

Essays on Functions and Organisations of Political Parties

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Abstract

This thesis consists of the three papers that present new formal models of functions and organisations of political parties. The models begin with a particular function or organisational feature of political parties and integrate it with the related issues that the formal literature in political science has either discussed separately or has not paid sufficient attention to. The first paper analyses the strategic interactions between parties and their candidates in elections. It answers the question of why parties provide greater campaign support toward open-seat races than reelection races; to what extent campaign support of parties influences and incentivises valence investment of individual candidates. It also identifies and distinguishes party and personal attributes to an incumbency advantage and discovers a ‘multiplying’ effect that the sequential nature of reelection race has on the advantage. The second paper discusses intraparty competition between factions. It identifies a trade-off between collective and individual benefits in faction members’ choice between intraparty factions and provides a theoretical explanation for factional splits and merges observed in politics. It differentiates itself from the small literature of factions, which is often found to be insufficient to analyse the dynamics of intraparty factions, by incorporating a hierarchical structure of party organisations. The third paper integrates different types of organisational hierarchies, in power, as the second paper does, and in decision procedures and connects them to the longevity of political power. It analyses endogenous allocation of power that gives rise to a specific pattern of power hierarchy that best serves the two objectives of political power, the absolute size and longevity of power. It also shows that the optimal power hierarchy differs across the types of decision hierarchies, indicating the decision-making procedures adopted by a parties. It offers a theoretical explanation to why some parties have undergone more frequent leadership turnover.

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Introduction

Traditionally political parties are viewed as a group of individuals whose common interest is to win in elections as Downs defines

“a team of men seeking to control the governing apparatus by gaining office in a duly constituted election.” (Downs 1957)

The definition specifies a particular organisational objective of political parties. It implies a number of functions that parties carry over the course of elections, e.g. candidate recruit and nomination and campaign support. In defining political parties, some have focused on another function, or interest, of parties, that parties represent and advocate particular ideologies through different channels, including party manifesto (La Palombara and Weiner 1966). Others also have paid attention at organisational features (Blondel 1978; La Palombara and Weiner 1966). They have distinguished political parties from other types of organisations in politics. According to their view, parties aim to extend their continuity, not serving as a temporary coalition with ideological and office-seeking motivations. In order to achieve this, parties have established a “permanent organisation”.

When analysing strategic behaviour of parties, it is necessary to consider features addressed in the functional and organisational definition of political parties. One should also acknowledge that a party is an organisation, in which individuals and groups with different characteristics and motivations interact. There exists a large volume of the formal literature that analyses different functions, especially focusing on election-related roles of parties. The literature has frequently adopted an assumption that parties are unitary actors, particularly for the simplicity of analysis. The assumption implies that parties are groups of similar-minded individuals that act to advance their collective interests. Although the formal models based on the assumption have greatly contributed to the analysis of the

strategic behaviour of parties, it is also true that the assumption limits the scope of analysis.

For example, there are a large number of formal models that analyse parties' involvement in electoral campaign or redistributive politics. They provide a theoretical explanation for different patterns of campaign spending and redistributive benefits (Brams and Davis 1973, 1974; Snyder 1989; Stromberg 2008; Lindbeck and Weibull 1987; Dixit and Londregan 1996). They have explained why parties spend more resources, both financial and non-materialistic, e.g. district visits of key politicians of parties during the election, in marginal or swing constituencies and why better-resourced parties show a different pattern of spending, compared to the less-resourced ones. However, these models do not explain whether and how strategic motivations of parties affect the way their candidates behave and perform. They insufficiently answer related questions such as whether the candidates of a better-resourced party act differently from those of a less-resourced party, or whether the tendency of outspending in marginal constituencies discourages or motivates candidates running in constituencies with a different degree of competition.

Similarly, the formal models of valence have primarily focused on individual candidates. They have identified the incentives for valence investment during elections and shown that candidate with different characteristics invest in valence differently (Meiowitz 2008; Ashworth and Bueno de Mesquita 2009; Serra 2010). Some have extended the discussion to an incumbency advantage, identifying sources of an incumbency advantage, such as campaign resources and quality (Erikson and Palfrey 2000; Ansolabehere et al. 2000; Hirano and Snyder 2009; Ashworth and Bueno de Mesquita 2008). As much as the models of parties in elections, these models portray political actors of one type and overlook the interdependence between parties and their candidates.

This lack of attention motivates the first paper of the thesis, titled **Campaign Support of Parties and Candidate Performance**, in Chapter 2. The paper extends Snyder (1989) and modifies his framework of a two-party multi-district election, in which the parties decide the amount of campaign resources they allocate to each district. The framework

in the paper considers two parties competing in three districts. Previously Snyder (1989) shows that the parties allocate more resources to a district with “natural advantage”, which indicates their relative strength in the district, depending on their relative advantage in campaigns. The paper rejects the idea that a party’s relative strength in a district is exogenously given. Instead, it integrates valence investment of candidates and campaign support of parties and assumes that the electoral race in each district involves a two-stage contest. The “natural advantage” is reinterpreted as an endogenous factor, which is determined by valence investment of individual candidates during the first stage of the game. Their parties, after having observed the first-stage performance of their candidates, allocate campaign resources across their candidates in the second stage.

The framework also considers different types of candidate-level asymmetries, explicitly comparing the candidates in a reelection race, the current incumbent and the relatively inexperienced and lower-quality challenger, and those in an open-seat race, sharing similar characteristics. The paper answers the questions overlooked in the previous work, showing that the party-level (dis)advantage influences candidates to a different extent, depending on their personal-level (dis)advantage over their opponent. This interaction between asymmetries between the parties and between the candidates derives the difference in the candidates’ valence investment and thus the parties’ relative strength in each district. The result hence explains the empirically observed variation in the degree of competition between reelection and open-seat races. The variation, in turn, motivates the parties to differentiate their campaign support between the two types of electoral races.

The paper further contributes to the formal literature of an incumbency advantage by identifying and distinguishing the party and personal attributes to an incumbency advantage. It also identifies the sources of an incumbency disadvantage, which is again an electoral outcome derived by the interaction of party- and candidate-level characteristics.

The frequent adoption of the ‘unitary-actor’ assumption in the formal literature can also explain for the relative under-development in some areas of the party literature, such as

intraparty factions despite their extensive presence in politics. Furthermore, the assumption has been found effective to analyse how parties interact with external organisations, e.g. opponent parties, in elections and, parliament or legislature. While there are a wide range of both formal and empirical studies that discuss ‘party systems’, insufficient attentions have been paid to the internal structure and organisation of political parties (Wolinetz 2002).

In order to fulfil their functions and maintain the continuity, parties have adopted different types of structures and organisations. The formal organisation of political parties may evolve, not only because an alternative system replaces the previous one, but also because the formal organisation motivates or facilitates formation of informal organisations. The former is a change made for organisational reasons. On the contrary, the latter often takes place, as the formal organisation of parties incentivises and disincentivises their politicians with conflicting interests. An example of such informal organisations is intraparty factions.

There have been a number of attempts in the formal literature to study formation of factions in political organisations. The earlier models of factions have made a departure from the ‘unitary-actor’ assumption and recognised the presence of intraparty groups that have interests, mostly ideological, conflicting those of the party (Eguia 2011a, 2011b; Mutlu-Eren 2010). Instead, the models have adopted an assumption that factions are unitary actors, implying that the individuals in a faction share the common interest and act to achieve the collective objectives. Recently, new models of factions recognise the presence of individuals with a conflict of interest in factions (Persico et al. 2011; Morelli and Park 2016; Dewan and Squintani 2015). Some of them also incorporate the organisational features of formal and informal organisations in political parties and try to better explain factional politics.

However, each of the existing models focuses on a particular organisational feature or a motivation of politicians. They do not provide an integrated model that analyses the

dynamics of intraparty factions, such as splits and mergers between factions. More specifically, the existing models explain why individuals form a faction, how they negotiate and bargain with other members and with other factions. The questions of, exactly what motivates politicians of a faction to join another or to merge with another, and why a faction prevails longer than the other, are not sufficiently answered.

The empirical observations and the existing literature motivate the second paper of the thesis, “**Factions Explained with Power Hierarchies**”, in Chapter 3. The paper, as the title indicates, incorporates a hierarchical structure in intraparty factions. It is one of the features in party organisation that distinguishes parties from other types of temporary organisation. As for parties, intraparty factions often have substantial continuity and have a structured organisation which defines roles and responsibilities of their members (Zuckerman 1975). The characteristics of intraparty factions present opportunities and incentives to faction members, which are absent in other types of factions that are relatively temporary and unstructured, e.g. legislative and parliamentary coalitions.

The paper presents a model in which the two intraparty factions with asymmetric strength compete with each other for greater party resources. Party resources to which each faction has access serve as a ‘club good’, exclusively available to its member. Members in each faction are ranked according to the size of influence they exercise. They want to be part of a faction offering a greater club good, and to hold greater power. The game begins as each member of the factions decides whether to leave for the opposition and their decision determines the winner of intraparty competition. The model identifies a trade-off between collective and individual benefits in the members’ choice between the factions. The members assess the relative gain or loss in the two types of benefits they anticipate when joining each faction and choose a faction with the greater relative gain (or the smaller relative loss).

It also shows a variation in the trade-off across the members of different ranking. The high-ranked members of any faction suffer from a smaller loss in their power, whichever faction they choose. The opposite is true for the lower-ranked. This causes a variation

in the members' decision. The higher-ranked members' decision then rests purely on the collective benefits from each faction. The relatively lower-ranked members balance between the two types of benefits, and some of them in the stronger or major faction choose a weaker or minor faction forgoing the collective benefits for a higher ranking.

The results provide a theoretical explanation for a number of empirical observations in factional splits and mergers. They also provide implications for the question: why some factions prevail, whereas others decline. As party resources are highly disproportionately distributed across strong and weak factions, the model predicts that, more members will rest their decision on the absolute size of collective benefits from each faction, inducing a greater number of departures from a weak faction and a smaller number of defection from a strong faction. The disproportionality is also shown to encourage mergers between smaller factions, further facilitating their decline.

It is also noted that different formal organisations adopted by political parties create contrasting incentives to politicians. At the same time, as much as parties care about their influence and continuity, politicians have similar motivations. In the second paper of the thesis, the 'power' motivation of politicians, combined with a hierarchical structure of party organisation, is addressed. In the third paper, titled "**Hierarchies of Power and Decisions**" in Chapter 4, another aspect of political power, its longevity, is analysed in a framework that incorporates different types of organisational hierarchies.

The paper explores an interaction between the two organisational hierarchies. It answers the question of, whether a particular pattern of power allocation better serves the interests of political leadership, i.e. secures greater influence over a longer period, and to what extent the decision procedure affects endogenous formation of power hierarchy in an organisation. Some empirical observations and implications on the related issues have been made (Quinn 2005, 2012), when comparing the leader selection and eviction rules adopted in the British political parties. The relatively more frequent leadership changes in the Conservative Party are argued to be attributable to the concentration of decision-making power

and the procedure that further strengthens the influence of those with substantial power.

While the formal literature has not actively explored internal organisations and decision procedures of political parties, the existing models have discussed some interactions between decision procedures and bargaining in a legislative setting (Baron and Ferejohn 1989, Krehbiel 1998; McCarty 2000; Denzau and Robert 1983; Krehbiel 2004; Diermeier et al. 2014). Their findings could have some implications to power allocation within a party to some extent, but a limitation is acknowledged. The motivation in these models is rather single faceted, portraying the players whose primary concern is on the absolute size of power they exercise.

The paper presents a simple model of power allocation in an organisation, in which the leader determines how much decision-making power the members and she exercise in organisational decisions. Her decision over power may increase her risk of removal from office, as the members can initiate a coup and overthrow her when realising redistribution of power after her departure benefits them. When deciding over a coup, the members reach the decision according to the organisation's procedure, which specifies the right to initiate a coup and the process showing how an initiative is passed onto the members. For given allocation of power across the members, a decision procedure may grant greater bargaining power to some of the members, more than the absolute size of power they are allocated to. Furthermore, the paper shows that some procedures produce a greater number of members with a procedural power than what they initially specify. Under such a procedure, the leader has to compensate those affected more by allocating more power to them in order to prevent a coup, which, in turn, reduces her share of power. The result therefore implies a trade-off between the absolute size and longevity of power in the leader's allocation and indicates that different procedures, or decision hierarchies, increase or reduce the trade-off.

In terms of the theoretical approaches taken in the three papers, the second and the third paper adopt a cooperative solution concept, a stability concept in the former and the Shapley value in the latter. Whereas both approaches are effective to analyse political science problems, including coalition formation, not many have attempted. Both papers involve

a relatively simple, but well-integrated, framework, with the potential of extensions and modification for different types of political problems, other than those discussed in this thesis. This would be one of the contributions that this thesis could offer to the formal literature in political science.

Each of the three papers begins with a particular function or organisational feature of political parties, some of which have not been sufficiently addressed or studied in the literature. Another common feature in the three papers is their departure from the common assumption of ‘unitary parties’. Each of them shows how individuals with different characteristics and interests interact within the ‘framework’ political parties provide and connects the strategic interaction between the individuals to a function or organisational aspect of parties. They answer the questions that the existing literature has insufficiently answered or failed to cover and identify the key attributes and trade-offs that derive empirical phenomena observed in politics. The thesis contributes to the literature by expanding the scope of analysis and by providing a new insight into a number of under-developed areas in party politics.

2. Campaign Support of Parties and Candidate Performance

Chapter Abstract

This paper presents a model of electoral campaigns, extending Snyder (1989). The model takes and modifies the framework in Snyder (1989), in which the two parties with the (a)symmetric capacity of campaign and district-level electoral strength compete in a multi-district elections. The framework is reinterpreted as a two-stage contest between the parties and between their candidates. The model analyses the interaction between the parties and the candidates, given their interdependence. While many have studied the strategic motivations for campaign support of parties and valence investment of candidates, they have not sufficiently paid attention to the strategic interaction between them. This paper tries to uncover a mechanism that how strategic behaviour of the parties and their candidates affect each other. In the model, each candidate of the (a)symmetrically resourced parties exerts effort during the first stage, which determines their electoral strength. The parties allocate their campaign resources across the districts during the second stage. One of the departures from Snyder (1989) is to endogenise the district-level electoral strength of the parties, by introducing the first-stage, in which the candidates with different characteristics compete. The model compares between a reelection race, in which the current incumbent is challenged by a freshman candidate, to whom effort is more costly, and an open-seat race, in which two freshman candidates with a similar cost of effort compete. The framework reflects the election cycle in politics, by assuming sequential investment of effort in the reelection race and simultaneous investment in the open-seat race. The results provide a theoretical explanation to a number of empirical phenomena. The parties, whether or not asymmetric in campaign capacity, provide greater resources to their open-seat candidate, whereas the incumbent leads the race from an early stage of the election. A contribution of this paper, especially to the formal models of incumbency advantage, is that it identifies and distinguishes between party and personal attributes to the advantage, and discovers an additional aspect of the advantage. The sequential nature of the

reelection race works as a ‘multiplier’ of the advantage. Whenever an incumbency advantage exists, it strengthens the advantage. The model also shows that when an incumbency disadvantage emerges, the sequential nature works against an incompetent incumbent.

1. Introduction

Political parties are actively involved in electoral campaigns of individual candidates. They provide their candidates with campaign support. They allocate party resources including campaign funds and party leaders visit individual constituencies. It is observed that parties in a multi-district election disproportionately distribute their resources across their candidates. During the UK general election in 2010, the major party leaders more frequently visited areas which were more densely populated with a large number of constituencies or more competitive. The US party committees make larger financial contributions to candidates who have been party loyalists (Leyden and Borrelli 1990) or who compete in a more competitive race (Damore and Hansford 1999); and incumbents or those with previous campaign experiences (Jacobson 1993; Herrnson 1988, 1989).

The theoretical literature has identified different incentives for parties when they support the campaigns of individual candidates. Snyder (1989) illustrates a two-party multi-district election in a framework in which the parties differ in their strength in each district and marginal cost of campaign. He shows that the asymmetries between the parties affect their allocation of campaign resources.

However, the strategic allocation of campaign resources should be examined in a wider scope, integrating players at different levels. A party's strength in a district is determined by how voters in the district perceive not only the party but also its candidate. When a party's resource allocation depends on how competitive its candidates are perceived to be, the candidates are incentivised to act and perform in a certain way. The incentives also potentially vary across the candidates with different characteristics.

This paper uses the possible interaction between parties and their candidates as a motivation and develops an integrated framework of multi-district electoral competition. The framework departs from the common assumption in literature that parties are unitary actors. It considers not only when parties compete in a multi-district election, but also when individual candidates compete in each district. The framework reinterprets Snyder (1989)

as a two-stage rent-seeking contest in which a different type of a player invests in a different type of valence at each stage of the game. During the first stage, the candidates in each district exert effort, which determines their electoral strength. One of the departures this paper makes from Snyder (1989) is therefore to endogenise the parties' district-level strength. In the second stage, the parties allocate campaign resources across their candidates.

The model assumes that there are asymmetries between not only the parties, but also the candidates. The parties are endowed with campaign budget of a fixed, but different size. Snyder (1989) considers parties with a different marginal cost of campaign. There are formal models that discuss a similar theme but assume an asymmetry in campaign budget (Brams and Davis 1973; Lindbeck and Weibull 1987). Despite the difference in assumption, they deliver similar results. Whether they adopt an asymmetry in marginal cost or in budget, the existing models examine the strategic behaviour of parties with asymmetric capacity of campaign. When a party's marginal cost is relatively lower, it can provide greater resources than its rival. The same applies to a party with a bigger budget. This paper extends the earlier models and examines whether and to what extent an asymmetry between the parties affects their candidates as well as themselves in equilibrium.

The model also complements the formal models of endogenous valence by considering various types of candidates who differ in their ability. In the framework, the electoral districts differ in the type of race that takes place. A *reelection* race of an incumbent against a freshman takes place in one district, whereas two freshman candidates compete (an *open-seat* race) in the other. A variation in the sequence of the candidates' decisions across the districts is introduced. The candidates in the reelection race *sequentially* invest in valence. However, those in the open-seat race decide on valence simultaneously.

Some models of endogenous valence have reflected an incumbent's relative advantage in valence. A similar assumption is adopted. The marginal cost of effort differs between the candidates and the incumbent's cost of effort is lower than any freshman candidate. While

many of the existing models adopt a simultaneous game, a few exceptions exist. Wiseman (2006) shows that sequential investment of valence gives a first-mover advantage and the first mover can effectively preempt the second mover(s). The attempt to consider a sequential game in the reelection race is reasonable, given the election cycle in politics. An incumbent who plans to run for reelection essentially has the entire term to invest in valence. On the contrary, a challenger often remains undetermined until the beginning of campaign. Once nominated, a challenger who has observed the incumbent starts investing in valence.

The equilibrium results confirm the previous findings. When the parties are symmetric in campaign budget, the equilibrium amount of resources each party allocates to each district is symmetric. The symmetric parties allocate more resources to a relatively marginal district. When they differ in campaign budget, the better-resourced always outspends the less-resourced in any district. The better-resourced allocates more resources to a weaker district, whereas the less-resourced gives more to a stronger district. The target district of each party is determined by the marginal gain of additional resources. The marginal gain varies with the relative size of the parties' campaign budget. For example, if a symmetric party deviates and allocates more to a less marginal district, it loses the marginal district for sure, whereas winning in the other is not guaranteed.

It is shown that the equilibrium effort of the candidates is affected by an interaction of multi-level factors. The extent to which the asymmetry in campaign budget affects the candidates depends on the characteristics of each candidate and his opponent. A candidate may increase or decrease his effort, depending on his relative advantage over his opponent, in effort and in campaign. Furthermore, a change in a candidate's relative advantage affects him and his opponent disproportionately if they exert effort sequentially and symmetrically if they choose effort simultaneously.

For instance, the relative advantage of the incumbent strengthens whenever his party becomes better resourced. When the challenger and he exert effort simultaneously, both candidates respond to the change in budget by decreasing their investment. The challenger

who is discouraged by the widened disadvantage invests less and the incumbent matches the decrease. The sequential nature of the reelection race incentivises the incumbent to exert greater effort so that he can preempt, or deter, the challenger completely.

The type of a race for a candidate, that determines who he competes against and in what sequence his opponent and he exert effort, therefore influences his first-stage performance. The candidates in the open-seat race perform similarly and the race tends to be marginal, whereas the incumbent leads the race from the first stage and the reelection race becomes lop-sided. Combined with the optimal strategies of the parties, it is concluded that the parties always provide greater resources to the open-seat. When they are symmetric in budget, they focus on a more marginal race. When they differ in budget, the better-resourced incumbent party allocates more to the relatively weaker district, in which the open-seat race takes place.

The results address an incumbency advantage, indicating an incumbent is more likely to be (re)elected than a challenger. The model identifies an interaction of personal and party attributes as a potential source of an incumbency advantage. The findings are distinguished from a majority of the existing formal models that often focus explicitly on personal attributes. In the model, the sequentiality of the reelection race serves as a ‘multiplier’ of the advantage whenever it exists. When an incumbent lacks either of the two attributes or both, the sequentiality worsens his electoral performance and intensifies an incumbency ‘disadvantage’.

The remaining sections are organised as follows. After a brief review of the related literature in the next section, Section 3 introduces the model. Section 4 provides the equilibrium analysis and offers a theoretical explanation for the empirical patterns of party support to candidate campaigns. Section 5 examines (i) an an incumbency advantage in the model; (ii) the effect of a campaign spending cap. It identifies the sources of an incumbency advantage in the model and analyses the effect of a campaign-spending limit on an incumbency advantage. The paper concludes with the summary and discussion of the results,

concludes.

2. Related Literature

The model incorporates a number of features often observed in elections: campaign support of parties, valence investment of candidates and incumbency advantage. These are the issues that have been rather separately addressed in the political science literature. The model integrates them into a simple framework that employs a two-stage contest in which a different type of player makes strategic decisions during each stage. The framework addresses the interdependence between parties and their candidates and makes it possible to analyse strategic interactions between them. With regard to the latter aspect, the model complements the existing literature that has analysed these issues separately and suggests a new perspective on the issues.

As briefly mentioned, the model extends Snyder (1989), in which the two office-seeking parties run costly campaign in a multi-district election. The parties have a “natural advantage” in each district and a different marginal cost of campaign. When they are symmetric in marginal cost and maximising the number of seats they win in the election, they allocate more resources to swing districts in which their natural advantages are similar. When their marginal costs are different, the party with the lower cost allocates more to relatively advantaged districts in which its natural advantage is smaller than the rival. The opposite is true for the party with the higher cost.

When an endogenous “natural advantage” of the parties is considered, the results in Snyder (1989) remain robust. This paper first answers why the parties enjoy a greater advantage in some districts. The results further complements Snyder (1989) by explaining why party contributions vary across different types of races, e.g. reelection races and open-seat races. While this observation has been addressed in the empirical literature of campaign finance, the existing formal models have rather overlooked it, although some related comparisons

were incorporated, such as a variation in campaign funds raised by incumbents and challengers.

A large volume of formal literature has discussed the allocation of campaign resources. Brams and Davis (1973, 1974) illustrate a presidential race between the two candidates, each of whom is elected in a district with probability equivalent to his relative campaign spending in the district. The election outcome is determined by the aggregate votes won in all districts. The candidates hence spend more in large and populated districts. Snyder (1989) modifies the framework of Brams and Davis (1973, 1974) that for a party winning a district depends not only on the relative spending but also on its natural advantage. Stromberg (2008) shares a similar motivation with Snyder (1989) in a model in which the two presidential candidates run campaign tours. He asserts, in reference to empirical results, that that in swing districts, candidates run larger campaigns, allocate more resources, and pay more frequent or longer visits.

The notion of “natural advantage” in Snyder (1989) is adopted and interpreted as ideological predisposition of voters by Stromberg (2008) and Case (2001). These models also treat it as exogenously given. Cadigan (2007) modifies Snyder’s framework to a two-stage rent-seeking game, which resembles the framework of this paper. In each stage, the players invest simultaneously. Their first-stage investment determines what Snyder calls “natural advantage” and sequentially their second-period investment. The extension in this paper uses a similar functional form introduced by Cadigan (2007), but the two differ that this paper considers a two-stage game in which a different set of players invest in each of the stage. The modification makes it possible to identify and analyse the interaction between different types of players, such as parties and their candidates, whose payoffs are interdependent.

Building on a similar motivation, there are a number of formal studies that address the electorally motivated allocation of redistributive benefits (Lindbeck and Weibull 1987; Dixit and Londregan 1996; Lizzeri and Persico 2001). Lindbeck and Weibull (1987) and Dixit and Londregan (1996) consider an environment, in which the parties with a fixed

budget promise and provide redistributive transfers to groups of voters with an ideological predisposition to the parties. The parties allocate larger transfers to groups with a vast number of swing voters. Lizzeri and Persico (2001) show that different electoral systems provide politicians with contrasting incentives over targeted redistribution. As for the models of campaign resource allocation, they recognise an asymmetry in “natural advantage”, but do not answer how and why the asymmetry emerges.

This paper also shares some characteristics with the formal models of endogenous valence. They analyse when office-seeking politicians, who are often assumed to be *ex-ante* homogeneous, invest in valence in order to improve their electoral competitiveness (Meiowitz 2008); to complement or compensate for their policy choice (Ashworth and Bueno de Mesquita 2009; Serra 2010; Carrillo and Castanheira 2008), or to win candidacy (Cail-laud and Tirole 2002; Castanheira et al. 2010).

This paper shows that valence investment of an election-motivated candidate is influenced not only by a single characteristic of the candidate, but also a combination of factors at different levels. An asymmetry in personal attributes such as marginal cost of effort is often incorporated into the formal models of endogenous valence, but an interaction between the different attributes of valence investment is rarely addressed. This would offer a new perspective by identifying and analysing the interaction.

The result of the model is further extended to the issue of an incumbency advantage. An incumbency advantage, i.e. incumbents are more likely to be elected, often with a large margin, is a well-recognised empirical regularity in the literature (Erikson 1971; Cox and Katz 1996; Ansolabehere and Snyder 2002). The literature has identified various sources that give rise to the advantage. Incumbents are more or better resourced than challengers (Erikson and Palfrey 2000; Ansolabehere et al. 2000; Hirano and Snyder 2009), and have better quality deterring challengers (Cox and Katz 1996; Ashworth and Bueno de Mesquita 2008).

In identifying sources of an incumbency advantage, this paper makes a distinction between personal and party attributes of incumbents that potentially facilitate the advantage to a different extent. Additionally, it is shown that the sequential nature in reelection races, that an incumbent is almost always the first mover, followed by his challengers, intensifies or multiplies an incumbency advantage, although it does not give rise to the advantage. The results also address a possibility of an incumbency disadvantage, which is a topic relatively less frequently addressed in the literature.

3. Model

Consider a society with three electoral districts, $i \in \{1, 2, 3\}$. Each district has one representative voter. Two parties, $P \in \{A, B\}$ compete in the upcoming election. A and B own one and $\tau \in [1, 2)$ units of resources respectively that they can spend on campaigning in the two districts. They maximise the expected number of seats won by their candidates in the election. If a candidate, denoted by i^P , wins, his party receives Δ and zero otherwise. In district 1 and 3, the incumbent affiliated respectively with B and with A runs for reelection. An open-seat race takes place in district 2. B is the *incumbent* party and A is the *challenging* party.

Therefore, there are four freshman candidates running for a seat: the challengers nominated by A in district 1 and by B in district 3, and the two candidates nominated by each party in district 2. Each freshman candidate has political capital, $\lambda \in (0, 1)$. λ 's as well as τ are publicly known.

The game begins as the incumbents in district 1 and district 3 and exert effort, $e_1^B \in [0, 1]$ and $e_3^A \in [0, 1]$, both with marginal cost, 1. The freshmen candidates then exert effort $e_i^P \in [0, 1]$ with the marginal cost, $\frac{1}{\lambda}$ for $i \in \{1, 2, 3\}$. Whereas the candidates in districts 1 and 3 move sequentially, those in district 2 decide and exert effort simultaneously. The representative voter in each district observes effort invested by the candidates running in

her district and perceives the electoral strength of each candidate,

$$(2.1) \quad \alpha_i^P = \frac{e_i^P}{e_i^P + e_i^Q}$$

for any $P \neq Q \in \{A, B\}$ where $\alpha_i^Q = 1 - \alpha_i^P$.

The parties simultaneously allocate campaign resources to their candidates, (r_1^P, r_2^P, r_3^P) . Resources that remain unspent after the election have no value to the parties. The candidates spend all of the resources received to run the campaign¹. As for the parties, the candidates do not receive any benefit from unspent resources. The voters observe campaigns in their district. In the election the voter in district i elects candidate i^P with probability,

$$(2.2) \quad \pi_i^P = \frac{\alpha_i^P r_i^P}{\alpha_i^P r_i^P + (1 - \alpha_i^P) r_i^Q}.$$

Payoffs are distributed accordingly and the game ends.

All the candidates are office-seeking. A candidate receives w , if he wins and nothing otherwise. Party P receives a payoff, U_P for any $P \in \{A, B\}$,

$$U_P = n_p \Delta,$$

where n_p is the number of seats P 's candidates win. Candidate i^P for any $i \in \{1, 2, 3\}$ and any P , receives

$$U_i^P = \begin{cases} w - C(e_i^P) \cdot e_i^P & \text{if } i^P \text{ wins} \\ -C(e_i^P) \cdot e_i^P & \text{otherwise,} \end{cases}$$

where $C(e_i^P)$ is the marginal cost of effort for i^P with

$$C(e_i^P) = \begin{cases} 1 & \text{if } i^P \text{ is the incumbent in district 1 and 3} \\ \frac{1}{\lambda} & \text{if } i^P \text{ is a freshman candidate.} \end{cases}$$

¹Snyder (1989) assumes that resources allocated to a district, r_i^P produce a campaign of size, $(r_i^P)^b$ with $b \leq 1$. In this paper, it is assumed that $b = 1$.

Note that electoral strength, α_i^P also quantifies how competitive the race in district i is, at least initially. When the candidates in a district exert similar effort, the district becomes marginal with $\alpha_i^P \simeq \alpha_i^Q \simeq 0.5$. As α_i^P moves away from $\frac{1}{2}$, the race becomes lop-sided. π_i^P in (2.2) suggests that, at the two extreme values of $\alpha_i^P = 0$ or 1 , one candidate wins for sure as the seat is essentially not contested.

To further clarify the timing of effort investment, it is assumed that the incumbent in district 1 exerts effort first and that the remaining candidates choose effort simultaneously. The timing of the game is summarised:

- 1) The incumbents in district 1 and 3 exert effort, e_1^B and e_3^A
- 2) The remaining candidates exert e_1^A , (e_2^A, e_2^B) and e_3^B
- 3) Electoral strength, $\alpha_i^P = \frac{e_i^P}{e_i^P + e_i^Q}$ for each candidate is evaluated
- 4) Parties allocate resources, (r_1^P, r_2^P, r_3^P) and campaign takes place
- 5) In the election, a candidate is elected with probability, $\pi_i^P = \frac{\alpha_i^P r_i^P}{\alpha_i^P r_i^P + (1 - \alpha_i^P) r_i^Q}$
- 6) Payoffs are distributed according to the election outcome.

4. Equilibrium of the Game

The notion of equilibrium is a subgame perfect equilibrium of the two-stage game illustrated in the previous section. As in Snyder (1989), the analysis focuses on and looks for pure-strategy equilibria are analysed, and assume that resource allocations are nonnegative, i.e. $r_i^P \geq 0$ for any $i \in \{1, 2, 3\}$ and $P \in \{A, B\}$. Each party's allocation of resources, $r^P = (r_1^P, r_2^P, r_3^P)$ is optimal, given the strategies of the other party and the strategies of the candidates, $e = (e^A, e^B)$, where $e^P = (e_1^P, e_2^P, e_3^P)$. Formally,

DEFINITION .1. Given $e = (e^A, e^B)$, where $r^P = (r_1^P, r_2^P, r_3^P)$, a strategy profile of the parties $r = (r^A, r^B)$ is optimal if $U^P(e, r) > U^P(e, r^{P'}, r^Q)$ where $r^P \neq r^{P'}$ for any $P \neq Q \in \{A, B\}$.

The candidates' effort e is optimal in a subgame perfect equilibrium given the optimal strategies of the parties, denoted by r^* :

DEFINITION .2. The subgame perfect effort of the candidates, $e^* = (e^{A^*}, e^{B^*})$ is consistent with r^* and satisfies $U_i^P(e^*, r^*) > U_i^P(e_i^P, e_{-i}^{P^*}, e_{-i}^{Q^*}, r^*)$ for any $e_i^{P^*} \neq e_i^P$, $i \in \{1, 2, 3\}$ and $P \neq Q \in \{A, B\}$.

The subgame perfect equilibrium is solved using backward induction. The equilibrium is unique in pure strategies. The analysis begins with the optimal allocation of the parties.

4.1. Resource Allocation. The parties simultaneously allocate resources across the candidates, (r_1^P, r_2^P, r_3^P) . Party $P \neq Q \in \{A, B\}$ solves the following optimisation problem to determine the size of resources allocated to each district,

$$\max_{r_i^P} \Delta \sum_{i=1}^3 \frac{\alpha_i^P r_i^P}{\alpha_i^P r_i^P + (1 - \alpha_i^P) r_i^Q}$$

subject to

$$\begin{cases} \sum_{i=1}^3 r_i^A \leq 1 \\ \sum_{i=1}^3 r_i^B \leq \tau, \end{cases}$$

where α_i^P is the electoral strength for candidate i^P , nominated by P in district i , and T^P is the size of P 's campaign budget ($T^A = 1$ and $T^B = \tau \geq 1$).

First-order conditions of the maximisation problem above imply that

$$(2.3) \quad r_i^{B^*} = \tau r_i^{A^*}$$

for any i . In equilibrium, the relative amount of resources allocated by the parties in each district corresponds to the relative size of their campaign budget. Substituting this back

into the first-order conditions yields the equilibrium allocation.²

PROPOSITION .1. *In subgame perfect equilibrium³, the symmetric ($\tau = 1$) parties spend more in a relatively more marginal district with $\alpha_i^P \approx \frac{1}{2}$, where the candidate have performed similarly in the first stage. When they are asymmetric ($\tau > 1$), each party allocates more resources to a district, where its candidate's first-stage performance is closer to the threshold, $\bar{\alpha}_{\tau>1}^P$, where*

$$\begin{cases} \bar{\alpha}_{\tau>1}^A = \frac{\tau}{\tau+1} > \frac{1}{2} \\ \bar{\alpha}_{\tau>1}^B = \frac{1}{\tau+1} < \frac{1}{2}. \end{cases}$$

The better-resourced party, B allocates more resources to a district which is relatively poorly-performing, whereas the opposite is true for the less-resourced, A.

In equilibrium, any party spends most of its resources in a district, in which the electoral strength of its candidate, α , is closer to some threshold level: denote such α by $\bar{\alpha}^P$. When the parties are equally endowed ($\tau = 1$) they spend more in a district, where the candidates have performed similarly, i.e., a relatively marginal district. Note that when $\tau = 1$, the amount of resources allocated to any district is also symmetric across the parties with $r_i^* = r_i^{A*} = r_i^{B*}$ for any i .

When the parties are asymmetric ($\tau > 1$), the better-resourced party, B , allocates more resources in a relatively *weaker* district, in which its candidate has not performed well during the previous stage. Specifically, it is a race with α_i^B closer to $\frac{1}{\tau+1} < 0.5$. On the contrary, the less-resourced party, A , spends more in a relatively *safer* district in which its

²All proofs are found in the Appendix.

³In subgame perfect equilibrium, the parties allocate

$$\begin{cases} r_i^{A*} = \frac{\alpha_i^A(1-\alpha_i^A)}{(\alpha_i^A + (1-\alpha_i^A)\tau)^2}, \\ r_i^{B*} = \tau r_i^{A*} \end{cases}$$

in district i , for any $i \in \{1, 2, 3\}$.

candidate has performed relatively well with α_i^A closer to $\frac{\tau}{\tau+1} > \frac{1}{2}$.

Why do the parties alter their target districts depending on the relative size of their campaign budget? To answer this, examine the marginal effect of party resources on a candidate's winning probability, by looking at

$$\frac{\partial \pi_i^P}{\partial r_i^P} = \frac{\alpha_i^P (1 - \alpha_i^P) r_i^Q}{(\alpha_i^P r_i^P + (1 - \alpha_i^P) r_i^Q)^2}.$$

First note that, in equilibrium, $\frac{\partial \pi_i^P}{\partial r_i^P}$ is maximised at $\bar{\alpha}_i^P$ where

$$\bar{\alpha}_i^P = \frac{1}{2}$$

for any $P \in \{A, B\}$ if $\tau = 1$ and

$$\begin{cases} \bar{\alpha}_i^A = \frac{\tau}{\tau+1} \\ \bar{\alpha}_i^B = \frac{1}{\tau+1} \end{cases}$$

if $\tau > 1$. Specifically, the parties in equilibrium allocate greater resources to the race where a larger campaign would improve their candidate's electoral prospects the most. When the candidates have exerted similar effort, the representative voter finds it difficult to distinguish between them ($\bar{\alpha}_i^P \approx \frac{1}{2}$). Combined with (2.3), the symmetric parties ($\tau = 1$) have a strong incentive to differentiate and run a larger campaign in such a district. The smallest deviation in a marginal district costs a party more in terms of the candidate's probability of winning than in a safe or weaker race.

