# Asymmetries of Information in Financial Markets with Applications to Debt Renegotiation and Financial Certification

by

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# Abstract

This thesis investigates a number of issues caused by informational asymmetries between firms and investors. It looks at two situations where problems arise because of asymmetric information, and examines possible solutions, and presents one case where asymmetric information is the solution to a problem.

The first situation developed in chapter two provides a theoretical explanation based on reputational concerns for why certification intermediaries like rating agencies may exhibit excess sensitivity to the business cycle and for differences in ratings across agencies. It also analyses how competition in this industry affects the behaviour of these intermediaries and how this depends on reputational disparities among the different competitors.

Chapter three looks at the impact of auditor rotation on the gathering and disclosure of information about projects taking into account that longer auditor tenures can generate substantial savings in information collection costs but also make auditors more willing to preserve future expected rents and private benefits. It also shows that the regulation of auditing procedures becomes less relevant if accounting transparency increases.

In contrast with the previous two cases, the next chapter finds that co-ordination failures among small creditors during debt renegotiation can be mitigated by the presence of a large and more informed creditor or by a voting requirement. It examines how the strength of these effects depends on the relative precision of private information of the small and large creditors and provides a rationale for a diversified capital structure based on the informational role that some creditors might have in case of financial distress.

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# Chapter 1

# Introduction

Akerlof's "lemons" paper sparked renewed interest on the implications of asymmetric information in market efficiency. The issue of asymmetric information was before addressed by Vickrey (1961) and Mirrlees (1971) for example, but it was Akerlof's paper that showed that informational asymmetries can give rise to adverse selection in markets. The idea is very simple: the fact that the owner of a car knows more about it than any potential buyer, leads the potential buyer to assume that any used car has a high probability of being low quality, i.e. a lemon. This causes them to bid down the price of used cars in general which ends up driving high quality used cars out of the market.

A key insight of the "lemons" paper is that economic agents may have strong incentives to offset the undesirable effects of information problems in market efficiency. In fact, Akerlof argues that many market institutions may be regarded as emerging from attempts to resolve problems due to asymmetric information.

Given this it is very important to assess whether this type of institutions reach the goals for which they were created. Hence, this thesis looks at two institutions, more precisely rating agencies and audit firms, and assesses whether the way they function impacts on the objective to overcome asymmetries of information between firms and investors.

On the other hand, asymmetric information between agents can also be used to solve problems that arise when agents are imperfectly informed. The last chapter focus on the role of a large player in solving or helping to attenuate co-ordination problems between the remaining players when these are small and the large player is better informed.

**Research Agenda**

Berle and Means (1933) were among the first to identify asymmetric information as a problem for firm management. They labelled it the "problem of the separation of ownership and control" and provided the basis for the principal-agent models developed by Ross (1973) and Townsend (1979) among others.

In reality, the ownership structure of firms nowadays is a lot more complex than what these models implicitly assume and investors are in general dispersed and more often than not do not even know the people in charge of the firms they own or are planning to invest in. Hence, institutions where created with the purpose of transmitting timely and reliable information about firms to investors with the purpose of helping them in their investment decisions. These are financial certification intermediaries such as rating agencies and audit firms.

The role of intermediaries in general is discussed by Leland and Pyle (1977) that presents them as a way to facilitate transactions between buyers and sellers that are asymmetrically informed. Allen (1990), on the other hand, points out that because the original seller only captures a portion of his information's value due to lack of reliability, there is an opportunity to make profits on the remaining value and this is captured by an intermediary.

Papers that are closest to the way this thesis approaches the topic of financial intermediation are for example Millon and Thakor (1985), Ramakrishnan and Thakor (1984), Biglaiser (1993) and Biglaiser and Friedman (1994), where intermediaries are agents with expertise in certification, screening and monitoring.

Some of these papers present this type of intermediaries as a solution to solve informational asymmetries but do not explicitly address the incentive problems and reputational issues they are faced with. As Crawford and Sobel (1984) demonstrates, a sender of information has problems convincing the receivers of the reliability of the information provided if their interests are not perfectly aligned[1]. Therefore, chapters two and three of this thesis contribute to this literature by looking at how incentives and reputational issues affect certification intermediary in their role of transmitting reliable information to investors.

- **Financial Certification via Ratings**

Firms hire certification intermediaries like rating agencies if they expect investors to use the information they provide in their investment decisions. As a consequence, rating agencies worry about how they are perceived by the market, i.e., worry about reputation. One way to consolidate reputation is to avoid making mistakes, and hence to use all available information, both public (e.g. accounting statements) and non-public (e.g. confidential interviews) in their rating decisions. But empirical evidence seems to suggest that ratings tend to reflect market sentiment. Thus, reputational

[1] In contrast with Crawford and Sobel (1984), where reports are not verifiable, strategic information transmission with verifiable reports is discussed in Milgrom and Roberts (1986) and Okuno-Fujiwara, Postlewaite and Suzumura (1990).

concerns seem to generate conflicting incentives for certification intermediaries: an accurate report should incorporate private information but when an intermediary is unsure of her private information reporting according to public consensus might be regarded as the best way to prevent mistakes and consequently, to preserve reputation. The second chapter of this thesis takes into account this trade-off and assesses whether certification intermediaries that worry about reputation transmit reliable information, and in what way the structure of the certification industry affects information transmission.

In this model there is a different firm each period trying to raise funds for a new project that hires a certification intermediary to emit an opinion about the quality of this project. This opinion is based on private information collected by the intermediary and is going to affect the price of the new funds raised by the firm. Market conditions and the firm's past history determine the existing public information about the quality of the project that complements the message sent by the intermediary. However, both investors and firms are unsure about how much to trust this message as the intermediary might, or might not, make mistakes when assessing the firm, i.e. might be untalented or talented. At the end of the first period, investors have the chance to update their belief about the intermediary's type by comparing the message sent with the true state in case the project is undertaken. Reputation in this context translates the beliefs of investors about the certification intermediary's ability.

This chapter shows that in some situations a certification intermediary that has reputational concerns and is unsure about her private signal chooses to ignore it and *conform* instead to the public information available about the project if these two pieces of information contradict each other. This can happen whenever public information is extreme, i.e. when public information is predominantly very good or very bad. For example, if investors expect a firm to be good and the intermediary private information indicates that the firm is bad, there are situations where she chooses to report that the firm is good. However, an intermediary that is sure of her private signal always reports truthfully.

Intermediaries perceived by the market as more talented, tend to issue less favourable reports with greater frequency than more favourable ones. This happens because there is an asymmetry of observability in the model: a project issued with an unfavourable report is not undertaken and this limits the learning process about the certification intermediary's type. This combined with the fact that the more reputable an intermediary is, the less she benefits from issuing a report that turns out to be correct and the more she loses when proven to be wrong, results in more reputable intermediary being more prone to sending unfavourable reports when the prior is relatively low or relatively high than a less reputable one.

Finally, competition in the certification industry forces an intermediary to issue more favourable reports. The difference is that in the new setting sending an unfavourable report also carries disadvantages: reputation might decrease and this might compromise the chances of being hired next period. This happens because, even though the project is not undertaken, investors have an initial opinion about its quality. Hence, if they are faced with an unfavourable report when they believe very strongly that the project is good they suspect that the intermediary has made a mistake and consequently, that they are more likely to be dealing with an untalented intermediary. And it can even happen that only favourable reports are issued. This is the case if the reputational levels between two competitors are very similar and sending anything else other than a correct favourable report decreases reputation below that of the competitor resulting in the latter being hired in the following period.

The issues addressed in this chapter are relevant for policy-makers. The Basel Committee on Banking Supervision has put forward regulatory proposals that would require banks to use a standardised approach to calculating their minimum required capital based on the credit ratings assigned to the companies to which they lend. However, the results derived in this chapter cast doubts about how appropriate the use of ratings is as a device for risk management: if rating agencies rely excessively on public information they tend to behave in a pro-cyclical way and as a consequence, bank capital requirements will tend to be higher during downturns, further reducing credit supply during these periods.

- **Accounting Audits**

Auditors are certification intermediaries that are hired by firms to make sure that the information given to investors about the firm's financial accounts during a certain period of time is correct. Basically, an auditor's job is to validate the manager's report about the firm's accounts and this is necessary because managers might have incentives to bias reports in their own favour. In particular, they might derive private benefits of control and therefore be biased towards continuing projects that should be terminated.

However, in recent years many accounting irregularities have been left undetected for long periods of time which raises questions about current auditing procedures. In order to avoid further accounting scandals the Sarbanes-Oxley Act established, among other things, that partners of accounting companies supervising the external audit have to rotate regularly.

Empirically, several studies have concluded that the frequency of accounting re-statements increases with the length of auditor tenure. This might be the case because if an auditor knows she is going to remain on the firm for a sufficiently long period she feels tempted to delay the announcement of "bad news" in order to protect the expected rents from a continued engagement with the firm. But on the other hand, in a situation where auditors can make mistakes and have the possibility to revise their initial audit reports, having an auditor that is more familiar with the firm means that she is more efficient when looking for genuine mistakes.

Hence, the model developed in this chapter takes into account this trade-off while looking at how rotating an auditor affects the gathering and disclosure of information about the quality of a project.

Auditing requires a certain level of effort to collect private information and results in either a good report, implying that the project should continue, or a bad report meaning that the project should be terminated. Auditors make a mistake when they incorrectly identify a bad project as a good one and as a result, an audit report can be subject to a revision by either the same or a different auditor. In addition, managers can tempt auditors with private benefits that will be received by one auditor only provided that the project continues. This makes it harder for the firm's shareholders to convince an auditor to be truthful about a bad private signal. Hence, the firm's shareholders need to provide incentives for the gathering and disclosure of information in two consecutive audits (if needed) bearing in mind that auditors should report bad private signals immediately rather than delaying its announcement and adjusting the transfer made to auditors for the existence of private benefits.

The optimal contract rewards auditors when their reports are correct and the reward is higher when they contradict previous reports or make correct announcements that ex-ante appear less likely given what is publicly known about the quality of the project. Private benefits decrease the transfers made to auditors for correct good reports but increase the transfers for correct bad reports as in this case the auditor clearly foregoes the private benefit. Limited liability determines that if the private benefit exceeds a certain threshold, auditors extract extra rents because transfers cannot be negative.

Familiarity with the firm's accounting procedures establishes that if the same auditor performs a second audit, the cost to gather information for the second time is lower. As a result, and given that auditors are paid to be compensated for effort exertion, one would expect these payments to decrease the lower the effort cost is. However, the model concludes that this effect is limited and when the new cost of effort is lower than a certain threshold some of the payments need to increase because if they continue adjusting to even lower effort costs in the second audit, an auditor no longer finds it worth it to exert effort the first time she audits the firm's accounts and as a result, effort is never exerted.

Rotating auditors is preferred by the firm's shareholders when their initial opinion about the quality of the project is lower than a certain threshold. This happens because if they expect the project to be bad, transfers for correct good reports, which are lower in the single auditor case, are less likely to be paid and auditors are instead more likely to be rewarded for a correct bad report in the first audit, which is higher in the single-auditor case to prevent her from delaying the announcement of bad reports. If familiarity with the firm generates savings in information collection, the threshold is lower than before but the existence of private benefits crowds out this positive effect.

- **Renegotiation Procedures**

Finally, chapter four looks at a slightly different situation. The focus is still on asymmetric information between firms and investors, but a further level of asymmetry of information is introduced, i.e., there is asymmetric information between two different types of investors. This chapter shows how having a large and better informed investor/creditor can help resolving co-ordination problems among a group of small investors/creditors, when they are faced with the need to renegotiate their debt claims. In some sense, the large creditor can, in some circumstances, act as a *certifier* of the value of the firm towards the remaining creditors.

The investors base available to provide financing to a firm is significantly broadened when bond issues as well as bank loans are used to issue capital. The downside is that co-ordination problems might arise in case a renegotiation is needed. For example, in the situation where a debtor fails if an insufficient number of bondholders agrees to accept an exchange offer, each bondholder may only want to accept the offer provided that the other bondholders accept it as well. This can lead to a situation where restructuring fails, even though creditors would be best served by agreeing to it.

The purpose of this chapter is to assess the role of voting requirements that are widely attached to public debt restructurings and the impact of a large creditor, for example a bank, in the resolution of financial distress, in particular, how they perform as co-ordination devices in the renegotiation of public debt.

In this model there is a financially distressed firm with outstanding private and public debt and each creditor must decide independently whether or not to agree with a reorganisation plan proposed by the firm. The reorganisation plan is similar to a debt-to-equity exchange offer and both the large creditor and a critical majority of small creditors need to restructure their claims or the firm will otherwise be liquidated. If renegotiation succeeds, the firm cannot continue its activity unless it replaces each claim that was withdrawn by new and more expensive funds.

This chapter argues that the existence of a large creditor can facilitate the reorganisation of public debt but this depends on how informed she is relative to small creditors. Hence, when renegotiation takes place *simultaneously* and the large creditor is perfectly informed, and more informed than the remaining creditors, small creditors *always agree* to renegotiate and let the decision of the large creditor determine the outcome of renegotiation. This happens because renegotiation is successful if the large creditor also accepts to exchange her claim and small creditors know that because she is precisely informed she will not let the firm continue for it to default in the last period. When the large creditor decision is announced before renegotiation takes place, small creditors simply mimic the large creditor's behaviour. On the other hand, the large creditor plays an insignificant role when small creditors are those whose information is precise.

As far as the voting requirement is concerned, it becomes more relevant as a coordination device the lower the informational advantage of the large creditor is. In fact, it is derived that renegotiation is always easier in a game with voting requirements than in a game without, for any level of relative precision of private information between the two groups of creditors (except when the large creditor is the only creditor that is precisely informed). This happens because the existence of the voting requirement puts a limit to the number of creditors withdrawing their claims, limiting as well the number of claims replaced by new and more expensive funds.

These results suggest that the mix of private and public debt is an important determinant of a distressed firm's ability to restructure out-of-court but it shows that a capital structure with multiple investors and different debt classes does not necessarily make renegotiation more difficult than with only one type of claim and provides an rationale for a diversified capital structure based on the *informational role* that some creditors might have in case of financial distress.

These chapters add to the discussion of how to mitigate asymmetric information between a firm and its investors. The first two situations discuss some of the problems caused when markets attempt to overcome this asymmetric information but in the last one it is presented as the solution to reach a desirable outcome.

A summary of conclusions and outlook for future research is given in chapter 5.

# Chapter 2

# Conformity and Competition in Financial Certification

## 2.1 Introduction

Certification intermediaries in financial markets provide information to investors about the value of firms or other economic entities that approach them. Examples of such intermediaries are credit rating agencies and auditing firms. Reputation is the main asset of these intermediaries, since it confers credibility to their announcements and consequently makes firms hire their services. The *Economist*[1] summarises the importance of reputation for rating agencies as follows:

> "Even more than for accountants and lawyers, rating agencies must trade on their reputations. If, for example, bond investors lose faith in the integrity of rating agencies' judgements, they will no longer pay attention to their ratings; if rating agencies' opinions cease to affect the price that

[1] "Use and Abuse of Reputation", Economist, April 6, 1996.

> borrowers pay for capital, issuers will not pay their fees. So market forces should make rating agencies careful of their good names".

Therefore, one would expect reputational concerns to be a strong motive for them to try hard not to make mistakes and to use all available information, both public (e.g. accounting statements) and non-public (e.g. confidential interviews) when reporting their judgements to investors. But in reality, reputational concerns seem to generate conflicting incentives for certification intermediaries: an accurate report should incorporate private information but reporting according to public beliefs might be the best strategy for intermediaries whose private information is imprecise. This chapter takes into account this trade-off and assesses whether certification intermediaries that worry about reputation transmit reliable information, and in what way the structure of the certification industry affects information transmission.

In 2001, credit raters failed to downgrade Enron to below-investment grade until four days before the company filed for bankruptcy. In fact, by the time investors services like Moody's began cutting Enron's ratings, bond traders had already been trading Enron at junk levels for several weeks and common stock had dramatically fallen to a seven year low. Quoting Chairman Joe Lieberman:

> "In the Enron case (...) credit raters appear to have been no more knowledgeable about the company's problems than anyone else who was following its fortunes in the newspapers."

WorldCom bonds had also collapsed to junk levels weeks before the company's rating was downgraded and this happened only about a month before the company disclosed nearly $4 billion in improper accounting.

In the context of the Asian crisis, Reinhart (2002) and Ferri, Lui and Stiglitz (1999) describe how agencies also failed to give warning signals until after the turbulence in the Asian markets had begun. However, when the crisis was actually spreading, there was widespread downgrading of the Southeast-Asian issuers.

These facts raise several questions regarding the informational value of ratings. Downgrades seem to have reflected information that market participants had already previously incorporated in the pricing process and in some cases, they occurred after the rated entities had themselves disclosed substantially increased risk. Nonetheless, the information rating agencies provide is widely used for purposes that reach far beyond the intention to mitigate asymmetric information among market participants. For example, there are proposals to use ratings for regulatory purposes: the Basel Committee on Banking Supervision intends to see borrowers' credit ratings included in assessments of the adequacy of bank's capital. For this reason, it is of foremost importance to understand how rating agencies behave and which mechanisms can be put into practice to increase the credibility of their announcements.

In the model developed below there exists public as well as private information about the quality of the firm and both investors and firms are unsure about the (monopolistic) certification intermediary's type: she might, or might not, make mistakes when assessing the firm (be untalented or talented). This chapter shows that in some situations an untalented certification intermediary chooses to *conform* to the public information going against what her private information indicates because of fears of being wrong, in which case she would have to bear a heavy reputational cost. As a result, this can happen whenever public information is extreme, i.e. when public information is predominantly very good or very bad. For example, if investors expect a firm to be good and the intermediary's private information indicates that the firm is bad, there are situations where she chooses to report that the firm is good and vice-versa.

Moreover, whenever the prior belief is not very informative, i.e. for medium values of the prior, conservatism might arise as an untalented intermediary prefers issuing bad reports even though her private signals was positive. And more reputable certification intermediaries, i.e. intermediaries perceived by the market as more talented and that are in fact untalented, tend to issue less favourable reports with greater frequency than more favourable ones for a given prior. This happens because there is an asymmetry of observability in the model: a project issued with an unfavourable report is not undertaken and this limits the learning process about the certification intermediary's type, which makes sending unfavourable reports a *safer* option. In addition, the more reputable an intermediary is the less she benefits from issuing a report that turns out to be correct and the higher the loss she incurs into when proven to be wrong.

Finally, the model concludes that the presence of a potential competitor forces a certification intermediary to issue more favourable reports: it makes her more aggressive and opt for the *riskier* option more frequently. It can also force more reputable certification intermediaries to abandon their conservative behaviour. The difference is that in the new setting sending an unfavourable report also carries disadvantages: reputation might decrease and this might compromise the chances of being hired next period.

All these conclusions hold even though the model abstracts from conflicts of interest, communication between firms and certification intermediaries, repeated relationships between firms and intermediaries and bribes.

Empirically, several studies addressed the informational value of ratings but the results have been inconclusive. Looking at the US corporate bond market, Katz

(1974) finds that bond prices adjust to rating changes and that there is no price movement prior to the announcement of a rating change, suggesting that this change is not anticipated by investor. In contrast, Hettenhouse and Sartoris (1976) and Weinstein (1977) conclude that bond prices react to other information released prior to the rating change. More recently, Heinke and Steiner (2001) examine daily excess Eurobond returns associated with announcements of watchlistings and rating changes by S&P and Moody's. They find significant price changes up to 100 trading days prior to the rating change. Moreover, bond prices still react to the actual announcements of downgrades but upgrades do not seem to cause any effect in prices. Finally, Amato and Furfine (2003) find that, for a set of observations where a rating has either just been issued or changed, ratings exhibit excess sensitivity to the business cycle.

The model developed here is also related to the literature on reputational concerns and information transmission, in which Benabou and Laroque (1992) and Morris (2001) are major contributors. Both papers build on Crawford and Sobel (1982) and Sobel's (1985) papers by developing repeated cheap talk models where there is a sender of information, i.e. the equivalent to the certification intermediary in this model, whose type (honest or strategic) is unknown to receivers. Benabou and Laroque (1992) assume the honest sender always reports her signal and, because private information is noisy, they conclude that a strategic sender can manipulate information without risking losing all her credibility as predictions which turn out to be incorrect can always be attributed to an honest mistake. Morris (2001) endogenises the behaviour of the honest sender and shows that she can also have incentives to *lie* in order to enhance reputation. However, both papers abstract from the role of public information. Moreover it seems more suitable to assume a sender that is primarily concerned with maximising profits as rating agencies and auditing firms are private companies[2]. Reporting a message that differs from the private signal in this model originates from the fact that the intermediary wants to maximise profits, and therefore her reputation, but is unsure about how much she can trust her private signals.

Reputational concerns and conflicts of interest for investment banks and equity analysts have been covered by Chemmanur and Fulghieri (1994b) and Morgan and Stocken (2003). The former model reputation by investment banks in the equity market, while the latter, develop a static cheap talk model of information transmission for financial analysts. They both assume that compensation is contingent on the message sent, unlike the model developed below where the intermediary fee is paid upfront and before any assessment is performed by the certification intermediary.

There are also papers that address competition and information transmission. Examples are the models by Lizzeri (1999) and Bolton, Freixas and Shapiro (2004). Lizzeri (1999) discusses the role of intermediaries who search out the information of privately informed agents and then decide what to disclose to the uninformed ones. Bolton, Freixas and Shapiro (2004) look at competition between investment banks that help clients, whose type is only known by the bank, to choose the appropriate financial product. Both models abstract from reputational issues whereas the model developed here explicitly models reputation using Bayesian updating.

Finally, this chapter is also related to the literature on career concerns, whose seminal papers are Holmstrom (1999) and Holmstrom and Ricart i Costa (1986). Later developments are Scharfstein and Stein (1990) on career concerns and herd behaviour, Prat (2003) on career concerns and transparency and Boot, Milbourn and

[2]For example, Moody's is a public company and was until 2000 a subsidiary of Dun&Bradstreet, Standard&Poors is a subsidiary of MacGraw-Hill and Fitch , which resulted from the merge of Fitch IBCA Investors Service, Inc. and Duff&Phelps Credit Rating (DCR), is owned by a French conglomerate, FIMALAC SA.

Thakor (2002) on the delegation of ideas.

The rest of the chapter is organised as follows. Section 2.2 describes the basic characteristics of the monopolistic model and section 2.3 looks at a benchmark case. Section 2.4 contains the equilibrium analysis and comparative statics. In Section 2.5 competition is introduced and Section 2.6 concludes. Some proofs are in the Appendix.

## 2.2 The Model

In this economy, there are three different classes of risk-neutral agents: investors (the market), a certification intermediary (she) and a firm or its manager. The model lasts for two periods and the risk-free interest rate is zero. At each date, there is a firm that needs to undertake an investment project that lasts for one period. At the end of the period the project either succeeds, and its proceeds are distributed to the firm's holders, or it fails and the firm is liquidated. Market conditions determine the expected liquidation value that to simplify is normalised to zero. The current holders of the firm are liquidity/credit constrained thus the firm cannot undergo the new project unless it succeeds in obtaining financing in the form of an extra loan. In order to obtain this loan the firm needs to be evaluated by a certification intermediary. This may be because creditors are small and dispersed and information is difficult to gather on an individual basis or it might constitute an institutional requirement[3]. The

[3]Ratings are used in prudential supervision in a large number of countries. For example, of the 12 BIS Basel Committee in Banking Supervision countries, 11 did so in 2000. In the US it goes back to 1931 when regulators either banned some institutional investors from holding securities that fell below a certain grade or specified capital requirements for holding securities that were geared to their ratings. At present, institutional investors, pension and mutual funds and insurance companies, that are among the largest purchasers of fixed-income securities, all use credit ratings to comply with regulatory requirements that require them to maintain certain minimum credit ratings for investments. Financial regulators also use ratings in a similar way for safety regulation of broker-

certification intermediary sends a message to investors based on public information and on the information she collects about the quality of the firm. The model studies the distortions to the report that the intermediary issues each period as she considers how it affects her future reputation and profits.

### 2.2.1 Agents and Basic Set-up

The firm's initial market value is $V$, with $V > 1$[4] and the firm needs a loan of $\frac{1}{2}$ to invest in a project essential to the continuation of its activity. The project can be of two types $f$, Good ($f = G$) or Bad ($f = B$). For simplicity, it is assumed that a $G$ project has a payoff of 1, while a $B$ project pays off 0.

The debt market is characterised by asymmetric information. The firm's manager, that acts on behalf of the current holders of the firm, knows the project's type but also that the firm will only continue provided that the project is undertaken and that he will be unemployed otherwise . Therefore, even if a project is of the $B$ type (and generates 0) the manager has incentives to persuade external investors to participate. Investors however, cannot tell good firms from bad ones. The firm's previous history and general conditions of the economy determine the common prior over the quality of the project. Therefore $\text{pr}(G)$ equals $\theta$, with $\theta \in (0, 1)$ and for simplicity, it is constant over time. In period 1, before another firm requires certification, the true type of the firm certified in period 0 is revealed.

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dealers and creditors can demand "ratings triggers" in financial contracts in order to accelerate repayment of an outstanding loan or to secure collateral if the borrower's rating falls below a certain level.

[4]Given that the fees are paid upfront, this assumption is necessary to make sure the firm is has enough resources to pay the intermediary's fees.

### 2.2.2 Intermediary's Private Signals

At time $t$, with $t \in \{0,1\}$, the certification intermediary receives a request for an assessment. She cannot *a priori* distinguish between Good and Bad firms but by conducting an evaluation of the firm she receives additional noisy information. The intermediary can be of two types: Talented (T) and Untalented (U). A talented intermediary identifies the project's type with probability 1 (a.s.), while if untalented (U) she only observes a noisy signal about the project. The certification intermediary knows her own type but investors and firms are uncertain about the intermediary's ability denoted by $a$, where $a = \{\mathrm{T}, \mathrm{U}\}$, and must learn about it over time. The intermediary's private information is given by $s_f$, where $s_G$ is a signal indicating a Good project and $s_B$ is a signal indicating a Bad project (the time subscript is omitted in order to simplify the notation). It is assumed that if the intermediary is talented, which at date 0 occurs with probability $\alpha_0$, with $\alpha_0 \in (0,1)$, then:

$$\Pr(s_G \mid G, T) = \Pr(s_B \mid B, T) = 1,$$

and

$$\Pr(s_G \mid B, T) = \Pr(s_B \mid G, T) = 0. \tag{2.1}$$

If untalented, which at date 0 occurs with probability $1 - \alpha_0$, the signal-generation process is given by:

$$\Pr(s_G \mid G, U) = \Pr(s_B \mid B, U) = 1 - \varepsilon$$

and

$$\Pr(s_G \mid B, U) = \Pr(s_B \mid G, U) = \varepsilon, \tag{2.2}$$

where $\varepsilon \in \left(0, \frac{1}{2}\right)$ and is common knowledge.

After observing the private signal the certification intermediary uses Bayes' rule to revise her estimate that the project is good. Thus, an intermediary forms her posterior belief using the prior about the project's type and (2.1) or (2.2) depending on her type, according to

$$\Pr\left(G \mid s_G, T\right) = 1, \ \Pr\left(G \mid s_B, T\right) = 0 \tag{2.3}$$

and

$$\Pr\left(G \mid s_G, U\right) = \frac{\left(1-\varepsilon\right)\theta}{\left(1-\varepsilon\right)\theta + \left(1-\theta\right)\varepsilon}, \ \Pr\left(G \mid s_B, U\right) = \frac{\varepsilon\theta}{\varepsilon\theta + \left(1-\theta\right)\left(1-\varepsilon\right)}. \tag{2.4}$$

Certification intermediaries charge a fee determined endogenously and paid up-front, like common practice with rating agencies. Before evaluating the firm the intermediary is at the same informational level as any potential investor of the firm, i.e. she cannot ex-ante distinguish between the two projects.

Investors value certified firms based on the intermediary's report, the belief they have that her report is correct, and the other variables that are common knowledge. At the end of the period the state of nature is realised and publicly observed. Investors have then the chance to update their beliefs about the intermediary's type, by comparing the message sent with the true state in case the project is undertaken. The game is then repeated for one more period with the same intermediary and investors but with a new firm. This concludes the game. Reputation in this context translates the beliefs of investors about the certification intermediary ability, given the message that she sent and the true project's type or the investor's initial belief about the project's true type in case this is not undertaken.

### 2.2.3 Investors

At each date, the investors' required repayment to invest in certified debt is derived after a message has been sent by the certification intermediary. The required repayment at time $t$ depends on the true type, on the intermediary's message, and on the confidence investors have in her message, as captured by her reputation $\alpha_t$. This message is given by $m_f$, where $f \in \{G, B\}$, $m_G$ corresponds to a favourable report and $m_B$ to an unfavourable report (again the time subscript is omitted in order to simplify the notation). The required repayment at time $t$, for a message $m_f$ and provided that the firm is of type $f$, is denoted by $r^t_{ff}$, with $r^t_{ff} \in \left[\frac{1}{2}, 1\right]$. Using Bayes' rule to evaluate the various conditional probabilities and given that the financial market is competitive and risk neutral, $r^t_{ff}$ needs to satisfy the investors participation constraints for each $m_f$:

$$\Pr\left(G \mid m_f, \{\Omega_t\}\right) r^t_{fG} + \Pr\left(B \mid m_f, \{\Omega_t\}\right) r^t_{fB} = \frac{1}{2}$$

where $\{\Omega_t\}$ represents the investors' information set at time $t$.

Whenever the project is undertaken and fails the liquidation value is zero meaning that $r^t_{GB} = r^t_{BB} = 0$. Additionally, in order to simplify the model and focus on the most interesting case it is assumed that investors cannot become involved in a project whose report has been unfavourable[5]. As a result, $r^t_{GG}$ is derived to be equal to

$$\frac{1}{2}\left(1+\frac{\Pr(B)\Pr(m_G \mid B,\{\Omega_t\})}{\Pr(G)\Pr(m_G \mid G,\{\Omega_t\})}\right).$$

Furthermore, $r^t_{GG}$ needs to be lower than 1 to make sure that firms would like to undertake the project. Hence,

**Lemma 1.** *A necessary condition for investment to happen is*

$$\Pr(G)\Pr(m_G \mid G,\{\Omega_t\}) > \Pr(B)\Pr(m_G \mid B,\{\Omega_t\}). \tag{2.5}$$

For a given prior belief, the intermediary message needs to be informative. The remainder of this chapter considers that investment only takes place if a favourable report is issued, provided that (2.5) holds.

### 2.2.4 The Certification Intermediary Fee

The objective of the certification intermediary is to maximise the expected value of her future profits (fee net of any certification costs). The fee is derived as follows. The firm's manager acts on behalf of the shareholders and knows the project's type but enjoys private benefit of control. Hence, he is willing to pay any fee to undertake the project and not to reveal the true type in the Bad-project's case. In particular, the manager of Bad project is willing to pay as much as the manager of a Good project. Because certification is compulsory, at time $t$ a Good firm is willing to pay a fee $F_t(\alpha_t)$ up to the amount for which its participation constraint is binding. A

[5]This can be an equilibrium condition for certain values of the parameters. But otherwise it can be justified by institutional reasons; for example pensions funds and insurance companies are not allowed to invest in securities rated with a non-investment grade. For other examples *see supra* note 3.

higher fee cannot be extracted from Good firms as shareholders would veto it. The surplus for a good message is $1 - r_{GG}^t - F_t(\alpha_t)$ and for a bad message $-F_t(\alpha_t)$. Thus, the Good firm's participation constraint at $t$ is the following:

$$\Pr(m_G \mid G, \{\Omega_t\}) \left(1 - r_{GG}^t - F_t(\alpha_t)\right) + \Pr(m_B \mid G, \{\Omega_t\}) \left(-F_t(\alpha_t)\right) = 0.$$

Looking at the firm participation constraint, two conflicting interests can be identified for the firm (and indirectly for the intermediary): the firm wants the repayment to investors to be as low as possible, and this happens if a more reputable intermediary sends a good message but, on the other hand, a less reputable intermediary is more likely to send a good message necessary for the project to be undertaken.

Given $r_{GG}^t$, the fee at time $t$, $F_t(\alpha_t)$ can be set up to

$$\frac{\Pr(G)\Pr(m_G \mid G, \{\Omega_t\}) - \Pr(B)\Pr(m_G \mid B, \{\Omega_t\})}{2\Pr(G)}. \tag{2.6}$$

By Lemma 1, this is always positive.

However, for the firm no certification means no project but for the intermediary no certification also means no fee. Both parties have something to lose if the project is not undertaken and this implies that the intermediary might not extract the full surplus of certification from the firm. Hence, the certification intermediary knows she can charge a fraction $\kappa$, with $\kappa \in (0, 1]$, can be thought of as the outcome of bargaining, exogenous to the model, between the intermediary and the firm.

