Cultural transmission, public goods, and institutions

thesis submitted by Emmanuel Frot,

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Declaration

- No part of this doctoral dissertation has been presented to any University for any degree.
- The work presented in the thesis is my own, and no chapter was undertaken as joint work.

Abstract

This thesis discusses the consequences of different institutional forms in various settings, with a particular focus on the interactions between institutions, cultural transmission, and public goods. Chapter 1 introduces the main ideas, motivation, and results of the subsequent chapters. It provides a detailed summary of the thesis. Chapter 2 considers how institutions that modify behaviors affect the transmission of cultural traits. It argues that they create an environment that crowds out the behavior they were trying to promote. When applied to a model of public good provisions it illustrates how institutions that reduce free riding may decrease the level of public good in the long run. Chapter 3 extends this framework to make institutions endogenous. Individuals vote for their preferred institutional arrangement and the outcome is determined by majority voting. The crowding out of behaviors imply that agents have an incentive to affect strategically the transmission of preferences through collective socialization. Institutions can induce the formation of additional institutions such as schools in order to guarantee their sustainability. Chapter 4 considers that children acquire preferences through the choice of friends in the population, and that parents try to influence this choice. It shows how this creates a game between parents where their efforts to socialize their children to a particular cultural trait constitutes a public good. It studies the consequences for cultural groups of being intolerant and how they can survive cultural transmission. Chapter 5 uses the important example of commons as an institutional failure. It examines the case for privatization in an environment with different resources that may not be all privatized. It shows that labor reallocation reduces the gains of privatization, potentially to the point of reducing welfare. First best institutions may fail in a second best environment.

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Chapter 1

Overview

Institutions are an important determinant of economic performance. They constrain individual choices and can promote welfare-enhancing behaviors, such as cooperation in games, altruism, strong reciprocity, or policies such as the provision of public goods. North (1990, p.3) states that "institutions are the rules of the game in a society or, more formally, are the humanly devised constraints that shape human interaction". Bowles (2004, p.47) similarly defines them as "the laws, informal rules, and conventions that give a durable structure to social interactions among the members of a population". These are crucial to enforce behaviors that unconstrained, or not fully informed individuals would not undertake. The economics literature has provided numerous examples of the impact of institutions on economic outcomes, not only theoretically but also using historical and econometric analysis (Greif 2006 provides a recent review of the main contributions).

By considering institutions as fixed rules (North 1990), part of the literature tries to understand how they influence behavior. This helps to appreciate the consequences of an exogenous change in institutions. A prominent example is the tragedy of the commons described by Hardin (1968). Individuals who do not coordinate their actions deplete the resource they depend upon. It is an institutional failure where the rules of the game are not "right". Institutions which create property rights by privatizing the resource avert the tragedy and increase welfare. The focus in this literature is on the outcomes generated by two different institutional forms. The last chapter of this thesis places itself in this perspective and addresses the issue of the commons by assessing how apparently inefficient institutional arrangements can be justified in a second best world.

However this perspective of exogenous rules is ill-equipped to apprehend the emergence, persistence, and change of institutions. Greif (2006) argues that we must understand the processes through which institutions reinforce or undermine themselves. Institutions influence their own future by changing individuals' behaviors. Chapters 2, 3, and 4 relate to this idea and present different theoretical studies to explain these mechanisms by focusing on cultural transmission.

This chapter gives an overview of the thesis, and provides a summary of each of its sections.

1.1 Intergenerational crowding out

Chapter 2 is a first step towards the understanding of the processes through which institutions can ensure the pereniality of what they were designed to promote. It considers that constraints put on individual choices by institutions determine the evolution of preferences in the society. It investigates in an example the consequences of this shift in constraints for behaviors, but not for institutions. So strictly speaking this chapter still looks at institutions as fixed and exogenous entities. However it argues that institutions designed to promote a behavior can lead do its unraveling in the long run and as such looks at a changing environment that affects economic outcomes. There is only a small step from this conclusion to the dynamics of institutional change. If an institutional constraint fails to fulfill its goal to the point that it becomes irrelevant and obsolete, people may vote to remove it, or revolt against it, in order to design new rules.

An important field of economics has already studied the dynamics of institutions. Evolutionary institutionalism assumes that individuals are not fully rational and take decisions based on limited knowledge. They learn from experimentation by interacting with other agents. Institutions, as a stable pattern of expectations and beliefs, are shaped by these evolutionary forces (Young 1998). I depart from this line of research by studying the evolution of preferences, instead of behaviors and expectations. The distribution of preferences in the population then determines behaviors, but the dynamics are created by the transmission of preferences. Bisin and Verdier (2001) have developed an economic model of cultural transmission that I extend and rely on in Chapters 2 and 3.

Bisin and Verdier (2001) model assumes that preferences are not inherited genetically, or by conformism (Bowles 2004 provides examples of both). They claim instead that cultural traits are acquired through a process of socialization initiated by parents. These evaluate their children's actions and decide how to socialize them. This choice depends on the environment and is made through the lens of paternalistic altruism whereby parents use their own preferences to evaluate their children's behavior. Consider a society where individuals have different cultural values, in the form of different preferences. Cultural transmission can be seen as a two step process after the birth of the naive child. First, the parent tries to socialize his child to a cultural trait, and this action is costly. This constitutes the vertical transmission, and takes place inside the family. If the parent is successful then his child adopts the preferences corresponding to the cultural trait. If he fails, the child is randomly paired to an adult in the population, and takes his preferences. This is the oblique transmission, and takes place outside of the family. The vertical and oblique channels provide transition probabilities for given socialization efforts. Aggregation of parents' decisions yields the dynamics of preferences.

The main result of the chapter comes from how parents choose to socialize their children, so I give a detailed, albeit very simplified, account of their decision rule here. Consider that there are two types of agents in the society, with respective utility functions $u_a(x)$ and $u_b(x)$. Adults choose x that maximizes their utility function, and the optimal choice for type a and type b agents is respectively denoted x_a and x_b. The cultural transmission mechanism implies probabilities for the child of a parent of either type to adopt either type of preferences. Let P^{ij} be the probability that a child of a type i parent becomes a type j agent, with $i,\ j \in \{a, b\}$. Type i parents choose their socialization effort τ_i maximizing $P^{ii}u_i(x_i) + P^{ij}u_i(x_j) - C(\tau_i)$, where $C(\tau_i)$ is the cost of effort. Parents are altruistic but biased because they use their own preferences to evaluate their children's actions. This maximization (under suitable assumptions) implies that socialization effort increases with the difference $u_i(x_i) - u_i(x_j)$. The intuition is clear: a large gap in utility from parents' point in view between being a type i and a type j means that they should invest heavily in socialization. That can be summarized by saying that intolerance intensify efforts to transmit cultural traits.

How can we formally include institutions in this set-up? I gave a definition of institutions as entities that devise constraints and shape human interactions. I assume that institutions constrain individuals' choices x. For instance, the two types could be labeled as cooperative and selfish and decide how much to contribute to a public good, with cooperative agents giving a higher contribution to the public good than selfish agents. The institutional arrangements would consist for instance in a fine for giving a too low contribution. This constraint forces selfish agents to participate more to the public good than they would otherwise do. Institutions do not necessarily impose material penalties, it could also be social stigma associated to a behavior, or punishment by forbidding future interactions with the agent, etc.

The question the chapter addresses is whether an institution that promotes a behavior makes it sustainable in the long run. The perspective adopted is close to what Greif (2006) argues to be important to understand institutional change. He claims that we should "study the interplay between micromechanisms through which institutions influence behavior and their implications", and "how an institution cultivates the seeds of its own demise". I make use of the fact that institutions affect behaviors, which in turn change cultural transmission and so the prevalence of traits in the population, and that this eventually feeds back to behaviors. While I am not the first to attempt such an exercise (see Bisin and Verdier 2004 who study the implication of cultural transmission on welfare state, and particularly Francois 2006 who looks at the dynamics of institution formation), the result that institutions may "cultivate the seeds of [their] own demise" through cultural transmission is, up to my knowledge, new to the literature.

To understand this result, come back to the preceding public good example, and consider that there is an (exogenous) institutional change that induces selfish agents to increase their participation to the public good, in such a way that contributions of both types are now closer. The effect on socialization effort is usually that they fall. The intuition is that parents feel less intolerant towards a behavior close to theirs. After the institutional change cooperative parents are in an environment where everyone looks "more cooperative" than before, because even selfish agents now contribute a lot to the public good. Parents make a smaller effort to transmit their preferences because even if their children adopt selfish preferences, their behavior will not be that far from a cooperative behavior. Less investment in socialization means that cooperative preferences are less transmitted, and so that there are fewer cooperative agents next period. The confidence in the present environment is what may make the society evolve towards one with a large number of selfish agents.[1] The result we reached is the following: if institutions are built to promote public good provision, they may lead to a society where most people contribute very little. I call this property intergenerational crowding out. This counterintuitive result occurs because the promoted behavior becomes very common, such that people do not invest in the transmission of preferences that sustain it.

I now illustrate this property of cultural transmission with some examples. The first one is developed in Chapter 2 and is directly related to the public good provision example used here. Individuals can be cooperative or selfish but in addition they interact with a group of peers that influence their contribution decision. A selfish agent interacting only with copperative individuals contributes more than if he interacted only with selfish agents. Institutions shape these interactions by affecting their importance in decision making because they modify the cost of departing from the behaviors of one's peers. For instance institutions may make contributions either public information, and agents suffer a large utility loss from looking selfish, or keep contributions private, and agents do not take into account the decisions of their peers. The model is designed such that, everything else being equal, stronger interactions imply a larger provision of public good.[2] Institutionally it seems a good idea to reinforce these interactions. However I show that in the long run it unambiguously leads to a smaller proportion of cooperative agents in the population, and, somewhat more importantly, that it may result in lower levels of pub-

[1] Parents have rational expectations and so the confidence is rational. Individuals are not surprised by what happens next period and their confidence is perfectly justified. However repeated on a large number of periods it slowly erodes public good provision.

[2] Stronger interactions mean that the individual prefers his behavior to be closer to his peers' behaviors.

lic good. An institutional rule supposed to support public good provision is successful in the short run, but fails in the long run. That reasoning can be applied to different issues. One is the level of trust between individuals in the US. Individuals exhibit stable trust levels over their life cycle, but trust has dramatically fallen between generations during the last century. As documented by Putnam (2000), people born around 1930 were more civic than their elders, and than their subsequent offsprings. Intergenerational crowding out suggests that the upsurge in civic attitudes caused their own decline. Similar examples are discussed in Chapter 2, but I casually offer here an additional application of the model. The interaction between welfare state and the evolution of preferences has already been studied (Lindbeck, Nyberg and Weibull 1999, Bisin and Verdier 2004) but intergenerational crowding out highlights a new feature that could explain the present tensions in various countries about the generosity of welfare state systems. These were mainly established at the peak of the "civic generation" when one could argue that some incentives compatibility constraints were satisfied because of civic mindedness (basically, not cheating, or claim undue benefits). This cultural trait was present because it gave some advantage before the welfare state provided a systematic safety net, by sustaining cooperation, or reciprocity in a risky environment. Welfare state made individual economic outcomes less related to civic attitudes, as it compensated for bad outcomes. In this environment the civic cultural trait has been less transmitted, increasing the proportion of people relying on benefits. The welfare state system is weakened by this change of attitudes, and many countries now focus on ways to control benefit claims and make sure incentive compatibility constraints are satisfied. This long run evolution corresponds to what Greif (2006) calls an undermining institution.

Intergenerational crowding out is the main result of Chapter 2, but it also brings new developments to the theory of cultural transmission. It includes in particular social interactions into preferences. This requires a formal extension of Bisin and Verdier (2001) to show how it affects their equilibrium conditions, and can generate multiple equilibria. Apart from the theoretical argument, it underlines how interactions can make steady states dependent on past history in the cultural transmission framework. I illustrate in the public good example how the composition of peer group is crucial to understand how public good provision changes with stronger interactions. This motivates future research to understand the link between peer choice and cultural transmission, and possible implications, among others, for residential choice and neighborhood segregation, or society polarization into culturally homogeneous groups.

In this chapter I considered that institutions were exogenously given, and examined the implications of different institutional settings. The next chapter builds on these results to understand the change in institutions.

1.2 Institutional crowding out

Chapter 3 directly uses the conclusions of Chapter 2 to endogenize institutions. It considers a case where there is intergenerational crowding out and investigates its implications in a model with voting. Bisin and Verdier (2000, 2004) also look at a political equilibrium in a cultural transmission framework. However their results hold precisely because there is no intergenerational crowding out. Francois (2006) is also closely related to this chapter. Although he does not consider voting, he looks at the simultaneous evolution of institutions, and norms. He claims, as a result of his model, that good institutions make good agents, and that good agents are also a requirement for good institutions to arise. Chapter 3 somehow challenges the first affirmation. Intergenerational crowding out implies that good institutions may not make good agents, but may substitute for good agents to the point of not being sustainable.

The argument is a direct consequence of Chapter 2, and it can be exposed using the public good model with cooperative and selfish agents. Instead of having exogenous institutions, individuals choose institutions in each period. The outcome is determined by majority voting. Assume that selfish agents vote for institutions that make their contributions low (they may not benefit from the public good, and so decide to scrap the institutional arrangements ensuring a minimum contribution from each type), while cooperative agents opt for some enforcing mechanisms that make selfish agents' contributions high. Assume in addition that there is an extreme case of intergenerational crowding out, whereby the steady state distribution of preferences with the "selfish" institutions implies that cooperative agents represent a majority, and a minority with the "cooperative" institutions. As a consequence none of these steady states exists, because of majoritarian voting. More disturbingly, I show that there is no rational expectations paths as agents are bound to be wrong in their expectations. If agents believe that institutions next period will be "selfish" then they are "cooperative", and reciprocally. The decision problem cannot be solved at the individual level, as long as agents have rational expectations. It requires some degree of coordination between individuals. This dilemma exists because agents do not take into account the consequences of their own socialization choice on the dynamics of preferences. One way for individuals to make rationally consistent choices is to coordinate, that is to build some device to act as a group. A collective institution of preference formation (schools, state education, ideology) would emerge to solve the decision puzzle. This provides a rationale for collective socialization: it allows rational expectations paths to exist. Bisin and Verdier (2000) also study this issue but they find that institutions emerge when it allows a cultural trait to thrive in an environment where it would otherwise disappear. While the same effect is present in my model, there is a second complementary argument: when an institution produces intergenerational crowding out, agents can organize themselves to decide collectively about socialization in order to have rational beliefs. People do realize that their beliefs are not supported by their actions and consequently decide to act collectively.

The model also shows that when agents are able to set up such collective institutions they do so only for not too small minorities, and small majorities. Collective socialization is not a profitable option for a group whose size is too small, or too large. For intermediate sizes agents organize themselves to affect cultural transmission. If they represent a minority, it is to be a majority next period. This is feasible only if they are not a small minority. If they are already majoritarian (but not by a large quantity), it is to still be next period. This acts as a counteracting force to crowding out. Thus this chapter extends the model of Chapter 2 by recognizing that individuals shape institutions, as much as institutions shape individuals. It continues to offer an illustration of the mechanisms of institutional change described by Greif (2006). Institutions modify behaviors, and that reinforces or undermines themselves. Crowding out is an undermining process. According to Greif this dynamic process makes the institutions more or less stable when confronted to a new environment. In the model institutions indirectly change cultural group sizes and it makes the institutional arrangement not viable because not supported by the majority. However this undermining can be avoided if the initial institution is complemented by a collective socialization institution. A general conclusion of this chapter is that institutions crowd themselves out, but may come to realize this when on the brink of falling apart and maintain conditions for their existence. According to this argument public education can be seen as a collective device in order to support rules that make existing institutions viable.

1.3 Cultural transmission through network formation

In Chapter 2 and 3 the transmission of preferences follows the Bisin and Verdier (2001) model that uses vertical transmission and then oblique transmission but only if the child has not been socialized vertically. Children are merely passive and do not play any role in this process. Furthermore they are influenced by other adults only if their parents fail to socialize them. Chapter 4 relaxes this assumption and makes both parents and children actors in the transmission of preferences. It assumes that children build a network of friends and that they are influenced by them in the socialization process. Parents anticipate this and try to influence their choice of friends by altering the cost of building network links. Unlike Bisin and Verdier (2001), vertical transmission does not result directly in socialization to a cultural trait, but more subtly shape the environment to make oblique transmission favorable to parents. The assumption that parents are imperfectly altruistic is maintained. The way parents influence network formation can be through residential choice in a segregated area, or school choice. These come at a cost for parents. An important feature of the model is that children may not be able to build the network that maximizes their utility. Friendship requires the consent of both children who take part in the relationship. If one child has already built his optimal network he does not desire to have further friends, and the other is constrained in his choice. Consequently parents may not have to exert costly effort if they know that their child is going to be constrained in his friendships. To understand this consider the extreme case of a population made of a highly predominant cultural group, and of a very small minority. Children from the largest group cannot find many friends from the very small community, simply because there are very few. It is likely that they will not be able to build their desired network of friends, but in that case their parents should not worry about affecting network formation. On the other hand parents from the minority group are surrounded by individuals with different preferences, and they know that their children will always be able to find as many friends as they want from the majority group. These parents are therefore willing to spend resources (time, money, etc.) to limit the number of friendships. This creates a game between parents, where it is always in the interest of one cultural group not to make any effort to curb their children's friendships. Socialization is a public good and the group which values it most provides it, while the other free rides.

I show that the game in socialization efforts does not always have a unique Nash equilibrium. When it is not unique cultural groups disagree on which should be implemented: each group prefers the equilibrium where it free rides on the other. In order to study the dynamics of preferences in the population, I assume that the equilibrium is chosen by majority voting. I study first the outcome of the political process in each generation and how it depends on the intolerance of each group towards the other. It appears that the most intolerant group always sees its preferred equilibrium implemented. In this sense one can claim that intolerance confers an advantage. However this comes at a cost: everything else being equal parents' welfare increases with the intolerance of parents from the other cultural group. It is always better to face a very intolerant group because it allows free riding for a larger range of group sizes. Finally, the political process not only selects the equilibrium preferred by the majority, but it also affects the existence of steady states. It may eliminate Nash equilibria that would be steady states, if not for the majority voting. It can result in a situation without any steady states because the majority never votes for them. That creates cycles between states with alternating majorities.

One could expect that intolerance is not welfare improving, but that it has an evolutionary advantage. More intolerant groups are better able to preserve their cultural traits by limiting external contacts. This is only partly true. Intolerance avoids damaging contacts with other cultures but it also prevents conversion of children from families of different cultures. The most intolerant group may not survive cultural transmission, however this happens only if its children establish only a small number of links in their own cultural group. An intolerant and poorly internally connected group is bound to disappear, whereas a similar group with the same intolerance level but with numerous internal connections does survive evolutionary pressures. This result sheds some light on how cultural groups can organize to reproduce themselves. It helps to understand how institutions specific to each group play a crucial role in the dynamics of culture. Institutions should underline the importance of the internal group structure, and focus on intragroup friendships. This always increases group size in steady state. On the other hand, to reinforce intolerance is usually not a solution, and can even lead to the death of the cultural group.

1.4 Common property resource privatization and labor allocation

The tragedy of the commons (Hardin 1968) is a famous example of institutional failure. Common resources are overexploited because of their lack of clearly defined property rights and it is argued that privatization, by defining these, would avert the tragedy and restore the first best outcome. This logic has been criticized on different grounds. Weitzman (1974) shows that privatization reduces the return to labor, and that labor must be made worse off. This result has been used recently by Baland and Francois (2005). They show that commons offer insurance properties that cannot be replicated after privatization, because of incomplete markets, such that the commons Pareto dominate private property. Chapter 5 considers the effect of a privatization reform on labor allocation, and welfare. The motivation comes from Jodha (1985, 1995). He observes that following the privatization of common property resources due to a change in institutions in India, the remaining commons are congested and dangerously depleted. Furthermore workers have seen their wages fall. The model described in the chapter considers two resources. One is privatized while the other is common property. One can think of two different situations that the model illustrates. First, some land that is privatized, while the surrounding forest, more difficult to privatize, is commonly exploited. Second, the agricultural land is privatized, and individuals can migrate to a labor market where they are employed. The important feature of the model is that the outside opportunity after the land has been privatized is congested.

Following Weitzman (1974), privatization reduces labor allocation, and as a result shifts workers to their outside (congested) opportunity. First, the return to labor on the privatized resource falls. Second, the return to labor on the congested resource falls as well. In equilibrium these returns must be equal, such that workers lose twice: first because of privatization, second because of congestion. I show that under these conditions welfare can fall, even though there used to be a full tragedy of the commons, and the reform is perfectly egalitarian.

The model is then extended to include heterogeneity in skills with respect to the privatized resource. Individuals are more or less talented and I study the consequences of reform design on incomes and welfare. Land can be given to the most skilled agents because they have a larger labor demand, or a more egalitarian reform can be implemented. It appears that welfare is maximized when only the most able individuals receive some land. Finally the article draws some conclusions on the distribution of skills. The model is an illustration of the second best theorem where a first best reform may not be beneficial in a non first best environment. Good institutions (private property rights) may not achieve welfare maximization when other distortions are present.

Chapter 2

Intergenerational crowding out

2.1 Introduction

In his book Bowling Alone (2000), Robert D. Putnam documents the loss of social capital in the American community. Using a wide range of indicators, he shows how people in the US are less and less civically engaged. Life-cycle effects do not account for this pattern, while intergenerational differences appear to be dramatic. Social capital has declined not because individuals have changed over their lifetime but because generations have. In the words of Putnam "all these forms of civic involvement and more besides have declined largely, if not exclusively, because of the inexorable replacement of a highly civic generation by others that are much less so" (p. 250). He identifies a "long civic generation" born between 1910 and 1940 that has been followed by cohorts whose contributions to social capital have decreased continuously. Indicators of trust, volunteering, organization membership, and voting turnout have declined over time. Figure 2.1 uses data about trust from the General Social Survey to illustrate this fall across cohorts. Age seems to have a much smaller influence, as trust is pretty stable over the lifetime. Using the same data and controlling for various individual characteristics, Glaeser et al. (2000)

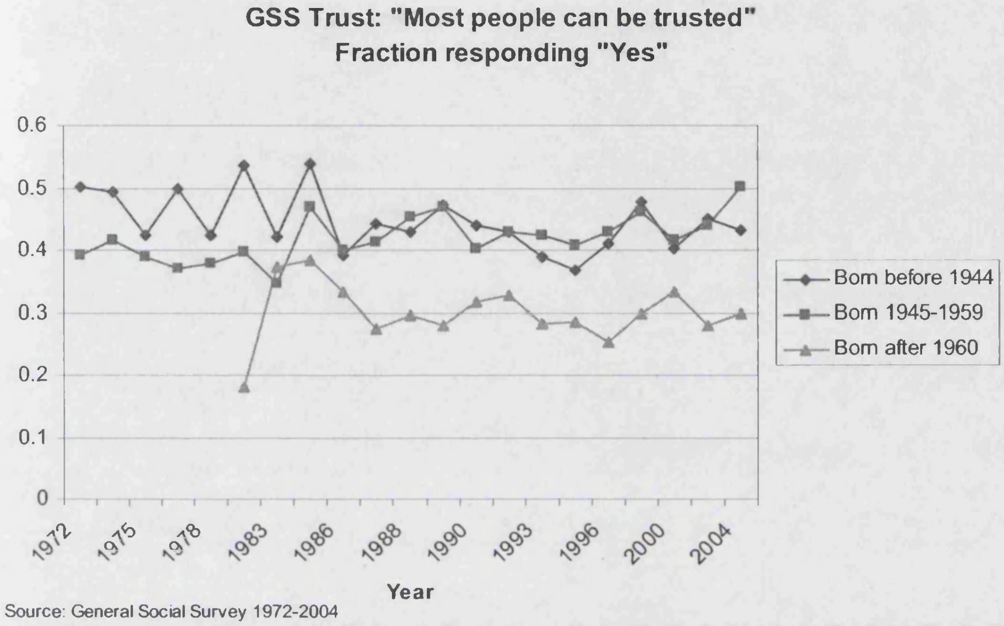

Figure 2.1: Trust over time, by cohort

indicate that the gap in the level of trust between individuals born before 1915 and those born after 1959 is larger than 20 percent and find an insignificant effect of age. Robinson and Jackson (2001) provide an analysis of trust by cohorts in the US and confirm the finding that successive generations are less and less trusting.

According to Putnam (2000) all the decline in voting in the US can be explained by generational change. He reports that only 54 percent of adults born in the seventies feel guilty when they do not vote, as compared with over 70 percent of the older generations.[1] Miller (1992) demonstrates that the change in the US turnout is due to generational change and that young generations vote less than their parents, who themselves vote more than their parents. Intergenerational changes are a common feature in social sciences

[1] While the methodology and conclusions of Putnam have not remained unchallenged (see Durlauf 2002 and Sobel 2002), authors agree that he provided convincing data on the decline of participation in voluntary organizations in the US. The results about voting have been extensively confirmed and had been established before Putnam's book.

and are observed for a large range of characteristics by sociologists. However the lacuna in this research is a general framework to understand why these transformations occur. Incentives have certainly evolved but it is less clear why these changes would affect cohabiting cohorts differently. In this chapter I use an economic model of cultural transmission from parents to children to show how widespread attitudes create an environment that leads to their own demise. Parents fail to transmit a common behavior simply because it is too common.

In order to spell out this mechanism I use a model of cultural transmission developed by Bisin and Verdier (2001), where the cultural trait to be transmitted are the preferences for a certain behavior. To address the issue of intergenerational change I extend this model to allow for social interactions. It has two consequences: first it generates multiple equilibria and this may explain large differences between otherwise similar societies; second, it creates a link between individuals and with who they interact. Peer groups depend on what types of individuals are in the population and this is determined by cultural transmission. Therefore interactions establish a connection between individual choices and cultural transmission.

I characterize how a policy or an exogenous institutional shock has unintended consequences in the long run such that it is crowded out. The mechanism is that when behaviors become more homogeneous across individuals, parents put less effort into transmitting their own preferences: from what they observe any preferences lead to a similar behavior. Parents therefore neglect that preferences drive choices. This "neglect effect" in education ultimately leads to the decline of these preferences and it affects their population distribution. Finally individual choices respond to this new distribution through interactions. When the new equilibrium is eventually reached, behaviors and the distribution of preferences have changed. Using utility functions satisfying standard assumptions I show that crowding out is a general result for cultural transmission models based on Bisin and Verdier (2001). The endogenization of preferences allows me to disentangle the short run from the long run effect of changing incentives. The driving force leading to crowding out is the increased similarity between behaviors after incentives have been applied, and social interactions. To my knowledge, this model is the first to introduce explicit social interactions in a cultural transmission framework.

The exogenous shock assumption should not be taken too literally. The main point of the model is to study the long run outcome of different institutional arrangements characterized as constraints on behaviors. The crowding out result demonstrates that short and long run objectives are contradictory. Institutions that effectively promote a behavior in the short run have opposite consequences in the long run. There is a slow but ineluctable erosion of the values the institution supported in the first place. Institutional constraints make behaviors similar across types of preferences, but this comes at the cost of making the transmission of values less compelling since the institution provides the right constraints. Institutions potentially fail to foster the behavior which they were designed for and this clearly challenges their sustainability. The important study of how this creates pressure for institutional change is done in Chapter 3.

I apply this model to study the dynamics of voluntary contribution to a public good. Volunteering, voting, church attendance and trust usually suffer from underinvestment, free riding and are, to a certain extent, non rival. A public good model captures these characteristics. The population is divided into two types: conditional and strong cooperators. Cooperative individuals follow an internal norm, such that they are insensitive to the distribution of types in the society. These could be described as ethical, or strong cooperators. The rest of the population is composed of conditional cooperators whose contributions are driven by conformity to their reference group contributions. I show first that cultural transmission decrease a strong cooperators' population size when conditional cooperators contribute more because of a higher concern for conformity. Second, I investigate how the composition of the group individuals interact with determine the equilibrium provision of public good. I show that to reduce the differences between contributions is counterproductive and detrimental to the level of public good. This suggests that a split society with an elite, following its own internal norm, can exhibit higher levels of cooperation than an egalitarian society. Conformity has only short run benefits that may be cancelled out by cultural transmission.

This work is related to a growing literature on cultural transmission (see Bisin and Verdier (forthcoming) for a recent survey). However these contributions do not specifically look at the effects of incentives, and do not underline the opposition between beneficial short run effects and long run consequences, partly because they do not allow for interactions. My work also has connections with the vast literature on social interactions (see Brock and Durlauf 2001, Glaeser and Scheinkman 2003) where an individual decision is dependent on the decisions made by his peers. Interaction models characterize a static equilibrium that individuals can reach through some class of dynamics. This article, rather than solving for a static equilibrium and then to think of how people can reach it, provides the microfoundations for the dynamics in the presence of interactions and then characterize the equilibrium.

The crowding out of norms is now well documented in micro studies (Frey 1997 and Bénabou and Tirole 2006 provide surveys) following the work of Gneezy and Rusticchini (2000). In this literature crowding out takes place because extrinsic incentives cast doubt on the real motives of an action. The mechanism underlined in this chapter is very different since individuals do not have any reputational concerns but care about transmitting preferences to the next generation.

Finally, generational change and the decline of values is a popular theme that has not received any attention from the economics literature. Putnam (2000) and Fukuyama (1999) document these changes in the US after the Second World War. Putnam's book initiated a large literature on social capital. Its fall has been confirmed and its causes evaluated by Costa and Kahn (2003). The importance of social capital for growth has been documented in a cross-country study by Knack and Keefer (1997). The literature on institutions and on how they create the conditions of their demise has been reviewed in Chapter 1.

The remainder of the chapter is organized as follows: Section 2.2 gives the details of the general model. It must first extend the Bisin and Verdier (2001) model to incorporate social interactions. It shows that crowding out is a general property. Section 2.3 uses a simple model to illustrate the result and looks at the effect of interaction on the average behavior in the society. Section 2.4 provides related evidence that fit into the framework of the model. Section 2.5 concludes.

2.2 A general model

This section shows how crowding out occurs in a general overlapping generations model with cultural transmission process. There are two types of agents identified by $i \in \{1, 2\}$. Each agent has to choose an action a_i and he is influenced in his choice by a weighted average S_i of the actions of a group of peers. All the type i agents have the same utility function U^i and the same structure of peer groups. I consider only symmetric equilibria, such that in equilibrium S_i can be written $S_i = \gamma_{i1}a_1 + \gamma_{i2}a_2$, where $\gamma_{i1}, \gamma_{i2} \geq 0$, $\gamma_{i1} + \gamma_{i2} = 1$ and

a_1 and a_2 are the equilibrium actions. The weights represent the importance given to type i individuals actions in the peer group. Because of the dynamic nature of the model these weights depend on the proportions of each type in the population. Interactions, and so the weights given to different types, evolve with population characteristics. A simple example is when individuals have a reference equal to the average action in the society.

