right)'\mu + \frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right]_{|\mu=\tilde{\mu}^I} = 0$$

S.O.C.:

$$\frac{\partial^2 log\left(\frac{V(e=1,\mu)-V(e=0,0)}{\mu-0}\right)}{\partial \mu^2}$$
$$= \left(-(p(1,B) - p(0,B))(y_H - y_L) + \left(-\frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right)'\mu + \frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right)'$$
$$= \left(-\frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right)' + \left(\frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right)' + \left(-\frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right)''\mu < 0$$

,when $V(e=1,\mu)$ is concave.

$-(p(1,B) - p(0,B))(y_H - y_L) + \left[\frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]}{(E_\mu \Delta p)^2}\mu + \frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right]_{|\mu=0} < 0$ iff $(y_H - y_L) < \frac{cE_\mu p(e=1)}{(E_\mu \Delta p)^2}_{|\mu=0}$ However, this is impossible if $V(e=1,\mu)$ is concave and $V(e=1,\mu) > V(e=0,\mu)$ for at least some μ.

Therefore, $-(p(1,B)-p(0,B))(y_H-y_L)+\left[\frac{c[p(1,G)p(0,B)-p(1,B)p(0,G)]}{(E_\mu \Delta p)^2}\mu + \frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right]_{|\mu=0} > 0$.

Therefore, $\tilde{\mu} = \min\left\{\tilde{\mu}^I, 1\right\} = \text{argmax}_\mu\, log\left(\frac{V(e=1,\mu)-V(e=0,0)}{\mu-0}\right) = \text{argmax}_\mu \left(\frac{V(e=1,\mu)-V(e=0,0)}{\mu-0}\right)$.

It is then easy to show that

$$\{\tau*, \mu*\} = \begin{cases} \left\{\tau(\mu_G) = \frac{\mu_0}{\tilde{\mu}}, \tau(\mu_B) = 1 - \tau(\mu_G), \mu_B = 0, \mu_G = \tilde{\mu}\right\}, \text{if } \mu_0 \leqslant \tilde{\mu} \\ \left\{\tau(\mu_G) \in [0,1], \tau(\mu_B) = 1 - \tau(\mu_G), \mu_B = \mu_G = \mu_0\right\}, \text{if } \mu_0 > \tilde{\mu} \end{cases}$$

is indeed the distribution of posteriors under any optimal experiment:

Step 1: for any $\mu \in (0, \bar{\mu}]$, it is easy to show that for any $\tau_1, \tau_2, \mu_1, \mu_2$ such that $0 \leqslant \tau_1, \tau_2, \mu_1, \mu_2 \leqslant 1$, $\mu_1 < \mu < \bar{\mu} < \mu_2$, $\tau_1 + \tau_2 = 1$ and $\tau_1\mu_1 + \tau_2\mu_2 = \mu$, we have $V(\mu) \leqslant \tau_1 V(\mu_1) + \tau_2 V(\mu_2)$. This is because that $V(\mu) = V(e = 0, \mu)$, $V(\mu_1) = V(e = 0, \mu_1)$, $V(\mu_2) = V(e = 1, \mu_2) > V(e = 0, \mu_2)$, $\tau_1 V(\mu_1) + \tau_2 V(\mu_2) > \tau_1 V(e = 0, \mu_1) + \tau_2 V(e = 0, \mu_2) = V(e = 0, \tau_1\mu_1 + \tau_2\mu_2) = V(e = 0, \mu) = V(\mu)$.

Step 2: following step 1, it can be show that for those μ, it is optimal to set $\mu_1 = 0$. $\tau_1\mu_1 + (1 - \tau_1)\mu_2 = \mu$, therefore $\tau = \frac{\mu_2 - \mu}{\mu_2 - \mu_1}$. $\tau_1 V(\mu_1) + \tau_2 V(\mu_2) = \frac{\mu_2 - \mu}{\mu_2 - \mu_1}(V(\mu_1) - V(\mu_2)) + V(\mu_2)$

Take the derivative over μ_1, then we have $\frac{\mu_2 - \mu}{(\mu_2 - \mu_1)^2}\left[V(\mu_1)'(\mu_2 - \mu_1) - (-1)(V(\mu_1) - V(\mu_2))\right] = \frac{\mu_2 - \mu}{(\mu_2 - \mu_1)^2}\left[V(\mu_1)'(\mu_2 - \mu_1) - (V(\mu_2) - V(\mu_1)\right] < 0$. Therefore it is optimal to set $\mu_1 = 0$

Step 3: after step 2, it is optimal to set $\mu_2 = \tilde{\mu}$. Given $\mu_1 = 0$, $\tau_1 V(\mu_1) + \tau_2 V(\mu_2) = \frac{\mu - \mu_1}{\mu_2 - \mu_1}(V(\mu_2) - V(0)) + V(0) = \frac{\mu}{\mu_2}(V(\mu_2) - V(0)) + V(0)$ which is maximized at $\tilde{\mu}$ as we shown before.