To sum up, the fee charged by the certification intermediary is unique. According to what happens in reality, it is not possible for the certification intermediary to screen among firms by offering a menu of fees $\{F_t(\alpha_t)\}$ and let each firm choose a fee according to its type. This would mean that by choosing a fee the firm would reveal its type and there would be no need for the firm to be assessed and for the intermediary to worry about reputation. The Bad-firm has a monetary surplus associated to the project that equals zero but its manager is ready to pay as much as the Good-firm is paying, in order not to reveal its type, at the expense of the existing holders of the firm[6]. Basically, in this model the firm has a passive role. Certification is compulsory, therefore the firm only chooses whether to obtain certification and indirectly whether to undertake the project.

It is also assumed that the firm cannot refuse to make use of the information collected by the intermediary after learning the evaluation that she will report to investors.

### 2.2.5 The Certification Intermediary Behaviour

After being hired, the certification intermediary collects information about the firm in the form of the private signal $s_f$. She then balances out the costs and benefits of sending a report that is contrary to the private signal, i.e. she chooses $\Pr(m_G \mid s_B)$ and $\Pr(m_B \mid s_G)$. In order to ease notation the time subscript is omitted and henceforth $\Pr(m_G \mid s_B)$ is denoted by $\overline{\gamma}$ and $\Pr(m_B \mid s_G)$ by $\underline{\gamma}$. There is however an arbitrarily small cost from deviating from the private signal given by $c_t$ that includes, for example, the cost in terms of extra time and effort of commissioning a report where a financial analyst has to disguise private information about the firm[7]. This cost ensures that

[6]Remember that the manager will be unemployed unless the project is undertaken.

[7]It can also include litigation costs, i.e. $c_t$ can be interpreted as the legal cost in case misreporting is discovered times the probability of legal action. Even though legal action is relatively common for auditing firms, lawsuits against rating agencies seem to be quite infrequent. There have however been some cases where rating agencies have been accused of fraud in misreporting or omitting certain facts in their ratings (e.g. the Jefferson County, Colorado, School District case against Moody's, the Orange County, California case against S&P or the LaSalle Nat'l Bank v. Duff&Phelps Co. case).

the intermediary has incentives to care about reputation even in the last period of the game.

## 2.3 Equilibrium Analysis

In a first best world, an intermediary should simply report her private signal. In a world with reputational concerns and where a certification intermediary seeks to maximise expected profits, an equilibrium consists of choices by the intermediary of $\overline{\gamma}$ and $\underline{\gamma}$ specifying the probability of sending a message different from the signal received. It also consists of choices by the firm of whether to hire or not the certification intermediary (and indirectly whether to undertake the new project) based on $\alpha_t$, $\overline{\gamma}$ and $\underline{\gamma}$ and a system of beliefs formed by investors. Investors choose whether to provide investment funds and the expected repayment based on $\theta$, $\alpha_t$, $\overline{\gamma}$ and $\underline{\gamma}$. The model is solved by backwards induction.

### 2.3.1 Period 1

#### Certification Intermediary Optimal Behaviour and Fee

In period 1, since deviating from the private signal is costly and there is no reputational benefit to consider (as the game is over at the end of period 1), the certification intermediary minimises costs by always reporting her signal. Therefore,

**Proposition 1.** *The certification intermediary never misreports in the last period, regardless of her type.*

In addition, agencies have also been investigated by the SEC and the US Department of Justice.

Alternatively, it can been seen as a short-cut to capture in a two period model the impact over the future reputation which would happen in repeated relationships.

Given Proposition 1, probabilities $\Pr(m_G \mid G, \{\Omega_1\})$ and $\Pr(m_G \mid B, \{\Omega_1\})$ are equal to $\alpha_1 + (1-\alpha_1)(1-\varepsilon)$ and $(1-\alpha_1)\varepsilon$ respectively. And using (2.6) the fee charged in period 1 is

$$F_1(\alpha_1) = \kappa \left( \frac{\alpha_1 \varepsilon}{2\theta} + \frac{(\theta - \varepsilon)}{2\theta} \right). \tag{2.7}$$

A higher reputational level of the certification intermediary generates a higher fee as $\frac{\partial F_1(\alpha_1)}{\partial \alpha_1}$ is positive. As a result, when hired in period 0, the intermediary acts to maximise $\alpha_1$. When investors are unsure about the intermediary's type they require a higher repayment because they want to be compensated for the probability of an untalented intermediary making a mistake by sending a favourable report for a bad project. In addition to bearing a higher repayment good firms are also uncertain about which message the intermediary is going to send. As a result a lower fee is derived from the firm's participation constraint.

### Posterior beliefs

So far reputation in period 1 has been generally denoted by $\alpha_1$ but in fact it varies depending on the message sent and how it relates to the outcome of the project. Therefore, henceforth $\alpha_{GG}$ denotes the posterior belief that the certification intermediary is talented given that she sent $m_G$ and the true firm's type was indeed G, i.e. to denote $\Pr(T \mid m_G, G)$; $\alpha_{GB}$ is used to identify the probability that the certification intermediary is talented given that she sent $m_G$ but the true type turned out to be B, i.e. $\Pr(T \mid m_G, B)$; and finally, $\alpha_B$ denotes the posterior belief that the certification intermediary is talented given that she sent $m_B$, i.e. $\Pr(T \mid m_B)$. In this case the project is not undertaken and therefore investors and the new firm cannot compare the certification intermediary's report to the project realisation. The analytical expressions for these probabilities are derived below.

### 2.3.2 Period 0

#### Certification Intermediary Optimal Behaviour

At date 0, a certification intermediary is hired, paid $F_0(\alpha_0)$ and collects a private signal about the project. After observing the signal, the intermediary comes up with a posterior belief about the value of the project and must make a decision about what message to send. She makes this decision such that her expected fee in period 1, that depends on her reputational level at the end of period 0, is maximised. The link between periods 0 and 1 is therefore the intermediary's reputation, which is revised at the end of period 0 in view of whether her forecast was realised or not.

After observing the private signal the certification intermediary uses the Bayes' rule to revise her estimate about the project's type according to (2.3) and (2.4). A certification intermediary with ability $a$ and a private signal $s_G$ has an expected profit from reporting her signals of:

$$\Pr(G \mid s_G, a) F_1(\alpha_{GG}) + \Pr(B \mid s_G, a) F_1(\alpha_{GB}).$$

But it may be that she decides to send a message different from her private signal even though this implies an extra cost of $c_0$. In this case, the expected profit in period 1 is $F_1(\alpha_B) - c_0$. The intermediary sends the message that generates a higher expected profit in period 1. Looking at the following expression:

$$\underline{\pi}^e(a) = \Pr(G \mid s_G, a) F_1(\alpha_{GG}) + \Pr(B \mid s_G, a) F_1(\alpha_{GB}) - F_1(\alpha_B) + c_0, \tag{2.8}$$

in equilibrium, if $\underline{\pi}^{e}(a) > (<) 0$ the intermediary follows (contradicts) the private signal and if $\underline{\pi}^{e}(a) = 0$ there is an equilibrium in mixed strategies.

On the other hand, if the private signal indicates that the project is bad and the message coincides with this private signal, the expected profit in period 1 is simply $F_1(\alpha_B)$, but if the certification intermediary decides to go against her private signal the expected profit is

$$\Pr(G \mid s_B, a) F_1(\alpha_{GG}) + \Pr(B \mid s_B, a) F_1(\alpha_{GB}) - c_0.$$

Once more the intermediary looks at

$$\overline{\pi^e}(a) = \Pr(G \mid s_B, a) F_1(\alpha_{GG}) + \Pr(B \mid s_B, a) F_1(\alpha_{GB}) - c_0 - F_1(\alpha_B). \quad (2.9)$$

There is no deviation from the private signal for $\overline{\pi^e}(a) < 0$ and there is an equilibrium in mixed strategies if $\overline{\pi^e}(a) = 0$. Otherwise, the intermediary contradicts the private signal.

It can also be proven that the talented certification intermediary never misreports. In particular, there cannot be an equilibrium in which the talented certification intermediary always sends a message that goes against her private signal. Looking at pure strategies only, observe that a talented intermediary always has less of an incentive to misreport than the untalented one. So if an intermediary observes $s_G$ and reports $m_B$ the expected profit is $F_1(\alpha_B) - c_0$ regardless of the type. But when a talented intermediary observes $s_B$ and reports $m_G$ her expected profit is $F_1(\alpha_{GB}) - c_0$ which is lower than the expected profit an untalented intermediary, $\Pr(G \mid s_B, a) F_1(\alpha_{GG}) + \Pr(B \mid s_B, a) F_1(\alpha_{GB}) - c_0$ as the fee is increasing in the reputational level and the reputational level increases when the intermediary is correct and decreases otherwise i.e., $\alpha_{GG}$ exceeds $\alpha_{GB}$. Consequently, an untalented intermediary misreports whenever a talented intermediary does so. Secondly, it can be proven by contradiction that a talented intermediary never misreports. If for signal $s_G$ the talented certification intermediary sends $m_B$, then the untalented certification agent would also choose to send $m_B$. The firm then decides not to hire an intermediary because certification is costly and an unfavourable report implies no investment. And if whenever the signal is $s_B$ the talented certification intermediary sends $m_G$, the untalented certification intermediary would also chooses to send $m_G$. If the talented intermediary decides to deviate and be truthful, the untalented might or might not deviate from $m_G$. If she does not, the talented intermediary prefers being truthful because when sending $m_B$ she reveals her type whereas before investors could not distinguish between the two types. In fact, if both types behaved alike investors would be unable to update their prior belief about the intermediary's type. This would lead to a lower reputational level and consequently to a lower fee. Hence, if the untalented type does not follow the talented type deviation, she always prefers to deviate. If she also sends the true signal $m_B$, then the talented intermediary reconsiders what to do: she can either keep sending $m_B$ or not. But if not the untalented type will again follow because as it was stated in the beginning of this proof a talented intermediary always has less of an incentive to misreport than the untalented one, so if she misreports the other does it as well. And in such case, it is better to be truthful. Hence, in equilibrium the talented intermediary reports her private signal.

This also does not mean that the talented certification intermediary follows a mixed strategy in equilibrium. In this case if a signal $s_G$ is received, the talented certification intermediary is indifferent between sending $m_G$ and $m_B$ and therefore randomises between the two, i.e. $\underline{\pi}^e(T) = 0$. As a result, the untalented certification intermediary strictly prefers to send $m_B$ as $\underline{\pi}^e(U) < 0$. This follows from the fact that the noisy private signal makes sending $m_G$ strictly worse for the untalented intermediary than for the talented one (sending $m_B$ gives both types the same profit; i.e. $F_1(\alpha_B) - c_0$ and $F_1(\alpha_{GG})$ exceeds $F_1(\alpha_{GB})$). Hence, every time the talented intermediary chooses to randomise, the untalented intermediary strictly prefers to send $m_B$. This implies that only the talented certification intermediary ever sends $m_G$ and as a result, a favourable report allows firms and investors to identify the intermediary's type with certainty. Consequently, when this happens she is able to extract the maximum fee in period 1. But this also contradicts the conjectured indifference of the talented certification intermediary between sending $m_G$ and $m_B$. Thus, in equilibrium the talented certification intermediary cannot play a mixed strategy. A similar proof holds when the private signal is $s_B$. However in this case, the talented certification intermediary is the only one sending $m_B$ as $\overline{\pi^e}(U) > 0$.

As far as the untalented intermediary is concerned, a mixed strategy independent of $\theta$ cannot be an equilibrium. This can be proven by contradiction. If there is a mixed strategy such as $\overline{\pi^e}(U) = 0$ this implies that $\underline{\pi}^e(U) > 0$ because $F_1(\alpha_{GG})$ is higher than $F_1(\alpha_{GB})$ and $\Pr(G \mid s_G, U)$ exceeds $\Pr(G \mid s_B, U)$[8]. This means that there is set of priors that makes the untalented intermediary indifferent between reporting favourably or unfavourably when she receives a bad private signal but that makes her report the private signal when this is positive. A similar result holds for $\underline{\pi}^e(U) = 0$. Therefore, it is obvious that the certification intermediary's optimal behaviour is going to be affected by the prior $\theta$. In fact, it can be proven that the equilibrium is characterised by two values of $\theta$, given by $\theta_L$ and $\theta_H$, with $0 < \theta_L < \theta_H < 1$, such that

[8] And hence $\underline{\pi}^e(U) - \overline{\pi}^e(U)$ is always positive.

for $\theta < \theta_L$ the untalented certification intermediary reports $m_B$ when the signal is $s_B$ but plays a mixed strategy if the private signal is $s_G$; on the other hand, for $\theta > \theta_H$ the untalented certification intermediary reports $m_G$ when the signal is $s_G$ but plays a mixed strategy when the private signal if $s_B$. For the remaining set of priors, i.e. $\theta_L < \theta < \theta_H$, the intermediary reports her private signal. It is then proved that this is indeed the unique equilibrium.

In order to prove this, the way reputation evolves between date 0 and 1 needs to be examined. If $\theta < \theta_L$ it is above conjectured that a talented certification intermediary always reports her signal, whereas an untalented certification intermediary is expected to report $m_B$ if $s_B$ is observed but plays a mixed strategy if $s_G$ is observed, i.e. reports $m_B$ with probability $\underline{\gamma}$ and $m_G$ with probability $1-\underline{\gamma}$. If $m_B$ is sent, the posterior assessment of her ability is given by

$$\underline{\alpha}_B = \frac{\alpha_0 (1-\theta)}{\alpha_0 (1-\theta) + (1-\alpha_0)\left(((1-\varepsilon)\theta + \varepsilon(1-\theta))\underline{\gamma} + (\varepsilon\theta + (1-\varepsilon)(1-\theta))\right)}. \tag{2.10}$$

And if the certification intermediary reports $m_G$, her date 1 reputation varies depending on whether the project pays off 1 or 0. These two reputational levels are given by

$$\underline{\alpha}_{GG} = \frac{\alpha_0}{\alpha_0 + (1-\alpha_0)(1-\varepsilon)\left(1-\underline{\gamma}\right)} \tag{2.11}$$

and $\underline{\alpha}_{GB} = 0$ respectively. Moreover, $\underline{\alpha}_{GB} < \underline{\alpha}_B < \underline{\alpha}_{GG}$ and $\underline{\alpha}_{GG} > \alpha_0$ but $\underline{\alpha}_B$ only exceeds $\alpha_0$ when the prior belief $\theta$ is relatively low and definitely lower than $\frac{1}{2}$. Obviously investors need to be very convinced about the bad quality of the project to be confident about an intermediary's judgement that is not verifiable.

For projects whose $\theta$ exceeds $\theta_H$, the talented certification intermediary reports her signal, whereas the untalented certification intermediary is conjectured to always send $m_G$ when $s_G$ is observed, but sends $m_G$ with probability $\overline{\gamma}$ and $m_B$ with probability $1-\overline{\gamma}$ if $s_B$ is observed. When $m_B$ is sent, the posterior assessment of her ability is

$$\overline{\alpha}_B = \frac{\alpha_0 (1-\theta)}{\alpha_0 (1-\theta) + (1-\alpha_0)(\varepsilon\theta + (1-\varepsilon)(1-\theta))(1-\overline{\gamma})}. \tag{2.12}$$

And if the certification intermediary sends $m_G$ her reputation is

$$\overline{\alpha}_{GG} = \frac{\alpha_0}{\alpha_0 + (1-\alpha_0)((1-\varepsilon)+\varepsilon\overline{\gamma})} \tag{2.13}$$

or $\overline{\alpha}_{GB} = 0$, depending on whether the project pays off 1 or 0 respectively. In this case $\overline{\alpha}_{GG}$ exceeds $\overline{\alpha}_B$ if $\theta > \frac{\overline{\gamma}}{\varepsilon(1-\overline{\gamma})+\overline{\gamma}}$. However, this is always the case because if $\theta$ is lower than that threshold, $\overline{\gamma}$ equals zero for any $c_0$ . But it is also the case there is no $\theta$ compatible with such a $\overline{\gamma}$. Also, $\overline{\alpha}_{GG} > \alpha_0$ and $\overline{\alpha}_B > \alpha_0$ if $\theta$ is relatively low and always happens for $\theta$ lower than $\frac{1}{2}$. In addition, $\underline{\alpha}_B$ is always lower than $\overline{\alpha}_B$ and $\underline{\alpha}_{GG}$ is always higher than $\overline{\alpha}_{GG}$.

The final step of the proof, i.e. to show that the mixed strategies $(\underline{\gamma}, 1-\overline{\gamma})$ and $(\overline{\gamma}, 1-\underline{\gamma})$ do in fact exist, is relegated to the Appendix. The results can be generalised by Proposition 2:

**Proposition 2.** *The behaviour of the certification intermediary in period 0 is such that:*

1. *A talented certification intermediary always reports her signal. This means that she reports $m_G$ whenever $s_G$ is observed, and reports $m_B$ whenever $s_B$ is observed.*

2. *For the untalented certification intermediary, there are $\theta_L$ and $\theta_H$, with $\theta_L > \frac{1}{2}$,*

*such that, for $\theta \in (\theta_L, \theta_H)$, she always reports her signal if $c_0$ is arbitrarily small. Otherwise, and provided that the same condition on $c_0$ is satisfied, the certification intermediary behaves as follows:*

- *For $\theta$ equal to $\theta_L$ she reports $m_B$ whenever $s_B$ is observed, and reports $m_B$ with probability $\underline{\gamma}$ and $m_G$ with probability $1-\underline{\gamma}$ whenever $s_G$ is observed and for $\theta \in [0, \theta_L)$ she always sends a bad report;*
- *For $\theta$ equal to $\theta_H$, she reports $m_G$ whenever $s_G$ is observed, and reports $m_G$ with probability $\overline{\gamma}$ and $m_B$ with probability $1-\overline{\gamma}$ whenever $s_B$ is observed and for $\theta \in (\theta_H, 1]$ she always sends a good report.*

This proposition establishes that an untalented intermediary may ignore her private signal and decide instead to send a report that fits the expectations created by the public signal. This result is directly related to the issue of *conformity.* A number of papers such as Bernheim (1994) and Prendergast (1993) discuss this topic. By behaving in this particular way, an agent is basically trying to differentiate himself from the type that he wishes not to be identified with. The goal of the untalented intermediary is to mimic the talented type as by doing so she diminishes the chances of revealing her type. If for example $\theta$ is sufficiently low, there is a relatively high probability that the talented intermediary has received a bad signal and that she will send an unfavourable report. Given that the untalented intermediary cannot trust completely her private signal, there is a critical level of $\theta$, such that she chooses to ignore it with positive probability if it indicates that the firm is good.

On the other hand, it was proven in the Appendix that the threshold $\theta_L$ is higher than $\frac{1}{2}$. For medium values of the prior belief about the project quality, i.e when the prior is less informative, one would expect the untalented intermediary to report truthfully but in fact, she chooses to report $m_B$ even when this contradicts her private information, i.e. there is an excessive number of bad reports. This happens because there is an asymmetry of observability in the model: a project issued with an unfavourable report is not undertaken and this limits the learning process about the certification intermediary's type, which makes sending unfavourable reports a *safer* option.

### Comparative Statics

A number of interesting results are derived when performing comparative statics in the equilibrium values of $\underline{\gamma}$ and $\overline{\gamma}$.

**Proposition 3.** *The equilibrium probabilities $\overline{\gamma}$ and $\underline{\gamma}$ are monotonic in $\theta$ and greatest for extremely high and low values of $\theta$, i.e. $\frac{\partial \overline{\gamma}}{\partial \theta} > 0$ for $\theta \in (\theta_H, 1)$, and $\frac{\partial \underline{\gamma}}{\partial \theta} < 0$ for $\theta \in (0, \theta_L)$, with $\overline{\gamma}\mid_{\theta=\theta_H} = 0$ and $\underline{\gamma}\mid_{\theta=\theta_L} = 0$.*

This means that the more extreme the prior belief $\theta$ is, the higher the probability of deviation from the private signal. The lower the prior belief, the higher the probability of sending an unfavourable report when facing a good private signal. On the contrary, the higher the prior belief, the higher the probability of sending a favourable report when facing a bad private signal. If the public signal is uninformative, i.e. $\theta = \frac{1}{2}$, an untalented intermediary can either be truthful or conservative, i.e. always reports an unfavourable private signal but reports a favourable private signal with a positive probability only, depending on the reputational level.

**Proposition 4.** *The equilibrium probabilities $\overline{\gamma}$ and $\underline{\gamma}$ are monotonic and increasing in $\varepsilon$, i.e. $\frac{\partial \overline{\gamma}}{\partial \varepsilon} > 0$ and $\frac{\partial \underline{\gamma}}{\partial \varepsilon} > 0$.*

This is an intuitive result: the less talented an intermediary is, the less she trusts her private signal and the higher the incentive to ignore it and convey with the public information.

**Proposition 5.** *The equilibrium probability $\overline{\gamma}$ is concave and decreasing in $\alpha_0$ for sufficiently small levels of $c_0$.*

As reputation increases, the intermediary's difference in future profits from sending the two different messages becomes lower, i.e. $\frac{\partial \overline{\pi^e}}{\partial \alpha_0}$ is negative. For high levels of $\theta$, the higher the initial reputational level $\alpha_0$ the lower the increase in reputation if a favourable report is correct, and the higher the decrease in reputation if it turns out to be wrong. Hence, the more reputable an intermediary is the less incentives she has for gambling by deviating from an unfavourable private signal that has the advantage of generating a non-random level of future profits.

**Proposition 6.** *The equilibrium probability $\underline{\gamma}$ is always increasing and convex in $\alpha_0$.*

The same reasoning as before applies here. Reporting a favourable private signal increases reputation and future profits if the report turns out to be right but if it turns out to be wrong the intermediary's type is revealed and this generates a high cost in terms of profits next period. Thus because the benefit from deviating is not certain, an intermediary tends to deviate from her private information and send an unfavourable report instead as this turns out to be less risky. As reputation increases, the intermediary's difference in future profits from sending the two different messages becomes lower, i.e. $\frac{\partial \underline{\pi^e}}{\partial \alpha_0}$ is negative. This means that the higher the reputational level the lower the benefits from reporting the private signal and hence, the stronger the incentives to deviate.

This leads to the following corollary:

**Corollary 1.** *The behaviour of the untalented certification intermediary is such that as her reputational level increases, she tends to issue unfavourable reports more often than favourable ones.*

The asymmetry of observability arising from the fact that a good report is always verifiable whereas a bad one is not, combined with the fact that a more reputable intermediary benefits less from gambling, results in a more reputable intermediary being more prone to sending unfavourable reports when the prior is relatively low or relatively high than a less reputable one[9].

This result is in line with some empirical evidence that suggests that smaller agencies, which are usually regarded as less reputable by investors and firms, tend to rate in a more favourable way. For instance, Cantor and Packer (1997) reveal that, in their sample, DCR and Fitch give systematically higher ratings on jointly rated issues than Moody's and S&P. They test whether this fact reflects different rating scales or results from selection bias, i.e. only higher-rated firms seek DCR and Fitch ratings. They found limited evidence of selection bias and concluded that observed differences in average ratings seem to reflect differences in rating scales and standards. Also Jewell and Livingston (2000) find that DCR ratings are higher than S&P's or Moody's.

### Fees and Investors

The behaviour described above constitutes the only credible behaviour from the intermediary point of view. And this is going to be useful in period 0 when investors price debt and the firm makes its hiring decision and decides on the fee. Thus, for $\theta \in$

[9]Conservatism is also discussed by Zwiebel (1995). The idea is that reputational concerns may lead managers to refrain from undertaking innovations that are first order stochastically dominant because of the downside risk of being fired.

$[\theta_L, \theta_H]$ the conditional probabilities, repayment to investors and fee remain the same as in period 1 because intermediaries are truthful. If $\theta \in [0, \theta_L)$, $\text{pr}(m_G \mid G, \{\Omega_0\}) = \alpha_0 + (1-\alpha_0)(1-\varepsilon)\left(1-\underline{\gamma}\right)$ and $\text{pr}(m_G \mid B, \{\Omega_0\}) = (1-\alpha_0)\,\varepsilon\left(1-\underline{\gamma}\right)$. And using (2.6) the fee charged in period 0 is

$$F_0(\alpha_0) = \kappa \frac{\alpha_0 \varepsilon + (\theta - \varepsilon) - (1-\alpha_0)\,\underline{\gamma}\,(\theta - \varepsilon)}{2\theta}.$$

There are two effects that need to be considered in this case. The firm knows that an intermediary is likely to conform to the public information and that a favourable report is less likely to occur. This has a negative effect on the fee. On the other hand, when the report is indeed favourable, the intermediary is choosing to contradict the public signal so investors believe the intermediary is more likely to be telling the truth but only if the probability that she is making a mistake is not very high, i.e. $\varepsilon < \theta$. The required repayment to investors is lower and the intermediary can extract a higher fee than if there had not been deviation from the private signal, i.e. $\underline{\gamma} = 0$. The lower the difference between $\theta$ and $\varepsilon$, i.e. the higher the prior belief $\theta$ or the lower the $\varepsilon$, the lower the repayment to investors and the higher the fee. If on, the other hand, $\theta$ decreases and $\varepsilon$ increases, investors tend to attribute a favourable message to an honest mistake. They require a higher compensation for this extra risk and consequently the fee is going to be lower.

Finally, if $\theta \in (\theta_H, 1]$,

$$\Pr(m_G \mid G, \{\Omega_0\}) = \alpha_0 + (1-\alpha_0)(1 - \varepsilon + \varepsilon\overline{\gamma})$$

and

$$\Pr\left(m_G \mid B, \{\Omega_0\}\right) = (1-\alpha_0)\left(\varepsilon + (1-\varepsilon)\overline{\gamma}\right).$$

It follows that the fee in period 0 is

$$F_0\left(\alpha_0\right) = \kappa \frac{\alpha_0\varepsilon + (\theta-\varepsilon) + (1-\alpha_0)\overline{\gamma}\left(\theta+\varepsilon-1\right)}{2\theta}.$$

If $\theta$ exceeds $1-\varepsilon$ the fee is higher than if the intermediary had followed the private signal, i.e. $\overline{\gamma} = 0$. The same logic applies here. As the intermediary is more likely to conform with the (good) public information, the firm and investors can expect a favourable report more frequently. But it turns out that $\theta_H$ is always higher than $1-\varepsilon$[10], hence deviations from the private signal only happen when $\theta$ is very high or the probability of making a mistake $\varepsilon$ is very low. Therefore, investors are more likely to believe that a favourable report does translate the intermediary private information. They require a lower repayment and the surplus that the firm is going to share with the intermediary is higher.

## 2.4 Bertrand Competition

So far the focus of this chapter has been on the strategic information revelation of a monopolistic certification intermediary. However, it is common to observe in many markets intermediaries competing among each other. In the credit rating industry this trend is very likely to increase in the future given the likely increase in the demand for ratings for regulatory purposes. To reflect this situation consider again the framework

[10]See Proof of Proposition 2 (part (i) Type $U_B$ randomises for high values of $\theta$) in the Appendix.

developed in Section 2.2 and introduce a second certification intermediary[11]. Thus at each date the existing certification intermediary $i$ faces potential competition of an incoming certification intermediary $j$. The incoming competitor has entry costs of zero and to simplify the analysis and limit the number of cases to consider they only differ in terms of the initial reputation, i.e. $\alpha_{i0} \neq \alpha_{j0}$[12] and $\alpha_{j0}$ is positive. At each date, intermediaries make simultaneous offers and the firm accepts or refuses the proposals simultaneously.

The repayment to investors is calculated as before but that is not the case for the fee paid by the firm. At each date, the firm's expected surplus from being certified by intermediary $\phi$ with $\phi = i, j$ is

$$\Pr\left(m_G \mid G, \{\Omega_t\}, \phi\right)\left(1 - r_{\phi t}\left(m_G, G\right)\right) - F_{\phi t}\left(\alpha_{it}, \alpha_{jt}\right).$$

### 2.4.1 Period 1

**Certification Intermediary Optimal Behaviour and Fee**

In this period no one misreports because this implies an additional cost that is not matched by an additional benefit. Thus,

**Proposition 7.** *With competition, a certification intermediary continues reporting her private signal in the last period.*

Intermediaries make offers to the firm regarding the fee to be charged for certifi-

[11]Most companies are rated by more than one rating agency. Therefore this can be reinterpreted as a firm seeking for an additional rating that is going to be attributed by one of the remaining rating agencies. For example, according to Jewell and Livingston (1999 and 2000), show that the DCR and Fitch rating serves as a tie-break in case of split ratings between Moody's and S&P.

[12]You can think of a situation where there is a pool of analysts whose ability is fixed: it can either be 1 or $1 - \varepsilon$ but the entity that hires them might (or might not) be able to distinguish between the two types of analysts with probability $\alpha_{i0}$ $(1 - \alpha_{i0})$.

cation and this process simply results in the certification intermediary that generates the highest expected surplus setting the price by forcing the other intermediary to set a zero fee. Using the result derived in Proposition 7 and assuming that $i$ generates the highest expected surplus for the firm, and is consequently hired, the fee charged in period 1 is

$$\Pr\left(m_G \mid G, \{\Omega_t\}, i\right)\left(1 - r_{it}\left(m_G, G\right)\right) - F_{it}\left(\alpha_{it}, \alpha_{jt}\right) = \Pr\left(m_G \mid G, \{\Omega_t\}, j\right)\left(1 - r_{jt}\left(m_G, G\right)\right)$$

or

$$F_{i1}\left(\alpha_{i1}, \alpha_{j1}\right) = \left(\frac{\alpha_{i1}\varepsilon}{2\theta} + \frac{(\theta - \varepsilon)}{2\theta}\right) - \left(\frac{\alpha_{j1}\varepsilon}{2\theta} + \frac{(\theta - \varepsilon)}{2\theta}\right)$$

that simplifies to

$$F_{i1}\left(\alpha_{i1}, \alpha_{j1}\right) = \frac{\varepsilon}{2\theta}\left(\alpha_{i1} - \alpha_{j1}\right). \tag{2.14}$$

After setting a fee such that the fee of the competitor is driven to zero, the firm still has the option not to undertake the project. Therefore, the surplus is divided between the firm and the intermediary according to their bargaining power resulting in the intermediary charging a proportion $\kappa$ of $F_{i1}$. Of course, certification intermediary $i$ is hired because $\alpha_{i1} > \alpha_{j1}$ (initial assumption). In addition, the fee charged now is lower than in the monopolistic case, i.e. $F_{i1}\left(\alpha_{i1}, \alpha_{j1}\right) < F_1\left(\alpha_1\right)$ when $\alpha_{i1} = \alpha_1$, as the fee is now the difference between the expected surplus generated by certification by $i$ minus the surplus generated if $j$ had been hired by the firm in proportion $\kappa$.

Negative fees are ruled out in this model, however if this was not the case the results about the fee in period one would not change. If fees could in fact be negative, the intermediary that is not hired in period 0, which is the one with the lowest reputational level, would be willing to pay to have the chance to certify the firm in this period hoping to recover this amount in period one by being hired again. Of course this would only happen if in period one her new reputational level would be higher than her competitor's, at least in some situations, and consequently, the firm would prefer to hire her instead i.e., if there were chances to recover the negative fee by charging a positive fee in the last period. The competitor, on the other hand, would never set negative fees in the last period as there are no future periods when to recover this "investment". Hence, in period one the intermediary with the highest reputational level would extract as much as possible from the firm. But then the intermediary with the highest reputational level would also be willing to pay to certify in period zero and given that she has more to lose[13], she would be willing to pay even more, and up to the expected profit in period one and she would end up being hired in period zero. At this point she decides whether to misreport by looking at the expected profit in period one. She would try to extract as much as possible from the firm in the cases where she is the intermediary with the highest reputational level[14] given that her competitor would have no incentives to set negative fees. Hence, in period one intermediaries would always behave according to the differentiated Bertrand competition set-up described above.

Moreover, one could say that because there is competition $\kappa$ could perhaps be lower than in the monopolistic case. But once negative fees are ruled out, one intermediary sets the fee such that the competitor is out of the market even when she sets a zero-fee, i.e. even if all the surplus goes to the firm. So the intermediary with the highest reputational level charges a positive fee that generates at least the same surplus for the firm than the competitor is able to generate with a zero-fee. But even at this point,

[13]Remember that the fee increases with the reputational level.

[14]And this will happen for sure at least for the case where she sends a good report in period zero that turns out to be correct.

the firm has the option of not undertaking the project. Therefore, intermediary and firm can bargain on the fee that may decrease even further. But the intermediary that was already out of the market cannot interfere in this bargaining process, since she had already set the fees at the minimum possible level before.

Therefore, $\kappa$ could in fact remain the same or not. In fact, since in competition the intermediary is bargaining over a lower surplus, it could even be the case that she bargains for a higher $\kappa$ to ensure a profit closer to monopoly but, for simplicity, it was assumed that $\kappa$ remains the same.