Agents choose the action $a_i(S_i, \alpha)$ that maximizes $U^i(a_i, S_i, \alpha)$ for a given S_i, α being a vector of parameters that will be affected by institutions.[2] Concavity of utility in a_i ($U^i_{11} < 0$) implies that there is a well defined solution to this problem. Glaeser and Scheinkman (2001) give conditions for existence and uniqueness of equilibrium for this type of model. In particular there is a unique equilibrium under moderate social influence (MSI), i.e. $\left|\frac{U^i_{12}(a_i,S_i,\alpha)}{U^i_{11}(a_i,S_i,\alpha)}\right| < 1$. MSI is assumed to be satisfied in the rest of the chapter. I also usually assume strategic complementarity $U^i_{12} > 0$, as is common in models of interactions, such that there is complementarity between individual and reference group actions.

I assume that S_i is a continuously differentiable function of actions and proportions, such that it can be written as $S_i = S_i(a_1, a_2, p)$, with p being the proportion of type 1 agents in the population. The dependence on p is made through the weight γ. In this article I allow for all kinds of interactions that boil down to a weighted average. It is not useful at this point to specify the function γ but, as we will see in the next sections, it affects the equilibrium properties.

[2]Institutions act as constraints on individuals when they maximize their utility. This is represented by the parameter α.

2.2.1 Education choice

Cultural transmission mechanism

In this section we integrate the static setting into an overlapping-generations model with continuous time. In each period the proportion of agents with type i preferences is p_t^i. Agents are born and die following a Poisson process. The size of the population is therefore constant. During a time interval dt, a proportion λdt of all individuals die. Just before dying, they give birth to a child without any preferences who is instantaneously socialized into one of two traits. Socialization is a product of the family (what Bisin and Verdier (2001) call vertical transmission) and of the social environment (oblique transmission). Parents are altruistic towards their children and want to socialize them to the preferences that increase their welfare. However, they use their own subjective evaluation that may differ from their children utility.

I follow the Bisin and Verdier (2001) model in this section but, contrary to their model, I let agents interact: actions depend on reference groups. This extension has not been formally discussed in the literature. I am therefore able to give stronger conditions for what they define as "cultural substitution".

Education is done in the following way: a naive child is educated by his parents, say of type i, with probability $d_i(p_t^i)$. This defines the probability with which vertical transmission is successful, such that the child takes on the parent's type. Otherwise, he remains naive and is matched randomly with an individual in the population. He adopts his trait through oblique transmission. In other words, when the family fails to socialize the child, he is influenced by a role model (friend, teacher, peer, etc.) chosen randomly in the population.

Let P_t^{ij} denote the probability that a child from a family with type i preferences is socialized to cultural trait j, then

$$P_t^{ii} = d_i(p_t^i) + \left[1 - d_i(p_t^i)\right] p_t^i \qquad P_t^{ij} = \left[1 - d(p_t^i)\right](1 - p_t^i) \tag{2.1}$$

where P_t^{ii} is computed in the following way: with probability $d(p_t^i)$ vertical transmission succeeds and the child adopts trait i. If education fails then he can still adopt type i preferences by being matched with a random individual who is type i. This occurs with probability $[1 - d(p_t^i)]\, p_t^i$.

p_{t+dt}^i is given by:

$$p_{t+dt}^i = p_t^i(1 - \lambda dt) + \lambda dt p_t^i P_t^{ii} + \lambda dt(1 - p_t^i) P_t^{ji} \tag{2.2}$$

Using (2.1), and $dt \to 0$ (the time index is dropped for notational clarity), (2.2) becomes

$$\dot{p}^i = \lambda p^i(1 - p^i)(d_i(p^i) - d_j(1 - p^i)) \tag{2.3}$$

I define cultural substitution as in Bisin and Verdier (2001).

Definition: *Vertical cultural transmission and oblique cultural transmission are cultural substitutes for agent i (or, equivalently, $d_i(p^i)$ satisfies the cultural substitution property) if $d_i(p^i)$ is a continuous, strictly decreasing function in p^i, and, moreover, $d_i(1) = 0$.*

Bisin and Verdier show that when vertical and oblique cultural transmission are cultural substitutes for both groups, then $0, 1$, and p^{i*} are stationary states of (2.3), with $0 < p^{i*} < 1$, and p^{i*} is stable, with its basin of attraction being $(0, 1)$. The next section gives a sufficient condition for cultural substitution.

Endogenous education choice

In this section $d_i(p^i)$ is chosen endogenously. We want to identify conditions under which there is cultural substitutability since it ensures stability of the interior equilibrium. Parents evaluate their children's actions and then try to socialize them into their own trait. V^{ij} is the utility a parent of type i derives from having a child of type j. Two things differentiate this framework from Bisin and Verdier (2001): first, they assume imperfect empathy, i.e. $V^{ij} = U^i(a_j, S_i, \alpha)$, such that parents evaluate the actions of their child using their own preferences. I do not impose such structure on V^{ij}, but instead consider the general case where $V^{ij} = V^i(a_j, S_i, \alpha)$. Second, since agents interact V^{ij} is a function of S_i, and so indirectly of p^i. Peer groups evolve with p^i. This distinction is important as it potentially generates multiple equilibria and introduces instability in the dynamics.

Parents of type i have to choose a vector $\tau_i \in \mathbb{R}_+^n$ of inputs to educate their children. These inputs and the proportion of types in the population determine the probability for parents i to have a child of their own type according to the map $D : \mathbb{R}_+^n \times [0,1] \to [0,1]$,

$$d_i = D(\tau_i, p^i) \tag{2.4}$$

Education has a cost $C(\tau_i)$. C and D satisfy the assumptions found in Bisin and Verdier:

- D is C^2, strictly increasing and strictly quasi-concave in τ_i. $D(0, p^i) = 0$, $\forall p^i \in [0,1]$.
- C is C^2, strictly increasing and strictly quasi-convex. $C(0) = 0$ and $C'(0) = 0$.

Dropping the time subscripts, parents choose $\tau_i \in [0,1]$ maximizing

$$P^{ii}V^{ii} + P^{ij}V^{ij} - C(\tau_i) \tag{2.5}$$

Parents have rational expectations and so perfectly anticipate the new a_i and the proportion of type 1 agents next period.

The solution to the maximization of (2.5) is given by a continuous map $d_i = d(p^i, \Delta_i)$, where $\Delta_i \equiv V^{ii} - V^{ij}$ measures the intolerance of parents towards the other trait. While I assume $\Delta_i \geq 0$ in the general proof to rule out special cases, one could imagine situations where parents actually prefer their child to adopt the other cultural trait. In this case their optimal education effort would be zero.

Bisin and Verdier show that the probability of success of vertical transmission $d(p^i, \Delta_i)$ satisfies the cultural substitution property if the "vertical transmission technology" is decreasing with p^i, $\frac{\partial D(\tau_i, p^i)}{\partial p^i} \leq 0$. When Δ_i depends on p^i, this is not true any more, and a stronger condition is required. Let η_{Δ_i} be the elasticity of Δ_i with respect to p^j, $\eta_{\Delta_i} = \frac{p^j}{\Delta_i} \frac{d\Delta_i}{dp^j}$.

Proposition 2.1 *$d(p^i, \Delta_i)$ satisfies the cultural substitution property if $\frac{\partial D(\tau_i, p^i)}{\partial p^i} \leq 0$ and $\eta_{\Delta_i} > -1$.*

The Bisin and Verdier result that $\frac{\partial D(\tau_i, p^i)}{\partial p^i} \leq 0$ implies a stable interior equilibrium no longer holds. Consider an equilibrium and then increase slightly p^i. When Δ_i does not depend on p^i, then the equilibrium is stable if type i parents reduce their education effort, while type j parents increase theirs. In practice, type i parents reduce their effort when p^i increases because they can rely on oblique transmission. In other words if they fail to educate their child, he is likely to be educated by the society. It brings back the system to its former equilibrium. This is the intuition behind $\frac{\partial D(\tau_i, p^i)}{\partial p^i} \leq 0$.

But if Δ_i is a function of p^i, the perturbation directly affects intolerance. If, for instance, type i parents become much more intolerant because of the fall in p^j (implying $\eta_{\Delta_i} \leq -1$), then, even if $\frac{\partial D(\tau_i, p^i)}{\partial p^i} \leq 0$, the system can diverge to another equilibrium. In order to still have cultural substitution, intolerance should not change too greatly in a direction opposed to $D(\tau_i, p^i)$. In particular, we always have cultural substitution when $\frac{d\Delta_i}{dp^j} \geq 0$, that is when parents are more intolerant the more they represent a minority. It is not evident that it is the case. One can easily figure out cases where minorities are willing to integrate and so are less intolerant.

The Bisin and Verdier result is complemented by a condition on the elasticity of intolerance. This additional term somehow follows the same logic. Bisin and Verdier established that the vertical transmission probability for individuals of type i must decrease with the group size of type i to have stable equilibria. The new elasticity term implies that intolerance must also decrease with the size of the group (in the case $\eta_{\Delta_i} \geq 0$), or more precisely that it may increase but not too much (case $0 > \eta_{\Delta_i} > -1$).

If the sufficient (but not necessary) conditions of Proposition 2.1 are not satisfied then (2.3) may have multiple equilibria. They occur because reference groups depend on proportions. If groups were fixed over time (for instance $S_i = \gamma a_1 + (1-\gamma)a_2$ with γ a positive constant smaller than 1) multiple equilibria would not arise.

Example This example illustrates how population dependent reference groups generate multiple equilibria. It uses the transmission technology $D(\tau_i, p^i) = \tau_i$ that satisfies $\frac{\partial D(\tau_i, p^i)}{\partial p^i} \leq 0$, and cost function $C(\tau_i) = \frac{\tau_i^2}{2}$ that satisfies the convexity and monotonicity assumptions. It also assumes imperfect empathy $V^{ij} = U^i(a_j, S_i, \alpha_i)$. Without social interactions the dynamic equation would have a unique stable equilibrium and it would be interior. Allowing for interactions, it generates multiple equilibria.

The two utility functions are

$$\begin{aligned} U^1(a_1, S_1, \beta) &= -\frac{a_1^2}{2} + a_1^\rho - \frac{\beta}{2}(a_1 - S_1)^2 \\ U^2(a_2, S_2, \delta) &= -\frac{a_2^2}{2} - \frac{\delta}{2}(a_2 - S_2)^2 \end{aligned}$$

Both types have a concern for conformity with respect to their reference group and have the same cost $-\frac{a_i^2}{2}$. However, only type 1 individuals benefit from contributing a_1. Their payoff is a_1^ρ with $0 < \rho < 1$.

While deriving endogenously the reference groups is beyond the scope of this example, I briefly try to provide some intuition for these. A small minority is expected to be closed to influences outside its own group. For p small, the reference group of type 1 individuals is only composed of individuals of their own type. As p increases, the community of type 1 becomes more integrated and starts mixing with type 2 individuals who enter in their reference group. However after some threshold, their influence decreases, possibly from two effects: first, type 1 are now a large majority of the population and they become more intolerant towards marginal agents; second, type 2 people, now a small minority, are less open to influences, as type 1 was when it was a minority. Figure 2.2 illustrates the weights for a_1 and a_2 in S_1.[3]

The upper curve is the weight γ_{11}, the lower curve is γ_{12}. For $p = 0.6$ the two weights are equal: type 1 consider equally all individuals when their own group represents a small majority of the population.[4] The weights in S_2 are defined symmetrically, such that they are equal for $p = 0.4$.

$D(\tau_i, p^i) = \tau_i$ implies that stationary states are such that $\tau_1 = \tau_2$. Maximization of (2.5) yields that optimal efforts are such that $\tau_i = p^j \Delta_i$. Equilibria

[3]The exact functions are available upon request. All the functions of the problem are continuously differentiable such that multiple equilibria cannot be attributed to discontinuities.

[4]In other words, a type is "perfectly" tolerant, i.e. the two weights are equal, when he is not at risk of becoming a minority.

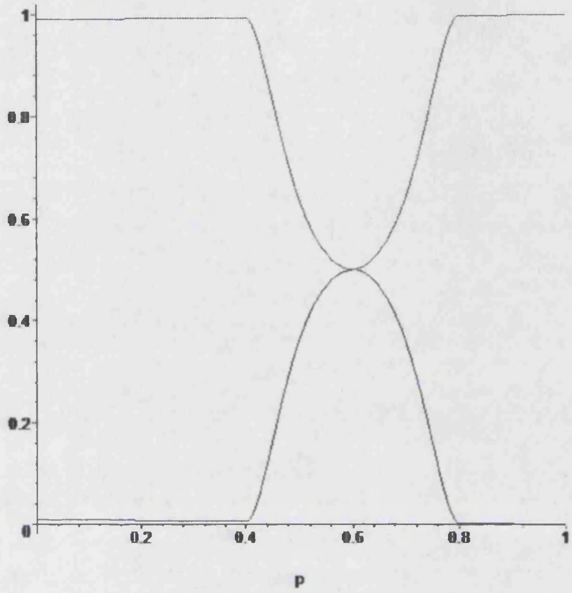

Figure 2.2: Weights

are therefore solutions of $p = \frac{\Delta_1}{\Delta_1+\Delta_2}$.

Figure 2.3 shows the two curves p and $\frac{\Delta_1}{\Delta_1+\Delta_2}$. For the chosen parameters[5], there are three interior equilibria $p_1^* < p_2^* < p_3^*$. p_1^* and p_3^* are stable with respective basin of attraction $(0, p_2^*)$ and $(p_2^*, 1)$. p_2^* is unstable. p_1^* represents a rather homogeneous society, with two large groups whose behaviors are quite similar. The type 2 community who does not value the action a_2 in itself is however willing to take it by conformity. p_3^* corresponds to an economy with a minority whose behavior is markedly different (a_2 is almost zero). This example illustrates how initial conditions matter and how an exogenous shock can induce a switch between two different equilibria. For instance, a sudden immigration of type 1 individuals potentially changes the long run equilibrium from a homogeneous to a split society.

[5] $\rho = 0.2$, $\beta = 1$, $\delta = 3$.

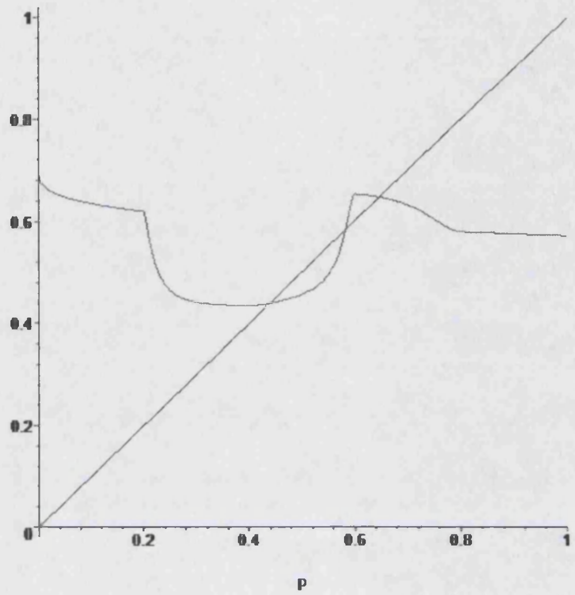

Figure 2.3: Equilibria

2.2.2 Crowding out

I show in this section that a change in the parameter α has a crowding out effect on any stable equilibrium p^*. The main aim is to show that intergenerational change is triggered by incentives that have an unexpected negative effect in the long run. The parameter α should be understood as a constraint that agents must take into account when maximizing their utility function. D is constrained to be only a function of τ_i such that equilibrium p^* is determined by

$$p^* = \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)(p^*) \tag{2.6}$$

Assume that α is a scalar and that it has a positive impact on a_2, $\frac{\partial a_2}{\partial \alpha} > 0$.[6] A policy increasing α is beneficial for the action, at least in the short run. In particular, if a_1 depends (weakly) positively on α, then the overall

[6]Remember that a_1 and a_2 are the short run equilibrium actions of respectively any type 1 and any type 2 individual. $\frac{\partial a_2}{\partial \alpha}$ must be understood as the partial change in the short run of $a_2(A_2, \alpha)$, holding A_2 constant. The total change in $\frac{da_2}{d\alpha}|_p$ in the short run is proportional to and of the sign of $\frac{\partial a_2}{\partial \alpha}$. This is a consequence of moderate social influence.

effect is unambiguous. But the change in a_1 and a_2 implies that the long run equilibrium is modified, since it alters the education efforts.

The derivative of (2.6) with respect to α tells us how p^* changes in the long run. I focus here on a simple case to illustrate the different effects at play, but the general case is given in the appendix. Assume $\frac{\partial U^1}{\partial \alpha} = 0$ and $\frac{\partial S_1}{\partial a_2} = 0$: type 1 utility is independent of α, and more importantly, S_1 is independent of a_2. This implies that a_1 is independent of α. Type 1 individuals do not take into account the behavior of type 2 agents when deciding on their own action. This restriction is helpful in understanding the different effects because it holds a_1 constant such that we can concentrate only on a_2. Finally, parents exhibit imperfect empathy when choosing education. The result does not hinge on any of these assumptions.

From (2.6), it follows that

$$
\begin{aligned}
K\frac{dp^*}{d\alpha} &= -(1-p^*)\,U_1^1(a_2,S_1,\alpha)\frac{1}{1-M_2}\frac{\partial a_2}{\partial \alpha} \\
&\quad +p^*\left[U_2^2(a_1,S_2,\alpha)-U_2^2(a_2,S_2,\alpha)\right]\frac{\partial S_2}{\partial a_2}\frac{1}{1-M_2}\frac{\partial a_2}{\partial \alpha} \\
&\quad +p^*\left[U_3^2(a_1,S_2,\alpha)-U_3^2(a_2,S_2,\alpha)\right]
\end{aligned}
\tag{2.7}
$$

where $K > 0$, U_k^i is the derivative of U^i with respect to its k-th variable and $M_2 = \frac{\partial a_2}{\partial S_2}\frac{\partial S_2}{\partial a_2} < 1$ by MSI and because S_2 is a convex combination of a_1 and a_2.

Crowding out results from the first two terms on the right hand side of (2.7). Given that $U_{11}^1 < 0$ and the first order condition $U_1^1(a_1,S_1,\alpha) = 0$, $-U_1^1(a_2,S_1,\alpha)$ has the sign of $a_2 - a_1$.

On the other hand, $U_2^2(a_1,S_2,\alpha) - U_2^2(a_2,S_2,\alpha)$ has the sign of $a_1 - a_2$ because of strategic complementarity. Therefore the first two terms (call them respectively E_1 and E_2) have opposite signs.

Assume that $a_2 < a_1$, then E_1 is negative, while E_2 is positive. The total effect can be of any sign. The interpretation is the following: E_1 is the change in type 1 parents education effort due to the change in a_2, holding everything else constant. Since S_1 is not affected by the change in a_2, a_1 is constant. It implies that a_2 is closer to a_1 than before and Δ_1 falls. Type 1 parents decrease their effort because if they fail to transmit their trait, their children will still have a behavior quite close to theirs. As a consequence, it has become less important to educate children. This is what I refer to as a "neglect effect". Parents neglect to transmit their cultural trait because behaviors are quite homogeneous in the population, but they fail to understand that this relies on the current distribution of traits in the population.

Secondly, E_2 is minus the change in Δ_2 due only to the change in S_2. To understand its sign, imagine that both a_1 and a_2 are fixed and that S_2 increases. Since $a_2 \leq S_2 \leq a_1$ then it must be that S_2 gets closer to a_1. Given strategic complementarity, the marginal impact on U^2 is larger for a_1 than for a_2. Given the higher reference point, type 2 parents decrease their effort because type 1 action is (marginally) closer to the reference. This has a positive impact on p^*.

The change in a_2 has similar consequences on type 1 and 2 education efforts, and so opposite consequences on p^*.

Finally, there is a third effect, E_3, that is the change in Δ_2 due to the change in α. Without any further specification, this last term has not a well defined sign and may reinforce any of the first two effects. It is not, strictly speaking, related to the change in a_2 as it would not be present had we considered $\frac{dp^*}{da_2}$. The strict consequence of increasing a_2 is embedded in E_1 and E_2. However, a policy changing α affects education directly through E_3.[7]

[7] It is not true without imperfect empathy when V^{ij} is independent of α, whereas E_1 and E_2 are always present.

As a conclusion, when a_2 increases, first it has a negative "private" effect on education decision: type 1 decrease their effort because type 2 individuals are less different from them than they used to be. Second, it has a positive "social" effect through social interactions, because the reference point is increased for type 2, such that type 2 parents feel type 1 individuals are now closer to this reference point.

Proposition 2.2 summarizes these results.[8]

Proposition 2.2 *If interactions exhibit strategic complementarity, an increase in action a_j, with $a_j < a_i$, has both a positive "social" and a negative "private" effect on p^*.*

From equation (2.7) there is always crowding out when there is no social effect. If there are no social interactions, then p^* decreases without any ambiguity. Incentives of a purely private activity that parents want to transmit are always crowded out. It seems however difficult to think of an activity completely independent of interactions and culturally transmitted. This particular case makes the result trivial. I focus in this article on situations with social interactions precisely because they make the problem more interesting, as well as more realistic.

We will say that there is no crowding out when p^* increases with α. This occurs (abstracting from E_3) if the social effect is larger than the private effect. Behaviors such that a change in the peer group action affects considerably (other things being equal) the marginal utility of one's own action are less prone to crowding out. When private and peer group behaviors are close to being perfect complements, then the social effect E_2 is large. Strategic complementarity is crucial in avoiding crowding out. Highly social behaviors, like playing a team sport, enter into this category.

[8] As shown in the appendix, they hold under general assumptions.

Without strategic complementarity (i.e. $U_{12} \leq 0$) there is always crowding out. A classical example is the contribution to a public good where utility depends on the quantity of public good $G = \sum_i g_i$ and on the cost of one's own contribution g_i. There are n individuals in the population. The reference group of individual i is formed by everyone but himself and $S_i = \frac{1}{n-1}\sum_{j \neq i} g_j$. Two types of individuals $i \in \{1,2\}$ with utility functions U^1 and U^2 cohabit. $U^i = u^i(g_i + (n-1)S_i) - c^i(g_i)$, with u^i concave and c^i convex. $U^i_{12} = (n-1)\left(u^i\right)''(G)$ is negative and so incentives must be crowded out. The interpretation of E_1 is not affected but E_2 means that when public good provision becomes larger free riding is seen as making more sense (substitution between g_i and S_i).

Throughout this discussion, the point emphasized is that private and social behaviors are complementary. Crowding out occurs because there is substitution in the transmission process between vertical and oblique channels. More powerful incentives impose a negative externality on cultural transmission. Strategic complementarity acts as an opposing force against this mechanism by creating a positive externality.

When $\frac{dp^*}{d\alpha} < 0$, there is crowding out on different grounds.

First, an intervention increasing a_2 in the short run has a negative effect in the long run (as long as $\frac{\partial S_2}{\partial p} > 0$) :

$$\frac{da_2}{d\alpha} = \underbrace{\frac{1}{1-M_2}\frac{\partial a_2}{\partial \alpha}}_{\text{short run}} + \underbrace{\frac{1}{1-M_2}\frac{\partial a_2}{\partial S_2}\frac{\partial S_2}{\partial p}\frac{dp^*}{d\alpha}}_{\text{long run}} \tag{2.8}$$

If the policy aimed at increasing a_2, then it is crowded out. Note that, as is typical with social interactions, the social multiplier $1 - M_2$ amplifies the changes. Social interactions also imply that the change in p^* affects a_2. If $\frac{\partial a_2}{\partial S_2} = 0$ then there is no long run effect. Social interactions have a dual role in crowding out. First they mitigate the fall in p^*, as shown above. Second they exacerbate how the fall in p^* affects a_2. They may imply that a_2 actually decreases because of the modification in the peer group structure. It never happens without interactions because $\frac{da_2}{d\alpha}$ would be equal to $\frac{1}{1-M_2}\frac{\partial a_2}{\partial \alpha} > 0$. This property of social interactions drives the intergenerational change because it says that a_2 decreases over time.

Second, when $a_2 < a_1$, incentives tend to homogenize behaviors towards type 1 preferences. However, by promoting their behavior, it actually decreases their proportion in the population. It can be self defeating for a group of people to make individuals outside their group behave in a similar way.

Third, if one takes the average action $\bar{a}$ in the society as a measure of how much it is followed, then again, the intervention is crowded out.

$$\frac{d\bar{a}}{d\alpha} = (1-p^*)\frac{da_2}{d\alpha} + (a_1 - a_2)\frac{dp^*}{d\alpha} \tag{2.9}$$

Even if $\frac{da_2}{d\alpha} > 0$, the average behavior can fall as the proportion of types shifts in favor of type 2 individuals. Once again social interactions have opposing effects on $\bar{a}$.

The next section illustrates these different effects in a model of public good provision but first it is shown that the example above may exhibit crowding out in p^* when δ is increased. Figure 2.4 displays the different equilibria of the model when α varies on a certain interval.[9] It is not possible to find the closed forms of p^* and so numerical computation is used.

The upper line of Figure 2.4 corresponds to p_3^*, the upward sloping part of the parabola to p_2^*, the downward sloping part to p_1^*. Proposition 2.2 does not apply to unstable equilibria and so we should look only at the evolution of p_1^* and p_3^*. They both decrease with δ because a larger δ tends to homogenize

[9] This interval is chosen such that it corresponds to the situation with 3 equilibria that Figure 3 illustrates.

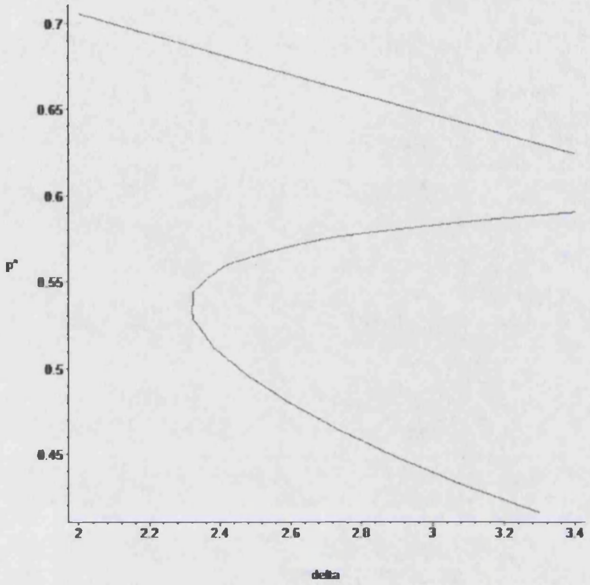

Figure 2.4: p^* as a function of δ

behaviors towards type 1 action. This model does not allow tractable forms and so we cannot make sharp predictions on the influence of parameters. The next section remediates to this issue.

2.3 Average behavior and the shape of interactions

In this section I use the framework developed in Section 2 to build up a simple model that illustrates how interactions in the form of the function γ affect the crowding out results. I assume that utility functions are quadratic. It turns out that in this special case we can always solve explicitly for the equilibrium and study how the shape of interactions influence the equilibrium actions. I investigate in particular the crowding out of the average action $\bar{a}$ when the desire to conform to the reference group increases. This happens for instance if behaviors become publicly observable, or there is praise for the reference action, or social stigma associated with "deviant" behaviors is stronger, etc. These institutional arrangements are common to ensure cooperation in a public good game. Two main results emerge from this analysis. First, when the reference action is the average action in the population, then $\bar{a}$ does not depend on conformity. When the reference group is perfectly representative of the population then crowding out cancels exactly any short run positive effect. Second, if the reference action is not the average action, then how interactions are affected by changes in type proportions determines whether $\bar{a}$ increases or not with conformity.

In order to fix ideas, I consider that agents have to choose a contribution to a public good but this is merely for expositional purpose. The average contribution $\bar{a}$ is proportional to the provision of public good in the society. Type 1 individuals are strong cooperators and, as in the last section, always choose the same contribution a_1. Their utility function U^1 does not need to be specified, but I assume that it is concave and quadratic in a_1, $U^1_{11} = -\beta < 0$ with β being a constant. Type 2 individuals are conditional cooperators and are influenced by a reference contribution. Their utility $U^2(a_2, S_2, \alpha)$ is assumed to consist of two components

$$U^2(a_2, S_2, \alpha) = u(a_2) - \alpha(a_2 - S_2)^2 \tag{2.10}$$

$u(a_2)$ is the private utility associated with a contribution and is such that $u''(a_2) = -\omega < 0$ with ω a constant. The interactions term in the form of $-\alpha(a_2 - S_2)^2$ captures a pure conformity effect. This functional form with conformity and quadratic private utility function is common in the social interactions literature (Brock and Durlauf 2001, Glaeser and Scheinkman 2003). The private utility is less restrictive than it may seem. It encompasses cases where conditional cooperators do not enjoy any benefits from the public good but incur a linear cost $u(a_2) = -a_2$, and cases where they do enjoy linear benefits and incur a linear or quadratic cost of contributing: $u(a_2) = -a_2 + G$ and $u(a_2) = -\frac{a_2^2}{2} + G$, where the public good G is the sum of all the contributions. Finally, I assume that $a_1 > a_2$ is always satisfied.[10] The parameter α measures the strength of conformity. A higher α means that for a given proportion of strong cooperators the difference between conditional and strong cooperators contributions is smaller. The weight $\gamma(p)$ is not specified in order to understand how it influences the equilibrium properties, and in particular public good provision.

It is shown in Appendix A that quadratic utility functions ensure that there is only one interior equilibrium p^*, and that it is stable. It is such that, omitting the variables U^1 and U^2 depend on,

$$p^* = \frac{U_{11}^1}{U_{11}^1 + U_{11}^2} = \frac{\beta}{\beta + \omega + \alpha} \tag{2.11}$$

It is immediate that the equilibrium proportion of strong cooperators falls with conformity α. There is always crowding out in proportions. Using the terminology developed in the preceding section, it can be shown that the private effect is always larger than the social effect. Conformity implies that the complementarity between private and reference contribution is not large enough to compensate for the fall in strong cooperators education effort. However a smaller p^* does not imply that public good provision is smaller and higher conformity may actually be beneficial to the level of public good in the population. As long as conformity increases contributions there is a tradeoff between having a large number of strong cooperators but conditional cooperators giving small amounts, and a smaller strong cooperators population but conditional

[10]The derivative of u can be written $u'(a_2) = \omega a_2 + v$. The condition $a_1 > a_2$ is equivalent to $a_1 > \frac{v}{\omega}$. It is always satisfied when $v \leq 0$, even if $\omega = 0$.

cooperators contributing more.

Calculation of the derivative of $\bar{a}$ gives necessary and sufficient conditions on $\gamma(p)$ for public good provision to increase with conformity. All the functions $\gamma(p)$ such that $\frac{d\bar{a}}{d\alpha} = 0$ can be found, and they will be useful to understand how the shape of γ determines the variations of public good level.

Proposition 2.3 *Public good provision is not affected by conformity if and only if for every p such that* $0 \leq p < \frac{\beta}{\omega+\beta}$ $\gamma(p) = \gamma_C(p) = \frac{C(1-p)-\omega}{\beta-p(\omega+\beta)}p$ *where C is a constant.*

The only function γ *that satisfies* $0 \leq \gamma(p) \leq 1$ *and* $\frac{d\bar{a}}{d\alpha} = 0$ *is* $\gamma(p) = \gamma_{\omega+\beta}(p) = p$.