When $\tau > 1$, (2.3) implies that B always outspends A in any district. A 's candidate who has performed poorly in the first stage cannot overturn the race during the campaign, as he is forced to run a smaller campaign than his rival. Conversely, strong candidates from the party may have a better chance of competing, if given additional resources. Similarly, B forwards more resources to weaker candidates than to those in safe or marginal races who expect to win with relative ease.

PROPOSITION .2. *Equilibrium strategies of parties are unique in pure strategies.*

One could well question whether the parties may find it profitable to spend all of their resources in one district. The winning probability of the candidates in (2.2) suggests that if a party allocates all of its resources to one district, it loses in the other districts for sure for any α_i^P , as long as the other party allocates any positive amount to all three districts. A deviation from the equilibrium strategies, including spending all the resources available in one district, is not optimal. This is primarily because an additional unit of resources allocated to a district does not improve the probability of winning in the district to the same extent. $\frac{\partial \pi_i^P}{\partial r_i^P}$ is always smaller than 1. Especially for a symmetric party or the less-resourced party, allocating all of its resources to a district is never profitable and does not even guarantee an election in the district. Whereas the strategy improves the probability of winning to some extent, it costs the party a seat in the district it abandons.

4.1.1. Fixed-Budgeted Parties versus Costly Campaign. Whereas Snyder (1989) assumes that providing campaign resources is costly for the parties, the parties are endowed with a fixed amount of campaign budget in the framework. Snyder (1989) has asserted that his results would remain robust for budget-constrained parties. Proposition 1 confirms this. When the parties are symmetric (asymmetric) in their budget, the results are comparable to when the marginal cost of campaign is symmetric (asymmetric) across the parties. The better-resourced, in the model, corresponds to the party with the lower marginal cost in Snyder (1989).

Both in Snyder and in Proposition 1, when the parties are symmetric either in their campaign budget or in their marginal cost of campaign, they spend an equal amount in each district. When they are asymmetric, the party, either with the larger budget or with the lower marginal cost, outspends its rival in every district. Snyder also shows that the parties would spend more in a district than the other(s). **Comment 3.1** and **3.2** in Snyder (1989) are rewritten with the notations of the model:

(in a unique pure-strategy Nash equilibrium) $r_i^{P^*} > r_j^{P^*}$ if and only if
 $|\pi_i^{P^*} - \frac{1}{2}| < |\pi_j^{P^*} - \frac{1}{2}|$,

where $\pi_i^{P^*} = \alpha_i^P$ for a symmetric party and $\pi_i^{P^*} = \frac{\alpha_i^P}{\alpha_i^P + (1 - \alpha_i^P) \frac{c_P}{c_Q}}$ for an asymmetric party. c_P and c_Q indicate party P and Q 's marginal cost of campaign.

If it is assumed that $c_A = 1$ and $c_B = \frac{1}{\tau}$ with $\tau \geq 1$, the equilibrium probability of winning above becomes identical to (2.4) below. Substituting $\pi_i^{P^*}$ into the condition in Snyder's Comments readdresses the results of this model:

- a symmetric party allocates more resources to a relatively more marginal race with $\alpha_i^P \rightarrow \frac{1}{2}$;
- the less efficient party with the higher marginal cost of campaign allocates more resources to a relatively stronger candidate with $\alpha_i^A \rightarrow \frac{\tau}{1+\tau}$;
- the more efficient party allocates more to a relatively weaker candidate with $\alpha_i^B \rightarrow \frac{1}{1+\tau}$,

given $\alpha_i^B = 1 - \alpha_i^A$, $\pi_i^{B^*} = 1 - \pi_i^{A^*}$ and $r_i^{B^*} = \tau r_i^{A^*}$ for any i . The equilibrium strategies of the candidates also remain robust if it is assumed that the parties run a costly campaign.

4.2. Candidate Effort in Equilibrium. Substituting the equilibrium allocation condition, (2.3) into (2.2), the equilibrium probability of election for a candidate is obtained,

$$(2.4) \quad \pi_i^{P^*} = \frac{T^P \alpha_i^P}{T^P \alpha_i^P + T^Q (1 - \alpha_i^P)}$$

where again T^P is the size of P 's campaign budget with $T^A = 1$ and $T^B = \tau \geq 1$.

Recall the timing of the game. During the first stage of the game, the incumbent in district 1 is the first to exert effort, $e_1^B \in [0, 1]$. The challenger, after observing e_1^B , exerts effort, $e_1^A \in [0, 1]$. With (2.4), the candidates in district 1 solve the following maximisation problems

to determine the optimal effort, $e_1^{P^*}$:

$$\begin{aligned} e_1^{A^*}(e_1^B) &\in \arg \max_{e_1^A} w \frac{\alpha_1^A}{\alpha_1^A + (1 - \alpha_1^A)\tau} - \frac{e_1^A}{\lambda} \\ e_1^{B^*} &\in \arg \max_{e_1^B} w \frac{(1 - \alpha_1^{A^*}(e_1^B))\tau}{\alpha_1^{A^*}(e_1^B) + (1 - \alpha_1^{A^*}(e_1^B))\tau} - e_1^B \end{aligned}$$

where $\alpha_1^A = \frac{e_1^A}{e_1^A + e_1^B}$ is the challenger's electoral strength. Solving the challenger's problem on the top, $e_1^{A^*}(e_1^B)$ is derived as a function of the incumbent's effort, e_1^B . $\alpha_1^{A^*}(e_1^B) = \frac{e_1^{A^*}(e_1^B)}{e_1^{A^*}(e_1^B) + e_1^B}$ in the incumbent's problem on the bottom, is the challenger's strength as a function of e_1^B .

In district 2, the candidates simultaneously solve the following problem to decide their optimal level of effort, $e_2^{P^*}$. For any $P \neq Q \in \{A, B\}$

$$e_2^{P^*} \in \arg \max_{e_2^P} w \frac{T^P \alpha_2^P}{T^P \alpha_2^P + T^Q (1 - \alpha_2^P)} - \frac{e_2^P}{\lambda}$$

where $\alpha_2^P = \frac{e_2^P}{e_2^A + e_2^B}$ is the electoral strength of P 's candidate given (e_2^A, e_2^B) .

Finally in district 3, the candidates decide their effort in the sequence, identical to district 1. The incumbent affiliated with party A first exerts effort, $e_3^A \in [0, 1]$. Then, the challenger from party B chooses $e_3^B \in [0, 1]$. The candidates in district 3 choose the optimal effort, $e_3^{P^*}$, which is the solution to

$$\begin{aligned} e_3^{A^*} &\in \arg \max_{e_3^A} w \frac{1 - \alpha_3^{B^*}(e_3^A)}{1 - \alpha_3^{B^*}(e_3^A) + \alpha_3^{B^*}(e_3^A)\tau} - e_3^A \\ e_3^{B^*}(e_3^A) &\in \arg \max_{e_3^B} w \frac{\alpha_3^B \tau}{1 - \alpha_3^B + \alpha_3^B \tau} - \frac{e_3^B}{\lambda} \end{aligned}$$

where $\alpha_3^B = \frac{e_3^B}{e_3^A + e_3^B}$ is strength of the challenger. Solution to the challenger's problem, $e_3^{B^*}(e_3^A)$ is a function of the incumbent's effort, e_3^A . The challenger's strength is rewritten as a function of e_3^A , i.e. $\alpha_3^{B^*}(e_3^A) = \frac{e_3^{B^*}(e_3^A)}{e_3^A + e_3^{B^*}(e_3^A)}$ and used to solve the incumbent's problem. The optimisation problems of the candidates in district 3 are similar to those in district 1.

PROPOSITION .3. *In equilibrium, the challenger and the incumbent in district 1 exert*

$$\begin{cases} e_1^{A^*} = \frac{w\tau}{2} \left(1 - \frac{\tau}{2\lambda}\right) \\ e_1^{B^*} = \frac{w\tau}{4\lambda}. \end{cases}$$

The challenger finds it optimal to exert no effort at all whenever $1 < \frac{\tau}{2\lambda}$. The optimal effort of an open-seat candidate in district 2 is

$$e_2^{P^*} = w\tau \frac{\lambda}{(1+\tau)^2}.$$

for any $P \in \{A, B\}$. The candidates in district 3 exerts

$$\begin{cases} e_3^{A^*} = \frac{w}{4\lambda\tau} \\ e_3^{B^*} = \frac{w}{2\tau} \left(1 - \frac{1}{2\lambda\tau}\right). \end{cases}$$

The challenger in district 3 exerts zero effort if and only if $1 < \frac{1}{2\lambda\tau}$.

Let's begin the analysis with the equilibrium behaviour of the candidates in district 1. Whenever the incumbent has exerted positive effort ($e_1^B > 0$), if $e_1^A = 0$, the challenger in district 1 is perceived as completely 'incompetent' with $\alpha_1^A = \frac{0}{0+e_1^B}$. Proposition 1 implies that the challenging party, A , allocates him $r_1^A = 0$. He loses the election for sure. For any $\tau \geq 1$, if the challenger needs to bear a very high cost of effort with $1 < \frac{\tau}{2\lambda}$, the benefit of election (w) may not be sufficient to induce him to exert positive effort. Note that if A is extremely under-resourced with $\tau > 2$, the challenger could also be incentivised to exert zero effort even if he has a substantially large amount of capital ($\lambda \rightarrow 1$). The following analysis explicitly focuses on the case in which $2\lambda > \tau$ and the challenger exerts a positive level of effort in equilibrium.

$e_1^{B^*}$ in Proposition 3 suggests that the incumbent always invests positive effort. Sequential effort provides the incumbent with a 'first-mover' advantage. The incumbent preempts the challenger in the sequential game of effort. The challenger in equilibrium exerts lower effort than when they decide effort simultaneously. When the incumbent exerts effort

greater than \bar{e}_1^B , the challenger's effort decreases. Given $\tau \geq 1$ and $\lambda \in (0, 1)$,

$$(2.5) \quad \frac{\partial e_1^{A^*}(e_1^B)}{\partial e_1^B} < 0 \Leftrightarrow e_1^B > \frac{w\lambda}{4\tau} \equiv \bar{e}_1^B.$$

The condition above also suggests that for any $\tau \geq 1$, whenever $e_1^B < \bar{e}_1^B$, the challenger responds to an increase in e_1^B with higher effort if and only if his marginal cost of effort is sufficiently low (λ high).

If the incumbent and the challenger in district 1 exert effort, $(\hat{e}_1^A, \hat{e}_1^B)$ simultaneously, their equilibrium effort becomes

$$(2.6) \quad (\hat{e}_1^{A^*}, \hat{e}_1^{B^*}) = (w\tau \frac{(\lambda)^2}{(\lambda + \tau)^2}, w\tau \frac{\lambda}{(\lambda + \tau)^2}).$$

Comparing $\hat{e}_1^{B^*}$ to $e_1^{B^*}$ in Proposition 3, the incumbent exerts higher effort under sequential investment. Under simultaneous investment, the incumbent cannot 'credibly threaten' the challenger. The candidates under simultaneous investment choose effort, unaware of the opponent's choice. The incumbent cannot affect and incentivise the challenger to act in a certain way. His choice of effort rests purely on the relative advantage he holds over the challenger in terms of the marginal cost of effort and the campaign budget of the parties. This results in a smaller efforts than under sequential investment. The challenger with λ_1^A always exerts lower effort under sequential investment than he would under simultaneous investment, i.e. $e_1^{A^*} < \hat{e}_1^{A^*}$ for any $\tau \geq 1$ for a similar reason.

The 'preemptive' behaviour of the sequentially-moving incumbent in district 1 is also reflected in the equilibrium level of effort in the district. Proposition 3 also implies that $e_1^{B^*} > e_1^{A^*}$ for any $\tau \geq 1$ and $\lambda < 1$. The incumbent in district 1 always exerts a higher level of effort in equilibrium. The same is true for the simultaneously-moving incumbent with $\hat{e}_1^{B^*} > \hat{e}_1^{A^*}$ for any τ and $\lambda < 1$. It is also noted that

$$\frac{e_1^{B^*}}{e_1^{A^*}} = \frac{1}{2\lambda - \tau} > \frac{\hat{e}_1^{B^*}}{\hat{e}_1^{A^*}} = \lambda.$$

The relative size of the candidate effort in district 1 is greater under sequential investment than under simultaneous investment. It is indicated that the first-mover position under sequential investment further motivates the incumbent who is more efficient in effort with the lower marginal cost to exert higher effort.

The incumbent in district 3, affiliated with A , is less-resourced than the challenger. Whenever $2\lambda > \tau$, $2\lambda > \frac{1}{\tau}$. This implies that whenever the challenger in district 1 exerts positive effort, so does the challenger in district 3. As in district 1, the incumbent effort in district 3, e_3^{A*} is always positive for any τ and λ , but is always smaller than e_1^{B*} . A similar condition to (2.5) is also derived

$$(2.7) \quad \frac{\partial e_3^{B*}(e_3^A)}{\partial e_1^B} < 0 \Leftrightarrow e_3^A > \frac{w\tau\lambda}{4} \equiv \bar{e}_3^A.$$

Whenever the incumbent exerts effort greater than \bar{e}_3^A , an increase in incumbent effort discourages the challenger, whose effort then decreases. For $\tau > 1$, the incumbent in district 3 needs to exert much greater effort to discourage and preempt the challenger than the incumbent in district 1, i.e. $\bar{e}_3^A > \bar{e}_1^B$. It is also noted that the incumbent may exert lower effort than the challenger if $\frac{1}{\lambda} < \frac{\tau}{2(\tau+1)}$. It is when the challenger's party is substantially better-resourced ($\tau \uparrow$), or when the challenger is sufficiently efficient in effort ($\lambda \downarrow$). When comparing the equilibrium candidate effort under sequential and simultaneous investment,

$$\frac{e_3^{A*}}{e_3^{B*}} = \frac{\tau}{2\lambda\tau - 1} \geq \frac{\bar{e}_3^A}{\bar{e}_3^B} = \frac{1}{\lambda},$$

if and only if $\frac{1}{\lambda} \geq \tau$. It implies that the less-resourced incumbent under sequential investment tries to preempt the challenger by exerting greater effort than under simultaneous investment, only when the challenger's advantage in campaign, represented by τ , is overshadowed by his inefficiency in effort, represented by $\frac{1}{\lambda}$.

Proposition 3 indicates that when $\lambda = 1 = \tau$, the incumbent and the challenger in the two reelection races exert the same level of effort in equilibrium, i.e. $e_j^{A*} = e_j^{B*}$ for any $j \in \{1, 3\}$. The incumbents cannot preempt the challenger by exerting higher effort. The challenger will match whatever effort the incumbent exerts. When $\tau = 1$, the equilibrium probability of winning for any candidate depends on his performance during the first stage

of election, $\frac{e_i^P}{e_i^P + e_i^Q}$. The contest function implies that whenever both candidates in a district exert the same effort whether they decide sequentially or simultaneously, their electoral strength will be identical at $\frac{1}{2}$. The challenger may be discouraged and even unable to match against the incumbent when the incumbent has exerted sufficiently high effort and the challenger is inefficient in effort. On the other hand, if the challenger does not match the incumbent when he is capable, it would only deteriorate his electoral prospects. Specifically it is noted that

$$\frac{\partial e_1^{A*}(e_1^B)}{\partial e_1^B} = \frac{\partial}{\partial e_1^B} \sqrt{w\lambda e_1^B} - e_1^B = \frac{1}{2} \sqrt{\frac{w\lambda}{e_1^B}} - 1.$$

At $e_1^B = e_1^{B*}$, the partial derivative is zero if $\lambda = 1$ and negative for any $\lambda < 1$. It shows that the incumbent cannot effectively preempt the challenger when $\lambda = 1 = \tau$. Proposition 3 and (2.6) show that when $\lambda = 1 = \tau$, the equilibrium effort for the candidates in the reelection races is the same under both sequential and simultaneous investment, implying that no first-mover advantage whenever there is no relative advantage in campaign and effort across the candidates. Furthermore, (2.5) suggests that when $\tau > 1$ but $\lambda < 1$, $\frac{\partial e_1^{A*}(e_1^B)}{\partial e_1^B} < 0$ at $e_1^B = e_1^{B*}$. A similar result is derived for the incumbent who is more efficient in effort, but not in campaign. (2.7) indicates that when $\tau = 1$ but $\lambda < 1$, $\frac{\partial e_3^{B*}(e_3^A)}{\partial e_3^A} < 0$ at $e_3^A = e_3^{A*}$, whereas when $\tau > 1$ and $\lambda = 1$, $\frac{\partial e_3^{B*}(e_3^A)}{\partial e_3^A} > 0$ if at $e_3^A = e_3^{A*}$. In the model, the first-mover advantage in the reelection races of the model is an outcome of the relative advantages that the incumbent holds over the challenger. The first-mover position itself does not give the incumbent an advantage.

Proposition 3 shows that the open-seat candidates in district 2 exert the same level of effort. In district 2, the open-seat candidates move simultaneously. They cannot observe their opponent's, effort but exert effort taking the rival's action as given. Their equilibrium effort, e_2^{P*} in Proposition 3 takes a functional form similar to (2.6), the equilibrium effort of the candidates in district 1 when they decide simultaneously. The earlier discussion shows that the challenger adjusts his effort, responding to the incumbent's choice of effort under sequential investment. A similar pattern of behaviour is not observed in the open-seat race, in which the candidates decide simultaneously and make a decision with the relative

(dis)advantage in campaign they hold over the opponent internalised. Provided this, the two open-seat candidates who have the same marginal cost of effort exert the same level of effort in equilibrium.

It is found that the effect of the party-level asymmetry, indicated by τ , on the equilibrium effort varies across the candidates. It depends on the type of the candidate and his opponent. Differentiating the equilibrium effort of the incumbent and the challenger in Proposition 3 with respect to τ derives

$$\begin{cases} \frac{\partial e_1^{A*}}{\partial \tau} = \frac{w}{2}(1 - \frac{\tau}{\lambda}) < 0 \\ \frac{\partial e_1^{B*}}{\partial \tau} = \frac{w}{4\lambda} > 0. \end{cases}$$

The incumbent exerts higher effort when his party is better-resourced ($\tau > 1$) than when the parties are equally resourced. The opposite is true for the challenger. The condition in (2.5) also suggests that the preemptive effect of the incumbent's effort intensifies as the asymmetry in the campaign budget widens (τ high). As the campaign advantage, reflected in τ , becomes greater, the incumbent can induce the challenger to exert lower or almost zero effort even by slightly increasing his own effort, e_1^B .

The less-resourced incumbent in district 3 also exerts smaller effort when $\tau > 1$ than $\tau = 1$, as

$$\frac{\partial e_3^{A*}}{\partial \tau} = -\frac{w}{4\lambda\tau^2} < 0.$$

However, the better-resourced challenger does not always react to the asymmetry in party resources ($\tau > 1$ with higher effort, unlike the incumbent in district 1, i.e.

$$\frac{\partial e_3^{B*}}{\partial \tau} = \frac{w}{2\tau^2}(\frac{1}{\lambda\tau} - 1) \gtrless 0,$$

if and only if $\frac{1}{\lambda} \gtrless \tau$. The asymmetry in party resources may motivate the challenger who is sufficiently inefficient in effort to exert higher effort than when $\tau = 1$. It is also noted that whether the challenger exerts higher effort in response to a rise in τ or not,

$$\frac{\partial}{\partial \tau} \frac{e_3^{A*}}{e_3^{B*}} = \frac{\partial}{\partial \tau} \frac{\tau}{2\lambda\tau - 1} = -\frac{1}{(2\lambda\tau - 1)^2} < 0.$$

When τ becomes higher, the relative effort exerted by the incumbent and the challenger in district 3 becomes smaller, as the less-resourced incumbent is affected by the rise to a greater extent than the challenger.

Proposition 3 also implies

$$\frac{\partial e_2^{P^*}}{\partial \tau} \leq 0 \Leftrightarrow 1 \leq \tau$$

for any $P \in \{A, B\}$. A change in τ affects the open-seat candidates symmetrically. Whenever their relative marginal cost of effort, $\frac{\lambda}{\lambda} = 1$ is smaller than τ , an increase in τ has a decreasing effect on the effort of the open-seat candidates, which is always the case in the model assuming $\tau \geq 1$. The open-seat candidates have the same marginal cost of effort. At the personal level, they are not disadvantaged, nor advantaged, over their opponent. When A 's candidate expects that the relative disadvantage in campaign, reflected in τ , increases, his effort is discouraged, which, in turn, incentivises his opponent to match the change in effort.

When the marginal cost of effort differs across the open-seat candidates, such as the candidate of A has $\lambda^A \geq \lambda^B$, where λ^B is the marginal cost of B 's open-seat candidate, then their equilibrium effort becomes $w\tau \frac{(\lambda^P)^2 \lambda^Q}{(\lambda^A + \tau \lambda^B)^2}$, for any $P \neq Q \in \{A, B\}$, which increases in τ whenever $\frac{\lambda^A}{\lambda^B} > \tau$. The intuition for this result is as follows. When the open-seat candidate of A has the relative advantage over his opponent, that effort is less costly for him, he would try to offset or outweigh the campaign advantage of the opponent, by exerting greater effort, which leads that his opponent also increases the effort.

Further implications can be made on the effect of the asymmetry in campaign budget on the candidates in the reelection races, when they exert effort simultaneously. Differentiating the equilibrium effort in (2.6) with respect to τ derives a condition similar to that from the open-seat race above:

$$\frac{\partial \hat{e}_1^{P^*}}{\partial \tau} < 0 \Leftrightarrow \lambda < \tau.$$

Given $\tau \geq 1$ and $\lambda \in (0, 1)$, $\lambda < \tau$. Hence, whenever there is an increase in τ , the candidates in the reelection race also exert smaller effort, if they invest in effort simultaneously.

Their motivation becomes similar to that of the open-seat candidates above.

Wiseman (2006) considers a sequential game of electoral competition. The incumbent party announces a policy and provides campaign support for its incumbent. After observing the incumbent party's move, the challenger decides whether to run, and if he runs, he announces a policy. It is shown that the incumbent party can effectively preempt the challenger, deterring the entry with campaign or a combination of campaign and policy. The results above provide similar implications and identify the sources of an incumbent advantage. When the parties are symmetric in campaign budget, the sequential nature of the game, combined with the incumbent's relative advantage in effort (lower marginal cost of effort), helps the incumbent secure a safe position from the early stage of the race. In district 1, the preemptive pressure on the less-resourced challenger increases with τ . As the asymmetry in campaign budget widens, the incumbent more easily preempts the challenger. The opposite is true for the less-resourced incumbent in district 3. The incumbent's ability to preempt the challenger deteriorates as τ rises and may even disappear.

Using Proposition 3, the candidates' electoral strength in equilibrium is derived. Each of the candidates in district 1 is perceived with,

$$(2.8) \quad \begin{cases} \alpha_1^{A^*} = \frac{2\lambda - \tau}{2\lambda - \tau + 1} \\ \alpha_1^{B^*} = \frac{1}{2\lambda - \tau + 1}. \end{cases}$$

(2.8) shows that $\alpha_1^{A^*} < \frac{1}{2} < \alpha_1^{B^*}$ for any $\lambda < 1$ and $\tau \geq 1$. The first-stage performance of the candidates in district 3 is,

$$(2.9) \quad \begin{cases} \alpha_3^{A^*} = \frac{1}{2\lambda - \frac{1}{\tau} + 1} \\ \alpha_3^{B^*} = \frac{2\lambda - \frac{1}{\tau}}{2\lambda - \frac{1}{\tau} + 1}. \end{cases}$$

In (2.9), $\alpha_3^{A^*} \geq \alpha_3^{B^*}$ if and only if $\frac{1}{\lambda} \geq 2\frac{\tau}{1+\tau}$. It is when the challenger is substantially inefficient in effort ($\frac{1}{\lambda}$) to an extent that his relative advantage in campaign ($\frac{\tau}{1+\tau}$) is overshadowed. The equilibrium electoral strength for each of the open-seat candidates is

$$(2.10) \quad \alpha_2^{P^*} = \frac{1}{2},$$

for any $P \in \{A, B\}$. A number of differences in electoral strength between the candidates exist. (2.8)–(2.10) show that the first-stage performance of the candidates in the reelection race is affected by τ , the relative size of the parties' campaign budget. This is not the case for the open-seat candidates. In fact, when it is assumed that their marginal cost differs with $\lambda^A \geq \lambda^B$, an open-seat candidate's electoral strength corresponds to his relative cost of effort, $\frac{\lambda^P}{\lambda^P + \lambda^Q}$ for any $P \neq Q \in \{A, B\}$. The candidate with the lower marginal cost will be perceived with the greater α , i.e. when $\lambda^P > \lambda^Q$, candidate P is perceived with $\frac{\lambda^P}{\lambda^P + \lambda^Q} > \frac{\lambda^Q}{\lambda^P + \lambda^Q}$, better than his rival, candidate Q .

It has been noted that the relative disadvantage of the challenger becomes greater, when τ increases, and when the candidates in the reelection race choose effort sequentially. It is thus implied that

$$\frac{\lambda}{\lambda + 1} > \frac{2\lambda - 1}{2\lambda} > \frac{2\lambda - \tau}{2\lambda - \tau + 1} \equiv \alpha_1^{A^*},$$

for any $\lambda \in (0, 1)$ and $\tau > 1$. In the inequality above, $\frac{\lambda}{\lambda + 1}$ indicates the challenger's equilibrium strength under simultaneous investment. As for an open-seat candidate, his strength depends only on the marginal cost of effort for the incumbent and himself. $\frac{2\lambda - 1}{2\lambda}$ is his strength under sequential investment but under the symmetric parties ($\tau = 1$). In equilibrium, the challenger is perceived much less competent than he would be under simultaneous investment and under the symmetric parties. Also note that the challenger is always perceived as less competent than the incumbent, implying that $\alpha_1^{A^*} < \frac{1}{2}$, for any λ and any $\tau \geq 1$.

The equilibrium strength of the challenger in district 3 is identical to that of the challenger in district 1, when the candidates in a reelection race simultaneously decide effort ($\frac{\lambda}{\lambda + 1}$); and when the parties are equally resourced with $\tau = 1$ ($\frac{2\lambda - 1}{2\lambda}$). Under sequential investment, the challenger is always perceived higher when $\tau > 1$ than when $\tau = 1$ with $\frac{2\lambda - \frac{1}{\tau}}{2\lambda - \frac{1}{\tau} + 1} > \frac{2\lambda - 1}{2\lambda}$. However, whether the challenger is better perceived under sequential or simultaneous investment when $\tau > 1$ depends on the relative disadvantage of the challenger, i.e.

$$\frac{2\lambda - \frac{1}{\tau}}{2\lambda - \frac{1}{\tau} + 1} \geq \frac{\lambda}{\lambda + 1} \Leftrightarrow \tau \geq \frac{1}{\lambda}.$$

It is noted earlier that if $\tau > 1$, sequential investment discourages the incumbent and forces him to fail to preempt the challenger with the greater effort. If the challenger is substantially efficient (λ high), the incumbent is further affected. The condition, $\tau \geq \frac{1}{\lambda}$, also reflects the earlier results. Whenever $\tau \geq \frac{1}{\lambda}$, the better-resourced who is sufficiently efficient in effort is better perceived under sequential investment than under simultaneous investment.

The relative performance of the open-seat candidates after the first stage is determined only by their relative advantage in effort. In the model, in which their marginal cost is symmetric, their equilibrium strength is also symmetric at $\frac{1}{2}$. If $\lambda^A > \lambda^B$, A 's candidate is considered more electable than his rival and $\alpha_2^{A*} > \frac{1}{2}$, and vice versa. The electoral strength of *any* candidate is by assumption a contest function of the effort exerted by his opponent and himself. The effect of τ on the open-seat candidates is cancelled out in their equilibrium strength. Proposition 3 shows that τ influences the equilibrium effort of the open-seat candidates symmetrically⁴.

4.3. ‘Target Race’ in Equilibrium. Proposition 1 shows that the relative size of campaign budget influences the parties. They adopt different campaign strategies when their relative advantage in campaign budget changes. The symmetric parties allocate more resources to a relatively more marginal race with $\alpha_i^P \approx \frac{1}{2}$. When $\tau > 1$, the better-resourced party allocates more resources to a candidate who is relatively weaker than his co-partisan in the other district. The opposite is true for the less-resourced party. A candidate whose first-stage performance is stronger than his co-partisan receives more. The parties are motivated to allocate more resources to a race in which the additional resources improve or protect the candidate’s electoral prospect more than elsewhere.

Recall the equilibrium electoral strength of the candidates in (2.8)–(2.10). The representative voter in district 1 always finds the incumbent more competent with $\alpha_1^{B*} > \frac{1}{2}$ for any

⁴The relative effect of the asymmetry in campaign budget across the open-seat candidates, $\frac{\frac{\partial \alpha_2^{P*}}{\partial \tau}}{\frac{\partial \alpha_2^{Q*}}{\partial \tau}}$, for any $P \neq Q \in \{A, B\}$ is independent of τ , only depending on λ^P and λ^Q , and is hence 1, whenever $\lambda_P = \lambda^Q$.

$\lambda \in (0, 1)$. Each open-seat candidates is perceived with $\alpha_2^{P^*} = \frac{1}{2}$, for any $P \in \{A, B\}$, implying a very marginal race. The electoral strength of the candidates in district 3 depends on the size of λ and τ . Combined with Proposition 1, it is predicted that the symmetric parties with $\tau = 1$ allocate more resources to their open-seat candidates than those in reelection races, and allocate an equal amount to each of the reelection races. To see this, let's consider for party A when $\tau = 1$ and rewrite $\alpha_1^{A^*}$ and $\alpha_3^{A^*}$ as

$$\alpha_1^{A^*} = \frac{2\lambda - 1}{2\lambda} < \frac{1}{2} < \frac{1}{2\lambda} = \alpha_3^{A^*}.$$

It also implies that

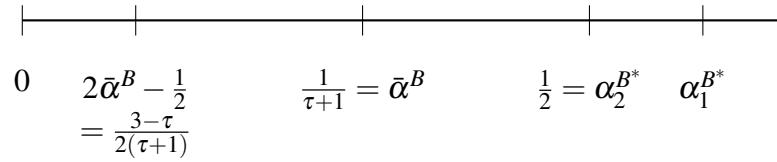
$$\frac{1}{2} - \alpha_1^{A^*} = \frac{1}{2} - \frac{2\lambda - 1}{2\lambda} = \frac{1 - \lambda}{2\lambda} = \frac{1}{2\lambda} - \frac{1}{2} = \alpha_3^{A^*} - \frac{1}{2}.$$

As the equilibrium strength of each candidate in the reelection races is equidistant from $\frac{1}{2}$, i.e. $\frac{1}{2} - \alpha_1^{A^*} = \alpha_3^{A^*} - \frac{1}{2}$, both parties will allocate the same amount, which is smaller than that to district 2, to each of the reelection races.

When $\tau > 1$, it is noted earlier that $\alpha_1^{B^*} > \frac{1}{2} = \alpha_2^{B^*}$ and $\alpha_1^{B^*} > \alpha_3^{B^*}$ for any λ . Proposition 1 shows that the better-resourced party, B , allocates more resources to a district with electoral strength closer to $\frac{1}{\tau+1}$. Firstly, $\alpha_1^{B^*} > \alpha_3^{B^*} > \frac{1}{2}$, if and only if

$$\frac{2\lambda - \frac{1}{\tau}}{2\lambda - \frac{1}{\tau} + 1} > \frac{1}{2} \Leftrightarrow \lambda > \frac{\tau + 1}{2\tau},$$

and then B allocates $\alpha_2^{B^*} > \alpha_3^{B^*} > \alpha_1^{B^*}$, as $\frac{1}{1+\tau} < \frac{1}{2} = \alpha_2^{B^*} < \alpha_3^{B^*} < \alpha_1^{B^*}$.



$2\bar{\alpha}^B - \frac{1}{2} = \frac{3-\tau}{2(\tau+1)}$ and $\frac{1}{2}$ are equidistant from the threshold, $\bar{\alpha}^B$. If $2\bar{\alpha}^B - \frac{1}{2} < \alpha_3^{B^*} < \frac{1}{2}$, B allocates more resources to district 3 than to district 2, and the least to district 1. So does

A. When $0 < \alpha_3^{B^*} < 2\bar{\alpha}^B - \frac{1}{2}$, i.e.

$$(2.11) \quad \begin{aligned} \frac{2\lambda - \frac{1}{\tau}}{2\lambda - \frac{1}{\tau} + 1} &< \frac{3 - \tau}{2(\tau + 1)} \\ \tau &< -(2\lambda - \frac{1}{\tau}) + 6(1 - \lambda\tau), \end{aligned}$$

the open-seat candidates receive the most resources from their parties than those in reelection races. The inequality above shows that as the challenger in district 3 is substantially inefficient in effort (λ low) or the asymmetry in party resources is not wide (τ low), the asymmetric parties become likely to support the open-seat candidates more. The result is summarised and formally stated in the following Proposition.

PROPOSITION 4. *When the parties are symmetric in their campaign resources with $\tau = 1$, they allocate more resources to the open-seat race in district 2 than any of the reelection races. When they are asymmetric with $\tau > 1$, they continue to do so as long as $\alpha_3^{B^*} < 2\bar{\alpha}^B - \frac{1}{2}$ ⁵, satisfying (2.11); otherwise they allocate the largest amount of resources to the reelection races in district 3.*

Proposition 4 provides theoretical explanations for the empirical data of campaign spending. Table 2.1 illustrates campaign contributions of the US Democratic and Republican party to individual candidates during the 2010 election cycle. During the electoral cycle, the Republican candidates received larger financial contributions than the Democratic counterparts. The data from the Federal Election Commission shows that the Republican candidates for the House of Representatives received USD 588,959,746 (in total) whereas their Democratic counterparts did USD 510,778,401. Using this as a proxy for the campaign budget of the parties, the threshold level of electoral strength $\bar{\alpha}^P$ for the Republican would be slightly lower than that for the Democrats. If the parameters of the model are applied, it is derived that $\tau \simeq 1.2$, with $\bar{\alpha}^R = 0.46$ for the Republicans and $\bar{\alpha}^D = 0.54$ for the Democrats. If the post-nomination poll of the candidates is used as a proxy for α in

⁵i.e. $\alpha_3^{A^*} > 2\bar{\alpha}^A - \frac{1}{2}$

the model, what Table 2.1 shows reflects what the model predicts.

Table 2.1. Party Committee Contribution¹

	Democratic		Republican	
	Poll (%) ¹	Receipts (\$) ²	Poll (%) ¹	Receipts (\$) ²
Open-Seat	42.6	3,664.0	54.7	4,093.1
Challenger	33.0	1,427.6	35.3	4,070.2
Incumbent	61.1	2,832.6	63.5	1,269.2

¹ Poll results taken after primaries or nomination (New York Times, <http://elections.nytimes.com/2010/results/house>)

² Contributions from party committees, (Federal Election Commission, <http://www.fec.gov>)

Both the Democrats and the Republicans made larger contributions to open-seat races on average. The average poll rating for the open-seat candidates of both parties approximately matches $\bar{\alpha}^D$ and $\bar{\alpha}^R$. The average poll rating for the Republican challengers was closer to the threshold than that of the Republican incumbents, whereas the opposite is true for the Democratic challengers and incumbents. Hence, the Republican challengers received a greater amount of contribution from their party committees than the Republican incumbents, and the opposite is true for the Democratic challengers and incumbents.

The next section extends the discussion to a number of related issues: incumbency advantage and campaign spending limit. The robustness of the model is also examined by considering an alternative functional form for the probability of election.

5. Discussion

5.1. Inc incumbency Advantage. Incumbents are more likely to be elected and often win with a large margin. 85% of the US House incumbents running for reelection in 2010 were successful. Incumbents may deter challenger entry (Gordon et al. 2007; Ashworth and Bueno de Mesquita 2008); receive enhanced media coverage or endorsement (Ansolabehere et al. 2006); and have better reputation (Ashworth 2005; Zaller 1998). In the literature, an incumbency *disadvantage* is also addressed. Incumbents who fail to

deliver distributive benefits (Serra and Moon 1994; Sellers 1997) or public goods (Uppal 2009) can be electorally punished, or held responsible following government failure (Uppal 2009).

The model assumes that the incumbent's marginal cost of effort is lower than the challenger's. In district 1, the incumbent's party, B , is at least as well-resourced as its rival. Sequential investment of effort further helps the incumbent lead the race from an early stage. Substituting α_1^{A*} and α_1^{B*} in Proposition 3 into the equilibrium probability of election in (2.4) gives

$$\pi_1^{B*} = \frac{\tau}{2\lambda}.$$

Given $\tau \geq 1$ and $\lambda \in (0, 1)$, the incumbent is always reelected with probability higher than $\frac{1}{2}$ in equilibrium. The assumptions of the model reflect a number of attributes identified in the existing literature on incumbency advantage. Incumbents tend to attract larger campaign contributions. They can forward official resources to their reelection campaign. They can also start making electoral appeals or promises earlier than their challengers. On the contrary, challengers usually get their candidacy later than incumbents, as the challenger in the model, who is a second-mover. However, this is not always the case in district 3, where the incumbent's party, A , may be less-resourced. The earlier results imply,

$$\pi_3^{A*} = \frac{1}{2\lambda\tau},$$

which is greater than $\frac{1}{2}$ if and only if $\frac{1}{\lambda} > \tau$. The incumbent in district 3 is reelected with probability higher than $\frac{1}{2}$ if the challenger's relative disadvantage in effort, $\frac{1}{\lambda}$, overshadows the relative advantage in campaign, τ . This happens whenever:

- the challenger's cost of effort, $\frac{1}{\lambda}$ is substantially high ($\lambda \uparrow$);
- the asymmetry in campaign budget is not very wide ($\tau \downarrow$);

or both. In either case, the effect of the campaign disadvantage on the incumbent is limited. With λ very small, thus $\frac{1}{\lambda}$ very large, the challenger is unable to exert any substantial effort with or without the relative advantage in campaign. Regardless of the campaign's scale, this does not compensate for his lack of effort in the first stage.