#### Posterior Beliefs

The posterior beliefs about the certification intermediaries are calculated as before and to distinguish between both intermediaries a subscript $i$ is added to the previous notation. The only difference relative to the previous case is that updating only occurs when an intermediary is hired. Otherwise, her reputational level remains equal to the prior. Also note that $\overline{\alpha}_{iB}$ and $\underline{\alpha}_{iB}$ are higher than $\alpha_{i0}$ for low levels of $\theta$ and definitely if $\theta < \frac{1}{2}$, i.e. sending an unfavourable report results in an increase in reputation only if investors expect the project to be bad, and $\overline{\alpha}_{iB}$ is always higher than $\underline{\alpha}_{iB}$. On the other hand, when they issue favourable reports that turn out to be incorrect the reputational level becomes 0 and increases relative to $\alpha_{i0}$ if they are correct.

### 2.4.2 Period 0

#### Certification Intermediary Optimal Behaviour

Assume that $i$ was hired in period 0 and therefore, $\alpha_{j1} = \alpha_{j0}$. The difference in expected profits (in period 1, from sending a favourable and an unfavourable report) when she receives $s_G$ is

$$\underline{\pi^{ei}}(a) = \Pr(G \mid s_G, a) F_{i1}\left(\underline{\alpha}^i_{GG}, \alpha_{j0}\right) + \Pr(B \mid s_G, a) F_{i1}\left(\underline{\alpha}^i_{GB}, \alpha_{j0}\right) - F_{i1}\left(\underline{\alpha}^i_B, \alpha_{j0}\right) + c_0. \tag{2.15}$$

and the difference in expected profits when she receives $s_B$ is

$$\overline{\pi^{ei}}(a) = \Pr(G \mid s_B, a) F_{i1}\left(\overline{\alpha}^i_{GG}, \alpha_{j0}\right) + \Pr(B \mid s_B, a) F_{i1}\left(\overline{\alpha}^i_{GB}, \alpha_{j0}\right) - F_{i1}\left(\overline{\alpha}^i_B, \alpha_{j0}\right) - c_0. \tag{2.16}$$

Agent $i$ is going to issue a favourable report (or play a mixed strategy) if $\underline{\pi^{ei}}(a)$ and $\overline{\pi^{ei}}(a)$ are positive (or equal to zero). Otherwise she issues an unfavourable report. Following the same logic as before the following result can be stated:

**Proposition 8.** *With competition, the talented certification intermediary never misreports in period 0.*

One of the main differences in relation to the previous Section is that now whenever an untalented intermediary is confronted with a realised project type that differs from the report issued, her reputation is driven down to zero and she is not going to be hired in the following period. This happens because if confronted with two intermediaries, one with a reputational level of $\alpha_{j0}$ and the other one with 0, the firm chooses the one with higher posterior about the ability. Therefore:

**Proposition 9.** *If investors and firms are sure a certification intermediary is untalented, she is never hired independently of the reputational level of her competitor.*

On the other hand, if an intermediary issues a correct good report her reputation increases, which means that given that she was hired in period 0, she is necessarily going to be hired in period 1. However, issuing a bad report is no longer a riskless strategy. For high values of $\theta$, issuing a bad report worsens the intermediary's reputation as the remaining agents believe she is making a mistake. This might or might not affect the intermediary's possibilities of being hired in the following period depending on how close the initial reputations are.

Summing up, compared with the monopolistic case, the reputational cost of a mistake is now much higher and intermediaries face a considerably lower probability of being hired in the last period when a mistake is discovered. This first effect encourages truth-telling. But there is a second effect: the probability of being hired is determined by the first period announcement and a truthful report that cannot be verified might also affect the intermediary's chances of being hired, which may in turn lead to less truth-telling. Hence, the crucial point is to study what happens when a bad report implies a decrease in reputation that might compromise future commitments. If not, the following proposition is derived:

**Proposition 10.** *Competition changes the set of prior beliefs about the quality of the project for which an untalented certification intermediary deviates from the private signal in period 0, when $\alpha_{j0} < \underline{\alpha}_B^i < \overline{\alpha}_B^i$: she becomes more aggressive and issues favourable reports more frequently than in the monopolistic case.*

**Proof.** In the Appendix. ■

In this case, the intermediary is always hired next period except when a good report is found to be incorrect. However, there is also a monetary effect to consider with competition: the fee is now lower by an amount equivalent to proportion $k$ of the surplus of the competitor relative to the case without competition. But if the intermediary reports an incorrect report she is not going to hired next period and in this case the future payoff is simply zero. Without competition, the intermediary's type would be revealed but, because the firm has no outside option, she would still be hired but receiving a fee in accordance to her type. So it turns out that the difference between the payoffs in this particular scenario is lower than proportion $k$ of the surplus of the competitor and this makes the decrease in the expected fee relative to the case with no competition lower when a good report rather than a bad report is issued. Hence $\overline{\pi}^{ei}(U) > \overline{\pi}^{e}(U)$ and $\underline{\pi}^{ei}(U) > \underline{\pi}^{e}(U)$ and reporting favourable messages becomes more frequent than before.

To sum up, the dominant effect in this case is the first one: a lower probability of being hired encourages truth-telling. Intermediaries still conform with public information and ignore private signals when the prior about the quality of the project is extreme, but conservatism is attenuated.

For a low degree of differentiation, i.e. $\alpha_{j0} > \overline{\alpha}_{iB}$ and/or $\alpha_{j0} > \underline{\alpha}_{iB}$, competition becomes very aggressive and only when positive reports turn out to be correct the intermediary is hired in the following period. In fact, the intermediary no longer behaves conservatively and does not take into account the effect of the initial prior about the quality of the project when issuing her report. Thus,

**Proposition 11.** *When $\alpha_{j0} > \overline{\alpha}_{iB} > \underline{\alpha}_{iB}$, untalented certification intermediaries always issue favourable reports in period 0.*

The second effect is now the dominant one: in order to maximise the probability of being hired in the future the intermediary compromises truth-telling and in the limit only issues favourable reports regardless of her private information.

In the monopolistic case the asymmetry in payoffs observability make more reputable intermediary more prone to sending unfavourable reports for relatively lower or relatively higher priors. A duopolistic structure in the certification industry mitigates this result by introducing more symmetry between sending favourable and unfavourable reports. Sending unfavourable reports is now less beneficial and this affects any untalented intermediary regardless of her reputation.

## 2.5 Conclusion

This chapter studies the behaviour of certification intermediaries, in particular it looks at their incentives to report a message that differs from their private signal in a framework where they value reputation. The model identifies a source of incentive conflicts for certification intermediaries. It finds that reputational concerns are not enough to prevent deviations from the private signal, in fact these concerns might end up being the driving force being them. Intermediaries that are sure of their signals always report truthfully but those that cannot trust their private signals may end up ignoring them and sending the report that investors and firms anticipated based on public information, in particular when the public signal is extreme, in an attempt to avoid reputational costs. Despite its simplicity, the model can motivate several patterns of behaviour, in particular, this results provide a theoretical explanation for empirical findings that suggest that ratings agencies exhibit excess sensitivity to the business cycle and in some cases adjust their ratings after market participants have already adjusted their perceptions of creditworthiness.

In the monopolistic setting, the intermediaries with a higher reputation tend to be conservative when issuing their reports but competition forces them to be more aggressive in order to be hired in the following period.

This is relevant for policy-makers. Under proposed revisions to bank capital requirements advanced by the Basel Committee on Banking Supervision, banks using a standardised approach to calculating their minimum required capital will base such requirements, whenever possible, on the credit ratings assigned to the companies to which they lend. To the extent that rating agencies might behave pro-cyclically, bank capital requirements will tend to be higher during downturns, further reducing credit supply during downturns. In addition, the Basel proposal will increase the demand for ratings as will definitely have an impact on the market structure of the industry. The increasing importance of ratings agencies in financial market as a result of regulatory measures demands that these issues should be identified and tackled appropriately.

# Chapter 3

# In Pursuit of Honesty in Auditing

*The detection of fraud is the most important portion of the Auditor's duties, and there will be no disputing the contention that the Auditor who is able to detect fraud is - other things being equal - a better man than the Auditor who cannot.*

Dicksee (1892)

## 3.1 Introduction

In recent years many accounting irregularities have been left undetected for long periods of time and investors have not been timely warned about the implications of some activities that were being pursued by the firms they owned. An indication of this is the large number of times corporations restated their disclosed financial statements and how far back in time these restatements go. According to the U.S. General Accounting Office, there were 225 restatements in 2001 in the U.S. compared to 92 in 1997. Examples are Enron that made a restatement in the fall of 2001 of the net income back to 1997 that generated a $586 million reduction. More recently, Fannie Mae and AIG accounting mistakes that took place between 2001-2003 were only disclosed in 2005.

As a result of some of these accounting irregularities, most notably those related to Enron, the Sarbanes-Oxley Act introduced a number of measures in an attempt to strengthen investors confidence in auditors' integrity and honesty. For instance, the Act established the introduction of an independent regulatory body that is in charge of supervising accounting firms and requires that partners of accounting companies supervising the external audit have to rotate regularly as to undermine the development of personal relationships between the partner and his client.

Empirical evidence[1] suggests that there is a connection between lack of auditor rotation or longer auditor tenures and financial restatements as *unexpected* auditor replacements tend to lead to restatements of a firm's earnings. Also both Myers et al. (2004) and Choi and Doogar (2005) find that the likelihood of certain earnings misstatements (misstatements that increase earnings, misstatements for core earnings components and in quarterly financial reports) increases with auditor tenure. Basically, in a situation where auditors can make mistakes because for example, the firm's accounting procedures are not transparent enough, if an auditor remains in the firm long enough she can simply delay the announcement of bad news about it in order to protect expected rents from a continued engagement with the firm. On the other hand, the fact that honest mistakes can be made gives the firm no way to distinguish between a genuine mistake or simple manipulation: even if shareholders suspect something is wrong with the firm and demand an extra audit that ends up proving their suspicions, the auditor can always argue against manipulation. However, keeping the

[1] See for example, Lazer et al. (2004).

same auditor might also be advantageous for the firm as the longer an auditor stays with a firm the more familiar she becomes with the firm's accounting procedures and activities and consequently, more efficient when looking for genuine mistakes.

Given this trade-off, it is important to study how rotating auditors affects the gathering and disclosure of information about the firm, in a context where audits are not perfect[2] and auditors can enjoy private benefits and have limited liability.

In the model developed below, auditors are hired to validate the manager's report about the financial accounts of a certain firm. This is necessary because management typically has an incentive to bias reports in its own favour. In particular, managers might derive private benefits of control and therefore be biased towards continuing projects and this creates a demand for confirmation of the financial reports that they provide. Thus, an audit can generate a good report (good news), implying that the project should continue, or a bad report (bad news) meaning that the project should be terminated. Auditors make a mistake when they incorrectly identify a bad project as a good one. As a result, a good audit report can be subject to a revision by either the same or a different auditor. The best example of such situation is the revision of quarterly financial statements during the annual audit.

In order to perform an audit the auditor needs to exert effort. Therefore, the objective of this chapter is to characterise the optimal contract between a firm's shareholders and the firm's auditor (or auditors) that overcomes the moral hazard problem that affects her (or them). In addition, the firm's manager might try to tempt the auditor (or one of them in case there is auditor rotation) to hide unfavourable reports by offering her private benefits such as access to other clients, or leisure related benefits like paid vacations, golf club memberships, etc., in case the project

[2] I.e., mistakes can be made.

continues. As a result, the auditors' actions can lead to report manipulation, which is translated into either the delay of the announcement of bad news or the lack of announcement of bad news and consequent inefficient continuation of the project. The former happens when the auditor continues auditing the firm and knows that she will have other opportunities to report about the real situation of the firm. The latter results from the existence of private benefits. Hence, the firm's shareholders need to provide incentives for the auditor to report failure immediately rather than delaying its announcement as well as to adjust the transfer made to auditors for the existence of private benefits.

This chapter concludes that the optimal contract is such that auditors are rewarded if and only if their reports are confirmed by facts and they tend to receive a higher reward (transfer) when they correctly contradict previous reports or make correct announcements that ex-ante appear less likely given what is publicly known about the quality of the project. Private benefits decrease the transfers made to auditors for correct good reports but increase the transfers for correct bad reports as in this case the auditor foregoes the private benefit. If the private benefit exceeds a certain threshold, auditors extract extra rents because limited liability determines that transfers cannot be negative.

Familiarity with the firm due to repeated audits takes the form of a lower cost of effort to collect private information in the second audit. One interesting conclusion regarding this is that, even though transfers are paid to reward auditors for their effort and therefore one would expect them to decrease if the cost of effort decreases, this effect is limited. The intuition is the following. Shareholders need to reward the single auditor such that she collects and truthfully reports her private signal in both periods (when a report in the second period is needed). In addition, the auditor needs to be rewarded for the immediate announcement of correct bad news in the first audit in order to avoid manipulation. As a consequence, this transfer is going to be equal to the transfer for a correct bad report in the last period and both depend on the new cost of effort. Because sending correct bad reports in the first audit is so highly rewarded, sending two effortless consecutive good reports is going to be always dominated and the auditor is going to be indifferent between collecting and truthfully reporting information in both periods or in the last period only after having sent an initial good report. However, as the cost of effort for repeated audits decreases, collecting and revealing private information in both periods becomes increasingly preferred to behaving this way in the last period only[3]. And because the transfer for sending a correct bad report in the first audit depends on the new cost of effort it will also happen that this transfer is constantly decreasing and at some point sending two effortless good reports becomes a dominant strategy. Hence, in order to avoid this the transfer for a correct bad report in the first audit needs to increase and is going to be determined by the *initial* rather than by the new cost of effort: the firm needs to make the auditor exert the initial effort as once this has been done exerting effort in the second period is always preferred. The transfer for two correct good reports is also going to be independent of the new cost of effort as well as the expected transfer.

Shareholders' initial opinion about the quality of the project determines whether the firm prefers rotating auditors or not when there is the need for confirmation of the first report. Not rotating is preferred if shareholders' initial opinion about the project's quality is higher than a certain threshold, i.e. in good times. In this case, the "more expensive" transfer for a correct bad report in the first audit[4] is less likely

[3]For example, think of the case where the new cost of effort is simply equal to zero.
[4]"More expensive" in order to avoid manipulation as explained above.

to be paid and auditors are instead more likely to be rewarded for correct good reports whose corresponding transfers are lower in the case of a single auditor. If familiarity with the firm generates savings in information collection, the threshold is lower than before but the existence of private benefits crowds out this positive effect.

Finally, audit regulation seems to be often seen as a substitute for corporate governance and accounting transparency. Better governance and more accounting transparency, that can be translated in the model for example, in terms of a lower probability of misidentification of failures when effort is exerted, would make auditing easier and cheaper and consequently, auditing regulation less necessary.

Information acquisition and transmission in principal-agent relationships have been widely studied. In Demski and Sappington's (1987) the principal must choose from a range of available projects whose payoffs depend on an unknown state of the world. The risk-averse agent acquire a costly private signal about the state. They study the properties of optimal contracts for information acquisition and subsequent project selection in a series of examples and under fairly general information structures. More recently, a paper by Gromb and Martimort (2005) takes a setting with a simpler information structure and risk-neutral agents with limited liability and looks at the optimal organisation of expertise in a situation where experts advise on whether a firm should pursue a certain project. Depending on the experts' reports, that can make symmetric mistakes when identifying a bad project as a good one and vice-versa, the project is undertaken or not. The model below differs from Gromb and Martimort (2005) as it considers sequential reporting in a situation where the outcome is fully observable ex-post, regardless of the auditors' report, auditors only make mistakes when they incorrectly identify bad projects, they can derive private benefits and there are savings in costs of information collection from repeated audits to the firm by the same auditor[5]. This is more in line with what happens in the auditing profession. For example, outcome observability is assumed because auditors certify the firm's accounts that translate what has already been done on the firm and that it is going to be observable once the projects that the firm is undertaking are completed. And the motivation behind the assumption that auditors make asymmetric mistakes is the fact that managers often try to disguise unfavourable information and hide it from shareholders but they rarely try to hide favourable information. Hence, it is more likely that auditors fail to identify potential problems with the firm rather than miss to spot favourable developments.

This chapter also contributes to the literature on information acquisition and disclosure with multiple agents. Related papers are Krishna and Morgan (2001), Wolinsky (2002) and Hirao (1994). Krishna and Morgan (2001) extends Crawford and Sobel (1982) allowing for a decision-maker to sequentially consult two biased experts. Wolinsky (2002) also considers a problem in which experts have noisy information and their preferences differ significantly from those of the principal but he is interested in the optimal organisation of communication procedures among the experts. Kofman and Lawarrée (1993) also looks at the case where there are multiple auditors. They derive the optimal contract when both an internal and external auditor are available but their focus is on the effect of collusion on the agent's incentives to exert effort.

The issue of how private benefits affect the decision to whom to delegate a project-valuation task has been studied by Laux (2001). Private benefits in the model below are derived from sending a certain message, i.e. the auditor enjoys the benefit for sure if a certain message is sent, whereas in Laux it is outcome-related.

[5]An older version of this paper, Gromb and Martimort (2003), does look at the case of sequential reporting but all the other differences between the two set-ups still hold even in this version.

The model assumes limited liability however it also explores the impact of this assumption. This topic is also discussed in Dye (1993) which explicitly deals with the impact of legal liability on audit quality and the degree of care of auditors and Narayanan (1994) that looks at the effect of different liability rules.

This chapter is organised as follows. The model is outlined in the next section. Section 3.3 derives the optimal contract for a two auditor/two reports case and the following section compares it with the optimal contract for a one auditor/two reports case. Section 3.5 presents several extensions and policy implications and section 3.6 concludes. Some proofs are in the Appendix.

## 3.2 The Model

### 3.2.1 Setup

In this economy, there are two different classes of risk-neutral agents: two auditors (she) and a client-firm represented by a group of homogenous shareholders. The model lasts for two periods and the risk-free rate is zero. The firm is in the process of undertaking an investment project that is developed in two stages and the final outcome, which is the sum of the outcomes in both stages, is publicly observable in the last period[6]. In period 0 (today) shareholders have to decide whether to invest an extra $I$ to undertake the second stage or alternatively they can scale down the investment project and decide not to invest the extra funds. The firm's shareholders (principal) own a productive technology but lack the ability or time necessary to supervise it

[6]For example, think about the decision of whether to increase the scale of the initial investment or of whether to invest in a complementary product.

and must hire an auditor or auditors (agent) for that purpose[7]. Hence, shareholders require the services of an auditor (or auditors) and based on the information provided by her (them) they make the interim investment decision and decide to go ahead only if there is clear indication that the first stage was successful.

The firm's shareholders want to minimise expected transfer costs of inducing acquisition and revelation and the auditors, that have limited liability, want to maximise their income. As a result shareholders offer a contract that consists of a payment schedule for the auditors based on all jointly observed variables. It is assumed that shareholders can precommit to implementing the agreed-upon contract.

### 3.2.2 Project and Information Structure

In the beginning of period 0, the first stage of the project has been completed. The outcome $\Phi$ can be either a success $\overline{S}_1$ with probability $v$ or a failure $\underline{S}_1$ with probability $1-v$, and $\underline{S}_1 = -\overline{S}_1$. The second stage is undertaken if investors provide the extra investment at the end of period 0. The outcome of this stage is $\overline{S}_2$ or $\underline{S}_2$ with probabilities $u$ and $1-u$ respectively if the first period outcome is $\overline{S}_1$ and probabilities $1-u$ and $u$ respectively if $\underline{S}_1$ has been generated in the first period, with $\underline{S}_2 = -\overline{S}_2$ and $u > \frac{1}{2}$.

By exerting effort at a personal cost $\psi$, the auditor privately observes a signal $\sigma \in \{\underline{\sigma}, \overline{\sigma}\}$ about the project. Denote by $\theta \in \left(\frac{1}{2}, 1\right]$ the signal's precision defined as $Pr\left(\underline{\sigma} \mid \underline{S}_1\right) = \theta$ and $Pr\left(\overline{\sigma} \mid \overline{S}_1\right) = 1$. Hence, $\overline{\sigma}$ is "good news" and $\underline{\sigma}$ "bad news" about the project and mistakes arise from wrongly identifying a $\underline{S}_1$- project as a $\overline{S}_1$-project. The signal $\sigma$ is assumed to be soft information, i.e., the auditor can fully

[7]Alternatively, assume that shareholders are small and dispersed and individual monitoring is very costly or even that auditing is compulsory.

manipulate it when reporting to shareholders. The project's outcome is verifiable in the long-run even when the second stage is not undertaken.

Note that the manager is a passive agent in this model and the focus is on the behaviour of auditors, however this information structure shows the incentives that managers might face: they might have preferences for "empire building" in the sense that they derive utility from overseeing large investment projects and hence might try to hide failures but have no incentive to disguise successes. In addition, they can commit to provide *one* auditor with a private non-monetary, non-divisible and non-transferable benefit $B$ whenever the second stage of the project is undertaken. As a result, auditors' interests are not perfectly aligned with those of the shareholders.

The private benefit enjoyed by the auditor is closely tied to the private benefits of control derived by the manager in case the project's second stage is undertaken and this explains why the auditor's private benefit is independent of the state of the world and depends solely on the continuation of the project. When both auditors are hired they are equally likely to receive private benefit $B$. This happens for a non-divisible private benefit and if the resources available to the manager for this purpose are limited. Consequently, he cannot credibly commit to provide a private benefit in excess of $B$. Example of such benefits are manager's promises to introduce the auditor to a new client that is looking to hire one auditor or to involve one auditor in a new project that will only be undertaken in case of continuation. If the private benefit is in fact a promise to be fulfilled in the future, then it makes sense that when two auditors are hired by the firm they are equally likely to receive it since the promise is fulfilled after the shareholders' decision to continue the project has been taken and ex-post the manager has no reason to prefer one auditor over the other.

It is assumed that shareholders prefer, whenever necessary, to receive two reports, the audits take place sequentially and only if both audits are "good news" the second stage of the project goes ahead. Note that in equilibrium, if effort is exerted and the report is $\underline{\sigma}$ the project's first stage has failed and there is no need for an extra audit. Given this, the firm's expected profit, $(1-u)\overline{S}_2 + u\underline{S}_2 - I$, is always negative and the project should be terminated.

Shareholders can either rotate auditors when a second audit is needed to confirm the first report[8] or remain with the same auditor that can make an audit revision and issue a restatement if she finds the first audit to be inaccurate.

Shareholders' expected profit without a report is:

$$W = (vu + (1-v)(1-u))\overline{S}_2 + (v(1-u) + (1-v)u)\underline{S}_2 - I$$

and it is assumed to be negative. The condition that ensures that shareholders' expected utility with two reports $W^2$, exceeds $W^1$, the expected utility with one report only, which is derived below.

To simplify, it is assumed that the second-stage investment is undertaken after the second audit, however the results would not change if investment starts immediately after the first audit provided that it is concluded after the second audit: some investment is needed after the second audit as otherwise this would be worthless for the firm.

[8]An auditor coming anew to a firm would always check whether previous reports are correct as she does not want to be responsible for mistakes that took place while she was not in charge of auditing the firm.

## 3.3 Optimal Contract with Two Auditors and Two Signals

The primary focus of this section is on characterising the optimal contractual relationship between shareholders and two different auditors. The starting point of this approach is a simple model of auditing in which contractual relationships are not restricted. We then solve for the optimal relationship.

A contract for each auditor consists of transfers from the firm's shareholders based on her report $\widehat{\sigma}_t$ of the private signal $\sigma_t$ collected during audit $t$, with $t = \{1, 2\}$. Formally, a contract is a menu of lotteries on transfers depending on the auditor's report $\widehat{\sigma} \in \{\underline{\sigma}, \overline{\sigma}\}$. It is summarised by the transfers to the first auditor $\{t(\widehat{\sigma}_1, \Phi)\}$ where $\widehat{\sigma}_1$ is the report and $\Phi$ the real outcome, i.e., $t(\underline{\sigma}, \underline{S}_1)$ and $t(\overline{\sigma}, \overline{S}_1)$, if the auditor is correct and $t(\underline{\sigma}, \overline{S}_1)$ and $t(\overline{\sigma}, \underline{S}_1)$ if the auditor is incorrect; and the transfers to the auditor that performs the second audit, $\{t(\widehat{\sigma}_2, \Phi \mid \widehat{\sigma}_1)\}$, where $\widehat{\sigma}_2$ is her report and $\widehat{\sigma}_1$ the report sent by the first auditor, i.e., $t(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma})$ and $t(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma})$ if the auditor is correct and $t(\underline{\sigma}, \overline{S}_1 \mid \overline{\sigma})$ and $t(\overline{\sigma}, \underline{S}_1 \mid \overline{\sigma})$ if the auditor is incorrect.

The purpose of these transfers is to reward auditors for effort exertion, given all the information that is available about the project at the time auditors collect their private signal. This explains why the first auditor's transfers are not conditioned on the second auditor's report, whereas the second auditor's transfers are conditioned on the first auditor's report. The first auditor only has one piece of information at the time she collects her private signal, i.e. she only knows the prior about the project's quality. Moreover the project's outcome is always observable ex-post, regardless of the second auditor's report. Hence, the second auditor's report does not have an impact on the first auditor decision to exert effort and on how costly it is to make her exert effort. The second auditor, on the other hand, starts with two pieces of information: the prior and the first auditor's report. Consequently, her belief about the quality of the project before collecting the private signal is no longer the prior belief but has been updated given the first auditor's report. This has an impact on how costly it is to induce him to exert effort. Moreover, since $S_1$ is perfectly observable, the transfers solely depend on $S_1$ and not on $S_2$.

Note that if shareholders can perfectly observe auditor's effort, the incentive problem could be solved with a flat transfer equal to the cost of effort. The intuition is the following. In such a situation, an auditor receives either 0, if she exerts effort, or the cost of effort, otherwise. An auditor maximises profits but in both cases the auditor would make zero profits. However, when indifferent between exerting effort or not, auditors choose to exert effort[9] consequently, a flat fee equal to the cost of effort guarantees effort exertion [10]. Here, however the auditor may be tempted not to gather information and instead report the signal implying the highest expected transfer. Therefore the auditor must be given incentives not only to gather information, but also to report the private signal accurately. As effort is not perfectly observable, a contingent contract is needed to motivate the desired level of audit effort, with transfers dependent on the ex-post correctness of the audit report. The first-best contract is the least (expected) cost contingent contract that rewards auditors when they are right and imposes a penalty when they are incorrect.

In reality, auditors are compensated with a fixed fee as stated in Rule 302 on "Contingent Fees" of the AICPA Code of Professional Ethics. This rule determines that "Professional services shall not be offered or rendered an arrangement whereby no fee

[9]This is the assumption behind the optimal contract and made throughout the paper.

[10]Or alternatively, paying $\varepsilon$ higher than the cost of effort, with $\varepsilon \to 0$, would guarantee effort exertion.

will be charged upon the findings or results of such services. However, a member's fee may vary depending, for example, on the complexity of the service rendered...Fees are not regarded as being contingent if fixed by courts or other public authorities, or, in tax matters, if determined based on the results of judicial proceedings or the findings of governmental agencies". The analysis here abstracts from this and examines how auditors respond to the introduction of report-contingent audit contracts. Dye, Balachandram and Magee (1990) and Baiman, Evans and Noel (1987), among others, followed a similar approach.

Collusion between auditors is also ruled out.

### 3.3.1 Posterior Beliefs

The likelihood of $\sigma \in \{\underline{\sigma}, \overline{\sigma}\}$ is given by $p(\sigma)$ and $p(\Phi \mid \sigma)$ is the probability of outcome $\Phi$ conditional on $\sigma$. For example, $p(\overline{\sigma}) = v + (1-v)(1-\theta)$ and $p(\overline{S}_1 \mid \overline{\sigma}) = \frac{v}{v+(1-v)(1-\theta)}$. In addition:

$$p(\underline{\sigma}) = p(\underline{S}_1) - p(\overline{\sigma})p(\underline{S}_1 \mid \overline{\sigma}) = (1-v)\,\theta. \tag{3.1}$$

This equivalence will be used below.

Also note that the probability of signal $\sigma$ for any given signal $\sigma$ is denoted by $p(\sigma \mid \sigma)$, in particular the probabilities of a good and bad signal in the second audit given that the first audit generated a good signal are given by $p(\overline{\sigma} \mid \overline{\sigma}) = \frac{(1-\theta)^2(1-\nu)+v}{p(\overline{\sigma})}$ and $p(\underline{\sigma} \mid \overline{\sigma}) = \frac{(1-\theta)\theta(1-v)}{p(\overline{\sigma})}$ respectively. The conditional probability of outcome $\Phi$ given $(\sigma, \sigma)$, where the first $\sigma$ is related to the second audit and the second to the first audit, is $p\left(\Phi \mid \sigma, \sigma\right)$, i.e.:

$$p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) = \frac{\nu}{\left(1-\theta\right)^2\left(1-\nu\right)+v} = \frac{p(\overline{S}_1 \mid \overline{\sigma})}{p\left(\overline{\sigma} \mid \overline{\sigma}\right)} \tag{3.2}$$

and

$$p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) = \frac{\left(1-\theta\right)^2\left(1-\nu\right)}{\left(1-\theta\right)^2\left(1-\nu\right)+v} = \left(1-\theta\right)\frac{p\left(\underline{S}_1 \mid \overline{\sigma}\right)}{p\left(\overline{\sigma} \mid \overline{\sigma}\right)}. \tag{3.3}$$

Finally, $p\left(\underline{S}_1 \mid \underline{\sigma}, \overline{\sigma}\right) = 1$ and $p\left(\overline{S}_1 \mid \underline{\sigma}, \overline{\sigma}\right) = 0$. It will also be useful to note that:

$$p\left(\underline{\sigma} \mid \overline{\sigma}\right) = p\left(\underline{S}_1 \mid \overline{\sigma}\right) - p(\overline{\sigma} \mid \overline{\sigma})p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) = \frac{\theta\left(1-\theta\right)\left(1-v\right)}{p\left(\overline{\sigma}\right)}. \tag{3.4}$$

### 3.3.2 Optimal Contract

There are two adverse selection incentive constraints. Firstly, given that the first signal was $\overline{\sigma}$, the auditor should not prefer reporting $\underline{\sigma}$ after having observed $\overline{\sigma}$, taking into account that when a $\overline{\sigma}$-report is issued the auditor receives private benefit $B$ with probability $\frac{1}{2}$, i.e.:

$$p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right) + p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) + \frac{B}{2}$$

$$\geq p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\underline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right) + p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right). \tag{3.5}$$

On the other hand, the auditor should not prefer reporting $\overline{\sigma}$ after having observed $\underline{\sigma}$:

$$t\left(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) \geq t\left(\overline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) + \frac{B}{2} \tag{3.6}$$

If the first signal is $\underline{\sigma}$, there is no second audit and investment $I$ is not undertaken.

The contract must also satisfy a moral hazard incentive constraint to induce the auditor to gather information. If the auditor does not gather information, she can pretend she did and report the signal yielding the highest expected transfer. In a Bayesian-Nash equilibrium, each auditor anticipates that the other gathers information and reports it truthfully. Thus the moral hazard constraint is:

$$p(\overline{\sigma} \mid \overline{\sigma}) \left[ p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right) + p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) + \frac{B}{2} \right]$$
$$+p(\underline{\sigma} \mid \overline{\sigma}) \left(t\left(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right)\right)$$
$$-\psi \geq \max \left\{ \begin{array}{c} p\left(\overline{S}_1 \mid \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right) + p\left(\underline{S}_1 \mid \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) + \frac{B}{2}, \\ p\left(\overline{S}_1 \mid \overline{\sigma}\right) t\left(\underline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right) + p\left(\underline{S}_1 \mid \overline{\sigma}\right) t\left(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) \end{array} \right\}.$$

Note that expectations on the right-hand-side (RHS) are based on the prior belief $v$ and the message sent by the auditor in the first period. The RHS considers the two options available to auditors when effort is not exerted: to send a $\overline{\sigma}$- report and receive the private benefit with probability $\frac{1}{2}$ or send a $\underline{\sigma}$- report. The moral hazard constraints can be rewritten as follows:

$$p\left(\underline{\sigma} \mid \overline{\sigma}\right) t\left(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) - \psi \geq \left(p\left(\underline{S}_1 \mid \overline{\sigma}\right) - p(\overline{\sigma} \mid \overline{\sigma}) p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right)\right) t\left(\overline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) + p\left(\underline{\sigma} \mid \overline{\sigma}\right) \frac{B}{2} \tag{3.7}$$

and

$$p(\overline{\sigma} \mid \overline{\sigma}) \left[ p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right) + p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) \right] - \psi \geq$$
$$p\left(\overline{S}_1 \mid \overline{\sigma}\right) t\left(\underline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right) + \left(p\left(\underline{S}_1 \mid \overline{\sigma}\right) - p(\underline{\sigma} \mid \overline{\sigma})\right) t\left(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) - p(\overline{\sigma} \mid \overline{\sigma}) \frac{B}{2}. \tag{3.8}$$

Using (3.2) and (3.4), these two constraints imply the adverse selection constraints (3.5) and (3.6), therefore they can be ignored. This guarantees that report's manipulability is irrelevant once the auditor is uninformed as otherwise remaining uninformed would be optimal. Expected transfers are derived from these constraints and limited liability: whenever the RHS of a constraint is positive, the corresponding transfer has to be positive; if it is negative the transfer is zero. Constraint (3.7) assures that the auditor has no incentive to issue a good report without acquiring information. From this constraint it is noted that the auditor has to be compensated for the foregone private benefits in case a bad signal is received when effort is exerted. Correspondingly, constraint (3.8) assures that no bad reports are issued without private information. If the auditor collects a good report when effort is exerted, she enjoys from private benefits with probability $\frac{1}{2}$, thus the transfer that persuades auditors to exert effort can be lowered accordingly.