Step 4: for any $\mu \in (\bar{\mu}, \tilde{\mu}]$, it is straightforward to check that it is also optimal to set $\mu_1 = 0$ and $\mu_2 = \tilde{\mu}$ using exactly the same approach as Step 2 and Step 3.

Step 5: for any $\mu \in (\tilde{\mu}, 1)$, it is optimal to set $\mu_1 = \mu_2 = \mu$. $V(\mu)$ is above the line connecting $V(0)$ and $V(\mu_2)$, such that $0 < \tilde{\mu} < \mu < \mu_2$. It is easy to proof by the definition of $\tilde{\mu}$.

□

Proof of Proposition 3.3. From Proposition 3.2 we know if $p(1,G)p(0,B) > p(1,B)p(0,G)$ and $V(\mu) = V(e = 0, \mu)$ for some μ and $V(\mu) = V(e = 1, \mu)$ for some other μ, $\mu_G = \max\{\tilde{\mu}, \mu_0\}$. $Prob(e^* = 1) = \min\left\{\frac{\mu_0}{\tilde{\mu}}, 1\right\}$. It is obvious then both μ_G and $Prob(e^* = 1)$ are (weakly) increasing in μ_0.

We know from previous proposition that $\tilde{\mu} = \min\left\{\tilde{\mu}^I, 1\right\}$, in which $\tilde{\mu}^I$ is define as the level of μ such that $-(p(1,B) - p(0,B))(y_H - y_L) + \left[\frac{c[p(1,G)p(0,B) - p(1,B)p(0,G)]}{(E_\mu \Delta p)^2}\mu + \frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right]_{|\mu = \tilde{\mu}^I} = 0$.

By Implicit Function Theorem, $\tilde{\mu}^I$ is strictly decreasing in $y_H - y_L$ and strictly increasing in c. Therefore, μ_G is weakly decreasing in $y_H - y_L$ and weakly increasing in c. Therefore, $Prob(e^* = 1)$ is weakly increasing in $y_H - y_L$ and weakly decreasing in c.

For two production functions, $\{\tilde{p}(e,\theta)\}_{(e,\theta) \in \{(0,0),(1,0),(0,1),(1,1)\}}$ and $\{p(e,\theta)\}_{(e,\theta) \in \{(0,0),(1,0),(0,1),(1,1)\}}$, such that $\tilde{p}(e,\theta) =$

$ap(e,\theta) + (1-b)(1-p(e,\theta)) = (a+b-1)p(e,\theta) + (1-b)$, $1 > a, b > \frac{1}{2}$,
hence for any (e,θ) and (e',θ'), $\tilde{p}(e,\theta) - \tilde{p}(e',\theta') = (a+b-1)(p(e,\theta) - p(e',\theta'))$.

Therefore, $\tilde{p}(1,B) - \tilde{p}(0,B) = (a+b-1)((p(1,B) - p(0,B))$,
$E_\mu \Delta\tilde{p} = \mu\left[(\tilde{p}(1,G) - \tilde{p}(0,G)) - (\tilde{p}(1,B) - \tilde{p}(0,B))\right] + (\tilde{p}(1,B) - \tilde{p}(0,B)) = (a+b-1)E_\mu \Delta p$,
$E_\mu \tilde{p}(e=1) = \mu\left[\tilde{p}(1,G) - \tilde{p}(1,B)\right] + \tilde{p}(1,B) = (a+b-1)\mu\left[p(1,G) - p(1,B)\right] + (a+b-1)p(1,B) + (1-b) = (a+b-1)E_\mu \tilde{p}(e=1) + (1-b)$,
$\tilde{p}(1,G)\tilde{p}(0,B) - \tilde{p}(1,B)\tilde{p}(0,G) = ((a+b-1)p(1,G) + (1-b))((a+b-1)p(0,B) + (1-b)) - ((a+b-1)p(1,B) + (1-b))((a+b-1)p(0,G) + (1-b))$
$= (a+b-1)^2(p(1,G)p(0,B) - p(1,B)p(0,G)) + (a+b-1)(1-b)((p(1,G) - p(0,G)) - (p(1,B) - p(0,B)))$