Note that whenever $\tau = \lambda = 1$, i.e. when there are no asymmetries across the parties and the candidates, both $\pi_1^{B^*}$ and $\pi_3^{A^*}$ become $\frac{1}{2}$ and the incumbent and the challenger in district 1 as well as in district 3 are equally likely to get elected. This implies no incumbency advantage, as well as no first-mover advantage which has been discussed earlier. When $\tau = 1$, the probability of winning for a candidate becomes equivalent to his electoral strength after the first stage, $\alpha_i^P = \frac{e_i^P}{e_i^P + e_i^Q}$ for any $i \in \{1, 2, 3\}$ and $P \in \{A, B\}$. Earlier, it has been discussed that whenever $\tau = \lambda = 1$, $e_i^P = e_i^Q$ in a reelection race, which leads to the result here.

In the previous section, some modification to the assumptions of the model such as simultaneous investment of effort in the reelection races been briefly considered. This section considers a modification to the marginal costs of incumbents and examines whether the incumbent would still enjoy an incumbency advantage, trying to identify the factors determining the incumbency advantage in the model.

Suppose that the incumbent's marginal cost of effort, denoted by ψ , is higher than the challenger's in district 1 and 3. In the model, it is assumed that $\psi = 1$. The incumbents are now relatively disadvantaged in effort or inefficient than in the Main model. Continue to assume that B remains at least as well-resourced as A with $\tau \geq 1$. In equilibrium, the incumbent in district 1 exerts effort $e_1^{B(\psi)^*}$, which is the solution to the following problem:

$$e_1^{B(\psi)^*} \in \arg \max_{e_1^B} w \frac{(1 - \alpha_1^A(e_1^B))\tau}{\alpha_1^A(e_1^B) + (1 - \alpha_1^A(e_1^B))\tau} - \psi e_1^B,$$

where $\alpha_1^A(e_1^B)$ is the challenger's electoral strength as a function of the incumbent's effort. The solution to this maximisation problem suggests that in equilibrium, the challenger is perceived with $\alpha_1^{A(\psi)^*} = \frac{2\psi\lambda - \tau}{2\psi\lambda - \tau + 1}$ after the first stage⁶. The inefficient incumbent is then reelected with probability

$$\pi_1^{B(\psi)^*} = \frac{\tau}{2\psi\lambda},$$

⁶The proof is in the Appendix

which is greater than $\frac{1}{2}$ if and only if

$$(2.12) \quad \frac{1}{\lambda} > \frac{\psi}{\tau}.$$

Similarly the incumbent in district 3 is reelected with probability

$$\pi_3^{A(\psi)*} = \frac{1}{2\psi\lambda\tau},$$

and is greater than $\frac{1}{2}$ if and only if

$$(2.13) \quad \frac{1}{\lambda\tau} > \psi.$$

(2.13) implies that the less-resourced incumbent is more likely to get elected than the challenger, when the challenger's relative disadvantage in effort, $\frac{1}{\lambda\tau}$, exceeds his relative advantage in campaign, τ . Note that whenever (2.13) holds, so does (2.12)

Result 1. *Suppose that $\psi > 1$ is the incumbents' marginal cost of effort. The incumbent in district 3 is reelected with probability higher than $\frac{1}{2}$ if and only if:*

$$\frac{1}{\lambda\tau} > \psi.$$

Whenever this takes place, the incumbent in district 1 is also reelected with probability higher than $\frac{1}{2}$.

It has been found that when the candidates in the reelection race decide their effort simultaneously, the electoral strength of the incumbent in district 1 after the first stage is $\hat{\alpha}_1^{B*} = \frac{1}{\lambda+1}$ in equilibrium. It implies that

$$\hat{\pi}_1^{B*} = \frac{\tau}{\lambda + \tau}.$$

Although $\hat{\pi}_1^{B*}$ is lower than $\pi_1^{B*} = \frac{\tau}{2\lambda}$, the incumbent still enjoys an incumbency advantage. It is possible to consider modifications on the relative advantage of the district-1 incumbent, in effort and in campaign, when the candidates in the reelection race move

simultaneously.

If $\tau < 1$, $\hat{\pi}_1^{B^*} > \frac{1}{2}$ if and only if $\frac{1}{\lambda} > \frac{1}{\tau}$. Similarly, when $\psi > 1$ and/or $\tau < 1$, the condition for an incumbency advantage remains the same as (2.12). Whether there is an incumbency advantage for the incumbent in district 1 depends on the relative advantage in effort he holds over the challenger, in relation to the relative disadvantage in campaign. This is also noted for the incumbent in district 3 above. A particularly interesting observation is, whenever these conditions do not hold and the incumbent is reelected with probability lower than $\frac{1}{2}$, simultaneous investment alleviates the damage to his electoral prospects. Specifically, $\hat{\pi}_1^{B(\psi)^*} > \pi_1^{B(\psi)^*}$ where $\psi \geq 1$ whenever

$$\frac{1}{\lambda} < \frac{\psi}{\tau}.$$

Whenever the incumbent is disadvantaged over the challenger, his position as the first mover further worsens his electoral prospects. Then, a ‘second-mover’ advantage emerges. The disadvantaged incumbent cannot credibly preempt the challenger. For example, even if he tries to discourage the challenger with a higher level of effort, it could easily be counteracted by the advantaged challenger.

Suppose that $\tau < 1$, but $\psi = 1$. From the previous results,

$$\frac{\partial e_1^{A^*}}{\partial e_1^B} = \frac{1}{2} \sqrt{\frac{w\tau\lambda}{e_1^B}} - \tau.$$

Let’s assume $w = 4$ for simplicity. The partial derivative is positive, $\frac{\partial e_1^{A^*}}{\partial e_1^B} > 0$ if $\frac{\lambda}{\tau} \geq e_1^B$. Substituting $e_1^{B^*}$ in Proposition 3, it shows that as long as $\lambda > \tau$, the district-1 challenger in equilibrium responds to an increase in e_1^B by increasing e_1^A . A similar conclusion is derived when $\psi > \frac{1}{\lambda_1^A}$ and/or $\tau < 1$.

It is then argued the sequential nature of the reelection race enhances an incumbency advantage. It alone is not a factor that generates the advantage. Instead, sequential investment further helps the district-1 incumbent with a relative advantage in effort and campaign secure a strong position from an early stage of the race. The ‘multiplying’

effect of sequential investment is hence present only when the incumbent is relatively advantaged at the individual or at the party level, or both. Otherwise, the multiplying effect works against the incumbent. The first-mover position further deteriorates his electoral prospects.

In the model, the first-stage performance of the *less-resourced* candidates is damaged during the campaign stage, as he is unable to run a campaign as large as their opponents. This is a *direct effect* of the asymmetry in campaign budget on candidate performance. In the reelection races, the candidates are also influenced by an *indirect effect* of the asymmetry. The modifications show that even a current office-holder may not be able to materialise or forge their personal advantage if his party is heavily disadvantaged over its rival. The open-seat candidates are, however, neither punished nor do they benefit from this indirect effect, as suggested by their first-stage performance, $\alpha_2^{P^*}$.

Some previous studies have pointed that an incumbent advantage is derived from *partisan* and *personal* attributes. Lee (2008) estimates that partisan attributes have a greater influence on an incumbent advantage in the US House election than personal attributes. In Oppenheimer (2005), a similar conclusion is drawn. The effect of partisan attributes becomes greater as geographic polarisation intensifies, with candidates tending to rely less on their “personal constituency”. Ansolabehere et al. (2000) find that candidates running in more partisan districts enjoy a smaller incumbency advantage.

In the model, $\tau > 1$ and λ respectively indicate the partisan and the personal attribute of an incumbency advantage in district 1. In addition, sequential investment works as a ‘multiplier’ of the advantage. The effect of these attributes, either alone or combined, on the incumbent’s electoral prospects, is examined. The incumbent’s equilibrium probability of reelection when each or a combination of these attributes is available in Table 2.2.

Whenever only one of the attributes, either $\tau > 1$ (but $\psi = \frac{1}{\lambda_1^A} = 1$) or $\psi = 1 < \frac{1}{\lambda_1^A}$ (but $\tau = 1$), there exists an incumbency advantage. The size of the incumbency advantage attributable to each of the factors depends on the relative size of the attributes, whether

Table 2.2. Equilibrium Probability of Reelection

Party Attribute (τ)	Personal Attribute (λ_1^A)	Sequential	π_1^{B*}
✓			$\frac{\tau}{\tau+1}$
	✓		$\frac{1}{1+\lambda_1^A}$
✓	✓	✓	$\frac{1}{\frac{2}{\tau} + \lambda_1^A}$
	✓	✓	$\frac{1}{2\lambda_1^A}$
✓		✓	$\frac{\tau}{2}$
✓	✓	✓	$\frac{\tau}{2\lambda_1^A}$

$\tau > \frac{1}{\lambda_1^A}$. When $\tau > \frac{1}{\lambda_1^A}$, the party attribute generates a greater incumbency advantage than the personal attribute.

When the incumbent has access to both attributes, sequential investment further enhances the incumbent's electoral prospects, reiterating the earlier result. Whereas a first-mover position alone does not generate an incumbency advantage, it helps the incumbent win with a greater margin. If the candidates move sequentially, the party attribute tends to contribute to a larger extent than the personal attribute whenever $\tau > \frac{1}{\lambda_1^A}$. Furthermore, multiple attributes yield a higher probability of reelection than when only a single attribute is available to the incumbent.

It is possible to connect the findings to the previous discussion regarding the effect of party and personal attributes on an incumbency advantage. The results suggest that the relative importance of party and personal attributes to an incumbency advantage depends on the *relative* size of the two attributes. They thus reiterate some of the previous findings related to the party attribute. τ reflects the partisan alignment in the incumbent's constituency. As geographical polarisation or partisan alignment becomes more significant, for instance in favour of the incumbent party and thus τ increases, an incumbency advantage would owe more to the party attribute, than to his personal attribute.

5.2. The Effect of a Spending Cap. The framework can be extended to examine the effect of a campaign spending cap on the strategic behaviour of parties and individual candidates. Suppose that the amount of resources the parties can spend is limited to $k > 0$. Similar regulations are adopted in the United Kingdom. There are countries that restrict campaign spending of individual candidates. The US laws do not restrict the expenditures of individual candidates. They do, however, regulate the amount of contributions that parties allocate to the campaign of an individual politician.

Let's continue to assume that $\tau > 1$. It is straightforward to see that when $k \leq 1$, the equilibrium strategy of the parties is symmetric to when $\tau = 1$. Whether their budget is larger or smaller, the parties can spend as much as k . The equilibrium allocation in Proposition 1 remains robust, Both of them spend the same amount of resources in each district such that $r_i^{A(k \leq 1)*} = r_i^{B(k \leq 1)*} = r_i^{(k \leq 1)*} = kr_i^*$ for any $i \in \{1, 2, 3\}$ and $r_1^{(k \leq 1)*} + r_2^{(k \leq 1)*} + r_3^{(k \leq 1)*} = k$. They also allocate more resources to the more marginal district of the three. When $k \geq \tau$, the spending cap has no effect on the parties. They can use as much as their budget permits. Proposition 1 again remains robust.

When $k \leq 1$, the spending cap is welfare-improving for the less-resourced party (A) and its candidates. For instance, it benefits the challenger in district 1. The spending cap not only prevents the incumbent from running a larger campaign, but also limits the incumbent's preemptive behaviour. The challenger is still less likely to get elected than the incumbent, but his overall electoral performance indicated, by π_1^{A*} , enhances when $k \leq 1$. It is also noted that, despite the spending cap, the challenger who is relatively disadvantaged in effort is still elected with probability less than $\frac{1}{2}$.

When $k \in (1, \tau)$, the equilibrium condition in (2.3) changes to $r_i^{B*} = kr_i^{A*} < \tau r_i^{A*}$. The relative advantage of B 's candidates in campaign still exists, but becomes smaller. In equilibrium, A 's candidate in any district i for any given $\tilde{\alpha}_i^A$ is elected with probability

$$\pi_i^{A(k \leq 1)*} > \pi_i^{A(k \in (1, \tau))*} = \frac{\tilde{\alpha}_i^A}{\tilde{\alpha}_i^A + (1 - \tilde{\alpha}_i^A)k} > \pi_i^{A*}.$$

There exists a welfare-improving effect of the spending cap whenever $k < \tau$ and the effect is greatest when $k \leq 1$.

Meiowitz (2008) and Pastine and Pastine (2010) confirm that a campaign spending cap in general is welfare-improving. They also show that a restrictive cap that forces all candidates to spend an equal amount benefits candidates who are advantaged in non-campaign attributes more. When a restrictive cap with $k \leq 1$, each candidate in each district is allocated the same amount of resources as his opponent. Given this, the voters find it difficult to distinguish between the candidates in their district based on campaign. Instead they assess the non-campaign attributes of each candidates.

Sahuguet and Persico (2006) present a theoretical model with similar motivations to the model in this paper, but offers an alternative view on spending caps. The parties compete in multiple districts by promising redistributive benefits. They differ in voter predisposition across the districts. The authors assert that spending caps are “anti-competitive,” limiting campaign opportunities for candidates with disadvantages. Empirical studies also find that spending caps may hinder challengers more than incumbents. Campaign is known to raise a candidate’s popularity or name recognition. They are the types of valence that challengers who are relatively less experienced than incumbents, normally do not hold (Jacobson 1978; Palda 1992; Levitt 1994). Jacobson (1978) shows that the campaign spending of incumbents is less efficient and exhibits a ‘diminishing return’. Additional campaign spending generates less votes for incumbents. Palda (1992) illustrates similar results with Canadian data. A spending cap therefore reduces campaign opportunities for challengers to raise their popularity among voters.

An interesting empirical regularity regarding campaign contribution is that incumbents receive larger contributions than challengers. There is a view that spending cap can balance out the funding disparity between challengers and incumbents. In practice, the result is somewhat mixed. Incumbents have access to resources that are potentially non-financial and official and that are unavailable to challengers. Therefore, the effect of a restrictive

spending cap on the funding and resource disparity is limited (Benoit and Marsh 2008).

It has been shown that $k \leq 1$ has a welfare-improving effect for the challenger by restricting the incumbent's campaign. However, an incumbency advantage still prevails. The incumbent is elected with probability greater than $\frac{1}{2}$ for any $k > 0$. The results reiterate the findings of Benoit and Marsh (2008). Spending regulation will not completely remove the disadvantages of challengers against incumbents.

Suppose that $\tau < 1$ and the incumbent party, B , is the less-resourced. A spending cap, $k \leq \tau$, may be anti-competitive. It limits the challenger's chance to compete, as noted in the earlier studies mentioned above. The players of the model would behave as if under the symmetric parties. The challenger is unable to enjoy his relative advantage in campaign. As the earlier discussion regarding an incumbency advantage shows, when $\tau < 1$ and the challenger is sufficiently efficient (λ_1^A high), he can win the election with probability greater than $\frac{1}{2}$. $k \leq \tau$ deteriorates the electoral prospects of the challenger and his equilibrium probability of winning decreases to $\frac{2\lambda_1^A - 1}{2\lambda_1^A} < \frac{1}{2}$ for any $\lambda_1^A \in (0, 1)$. In this case, the spending cap facilitates an incumbency advantage.

5.3. A Short note on the Validity of the Contest Function. In the framework, the probability of election for a candidate is determined by a contest function of effort (e_i^A, e_i^B) and campaign (r_i^A, r_i^B) of his opponent and himself, for any $i \in \{1, 2\}$. If a candidate, i^P , either exerts zero effort or runs no campaign, given his opponent's strategy, (e_i^Q, r_i^Q) , his probability of winning based on the contest function becomes zero and he loses the election for sure.

The contest function implies that even if a candidate has led the race with a large margin, he loses the race for sure if there is no campaign support from his party. The 'multiplicativity' of the contest function can be seen as unrealistic and unnatural.

Some empirical observations support the framework and results in this paper. The US party committees have provided little financial support, (almost none) to those performing very poorly at the initial stage. These candidates are often challengers competing in reelection races, receiving poor initial poll ratings and usually losing the race in the end. Table 2.4 provides a set of related empirical evidence. The table shows the number of challengers and incumbents running in the 2010 US House election who received no contribution and any positive contribution from their party.

Table 2.3. Party Support: 2010 US House Election¹

	Party Support=0 #. candidates	Party Support>0 #. candidates
Incumbent	104	262
Challenger	205	241
	309	503

¹ Source: Federal Election Commission, <http://www.fec.gov>

Whereas more than two thirds of the incumbents in the election received some contribution from their party, almost half of the challengers received no support from their party. Given that 85% of the incumbents who ran in the election succeeded in reelection, the data reflects what has been found in the model. It is identified that in equilibrium, a candidate may receive no resources from his party. It takes place when the challenger exerts (almost) zero effort and is therefore perceived with $\alpha_1^A \rightarrow 0$.

Suppose that the winning probability of a candidate in the model is modified slightly, such that

$$\check{\pi}_i^P = \gamma H(e_i^P, e_i^Q) + (1 - \gamma) \pi_i^P,$$

where $\gamma \in (0, 1)$ and π_i^P is the probability of winning for any candidate assumed in the model. $H(e_i^P, e_i^Q)$ is defined as

$$H(e_i^P, e_i^Q) = \begin{cases} 1 & \text{if } e_i^P - e_i^Q \geq \frac{1}{2} \\ \frac{1}{2} + e_i^P - e_i^Q & \text{if } e_i^P - e_i^Q \in (-\frac{1}{2}, \frac{1}{2}) \\ 0 & \text{otherwise.} \end{cases}$$

Solving for the equilibrium backward, the parties behave the same as what Proposition 1 shows. The equilibrium probability of winning changes to

$$\check{\pi}_i^{P*} = \gamma\left(\frac{1}{2} + e_i^P - e_i^Q\right) + (1 - \gamma)\frac{T^P \alpha_i^P}{T^P \alpha_i^P + T^Q (1 - \alpha_i^P)} \equiv \gamma\left(\frac{1}{2} + e_i^P - e_i^Q\right) + (1 - \gamma)\pi_i^{P*}.$$

$\check{\pi}_i^{P*}$ changes the maximisation problem of the candidates accordingly⁷. Furthermore for simplicity, without loss of generality, assume that $w = 1$, a candidate's payoff from election. Then,

$$\check{\alpha}_1^{B*} = \frac{1}{1 - \tau + 2\frac{\lambda}{1 - \lambda\gamma}(1 - \gamma)},$$

which is greater than α_1^{B*} for any $\gamma > 0$. For the open-seat candidates,

$$\check{\alpha}_2^{P*} = \frac{1}{2},$$

for any $P \neq Q \in \{A, B\}$, which is identical to α_2^{P*} in the model. Therefore, the results of the model, in particular with regard to the patterns of party supports in equilibrium, remain robust.

$\gamma > 0$ further highlights the importance of the candidates' first-stage in determining their final outcome. Therefore, it further deteriorate the electoral prospects of those who perform relatively poorly during the first stage. Specifically, the challenger in the reelection race is hit hardest. With the increasing importance of the first stage with $\gamma > 0$, one may well think that the candidates, especially when nominated by the less-resourced party, would be motivated to exert higher effort than he would previously. However, the challenger, by assumption, less efficient in effort is further discouraged, primarily because of the incumbent's strengthened preemptive action, i.e. higher e_1^B .

The equilibrium patterns of party supports in Proposition 4 remain unchanged. The parties will still find an incentive to allocate more resources to the open-seat race than the reelection race, which has become more lop-sided with $\gamma > 0$.

⁷The proof is available in the Appendix.

6. Conclusion

This paper presents and analyses a model of resource allocation in a three-district electoral competition. The model illustrates a two-stage rent-seeking contest in which different types of players, parties and candidates invest in different types of valence at each stage. It addresses asymmetries at the party and the candidate level, showing how these asymmetries influence the equilibrium behaviour of the parties and the candidates in a two-district setting.

The equilibrium of the model explains a range of electoral phenomena. It provides theoretical explanations for (i) allocation of greater party resources to open-seat races and (ii) a variation in candidate performance at different stages of an election and in different types of races. A candidate's performance during the first stage is affected by his relative advantage in effort and campaign and the incumbency status of his opponent and himself. The initial performance of a candidate is further reinforced or damaged by the relative size of the resources allocated by his party. It is also illustrated that the timing of effort investment, i.e. whether or not candidates move *sequentially* has an effect on their initial performance.

The results reconfirm and complement Snyder (1989) with respect to the equilibrium behaviour of the parties. The symmetric parties offer greater support to marginal races where the candidates perform similarly during the first stage. The asymmetric parties however choose to protect strong-performing candidates or to bail-out poor-performing ones. In this model, a three-district election takes place, consisting of two reelection races and an open-seat race between two freshmen. The incumbent of the better-resourced party always performs better than his co-partisan who is running in the open-seat race. Following the first-stage performance of the candidates, the better-resourced party helps its open-seat candidate differentiate, by offering him more resources than the incumbent. The better-resourced party would favour its open-seat candidate over its challenging candidate in a reelection race, as long as the challenger is substantially disadvantaged in effort.

The model identifies different attributes to an incumbency advantage, one of the well-documented political phenomena in the literature. The paper contributes to the formal models of incumbency advantage, by distinguishing the effect of party and personal attributes, and examining how the two types of attributes interact. Furthermore, the results identifies the sequential nature of reelection races, that decisions made by the candidates in a reelection race tend to be made sequentially, with the incumbent moving first, as an incumbency-advantage ‘multiplier’. Whenever an incumbency advantage exists, the sequential nature of the game strengthens the advantage. It is a new insight to complement the studies, especially those that have predicted challenger deterrence as a type, or an outcome, of an incumbency advantage. The results also identify an interaction of the incumbent’s party and personal attributes, that gives rise to an incumbency disadvantage. They further shows that, when an incumbency disadvantage exists, the sequentiality works against the incumbent.

The impact of campaign spending caps on the equilibrium behaviour of the players has been also examined using the framework of the model. The results address and confirm the findings, both supporting and rejecting the welfare-improving effect of spending caps. It is shown that although spending caps prevent the better-resourced party or candidates from outspending the less-resourced to some extent, thus having *some* welfare-improving effect, an incumbency advantage remains to exist.

The model has prospects to be extended to a multi-period model that offers a theoretical explanation for another electoral phenomenon: *rematches*. Although rarely explored in the literature, 13-4% of the races in every US House election during the 2000’s were between the candidates who had fought together in the previous election(s). Rematches can be seen as a phenomenon which emerges from political motivations of parties and candidates, potentially different from those behind open-seat or reelection races. The current framework can be extended over multi-periods, hopefully further expanding the understanding of the interactions between multi-level asymmetries and presenting new insights for a wider range of electoral phenomena.

3. Intraparty Factions Explained with Power Hierarchy

Chapter Abstract

This paper presents a new model of intraparty faction, incorporating a power hierarchy, integrating the two areas of the literature that are relatively under-developed, despite their extensive presence in politics. The model is further distinguished from the small literature of factions. It tries to reflect the characteristics of intraparty factions that distinguish themselves from temporary and relatively less structured factions, e.g. parliamentary or legislative coalitions, by assuming a hierarchical organisation in intraparty factions. The framework considers two intraparty factions that compete each other for party resources, which serve as a ‘club good’, or an exclusive benefit to their faction members. Individuals in a faction are ordered according to the size of power they exercise in the faction, and have two potentially conflicting interests. As much as they want to be part of a strong faction that provides greater collective benefits, i.e. party resources allocated to each faction, they want to exercise greater influence and be placed at a higher rank in the faction. The model begins as the members of each faction decide whether to stay in their initial faction, or to leave for the opposition. After the members have made a decision, the winner of intraparty competition is determined, depending on the relative number of high- and low-quality members in each faction. The results of the model identify a trade-off between collective and individual benefits as the key determinant of the members’ decision, which varies across the members with different initial ranking and quality. It shows that the trade-off is relatively small for the higher-ranked, whose loss in ranking is limited or minimal, whichever faction they choose. In turn, they rest their decisions on the relative size of the collective benefits each faction offers. This explains the departures from a relatively weaker faction to a relatively stronger one, which are often observed in factional politics. The result also provides a logic for the more ‘puzzling’ dynamics of factions, the departure of the members in a stronger one to a minor or weaker one. It identifies that the relatively lower-ranked members with high quality would be incentivised to do so, when

they find a better career opportunity in the other, e.g. higher-ranked positions, and the individual benefits in the other faction outweigh the collective benefits from the stronger faction. As the size of resources the winning faction gets entitled to increases, it is predicted that the latter type of departure becomes less frequent. This paper also considers a number of modifications to the model. When the winner selection rule for intraparty faction, which determines the relative advantage of the factions, changes, the key results of the model remain robust. The identities of the members who move between the factions are more or less identical and the direction of departure remains unchanged. That is, the higher-ranked members are able to choose the winning faction with relative ease, whereas there are some lower-ranked members who move to the less advantaged faction for better career opportunities. It is also shown that even if a new faction is created by split of a minor faction, it does not sustain and last long, explaining for a stabilised system of intraparty factions, notably in the Liberal Democratic Party of Japan.

1. Introduction

Intraparty factions are criticised because self-interested individuals can hinder collective action of the party by prioritising the interests of their faction, which potentially conflict with those of the party. The existing literature, while distinguishing different motivations of faction formation, such as factions from interest and from principle (Sartori 1976), largely assumes that faction members are unitary actors in achieving the collective objectives of their factions. However, it is found that the postulation may not be sufficient to explain the dynamics of intraparty factions in politics. As much as a party does not, an intraparty faction does not act as a unitary actor.

Let's take an example of factions in Japan's Liberal Democratic Party (LDP). Although there were occasional splits of factions, LDP factions had performed as unitary actors advancing their collective interests. The internal organisation of the party had remained stable with five major factions. The major factions had maintained the overall control of the party, providing greater benefits to their members. Following a political scandal that hit the party and its key politicians in the early 1990s, a series of splits between the major factions started. There are different interpretations on what caused the splits. Some see that the splits were inflicted by disagreement over political ideologies. Others argue that at least some of them were triggered by a power struggle between faction members and some members chose to leave their faction for a better 'career' opportunity such as promotion to a higher-ranked position or a prospect of it.

The example suggests that when collective and personal interests conflict, factions can potentially stop acting as unitary actors. This paper focuses on this possibility and departs from the common view on factions in the literature. It presents a model of intraparty factions that offers a theoretical explanation for the dynamics of factions, often observed in politics. Different interests of politicians are incorporated in the framework. They want to exercise greater power in their faction as well as to be part of a faction that brings greater exclusive benefits to them. The framework makes it possible to analyse the strategic behaviour of faction members, particularly when the personal interests conflict with

collective goals of their faction.

The model introduces features that distinguish intraparty factions from other types of factions in the framework. Following the definition in Zuckerman (1975), also acknowledged in Persico et al. (2011), this paper discusses and analyses intraparty groups that are “structured” with “established patterns of behaviour and interaction for their members over time”. They differ from other types of factions in politics, e.g. interparty factions and legislative coalitions, not only in terms of their organisational durability but also in terms of their organisational hierarchy. Whether the focus is on ideological or non-ideological motivations for faction formation, it is then essential to examine the dynamics of intraparty factions along with their institutional features.

The framework considers a party, in which each member of the party belongs to one of two intraparty factions. The members in each faction are ranked according to the size of power they exercise in the faction. Initially the factions differ in their quality. Each member in the higher-quality faction has higher quality than the same-ranked member in the other. Each of the members decides whether to remain in his initial faction or to move to the other. Once the members have made a decision, the winner of intraparty competition is determined, depending on the relative number of high- and low-quality members in each faction. The faction with the higher quality wins. The winner is given additional party resources, transferred from the losing faction. The party resources each faction has are available exclusively to its members, as a ‘club good’. Furthermore, it is assumed that when a member joins a faction, his ranking in the faction, influenced by his ranking in the initial faction and his quality, may change.

One could expect that the members of the lower-quality faction would move to the higher-quality one. It is a reasonable speculation as such dynamics are observed in factional politics. The analysis of the model indicates that while the greater ‘club good’ in the winning faction is definitely an incentive for the members in the lower-quality faction, not every member makes such a decision. It also predicts that a different type of departure can take place. As in the LDP example, some members of the higher-quality faction may

choose to leave for the lower-quality faction.

The members' choice over the factions is determined by a trade-off between the collective and individual benefits that a member expects from each faction. If a faction provides a member with greater benefits of both types than the other, his decision is straightforward. Suppose instead that there is a conflict between the two types of benefits for a member when joining a faction. A member of the low-quality faction moves to the high-quality faction as long as the benefit from the additional resources outweighs the loss in ranking. Similarly, a high-quality member chooses to give up the additional resources for a higher ranking in the opposite faction if the gain in ranking is substantial.

The trade-off varies across the members of different ranking, given a size of additional resources awarded to the winner. The high-ranked members from the lower-quality faction are more likely to leave than those ranked below them. Their loss in ranking in the higher-quality faction is relatively smaller. The members whose ranking is relatively low may leave the higher-quality faction. They could take the 'vacancy' in the lower-quality faction, created by the departure of the high-ranked, and improve their ranking. If they perceived the gain from a higher ranking outweighing the collective benefit in the higher-quality faction, they would leave.

As greater resources are awarded to the winning faction, more members would choose the higher-quality faction. This happens in two directions. The relative loss in ranking the members from the lower-quality faction bear becomes smaller, that more move to the higher-quality faction. At the same time, the relative gain in ranking the members from the higher-quality faction anticipate becomes smaller, that fewer move to the lower-quality faction.

A stability concept (Ray and Vohra 1997, 1999; Levy 2004) is adopted to study the members' optimal behaviour. The concept has been found effective to analyse endogenous formation of coalitions. The members' choice over the factions gives rise to a 'coalition structure', replacing the initial structure of the two factions. A structure is replaced over

and over by an alternative as long as there is a member who finds an incentive to move between the two factions. As a solution to the model, a stable structure, in which no member is incentivised to leave the faction to which he belongs, is found. In a stable structure, the optimal strategies of the players are identified, answering questions such as: how many members are leaving their initial faction, and whether members of particular ranks are more likely to leave. The cooperative concept is also ‘less restrictive’. It is not necessary to limit ourselves, for example, to a particular sequence of the players’ decision and are thus able to develop a more general model of endogenous coalition formation.

A number of modifications to the model are also considered. The modifications consider when the winner-selection rule changes, when the members can form a new faction, and when the benefits to the winning faction are endogenous. In the modifications, the trade-off between collective and individual benefits remains as a key determinant for the members’ decision.

When the faction with the greater number of members wins, the initial advantage of having the greater quality disappears and the factions are equal. The members still find similar incentives. The higher-ranked are more likely to move and those ranked relatively low may move to take up the higher-ranked positions in the opposite faction, giving up the collective benefits. However, now the higher-ranked members of *any* faction are incentivised to move. Depending on which faction is the first to get split, a different stable structure in which a different faction wins in intraparty competition is found.

When the members choose to join one of the two factions or to form and join a new faction, the decisions made by the high-ranked remain unchanged from what is observed in the model. The option of forming and joining a new faction may be taken by the lower-ranked, that those previously choosing the lower-quality faction are divided between the lower-quality faction and the new one. However, a new faction is formed only when it could provide sufficient collective benefits. A new faction also lasts only *temporarily*. In a stable structure, even if formed, a new faction would be merged into another ‘minor’ or

losing faction.

Despite their presence in politics, the subjects of intraparty factions and power hierarchy have not been paid sufficient attention in the formal literature. A few models have discussed endogenous formation of factions. They are, however, insufficient to explain the dynamics of factions, involving splits and mergers between factions. This paper not only incorporates a hierarchical structure, but also analyses endogenous formation of power hierarchy which changes and replaces the initial structure. The latter is an aspect rarely explored in the existing models and this paper would make a novel contribution.

The results are presented in the following order. The next section provides a summary of the related literature. Section 3 introduces the framework and briefly discuss factional politics in Japan. The discussion shows that the framework reflects the features of intraparty factions in politics. In section 4, the stability concept is defined and illustrated with an example of a six-member party. Section 5 includes the analysis of the results, followed by the discussion on the framework and the related empirical cases in Section 6. A number of modifications are considered and discussed in section 7, and then conclude.

2. Related Literature

In the framework of this paper, a number of organisational features in political parties are integrated: intraparty factions and hierarchical relationships between party members. Such an attempt has been rarely made in the formal literature. Although relatively more frequently analysed than (power) hierarchies, intraparty factions in the existing models are often insufficient to reflect how they function and evolve in politics. For instance, formal models perceiving intraparty factions as unitary actors and focusing on ideological motivations for faction formation often predict what would be expected from a model of policy and coalition bargaining in legislature. This paper tries to overcome this and provide a new perspective on the dynamics of intraparty factions by incorporating a hierarchical structure and recognising different motivations for faction membership.

The earlier models of factions (Eguia 2011a, 2011b; Mutlu 2010) have made a departure from the assumption that parties are unitary actors. They assume that parties consist of factions with a conflict of ideological interests, whereas each faction acts as a unitary actor. Eguia (2011a, 2011b) presents a framework where individual legislators can form a voting bloc and the members of a bloc vote together according to their agreement on policy issues. Mutlu (2010) considers when ideologically motivated factions in a majority party bargain over policies. This paper takes a further departure and considers an environment in which there is a conflict of interest between the members in a faction.

There are a few formal models addressing an organisational hierarchy, similar to the organisational features that this paper highlights. These models also recognise that factions may be motivated by non-ideological benefits serving roles other than policy coordination, and that faction members pursue personal interests as well as collective ones (Persico et al. 2011; Morelli and Park 2016). Both Persico et al. (2011) and Morelli and Park (2016) consider a hierarchical structure in which the players are ordered according to ranking in a faction and they want to be promoted to a higher rank.

In Persico et al. (2011), politicians in each faction contribute costly effort to help a candidate their faction supports win in an election. Each of them is promoted to a higher rank when the candidate is elected. Thus, the politicians' collective and individual benefits do not conflict, which is one of the key differences from this paper. Another difference is that this paper explicitly studies endogenous formation of factions and hierarchy. Persico et al. (2011) adopt a hierarchical structure, but do not consider, for instance, when the initial hierarchy changes by the players' strategic behaviour. They also treat that the politicians are assigned to a faction in each period of the game, rather than making a strategic choice over the factions.

Persico et al. (2011) show that factional competition encourages effort investment among career-motivated politicians. Politicians' effort facilitates election of the party's candidates, improving the welfare of the party. The result primarily rests on their framework that does not address a conflict between collective and individual interests. The formal

models identifying a negative effect of “collusion in hierarchies” better reflect the conflict. Tirole (1986) and Carillo (2000) consider a hierarchical organisation in which an information asymmetry exists across the players and some players may collude to advance personal interests over the organisation’s interests.

Morelli and Park (2016) present a framework which is the most similar to the one in this paper among those discussing endogenous formation of factions and/or hierarchy. In their model, the players with a different level of ability decide whether to join a coalition, in which they are ranked according to their ability. Their decisions are influenced by a trade-off between collective and individual benefits, similar to the one identified in this paper. Whereas joining a coalition with the high-ability players increases the collective benefit to a player, it lowers his individual benefit as he is placed at a lower rank in the coalition. Their results, also based on a stability concept, identify and explain the coalition structure derived by the players’ strategic behaviour.

However, Morelli and Park (2016) differ in a number of aspects. The model in this paper begins with a coalition structure and a hierarchical relationship between the players, whereas they consider a group of individual players initially ‘unaffiliated’. It analyses the endogenous changes to the exogenous structure and hierarchy made by the players. The results extend and complement what is implied in Morelli and Park (2016), specifically answering what would happen to coalitions or factions once they are formed.

Another aspect that distinguishes this paper from Morelli and Park (2016) is that the players in this model can be restricted to choose and move to a faction, even if it improves his payoff. On the contrary, in Morelli and Park (2016), no such mechanism is present. Their players can join and leave a coalition as long as there is a better option. In this model, whenever a player’s move changes the winner of intraparty competition, the relatively higher-ranked will ‘sequentially block’ such a move and this possibility restricts, and in some cases prohibits, splits and mergers of the factions. The result captures and reflects one of the critical features in factional dynamics observed in politics, possibly better than

the existing models of faction or coalition formation.

Dewan and Squintani (2015) also analyse endogenous formation of factions, focusing on a role of information aggregation. They consider ideological politicians choosing whether to join a faction or to stay unaffiliated. When they join a faction, they increase the influence that their faction leader exercises in determining the party policy, though their individual bargaining power over the policy may be weakened. The authors, while identifying a trade-off in the players' decision, also identify a stable structure of factions.