Consider now the auditor's incentive to report the first signal truthfully keeping in mind that only when both reports are positive the project continues and only then the private benefit plays a role but that, in any case, the outcome is always verifiable in the last period. If the private signal $\sigma$ equals $\overline{\sigma}$,

$$p\left(\overline{S}_1 \mid \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1\right) + p\left(\underline{S}_1 \mid \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1\right) + p\left(\overline{\sigma} \mid \overline{\sigma}\right) \frac{B}{2}$$

$$\geq p\left(\overline{S}_1 \mid \overline{\sigma}\right) t\left(\underline{\sigma}, \overline{S}_1\right) + p\left(\underline{S}_1 \mid \overline{\sigma}\right) t\left(\underline{\sigma}, \underline{S}_1\right). \tag{3.9}$$

On the other hand, the auditor should not prefer reporting $\overline{\sigma}$ after having observed $\underline{\sigma}$. Thus:

$$t\left(\underline{\sigma}, \underline{S}_1\right) \geq t\left(\overline{\sigma}, \underline{S}_1\right) + p\left(\overline{\sigma} \mid \underline{\sigma}\right) \frac{B}{2} \tag{3.10}$$

The moral hazard constraint is:

$$p(\overline{\sigma}) \left(p\left(\overline{S}_1 \mid \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1\right) + p\left(\underline{S}_1 \mid \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1\right)\right) + p(\underline{\sigma}) t\left(\underline{\sigma}, \underline{S}_1\right) + \frac{B}{2} p(\overline{\sigma}) p\left(\overline{\sigma} \mid \overline{\sigma}\right) - \psi \geq$$

$$\max\left\{\begin{array}{c} p\left(\overline{S}_1\right)t\left(\overline{\sigma},\overline{S}_1\right)+p\left(\underline{S}_1\right)t\left(\overline{\sigma},\underline{S}_1\right)+p\left(\overline{\sigma}\right)\frac{B}{2}, \\ p\left(\overline{S}_1\right)t\left(\underline{\sigma},\overline{S}_1\right)+p\left(\underline{S}_1\right)t\left(\underline{\sigma},\underline{S}_1\right) \end{array}\right\}.$$

The moral hazard constraints become:

$$p(\underline{\sigma})t\left(\underline{\sigma},\underline{S}_1\right)-\psi\geq\left(p\left(\underline{S}_1\right)-p(\overline{\sigma})p\left(\underline{S}_1\mid\overline{\sigma}\right)\right)t\left(\overline{\sigma},\underline{S}_1\right)+p\left(\overline{\sigma}\right)p\left(\underline{\sigma}\mid\overline{\sigma}\right)\frac{B}{2} \tag{3.11}$$

and

$$p(\overline{\sigma})\left(p\left(\overline{S}_1\mid\overline{\sigma}\right)t\left(\overline{\sigma},\overline{S}_1\right)+p\left(\underline{S}_1\mid\overline{\sigma}\right)t\left(\overline{\sigma},\underline{S}_1\right)\right)-\psi$$

$$\geq p\left(\overline{S}_1\right)t\left(\underline{\sigma},\overline{S}_1\right)+\left(p\left(\underline{S}_1\right)-p(\underline{\sigma})\right)t\left(\underline{\sigma},\underline{S}_1\right)-\frac{B}{2}p(\overline{\sigma})p\left(\overline{\sigma}\mid\overline{\sigma}\right) \tag{3.12}$$

respectively. Again Bayes' rule, expression (3.1) and the fact that $p\left(\overline{\sigma}\right)p\left(\underline{\sigma}\mid\overline{\sigma}\right)$ equals $p(\underline{\sigma})p\left(\overline{\sigma}\mid\underline{\sigma}\right)$ guarantee that both (3.11) and (3.12) imply the adverse selection constraints (3.10) and (3.9). Once more this rules out that remaining uninformed would be optimal. The constraints are interpreted as before but it is worth pointing out that according to (3.11) the optimal $t\left(\underline{\sigma},\underline{S}_1\right)$ is going to compensate the auditor for the foregone benefit in case the second auditor issues a $\underline{\sigma}$-report and she is going to be penalised by $\frac{B}{2}p(\overline{\sigma})p\left(\overline{\sigma}\mid\overline{\sigma}\right)$ in case both auditors issue a $\underline{\sigma}$-report.

Finally, the auditor's participation and limited liability constraints must be satisfied. As all transfers need to be positive or equal to zero, the participation constraints are automatically satisfied and are omitted henceforth.

To implement the first-best decisions at minimum costs, shareholders solve:

$$\min_{\{t(.)'s\}}p(\overline{\sigma})\left(p\left(\overline{S}_1\mid\overline{\sigma}\right)t\left(\overline{\sigma},\overline{S}_1\right)+p\left(\underline{S}_1\mid\overline{\sigma}\right)t\left(\overline{\sigma},\underline{S}_1\right)\right)+p(\underline{\sigma})t\left(\underline{\sigma},\underline{S}_1\right)$$

$$+p\left(\underline{\sigma}\mid\overline{\sigma}\right)t\left(\underline{\sigma},\underline{S}_1\mid\overline{\sigma}\right)+p(\overline{\sigma}\mid\overline{\sigma})\left(p\left(\overline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\overline{S}_1\mid\overline{\sigma}\right)+p\left(\underline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\underline{S}_1\mid\overline{\sigma}\right)\right)$$

subject to (3.7), (3.8), (3.11), (3.12) and all $t(.) \geq 0$.

Note that $t\left(\underline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right)$ and $t\left(\underline{\sigma}, \overline{S}_1\right)$ are not included in the expected audit cost as in both cases it is obvious that no effort has been exerted.

One could think of alternative forms that guarantee effort exertion at the minimum cost for shareholders. For example, one can wonder if it is better not to reveal the first report before the second auditor reveals her report, i.e., the first auditor's report would only be known to shareholders and announced in simultaneous with the second report. But given the setup of the model, the second auditor can always anticipate that she is hired because the first report is good: a bad report in the first period is necessarily a bad project and since auditing is costly, shareholders only require a second report when the first report is good. This happens provided that the second auditor knows that a report has been issued already and that she has been hired as a second auditor. If auditors do not know whether a report has been issued, each auditor will find herself in a "first auditor" situation.

The derivation of the optimal transfers is relegated to the Appendix and the main results are summarised in the following proposition:

**Proposition 12.** *The optimal incentive contract with two auditors is as follows:*

1. *If $B < \frac{2p(\overline{\sigma})}{\theta v}\psi$, the optimal transfers are:*

$$t\left(\underline{\sigma}, \underline{S}_1\right) = \frac{\psi}{\theta(1-v)} + \frac{(1-\theta)}{2}B;\ t\left(\overline{\sigma}, \overline{S}_1\right) = \frac{\psi}{\theta v} - \frac{B}{2};$$

$$t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right) = \frac{\psi p(\overline{\sigma})}{\theta v} - \frac{B}{2} \text{ and } t\left(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) = \frac{\psi p(\overline{\sigma})}{(1-\theta)\theta(1-v)} + \frac{B}{2}.$$

*The remaining transfers are equal to zero.*

*2. If $\frac{2p(\overline{\sigma})}{\theta v}\psi \leq B < \frac{2}{\theta v}\psi$, then $t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right)$ equals zero. All the other transfers remain unchanged.*

*3. If $B \geq \frac{2}{\theta v}\psi$, both $t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right)$ and $t\left(\overline{\sigma}, \overline{S}_1\right)$ equal zero. All the other transfers remain unchanged.*

**Proof.** In the Appendix. ∎

The first result of interest is that whenever auditors are wrong or commit fraud the optimal transfer is zero. The non-zero transfers have two components: one related to the cost of effort and one related to the private benefit. The higher the cost of effort the higher the transfer as auditors need to be rewarded for their effort. The private benefit increases the transfer when a bad report that is correct is issued but decreases it when a good report that is correct is issued. This makes sense as incentives to report $\overline{\sigma}$ without private information are now stronger. However $t\left(\underline{\sigma}, \underline{S}_1\right)$ adjustment to the private benefit in lower than $\frac{B}{2}$: this happens because a $\overline{\sigma}$-report gives the first auditor the possibility of enjoying the private benefit *if* the second auditor also sends a $\overline{\sigma}$-report. Hence, the opportunity cost of sending a $\underline{\sigma}$-report is lower for the first auditor[11]. For the same reason one would expect $t\left(\overline{\sigma}, \overline{S}_1\right)$ to decrease by less than $\frac{B}{2}$ but this is not the case. Because $t\left(\underline{\sigma}, \underline{S}_1\right)$ is less affected by the private benefit, a lower $t\left(\overline{\sigma}, \overline{S}_1\right)$ is also needed in order to convince the first auditor to exert effort rather than sending a effortless $\underline{\sigma}$-report.

It is also derived that in equilibrium auditors are indifferent between gathering and truthfully disclosing information and not gathering information, and in this case they are indifferent between both reports, but only when the private benefit is low enough. As the private benefit increases, the transfers when a $\overline{\sigma}$-report is issued can decrease

[11] The second auditor knows that a $\overline{\sigma}$-report always gives her the possibility of enjoying the private benefit.

as auditors have the extra compensation derived from $B$. If $B$ is very large, the transfers generated by correct $\overline{\sigma}$-reports can even become negative. However, this is impossible due to limited liability and $t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right)$ and $t\left(\overline{\sigma}, \overline{S}_1\right)$ increase just enough to satisfy the limited liability constraints but the other transfers are not modified as this violates the remaining constraints. Therefore, when the private benefit is high enough, auditors are indifferent between acquiring information and sending a $\overline{\sigma}$-report, but prefer acquiring information to sending a $\underline{\sigma}$-report. This also means that if uninformed, auditors prefer sending a $\overline{\sigma}$-report.

The higher the prior belief $v$, the lower the private benefit needed to generate a situation where the transfers from sending a correct $\overline{\sigma}$-report are set equal to zero. This happens because a higher prior implies a lower cost of effort related component when a correct good report is issued. Hence a lower private benefit is necessary to convince auditors not to exert effort, send a good signal instead and receive the private benefit.

In addition, and independently of the private benefit $B$, $t\left(\underline{\sigma}, \underline{S}_1\right)$ is higher than $t\left(\overline{\sigma}, \overline{S}_1\right)$ if $v$ exceeds $\frac{1}{2}$, i.e., a higher transfer is received whenever a good report is expected but a correct bad report is issued than when a correct good report is issued. The higher the private benefit the lower the threshold $v$ for which $t\left(\underline{\sigma}, \underline{S}_1\right)$ exceeds $t\left(\overline{\sigma}, \overline{S}_1\right)$. In addition, $t\left(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right)$ is always higher than $t\left(\underline{\sigma}, \underline{S}_1\right)$ regardless of $B$. Such results mean that the higher the prior the higher the reward for "contradicting" it is going to be. This reflects the fact that because of moral hazard, agent's rewards are linked with the outcomes that are most informative about effort exertion, here about having gathered information. This results are summarised in the Lemma 1:

**Lemma 2.** *The optimal transfers are such that:*

1. *If the prior belief* $v$ *is lower than* $\frac{1}{2}$, *then* $t\left(\overline{\sigma},\overline{S}_1\right) > t\left(\underline{\sigma},\underline{S}_1\right)$ ;

2. *For any prior belief* $v$, $t\left(\underline{\sigma},\underline{S}_1 \mid \overline{\sigma}\right) > t\left(\underline{\sigma},\underline{S}_1\right)$ *and* $t\left(\overline{\sigma},\overline{S}_1\right) \geq t\left(\overline{\sigma},\overline{S}_1 \mid \overline{\sigma}\right)$;

3. *There exists a* $v^* < \frac{1}{2}$ *such that,* $t\left(\underline{\sigma},\underline{S}_1 \mid \overline{\sigma}\right) > t\left(\overline{\sigma},\overline{S}_1 \mid \overline{\sigma}\right)$ *if* $v > v^*$;

*regardless of the private benefit.*

Trueman(1994) develops a model that studies the behaviour of analysts and concludes that they exhibit herding behaviour in the sense that they release forecasts similar to those previously announced by other analysts. The results derived in this section go in the same direction: the optimal contract tends to reward auditors for contradicting the prior belief and previous reports as this constitutes evidence that effort was indeed exerted.

It is also useful to look at what would have happened if shareholders only required a single report. Without private benefits this is simply equal to the first auditor problem in the previous case but if private benefits are present, then the auditor is sure than when a $\overline{\sigma}$-report is issued, the project continues and she receives the private benefit. The optimal transfers are therefore changed accordingly.

**Lemma 3.** *The single audit optimal transfers are:*

1. *If* $B < \frac{\psi}{\theta v B}$, *the optimal transfers are:*

$$t\left(\underline{\sigma},\underline{S}_1\right) = \frac{\psi}{\theta\left(1-v\right)} + B; t\left(\overline{\sigma},\overline{S}_1\right) = \frac{\psi}{\theta v} - B;$$

   *The remaining transfers are equal to zero.*

2. *If* $B \geq \frac{\psi}{\theta v B}$, *then* $t\left(\overline{\sigma},\overline{S}_1\right)$ *equals zero. All the other transfers remain unchanged.*

### 3.3.3 Expected Utilities

This section calculates the expected utility or expected transfer of both auditors and of the firm's shareholders. The first auditor's expected utility is[12]:

$$W_1^A = p(\overline{\sigma})p\left(\overline{S}_1 \mid \overline{\sigma}\right) t\left(\overline{\sigma},\overline{S}_1\right) + p(\underline{\sigma})t\left(\underline{\sigma},\underline{S}_1\right) + p\left(\overline{\sigma}\right) p\left(\overline{\sigma} \mid \overline{\sigma}\right) \frac{B}{2} - \psi,$$

and the second auditor's expected utility is given by:

$$W_2^A = p(\overline{\sigma} \mid \overline{\sigma})p\left(\overline{S}_1 \mid \overline{\sigma},\overline{\sigma}\right) t\left(\overline{\sigma},\overline{S}_1 \mid \overline{\sigma}\right) + p\left(\underline{\sigma} \mid \overline{\sigma}\right) t\left(\underline{\sigma},\underline{S}_1 \mid \overline{\sigma}\right) + p\left(\overline{\sigma} \mid \overline{\sigma}\right) \frac{B}{2} - \psi$$

and is only paid with probability $p(\overline{\sigma})$. In the absence of private benefits the first auditor receives on average[13] $\frac{\psi}{\theta}$ and the second $p(\overline{\sigma})\frac{\psi}{\theta}$. The expected transfer depends on the precision of the auditor's report. The higher the precision the lower the expected profit: the higher the precision the lower the incentives needed to make an auditor exert effort.

For the lowest interval of private benefits, i.e. $B \leq \frac{2p(\overline{\sigma})}{\theta v}\psi$, the first auditor expected transfer increases to $\frac{\psi}{\theta} + \frac{(1-\theta)(1-v)B}{2}$ and the second auditor expected transfer is now $p(\overline{\sigma})\left(\frac{\psi}{\theta} + \frac{B(1-v)(1-\theta)}{2p(\overline{\sigma})}\right)$. In the middle range of benefits, the expected transfers are $\frac{\psi}{\theta} + \frac{(1-\theta)(1-v)B}{2}$ and $p(\overline{\sigma})\frac{B}{2}$ respectively. Lastly, when private benefits are very high both auditors expect to receive $p(\overline{\sigma})\frac{B}{2}$.

Hence, the higher the private benefit the higher the expected transfer. When the private benefit is very high, $W_1^A$ equals $W_2^A$, because some transfers are capped to zero ($t\left(\overline{\sigma},\overline{S}_1 \mid \overline{\sigma}\right)$ and $t\left(\overline{\sigma},\overline{S}_1\right)$) and the remaining transfers, in particular the transfer of the first auditor has already been adjusted for the fact that she receives

[12]Note that $p(\overline{\sigma})p\left(\overline{S}_1 \mid \overline{\sigma}\right)$ equals $p(\overline{\sigma})p(\overline{\sigma} \mid \overline{\sigma})p\left(\overline{S}_1 \mid \overline{\sigma},\overline{\sigma}\right)$.

[13]That is also equal to the expected fee in the single auditor case.

$B$ with probability $\frac{1}{2}$ when she and the second auditor both issue a $\overline{\sigma}$-report. Hence the expected transfer increases less than proportionally with $\frac{B}{2}$, contrarily to what happens with the expected transfer of the auditor that is in charge of the second audit. The following lemma summarises the results:

**Lemma 4.** *With no private benefits an auditor's expected fee is $\frac{\psi}{\theta}$ if she is hired for the first audit and $p(\overline{\sigma})\frac{\psi}{\theta}$ if she is hired for the second audit. This expected fee increases with the private benefit and when $B$ exceeds $\frac{2\psi}{\theta v}$ she expects to receive $p(\overline{\sigma})\frac{B}{2}$ independently of when she is hired.*

Finally, shareholders' expected profit for the extra investment when one report is issued

$$W^1 = p(\overline{S}_1 \mid \overline{\sigma})p(\overline{\sigma})\left(u\overline{S}_2 + (1-u)\underline{S}_2 - t\left(\overline{\sigma}, \overline{S}_1\right) - I\right)$$

$$+p(\underline{S}_1 \mid \overline{\sigma})p(\overline{\sigma})\left((1-u)\overline{S}_2 + u\underline{S}_2 - I\right) - p(\underline{\sigma})t(\underline{\sigma}, \underline{S}_1)$$

and with two reports

$$W^2 = p(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma})p(\overline{\sigma}, \overline{\sigma})\left(u\overline{S}_2 + (1-u)\underline{S}_2 - t\left(\overline{\sigma}, \overline{S}_1\right) - t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right) - I\right)$$
$$+p(\underline{S}_1 \mid \overline{\sigma})p(\overline{\sigma})\left((1-u)\overline{S}_2 + u\underline{S}_2 - I\right) - p(\underline{\sigma})t(\underline{\sigma}, \underline{S}_1) - p(\underline{\sigma} \mid \overline{\sigma})p(\overline{\sigma})\,t(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}).$$

These expressions simplify to,

$$W^1 = v\left(u\overline{S}_2 + (1-u)\underline{S}_2 - \frac{\psi}{\theta v} + B\right) + (1-\theta)(1-v)\left((1-u)\overline{S}_2 + u\underline{S}_2\right)$$
$$-\psi - p(\underline{\sigma})B - p(\overline{\sigma})\,I$$

and

$$W^2 = v\left(u\overline{S}_2 + (1-u)\underline{S}_2 - \frac{\psi}{\theta v} - \frac{\psi p(\overline{\sigma})}{\theta v} + B\right) + (1-\theta)^2(1-v)\left((1-u)\overline{S}_2 + u\underline{S}_2\right)$$

$$-\psi - \left(p(\underline{\sigma})(1-\theta)\right)B - \psi p(\overline{\sigma}) - p(\overline{\sigma},\overline{\sigma})I.$$

The difference between $W^2$ and $W^1$ can be written as $-\frac{\psi p(\overline{\sigma})}{\theta} - p(\underline{\sigma})(2-\theta)B - \psi p(\overline{\sigma}) - (1-\theta)^2(1-v)\theta\left((1-u)\overline{S}_2 + u\underline{S}_2\right) + I(1-v)(1-\theta)\theta$. It follows that $(1-u)\overline{S}_2 + u\underline{S}_2$ is negative as $u$ is lower than $\frac{1}{2}$ hence, shareholders always prefer two signals rather than one if the private benefit and the cost of effort are low relative to the project outcome and investment level.

## 3.4 Optimal Contract with One Auditor and Two Signals

Auditors are sometimes required to certify the same accounts twice. This happens for example with quarterly financial statements that are also revised during the annual audit[14]. For this reason, this section looks at situations where the same auditor is required to make two sequential reports about the quality of the project's first stage.

This auditor faces a conflict of interest: if she sees that the first stage is unsuccessful, investors will not want to invest any more and she will forgo the transfer next period. By manipulating information she still has the possibility of covering this up in the next period: a longer relationship with the firm gives the auditor the possibility of

[14]As explained in the introduction, Myers et al. (2004) and Choi and Doogar (2005) find that quarterly financial reports restatements increase with auditor tenures, i.e. the longer an auditor remains in the firm, the more frequent this type of restatements are.

revising previous reports. Shareholders want the auditor to be truthful and to reveal her information even if this goes against what she previously stated but on the other hand she should be truthful in the first period and not rely on the second period to fix her mistakes or incorrect reports.

A contract in this case consists of transfers from the firm's shareholders based on the auditor's reports $\widehat{\sigma} = (\widehat{\sigma}_1, \widehat{\sigma}_1)$ of her private signals for both audits or only one if the private signal is $\underline{\sigma}$. Formally, a contract is summarised by the following transfers: if the signal in the first period is $\underline{\sigma}$ and it is truthfully reported the auditor receives $\widetilde{t}\,(\underline{\sigma}, \underline{S}_1)$ and if she is not truthful the transfer is $\widetilde{t}\,(\overline{\sigma}, \underline{S}_1)$. If the signal in the first period is $\overline{\sigma}$ is followed by another $\overline{\sigma}$-report, the auditor receives $t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$ if she is correct and $t\,(\overline{\sigma}, \underline{S}_1, \overline{\sigma})$ if the auditor is incorrect. If the second report is $\underline{\sigma}$, the transfers are $t\,(\underline{\sigma}, \underline{S}_1, \overline{\sigma})$ if she is correct and $t\left(\underline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$ otherwise.

A difference relative to the previous case is that an auditor may now enjoy cost savings when exerting effort for the second time, i.e. familiarity with the firm implies a lower cost of effort, $\widetilde{\psi}$, in the second period. Also, the auditor is entitled to receive the private benefit after two consecutive good reports whereas before this would happen with probability $\frac{1}{2}$.

### 3.4.1 Optimal Contract

There are two adverse selection incentive constraints. Firstly, given that the first signal was $\overline{\sigma}$, the auditor should not prefer reporting $\underline{\sigma}$ after having observed $\overline{\sigma}$ i.e.:

$$p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right) + p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1, \overline{\sigma}\right) + B$$

$$\geq p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\underline{\sigma}, \underline{S}_1, \overline{\sigma}\right) + p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\underline{\sigma}, \overline{S}_1, \overline{\sigma}\right). \tag{3.13}$$

The auditor is also sure that two consecutive $\overline{\sigma}$-reports guarantee the extra investment and therefore the private benefit. On the other hand, the auditor should not prefer reporting $\overline{\sigma}$ after having observed $\underline{\sigma}$:

$$t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right) \geq t\left(\overline{\sigma},\underline{S}_1,\overline{\sigma}\right) + B \tag{3.14}$$

If the first signal is $\underline{\sigma}$, there is no second audit.

Again, the contract must also satisfy a moral hazard incentive constraint to induce the auditor to gather information, i.e.,

$$p(\overline{\sigma}\mid\overline{\sigma})\left[p\left(\overline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)+p\left(\underline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\underline{S}_1,\overline{\sigma}\right)+B\right]+p(\underline{\sigma}\mid\overline{\sigma})\left(t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)\right)$$

$$-\widetilde{\psi} \geq \max\left\{\begin{array}{c} p\left(\overline{S}_1\mid\overline{\sigma}\right)\left(t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)\right)+p\left(\underline{S}_1\mid\overline{\sigma}\right)t\left(\overline{\sigma},\underline{S}_1,\overline{\sigma}\right)+B, \\ p\left(\overline{S}_1\mid\overline{\sigma}\right)t\left(\underline{\sigma},\overline{S}_1,\overline{\sigma}\right)+p\left(\underline{S}_1\mid\overline{\sigma}\right)t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)\end{array}\right\}. \tag{3.15}$$

The moral hazard constraints can be rewritten as follows:

$$p(\underline{\sigma}\mid\overline{\sigma})t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)-\widetilde{\psi} \geq \left(p\left(\underline{S}_1\mid\overline{\sigma}\right)-p(\overline{\sigma}\mid\overline{\sigma})p\left(\underline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)\right)t\left(\overline{\sigma},\underline{S}_1,\overline{\sigma}\right)+Bp(\underline{\sigma}\mid\overline{\sigma}) \tag{3.16}$$

and

$$p(\overline{\sigma}\mid\overline{\sigma})\left[p\left(\overline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)+p\left(\underline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\underline{S}_1,\overline{\sigma}\right)+B\right]-\widetilde{\psi} \geq$$

$$p\left(\overline{S}_1\mid\overline{\sigma}\right)t\left(\underline{\sigma},\overline{S}_1,\overline{\sigma}\right)+\left(p\left(\underline{S}_1\mid\overline{\sigma}\right)-p(\underline{\sigma}\mid\overline{\sigma})\right)t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right). \tag{3.17}$$

Using (3.4) the first inequality implies the adverse selection constraint (3.14). Hence, it can be ignored and the second one implies (3.13) as $p\left(\overline{\sigma}\mid\overline{\sigma}\right)p\left(\overline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)$ equals

$p(\overline{S}_1 \mid \overline{\sigma})$.

Consider now the auditor's incentive to report the first signal truthfully. If $\sigma = \overline{\sigma}$, the firm's shareholders will ask for a second signal, i.e.,

$$p(\overline{\sigma} \mid \overline{\sigma}) \left(p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right) + p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1, \overline{\sigma}\right) + B\right) + p\left(\underline{\sigma} \mid \overline{\sigma}\right) t\left(\underline{\sigma}, \underline{S}_1, \overline{\sigma}\right)$$

$$\geq p\left(\overline{S}_1 \mid \overline{\sigma}\right) \widetilde{t}\left(\underline{\sigma}, \overline{S}_1\right) + p\left(\underline{S}_1 \mid \overline{\sigma}\right) \widetilde{t}\left(\underline{\sigma}, \underline{S}_1\right). \tag{3.18}$$

On the other hand, the auditor should not prefer reporting $\overline{\sigma}$ after having observed $\underline{\sigma}$. If the expert reports a good signal, she will find it optimal under condition (3.16) to report a bad second signal, hence she will not gather a second signal and reports the signal gathered in the first period. Thus:

$$\widetilde{t}\left(\underline{\sigma}, \underline{S}_1\right) \geq t\left(\underline{\sigma}, \underline{S}_1, \overline{\sigma}\right). \tag{3.19}$$

Finally, the moral hazard constraint needed to induce the auditor to gather the first signal can be written as:

$$\begin{gathered} p(\overline{\sigma}) p\left(\overline{\sigma} \mid \overline{\sigma}\right) \left(p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right) + p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1, \overline{\sigma}\right) + B\right) \\ + p(\overline{\sigma}) p\left(\underline{\sigma} \mid \overline{\sigma}\right) t\left(\underline{\sigma}, \underline{S}_1, \overline{\sigma}\right) - p(\overline{\sigma}) \widetilde{\psi} + p\left(\underline{\sigma}\right) \widetilde{t}\left(\underline{\sigma}, \underline{S}_1\right) - \psi \geq \end{gathered}$$

$$\max \left\{ \begin{array}{c} p(\overline{\sigma}) \left(p\left(\overline{S}_1 \mid \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right) + p\left(\underline{S}_1 \mid \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1, \overline{\sigma}\right) + B\right) + p(\underline{\sigma}) t\left(\underline{\sigma}, \underline{S}_1, \overline{\sigma}\right) - \psi, \\ p\left(\underline{S}_1\right) \widetilde{t}\left(\underline{\sigma}, \underline{S}_1\right) + p\left(\overline{S}_1\right) \widetilde{t}\left(\underline{\sigma}, \overline{S}_1\right), \; p\left(\overline{S}_1\right) t\left(\underline{\sigma}, \overline{S}_1, \overline{\sigma}\right) + p\left(\underline{S}_1\right) t\left(\underline{\sigma}, \underline{S}_1, \overline{\sigma}\right), \\ p\left(\overline{S}_1\right) t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right) + p\left(\underline{S}_1\right) t\left(\overline{\sigma}, \underline{S}_1, \overline{\sigma}\right) + B. \end{array} \right\}. \tag{3.20}$$

The RHS reflects that the relevant possibilities are to gather only one signal and report accordingly after having reported $\overline{\sigma}$, not to exert effort in both periods and report a bad signal in the first period or wait and do it in the second period, or always report a good signal. Finally the auditor's limited liability constraints must also be satisfied, i.e., all transfers need to be positive. It will be optimal to set $t\left(\underline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$ equal to zero as it relaxes the RHS of the third constraint of (3.20) and does not affect the expected transfer. Hence, this constraint can be ignored using (3.19). Therefore, (3.20) can be simplified as follows:

$$p\left(\underline{\sigma}\right)\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)-p(\overline{\sigma})\widetilde{\psi}-Bp(\overline{\sigma})p\left(\underline{\sigma}\mid\overline{\sigma}\right)\geq$$

$$p(\overline{\sigma})\left(p\left(\underline{S}_1\mid\overline{\sigma}\right)-p\left(\overline{\sigma}\mid\overline{\sigma}\right)p\left(\underline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)\right)t\left(\overline{\sigma},\underline{S}_1,\overline{\sigma}\right)+\left(p(\underline{\sigma})-p(\overline{\sigma})p\left(\underline{\sigma}\mid\overline{\sigma}\right)\right)t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right), \tag{3.21}$$

$$p(\overline{\sigma})p\left(\overline{\sigma}\mid\overline{\sigma}\right)\left(p\left(\overline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)+p\left(\underline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\underline{S}_1,\overline{\sigma}\right)+B\right)$$

$$+p(\overline{\sigma})p\left(\underline{\sigma}\mid\overline{\sigma}\right)t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)-p(\overline{\sigma})\widetilde{\psi}-\psi\geq\left(p\left(\underline{S}_1\right)-p\left(\underline{\sigma}\right)\right)\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)+p\left(\overline{S}_1\right)\widetilde{t}\left(\underline{\sigma},\overline{S}_1\right), \tag{3.22}$$

and the last one is

$$p(\overline{\sigma})p\left(\underline{\sigma}\mid\overline{\sigma}\right)t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)+p\left(\underline{\sigma}\right)\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$$

$$\geq\left(p\left(\underline{S}_1\right)-p(\overline{\sigma})p\left(\overline{\sigma}\mid\overline{\sigma}\right)p\left(\underline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)\right)t\left(\overline{\sigma},\underline{S}_1,\overline{\sigma}\right)+\psi+p(\overline{\sigma})\widetilde{\psi}+B\left(1-p(\overline{\sigma})p\left(\overline{\sigma}\mid\overline{\sigma}\right)\right). \tag{3.23}$$

Again, using Bayes' rule and (3.1), the adverse selection constraint (3.18) can be ignored as it is implied by constraint (3.22). However, by looking at (3.21) it is not immediately obvious that (3.19) is also satisfied hence this constraint is not going to be ignored.

The optimal contract is the solution to the problem:

$$\min_{\{t(.)'s\}} p(\overline{\sigma})p\left(\overline{\sigma},\overline{\sigma}\right)\left(p\left(\overline{S}_1 \mid \overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\overline{\sigma},\overline{S}_1\right)+p\left(\underline{S}_1 \mid \overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\overline{\sigma},\underline{S}_1\right)\right)$$

$$+p\left(\underline{\sigma},\overline{\sigma}\right)t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)+p(\underline{\sigma})\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$$

subject to (3.16), (3.17), (3.21), (3.22), (3.23), (3.19) and all $t(.)\geq 0$

Hence, the following result is derived:

**Proposition 13.** *If one auditor is to report twice the optimal incentive contract is:*

$$t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)=\frac{\widetilde{\psi}p\left(\overline{\sigma}\right)}{(1-\theta)\,\theta\,(1-v)}+B \text{ and}$$
$$\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)=\max\left\{\frac{\widetilde{\psi}p\left(\overline{\sigma}\right)}{(1-\theta)\,\theta\,(1-v)}+B,\frac{\psi}{\theta\,(1-v)}+B\right\}.$$

*If* $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)=\frac{\widetilde{\psi}p(\overline{\sigma})}{(1-\theta)\theta(1-v)}+B$ *then* $t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)=\max\left\{0,\frac{\widetilde{\psi}p(\overline{\sigma})}{v\theta}+\frac{\psi}{v}-B\right\}$*. If* $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)=\frac{\psi}{\theta(1-v)}+B$ *then* $t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)=\left\{0,\frac{\psi}{v\theta}-B\right\}$*. The remaining transfers are equal to zero.*

**Proof.** In the Appendix. ■

The analysis focuses initially on the case where there are no private benefits and no cost savings in information collection. Shareholders want to prevent an auditor from delaying bad news. For this to happen it is enough that the reward from a correct bad report issued in the first audit, $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$, equals the reward derived from a bad report in the second audit that corrects a good report issued in the first audit, given by $t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)$. In the two-auditor case timing manipulation is not an issue therefore the first auditor does not need to be so highly rewarded when she issues a correct bad report, whereas the reward for the second period audit that results in a correct bad report equals the equivalent reward in the single auditor case, i.e. $t(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma})$ equals $t(\underline{\sigma}, \underline{S}_1, \overline{\sigma})$. This happens because the incentives to exert effort and the informational level are exactly the same in both situations. Finally, two consecutive and correct good reports issued by the single auditor yield a higher reward than a second good report that turns out to be correct in the two-auditor case, i.e. $t(\overline{\sigma}, \overline{S}_1, \overline{\sigma})$ is higher than $t(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma})$. This is obviously the case as the latter rewards effort in two consecutive periods and the former rewards effort in the last period only. In addition, regarding a single auditor for two correct good reports is cheaper than rewarding two auditors with the same consecutive reports that are correct. This happens because in the first period, before any signal has been collected, the single auditor has an extra option relative to the two-auditor case: in the previous section when a $\overline{\sigma}$-report was collected the auditor could either be right or wrong but now she has the opportunity to collect another private signal and if wrong has an extra chance to fix her mistake.