Proposition 2.3 makes two statements. First all the γ_C functions let provision be unaffected by conformity. They are defined only on the interval $[0, \frac{\beta}{\omega+\beta})$ because the equilibrium proportion p^* is necessarily in this interval, such that the value of $\gamma(p)$ for $p \geq \frac{\beta}{\omega+\beta}$ does not matter in equilibrium. The tradeoff between higher contributions and smaller strong cooperators population size is perfectly balanced with these functions. If the constant C is greater than $\omega+\beta$ then they are positive on the whole interval. However, for every $C > \omega+\beta$ they are greater than 1 when p is large enough and so are not possible candidates for the weight γ. So the only admissible γ_C function is obtained for $C = \omega+\beta$ and in this case the reference group is simply the average contribution in the population, $S_2 = \bar{a}$. If agents give the same weight to all the individuals in the population then conformity has no effect on public good provision.

A more general question is: for a given function $\gamma(p)$, is it possible to know whether it guarantees that public good provision increases with conformity? It actually is and the criterion is graphically easy to observe. First, notice that there is a one to one mapping between conformity level α and equilibrium proportion p^*. This allows us to give conditions on $\gamma(p)$ for $p \in (0, \frac{\beta}{\omega+\beta})$

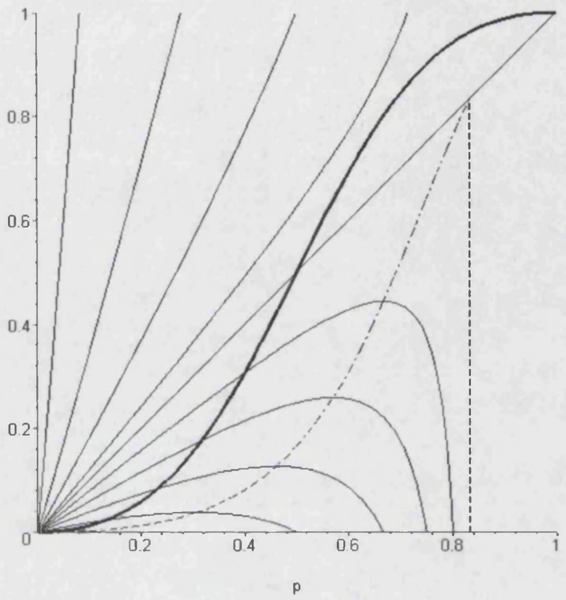

Figure 2.5: γ and γ_C curves

because p^* describes this interval when α goes from 0 to infinity. By looking at $\gamma(p)$ we are able to derive properties that work for all the possible values of α.

To get the answer, one has to draw the curve of γ as a function of p, and then all the γ_C curves[11]. At each point $\tilde{p}$ the γ curve crosses one (and only one) γ_C curve. If at this point the slope of γ is higher than the slope of γ_C then for the conformity level $\tilde{\alpha}$ that results in $\tilde{p}$ being the equilibrium, the quantity of public good is decreasing with conformity. It means that if we start in the equilibrium $\tilde{p}$, that corresponds to a conformity level $\tilde{\alpha}$, and we increase conformity then in the long run we reach an equilibrium with less public good.

Figure 2.5 shows how the variations of public good provision can be obtained by drawing the γ function. The bold curve is an arbitrary $\gamma(p)$. The other curves are some of γ_C curves, including the 45 degree line. The vertical dotted line is at the proportion $\frac{\beta}{\omega+\beta}$, that is the highest possible equilibrium

[11]These are actually "iso-public good" curves along which provision is constant.

proportion. The function γ has to be defined on the whole interval $[0, 1]$ but its values for proportions greater than $\frac{\beta}{\omega+\beta}$ do not affect the equilibrium. The dashed curve delimits the region where the γ_C curves are downward sloping. At each point of the set $(0, \frac{\beta}{\omega+\beta}) \times [0, 1]$ the curve of γ crosses only one γ_C curve. Graphically it is immediate to check when the γ curve is steeper than the γ_C curve it crosses. Another way of looking at Figure 2.5 is that provision is larger on higher γ_C curves. If an equilibrium lies on a higher γ_C curve than another one then public good provision is higher in that equilibrium.

The graph of all the γ_C curves shows that any increasing function γ exhibits decreasing public good provision as long as it is in the lower right region of downward sloping γ_C curves. It means that for γ small enough conformity always reduces public good provision in the society. In other words, agents must give some sufficient weight to strong cooperators in their reference group.

A general criterion for an increasing γ function to yield increasing public good provision is that its variations should be moderate, such that its slope is not very steep. From Figure 2.5, it appears as well that concave functions easily ensure that public good provision is reinforced by conformity, while it is more difficult for convex γ functions.[12] The intuition behind this result appears when we decompose the different steps of an increase in conformity. Consider that we start in a long run equilibrium and that conformity increases. This is beneficial in the short run and contributions increase. However in the long run the proportion of strong cooperators in the population shrinks. This in turn affects the reference groups such that the weight of strong cooperators falls in the reference contribution.[13] This change of balance between types tends to decrease contributions, because agents wants to conform to a smaller reference. This effect is particularly large when the weight γ varies a lot with

[12] Though it is not impossible. Consider for instance $\gamma(p) = \gamma_{\tilde{C}}(p)$ as long as $\gamma_C(p) \leq 1$ and 1 otherwise, where $\gamma_{\tilde{C}}$ is one of the convex γ_C functions.

[13] This assumes an increasing γ function.

proportion. In that case there is large shift in the composition of the reference group and it affects contributions dramatically.

$\gamma(p)$ somehow measures sorting in the society: if there is perfect segregation then type 2 people do not have any contact with type 1 people and it would make sense to assume $\gamma = 0$. If there is no segregation at all, such that reference groups are perfectly representative of the society, then $\gamma(p) = p$ and the quantity of public good provided is neutral to conformity. In many situations it seems natural to assume a less than perfect mixing of the population. Figure 2.5 provides an example where the largest group in the population is over-represented in the reference group. For this kind of γ function, $\bar{a}$ decreases unless the equilibrium proportion is high, that is conformity is low. For small values of α it is possible to increase public good provision by promoting conformity. However above some threshold it is counterproductive.

The particular example of quadratic utility functions allows also to understand how a change in interactions, that is a change in the γ function, affects the quantity of public good. Starting from an equilibrium and changing the function γ does not modify the equilibrium proportion in the long run but it changes $\bar{a}$ in equilibrium. If the new γ curve crosses a higher γ_C curve at the this equilibrium proportion then provision has risen. The γ function depicted on Figure 2.5 is of the type $\gamma(p) = \frac{1}{1+\left(\frac{1-p}{p}\right)^n}$ with n a positive number. The large is n the closer is γ to a step function equal to 0 on $[0, \frac{1}{2})$ and to 1 on $(\frac{1}{2}, 1]$. In other words the larger is n the closer is the reference contribution to the contribution of the majority group. For a given conformity level, a larger n implies that less public good provision if in the equilibrium proportion is smaller than half, and more public good if it is greater than half. Figure 2.6 shows that a higher n implies a higher curve, and so higher provision, only for proportions above one half.

Finally from (2.9) and (2.8) it appears that $\bar{a}$ falls before a_2 does. The

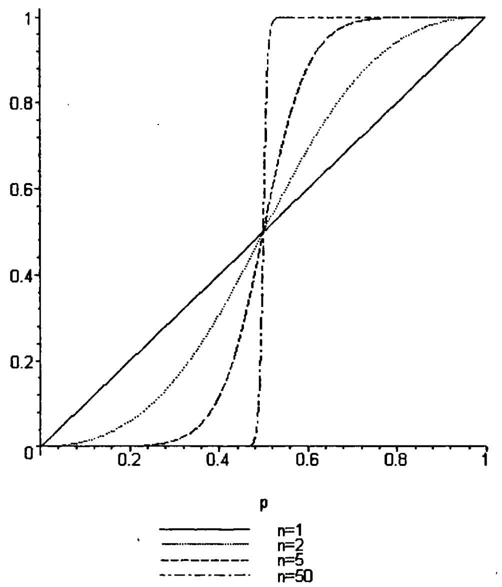

Figure 2.6: γ curves

conformity level that maximizes public good provision is always smaller or equal than the level that maximizes conditional cooperators contributions. In this sense a certain degree of heterogeneity, whereby the society is made of two groups rather distinct in their choices may actually be optimal when compared with a more homogeneous society.

The setup of this section can be used for various issues where interactions are determined endogenously. It is not the point of this chapter to build a model that would provide a structural form to the weights in the reference contribution. It underlines how interactions structure the equilibrium and offers a framework to understand how they interact with cultural transmission. It is then useful to think in terms of a particular issue where interactions are determined by sorting into neighborhoods, membership to organizations, etc. This is left to future research.

2.4 Related evidence

As mentioned in the introduction Putnam (2000) provides a wealth of evidence on intergenerational decline. He establishes that for many indicators individuals have not changed through their lives, and that the observed decline is mostly due to generational change. It is particularly striking for voting. The continuous decline in turnout rates has been investigated, among many social scientists, by Putnam, and is not restricted to the US (see Rattinger 1992 for Germany, Phelps 2004 for the United Kingdom, Blais et al. 2004 for Canada argue that it reflects a large cultural change). The continuous fall in voting is a concern for the political system (Highton 2001) and it has been argued that voting should be made compulsory in the US (Lijphart 1997). Voting theories have tried to explain the secular fall in American turnout rates but the voting paradox has severely complicated the task. The paradox emerges when one realizes that the probability for a voter to be pivotal decreases quickly with the number of voters. If there is some cost of voting, even low, then the expected benefits must be extremely large to exceed the cost. As a consequence no rational voter would vote. Riker and Ordershook (1968) resolved the paradox by introducing a taste for voting. They find that a high sense of citizen duty has a much larger impact on voting than high values of the probability of the election being close and high values of the benefits. Other studies find that the sense of duty appears to be the best predictor of voting (see Mueller 1989 for a review). This provides the rationale for introducing different types of individuals that differ in their preferences for voting. This taste is usually invoked to explain the fall in turnout rates (Aldrich 1993), but why it changed is not clear.[14] Sociologists have studied the generational nature of the problem but recognize that they fail to identify its causes (Miller 1992).

[14]However, see Castanheira (2003) for an explanation using the rational voter model without introducing any taste for voting.

Voting, even though done privately, is influenced by social pressure. I have already alluded to the guilt feeling reported by Putnam (2000) that young people feel less guilty than old people do when they do not vote. Quite interestingly Harbaugh (1996) shows that a quarter of non voters in American presidential elections lie and claim they did vote when asked. Models of voting cannot account for lying and Harbaugh develops a model where people get praise for voting. Non-partisan campaigns that emphasize citizen duty and responsibility for the community offer evidence that this effect is at play.

My model provides an explanation of how the fall in turnout may be due to generational change. However the model requires something to be crowded out. Aldrich (1993) argues that costs are an unlikely candidate to explain voting patterns because they have fallen over time. Cultural transmission modelling provides a very different answer. Voting costs have been reduced because of liberalized registration laws, elimination of poll taxes and more recently post and online voting. In 1993 the National Voter Registration Act, known as Motor Voter, was signed into effect by President Clinton. This act makes the voter registration process easier by reducing "the necessary and burdensome bureaucratic obstacles" (as quoted on the official website for the National Voter Registration Act of 1993). Registration must be made available at agencies that provide public assistance. At the department of motor vehicles it must be incorporated into the process of applying for or renewing one's driving license. Internet voting systems have already been used in the US, the UK, Ireland, Switzerland, and Estonia. In the rational voter framework this points unambiguously towards an increase in turnout rates. I argue that this is a short run result and that in the long run the opposite may be true. To facilitate voting actually makes it so easy that generations tend in a way to forget why people used to vote. From a policy perspective this is a rather frustrating result and calls for education campaigns.

Much of the literature on church attendance in the US suggests that there are no cohort effects. Furthermore attendance rates have been stable for the last thirty years. However Miller and Nakamura (1996) argue that it should not be the case because the baby boomers are aging, and religiosity of this cohort is usually found to increase with age. They logically conclude and then check that old generations are replaced by young generations who attend church less regularly. Such patterns have also been studied in Britain by Voas and Crockett (2005). They find a strong cohort effect in church attendance and religious affiliation.

Another body of research (Peele 1984, Sorman 1985) argues that the decline in traditional values has produced a religious backlash. This fits well the predictions of the model, whereby a shock to morals in the sixties would lead to a revival later on. A growing gap between liberals and conservatives reinforces conservatism.

Similarly, each individual cohort is as trusting as it ever was but the overall level of trust has fallen because the old generation is replaced by a much less trusting one. Robinson and Jackson (2001) indicate that the over-time decline in trust is partly explained by aging but that it is mainly a cohort effect initiated in the 40s. Putnam concludes is that half the decline observed in social capital and civic engagement can be traced back to generational change. The Second World War, and the strong focus on national unity and patriotism that accompanied it, may provide the exogenous change required in the model to have crowding out. After the war, people felt they had to be more civically engaged, or that voting was indeed a civic duty that one could not miss. In more egalitarian societies, where behaviors are less class specific, homogenization may have led to crowding out. The theory says that this upsurge backfired because these preferences were not transmitted strongly enough.

2.5 Conclusion

This chapter argues that cultural transmission can explain the decline of a broad range of values. Parents neglect to transmit preferences when behaviors are homogeneous. This result relies on two mechanisms: first, the substitutability between vertical and oblique transmission, second social interactions. In particular I emphasize how a policy has unexpected consequences in the long run that crowds it out.

This chapter also shows that the Bisin and Verdier (2000) cultural transmission process can be extended to situations where agents interact. As culture is usually characterized by peer effects and interactions, this is an interesting point. A subject for future research is to extend this framework to let agents choose their peer group. In many cases, people sort into neighborhoods where education is then made. Parents would have to make a residential choice. Although this is partially included in the model, since parents choose an education effort that can reflect a neighborhood choice, it would be fruitful to see how a precise formulation would affect reference groups and would make predictions on the geographic distribution of preferences.

Finally, the framework developed in this chapter serves to understand how institutional arrangements modify behaviors by affecting the distribution of preferences. Crowding out casts a shadow on the benefit of having good institutions. Strong beneficial constraints are defeated by socialization, such that it may be optimal to opt for weak constraints, maybe with smaller short run benefits, to preserve a "good" cultural trait. As argued in the introduction, this result leads to a dynamic theory of institutional change. If institutions designed to promote a behavior reach the opposite outcome then at some point in time they should be replaced by others. It requires to make institutions endogenous, and this is the purpose of the next chapter.

2.6 Appendix 1

2.6.1 Two "impossibility results"

The next two propositions prove two impossibility results for quadratic utility functions.

A simple sufficient condition for cultural substitutability is that $\frac{d\Delta_i}{dp^j} \geq 0$ for both types. Proposition 2.4 shows that under fairly general conditions, this is never satisfied for both i and j. V_k^i is the derivative of V^i with respect to its k-th variable.

Proposition 2.4 *If* $sign(V_1^i(a_j, S_i, \alpha)) = -sign(V_1^j(a_i, S_j, \alpha))$ *and* $V_{112}^i = V_{111}^i = V_{112}^j = V_{111}^j = 0$ *then* $sign\left(\frac{d\Delta_i}{dp_j}\right) = -sign\left(\frac{d\Delta_j}{dp_i}\right)$.

The first condition is always satisfied when imperfect empathy is assumed, i.e. when $V^{ij} = U^i(a_j, S_i, \alpha)$, as in Bisin and Verdier (2001) and most of the literature using this model. More generally, it holds whenever type i considers a_j to be too small in the sense that a higher a_j, holding everything else constant, would increase its valuation of being a type j, and on the contrary type j considers a_i to be too large.

The second condition is satisfied for all the utility functions quadratic in a_i ($V_{111} = 0$) and for most of the social interactions specifications used in the literature. A very common example is $-(a_i - S_i)^2$, another is $a_i S_i$ (both are given in Brock and Durlauf 2001). Both satisfy $V_{112} = 0$.

The most widely used utility functions yield that by allowing agents to interact, we cannot expect to rely on the simple conditions that $\frac{d\Delta_i}{dp^j} \geq 0$ for both types to have a unique stable equilibrium.

Proof of Proposition 2.4

Define the matrices $\frac{d\Delta}{dp} = \begin{bmatrix} -\frac{d\Delta_1}{dp} \\ \frac{d\Delta_2}{dp} \end{bmatrix}$, $\frac{\partial S}{\partial p} = \begin{bmatrix} \frac{\partial S_1}{\partial p} \\ \frac{\partial S_2}{\partial p} \end{bmatrix}$, $\frac{dA}{dp} = \begin{bmatrix} \frac{dA_1}{dp} \\ \frac{dA_2}{dp} \end{bmatrix}$ and similarly $\frac{da}{dp} = \begin{bmatrix} \frac{da_1}{dp} \\ \frac{da_2}{dp} \end{bmatrix}$. $\delta = \begin{bmatrix} -\frac{\partial\Delta_1}{\partial a_1} & -\frac{\partial\Delta_1}{\partial a_2} \\ \frac{\partial\Delta_2}{\partial a_1} & \frac{\partial\Delta_2}{\partial a_2} \end{bmatrix}$ and $\phi = \begin{bmatrix} -\frac{\partial\Delta_1}{\partial S_1} & 0 \\ 0 & \frac{\partial\Delta_2}{\partial S_2} \end{bmatrix}$. $D = \begin{bmatrix} \frac{1}{m_1} - M_{11} & -M_{12} \\ -M_{21} & \frac{1}{m_2} - M_{22} \end{bmatrix}$, $\frac{1}{m} = \begin{bmatrix} \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix}$ and $M = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$, where $m_i = \frac{\partial a^i}{\partial S^i}$ and $M_{ij} = \frac{\partial S^i}{\partial a^j}$.

Actions a_i, reference groups S_i and intolerance Δ_i are such that $a_i = a_i(S_i, \alpha)$, $S_i = S_i(a_1, a_2, \alpha)$, and $\Delta_i = \Delta_i(a_1, a_2, S_i, \alpha)$. Differentiation shows that $D\frac{da}{dp} = \frac{dA}{dp}$, hence $\frac{da}{dp} = D^{-1}\frac{dA}{dp}$ and $\frac{dA}{dp} = \frac{\partial S}{\partial p} + M\frac{da}{dp} = (I + MD^{-1})\frac{\partial S}{\partial p}$.

And therefore $\frac{d\Delta}{dp} = \delta\frac{da}{dp} + \phi\frac{dA}{dp} = \delta D^{-1}\frac{\partial S}{\partial p} + \phi\,(I + MD^{-1})\frac{\partial S}{\partial p}$.

Using $I + MD^{-1} = \frac{1}{m}D^{-1}$,

$$\frac{d\Delta}{dp} = \left(\delta + \phi\frac{1}{m}\right) D^{-1}\frac{\partial S}{\partial p} \tag{2.12}$$

When $V^i_{112} = V^i_{111} = V^j_{112} = V^j_{111} = 0$, using Taylor expansions of V^1 and V^2, $\delta + \phi\frac{1}{m} = \begin{bmatrix} -V^1_1(a_2, S_1, \alpha) & V^1_1(a_2, S_1, \alpha) \\ -V^2_1(a_1, S_2, \alpha) & V^2_1(a_1, S_2, \alpha) \end{bmatrix}$, and

$$\frac{d\Delta}{dp} = \omega \begin{bmatrix} V^1_1(a_2, S_1, \alpha) \\ V^2_1(a_1, S_2, \alpha) \end{bmatrix} \tag{2.13}$$

where ω is a scalar. If $\text{sign}(V^i_1(a_j, S_i, \alpha)) = -\text{sign}\big(V^j_1(a_i, S_j, \alpha)\big)$, then (2.13) shows that $\text{sign}\left(\frac{d\Delta_i}{dp^j}\right) = -\text{sign}\left(\frac{d\Delta_j}{dp^i}\right)$. QED.

Next, we focus on the case where $D(\tau_i, p^i) = D(\tau_i)$. Bisin and Verdier (2001) underline that when D does not depend on p^i, cultural substitutability is automatically satisfied. In the more general framework of this chapter, it is not true any more. It is interesting to study this case because by making D

directly independent of p^i, it allows us to isolate the effect of p^i on stability through the intolerance factors.

In this particular case and from the dynamic equation (2.3), the interior equilibrium is such that $\tau_i = \tau_j$, when parents exert the same education effort. (2.3) defines p^* implicitly $p^* = \frac{\Delta_1}{\Delta_1+\Delta_2}$.

Proposition 2.5 shows that imperfect empathy and quadratic utility functions in a_i produce a unique stable equilibrium.

Proposition 2.5 *With $D(\tau_i, p^i) = D(\tau_i)$, imperfect empathy and quadratic utility functions in a_i, the dynamic system defined by (2.3) has only one interior equilibrium, it is stable and such that $p^* = \frac{U^1_{11}(a_1,S_1,\alpha)}{U^1_{11}(a_1,S_1,\alpha)+U^2_{11}(a_2,S_2,\alpha)}$.*

To generate multiple equilibria with social interactions, we cannot use imperfect empathy and quadratic utility functions. We have to rely on more complex specifications. Proposition 2.5 makes Proposition 2.4 rather irrelevant when there is imperfect empathy as it says that we should not worry about any stability condition in this case. However, Proposition 2.4 still has some interest for more general V^i functions.

Proposition 2.5 shows how, under particular assumptions, the equilibrium is related to the characteristics of the utility functions. If U^1 becomes more concave than U^2 then p^* increases. Individuals with more concave utility functions are more sensitive to differences between a_i (or, alternatively, they are more intolerant) and so choose higher efforts. In equilibrium they represent a larger proportion of the population (as $p^* > \frac{1}{2}$ is equivalent to $|U^1_{11}| > |U^2_{11}|$). This proposition shows as well that parameters that do not enter into the second order derivatives have no influence on the equilibrium. By extension there is no crowding out for such parameters.

Proof of Proposition 2.5

We know that $p^* = \frac{\Delta_1}{\Delta_1+\Delta_2}$. Imperfect empathy implies that $\frac{\partial \Delta_i}{\partial a_i} = \frac{\partial U^i}{\partial a_i} = 0$

from the first order conditions of the maximization of U^i. Taylor expansions on $\Delta_i = U^i(a_i) - U^i(a_j)$ yield

$$p^* = \frac{U^1_{11}(a_1, S_1, \alpha)}{U^1_{11}(a_1, S_1, \alpha) + U^2_{11}(a_2, S_2, \alpha)} \tag{2.14}$$

The assumption $U^i_{11} < 0$ proves that $p^* > 0$. Quadraticity of the utility functions implies that $\frac{U^1_{11}(a_1,S_1,\alpha)}{U^1_{11}(a_1,S_1,\alpha)+U^2_{11}(a_2,S_2,\alpha)}$ is a constant and so that the equilibrium is stable. QED

2.7 Appendix 2: proofs of the propositions

Proof of Proposition 2.1

The proof of Proposition 2 in Bisin and Verdier (2001) provides the main part of the proof. Only the very end is different. First, the indirect cost function $H(d_i, p^i)$ of direct socialization is defined:

$$H(d_i, p^i) = \min_{\tau_i \in \mathbb{R}^n_+} C(\tau_i), \text{ s.t. } d_i = D(\tau_i, p^i) \tag{2.15}$$

Using the assumptions on C and D, the minimization problem is convex. As a consequence H is continuous in d_i and p^i and the argmin τ_i is a continuous mapping from $[0,1]^2$ into $\mathbb{R}^n$, $\tau(d_i, p^i)$. Therefore H is convex in d_i and satisfies $H(0, p^i) = 0 \; \forall p^i \in [0,1]$ and $\frac{\partial H}{\partial d_i} = 0$ when $d_i = 0$.

Let λ_i denote the Lagrange multiplier of the constraint in (2.15). The first order condition is

$$C'(\tau_i) = \lambda_i \frac{\partial D}{\partial \tau_i} \tag{2.16}$$

$$d_i = D(\tau_i, p^i) \tag{2.17}$$

Differentiation of (2.17) implies

$$1 = \frac{\partial D}{\partial \tau_i}\frac{d\tau_i}{dd_i} \tag{2.18}$$

$$0 = \frac{\partial D}{\partial \tau_i}\frac{d\tau_i}{dp^i} + \frac{\partial D}{\partial p^i} \tag{2.19}$$

Hence from (2.18) $\tau_i(d_i)$ is increasing in d_i. Differentiation of (2.16) yields with (2.19)

$$C''(\tau_i)\frac{\partial \tau_i}{\partial p^i} = \frac{\partial \lambda_i}{\partial p^i}\frac{\partial D}{\partial \tau_i} \tag{2.20}$$

(2.20) shows that sign $\left(\frac{\partial \lambda_i}{\partial p^i}\right)$ = sign$\left(\frac{\partial \tau_i}{\partial p^i}\right)$. But from (2.19) this has the sign of $-\frac{\partial D}{\partial p^i}$.

But, by the Envelope Theorem, $\frac{\partial H}{\partial d_i} = \lambda_i$, and hence

$$\text{sign}\left(\frac{\partial^2 H}{\partial p^i \partial d_i}\right) = -\text{sign}\left(\frac{\partial D}{\partial p^i}\right) \tag{2.21}$$

Individuals have to choose d_i:

$$\max_{d_i \in [0,1]} V^{ij} + \left[d_i + (1-d_i)\, p^i\right]\Delta_i - H(d_i, p^i) \tag{2.22}$$

Differentiation of the first order condition of the maximization problem implies that

$$\frac{\partial^2 H}{\partial d_i^2}\frac{\partial d_i}{\partial p^i} = \frac{\partial}{\partial p^i}\left[\left(1-p^i\right)\Delta_i\right] - \frac{\partial^2 H}{\partial p^i \partial d_i} \tag{2.23}$$

Hence $\frac{\partial d_i}{\partial p^i} < 0$ if $\frac{\partial^2 H}{\partial p^i \partial d_i} \geq 0$ and $\frac{\partial}{\partial p^i}\left[\left(1-p^i\right)\Delta_i\right] < 0$. This is satisfied if $\frac{\partial D}{\partial p^i} \leq 0$ and $\eta_{\Delta_i} > -1$. Finally, $d_i(1) = 0$ since $\left[d_i + (1-d_i)\, p^i\right]\Delta_i$ in (2.22) is independent of d_i at $p^i = 1$.

Proof of Proposition 2.2: Crowding Out

In this section we deal with the general case when there is a change in one component α_c of the vector of parameters α.

First, using (2.6), a necessary and sufficient condition for an equilibrium to be stable is $\frac{d}{dp}\left(\frac{\Delta_1}{\Delta_1+\Delta_2}\right) < 1$. But this can be written

$$\Delta_1 + \Delta_2 > (1-p^*)\frac{d\Delta_1}{dp} - p^*\frac{d\Delta_2}{dp} = -\begin{bmatrix} 1-p^* \\ p^* \end{bmatrix}' \frac{d\Delta}{dp} \tag{2.24}$$

Second, using the notations defined above and $\psi = \begin{bmatrix} -\frac{\partial\Delta_1}{\partial\alpha_c} & 0 \\ 0 & \frac{\partial\Delta_2}{\partial\alpha_c} \end{bmatrix}$:

$$\frac{d\Delta}{d\alpha_c} = \delta\frac{da}{d\alpha_c} + \phi\frac{dA}{d\alpha_c} + \psi \tag{2.25}$$

$$\frac{da}{d\alpha_c} = D^{-1}\frac{\partial S}{\partial p}\frac{dp}{d\alpha_c} + D^{-1}\frac{1}{m}\frac{\partial a}{\partial\alpha_c} \tag{2.26}$$

$$\frac{dA}{d\alpha_c} = \frac{\partial S}{\partial p}\frac{dp}{d\alpha_c} + M\frac{da}{d\alpha_c} \tag{2.27}$$

And

$$(\Delta_1+\Delta_2)\frac{dp^*}{d\alpha_c} = \frac{\Delta_2\frac{d\Delta_1}{d\alpha_c} - \Delta_1\frac{d\Delta_2}{d\alpha_c}}{\Delta_1+\Delta_2} = (1-p^*)\frac{d\Delta_1}{d\alpha_c} - p^*\frac{d\Delta_2}{d\alpha_c} \tag{2.28}$$

Combining all the derivatives with respect to α_c, we get the change in p^* due to the change in the parameter α_c.

$$(\Delta_1+\Delta_2)\frac{dp^*}{d\alpha_c} = -\begin{bmatrix} 1-p^* \\ p^* \end{bmatrix}' \frac{d\Delta}{d\alpha_c} \tag{2.29}$$

$$\Longrightarrow \underbrace{\left[(\Delta_1+\Delta_2) + \begin{bmatrix} 1-p^* \\ p^* \end{bmatrix}' \frac{d\Delta}{dp}\right]}_{\text{positive by stability}} \frac{dp^*}{d\alpha_c} \tag{2.30}$$

$$= -\begin{bmatrix} 1-p^* \\ p^* \end{bmatrix}' \left[(\delta+\phi M)\,D^{-1}\frac{1}{m}\frac{\partial a}{\partial\alpha_c} + \psi\right]$$

Assume $\frac{\partial a}{\partial \alpha_c} \geq 0$ and $a_1 > a_2$. The non zero entries of ϕ on row i is $V_2^i(a_2, S^i, \alpha) - V_2^i(a_1, S^i, \alpha)$ for $i = 1, 2$. These are negative by MSI. The term ϕM increases p^* (because it is multiplied by the negative term $-\begin{bmatrix} 1-p^* \\ p^* \end{bmatrix}$). This corresponds to the "social effect" whereby each group thinks it is better to be a type 1 because their behavior is closer to the reference S^i.

The non diagonal entries of δ are $V_1^1(a_2, S_1, \alpha)$ on the first row and $-V_1^2(a_1, S_2, \alpha)$ on the second row. They are positive by assumption (in other words, type 1 consider a_2 to be too low, and type 2 consider a_1 to be too high). This is always true when imperfect empathy is satisfied. These two terms decrease p^*. These are the "private" effects, whereby a higher a_2 decrease type 1 intolerance and a higher a_1 increase type 2 intolerance. The non diagonal terms can reinforce or not these effects. With imperfect empathy they are equal to zero.

Therefore in $(\delta + \phi M)$, there are negative "private" effects and positive "social" effects. The "private" terms are at the origin of crowding out. Interactions reduce the magnitude of crowding out.

Nothing can be said at this level of generality about $\psi \frac{d\alpha}{d\alpha_c}$.