Therefore, $-(\tilde{p}(1,B) - \tilde{p}(0,B))(y_H - y_L) + \left[\frac{c[\tilde{p}(1,G)\tilde{p}(0,B) - \tilde{p}(1,B)\tilde{p}(0,G)]}{(E_\mu \Delta\tilde{p})^2}\mu + \frac{cE_\mu \tilde{p}(e=1)}{E_\mu \Delta\tilde{p}}\right]$

$$= -(a+b-1)((p(1,B) - p(0,B))(y_H - y_L)$$
$$+ \frac{c\left[(a+b-1)^2(p(1,G)p(0,B) - p(1,B)p(0,G)) + (a+b-1)(1-b)((p(1,G) - p(0,G)) - (p(1,B) - p(0,B)))\right]}{((a+b-1)E_\mu \Delta p)^2}\mu$$
$$+ \frac{c(a+b-1)E_\mu \tilde{p}(e=1) + c(1-b)}{(a+b-1)E_\mu \Delta p}$$
$$= -(a+b-1)((p(1,B) - p(0,B))(y_H - y_L) + \frac{c[p(1,G)p(0,B) - p(1,B)p(0,G)]}{(E_\mu \Delta p)^2}\mu + \frac{cE_\mu p(e=1)}{E_\mu \Delta p}$$
$$+ \frac{c[(1-b)((p(1,G) - p(0,G)) - (p(1,B) - p(0,B)))]}{(a+b-1)(E_\mu \Delta p)^2}\mu + \frac{c(1-b)}{(a+b-1)E_\mu \Delta p}$$
$$> -(p(1,B) - p(0,B))(y_H - y_L) + \left[\frac{c[p(1,G)p(0,B) - p(1,B)p(0,G)]}{(E_\mu \Delta p)^2}\mu + \frac{cE_\mu p(e=1)}{E_\mu \Delta p}\right] \text{ for any } \mu \in [0,1].$$

Therefore, using argument similar to Implicit Function Theorem, $\tilde{\mu}^I(\{\tilde{p}(e,\theta)\}) > \tilde{\mu}^I(\{p(e,\theta)\})$. Therefore, μ_G is weakly higher and $Prob(e^* = 1)$ is weakly lower if the production process becomes noisier.

□

Proof of Corollary 3.1. Part 1 and 2 of this corollary are obvious given that $V(e=0,\mu)$ is linear in μ, and $V(e=0,\mu)$ is strictly concave in μ.

Part 3:

Step 1: for any $\mu \in (\bar{\mu}, 1]$, it is easy to show that for $\tau_1, \tau_2, \mu_1, \mu_2$ such that $0 \leqslant \tau_1, \tau_2, \mu_1, \mu_2 \leqslant 1$, $\mu_1 < \mu < \bar{\mu} < \mu_2$ and $\tau_1\mu_1 + \tau_2\mu_2 = \mu$, we have $V(\mu) \leqslant \tau_1 V(\mu_1) + \tau_2 V(\mu_2)$.

This is because that $V(\mu) = V(e=0,\mu)$, $V(\mu_2) = V(e=0,\mu_2)$, $V(\mu_1) = V(e=1,\mu_1) > V(e=0,\mu_1)$, $\tau_1 V(\mu_1) + \tau_2 V(\mu_2) > \tau_1 V(e=0,\mu_1) + \tau_2 V(e=0,\mu_2) = V(e=0, \tau_1\mu_1 + \tau_2\mu_2) = V(e=0,\mu) = V(\mu)$.

Step 2: following step 1, it can be show that for those μ, it is optimal to set $\mu_2 = 1$. $\tau_1 V(\mu_1) + \tau_2 V(\mu_2) = \frac{\mu - \mu_1}{\mu_2 - \mu_1}(V(\mu_2) - V(\mu_1)) + V(\mu_1)$ which is increasing in μ_2.

Step 3: for those μ, it is optimal to set $\mu_1 = \tilde{\mu} < \bar{\mu}$. Following step 2, $\tau_1 V(\mu_1) + \tau_2 V(\mu_2) = \frac{\mu_2 - \mu}{\mu_2 - \mu_1}(V(\mu_1) - V(\mu_2)) + V(\mu_2) = \frac{1-\mu}{1-\mu_1}(V(\mu_1) - V(1)) + V(1)$. $\mu_1 = \tilde{\tilde{\mu}}$ is thus the level that maximizes $\frac{V(\mu_1) - V(1)}{1-\mu_1} = \frac{V(e=1,\mu_1) - V(e=0,1)}{1-\mu_1}$.

Step 4: for any $\mu \in \left(\tilde{\tilde{\mu}}, \bar{\mu}\right]$, it is either to check that it is also optimal to set $\mu_2 = 1$ and

$\mu_1 = \tilde{\tilde{\mu}}$ using exactly the same approach as Step 2 and Step 3.