Moreover, in the case with no private benefits and no cost savings, the auditor is going to be indifferent between gathering information and reporting truthfully in both periods and gathering information in the second period only, and this is better than gathering no information and sending two consecutive good reports.

Hence,

**Lemma 5.** *With no private benefit and no cost savings in information collection, $t(\underline{\sigma}, \underline{S}_1, \overline{\sigma})$ equals $t(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma})$ and the remaining non-zero transfers $\widetilde{t}(\underline{\sigma}, \underline{S}_1)$ and $t(\overline{\sigma}, \overline{S}_1, \overline{\sigma})$ are higher than $t(\underline{\sigma}, \underline{S}_1)$ and $t(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma})$ respectively. In addition, $t(\overline{\sigma}, \overline{S}_1, \overline{\sigma})$ is lower than the sum of $t(\overline{\sigma}, \overline{S}_1)$ and $t(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma})$.*

With cost savings in information collection, the results change slightly. The intuition is the following: as the cost of the second audit, $\widetilde{\psi}$, decreases and eventually tends to zero, collecting information in both periods becomes increasingly preferred to gathering information in the second period only as with two signals the auditor issues a more reliable report at a very low incremental cost (or in the limit at no extra cost at all), and the relevant choice starts being whether or not to exert effort in the initial period, i.e. constraint (3.21) is relaxed and the binding constraint at some point becomes (3.23), meaning that the auditor is indifferent between gathering information in both periods or sending $(\overline{\sigma}, \overline{\sigma})$. As a result the reward for a correct bad report in the initial audit, $\widetilde{t}(\underline{\sigma}, \underline{S}_1)$, needs to increase to convince the auditor to exert the initial effort. But the second period problem remains the same and consequently this reward is now higher than sending a correct bad report in the second audit. However, the new cost of effort $\widetilde{\psi}$ can be positive in order to achieve this result. In fact, there is a $\widetilde{\psi}^*$ equal to $\frac{1-\theta}{p(\overline{\sigma})}\psi$ such that if $\widetilde{\psi} \geq \widetilde{\psi}^*$ the constraint (3.21) is binding but if $\widetilde{\psi} < \widetilde{\psi}^*$ constraint (3.23) becomes the relevant one.

The transfer that rewards correct bad reports in the first audit, $\widetilde{t}(\underline{\sigma}, \underline{S}_1)$, increases from $\frac{\widetilde{\psi}}{p(\underline{\sigma}|\overline{\sigma})}$ to $\frac{\psi}{p(\underline{\sigma})}$ and the transfer $t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$ that rewards two consecutive and correct good reports is no longer $\frac{\widetilde{\psi}p(\overline{\sigma})}{\theta v} + \frac{\psi}{v}$ and becomes instead $\frac{\psi}{v\theta}$, i.e. equals the reward that is given to the first auditor in the two-auditor case when she rightly sends a good report, $t\left(\overline{\sigma}, \overline{S}_1\right)$.

Note that the incentives to gather information in the first period once effort is assured in the second period, are the same as those faced by a single auditor that issues a unique report. This happens because mistakes only occur with the incorrect observation of a $\overline{\sigma}$-signal when the outcome is bad. Thus the joint probabilities $p(S, \sigma, \sigma)$ and $p(S, \sigma)$ are constant and equal to the prior belief $v$. Intuitively, as $\widetilde{\psi} \to 0$, there will always be information gathering in the second period and both auditors just need to decide whether to gather information once.

**Lemma 6.** *With cost savings in information collection, $t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)$ is lower than $t\left(\underline{\sigma},\underline{S}_1 \mid \overline{\sigma}\right)$. Transfer $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$ is either higher or equal to $t\left(\underline{\sigma},\underline{S}_1\right)$ and $t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)$ is still lower than the sum of $t\left(\overline{\sigma},\overline{S}_1 \mid \overline{\sigma}\right)$ and $t\left(\overline{\sigma},\overline{S}_1\right)$.*

Finally, with private benefits, the auditor has to be compensated for the private benefit foregone by sending a $\underline{\sigma}$-report that is correct: hence $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$ and $t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)$ both increase by $B$. On the other hand, the transfer for two correct good reports, $t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)$, decreases by $B$. As before this transfer needs to be positive, hence the private benefit cannot be too high.

**Lemma 7.** *If the private benefits are high enough $t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)$ and $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$ are higher than $t\left(\underline{\sigma},\underline{S}_1 \mid \overline{\sigma}\right)$ and $t\left(\underline{\sigma},\underline{S}_1\right)$ respectively. Transfer $t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)$ is still lower than the sum of $t\left(\overline{\sigma},\overline{S}_1 \mid \overline{\sigma}\right)$ and $t\left(\overline{\sigma},\overline{S}_1\right)$.*

In addition, the optimal transfers can be ordered in the following way:

**Lemma 8.** *The optimal transfers can be ordered as follows:*

$$\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right) \geq t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)$$

*The transfer $t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)$ can be higher than $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$ (and $t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)$) if $v$ and $B$ are low enough.*

The transfer $t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)$ can be higher than $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$ (and $t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)$) for a low private benefit, a high difference between the two effort costs and a low $v$. However as the difference in the cost of effort increase, a new $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$ is calculated and only a low private benefit and a low $v$ can assure a higher $t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)$ relative to the remaining non-zero transfers.

A curious conclusion is that if cost savings are very high, a lower private benefit is needed in order for the auditor to extract an extra rent due to limited liability.

### 3.4.2 Expected Utilities and Comparison Between the Two Cases

The auditor's expected utility is now given by the following expression:

$$W^A = p(\underline{\sigma})t\left(\underline{\sigma}, \underline{S}_1\right) - \psi + p(\overline{\sigma}) \left( \begin{array}{c} p(\overline{\sigma} \mid \overline{\sigma})p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right) \\ +p\left(\underline{\sigma} \mid \overline{\sigma}\right) t\left(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) + p\left(\overline{\sigma} \mid \overline{\sigma}\right) B - \widetilde{\psi} \end{array} \right).$$

When no private benefits and no cost savings are considered the auditor's expected transfer is $\frac{\psi p(\overline{\sigma})}{(1-\theta)} + \frac{\psi p(\overline{\sigma})}{\theta}$ that can be compared to the sum of the two auditors expected transfer, i.e. $\frac{\psi}{\theta} + \frac{\psi p(\overline{\sigma})}{\theta}$. Since $\theta$ is higher than $\frac{1}{2}$, the expected transfer for the single auditor is higher provided that $v$ is higher than $\frac{(1-\theta)^2}{\theta^2}$, i.e., that $t\left(\underline{\sigma}, \underline{S}_1\right)$, that increased in order to prevent manipulation, is high enough to overcome the fact that transfer $t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$ is lower than the sum of $t\left(\overline{\sigma}, \overline{S}_1\right)$ and $t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right)$.

Introducing cost savings, the expected transfer depends on the strength of the cost reduction:

$$W^A = \begin{cases} \frac{\widetilde{\psi} p(\overline{\sigma})}{(1-\theta)\theta} & \text{if } \widetilde{\psi} \geq \frac{1-\theta}{p(\overline{\sigma})}\psi \\ \frac{\psi}{\theta} & \text{if } \widetilde{\psi} < \frac{1-\theta}{p(\overline{\sigma})}\psi \end{cases}.$$

Obviously, a decrease in the cost of effort decreases the single auditor expected transfer, but it is capped at $\frac{\psi}{\theta}$ even if the new cost of effort becomes zero. Note that $\frac{\psi}{\theta}$ is just the expected transfer derived in single auditor/single report case. This happens because, as discussed above when cost savings are high enough both problems are equivalent. It is interesting to notice that the new cost of effort does not need to be zero for this to happen. Hence:

**Lemma 9.** *When cost savings are high enough, the single auditor's expected fee is independent of the level of cost savings and it is equal to the expected fee in the single auditor/single report case.*

Introducing private benefits but still without cost savings, changes the expected transfer as follows:

$$W^A = \begin{cases} \frac{\psi p(\overline{\sigma})}{(1-\theta)\theta} + (1-v)\,B & \text{if } B < \frac{\psi p(\overline{\sigma})}{\theta v} + \frac{\psi}{v} \\ \frac{\psi p(\overline{\sigma})}{(1-\theta)} - \psi + B & \text{if } B \geq \frac{\psi p(\overline{\sigma})}{\theta v} + \frac{\psi}{v} \end{cases}.$$

The introduction of the private benefit increases the transfer but as the private benefit increases and because of limited liability, $t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$ is capped at zero and the expected transfer suffers a jump: it increases directly because of the private benefit and because $t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$ cannot adjust (decrease) further. Note that $\frac{\psi p(\overline{\sigma})}{(1-\theta)\theta} + (1-v)\,B$ can be higher or lower than the sum of the expected transfers of the two auditors in the previous Section. However, in this case the highest expected transfers are achieved for B higher than $\frac{\psi p(\overline{\sigma})}{\theta v} + \frac{\psi}{v}$, whereas in the two-auditor case $B$ needs to exceed $\frac{2\psi}{\theta v}$, which is definitely higher than $\frac{\psi p(\overline{\sigma})}{\theta v} + \frac{\psi}{v}$. And if $B$ is very high, i.e., higher than $\frac{2\psi}{\theta v}$, $W^A$ exceeds $W_1^A + W_2^A$ for sure as the latter only equals $p(\overline{\sigma})B$. Thus, $W_1^A + W_2^A$ can start by exceeding $W^A$, but this is reversed as $B$ increases (although $W_1^A + W_2^A$ jumps twice $W_1^A + W_2^A$ and $W^A$ cross at most once).

Finally, when both cost savings and private benefits are available a number of intermediate cases arise:

$$W^A = \begin{cases} \frac{\widetilde{\psi}p(\overline{\sigma})}{(1-\theta)\theta} + (1-v)\,B & \text{if } \widetilde{\psi} \geq \frac{1-\theta}{p(\overline{\sigma})}\psi \text{ and } B < \frac{\widetilde{\psi}p(\overline{\sigma})}{\theta v} + \frac{\psi}{v} \\ \frac{\widetilde{\psi}p(\overline{\sigma})}{(1-\theta)} - \psi + B & \text{if } \widetilde{\psi} \geq \frac{1-\theta}{p(\overline{\sigma})}\psi \text{ and } B \geq \frac{\widetilde{\psi}p(\overline{\sigma})}{\theta v} + \frac{\psi}{v} \\ \frac{\psi}{\theta} + (1-v)\,B & \text{if } \widetilde{\psi} < \frac{1-\theta}{p(\overline{\sigma})}\psi \text{ and } B < \frac{\psi}{v\theta} \\ B & \text{if } \widetilde{\psi} < \frac{1-\theta}{p(\overline{\sigma})}\psi \text{ and } B \geq \frac{\psi}{v\theta} \end{cases}.$$

Looking at the first two cases, $W_1^A + W_2^A$ can also start by exceeding $W^A$, and again this is reversed as $B$ increases but a higher $B$ is needed for this to happen as the cost of effort related component of the expected transfer is now lower. And this happens for sure because when the private benefit is very high, the expected transfer is simply $B$ which is higher than $p(\overline{\sigma})B$. Hence, the private benefit can totally crowd out the effect of the cost saving.

**Lemma 10.** *If the private benefit is high enough, the single auditor/two reports expected net income is simply equal to $B$. In this case, no matter how strong the cost savings are, the expected fee is always higher than the sum of the individual expected net income.*

But which situation is going to be preferred by the firm's shareholders? The number of different cases is very high so the algebra is relegated to the Appendix and the main results are summarised below:

**Lemma 11.**

1. *With no private benefits and no cost savings in information collection, the firm's shareholders prefer a single auditor if $\frac{\psi}{\theta}$ exceeds $\frac{\psi p(\overline{\sigma})}{1-\theta}$. This happens when $v$ is sufficiently high.*

2. *Cost savings in information collection determine that shareholders prefer a single auditor when $\frac{\psi}{\theta}$ exceeds $\frac{\widetilde{\psi}p(\overline{\sigma})}{1-\theta} - \left(\psi - \widetilde{\psi}\right) p\left(\overline{\sigma}\right) \left(\frac{1+\theta}{\theta}\right)$, i.e., for a lower level of*

*$v$ relative to the previous case. If cost savings are sufficiently high, shareholders always prefers a single auditor.*

3. *With private benefits, $\frac{\psi}{\theta}$ needs to exceeds $\frac{\psi p(\overline{\sigma})}{1-\theta} + \theta (1 - v) \frac{B}{2}$ for shareholders not to rotate auditors, i.e., for a higher level of $v$ relative to case 1. When private benefits are sufficiently high, auditor rotation is always preferred.*

4. *When private benefits and cost savings coexist, a higher private benefit is needed for shareholders to switch from a single to a two-auditor situation.*

**Proof.** In the Appendix. ■

Transfer $t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$ is always lower than $t\left(\overline{\sigma}, \overline{S}\right) + t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right)$ and $\widetilde{t}\left(\underline{\sigma}, \underline{S}_1\right)$ increased relative to $t\left(\underline{\sigma}, \underline{S}_1\right)$ in order to avoid manipulation. Therefore, when $v$ is high shareholders are more likely to pay the transfers that reward correct good reports. This means that the single auditor case is preferred because shareholders are more likely to pay the "cheaper" $t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$ (relative to $t\left(\overline{\sigma}, \overline{S}\right) + t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right)$) and, on the other hand, they are less likely to pay the "expensive" $\widetilde{t}\left(\underline{\sigma}, \underline{S}_1\right)$ (relative to $t\left(\underline{\sigma}, \underline{S}_1\right)$). With cost savings a single auditor becomes increasingly preferred, but private benefits have a greater impact on the single auditor's transfers than on the transfers derived in the two-auditor case and end up undoing the gain that resulted from cost savings. Also note that a higher $\theta$ makes the effect from cost savings less important.

## 3.5 Extensions and Policy Implications

### 3.5.1 Auditor Liability

With limited liability the auditor receives a zero transfer whenever mistakes or fraud occurs. But in reality, auditors are subject to unlimited liability. Introducing unlimited liability in the model, would not dramatically change the results. The assumption of risk neutrality means that individual transfers would change but expected transfers would remain the same. However, it would definitely limit the number of cases to look at: with the introduction of the private benefit limited liability prevents some transfers from becoming negative, but this would not be an issue if there was unlimited liability and the auditor would be unable to extract extra rents.

### 3.5.2 Accounting Transparency and Corporate Governance

Most observers of the Enron debacle agree that its intention to circumvent existing accounting rules by hiding large amounts of debt from the public was one contributing factor of the final explosion. It achieved this by shifting its debts off its balance sheet. Therefore there is an ongoing debate about how to make accounting procedures more transparent. For example, the Washington Post has recently reported that the SEC has suggested that "(...) accounting practices related to real estate leases, employee pension plans and financial instruments known as derivatives remain particularly complex and opaque(...)" and "(...)urged standard-setters at the Financial Accounting Standards Board to determine whether changes are needed to eliminate possible trouble spots"[15].

In this model less confusing accounting rules could be translated in a higher probability of identifying failures, i.e., a higher $\theta$. For example, when $\theta$ tends to 1 shareholders only require a single audit in order to make the right investment decision and information gathering and disclosure becomes considerably less costly the higher $\theta$ is. Alternatively, accounting transparency could also imply a lower cost of effort for auditors. From Lemma 13, it can be seem that as the cost of effort tends to zero,

[15]"SEC Calls for Transparency" in the *Washington Post* of 15/06/2005.

shareholders are indifferent between rotating and not rotating auditors, unless there are private benefits. In this case, auditor rotation is always preferred. Hence, making accounting procedures more transparent makes the organisational design of auditing less relevant.

Good corporate governance can also be translated in a lower probability of incorrect audits in particular if one thinks that good corporate governance mechanisms make it more difficult for managers to disguise unfavourable information. But it can also result in lower levels of private benefit: shareholders monitor managers more closely which makes it more difficult for them to "bribe" auditors into not disclosing unfavourable information. In this case, better governance combined with more accounting transparency would make shareholders increasingly indifferent between rotating auditors or not. To put it in another way, more accounting transparency would, in the context of this model, make costs savings in information collection less important and therefore, the advantage of a having a single auditor would disappear. On the other hand, better governance makes the disadvantage of a single auditor also disappear: the fact that the private benefit is received for sure by the single auditor in case the project is undertaken and only with probability $\frac{1}{2}$ in the two-auditor case, makes auditing with a single auditor more costly; if private benefits disappear, this is no longer a problem. In the limit, both cases converge.

To sum up, regulation of the audit profession seem to be less relevant with better governance (lower $B$ and higher $\theta$) and more accounting transparency (higher $\theta$ or lowers $\psi$ and $\widetilde{\psi}$).

## 3.6 Conclusion

This chapter looks at a situation where shareholders need an auditor to certify the financial reports provided by the firm's managers. However, because auditors can make mistakes, an auditor's initial report can be revised. At this point, the firm's shareholders are faced with a dilemma as they can either keep or replace the auditor that produced the initial report. If an auditor knows she is going to remain on the firm for a sufficiently long period she feels tempted to delay the announcement of "bad news" if this results in a higher expected income but on the other hand, having an auditor that is more familiar with the firm means that she is more efficient when looking for genuine mistakes. Such a set-up incorporates some of the trade-offs that are present when there is auditor rotation.

The optimal contract between shareholders and auditors is such that auditors are rewarded if and only if their reports are proved to be correct and they tend to receive a higher reward when they correctly contradict previous reports or make correct announcements that ex-ante appear less likely given what is publicly known about the quality of the project. The model also assesses the effect of private benefits and it concludes that they lower the transfers made to auditors for correct good reports but increase the transfers when it is obvious that the auditor has forgone the private benefit, i.e. when correct bad reports are issued. If the private benefit exceeds a certain threshold, auditors extract extra rents because of limited liability.

Shareholders' initial opinion about the project determines whether the firm prefers keeping the same auditor or not when there is the need for confirmation of the first report. Having a single auditor is preferred if the prior belief is higher than a certain threshold. This happens because if shareholders believe the project is a success, a transfer for a correct bad report in the first audit, which is higher in the single-auditor case to avoid manipulation, is less likely to be paid and auditors are instead more likely to be rewarded for correct good reports whose corresponding transfers are lower in the case of a single auditor. Cost savings in information collection results in a lower threshold but the existence of private benefits crowds out the positive effect of the cost savings.

More accounting transparency, that takes the form of a lower likelihood of making mistakes, makes auditing cheaper and simplifies the auditing process. This is important for regulators as there is an on-going debate about how flexible accounting standards should be: some flexibility is necessary given the extreme complexity of some activities developed by firms but too much flexibility opens the door to subjectivity, manipulation and fraud.

# Chapter 4

# The Resolution of Co-ordination Failures in Financial Distress

## 4.1 Introduction

The way firms deal with financial distress is determined to a large extent by their debt structures. Debt structures are typically composed by several classes of debt, with multiple creditors in each class. This fact makes debt renegotiation a lengthy and complex process, as creditors try to position themselves strategically. For example, in the situation where a debtor fails if an insufficient number of bondholders agrees to roll over debt or accept an exchange offer, each bondholder may only want to accept the offer if the other bondholders accept it as well. This can lead to a situation where bondholders want to withdraw their support before others do so, akin to a bank run as in Diamond and Dybvig (1983), even though creditors - as a group - would be best served by agreeing to a restructuring.

However, there is empirical and anecdotal evidence[1] that suggests that banks and other large creditors can facilitate the resolution of financial distress as they can signal to the remaining (and possibly small and less informed) creditors how well they expect a firm to perform. Also, there is often an element of formal voting in the restructuring of distressed firm, i.e., firms use voting requirements as a way to ensure that at least a minimum number of creditors agree with the restructuring and in this way limit creditors' withdrawals to a level that does not compromise the firm's chances of recovery. This aims at making the decision of staying with the firm more appealing and at convincing the more doubtful that the firm is viable as there is a significant number of creditors whose optimism is enough to back the firm in such a difficult situation.

The purpose of this chapter is therefore to assess to what extent these two mechanisms (large creditors and voting requirements) can be used as co-ordination devices in the renegotiation of public debt.

This is a model of a financially distressed firm with outstanding private and public debt. Each creditor must decide independently whether or not to agree with a reorganisation plan proposed by the firm. The reorganisation plan is similar to a debt-for-equity exchange offer and consists of replacing existing debt by an equity claim on the firm. Both the large creditor (possibly a bank) and a critical majority of small creditors (possibly bondholders) need to restructure their claims or the firm will otherwise be liquidated. But crucially, any funds withdrawn from the firm during renegotiation need to be replaced by (or paid off using) new funds or the firm will otherwise be liquidated. Think of a situation where all the firm's funds are invested in machinery or other fixed assets that are crucial for the firm's productive activity and

[1] Discussed below.

cannot be sold off to generate funds to pay creditors that refuse to renegotiate. The firm has instead to raise new funds to cover for these withdrawals and it is very likely that this generates an extra cost to the firm. Hence, the more creditors withdraw the lower are the firm's chances of success. This in turn, discourages further acceptances of the reorganisation package and generates even more withdrawals of funds from the firm. This situation is analogous to Diamond and Dybvig (1983) and also here there are *impatient* creditors who withdraw their claims and prefer to invest in the risk free asset and *patient* creditors who agree to the renegotiation plan and stay with the firm for an extra period.

This chapter argues that the presence of a large creditor can facilitate the reorganisation of public debt but this depends on how informed she is relative to small creditors. For example, perfect co-ordination of the actions of small creditors is derived if she has precise information, regardless of how noisy the small creditors' information is, *even if* they do not learn the large creditor's decision prior to being confronted with the renegotiation plan. Small creditors always agree to renegotiate and let the decision of the large creditor determine the outcome of renegotiation. This happens because renegotiation is successful if the large creditor also agrees to exchange her claim and small creditors know that since she is precisely informed she will not let the firm continue for it to default in the last period. When the large creditor decision is announced before renegotiation takes place, small creditors simply mimic the large creditor behaviour. On the other hand, the large creditor plays a very insignificant role when small creditors are those whose information is precise.

As far as the role of the voting requirement is concerned, it does not affect renegotiation when the large creditor is the one that is precisely informed but it becomes more relevant as a co-ordination device as the informational advantage of the large creditor disappears. In fact, it is derived that renegotiation is always easier in a game with voting requirements than in a game without, for any other level of relative precision of private information between the two groups of creditors.

In addition, in order for the large creditor to make use of her private information, her initial claim needs to be secured otherwise she has nothing to gain by liquidating and consequently will always agree with renegotiation[2]. This result holds when she is pivotal for a successful renegotiation. This is not the case for a small creditor: the fact that his initial claim is secured does not affect his decision about renegotiation because an individual small creditor is not pivotal and his payoff in case of liquidation does not vary with the direction of his vote.

The framework of the model is one of an exchange offer but in practice, a firm that needs to restructure its debt is faced with two choices: it can either file for bankruptcy (Chapter 11 in the US) or attempt to reorganise with its creditors privately (exchange offer). Under Chapter 11, approval for a reorganisation plan is required from a specified majority of the creditors in each class of claims, and dissenting classes can be forced to comply with the plan under the Code's cram-down provision. When adopting a private debt restructuring, participation in the offer is voluntary but the success of the exchange offer is often conditional on a stipulated voting majority of bonds being tendered.

Empirically, both types of reorganisations are important. Gilson, John and Lang (1990) investigate the incentives of financially distressed firms to restructure their debt privately rather than through formal bankruptcy and conclude that about half successfully restructure their debt outside of Chapter 11. They also show that finan-

[2]The large creditor can also be indifferent if she is precisely informed and her private information indicates that the firm will default even if renegotiation succeeds.

cial distress is more likely to be resolved privately when firms have more intangible assets, owe more of their debt to banks and owe to fewer lenders. The implications of the capital structure for the renegotiation process are also documented in James (1995,1996) where he finds evidence that bank participation in debt restructurings facilitates public debt exchange offers and increases the likelihood of achieving minimum tendering rates.[3]

These issues have also been widely debated on a sovereign level as in recent years countries have turned increasingly from bank loans to bond issues to raise capital; this broadens the investor base available to provide financing for emerging market sovereigns but on the other hand creditors have become increasingly numerous, anonymous and difficult to co-ordinate in case a reorganisation is needed.

In fact, ever since the Mexican, Asian and Russian crises of the mid-1990's efforts have been made to find means for more effective prevention and resolution of financial crises. There are a number of proposals currently under consideration that acknowledge the importance of the firm's debt structure and co-ordination problems that arise when renegotiation takes place. One of these proposals directly addresses creditors' co-ordination problems by claiming that they can be mitigated via changing the voting requirements in restructurings, for instance by including so-called collective action clauses (CACs) in bond covenants, which stipulate a critical majority in the creditors' vote to restructure the claims.

There is a vast theoretical literature on debt renegotiation but it is mostly focused on the case of a "representative" creditor. An exception is for example, Detragiache and Garella (1996) that construct an analytical framework to analyse exchange offers

[3]See also Asquith, Gertner and Scharfstein (1994) and Frank and Torous (1994) for more empirical evidence on this topic.

that exploits the analogy between debt renegotiation and the problem of financing the provision of a public good through private contributions. The debtor engages in a form of "price discrimination" because creditors contribute to debt forgiveness by choosing to exchange a different fraction of their loan portfolio depending on the utility that they expect to receive in case of bankruptcy. More recently, Morris and Shin (2002b) model a contribution game using the global games methodology. However, neither considers the case where a firm has different debt claims or explicitly models the impact of the voting requirements.

An alternative approach that also looks at voting amongst creditors in case of financial distress is presented by Bolton and Scharfstein (1996), who frame the discussion in terms of ex-post bargaining problems in a cooperative game theory setting. In their model, lenders have the right to seize assets in case of default. A large number of lenders makes renegotiations more difficult. This can be an advantage, as it deters firms from strategic default (where the owners are in a position to service their debt but refuse to do so), but of course creates problems in the case of a liquidity default (where the firm is fundamentally viable, but has short-term liquidity problems). Even though they examine the role of voting requirements, it is arguable whether cooperative game theory is the ideal framework in which to examine a problem created by a lack of cooperation.

A recent paper by Bond and Eraslan (2005) also looks at the effect of different voting rules in debt restructuring in a situation where the firm has multiple creditors with informational differences. In particular, they find out that, because information aggregation fails under the unanimity rule, the debtor needs to offer a reorganisation package that is more favourable to creditors. Consequently, unanimity rules give creditors higher recovery rates. But if markets turn out to be illiquid, unanimity rules make reorganisation almost impossible, hurting creditors and the debtor alike[4]. However, they address neither coordination problems among creditors, nor the signalling role of large creditors as a coordination device.

Many have also focussed on the role of banks in reorganisation. Diamond (1984) points out that a financial intermediary (a bank) can solve the free-rider and information duplication problems in monitoring a firm. In Chemmanur and Fulghieri (1994a), banks are beneficial because they devote more resources to evaluating whether to liquidate or continue in a firm in Chapter 11. On the other hand, Rajan (1992) points at some problems linked with bank debt: bank debt is more flexible but banks have bargaining power and expropriative incentives over the firm's profits if a short-term crisis arises.

The role of a firm's debt structure as a determinant of how it fares in reorganisation is also explored by Gertner and Scharfstein (1991). They consider a model of a financially distressed firm with bank and public debt and focus on the investment inefficiencies that arise in reorganisation due to co-ordination problems among public debtholders. Berglöf and von Thadden (1994) also look at a firm with a complex debt structure and show that firms can use multiple creditors with different types of debt to commit to optimal ex post termination. If liquidation values are low, it is optimal that short-term and long-term claims be held by separate investors, and short-term claims be secured. This arrangement strengthens the ex post bargaining position of the short-term lenders and diminishes the firm's incentives to default strategically. Repullo and Suarez (1998) characterise the circumstances under which a mixture of

[4]In the paper this situation arises when creditors holding the same belief about the future cash-flow of the firm value securities received in reorganisation differently due to different opportunity costs of funds or different tax rates, for example. If securities issued by the reorganised firm cannot be easily traded, they will not end up in the hands of those that value them the most and voting becomes less informative of creditors' private information about the firm.

informed (bank) and uninformed (market) finance is optimal and explain why bank debt is typically secured, senior, and tightly held, taking into account that banks can credibly threaten to liquidate a firm. This issue is also explored by Park (2000) that, in addition, endogenises the monitoring incentives of lenders. Finally, Rajan and Winton (1995) look at the role of covenants and collateral as incentives to monitor in the context of the optimal design of debt contracts.

This chapter does not address many of these issues, namely those related to security design, but focuses instead on the renegotiation game itself and explicitly looks at the role of information asymmetries between creditors, considers a more realistic structure to the renegotiation game and clearly models co-ordination problems among multiple creditors.

On the technical side, this chapter is related to the rapidly growing literature on global games. Morris and Shin (2002) provide an overview stressing methodological issues and several applications can be found in Morris and Shin (2001) on the pricing of debt, Goldstein and Pauzner (2004) on bank runs and Rochet and Vives (2001) on banks' liquidity crises.

The rest of the chapter is organised as follows: section 4.2 presents the model, section 4.3 characterises the equilibrium of the simultaneous game and studies the properties of this equilibrium; section 4.4 looks at the sequential game and section 4.5 derives some policy implications and concludes the chapter.

## 4.2 The Model

### 4.2.1 The Framework

The general structure of the model is as follows. Time is divided into two periods, $t = 0$ and 1 and the risk-free rate is normalised to zero. There are two types of risk-neutral agents: a firm (or manager-owner of the firm) and its creditors. There are two classes of creditors with a total claim of $D$ on the firm: a large creditor (she) holds a position on the firm with face value $B$, and the remainder is held by a continuum $[0, 1]$ of small bondholders, each holding an identical claim with face value $b$. The firm has a fixed-scale investment project that should have matured at the beginning of period 0, but due to unforeseen circumstances it will take an extra period to generate monetary cash-flows. Hence, at the beginning of period 0, the firm makes a take-it-or-leave-it offer to creditors to exchange their loans in return for a equity claim on the firm to be paid in the following period (like a debt-for-equity exchange offer)[5]. Investors can therefore individually choose whether to call in (withdraw) their loans or accept the exchange offer made by the firm.

The chapter considers a renegotiation set-up that is common practice in exchange offers: the large creditor and a *minimum* proportion of creditors are required to accept the firm's offer[6]. The proportion of small creditors that withdraw their claims is defined as $\omega \in [0, 1]$ and $1 - \widehat{\omega} \in [0.5, 1)$ is the minimum proportion of creditors required to accept the plan.

All initial debt is collateralised, hence if more than $1 - \widehat{\omega}$ creditors and/or the large creditor disagree with the plan, the firm is liquidated and all creditors receive

[5]In reality, this happens quite frequently. See for example, Brown et. al (1993) and James (1995).

[6]Alternatively, assume that bank debt is so large that withdrawal by the bank implies immediate failure.

the liquidation value: the large creditor receives $L$, whereas each individual small debtholders $i$ is entitled to $l$, where $\sum_i l = LV_0 - L$ and $LV_0$ stands for liquidation value at time 0.

If the exchange offer goes through, the game continues to period 1. Small creditors that have decided to withdraw their claims are paid $b$. The lenders that decide to exchange their loans (including the large lender) receive the new claim proposed by the firm[7] provided that the project succeeds. The project's success depends on the realisation of the random variable $\theta$ that is known by the firm at the beginning of period 1. In order to generate $\theta$, the firm needs to incur a cost $\overline{K}$. This cost measures the reward expected by the firm as a compensation for the disutility of effort required to manage the project successfully. Hence, if the firm does not contribute $\overline{K}$, and this happens whenever $\theta$ is not high enough to compensate the firm after creditors have been paid[8], the project is worthless. In this case, the firm is liquidated and creditors share the liquidation value at time 1, $LV_1$ that simply equals zero.