In Section 2.2, we assumed imperfect empathy, $\frac{\partial U^1}{\partial \alpha_c} = 0$ and $\frac{\partial S_1}{\partial a_2} = M_{12} = 0$. In this case when expanded (2.30) becomes

$$\left[(\Delta_1 + \Delta_2) + \begin{bmatrix} 1-p^* \\ p^* \end{bmatrix}' \frac{d\Delta}{dp}\right] \frac{dp^*}{d\alpha_c} = -(1-p^*)\frac{1}{1-m_2 M_{22}} U_1^1(a_2, S_1, \alpha)\frac{\partial a_2}{\partial \alpha_c} \tag{2.31}$$

$$-p^* \frac{M_{22}}{1-m_2 M_{22}} \frac{\partial a_2}{\partial \alpha_c} \left[U_2^2(a_2, S_2, \alpha) - U_2^2(a_1, S_2, \alpha)\right]$$

$$-p^* \left[U_3^2(a_2, S_2, \alpha) - U_2^2(a_1, S_2, \alpha)\right]$$

$$K = \left[(\Delta_1 + \Delta_2) + \begin{bmatrix} 1 - p^* \\ \\ p^* \end{bmatrix}' \frac{d\Delta}{dp} \right] \text{ and } M_2 = m_2 M_{22} \text{ give (2.7).}$$

Proof of Proposition 2.3

The condition $\frac{d\bar{a}}{d\alpha} = 0$ yields a first order differential equation on γ.

$$\gamma' - \frac{\beta(1-p)^2 + p^2\omega}{p(1-p)[\beta - p(\omega+\beta)]}\gamma + \frac{p\omega}{(1-p)[\beta - p(\omega+\beta)]} = 0 \tag{2.32}$$

The solutions to this equation are the γ_C functions and it is easy to check that the only γ_C function that satisfies $\gamma_C(p) \in [0,1]$ for every $p \in (0, \frac{\beta}{\omega+\beta})$ is $\gamma_{\omega+\beta}(p) = p$.

The condition $\frac{d\bar{a}}{d\alpha} > 0$ is

$$\gamma' < \frac{1}{p(1-p)[\beta - p(\omega+\beta)]} \left[\left[\beta(1-p)^2 + p^2\omega \right] \gamma - p\omega \right] \tag{2.33}$$

The right hand side is equal to the slope of the γ_C curve and so $\frac{d\bar{a}}{d\alpha} > 0$ is equivalent to the slope of γ being flatter than the slope of γ_C.

Chapter 3

Institutional crowding out

3.1 Introduction

I now use the main result of the last chapter about the existence of crowding out in intergenerational transmission. I relax the strong assumption that the variable generating crowding out is exogenous, and consider instead that it is determined by majority voting. Crowding out is now endogenously generated. This extension of the model is quite natural. Individuals with different preferences are expected to prefer different institutions and try to influence the political process to change the institutional arrangements. In the public good model of Chapter 2 conditional cooperators would vote for a lower level of public goods than strong cooperators. The important point is that different institutions imply different dynamics, and so different steady states. The political process selects institutions that affect behaviors and population dynamics. Those subsequently change institutions through the political process. I underlined in the preceding chapter how institutions were self-defeating, in the sense that they crowded out the behaviors they were supposed to promote. The endogenization adds a second self-defeating effect: not only they crowd out behaviors, they also crowd themselves out by reducing their political support in the population. Greif (2006) argues that institutions can be self-reinforcing or undermining by modifying parameters that lead them to thrive or decline. Crowding out is an undermining process because it slowly erodes the political majority that votes for the institution.

Cultural transmission has already been used in models with voting. Bisin and Verdier (2000) study the provision of public goods through majority voting, and Bisin and Verdier (2004) look at the evolution of work ethics that determine the generosity of the welfare state. None of these models consider crowding out.[1] They instead focus on the beliefs that support a given steady states and find that optimistic beliefs can lead a minority to become eventually the only existing group in steady state. This chapter adopts a similar approach but with crowding out. I show that there are dynamics where no steady state and, more importantly, no rational expectations path in pure strategies exists. Agents are bound to be wrong in their rational expectations. Decisions cannot be based on beliefs consistent with the future. Myopia would make choice possible but at the cost of rationality. I instead study why rational expectations fail to produce consistent dynamics. Agents do not take into account the effect of their choice on the evolution of preferences. One way to restore rational expectations is to let agents create some institution of collective decision that internalizes the effect of individual socialization efforts on preferences dynamics. By doing so agents' expectations are rational. That creates a rationale for the existence of collective socialization institutions that coordinate decisions. Bisin and Verdier (2000) also provide conditions for such institutions to exist and I am close to their perspective of institutions as strategic devices to select particular cultural paths. However in their model the motive for agents is only

[1]Without going too much into technical details, these models only have two possible steady states, either at 0 or 1. Parameters change the dynamics to reach these, but do not modify their value. This peculiarity is usually due to the fact that parents with different cultural traits do not make efforts simultaneously.

to shift political power in their favor. I find similarly that cultural groups use institutions to be majoritarian in the future, but I provide a second justification setting them up. They solve the puzzle of the impossibility of having rational expectations. Agents do realize that their decisions cannot be consistent with their expectations and find a mechanism in the form of institutions to organize their uncoordinated actions.[2]

3.2 Endogenous institutions

There are two types of individuals, identified by $i \in \{1, 2\}$, with preferences represented by their utility functions $u_i(a_i)$. Agents maximize their utility with respect to a_i subject to some constraint $g_i(\alpha)$ where α is a parameter, and this yields their optimal choice $a_i^*(\alpha)$.[3] I make the assumption that a_i^* is an increasing function of α.

I assume that preferences are transmitted from parents to children but unlike Chapter 2, parents vote over the value of α. It could be the level of taxation, school funding, unemployment benefits, etc. The outcome is determined by majority voting. I assume that type 1 parents prefer a high level of α, denoted α_1. On the other hand type 2 parents' optimal choice is α_2, with $\alpha_2 < \alpha_1$. p represents the proportion of type 1 parents in the population, and so when $p < \frac{1}{2}$, $\alpha = \alpha_2$, and $\alpha = \alpha_1$ otherwise.[4] To fix ideas, one can say that when type 1 agents are a majority in the population they vote for a high level of taxes, and when not the voting outcome is a low level of taxes.

Chapter 2 established that under these assumptions, the steady state $p^*(\alpha)$ of the distribution of cultural traits in the population may be a decreasing func-

[2] An alternative approach would look at the possibility of mixed strategy equilibrium. I study only pure strategies equilibria here.

[3] Note that compared to chapter 2 and for simplicity, these preferences do not include social interactions. The framework can be extended to include them.

[4] In the limit case $p = \frac{1}{2}$, I assume that α_1 is implemented.

tion of α. It provided examples where it is satisfied for all the possible values of the parameters. I impose the following assumption: because of crowding out $p^*(\alpha_1) < \frac{1}{2} < p^*(\alpha_2)$. It can be expressed with the intolerance parameters $\Delta_i(\alpha) \equiv u_i(a_i^*(\alpha)) - u_i(a_j^*(\alpha))$. The condition is equivalent to $\Delta_1(\alpha_1) < \Delta_2(\alpha_1)$ and $\Delta_1(\alpha_2) > \Delta_2(\alpha_2)$. It means that type 1 parents are more intolerant than type 2 parents when taxes are low, and that type 1 parents are less intolerant than type 2 parents when taxes are high. In other words, individuals are less intolerant when their preferred policy is implemented.

The fraction p_{t+1} of individuals of type 1 in period $t+1$ is given by

$$p_{t+1} - p_t = p_t(1-p_t)\left[\tau_1 - \tau_2\right] \tag{3.1}$$

τ_i is the socialization effort of parents of type i. Because of the absence of social interactions, we know that for a given value of α equation (3.1) has only one interior stationary state, and that it is stable.

3.3 The socialization problem

Type i parents choose τ_i to maximize their utility in period t

$$P_t^{ii} V^{ii}(p_{t+1}^e) + P_t^{ij} V^{ij}(p_{t+1}^e) - \frac{\tau_i^2}{2} \tag{3.2}$$

where P_t^{ii} and P_t^{ij} are the transition probabilities at time t, V^{ij} is the utility of a type j child as perceived by a type i parent. p_{t+1}^e is the expected proportion of type i agents next period, associated with the corresponding political equilibrium. For $j \in \{1,2\}$

$$V^{1j} = \begin{cases} u_1(a_j^*(\alpha_1)) & \text{if } p_{t+1}^e \geq \frac{1}{2} \\ u_1(a_j^*(\alpha_2)) & \text{if } p_{t+1}^e < \frac{1}{2} \end{cases} \tag{3.3}$$

$$V^{2j} = \begin{cases} u_2(a_j^*(\alpha_1)) & \text{if } p_{t+1}^e \geq \frac{1}{2} \\ u_2(a_j^*(\alpha_2)) & \text{if } p_{t+1}^e < \frac{1}{2} \end{cases} \tag{3.4}$$

Optimal socialization effort can be derived using the intolerance parameters

$$\tau_1 = \begin{cases} (1-p_t)\Delta_1(\alpha_1) & \text{if } p_{t+1}^e \geq \frac{1}{2} \\ (1-p_t)\Delta_1(\alpha_2) & \text{if } p_{t+1}^e < \frac{1}{2} \end{cases} \tag{3.5}$$

$$\tau_2 = \begin{cases} p_t\Delta_2(\alpha_1) & \text{if } p_{t+1}^e \geq \frac{1}{2} \\ p_t\Delta_2(\alpha_2) & \text{if } p_{t+1}^e < \frac{1}{2} \end{cases} \tag{3.6}$$

3.4 Rational expectations failure

Population dynamics depend on agents' expectations. I now characterize the rational expectations path where $p_{t+1}^e = p_{t+1}$. The dashed curve on Figure 3.1 gives the value of $p_{t+1} - p_t$ as a function of p_t when expectations are that $p_{t+1}^e < \frac{1}{2}$, and so that $\alpha = \alpha_2$. The other curve depicts the evolution of p_t when $p_{t+1}^e \geq \frac{1}{2}$, and so $\alpha = \alpha_1$.

For some values of p_t agents with rational expectations are bound to be wrong. Assume that $p_t = \frac{1}{2} + \varepsilon$, with $\varepsilon > 0$ and small. If agents think that $p_{t+1}^e \geq \frac{1}{2}$, we can read on the non-dashed curve that $p_{t+1} - p_t < 0$. For ε small enough it implies that $p_{t+1} < \frac{1}{2}$. These expectations are ruled out, because they are not rational. So it must be that $p_{t+1}^e < \frac{1}{2}$. But in that case, using now the dashed curve, $p_{t+1} - p_t > 0$, and so $p_{t+1} > \frac{1}{2}$.[5]

[5]This is true only for the case where the two equilibria are on either sides of $\frac{1}{2}$. This does

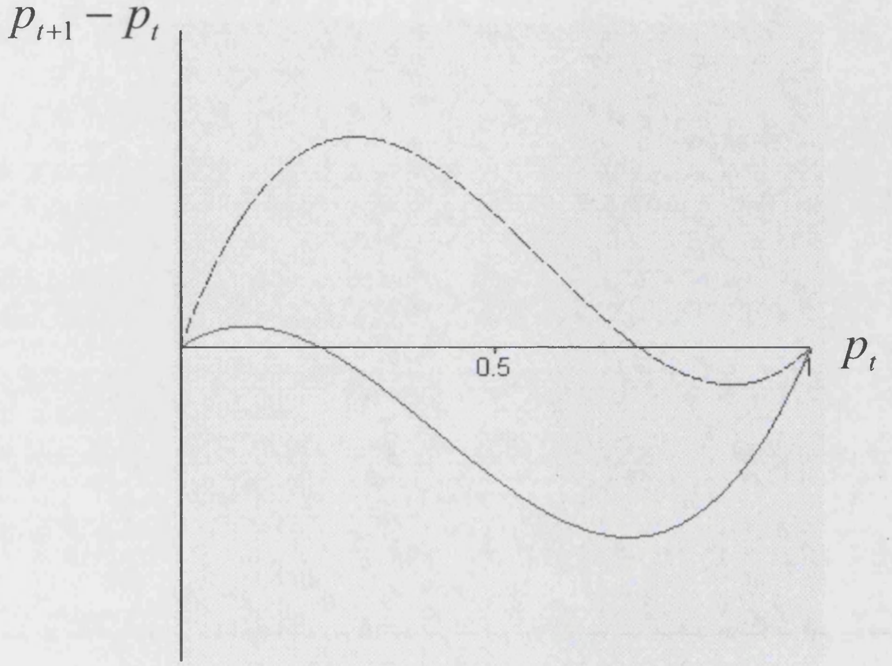

Figure 3.1: Dynamics under different institutions

Whatever their expectations are, agents must be wrong. There is no rational expectations path, at least in some interval that includes $\frac{1}{2}$. Agents are unable to make a rational choice. This surprising result is driven by the fact that individuals take isolated, uncoordinated decisions. Despite their rational expectations, they do not take into account the influence of their effort on next period population state. In technical terms, they do not use equation (3.1) when maximizing their utility.

Myopic agents with $p^e_{t+1} = p_t$ would not suffer from this impossibility of choosing. However it does not imply that the population converges to a steady state. We would actually observe cycles around $\frac{1}{2}$ (but not necessarily centered on $\frac{1}{2}$). Assume that initially $p_0 > \frac{1}{2}$. As long as p_t is above $\frac{1}{2}$, $p_{t+1} - p_t < 0$ and at some point in time p_t falls below $\frac{1}{2}$. It implies subsequently that $p_{t+1} - p_t > 0$,

not have to be satisfied. Assume for instance that both are above $\frac{1}{2}$. Crowding out means that the institutions chosen by the majority lead to the lowest steady state. Institutions make the majoritarian group smaller than it could be.

and p_t increases until it is above $\frac{1}{2}$, and we enter a new phase of falling p_t, and so on. There is no equilibrium, not even $\frac{1}{2}$.[6] The policy implemented changes with the cycles of p_t and there are alternate political majorities.

3.5 Collective decision

The failure of individual rational expectations calls for a device that preserves the assumption of rationality. When agents coordinate, or leave the choice of effort to a collective institution (school for instance), they anticipate the consequence of their choice on p_{t+1}. Type 1 institution maximizes

$$\begin{aligned} & P_t^{11}V^{11}(p_{t+1}^e) + P_t^{12}V^{12}(p_{t+1}^e) - \frac{\tau_1^2}{2} \qquad (3.7) \\ = \; & (1-p_t)V^{12}(p_{t+1}) + (1-p_t)\Delta_1(p_{t+1}) - \frac{\tau_1^2}{2} \\ \text{s.t } V^{12}(p_{t+1}) \; = \; & u_1(a_2(\alpha_1)) \quad \text{if } p_{t+1} \geq \frac{1}{2} \\ V^{12}(p_{t+1}) \; = \; & u_1(a_2(\alpha_2)) \quad \text{if } p_{t+1} < \frac{1}{2} \\ \Delta_1(p_{t+1}) \; = \; & \Delta_1(\alpha_1) \;\; \text{if } p_{t+1} \geq \frac{1}{2} \\ \Delta_1(p_{t+1}) \; = \; & \Delta_1(\alpha_2) \;\; \text{if } p_{t+1} < \frac{1}{2} \\ p_{t+1} - p_t \; = \; & p_t(1-p_t)\left[\tau_1 - \tau_2\right] \end{aligned}$$

The solution to the maximization problem (3.7) is a function of τ_2, where it is assumed that type 2 agents do not coordinate. [7] Type 1 parents are able to adjust their effort in order to force the proportion p_{t+1} to be above $\frac{1}{2}$. They do so only if it makes them better off. The point is that their expectations will now be rational because they take into account the dynamics of cultural traits when making their choice. I provide an example to illustrate the argument.

[6] However dynamics can converge to a stable cycle where $p_{t+2} = p_t$.

[7] If type 2 individuals follow a similar program, it creates a game between institutions whose reaction functions are given by their respective maximizations.

Example There are two types of agents in the population. They value consumption and two different types of goods that can be publicly or privately provided. Utility functions are

$$\begin{cases} u_1(c, h_1) = c + 2\sqrt{h_1} \\ u_2(c, h_2) = c + 2\sqrt{h_2} \end{cases} \tag{3.8}$$

Type i agents' preferences depend on consumption c and good h_i. At the beginning of each period agents receive an income I. Good h_i can be publicly or privately provided. When it is done publicly all individuals have to contribute a fixed quantity, and the state provides it as a public good. Private provision is done at the individual level, it is more expensive (for instance because of economies of scale) and the good is consumed privately. An alternative interpretation is that good h_i is either a public good at the state level, with compulsory contributions through taxes, or a club good.[8] Because of economies of scale in its production it is more expensive when consumed as a club good. Individuals vote for which of h_1 or h_2 is publicly provided. Since type i agents do not consume h_j they always vote for h_i to be provided by the state, to benefit from its lower price. The majority group imposes its choice to the minority, which can still provide its own club good, but at a larger cost. The price of a publicly provided good is assumed to be equal to 1, the price of a privately provided good is $\theta > 1$.

This model captures situations where two groups have conflicting preferences over the nature of the public good. For instance one group may favor religious schools, while the other prefers secular schools. When the secular agents are majoritarian the state raises taxes to finance secular public schools.

[8]The public good is non excludable. The conflict between the two types of agents is about which good should be available to everyone and publicly financed. Agents do not prefer to enjoy private goods because public finance implies a smaller cost.

This does not prevent the religious minority to open religious schools, however it is more costly.

Assume that type i agents represent the largest group. They maximize their utility $u_i(I - h_i, h_i)$, and vote for the quantity h_i^* to be provided by the state through taxation. Type j agents have to pay their taxes and so maximize their utility $u_j(I - h_i^* - \theta h_j, h_j)$.

$$\begin{cases} h_i^* = 1 \\ h_j^* = \frac{1}{\theta^2} \end{cases} \tag{3.9}$$

The intolerance functions are

$$\begin{aligned} p_{t+1}^e &\geq \frac{1}{2} \begin{cases} \Delta_1 = \frac{1}{\theta^2} \\ \Delta_2 = \frac{2\theta-1}{\theta^2} \end{cases} \\ p_{t+1}^e &< \frac{1}{2} \begin{cases} \Delta_1 = \frac{2\theta-1}{\theta^2} \\ \Delta_2 = \frac{1}{\theta^2} \end{cases} \end{aligned} \tag{3.10}$$

The two possible states p_1^* and p_2^* are

$$\begin{aligned} p_{t+1}^e &\geq \frac{1}{2} \Longrightarrow p_1^* = \frac{1}{2\theta} \\ p_{t+1}^e &< \frac{1}{2} \Longrightarrow p_2^* = 1 - \frac{1}{2\theta} \end{aligned} \tag{3.11}$$

There is intergenerational crowding out and $p_1^* < \frac{1}{2} < p_2^*$. A high level of h_i results in a small population of agents with type i preferences. The political process implies that there is no steady state. Each majority implements a policy that decreases its size. As argued above there exists an interval K that includes $\frac{1}{2}$ such that no rational expectations are possible. Any belief that p_{t+1} is larger than $\frac{1}{2}$ leads to efforts that make it smaller than $\frac{1}{2}$. Agents cannot have correct expectations when they decide their efforts individually.

A collective institution can be set up to choose the optimal effort by solving (3.7).

In this example, when $p_t \notin K$ rational expectations impose that $p_{t+1} > \frac{1}{2}$ if and only if $p_t > \frac{1}{2}$. So outside K the rational expectations path is unique, and well defined when agents act individually. Assume now that $p_t \in K$ and $p_t \geq \frac{1}{2}$ and that type 1 agents decide collectively their socialization effort. If expectations are that $p^e_{t+1} \geq \frac{1}{2}$, type 1 parents would optimally choose $\tau_1 = \frac{1-p_t}{\theta^2}$. However this is not consistent with their beliefs. Given τ_2, their effort is too low. In order to fulfill $p_{t+1} \geq \frac{1}{2}$, they must choose a higher effort. It can be shown that their optimal choice is such that p_{t+1} is exactly $\frac{1}{2}$. The alternative is to decide collectively to switch to a regime with $p_{t+1} < \frac{1}{2}$. But this is never optimal: parents prefer their cultural trait to be majoritarian next period.

The point of this example is to show how collective socialization allows rational expectations to be maintained. A more complete analysis would study the range of group sizes that support the emergence of institutions that affect socialization. I keep on assuming that only type 1 individuals consider this opportunity. For every $p_t \notin K$ and $p_t \geq \frac{1}{2}$ it is interesting to see that there are no benefits to such institutions. It would only replicate individual decisions that maximize utility with rational expectations. So institutions emerge only for relatively small group sizes. Second, a type 1 institution always tries to give the majority to type 1 individuals next period. It is never profitable to set up an institution that collectively decides to lose, or avoid winning, the majority. Third, there is a threshold $\tilde{p} \notin K$, and between 0 and $\frac{1}{2}$, such that for every p_t larger than $\tilde{p}$ agents of type 1 gain from deciding collectively their socialization efforts, and are able to be majoritarian next period. These new dynamics lead to a unique steady state with equal group sizes.[9]

Institutions affecting cultural change are therefore likely to be created for not too small minorities and for small majorities. Outside this group size range, either the group is too small to be majoritarian next period (it would require a too large effort), or it is large enough to be still a majority next period.

Finally, I assumed that only one group was able to create an institution, but one could also study the case where both groups can do so. There would be competition between the two groups, as each one would try to be majoritarian next period. I leave this possibility for future research.

3.6 Conclusion

Crowding out undermines the purpose of institutions but also their foundations. Individuals can willingly create good institutions, and it makes good behavior widespread. However eventually it leads to the demise of the institution by eroding the political support required for its existence. This is a dismal story for anyone considering how institutions can induce good behavior. This chapter shows that individuals can counterbalance this undermining trend by organizing themselves and promote their culture collectively, instead of relying on uncoordinated actions. Institutions can be accompanied by other institutions to avoid this effect. Public education is a collective device that complements private education in order to support rules that makes existing institutions viable.

[9] The assumption that type 1 agents win the election when $p_t = \frac{1}{2}$ is important for this point to be a steady state. It should be thought of as a limit: the collective decision should lead to the proportion $p_t + \eta$, with η infinitely small as there is no gain to constitute a large majority.

Chapter 4

Cultural transmission through network formation

4.1 Introduction

This chapter is a first attempt to bring some elements of network theory into the framework of the intergenerational transmission of preferences. A recent strand in the literature has been initiated by Bisin and Verdier (2001, denoted hereafter BV) who developed a new model of transmission of preferences from parents to children. The transmission mechanism can be decomposed into two components: vertical and oblique. Parents prefer their children to have the same preferences because they are imperfectly altruistic. They can socialize their children to their cultural trait, at the cost of some effort. This describes the vertical transmission and it takes place inside the family. The oblique transmission occurs only if the vertical failed. Children are influenced by individuals outside of their family and are socialized to their culture. In the BV model, and in most models based on it, oblique transmission takes the form of a random matching with subsequent successful cultural transmission. Children are merely passive in this process.

I relax this assumption and consider instead that children build a network of friends from families of different cultures. They are then influenced by the composition of their network. If a child has most of his friends from one particular cultural group then he is very likely to adopt the preferences of this group because he is often exposed to them. Parents can intervene by affecting network formation. They can influence the cost of an intergroup link (as opposed to an intragroup link). This action can take different forms: parents can choose to live in a segregated neighborhood, put their child in a religious school, or use verbal recommendations ("I do not want you to play with these kids", "I do not want you to go to this area of the city", etc.). This action is costly: rent for the residential choice, fees for the school, time, or even resentment from the child for the verbal recommendations. Parents choose their optimal effort anticipating network formation, and then children choose their friends given the costs and benefits of friendships. Vertical (parents' effort) and oblique (socialization through the network) transmissions are intertwined and are the result of choices from both parents and children.

Network formation creates a game between parents from different cultural groups. Because a link between two agents can only be created when both consent, children from one cultural group are constrained in their network building: they cannot find enough individuals willing to accept their friendship. If parents from group A make a high effort, their children create few intergroup links and children from group B are constrained. Group B parents can free ride on this effort and optimally choose a zero socialization effort. Socialization is a public good and parents who value it less free ride on the contributions of parents with a high valuation. I derive the Nash equilibria of this game with two cultural groups and show that for a range of group sizes there always exist two equilibria in effort, each with a different group choosing a zero effort. I investigate the case where majority voting is used as an equilibrium selection mechanism.

I consider the effect of intolerance on the political equilibrium. I show first that, depending on the parameters, the political equilibrium may be the outcome always preferred by the most intolerant group. This is never true for the least intolerant group, regardless of the parameters. There seems to be an advantage to intolerance in the political process. However I then show that parents' welfare increases with the intolerance of parents from the other cultural group. The intuition follows from the public good nature of socialization: highly intolerant parents choose a high effort and prevent friendships with children from other groups. More tolerant parents benefit from this high effort at no cost, and free ride on the intolerant group.

I then study the dynamics of preference transmission to find the steady states in cultural group sizes. The political process "eliminates" some steady states. When parents have the choice between two equilibria the majority never chooses to implement one of them, thereby avoiding the steady state it does not prefer (at the disadvantage of the minority). This political selection may culminate in the absence of steady states and induce political cycles with alternating majorities and equilibrium in efforts.

Another important feature is the role of intolerance and of network characteristics. First, the cultural group whose children have the smaller number of friends is the group having to exert some effort in the steady state. This "least connected type" does not free ride in steady state and so there is an advantage in having many connections, both inter and intra group. Second, intolerance is detrimental to the survival of a cultural group. There are two consequences to intolerance. It prevents intergroup friendships, and so socialization to other cultural traits. But it also prevents socialization of individuals external to the group. A very closed cultural group is safe from external influences, but it also fails to spread its culture to other cultural groups. It is not present in steady state, but only if it is weakly connected internally. A highly intolerant group does survive if its children have many friends from their own group. This second property underlines the importance of the structure of the intragroup network. It also sheds some light on the measures a shrinking cultural group may take in order to survive cultural transmission: it should focus on the importance of internal links, and on its internal identity. The model shows how intolerance preserves a cultural group from external influences and so contributes to its pereniality, but at the same time prevents its expansion.

Few articles in the economics literature relate to this work, apart from those on cultural transmission initiated by BV and surveyed in Bisin and Verdier (forthcoming). One exception is Pattachini and Zenou (2006) who consider a somewhat similar process of network formation because children choose their percentage of same-race friends. However the authors assume that choices of friends from children of the two groups are always compatible, and do not integrate their model in the cultural transmission framework. Although I am not providing an explicit model of network formation (see Jackson 2006 for a review), I am going a step further in recognizing that these choices are usually not compatible and that this determines equilibrium outcomes.

4.2 The model

Cultural transmission is seen as a two stage game where first parents choose their socialization effort, and then children decide who to establish links with. There are two types of cultural traits $\{1, 2\}$ in the population. The fraction of individuals with trait i is p_i.

4.2.1 Network formation

Children choose how many links with individuals of each type to build. As in BV children are assumed to be naive and so they cannot influence each other. They can form links with children growing in families of different types, or with adults. For instance a child from an upper class family can choose friends with a lower class background and be exposed to their parents' cultural traits. I retain this interpretation here to avoid the creation of a group of non-naive children.

I assume that children always build the same number of links with individuals of their parents' type. This simplifies the analysis but it can be relaxed in different ways. It implies that children do not substitute between friends of different types. They rather choose how many friends of the other type to have on top of their friends with the same background. Children of the same cultural group can be considered to live in the same area and to know each other, maybe costlessly, and to decide how many friends of the other neighborhood to have. This does not affect their number of friends from their own neighborhood.

The benefit of a new link is constant and independent of the total number of links. However the cost of forming a new link increases with the existing number of links. As in BV parents prefer their children to have their preferences. They can influence network formation by modifying the cost of creating a link with someone of the other cultural group. Children take this cost as given but it will be chosen by parents in the first stage of the game.

The creation of n_i^j direct links with individuals of the other cultural group has a benefit $n_i^j v_i$ and a cost $h(\tau_i)\frac{(n_i^j)^2}{2}$ where τ_i is the socialization effort of the parents and h is some concave continuously differentiable function with $h(0) = 1$ and $h'(0) = +\infty$. The friend of a friend does not bring any benefit.

Children choose their number of friends by maximizing their "network" utility function, for all $i,\ j \in \{1, 2\}$

$$u_i\left(n_i^i, n_i^j\right) = f_i(n_i^i) + n_i^j v_i - h(\tau_i)\frac{\left(n_i^j\right)^2}{2} \tag{4.1}$$

The function f_i is continuously differentiable and concave. The important point is that u_i is separable in n_i^i and n_i^j. Note also that parents could modify the benefit v_i (or its perception), and it would yield equivalent results. Optimally we have

$$n_i^j = \frac{v_i}{h_i} \tag{4.2}$$

where $h_i = h(\tau_i)$. Henceforth I use the expression "type i children" for "children from a family of type i". Type 1 children want to form n_1^2 links with type 2 children, who themselves want to form n_2^1 links with type 1 children. However the formation of a link requires the consent of both children. It may not be possible to build the desired network.

There are N_i type i children, each of them willing to build n_i^j links with the other group. Cultural group i wants to form a total of $N_i n_i^j$ links with group j, and similarly group j wants to form $N_j n_j^i$ links with group i. It is feasible if and only if $N_i n_i^j = N_j n_j^i$. If $N_i n_i^j > N_j n_j^i$ then (at least some) type i children must be constrained in their number of links and cannot achieve their optimal network size. I will say that type i is constrained when $N_i n_i^j > N_j n_j^i$, and write indifferently that parents and children are constrained, but it must be understood that children are the ones constrained in their choice. I assume that the equilibrium is symmetric such that all the individuals in the same group establish the same number of links with the other group. This raises the possibility of a number of links that is not an integer. I abstract from this difficulty, keep the requirement of a symmetric equilibrium and treat n_i^j as a continuous variable. n_i^i is assumed to be always feasible. In Proposition 4.1 $\tilde{n}_i^j$ indicates the unconstrained choice and n_i^j the constrained optimum.

Proposition 4.1 *For $j \neq i$, if $\tilde{n}_i^j \leq \frac{1-p_i}{p_i}\tilde{n}_j^i$ then $n_i^j = \tilde{n}_i^j = \frac{v_i}{h_i}$ and $n_j^i = \frac{p_i}{1-p_i}\frac{v_i}{h_i}$.*
For $j \neq i$, if $\tilde{n}_i^j > \frac{1-p_i}{p_i}\tilde{n}_j^i$ then $n_i^j = \frac{1-p_i}{p_i}n_j^i$ and $n_j^i = \tilde{n}_j^i = \frac{v_j}{h_j}$.

Proposition 4.1 only rephrases the fact that the constrained group is the group that wants to create the largest number of links. Network formation is constrained by the decision of the other group. If group i is large and wants to build many links with group j then it is constrained because group j is not willing to do so. In reality some type i children would form links while others would not, but in the model the symmetric equilibrium rules out this possibility. A group is constrained under two conditions: it is not sufficient that individuals want to form many links, the group must be large enough as well. A small group very "open" is unlikely to be constrained in its network formation.

Once children have formed their network they are socialized to a cultural trait by interacting with individuals of their network. Unlike BV I make oblique transmission the result of a decision, instead of being a passive process. I assume that type i children are socialized to cultural trait i with a probability q_i equal to the proportion of type i children in their network.[1] It captures the fact that by having friends of their own type they are confronted to their own environment and to other type i parents, or maybe that they stay in the same area and do not learn about other cultures.

Proposition 4.2 *If $\tilde{n}_i^j \leq \frac{1-p_i}{p_i}\tilde{n}_j^i$ then $q_i = \frac{n_i^i}{n_i^i+\tilde{n}_i^j}$.*
If $\tilde{n}_i^j > \frac{1-p_i}{p_i}\tilde{n}_j^i$ then $q_i = \frac{n_i^i}{n_i^i+\frac{1-p_i}{p_i}\tilde{n}_j^i}$.