Additionally, this paper contributes to the literature on endogenous party formation and defection. The existing models have addressed different motivations for party formation. Snyder and Ting (2002) focus on the informative benefit that parties provide, enhancing electoral performance of politicians. Levy (2004) shows that parties are stable coalitions of individuals with diverging ideologies. The stability concept in Levy (2004) is used and modified. The next section provides more detailed discussion on the approach. Morelli (2004) finds that electoral systems provide different incentives for politicians, resulting in a variation in party systems.

The literature has also identified the motivations for party defection, such as ideological conflicts (Reed and Scheiner 2003; Mutlu 2010); changes in electoral environment (Heller and Mershon 2005; Cox and Rosenbluth 1995; Cox et al. 1999); and party resources or a lack of them (Aldrich and Bianco 1992; Desposato 2006). In particular, Cox and Rosenbluth (1995) analyse party defection of the Japanese LDP members and identify the types of politicians who are more or less prone to defection. Some of their findings are in line with the results in this paper, which will be discussed with detail in the following sections.

This paper can also be related to the formal models that study intraparty competition (Cail-laud and Tirole 2002; Castanheira et al. 2010). They examine the effect of intraparty competition on the quality of a party. and expand to consider different decision rules. A number of modifications that address these aspects are provided. For instance, it is shown that when the winner-selection rule changes, some changes in the optimal strategies of

players, and thus in the stable structures, are observed, although they do not affect the robustness of the model to a great extent.

3. Model

Consider a party of $2n$ politicians, where $n \geq 3$. Politicians belong to one of the two factions. Each faction $i \in \{a, b\}$ has n members. Members in each faction are ranked according to the size of power they exercise in the faction. Member i^j for any $j \in \{1, \dots, n\}$ holds greater power than member i^k for any integer k with $j < k \leq n$.

A faction's quality, denoted by Q_i , is defined as the average of political capital owned by its members:

$$Q_i = \frac{\text{sum of capital held by members in } i}{\text{number of members in } i}.$$

Each member in a owns political capital $q_{aj} = 1$, whereas each in b has $q_{bj} = 0$ ¹. Each faction is endowed with n units of party resources. The party resources in a faction are a *club good*, exclusively available to the faction members.

The politicians want to be part of a faction that provides a greater amount of party resources. They also want to be ranked higher in their faction. A member's ranking in a faction may change with the decisions made by the $2n$ members including himself, determined by his initial ranking and by his political capital.

Consider member i^j . Initially there are $2(j-1)$ members ranked higher than him: $j-1$ members each in his initial faction, i and the other. Those among the $2(j-1)$ members who choose the same faction as him are ranked above him. The ranking of the members initially at the same rank, i.e. a^j and b^j for any $j \in \{1, \dots, n\}$, when choosing the same faction is determined by their political capital. The two members own a different amount of political capital with $q_{aj} = 1 > q_{bj} = 0$. Whenever in the same faction, a^j with the greater capital is ranked above b^j . To further illustrate this, a simple example is given

¹The initial quality of a and b therefore are 1 and 0 respectively.

below.

Example) Consider a six-member party. Each faction has three members, $a = \{a^1, a^2, a^3\}$ and $b = \{b^1, b^2, b^3\}$. The members are lined in a descending order of power in their faction. Suppose that a^2 moves to b , whereas the other five are not moving. This changes the ranking of the members in each faction in the following way:

$$\begin{cases} a = \{a^1, a^3\} \\ b = \{b^1, a^2, b^2, b^3\}. \end{cases}$$

a^2 's departure improves a^3 's ranking in a , but deteriorates the ranking of b^2 and b^3 .

The game begins as the politicians in each faction decide whether to move to the other faction. Their decisions determine the winner of intraparty competition. The faction with the greater quality wins. Faction i wins if $Q_i > Q_{\sim i}$, where $i, \sim i \in \{a, b\}$. If $Q_i = Q_{\sim i}$, the winner is chosen randomly. The winning faction is awarded additional party resources, denoted by $\omega \in [0, n]$. ω is transferred from the losing faction. The game ends and the payoffs are distributed accordingly across the members.

U_{ij} is defined as the payoff which i^j receives when he chooses \hat{i}_{ij} . It is a combination of his payoff from a change in his ranking, which is positive or negative and denoted by \hat{i}_{ij} , and the benefit or loss from the chosen faction. Specifically,

$$U_{ij}(\omega; \hat{i}_{ij}) = \begin{cases} \Delta(i^j; \hat{i}_{ij}) + \omega & \text{if } \hat{i}_{ij} \text{ is the winner} \\ \Delta(i^j; \hat{i}_{ij}) - \omega & \text{otherwise,} \end{cases}$$

for any $i, \hat{i}_{ij} \in \{a, b\}$. $\Delta(i^j; \hat{i}_{ij})$ is the change in i^j 's ranking when choosing \hat{i}_{ij} , i.e. $j - \hat{i}_{ij}$'s ranking in \hat{i}_{ij} . $\omega \in [0, n]$ is the size of party resources the members in the winning faction enjoy, or the members in the losing faction lose. The members' decision over the factions is perfectly observed by everyone else in the party. When indifferent, they stay in

their initial faction.

4. Definition of a Stable Structure

4.1. Stability Concept. The $2n$ politicians in the model are treated as organised in a ‘coalition structure’. The game begins with a structure, in which there are two *coalitions*, a and b , each with n members. In a coalition structure, if a member is not satisfied with his current faction and finds an incentive to move to the other, he would do so. Such a departure gives rise to an alternative structure which replaces the previous one.

A stability concept in Ray and Vohra (1997, 1999) and Levy (2004) is adopted to analyse the model. Firstly, let us denote a coalition structure by $\pi(\omega)^{m-l}$:

DEFINITION 1. $\pi(\omega)^{m-l}$ for $\omega \in [0, n]$ is a partition on the set of $2n$ in which faction a and b have $(n+m-l)$ and $(n-m+l)$ members respectively where $m, l \in \{0, \dots, n\}$. It is formed when m of the initial members in b move to a and l of the initial members in a move to b .

In identifying the optimal strategies of the players, *stable* partitions, or a stable coalition structure, is found. Ray and Vohra (1997, 1999) study endogenous coalition formation in which “individual (and group) payoffs depend on the entire coalition structure that might form”. They consider a framework in which the players in the grand coalition are allowed to leave the existing coalition and form smaller coalitions. Levy (2004) considers a similar setting in a citizen-candidate framework. Each player who has ideological preferences decides whether to coalesce with some of the others in the grand coalition. A deviation to a smaller coalition continues as long as there is a player or a group of players with an incentive to break the current one. A stable structure is therefore defined as a coalition structure free from any further deviation by a player or a group of players. A structure is

blocked if there is a structure preferred to it by all the players involved.

In the model of this paper, a member's payoff in a coalition structure depends on his own choice over the factions as well as the decision made by the rest of the members. The initial coalition structure, $\pi(\omega)$, changes whenever individual members move between the two factions. A newly emerged structure is replaced by another, as long as a member or a group of members makes a move. By finding a stable structure, it is possible to identify the optimal strategies of the $2n$ members. The stability concept also helps us analyse a simple model without arbitrary assumptions and restrictions, e.g. on the sequence of the players' actions².

The definition of π in Levy (2004) is adopted and modified. Denote a set of coalition structures 'induced from $\pi(\omega)^{m-l}$ by $S(\pi(\omega)^{m-l})$ '. A structure is induced from $\pi(\omega)^{m-l}$ if it is formed when a member or a group of members in $\pi(\omega)^{m-l}$ moves to the other faction. When additional l' members leave a after m and l members have left b and a respectively, $\pi(\omega)^{m-l-l'}$ is formed and is induced from $\pi(\omega)^{m-l}$. Similarly, $\pi(\omega)^{m+m'-l}$ which emerges when additional m' members has moved to a , is also induced from $\pi(\omega)^{m-l}$. A payoff to member i^j in a structure, $\pi(\omega)^{m-l}$, is denoted by $U_{ij}(\pi(\omega)^{m-l})$.

DEFINITION 2. $\pi(\omega)^{m-l}$ is *sequentially blocked* when there are a sequence of structures induced from $\pi(\omega)^{m-l}$ in which there is always a member finding an incentive to move between the factions. $\pi(\omega)^{m-l}$ is stable if there is no alternative structure that *sequentially blocks* it.

Suppose that $\pi(\omega)^{(m',l')} = \pi(\omega)^{m+m'-l-l-l'} \neq \pi(\omega)^{m-l}$ is stable, i.e. $\pi(\omega)^{(m',l')}$ is induced from $\pi(\omega)^{m-l}$, when m' members move to a and l' leave a . When $m' = 0 = l'$, no structure is induced. $\pi(\omega)^{m-l}$ is 'sequentially blocked', whenever

²It is not necessary to consider assumptions such as: whether the players are making decisions simultaneously; if they decide sequentially, who is to decide first or before anyone, and whether there is a group of the players deciding simultaneously before the rest, etc.

- in $\pi(\omega)^{m-l}$, there is a member who has an incentive to move between the faction, giving rise to a new structure, i.e. *induced from* $\pi(\omega)^{m-l}$, denoted by $\pi(\omega)^{(1,0)}$ or $\pi(\omega)^{(0,1)}$;
- in the induced structure, there is also a member with an incentive to move between the faction and another structure, $\pi(\omega)^{(2,0)}$, $\pi(\omega)^{(0,2)}$, or $\pi(\omega)^{(1,1)}$, also induced from $\pi(\omega)^{m-l}$ emerges;
- similarly, in each of $\pi(\omega)^{(1,0)}, \pi(\omega)^{(0,1)}, \dots, \pi(\omega)^{(m'-1,l'-1)}$, there will be a member moves between the factions, leads to a new structure induced.

In both Ray and Vohra, and Levy, the players decide to form and join a coalition based on a binding agreement. All the players in a coalition jointly agree and precommit to it costlessly. In the framework of this paper, members are free to move between the factions and hence they have an incentive to do so only when their move is not ‘sequentially blocked’. They continue to move as long as the subsequent moves inflicted by others do not make them worse off. An implicit assumption embedded in the framework is that whenever a coalition structure emerges, members in each faction jointly agree on a binding agreement.

4.2. A Party of Six: An Illustration. To illustrate the application of the stability concept defined above and the intuitions for the model, a party of six politicians is considered. Factions a and b have three members each. The game starts with the two grand coalitions, $a = \{a^1, a^2, a^3\}$ and $b = \{b^1, b^2, b^3\}$. Let’s first check if faction b can break into smaller coalitions, by a member or a group of members moving to a , given $a = \{a^1, a^2, a^3\}$.

Suppose only one of the three members is leaving b . Recall that b is a group of politicians lacking capital with $q_{bj} = 0$ for all $j \in \{1, 2, 3\}$. In a , b^j is placed at the $(j+1)^{\text{th}}$. However, he has access to additional resources, $\omega \in [0, 3]$. By b^j ’s move, a ’s quality becomes $Q_a = \frac{3}{4}$, still higher than $Q_b = 0$, and a wins. A single-member deviation from b to a is profitable as long as $-1 + \omega > -\omega \Leftrightarrow \omega > \frac{1}{2}$.

If $\omega \leq \frac{1}{2}$, deviations by two or more members are not profitable either. The players are placed at the $(2j)^{\text{th}}$ in a . $\omega \leq \frac{1}{2}$ is insufficient to offset a loss of one rank. No member leaves b . Provided $b = \{b^1, b^2, b^3\}$ unchanged, the members in a do not move either. In

addition to losing ω , they are not ranked higher in b . In fact, the result holds for any n :

PROPOSITION .1. *If $\omega \leq \frac{1}{2}$, there exists the unique stable structure in a party of $2n$, in which*

$$\begin{cases} \hat{a} = \{a^1, \dots, a^n\} \\ \hat{b} = \{b^1, \dots, b^n\}. \end{cases}$$

The initial structure remains unchanged.

When $\omega > \frac{1}{2}$ a single-member deviation from b is always profitable. It implies,

LEMMA .1. *When b^j is incentivised to move to a , so is b^{j-k} , for any $k \in \{1, \dots, j-1\}$.*

Proof of Lemma 1. It is established that if $\omega > \frac{1}{2}$, a single-member deviation from b is profitable given the grand coalition of a . Suppose that only b^2 moves to a . b^1 receives a payoff of $-\omega$ in b , smaller than his payoff in a , $-1 + \omega$. b^1 has an incentive to move to a : a contradiction. Whenever b^2 moves, so does b^1 . Similarly, when b^j moves, so do the members ranked higher than him. ■

By Lemma 1, whenever $\omega > \frac{1}{2}$, the number of members who move to $a = \{a^1, a^2, a^3\}$ depends on the size of ω . b^2 also wants to do so if and only if:

$$-2 + \omega > 1 - \omega.$$

$\omega > \frac{3}{2}$ is sufficient to cover the loss in b^2 's ranking in a . b^1 's departure places b^2 at a rank higher at the 1st in b . In a , he is placed at the 4th after a^1, b^1 and a^2 , two ranks down from the initial placement. Similarly, b^3 moves to a if and only if:

$$-3 + \omega > 2 - \omega.$$

b^3 is placed at the 6th in a after the three initial members of a and the two others from b . If he stays in b , he is at the top. In order to compensate his loss in ranking, he needs $\omega \geq \frac{5}{2}$.

For $\omega > \frac{1}{2}$, the grand coalition of b is divided and only the following members stay in b :

$$b = \begin{cases} \{b^2, b^3\} & \text{for } \omega \in (\frac{1}{2}, \frac{3}{2}] \\ \{b^3\} & \text{for } \omega \in (\frac{3}{2}, \frac{5}{2}] \\ \emptyset & \text{for } \omega \in (\frac{5}{2}, 3]. \end{cases}$$

Let's now check if a after the arrivals from b is divided for each range of ω identified above.

For $\omega \in (\frac{1}{2}, \frac{3}{2}]$, b^1 's arrival puts a^2 and a^3 at a lower rank. a^2 moves to b if and only if:

$$-1 + \omega < 1 - \omega.$$

When $\omega < 1$, he finds that the relative gain in b is greater than that in a . When $\omega < 1$ and a^2 moves to b , a^3 finds it profitable to stay in a . After a^2 's departure, a^3 does not move. His move will be blocked by a^1 . The departure of a^2 and a^3 equalises the quality of both factions as $a = \{a^1, b^1\}$ and $b = \{a^2, b^2, a^3, b^3\}$. The winner is chosen randomly. a^1 can improve his payoff from 0 to ω by moving to b . He then moves to b . b^1 will follow a^1 and move to b .

In $b = \{a^2, b^2, b^3\}$, b^2 may consider moving to a , but his move will be sequentially blocked. His departure equalises the two factions. The higher-ranked members, a^1 and b^1 , find an incentive to move to b , which triggers a^2 and b^3 to move to a . b^2 's payoff then is identical to what he receives in $b = \{a^2, b^2, b^3\}$. No further deviation is made.

When $\omega \geq 1$, no member in $a = \{a^1, b^1, a^2, a^3\}$ wants to move 'alone' to b . Consider deviations by two or more members. Note that any a^j never finds a deviation with b^1 profitable. Such deviations put the quality of a back at 1. When moving to b , a^j loses ω and is placed at a lower rank in some cases. a^1 is also not incentivised to move to b with either

a^2 or a^3 . The deviations equalise the two factions in quality. He receives an expected payoff of zero from the deviations, whereas staying in a yields a payoff of ω . Finally, consider when a^2 and a^3 move together. The factions become equal in their quality. As above, the deviation is sequentially blocked by the higher-ranked. The options available to the lower-ranked are relatively limited. Each of b^1 , b^2 and b^3 wants to move to a for any $\omega > \frac{1}{2}$. The departure of higher-ranked may discourage the lower-ranked (Lemma 1). When ω is sufficiently small but not too small, the arrival from b may also motivate some members in a to move to b .

It is indicated that b^1 and b^2 move to a for $\omega \in (\frac{3}{2}, \frac{5}{2}]$, and b is completely dissolved for $\omega > \frac{5}{2}$. When $\omega \in (\frac{3}{2}, \frac{5}{2}]$, no departure from $a = \{a^1, b^1, a^2, b^2, a^3\}$ is made. For b^1 , a^2 and b^2 , ω is sufficient to offset the loss in ranking as discussed above. a^3 's ranking is down by two ranks after the arrival of b^1 and b^2 . If $\omega < 2$, he wants to move to b as:

$$-2 + \omega < 2 - \omega.$$

The deviation again equalises both factions and is sequentially blocked by higher ranked members.

Let's further illustrate how a^3 's departure is sequentially blocked. Suppose that a^3 has moved to b . There are $a = \{a^1, b^1, a^2, b^2\}$ and $b = \{a^3, b^3\}$ in structure $\pi(\omega)^{2-1}$. Consider the induced structures that emerge when a is further divided:

$$a = \begin{cases} \{a^2, b^2\} & \text{in structure } \pi(\omega)^{(2-1)-2} \\ \{b^2\} & \text{in structure } \pi(\omega)^{(2-1)-3} \\ \emptyset & \text{in structure } \pi(\omega)^{(2-1)-4}. \end{cases}$$

After a^3 moves to b , the remaining politicians in a receive a lower payoff than before. a^1 improves his payoff by moving to b , which is followed by b^1 . In $\pi(\omega)^{(2-1)-2}$, a^2 also follows the move as $1 - \omega < -1 + \omega$, giving rise to $\pi(\omega)^{(2-1)-3}$, in which b^2 also finds an incentive to leave a as $1 - \omega < -2 + \omega$. $\pi(\omega)^{(2-1)-4}$, in which all the six politicians are in b is not stable either. b^3 can move to a receiving $2 - \omega$, which is greater than the

payoff of $-3 + \omega$ in b .

When $2 \leq \omega \leq \frac{5}{2}$, a^3 does not find an incentive to move to b . When $\omega > \frac{5}{2}$, all the six members are in a . The results for $\omega \in (\frac{1}{2}, 3]$ is summarised below

Result 1. For $\omega \in (\frac{1}{2}, 3]$, the following members choose faction b

$$\hat{b} = \begin{cases} \{a^2, b^2, b^3\} & \text{for } \omega < 1 \\ \{b^2, b^3\} & \text{for } \omega \in [1, \frac{3}{2}] \\ \{b^3\} & \text{for } \omega \in (\frac{3}{2}, \frac{5}{2}] \\ \emptyset & \text{for } \omega > \frac{5}{2}, \end{cases}$$

in a stable structure. For each sub-range of ω above, the rest of the players choose a .

The ‘party of six’ example has illustrated how the stability concept works and the intuitions of the game. The optimal strategies of each member and the corresponding stable structures are identified. The results indicate that the members’ choice between the factions which, in turn, changes the organisational structure of the party is influenced by the relative gain (or loss) in ranking and ω . It is also found that the relative gain varies across the members. In the next section, the results from the example is extended to analyse the stable structures in a party of $2n$.

5. General Results

In this section, a party of $2n$ is considered. The intuitions illustrated in the ‘party of six’ example to find the stable structures in the $2n$ -player model are applied. Firstly, let’s define:

DEFINITION .3. For $y \in \mathbb{R}^+$, $\lfloor y \rfloor$ is the nearest integer of y , i.e. $\lfloor y \rfloor = x \in \mathbb{Z}$ such that $y \in (\frac{2x-1}{2}, \frac{2x+1}{2}]$.

The optimal choices of the politicians when $\omega \leq \frac{1}{2}$ are discussed in Proposition 1. For $\omega \leq \frac{1}{2}$, there is no deviation from the initial structure and this holds for any $n > 3$. The benefit from the winning faction is not big enough to compensate for the loss in ranking for any b^j when moving to a . Provided that the grand coalition of b remains undivided, if any a^j moves to b , he loses ω and, in some cases, is placed at a lower rank. The grand coalition of a also remains unchanged.

For $\omega > \frac{1}{2}$, Result 1 is extended to a party of $2n$ and the following Proposition summarises the result in a party of $2n$:

PROPOSITION .2. For $\omega \in (\frac{1}{2}, n - \frac{3}{2}]$, $b^1, \dots, b^{\lfloor \omega \rfloor}$ always move to a . If $\omega < \lfloor \omega \rfloor$ and

- (i) $n - \lfloor \omega \rfloor$ is even: $i^{(\lfloor \omega \rfloor + 1)}, \dots, i^{(\lfloor \omega \rfloor + \frac{n - \lfloor \omega \rfloor}{2} - 1)}$ and $a^{(\lfloor \omega \rfloor + \frac{n - \lfloor \omega \rfloor}{2})}$
- (ii) $n - \lfloor \omega \rfloor$ is odd: $i^{(\lfloor \omega \rfloor + 1)}, \dots, i^{(\lfloor \omega \rfloor + \frac{n - \lfloor \omega \rfloor - 1}{2})}$

move to the opposite faction for any $i \in \{a, b\}$. If $\omega > n - \frac{3}{2}$, $b^1, \dots, b^{\lfloor \omega \rfloor}$ move to a , whereas no others move.

Proposition 2 indicates the number of members moving between the two factions and identifies their initial ranking. Recall that a initially holds the higher average quality than b . The Proposition implies that the ‘direction’ of departure is primarily from the lower-quality faction to the higher-quality one.

The first part of the Proposition is straightforward from the ‘party of six’ example. With Lemma 1 and given the grand coalition of a , b^j is placed at the $(2j)^{\text{th}}$ in a for any $j \in \{1, \dots, n\}$. If the $(j - 1)$ members who are ranked above him leave for a , b^j is placed at

the top in b . He moves to a if and only if:

$$\omega - j > -\omega + j - 1 \Leftrightarrow \omega > \frac{2j-1}{2}.$$

By Definition 3, if $\omega > \frac{2j-1}{2}$, $\lfloor \omega \rfloor \geq j$. Given any $\omega \in [0, n]$, $b^1, \dots, b^{\lfloor \omega \rfloor}$ always move to a .

a^x is then placed at the $(2x-1)^{\text{th}}$ in a for any $x \in \{2, \dots, \lfloor \omega \rfloor\}$. There are $(x-1)$ additional members ranked above a^x . If $\omega \geq x-1$, he stays in a and this always holds for any x , given Definition 3. For any x , the loss in ranking for b^x and a^x in a are x and $x-1$ respectively. b^x finds ω sufficient to cover his loss in ranking and so does a^x , whose loss is smaller.

Now consider $a^{\lfloor \omega \rfloor+y}$, where $y \in \{1, \dots, n - \lfloor \omega \rfloor\}$. As for $b^{\lfloor \omega \rfloor}$, $a^{\lfloor \omega \rfloor+y}$ is placed down by $\lfloor \omega \rfloor$ ranks and receives a payoff of $-\lfloor \omega \rfloor + \omega$ in a . By Lemma 1 and 2 below, he receives $\lfloor \omega \rfloor - (y-1) - \omega$ in b .

LEMMA 2. *An induced structure of $\pi(\omega)^{\lfloor \omega \rfloor}$ is always not stable if the winning faction in the structure does not include $a^1, \dots, a^{\lfloor \omega \rfloor}$.*

Proof of Lemma 2. Suppose that $b^1, \dots, b^{\lfloor \omega \rfloor}$ have moved to a . A structure, $\pi(\omega)^{\lfloor \omega \rfloor}$ emerges. Suppose that $l \leq n - \lfloor \omega \rfloor$ members among $a^{\lfloor \omega \rfloor+1}, \dots, a^n$ are *pivotal* in determining the winner of intraparty competition. If l members move to b , b wins. If the deviation takes place, there is a new structure $\pi^{\lfloor \omega \rfloor-l}$ in which the remaining $(n + \lfloor \omega \rfloor - l)$ members in a receive a negative payoff: [loss in ranking] $- \omega$. Each of $a^1, \dots, a^{\lfloor \omega \rfloor}$ is incentivised to move to b and their move is followed by $b^1, \dots, b^{\lfloor \omega \rfloor}$. The politicians continue moving till $\pi(\omega)^{-\lfloor \omega \rfloor}$, in which $b^{\lfloor \omega \rfloor+1}, \dots, b^n$ are in a and the remaining $(n + \lfloor \omega \rfloor)$ members are in b , emerges. $\pi(\omega)^{-\lfloor \omega \rfloor}$ is a ‘mirror image’ of $\pi(\omega)^{\lfloor \omega \rfloor}$. The composition of members in the winning faction is identical in both structures, but the winner of intraparty competition differs. Thus, the l -member deviation is sequentially blocked. ■

If $\omega \geq \lfloor \omega \rfloor$, all of $a^{\lfloor \omega \rfloor+y}$ stays in a . $a^{\lfloor \omega \rfloor+1}$ is placed down by $\lfloor \omega \rfloor$ ranks in a . His ranking rises to the same extent in b . Whenever $\omega \geq \lfloor \omega \rfloor$, he receives a strictly positive payoff in a and a strictly negative payoff in b . Provided that $a^{\lfloor \omega \rfloor+1}$ stays, $a^{\lfloor \omega \rfloor+2}$ finds that his ranking changes to the same extent in both factions. As $\omega \geq \lfloor \omega \rfloor$, he also stays in a and so does $a^{\lfloor \omega \rfloor+y'}$, for any $y' \in \{3, \dots, n - \lfloor \omega \rfloor\}$.

Suppose $\omega < \lfloor \omega \rfloor$. $a^{\lfloor \omega \rfloor+1}$ then moves to b and a new structure, $\pi(\omega)^{\lfloor \omega \rfloor-1}$, is induced. Under the new structure, $b^{\lfloor \omega \rfloor+1}$ finds an incentive to move. He receives a payoff of $-\lfloor \omega \rfloor + \omega$ in a , greater than the payoff of $\lfloor \omega \rfloor - \omega - 1$ in b . Previously he was not allowed to move to a , primarily because of $a^{\lfloor \omega \rfloor+1}$ in a . $b^{\lfloor \omega \rfloor+1}$'s departure provides a room for $a^{\lfloor \omega \rfloor+2}$ who previously was discouraged from moving to b , which, in turn, allows $b^{\lfloor \omega \rfloor+2}$ to move to a , and so on. The ‘stepwise’ deviation among $i^{\lfloor \omega \rfloor+y}$ for $i \in \{a, b\}$ continues as long as it is not sequentially blocked by Lemma 2.

It is established that for any $\omega \in [0, n]$, $b^1, \dots, b^{\lfloor \omega \rfloor}$ always move to a whereas $a^1, \dots, a^{\lfloor \omega \rfloor}$ always stay in a . Suppose that l' members and m' members, among $i^{\lfloor \omega \rfloor+1}, \dots, i^n$, respectively move to b and to a , for any $i \in \{a, b\}$ and $l', m' \in \{1, \dots, n - \lfloor \omega \rfloor\}$. Unless

$$\frac{n - l'}{n - l' + \lfloor \omega \rfloor + m'} > \frac{l'}{n - \lfloor \omega \rfloor - m' + l'} \\ n - \lfloor \omega \rfloor > l' + m',$$

any additional deviation from $\pi(\omega)^{\lfloor \omega \rfloor}$ is sequentially blocked.

The earlier discussion on the stepwise deviation among $i^{\lfloor \omega \rfloor+y}$ where for $y \in \{1, \dots, n - \lfloor \omega \rfloor\}$ implies that $l' \geq m'$. Some of $b^{\lfloor \omega \rfloor+y}$ stay in b even if his counterpart, $a^{\lfloor \omega \rfloor+y}$, has moved to b . When $b^{\lfloor \omega \rfloor+y}$'s move to a changes the winner of intraparty competition from a to b , the higher-ranked will leave a by Lemma 2. Combined with $n - \lfloor \omega \rfloor > l' + m'$, as long as a majority of $a^{\lfloor \omega \rfloor+1}, \dots, n$ stay in a , the higher-ranked will not block the subsequent structure(s) having emerged after some of them have moved to b .

This leads to the second part of Proposition 2. When $n - \lfloor \omega \rfloor$ is odd, $l' = m'$ and the top $\frac{n - \lfloor \omega \rfloor - 1}{2}$ members among $i^{\lfloor \omega \rfloor + 1}, \dots, i^n$ for any $i \in \{a, b\}$ move to the opposite faction. When $n - \lfloor \omega \rfloor$ is even, $l' = m' + 1$. The top $\frac{n - \lfloor \omega \rfloor}{2} - 1$ members among $i^{\lfloor \omega \rfloor + 1}, \dots, i^n$ as well as $a^{(\lfloor \omega \rfloor + \frac{n - \lfloor \omega \rfloor}{2})}$ move to the opposite faction.

Lastly, when $\omega > n - \frac{3}{2}$, the top $(n - 1)$ or all the members in b move to a . Particularly for $\omega \in (n - \frac{3}{2}, n - \frac{1}{2}]$, only the n^{th} members stay in their initial faction. Even if a^n finds an incentive to move to b , his move is sequentially blocked by Lemma 2. Whenever $\omega > n - \frac{3}{2}$, there exists the unique structure in which a has $n + \lfloor \omega \rfloor$ members.

Let's revisit the 'party of six' example. For $\omega \in (\frac{1}{2}, 1)$, after b^1 has moved to a , a new structure, π^{n+1} , emerges but is replaced by $\pi^{(n-1)+1}$ as a^2 moves to b . b^2 is forced to stay in b as his move to a is sequentially blocked. Proposition 2 confirms that for $\omega \in (\frac{1}{2}, 1)$, $\lfloor \omega \rfloor = 1$ and that $\frac{n - \lfloor \omega \rfloor}{2} = \frac{3-1}{2} = 1$ member moves from a to b , whereas none apart from b^1 moves from b to a .

Proposition 2 leads to predict the unique stable structure for each range of $\omega \in (\frac{1}{2}, n]$. Formally:

PROPOSITION .3. *For $\omega \in (\frac{1}{2}, n - \frac{3}{2}]$, the following stable structures exist*

$$\begin{cases} \pi^*(\omega)^{n+\lfloor \omega \rfloor} & \text{if } \omega \geq \lfloor \omega \rfloor; \text{ or if } \omega < \lfloor \omega \rfloor \text{ and } n - \lfloor \omega \rfloor \text{ is odd,} \\ \pi^*(\omega)^{n+\lfloor \omega \rfloor - 1} & \text{if } \omega < \lfloor \omega \rfloor \text{ and } n - \lfloor \omega \rfloor \text{ is even.} \end{cases}$$

If $\omega > n - \frac{3}{2}$, $\pi^(\omega)^{n+\lfloor \omega \rfloor}$ is the unique stable structure.*

Proposition 2 shows and proves that for any $\omega > \frac{1}{2}$, the top $b^1, \dots, b^{\lfloor \omega \rfloor}$ always move to a , whereas their counterparts, $a^1, \dots, a^{\lfloor \omega \rfloor}$ never leave their initial faction, a . The Proposition further indicates that when $\omega < \lfloor \omega \rfloor$ and $n - \lfloor \omega \rfloor$ is odd, an equal number of members ranked below the $\lfloor \omega \rfloor^{\text{th}}$ move to the opposite faction, cancelling out the additional inflow

of members in a . When $\omega < \lfloor \omega \rfloor$ and $n - \lfloor \omega \rfloor$ is even, the number of additional members moving to a is smaller than that moving to b .

Propositions 1–3 imply that ω determines the structure of factions at the *aggregate* level. As ω grows, there will be a greater number of members choosing a , and fewer leaving a . At the *individual* level, the size of ω affects the relative gain and loss in ranking each member receives in each faction.

For any given $\omega \in [0, n]$, a relative loss in ranking in a faction is relatively smaller for the higher-ranked than for the lower-ranked. It is reflected in the variation in the strategies taken by the members at different ranks. The higher-ranked's decision is relatively straightforward, primarily resting on the collective benefits. On the contrary, the lower-ranked members balance between the collective and the individual benefits. If the relative loss in ranking is too great, they are forced to give up the collective benefits. This happens in two directions. The members ranked relatively low in b are discouraged from moving to a . The lower-ranked in a are incentivised to move to b . As ω grows, more members find a relative loss in ranking between the factions small and more members choose a .

The results resonate the splits between LDP factions³ in the 1990s. A major faction (Takeshita) was divided after a group of its members, led by Hata, who allegedly fell behind in competition over the faction head, left. Another split of a major faction (Kato) was triggered when its members' interests conflicted. Some members took sides with the other major factions, instead of their own, incentivised by greater electoral supports from the major factions. Both cases reflect a similar trade-off between the collective and individual benefits in the model. Some choose a minor, 'losing' faction for greater power. Others choose a major, 'winning' faction for greater factional benefits, despite smaller power they exercise in such a faction.

Cox and Rosenbluth (1995) study the LDP members' decision to defect their party after an electoral reform in 1993. They identify the 'types' of politicians more prone to party

³With regard to this, the following section provides detailed discussions.

defection. Politicians who were more “electorally marginal⁴”, “ideologically compatible” with the opposition and less likely to receive spoils from the party were more likely to leave. The individual strength of politicians also mattered, that the members who had previously won elections without an LDP endorsement were more likely to defect. More senior members who had served longer terms were less likely to defect.

Some of these findings are in line with the Propositions. When a party is weakened, suffering from a lack of public support, as the LDP was in the early 1990s, some members are hit more than the others and thus more likely to defect. The effect of ω or a change in ω varies across the members who differ in individual ‘advantages’. A similar result to Cox and Rosenbluth (1995) is derived, predicting member departures from the less-quality factions. The higher-ranked, whose relative loss between the payoff from the factions is greater than the lower-ranked, move to a .

Similarly, the members who expect to perform well without the party brand rely less on the collective good of the party, e.g. party endorsement and campaign support. This reminds us of the decision made by the members in a when leaving for b . Their ranking improves greatly, having an ‘individual-level’ advantage if they move to b . Whenever their faction does not provide sufficient resources ($\omega < \lfloor \omega \rfloor$), they leave. The same option is not available for those ranked lower or at the bottom.

6. Discussion

As briefly introduced, the model studies politics of intraparty factions following Zuckerman’s (1975) definition. He highlights the structures in factions established and maintained to enhance “durability of factions (Persico et al. 2011), and argues that the established structure differentiates intraparty factions from temporary coalitions. Motivated by this, the model focuses on the organisational hierarchies in factions, represented by rankings of faction members. Such hierarchies are observed in practice, especially distinct in

⁴Based on the vote share in the previous election

long prevailing factions such as those in the LDP (Liberal Democratic Party) of Japan. The model extends the organisational characteristics and envisages that the organisational hierarchies provide an additional motivation for members when choosing between intra-party factions, i.e. a higher ranking in the chosen faction.

The model also assumes that the number of factions in the party is fixed at two and that the members cannot create a new faction. In politics, new factions may emerge, for instance, when an existing faction is divided and some of its members leave. At the same time, the faction structure in a party stabilises over time.

A notable example is the Japanese LDP. After the party was founded in 1955, LDP factions underwent a series of splits and mergers competing over party leadership. The number of major factions in the party reduced to five by the early 1970s (Kohno 1997). The five ‘major’ factions continued to prevail over two decades till the early 1990s. Following a political scandal, one of the major factions broke up in 1993. In the midst of public pressure for political reform, the major factions continued to split. The long-maintained structure of LDP factions became volatile. By the end of 1998, the number of factions in the party increased to eight. Soon after, some of the factions started merging and the factions were reorganised into the pre-1993 structure.

Currently there are still five major factions in the party. The major factions in Table 3.1 have prevailed apart from Komoto⁵; some of them have been renamed following changes in faction leadership. Table 3.1 also shows whether and how newly emerged factions survive. Some reduced to minor groups failing to gain strength, such as Kono and Yamazaki factions, or even disappeared, as did Hata faction. Others remained as a major faction by merging. The merger between Watanabe-Nakasone and Kamei factions led to the birth of Murakami-Kamei faction, which became a major group.

⁵Komoto faction has continued and still exists under the leadership of Komura, but has lost strength.

Table 3.1. Major Factions of the LDP in the 1990s

Pre-93	1993	1998	1999
Takeshita	Obuchi Hata	Obuchi × ¹	Obuchi ×
Miyazawa	Miyazawa	Kato Kono	Kato Kono
Mitsuzuka	Mitsuzuka	Mori Kamei	Mori ² Murakami-Kamei
Watanabe	Watanabe	Watanabe-Nakasone Yamazaki	Yamazaki
Komoto	Komoto	Komoto	Komoto

* Major factions before 1993 and in 1999 are typed in **bold**.

¹ Hata faction soon left the party.

² Murakami-Kamei faction was formed by the merger between Watanabe-Nakasone and Kamei factions.

LDP factions often form a coalition to gain a majority in the party. Coalitions of intra-party factions are also found in other countries. In the former Italian Christian Democrats, the number of factions increased over time, from three in 1947 to its peak of 12 in 1982 (Bettcher 2005; Boucek 2012). New factions were created by splits of existing ones. At the same time, “inter-factional blocs” started emerging.

During the period of LDP dominance, the party chairman was selected and “rotated” among the leaders of the major factions (Shinoda 2000). Cabinet portfolios were allocated “proportionally” across the members of the major factions, depending on each faction’s influence (Pekkanen et al. 2014). It implies that the benefits, including cabinet posts, had been concentrated within the major factions or the coalitions of them.

The assumption that limits the number of factions at two is reasonable given the dynamics of factional politics. The structure of intraparty competition gets eventually stabilised. It also incorporates the discrepancy in benefits between the winning (major) and the losing (minor) factions, as evident in the LDP. The model begins with a framework where there is a stronger faction (*a*) of the two in terms of the criteria set by the selection rule adopted by the party. The framework proceeds with the assumption that the members from the

initially stronger faction are advantaged at the individual level over their counterparts in the other faction, i.e. those at the same ranks. Whenever in the same faction, a^j is ranked higher than b^j for any $j \in \{1, \dots, n\}$. As discussed below, the members of the major LDP factions benefited from greater political resources including cabinet posts and these benefits helped them further advance their individual political career. The assumption hence reflects the individual strengths members can potentially develop by being part of a stronger faction.