The following table summarises the small creditors' payoffs under the different scenarios:

| | $U$ | $S$ and $ND$ | $S$ and $D$ |
|---|---|---|---|
| Accept Offer | $l$ | $\alpha b$ | 0 |
| Withdraw Claim | $l$ | $b$ | $b$ |

where ND stands for "no default", D stands for "default", and S and U for "successful" and "unsuccessful" renegotiation respectively and $\alpha > 1$. The large creditor receives

[7]If it is credible that a group of numerous and uncoordinated creditors, with little or no bargaining power, would accept such a proposal put forward by the firm and would not demand a higher payoff for accepting to renegotiate. But it would not be difficult to assume that a large bank would exert its bargaining power and try to extract the best deal possible from the firm (cf. Rajan (1992)). For simplicity, the paper abstracts from this issue at this stage.

[8]Alternatively, the residual payoff (by priority of claims rules) to the manager-owner of the firm has to be at least $\overline{K}$.

$\alpha B$ in case renegotiation and project succeed.

For simplicity, it is assumed that if accepting the exchange offer yields the same expected payoff as withdrawing the loan, then a creditor prefers to withdraw.

The firm's investment opportunity returns a stochastic cash flow of $\theta$ in period 1, which has an (improper) uniform prior over the real line. However, in order to finish the project the firm needs to obtain refinancing from new lenders (or otherwise forego economies of scale) to cover for the withdrawals of funding of those lenders that decide to reject the exchange offer. The marginal cost of an outflow of funds is $k$, where $k = \alpha_k b$ and $\alpha_k > \alpha$. Such cost may reflect the fact that the new creditors require a mark-up to refinance the firm, or that the firm incurs in a extra cost to find new financiers. Alternatively, it can result from the use of more expensive sources of financing such as trade credit or over-drafts. Hence the firm's payoff from continuation is $\theta - \alpha B - \alpha b - \omega (k - \alpha b)$, where the last term is negative and indicates the extra cost a firm has to bear due to creditors' withdrawals.

As a consequence, the value of the firm at maturity depends on two factors: the randomly determined fundamental state $\theta$ and the severity of the extra cost $k - \alpha b$ caused by withdrawal of creditors.

As in Morris and Shin (2001) creditors have the option to withdraw their funds or accept the new offer put forward by the firm but in this model creditors vote on the reorganisation plan. Contrarily to their paper, there is a large creditor and the firm is allowed to continue as extra funds are injected on it but this generates an extra cost that affects the probability of failure in the last period (cf. Figure 4.1 and 4.2).

The model is also closely related to Hubert and Schäfer (2001) but they compare the situation of multiple bondholders to the case of a single lender and do not analyse the effects of a more complex debt structure that combines the two types of debt.

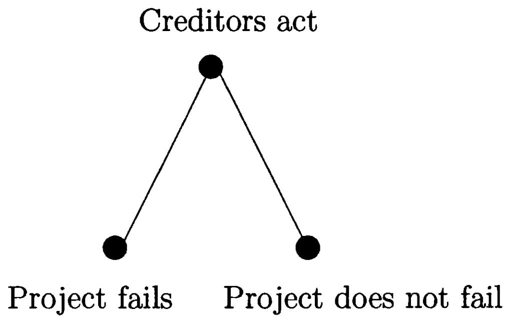


Figure 4.1: Morris and Shin (2001) game.

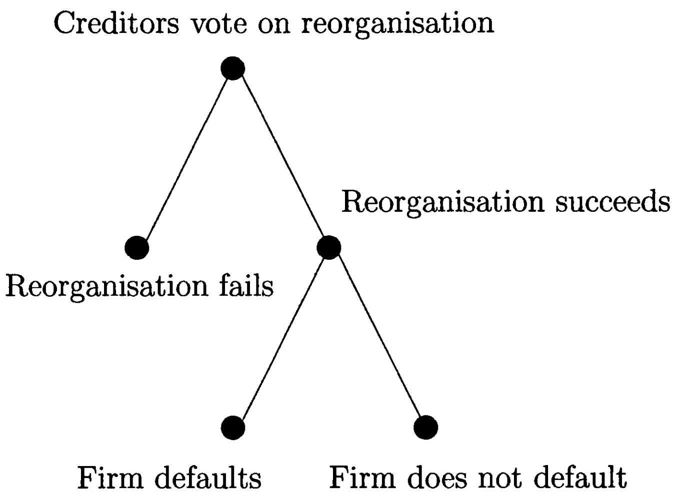


Figure 4.2: Game with extra stage.

The element of formal voting is also absent from their paper.

To summarise, the timing of the events is as follows:

- Period 0: The firm makes a take-it-or-leave-it offer to creditors. Creditors observe private signals on $\theta$ and vote on the offer. If the voting is successful, creditors who choose to withdraw receive $b$; if unsuccessful, the firm is liquidated.

- Period 1: The firm observes $\theta$ and decides whether to incur the cost $\overline{K}$. If so new funds are injected in the firm. The project matures and yields a return $\theta$. Creditors remaining with the firm are paid the new claim.

### 4.2.2 The Information Structure

In order to make an informed voting decision, creditors obtain an (imperfect) signal about the firm. Each creditor $i$ receives a noisy *private signal:*

$$x_i = \theta + \sigma\varepsilon_i$$

revealing some information about the fundamentals, where $\varepsilon_i$ is normally distributed with mean *zero* and unit variance (and density $f(.)$ and c.d.f. $F(.)$) and $\sigma > 0$ is a constant. The noise terms are *i.i.d.* across creditors.

The large creditor observes the realisation of the random variable:

$$y = \theta + \tau\eta$$

where $\tau > 0$ is a constant and $\eta$ is a normally distributed random variable with mean *zero* and unit variance (and density $f(.)$ and c.d.f. $F(.)$) and each $\varepsilon_i$ is independent of $\eta$. The distributions of the fundamentals and private signals are assumed to be known by all participants.

Note that in the last stage failure will occur if $\overline{K} \geq \theta - \alpha B - \alpha b - \omega\,(k - \alpha b)$. Hence, if creditors know the value of $\theta$ perfectly before deciding on whether to accept the exchange offer and there is no voting, the decision is trivial. If $\theta > \overline{K} + \alpha B + k \equiv \overline{\theta}$, then it is optimal to continue the project irrespective of the actions of the other creditors. Conversely, if $\theta < \overline{K} + \alpha B + \alpha b \equiv \underline{\theta}$, failure is certain no matter what the other lenders do. However, because there is a voting requirement, the project proceeds if and only if $\widehat{\omega}$ or fewer creditors withdraw their funding. Hence, if exactly $1 - \widehat{\omega}$ creditors renegotiate $\theta$ needs to exceed $\overline{K} + \alpha B + \alpha b + \widehat{\omega}\,(k - \alpha b) \equiv \widehat{\theta}$ for the project to succeed. Otherwise, the firm defaults for sure. It is the intermediate range $\theta \in \left(\widehat{\theta}, \overline{\theta}\right]$ that is critical. Hence, the firm's success depends on the action of its debtholders.

With imperfect information, agents form a posterior about the level of the fundamentals in period 1. Since $x_i$ is normally distributed, a creditor $i$ posterior of $\theta$ upon observing signal $x_i$ is normal with mean and precision $x_i$ and $\sigma$ respectively. In equilibrium, every small creditor follows the switching strategy around the critical value $x^*$. The large creditor, on the other hand, follows a switching strategy around $y^*$.

Therefore, a creditor's strategy is a rule of action which maps each realisation of her signal to one of the actions - to accept the exchange offer or to withdraw the loan. The equilibrium is a Bayesian Nash equilibrium in which, conditional on each creditor's signal, the action prescribed by this creditor maximises her conditional expected payoff when all other creditors follow their strategies in equilibrium.

## 4.3 Equilibrium of the Simultaneous Move Game

Using the global games methodology, the agents follow a switching strategy around a certain posterior belief. Given this posterior belief the number of agents that accept reorganisation is determined. Additionally, the critical next period value of the fundamentals for which the firm will default is computed and depends on the belief in this period which makes agents switch. When deciding whether to switch, creditors compute what happens if there is continuation and this is then compared with what results from liquidation. There are *three* equations and *three* unknowns that define the unique equilibrium of the game.

It is also important to point out that in this game there are no global strategic complementarities. This property requires that an agent's incentive to take an action increases with the number of other agents taking that same action. This property does not hold in this model since when the proportion of creditors that withdraws is between $\widehat{\omega}$ and 1, the effect on the incentive to withdraw of an individual creditor is constant as renegotiation is going to be unsuccessful and regardless of what he does a creditor always receives the liquidation value. Only when less than $\widehat{\omega}$ withdraw does the game exhibits strategic complementarities. Goldstein and Pauzner (2004) prove that in such a setting there are one-sided strategic complementarities and show that the equilibrium is still unique and in switching strategies.

### 4.3.1 Firm's Default Point and Ex-ante Probability of a Successful Renegotiation

It is initially conjectured that all agents optimally follow a trigger strategy: they accept the offer if and only if their signal is above some optimally selected threshold

$x^*$ for bondholders and $y^*$ for the large creditor respectively; otherwise they withdraw their loan. It is later shown that in fact the unique equilibrium of this game is characterised by a critical value $\theta^*$, for which the project is on the margin of success and failure, and a critical value for the signals $x^*$ and $y^*$. Given this, conditional on state $\theta$, the distribution of $x$ is normal with mean $\theta$ and variance $\sigma^2$. So the ex-ante probability that any agent refuses reorganisation is equal to

$$\omega\left(x^*, \theta\right) = \Pr\left(x_i < x^* \mid \theta\right) = F\left(\frac{x^* - \theta}{\sigma}\right). \tag{4.1}$$

As the number of agents tends to infinity[9], the proportion of agents that reject reorganisation is deterministic and will be equal to this ex-ante probability by the Law of Large Numbers (see Judd(1985)). Therefore, when deciding whether to agree with renegotiation a small creditor not only takes into account how the other players are going to vote (and that they also follow a trigger strategy) but is also able to anticipate precisely the proportion of small creditors that votes for reorganisation. For this reason, even if voting is not binding, agents do not want to revise their decisions based on the outcome of the voting stage.

The default point $\theta^*$ is such that the expected profit of the firm is equal to zero and it is implicitly given by the following expression:

$$\theta^* = \overline{K} + \alpha B + \alpha b + \omega\left(x^*, \theta^*\right)(k - \alpha b). \tag{4.2}$$

The right-hand side of (4.2) represents the firm's liabilities and the left hand side the firm's assets in period 1 after paying its creditors. Substituting (4.1) evaluated at $\theta^*$

[9] And this is what happens here in the case of small creditors that are continuously distributed in the interval $[0, 1]$.

in (4.2), the first equilibrium condition is derived as:

$$F\left(\frac{x^{*}-\theta^{*}}{\sigma}\right)=\frac{\theta^{*}-\overline{K}-\alpha B-\alpha b}{k-\alpha b}. \tag{4.3}$$

On the other hand, if the true state is $\theta$, the proportion of players observing a signal lower than $x^*$ is $F\left(\frac{x^*-\theta}{\sigma}\right)$. Renegotiation is successful if $\omega<\widehat{\omega}$, that is equivalent to $F\left(\frac{x^*-\theta}{\sigma}\right)<\widehat{\omega}$ or rearranging, $\theta>x^*-\sigma F^{-1}(\widehat{\omega})\equiv\widehat{\theta}$. Thus, the probability that a player assigns a proportion lower than $\widehat{\omega}$ of the other players observing a signal lower than $x^*$, if he has observed $x^*$ is defined as

$$\Pr\left(\omega<\widehat{\omega}\mid x^{*}\right)=1-\Pr\left(\theta<x^{*}-\sigma F^{-1}(\widehat{\omega})\mid x^{*}\right)=\widehat{\omega}. \tag{4.4}$$

This result implies that the density will be *uniform* and it depends on the fact that $\theta$ follows an improper prior[10].

### 4.3.2 Large Creditor Switching Point

If the large creditor disagrees renegotiation will automatically fail. If she agrees, renegotiation succeeds if more than $1-\widehat{\omega}$ bondholders agree with the renegotiation plan. This is in line with the empirical evidence that asserts that a firm in financial distress does generally restructure all outstanding debt. In fact, Gilson, John and Lang (1990) state that 90.0 percent of firms in the sample with bank debt (large creditor) outstanding, and 69.8 percent of firms with publicly traded debt, do restructure that particular type of debt.

She knows that, given the small creditors switching point $x^*$, if $\theta\leq\widehat{\theta}$, renegotia-

[10]See Morris and Shin (2002b) for a detailed discussion of this result.

tion is unsuccessful because a proportion lower than $1-\widehat{\omega}$ of small creditors decides to accept renegotiation, regardless of the large creditor's actions. In this case, the firm is liquidated and the large creditor receives the liquidation payoff $L$. If a proportion higher than $1-\widehat{\omega}$ accepts the renegotiation package, renegotiation is successful provided that large creditor agrees. Note that the signals $x$ and $y$ are independent, therefore the large creditor needs to take into account the probability that the small creditors renegotiate given the small creditor's switching point $x^*$. But even if renegotiation is indeed successful, only when $\theta \geq \theta^*$ does the firm not default in the last period and only then the large creditor receives $\alpha B$. Refusal to renegotiate by the large creditor results in liquidation and a payoff of $L$. Thus, the large creditor restructures her debt, conditional on small creditors' acceptance when,

$$\Pr\left(\theta \leq \widehat{\theta} \mid x^*\right) L + \Pr\left(\theta \geq \theta^* \mid y^*\right) \Pr\left(\theta > \widehat{\theta}, \mid x^*\right) \alpha B = L$$

or

$$\frac{1}{\sigma}\int_{-\infty}^{\widehat{\theta}} f\left(\frac{\theta - x^*}{\sigma}\right) d\theta L + \frac{1}{\tau}\int_{\theta^*}^{+\infty} f\left(\frac{\theta - y^*}{\tau}\right) d\theta \left(\frac{1}{\sigma}\int_{\widehat{\theta}}^{\infty} f\left(\frac{\theta - x^*}{\sigma}\right) d\theta\right) \alpha B = L.$$

Rearranging the expression, it can be written as follows:

$$F\left(\frac{\theta^* - y^*}{\tau}\right) = 1 - \frac{L}{\alpha B}.$$

The second equilibrium condition determines the level of $y^*$ such that the large creditor restructures her debt if her signal is above $y^*$, and does not restructure otherwise. The threshold $y^*$ is defined as follows:

$$y^* = \theta^* - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right). \tag{4.5}$$

Note that $\theta^*$ is in fact a function of $x^*$, and $x^*$ depends on the large creditor's switching point $y^*$.

### 4.3.3 Small Creditors Switching Point

Consider now a small creditor's problem: he knows that for $\theta \leq \widehat{\theta}$, renegotiation is unsuccessful, regardless of the large creditor's actions and in this case the firm is liquidated and he receives his share of the liquidation value, $l$. If $\theta \in \left(\widehat{\theta}, \theta^*\right]$, renegotiation is successful provided that the large creditor agrees to renegotiate [11] but the firm defaults. In this case, a small creditor that accepted renegotiation receives zero but and he would have received $b$, had he withdrawn his funds from the firm. If $\theta > \theta^*$ the project succeeds when the large creditor agrees to renegotiate. A small creditor that remains with the firm receives $\alpha b$ and he would have received $b$ otherwise. Since a small creditor's optimal strategy is to continue lending if and only if his expected payoff from continuing lending conditional on $x$ exceeds his payoff from stopping lending, the switching point $x^*$ equates the payoff when a creditor consents to the payoff when he refuses[12]:

$$\Pr\left(\theta \leq \widehat{\theta} \mid x^*\right) l + \Pr\left(\widehat{\theta} < \theta < \theta^*, y < y^* \mid x^*\right) l + \Pr\left(\widehat{\theta} < \theta < \theta^*, y > y^* \mid x^*\right) 0$$

$$\Pr\left(\theta \geq \theta^*, y < y^* \mid x^*\right) l + \Pr\left(\theta \geq \theta^*, y > y^* \mid x^*\right) \alpha b$$

[11] And this happens when her private signal $y$ exceeds the switching point $y^*$.
[12] This possibly excessive notation will be useful later on.

$$= \Pr\left(\theta \leq \widehat{\theta} \mid x^*\right) l + \Pr\left(\widehat{\theta} < \theta < \theta^*, y < y^* \mid x^*\right) l + \Pr\left(\theta \geq \theta^*, y < y^* \mid x^*\right) l$$

$$+ \Pr\left(\widehat{\theta} < \theta < \theta^*, y > y^* \mid x^*\right) b + \Pr\left(\theta > \theta^*, y > y^* \mid x^*\right) b$$

that can be simplified to

$$\Pr\left(\theta \geq \theta^*, y > y^* \mid x^*\right) \alpha = \Pr\left(\theta > \widehat{\theta}, y > y^* \mid x^*\right)$$

and re-written as [13]:

$$\int_{\theta^*}^{\infty} \frac{1}{\sigma} f\left(\frac{\theta - x^*}{\sigma}\right)\left(1 - F\left(\frac{y^* - \theta}{\tau}\right)\right) d\theta \alpha = \int_{\widehat{\theta}}^{\infty} \frac{1}{\sigma} f\left(\frac{\theta - x^*}{\sigma}\right)\left(1 - F\left(\frac{y^* - \theta}{\tau}\right)\right) d\theta. \tag{4.6}$$

Creditors receive a sure amount $b$ if renegotiation is successful but they withdraw their loans whereas acceptance of the firm's offer confers a risky output, $\alpha b$ or 0.

There is a unique $x^*$ that solves this equation. In order to simplify this expression it is helpful to introduce a change of variables in the integrals. Let

$$z = \frac{\theta - x^*}{\sigma}$$

and denote

$$\widehat{\delta} = \frac{\widehat{\theta} - x^*}{\sigma} \text{ and } \overline{\delta} = \frac{\theta^* - x^*}{\sigma}.$$

[13]Note that $\Pr\left(\theta \geq \theta^*, y > y^* \mid x^*\right)$ is the joint probability of no default and renegotiation by the large creditor given $x^*$ and the probability that the large creditor renegotiates for a given $\theta$ is $\Pr\left(y > y^* \mid \theta\right) = 1 - F\left(\frac{y^* - \theta}{\tau}\right)$.

and using (4.5), the expression can be rewritten as (see Appendix for details):

$$\int_{\overline{\delta}}^{+\infty} f(z)\left(1-F\left(\frac{\sigma}{\tau}\left(\overline{\delta}-z\right)-F^{-1}\left(1-\frac{L}{\alpha B}\right)\right)\right)dz\alpha$$

$$=\int_{\widehat{\delta}}^{+\infty} f(z)\left(1-F\left(\frac{\sigma}{\tau}\left(\overline{\delta}-z\right)-F^{-1}\left(1-\frac{L}{\alpha B}\right)\right)\right)dz \qquad (4.7)$$

Note that $\widehat{\delta}$ and $\overline{\delta}$ are invariant and strictly decreasing in $x^*$[14] respectively, since

$$\frac{\partial\widehat{\delta}}{\partial x^*}=0 \text{ and } \frac{\partial\overline{\delta}}{\partial x^*}=-\frac{1}{(k-\alpha b)\,f\left(\overline{\delta}\right)+\sigma}<0.$$

Both sides of (4.7) are continuous and strictly increasing in $x^*$. Hence, as $\alpha \longrightarrow 1$, the RHS is always higher than the LHS for any $x^*$ but both converge to the same value as $x^*$ increases (as $\overline{\delta}$ decreases and approaches $\widehat{\delta}$). Consequently, as $\alpha$ increases, the LHS suffers a parallel upward shift and there will be a unique solution to (4.7) when both sides cross. The higher the $\alpha$, the lower the value of $x^*$ that satisfies expression (4.7). This makes sense as a higher payoff from accepting the exchange-offer makes it the optimal choice for a higher number of creditors.

Also note that the optimal $x^*$ decreases with the relative precision of private information of the small and the large creditors, $\frac{\sigma}{\tau}$, and with the voting requirement established by the firm. On the other hand, the payoff received in case of liquidation, i.e., the value of the collateral backing up the initial claim does not affect the decision to renegotiate. This happens because a small creditor receives $l$, when the exchange offer does not succeed, independently of how he voted before and the vote of an

[14] By total differentiating $\widehat{\delta}(.)$ and $\overline{\delta}(.)$ and using expression (4.3).

individual small creditor is not pivotal.

In contrast, looking at the large creditor equilibrium condition, a higher $L$ makes her acceptance of the renegotiation plan more difficult and this increases if her private signal becomes less precise. When she is perfectly informed, i.e. $\tau \rightarrow 0$, then $y^* \rightarrow \theta^*$ and the large creditor rejects renegotiation when the firm defaults and agrees to renegotiate otherwise. As $\tau$ increases, the switching point $y^*$ increases: she becomes more conservative and acceptance of renegotiation happens for higher levels of her private signal. But this only happens if $L$ is high enough[15]. For example, if $L$ is equal to zero, the large creditor always renegotiate for any positive $\tau$, whereas having something (high enough) to lose in case of liquidation makes the large creditor more cautious and use her private information when deciding on whether to renegotiate. Moreover, this result depends crucially on the fact that the large creditor is pivotal. This is in line with the fact that bank debt (large creditor) is often senior and collateralised (cf. Mann(1997) and Schwartz(1997)) and public debt is in general junior and uncollateralised. Small creditors do not use the value of the collateral (liquidation value) when deciding whether to renegotiate. On the other hand, the value of the collateral forces the large creditor to be careful when considering whether to agree with renegotiation and in doing she makes use of her private information. But note that this is beneficial if large creditors are well-informed institutions. If not, this generates excessive failures to renegotiate as the switching point $y^*$ becomes increasingly higher. In addition, as Rajan (1992) demonstrates this potential benefit has to be weighed against the monopoly bargaining power of the single lender derived from its monitoring function[16].

[15]The function $F^{-1}\left(1-\frac{L}{\alpha B}\right)$ is negative when $L > \frac{\alpha B}{2}$.

[16]This model is based on the assumption that the large creditor/bank is benevolent and does not try to extract more money from the firm. Of course the bank could try to behave in an opportunistic

Some other results can be analysed by focusing on the limiting cases of the relative precision of private information, i.e. by letting agents become arbitrarily well informed about the fundamentals. The empirical evidence and the theoretical literature on this area seem to agree with the fact that large creditors are in general better informed than small creditors. Hence, the focus is now on the case in which the large player is more informed than the rest of the market and has arbitrarily precise private information, that is $\lim \frac{\sigma}{\tau} = \infty$. In this case, the probability that the large creditor renegotiates when the firm defaults from the small creditors' point of view is

$$\Pr\left(\theta < \theta^*, y \geq y^* \mid x^*\right)$$

and, given the information structure of the game and the definition of the large creditor signal, i.e.,

$$y = x_i - \sigma\varepsilon_i + \tau\eta$$

and

$$y^* = \theta^* - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right).$$

this probability can be expressed as

$$\Pr\left(\varepsilon_i > \frac{x^* - \theta^*}{\sigma}, \tau\eta - \sigma\varepsilon_i \geq \theta^* - x^* - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right).$$

way and renegotiate a higher $\alpha$. This would have to be made public in the reorganisation plan and the bank would therefore reveal her information to the remaining creditors. This is just equivalent to the model developed in the next section where the bank moves first. The optimal thing for the bank to do would be to extract the maximum after small creditors and the firm are paid. The bank would still act as a co-ordination device but at a higher cost for the firm's shareholders.

When $\lim \frac{\sigma}{\tau} = \infty$ (or $\lim \frac{\tau}{\sigma} = 0$) the probability can be rewritten as,

$$\Pr\left(\varepsilon_i > \frac{x^* - \theta^*}{\sigma}, \frac{\tau}{\sigma}\eta - \varepsilon_i \geq \frac{\theta^* - x^*}{\sigma} - \frac{\tau}{\sigma}F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)$$

and it is simply equal to

$$\Pr\left(\varepsilon_i > \frac{x^* - \theta^*}{\sigma}, -\varepsilon_i \geq \frac{\theta^* - x^*}{\sigma}\right)$$

that is just equal to zero. Hence, small creditors believe that when the large creditor is a.s. (almost surely) perfectly informed, she does not renegotiate for values of the fundamentals where the firm defaults. Consequently if renegotiation succeeds there is no default. Thus, given that the large creditor does not renegotiate $\Pr(\theta < \theta^*, y < y^* \mid x^*)$ equals $\Pr(\theta < \theta^* \mid x^*)$ and the small creditor indifference condition is modified as follows:

$$\int_{-\infty}^{\overline{\delta}} f(z)\, dzl + \int_{\overline{\delta}}^{+\infty} f(z)\left(1 - F\left(\frac{\sigma}{\tau}\left(\overline{\delta} - z\right) - F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)\right) dz\alpha$$

$$= \int_{-\infty}^{\overline{\delta}} f(z)\, dzl + \int_{\overline{\delta}}^{+\infty} f(z)\left(1 - F\left(\frac{\sigma}{\tau}\left(\overline{\delta} - z\right) - F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)\right) dz.$$

If $\theta \geq \theta^*$ (that is, for any $z \geq \overline{\delta}$) the probability that a precisely informed large player chooses to renegotiate is equal to one and the probability of a successful renegotiation is precisely equal to the probability of successful renegotiation on the small creditors' group only. In other words, if the large creditor is very well informed there is no uncertainty on her side only on the small creditor side and the expression for the switching point becomes

$$\int_{\overline{\delta}}^{+\infty} f(z)\, dz\alpha = \int_{\overline{\delta}}^{+\infty} f(z)\, dz$$

which means that small creditors always renegotiate, i.e., $x^* \to -\infty$. Expression (4.2) defines the default point:

$$\theta^* \to \overline{K} + \alpha B + \alpha b$$

and the switching point of the large creditor is derived from (4.5):

$$y^* \to \overline{K} + \alpha B + \alpha b - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right).$$

These results are summarised in the following proposition[17]:

**Proposition 14.** *When $\frac{\sigma}{\tau} \to \infty$, there is a unique trigger equilibrium where small creditors always agree to renegotiate and the level of fundamentals that implies default by the firm and indifference between accepting and rejecting renegotiation by the large creditor are:*

$$\theta^* \to \overline{K} + \alpha B + \alpha b$$

*and*

$$y^* \to \overline{K} + \alpha B + \alpha b - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right).$$

Basically, small creditors have nothing to lose by accepting to renegotiate because if the firm is going to default, the large creditor does not renegotiate and this implies immediate liquidation. This is the case because liquidation produces the same payoff to all small creditors independently of the direction of their vote. Also, the voting

[17]Morris and Shin (2001) also prove in a similar model that this is the only equilibrium that survives iterative elimination of strictly dominated strategies.

requirement does not play any role in this case.

As the noise goes to zero, i.e. the large creditor becomes more confident about the information content of her signal, $y^* \longrightarrow \theta^*$. Also, although the precision of the large creditor's private signal, rather then her size, was fundamental to derive the small creditor's switching point, size does have an impact precisely on the trigger point of the large creditor. Given that the sum of the firm's total claims is equal to $D$, an increase in $B$ implies a reduction in $b$. Hence, $\frac{\partial \theta^*}{\partial B} = 0$ and $\frac{\partial y^*}{\partial B} = -\tau \frac{\partial F^{-1}\left(1-\frac{L}{\alpha B}\right)}{\partial\left(1-\frac{L}{\alpha B}\right)} \frac{L}{\alpha B^2}$ and is always negative: the higher the claim from the large creditor relative to small creditors, the lower her switching point and the easier it is for her to accept renegotiation.

It is interesting to compare the previous limiting case with the case where $\frac{\sigma}{\tau} \longrightarrow 0$, i.e. the case where the large creditor is less informed. This might be implausible if the large creditor is a bank but can be the case if the large creditor is instead a pool of uniformly uninformed creditors. When taking the limit, because the probability that the large creditor renegotiates is constant, (4.7) becomes:

$$\int_{\overline{\delta}}^{+\infty} f(z)\, dz\alpha = \int_{\widehat{\delta}}^{+\infty} f(z)\, dz$$

or,

$$F\left(\frac{\theta^* - x^*}{\sigma}\right) = 1 - \frac{\widehat{\omega}}{\alpha}.$$

The switching point is therefore implicitly given by the following expression:

$$x^* = \theta^* - \sigma F^{-1}\left(1 - \frac{\widehat{\omega}}{\alpha}\right). \tag{4.8}$$

The expressions (4.5), (4.3) and (4.8), represent a system of three equations and three unknowns that completely characterise the equilibrium in this limiting case. The system yields a closed form solution such as:

**Proposition 15.** *When $\frac{\sigma}{\tau} \to 0$, there is a unique trigger equilibrium with:*

$$\theta^* \to \overline{K} + \alpha B + \alpha b + \frac{\widehat{\omega}(k - \alpha b)}{\alpha}$$

$$x^* \to \overline{K} + \alpha B + \alpha b + \frac{\widehat{\omega}(k - \alpha b)}{\alpha} - \sigma F^{-1}\left(1 - \frac{\widehat{\omega}}{\alpha}\right)$$

*and*

$$y^* \to \overline{K} + \alpha B + \alpha b + \frac{\widehat{\omega}(k - \alpha b)}{\alpha} - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right).$$

The result is quite different from the previous case. When small creditors decide whether to renegotiate the large creditor only plays a role through $\theta^*$. This happens because it is assumed that all different debt classes need to agree with the plan and the probability that the large creditor renegotiates is constant. Moreover, the voting requirement now plays a role.

Before the analysis of the comparative statics in this case, it is worth looking at the case where there is no voting requirement.

### 4.3.4 No Voting Requirements

This is simply the case where voting takes place but the firm does not set a majority requirement, i.e. $\widehat{\omega} = 1$. If the firm does not impose voting requirements than there is no forced liquidation unless the large creditor decides not to renegotiate[18], but on the other hand, there is no upper limit on the amount of funds that need to be replaced at a cost when the existing small creditors foreclose. The "no-renegotiation point" is

[18] The case where the large creditor is not pivotal is discussed below.

now $\underline{\theta}$, i.e. it the point such that the level of fundamentals is lower than $\overline{K}+\alpha B+\alpha b$: even if all creditors accept to exchange their claims the firm defaults. This point is lower than $\widehat{\theta}$, i.e, the threshold point that determines default when $1-\widehat{w}$ creditors agree with renegotiation. Making the same change of variable as before:

$$\underline{\delta}=\frac{\underline{\theta}-x^*}{\sigma},$$

hence the expression (4.7) is modified as follows,

$$\int_{\overline{\delta}}^{+\infty} f(z)\left(1-F\left(\frac{\sigma}{\tau}\left(\overline{\delta}-z\right)-F^{-1}\left(1-\frac{L}{\alpha B}\right)\right)\right)dz\alpha$$

$$=\int_{\underline{\delta}}^{+\infty}\left(1-F\left(\frac{\sigma}{\tau}\left(\overline{\delta}-z\right)-F^{-1}\left(1-\frac{L}{\alpha B}\right)\right)\right)dz. \tag{4.9}$$

The RHS of (4.9) is always higher than the RHS of (4.7) (because $\underline{\theta}$ is lower than $\widehat{\theta}$), hence, agreeing to renegotiate becomes more difficult. Thus, a higher $x^*$ is needed for indifference (remember the LHS of (4.9) is increasing in $x^*$). As $\frac{\sigma}{\tau}\rightarrow\infty$, $x^*\rightarrow-\infty$ as before and the equilibrium coincides with the corresponding equilibrium discussed above (Proposition 14). However, when $\frac{\sigma}{\tau}\rightarrow 0$, $x^*$ can be defined as

$$x^*=\theta^*-\sigma F^{-1}\left(1-\frac{\underline{\delta}}{\alpha}\right),$$

and

$$\theta^*=\overline{K}+\alpha B+\alpha b+\frac{k-\alpha b}{\alpha}.$$

Finally, the thresholds $y^*$ and $x^*$ become:

$$y^* = \overline{K} + \alpha B + \alpha b + \frac{k - \alpha b}{\alpha} - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right)$$

and

$$x^* = \overline{K} + \alpha B + \alpha b + \frac{k - \alpha b}{\alpha} - \sigma F^{-1}\left(1 - \frac{\delta}{\alpha}\right).$$

The comparison with the results derived in Proposition 15 is performed below.

### 4.3.5 Comparative Statics

This subsection looks at the comparative statics of the equilibrium levels, $\theta^*$, $x^*$ and $y^*$, of the simultaneous game where small creditors are better informed than the large creditor, derived in Proposition 15. Looking at the expressions for the equilibrium values, $\theta^*$ increases with the cost related parameters $\overline{K}$ and $k$. A higher $\alpha$ decreases $\theta^*$ by increasing the number of creditors that accept renegotiation but $\theta^*$ needs to increase to meet the higher claims to be paid next period to the creditors that remain on the firm. The overall effect is negative if $k$ is high enough (see the Appendix for more details). A more lenient voting requirement also means that renegotiation is more likely to succeed but at a higher cost of replacing funds. Hence, it increases the default point, although this effect is lower the lower $k$ and the higher $\alpha$ are: as $k$ decreases the cost of replacing the creditors that withdraw decreases and an increase in $\alpha$ discourages withdrawal of funds from the firm.

The equilibrium level of the switching point for the large creditor[19] changes with $\theta^*$ and from the extra term the same as in the previous limiting case can be concluded

[19]That is simply $y^* = \theta^* - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right)$.

about the role of seniority in bank debt.