[1]Instead of being equal to q_i the probability could be some increasing function of q_i and the results would go through.

Proposition 4.2 follows directly from Proposition 4.1. The important point is that conditional on being constrained, the proportion of same type friends is independent on children's optimal choice.

4.2.2 Parents socialization choice

I follow BV and assume that parents do care about the welfare of their children but that they use their own preferences to evaluate their children's actions. Parents exhibit imperfect empathy. They consider that an individual of their own type i gets a utility V^{ii} and someone of type j gets V^{ij}. The quantity $\Delta_i \equiv V^{ii} - V^{ij}$ represents their intolerance towards the other type and it is positive: parents always prefer their children to be of their own type.

Vertical transmission is summarized by the effort parents exert to shape the network built by their children. Families are composed of one parent and one child, and the parent maximizes

$$P^{ii}V^{ii} + P^{ij}V^{ij} - C(\tau_i) \tag{4.3}$$

where P^{ij} is the probability that a child from a type i family is socialized to trait j. C is continuously differentiable, increasing, convex, and $C(0) = C'(0) = 0$.

Given the assumption about cultural transmission in the network $P^{ii} = q_i$ and $P^{ij} = 1 - q_i$. When type i children are unconstrained these probabilities depend on τ_i. Otherwise they do not. Parents anticipate this and therefore face two possible situations: either their children are constrained in the number of friends of the other type they can get, and in this case their effort does not influence the probability of socialization to the other trait and any effort is useless, or their children are unconstrained and they choose a strictly positive effort in order to influence network building.

Assume first that type i children are unconstrained, the optimal effort from parents is given by

$$\begin{aligned} C'(\tau_i) &= \Delta_i q_i'(\tau_i) \\ &= \Delta_i \frac{v_i n_i^i}{(n_i^i + \frac{v_i}{h(\tau_i)})^2} \frac{h'(\tau_i)}{h(\tau_i)^2} \end{aligned} \tag{4.4}$$

Because of the convexity of C and the concavity of h, (4.4) implicitly defines a unique τ_i. Secondly when type i children are constrained the optimal effort is $\tau_i = 0$. Note that the unconstrained effort τ_i is a constant independent of p_i.

In the first stage of the game parents have to anticipate whether their children will be constrained or not, and this depends on the decisions of parents from the other group. This creates an incentive to free ride on the effort of the other group in order to make no effort. Parents play a game where their optimal action depends on their opponents' actions. The next section presents results when parents do not perfectly understand this game and act in a "naive" way. I then investigate how the results must be modified to take into account the free riding induced by the game.

In the rest of the article group i is said to be "more intolerant" than group j when $\tilde{n}_i^j = \frac{v_i}{h_i} < \frac{v_j}{h_j} = \tilde{n}_j^i$.[2]

4.3 Naive parents

By naive parents I mean that they compute their optimal unconstrained effort, the unconstrained effort of parents from the other group, and check whether

[2] It is not always true that $\Delta_i < \Delta_j$ implies $\tau_i < \tau_j$. It depends on the other parameters v_i, v_j, n_i^i, and n_j^j. The definition used in the article means that an intolerant group is less willing to form links with the other group. It disregards whether it results from parents' effort or children tastes. In Section 4.1 I use some parameter restrictions that link directly intolerance to Δ_i.

their children are constrained. If they are then parents optimally do not make any effort. Proposition 4.3 describes the equilibrium.

Proposition 4.3 $n_i^j = \frac{v_i}{h_i}$, $q_i = \frac{n_i^i}{n_i^i + \frac{v_i}{h_i}}$, *and* $\tau_i > 0$ *if* $\frac{p_i}{1-p_i} \leq \frac{v_j}{v_i}\frac{h_i}{h_j}$.

$n_i^j = \frac{1-p_i}{p_i} n_j^i$, $q_i = \frac{n_i^i}{n_i^i + \frac{1-p_i}{p_i}\frac{v_j}{h_j}}$, *and* $\tau_i = 0$ *otherwise.*

Proposition 4.3 states that when $\frac{p^i}{1-p^i} \leq \frac{v_j}{v_i}\frac{h_i}{h_j}$ type i children are unconstrained (and so type j children are constrained). It means that when both parents choose their optimal unconstrained effort, type i children are unconstrained and $\tau_i > 0$ is indeed optimal for type i parents. Type j parents do not make any effort: given type i effort it is optimal to do so.[3] The intuition driving this result is that when cultural group i represents a small fraction of the population (p_i small) and type j children are willing to form more links than type i children ($\frac{v_j}{v_i}\frac{h_i}{h_j}$ large, this is true in particular when type i parents are more intolerant) then type j children are not able to get as many type i friends as they would like. Type j parents anticipate this outcome in the second stage and do not make any effort.

Parents try to curb their children's decisions if they are part of a minority. They fear that their children have a large number of friends from the other cultural group and consequently are influenced by them. On the other hand the majority feels safe and can rely on the minority's efforts to avoid undesired friendships. This result is similar to an important characteristic of BV model that they call cultural substitution between vertical and oblique transmissions. Put simply, parents' effort decreases with the population proportion of their cultural group.

Figure 4.1 illustrates the optimal effort and the resulting proportion q_i as functions of type i individuals in the population. $\tilde{p}_i$ satisfies $\frac{\tilde{p}_i}{1-\tilde{p}_i} = \frac{v_j}{v_i}\frac{h_i}{h_j}$.

[3]What is examined in the next section is that type i parents could also find it optimal to choose a zero effort if that induces type j parents to choose a positive effort.

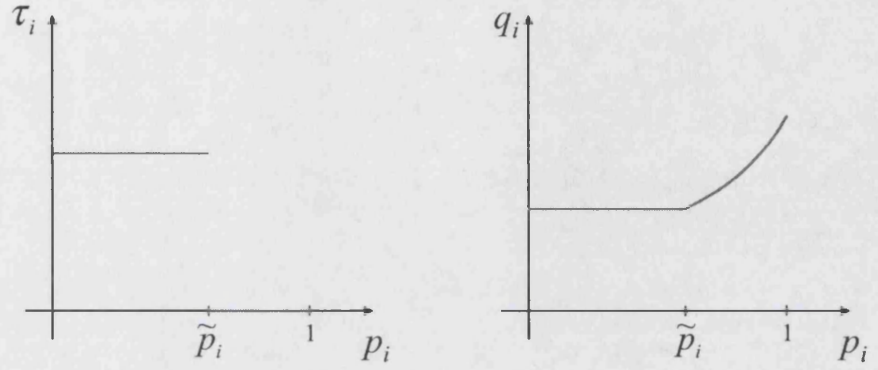

Figure 4.1: Effort and proportion of type i friends

The proportion q_i of type i friends, and so the probability of staying a type i individual is strictly increasing when p_i is large enough (when $\frac{p_i}{1-p_i}$ is larger than $\frac{v_j}{v_i}\frac{h_i}{h_j}$, or equivalently when $p_i > \tilde{p}_i$). There are few type j children and through simple shortage the proportion increases. This would occur even without parents making any effort. The consequence of parents trying to affect the costs and benefits of friendships is to modify q_i in their interest. Without socialization the proportion q_i would be lower and would start increasing at a different threshold (for p_i is larger than $\frac{v_i}{v_j}$) that can be smaller or greater than the threshold with socialization. On Figure 4.2 is drawn the proportion q_i when $\frac{h_i}{h_j} < 1$. This is equivalent to $\tau_i < \tau_j$, in particular this is the case when $v_i = v_j$ and $n_i^i = n_j^j$, but type j parents are more intolerant $\Delta_j > \Delta_i$. The dashed curve corresponds to the regime with socialization. q_i is higher with socialization than without. Of course this comes at the cost of effort.

If we consider the change from a situation where socialization does not exist, or when it is forbidden, then all the parents are better off.[4] However less intolerant parents benefit from the effort of the most intolerant cultural group: for some values of p_i they enjoy a much higher proportion q_i at no cost. This

[4]They must be because to allow socialization expands their choice set. They could always choose $\tau_i = 0$ if that was optimal.

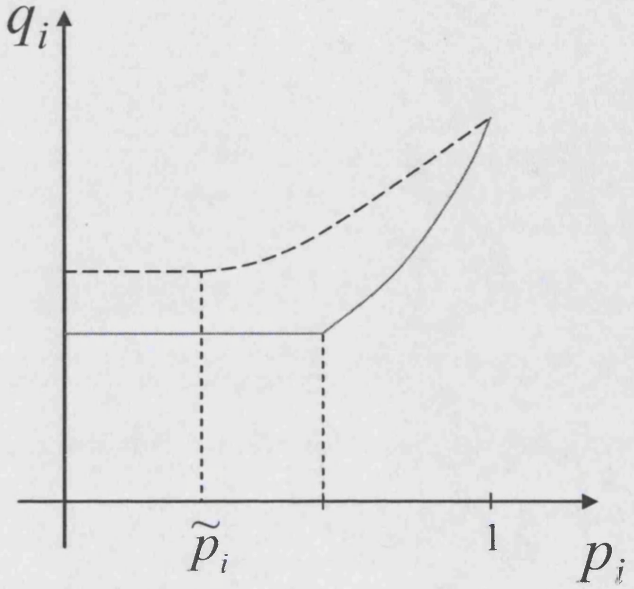

Figure 4.2: q_i with and without socialization

is never the case for type j parents. It can be formalized by looking at type i parents' welfare for different values of type j parents' intolerance. Their welfare increases with intolerance, so parents always prefer to face a highly intolerant cultural group because they do not have to make any socialization effort. Their children almost never meet the other cultural group. With infinitely intolerant parents they would never try to influence network formation.

Proposition 4.4 *Everything else being equal parents of cultural group i are better off when parents of cultural group j are more intolerant. Type i parents' welfare increases with* $\frac{v_j}{h_j}$.

4.3.1 Dynamics

Given the transition probabilities P^{ii} and P^{ij}, the evolution of p_{it} is

$$p_{it+1} = p_{it}q_{it} + (1 - p_{it})(1 - q_{jt}) \tag{4.5}$$

Or equivalently

$$p_{it+1} - p_{it} = (1 - p_{it})(1 - q_{jt}) - p_{it}(1 - q_{it}) \tag{4.6}$$

In order to characterize the steady states of equation (4.6), define first "the least connected type" as being type i if and only if $n_i^i + \frac{v_i}{h_i} < n_j^j + \frac{v_j}{h_j}$. Children of the least connected type form fewer links than children from the other cultural group.

Proposition 4.5 *Assume type i is the least connected type.*

If $n_i^i - n_j^j + \frac{v_i}{h_i} > 0$ then there is a unique steady state p_i^* with $0 < p_i^* < 1$; its basin of attraction is $(0,1)$. When $p_i = p_i^*$ the least connected type chooses a strictly positive effort. 0 and 1 are unstable steady states.

If $0 > n_i^i - n_j^j + \frac{v_i}{h_i}$ then $(0,1)$ are steady states and the basin of attraction of $p_i = 0$ is $[0,1)$.

Proposition 4.5 states different points. First there are two possible interior steady states. One where type 1 parents do not make any effort, and one where type 2 parents do not, depending on who is the least connected type. Second, these two equilibria do not coexist as there is only one least connected type in the population. Equation (4.6) has at most one interior stationary state.[5] Third, the interior equilibrium is globally stable when it exists, while 0 and 1 are unstable. When there is no interior steady state the least connected type does not survive cultural transmission. Finally the most connected group always free rides on the socialization effort of the least connected group in steady state.

The exercise of increasing type j intolerance while holding everything else

[5] When $n_i^i + \frac{v_i}{h_i} = n_j^j + \frac{v_j}{h_j}$ there are actually two stable interior equilibria. In each of them $\frac{p_i}{1-p_i} = \frac{v_j}{v_i}\frac{h_i}{h_j}$ but in the first equilibrium only type i parents choose a strictly positive effort, only type j parents do so in the second.

constant is repeated. Assume that type i is the least connected type. Two cases have to be considered. First if $0 > n_i^i - n_j^j + \frac{v_i}{h_i}$, type i is the least connected type for every intolerance level and the only stable steady state is $p_i = 0$. Second if $n_i^i - n_j^j + \frac{v_i}{h_i} > 0$, for large enough intolerance type j is the least connected type. As long as i is the least connected type the steady state is fixed and type i parents' welfare is constant. Above some intolerance threshold type j is the least connected type. In steady state type i parents stop making any effort and their proportion in the steady state increases with $\frac{v_j}{h_j}$. From Proposition 4.4 they also enjoy a higher welfare for a given group size. Eventually it could be that $0 > n_j^j - n_i^i + \frac{v_j}{h_j}$ and the only stable steady state would be $p_i = 1$. Thus it is better to face a highly intolerant group for two reasons: not only it increases welfare but in the long run that group might disappear. However intolerant cultural group can survive by being well connected internally (n_j^j large), such that the condition $0 > n_j^j - n_i^i + \frac{v_j}{h_j}$ is never satisfied. A group whose size shrinks may actually take measures to make its members more connected, focus on its internal identity and on the importance of establishing links among its members in order to survive cultural transmission.[6]

There seems to be a paradox in Proposition 4.5 as a highly intolerant group may disappear. It contrasts with the typical BV model where intolerance usually increases group size in equilibrium because it induces a high socialization effort in each period. This result actually underlines the importance of the total number of links in this type of model. An intolerant group with poor intra-connections is bound to disappear, whereas a group similar but highly connected internally is present in the steady state. Two mechanisms explain

[6]Two last cases have to be mentioned. When $n_i^i = n_j^j$ the steady state $p_i = \frac{1}{2}$ always exists and is stable. Intolerance only determines which group makes some positive effort in steady state. Second when $v_i = 0$ it implies that $q_i = q_j = 1$ and so any initial point is a steady state.

this result. Intolerance, everything else being equal, increases socialization effort and so the proportion of intragroup friends. But it also brings some impediment to cultural transmission because it prevents socialization of children from families of the other group. A group closed to cultural influences is able to transmit efficiently its own set of cultural traits but it is unable to disseminate it to other cultural groups. It relies mostly on intra cultural transmission to survive, but this fails when intra connections are weak. The risk for a group of being too closed is expressed by the condition $0 > n_i^i - n_j^j + \frac{v_i}{h_i}$. It requires in particular that $n_j^j > n_i^i$, such that type i children not only have a smaller total number of links, but also a smaller number of internal connections. This is the condition for a group to disappear. The condition $0 > n_i^i - n_j^j + \frac{v_i}{h_i}$ also implies that a group is less likely to vanish *without* socialization effort (when $h_i = 1$). Intolerance from parents can be damaging for the culture they want to transmit: they can always fail with their own children (q_i is always smaller than one), and they fail to transmit it to people outside their group.

4.4 Rational parents

In this section parents perfectly understand the nature of the game they play with the other cultural groups. In the first stage of the cultural transmission game they do not only consider the situation where both types of parents choose their optimal effort to understand which children are constrained in their network formation. They realize that it may be possible for them to "force" the other cultural group to choose a positive effort. Regardless of the equilibrium parents' efforts, they must be in a situation where only one cultural group is constrained. We know that optimally the constrained parents must choose a zero effort. If they are unconstrained then the optimal choice is τ_i. The strategy space in equilibrium is therefore restricted to $\{\tau_i, 0\}$. Equilibrium symmetry allows us to denote an equilibrium (τ_i, τ_j) where τ_i is the equilibrium strategy of all the type i parents.

I now consider the Nash equilibrium in socialization efforts when $\frac{h_i}{h_j} > 1$. It is equivalent to $\tau_i > \tau_j$.

Proposition 4.6 *There exist p_1 and p_2 in $(0,1)$, with $\frac{v_j}{v_i}\frac{1}{h_j} < \frac{p_1}{1-p_1} < \frac{v_j}{v_i}\frac{h_i}{h_j}$, $\frac{v_j}{v_i} < \frac{p_2}{1-p_2} < \frac{v_j}{v_i}h_i$, and $p_1 < p_2$ such that:*

- *When $p_i < p_1$ the only Nash equilibrium in pure strategies is $(\tau_i, 0)$.*
- *When $p_1 \leq p_i \leq p_2$ both $(\tau_i, 0)$ and $(0, \tau_j)$ are Nash equilibrium in pure strategies.*
- *When $p_2 < p_i$ the only Nash equilibrium in pure strategies is $(0, \tau_j)$.*

With naive parents $(0, \tau_i)$ is the equilibrium when $p_i < \tilde{p}_i$. It is $(0, \tau_j)$ otherwise. Proposition 4.6 establishes that with rational parents $(0, \tau_j)$ is always a Nash equilibrium (and may be the only one) on some interval where $p_i < \tilde{p}_i$. There always exists a zero effort equilibrium for group i below the naive threshold $\tilde{p}_i$ when $\tau_i > \tau_j$. It is not necessarily the case for the other cultural group (p_2 can be below $\tilde{p}_i$ and if it is then $(\tau_i, 0)$ is never an equilibrium when $p_i > \tilde{p}_i$).

Figure 4.3 pictures the equilibria for the different values of p_i. Compared to the naive case, the game between rational parents always expands the range of values where $(0, \tau_j)$ is an equilibrium, but can either expand or shrink the range where $(\tau_i, 0)$ is an equilibrium. The important conclusion is that the game must have two Nash equilibria on a non-empty interval.

Rational parents make use of the switch between being constrained or not and they swicth between zero and positive effort. Consider p_i smaller than but close to the naive threshold. Type i parents may prefer to deviate from

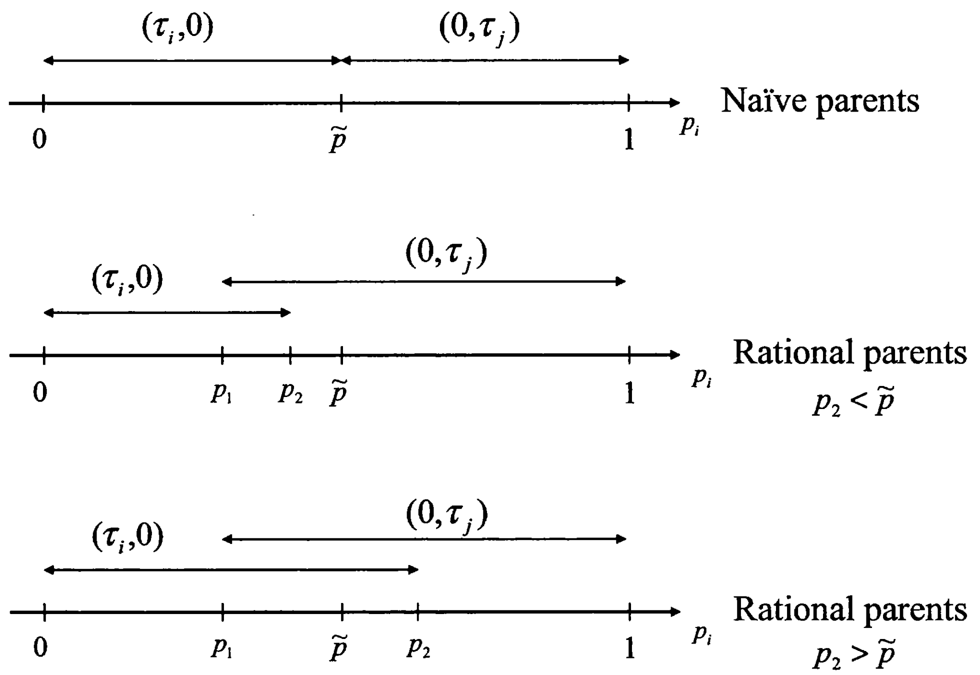

Figure 4.3: Nash equilibrium in pure strategies when $\tau_i > \tau_j$.

the naive equilibrium $(\tau_i, 0)$ by choosing a zero effort. That decreases the probability that their children adopt their trait. They also become constrained. Had they stayed unconstrained, the fall in q_i would have been larger. The benefit of the deviation is that parents do not suffer any cost. If the cost of τ_i is large then the deviation can be profitable. Naive parents do no understand the switch constrained/unconstrained associated with zero/positive effort, and so never see a deviation as profitable. I provide in the appendix a longer explanation in the appendix, along with the proof of Proposition 4.6.

Cultural group i is assured of the existence of a zero effort level below the naive threshold, but we have not said anything about the preferred equilibrium for each type of parents. The zero effort equilibrium exists but it may not be desirable for any group. Proposition 4.7 states that parents always prefer the zero effort equilibrium.

Proposition 4.7 *If there are two Nash equilibria then parents prefer the equi-*

librium where they make zero effort.

If there is only one Nash equilibrium then it maximizes the payoffs for both types of parents.

When there are two equilibria there is a conflict of interest between the two cultural groups as they prefer the other group to make some effort and to free ride on it. If parents could choose the equilibrium when both exist then they would not agree. We need some mechanism to select the equilibrium. I investigate below the case where the equilibrium is chosen by majority voting.

The second part of Proposition 4.7 states that when it is unique the Nash equilibrium is payoff maximizing for all parents. They do not prefer any other non equilibrium outcome.

4.4.1 Political equilibrium

I restrict children of both types to be identical, with $n_i^i = n_j^j$ and $v_i = v_j$. In the absence of socialization type i children would be unconstrained for $p_i < 1/2$ and constrained otherwise. Type i parents are assumed to be more intolerant than type j, $\Delta_i > \Delta_j$. It implies that they choose a higher socialization effort when they are unconstrained $\tau_i > \tau_j$.[7]

When $(\tau_i, 0)$ and $(0, \tau_j)$ are Nash equilibria the largest group in the population chooses its preferred equilibrium. The outcome is chosen by majority voting. Alternatively the majority may be able to commit to a strategy in order to pick up its preferred equilibrium (type i parents would credibly commit to a zero effort), or to play first in a two stage game. If there is only one Nash equilibrium then it is automatically implemented.

[7] The general case without any restrictions on the paramaters is similar and would consider that group i is said to be more intolerant when $\frac{v_i}{h_i} < \frac{v_j}{h_j}$.

When $\frac{p_i}{1-p_i}$ is smaller than $\frac{v_i}{v_j} = 1$, group j is the largest ($p_i < \frac{1}{2}$) and it can choose the equilibrium when there are two Nash equilibria. Using Proposition 4.6, for p_i small enough $(\tau_i, 0)$ is the only Nash equilibrium. For larger values of p_i it may be that $(0, \tau_j)$ is an equilibrium but it is never chosen by the majority. $(\tau_i, 0)$ is the equilibrium for every $p_i < \frac{1}{2}$.

When $p_i > \frac{1}{2}$ group i has the majority. When it is a short majority (p_i close enough to $\frac{1}{2}$) either $(\tau_i, 0)$ and $(0, \tau_j)$ are equilibrium or only $(\tau_i, 0)$. If parents have the choice they go for the zero effort equilibrium $(0, \tau_j)$. If not, $(\tau_i, 0)$ is implemented even though type i group represents the majority. Finally when p_i is large the only Nash equilibrium is $(0, \tau_j)$. Proposition 4.8 summarizes these results.

Proposition 4.8 *There exists p_3, with $\frac{1}{2} \leq p_3 < \tilde{p}_i$, such that the political equilibrium is $(\tau_i, 0)$ when $p_i < p_3$, and $(0, \tau_j)$ otherwise.*

Corollary: *When $p_3 > \frac{1}{2}$ the outcome preferred by group i is always implemented.*

Consider first the corollary. If $p_3 > \frac{1}{2}$ then $(0, \tau_j)$ is never a Nash equilibrium when group j constitutes a majority. There is a conflict of interest between the two groups only when group i is a majority and so $(0, \tau_j)$ is always implemented when groups disagree. The outcome preferred by group i is always the political equilibrium. In the case with $p_3 = \frac{1}{2}$, $(0, \tau_j)$ starts being an equilibrium when $p_i < \frac{1}{2}$ and group j chooses not to implement it. When $p_i > \frac{1}{2}$ group i is a majority and chooses $(0, \tau_j)$ while $(\tau_i, 0)$ is still an equilibrium. In other words there is an interval I such that $\frac{1}{2} \in I$ and when $p_i \in I$ the majority chooses the equilibrium not preferred by the minority. Both groups "suffer" from the political process, while with $p_3 > \frac{1}{2}$ only group j does. I will come back later to this point when looking at welfare.

Two possibilities emerge from Proposition 4.8, either with $p_3 = \frac{1}{2}$ or with $p_3 > \frac{1}{2}$. First, the majority group implements its zero effort equilibrium. It free rides on the other group. Second, the less intolerant group always implements its zero effort equilibrium but the most intolerant group does only so when it represents a large majority ($p_i > p_3 > \frac{1}{2}$). It prefers to make a positive socialization effort for p_i close to $\frac{1}{2}$ precisely because it is more intolerant. If it relies on group j to make some effort it is not satisfied by the outcome. Type j parents make a too small effort from type i parents' point of view and as a consequence their children have too many friends from the other cultural group. For p_i large, type i group size makes contacts with people from the other group unlikely and so parents can stop making any effort. The next proposition follows directly from this argument.

Proposition 4.9 *p_3 is a decreasing function of Δ_j.*

When group j becomes more intolerant group i can free ride on its effort for smaller group sizes. It can do so eventually as soon as it is a majority ($p_3 = \frac{1}{2}$).

4.4.2 Welfare

I repeat the analysis of the last section by considering whether group i would like to face a more intolerant group than it is itself. I am still using the restrictions made to study the political equilibrium, with children of both cultural groups being identical. Consider a type i parent and increase Δ_j for a given p_i. Compared to the case with naive parents we now have to take into account the political equilibrium, but actually it does not affect the result. If before and after the change the equilibrium is $(\tau_i, 0)$ then welfare is unchanged. If the equilibrium is $(0, \tau_j)$ then welfare increases, because q_i does. If the equilibrium switches from $(\tau_i, 0)$ to $(0, \tau_j)$, then again welfare increases, as in the naive case. Welfare would fall if equilibrium switched from $(0, \tau_j)$ to $(\tau_i, 0)$ but because of p_3 decreasing with Δ_j it cannot happen.

Therefore group i always prefers to face a highly intolerant group. I claimed above that the most intolerant group does not necessarily "suffer" from the political process while the other always does. It is still correct in the sense that when group i is much more intolerant than group j its preferred outcome is always the political equilibrium. This is not true when it is less intolerant but the outcome still makes group i parents better off. They may be disappointed that their preferred outcome is not implemented but their welfare cannot be lower than if group j was less intolerant.

Proposition 4.10 *Everything else being equal the more intolerant cultural group j parents are, the higher is cultural group i parents' welfare.*

Intolerance from the other group allows free riding and this always increases welfare.[8]

4.4.3 Dynamics

Equation (4.6) is still valid. In the general case where $v_i \neq v_j$ and $n_i^i \neq n_j^j$ Proposition 4.8 holds but the result $\frac{1}{2} \leq p_3 < \tilde{p}_i$ is replaced by a weaker version $\frac{v_j}{v_i}\frac{1}{h_j} < \frac{p_3}{1-p_3} < \frac{v_j}{v_i}h_i$. With naive parents equation (4.6) has only one stable steady state. However with rational parents and the political process there can be two, one, or even no stable steady states.

Define $p_i^* = \frac{\frac{v_i}{h_i}+n_i^i-n_j^j}{2\frac{v_i}{h_i}+n_i^i-n_j^j}$ if $\frac{v_i}{h_i} + n_i^i - n_j^j > 0$ and $p_i^* = 0$ otherwise, and $p_i^{**} = \frac{\frac{v_j}{h_j}}{2\frac{v_j}{h_j}+n_j^j-n_i^i}$ if $\frac{v_j}{h_j} + n_j^j - n_i^i > 0$ and $p_i^{**} = 1$ otherwise . These type i proportions are the two possible interior steady states of equation (4.6). p_i^*

[8]Note also that children otherwise identical prefer to be in a family from the least intolerant group.

is such that the effort equilibrium is $(\tau_i, 0)$, and p_i^{**} such that it is $(0, \tau_j)$. Proposition 4.11 describes all the possible steady states.

Proposition 4.11 *Assume that $\frac{v_i}{h_i} < \frac{v_j}{h_j}$.*

- *When type i is the least connected type, the naive steady state would be p_i^*. With rational parents and the political process:*
  - *If $n_i^i < n_j^j$:*
    * *If $p_3 < p_i^*$ the steady state is $p_i^{**} < \frac{1}{2}$.*
    * *If $p_i^* < p_3 < p_i^{**}$ there are two steady states: p_i^* and p_i^{**}, with $p_i^* < p_i^{**} < \frac{1}{2}$.*
    * *If $p_i^{**} < p_3$ the steady state is $p_i^* < \frac{1}{2}$.*
  - *If $n_i^i > n_j^j$:*
    * *If $p_3 < p_i^{**}$ the steady state is $p_i^{**} > \frac{1}{2}$.*
    * *If $p_i^{**} < p_3 < p_i^*$ there is no steady state.*
    * *If $p_i^* < p_3$ the steady state is $p_i^* > \frac{1}{2}$.*
- *When type j is the least connected type, it must be that $n_i^i > n_j^j$ and the steady state is $p_i^{**} > \frac{1}{2}$ (and it is also the naive steady state).*

The role of the political process can be clarified for the cases where it affects the outcome. Assume first that type i is the least connected type. If $n_i^i < n_j^j$, the political equilibrium may rule out a steady state when $p_i^{**} < p_3$. p_i^{**} may be a steady state, with Nash equilibrium $(0, \tau_j)$. However p_i^{**} is below $\frac{1}{2}$ such that group i does not represent the majority in this equilibrium. If both $(\tau_i, 0)$ and $(0, \tau_j)$ are Nash equilibria the majority will select $(\tau_i, 0)$ and p_i^{**} is not an equilibrium. By doing so the j majority avoids the steady state where its size is smaller (as $p_i^* < p_i^{**}$) and picks up its preferred steady state.

Assume still that type i is the least connected type but that $n_i^i > n_j^j$. When $p_i^{**} < p_3 < p_i^*$ there is no steady state because of the political process. If the majority of type i individuals chose the Nash equilibrium $(\tau_i, 0)$ instead of $(0, \tau_j)$, p_i^* would be a steady state. Instead it prefers the zero effort Nash equilibrium $(0, \tau_j)$ and this blocks the emergence of a steady state.

When there are two steady states initial conditions matter. When there is none, cycles appear around p_3. For p_{it} smaller than p_3, $p_{it+1} - p_{it} > 0$, but when p_{it} becomes larger than p_3 it decreases, until it falls below p_3, and increases again. The two Nash equilibria are alternatively implemented and these cycles can also induce changes in the majority in each period.

The last bullet point of the proposition refers to the situation where type j is the least connected type and is also less connected "internally". In the steady state the political equilibrium is $(0, \tau_j)$ and there is a majority of type i individuals. Cultural group i, even though it is less tolerant $(\frac{v_i}{h_i} < \frac{v_j}{h_j})$, can free ride on group j because it is poorly connected.