Consider when the selection criteria, that the faction with the greater average quality wins, is modified, such that the faction with the greater number of members with positive political capital. Now the faction joined by $\frac{n+1}{2}$ (for n odd, and $\frac{n}{2} + 1$ for n even) or more members from a wins the intraparty competition. The key results in Proposition 1–3 do not change. Whenever $\omega \leq \frac{1}{2}$, Proposition 1 remains robust and the unique stable structure is unchanged from the initial structure. For $\omega > \frac{1}{2}$, the top $\lfloor \omega \rfloor$ members always move from b to a . Whether the members ranked at the $(\lfloor \omega \rfloor + 1)^{\text{th}}, \dots, n^{\text{th}}$ are allowed to move between the factions depends on the relative size of ω as before. Suppose that $\lfloor \omega \rfloor \geq \frac{n+1}{n}$ for odd n . Even if all of $a^{\lfloor \omega \rfloor + 1}, \dots, a^n$ move to b , a remains as the winner of intraparty competition and their moves will not be sequentially blocked by the members ranked above. Then, there will be ‘stepwise move’ of all members ranked at the $(\lfloor \omega \rfloor + 1)^{\text{th}}, \dots, n^{\text{th}}$ for any $i \in \{a, b\}$ to the opposite faction. When $\lfloor \omega \rfloor < \frac{n+1}{n}$, only $\frac{n-1}{2}$ top members among $i^{\lfloor \omega \rfloor + 1}, \dots, i^n$ are allowed to move to the opposite faction.

The splits and mergers between LDP factions illustrated in Table 3.1 also reflect the incentives and motivations in the framework. In 1992, Kanemaru resigned as the head of Takeshita faction after a major political scandal⁶. The faction was then divided by disagreement over political reform and new leadership and soon it split into Obuchi and Hata factions.

Some view that the split was an outcome of a power struggle (Curtis 1999). Obuchi was considered to head the faction, while Hata was to succeed Obuchi. Hata was sceptical

⁶It was revealed that a logistics company had lobbied politicians, including Kanemaru.

about his post-Obuchi prospects and instead joined Ozawa to form a new faction where he could be the head and potentially the party chairman when the reform Ozawa had initiated was successful. Hata faction, without gaining much support, left the party and formed a new party.

It is also noted that the ‘winning benefit’ from a major faction was uncertain given the party’s unpopularity following the scandal. The split from Takeshita faction and subsequently from the party helped Hata faction join the non-LDP coalition government that put an end to the LDP reign.

Another relevant example is when a motion of no confidence against Mori, back then the Prime Minister, was introduced in 2000. Kato tried to persuade Yamazaki faction and his own in favour of the motion. He was believed to go against Mori to seek the party leadership. Against Kato’s attempt, the major factions were rumoured to offer the members of Kato and Yamazaki factions campaign funds⁷ and threatened them with expulsion (Scheiner 2006). The motion failed and Kato faction was split.

The faction was divided into two groups: pro-Mori (or anti-Kato) and pro-Kato. After the motion failed, pro-Kato group reduced to a minor faction in the party. The cabinet members previously in Kato faction were not selected in the cabinet reshuffle following the motion, even though not having gone against Mori. It should also be noted that not all suffered. Koga, previously the closest aide of Kato, was soon appointed as the Chief Secretary, a number 2 position in the party. As Koga’s move reflects the departure of the top-ranked members from *b*, a similarly motivated merger took place. The divided groups of the Kato faction were reunited when the previously pro-Kato group was merged into the pro-Mori group, one of the major factions in 2007 (Krauss and Pekkanen, 2011).

The splits of LDP factions that have been briefly illustrated above show that when politicians choose between factions, they are influenced by a combination of different incentives. At the same time, they suggest that the extent to which the incentives affect a

⁷Economist, November 23, 2000

politician's decision may vary across politicians at different ranks.

So far it has been assumed that a faction with the greater average quality wins in intraparty competition. The analysis is also based on an assumption that the size of ω is exogenous and that the players do not form a new faction. In the next section, the modifications of these assumptions are considered to check if the results in Propositions 1–3 remain robust.

7. Modifications to the Model

In this section, a number of modifications to the model are introduced and analysed. The following are considered: an alternative selection rule that a faction with the greater number of members wins; and expand the members' options, that they choose between the two existing factions and a new faction, if formed. The results in this section are illustrated with the 'party of six' examples.

7.1. A Change in the 'Selection Rule'. The assumption on who wins in intraparty competition is modified. Now a faction with the greater number of members wins and receives $\omega \in [0, n]$. If the two factions have the same number of members, the winner is chosen randomly. The remaining assumptions are the same as in the model. As before, stable structures are analysed given ω , again denoted by $\pi(\omega)^{m-l}$, where $m \in \{0, \dots, n\}$ is the number of members moving from b to a and $l \in \{0, \dots, n\}$ is the number of members moving from a to b .

Let's check if the grand coalition of $b = \{b^1, b^2, b^3\}$ is divided, given $a = \{a^1, a^2, a^3\}$. If nobody moves, b^j for any $j \in \{1, 2, 3\}$ receives an expected payoff of $\frac{1}{2}(\omega - \omega) = 0$. b^1 moves if and only if $-1 + \omega > 0 \Leftrightarrow \omega > 1$. By Lemma 1, b^k for any $k \in \{2, 3\}$ moves to

a if $-k + \omega > k - 1 - \omega \Leftrightarrow \omega > \frac{2k-1}{2}$. Given $a = \{a^1, a^2, a^3\}$, b is divided such that

$$b = \begin{cases} \{b^1, b^2, b^3\} & \text{if } \omega \leq 1 \\ \{b^2, b^3\} & \text{for } \omega \in (1, \frac{3}{2}] \\ \{b^3\} & \text{for } \omega \in (\frac{3}{2}, \frac{5}{2}] \\ \emptyset & \text{for } \omega > \frac{5}{2}. \end{cases}$$

The change in the selection rule eliminates the relative disadvantage at the faction level, at least initially. Both factions in the initial structure are equally likely to win. b^1 now demands greater compensation as the relative loss in ranking when he moves to a becomes greater than before. Once b is divided by b^1 's departure, a 'restores' the relative advantage, which makes the relative benefit and loss b^k would expect in both factions the same as before. The range of ω that makes b^k move to a is therefore unchanged from the previous result.

When $\omega \leq 1$, given $b = \{b^1, b^2, b^3\}$, a^1 is the only member who moves to b . By Lemma 1, a^k receives $-k + 1 + \omega$ in b , which is smaller than $k - 1 - \omega$ in a . $\pi(\omega)^{-1}$ emerges. b^1 is then placed down by a rank and receives a negative payoff of $-1 + \omega < 0$. If moving to a , he can keep his initial ranking but his move equalises the two factions in numbers. By Lemma 2, it is sequentially blocked. a^1 follows the move. A stable structure, $\pi(\omega)^{-1}$, is found.

When $\omega \in (1, \frac{3}{2}]$, b^1 is the only member who leaves b and then, $\pi(\omega)^1$. All a^k stay in a , as the payoff in a is strictly positive, whereas the payoff in b is strictly negative. Then, there exists a stable structure, in which only b^1 joins a . When $\omega \in (\frac{3}{2}, \frac{5}{2}]$, $\pi(\omega)^2$ emerges. Whereas i^1 and i^2 for any $i \in \{a, b\}$ receive a strictly greater payoff in a than in b , a^3 finds an incentive to move to b if $-2 + \omega < 2 - \omega \Leftrightarrow \omega < 2$, which is not sequentially blocked and allows b^3 to join a . A stable structure, $\pi(\omega)^{3-1}$, emerges. If $\omega \geq 2$, $\pi(\omega)^2$ remains unchanged and is stable. When $\omega > \frac{5}{2}$, there is $\pi(\omega)^3$ in which all b^j move to a .

In the previous section(s), the analysis begins to examine whether b is divided given the grand coalition of a . It then proceeds to examine whether a after the arrival of members from b breaks. It is not necessary to check and see if the same result is obtained when the analysis begins with the grand coalition of a given the grand coalition of b . Given the initial coalition of b , no member or group of members in a wants to move to b for any ω .

It holds true when $\omega \leq 1$ under the new selection rule. Any a^j receives a zero payoff in a , given $b = \{b^1, b^2, b^3\}$. He is placed at the $(2j-1)^{\text{th}}$ in b by Lemma 1 and obtaining ω . If $\omega > j-1$ he will move to b . For $\omega \leq 1$, a^1 is the only member who moves to b . In the resulting structure, $\pi(\omega)^{-1}$, a move to a by any b^j is sequentially blocked. When $\omega \leq 1$, the unique stable structure, $\pi(\omega)^{-1}$, exists.

This is not the case when $\omega > 1$. Both a^1 and a^2 move to b for $\omega \in (1, 2]$, given $b = \{b^1, b^2, b^3\}$. In $\pi(\omega)^{-2}$, b^2 finds an incentive to move to a if $\omega < \frac{3}{2}$ as $1 + \omega > -2 + \omega$, which allows a^3 to move to b , followed by b^3 moving to a . A stepwise deviation observed earlier in the model also takes place, till $\pi(\omega)^{2-3}$ emerges. If $\omega \geq \frac{3}{2}$, $\pi(\omega)^{-2}$ remains unchanged.

When $\omega \in (2, 3]$, a^1, a^2, a^3 move to b and $\pi(\omega)^{-3}$ emerges. If $\omega < \frac{5}{2}$, b^3 moves to a with the same incentive as for b^2 above. If $\omega \geq \frac{5}{2}$, all the six members are in b in a stable structure. The results for a party of $2n$ is formally stated:

PROPOSITION .4. *Suppose that the greater-numbered faction wins. If $\omega \leq 1$, there exists the unique stable structure in which only a^1 moves to b . In the ranges of $\omega \in (1, n]$, there are multiple stable structures which are ‘mirror images’ of each other in which the winning faction consists of the following members*⁸

⁸The result implies the following compositions of members in the winning faction, which can be either a or b , for $\omega \in (1, 3]$

$$\begin{cases} \{a^1, b^1, a^2, a^3\} & \text{if } 1 < \omega \leq \frac{3}{2} \\ \{a^1, b^1, a^2, b^2, b^3\} & \text{if } \frac{3}{2} \leq \omega \leq 2 \\ \{a^1, b^1, a^2, b^2, a^3\} & \text{if } 2 \leq \omega \leq \frac{5}{2}, \end{cases}$$

- (i) $a^1, b^1, \dots, a^{\lfloor \omega \rfloor}, b^{\lfloor \omega \rfloor}, a^{\lfloor \omega \rfloor+1}, \dots, a^n$, when $\omega \geq \lfloor \omega \rfloor$
- (ii) $a^1, b^1, \dots, a^{\lfloor \omega \rfloor}, b^{\lfloor \omega \rfloor}, b^{\lfloor \omega \rfloor+1}, \dots, b^n$, when $\omega \leq \lfloor \omega \rfloor$

Proposition 4 shows that when $\omega \geq 1$, there are multiple stable structures which are mirror images of each other. For a range of ω , there are stable structures in which the winning faction has the same set of members but the winning faction differs. Under the new selection rule, a relative advantage at the faction level disappears. In the model, the initial average quality of the two factions is assumed to differ. a has the relative advantage. Given $b = \{b^1, b^2, b^3\}$, any a^j does not want to leave a . In the modification, each faction initially has an equal chance of winning. a can be the first one to split. However, the individual-level advantage still prevails. In particular, a^1 with the relative advantage over any player can place himself in the winning faction without a loss in ranking. In fact, which faction wins largely depends on whether he stays in a or moves to b , and whether he is the first one to split the initial faction. The members have the same motivations, resting their decisions on a trade-off between ω and their ranking in each faction. Combined together, they give rise to multiple stable structures as described in the Proposition.

Motivated by Proposition 4, another case is considered. Suppose that one of the two has a clear advantage under the new selection rule. It initially has the greater number of members. Now consider a party of seven with $a = \{a^1, a^2, a^3, a^4\}$ and $b = \{b^1, b^2, b^3\}$ with $\omega \in [0, 3]$.

Result 6-1. *Under the new selection rule, consider an initial structure in which $a = \{a^1, a^2, a^3, a^4\}$ and $b = \{b^1, b^2, b^3\}$. For any range of $\omega \in [0, 3]$, there exists the unique stable structure, in which $b^1, \dots, b^{\lfloor \omega \rfloor}$ always move to a . If $\omega < \lfloor \omega \rfloor$, $i^{\lfloor \omega \rfloor+1}, \dots, i^3$ and a^4 also move to the opposite faction for any $i \in \{a, b\}$.*

Given $b = \{b^1, b^2, b^3\}$, no member leaves the grand coalition of a as in the Main model. b^j , $j \in \{1, 2, 3\}$ moves to a if $\omega > \frac{j}{2}$ by Lemma 1. When $\omega \leq \frac{1}{2}$, no deviation from the and when $\frac{5}{2} \leq \omega \leq 3$, all the six members choose $\sim i$.

initial structure is made and the unique stable structure is $\pi(\omega)$. For $\omega \in (\frac{1}{2}, \frac{3}{2}]$, b^1 moves to a . If $\omega < 1$, the lower-ranked move between the factions in a stepwise order. If $\omega \geq 1$, no further move is made and $\pi(\omega)^1$ is stable.

Similarly for $\omega \in (\frac{3}{2}, 2)$, in $\pi(\omega)^2$, a^3 and a^4 move to b and b^3 moves to a . For $\omega \in [2, \frac{5}{2}]$ the stable structure involves only b^1 and b^2 moving to a . Then, there exists $\pi(\omega)^{3-1}$ for $\omega \in (\frac{5}{2}, 3)$ and $\pi(\omega)^3$ for $\omega = 3$ as the stable structure.

Proposition 4 and Result 6-1 suggest that the selection rule adopted by the party determines the relative (dis)advantage each faction initially holds, which, in turn, determines the ‘direction’ of the members’ departure, i.e. which faction a majority of the party choose. Combined with the results from the model, the faction with the relative advantage over the other never breaks first, at least before the other gets split. Occasionally there might be member departures from the relatively advantaged faction, but a majority of the party gather around the relatively advantaged. The trade-off between the collective and individual benefits when joining a faction remains as the key determinant for the players’ decision.

The key results from the model still apply. The members base their decisions on a trade-off between the collective and individual benefits in each faction. A higher-ranked member expects a smaller trade-off than a lower-ranked one. It is relatively easier for a higher-ranked member to move to the winning faction or the faction with a winning prospect as the loss in his ranking in any faction is limited. Under any selection rule, it would be expected that the winning faction will have a similar set of high-ranked members for given any ω , and as ω rises, more members would choose the winning faction.

A selection rule can be more or less ‘restrictive’ for the relatively lower-ranked members. From Proposition 2, it is implied that some of the members ranked lower than the $\lfloor \omega \rfloor^{\text{th}}$, even if they do not want to, are forced to stay in their initial faction. Their move, which equalises both factions in quality, is sequentially blocked by the higher-ranked. From both Proposition 4 and Result 6-1, it is possible to see under the alternative selection rule, all

of those ranked below the $[\omega]^{\text{th}}$ can move to the faction they want without being restricted.

When there is no relative advantage at the faction level, as reflected in Proposition 4, it becomes difficult to predict the outcome of the factional competition. In the multiple stable structures that have been identified, the faction that gets divided before the other suffers from a greater loss of members and subsequently loses in intraparty competition. Unless the sequence of the players' decision is restricted in some way, any faction can be the winner in a stable structure.

It can be implied that different patterns of splits and mergers can take place when the factions are of similar strength and can continue at least till there emerges a major or a relatively stronger faction. It has been noted earlier that LDP factions underwent a long series of splits and mergers till a group of major factions were established and stabilised. A similar observation is made in the former Italian Christian Democrats. The number of intraparty factions grew from three in 1947 to its peak of 12 in 1982 (Bettcher 2005; Boucek 2012) after the existing factions were divided. Over time “inter-factional blocks” were created as the factions coalesced.

The LDP case also indicates that a group of major factions once established in a party can prevail over a long period of time. Proposition 2 and Result 6-1 suggest how a major faction or a group of major factions can sustain their strength and maintain their reign. The initially stronger faction, determined by the selection rule adopted by the party, continues to hold the relative strength over the rival through intraparty competition. Although it may lose some of its initial members, such departures are kept limited or minimal to the extent that they do not threaten the strength of the faction.

The previous sections have discussed a number of factional splits in the Japanese LDP. Despite the split in the early 1990s, Takeshita faction remained as a major faction of the party. Those who left the faction formed a new faction, but did not gain sufficient support and strength to be a major faction. The model's prediction is in line with this, that when a group of members leave a , they are unable to win in intraparty competition. With respect

to this, the framework can be modified to provide an additional insight.

7.2. A Rise of a New Faction. Suppose that the members also decide to form or join a new faction, denoted by c . The members in the winning faction are awarded party resources of $n + \omega$ units as before. The members in the first runner-up faction enjoy $\lambda(n - \omega)$ units where $\lambda \in (\frac{1}{2}, 1]$, whereas those in the second runner-up get $(1 - \lambda)(n - \omega)$ units. If a new faction is not formed and there is no second runner-up, the losing faction is endowed with party resources of $n - \omega$. The faction with the greater average quality is chosen as the winner. $\pi(\omega)^{x,y,z}$ is defined as a faction structure in which x , y and z members are in a , b and c where $x, y, z \geq 0$ and $x + y + z = 2n$.

The result is illustrated with a party of six. For any $\omega \in [0, 3]$ and $b = \{b^1, b^2, b^3\}$, the grand coalition of a does not break as before. A move to b by a member or a group of members does not take place as it is not profitable. a^2 and a^3 may want to form and join a new faction, c . They can improve their ranking. With probability $\frac{1}{2}$, c wins as a and c both would hold the average quality of 1. However, by Lemma 2, such a move is sequentially blocked.

Let's check if $b = \{b^1, b^2, b^3\}$ breaks, given $a = \{a^1, a^2, a^3\}$. b^1 moves to a if $\omega > \frac{1}{2}$. When $\omega \leq \frac{1}{2}$ and b^1 stays in b , b^k for any $k \in \{2, 3\}$ now has an option to leave and form a new faction. With Lemma 1, b^k receives $n - \omega$ if staying in b and

$$1 + \frac{1}{2}\lambda(3 - \omega) + \frac{1}{2}(1 - \lambda)(3 - \omega) = 1 + \frac{1}{2}(3 - \omega) = 1 + \frac{1}{2}(3 - \omega)$$

if in c . For any $\omega \leq \frac{1}{2}$, $3 - \omega$ is greater. Thus, the initial coalition of b does not break and the initial structure remains unchanged.

When $\omega > \frac{3}{2}$, Results 1 and 2 remains robust. For $\omega \in (\frac{3}{2}, \frac{5}{2}]$, b^1 and b^2 move to a . a^3 , even if wanting to move to b or create c , stays in a by Lemma 2. b^3 is the only member staying in b . When $\omega > \frac{5}{2}$, there is one 'grand coalition' where all the six players choose a .

Previously it was found that

$$b = \begin{cases} \{a^2, b^2, b^3\} & \text{for } \omega \in (\frac{1}{2}, 1) \\ \{b^2, b^3\} & \text{for } \omega \in [1, \frac{3}{2}] \end{cases}$$

in the unique stable structure for each range of ω . For $\omega \in (\frac{1}{2}, 1)$, when b^1 moves to a , b^3 is now left with an option of staying in b and of forming and moving to c . He receives a payoff of $1 + (3 - \omega)$ in b , whereas $2 + \frac{1}{2}(3 - \omega)$. For $\omega \in (\frac{1}{2}, 1)$, his expected payoff in c never exceeds what he receives for sure in b . No new faction is formed. When a^2 moves to a , $\pi(\omega)^{3,3,0}$ emerges. b^2 does not form a new faction as $3 - \omega > 1 + (1 - \lambda)(3 - \omega)$. b^3 forms a faction if and only if:

$$3 - \omega < 2 + (1 - \lambda)(3 - \omega),$$

$$\lambda(3 - \omega) < 2.$$

As $\lambda \rightarrow \frac{1}{2}$, or $\omega \rightarrow 1$, a new faction forms. Suppose that c has formed and b^2 also joins c . Then a^3 is left alone in b and b wins in intraparty competition. By Lemma 2, the resulting structure $\pi(\omega)^{3,1,2}$ is sequentially blocked. To prevent this, a^2 will also join c . Provided this, whenever c is formed, it is optimal for b^2 to join as his move will lead to a structure, $\pi(\omega)^{3,0,3}$, in which he receives a payoff of $3 - \omega$ which is greater than $\lambda(3 - \omega)$. Even if $\lambda(3 - \omega) < 2$, b^3 does not form and join a new faction.

Similarly for $\omega \in [1, \frac{3}{2}]$, in $\pi(\omega)^{4,2,0}$, even if b^3 forms a new faction, he will be soon joined by b^2 . b^2 receives a payoff of $1 + \frac{1}{2}(n - \omega)$ in b and $1 + n - \omega$ in c . For $\omega \in (\frac{1}{2}, \frac{3}{2}]$, there is the unique stable structure identical to what Proposition 3 shows. Summarising the findings from the modification,:

Result 6-2. *When the players can form and join a new faction in a party of six, the results of the model remain robust. In a stable structure, a new faction never forms.*

Result 6-2 shows that even if a new faction is formed and joined by a group of members, it never sustains and exists in a stable coalition. It also indicates that when party resources are distributed less disproportionately across the losing factions, the relatively lower-ranked in the losing faction may find an incentive to create a new faction. The incentive is particularly strong when $\lambda \rightarrow \frac{1}{2}$, that the losing factions share roughly the same size of resources. The relatively lower-ranked in the losing faction can improve his ranking without losing too much of the collective benefit in a new faction. However, the formation of a new faction reduces the collective benefits of the (initial) losing faction as now they have to share the resources with those in another faction. This works as an incentive for a merger between the losing factions. This is reflected when a decision to form a new faction is sequentially blocked by the higher-ranked in the losing faction.

Whether a new faction is formed following a split of the existing factions, the members' decisions are still based on a trade-off between the collective and individual benefits. The relatively lower-ranked form and join a new faction when choosing the individual gain over the collective benefit. The higher-ranked can overturn the newly emerged structure, as it lowers their collective benefits.

As ω rises, the number of members choosing a increases and no new faction is created. Result 6-2 suggests that when party resources are distributed highly disproportionately not only across the losing factions, as it has been discussed above ($\lambda \rightarrow 1$), but also between the winner and the rest, the incentive for a new faction diminishes. For given λ , as ω rises, the relative benefit from a new faction decreases. With the more higher-ranked members leaving for a , the gain in ranking in a new faction reduces. The share of party resources allocated to a new faction also gets smaller.

The result could be further analysed again in connection with the dynamics of LDP factions. Kohno (1997) argues that the electoral system (single non-transferable vote in multi-member districts) had given members of major factions strong electoral advantages over those in minor factions or unaffiliated. The party under the electoral system limited candidate endorsement to maximise the seats won and endorsement was primarily given to

those from the major factions. Other benefits, including cabinet posts and campaign funding, were also allocated mainly to them. This helped the major factions to maintain their strength without experiencing significant defects and splits. The five major factions established and kept a system of power-sharing. The party leadership was rotated between the faction leaders and the cabinet posts were allocated across their members.

Table 3.2. Faction Affiliation of LDP Diet Members

	1969	1972	1976	1980	1984	1989
Major Faction Membership (%)	74.3	81.7	85.6	82.9	89.1	92.6
Unaffiliated (%)				13.2	8.5	3.7

* Calculated based on the data in Masumi (1995)

Table 3.2 shows the proportion of the LDP members in the House of Representatives (the lower chamber) who were affiliated with one of the major factions and those that were not affiliated with any intraparty factions over time. Since the early 1970s, when the five major factions emerged, membership to these factions increased, whereas fewer and fewer members chose to be unaffiliated. As the major factions expanded their power within the party and became able to offer greater benefits to their members, the minor factions reduced both in numbers and in membership and were merged into major factions (Kohno 1997).

Table 3.3. Number of LDP Diet Members in Minor Factions, 1974–1980

	Miki ¹ (74–76)	Fukuda ¹ (76–78)	Ohira ¹ (78–80)
Shiina	18	9	
Ishii	9	4	
Funada	9	8	
Mizuta	13	11	5
Nakagawa ²			10

* Masumi (1995)

¹ The Prime Ministers over the periods indicated.

² Nakagawa faction includes a number of former members from Shiina, Ishii and Mizuta factions.

In Table 3.3, the number of Diet members affiliated with minor factions of the party and the number of minor factions in the party decreased throughout the 1970s, when the system

of the five major factions started to stabilise. Result 6-2 provides a theoretical explanation for this. Minor factions compete against each other for the party resources remaining for them. As the major factions dominated party resources (ω high), and mergers of several minor factions gave rise to a relatively stronger group (λ high), the presence of minor factions further diminished.

A number of modifications to the model have been considered. The modifications indicate that the key determinant for the members' decision remains robust. A trade-off between the different types of benefits in each faction influences their choice over the factions. It is relatively smaller for the high-ranked. In fact, both modifications affect mostly the decisions of the lower-ranked to a limited extent, that their decisions remain relatively restricted, as in the second modification in which a new faction, even if formed by the bottom-ranked, is merged into another. As ω rises, the effect of the changes in the modifications on the (lower-ranked) members becomes smaller or disappears.

8. Conclusion

This paper have presented and analysed a new model of intraparty factions. It considers the factions competing over party resources and consisting of politicians who decide to move between the factions. Their decision over the factions determines the winner in intraparty competition. The members of the winning faction enjoy greater party resources. The model is sufficiently simple and does not rely on too many arbitrary assumptions, using a stability concept. The stability concept is effective to identify the optimal decisions of each player and predict the resulting structure of the intraparty factions.

In identifying the strategic behaviour of the politicians, this paper focuses on the collective and individual benefits that the politicians expect when joining a faction. While the politicians want to be in a faction that brings greater resources, they want to be ranked higher and exercise greater power in their faction. The results show that a trade-off between the two types of benefit determines each politician's choice over the factions.

If a politician can enjoy greater resources while exercising greater power in a faction than the other, his decision is straightforward. This is often not the case both in politics as well as in the model. A politician chooses a faction over the other if the faction better compensates for a loss in a type of benefit, collective or individual, than the other. The results have proven particularly helpful to identify and understand the motivations for politicians leaving a mainstream faction for a minor one, a slightly puzzling phenomenon in comparison to the arrival of politicians into a strong faction.

A number of modifications to the model are considered. The trade-off that the model identifies continues to influence the politicians' decision over the faction. Whichever selection rule is adopted to choose the winner of intraparty competition, if the selection rule favours a faction over the other in the initial structure of factions, the initially advantaged faction prevails. The higher-ranked politicians, whose relative loss in ranking is relatively limited in any faction, manage to belong to the winning faction. Among the relatively lower-ranked, there are some politicians who choose to leave the winning faction for a rise in ranking under any selection rule. Although there is some variation in the decisions of the relatively lower-ranked between the selection rules, as the collective benefits from the winning faction increase, the politicians' decision under the different rules becomes more or less similar.

The model restricts that the politicians choose one of the existing factions, but cannot form and join a new faction. A modification to this assumption is also analysed. It is shown that even if formed, a new faction lasts only temporarily and does not 'sustain' in a stable structure. The relatively lower-ranked politicians may be incentivised to leave an existing faction and to form a new, but minor, faction. As party resources are more disproportionately between the winning and the losing faction(s), such an incentive diminishes. Furthermore, a new faction, soon after its formation, gets merged into another minor faction.

The results of the model are shown to provide a theoretical explanation for a number of empirical phenomena observed in factional politics. They contribute to the relatively

small literature of factions and hierarchies in formal political science. The key contributions include that the model not only explains endogenous formation of factions but also dynamics of factions reflected in splits and mergers of factions. It clearly addresses the features that distinguish intraparty factions from other types of factions and that give rise to a conflict of interest between faction members, resulting in mergers and splits between factions.

There are potential limitations of the model. Politicians' choices over intraparty factions are influenced by a variety of factors other than those in the model. Factors such as ideological or policy preferences of individual politicians, as addressed in some of the existing formal models, could be the key determinant in some politicians' decision. It is also possible that there are more complex interactions of the types of factors in the model than how they are envisaged in the model.

Another possible limitation could be the way that the ranking of the politicians is determined. The simple framework, in which every initial member of a faction has a relative advantage over the member of the same ranking in the other, has been proven effective and sufficient to predict and explain a number of observations in politics. The model also does not provide any welfare implication of factional competition which is one of the themes in Persico et al. (2011) and Dewan and Squintani (2015).

The limitations present directions for future research. The extension of the model could further incorporate the multifaceted motivations of politicians and modify the patterns of the organisational hierarchy in the framework to better reflect the complexity of decisions over intraparty factions.

4. Hierarchies of Power and Decisions

Chapter Abstract

This paper presents a simple model of endogenous allocation of power, integrating different types of organisational hierarchies. It connects power allocation to the longevity of power. It portrays a leader who wants to exercise greater control over her organisation, while extending the longevity of her leadership. Fulfilling the two interests simultaneously is not straightforward. The greater the power she holds, the members of the organisation are more incentivised to remove her and reallocate power across them. In identifying the optimal power structure that alleviate the leader's trade-off between power and survival, the model considers decision 'procedures' in the organisation. A decision procedure specifies who has the right to initiate or propose an issue, how an initiative is passed onto the members, who is involved in decision-making and how a decision is reached. A procedure is more hierarchical, if it limits the 'proposal right' to a few members and if it imposes multi-level decision making. The game starts as the leader allocates power across the members and herself, followed by the members deciding over a coup against her according to the organisation's procedure. The results show that as the number of 'de facto proposers' increases, the leader allocates power in a more hierarchical manner. While a democratic procedure grants the proposal right essentially to every member, an extremely hierarchical procedure, although the proposal right per se is given to a very few, produces additional members who are granted a similar right. In such an organisation, an initiative is passed onto the members in sequence and whether it is passed down to the others is determined by each member. The members with a procedure power hold greater bargaining power than those without it. When increasing the number of such members and making them too powerful, the leader has to give up substantial power to disincentivise a coup. Then only a few are involved in decisions, exercising positive power. The results predict that the leader's trade-off between power and survival is minimised in an organisation, in which the number of 'de facto proposers' is kept minimum. This become more significant

in a large organisation. They reiterate findings in the literature of legislative bargaining focusing on procedure rights, as well as some empirical observations on party leadership

1. Introduction

After the resignation of Ed Miliband in 2015 the UK Labour Party elected its new leader under a new set of rules. Changes were made in nomination and voting regulations. At least 15% of the Labour MPs, a rise from 12.5%, needed to support the nomination of a candidate and the ‘three-way electoral college’ system was removed. Previously, the final number of votes to each candidate was calculated as a weighted average of the votes from the three groups: the MPs¹, the remaining party and affiliated societies including trade unions. Votes from each of the three groups carried an equal weight. Under the new rule, each and every registered member and supporter of the party became eligible to vote and each vote carried the same weight.

The UK Conservative Party also operates a set of rules, sharing similarities and differences with that of the Labour Party, to elect and remove its leaders (Quinn 2012; Cross and Blais 2012). Conservative MPs have the right to nominate, and to introduce a vote of confidence in the current leader and decide on the motion if introduced². The Conservative MPs exercise greater power when removing a leader than their counterparts in the Labour party. In the Labour, the decision of removal is determined by a new leadership election with the incumbent leader contested by a challenger.

The rules adopted by the British parties when electing and removing their leaders reflect different types of organisational hierarchies. Some of the rules are related to the issue of *how much influence each party member can exercise* in (dis)electing party leaders. Others are concerned with *who has the right to make an initiative* on leadership issues, such as candidate nomination and a proposal for a vote of (no) confidence. Whereas the former indicates a ‘hierarchy of power’, i.e. how decision-making power is allocated across the members of a party, the latter reflects a ‘hierarchy of decisions’, or of decision procedures, i.e. whether the proposal right is limited to a few or how structured the decision process is.

¹As well as Labour members of the European Parliament

²A recent incident of a vote of confidence took place when Iain Duncan Smith back then the Conservative leader was replaced in 2003.

This paper further explores the two organisational hierarchies, in power and decisions, and analyses their interaction by connecting them to the longevity of leadership. Specifically, it answers the question of, whether a particular pattern of power allocation better serves the interests of a political leader, that she wants to exercise greater power while extending the longevity of her leadership. It also examines if there is a variation in the optimal allocation of power when the organisation adopts a different decision procedure.

Some have also acknowledged the need to analyse an interaction between the two organisational hierarchies, especially when studying political leadership. Quinn (2005) who has examined the effect of eviction rules on the longevity of leadership in British parties, finds that the risk of removal for a leader grows with an asymmetric power distribution. He addresses that eviction rules adopted by a party should be understood in connection with the distribution of power in the party. He concludes that concentration of power on a few, as reflected in the procedures of a vote of confidence and nomination, is a potential cause for the relatively more frequent “plots” against the Conservative leadership.

In exploring the strategic interaction of the two hierarchies, this paper considers a model of endogenous power allocation. In the framework, the leader of an organisation who has an interest of maximising her power and survival in office, allocates decision-making power across the members and herself. The members then decide whether to initiate and support a coup against the leader. The coup decision is determined by a weighted majority. If the sum of power held by the members supporting exceeds the threshold, the leader is removed.

In allocating power, the leader balances between different factors. Firstly, her risk of removal increases with the power she exercises. If the leader holds a greater share of power, the members expect a gain by overthrowing her and redistributing power. She seeks to allocate power across the organisation in order to reduce the members’ incentive to overthrow her. The model finds that the leader’s trade-off between power and survival varies across the decision procedures of the organisation. This, in turn, affects the pattern of a

power hierarchy that the leader chooses in equilibrium.

The model considers three different types of decision procedures. They differ in who have the right to initiate a coup, and how the initiative is passed onto the members. In a democratic organisation, any member can initiate a coup. The entire group of the members decide over a coup simultaneously. In hierarchical organisations, only the highest-ranked member, holding the greatest power among the members, can initiate. In a ‘flat-pyramid’ organisation, when initiating a coup, the highest-ranked passes the initiative onto the rest of the members simultaneously. The rest of the members decide whether to support the initiative simultaneously. In a ‘top-down’ organisation, when an initiative is made, the members decide in sequence, according to the size of power they exercise. The initiative is passed onto the higher-ranked first. The model defines that a procedure is hierarchical, when only a few can initiate and when decisions are made in a predetermined sequence. The opposite is true for a democratic procedure.

In equilibrium, the power hierarchy that the leader adopts is hierarchical, when the decision procedure becomes more ‘de facto’ democratic. Power is allocated in a relatively centralised manner and a few hold much greater power than the rest of the members. One of the key elements for the result lies in the variation in expected payoffs of the members in a ‘coalition’, against and in favour of the leader, across the procedures. Following a coup, whether or not it succeeds, power is redistributed across the members, corresponding to their marginal contribution in overthrowing or keeping the leader. If a member is pivotal in such a coalition, he anticipates a greater gain in power in the post-coup organisation.

With regard to this, a variation in the number of ‘de facto proposers’ is observed across the procedures. Whereas the democratic procedure allows any member to initiate a coup, the two hierarchical procedures specify that only the most powerful member can do so. However, the two hierarchical procedures produce a different number of ‘de facto proposers’. When the organisation has adopted the most hierarchical procedure, a coup decision can be reached by a few powerful members, each of whom decides not only whether to support the initiative, but also whether to pass the initiative onto the remaining member(s).

Compared to the less hierarchical one, the procedure allows additional members to hold a ‘procedure right’, similar to the proposal right it specifies. Hence, the most hierarchical procedure is shown to be ‘de facto’ democratic.

The members prefer being part of a successful coalition, whether or not against the leader, even if they exercise no power, to being removed from organisational decisions. The motivation leads that the members, whenever deciding simultaneously, form a ‘grand coalition’. In a democratic organisation, if the leader delegates a large share of power to a few members, as much as they anticipate in the new regime after overthrowing the leader, they are disincentivised to support the removal of the leader and the rest of members coordinate accordingly. It subsequently implies that the leader has to give up substantial power for survival.

In a flat-pyramid organisation, when an initiative is made, the remaining members compare between power redistributed under the leader and the ‘proposer’, and choose between the two simultaneously. In equilibrium, The leader makes contribution each of the remaining members necessary, but minimal in a successful coup. The proposer still holds substantial power. Each of the remaining members then finds greater power redistributed after he is removed and opposes the initiative, which, in turn, prevents any initiative. The leader manipulates the discrepancy between power redistributed to the remaining members and does not need to give up power as much as in a democratic organisation.

In a top-down organisation, the members engage in sequential bargaining. They form a ‘minimal winning coalition’ in equilibrium, and their contribution to a successful coalition becomes greater than under the other types of organisation. Compared to a flat-pyramid organisation, the presence of the greater number of ‘de facto proposers’ reduces the leader’s power. The leader finds similar incentives to those in a perfectly democratic organisation and limits the ‘de facto proposal right’ to only a few. The optimal allocation of power resembles that under perfect democracy.