The switching point $x^*$[20] is also affected by the way $\theta^*$ changes and, even taking into account the extra term, $x^*$ still increases with a more lenient voting requirement. This effect is exacerbated for higher values of $\sigma$: as creditors' opinions become more dispersed the co-ordination effect of the voting requirement becomes stronger. In the *no voting requirement* case, i.e., in the case where there is no public debt voting on the reorganisation plan, $x^*$ is always higher or equal than when a voting requirement is imposed by the firm, regardless of the relative precision of private information between the two groups of creditors. From this it can be concluded that the existence of a voting requirement does act as a co-ordination device. But some caution should be exerted in order not to set a requirement so high that makes renegotiation impossible. Setting it too low means that extra funds need to be borrowed at a higher cost.

The effect of $\alpha$ in $x^*$ and $y^*$ is still ambiguous but it can be said that the change in $x^*$ is lower than the change in $\theta^*$ (see Appendix for details).

The effect of the large creditor size in this case is also different from before (see the Appendix for more details). The default point $\theta^*$ increases with $B$. A higher claim to the large creditor is equivalent to a lower claim to each small creditor so this effect cancels out as before. But in this case there is an additional effect: a lower claim for each small creditor, means that there is less to gain from gambling which implies that the number of creditors that renegotiate decreases and the failure point increases. This increase is higher the higher $k$ is.

The effect on $x^*$ is exactly the same but the effect on $y^*$ is ambiguous. It depends on which effect dominates: the one related to $\theta^*$ or the one that is linked to the precision of information. As the precision increases (i.e., $\tau$ decreases) the effect of $\theta^*$

[20]That is defined as $x^* = \theta^* - \sigma F^{-1}\left(1 - \frac{\hat{\omega}}{\alpha}\right)$

becomes more important and $\frac{\partial y^*}{\partial B}$ increases. This effect is obvious as the higher the large creditor debt the higher the fundamentals required to repay it.

## 4.4 Discussion

### 4.4.1 Equilibrium of the Sequential Move Game

Asquith, Gertner and Scharfstein (1994) find evidence that banks are reluctant to bear the full cost of a reorganisation. James (1995) find that for firms with public debt outstanding, banks never make concessions unless public debtholders also restructure their claims. Hence, since banks rarely make unilateral concessions, how does the announcement of the large creditor decision to renegotiate affects the behaviour of small creditors?

By agreeing to renegotiate, a well-informed large creditor signals that, based on her (more precise) information, she finds the fundamentals to be strong. The result is similar to the simultaneous game and in the limit there is acceptance of the exchange offer by all the firm's small creditors independently of their private signals and of the value and terms of the exchange offer made by the firm. But there is a difference between the two cases: the result of the simultaneous depends on the fact that liquidation gives the same payoff to a small creditor that agrees with renegotiation and to a small creditor that refuses to renegotiate [21], whereas this is not necessary in the sequential case.

The large creditor claim, that can be interpreted as a bank loan, acts as a disciplining device on the firm because default allows this creditor to exercise the option

[21] It can be argued that in some circumstances these payoffs might not be the same: rejecting the renegotiation plan might mean "doing nothing" but accepting it might entail costs in terms of time/effort to attend creditors' meetings or can even include legal costs.

to force the firm into liquidation and generates information useful to all the remaining and less informed creditors. The agreement by a better informed creditor to renegotiate, even though her debt might be senior and secured and she would be able recoup her investment in case of liquidation, provides a strong signal to the remaining creditors and constitutes a strong co-ordination device. Hence,

**Proposition 16.** *When $\frac{\sigma}{\tau} \to \infty$ and the large creditor announces she agrees to renegotiate there is a unique trigger equilibrium with*

$$\widetilde{x} \to -\infty,$$

$$\widetilde{\theta} \to \overline{K} + \alpha B + \alpha b$$

*and*

$$\widetilde{y} \to \overline{K} + \alpha B + \alpha b - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right).$$

*Large creditor's refusal to renegotiate leads to liquidation.*

Proof: See Appendix.

If the large creditor is less informed than small creditors, the problem is also similar to the case of simultaneous renegotiation when $\frac{\sigma}{\tau} \to \infty$. Because small creditors are better informed, the large creditor's decision is not taken into account on the small creditor's decision to renegotiate. Therefore,

**Proposition 17.** *When $\frac{\sigma}{\tau} \to 0$, there is a unique trigger equilibrium that coincides with the one in the simultaneous game.*

Proof: See Appendix.

Obviously, if small creditors move first and are precisely informed, whereas the large creditor's private information is noisy, the large creditor mimics the result of the voting stage where small creditors follow a trigger strategy. If they are worse informed than a well-informed large creditor they choose as in the equivalent simultaneous game: all small creditors agree with the renegotiation package proposed by the firm and wait for the large and well-informed to decide on the renegotiation outcome. This happens because the large creditor is pivotal and without her approval renegotiation does not succeed.

### 4.4.2 Large Creditor not Pivotal

In reality, when a firm seeks to renegotiate its debt it is common that all different types of debt claims are affected. Thus, the firm usually proposes a renegotiation package that needs to be approved by all classes of creditors. Since bank debt (represented by the large creditor) and bonds (represented by small creditors) belong to different classes, it was assumed throughout this chapter that the large creditor was pivotal in renegotiation. When this assumption is relaxed and the large creditor is not pivotal for the success of the reorganisation game but only a critical majority of all creditors is needed, her role as a coordination device is obviously weakened. Consider the case where the large creditor is perfectly informed. In subsection 4.3.3. it was derived that small creditors would always agree to renegotiate and let the large creditor determine the outcome of the renegotiation game. However, now if all small creditors agree but the large creditor refuses to renegotiation, renegotiation is successful and the firm continues to period 1. Before small creditors ignored their private information and relied on the large creditor but they cannot afford to do so if the large creditor is not pivotal. Since small creditors take into account their private information, creditors with low private signals are not going to participate in renegotiation and only creditors that are optimistic enough about the firm are going to agree with the renegotiation package. Hence, the switching point $x^*$ is no longer $-\infty$ but is instead a positive finite value the large creditor participates in reorganisation for a high enough level of the private signal.

The simultaneous game where small creditors are better informed is also going to change. In subsection 4.3.3 small creditors decided whether to renegotiate based on their private information but taking into account that the large creditor would also have to agree for renegotiation to succeed. If the large creditor is not pivotal, the switching point is going to increase because small creditors need to bear in mind that if the large creditors does not participate in renegotiation, extra (and more expensive) funds need to be raised in case renegotiation succeeds[22]. Thus, the firm needs to generate a higher outcome to avoid default in the last period of the game. Consequently, small creditors renegotiate for higher levels of the private signals about the project's outcome.

In the sequential game, the less informed class always mimics the more informed one, if the latter moves first and whether or not each class is pivotal for renegotiation. If the class that moves first is uninformed, for example, if small creditors are uninformed and move first, they can no longer rely on the large creditor to determine the outcome of renegotiation. Hence, they start taking into account their private signals in their decision to renegotiate.

[22]All funds that are withdrawn from the firm need to be replaced, including the funds withdrawn by the large creditor

## 4.5 Concluding Remarks

This chapter studies the process of renegotiation between a firm and its claimants. More specifically, it focuses on the role of voting requirements that are widely attached to public debt restructurings and on the impact of a large creditor, for example a bank, in the restructuring of public debt. It also contrasts the outcome of reorganisation when the voting is simultaneous with what happens when the large creditor's decision is announced in advance.

Both the voting requirement and the relative precision of private information of the small and large creditors are found to affect the small creditors decision to renegotiate. The key results can be explored by looking at limiting cases. Hence, when renegotiation takes place simultaneously and the large creditor is arbitrarily more informed than the remaining creditors and a.s. perfectly informed, small creditors always agree to renegotiate. This happens because renegotiation is successful if all debt classes agree with the renegotiation plan proposed by the firm. Thus, as liquidation provides small creditors with the same payoff independently of how they voted, they have nothing to lose by accepting to renegotiate as they believe the large creditor will not allow the firm to continue for it to default in the last period: they believe default will not occur if renegotiation succeeds. The voting requirement does not play a role in this case, however as small creditors become more informed relative to the large creditor, the more important the voting requirement is as a co-ordination device. In the opposite limiting case, a more lenient voting requirement is translated into an higher expected number of creditors foreclosing their claims which forces the firm to replace their claims by new and more expensive funds. However, the existence of voting requirements always facilitates renegotiation relative to the case where they are absent from the renegotiation process.

When the large creditor is able to signal her position by announcing that she agrees (or does not agree) to renegotiate before small creditors have voted on the renegotiation plan, the results from the limiting cases are very similar but more general than in the previous case. Before small creditors always agreed to renegotiate, and now they mimic the large creditor's behaviour. Also in the previous case, perfect co-ordination requires the payoff from liquidation to be the same regardless of how a small creditor votes and in this case such a condition is not required.

These results suggest that the mix of private and public debt is an important determinant of a distressed firm's ability to restructure out-of-court but it shows that a capital structure with multiple investors and different debt classes does not necessarily make renegotiation more difficult than with only one type of claim and provides an rationale for a diversified capital structure based on the *informational role* that some creditors might have in case of financial distress.

From a technical point of view, the results derived here differ significantly from the existing literature on global games with large and small players, as perfect co-ordination of small players is usually closely related to the signalling role of the large player and in this model it happens even when the large player's actions are not publicly known in advance.

# Chapter 5

# Conclusion

Firms and investors are in general asymmetrically informed. The role of financial certification intermediaries, such as audit firms and rating agencies, is to fill the informational gap between these two groups of agents. However, these intermediaries have their own objectives (they are profit maximisers or have reputational concerns, for example) which results in a misalignment of interests between them and the users of the information they collect and provide.

This thesis assesses to what extent their private interests interfere with their objective to mitigate asymmetries of information. This is particularly important in light of recent financial scandals and regulatory developments. For example, there are proposals to use ratings for regulatory purposes: the Basel Committee on Banking Supervision intends to see borrowers' credit ratings included in assessments of the adequacy of bank's capital. On the other hand, the Sarbanes-Oxley Act introduced a number of the reforms to auditing procedures and established, among other things, that partners of accounting companies supervising the external audit have to rotate regularly.

Chapter two concludes that when an intermediary is concerned about reputation and there is asymmetric information about her ability, the intermediary may ignore private information and simply confirm investors' opinions about a certain firm. However, and bearing in mind that there is an on-going debate about how competitive the rating industry should be, incentives to conform with public information are mitigated by competition but with "too much" competition intermediaries end up being always too lenient in their assessments.

And according to chapter three, the length of auditor tenure and the existence of private benefits do affect incentives to gather and disclose information. But on the other hand, longer tenures improve the activity of information collection and consequently auditor rotation might be beneficial in some circumstances or for some industries and not for others. Once more the Enron case provides an example for this: Enron expanded its activities from energy production to energy derivatives trading, but the task to audit the former is far easier than the more complex and specialised audits needed for a firm that does the latter.

In contrast to the previous two chapters, chapter four looks at asymmetric information as a way to achieve a desirable outcome. In a situation where bondholders are voting for the reorganisation of a viable firm that will otherwise be liquidated, co-ordination problems might arise. But the existence of a large and more informed creditor can facilitate the reorganisation of public debt even if her decision on whether to renegotiate is not known in advance by the remaining creditors. Voting requirements can also act as co-ordination devices.

There are obviously many questions left unanswered. This thesis has not looked at the role of managers although it was assumed that managers were also interested in pursuing their own interests. One interesting project would be to explore the impact of recent measures that make managers liable for the accuracy of financial reports and to what extent this interferes with the quality of the work performed by certification intermediaries. Or for example, to look at the impact of auditing standards in earnings manipulation by managers and to how important are other variables such as competitiveness of a firm's industry or managers career concerns.

Finally, it would also be interesting to assess whether the new regulatory regime that establishes mandatory rotation of auditors is going to restrain competition among auditors and if it will end up strengthening the oligopoly structure for audit services.

These are possible avenues for future research.

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# Appendix A

# Auxiliary Calculations for Chapter 2

## A.1 Proof of Proposition 2

Similar steps to the ones used in Boot, Milbourn and Thakor (2002) are used to solve for the equilibrium.

### A.1.1 The Equilibrium Behaviour of the Untalented Intermediary

Define $\tau \in \{T_G, T_B, U_G, U_B\}$ as the set of possible types, where T and U indicate talented or untalented, and G and B designate the signal received, e.g. $T_G$ is a talented certification intermediary that received a good signal. The set of possible actions is binary: send a favourable report ($m_G$) or send an unfavourable report ($m_B$). Types $U_G$ and/or $U_B$ may randomise across these two actions depending on the value of the prior $\theta$, but $\mathrm{T}_G$ and $\mathrm{T}_B$ prefer to follow a pure strategy where they always report their private signals. This is proved by identifying the mixed strategy for high and low values of $\theta$ and by proving that the $\theta$ ranges do not overlap.

**(i) Type $\mathrm{U}_B$ randomises for high values of $\theta$ :** Let $\mathrm{U}_B$ send $\mathrm{m}_G$ with probability $\overline{\gamma}$ and $\mathrm{m}_B$ with probability $1-\overline{\gamma}$ and assume that the remaining types follow their conjectured equilibrium strategies. The following equation should therefore hold as $\mathrm{U}_B$ should be indifferent between sending $\mathrm{m}_B$ and $\mathrm{m}_G$

$$\Pr\left(G \mid s_B, U\right) F_1\left(\overline{\alpha}_{GG}\right) + \Pr\left(B \mid s_B, U\right) F_1\left(\overline{\alpha}_{GB}\right) = F_1\left(\overline{\alpha}_B\right) + c_0. \tag{A.1}$$

The expression becomes clearer by replacing (2.4), (2.7) and the different values for $\alpha_1$, that are $\overline{\alpha}_{GG}$, $\overline{\alpha}_{GB}$ and $\overline{\alpha}_B$ and whose expressions are derived above, in (A.1). It is easily shown that the LHS of (A.1) is monotonically decreasing in $\overline{\gamma}$, while the RHS is monotonically increasing in $\overline{\gamma}$. Moreover, it can be showed that the equality in (A.1) can only hold for a interior $\overline{\gamma} \in (0,1)$ provided that $\theta$ is sufficiently high.

Firstly, observe that at $\overline{\gamma} = 0$ the LHS exceeds the RHS provided that $\theta$ is sufficiently high. After straightforward manipulation the expression becomes

$$\begin{aligned} &\kappa \frac{\varepsilon\theta}{\varepsilon\theta + (1-\varepsilon)(1-\theta)} \frac{\alpha_0 \varepsilon}{2\theta\left(\alpha_0 + (1-\varepsilon)(1-\alpha_0)\right)} - c_0 \\ &= \kappa \frac{\alpha_0 (1-\theta)\varepsilon}{2\theta\left(\alpha_0(1-\theta) + (1-\alpha_0)\left(\varepsilon\theta + (1-\varepsilon)(1-\theta)\right)\right)}. \end{aligned}$$

In order for the LHS to exceed the RHS it is necessary that

$$(1-\theta)\left(\varepsilon\theta + (1-\varepsilon)(1-\theta)\right)\left(\alpha_0 + (1-\varepsilon)(1-\alpha_0)\right)$$

$$< \varepsilon\theta \left(\alpha_0 (1-\theta) + (1-\alpha_0)(\varepsilon\theta + (1-\varepsilon)(1-\theta))\right),$$

and the expression can be simplified as follows

$$(1-2\theta)(1-\varepsilon)(\alpha_0 + (1-\varepsilon)(1-\alpha_0)) < \theta^2 \left((1-\alpha_0)(2\varepsilon - 1) - (1-\varepsilon)\alpha_0\right). \quad \text{(A.2)}$$

Secondly, it can be proven that $\theta$ needs to be higher than $\frac{1}{2}$ and more precisely higher than $1-\varepsilon$. Noting that as $\varepsilon < \frac{1}{2}$ the RHS of (A.2) is always negative, $1-2\theta$ needs to be negative to transform the LHS in a negative number and $\theta$ needs to be high enough for the inequality to occur. But if $\theta$ equals $1-\varepsilon$ the expression becomes

$$1 - 2\varepsilon > (1-\varepsilon)^2$$

and this is impossible since $\varepsilon < \frac{1}{2}$.

Consequently, equality (A.1) requires that $\overline{\gamma} > 0$ provided that

$$c_0 < \kappa \frac{\varepsilon\theta\overline{\alpha}_{GG}(\overline{\gamma}=0)}{2\theta(\varepsilon\theta + (1-\varepsilon)(1-\theta))} - \kappa \frac{\overline{\alpha_B}(\overline{\gamma}=0)}{2\theta} \equiv \overline{c}_{\max}$$

Now, evaluate (A.1) at $\overline{\gamma} = 1$. It immediately follows that, independently of $c_0$, the LHS of (A.1) is always smaller than the RHS as the expression simplifies to:

$$\kappa \left(\frac{\alpha_0\varepsilon}{2\theta}\right)\left(\frac{\varepsilon\theta}{\varepsilon\theta + (1-\varepsilon)(1-\theta)}\right) - c_0 = \kappa \frac{\varepsilon}{2\theta}$$

Thus, there exists $\overline{\gamma}$, with $0 < \overline{\gamma} < 1$. Finally, the posterior beliefs about the certification intermediary need to satisfy a technical condition that ensures $\overline{\alpha}_B < \overline{\alpha}_{GG}$ and allows for the existence of a mixed strategy no matter how arbitrarily small $c_0$ is (otherwise the RHS of (A.1) would always be higher for $0 < \overline{\gamma} < 1$). This condition states that

$$1-\varepsilon+\varepsilon\overline{\gamma}\leq\left(\varepsilon\theta+\left(1-\varepsilon\right)\left(1-\theta\right)\right)\left(1-\overline{\gamma}\right). \tag{A.3}$$

**(ii) Types $\mathrm{T}_G$, $\mathrm{T}_B$ and $\mathrm{U}_G$ recommend according to their respective signals for high values of $\theta$ :** It can be easily shown that $\mathrm{T}_B$ strictly prefers to follow her signal (i.e. send an unfavourable report) just by looking at the indifference condition (A.1) for $\mathrm{U}_B$. Since $\mathrm{pr}(G \mid s_B, T) < \mathrm{pr}(G \mid s_B, U)$ and $F_1\left(\overline{\alpha}_{GG}\right) > F_1\left(\overline{\alpha}_{GB}\right)$ (because the fee $F_1\left(\alpha_1\right)$ is increasing in $\alpha_1$), $\mathrm{T}_B$ has strictly less to gain from sending a favourable report than $\mathrm{U}_B$. The remaining types, $\mathrm{T}_G$ and $\mathrm{U}_G$, always send a favourable report as $\mathrm{pr}(G \mid s_G, T) > \mathrm{pr}(G \mid s_G, U) > \mathrm{pr}(G \mid s_B, U) > \mathrm{pr}(G \mid s_B, T)$ by looking at

$$\Pr\left(G \mid s_G, a\right) F_1\left(\overline{\alpha}_{GG}\right) + \Pr\left(B \mid s_G, a\right) F_1\left(\overline{\alpha}_{GB}\right) - F_1\left(\overline{\alpha}_B\right) - c_0$$

and realising that it always exceeds

$$\Pr\left(G \mid s_B, a\right) F_1\left(\overline{\alpha}_{GG}\right) + \Pr\left(B \mid s_B, a\right) F_1\left(\overline{\alpha}_{GB}\right) - F_1\left(\overline{\alpha}_B\right) - c_0.$$

**(iii) Type $\mathrm{U}_B$ randomises for low values of $\theta$:** For low values of $\theta$, there are two cases: $\theta > \varepsilon$ (Case 1) and $\theta < \varepsilon$ (Case 2).

**Case 1:** This proof mirrors the previous arguments. $\mathrm{U}_G$ now sends an unfavourable report with probability $\underline{\gamma}$ when $\theta$ in the interval $(0,1)$ is sufficiently low, i.e.

$$\Pr\left(G \mid s_G, U\right) F_1\left(\underline{\alpha}_{GG}\right) + \Pr\left(B \mid s_G, U\right) F_1\left(\underline{\alpha}_{GB}\right) = F_1\left(\underline{\alpha}_B\right) - c_0. \tag{A.4}$$

As before (2.4), (2.7) and $\underline{\alpha}_{GG}$, $\underline{\alpha}_{GB}$ and $\underline{\alpha}_B$ (whose expressions are derived above) are used to rewrite expression (A.4). Following arguments similar to the previous case, it is shown that $0 < \underline{\gamma} < 1$ provided that $\theta$ is sufficiently low. It can be easily demonstrated that the LHS of (A.4) is monotonically increasing in $\underline{\gamma}$, while the RHS is monotonically decreasing in $\underline{\gamma}$. It needs to be shown that the equality in (A.4) can only hold for a interior $\underline{\gamma} \in (0,1)$ provided that $\theta$ is sufficiently low.

Firstly, observe that at $\underline{\gamma} = 0$ the RHS exceeds the LHS. After straightforward manipulation the expression becomes

$$\begin{aligned}
&\kappa \frac{\alpha_0 \varepsilon}{2\theta\left(\alpha_0 + (1-\alpha_0)(1-\varepsilon)\right)} \frac{(1-\varepsilon)\theta}{(1-\varepsilon)\theta + \varepsilon(1-\theta)} \\
&= \kappa \frac{\alpha_0 (1-\theta)\varepsilon}{2\theta\left(\alpha_0(1-\theta) + (1-\alpha_0)(\varepsilon\theta + (1-\varepsilon)(1-\theta))\right)} - c_0.
\end{aligned}$$

In order for the RHS to exceed the LHS it is necessary that

$$(1-\varepsilon)\theta\left(\alpha_0(1-\theta) + (1-\alpha_0)(\varepsilon\theta + (1-\varepsilon)(1-\theta))\right)$$

$$< (1-\theta)\left((1-\varepsilon)\theta + \varepsilon(1-\theta)\right)\left(\alpha_0 + (1-\varepsilon)(1-\alpha_0)\right).$$

This expression can be simplified as

$$-\theta^2\alpha_0 < (1-2\theta)\left(\alpha_0 + (1-\varepsilon)(1-\alpha_0)\right),$$

and as $\alpha_0 + (1-\varepsilon)(1-\alpha_0) > \alpha_0$ and $-\theta^2 < 1 - 2\theta$, the inequality is always satisfied for relatively low values of $\theta$ and always if $\theta$ is lower than $\frac{1}{2}$. Moreover, if $\theta < \varepsilon$, the equality is always satisfied for any values of the remaining parameters. Consequently, equality (A.4), requires that $\underline{\gamma} > 0$ provided that

$$c_0 < \kappa \frac{(1-\varepsilon)\theta\underline{\alpha}_{GG}(\overline{\gamma} = 0)}{2\theta\left((1-\varepsilon)\theta + \varepsilon(1-\theta)\right)} - \kappa \frac{\underline{\alpha}_B(\overline{\gamma} = 0)}{2\theta} \equiv \underline{c}_{\max}$$

Now, evaluate (A.4) at $\underline{\gamma} = 1$. It follows that:

$$\kappa \frac{\varepsilon}{2\theta} \frac{(1-\varepsilon)\theta}{(1-\varepsilon)\theta + \varepsilon(1-\theta)} = \kappa \frac{\alpha_0 \varepsilon (1-\theta)}{2\theta(1-\theta\alpha_0)} - c_0 \quad .$$

The LHS exceeds the RHS, regardless of $c_0$, if:

$$\frac{(1-\varepsilon)\theta}{(1-\varepsilon)\theta + \varepsilon(1-\theta)} > \frac{\alpha_0(1-\theta)}{(1-\theta\alpha_0)} \quad .$$

Given that the LHS is increasing in $\theta$, the RHS is decreasing in $\theta$ and when $\theta \longrightarrow 1$ the LHS exceeds the RHS and otherwise when $\theta \longrightarrow 0$, for a given $\varepsilon$ and $\alpha_0$ there exists a $\underline{\theta}$ lower than 1 such that the for $\theta > \underline{\theta}$ the LHS exceeds the RHS and otherwise for $\theta < \underline{\theta}$. For example if $\theta = \frac{1}{2}$, the relationship holds for $\alpha_0 < \frac{2(1-\varepsilon)}{(2-\varepsilon)}$.

Thus, if $\theta$ is low enough there exists $\underline{\gamma}$ such that $0 < \underline{\gamma} < 1$, provided that $c_0 < \underline{c}_{\max}$. Finally, the posterior beliefs about the certification intermediary needs to satisfy a technical condition that ensures $\underline{\alpha}_B < \underline{\alpha}_{GG}$ and allows for the existence of a mixed strategy no matter how arbitrarily small $c_0$. This condition states that

$$(1-\varepsilon)(1-\theta)\left(1-\underline{\gamma}\right) \leq ((1-\varepsilon)\theta + \varepsilon(1-\theta))\underline{\gamma} + (\varepsilon\theta + (1-\varepsilon)(1-\theta)) \qquad \text{(A.5)}$$

and is always satisfied for any values of the parameters.

**Case 2:** When $\theta < \varepsilon$ the untalented certification intermediary is not hired in period 1 if her type is revealed at the end of period 0 or her fee simply equals zero (this does not happen for high values of $\theta$ because it was shown before that "high values of $\theta$" means higher than $\frac{1}{2}$). $U_G$ now recommends rejection with probability $\underline{\underline{\gamma}}$, and this is in the interior of $(0, 1)$ if $\theta$ is sufficiently low. Firstly, observe that at $\underline{\underline{\gamma}} = 0$ the RHS exceeds the LHS provided that $\theta$ is sufficiently low. After straightforward manipulation the expression becomes

$$\kappa\left(\frac{\theta-\varepsilon}{2\theta}+\frac{\alpha_0\varepsilon}{2\theta\left(\alpha_0+(1-\alpha_0)(1-\varepsilon)\right)}\right)\left(\frac{(1-\varepsilon)\theta}{(1-\varepsilon)\theta+\varepsilon(1-\theta)}\right)$$
$$=\kappa\frac{\theta-\varepsilon}{2\theta}+\kappa\frac{\alpha_0\varepsilon(1-\theta)}{2\theta\left(\alpha_0(1-\theta)+(1-\alpha_0)\left((\varepsilon\theta+(1-\varepsilon)(1-\theta)\right)\right))}-c_0$$

In order for the LHS to exceed the RHS it is necessary that

$$(\theta-\varepsilon)\varepsilon(1-\theta)\left(\alpha_0+(1-\alpha_0)(1-\varepsilon)\right)\left(\alpha_0(1-\theta)+(1-\alpha_0)\left((\varepsilon\theta+(1-\varepsilon)(1-\theta)\right)\right))$$

$$<(1-2\theta)\left(\alpha_0+(1-\varepsilon)(1-\alpha_0)\right)+\theta^2\alpha_0,$$

The relationship holds because $\theta<\varepsilon$ and therefore $\theta<\frac{1}{2}$. Now, evaluate (A.4) at $\underline{\underline{\gamma}}=1$. It follows that:

$$\kappa\frac{\varepsilon}{2\theta}\frac{(1-\varepsilon)\theta}{(1-\varepsilon)\theta+\varepsilon(1-\theta)}-\kappa\frac{(\theta-\varepsilon)\varepsilon(1-\theta)}{(1-\varepsilon)\theta+\varepsilon(1-\theta)}=\kappa\frac{\alpha_0\varepsilon(1-\theta)}{2\theta(1-\theta\alpha_0)}-c_0$$

As the LHS is now higher than before, the conditions derived in the previous case also apply and $\theta$ can be even lower. Thus, if $\theta$ is low there exists $\underline{\underline{\gamma}}$, with $0<\underline{\underline{\gamma}}<1$, provided that $c_0<\underline{\underline{c}}_{\max}$.

**(iv) Types $\mathrm{T}_G$, $\mathrm{T}_B$ and $\mathrm{U}_B$ follow their respective signals for low values of $\theta$ :** Given the equality for $\mathrm{U}_G$ in (A.4), $\mathrm{T}_G$ strictly prefers to send a favourable report as $\mathrm{pr}(G\mid s_G,T)>\mathrm{pr}(G\mid s_G,U)$ and $F_1(\underline{\alpha}_{GG})>F_1(\underline{\alpha}_{GB})$. Similarly, $\mathrm{T}_B$ and $\mathrm{U}_B$ always send an unfavourable report because $\mathrm{pr}(G\mid s_G,U)>\mathrm{pr}(G\mid s_B,U)>\mathrm{pr}(G\mid s_B,T))$.

### A.1.2 Establishing the Distinct $\theta$ Ranges (and Proof of Proposition 3)

Defining $\theta = \theta_H$ as the value of $\theta$ for which (A.1) holds for $\overline{\gamma} = 0$ and $\theta = \theta_L$ as the value of $\theta$ for which (A.4) holds for $\underline{\gamma}= 0$, it can be demonstrated that $\frac{\partial \underline{\gamma}}{\partial \theta} < 0$ and $\frac{\partial \overline{\gamma}}{\partial \theta} > 0$. Taking the expressions $\overline{\pi^e}$ and $\underline{\pi^e}$ it can be shown that $-\frac{\frac{\partial \overline{\pi^e}}{\partial \theta}}{\frac{\partial \overline{\pi^e}}{\partial \overline{\gamma}}} > 0$. Simple algebra shows that $\frac{\partial \overline{\pi^e}}{\partial \theta}$ is positive and that $\frac{\partial \overline{\pi^e}}{\partial \overline{\gamma}}$ is negative. Computing $-\frac{\frac{\partial \underline{\pi}^e}{\partial \theta}}{\frac{\partial \underline{\pi}^e}{\partial \underline{\gamma}}}$ follows the same logic and is derived to be negative; both $\frac{\partial \underline{\pi}^e}{\partial \underline{\gamma}}$ and $\frac{\partial \underline{\pi}^e}{\partial \theta}$ are positive.

From (A.1) and (A.4) as $\theta \longrightarrow 1$, $\overline{\gamma} = 1$, and on the other hand, as $\theta \longrightarrow 0$, $\underline{\gamma}= 1$, for $c_0$ sufficiently low. Thus, when $\theta \in (\theta_H, 1)$, there is excessive favourable reports ($\overline{\gamma} > 0$) and when $\theta \in (0, \theta_L)$, there is excessive unfavourable reports ($\underline{\gamma}> 0$). It remains to be shown that $\theta_L < \theta_H$. Only then can be stated that there is a region $[\theta_L, \theta_H]$ where there is no deviation from the private signal by the untalented certification intermediary. The equality (A.4) evaluated at $\theta = \theta_L$ (or $\underline{\gamma}= 0$) is identical to (A.1) when this last equality is evaluated at $\theta = \theta_H$ (or $\overline{\gamma} = 0$), except for the probabilities $\mathrm{pr}(G \mid s_G, U)$ and $\mathrm{pr}(G \mid s_B, U)$. Since for a given $\theta$ $\mathrm{pr}(G \mid s_G, U) > \mathrm{pr}(G \mid s_B, U)$ and these probabilities are increasing in $\theta$, the equalities (A.4) and (A.1)) require $\theta_L$ to be lower than $\theta_H$ for $c_0$ sufficiently small.

## A.2 Proof of Proposition 4

The proof is done by implicit differentiation. Starting with $\frac{\partial \overline{\gamma}}{\partial \varepsilon}$, straightforward differentiation, using (A.1) and the fact that $\theta_H$ is always higher than $\frac{1}{2}$, it can be shown that $\frac{\partial \overline{\pi^e}}{\partial \varepsilon}$ is always positive. As $\frac{\partial \overline{\pi^e}}{\partial \overline{\gamma}}$ is always negative $\frac{\partial \overline{\gamma}}{\partial \varepsilon} = -\frac{\frac{\partial \overline{\pi^e}}{\partial \varepsilon}}{\frac{\partial \overline{\pi^e}}{\partial \overline{\gamma}}}$ is positive. Turning to $\frac{\partial \underline{\gamma}}{\partial \varepsilon}$, it can be proven by simple algebra that $\frac{\partial \underline{\pi}^e}{\partial \varepsilon}$ is negative if $\theta \leq \frac{1}{2}$. Otherwise, we need to use (A.5) in the proof; $\frac{\partial \underline{\pi}^e}{\partial \underline{\gamma}}$ is positive thus $\frac{\partial \underline{\gamma}}{\partial \varepsilon} = -\frac{\frac{\partial \underline{\pi}^e}{\partial \varepsilon}}{\frac{\partial \underline{\pi}^e}{\partial \underline{\gamma}}}$ is positive.