Step 5: for any $\mu \in (0, \tilde{\mu})$, it is optimal to set $\mu_1 = \mu_2 = \mu$. $V(\mu)$ is above the line connecting $V(\mu_1)$ and $V(1)$, such that $0 < \mu_1 < \tilde{\tilde{\mu}} < \mu$. It is easy to proof by the definition of $\tilde{\tilde{\mu}}$.

□

Proof of Proposition 3.4. Constrained Social Optimal Experiment:

Define $\tau\{\mu|\theta\} = \sum_{\mu(s)=\mu} \pi\{s|\theta\}$.

$$\begin{aligned}
&\max_{\{\tau\{\mu|\theta\}\}} E_\theta E_{\mu|\theta} E_{y|\theta, e^*(\mu, \{w^*(y,\mu)\})} \{y - c(e^*(\mu, \{w^*(y,\mu)\}))\} \\
= &\max_{\{\tau\{\mu|\theta\}\}} y_L + E_\theta E_{\mu|\theta} \{p(\theta, (e^*(\mu, \{w^*(y,\mu)\}))(y_H - y_L) - c((e^*(\mu, \{w^*(y,\mu)\}))\} \\
= &\max_{\{\tau\{\mu|\theta\}\}} y_L + E_\theta E_{\mu|\theta} \left\{\max_e \{p(\theta, e)(y_H - y_L) - c(e)\} - \Delta(\theta, e^*(\mu, \{w^*(y,\mu)\}))\right\} \\
= &\max_{\{\tau\{\mu|\theta\}\}} y_L + E_\theta \max_e \{p(\theta, e)(y_H - y_L) - c(e)\} - E_\theta E_{\mu|\theta} \{\Delta(\theta, e^*(\mu, \{w^*(y,\mu)\}))\}
\end{aligned}$$

,in which $\Delta(\theta, e)$ is the efficiency loss from not implementing the first-best effort, and it is defined as:

$$\Delta(\theta, e) \equiv \max_e \{p(\theta, e)(y_H - y_L) - c(e)\} - \{p(\theta, e)(y_H - y_L) - c(e)\}$$

From the definition we know that $\Delta(\theta, e) \geqslant 0$ Now consider three different cases:

Case I: if $\Delta p(\theta)(y_H - y_L) - c < 0$ for both $\theta \in \{G, B\}$, it is socially inefficient to implement high effort under both states. Then due to agency costs, it would not be individually profitable to implement high effort under both state, i.e., $V(e = 1, \mu) < V(e = 0, \mu)$ for $\mu = 0, 1$. Therefore, from previous results we know $V(e = 1, \mu) < V(e = 0, \mu)$ for any μ, and $V(\mu) = V(e = 0, \mu)$ for any μ. Then high effort is never implemented in this scenario, and any experiment is both constrained social optimal and individually rational.

Case II: if $\Delta p(\theta = G)(y_H - y_L) \geqslant c \geqslant \Delta p(\theta = B)(y_H - y_L)$, it is socially efficient to implement high effort only in good state but not in bad state. Due to agency problem, those are possible scenarios about the individual profitability of implementing effort:

II.1 $V(e = 1, \mu) < V(e = 0, \mu)$ for both $\mu = 0, 1$

II.2 $V(e = 1, \mu = 0) < V(e = 0, \mu = 0)$, $V(e = 1, \mu = 1) < V(e = 0, \mu = 1)$, and $V(e = 1, \mu = 1)$ is weakly convex

II.3 $V(e = 1, \mu = 0) < V(e = 0, \mu = 0)$, $V(e = 1, \mu = 1) < V(e = 0, \mu = 1)$, and $V(e = 1, \mu = 1)$ is concave

Under Case II.1, from previous results we know $V(e = 1, \mu) < V(e = 0, \mu)$ for any

μ, and $V(\mu) = V(e = 0, \mu)$ for any μ. Then the high effort is never implemented in this scenario, and any experiment is both constrained social optimal and individually rational.

Under Case II.2, first-best can be reached by having a full revealing signal, i.e., $\tau\{\mu = 0|\theta = B\} = 1, \tau\{\mu = 1|\theta = G\} = 1$. From previous results, we know that the e-quilibrium experiment under Case II.2 is also fully revealing. High effort is implemented if and only if the state is good.

Under Case II.3, first-best can be reached by having a full revealing signal, i.e., $\tau\{\mu = 0|\theta = B\} = 1, \tau\{\mu = 1|\theta = G\} = 1$. From previous results, we know that the equilibrium experiment under Case II.2 is either full revealing or partial obfuscating. When there is partial obfuscation, the bad signal is fully revealing but the good signal is not. There will be over-implementing of high effort sometimes when the state is actually bad.