The results contribute to the formal literature on hierarchies, which is relatively unexplored. They address the interaction between different types of organisational hierarchies. A new insight into the issues of power and leadership is provided. It has been often enough the case that formal models focus on the absolute size of power that political actors can exercise. An additional dimension is added to the analysis by addressing the issue of longevity. The model shows that it is a trade-off between the two attributes of power that influences the strategic behaviour of political actors. The results also offer a theoretical explanation for organisational hierarchies, of power and of decisions, observed in politics.

The approach of using a non-cooperative framework with a cooperative solution concept is relatively unique in the political science literature. Shapley value, a cooperative solution concept, is adopted to calculate marginal contributions of the players in the model. It is not used in political science as widely as in other fields, despite its applicability to political problems. Applying Shapley value to a non-cooperative setting, it is possible to derive a unique set of equilibria, while identifying strategic interactions between players.

The paper is organised as follows. In Section 2, the related literature is discussed. Section 3 introduces the model. In Section 4, the equilibrium is defined and analysed. The intuitions for the equilibrium are also illustrated with examples. Section 5 discusses the motivations for the framework including some of the assumptions and considers a simple modification of the model. Section 6 concludes the paper.

2. Related Literature

The paper addresses a number of issues that are part of political life but are rarely discussed in the formal literature. It presents a framework that links the allocation of decision-making power in an organisation, first to the decision procedure of the organisation, and then to longevity of leadership. There are a few studies that have analysed each of these aspects separately. The results of the model offer a new insight into the related issues by integrating them in a framework and can contribute to the relatively underdeveloped areas

in the literature.

A few formal models have discussed hierarchies of power in organisations (Persico et al. 2011; Morelli and Park 2016). Persico et al. (2011) look at intraparty factions in which the faction members are ordered according to their ranking in the faction and motivated by a rise in ranking. Promotion depends on the election of a candidate the faction supports. The candidate's electoral prospect improves as greater party resources are allocated to his district. The faction members can exert effort to secure this. In Morelli and Park (2016), the players, who differ in ability, decide whether to join a coalition. They want to obtain a high-ranked position in the coalition they choose. A player's ranking in a coalition is determined by how many players, with ability higher than his, also choose the coalition, which subsequently affects his share of benefit from the coalition.

In Persico et al. (2011), the factions compete against each other over party resources. The faction members, who are exogenously assigned to the faction, are motivated by a common objective. There is no power struggle between them. Each of them is promoted to a higher rank when their candidate is elected and expects their faction to be dissolved otherwise. Undivided cooperation between the faction members is thus anticipated. The authors also incorporate a hierarchical structure, which is exogenous. On the contrary, this paper considers a framework which explicitly addresses endogenous formation of hierarchy and of coalitions.

In Morelli and Park (2016), each player's payoff in a coalition increases in the value of his coalition, which is determined by the ability of the coalition's members, and in his ranking in the coalition. There exists a trade-off between collective and individual benefits in the player's decision. The players want to be part of a coalition with high-ability players, but their ranking may be lower in such a coalition. The players compete against each other in order to secure greater power and a membership to a coalition is not exogenous. Although sharing some similarities such as endogenous formation of coalitions, the presence of endogenous 'bargaining power' distinguishes this paper from Morelli and Park (2016). In the model of this paper, the ranking and share of benefit each player expects when joining

a coalition are determined not only by the other players' strategic behaviour, which is the case in Morelli and Park (2016), but also by endogenous allocation of initial power.

Another aspect in the setting that differentiates the framework from Morelli and Park (2016) is the presence of the procedure that specifies the process of coalition formation, which subsequently determines the form of a coalition. The results of the model shows that depending on the procedure adopted by the organisation, and on the leader's allocation of power, some members, even if they want to join, are not even invited into or allowed to join a coalition. Furthermore, one of the features that derive the equilibrium result in this paper is, that some procedures grant the 'de facto proposal right' to additional members. Such features do not exist in Morelli and Park (2016), in which the players' decision to join a coalition rests purely on the trade-off between collective and individual benefits from the coalition.

This paper is also related to the literature on bargaining over budget or redistributive benefit (Romer and Rosenthal 1978; Baron and Ferejohn 1989), cabinet portfolios (Austen-Smith and Banks 1988, 1990) and decision-making power (Dewan et al. 2015). These models have often analysed bargaining under different decision rules, such as a simple majority (Baron and Ferejohn 1989), a weighted voting (Banks and Duggan 2000; Snyder et al. 2005) and a unanimity (Merlo and Wilson 1995; Eraslan and Merlo 2014). However, the focus of this paper is not on the rules that determine how individual decisions are translated into the final outcome. It instead looks at the effect of the procedure that the players follow before and when making the decision, when the final outcome is reached by an exogenous decision rule (a weighted majority).

Some of the existing models have considered different legislative procedures, such as a committee rules (open versus closed rule, notably in Baron and Ferejohn 1989), veto power (Krehbiel 1998; McCarty 2000) and procedural power, including proposal and gatekeeping power (Romer and Rosenthal 1978; Denzau and Robert 1983; Krehbiel 2004; Diermeier et al. 2014). They have analysed their effect on bargaining and policy outcomes.

The procedures that this paper considers share some similarities with those adopted in legislative bargaining. Each procedure specifies who can initiate a coup against the leader, and how such an initiative is passed onto the players and is processed toward an agreement. The result that the leader has to give up greater power in a more ‘de facto democratic’ organisation than in a hierarchical one, reiterates what the theories of veto players and agenda setting have predicted. It reaffirms the findings from the models of legislative bargaining that even if allocated an equal share of decision-making power, those who have the procedure power have greater bargaining power. A departure of this paper from these models could be that it adds to an additional dimension to the analysis of bargaining procedures. It considers an interaction of decision-making power and procedural power and analyses its effect on coalition bargaining and distribution, of power.

Dewan et al. (2015)³ examine endogenous decision-making power across the ideological players, who communicate publicly or privately. The key difference between Dewan et al. and this paper lies in the motivation and objective of the players, and the hierarchy in communication or decision procedures. The former analyses the optimal power allocation that facilitates effective information sharing in an unrestricted setting, where the players choose whether and who to truthfully reveal. In this paper, who and what the players ‘communicate’ are predetermined and restricted by the procedure adopted in the organisation. They ‘communicate’ their decisions according to the procedure, but do not share or exchange information.

This paper considers a framework, in which the leader delegates power to the members, to prevent a coup against her. She can eliminate the risk of removal from office. Similar motivations are found in the formal models that discuss power-sharing under a dictatorship or an oppressive regime (Bueno de Mesquita et al. 2003; Gandhi and Przeworski 2007; Boix and Svolik 2013). They show that power-sharing between the leader and a selected

³There are formal models built on motivations similar to Dewan et al. (2015), analysing truthful communication of information and/or effective information aggregation in a group. Harris and Yariv (2005) study endogenous allocation of decision-making power in investment decisions with a framework of information transmission. The choice in Harris and Yariv is binary, between “centralisation” and “delegation”, different from this paper, which assumes a specific level of influence each player exercises in organisational decisions.

group is adopted to extend the longevity of leadership, and this is done by providing private benefits to a “minimal winning coalition” (Bueno de Mesquita et al. 2003), or to a group of potential opposition (Gandhi and Przeworski 2007). The leader may solve the credibility problem through establishing institutions including elections (Boix and Svolik 2013; Myerson 2008).

Finally, this paper adopts a methodological approach, that incorporates a cooperative solution concept into a non-cooperative framework. Such attempts have been made mostly in economics literature to explore multi-player bargaining in a general framework (Gul 1989; Krishna and Serrano 1995; Perez-Castrillo and Wettstein 2001), or in an economic or business setting, such as transactions within and between firms (Hart and Moore 1990). An illustration of the approach is provided in Section 4, along with the definition of the equilibrium is provided after introducing the model in the next section.

3. Model

The paper considers a simple framework of a n -person organisation, where $n \geq 4$. The organisation consists of the leader and $(n - 1)$ members. The leader decides how much power the members and she exercise in organisational decisions. She holds a share of power, $\alpha_1 \in [0, 1]$, and allocates the remaining across the members. Member $i \in \{2, \dots, n\}$ exercises power, $(1 - \alpha_1)\alpha_i$, where

$$\begin{cases} \alpha_2 \geq \alpha_3 \geq \dots \geq \alpha_{n-1} \geq \alpha_n & \text{and} \\ \alpha_2 + \dots + \alpha_n = 1. \end{cases}$$

Member i holds greater power than member $j > i$. The members are ranked according to the size of their power. Member i is ranked at the i^{th} in the organisation, with the leader at the top. If two or more members have the same share of power, their ranking is randomly determined.

Coup Initiative. The members decide whether to initiate and support a coup against the leader. The decision of member $i \in \{2, \dots, n\}$ is denoted by

$$x_i = \begin{cases} 0 & \text{if } i \text{ opposes a coup} \\ 1 & \text{otherwise.} \end{cases}$$

A coup-initiative succeeds, if the sum of power held by the coup-supporting members is greater than $(1 - \alpha_1) \frac{1}{2}$. For $i \in \{2, \dots, n\}$,

$$(4.1) \quad \sum_{i|x_i=1} \alpha_i > \frac{1}{2}.$$

Otherwise, the initiative fails. Therefore, the members in the set, \mathcal{N} , play a *cooperative* game, $\mathcal{G} = (w, q)$, when deciding whether to go against the leader, where $w = (\alpha_2, \dots, \alpha_n)$ is a vector of weights and $q = \frac{1}{2}$ is a quota. A coalition in the set of the members, $\mathcal{S} \subseteq \mathcal{N}$, is successful with $v(\{\mathcal{S}\}) = 1$, if $\omega(\mathcal{S}) := \sum_{j \in \mathcal{S}} \alpha_j > \frac{1}{2}$. Otherwise it is unsuccessful with $v(\{\mathcal{S}\}) = 0$. Note that $v(\emptyset) = 0$ and $v(\mathcal{N}) = 1$.

Who can initiate a coup depends on the type of procedure, denoted by c , in the organisation. The procedure of the organisation is exogenous. Specifically, the model considers,

- (a) when $c = d$ (**perfect democracy**), the members simultaneously decide whether to support a coup. If (4.1) is satisfied, a coup is successful. Essentially, under perfect democracy, any member can initiate a coup;
- (b) when $c = h_1$ (**flat-pyramid**), the 2nd-ranked in the organisation decides whether to initiate a coup. If he does so, he passes the initiative onto the remaining members simultaneously. The remaining members decide whether to support the initiative simultaneously;
- (c) when $c = h_2$ (**top-down**), the 2nd-ranked still decides whether or not to initiate a coup. When initiating a coup, he passes the initiative first onto the 3rd-ranked. If he does not receive support from the 3rd-ranked, the 2nd-ranked continues and passes the initiative onto a member ranked at the 4th or below until a member supports the initiative. When supporting the initiative, the 3rd-ranked then passes it onto the 4th-ranked or a member ranked below till the initiative is supported by a member, and so on.

Under any procedure, the members if indifferent choose to oppose a coup. They also prefer being part of organisational decisions even if exercising no power to being excluded from decisions. The members' decision is perfectly observed by everyone in the party.

When a coup succeeds, a coup-supporting member receives a share of power, equivalent to his contribution to the success. The non-supporting members and the leader lose their power and are not removed from organisational decisions.

When a coup fails, the coup-supporting members are removed. A non-supporting member is redistributed power, $(1 - \alpha_1)\hat{\alpha}_j$, where $\hat{\alpha}_j$ is his contribution to the failure of the coup. The leader continues to exercise α_1 . If no coup is initiated, the initial allocation, $\alpha_1, \dots, \alpha_n$, prevails.

Payoffs. Member $i \in \{2, \dots, n\}$, when not removed from decisions, receives an expected payoff,

$$U_i = \begin{cases} (1 - \alpha_1)\alpha_i & \text{if no coup takes place} \\ (1 - \alpha_1)\hat{\alpha}_i & \text{if a coup has failed and } i \text{ is not part of it} \\ \psi_i & \text{if a coup has succeeded and } i \text{ is part of it.} \end{cases}$$

Whereas α_i is member i 's share in $(1 - \alpha_1)$ allocated by the leader, $\hat{\alpha}_i$ and ψ are his Shapley values in a coalition respectively in favour of and against the leader. The Shapley values, $\hat{\alpha}_i$ and ψ , calculate member i 's *marginal* contribution to the success of a coalition. Consider a simplest example, in which there are two players and the only successful coalition is such that both players join. Each player's contribution to the success of the coalition would be the same and their Shapley value would be $\frac{1}{2}$.

Formally, given $\alpha_2, \dots, \alpha_n$, the Shapley value of member $i \in \{2, \dots, n\}$ in a coalition against the leader is

$$\psi_i(\mathcal{N}, v) = \sum_{\mathcal{S}, i \in \mathcal{S}} \frac{(|\mathcal{S}| - 1)!(|\mathcal{N}| - |\mathcal{S}|)!}{|\mathcal{N}|!} [v(\mathcal{S}) - v(\mathcal{S} \setminus \{i\})]$$

with $\sum_{i \in \mathcal{N}} \psi_i(\mathcal{N}, v) = 1$. ψ_i is the sum of the values that member i adds to the success of each possible coalition against the leader, weighted by the likelihood of each of such coalitions. $[v(\mathcal{S}) - v(\mathcal{S} \setminus \{i\})]$ indicates the value member i adds to a coalition, \mathcal{S} , when joining it and compares the success of the coalition with and without him. Therefore,

$$[v(\mathcal{S}) - v(\mathcal{S} \setminus \{i\})] = \begin{cases} 1 & \text{if } \mathcal{S} \text{ is successful and fails if } i \text{ leaves } \mathcal{S} \\ 0 & \text{if } \mathcal{S} \text{ is successful without } i \\ 0 & \text{if } \mathcal{S} \text{ is unsuccessful with or without } i. \end{cases}$$

$\frac{(|\mathcal{S}|-1)!(|\mathcal{N}|-|\mathcal{S}|)!}{|\mathcal{N}|!}$ denotes the likelihood of each possible coalition with and without i .

Similarly, $\hat{\alpha}_i$ is derived to calculate member i 's marginal contribution to a coalition in favour of the leader. An example is given below to illustrate how the Shapley values are derived.

Example) Consider a five-person organisation with the leader and four members. Suppose that the leader has assigned $w = (\alpha_2, \alpha_3, \alpha_4, \alpha_5) = (0.4, 0.3, 0.2, 0.1)$. The outcomes of possible coalitions against the leader, $v(\mathcal{S})$ are as follows:

$$\begin{cases} v(\{2, 3\}) = v(\{2, 4\}) = 1 \\ v(\{2, 5\}) = v(\{3, 4\}) = v(\{3, 5\}) = v(\{4, 5\}) = 0 \\ v(\{2, 3, 4\}) = v(\{2, 3, 5\}) = v(\{2, 4, 5\}) = v(\{3, 4, 5\}) = v(\{2, 3, 4, 5\}) = 1, \end{cases}$$

and $v(\{i\}) = 0$, for any $i \in \{2, 3, 4, 5\}$. Successful coalitions, that involve the 2nd-ranked member ($i = 2$) are:

$$v(\{2, 3\}), v(\{2, 4\}), v(\{2, 3, 4\}), v(\{2, 3, 5\}), v(\{2, 4, 5\}), v(\{2, 3, 4, 5\}).$$

Coalitions, $v(\{2, 3\})$, $v(\{2, 4\})$, $v(\{2, 3, 4\})$, $v(\{2, 3, 5\})$ and $v(\{2, 4, 5\})$, are no longer successful if the 2nd-ranked is not part of the coalition, i.e. $v(\mathcal{S} \setminus \{2\}) = 0$. The grand coalition, $v(\{2, 3, 4, 5\})$, is still successful, even if he leaves. The Shapley value for the

2nd-ranked in a coalition against the leader is:

$$\begin{aligned}\psi_2(\mathcal{N} = 4, v) &= \sum_{\mathcal{S}, i=2 \in \mathcal{S}} \frac{(|\mathcal{S}| - 1)! (|\mathcal{N}| - |\mathcal{S}|)!}{\mathcal{N}!} [v(\mathcal{S}) - v(\mathcal{S} \setminus \{2\})] \\ &= 2 \frac{1!2!}{4!} + 3 \frac{2!1!}{4!} = \frac{5}{12},\end{aligned}$$

where $|\mathcal{S}|$ and $|\mathcal{N}|$ are respectively the number of the members in coalitions, in which he is involved, and the number of the members in the game. The Shapley values for the remaining members are also derived:

$$\Psi = (\psi_2, \psi_3, \psi_4, \psi_5) = \left(\frac{5}{12}, \frac{3}{12}, \frac{3}{12}, \frac{1}{12} \right).$$

The leader's expected payoff, when she stays in office, corresponds to her power, α_1 . Any-one who is removed from decisions receives a zero payoff. Removal from decisions is the worst outcome for the members and the leader.

The timing of the game is summarised:

- 1) The leader allocates $(\alpha_1, \dots, \alpha_n)$
- 2) The members decide whether to initiate and support a coup
- 3) The game ends and the payoffs are distributed.

The following section defines the equilibrium and introduces the solution concept this paper adopts.

4. Definition and Analysis of Equilibrium

The framework introduced in Section 3, involves a sequential game, in which the players make strategic decisions over the coalitions, against and in favour of the leader, given the set of power allocated by the leader, $(\alpha_1, \dots, \alpha_n)$, and according to the procedure of the organisation, $c = \{d, h_1, h_2\}$. In finding a solution to the leader's allocation problem, the Shapley value (Shapley 1953) is adopted. The Shapley value is often used as a solution

concept for a cooperative game. It quantifies the marginal contribution that each individual player makes to the success of a coalition. The individual contribution determines the player's share of benefits from the success.

As Gul (1989) points out, a cooperative approach to a bargaining problem may not sufficiently analyse strategic interactions between the players. In response to this, a number of studies have addressed the possibility of applying the solution concept to a non-cooperative framework (Gul 1989; Hart and Moore 1988, 1990; Hart and Mas-Colell 1995a, 1995b).

Gul (1989) is one of the first to consider this. In his model, two of N players in a market are chosen in each period and decide whether or not to form a coalition. If they reach an agreement, they receive a corresponding payoff and leave the market. The game continues to the next period in which a new set of players negotiates and till there is no player remaining in the market. The model has a non-cooperative (stationary subgame perfect Nash) equilibrium, in which the expected payoff of each player corresponds to his/her Shapley value.

Hart and Moore (1988, 1990) analyse models similar to mine. In a multi-stage game (Hart and Moore 1988), the i^{th} player makes an offer to the $(i + 1)^{\text{th}}$ player, who then moves into the next stage and makes an offer to the $(i + 2)^{\text{th}}$ player, and so on. This continues until stage I , when the outcome of the previous $(I - 1)$ offers, made by the $1^{\text{st}}, \dots, (I - 1)^{\text{th}}$ player, is realised. Each player receives a payoff equal to his/her Shapley value. Hart and Moore (1990) also consider a framework, in which the players form a coalition over profits from physical assets. The players contribute to profits through their ownership over assets and through endogenous investment in effort. Profits are shared by coalition members according to their Shapley value.

Building on a framework, similar to the earlier models that have proposed “a non-cooperative foundation for cooperative solution concepts” (Serrano 2005), it is possible to derive the unique solution to the leader's allocation problem. In the n -person organisation of the

model, the members make decisions over a coup initiative against the leader, given the organisation's procedure, $c = \{d, h_1, h_2\}$, and the leader's allocation of decision-making power, $\alpha = (\alpha_1, \dots, \alpha_n)$. The notion of equilibrium is subgame perfect equilibrium. A strategy profile of the members in the $(n - 1)$ -tuple, $x = (x_2, \dots, x_n)$ is subgame perfect, if and only if the strategy of member i is optimal, after every history of the game, given (i) the strategies of the other $(n - 2)$ members, and (ii) $\alpha = (\alpha_1, \dots, \alpha_n)$. Formally,

DEFINITION .1. Given $c = \{d, h_1, h_2\}$ and $\alpha = (\alpha_1, \dots, \alpha_n)$, a strategy profile of the members, $x = (x_2, \dots, x_n)$, where

$$x_i = \begin{cases} 0 & \text{if } i \text{ opposes a coup initiative} \\ 1 & \text{otherwise} \end{cases}$$

is optimal, if $U_i(\alpha, x) > U_i(\alpha, x'_i, x_{-i})$, where $x_i \neq x'_i$, for any $i \in \{2, \dots, n\}$.

Similarly, α is optimal in a subgame perfect equilibrium, given the optimal strategies of members, denoted by x^* , under each procedure. That is,

DEFINITION .2. Under any procedure, c , the subgame perfect allocation of power, $\alpha^* = (\alpha_1^*, \dots, \alpha_n^*)$, is consistent with x^* and satisfies $U_1(\alpha^*, x^*) > U_1(\alpha, x^*)$, for any $\alpha^* \neq \alpha$.

Following the definition of the equilibrium in the previous section, this section provides the equilibrium analysis for each type of the procedure. The results are illustrated along with examples. The section starts with a perfectly democratic organisation

4.1. Perfect Democracy. The members simultaneously decide whether to support a coup against the leader in a perfectly democratic organisation. Essentially, each of them has the right to initiate a coup. When a coup succeeds, those that have opposed are removed. When it fails, the supporters are removed. The decision made by each member is perfectly observed by everyone else in the organisation. In equilibrium, each of them has the following optimal strategy.

LEMMA .1. *The optimal behaviour of the members who make a decision simultaneously is symmetric and identical.*

Proof of Lemma 1. Let (x_2, \dots, x_n) be the set of optimal strategies of the members, where $x_2 = \dots = x_n \in \{0, 1\}$. Suppose that the leader has allocated $\alpha_2 > \frac{1}{2}$. If the j^{th} -ranked, for any $j \in \{3, \dots, n\}$, deviates to $x'_j \neq x_j$, leading to (x'_j, x_{-j}) , he is the only one outside the successful coalition, either against or in favour of the leader, and is removed. Even if $\psi_j = 0 = (1 - \alpha_1)\hat{\alpha}_j$, member j prefers to be in the ‘winning’ coalition. Given $x_2 > \frac{1}{2}$, the coalition chosen by the 2^{nd} -ranked, whether against or in favour of the leader, will be the winning faction. Similarly, when $m' \in \{2, \dots, n - 2\}$ members, other than the 2^{nd} -ranked, simultaneously deviate from x_j , for any $j \in \{2, \dots, n\}$, they are removed, as their coalition never succeeds. Playing $x_j = x_2$ is strictly preferred. Suppose that $\alpha_2 \leq \frac{1}{2}$. Consider a successful coalition of $m < n - 1$ members against the leader which fails if one of m members leaves the coalition. Suppose that the coalition takes place. Each of the remaining $n - m - 1$ members finds an incentive to join the coalition, as they least prefer being removed. The m -member coalition expands till all n members support and the grand coalition forms. Finally suppose that the grand coalition against the leader is formed. If there is a member or a group of members who finds a profitable deviation, the grand coalition breaks. The remaining members also deviate. Therefore, whenever the members simultaneously decide, their optimal strategies are symmetric and identical. ■

Lemma 1 shows that whenever a coup is expected to be initiated and successful, even if a member anticipates smaller power in the post-coup organisation, he also supports. Otherwise, he is removed in the post-coup organisation completely, which is the worst outcome. Therefore, in equilibrium, the only possible coalition is the grand coalition of the $(n - 1)$ members, either against or in favour of the leader.

Suppose that $\alpha_2 > \frac{1}{2}$. A coup is always successful, if the 2nd-ranked supports. He receives $\psi_2 = 1$ following a successful coup, as $v(\{2\}) = 1$, but $v(\{3\}) = \dots = v(\{n\}) = v(\mathcal{S} \setminus \{2\}) = 0$. He opposes a coup if and only if $(1 - \alpha_1)\alpha_2 = 1$, i.e. $\alpha_1 = 0$ and $\alpha_2 = 1$.

For $\alpha \leq \frac{1}{2}$, consider a four-person organisation, in which there are three members. Given $\alpha_2 \leq \frac{1}{2}$, and thus $v(\{i\}) = 0$, for any $i \in \{2, 3, 4\}$, the following two-member coalitions against the leader are feasible

- (i) $v(\{2, 3\}) = 1$, but $v(\{2, 4\}) = v(\{3, 4\}) = 0$
- (ii) $v(\{2, 3\}) = v(\{2, 4\}) = 1$, but $v(\{3, 4\}) = 0$
- (iii) $v(\{2, 3\}) = v(\{2, 4\}) = v(\{3, 4\}) = 1$.

In each of (i)–(iii), if any successful two-member coalition takes place, $v(\{i, j\}) = 1$, for any $i \neq j \in \{2, 3, 4\}$, the marginal contribution that each member makes to the two-member coalition is $\frac{1}{2}$. In order to prevent any of such coalitions from taking place, the leader needs *both* of the members to receive greater power by opposing, i.e.

$$(1 - \alpha_1)\hat{\alpha}_{i(j)} \geq \frac{1}{2},$$

where $\hat{\alpha}_{i(j)}$ is the redistributed power for member i or j , when he leaves the two-member coalition and takes the side of the leader. The inequality implies that as $\hat{\alpha}_{i(j)}$ rises, the leader can reduce her loss of power, $(1 - \alpha_1)$. In other words, the leader has an incentive to make the ‘pivotality’ of member i or j in failing the two-member coalition as great as possible.

To see this, consider case (i), in which substantial power has been allocated to the 2nd- and the 3rd-ranked. If one of them deviates from $\{2, 3\}$, the coalition fails and the deviating member receives redistributed power, $1 - \alpha_1$. His deviation changes which ‘coalition’,

against or in favour of the leader wins. Hence, the leader can keep $\alpha_1 = \frac{1}{2}$. The grand coalition against the leader does not take place, either, as long as $\alpha_1 = \frac{1}{2}$. If any of the two leaves the grand coalition, he gets removed and the remaining member receives $(1 - \alpha_1)$.

The 3-member strategic game is illustrated in Figure 4.1, where

$$x_i = \begin{cases} 0 & \text{if the } i^{\text{th}}\text{-ranked opposes} \\ 1 & \text{otherwise,} \end{cases}$$

for any $i \in \{2, 3, 4\}$. Each member's power is indicated in the brackets. Figure 4.1 shows

	$x_3 = 0$	$x_3 = 1$
$x_2 = 0$	$(1 - \alpha_1)\alpha_2, (1 - \alpha_1)\alpha_3, (1 - \alpha_1)\alpha_4$	$1 - \alpha_1, 0, 0$
$x_2 = 1$	$0, 1 - \alpha_1, 0$	$\frac{1}{2}, \frac{1}{2}, 0$
$x_4 = 0$		
	$x_3 = 0$	$x_3 = 1$
$x_2 = 0$	$\frac{1 - \alpha_1}{2}, \frac{1 - \alpha_1}{2}, 0$	$1 - \alpha_1, 0, 0$
$x_2 = 1$	$0, 1 - \alpha_1, 0$	$\frac{1}{2}, \frac{1}{2}, 0$
$x_4 = 1$		

Figure 4.1. Case (i) in a 4-Person Organisation under Perfect Democracy

that whenever $(1 - \alpha_1) \geq \frac{1}{2}$, the 2nd- and the 3rd-ranked always choose to oppose. The 4th-ranked also chooses to oppose in equilibrium by Lemma 1.

Consider case (ii), a special case in which $\alpha_2 = \frac{1}{2}$ and $\alpha_3 + \alpha_4 = \frac{1}{2}$ with $\alpha_3 \geq \alpha_4 > 0$. Any two-member coalition with the 2nd-ranked, $\{2, 3\}$ or $\{2, 4\}$, is prevented with $1 - \alpha_1 \geq \frac{1}{2}$. Suppose that the 2nd-ranked deviates from $\{2, 3\}$. Now the coalition in favour of the leader has two members, 2nd- and 4th-ranked, with $v(\{2\}) = v(\{2, 4\}) = 1$. As $\alpha_3 + \alpha_4 = \frac{1}{2}$ and (4.1) does not hold even if the 4th-ranked joins the 3rd-ranked. Therefore, $v(\{4\}) = 0$. The 2nd-ranked receives the redistributed power, $\hat{\alpha}_1 = 1$, as in case (i). The same applies to $\{2, 4\}$. However, setting $\alpha_1 = \frac{1}{2}$ does not prevent the grand coalition against the leader. The 2nd-ranked receives $\psi_2 = \frac{2}{3} > \frac{1}{2}$, greater than his marginal contribution to a two-member coalition against the leader. Unless the leader sets $1 - \alpha_1 = \frac{2}{3} \Leftrightarrow \alpha_1 = \frac{1}{3}$, the grand coalition against the leader takes place and succeeds. Figure 4.2 shows that the best-response for the 3rd- and 4th-ranked is to choose the same strategy that the 2nd-ranked chooses, by the logic in Lemma 1. When $1 - \alpha_1 \geq \frac{1}{2}$, the two-member coalition against

		$x_3 = 0$	$x_3 = 1$	
$x_2 = 0$		$(1 - \alpha_1)\alpha_2, (1 - \alpha_1)\alpha_3, (1 - \alpha_1)\alpha_4$	$1 - \alpha_1, 0, 0$	
$x_2 = 1$		$0, \frac{1 - \alpha_1}{2}, \frac{1 - \alpha_1}{2}$	$\frac{1}{2}, \frac{1}{2}, 0$	
		$x_4 = 0$		
		$x_3 = 0$	$x_3 = 1$	
$x_2 = 0$		$1 - \alpha_1, 0, 0$	$1 - \alpha_1, 0, 0$	
$x_2 = 1$		$\frac{1}{2}, 0, \frac{1}{2}$	$\frac{2}{3}, \frac{1}{6}, \frac{1}{6}$	
		$x_4 = 1$		

Figure 4.2. Case (ii) in a 4-Person Organisation under Perfect Democracy

the leader does not take place. However, unless $1 - \alpha_1 \geq \frac{2}{3}$, there exist multiple equilibria, in which two grand coalitions take place, one against the leader, and the other in favour of the leader. If $1 - \alpha_1 \leq \frac{2}{3}$, then the leader needs to allocate $(1 - \alpha_1)\alpha_i = \psi_i$ to prevent a coup. It is impossible to achieve, even if $\alpha_1 = 0$, given $\frac{1}{2} \geq \alpha_2 > \alpha_3 \gtrsim \alpha_4$.

Finally, in case (iii), any two-member coalition is successful. Given $i \neq j \neq k \in \{2, 3, 4\}$, member i receives the redistributed power, $(1 - \alpha_1)\hat{\alpha}_i = (1 - \alpha_1)\frac{1}{2}$, when deviating from $\{i, j\}$. The deviation leads to another coalition, $\{i, k\}$, now in favour of the leader. In $\{i, k\}$, $v(\{i\}) = v(\{k\}) = 0$, whereas $v(\{i, k\}) = 1$, leading to $\hat{\alpha}_i = \hat{\alpha}_k = \frac{1}{2}$. Such a deviation is profitable if and only if $(1 - \alpha_1)\hat{\alpha}_i = \frac{1}{2} = \psi_i$, implying that $\alpha_1 = 0$.

The discussion above indicates whenever the leader makes one member too powerful, even if $\alpha_2 \leq \frac{1}{2}$, it costs her greater loss. This is evident in case (ii). Furthermore, making all of them equally pivotal in failing a coup, as in case (iii), is not profitable, either. It reduces $\hat{\alpha}_i$ for those opposing. Instead, it is shown to be optimal for the leader to allocate similar power across the top two members and to establish a system of ‘checks and balances’. Both of them are interdependent on each other in succeeding a coup, but each of them is powerful and pivotal enough to fail the other’s attempt to initiate a coup. The results are formally stated in the following Proposition.

PROPOSITION .1. *Consider a perfectly democratic ($c = d$) n -person organisation, in which the members decide on a coup simultaneously. In equilibrium, the leader prevents*

a coup and she allocates

$$\begin{cases} \alpha_2^{d^*} = \alpha_3^{d^*} = \frac{1}{2} \\ \alpha_4^{d^*} = \cdots = \alpha_n^{d^*} = 0 \end{cases}$$

across the members and $\alpha_1^{d^*} = \frac{1}{2}$ to herself.

Proof of Proposition 1. Whenever $v(\{2\}) = 1$, but $v(\{j\}) = 0$, for any $j \in \{3, \dots, n\}$, the leader has to compensate the 2nd-ranked with $1 - \alpha_1 = 1 \Leftrightarrow \alpha_1 = 0$. Suppose $v(\{i\}) = 0$, for any and all $i \in \{2, \dots, n\}$. Let \mathcal{S}' as a set of coalitions among the $(n - 2)$ members who are ranked below the 2nd. If $v(\{2, \mathcal{S}'\}) = 1$, but $v(\mathcal{S} \setminus \{2\}) = 0$, then each j holds $\psi_j = \frac{(2-1)!(n-2)!}{n!} = \frac{1}{(n-1)(n-2)}$, as the only positive contribution they make is to the two-member coalition, $\{2, j\}$, in the coalition against the leader. The 2nd-ranked keeps $\psi_2 = 1 - (n - 2)\psi_j = \frac{n-2}{n-1}$. Suppose that the leader prevents the $(n - 2)$ -member coalitions in which the 2nd-ranked is part of, in which he receives $1 - (n - 3)\frac{1}{(n-2)(n-3)} = \frac{n-3}{n-2}$, by allocating $(1 - \alpha_1) = \frac{n-3}{n-2}$. In order to prevent the grand coalition against her, she has to disincentivise the 2nd-ranked by $(1 - \alpha_1)\alpha_2 = \frac{n-2}{n-1} > \frac{1}{2}$: a contradiction to the assumption, $v(\{2, \mathcal{S}'\}) = 1$, but $v(\mathcal{S} \setminus \{2\}) = 0$. Finally, suppose that n' two-member coalitions with the 2nd-ranked are successful, where $n' \in \{1, \dots, n - 2\}$. Each of the n' members needs to get compensated with $(1 - \alpha_1)\frac{1}{n'} \geq \frac{1}{2}$, when deviating from the two-member coalition with the 2nd-ranked, which never satisfy for $n' \geq 3$, even if $\alpha_1 = 0$. The leader keeps greater power when $n' = 1$ than when $n' = 2$. Hence, she allocates power such that $v(\{2, 3\}) = 1$, but $v(\mathcal{S} \setminus \{2\}) = v(\mathcal{S} \setminus \{3\}) = 0$, as in the Proposition. The allocation that satisfies this would be $\alpha_2 = \alpha_3 = \frac{1}{2}$ and $\alpha_4 = \cdots = \alpha_n = 0$. Whenever there is a successful coup taking place, the top two members will always be part of the coup and each of them expects a Shapley value, $\frac{1}{2}$. If a member of the two deviates and opposes the coup, his marginal contribution to failing the coup, $\hat{\alpha}_2$ or $\hat{\alpha}_3$, becomes 1 and exercises power equal to $1 - \alpha_1$. If the leader allocates $1 - \alpha_1 = \frac{1}{2}$, the leader can effectively prevent a coup and exercises $\alpha_1 = \frac{1}{2}$ herself. ■

Proposition 1 provides a number of implications, along with the ‘4-person organisation’ example. By Lemma 1, the leader in equilibrium has the grand coalition of the $(n - 1)$

members, either against or in favour of her. Even if she manages to prevent coalitions of a smaller size, unless she prevents the grand coalition, she is removed. This is evident in case (ii) in the 4-person organisation. Combined with the outcome when $\alpha_2 > \frac{1}{2}$, case (ii) implies that making a single member too powerful is never profitable. It raises his bargaining power much greater than what he is initially allocated, even in the grand coalition. She has to give up greater control of the organisation to such a member. Making too many members ‘equally’ pivotal is found to be not profitable, either, as in case (iii). It reduces the pivotality of the effected members. The discrepancy between the redistributed power in the coalitions against and in favour of her, which helps the leader greater power in case (ii), decreases with the number of pivotal player.

To further illustrate the latter point, consider a n -person organisation, where n is sufficiently large. Suppose that the leader has allocated $\frac{1}{n'} < \frac{1}{2}$ to $n' \leq n$ members and zero to the rest. Assume that all $1, \dots, (n' - 1)$ -member coalitions are prevented. In the grand coalition against the leader, each of n' members holds $\frac{1}{n'}$ and each of the rest holds 0. Given $\frac{1}{n'} < \frac{1}{2}$ and $(n - 2)$ members in the coalition, a deviation never takes place. A member who deviates is removed for sure. Unless $(1 - \alpha_1)\frac{1}{n'} \geq \frac{1}{n'}$, which holds with equality if and only if $\alpha_1 = 0$, she cannot prevent a coup.

Instead, if she allocates power according to Proposition 1, each of the two member is sufficiently compensated when deviating from the two-member coalition, $\{2, 3\}$, as $\hat{\alpha}_2 = \hat{\alpha}_3 = 1$. The grand coalition against the leader never sustains. The Proposition also shows that the optimal allocation of power in a perfectly democratic organisation is highly centralised, or hierarchical. Only a few exercises substantial power, whereas the rest of the members exercise power almost zero.

4.2. Flat-Pyramid. In any hierarchical organisation, the members move *sequentially*. When $\alpha_2 > \frac{1}{2}$, the 2nd-ranked never passes an initiative, even if he makes one. The analysis for the hierarchical procedures proceeds with the following assumption.