## A.3 Proof of Propositions 5 and 6

The way the equilibrium values for $\overline{\gamma}$ and $\underline{\gamma}$ vary with $\alpha_0$ is determined as follows. Starting with $\overline{\gamma}$, $\frac{\partial \overline{\pi}^e}{\partial \alpha_0}$ is found to be negative if $c_0$ is sufficiently small and making use of condition (A.3) in the derivation. The second derivative $\frac{\partial^2 \overline{\pi}^e}{\partial \alpha_0^2}$ is always negative. On the other hand, $\frac{\partial \overline{\pi}^e}{\partial \overline{\gamma}}$ is always negative hence, $\frac{\partial \overline{\gamma}}{d\alpha}$ is decreasing and it can also be proven to be concave in $\alpha_0$. Looking at $\underline{\gamma}$, $\frac{\partial \underline{\pi}^e}{\partial \alpha_0}$ is found to be negative. This can be proven by straightforward derivation, summing and subtracting $\left(\frac{(1-\varepsilon)\theta}{(1-\varepsilon)\theta+(1-\theta)\varepsilon}\right)\left(\frac{\varepsilon \underline{\alpha}_{GG}}{2\theta}\right)\left(\frac{(1-\theta-(((1-\varepsilon)\theta+\varepsilon(1-\theta))\gamma_B+(\varepsilon\theta+(1-\varepsilon)(1-\theta))))}{(\alpha_0(1-\theta)+(1-\alpha_0)(((1-\varepsilon)\theta+\varepsilon(1-\theta))\gamma_B+(\varepsilon\theta+(1-\varepsilon)(1-\theta))))}\right)$ and making use of equilibrium condition (A.4) and of technical condition (A.5). The second derivative $\frac{\partial^2 \underline{\pi}^e}{\partial \alpha_0^2}$ is negative and the higher the $c_0$, the steeper is the slope. As $\frac{\partial \underline{\pi}^e}{\partial \underline{\gamma}}$ is always positive, $\frac{\partial \underline{\gamma}}{\partial \alpha_0}$ is respectively increasing and convex in $\alpha_0$.

## A.4 Proof of Propositions 10 and 11

In order to prove how the set of prior beliefs for which there is deviation from the private signal changes there needs to be a comparison between the expected profits functions with and without competition. With competition intermediaries decide whether to announce their private signals by looking at

$$\underline{\pi}^{ei}(U) = \Pr(G \mid s_G, U)\, F_{i1}\left(\underline{\alpha}^i{}_{GG}, \alpha_{j0}\right) + \Pr(B \mid s_G, U)\, F_{i1}\left(\underline{\alpha}^i{}_{GB}, \alpha_{j0}\right) - F_{i1}\left(\underline{\alpha}^i{}_B, \alpha_{j0}\right) + c_0$$

and

$$\overline{\pi^{ei}}(U) = \Pr(G \mid s_B, U) F_{i1}\left(\overline{\alpha}^i_{GG}, \alpha_{j0}\right) + \Pr(B \mid s_B, U) F_{i1}\left(\overline{\alpha}^i_{GB}, \alpha_{j0}\right) - F_{i1}\left(\overline{\alpha}^i_B, \alpha_{j0}\right) - c_0.$$

The fee in period 1 for the case without competition is $F_1(\alpha_1) = \kappa\left(\frac{\alpha_1 \varepsilon}{2\theta} + \frac{(\theta-\varepsilon)}{2\theta}\right)$.

**(i) Type $\mathbf{U}_B$ randomises for $\theta > \theta^H$ in the monopolistic case**

The different fees with competition are $F_{i1}\left(\overline{\alpha}^i_{GB}, \alpha_{j0}\right) = 0$,

$$F_{i1}\left(\overline{\alpha}^i_{GG}, \alpha_{j0}\right) = F_1\left(\overline{\alpha}_{GG}\right) - k\left(\frac{\alpha_{j0}\varepsilon}{2\theta} + \frac{(\theta-\varepsilon)}{2\theta}\right)$$

and

$$F_{i1}\left(\overline{\alpha}^i_B, \alpha_{j0}\right) = F_1\left(\overline{\alpha}_B\right) - k\left(\frac{\alpha_{j0}\varepsilon}{2\theta} + \frac{(\theta-\varepsilon)}{2\theta}\right)$$

in case $\overline{\alpha}^i_B > \alpha_{j0}$ and zero otherwise. The remaining probabilities remain the same with and without competition. Consequently, when $\overline{\alpha}^i_B > \alpha_{j0}$

$$\overline{\pi}^{ei}(U) - \overline{\pi}^{e}(U) = k\left(\frac{\alpha_{j0}\varepsilon}{2\theta}\right)\left(\Pr(B \mid s_B, U)\right) > 0.$$

Therefore, when for a given $\theta$ there is an equilibrium in mixed strategies for the monopolistic case, a favourable report is issued with competition. Because $\overline{\pi}^{ei}(U)$ is increasing in $\theta$, an equilibrium in mixed strategies occurs for a lower $\theta$.

If $\overline{\alpha}^i_B < \alpha_{j0}$, $\overline{\pi^{ei}}(U) = \Pr(G \mid s_{iB}, U) F_{i1}\left(\overline{\alpha}^i_{GG}, \alpha_{j0}\right) - c_0$. For an arbitrarily small $c_0$, $\overline{\pi^{ei}}(U)$ is also positive.

**(ii) Type $\mathbf{U}_B$ randomises for $\theta < \theta^L$ in the monopolistic case**

In this case, the different fees with competition become $F_{i1}\left(\underline{\alpha}^{i}{}_{GB}, \alpha_{j0}\right) = 0$,

$$F_{i1}\left(\underline{\alpha}^{i}{}_{GG}, \alpha_{j0}\right) = F_1\left(\underline{\alpha}_{GG}\right) - k\left(\frac{\alpha_{j0}\varepsilon}{2\theta} + \frac{(\theta - \varepsilon)}{2\theta}\right)$$

and

$$F_{i1}\left(\underline{\alpha}^{i}{}_{B}, \alpha_{j0}\right) = F_1\left(\underline{\alpha}_{B}\right) - k\left(\frac{\alpha_{j0}\varepsilon}{2\theta} + \frac{(\theta - \varepsilon)}{2\theta}\right)$$

in case $\underline{\alpha}^{i}{}_{B} > \alpha_{j0}$ and zero otherwise. The remaining probabilities remain the same with and without competition. A similar argument is applied here. When $\underline{\alpha}^{i}{}_{B} > \alpha_{j0}$

$$\underline{\pi}^{ei}(U) - \underline{\pi}^{e}(U) = k\left(\frac{\alpha_{j0}\varepsilon}{2\theta}\right)\left(\Pr\left(B \mid s_G, U\right)\right) > 0.$$

Hence, given $\theta$ if $\underline{\pi}^{e}(U) = 0$ then $\underline{\pi}^{ei}(U) > 0$. Because $\underline{\pi}^{ei}(U)$ is increasing in $\theta$, a mixed strategy with competition occurs for a lower $\theta$.

If $\underline{\alpha}^{i}{}_{B} < \alpha_{j0}$, $F_{i1}\left(\underline{\alpha}^{i}{}_{B}, \alpha_{j1}\right) = 0$ which means that

$$\underline{\pi}^{ei}(U) = \Pr\left(G \mid s_G, U\right) F_{i1}\left(\underline{\alpha}^{i}{}_{GG}, \alpha_{j0}\right) + c_0 > 0$$

meaning that a good report is always sent.

# Appendix B

# Auxiliary Calculations for Chapter 3

## B.1 Proof of Proposition 12

To implement the first-best decisions at minimum cost (or expected transfer), shareholders solve:

$$\min_{\{t(.)'s\}} p(\overline{\sigma})\left(p\left(\overline{S}_1 \mid \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1\right) + p\left(\underline{S}_1 \mid \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1\right)\right) + p(\underline{\sigma}) t\left(\underline{\sigma}, \underline{S}_1\right)$$

$$+p\left(\underline{\sigma} \mid \overline{\sigma}\right) t\left(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right) + p(\overline{\sigma} \mid \overline{\sigma})\left(p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right) + p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right)\right)$$

subject to (3.7), (3.8), (3.11), (3.12) and all $t(.) \geq 0$.

It follows immediately that $t\left(\underline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right)$ and $t\left(\underline{\sigma}, \overline{S}_1\right)$ should be set to zero as this relaxes constraints (3.8) and (3.12) without affecting the expected transfer. Intuitively this makes sense as it is obvious that fraud has been committed by the auditor.

Obviously, in this case the auditor should be penalised but because of limited liability the minimum transfer is in fact zero. In addition, the optimal $t\left(\overline{\sigma},\underline{S}_1\mid\overline{\sigma}\right)$ and $t\left(\overline{\sigma},\underline{S}_1\right)$ are also zero as (3.7) and (3.11) are relaxed keeping the expected transfer constant, i.e., increasing $t\left(\overline{\sigma},\overline{S}_1\mid\overline{\sigma}\right)$ and $t\left(\overline{\sigma},\overline{S}_1\right)$ respectively.

The constraints become

$$p\left(\underline{\sigma}\mid\overline{\sigma}\right)t\left(\underline{\sigma},\underline{S}_1\mid\overline{\sigma}\right)-\psi\geq p\left(\underline{\sigma}\mid\overline{\sigma}\right)\frac{B}{2}$$

$$p(\overline{\sigma}\mid\overline{\sigma})p\left(\overline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\overline{S}_1\mid\overline{\sigma}\right)-\psi\geq\left(p\left(\underline{S}_1\mid\overline{\sigma}\right)-p(\underline{\sigma}\mid\overline{\sigma})\right)t\left(\underline{\sigma},\underline{S}_1\mid\overline{\sigma}\right)-p(\overline{\sigma}\mid\overline{\sigma})\frac{B}{2}$$

$$p(\underline{\sigma})t\left(\underline{\sigma},\underline{S}_1\right)-\psi\geq p\left(\overline{\sigma}\right)p\left(\underline{\sigma}\mid\overline{\sigma}\right)\frac{B}{2}$$

$$p(\overline{\sigma})p\left(\overline{S}_1\mid\overline{\sigma}\right)t\left(\overline{\sigma},\overline{S}_1\right)\geq\left(p\left(\underline{S}_1\right)-p(\underline{\sigma})\right)t\left(\underline{\sigma},\underline{S}_1\right)+\psi-\frac{B}{2}p(\overline{\sigma})p\left(\overline{\sigma}\mid\overline{\sigma}\right).$$

The constraints define two cones in the positive quadrant of spaces $\left(t\left(\overline{\sigma},\overline{S}_1\right),t\left(\underline{\sigma},\underline{S}_1\right)\right)$ and $\left(t\left(\overline{\sigma},\overline{S}_1\mid\overline{\sigma}\right),t\left(\underline{\sigma},\underline{S}_1\mid\overline{\sigma}\right)\right)$, and the optimum is reached when all constraints are binding.

The optimum transfers are derived to be: $t\left(\underline{\sigma},\underline{S}_1\mid\overline{\sigma}\right)=\frac{\psi}{p(\underline{\sigma}\mid\overline{\sigma})}+\frac{B}{2}$, $t\left(\overline{\sigma},\overline{S}_1\mid\overline{\sigma}\right)=\frac{\psi p(\overline{\sigma})}{\theta v}-\frac{B}{2}$, $t\left(\underline{\sigma},\underline{S}_1\right)=\frac{\psi}{p(\underline{\sigma})}+\frac{(1-\theta)B}{2}$ and $t\left(\overline{\sigma},\overline{S}_1\right)=\frac{\psi}{\theta v}-\frac{B}{2}$.

Because of limited liability all transfers need to be positive. If $B<\frac{2p(\overline{\sigma})\psi}{\theta v}$, i.e. the private benefit is not too high relative to the cost of effort, all transfers are indeed positive. If $B=\frac{2p(\overline{\sigma})\psi}{\theta v}$ then $t\left(\overline{\sigma},\overline{S}_1\mid\overline{\sigma}\right)$ equals zero but when $\frac{2p(\overline{\sigma})\psi}{\theta v}<B<\frac{2\psi}{\theta v}$, without limited liability, $t\left(\overline{\sigma},\overline{S}_1\mid\overline{\sigma}\right)$ is negative and all constraints are binding. With limited liability, $t\left(\overline{\sigma},\overline{S}_1\mid\overline{\sigma}\right)$ needs to increase to zero. As it increases, the left-hand-side of constraint (3.8) increases, meaning that there is no need to vary the right-hand-side as the constraint is still satisfied but no longer binding. Decreasing $t\left(\underline{\sigma},\underline{S}_1\mid\overline{\sigma}\right)$

would mean that (3.8) could be binding again but (3.7) would be violated. If $B > \frac{2\psi}{\theta v}$, $t\left(\overline{\sigma}, \overline{S}_1\right)$ also becomes negative. Using the same reasoning as before, $t\left(\overline{\sigma}, \overline{S}_1\right)$ increases to zero and $t\left(\underline{\sigma}, \underline{S}_1\right)$ remains unchanged.

## B.2 Proof of Proposition 13

The optimal contract is the solution to the problem:

$$\min_{\{t(.)'s\}} p(\overline{\sigma})p\left(\overline{\sigma}, \overline{\sigma}\right)\left(p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{\sigma}, \overline{S}_1\right) + p\left(\underline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{\sigma}, \underline{S}_1\right)\right)$$

$$+p\left(\underline{\sigma}, \overline{\sigma}\right) t\left(\underline{\sigma}, \underline{S}_1, \overline{\sigma}\right) + p(\underline{\sigma})\widetilde{t}\left(\underline{\sigma}, \underline{S}_1\right)$$

subject to (3.16), (3.17), (3.21), (3.22), (3.23), (3.19) and all $t(.) \geq 0$

This problem is solved as before. It follows immediately that $t\left(\underline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$ and $\widetilde{t}\left(\underline{\sigma}, \overline{S}_1\right)$ should be set to zero as this relaxes the constraints without affecting the expected transfer. By setting $t\left(\overline{\sigma}, \underline{S}_1, \overline{\sigma}\right)$ equal to zero, due to limited liability, constraints (3.16), (3.21) and (3.23) are relaxed. As far as the the remaining relevant constraints are concerned one can keep the expected transfer constant by decreasing $t\left(\overline{\sigma}, \underline{S}_1, \overline{\sigma}\right)$ and increasing $t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$ without affecting the remaining constraints. The relevant constraints can then be simplified to:

$$t\left(\underline{\sigma}, \underline{S}_1, \overline{\sigma}\right) \geq \frac{\widetilde{\psi}}{p(\underline{\sigma} \mid \overline{\sigma})} + B,$$

$$p(\overline{\sigma} \mid \overline{\sigma})\left[p\left(\overline{S}_1 \mid \overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right) + B\right] - \widetilde{\psi} \geq \left(p\left(\underline{S}_1 \mid \overline{\sigma}\right) - p(\underline{\sigma} \mid \overline{\sigma})\right) t\left(\underline{\sigma}, \underline{S}_1, \overline{\sigma}\right),$$

$$p\left(\underline{\sigma}\right) \widetilde{t}\left(\underline{\sigma}, \underline{S}_1\right) - p(\overline{\sigma})\widetilde{\psi} - Bp(\overline{\sigma})p\left(\underline{\sigma} \mid \overline{\sigma}\right) \geq \left(p(\underline{\sigma}) - p(\overline{\sigma})p\left(\underline{\sigma} \mid \overline{\sigma}\right)\right) t\left(\underline{\sigma}, \underline{S}_1, \overline{\sigma}\right),$$

$$p(\overline{\sigma})p\left(\overline{\sigma}\mid\overline{\sigma}\right)\left(p\left(\overline{S}_1\mid\overline{\sigma},\overline{\sigma}\right)t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)+B\right)$$

$$+p(\overline{\sigma})p\left(\underline{\sigma}\mid\overline{\sigma}\right)t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)-p(\overline{\sigma})\widetilde{\psi}-\psi\geq\left(p\left(\underline{S}_1\right)-p\left(\underline{\sigma}\right)\right)\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right),$$

$$p(\overline{\sigma})p\left(\underline{\sigma}\mid\overline{\sigma}\right)t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)+p\left(\underline{\sigma}\right)\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)\geq\psi+p(\overline{\sigma})\widetilde{\psi}+B\left(1-p(\overline{\sigma})p\left(\overline{\sigma}\mid\overline{\sigma}\right)\right)$$

and

$$\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)\geq t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right).$$

From the constraints it immediately follows that $t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)$ can decrease still keeping the expected transfer constant (by increasing $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$ and $t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)$). Thus, $t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)=\frac{\widetilde{\psi}}{p(\underline{\sigma}\mid\overline{\sigma})}+B$. The constraints simplify to

$$t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)\geq\frac{\widetilde{\psi}p(\overline{\sigma})}{\theta p\left(\overline{S}_1\right)}-B,$$

$$\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)\geq\frac{\widetilde{\psi}}{p(\underline{\sigma}\mid\overline{\sigma})}+B,$$

$$t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)\geq\frac{\psi}{p\left(\overline{S}_1\right)}+\frac{\left(p\left(\underline{S}_1\right)-p\left(\underline{\sigma}\right)\right)}{p\left(\overline{S}_1\right)}\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)-\frac{p(\overline{\sigma})}{p\left(\overline{S}_1\right)}B,$$

$$\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)\geq\frac{\psi}{p\left(\underline{\sigma}\right)}+B$$

and,

$$\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)\geq\frac{\widetilde{\psi}}{p(\underline{\sigma}\mid\overline{\sigma})}+B.$$

The transfer $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$ should also be set equal to $\max\left\{\frac{\widetilde{\psi}}{p(\underline{\sigma}\mid\overline{\sigma})}+B,\frac{\psi}{p(\underline{\sigma})}+B\right\}$ as minimising $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$ relaxes the constraints. If no cost savings, i.e., $\widetilde{\psi}=\psi$ or ,in fact, for any $\widetilde{\psi} > \frac{1-\theta}{v+(1-v)(1-\theta)}\psi$, $\widetilde{t}\left(\underline{\sigma}, \underline{S}_1\right) = \frac{\widetilde{\psi}}{p(\underline{\sigma}|\overline{\sigma})} + B$. If $\widetilde{t}\left(\underline{\sigma}, \underline{S}_1\right) = \frac{\widetilde{\psi}}{p(\underline{\sigma}|\overline{\sigma})} + B$, it follows that

$$t\left(\overline{\sigma}, \overline{S}, \overline{\sigma}\right) \geq \frac{\widetilde{\psi} p(\overline{\sigma})}{\theta v} + \frac{\psi}{v} - B.$$

If $t\left(\underline{\sigma}, \underline{S}_1\right) = \frac{\psi}{p(\underline{\sigma})} + B$ the remaining constraint simplifies to

$$t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right) \geq \frac{\psi}{v\theta} - B.$$

Finally, the transfers need to be positive: $\widetilde{\psi}$ is lower than $\psi$ and both need to be high enough relative to the private benefit to ensure that happens.

## B.3 Proof of Lemma 11

Shareholders expected utility when a single auditor is hired, $W^S$, is the following:

$$p(\overline{S} \mid \overline{\sigma}, \overline{\sigma}) p\left(\overline{\sigma}, \overline{\sigma}\right) \left(u\overline{S}_2 + (1-u)\, \underline{S}_2 - I\right) - p(\overline{S} \mid \overline{\sigma}, \overline{\sigma}) p\left(\overline{\sigma}, \overline{\sigma}\right) t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right)$$

$$+ p(\underline{S} \mid \overline{\sigma}) p\left(\overline{\sigma}\right) \left((1-u)\, \overline{S}_2 + u\underline{S}_2 - I\right) - p(\underline{\sigma}) \widetilde{t}\left(\underline{\sigma}, \underline{S}_1\right) - p\left(\overline{\sigma}\right) p(\underline{\sigma} \mid \overline{\sigma}) t\left(\underline{\sigma}, \underline{S}_1, \overline{\sigma}\right),$$

whereas in the multiple auditor case, $W^2$ is:

$$p(\overline{S} \mid \overline{\sigma}, \overline{\sigma}) p\left(\overline{\sigma}, \overline{\sigma}\right) \left(u\overline{S}_2 + (1-u)\, \underline{S}_2 - I\right) - p(\overline{S} \mid \overline{\sigma}, \overline{\sigma}) p\left(\overline{\sigma}, \overline{\sigma}\right) \left(t\left(\overline{\sigma}, \overline{S}_1\right) + t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right)\right)$$
$$+ p(\underline{S} \mid \overline{\sigma}) p\left(\overline{\sigma}\right) \left((1-u)\, \overline{S}_2 + u\underline{S}_2 - I\right) - p(\underline{\sigma}) t\left(\underline{\sigma}, \underline{S}_1\right) - p\left(\overline{\sigma}\right) p(\underline{\sigma} \mid \overline{\sigma}) t\left(\underline{\sigma}, \underline{S}_1 \mid \overline{\sigma}\right)$$

and the difference equals:

$$\Delta = -p(\overline{S} \mid \overline{\sigma}, \overline{\sigma}) p\left(\overline{\sigma}, \overline{\sigma}\right) \left(t\left(\overline{\sigma}, \overline{S}_1, \overline{\sigma}\right) - t\left(\overline{\sigma}, \overline{S}_1\right) - t\left(\overline{\sigma}, \overline{S}_1 \mid \overline{\sigma}\right)\right)$$

$$-p(\underline{\sigma})\left(\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)-t\left(\underline{\sigma},\underline{S}_1\right)\right)-p\left(\overline{\sigma}\right)p(\underline{\sigma}\mid\overline{\sigma})\left(t\left(\underline{\sigma},\underline{S}_1,\overline{\sigma}\right)-t\left(\underline{\sigma},\underline{S}_1\mid\overline{\sigma}\right)\right).$$

With no private benefits and no cost savings:

$$\Delta=\frac{\psi}{\theta}-\frac{\psi p\left(\overline{\sigma}\right)}{1-\theta}$$

It happens that $t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)$ is always lower than $t\left(\overline{\sigma},\overline{S}\right)+t\left(\overline{\sigma},\overline{S}_1\mid\overline{\sigma}\right)$ and if $v$ is high enough, the probability of paying the "cheaper" $t\left(\overline{\sigma},\overline{S}_1,\overline{\sigma}\right)$ rather than transfer $t\left(\overline{\sigma},\overline{S}\right)+t\left(\overline{\sigma},\overline{S}_1\mid\overline{\sigma}\right)$ is higher (think for example as $v\to 1$) and on the other hand it is less likely to pay the "expensive" $\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)$: in fact, the term $p(\underline{\sigma})\left(\widetilde{t}\left(\underline{\sigma},\underline{S}_1\right)-t\left(\underline{\sigma},\underline{S}_1\right)\right)$ becomes less significant.

With low cost savings only:

$$\Delta=\frac{\psi}{\theta}-\frac{\widetilde{\psi}p\left(\overline{\sigma}\right)}{1-\theta}+\left(\psi-\widetilde{\psi}\right)p\left(\overline{\sigma}\right)\left(\frac{1+\theta}{\theta}\right),$$

that is definitely higher than $\frac{\psi}{\theta}-\frac{\psi p(\overline{\sigma})}{1-\theta}$. With high cost savings:

$$\Delta=\frac{\psi p\left(\overline{\sigma}\right)}{\theta}+\left(\psi-\widetilde{\psi}\right),$$

which is always positive.

With private benefits only:

$$\Delta=\frac{\psi}{\theta}-\frac{\psi p\left(\overline{\sigma}\right)}{1-\theta}-\theta\left(1-v\right)B,$$

that is definitely lower than $\frac{\psi}{\theta}-\frac{\psi p(\overline{\sigma})}{1-\theta}$. When the private benefit is very high:

$$\Delta=\psi-\frac{\psi p\left(\overline{\sigma}\right)}{1-\theta}-\theta\left(1-v\right)B,$$

which is definitely lower than before and always negative as $\psi < \frac{\psi p(\overline{\sigma})}{1-\theta}$. Hence, as the private benefit increases, and with limited liability, $\Delta$ is gradually decreasing.

Finally, with both private benefits and cost savings, there is a combination of the individual effects of the two previous cases. The bottom line is that, with cost savings, a higher private benefit is needed to turn $\Delta$ negative.

# Appendix C

# Auxiliary Calculations for Chapter 4

## C.1 Simplification of Expression (4.6)

Expression (4.6) can be simplified as follows. Making the change of variable

$$z = \frac{\theta - x^*}{\sigma},$$

denoting

$$\widehat{\delta} = \frac{\widehat{\theta} - x^*}{\sigma} \text{ and } \overline{\delta} = \frac{\theta^* - x^*}{\sigma},$$

and taking into account the fact that

$$y^* = \theta^* - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right)$$

the expression can be rewritten as:

$$\int_{\frac{\theta^*-x^*}{\sigma}}^{\infty} f(z)\left(1-F\left(\frac{\theta^*-\theta}{\tau}-F^{-1}\left(1-\frac{L}{\alpha B}\right)\right)\right)dz\alpha$$
$$=\int_{\frac{\widehat{\theta}-x^*}{\sigma}}^{\infty} f(z)\left(1-F\left(\frac{\theta^*-\theta}{\tau}-F^{-1}\left(1-\frac{L}{\alpha B}\right)\right)\right)dz$$

or

$$\int_{\frac{\theta^*-x^*}{\sigma}}^{\infty} f(z)\left(1-F\left(\frac{\theta^*-x^*-\sigma z}{\tau}-F^{-1}\left(1-\frac{L}{\alpha B}\right)\right)\right)dz\alpha$$
$$=\int_{\frac{\widehat{\theta}-x^*}{\sigma}}^{\infty} f(z)\left(1-F\left(\frac{\theta^*-x^*-\sigma z}{\tau}-F^{-1}\left(1-\frac{L}{\alpha B}\right)\right)\right)dz$$

and expression (4.7) is obtained by straightforward manipulation.

## C.2 Comparative Statics

The failure point $\theta^*$, varies with $\alpha$ as follows:

$$\frac{\partial\theta^*}{\partial\alpha}=B+b-\frac{\widehat{\omega}k}{\alpha^2}.$$

Hence,

$$\frac{\partial x^*}{\partial\alpha}=B+b-\frac{\widehat{\omega}k}{\alpha^2}-\sigma\frac{\partial F^{-1}(.)}{\partial\left(1-\frac{\widehat{\omega}}{\alpha}\right)}\frac{\widehat{\omega}}{\alpha^2}$$

and

$$\frac{\partial y^*}{\partial\alpha}=B+b-\frac{\widehat{\omega}k}{\alpha^2}-\sigma\frac{\partial F^{-1}(.)}{\partial\left(1-\frac{L}{\alpha B}\right)}\frac{L}{\alpha^2 B}$$

Since, $\frac{\partial F^{-1}(.)}{\partial(.)}$ is always positive the effect of a change in $\alpha$ is lower in $\frac{\partial x^*}{\partial \alpha}$ and $\frac{\partial y^*}{\partial \alpha}$ than in $\frac{\partial \theta^*}{\partial \alpha}$. On the other hand, $\theta^*$ also varies with the size of the large creditor loan $B$ as:

$$\frac{\partial \theta^*}{\partial B} = \alpha + \alpha \frac{\partial b}{\partial B} - \frac{\partial b}{\partial B}\widehat{\omega}$$

since the sum of both types of debt has to equal D, and increase of $B$ implies a decrease of $b$, therefore,

$$\frac{\partial \theta^*}{\partial B} = \widehat{\omega}.$$

Hence,

$$\frac{\partial x^*}{\partial B} = \frac{\partial \theta^*}{\partial B}$$

and,

$$\frac{\partial y^*}{\partial B} = \frac{\partial \theta^*}{\partial B} - \tau \frac{\partial F^{-1}\left(1 - \frac{L}{\alpha B}\right)}{\partial \left(1 - \frac{L}{\alpha B}\right)} \frac{L}{\alpha B^2}.$$

Since $\frac{\partial F^{-1}(.)}{\partial(.)}$ is always positive, the signal of $\frac{\partial y^*}{\partial B}$ is ambiguous.

## C.3 Sequential Game

Knowing that if the large creditor fails to renegotiate, the firm defaults and liquidates, the default point $\widetilde{\theta}$ in case of acceptance by the large creditor, is given by the same expression as before:

$$\widetilde{\theta} = \overline{K} + \alpha B + \alpha b + \omega\left(\widetilde{x}, \widetilde{\theta}\right)(k - \alpha b)$$

and the critical point that is now denoted by $\widetilde{y}$ is also defined as,

$$L = \Pr\left(\theta > \widehat{\theta} \mid \widetilde{x}, y > \widetilde{y}\right) \Pr\left(\theta > \widetilde{\theta} \mid \widetilde{y}\right) \alpha B + \Pr\left(\theta \leq \widehat{\theta} \mid \widetilde{x}, y \geq \widetilde{y}\right) L$$

that can be rewritten as,

$$\widetilde{y} = \widetilde{\theta} - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right).$$

The small creditors critical point, denoted in this section by $\widetilde{x}$, equates the payoff of agreeing

$$\Pr\left(\theta < \widehat{\theta} \mid y \geq \widetilde{y}, \widetilde{x}\right) l + \Pr\left(\theta \geq \widetilde{\theta} \mid y \geq \widetilde{y}, \widetilde{x}\right) \alpha b$$

to the payoff when he refuses

$$\Pr\left(\theta < \widehat{\theta} \mid y \geq \widetilde{y}, \widetilde{x}\right) l + \Pr\left(\theta \geq \widehat{\theta} \mid y \geq \widetilde{y}, \widetilde{x}\right) b.$$

Using the same transformations as before, the small creditor's posterior probability assessment of default conditional upon observing the large creditor decision for $x^*$ can be expressed as

$$\frac{\Pr\left(\varepsilon_i \geq \frac{\widetilde{x}-\widetilde{\theta}}{\sigma}, \tau\eta - \sigma\varepsilon_i \geq \widetilde{\theta} - \widetilde{x} - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)}{\Pr\left(\tau\eta - \sigma\varepsilon_i \geq \widetilde{\theta} - \widetilde{x} - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)}.$$

Rewriting as

$$\frac{\Pr\left(\varepsilon_i \geq \frac{\widetilde{x}-\widetilde{\theta}}{\sigma}, \frac{\tau}{\sigma}\eta - \varepsilon_i \geq \frac{\widetilde{\theta}-\widetilde{x}}{\sigma} - \frac{\tau}{\sigma}F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)}{\Pr\left(\frac{\tau}{\sigma}\eta - \varepsilon_i \geq \frac{\widetilde{\theta}-\widetilde{x}}{\sigma} - \frac{\tau}{\sigma}F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)}$$

and as $\frac{\tau}{\sigma} \to 0$ it simplifies to

$$\frac{\Pr\left(\varepsilon_i \geq \frac{\widetilde{x}-\widetilde{\theta}}{\sigma}, -\varepsilon_i \geq \frac{\widetilde{\theta}-\widetilde{x}}{\sigma}\right)}{\Pr\left(-\varepsilon_i \geq \frac{\widetilde{\theta}-\widetilde{x}}{\sigma}\right)} = 0$$

and again this implies that $\widetilde{x} \to -\infty$. But note that in this case, the payoff received in case of liquidation never plays any role (even if it did not cancel out) because if $\Pr\left(\theta < \widehat{\theta} \mid y \geq \widetilde{y}, \widetilde{x}\right)$ equals zero, $\Pr\left(\theta < \widehat{\theta} \mid y \geq \widetilde{y}, \widetilde{x}\right)$ will also necessarily be equal to zero. And in the previous case small creditors always agree to renegotiate and wait for the large creditor to decide what is best whereas here they always mimic the large creditor behaviour.

If the large creditor is less informed, the problem becomes similar to the case of simultaneous renegotiation when $\frac{\sigma}{\tau} \to \infty$. This is because the probability of default given the large creditor's acceptance decision from the small creditors point of view is:

$$\frac{\Pr\left(\varepsilon_i \geq \frac{\widetilde{x}-\widetilde{\theta}}{\sigma}, \tau\eta - \sigma\varepsilon_i \geq \widetilde{\theta} - \widetilde{x} - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)}{\Pr\left(\tau\eta - \sigma\varepsilon_i \geq \widetilde{\theta} - \widetilde{x} - \tau F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)},$$

or

$$\frac{\Pr\left(\varepsilon_i \geq \frac{\widetilde{x}-\widetilde{\theta}}{\sigma}, \eta - \frac{\sigma}{\tau}\varepsilon_i \geq \frac{\widetilde{\theta}-\widetilde{x}}{\tau} - F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)}{\Pr\left(\eta - \frac{\sigma}{\tau}\varepsilon_i \geq \frac{\widetilde{\theta}-\widetilde{x}}{\tau} - F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)}$$

and taking the limit the probability becomes:

$$\frac{\Pr\left(\varepsilon_i \geq \frac{\widetilde{x}-\widetilde{\theta}}{\sigma}, \eta \geq \frac{\widetilde{\theta}-\widetilde{x}}{\tau} - F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)}{\Pr\left(\eta \geq \frac{\widetilde{\theta}-\widetilde{x}}{\tau} - F^{-1}\left(1 - \frac{L}{\alpha B}\right)\right)}.$$

Given independence of $\varepsilon_i$ and $\eta$, the probability of default is simply $\Pr\left(\varepsilon_i \geq \frac{\widetilde{x}-\widetilde{\theta}}{\sigma}\right)$ or $\Pr\left(\theta < \widetilde{\theta} \mid \widetilde{x}\right)$ and $\Pr\left(\theta < \widehat{\theta} \mid y \geq \widetilde{y}, \widetilde{x}\right)$ equals $\Pr\left(\theta < \widehat{\theta} \mid \widetilde{x}\right)$.