When we studied the political equilibrium I assumed that children from both cultural groups were identical, with $n_i^i = n_j^j$ and $v_i = v_j$. It also implies that $p_i^* = p_i^{**} = \frac{1}{2}$, and the least connected type is the most intolerant group. Using Proposition 4.11, if $p_3 > \frac{1}{2}$, the steady state is at $p_i^* = \frac{1}{2}$ and $(\tau_i, 0)$ is implemented. On the other hand when $p_3 = \frac{1}{2}$ there are two steady states, at the same proportion $p_i = \frac{1}{2}$, but the Nash equilibrium in socialization efforts is either $(\tau_i, 0)$ or $(0, \tau_j)$. p_i^* is preferred by type i parents, p_i^{**} by type j parents. Because no group has a strict majority both outcomes are feasible and there is a tension between the two cultural groups as each wants to free ride on the other.[9]

[9]Mixed strategy equilibrium may arise in this situation.

4.5 Conclusion

The model has been voluntarily chosen to be very simple. However it allows some non trivial developments: it creates a game between parents that may have multiple Nash equilibria. I showed how intolerance plays an important role in determining the outcome of this game and how it influences the political equilibrium. In particular parents from the least intolerant group suffer from a disadvantage in equilibrium selection because they are not able to always enforce their preferred outcome. Despite this drawback, I also showed that parents' welfare increases with the intolerance of the other group. Finally the dynamics of cultural transmission yield different steady states for a population. The political process plays an important role by sometimes eliminating a potential steady state. It can lead to cycles that do not converge.

While this extension is very preliminary and does not build precisely upon the recent findings in network theory, it still underlines the importance of some characteristics in network formation such as the total number of links an individual forms, or the number of connections inside its own cultural group. These drive whether a type survives cultural transmission. The model shows how intolerance preserves a cultural group from external influences and so contributes to its pereniality, but at the same time prevents its expansion.

Many extensions are to be explored: non-separability in the choice of intra and intergroup connections, benefits of establishing a link as functions of group sizes, and shape of the network. More importantly, the main research agenda is to understand how social groups influence the formation of individual preferences, and how cultural groups structure themselves in order to be present in the next generations.

4.6 Appendix

Proofs of Propositions 4.1 and 4.2

When they maximize their own utility type i children want each to build $\tilde{n}_i^j$ links with group j children. This makes a total of $N_i\tilde{n}_i^j$ links. On the other hand type j children want to build a total of $N_j\tilde{n}_j^i$ links. This problem can be seen as having two sets, one with $N_i\tilde{n}_i^j$ points, and the other with $N_j\tilde{n}_j^i$. We want to find a bijective function between the two sets. This is feasible only if they have the same cardinal. If $N_i\tilde{n}_i^j < N_j\tilde{n}_j^i$ we can find an injective but non surjective function from the first to the second set. There is no injective function from the second to the first set of points. In this case type j is said to be constrained and some type j children cannot achieve their desired number of links. Since I consider "non-integer" links to have symmetric individuals, this proves Proposition 4.1. Proposition 4.2 follows directly.

Proof of Proposition 4.3

If type i is unconstrained then the optimal effort is given by (4.4). If it is constrained they maximize $P^{ii}V^i + P^{ij}V^j - C(\tau_i)$ with P^{ii} and P^{ij} being independent of effort. Their optimal choice is therefore a zero effort.

Proof of Proposition 4.4

A graphical illustration of the problem is helpful. Notice first that the utility of a type i parent can be written $V^{ij} + \Delta_i q_i - C(\tau_i)$. When type j intolerance, measured by $\frac{v_j}{h_j}$, varies it does not affect V^{ij}, Δ_i, and τ_i. Figure 4.4 displays $V^{ij} + \Delta_i q_i$ as a function of p_i. The dashed curve corresponds to a higher type j intolerance. We know that type i parents make a positive effort as long as $\frac{p_i}{1-p_i} < \frac{v_j}{v_i}\frac{h_i}{h_j}$. It corresponds to the flat part of the curves and the threshold for this to hold is decreasing with type j intolerance. This explains why the high intolerance curve steeps up before the low intolerance curve. Similarly higher intolerance implies that the dashed high intolerance curve is always above the low intolerance curve.

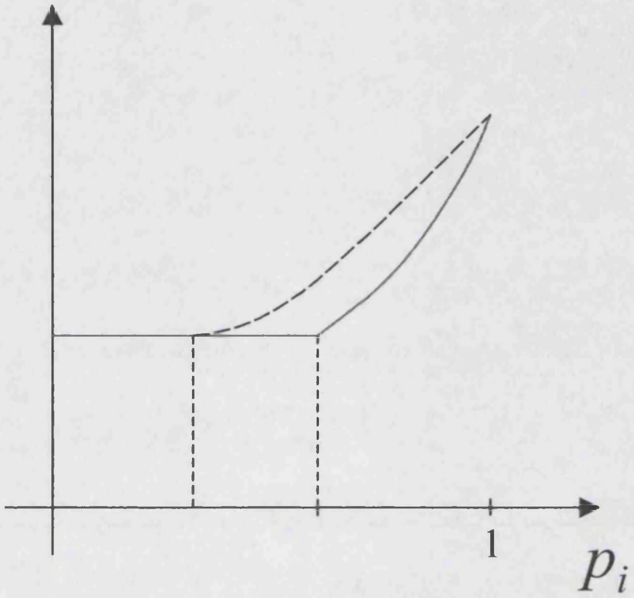

Figure 4.4: $V^{ij} + \Delta_i q_i$

$V^{ij} + \Delta_i q_i$ is therefore increasing with type j intolerance. The reason is that type i children are "more" constrained when type j parents are more intolerant. We have now to take into account the cost of effort $C(\tau_i)$. When the two curves are flat, or strictly increasing, the costs are the same under low and high intolerance. However when only the high intolerance curve is not flat the cost is strictly smaller under this regime, since there is none. It results that for every p_i type i parents welfare $V^{ij} + \Delta_i q_i - C(\tau_i)$ cannot be smaller under high intolerance from type j parents.

Proof of Proposition 4.5

Equation (4.6) is defined piecewise on $[0, 1]$, with different specifications on $[0, \tilde{p}_i]$ and $[\tilde{p}_i, 1]$. When we solve for stationary states, we find two possible interior equilibria: $p_i^* = \frac{\frac{v_i}{h_i} + n_i^i - n_j^j}{2\frac{v_i}{h_i} + n_i^i - n_j^j}$ that corresponds to a regime where type i

is unconstrained (so with positive type i effort), second $p_i^{**} = \frac{\frac{v_j}{h_j}}{2\frac{v_j}{h_j}+n_j^j-n_i^i}$ when type i is constrained and make zero effort.

p_i^* exists as a stationary states if and only if it is in $[0, \tilde{p}_i]$. This condition is equivalent to $0 \leq n_i^i - n_j^j + \frac{v_i}{h_i} \leq \frac{v_j}{h_j}$.

Similarly p_i^{**} exists if and only if it is in $[\tilde{p}_i, 1]$, or equivalently $0 \leq n_j^j - n_i^i + \frac{v_j}{h_j} \leq \frac{v_i}{h_i}$.

First, these two equilibria do not coexist and so there is at most a unique interior stationary state. Second, if it exists then the least connected type is unconstrained in steady state and so makes a strictly positive effort. Third, if $0 > n_i^i - n_j^j + \frac{v_i}{h_i}$ and i is the least connected type then it does not exist.

Finally, assume that there is an interior steady state p. From equation (4.6) $p_{it+1} - p_{it} > 0$ if and only if $0 < p_{it} < p < 1$, and so p has basin of attraction $(0, 1)$. 0 and 1 are unstable steady states. If there is no steady state then from (4.6) and assuming that i is the least connected type, $p_{it+1} - p_{it} < 0$ when $p_{it} > 0$. Hence 0 is a steady state with basin of attraction $[0, 1)$ and 1 is an unstable steady state.

Proof of Proposition 4.6

Parents choose either to make effort τ_i or 0, depending on whether they are constrained or not. First introduce the notation $C_i \equiv C(\tau_i)$ for $i \in \{1, 2\}$. C_i is positive and it can be bounded above by using the fact that τ_i is optimal when type i is unconstrained. Therefore it must be that type i parents get a higher utility with τ_i than with a zero effort when they are unconstrained:

$$\begin{aligned} V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{v_i}{h_i}} - C_i &> V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + v_i} \\ \iff C_i &< \Delta_i n_i^i \frac{v_i - \frac{v_i}{h_i}}{\left(n_i^i + \frac{v_i}{h_i}\right)\left(n_i^i + v_i\right)} \end{aligned} \tag{4.7}$$

Different cases have to be considered. Each time the only two possible

Nash equilibria are $(\tau_i, 0)$ and $(0, \tau_j)$.

- $\frac{p_i}{1-p_i} < \frac{v_j}{v_i}\frac{1}{h_j}$

Regardless of type i and j strategies, type i is always the constrained type (because $\frac{v_j}{v_i}\frac{1}{h_j} < \frac{v_j}{v_i} < \frac{v_j}{v_i}\frac{h_i}{h_j} < \frac{v_j}{v_i}h_i$). So it is a dominant strategy for type i parents to choose τ_i. Given that it is optimal for type j parents not to make any effort. The only Nash equilibrium is $(\tau_i, 0)$.

- $\frac{v_j}{v_i}\frac{1}{h_j} < \frac{p_i}{1-p_i} < \frac{v_j}{v_i} < \frac{v_j}{v_i}\frac{h_i}{h_j} < \frac{v_j}{v_i}h_i$

For these values of p_i type j is constrained only when the strategies are $(0, \tau_j)$. The payoff matrix is

	0	τ_j
0	$V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + v_i}$, $V^{ji} + \Delta_j \frac{n_j^j}{n_j^j + \frac{p_i}{1-p_i} v_i}$	$V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{1-p_i}{p_i}\frac{v_j}{h_j}}$, $V^{ji} + \Delta_j \frac{n_j^j}{n_j^j + \frac{v_j}{h_j}} - C_j$
τ_i	$V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{v_i}{h_i}} - C_i$, $V^{ji} + \Delta_j \frac{n_j^j}{n_j^j + \frac{p_i}{1-p_i}\frac{v_i}{h_i}}$	$V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{v_i}{h_i}} - C_i$, $V^{ji} + \Delta_j \frac{n_j^j}{n_j^j + \frac{p_i}{1-p_i}\frac{v_i}{h_i}} - C_j$

$(\tau_i, 0)$ is always a Nash equilibrium because type i parents prefer τ_i to 0 by a revealed preference argument. Type j parents are better off free riding and so do not deviate.

The conditions for $(0, \tau_j)$ for being a Nash equilibrium are that $\Delta_j n_j^j \frac{\frac{p_i}{1-p_i}v_i - \frac{v_j}{h_j}}{\left(n_j^j + \frac{v_j}{h_j}\right)\left(n_j^j + \frac{p_i}{1-p_i}v_i\right)} > C_j$ and that $C_i > \Delta_i n_i^i \frac{\frac{1-p_i}{p_i}\frac{v_j}{h_j} - \frac{v_i}{h_i}}{\left(n_i^i + \frac{1-p_i}{p_i}\frac{v_j}{h_j}\right)\left(n_i^i + \frac{v_i}{h_i}\right)}$. The first condition is never satisfied for $\frac{p_i}{1-p_i}$ close enough to $\frac{v_j}{v_i}\frac{1}{h_j}$, and using (4.7) it must be satisfied for $\frac{p_i}{1-p_i}$ close enough to $\frac{v_j}{v_i}$. The second condition is not satisfied for low values of p_i, and may be for higher values. $(0, \tau_j)$ may be an equilibrium, depending on the parameters.

- $\frac{v_j}{v_i}\frac{1}{h_j} < \frac{v_j}{v_i} < \frac{p_i}{1-p_i} < \frac{v_j}{v_i}\frac{h_i}{h_j} < \frac{v_j}{v_i}h_i$

	0	τ_j
0	$V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{1-p_i}{p_i} v_j}$, $V^{ji} + \Delta_j \frac{n_j^j}{n_j^j + v_j}$	$V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{1-p_i}{p_i}\frac{v_j}{h_j}}$, $V^{ji} + \Delta_j \frac{n_j^j}{n_j^j + \frac{v_j}{h_j}} - C_j$
τ_i	$V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{v_i}{h_i}} - C_i$, $V^{ji} + \Delta_j \frac{n_j^j}{n_j^j + \frac{p_i}{1-p_i}\frac{v_i}{h_i}}$	$V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{v_i}{h_i}} - C_i$, $V^{ji} + \Delta_j \frac{n_j^j}{n_j^j + \frac{p_i}{1-p_i}\frac{v_i}{h_i}} - C_j$

$(\tau_i, 0)$ is Nash equilibrium under the condition that $C_i < \Delta_i n_i^i \frac{\frac{1-p_i}{p_i}v_j - \frac{v_i}{h_i}}{\left(n_i^i + \frac{1-p_i}{p_i}v_j\right)\left(n_i^i + \frac{v_i}{h_i}\right)}$. It is always satisfied for $\frac{p_i}{1-p_i}$ close enough to $\frac{v_j}{v_i}$ (and may be on the whole interval).

$(0, \tau_j)$ is Nash equilibrium if $C_i > \Delta_i n_i^i \frac{\frac{1-p_i}{p_i}\frac{v_j}{h_j} - \frac{v_i}{h_i}}{\left(n_i^i + \frac{1-p_i}{p_i}\frac{v_j}{h_j}\right)\left(n_i^i + \frac{v_i}{h_i}\right)}$. It must be satisfied for $\frac{p_i}{1-p_i}$ close enough to $\frac{v_j}{v_i}\frac{h_i}{h_j}$.

So potentially there are two Nash equilibria, one, or none. It is easy to show that at least one of these conditions must be satisfied, such that there is always at least one Nash equilibrium. Similarly there always exist values for p_i such that both conditions are satisfied.

- $\frac{v_j}{v_i}\frac{1}{h_j} < \frac{v_j}{v_i} < \frac{v_j}{v_i}\frac{h_i}{h_j} < \frac{p_i}{1-p_i} < \frac{v_j}{v_i}h_i$

Similarly to the other cases, $(0, \tau_j)$ is always a Nash equilibrium. $(\tau_i, 0)$ is Nash if $C_i < \Delta_i n_i^i \frac{\frac{1-p_i}{p_i}v_j - \frac{v_i}{h_i}}{\left(n_i^i + \frac{1-p_i}{p_i}v_j\right)\left(n_i^i + \frac{v_i}{h_i}\right)}$ and $C_j > \Delta_j n_j^j \frac{\frac{p_i}{1-p_i}\frac{v_i}{h_i} - \frac{v_j}{h_j}}{\left(n_i^i + \frac{p_i}{1-p_i}\frac{v_i}{h_i}\right)\left(n_i^i + \frac{v_j}{h_j}\right)}$. The first condition may be satisfied for low values of p_i, but is never for high values. The second condition is always for low values, but never for high values of p_i.

- $\frac{v_j}{v_i}\frac{1}{h_j} < \frac{v_j}{v_i} < \frac{v_j}{v_i}\frac{h_i}{h_j} < \frac{v_j}{v_i}h_i < \frac{p_i}{1-p_i}$

$(0, \tau_j)$ is the only Nash equilibrium.

Putting all these results together, we obtain Proposition 4.6.

The existence of different equilibria with naive and rational parents is now explained with more details than in Section 4.4. When p_i is smaller but close enough to the naive threshold, $(\tau_i, 0)$ may not be a Nash equilibrium because type i parents may prefer to deviate and choose a zero effort. This deviation decreases the proportion q_i of type i individuals in their children network. However type i children are now constrained and so the fall in q_i is not as large as if they were unconstrained and with a zero effort from their parents. On the other hand parents do not have to suffer any cost. This move may be profitable if the cost of τ_i is large enough. The crucial feature for this situation to exist is that the deviation induces a switch between the two regimes type i / type j constrained, in other words because $\frac{v_j}{v_i} < \frac{p_i}{1-p_i} < \frac{v_j}{v_i}\frac{h_i}{h_j}$. Naive parents do not take this into account. Figure 4.5 explains graphically the maximization problem of type i parents for this range of p_i. Their utility function can be written $V^{ij} + \Delta_i q_i - C(\tau_i)$. On this figure $q_i(\tau_i) = \frac{n_i^i}{n_i^i + \frac{v_i}{h_i}}$ is the proportion of type i friends assuming type i is unconstrained. $q_i(0) = \frac{n_i^i}{n_i^i + \frac{1-p_i}{p_i}\frac{v_j}{h_j}}$ is the same proportion but with a zero effort. The optimal effort τ_i^* is such that the marginal cost equals the marginal benefit of effort, or graphically when $V^{ij} + \Delta_i q_i(\tau_i)$ and $C(\tau_i)$ have the same slope. The level of utility reached at this point is $U_i(\tau_i^*)$. When parents of both types do not make any effort, type i children are constrained and their parents get a utility $V^{ij} + \Delta_i q_i(0)$. It must be lower than $V^{ij} + \Delta_i q_i(\tau_i^*)$ because $\frac{v_j}{v_i} < \frac{p_i}{1-p_i} < \frac{v_j}{v_i}\frac{h_i}{h_j}$. However there is no cost attached to it. We can see on Figure 4.5 that it may be profitable for parents to choose a zero effort instead of τ_i^*. The horizontal line $V^{ij} + \Delta_i q_i(0)$ moves up with p_i while everything else is unaffected.[10] The higher p_i is, the more likely it is that type i parents deviate from $(\tau_i, 0)$. Had we assumed $\frac{p_i}{1-p_i} < \frac{v_j}{v_i} < \frac{v_j}{v_i}\frac{h_i}{h_j}$, the deviation would leave type i children unconstrained and

[10]For larger values of p_i cultural group i is larger, and cultural group j smaller. Type i children are therefore "more" constrained because they have to "share" a smaller number of type j friends. $q_i(0)$ increases with p_i.

$V^{ij} + \Delta_i q_i(0)$ would be at the same level than $V^{ij} + \Delta_i q_i(\tau_i)$ when no effort is made. Type i parents would never deviate by optimality of τ_i^*.

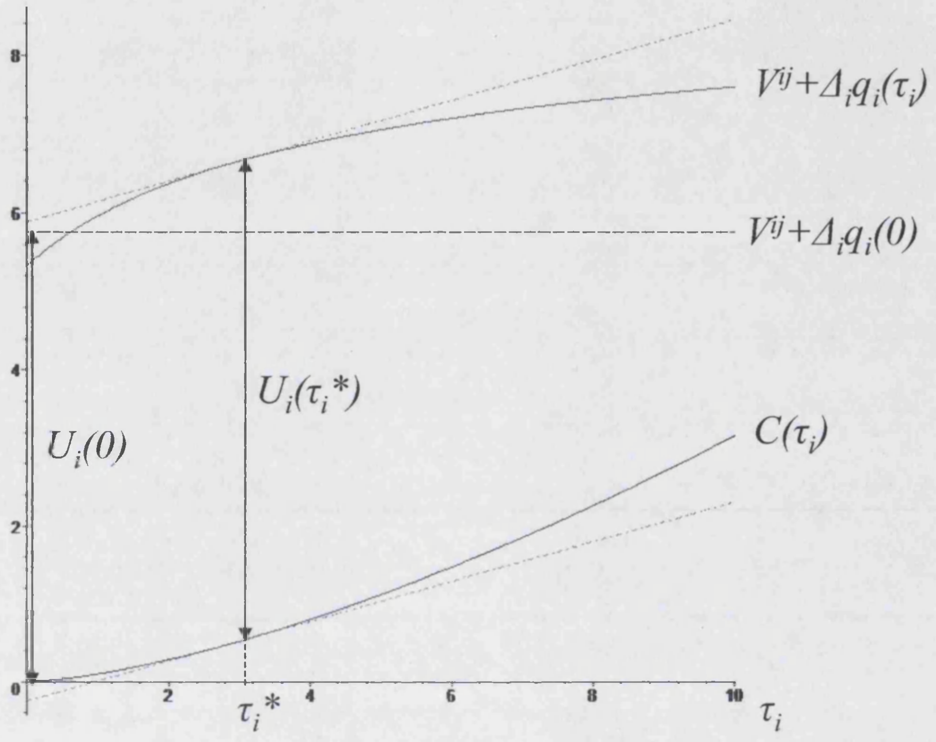

Figure 4.5: Utility levels with positive and zero efforts

Similarly $(0, \tau_j)$ is a Nash equilibrium for p_i close enough to the naive threshold. Type j parents do not want to deviate. If they do they are still constrained and their optimal choice in this regime is τ_j. Type i parents are willing to deviate if the rise in q_i compensates the cost of τ_i. If the cost of τ_i is large enough then the move is not profitable and $(0, \tau_j)$ is a Nash equilibrium. It can be shown that if p_i is smaller but close to the naive threshold then the cost must be large enough, because the rise in q_i is small and so the deviation is never profitable. Here again this situation exists because of the switch between constrained and unconstrained regime, or because $\frac{v_j}{v_i}\frac{1}{h_j} < \frac{p_i}{1-p_i} < \frac{v_j}{v_i}\frac{h_i}{h_j}$.

Proof of Proposition 4.7

Consider the case with two Nash equilibria and refer to the payoff matrix of the case $\frac{v_j}{v_i}\frac{1}{h_j} < \frac{v_j}{v_i} < \frac{p_i}{1-p_i} < \frac{v_j}{v_i}\frac{h_i}{h_j} < \frac{v_j}{v_i}h_i$ in the proof above. If type i parents were allowed to choose between the two equilibria, they would have to compare, $V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{v_i}{h_i}} - C_i$, their payoff in $(\tau_i, 0)$ to $V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{1-p_i}{p_i}\frac{v_j}{h_j}}$, their payoff in $(0, \tau_j)$. But that first payoff is also their payoff in (τ_i, τ_j) and because $(0, \tau_j)$ is a Nash equilibrium, it must be that $V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{v_i}{h_i}} - C_i < V^{ij} + \Delta_i \frac{n_i^i}{n_i^i + \frac{1-p_i}{p_i}\frac{v_j}{h_j}}$. A perfectly symmetric argument holds for type j parents. Parents consequently prefer the Nash equilibrium where they make zero effort, when two Nash equilibria exist.

When there is only one Nash equilibrium, for instance $(\tau_i, 0)$, type i parents are better off than in $(0, 0)$ because $(\tau_i, 0)$ is Nash, and than in $(0, \tau_j)$ because it is not an equilibrium. They are indifferent between $(\tau_i, 0)$ and (τ_i, τ_j). On the other hand type j parents must be better off than in $(0, \tau_j)$ and (τ_i, τ_j) because $(\tau_i, 0)$ is Nash, and than in $(0, 0)$ because $\frac{p_i}{1-p_i} < \frac{v_j}{v_i}h_i$. So a unique Nash equilibrium maximizes the payoffs of all the parents.

Proof of Proposition 4.11

The different solutions derive from noticing that $n_i^i < n_j^j$ iff $p_i^* < p_i^{**} < \frac{1}{2}$, and from using the definition of p_3.

Chapter 5

Common property resource privatization and labor allocation

5.1 Introduction

During the last decades, there has been much debate on common property resources (CPR). The seminal article by Hardin (1968) regards commons as a damaging way of exploiting a resource unless population density is very low. Under a growing population it ultimately leads to its ruin when the resource starts to become scarce. Common ownership strengthens depletion because users do not internalize the impact of their action on the return to others. This is a classical example of institution failure. Therefore according to Hardin, the tragedy of the commons can only be averted by a different institution, private property, that offers well defined property rights. Private ownership makes people internalize the social impact of their acts and allocates the optimal quantity of labor to the resource in order to reach the first best outcome.

By solving the tragedy of commons, privatization reduces labor allocation on the privately held resource. This chapter considers the impact of this labor shift in a complex environment with two resources, land and forest in the following example. Initially, both are under communal tenure and subject to the tragedy of commons. Due to state intervention private property rights are established on land. This reduces labor use on land and shifts some individuals to the forest, still under common property regime. This in turn worsens overexploitation of the remaining CPR and lowers the payoff individuals are able to extract from it. A crucial feature is that this payoff determines the equilibrium wage on land. This creates a link through labor supply between the two resources such that the return to labor decreases after privatization. The forest is more depleted and people working on it must be made worse off. Overexploitation in the forest may have also negative consequences on privatized land in an ecosystem (through, for instance, worse water supply, or erosion leading to increased risk of land slides). This technological link between resources imposes a negative externality on land that is exacerbated by privatization.

Even though labor is made worse off after privatization, the rents extracted from private ownership may be sufficient to compensate for the fall in labor return and thus increase welfare. If the distribution of property rights is equitable, such that each individual is entitled to receive an ownership rent, then privatization would be a Pareto improvement. The argument in this chapter is that this result does not necessarily hold when labor moves to the congested remaining CPR, even with an equal allocation of property rights. Welfare may decrease after privatization.

The chapter derives also results on privatization design by introducing heterogeneity among agents regarding their skills on land. Some have a higher productivity. Privatization is seen as a promoter of this skill, making it more productive under private tenure. This is to recognize that land titling has some beneficial effects that are not included otherwise in the model (tenure security, availability of a collateral for a loan). If skills are high enough, land reform may actually increase the return to labor. In any case, two results are proved. First, the depletion of the CPR (forest in the example) is minimized when land is distributed exclusively to skilled individuals. Being more productive, they have a larger labor demand than unskilled landowners and reduce the quantity of labor on the CPR. Second, welfare is maximized under the same land distribution, but it may still be smaller than before the land reform.

Finally inequality in skills is shown to reduce the benefits of privatization. This argues for measures designed to improve skills, in order to use them fully under private tenure.

The setting of the model applies to the numerous cases where a resource cannot be privatized, for instance because of economies of scale (Baland and Platteau 2003). The costs of defining and enforcing private property rights may be very high, particularly when the resource is extended spatially, or if it is part of an ecosystem. Forest, mangroves, and grazing meadows are good examples. Even when rights can be easily defined, risk pooling considerations may make division of the resource suboptimal when it has a low predictability (high variance of the value per unit of time per unit area, Dasgupta 1993). On the other hand, resources with high predictability and high average value per unit area tend to be held under private property.

Netting (1981) documents in his study of the Swiss Alps a situation where a resource is optimally private while the other is common: the summer pastures are communal while the fertile lands, more easily accessible, and concentrated, are privately owned.

The result that labor allocation and returns to labor decrease after privatization is well known in the literature (Weitzman 1974, Baland and Francois 2005, see De Meza and Gould, 1985, and Brito et al., 1997, for situations where this does not hold), even though it relies on slightly different grounds in this chapter because it is driven by the congestion effect. Weitzman and Cohen (1974) confirm the prediction of a fall in returns to labor by considering the enclosure movement in medieval England.

The fall in employment is also acknowledged in case studies, for instance in Kisamba-Mugerwa (1998). Looking at property rights in Uganda, it argues that privatization may increase unemployment. Jodha (1985), in his study of common property resources in India, documents how privatization of former CPRs led to over crowding and over exploitation of remaining CPRs, illustrating the fact that labor goes to a second, and non privatized, CPR subject to congestion.

The negative externality imposed on the privatized resource by the over-exploitation of the CPR is not necessary to get any of the results presented in this chapter but it serves as a simple device to exemplify the interconnection of environmental resources, and how it can reinforce or alleviate some effects. While one could give a much more complete survey of how resources can influence each other through their exploitation, it is not the point in this article. Two examples are briefly given as an illustration. Clarke, Reed and Shresta (1993) argue that forests convey beneficial externalities on both users and non users through their complementary role in facilitating agricultural production. Mangroves also constitute a good example of a resource usually under common property tenure that is part of an overall ecosystem. They reduce the effects of flooding, storm surges, and erosion of coastal land that may be used for agricultural purposes. More subtle interactions may also intervene. If people allocate their time between the two resources, and if more time must be spent on the remaining CPR for a given payoff because of overexploitation, then there may be a drop in agricultural productivity. Secondly, CPRs serve as a cushion when private resources fail to meet needs. If these common resources are more degraded, they lose this ability (Jodha (1995)) and people can be worse off. When landowners face negative shocks on their land, they turn to the CPR. But if these are more depleted because of the former privatization, they suffer a loss.

One does not have necessarily to consider a second CPR to make sense of the model. The determining feature is that the outside opportunity is congested. Another CPR is a possible and classic example but alternatives can be examined. If the flow of labor increases competition between workers in an outside manufacturing activity, wages can go down and the consequences are identical. The model also yields the same results when there is a positive externality on the privatized resource, and so covers many possible links between the two sectors.

The advantages of common property have already been underlined in the literature. Usually these rely on economies of scale (Baland and Platteau, 2003), risk pooling considerations when production is subject to idiosyncratic shocks (Carter, 1987), or insurance properties (Baland and Francois, 2005). The argument in this chapter does not build on these elements, but rather on the simple idea that by reducing labor, privatization forces people to use their outside opportunity and that this may respond negatively to the flow of new workers. Cohen and Weitzman (1975) are close to the results in this chapter but they consider an economy with two sectors, agriculture and manufacturing. Although privatization increases national product in the economy, workers do not benefit from it because they do not have access to the rents generated by the reform. This chapter makes a stronger case by allowing workers to be landowners as well, and so to extract the rent. Contrary to Weitzman and Cohen results, it is demonstrated that welfare (defined as the sum of all the incomes, so equivalent to national product) does not necessarily increases.

The remainder of the chapter is organized as follows. Section 2 outlines the framework for my analysis. Section 3 investigates the case where agents are homogeneous. Then section 4 presents the results with heterogeneous agents. Section 5 and 6 consider the influence of the privatization design. Section 7 examines the effect of skills inequality. Section 8 provides some further discussion on multiple equilibria in the model. Section 9 concludes and discusses the implications of the model for property rights reform.

5.2 The model

Consider a closed economy with $\overline{L}$ agents endowed with one unit of labor that is supplied inelastically. There are two CPRs R_1 and R_2 in the economy such that resource R_1 is privatizable whereas R_2 is not. Furthermore, R_2 is subject to congestion. labor is mobile between the two sites. The amount of CPR R_1 is fixed and equal to K. Good 1 can be produced from R_1 and labor L using a constant returns to scale production function $Y(K, L)$ that is strictly increasing, concave in L and continuously differentiable. Good 2 is produced with R_2 and requires only labor. The payoff for each agent working on R_2 is $g(L)$, with $g'(L) \geq 0$ where L is labor allocated to R_1(congestion effect).

The production of good 2 generates a negative externality $\psi(L)$ (ψ is a continuous function of L and $\psi'(L) > 0$)[1,2] on the production of good 1, such that the production function is altered to $Y(K, L)\psi(L)$. The crucial feature for the results is that g increases with L. If ψ is a constant, or even $\psi'(L) < 0$, all the results still hold.

[1] To keep a small number of variables, g and ψ are functions of L, the number of people working on R_1. This is why it is an increasing function of L. However g and ψ should be more rigorously considered as functions of $\bar{L} - L$, with $\frac{\partial g}{\partial(\bar{L}-L)} < 0$ and $\frac{\partial \psi}{\partial(\bar{L}-L)} < 0$.

[2] It may not be obvious that $\psi' > 0$. It could be imagined that whatever the number of workers on R_2 they always produce the same output and therefore create the same negative externality. A simple model with a congested CPR is developed in the Appendix to show that $\psi' > 0$ can be easily justified.