Case III: if $\Delta p(\theta = G)(y_H - y_L) \geqslant c$ for both $\theta \in \{G, B\}$, it is socially efficient to implement high effort under both cases. Due to agency problem, those are possible scenarios about the individual profitability of implementing effort:

III.1 $V(e = 1, \mu) < V(e = 0, \mu)$ for both $\mu = 0, 1$

III.2 $V(e = 1, \mu) > V(e = 0, \mu)$ for both $\mu = 0, 1$

III.3 $V(e = 1, \mu = 0) < V(e = 0, \mu = 0)$, $V(e = 1, \mu = 1) > V(e = 0, \mu = 1)$, and $V(e = 1, \mu = 1)$ is weakly convex

III.4 $V(e = 1, \mu = 0) > V(e = 0, \mu = 0)$, $V(e = 1, \mu = 1) < V(e = 0, \mu = 1)$, and $V(e = 1, \mu = 1)$ is strictly convex

III.5 $V(e = 1, \mu = 0) < V(e = 0, \mu = 0)$, $V(e = 1, \mu = 1) > V(e = 0, \mu = 1)$, and $V(e = 1, \mu = 1)$ is concave

Under Case III.1, from previous results we know $V(e = 1, \mu) < V(e = 0, \mu)$ for any μ, and $V(\mu) = V(e = 0, \mu)$ for any μ. Then the high effort is never implemented in this scenario, and any experiment is both constrained social optimal and individually rational.

Under Case III.2, there are two possible sub-cases: 1) $V(\mu) = V(e = 1, \mu)$ for any μ; 2) There exists two cut-off points, $\bar{\mu}_1$ and $\bar{\mu}_2$, such that $0 < \bar{\mu}_1 < \bar{\mu}_2 < 1$, $V(e = 1, \bar{\mu}_1) - V(e = 0, \bar{\mu}_1) = V(e = 1, \bar{\mu}_2) - V(e = 0, \bar{\mu}_2) = 0$, and $V(e = 1, \mu) < V(e = 0, \mu)$ if $\bar{\mu}_1 < \mu < \bar{\mu}_2$.

In the first sub-case, high effort is always implemented in this scenario, and any experiment is both constrained social optimal and individually rational.

In the second sub-case, this scenario is only possible if $V(e = 1, \mu)$ is convex: in this scenario, first-best (always implementing high effort) can be implemented, for example, by fully-revealing experiment. From previous results, we know the equilibrium experiment under this scenario is a fully-revealing experiment, which also always implement high effort. Therefore, there is no efficiency loss in this sub-case as well.

Under Case III.3 and III.5, there exist a single cut-off point $\bar{\mu} \in (0, 1)$, such that $V(e = 1, \mu) < V(e = 0, \mu)$ for $\mu < \bar{\mu}$, and $V(e = 1, \mu = 1) \geqslant V(e = 0, \mu = 1)$ for $\mu \geqslant \bar{\mu}$. Therefore, if $\mu_0 \geqslant \bar{\mu}$, high effort can be implemented with probability 1, for example, by an experiment that provides no additional information.

If $\mu_0 < \bar{\mu}$, the constrained social optimal experiment can be constructed in the following way: $\pi(s = G|\theta = G) = 1$, $\pi(s = G|\theta = B) = \frac{\mu_0(1-\bar{\mu})}{1-\mu_0}$. It is then obvious that regardless of $V(e = 1, \mu)$ being concave or convex, there is under-provision of high effort in equilibrium.

Under Case III.4, there exist a single cut-off point $\bar{\mu} \in (0, 1)$, such that $V(e = 1, \mu) < V(e = 0, \mu)$ for $\mu > \bar{\mu}$, and $V(e = 1, \mu = 1) \geqslant V(e = 0, \mu = 1)$ for $\mu \leqslant \bar{\mu}$. Therefore, if $\mu_0 \leqslant \bar{\mu}$, high effort can be implemented with probability 1, for example, by an experiment that provides no additional information.

If $\mu_0 > \bar{\mu}$, the constrained social optimal experiment can be constructed in the following way: $\pi(s = B|\theta = B) = 1$, $\pi(s = B|\theta = G) = \frac{\bar{\mu}(1-\mu_0)}{(1-\bar{\mu})\mu_0}$. We know from previous results, that equilibrium experiment is fully revealing, and probability of high effort being implemented is smaller than the social optimal. □

Bibliography

Abreu, Dilip and Faruk Gul, "Bargaining and reputation," *Econometrica*, 2000, *68* (1), 85–117.

Aghion, Philippe, Patrick Bolton, and Jean Tirole, "Exit options in corporate finance: Liquidity versus incentives," *Review of Finance*, 2004, *8(3)*, 327–353.