ASSUMPTION 1. The members are sequentially rational. In a hierarchical organisation, the j^{th} -ranked, for any $j \geq 2$, stops and does not pass the initiative to the lower-ranked, once sufficient support for a successful coup is gathered, with $\sum_{i \leq j} (\alpha_i | x_i = 1) > \frac{1}{2}$.

With regard to Assumption 1, recall Example 1, with $(\alpha_2, \alpha_3, \alpha_4, \alpha_5) = (0.4, 0.3, 0.2, 0.1)$. Suppose that the 2^{nd} -ranked has initiated and the 3^{rd} -ranked supports. If the 3^{rd} -ranked passes the initiative onto the 4^{th} -ranked, and the 4^{th} -ranked also supports, ψ_3 reduces from $\frac{1}{2}$ to $\frac{1}{3}$. To see this, whenever an initiative is made, the following coalitions against the leader are possible: $v(\{2, 3\}) = v(\{2, 4\}) = v(\{3, 4\}) = v(\{2, 3, 4\}) = 1$. In coalition, $\{2, 3\}$, each of the two members holds $\psi_2 = \psi_3 = \frac{1}{2}$. If the 3^{rd} -ranked passes the initiative to the 4^{th} -ranked who also supports, then $\psi'_3 = 2 \frac{(3-2)!(2-1)!}{3!} = \frac{1}{3}$, smaller than his marginal contribution to the 2-member coalition with the 2^{nd} -ranked. It is not profitable to invite members, whose support is not needed, into a coalition.

When $\alpha_2 > \frac{1}{2}$, the potential outcome remains unchanged from perfect democracy. The 2^{nd} -ranked receives $\psi_2 = 1$ in any successful coup. Unless the leader allocates $(1 - \alpha_1)\alpha_2 = 1$, i.e. $\alpha_2 = 1$ and $\alpha_1 = 0$, he initiates a coup and succeeds. The same applies to another type of a hierarchical organisation, ‘top-down’. Whenever he initiates a coup, he does not pass the initiative onto a member or a group of member. Unless he is allocate the entire control over the organisation, he initiates.

When $\alpha_2 \leq \frac{1}{2}$ and the 2^{nd} -ranked initiates a coup, he then passes it onto the remaining members simultaneously. The $(n - 2)$ members then decide whether to support the initiative simultaneously. Lemma 1 still applies to those ranked at the $3^{\text{rd}}, \dots, n^{\text{th}}$. Whenever the 2^{nd} -ranked has initiated, the $(n - 2)$ members form a grand coalition, either against the leader or against him. Let’s begin the analysis by formally stating the equilibrium result in a flat-pyramid organisation⁴:

⁴See Appendix for the proof.

PROPOSITION .2. Consider a flat-pyramid organisation ($c = h_1$), in which only the 2nd-ranked can initiate a coup and the remaining ($n - 2$) members decide whether to support, only when the 2nd-ranked passes the initiative onto them. The optimal power allocation across the members, that prevents a coup in equilibrium, is

$$\begin{aligned}\alpha_2^{h_1^*} &= \frac{1}{2} \\ \alpha_3^{h_1^*} = \cdots = \alpha_n^{h_1^*} &= \frac{1}{2(n-2)}.\end{aligned}$$

In equilibrium, the leader exercises $\alpha_1^{h_1^*} = \frac{n-2}{n-1}$.

When a coup is initiated, the ($n - 2$) members find it optimal to oppose if and only if:

$$(4.2) \quad (1 - \alpha_1) \hat{\alpha}_j \geq \psi_j,$$

for each and every $j \in \{3, \dots, n\}$. $\hat{\alpha}_j$ is the j^{th} -ranked's contribution to the grand coalition against the 2nd-ranked.

(4.2) implies that for given α_j , as $\frac{\psi_j}{\hat{\alpha}_j}$ decreases, the leader's share, α_1 , increases. It is in the leader's interest that the contribution, that each of the ($n - 2$) members makes to a coup, is limited to a minimum, but necessary for a success of a coup. Suppose that the leader has allocated the same amount of the power to each and every j . Let $\alpha_j = y$, for any $j \in \{3, \dots, n\}$, such that $\frac{1}{2} = \alpha_2 = 1 - (n - 2)y > y$ and $\alpha_2 + y > \frac{1}{2}$. Then $v(\{2, \mathcal{S}'\}) = 1$ for any $\mathcal{S} \neq \{\emptyset\}$ and $v(\mathcal{S} \setminus \{2\}) = 0$ for any \mathcal{S} . Each j holds power equal to,

$$\psi_j = \frac{(n-1-2)!(2-1)!}{(n-1)!}$$

in the coalition against the leader, where $\frac{(n-1-2)!(2-1)!}{(n-1)!}$, indicates j 's marginal contribution to the two-member coalition, $\{2, j\}$. Note that if $v(\{3, \dots, n\}) = 1$, then $\psi'_j = \frac{(n-1-2)!(2-1)!}{(n-1)!} + \frac{(n-1-(n-2))!(n-2-1)!}{(n-1)!} = \frac{2}{(n-2)(n-1)}$, greater than ψ_j above. $v(\{3, \dots, n\}) = 1$ increases j 's marginal contribution.

Similarly, each j 's contribution in the coalition against the 2nd-ranked is derived, i.e. $\hat{\alpha}_j = \frac{1}{n-2}$. Provided that initially, each j has been given y , where $(n-2)y = \frac{1}{2}$, their marginal contribution is also the same in the coalition against the 2nd-ranked. (4.2) is rewritten as

$$(1 - \alpha_1) \frac{1}{n-2} \geq \frac{1}{(n-1)(n-2)}$$

and holds with equality, if $\alpha_1 = \frac{n-2}{n-1}$. Provided that the $(n-2)$ members oppose, the 2nd-ranked does not initiate.

In equilibrium, the leader creates a power hierarchy, similar to the organisation's procedure, and less hierarchical than under perfect democracy. A single member is relatively powerful and more members exercise positive power. He cannot succeed in a coup alone, similar to the 2nd-ranked under perfect democracy.

In a flat-pyramid organisation, the leader also does not want to make the member who has the right to initiate a coup too powerful. Further strengthening his 'proposal power' by allocating him $\alpha_2 > \frac{1}{2}$ is never profitable. It forces the leader to give up the entire control over the organisation. If she allocates $\alpha_2 > \frac{1}{2}$ and exercises non-zero power, the earlier-mentioned case in the British Conservative Party can be anticipated. As Quinn (2005) has noted, if the power both to initiate a coup, and to determine the outcome of the initiative, is given to a member or a small group of members, the leader is either removed, or forced to give up substantial power. Furthermore, the $(n-2)$ members receive reallocated power, when the 2nd-ranked is removed for a failed coup. What is initially allocated to the 2nd-ranked is redistributed across those opposing him.

The leader can manipulate the organisational features and the incentives they provide. By allocating the same power across the $(n-2)$ members, such that their power is far smaller than the 2nd-ranked, but necessary for a successful coup, the leader creates a discrepancy between their redistributed power in a coalition, against and in favour of her. It, in turn, helps the leader reduce the trade-off between power and survival and assign herself greater power, as $\frac{n-2}{n-1} > \frac{1}{2}$, for any $n \geq 4$.

4.3. Top-Down. The discussion for a flat-pyramid organisation shows the outcome in a hierarchical organisation, whenever $\alpha_2 > \frac{1}{2}$. Now suppose that $\alpha_2 \leq \frac{1}{2}$.

If the 2nd-ranked initiates a coup and needs support from the others, he first passes it onto the member, who is ranked immediately below him. When he supports the initiative but needs support from an additional member, the 3rd-ranked passes the initiative onto the 4th-ranked. If the 3rd-ranked opposes, the 2nd-ranked passes the initiative onto the 4th-ranked, and so on. The members are approached by a higher-ranked member and when supporting, he passes the initiative onto the lower-ranked, if necessary. By Assumption 1, a successful coup in a top-down organisation takes the form of a ‘minimum winning coalition’, in which a departure of a member results in a failure of the coup.

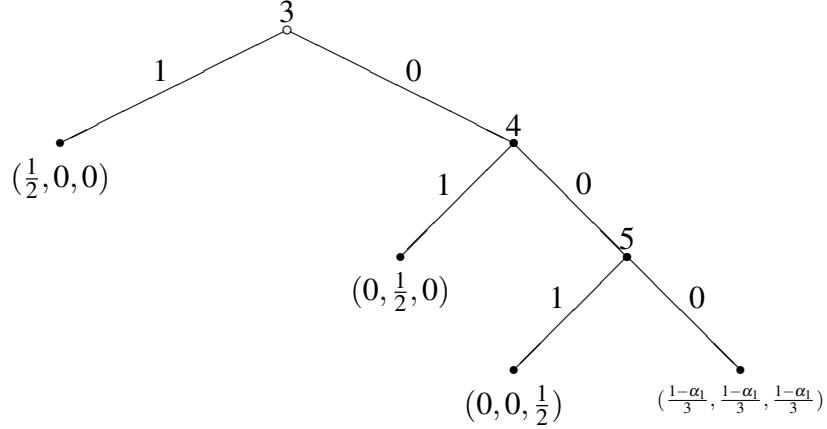
Suppose that the leader has allocated power across the members as described in Proposition 2, implying that $v(\{2, j\}) = 1$, for any $j \in \{3, \dots, n\}$. Even if all the members, ranked at the 3rd, \dots , $(n-1)$ th, have opposed, the 2nd-ranked still communicates with the n th-ranked. He stops as soon as a member at any rank supports. A successful coup involves the 2nd-ranked and another member, ranked at the j th, and $\psi_2 = \psi_j = \frac{1!0!}{2!} = \frac{1}{2}$.

Then, the leader needs each and all of the $(n-2)$ members to oppose, when the initiative is passed. To see this, consider a five-person organisation. Assume $v(\{2, j\}) = 1$, for any $j \in \{3, 4, 5\}$. The sequential game is illustrated in Figure 4.3, where

$$x_j = \begin{cases} 0 & \text{if opposes the initiative} \\ 1 & \text{otherwise} \end{cases}$$

for $j \in \{3, 4, 5\}$. The post-coup power redistributed to the j th-ranked, either $\hat{\alpha}_j$ or ψ_j , is indicated in brackets.

Figure 4.3 indicates that the leader needs to make each of the three members find his expected payoff, when opposing the 2nd-ranked, greater than their payoff in the two-person coup. It is found that if the 5th-ranked opposes, then the remaining two also oppose.

Figure 4.3. When $v(\{2, j\}) = 1 \forall j \in \{3, 4, 5\}$ in a Top-Down Organisation

However, he always finds,

$$(1 - \alpha_1) \frac{1}{3} < \frac{1}{2},$$

for any α_1 . Whenever an initiative is passed, he supports and so do the other members. A coup is always initiated and successful.

Similarly, in the n -person organisation, the redistributed power, $\hat{\alpha}_j$, that each of the $(n-2)$ members expects when opposing the initiative, is $\hat{\alpha}_j = \frac{1}{n-2}$. This leads to $(1 - \alpha_1)\hat{\alpha}_j = (1 - \alpha_1)\frac{1}{n-2} < \psi_j = \frac{1}{2}$, for any α_1 . In Propositions 1–2, the variation between ψ_j and $\hat{\alpha}_j$, which has helped the leader exercise positive power in the Propositions, does not exist. Moreover, the leader, even if she sets $\alpha_1 = 0$, cannot prevent a coup.

The ‘five-person organisation’ example above reiterates the leader’s motivation when allocating power in the previous type(s) of an organisation. She needs to minimise the bargaining power of the 2nd-ranked, who has the right to initiate, as well as each member’s contribution to a successful coup. However, her calculation is not as straightforward as before, as the latter becomes much greater in a top-down organisation. The example implies a minimal winning coalition, whenever a coup succeeds. If a member leaves such a coalition, the coalition fails and each member in the coalition holds an equal size of power, greater than ψ_j in a flat-pyramid organisation, for any $j \in \{3, \dots, n\}$. The example identifies an additional incentive for the leader. She needs to allocate $(\alpha_1, \dots, \alpha_n)$, such

that a successful coup does not involve a large number of the members and the members in a coup can be as pivotal as possible in failing a coup. Formally,

PROPOSITION .3. *Consider a top-down ($c = h_2$) organisation, in which when 2nd-ranked initiates a coup and needs support, he passes the initiative onto an additional member, who, in turn, passes it onto the one ranked below, if additional support is needed, and so on. In equilibrium, each member, $i \in \{2, \dots, n\}$, is allocated:*

$$\begin{cases} \alpha_2^{h_2^*} = \alpha_3^{h_2^*} = \frac{1}{2} \\ \alpha_4^{h_2^*} = \dots = \alpha_n^{h_2^*} = 0. \end{cases}$$

The leader allocates $\alpha_1^{d^} = \frac{1}{2}$ to herself. The results are identical to Proposition 1, under perfect democracy.*

Proposition 3⁵ shows that the equilibrium allocation of power is identical to what has been discussed under perfect democracy. The key element deriving this result is the number of ‘de facto proposers’ in a top-down organisation, which, essentially, can be as large as under perfect democracy. Under perfect democracy, each member is serving as a proposer or having the right to initiate a coup. The intuitions for Proposition 3 are further illustrated with the example of a five-person organisation. In discussing the intuition for the results under perfect democracy, different cases have been analysed with an example of a four-person organisation: when the 2nd-ranked is sufficiently capable of initiating and succeeding in a coup on his own ($\alpha_2 > \frac{1}{2}$); and when there could be a different number of two-member coalitions as represented in cases (i)–(iii). A similar approach is taken to analyse the top-down organisation with four members.

⁵The intuition for Proposition is the same as in Proposition 1. For the proof, refer to the proof of Proposition 1.

The earlier results for $\alpha_2 > \frac{1}{2}$ remain unchanged for the top-down organisation. The game illustrated in Figure 4.3 represents a case in which $v(\{2, j\}) = 1$ for any $j \in \{3, 4, 5\}$, given $\alpha_2 \leq \frac{1}{2}$. Provided that $\alpha_2 + \dots + \alpha_5 = 1$, additional cases are analysed⁶:

- (a) $v(\{2, 3\}) = 1$, but $v(\{2, 4\}) = v(\{2, 5\}) = 0$,
- (b) $v(\{2, 3\}) = v(\{2, 4\}) = 1$ but $v(\{2, 5\}) = 0$, and
- (c) $v(\{2, 3, 4\}) = v(\{2, 3, 5\}) = v(\{2, 4, 5\}) = 1$, but $v(\{2, j\}) = 0$ for any $j \in \{3, 4, 5\}$.

In *Case (a)*, it is assumed that $v(\{2, 3\}) = 1$ but $v(\{2, 4\}) = v(\{2, 5\}) = 0$. A successful coalition against the leader should include both the 2nd-ranked and the 3rd-ranked. When the 3rd-ranked opposes, the 2nd-ranked does not approach any member ranked below the 3rd. Even if any or both of the two remaining members supports, the coup initiative fails. Therefore, whether the 3rd-ranked supports or not, the game ends when he has decided. The outcome of a coup is solely determined by the 3rd and he receives $\hat{\alpha}_3 = 1$, when opposing a coup. He opposes the 2nd-ranked's initiative, if and only if:

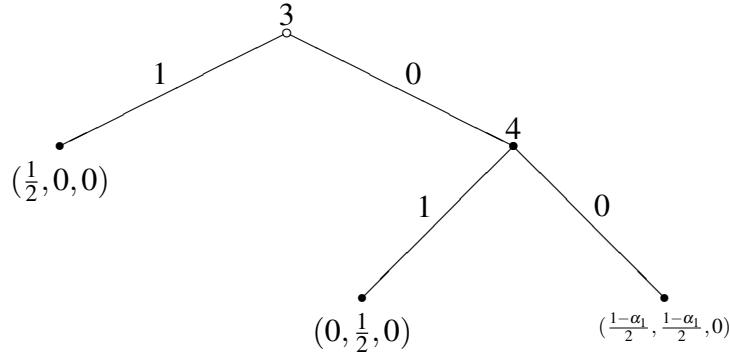
$$1 - \alpha_1 \geq \frac{1}{2}.$$

If the leader sets $\alpha_1 = \frac{1}{2}$, she can prevent a coup.

Case (b) considers when $v(\{2, 3\}) = v(\{2, 4\}) = 1$ but $v(\{2, 5\}) = 0$. The 2nd-ranked continues and passes the initiative onto the 4th-ranked, even if the 3rd-ranked has opposed. The game ends, when the 4th-ranked has made a decision. *Case (b)* is illustrated in Figure 4.4.

The leader needs both the 3rd and the 4th-ranked to oppose. In a successful coup, each of them receives $\psi_3 = \psi_4 = \frac{1}{2}$. Their marginal contribution to the failure of a coup is $\hat{\alpha}_3 = \hat{\alpha}_4 = \frac{1}{2}$, as the coalitions against the 2nd-ranked are successful if both the 3rd-ranked

⁶Provided that $\alpha_5 \leq \dots \leq \alpha_2 \leq \frac{1}{2}$ and $\alpha_2 + \dots + \alpha_5 = 1$, the feasible allocations are categorised into one of the four cases discussed in this section. For individual cases, refer to the Appendix. For $n > 5$, the feasible coalitions can be categorised in a similar manner.

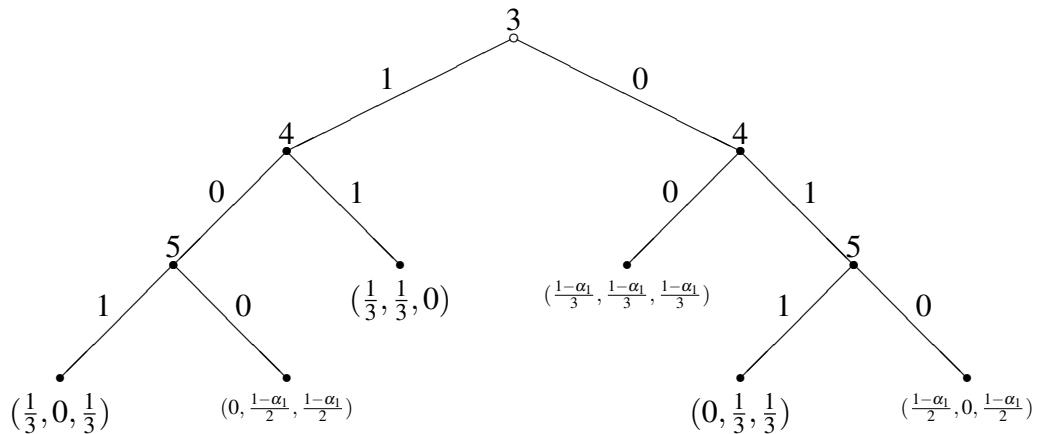
Figure 4.4. Case (b) in a 5-Person Top-Down Organisation

and the 4th-ranked join. As in the example with $v(\{2, j\}) = 1$ for any $j \in \{3, 4, 5\}$, if the 4th-ranked opposes, so does the 3rd-ranked. The 4th-ranked opposes, if and only if:

$$(1 - \alpha_1) \frac{1}{2} \geq \frac{1}{2}.$$

The inequality implies that in order to prevent a coup, the leader has to exercise no power at all, i.e. $\alpha_1 = 0$.

Finally in *Case (C)*, a successful coalition against the leader should include two other members than the 2nd-ranked. The 2nd-ranked needs an additional member, even if the 3rd-ranked support. Once the first two members have opposed, the game ends there and then. The game is illustrated in Figure 4.5:

Figure 4.5. Case (b) in a 5-Person Top-Down Organisation

When the 5th-ranked opposes, the 4th-ranked also opposes, independent of the decision made by the 3rd-ranked. In the subgame, following the history that the 3rd-ranked has opposed, if the 4th-ranked supports when the 5th-ranked opposes, he gets removed. In the subgame, following the history that the 3rd-ranked has supported, if the 5th-ranked opposes, then the condition that makes the 4th-ranked oppose also holds.

The 5th-ranked opposes, if and only if:

$$(1 - \alpha_1) \frac{1}{2} \geq \frac{1}{3}.$$

If $(1 - \alpha_1) \frac{1}{2} = \frac{1}{3}$, i.e. $\alpha_1 = \frac{1}{3}$, the condition above holds with equality and the 4th-ranked also opposes. Provided this, the 3rd-ranked opposes too, and no coup is initiated.

In cases (b) and (c), the number of ‘de facto proposers’ increases. It affects their bargaining power in a coup coalition, as well as in a coalition in favour of the leader. The increased number of ‘de facto proposers’ reduces each member’s redistributed power when opposing a coup and thus increases the share of power that the leader has to forgo. Instead, the leader limits the number of such members, by allocating power as she would under perfect democracy. Both under perfect democracy and in a top-down organisation, the number of ‘de facto proposers’ is limited to 2 in equilibrium. The two top-members maintain a system of ‘checks and balances’. In order to succeed in a coup, they are interdependent. However the two-member coalition is not straightforward. If the 2nd-ranked initiates a coup under either of the two procedures, the 3rd-ranked would not join, as he receives the redistributed power, transferred from the 2nd-ranked.

Propositions 1–3 show that power is spread out more evenly across the members in a flat-pyramid organisation. The optimal power allocation prevents any coup initiative in equilibrium, under any type of a procedure. A perfectly democratic and a top-down organisation exhibit a greater trade-off between power and survival for the leader, than a flat-pyramid organisation. She has to allocate substantial power to a few members and

relatively less to herself, in order to stay in office.

The Propositions imply that as the number of the members with the right to initiate a coup, or with a similar ‘procedure right’, increases, the optimal power allocation becomes more hierarchical. Only a few enjoy substantially greater power than the rest of the members. The number of the members with the right to initiate a coup is the greatest under perfect democracy. In a hierarchical organisation, only one member, the one with the greatest power among the members, has the right. However, the two hierarchical procedures produce a different number of the members with a right, similar to the right to initiate a coup. Recall that whenever $\alpha_2 \leq \frac{1}{2}$ in a top-down organisation, additional members are called into the decision process. Each of the additional members has the right to decide whether to add more members. As under perfect democracy, if the leader allocates power such that more members becomes pivotal in failing a coup, she has to compensate each of them sufficiently and this reduces her share of power. Therefore she limits the number of members having the ‘de facto’ proposal right to the minimum, by allocating power in an identical pattern to perfect democracy.

In equilibrium, the leader exercises positive power in any type of an organisation. Under any procedure, the leader allocates power in such a way that the ‘blame’ for holding greater power is dispersed between the leader and those that have initiated and supported a coup. The members are rewarded with redistributed power for not supporting a coup, according to their contribution. There is a transfer of power held by coup-supporter(s). This reduces the compensation the leader needs to provide and she exercises positive power in equilibrium. It is also found that the discrepancy between the redistributed power, $\hat{\alpha}_j$ and ψ_j , varies across the procedures and is the greatest in a flat-pyramid organisation.

The results point at the common notion of a scapegoat and blame-sharing in political science (notably, Fiorina 1986). Politicians delegate tasks and power to others to reduce their responsibility, when there is a negative outcome. The leader in the model finds a similar incentives. She delegates power to the members to reduce her risk of removal. The equilibrium results also show that some organisational features facilitate such motivations

better than others.

In a flat-pyramid organisation, the combination of the sequentiality and simultaneity in the members' decision helps the leader better 'share the blame'. This becomes more significant, as the organisation grows. Propositions 1–3 indicate that the leader exercises power, $\frac{1}{2}$, in a perfectly democratic and a top-down organisation, and $\frac{n-2}{n-1}$ in a flat-pyramid organisation, for any n . As the organisation grows, she exercises greater power in a flat-pyramid organisation:

$$\lim_{n \rightarrow \infty} \frac{n-2}{n-1} = 1.$$

As n grows ($n \rightarrow \infty$), the variation between ψ_j and $\hat{\alpha}_j$, for any $j \in \{3, \dots, n\}$ increases. The presence of each of the $(n-2)$ members in a successful coup becomes infinitesimal with $\lim_{n \rightarrow \infty} \psi_j = \frac{1}{(n-2)(n-1)} = 0$. Their bargaining power, ψ_j deteriorates much faster than $\hat{\alpha}_j$. The relative gain from opposing a coup, in terms of $\hat{\alpha}_i$, becomes much greater, as n is larger.

5. Discussions

Different patterns of hierarchies in power and procedures in politics are observed. The model focuses on the rules parties adopt when select and abandon their leader. It is argued that internal decisions reflect power held by party members. Most of internal decisions are unobserved and unknown with an exception of decisions over leadership. The procedures and rules for nomination, a motion of (no) confidence and election are well-documented and open to the public. How a leader is selected and replaced in a party shows hierarchies in communication and potentially in power present in the party and this is a motivation for the framework.

The constitution of the UK Conservative Party dictates the rules on leader selection and removal. When supported by 15% or more members of the Parliamentary Party, Conservative MPs may introduce a vote of confidence in the leader by "advising" the Chairman of its 1922 Committee, a committee of 'backbench' MPs⁷. If the leader fails to receive

⁷Although frontbench MPs are now allowed to attend meetings after a change in 2010.

support from a majority of the Parliamentary Party, he/she steps down. In selection of a new leader, candidates are nominated by MPs whereas party members (the Party Membership) are eligible to vote. A new leader is the candidate who is supported by more than 50% of the vote among the Party Membership⁸.

Green Party of the UK operates a different set of rules. There is a committee of regional representatives, Regional Council which is similar in nature to the Conservative's 1992 Committee. Whereas the Regional Council holds the power to "recall" the party leadership⁹, the same is also available to members. When 10% of the membership 'petitions' the Council, then the leadership is recalled. In nominating leadership candidates, any member of the party can be a candidate if supported by a minimum of twenty members. All members are eligible to vote.

A set of rules similar to that of the Conservative and the Labour whose case is discussed in Introduction, is often observed in parties of developed democracies, exhibiting features in a 'flat-pyramid' protocol of the framework. Green Party's rules share characteristics of a 'perfectly democratic' organisation in the model. Members can introduce a motion of (no) confidence or "recall" as capable of initiating a coup against the leader. Only a selected group of the Conservative party membership can initiate a coup whereas any member can do so in the Green.

The examples discussed above reflect how power is allocated in parties, especially across the members, denoted by $\alpha_2, \dots, \alpha_n$ in the model. The procedures adopted to elect and replace leaders in the parties do not indicate how much power leaders exercise in internal decisions, at least not as explicitly as they proxy some patterns of $\alpha_2, \dots, \alpha_n$. The characterisation of power allocated to each member, $(1 - \alpha_1)\alpha_i$, is hence based on the assertion that the hierarchies of power, and of procedures, in leader election and removal

⁸The recent incident of a motion of confidence, followed by an election of a new leader took place when Iain Duncan Smith back then the Conservative leader was replaced in 2003.

⁹When a "two-thirds majority of its voting membership" agrees, the Council recalls and suspends the (deputy) leader.

also indicate those in internal decisions of parties. It is useful to characterise different patterns of power hierarchies potentially adopted in parties as well as to analyse bargaining over a coup initiative between members in the model.

The equilibrium analysis indicates that the optimal allocation in a perfectly democratic and a top-down organisation would be identical (Propositions 1 and 3). However, a top-down procedure is more frequently observed than perfect democracy in practice. With regard to this, a simple modification is considered. The model assumes that the members under perfect democracy simultaneously decide their support for a coup initiative and when an initiative is not made, those that have supported are punished and removed from organisational decisions. Let's instead assume that whenever a majority of the members oppose an initiative, the initial allocation, $\alpha_1, \dots, \alpha_n$, prevails. That is, whenever no initiative is made under perfect democracy, the opposing members are not punished. The modification continues to assume that the members prefer being in the winning coalition and when indifferent, i.e. their decision does not determine the winning coalition and their power is the same whichever decision they make, they oppose.

The example of a four-member organisation is considered again. Firstly, suppose that the leader has allocated $\alpha_2 = \alpha_3 = \frac{1}{2}$ and $\alpha_4 = 0$, as in Proposition 1, with $\alpha_1 \in [0, 1]$. The game is represented in Figure 4.6. If $1 - \alpha_1 = 1$, i.e. $\alpha_1 = 0$, there is the unique equilib-

	$x_3 = 0$	$x_3 = 1$		$x_3 = 0$	$x_3 = 1$	
$x_2 = 0$	$\frac{1-\alpha_1}{2}, \frac{1-\alpha_1}{2}, 0$	$\frac{1-\alpha_1}{2}, \frac{1-\alpha_1}{2}, 0$		$\frac{1-\alpha_1}{2}, \frac{1-\alpha_1}{2}, 0$	$\frac{1-\alpha_1}{2}, \frac{1-\alpha_1}{2}, 0$	
$x_2 = 1$	$\frac{1-\alpha_1}{2}, \frac{1-\alpha_1}{2}, 0$	$\frac{1}{2}, \frac{1}{2}, 0$		$\frac{1-\alpha_1}{2}, \frac{1-\alpha_1}{2}, 0$	$\frac{1}{2}, \frac{1}{2}, 0$	
	$x_4 = 0$			$x_4 = 1$		

Figure 4.6. A 4-Person Organisation under Perfect Democracy

rium, in which all the three members oppose a coup initiative. The size of α_1 sufficient and necessary to prevent a coup reduces to zero from $\alpha_1^{d*} = \frac{1}{2}$. The results from cases (ii)–(iii) in the previous section remain robust under the modified assumption.

The equilibrium allocation in Proposition 1, especially the positive power the leader exercises, is made possible by creating ‘competition’ between the top two members, that the

possibility of punishment in the event of no coup hinders the coalition between the two members. In the modification, there is no such punishment and the leader, whatever the initial allocation has been, needs to make

$$(1 - \alpha_1)\alpha_i \geq \psi_i,$$

hold for any and all $i \in \{2, \dots, n\}$, where ψ_i is member i 's Shapley value in the grand coalition against the leader. As case (ii) in the previous section implies, making any member too pivotal results in multiple equilibria of the grand coalition, either against or in favour of the leader. Making all the members equally pivotal as in case (iii) results in zero power exercised by the leader, which is equivalent to $v(\{2, 3\}) = 1$ with $\alpha_4 = 0$.

The results imply that in order to prevent a coup under the modification, the leader would need to give up the entire control over the organisation while allocating $\alpha_i = \psi_i$ for any $i \in \{2, \dots, n\}$. The key factor deriving the results is the potential ‘anonymity’ or difficulty to control accountability under perfect democracy, hence explaining the relatively less frequent presence of perfect democracy in practice.

6. Conclusion

Motivated by a number of observations in parties, this paper presents a model that links different types of organisation hierarchies, hierarchy of power and decisions, to the longevity of political leadership. The model considers an organisation, in which the leader allocates decision-making power across the members and the members decide whether to initiate and support a coup against her. The members' decision process is specified by the organisation's procedure. The application of a cooperative solution concept into the non-cooperative framework has made it possible to identify specific patterns of power hierarchy that emerge in equilibrium.

The model considers different types of decision procedures, varying in the degree of hierarchy. They are more or less hierarchical depending on whether the ‘proposal right’, the

right to initiate a coup, is given to a few or many members, and whether the members make decisions simultaneously. The organisation's procedure is assumed to be exogenous and the paper compares endogenous allocation of decision-making power and subsequently, endogenous formation of a power hierarchy under each procedure.

It is shown that as the organisation's procedure involves a smaller number of 'de facto proposers', power is more evenly allocated across the members, creating a 'flatter' power hierarchy. Even if the organisation's procedure grants the 'proposal right' only to a few, it could grant a similar 'procedure right' to additional members, increasing the bargaining power of these members. In a top-down organisation, which is the most hierarchical among the three considered in the model, each member in the decision sequence has a 'procedure right', capable of deciding whether to pass the initiative onto the lower-ranked. The results show that as more members are given such a right, the size of compensation that the leader has to provide increases, reducing her own share. Instead, she limits the number of 'de facto proposers' by allocating substantial power to a few and creating a system of 'checks and balances'. With regard to the latter, the few hold power which is substantially large, but not sufficient to go against the leader without support of the other(s). However, a coalition between these few members is not easy. Each of them has an incentive to oppose and receive the greater power, transferred from the coup supporter.

The framework does not involve a specific binding or credibility mechanism for the players. The embedded assumption with respect to the payoffs is that the organisation makes a decision which generates a payoff of 1 and each member in the organisation, if not removed following a coup, receives a share of the payoff as an individual payoff, as in the existing models of legislative bargaining over budget. There could be potential criticism on this. As an extension of this paper, it is possible to extend the current framework over the (in)finite horizon and incorporate the possibility of a coup in each period when the previously 'agreed' allocation fails to be implemented. Then, it would be possible to examine and show if that the equilibrium results from the single-period game stay robust in a possible equilibrium, for instance a stationary SPE as attempted by Gul (1989).

Conclusions

This thesis has considered a number of functions and organisational features in political parties and demonstrated that they present different incentives to individual members of parties. The theoretical contribution of this thesis is to explain the strategic interactions between party members and their parties as well as organisational characteristics, that lead to empirical phenomena observed in politics. The thesis also contributes to the formal literature by shedding light on some of the relatively under-explored areas in party politics, such as organisational hierarchies and intraparty factions, and by expanding the scope of analysis.

In the following sections, the contributions each of the three papers makes to the literature are summarised. The possibility and directions for future research is also discussed.

1. Theoretical Contributions

The thesis presents three models of electoral campaign and candidate valence, intraparty factions, and endogenous power allocation. The first paper extends and modifies the framework in Snyder (1989), whereas the second and the third paper propose a new framework. In particular, the models in the second and the third paper take a cooperative approach. The second paper modifies a stability concept adopted in Ray and Vohra (1997, 1999), and Levy (2004). The third paper adopts the Shapley value, a cooperative solution concept, in a non-cooperative framework. With regard to the cooperative approaches taken in the two papers, the thesis makes a methodological contribution to the formal political science literature. Despite their applicability and simplicity, the approach is relatively less frequently adopted in political science. In fact, the frameworks in the two papers have the potential of modification as well as application to other political problems.

The first paper, “**Campaign Support of Parties and Candidate Performance**”, while confirming the existing findings on the patterns of party support in campaigns, connects them to the strategic behaviour of candidates. It identifies the interaction between party and candidate attributes to constituency-level electoral outcome and distinguishes between the two types of attributes and estimates their effect on electoral performance of candidates. It specifically explains how a type of the attribute compensates for the lack of the other, or strengthens the other. The result illustrates a reinforcing mechanism of the two attributes, explaining why some type of electoral races, e.g. reelection races between incumbents and challengers, tend to be lop-sided from an early stage of election, whereas the others, e.g. open-seat races between two challengers, tend to be marginal.

The existing models, including Snyder (1989), have predicted larger party spending in relatively marginal races, when they hold a similar campaign capacity. When there is an asymmetries in the capacity, the better-resourced party offer more to relatively poorly performing races, and the opposite is true for the less-resourced. The rationale for the predictions is that the relative marginal gain or loss, in terms of probability of winning for their candidates, varies depending on the party-level (a)symmetry in campaign. If a symmetric party abandons the marginal race, it could easily lose all races. One of empirical regularities in campaign spending is that open-seat races tend to receive more from the parties. The result of the first paper predicts that the symmetric parties will allocate more resources to marginal races such as open-seat races. It also derives conditions for greater allocations under the asymmetric parties. The reinforcing mechanism of the party and candidate attributes further strengthens the incumbent’s performance from an early stage, when the incumbent’s party is better-resourced. Compared to the reelection race, the incumbent’s party would find the marginal race relatively poorly performing. Therefore, the result shows that the pattern of party spending in campaign is an outcome derived through multiple channels, more than the simple calculus of the marginal gain or loss in votes as the previous models have reasoned.

The identification of party and personal attributes to electoral performance is also shown to be effective to analyse the sources of an incumbency advantage. Furthermore the result discovers that the sequential nature of reelection races multiplies an incumbency advantage, whenever it exists. Previously, there are formal models that have identified an incumbent's ability to deter or preempt challengers (Ashworth and Bueno de Mesquita 2008; Wiseman 2006) as a source of an incumbency advantage. The first paper clarifies the previous finding and explains that the sequential nature works in favour of the incumbent only when there is an incumbency advantage, i.e. he is relatively advantaged over the challenger in one of the two attributes or both. When the incumbent lacks an attribute or both, the sequentiality would further damage his electoral performance.

The first paper therefore expands the scope of analysis in the formal literature of campaign spending as well as incumbency advantage. It addresses the necessity and importance of incorporating multi-level players and asymmetries when analysing electoral phenomena, as they are the outcomes of strategic interactions between different types of players as well as between different types of asymmetries. Hence, it uncovers a mechanism or a channel through which parties and candidates affect each other's strategic behaviour.

The second paper, “**Factions Explained with Power Hierarchy**”, not only offers a methodological contribution, as discussed earlier, but also makes a contribution to the relatively less actively discussed areas in the literature. It is a new addition to the small number of formal models analysing endogenous formation of coalitions and power hierarchies. At the same time, it contributes to the literature of political parties, as it differentiates itself from the existing models of factions, by incorporating organisational features that distinguish intraparty factions from temporary and/or less structured types of factions.