A basic model with homogeneous agents based on Baland and Francois (2005) is first presented. Then a more general framework with heterogeneous agents is considered. R_1 is usually identified as land but it should be kept in mind that this is not necessarily the case.

5.3 Homogeneous Agents

5.3.1 Commons

When R_1 is not privatized, L agents working on R_1 are paid their average product $\frac{Y(K,L)\psi(L)}{L}$. This implies that there is a full tragedy of the commons on R_1 and that labor is overallocated to R_1.

In equilibrium the payoffs are equalized across resources. The equilibrium quantity of labor L^c solves the equation[3]:

$$\frac{Y(K,L^c)\psi(L^c)}{L^c} = g(L^c) \tag{5.1}$$

5.3.2 Private property

In this section, privatization where land is distributed equally among L individuals, with $0 < L \leq \bar{L}$, is investigated. Note that with homogeneous agents the value of L does not affect any result because of the constant returns to scale assumption. In particular the case where land is distributed only to individuals who used to work on R_1 (i.e. to L^c agents) leads to the same equilibrium wage than the perfectly egalitarian case where $L = \bar{L}$.

Each agent receives a parcel of size $\frac{K}{L}$ of R_1. Individuals working on R_1 (now privately owned) earn the wage w. Output on each parcel is given by

[3]This is true only for interior equilibrium. If there is only a corner equilibrium, it does not change the results, but labour allocation can be identical under both regimes.

$Y(\frac{K}{L}, l)\psi(L^p)$, with L^p the quantity of labor on the private resource and l the quantity of labor that each owner decides to hire. labor is allocated to each parcel such that the marginal product MP is equalized across the (identical) parcels, and the wage on the labor market is:

$$\begin{aligned} w(K, L^p) &= MP(K, l, L^p) = \frac{\partial}{\partial l}\left(Y\left(\frac{K}{L}, l\right)\psi(L^p)\right) \\ &= Y_L\left(\frac{K}{L}, l\right)\psi(L^p) = Y_L(K, L^p)\psi(L^p) \end{aligned} \tag{5.2}$$

Note that each owner does not internalize the effect of an additional unit of labor on the externality. The last equality comes from the homogeneity of degree 1 of Y and from $l = \frac{L^p}{L}$ since all the parcels are identical.

Each individual owns some land such that he receives the rent $Y(\frac{K}{L}, l)\psi(L^p) - wl$. Earnings of a resource 1 owner are $w(K,l) + \frac{1}{L}Y(K, L^p)\psi(L^p) - w(K,l)l$ and an interior equilibrium is such that the wage is the same on the two resources:

$$w(K, L^p) = g(L^p) \tag{5.3}$$

The land market is assumed to be complete. Because of the constant returns to scale assumption, homogeneous agents are indifferent between buying and selling land. The maximum price someone is willing to pay for land is equal to the minimum price a landowner is willing to sell his land for. One can innocuously (and realistically) assume that agents prefer to own some land rather than sell it, be landless and employed on the very same land. If it is the case then no transactions are observed on the land market. If it is not, then some transactions occur but apart from introducing inequality between landowners profits, it does not affect any other variables.

Equilibrium

Note that (5.1) and (5.3) can be rewritten:

$$\begin{aligned}\left(\frac{\psi}{g}\right)(L^c) &= \frac{L^c}{Y(K,L^c)}\\ \left(\frac{\psi}{g}\right)(L^p) &= \frac{1}{Y_L(K,L^p)}\end{aligned}$$

$\frac{L}{Y(K,L)} < \frac{1}{Y_L(K,L)}$ $\forall L$ by concavity of Y.

A major factor influencing equilibrium is the ratio $\left(\frac{\psi}{g}\right)(L)$. This measures the relative importance of the negative externality to the payoff on the common resource. Its characteristic affecting equilibrium is its variation with L. The next sections consider the two cases $\left(\frac{\psi}{g}\right)' < 0$ and $\left(\frac{\psi}{g}\right)' > 0$[4].

$\left(\frac{\psi}{g}\right)' < 0$

When people are allocated to the privatized resource, this decreases the negative externality and so increases the payoff on R_1 but at the same time it makes production on R_2 more profitable. This in turn rises the equilibrium wage, reducing profits on R_1. A key consequence in equilibrium of the interdependency between the two resources is that there is a tradeoff between a low level of externality and a cheap labor. If $\left(\frac{\psi}{g}\right)' < 0$, then the gain in production due to the reduced negative externality is always smaller than the increase in the payoff on the non-privatized resource. In other words, ψ is less elastic than g with respect to L. This implies that the rise in the equilibrium wage will offset the gains from the reduced externality when labor allocation on R_1 increases. Two important special cases of $\left(\frac{\psi}{g}\right)' < 0$ is first when ψ is constant, that is there is no externality on land of overexploitation on the remaining

[4]The case $\left(\frac{\psi}{g}\right)' = 0$ leads to the same conclusions.

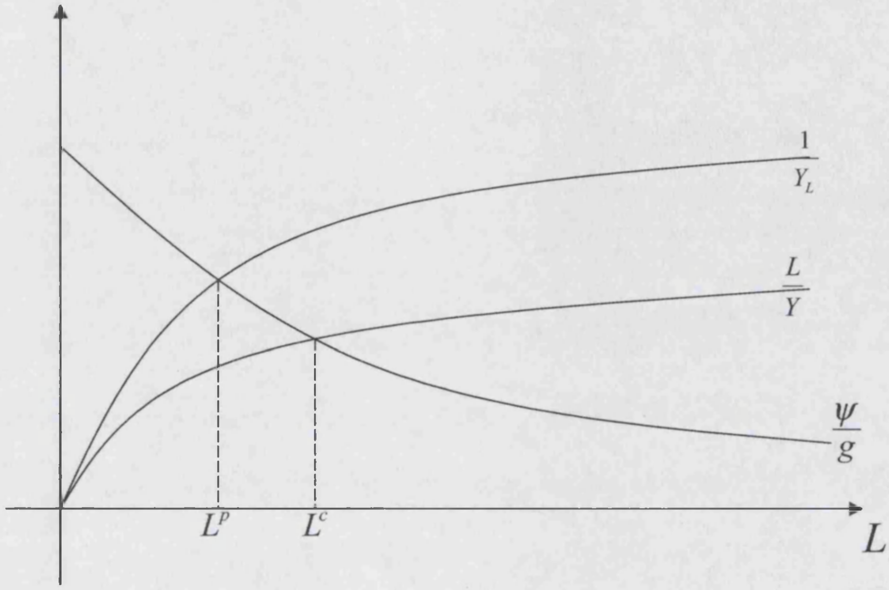

Figure 5.1: Equilibrium with homogeneous agents and $\left(\frac{\psi}{g}\right)' < 0$.

CPR. Second, if $\psi' > 0$ is relaxed and $\psi' < 0$ is allowed, then $\left(\frac{\psi}{g}\right)' < 0$. Thus this paragraph deals with negative but not too elastic (to have $\left(\frac{\psi}{g}\right)' < 0$) externalities and positive externalities.

Starting from the common equilibrium labor allocation, the corresponding marginal product of labor under private regime is too low compared to the payoff on R_2. This too low marginal product induces workers to move to R_2, until payoffs on the two resources are equalized.

The proof is straightforward from the equilibrium conditions. Graphically, we have the following situation[5]:

$\left(\frac{\psi}{g}\right)' > 0$

In this case, allocating labor to R_1 reduces proportionately more the negative externality than it increases the payoff on R_2. ψ is more sensitive to L than g. A possible situation is illustrated by Figure 5.2.

[5] Note that the convexity of the two increasing curves is not determined but that this does not affect the result.

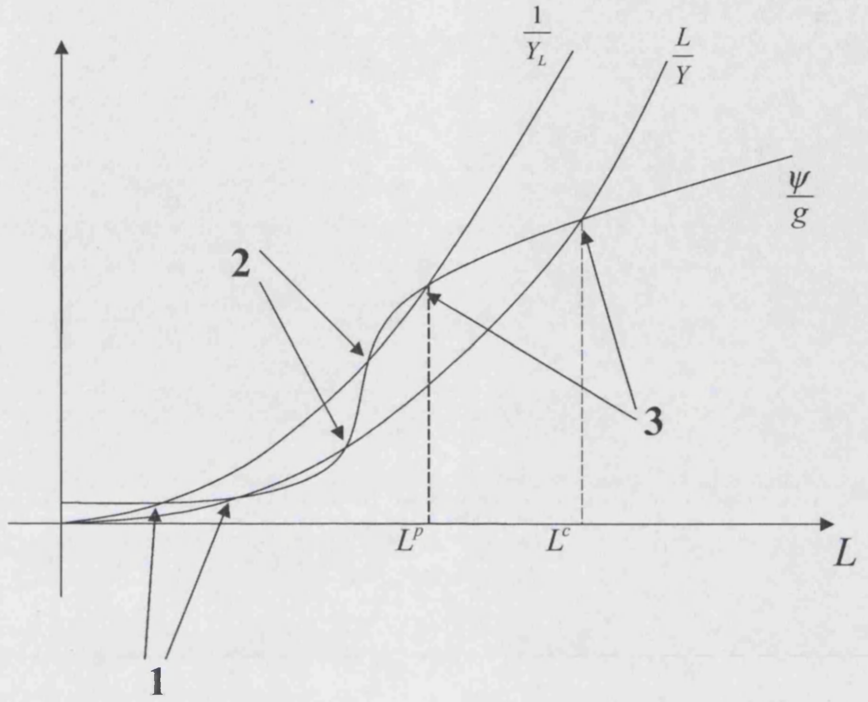

Figure 5.2: Equilibrium with homogeneous agents and $\left(\frac{\psi}{g}\right)' > 0$.

When $\left(\frac{\psi}{g}\right)' > 0$ unicity of the equilibrium is not guaranteed any more. On Figure 5.2, there are three equilibria but only two are stable. Consider the equilibrium labelled 2. If L increases from this equilibrium, payoff on R_1 is higher than payoff on R_2. In other words, the marginal (or average in the case of common held property) product of labor is higher than the wage given by $g(L)$. Therefore more labor is allocated to R_1 until the payoffs are equalized in equilibrium 3. The same analysis holds if L is decreased from equilibrium 2: the marginal product is too low and equilibrium 1 is finally reached.

Since $\left(\frac{\psi}{g}\right)' > 0$, a larger L means that the gain due to the reduced externality is always larger than the change in the market wage. Therefore as long as this relative gain compensates for the decreasing marginal (or average) product, it is better to increase N. This is what happens when moving from equilibrium 2 to 3. A similar reasoning holds for a move from 2 to 1: ψ falls relatively more than g, therefore L must keep on decreasing to reach a new equilibrium. This also implies that equilibria 1 and 3 are stable.

It is easy to construct graphically an example such that there is a stable equilibrium under private but not under common regime. In this case it cannot be argued that $L^p > L^c$. It must not be forgotten that L must be smaller than $\bar{L}$. If there is a stable equilibrium that does not exist under common regime, then $L^c = \bar{L}$ is a common equilibrium (corner solution with (5.1) satisfied as an inequality) and it is stable. This still yields $L^c \geq L^p$.

A final remark is that with multiple equilibria, it could be that the common equilibrium is in 1 whereas the private equilibrium is in 3. However, and without any formal argument, it is simply assumed that the population stays in the "same" equilibrium after the property rights change. Another argument could be that between two equilibria, the most Pareto efficient is always chosen. This would lead to a unique equilibrium where $L^c \geq L^p$.

More labor is allocated to R_1 under the common regime. Therefore the conclusion is similar to the case $\left(\frac{\psi}{g}\right)' < 0$ and this gives Proposition 5.1, valid for all values of $\left(\frac{\psi}{g}\right)'$.

Proposition 5.1 *More labor is allocated to the production of good 1 under CPR than under private regime and the return to labor is smaller after privatization.*

A direct consequence of Proposition 5.1 is that after privatization the common property resource is more depleted and income from this resource is smaller. However, all the agents now receive some income from the privatized resource and it may be that overall they are better off. This point is investigated in the next section.

Welfare analysis

Welfare W is defined in a utilitarian way as the sum of all the incomes in the population. Therefore under private regime $W^p = L\left[\frac{1}{L}Y(K,L^p)\psi(L^p) - w\frac{L^p}{L}\right] + w\bar{L} = Y(K,L^p)\psi(L^p) - wL^p + w\bar{L}$. Under common regime $W^c = \bar{L}\frac{Y(K^c,L^c)\psi(L^c)}{L^c}$.

Using (5.1) and (5.3),

$$W^p - W^c = \pi^p - \bar{L}\left[g(L^c) - g(L^p)\right]$$

where $\pi^p = Y(K,L^p)\psi(L^p) - wL^p$ is the total profit generated under private regime[6]. Privatization increases welfare for labor allocations such that

$$\pi^p > \bar{L}\left[g(L^c) - g(L^p)\right] \qquad (5.4)$$

This can be illustrated graphically by considering both sides of (5.4) as functions of L^p and taking L^c as given. It should be noted first that if R_2 was not congested, privatization would always increase welfare. It would create landownership rents and allocate optimally labor between the two resources, without changing the return to labor. This is why congestion is the driving force behind the results.

For some given values of parameters, L^c is fixed and it is known that $L^p < L^c$, but not exactly how far L^p is from L^c. All the values $L^p \in [0, L^c]$ are valid candidates for the optimal private labor allocation. The right hand side of (5.4) is decreasing in L^p and equal to zero when $L^p = L^c$. Furthermore it can be shown by differentiation that π^p increases with L^p[7], with $\pi^p(0) = 0$.

The two curves must intersect. This implies that there always exists a $\tilde{L}$

[6] Since L^p does not depend on L, it does not depend on land allocation.

[7] First, there is hère abuse of notation. L^p must be considered as a simple variable, and not as the private regime equilibrium. Second, this proof is straightforward $\frac{\partial \pi}{\partial L} = \frac{\partial}{\partial L}\left(Y\psi - Y_L\psi L\right) = \left(Y - LY_L\right)\psi' - LY_{LL}\psi > 0$ by concavity of Y.

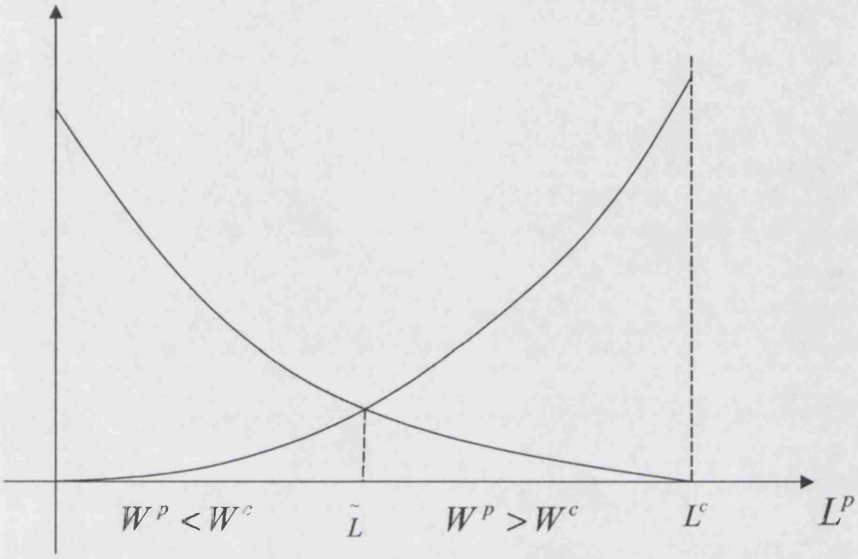

Figure 5.3: Welfare

such that $\forall L^p$ with $L^p < \tilde{L}$ (resp. $L^p > \tilde{L}$), $W^p < W^c$ is satisfied (resp. $W^p > W^c$). (5.4) means that welfare increases after privatization if the total landownership rent is large enough to compensate for the fall in all the wages. It cannot be argued that welfare increases if privatization does not reduce labor allocation on the privatized resource by a large amount, as could be wrongly inferred from the diagram (to change L^p one must change the model parameters and this in turn affects $\tilde{L}$). Actually a large difference between L^c and L^p can be beneficial if it increases profits a lot. Depending on the parameters of the model, there may be a tradeoff between a loss in wages and a larger profit. The appendix provides numerical examples to illustrate different cases.

Proposition 5.2 *The change in welfare is ambiguous, as the fall in wages may be compensated by larger profits. A privatization with a large impact on labor allocation, hence a large fall in wages, may actually increase welfare by having a large positive impact on profits.*

It has been proved that privatization changes labor allocation and thereby increases the depletion of R_2. Furthermore it may be that it decreases welfare, i.e. the sum of all the incomes in the population, even in the case where land distribution is perfectly equitable.

The assumptions of homogeneous agents and perfectly egalitarian privatization are quite restrictive and the next section relaxes them.

5.4 Heterogeneous agents

In this section, agents can have two skill levels γ_h and γ_l to produce good 1 (with $\gamma_h > \gamma_l$)[8]. There are N_h agents with high skills. This skill differential is only relevant for the production of good 1 and it affects both productivity as a worker and landowner talent. A highly skilled agent is not only more productive when working on R_1 but his landowning has also a positive effect on the production of the resource exploitation.

In this setting a low skilled individual may benefit from selling or renting out its plot to a more productive agent. However, these two possibilities are ruled out in the analysis. Bans on sales of redistributed land are actually part of some existing land reforms (see for instance Banerjee 2000, who also provides the motivation for these restrictions). The impossibility can be the result of contracting difficulties that imply a cost larger than the benefit of renting. However, if it is possible to rent land, the welfare analysis in Section 6 provides an indication of what welfare would be in that case.

Production function is changed to take into account skills $Y = K^{\alpha}H^{1-\alpha}$ where H is the amount of skills used to produce output, and $\alpha < 1$.

All the proposition proofs are in the appendix.

[8]These skills are due to some market imperfection not precised here. These may come from borrowing constraints on capital markets for instance.

5.4.1 Commons

In this part, the analysis is restricted to the case where in equilibrium all the γ_h agents and some of the γ_l agents work on R_1. The distribution of skills maximising welfare under common regime is usually not this one. L^c, and usually welfare, are decreasing with the number of γ_h types on R_1. The intuition is that a γ_h type is more productive on R_1 but equally productive on R_2. Equilibrium is reached with a smaller number of individuals on R_1 and this decreases the payoff of each agent[9]. Therefore to consider that all the γ_h agents work on R_1 actually reduces the benefit of the CPR regime. The optimal skill distribution would make the result that privatization may not be beneficial even stronger.[10]

When N_h agents with high skills and N agents with low skills work on good 1, the payoff for a γ agent on good 1 is given by:

$$\frac{\gamma}{N_h\gamma_h + N\gamma_l} Y(K, N_h\gamma_h + N\gamma_l)\psi(N_h + N) = \gamma K^\alpha (N_h\gamma_h + N\gamma_l)^{-\alpha}\psi(L) \quad (5.5)$$

Therefore a more skilled agent is able to extract a higher share of output from the total output.

In equilibrium, payoffs between R_1 and R_2 are equalized and

$$\gamma_l K^\alpha (N_h\gamma_h + N\gamma_l)^{-\alpha}\psi(L) = g(L) \Longleftrightarrow \gamma_l K^\alpha \left(\frac{\psi}{g}\right)(L) = (N_h\gamma_h + N\gamma_l)^\alpha \quad (5.6)$$

[9] This does not happen with certainty for γ_h agents as for a given L^c they lose earnings by moving to R_2 since they earn $\frac{\gamma_h}{\gamma_h} g(L^c)$ on R_1 and only $g(L^c)$ on R_2. However the increase in L^c to reach the new equilibrium compensates (partially or totally) this negative effect.

[10] To justify that all the γ_h agents are on R_1, one can think about the following argument: start with a given skills distribution across the two resources with some number $N > 0$ of γ_h individuals on R_2. The same number of γ_l individuals on R_1 would agree to switch with them. The low skilled types would not change their payoff, whereas the high skilled would (apparently a Pareto improvement). They would agree only if they do not anticipate that this shift of workers would change the equilibrium, resulting in a smaller L^c and consequently in a smaller payoff for the low skilled. If they do not, then the resulting distribution is that all the γ_h individuals work on R_1 (assuming that $N_h \leq L^c$).

5.4.2 Private Property

Skills influence both the productivity of the worker and of the landowner: a γ worker is paid γw and a γ landowner hiring labor maximizes $\pi = \gamma \left(\frac{K}{N}\right)^{\alpha} h^{1-\alpha}\psi(L) - wh$ with $h = \sum_i \gamma_i$ (this type of production function is similar to the one used in Murphy, Shleifer and Vishny 1991)[11]. Therefore the skill of the landowner influences the whole production. Private regime is thus assumed to give the opportunity to use the skill more productively. This is to allow for the productivity gains usually associated to privatization. A landowner hires his own labor, therefore he earns a wage and a landownership rent.

First is investigated privatization where only γ_h agents are landowners and such that in equilibrium $h > \gamma_h$ (i.e. landowners want to hire $n_h(\geq 0)$ γ_l workers). This makes heterogeneity in skills less relevant but helps to understand the main features of the model before turning to the general case.

The quantity of labor on R_1 is $L = N_h + N_h n_h$. Landowners maximize their profits such that:

$$\max_h \pi \implies \gamma_h(1-\alpha)\left(\frac{K}{N_h}\right)^{\alpha} h^{-\alpha}\psi(L) = w \tag{5.7}$$

By assumption $h = \gamma_h + n_h\gamma_l$, implying that the optimal n_h satisfies

$$\gamma_l w = g(L) \iff \gamma_l(1-\alpha)\gamma_h K^{\alpha}\left(\frac{\psi}{g}\right)(L) = (N_h\gamma_h + N_h n_h \gamma_l)^{\alpha} \tag{5.8}$$

(5.6) and (5.8) define the equilibrium labor allocation under both regimes.

[11] I could assume more generally that the production function is $m(\gamma)\left(\frac{K}{N}\right)^{\alpha} h^{1-\alpha}\psi(L)$ with $m'(\gamma) > 0$ but I take the symplifying assumption that $m(\gamma) = \gamma$. This does not change radically the results and makes them much simpler.

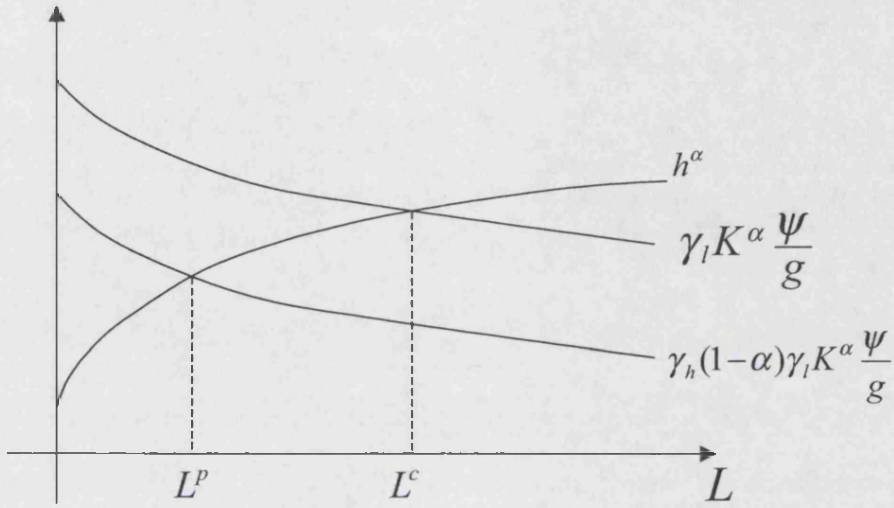

Figure 5.4: Equilibrium with heterogeneous agents, $\left(\frac{\psi}{g}\right)' < 0$ and $\gamma_h(1-\alpha) < 1$.

5.4.3 Comparison between private and common property equilibrium

$\left(\frac{\psi}{g}\right)' < 0$

This section and the next one are very similar to Sections 3.2.2 and 3.2.3 but the figures are presented in a slightly different way[12].

At the common equilibrium labor allocation, private equilibrium cannot be sustained because the marginal product is smaller than the wage given by payoff on R_2. Workers move to the commons. By doing so they reduce the wage by a greater amount than the externality and marginal product on R_1 increases. Private property equilibrium is reached when marginal product and wage are equal.

If the share of labor $1-\alpha$ is not too high and/or if the landowner is not

[12]Note that on the vertical axis of the next diagrams $h^\alpha > 0$. This is not true for $L = 0$ and these graphs should be seen as starting for $L = N_h$. The complete graph of h^α would have a discontinuity in the slope at $L = N_h$ as landowners start hiring γ_l workers for $L > N_h$. If the equilibrium is at $L < N_h$ this implies that no γ_l workers are hired but it does not affect the main result.

too skilled then γ_l workers are worse off because they earn a smaller wage and overexploitation of R_2 is more severe after privatization. The fall in labor demand is larger if the share of labor $1-\alpha$ is low. If landowners are highly skilled (γ_h high), this may compensate this effect. Finally, note that the equilibrium is stable: if L is increased from the equilibrium, the marginal product of labor will be lower than the wage on R_2 and workers are going to reallocate to this resource, coming back to the equilibrium. Looking at $\frac{\psi}{g}$, this means that when L rises, the increase is proportionately larger in g than in ψ. Therefore the fall in the negative externality is too small compared to the change in the market wage.

$\left(\frac{\psi}{g}\right)' > 0$

When $\left(\frac{\psi}{g}\right)' > 0$ multiple equilibria are possible. However, like in Section 3.2.3, instable equilibria can be ruled out and the result that privatization reduces labor allocation still holds.

On Figure 5.5, equilibrium 2 is unstable. Increasing L will induce more people to move to the privatized resource because congestion proportionately increases less than the benefits from the externality. Similarly, decreasing L will make more people work on the common property resource because payoff on R_2 falls less than the marginal product on R_1.

In a stable equilibrium it is always true that $L^p \leq L^c$.

These results are summarized in Proposition 5.2.

Proposition 5.3 *If landowners' skill and the share of labor are such that* $\gamma_l(1-\alpha) < 1$ *then more labor is allocated to a resource under common regime.*

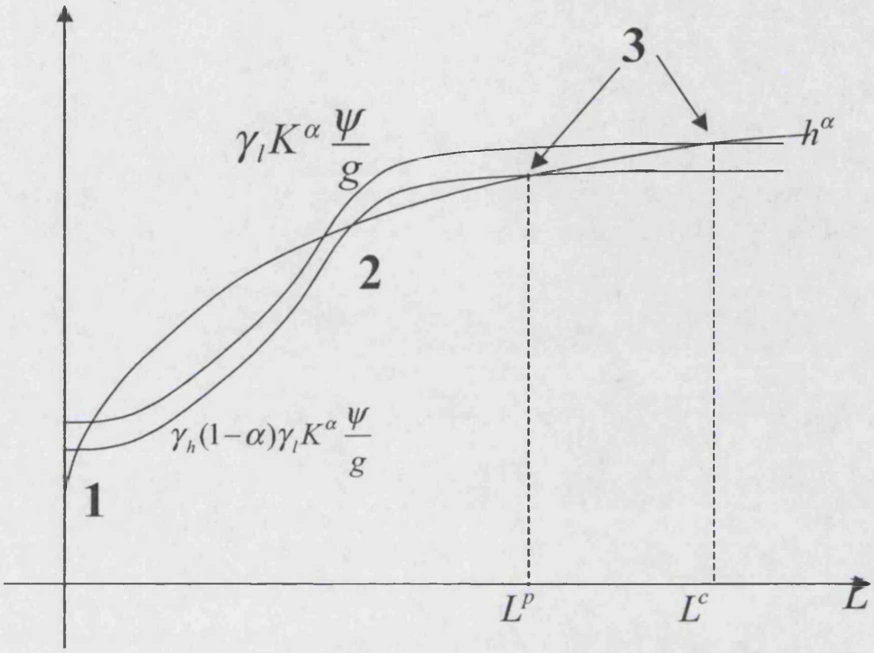

Figure 5.5: Equilibrium with heteregenous agents, $\left(\frac{\psi}{g}\right)' > 0$ and $\gamma_h(1-\alpha) < 1$.

Note that it is also possible to have an equilibrium under commons (resp. private property) that does not exist under private property (resp. commons) as in Figure 5.6. However using the same argument than in Section 3.2.3, the stable equilibrium with the highest labor allocation (if it exists) is always a common property equilibrium.

The main result of this section is that as long as labor share and skills are not too high privatization reduces wages and depletes the common property resource. This holds regardless of $\frac{\psi}{g}$. Consider also the case where $\frac{\psi}{g}$ is non-monotonic. There are multiple equilibria but stable equilibria are all such that $L^p \leq L^c$. This is shown on Figure 5.7 where only equilibrium 1 and 2 are stable.

However only a partial privatization with all γ_h being landowners and all γ_l being landless has been considered. A natural question to ask is whether a more or less egalitarian privatization can lead to better outcomes.

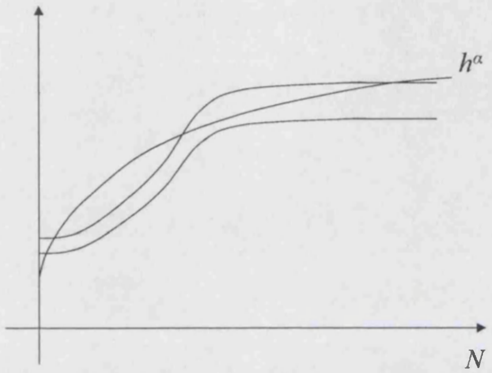

Figure 5.6: Alternative outcome

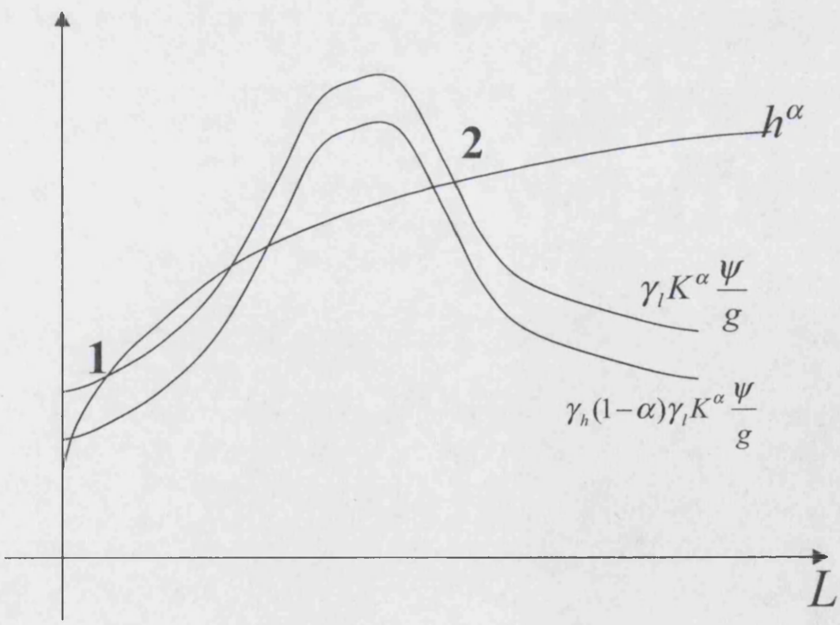

Figure 5.7: Multiple equilibria

5.5 More egalitarian privatization

In this section, all the γ_h types and N_l low skilled agents are landowners. Only locally stable equilibria are considered. This implies that $\frac{\phi'}{\phi}$, with $\phi = \frac{\psi}{g}$, can be negative or positive but not too large (as is illustrated by Figure 5.5, a formal condition for stability is given in the appendix). Before investigating the consequences of a more egalitarian privatization, a few important points must be made.