Alesina, Alberto and Bryony Reich, "Nation building," 2013.

Alonso, Ricardo, "Recruiting and Selecting for Fit," 2018.

_ **and Odilon Câmara**, "Persuading voters," *American Economic Review*, 2016, *106(11)*, 3590–3605.

_ **and** _ , "On the value of persuasion by experts," *Journal of Economic Theory*, 2018, *174*, 103–123.

Ashcraft, Adam and Til Schuermann, "The seven deadly Frictions of Subprime Mortgage Credit Securitization," in "The First Credit Market Turmoil of the 21st Century: Implications for Public Policy," World Scientific, 2009, pp. 325–343.

Aumann, Robert J, Michael Maschler, and Richard E Stearns, *Repeated games with incomplete information*, MIT press, 1995.

Azarmsa, Ehsan, "Public Disclosure and Price Fragility," *Working Paper*, 2017.

_ **and Lin William Cong**, "Persuasion in Relationship Finance," *Working Paper*, 2018.

Balakrishnan, Karthik, Mary Brooke Billings, Bryan Kelly, and Alexander Ljungqvist, "Shaping liquidity: On the causal effects of voluntary disclosure," *Journal of Finance*, 2014, *69(5)*, 2237–2278.

Battaglini, Marco, "Multiple referrals and multidimensional cheap talk," *Econometrica*, 2002, *70* (4), 1379–1401.

Bergemann, Dirk and Stephen Morris, "Bayes correlated equilibrium and the comparison of information structures in games," *Theoretical Economics*, 2016, *11* (2), 487–522.

— **and** — , "Information design: A unified perspective," *Journal of Economic Literature,*, 2018, p. forthcoming.

Berndt, Antje and Anurag Gupta, "Moral hazard and adverse selection in the originate-to-distribute model of bank credit," *Journal of Monetary Economics*, 2009, *56(5)*, 725–743.

Besley, Timothy and Andrea Prat, "Handcuffs for the grabbing hand? Media capture and government accountability," *American economic review*, 2006, *96* (3), 720–736.

Best, James and Daniel Quigley, "Persuasion for the long run," 2017.

Blackwell, David, "Comparison of experiments," *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, 1951, pp. 93–102.

Boleslavsky, Raphael and Christopher Cotton, "Grading standards and education quality," *American Economic Journal: Microeconomics*, 2015, *7* (2), 248–79.

— **and Kyungmin Kim**, "Bayesian persuasion and moral hazard," 2018.

— **, Bruce Carlin, and Christopher Cotton**, "Competing for Capital: Auditing and Credibility in Financial Reporting," *Working Paper*, 2017.

Bord, Vitaly and João Santos, "The rise of the originate-to-distribute model and the role of banks in financial intermediation," *Federal Reserve Bank of New York Economic Policy Review*, 2012, *18(2)*, 21–34.

Bruche, Max, Frederic Malherbe, and Ralf Meisenzahl, "Pipeline risk in leveraged loan syndication," *Working Paper*, 2018.

Bushman, Robert, Abbie Smith, and Regina Wittenberg-Moerman, "Price discovery and dissemination of private information by loan syndicate participants," *Journal of Accounting Research*, 2010, *48(5)*, 921–972.

Cai, Nianyun, Jean Helwege, and Arthur Warga, "Underpricing in the corporate bond market," *Review of Financial Studies*, 2007, *20(6)*, 2021–2046.

Chang, Chun and Xiaoyun Yu, "Informational efficiency and liquidity premium as the determinants of capital structure," *Journal of Financial and Quantitative Analysis*, 2010, *45(2)*, 401–440.

Che, Yeon-Koo, Wouter Dessein, and Navin Kartik, "Pandering to persuade," *American Economic Review*, 2013, *103* (1), 47–79.

Chemla, Gilles and Christopher Hennessy, "Skin in the game and moral hazard," *Journal of Finance*, 2014, *69(4)*, 1597–1641.

Ciorciari, John D and Jessica Chen Weiss, "Nationalist Protests, Government Responses, and the Risk of Escalation in Interstate Disputes," *Security Studies*, 2016, *25* (3), 546–583.

Crawford, Vincent P, "A theory of disagreement in bargaining," *Econometrica: Journal of the Econometric Society*, 1982, pp. 607–637.

_ **and Joel Sobel**, "Strategic information transmission," *Econometrica: Journal of the Econometric Society*, 1982, pp. 1431–1451.

Dahiya, Sandeep, Manju Puri, and Anthony Saunders, "Bank borrowers and loan sales: New evidence on the uniqueness of bank loans," *Journal of Business*, 2003, *76(4)*, 563–582.