The second paper identifies a trade-off between collective and individual benefits in the members' decision as a factor that derives the dynamics of intraparty factions observed in politics. There are a few models of coalition faction formation that address a similar trade-off in coalition formation (notably, Morelli and Park 2016), but their framework does not necessarily extend to potential splits and mergers between factions when the two types of

interests conflicts. In politics, mergers of small or minor factions into a large or major faction are relatively often observed. Although less frequently taking place, there are splits of a major faction, by some of its members choosing to leave for a minor faction or to form a new faction. The paper provides a theoretical explanation for both types of member departure, or defection, and identifies the types of faction members who are more likely to attempt each type of departure. The first type of departure is relatively straightforward. The greater size of exclusive benefits that a strong or major faction is an incentive to leave small and minor factions. However, such a departure is only limited to the relatively high-ranked politicians whose loss in power in a major faction is relatively smaller. The second type of departure, slightly puzzling if considering the collective benefit of a major faction, is attributed to the career interest or power motivation of the relatively lower-ranked members of such a faction.

The paper predicts that as party resources are distributed more disproportionately across the factions according to their influence within the party, the decline of small and minor factions is accelerated. The first type of departure increases, whereas the second type is discouraged. Furthermore, it will also motivate mergers between minor factions and hence, even if created, a minor faction would not sustain in a long-term. All of these are because as the size of benefits a major faction offers grows, the relative loss or gain in individual benefits becomes smaller. The predictions of the model reiterates a number of empirical observations in factional politics, notably in the Japanese LDP.

The paper also better reflects the characteristics of intraparty factions, first by incorporating an organisational structure, represented by power hierarchies, and by predicting an outcome that is absent in other models of coalition or faction formation. In the existing models of faction formation (Morelli and Park 2016; Dewan and Squintani 2015), the players are not restricted when they want to join a faction. A stable structure of the factions in the model indicates when no player finds a profitable deviation from the faction of which he is part in the structure. Even if a member finds a profitable departure, it can be ‘sequentially blocked’, mostly by those at higher ranks and in the subsequently emerging

structure(s), the player could be worse off than before. The result reflects informal or formal disciplines adopted to prohibit (party) defections, e.g. “anti-defection” laws in India and Israel (Janda 2009¹).

Whereas the second paper analyses endogenous changes within an existing, hence exogenous, hierarchical organisation, it does not answer the question of, why different patterns of power hierarchies are observed in political parties. The third paper, “**Hierarchies of Power and Decisions**” provides an answer to this question. It analyses endogenous allocation of power, that subsequently creates a power hierarchy in a political organisation, from a leader’s perspective. It argues that the leader of the organisation would choose a particular pattern of power hierarchy to fulfil her interests, that she exercises power as great as possible without a risk of removal.

The main result of the model is that there is a trade-off between the absolute size and longevity of power in the leader’s decision and this trade-off becomes smaller or greater depending on the organisation’s decision procedure. If a decision procedure specifies a limited number of members as having the ‘proposal right’ to initiate a coup against the leader and does not produce any more ‘de facto proposers’, the leader’s trade-off is also kept minimal. However, when either the procedure grants the proposal rights to a large number of members, or it grants a procedure right, similar to the proposal right, to additional members, she has to give up greater power to disincentivise a coup. She also minimises the sacrifice by allocating greater power to a few, so that the rest of the members who are allowed to initiate a coup by the procedure, or who can exercise a procedural power, whenever they exercise positive power, are blocked in the decision process. The result reiterates the findings in the literature of legislative bargaining, that those with procedural rights exercise greater influences over policy outcomes.

The result also predicts that as the proposal right, or a procedure right, is allocated ‘more democratically’, i.e. to a large number of the members, under a procedure, the leader finds

¹*Laws against Party Switching, Defecting, or Floor-Crossing in National Parliaments*, paper delivered at 2009 World Congress of the International Political Science Association, Santiago, Chile, 12-16 July 2009.

it optimal to create a more hierarchical power structure. Decision-making power is highly centralised. The opposite is true, when only a few are granted such a procedure right. The result implies a negative relationship between the two types of organisational hierarchies.

The third paper opens up new possibilities in the analysis of party politics. It integrates the issues of organisational hierarchies and political leadership. Both of them are still relatively insufficiently studied in the formal political science, despite their presence and importance in politics.

2. Future Research

The framework in each of the three papers has a potential for extension and modification to further analyse the related issues. In this thesis, each of the papers discusses a single-period, non-dynamic, game. Despite the nature, it is possible to analyse some characteristics of, for example, factional dynamics in the second paper, with the current framework. However, a multi-period or dynamic game of these framework could offer greater understanding in the issues that the three papers have discussed.

The framework of the first paper could be extended to a multi-period model, similar to Ashworth and Bueno de Mesquita (2008), and analyse another electoral phenomenon, ‘rematches’. Rematches, that indicate races between three candidates who have competed against each other in previous elections, have not received as much attention as reelection or open-seat races and the literature has rarely explored. However, it is a rather frequent phenomenon in elections. For instance, 13 – 14% of the races in every US House elections are rematches. Different strategic motivations are anticipated when parties nominate a candidate again to the opponent who has previously won against the candidate. It may also indicate some quality, which can be either positive or negative, about the candidate as well as the candidate’s party. Furthermore, parties may have different incentives when supporting a challenger in a rematch, from when supporting a freshman candidate. From the candidate perspective, running against the same opponent could provide different incentives as well. The suggested two-period model could uncover a mechanism for strategic

interactions between parties and candidates, that the single-period model in the first paper is not able to capture.

With regard to the second paper, the framework is sufficiently simple and capable of delivering the key results discussed and summarised earlier. However, it does not consider other factors that could potentially influence the members' choice between the factions, such as ideological motivations and potential political punishment following defection. Additional motivations of politicians can be incorporated into the current framework. Such modifications can provide an opportunity to examine the findings of the existing models that analyse the formation of ideologically motivated factions. For simplicity of the model, the members' ranking is assumed to change in the simplest possible way, while capturing the key aspect, but there is a room for improving this and better reflecting the reality. As for the first paper, the second one also has the prospects for a multi-period setting.

Finally, the third paper is the one with the greatest prospects for extension. Several extensions are possible, such as incorporating additional possibility of removal for the leader. For example, the organisation is hit by an external crisis with some probability and in the event of a crisis, the members and the leader are held responsible with probability corresponding to their power. Especially for the leader, the risk of removal increases as she exercises greater power as it incentivises a coup and it increases her share of blame following a crisis. A potential criticism for the current framework is the absence of a binding or credibility mechanism. The implicit assumption embedded is that the members and the leader, if not removed, exercise their power in some organisational decision, which generates a sure payoff of 1 to the organisation and at the end of the game, each receives his/her share of the payoff. It is indeed similar to the legislative bargaining models (Baron and Ferejohn 1989, Romer and Rosenthal 1978). The lack of a binding mechanism could potentially overcome by assuming a dynamic model, which would show a more explicit trade-off between the absolute size and longevity of power for the leader.

Appendix: Proofs

1. Propositions in Chapter 2

Proof of Proposition 1. The Parties' maximisation problems are rewritten as:

$$\max_{r_1^P, r_2^P} \Delta \left[\frac{\alpha_1^P r_1^P}{\alpha_1^P r_1^P + (1 - \alpha_1^P) r_1^Q} + \frac{\alpha_2^P r_2^P}{\alpha_2^P r_2^P + (1 - \alpha_2^P) r_2^Q} + \frac{\alpha_3^P (T^P - r_1^P - r_2^P)}{\alpha_3^P (T^P - r_1^P - r_2^P) + (1 - \alpha_3^P) (T^Q - r_1^Q - r_2^Q)} \right],$$

for any $P \neq Q \in \{A, B\}$. Differentiating the maximisation problems with respect to r_1^P and r_2^P , for any $P \in \{A, B\}$, the following first-order conditions are derived:

$$\begin{aligned} \Delta \left[\frac{\alpha_j^P (1 - \alpha_j^P) r_j^Q}{(\alpha_j^P (r_j^P) + (1 - \alpha_j^P) r_j^Q)^2} - \frac{\alpha_3^P (1 - \alpha_3^P) (T^Q - r_1^Q - r_2^Q)}{(\alpha_3^P (T^P - r_1^P - r_2^P) + (1 - \alpha_3^P) (T^Q - r_1^Q - r_2^Q))^2} \right] &= 0 \\ \Delta \left[\frac{\alpha_1^P (1 - \alpha_1^P) r_1^P}{(\alpha_1^P (r_1^P) + (1 - \alpha_1^P) r_1^Q)^2} - \frac{\alpha_3^P (1 - \alpha_3^P) (T^P - r_1^P - r_2^P)}{(\alpha_3^P (T^P - r_1^P - r_2^P) + (1 - \alpha_3^P) (T^Q - r_1^Q - r_2^Q))^2} \right] &= 0 \end{aligned}$$

for $j \in \{1, 2\}$. Dividing the first-order condition of P by that of Q leads to

$$(6.1) \quad \frac{r_j^Q}{r_j^P} = \frac{T^Q - r_1^Q - r_2^Q}{T^P - r_1^P - r_2^P},$$

which is simplified to

$$(6.2) \quad r_j^Q T^P - r_j^Q r_{\hat{j}}^P = r_j^P T^Q - r_j^P r_{\hat{j}}^Q$$

for $j \neq \hat{j} \in \{1, 2\}$. (6.1) implies $\frac{r_j^Q}{r_j^P} = \frac{r_{\hat{j}}^Q}{r_{\hat{j}}^P} \Leftrightarrow r_1^Q r_2^P = r_2^Q r_1^P$ and using this, (6.2) is simplified to $r_j^Q T^P = r_j^P T^Q$ for any $j \in \{1, 2\}$, implying $r_j^B = \tau r_j^A$. Provided $r_3^P = T^P - r_1^P - r_2^P$ for any $P \in \{A, B\}$, $r_3^B = \tau r_3^A$ and (2.3) is obtained.

Substituting (2.3) back into the first-order condition above for each $P \in \{A, B\}$ derives the equilibrium allocation of campaign resources, (r_i^{A*}, r_i^{B*}) for any $i \in \{1, 2\}$ in the Proposition. For the second part of the Proposition, first note that for the symmetric parties

$(\tau = 1)$, the equilibrium allocation of parties to any district is symmetric, that

$$r_i^* = r_i^{A^*} = r_i^{B^*} = \frac{\alpha_i^P(1 - \alpha_i^P)}{\alpha_1^P(1 - \alpha_1^P) + \alpha_2^P(1 - \alpha_2^P) + \alpha_3^P(1 - \alpha_3^P)}.$$

When we differentiate r_i^* with respect to α_i^P ,

$$\frac{\partial r_i^*}{\partial \alpha_i^P} = (1 - 2\alpha_i^P) \frac{\sum_{j \neq i} \alpha_j^P(1 - \alpha_j^P)}{[\alpha_1^P(1 - \alpha_1^P) + \alpha_2^P(1 - \alpha_2^P) + \alpha_3^P(1 - \alpha_3^P)]^2} \geq 0$$

if and only if $\alpha_i^P \leq \frac{1}{2}$ for any $\alpha_j^P > 0$ and $i \neq j \in \{1, 2, 3\}$. Suppose that the parties are asymmetric with $\tau > 1$. We begin the proof for the less-resourced party, A .

$$\frac{\partial r_i^{A^*}}{\partial \alpha_i^A} = \frac{[\tau - \alpha_i^A(\tau + 1)](\alpha_i^A + (1 - \alpha_i^A)\tau) \sum_{j \neq i} \frac{\alpha_j^A(1 - \alpha_j^A)}{(\alpha_j^A + (1 - \alpha_j^A)\tau)^2}}{\left(\sum_{k=1}^3 \frac{\alpha_k^A(1 - \alpha_k^A)}{(\alpha_k^A + (1 - \alpha_k^A)\tau)^2}\right)^2} \geq 0$$

if and only if $\alpha_i^A \leq \frac{\tau}{\tau + 1}$. Given $r_i^{B^*} = \tau r_i^{A^*}$, the result above suggests

$$\begin{aligned} \frac{\partial r_i^{B^*}}{\partial \alpha_i^A} &\geq 0 \Leftrightarrow \alpha_i^A \leq \frac{\tau}{\tau + 1} \\ &\Leftrightarrow \alpha_i^B \leq 1 - \frac{\tau}{\tau + 1} = \frac{1}{\tau + 1}. \blacksquare \end{aligned}$$

Proof of Proposition 2. Suppose that the parties are asymmetric with $\tau > 1$. Further suppose that A which is the less-resourced of the two spends all its budget, 1 unit of resources in district 1. A 's candidates running in district 2 and 3 then lose the race for sure, $\pi_j^A = \frac{\alpha_j^A \cdot 0}{\alpha_j^A \cdot 0 + (1 - \alpha_j^A)r_j^{B^*}} = 0$ for any $j \in \{2, 3\}$. B can improve its (expected) payoff by allocating a very small amount, $\varepsilon > 0$ to district 2 and the remaining, $\hat{\tau} = \tau - \varepsilon$ to district 1. The deviation by A from the equilibrium strategy is not profitable. Mathematically, let $r^* = \{(r_1^{A^*}, r_2^{A^*}, r_3^{A^*}), (r_1^{B^*}, r_2^{B^*}, r_3^{B^*})\}$ be the equilibrium allocation of the parties. $\hat{r} = \{(1, 0, 0), (\hat{\tau}, \varepsilon, \varepsilon)\}$ indicates the deviation we consider. with $U_A(r)$ indicating A 's (expected) payoff with the set of allocation across the candidates, $r = (r_1^A, r_2^A, r_3^A)$. Then

$$U_A(r^*) - U_A(\hat{r}) = \frac{\alpha_1^A}{\alpha_1^A + (1 - \alpha_1^A)\tau} - (\hat{\tau} - \tau) \sum_j \frac{\partial \pi_j^A}{\partial \tau} > 0$$

as

$$\frac{\partial \pi_j^A}{\partial \tau} = -\frac{\alpha_j^A(1-\alpha_j^A)}{(\alpha_j^A + (1-\alpha_j^A)\tau)^2} < |1|$$

Hence the deviation from the equilibrium strategy is not profitable. Suppose that the parties are symmetric with $\tau = 1$ and A deviates similarly: from the equilibrium allocation, $(r_1^{A*}, r_2^{A*}, r_3^{A*})$ to $(1, 0, 0)$. Again B can find a very small amount of resources, $\hat{\epsilon} > 0$ to be allocated to district 2 and 3 and win the seat for sure. B will allocate the remaining $(1 - 2\hat{\epsilon})$ to district 1. Although the probability of winning for B 's candidate running in district 1 may decrease from the equilibrium level, the party is better off by winning the seats in district 2 and 3 for sure. Therefore such a deviation is not profitable for a symmetric party either. ■

Proof of Proposition 3. The first-order condition of the maximisation problem for the challenger in district 1, is

$$\frac{w\tau e_1^B}{(e_1^A + \tau e_1^B)^2} - \frac{1}{\lambda} = 0$$

leading to $e_1^{A*}(e_1^B) = \sqrt{w\tau\lambda e_1^B} - \tau e_1^B$. The incumbent's choice of effort in equilibrium is then based on the following first-order condition:

$$\frac{w\tau}{(e_1^{A*}(e_1^B) + \tau e_1^B)^2} \cdot \left[\frac{\partial e_1^{A*}(e_1^B)}{\partial e_1^B} e_1^B - e_1^{A*} \right] - 1 = 0$$

With $\frac{\partial e_1^{A*}(e_1^B)}{\partial e_1^B} = \frac{1}{2} \sqrt{\frac{w\tau\lambda}{e_1^B}} - \tau$, the first-order condition is simplified to the incumbent's equilibrium effort, e_1^{B*} in the Proposition. Substituting e_1^{B*} back into $e_1^{A*}(e_1^B)$ derives the equilibrium level of effort exerted by the challenger. The first-order conditions for the candidates in district 3 are identical to those above when replacing τ with $\frac{1}{\tau}$, deriving the equilibrium effort in Proposition 3. The first-order conditions of an open-seat candidate's problem are respectively

$$\begin{cases} \frac{w\tau e_2^B}{(e_2^A + \tau e_2^B)^2} - \frac{1}{\lambda} = 0 & \text{for } A\text{'s candidate} \\ \frac{w\tau e_2^A}{(e_2^A + \tau e_2^B)^2} - \frac{1}{\lambda} = 0 & \text{for } B\text{'s candidate} \end{cases}$$

Simplifying the first-order conditions results in the equilibrium condition for the open-seat candidates (or their effort): $\frac{e_2^{B*}}{e_2^{A*}} = \frac{\lambda}{\lambda}$, implying that in equilibrium the relative size of

effort exerted in the open-seat race corresponds to the relative marginal cost of effort of the candidates. Substituting this equilibrium condition back into the first-order conditions, we obtain the equilibrium effort for each open-seat candidate in the Proposition. ■

Proof: Candidate Effort in District 1. Consider first for the incumbent's effort under sequential and simultaneous investment. As $\tau \geq 1 > \lambda$ for any $\lambda \in (0, 1)$, it is straightforward that the incumbent always exerts higher effort under sequential investment than under simultaneous investment, $e_1^{B^*} > \hat{e}_1^{B^*}$. For the challenger, $e_1^{B^*} \geq \hat{e}_1^{B^*}$ if and only if

$$\frac{2\lambda - \tau}{\lambda} \frac{(\lambda + \tau)^2}{4(\lambda)^2} \geq 1.$$

Whenever $2\lambda \leq \tau$, we can easily see that $e_1^{B^*} < \hat{e}_1^{B^*}$. Consider when $2\lambda < \tau$. Given that $\tau \geq 1$ this is when λ is substantially large, approaching to 1. Then the left-hand side of the inequality above becomes

$$\lim_{\lambda \rightarrow 1} \frac{2\lambda - \tau}{\lambda} \frac{(\lambda + \tau)^2}{4(\lambda)^2} < 1$$

for any value of $\tau \geq 1$. Therefore $e_1^{B^*} < \hat{e}_1^{B^*}$. ■

Proof of (2.8)–(2.10). The equilibrium level of electoral strength for the candidates in district 1 is derived as, (i) for the incumbent:

$$\begin{aligned} \alpha_1^{B^*} &= \frac{e_1^{B^*}}{e_1^{A^*} + e_1^{B^*}} \\ &= \frac{\frac{w\tau}{4\lambda}}{\frac{w\tau}{2} - \frac{w\tau}{4\lambda}(\tau + 1)} \\ &= \frac{1}{2\lambda - \tau + 1} \end{aligned}$$

and (ii) for the challenger, $\alpha_1^{A^*} = 1 - \alpha_1^{B^*} = \frac{2\lambda - \tau}{2\lambda - \tau + 1}$. As in Proof of Proposition 3, the equilibrium strength for the candidates in district 3 is derived by replacing τ with $\frac{1}{\tau}$ in

(2.8). In equilibrium, an open-seat candidate in district 2 is perceived with

$$\begin{aligned}\alpha_2^{P^*} &= \frac{e_2^{P^*}}{e_2^{P^*} + e_2^{Q^*}} \\ &= \frac{1}{2}\end{aligned}$$

for any $P \neq Q \in \{A, B\}$. ■

Proof: Candidate Effort in District 1 with ψ . The first-order condition for the incumbent's *new* maximisation problem is

$$\frac{w\tau}{(e_1^{A^*}(e_1^B) + \tau e_1^B)^2} \cdot \left[\frac{\partial e_1^{A^*}(e_1^B)}{\partial e_1^B} e_1^B - e_1^{A^*} \right] - \psi = 0.$$

As in the proof of Proposition 3 above, with $\frac{\partial e_1^{A^*}(e_1^B)}{\partial e_1^B} = \frac{1}{2} \sqrt{\frac{w\tau\lambda}{e_1^B}} - \tau$, the first-order condition is simplified to the incumbent's equilibrium effort, $e_1^{B(\psi)^*}$

$$e_1^{B(\psi)^*} = \frac{w\tau}{4\lambda\psi^2}.$$

Substituting $e_1^{B(\psi)^*}$ back into $e_1^{A^*}(e_1^B) = \sqrt{w\tau\lambda e_1^B} - \tau e_1^B$, we obtain

$$e_1^{A(\psi)^*} = \frac{w\tau}{2\psi} \left(1 - \frac{\tau}{2\lambda}\right).$$

We then derive the challenger's equilibrium strength, $\alpha_1^{A(\psi)^*} = \frac{2\psi\lambda - \tau}{2\psi\lambda - \tau + 1}$. The incumbent's strength is simply $\alpha_1^{B(\psi)^*} = 1 - \alpha_1^{A(\psi)^*}$. As before, the equilibrium effort for the incumbent in district 3 is derived by replacing τ in the proof above with $\frac{1}{\tau}$ ■

Proof: a Modification to the Winning Probability. The modification changes the first-order condition of the maximisation problem for the challenger in district 1 as:

$$\gamma + (1 - \gamma) \frac{w\tau e_1^B}{(e_1^A + \tau e_1^B)^2} - \frac{1}{\lambda} = 0$$

leading to $e_1^{A^*}(e_1^B) = \sqrt{w\tau \frac{\lambda(1-\gamma)}{1-\lambda\gamma} e_1^B} - \tau e_1^B$. The incumbent's choice of effort in equilibrium is then based on the following first-order condition:

$$\frac{w\tau}{(e_1^{A^*}(e_1^B) + \tau e_1^B)^2} \cdot \left[\frac{\partial e_1^{A^*}(e_1^B)}{\partial e_1^B} e_1^B - e_1^{A^*} \right] - 1 = 0$$

With $\frac{\partial e_1^{A^*}(e_1^B)}{\partial e_1^B} = \frac{1}{2} \sqrt{\frac{w\tau \frac{\lambda(1-\gamma)}{1-\lambda\gamma}}{e_1^B}} - \tau$, the first-order condition is simplified to the incumbent's equilibrium effort. Substituting $e_1^{B^*}$ back into $e_1^{A^*}(e_1^B)$ derives the new equilibrium level of effort exerted by the challenger. As shown in the first-order condition above, the electoral strength of the challenger and the incumbent is identical to (2.7) and (2.8) with λ replaced by $\frac{\lambda(1-\gamma)}{1-\lambda\gamma}$. ■

2. Propositions in Chapter 3

Proof of Proposition 1. Given $a = \{a^1, \dots, a^n\}$, a single-member deviation of b^j , for any $j \in \{1, \dots, n\}$ is not profitable. b^j is placed below a^j at $(j+1)^\ell h$ in a and receives a payoff $-1 + \omega$ whereas staying in b yields $-\omega$. The deviation is profitable if and only if $\omega > \frac{1}{2}$. In a k -member deviation for any $k \in \{2, \dots, n\}$, the highest-ranked among k members receives $-1 + \omega$ in a . The loss in ranking for the remaining $(k-1)$ members ranges from 2 to k and it depends on their ranking in relation to the other $(k-2)$ members. The payoff that $(k-1)$ members receive in a is smaller than $-1 + \omega$, e.g., $-2 + \omega, \dots, -k + \omega$ whereas each of the then receives $-\omega$ in b . Therefore any k -member deviation does not take place either.

- Recall our 6-member party example. When b^2 and b^3 move, they are placed at the 3rd and the 5th and the deviation leads to $a = \{a^1, a^2, b^2, a^3, b^3\}$.

Given $b = \{b^1, \dots, b^n\}$, consider a l -member deviation among a^1, \dots, a^n for any $l \in \{1, \dots, n-1\}$. Whereas the average quality of a remains at 1, the average quality of b , $\frac{l}{n+l}$ is never greater than 1 for any l . a still wins in intraparty competition. l members receive a strictly negative payoff in b , $-\omega + [\text{loss in ranking}] < 0$ and the deviation is never profitable. Finally when a^1, \dots, a^n move to b together, they secure ω but each member is placed $j-1$ ranks lower.

- In the 6-member party example, if everyone in a moves to b we have $b = \{a^1, b^1, a^2, b^2, a^3, b^3\}$ that each of the three members from a is placed at the 1st, the 3rd (a rank down) and the 5th (two ranks down).

The initial coalition of a is also unchanged. We have the unique stable structure in which no member moves, for $\omega \leq \frac{1}{2}$. ■

Proof of Proposition 2. As in the ‘party of six’ example, first check whether $b = \{b^1, \dots, b^n\}$ is divided given $a = \{a^1, \dots, a^n\}$. By Lemma 1, b^j for any $j \in \{1, \dots, n\}$ finds it profitable to move to a if and only if

$$-j + \omega > j - 1 - \omega,$$

simplified to $\omega > j - \frac{1}{2}$. Similarly b^{j+1} moves to a if and only if $\omega > j + 1 - \frac{1}{2} = j + \frac{1}{2}$. This implies that for $\omega \in (j - \frac{1}{2}, j + \frac{1}{2}]$, b^1, \dots, b^j move to a . With Definition 3, for any given $\omega > \frac{1}{2}$, the grand coalition of b is divided that the top $\lfloor \omega \rfloor$ members move to a . We check for $a = \{a^1, b^1, a^2, \dots, b^{\lfloor \omega \rfloor}, a^{\lfloor \omega \rfloor+1}, a^{\lfloor \omega \rfloor+2}, \dots, a^{n-1}, a^n\}$ in $\pi(\omega)^{\lfloor \omega \rfloor}$. a^k for any $k \in \{1, \dots, \lfloor \omega \rfloor\}$ never finds it profitable to move b . a^k receives a payoff of $k - 1 - \omega$ in b , which is smaller than his payoff of $-(k - 1) + \omega$ in a . This holds for any $k \in \{1, \dots, \lfloor \omega \rfloor\}$ as $\omega > \lfloor \omega \rfloor - 1$. Consider when $M > 1$ members among $a^1, \dots, a^{\lfloor \omega \rfloor}$ move to b together. First suppose the M -member deviation has no effect on the quality of the two factions and a remains holding the greater quality. Even the highest-ranked among the M members (say the m^{th}) finds that the gain in ranking in b , $m - 1$ is insufficient to offset the loss of ω in b . The gain in ranking is smaller for the 2nd, \dots , M^{th} -ranked members in the M -member deviation. Therefore there is no deviation by a member or a group of M members among $a^1, \dots, a^{\lfloor \omega \rfloor}$. A L -member deviation among $a^2, \dots, a^{\lfloor \omega \rfloor}$, where $L \geq n - \lfloor \omega \rfloor$ is however always sequentially blocked by Lemma 2. Now let’s examine the possibility of a deviation by a member or a group of members among $a^{\lfloor \omega \rfloor+1}, \dots, a^n$. If $\omega < \lfloor \omega \rfloor$, $a^{\lfloor \omega \rfloor+1}$ finds an incentive to move to b as:

$$-\lfloor \omega \rfloor + \omega < \lfloor \omega \rfloor - \omega \Leftrightarrow \omega < \lfloor \omega \rfloor.$$

This deviation is not sequentially blocked as long as

$$\frac{n-1}{n+\lfloor \omega \rfloor - 1} > \frac{1}{n-\lfloor \omega \rfloor + 1} \Leftrightarrow n - \lfloor \omega \rfloor > 1,$$

i.e., the quality of a remains greater than that of b after $a^{\lfloor \omega \rfloor + 1}$'s deviation by Lemma 2.

$a^{\lfloor \omega \rfloor + 2}$ does not want to move to b with $a^{\lfloor \omega \rfloor + 1}$ as:

$$-\lfloor \omega \rfloor + 1 + \omega > \lfloor \omega \rfloor - 1 - \omega \Leftrightarrow \omega > \lfloor \omega \rfloor - 1.$$

The deviation among $a^{\lfloor \omega \rfloor + 1}, \dots, a^n$ stops at $a^{\lfloor \omega \rfloor}$ and $\pi(\omega)^{\lfloor \omega \rfloor - 1}$ emerges. In the new structure, $b^{\lfloor \omega \rfloor + 1}$ moves to a as

$$-\lfloor \omega \rfloor + \omega > \lfloor \omega \rfloor - 1 - \omega \Leftrightarrow \omega > \lfloor \omega \rfloor - \frac{1}{2}.$$

This deviation is not sequentially blocked if and only if it does not change the quality of the two factions i.e.,

$$\frac{n-1}{n+\lfloor \omega \rfloor - 1 + 1} > \frac{1}{n-\lfloor \omega \rfloor + 1 - 1} \Leftrightarrow n - \lfloor \omega \rfloor > 2.$$

$b^{\lfloor \omega \rfloor + 1}$ arrival encourages $a^{\lfloor \omega \rfloor + 2}$ to move to b and if $a^{\lfloor \omega \rfloor + 2}$ moves to b , $b^{\lfloor \omega \rfloor + 2}$ moves to a and so on, as long as it is not sequentially blocked. Hence the stepwise deviation between the members ranked below the $\lfloor \omega \rfloor^{\text{th}}$ between the two faction continues as long as

$$(6.3) \quad \frac{n-x}{n+\lfloor \omega \rfloor - x + y} > \frac{x}{n-\lfloor \omega \rfloor + x - y} \Leftrightarrow n - \lfloor \omega \rfloor > x + y,$$

where x is the number of members in $a^{\lfloor \omega \rfloor + 1}, \dots, a^n$ moving from a to b and y is that in $b^{\lfloor \omega \rfloor + 1}, \dots, b^n$ moving from b to a . The stepwise deviation starts with a successful deviation by $a^{\lfloor \omega \rfloor + 1}$ which in turn creates a position in a for $b^{\lfloor \omega \rfloor + 1}$. This then provides a disincentive for $a^{\lfloor \omega \rfloor + 2}$ to stay in a and an incentive for him to move to b . (6.1) suggests that if x members ranked below $\lfloor \omega \rfloor^{\text{th}}$ move to the opposite faction and $x = \frac{n-\lfloor \omega \rfloor}{2}$, then the two factions are in tie in their quality. After $(\frac{n-\lfloor \omega \rfloor}{2} - 1)$ pairs ranked at the $\lfloor \omega \rfloor + 1^{\text{th}}$ and below move to their counterpart's faction, $a^{\frac{n+\lfloor \omega \rfloor}{2}}$ is allowed to move to b but not $b^{\frac{n+\lfloor \omega \rfloor}{2}}$ by Lemma 2. When $n - \lfloor \omega \rfloor$ is odd, then the number of a 's members moving to b becomes $\frac{n-\lfloor \omega \rfloor-1}{2}$. When $\omega \geq \lfloor \omega \rfloor$ each of the members, $a^{\lfloor \omega \rfloor + 1}, \dots, a^n$ who suffer from a loss of $\lfloor \omega \rfloor$ in ranking after $b^1, \dots, b^{\lfloor \omega \rfloor}$ finds no incentive to move to b . Consider $a^{\lfloor \omega \rfloor + k}$,

$k \in \{1, \dots, n - \lfloor \omega \rfloor\}$. If $\omega \geq \lfloor \omega \rfloor$, he receives a strictly positive payoff in a

$$- \lfloor \omega \rfloor + \omega > 0 > \lfloor \omega \rfloor - \omega,$$

and moving to b yields a strictly negative payoff. Therefore no member moves to b . ■

Proof of Proposition 3. Straightforward from the proof of Proposition 2.

Proof of Proposition 4. The first part is straightforward. Given $a = \{a^1, \dots, a^n\}$, b^j for any $j \in \{1, \dots, n\}$ receives a payoff of 0 in b and $-1 + \omega$ if moves alone to a . If $\omega \leq 1$, no single-member deviation is possible. As in the proof of for Proposition 1, any k -member deviation is profitable when $\omega \leq 1$ for any $k \in \{2, \dots, n\}$. The members ranked at the $2^{\text{nd}}, \dots, k^{\text{th}}$ bears a greater loss in ranking than the highest-ranked among the k members. Given $b = \{b^1, \dots, b^n\}$, a^j receives a payoff of $j - 1 + \omega$ if moves to b with Lemma 1. If $\omega \leq 1$, only a^1 moves to b . In $\pi(\omega)^{-1}$, any attempt among b^1, \dots, b^n to move to a is sequentially blocked. We have therefore the unique stable structure in which only a^1 moves to b . For $\omega \in (1, n]$, first consider when b^1, \dots, b^n . b^k moves to a if and only if $-k + \omega > k - 1 - \omega \Leftrightarrow \omega > k - \frac{1}{2}$. As in Proposition 2, if $\omega \in (k - \frac{1}{2}, k + \frac{1}{2}]$, b^1, \dots, b^k move to a where $k = \lfloor \omega \rfloor$. When they move to a , we have a new structure, $\pi(\omega)^{\lfloor \omega \rfloor}$. In the structure, each a^k receives a payoff of $-(k - 1) + \omega$ in a . If a^k alone moves to b , he receives a payoff of $k - 1 - \omega$ if $k \leq \lfloor \omega \rfloor$. $a^1, \dots, a^{\lfloor \omega \rfloor}$ receive a strictly greater payoff in a . For $k > \lfloor \omega \rfloor$, if $\omega < \lfloor \omega \rfloor$, the payoff from b when a^j could move alone. By Lemma 1 and the proof of Proposition 2, we then see a stepwise deviation among $i^{\lfloor \omega \rfloor + 1}, \dots, i^n$ for any $i \in \{a, b\}$. Note that under the new selection rule, for $\omega > 1$ therefore $\lfloor \omega \rfloor \geq 1$, such a stepwise deviation is never sequentially blocked: a wins even if all $a^{\lfloor \omega \rfloor + 1}, \dots, a^n$ are replaced by $b^{\lfloor \omega \rfloor + 1}, \dots, b^n$. When $\omega > \lfloor \omega \rfloor$, the payoff to each of $a^{\lfloor \omega \rfloor + 1}, \dots, a^n$ in a is strictly greater than his payoff when he moves alone to b . No further deviation is made in $\pi(\omega)^{\lfloor \omega \rfloor}$. Finally consider when a^1, \dots, a^n decide first, given $b = \{b^1, \dots, b^n\}$. a^k moves to b if and only if $-(k - 1) + \omega > k - 1 + \omega \Leftrightarrow \omega > j - 1$ by Lemma 1. With Definition 3, we can rewrite the condition as $\omega > \lfloor \omega \rfloor$. If $\omega \leq \lfloor \omega \rfloor$, then a^k does not move to b .

for any $k > \lfloor \omega \rfloor$ and moves to b if $k \leq \lfloor \omega \rfloor$. Provided this, in $\pi(\omega)^{-\lfloor \omega \rfloor}, b^{\lfloor \omega \rfloor+1}, \dots, b^n$ receive a payoff of $-\lfloor \omega \rfloor + \omega$ in b . If one of them moves to a , he receives a payoff of $\lfloor \omega \rfloor - 1 - \omega$. For $\omega \leq \lfloor \omega \rfloor$, they receive a strictly greater payoff in b . No further deviation is made in $\pi(\omega)^{-\lfloor \omega \rfloor}$. If $\omega > -\lfloor \omega \rfloor$, we first see that $a^1, \dots, a^{\lfloor \omega \rfloor+1}$ move to b , giving rise to $\pi(\omega)^{-(\lfloor \omega \rfloor+1)}$. In the new structure, $b^{\lfloor \omega \rfloor+1}, \dots, b^n$ receive a payoff of $-(\lfloor \omega \rfloor + 1) + \omega$ in b . If one of the moves to a , he receives a payoff of $\lfloor \omega \rfloor - \omega$. As $\omega > \lfloor \omega \rfloor - \frac{1}{2}$, we first see that $b^{\lfloor \omega \rfloor+1}$ which motivates $a^{\lfloor \omega \rfloor+2}$ to move to b , which then encourages $b^{\lfloor \omega \rfloor+2}$ to move to a , and so on: a stepwise deviation we have seen previously takes place till all of $b^{\lfloor \omega \rfloor+1}, \dots, b^n$ move to a . ■

3. Propositions in Chapter 4

Proof of Proposition 2. As in Proposition 1, if $\alpha_2 > \frac{1}{2}$, the only way to prevent a coup is to allocate $\alpha_2 = 1$ and $\alpha_1 = 0$. If $\alpha_2 \leq \frac{1}{2}$ and a coup is initiated, the $(n - 2)$ members decide over the initiative. Each of the $(n - 2)$ opposes if and only if (4.2) holds. Suppose that n'' two-member coalition with the 2nd-ranked are successful, where $n'' \in \{1, \dots, n - 2\}$. Each of the n' members needs to get compensated with $(1 - \alpha_1) \frac{1}{n''} \geq \frac{1}{n''(n''+1)}$, i.e. $1 - \alpha_1 \geq \frac{1}{n''+1}$ when deviating from the two-member coalition with the 2nd-ranked. This implies that $\alpha_1 = 1 - \frac{1}{n''+1}$, increasing in n'' . The maximum n'' is $n - 2$, indicating if she makes each of the $(n - 2)$ members equally pivotal in the grand coalition against her, she can keep greater power. ■

Individual Cases for a Four-Person Top-Down Organisations. In a four-person organisation, the following allocations of $\frac{1}{2} \geq \alpha_2 \geq \alpha_3 \geq \alpha_4$ can be considered. Each of the allocations belongs to one of the cases discussed in Chapter 4:

α_2	α_3	α_4	α_5	Case
0.5	0.5	0	0	a
0.5	0.4	0.1	0	b
0.5	0.3	0.2	0	b
0.5	0.3	0.1	0.1	$v(\{2, j\}) = 1$ for any $j \in \{3, 4, 5\}$
0.5	0.2	0.2	0.1	$v(\{2, j\}) = 1$ for any $j \in \{3, 4, 5\}$
0.4	0.4	0.2	0	b
0.4	0.4	0.1	0.1	a
0.4	0.3	0.3	0	b
0.4	0.3	0.2	0.1	b
0.4	0.2	0.2	0.2	$v(\{2, j\}) = 1$ for any $j \in \{3, 4, 5\}$
0.3	0.3	0.3	0.1	b
0.3	0.3	0.2	0.2	a
0.25	0.25	0.25	0.25	c

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