Debs, Alexandre and Jessica Chen Weiss, "Circumstances, domestic audiences, and reputational incentives in international crisis bargaining," *Journal of Conflict Resolution*, 2016, *60* (3), 403–433.

Edmans, Alex, "Blockholder trading, market efficiency, and managerial myopia," *Journal of Finance*, 2009, *64(6)*, 2481–2513.

_ **and Clifford Holderness**, "Blockholders: A survey of theory and evidence," *Handbook of the Economics of Corporate Governance*, 2017, pp. 541–636.

_ **and Gustavo Manso**, "Governance through trading and intervention: A theory of multiple blockholders," *Review of Financial Studies*, 2011, *24(7)*, 2395–2428.

_ **, Doron Levit, and Devin Reilly**, "Governance Under Common Ownership," *Review of Financial Studies,*, 2018, p. forthcoming.

_ **, Vivian Fang, and Emanuel Zur**, "The effect of liquidity on governance," *Review of Financial Studies*, 2013, *26(6)*, 1443–1482.

Ellison, Glenn and Sara Fisher Ellison, "Search, obfuscation, and price elasticities on the internet," *Econometrica*, 2009, *77* (2), 427–452.

Ellul, Andrew and Marco Pagano, "IPO underpricing and after-market liquidity," *Review of Financial Studies*, 2006, *19(2)*, 381–421.

Ely, Jeffrey C, "Beeps," *American Economic Review*, 2017, *107* (1), 31–53.

Eyerman, Joe and Robert A Hart Jr, "An empirical test of the audience cost proposition: Democracy speaks louder than words," *Journal of Conflict Resolution*, 1996, *40* (4), 597–616.

Faure-Grimaud, Antoine and Denis Gromb, "Public trading and private incentives," *Review of Financial Studies*, 2004, *17(4)*, 985–1014.

Fearon, James D, "Domestic political audiences and the escalation of international disputes," *American political science review*, 1994, *88* (3), 577–592.

— , "Rationalist explanations for war," *International organization*, 1995, *49* (3), 379–414.

Fudenberg, Drew and Jean Tirole, "Perfect Bayesian equilibrium and sequential equilibrium," *Journal of Economic Theory*, 1991, *53* (2), 236–260.

Gentzkow, Matthew and Emir Kamenica, "Costly persuasion," *American Economic Review P&P*, 2014, *104(5)*, 457–62.

Goldstein, Itay and Yaron Leitner, "Stress tests and information disclosure," *Journal of Economic Theory*, 2018, *177*, 34–69.

Gorton, Gary and George Pennacchi, "Financial intermediaries and liquidity creation," *Journal of Finance*, 1990, *45(1)*, 49–71.

— **and** — , "Banks and loan sales marketing nonmarketable assets," *Journal of Monetary Economics*, 1995, *35(3)*, 389–411.

Green, Jerry R and Nancy L Stokey, "A two-person game of information transmission," *Journal of economic theory*, 2007, *135* (1), 90–104.

Green, Richard, "Issuers, underwriter syndicates, and aftermarket transparency," *Journal of Finance*, 2007, *62(4)*, 1529–1550.

Grossman, Sanford J and Oliver D Hart, "An Analysis of the Principal-Agent Problem," *Econometrica: Journal of the Econometric Society*, 1983, pp. 7–45.

Hanley, Kathleen Weiss and Gerard Hoberg, "The information content of IPO prospectuses," *Review of Financial Studies*, 2010, *23(7)*, 2821–2864.

Hennessy, Christopher and Josef Zechner, "A theory of debt market illiquidity and leverage cyclicality," *Review of Financial Studies*, 2011, *24(10)*, 3369–3400.

Holmstrom, Bengt, "Moral hazard and observability," *Bell journal of Economics*, 1979, *10* (1), 74–91.

Holmström, Bengt, "Managerial incentive problems: A dynamic perspective," *The review of Economic studies*, 1999, *66* (1), 169–182.

— **and Jean Tirole**, "Market liquidity and performance monitoring," *Journal of Political Economy*, 1993, *101(4)*, 678–709.

Huang, Zeqiong, "Optimal Reporting Systems with Investor Information Acquisition," *Working Paper*, 2016.

Irani, Rustom and Ralf Meisenzahl, "Loan sales and bank liquidity management: Evidence from a us credit register," *Review of Financial Studies*, 2017, *30(10)*, 3455–3501.

Kambe, Shinsuke, "Bargaining with imperfect commitment," *Games and Economic Behavior*, 1999, *28* (2), 217–237.

Kamenica, Emir and Matthew Gentzkow, "Bayesian Persuasion," *American Economic Review*, 2011, *101(6)*, 2590–2615.

Keys, Benjamin, Tanmoy Mukherjee, Amit Seru, and Vikrant Vig, "Did securitization lead to lax screening? Evidence from subprime loans," *Quarterly Journal of Economics*, 2010, *125(1)*, 307–362.

Kyle, Albert, "Continuous auctions and insider trading," *Econometrica*, 1985, *53(6)*, 1315–1335.

Leland, Hayne and David Pyle, "Informational asymmetries, financial structure, and financial intermediation," *Journal of Finance*, 1977, *32(2)*, 371–387.

Lipnowski, Elliot and Doron Ravid, "Cheap talk with transparent motives," 2017.

Lummer, Scott and John McConnell, "Further evidence on the bank lending process and the capital-market response to bank loan agreements," *Journal of Financial Economics*, 1989, *25(1)*, 99–122.

Mansfield, Edward D, Helen V Milner, and B Peter Rosendorff, "Why democracies cooperate more: Electoral control and international trade agreements," *International Organization*, 2002, *56* (3), 477–513.

Maug, Ernst, "Large shareholders as monitors: Is there a trade-off between liquidity and control?," *Journal of Finance*, 1998, *53(1)*, 65–98.

Mirrlees, James A, "The optimal structure of incentives and authority within an organization," *The Bell Journal of Economics*, 1976, pp. 105–131.

_ , "The theory of moral hazard and unobservable behaviour: Part I," *The Review of Economic Studies*, 1999, *66* (1), 3–21.

Muthoo, Abhinay, "A bargaining model based on the commitment tactic," *Journal of Economic theory*, 1996, *69* (1), 134–152.

Osborne, Martin J and Ariel Rubinstein, *Bargaining and markets*, Academic press, 1990.

Pagano, Marco and Paolo Volpin, "Securitization, transparency, and liquidity," *Review of Financial Studies*, 2012, *25(8)*, 2417–2453.

Parlour, Christine and Guillaume Plantin, "Loan sales and relationship banking," *Journal of Finance*, 2008, *63(3)*, 1291–1314.

Passarelli, Francesco and Guido Tabellini, "Emotions and political unrest," *Journal of Political Economy*, 2017, *125* (3), 903–946.

Pennacchi, George, "Loan sales and the cost of bank capital," *Journal of Finance*, 1988, *43(2)*, 375–396.

Prévet, Antoine, "Information Management by a Budget constrained Principal," 2018.

Purnanandam, Amiyatosh, "Originate-to-distribute model and the subprime mortgage crisis," *Review of Financial Studies*, 2010, *24(6)*, 1881–1915.

Putnam, Robert D, "Diplomacy and domestic politics: the logic of two-level games," *International organization*, 1988, *42* (3), 427–460.

Rayo, Luis and Ilya Segal, "Optimal information disclosure," *Journal of Political Economy*, 2010, *118(5)*, 949–987.

Rodina, David, "Information Design and Career Concerns," 2016.

_ **and John Farragut**, "Inducing effort through grades," 2016.

Rogerson, William P, "The first-order approach to principal-agent problems," *Econometrica: Journal of the Econometric Society*, 1985, pp. 1357–1367.

Santos, João and Pei Shao, "Loan Ownership and Liquidity in the Secondary Loan Market," *Working Paper*, 2018.

Schelling, Thomas C, "The strategy of conflict," *Harvard University*, 1960, *1980*, 13.

Schultz, Kenneth A, "Do democratic institutions constrain or inform? Contrasting two institutional perspectives on democracy and war," *International Organization*, 1999, *53* (2), 233–266.

_ **and Barry R Weingast**, "The democratic advantage: institutional foundations of financial power in international competition," *International Organization*, 2003, *57* (1), 3–42.

Slantchev, Branislav L, "Politicians, the media, and domestic audience costs," *International Studies Quarterly*, 2006, *50* (2), 445–477.

Smith, Alastair, "International crises and domestic politics," *American Political Science Review*, 1998, *92* (3), 623–638.

Szydlowski, Martin, "Optimal Financing and Disclosure," *Working Paper*, 2016.

Weiss, Jessica Chen, "Authoritarian signaling, mass audiences, and nationalist protest in China," *International Organization*, 2013, *67* (1), 1–35.

_ , *Powerful patriots: nationalist protest in China's foreign relations*, Oxford University Press, 2014.

Westbrook, Ian, *Strategic Financial and Investor Communication: The Stock Price Story*, Taylor & Francis, 2014.

Wittenberg-Moerman, Regina, "The role of information asymmetry and financial reporting quality in debt trading: Evidence from the secondary loan market," *Journal of Accounting and Economics*, 2008, *46(2)*, 240–260.

Yang, Ming, "Optimality of debt under flexible information acquisition," *Review of Economic Studies,*, 2017, p. forthcoming.