As in the common regime, the skill distribution over the two resources is not neutral. Would welfare be increased if it was assumed that some high skill agents (but not all) and some strictly positive number of low skilled agents were landowners? For a given number of γ_l landowners, it can be shown that both L^p and welfare increase with the number of γ_h landowners. Thus optimally, each γ_h individuals should be given some land. It is what is assumed in the following section. Remember that under common property, the minimizing welfare skill distribution has been considered. Hence the worst (looking at welfare) common property regime is compared to the best private property regime.

5.5.1 Labor demands

Let n_l be the number of workers a γ_l landowner hires. Proposition 5.3 is derived by maximizing profits for the two types of landowners:

Proposition 5.4 *labor demand is increasing with skills and a more egalitarian privatization has a larger effect (in absolute terms) on highly skilled landowners labor demand.*

This proposition says how a more egalitarian privatization affects labor allocation. It suggests that adding one more γ_l landowner will have a stronger impact on the labor demand of γ_h landowners. However this does not give any indication of the total change in labor demand. More precisely, we do not know whether the labor demand of the new landowner can compensate for the loss in all the other labor demands. A careful analysis can provide the answer.

It is useful to notice first that without any inequality $\frac{dn_l}{dN_l} = -\frac{1+n_l}{N}$. This means that all the landowners adjust their labor demand such that the total labor allocation on R_1 does not change (i.e. $\frac{dL}{dN_l} = 0$). This is a consequence of the constant returns to scale assumption. Therefore in this case more egalitarian privatization does not affect wages and depletion of R_2. If $\gamma_h > \gamma_l$, this does not hold any more and different effects must be considered:

- When land is divided (N_l higher) the most direct effect is a pure negative size effect: parcel area is smaller. This reduces labor demand.
- The second effect is induced by the change in $\frac{\psi}{g}$ due to the establishment of a new landowner hiring people. This reflects the consequence on $\frac{\psi}{g}$ of taking one agent from R_2 and giving him some land where he hires a quantity n_l of γ_l agents. Abstracting from all the other effects, if $\left(\frac{\psi}{g}\right)' < 0$ then the benefit of reducing the negative externality is overcompensated by the increase in the wage on R_2 and this effect is negative. If $\left(\frac{\psi}{g}\right)' > 0$, it is positive.

These two effects can be added together to get the (partial) net impact of having one more landowner. It is the result of this new landowner hiring people minus the size effect. For a stable equilibrium, this sum is always negative. If $\left(\frac{\psi}{g}\right)' < 0$ then this is clear: by hiring new workers, the increase in wage is higher than the gain from the smaller externality and this decreases labor demand. If $\left(\frac{\psi}{g}\right)'$ is positive but small to ensure stability then it is not large enough to compensate for the size effect and the same result holds. This does not take into account inequality and simply reflects the additional labor demand of the new landowner, adjusted for the new parcel size.

After the more egalitarian privatization, each landowner sees his parcel size shrink and observes the new landowner hiring workers and therefore affecting L. This always decreases labor demand. Then, he notices that each landowner (including him) adjusts his labor demand because of the change in L. This in turn affects L and labor demands adjust, etc.... The process is very similar to a reaction function and provides a multiplier to the two first effects. This is the third effect.

- Since all the labor demands are affected, each landowner observes the change in L and changes again his labor demand until equilibrium is reached. If there is no inequality then this exactly compensates the second effect and the only observable consequence of a more egalitarian privatization is the size effect. With inequality, γ_h landowners react more than γ_l landowners and this introduces an asymmetry.

  If $\left(\frac{\psi}{g}\right)' < 0$, the fall in all the labor demands actually makes the change in L less harmful: L decreases, increasing $\frac{\psi}{g}$. This positive effect will decrease the size of the fall of each landowner labor demand since it makes the situation better compared to the one with only the two first effects. It mitigates the second negative effect because with inequality γ_h landowners react more strongly than γ_l landowners. Therefore the multiplier is smaller than 1.

  If $\left(\frac{\psi}{g}\right)' > 0$, the opposite occurs and this increases the fall in labor demand: a smaller L decreases ψ more than g and this implies a further fall in labor demands. Again, only inequality drives this result. γ_h landowners are more sensitive to a change in L. Therefore the effect is negative and completely cancels the benefits of the positive second effect.

The multiplier is greater than 1.

Proposition 5.5 *A more egalitarian privatization always decreases labor demand in a locally stable equilibrium.*

If $\left(\frac{\psi}{g}\right)' < 0$ *then inequality makes the fall smaller than it is without inequality.*

If $\left(\frac{\psi}{g}\right)' > 0$ *then inequality makes the fall greater than it is without inequality.*

5.5.2 Labor allocation on the privatized resource

More interesting is the change in the labor allocation L on R_1. Remember that without inequality L is not affected. Again with inequality, the total effect can be broken down in three parts similar to those used for the change in labor demands.

- Firstly, two negative parcel size effects have to be considered depending on the landowner skill. These are the two size effects already found in the labor demands n_l and n_h.

- Secondly, the new landowner hires labor and this increases labor allocation on R_1. This does not take into account the change in labor demands from all the landowners. This has a positive impact on L.

Overall, these two effects decrease L. This is only due to the skill differential between landowners. Starting from a given population of landowners composed of all the γ_h and of some of the γ_l individuals, if one more γ_l landowner is added, his labor demand does not compensate for the fall in all the labor demands due to the smaller parcel size. This happens because labor demand from highly skilled individuals falls more than labor demand from low skilled landowners. Note that this does not have anything to do with congestion or externality.

- Thirdly, the change in labor demands affects L through $\frac{\psi}{g}$. If $\left(\frac{\psi}{g}\right)' < 0$, this mitigates the negative effect by making it smaller in absolute value. This is very similar to the analysis for n_l. The first two effects decrease L and this is actually beneficial for all the landowners. Therefore they hire more labor and even though the total effect is still negative, it is smaller. On the other hand, if $\left(\frac{\psi}{g}\right)' > 0$ the fall in L is amplified.

In other words, an increase in the number of γ_l landowners first decreases L by reducing parcel size (the labor demand from the new landowners not compensating fully this effect). Then, if $\left(\frac{\psi}{g}\right)' < 0$ this fall in L reduces equilibrium wage relatively to the negative externality, reducing the total fall in L. If $\left(\frac{\psi}{g}\right)' > 0$ the fall in L increases equilibrium wage relatively to the negative externality, increasing the total fall in L.

Finally, if the skill ratio increases, the fall in L is larger. Again, if γ_h is much larger than γ_l the fall in γ_h landowners labor demands is large and cannot be compensated by the much smaller labor demand from the new γ_l landowner.

Proposition 5.6 *A more egalitarian privatization always decreases labor allocation and the non-privatized resource is more degraded. If there is no skill differential then L is fixed.*

If $\left(\frac{\psi}{g}\right)' < 0$ *then inequality makes the fall in L smaller than it is without inequality.*

If $\left(\frac{\psi}{g}\right)' > 0$ *then inequality makes the fall in L greater than it is without inequality.*

For a given γ_l*, the larger is the skill ratio, the larger is the decrease in L.*

Note that this proposition does not assume anything on $\gamma_h(1-\alpha)$. From Section 4 when $\gamma_h(1-\alpha) > 1$ and $N_l = 0$ labor allocation on R_1 is larger after privatization. However if the privatization is more egalitarian then L^p falls and may actually become smaller than L^c. Even in the case where privatization is profitable, this may not hold any more if land is allocated to many low skilled individuals.

5.5.3 Profits on the privatized resource

Profits are affected by a more egalitarian privatization. From profit maximization, using (5.7) and (5.8):

$$\pi_l = \frac{\alpha}{1-\alpha}(n_l+1)g(L^p) \qquad \textit{and} \qquad \pi_h = \left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}} \pi_l \tag{5.9}$$

When privatization is more egalitarian, it is not obvious whether profits increase or not. However (5.9) combined with Propositions 5.4 and 5.5 provides a clear answer. When the number of γ_l landowners rise, both n_l and $g(L)$ fall. Therefore profits fall as well[13].

Proposition 5.7 *A more egalitarian privatization always decreases profits on the privatized resource for both types of agents.*

To understand this, note that (5.9) can be written:

$$\pi_l = \gamma_l \left(\frac{K}{N}\right)^{\alpha} h_l^{1-\alpha}\psi(L) - wh_l \tag{5.10}$$

When N_l increases, $h_l = (1+n_l)\gamma_l$ falls as well by Proposition 5.4. Overall π_l falls because wh_l decreases more slowly than the first term in (5.10). This is basically due to the concavity of production in labor.

[13]Note however that income inequality between landowners and landless fall.

All the landowners see both their landownership rent fall and their incomes as workers fall. Consequently landowners and landless incomes fall but the income change for people who used to be landless and have been given some land must be also taken into account. This requires the welfare analysis of a more egalitarian privatization.

5.5.4 Welfare analysis

Welfare W^p is the sum of all the incomes in the population. Even though they fall after a more egalitarian privatization W^p could increase due to the allocation of landownership rents to a larger number of people. If it is the case, then land should be given to the largest number of people as this makes the population better off. However, it can be proved that this is never the case. To give land to low skilled landowners always decreases W^p.

Proposition 5.8 *A more egalitarian privatization always decreases welfare.*

When there is no inequality in skills then W^p does not depend on N_l and a more equal land distribution does not affect welfare.

Propositions 5.4 to 5.7 establish that a more egalitarian privatization intensifies depletion of the common property resource, reduces incomes of landless and landowners profits in such a way that new landowners would not be able to compensate former landowners and landless for the loss in their income. This emphasizes that if land is given without distinction in skills it may have some serious adverse effects.

5.6 Less egalitarian privatization

The case of a more egalitarian privatization has been examined and it has been proved that this may have negative consequences. Thus it is of interest to investigate the effects of a less egalitarian reform, where only some of the high skills agents receive a parcel of land. This case is actually very similar to the homogeneous agents situation of Section 3.

This section also gives an indication of welfare when land can be rented out to highly skilled individuals. In this case, welfare will be at most equal to welfare obtained with only high skills landowners. The rents paid are only transfers and do not affect welfare. It can be smaller if there is some efficiency loss due to the renting contract.

Only N γ_h-agents ($N < N_h$) are assumed to be landowners and it is assumed that $h > \gamma_h \frac{N_h}{N}$ $\forall N \in [1, N_h]$. This implies that γ_h landowners are always willing to hire γ_l workers. Finally, the simplifying assumption that each landowner hires the same quantity of γ_h workers is made. More precisely, this quantity is $\frac{N_h - N}{N}$ (or $\frac{N_h}{N}$ including himself). This makes the labor force composition identical across parcels.

Let's consider that N increases. Because of the constant returns to scale and the homogeneity of skills across landowners, labor demands decrease only by $\frac{n_h}{N}$. Similarly, the increase in N does not affect L.

Welfare is not affected by a less egalitarian privatization. Since L does not change, the equilibrium wage is constant and γ_l agents are neither better off nor worse off. On the other hand, to give land to a larger number of high skills landowners merely represents a transfer of money between these landowners. The former landowners have a smaller income but this fall is exactly compensated by the landownership rents of the new landowners.

Proposition 5.9 *A less egalitarian privatization affects neither labor allocation on the privatized resource nor welfare.*

Propositions 5.5 and 5.8 imply that the maximum L is reached for $N \leq N_h$

(and R_2 degradation is minimized, and wage is maximized) and Propositions 5.7 and 5.8 that W^p is maximized for $N \leq N_h$.

W^p for N_h landowners and W^c can be compared in a very similar fashion than in Section 3.2.4. This yields a comparable condition for W^p to be greater than W^c[14]:

$$N_h \pi_h(L^p) \geq [g(L^c) - g(L^p)] \left[\left(\frac{\gamma_h}{\gamma_l} - 1 \right) N_h + \tilde{L} \right]$$

The graph derived from this condition is as in Figure 5.3 and gives the same conclusion. Note that from Proposition 5.7 $\tilde{L}$ increases with N_l, implying that the larger is N_l, the more difficult it is to increase welfare by land titling.

5.7 Influence of inequality

The impact on labor allocation of a change in skills is now examined. Skills have been broadly defined and policies could influence them. For instance, they could be the consequence of borrowing constraints preventing individuals from buying high quality inputs, or of professional formation, access to new techniques, etc... The differential in γ between individuals could therefore be reduced (or increased) and this will have an influence on the equilibrium variables.

The effect on L is investigated.

Proposition 5.10 *A rise in γ_l or in γ_h increases labor allocation to the privatized resource.*

Looking at welfare, it can be easily established that a larger γ_h increases welfare W^p. However this is not necessarily true for a larger γ_l. This is because

[14]Note that for $\gamma_h = \gamma_l$ this is (5.4).

when γ_l increases skilled landowners are relatively less paid as workers (looking at the partial effect not accounting for the change in L) since they earn $\frac{\gamma_h}{\gamma_l} g(L)$. If the rise in L is small then the total effect on W^p may be negative, because high skills are relatively less paid. This does not occur when γ_h increases because this makes γ_h wages higher and does not affect γ_l wages that are always equal to $g(L)$ in equilibrium. On the other hand, a rise in γ_l or γ_h increases W^c. Therefore a positive change in skills is always beneficial under common property regime.

Proposition 5.9 shows that if privatization leads to an increase in γ_l then the result that $L^p < L^c$ may not hold. Therefore a strict privatization may not be profitable but may become an improvement if accompanied by other measures increasing the lowest skills.

5.8 Conclusion

This chapter argues that the privatization of a resource may have far reaching implications. Individual titling is beneficial when considering the resource in particular, but not necessarily if the whole environment is looked upon. One of the main result is that welfare does not necessarily increase after privatization, even when land reform is perfectly egalitarian. The lessons derived from this simple model are quite straightforward.

First, the beneficial nature of privatization in itself, to solve over exploitation, contains the seed of its potentially negative consequence. A shift of labor is not neutral. This calls for a careful and complete investigation of labor allocation when a change in property rights is advocated. When the outside opportunity is congested then it may be better to have a non optimal situation on the resource. This is a second best solution. To solve a distortion may lead to a worse outcome if there is another distortion (in this chapter the congestion externality on the remaining resource).

Second, the design of resource titling is not neutral. A conclusion of this chapter is that land should be given to the most able individuals. A more egalitarian distribution unambiguously decreases welfare. But in reality, it is usually difficult, or simply impossible, to identify skilled individuals when defining land rights. The market should then be allowed to allocate optimally labor through rental and sale. This article has precisely ruled out this possibility to have a clean analysis of a given land distribution. Banerjee (2000) has already underlined the absence of good reasons to restrict land rental, while he showed that there may be for sales. By allowing less able people to rent out their plot, land reform keeps its redistributive goal while maximising the gains from the reform. Looking at welfare, the use of a strictly utilitarian welfare function has the merit of being simple and to give some clear conclusions, however it has the serious drawback of disregarding inequality. Even though the sum of incomes is maximised by giving land only to skilled individuals, it also increases inequality compared to a more egalitarian distribution. A welfare function with inequality aversion may not support the strong result that low skilled people should not be given any land.

Third, the result that privatization reduces labor allocation does not hold when landowners skills are high. This assumes that privatization is potentially beneficial. Better ownership security, access to credit market through the use of land as a collateral, are arguments usually used in favour in privatization (Feder and Noronha 1987, Noronha 1997). This is why it is assumed in the model that privatization allows agents to use their skill both as a worker (as in the CPR situation) and a landowner. The model shows that these skills should be as high as possible to promote land reform. In the case where land is given to individuals of both types ("more egalitarian privatization"), the skill ratio should be minimized to reduce the fall in labor allocation. Hence the conclusion that land should be given only to skilled individuals does not mean that privatization must necessarily be inegalitarian. It rather stresses that skills should be made as equal and as high as possible across the population. Land reform should seek to promote these skills in order to fully exploit the benefits of land titling.

5.9 Appendix

5.9.1 A simple model of congested resource

This model shows that $g(L)$ and $\psi(L)$ increase with L. Consider that each agent i can work on R_2 by exerting effort e_i. Let's denote $E = \sum_i e_i$. Total output on R_2 is given by $(A - E)E$, where A is some positive constant.

Each agent receives the payoff $\frac{e_i}{E}(A - E)E = e_i(A - E)$. It is assumed for simplicity that the cost of extracting the resource is zero. In equilibrium all the individuals choose the same optimal level of effort e that maximizes $e(A - E)$.

Maximization yields $e = \frac{A}{\bar{L}-L+1}$ and $g(L) = e(A - E) = \left(\frac{A}{\bar{L}-L+1}\right)^2$.

Therefore $g'(L) > 0$ and the resource is congested.

Total effort is given by $E = A\frac{\bar{L}-L}{\bar{L}-L+1}$ and it is decreasing in L.

This shows that a larger number of people working on R_1 decreases total effort on R_2 and therefore decreases the negative externality, implying $\psi'(L) > 0$.

5.9.2 Welfare analysis

This section gives a simple example of welfare analysis in the case of homogeneous agents. It starts from the remark that in the limit case where $L^c = L^p$, $W^c = W^p$. In the general case where $L^c > L^p$, it is not obvious how Figure 5.3 changes with parameters and how this affects welfare. In particular, is it that the limit case is the only case where $W^c = W^p$? This example shows that it may be not.

A simple case with $Y(K, L) = K^\alpha L^{1-\alpha}$, $\psi(L) = L^\beta$, and $g(L) = L^\gamma$, with $\beta < \gamma$ is considered.

Solving for the equilibrium,

$$\begin{aligned} L^c &= K^{\frac{\alpha}{\alpha-\beta+\gamma}} \\ L^p &= (1-\alpha)^{\frac{1}{\alpha-\beta+\gamma}} L^c \end{aligned}$$

The parameter to be changed is the capital share α. When $\alpha = 0$, $L^c = L^p$ and therefore $W^c = W^p$. I will show that $W^c - W^p$ is not monotonic in α and that a large $L^c - L^p$ can be beneficial to privatization.

Two numerical examples are given, in both cases $\bar{L} = 35$ and $K = 50$. The graph of L^c and L^p is very similar in both cases and shows that $L^c - L^p$ (distance between the two curves) increases with α. .

In the first case, with $\beta = 0.2$ and $\gamma = 0.5$, privatization is always welfare damaging as is illustrated by the graph of $W^p - W^c$.

In this case, as $L^c - L^p$ increases, the gain in profits is not large enough to compensate for the fall in wages.

In the second one, with $\beta = 0.1$ and $\gamma = 0.2$, there are some values for α such that $W^c < W^p$. The graph of $W^p - W^c$ is

In this case, if the labor share $1 - \alpha$ is not too large and not too close

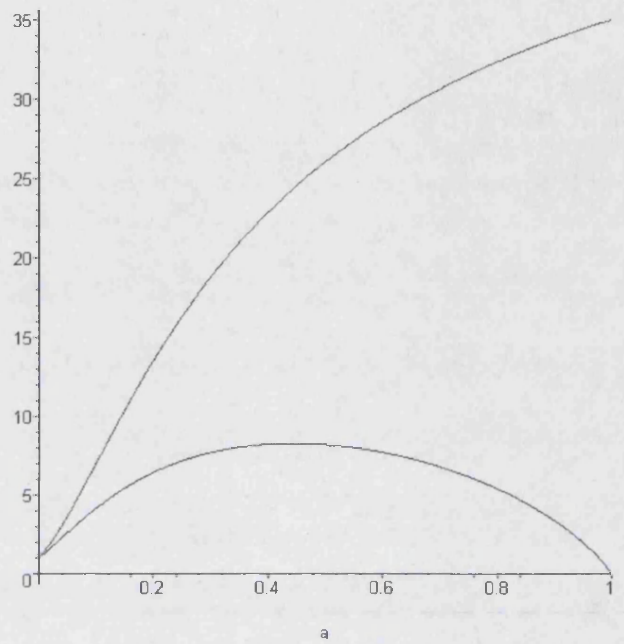

Figure 5.8: Labour allocations under private and common regime

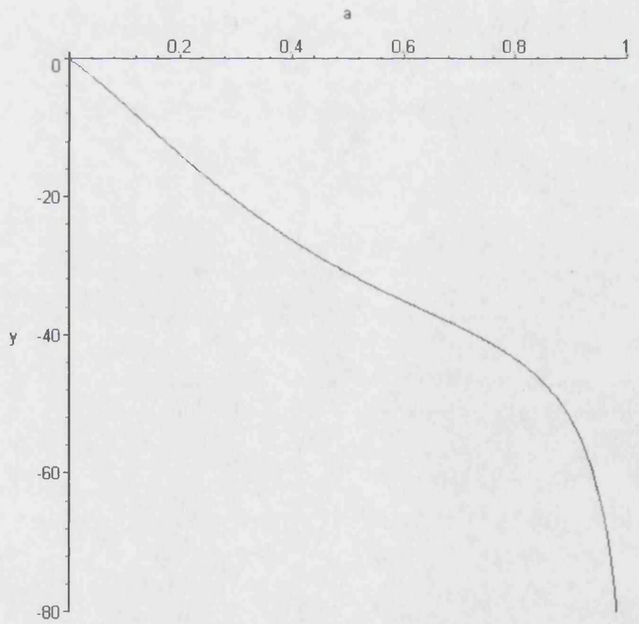

Figure 5.9: $W^p - W^c$

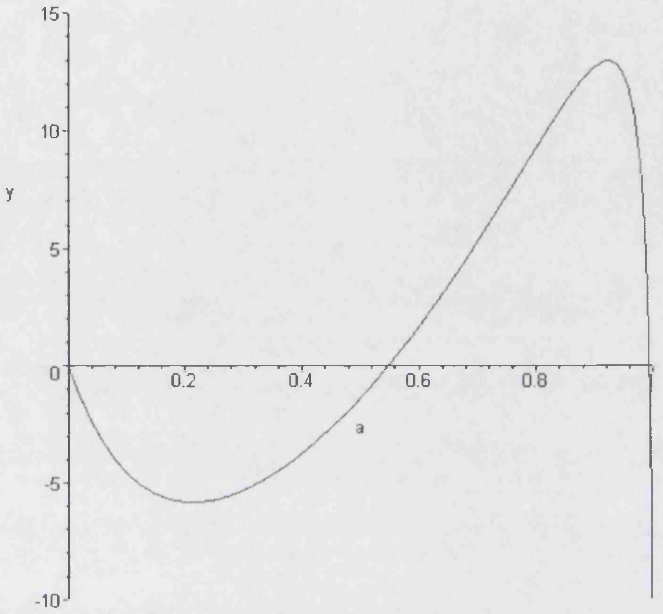

Figure 5.10: $W^p - W^c$

to zero, then privatization is beneficial. One could expect that a large labor share is necessary to have $W^p > W^c$ as it minimizes the transfer of labor to the congested resource. This is not true as a not too large labor share is good for profits. In this example, labor intensive activities should not be privatized while less intensive one should be (if the labor share is very close to 0 then privatization again loses its advantage because L^p and π^p converge to zero).

This simple example shows that privatization may or may not be welfare improving.

5.9.3 Proofs of the propositions

Proof of Proposition 5.3

From (5.7) skilled labor demand h_i for a γ_i landowner ($i = h, l$) is such that $\gamma_i(1-\alpha)\left(\frac{K}{N}\right)^{\alpha} h_i^{-\alpha}\psi(L) = w$. Hence $\frac{h_h^{\alpha}}{h_l^{\alpha}} = \frac{\gamma_h}{\gamma_l}$. By definition $h_i = \gamma_i + n_i\gamma_l$.

This yields (5.11). (5.12) follows immediately.

$$n_h = \left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}} n_l + \left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}} - \frac{\gamma_h}{\gamma_l} > n_l \tag{5.11}$$

$$\left|\frac{dn_h}{dN_l}\right| = \left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}} \left|\frac{dn_l}{dN_l}\right| > \left|\frac{dn_l}{dN_l}\right| \tag{5.12}$$

Note that since in equilibrium $\gamma_l w = g(L)$, the implicit expressions for n_l and n_h are:

$$n_l = \gamma_l^{\frac{2-\alpha}{\alpha}} \left[(1-\alpha)\frac{\psi}{g}\right]^{\frac{1}{\alpha}} \frac{K}{N} - 1 \tag{5.13}$$

$$n_h = \gamma_l^{\frac{1-\alpha}{\alpha}} \left[\gamma_h (1-\alpha)\frac{\psi}{g}\right]^{\frac{1}{\alpha}} \frac{K}{N} - \frac{\gamma_h}{\gamma_l} \tag{5.14}$$

Proof of Proposition 5.4

The condition under which an equilibrium is stable must be given. n_l is defined by the implicit equation (5.13). Stability requires that the derivative of the right hand side of (5.13) is smaller than 1. Differentiating and using $L = (1+n_h)N_h + (1+n_l)N_l$ yields

$$\frac{\phi'}{\phi} < \frac{1}{N}\frac{\alpha}{1+n_l}\frac{1}{\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}} N_h + N_l} \tag{5.15}$$

where $\phi(L) = \left(\frac{\psi}{g}\right)(L)$.

Proposition 5.4 can now be proven.

Differentiating (5.13) with respect to N_l,

$$\frac{dn_l}{dN_l} = (1+n_l)\left(\underbrace{\frac{1+n_l}{\alpha}\frac{\phi'}{\phi}}_{\text{second effect: new landowner hiring workers and therefore affecting } L} \underbrace{-\frac{1}{N}}_{\text{first effect: parcel size}}\right)$$

$$+ \underbrace{\frac{1+n_l}{\alpha}\left(N_h\frac{dn_h}{dN_l}+N_l\frac{dn_l}{dN_l}\right)\frac{\phi'}{\phi}}_{\text{third effect: change in all the labor demands affecting } L} \tag{5.16}$$

This explains the decomposition in three effects, with an always negative size effect, the influence of new landowner hiring people and finally the adjustment of all the labor demands affecting in return each labor demand.

Note that since by stability $\frac{\phi'}{\phi} < \frac{1}{N}\frac{\alpha}{1+n_l}\frac{1}{\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}}N_h+N_l} < \frac{1}{N}\frac{\alpha}{1+n_l}$, the sum of the first two effects is always negative.

(5.16) can be rewritten using (5.12):

$$\begin{aligned}\frac{dn_l}{dN_l} &= -\frac{1+n_l}{N}\frac{\frac{1}{N}-\frac{1+n_l}{\alpha}\frac{\phi'}{\phi}}{\frac{1}{N}-\frac{1+n_l}{\alpha}\left[\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}}\frac{N_h}{N}+\frac{N_l}{N}\right]\frac{\phi'}{\phi}} \\ &= A\left(\frac{dn_l}{dN_l}\right)_{\gamma_h=\gamma_l}\end{aligned} \tag{5.17}$$

with $A = \frac{\frac{1}{N}-\frac{1+n_l}{\alpha}\frac{\phi'}{\phi}}{\frac{1}{N}-\frac{1+n_l}{\alpha}\left[\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}}\frac{N_h}{N}+\frac{N_l}{N}\right]\frac{\phi'}{\phi}}$ is a positive multiplier for stable equilibria and $\left(\frac{dn_l}{dN_l}\right)_{\gamma_h=\gamma_l}$ the change in n_l when $\gamma_h=\gamma_l$. If $\frac{\phi'}{\phi} > 0$ (resp. < 0), or equivalently $\left(\frac{\psi}{g}\right)' > 0$ (resp. < 0), then $A > 1$ (resp. $A < 1$).

This proves Proposition 5.4.

Proof of Proposition 5.5

With a more egalitarian privatization, $L = N_h + N_h n_h + N_l + N_l n_l$.

Using the expressions for n_l and n_h gives:

$$\begin{aligned} L &= N_h + N_h \left[\gamma_l^{\frac{1-\alpha}{\alpha}} \left[\gamma_h (1-\alpha) \frac{\psi}{g} \right]^{\frac{1}{\alpha}} \frac{K}{N} - \frac{\gamma_h}{\gamma_l} \right] + \\ & \quad N_l + N_l \left[\gamma_l^{\frac{2-\alpha}{\alpha}} \left[(1-\alpha) \frac{\psi}{g} \right]^{\frac{1}{\alpha}} \frac{K}{N} - 1 \right] \end{aligned} \tag{5.18}$$

Taking the derivative of the last expression yields

$$\begin{aligned} \frac{dL}{dN_l} &= (n_l + 1) \left[\underbrace{1}_{\text{second effect: new landowner}} \underbrace{- \left(\frac{\gamma_h}{\gamma_l} \right)^{\frac{1}{\alpha}} \frac{N_h}{N} - \frac{N_l}{N}}_{\text{first effect: parcel size}} \right] \\ & \underbrace{+ \frac{1+n_l}{\alpha} \left[\left(\frac{\gamma_h}{\gamma_l} \right)^{\frac{1}{\alpha}} N_h + N_l \right] \frac{\phi'}{\phi} \frac{dL}{dN_l}}_{\text{third effect: change in labor demands affect } L} \\ &= (n_l + 1) \frac{N_h}{N} \left[1 - \left(\frac{\gamma_h}{\gamma_l} \right)^{\frac{1}{\alpha}} \right] + \frac{1+n_l}{\alpha} \left[\left(\frac{\gamma_h}{\gamma_l} \right)^{\frac{1}{\alpha}} N_h + N_l \right] \frac{\phi'}{\phi} \frac{dL}{dN_l} \end{aligned} \tag{5.19}$$

(5.19) can be written

$$\frac{dL}{dN_l} = -(n_l + 1) \frac{N_h}{N} \frac{\left(\frac{\gamma_h}{\gamma_l} \right)^{\frac{1}{\alpha}} - 1}{1 - \frac{1+n_l}{\alpha} \left[\left(\frac{\gamma_h}{\gamma_l} \right)^{\frac{1}{\alpha}} N_h + N_l \right] \frac{\phi'}{\phi}}$$

Proposition 5.5 holds since $\left(\frac{\gamma_h}{\gamma_l} \right)^{\frac{1}{\alpha}} > 1$ and $1 - \frac{1+n_l}{\alpha} \left[\left(\frac{\gamma_h}{\gamma_l} \right)^{\frac{1}{\alpha}} N_h + N_l \right] \frac{\phi'}{\phi} > 0$ from (5.15). Note also that $\frac{dL}{dN_l} = 0$ when $\gamma_h = \gamma_l$.

Finally, inequality in skills increases the fall in L,

$$\frac{\partial}{\partial \left(\left(\frac{\gamma_h}{\gamma_l} \right)^{\frac{1}{\alpha}} \right)} \left(\frac{dL}{dN_l} \right) = \frac{-1 + \frac{1+n_l}{\alpha} N \frac{\phi'}{\phi}}{\left[1 - \frac{1+n_l}{\alpha} \left[\left(\frac{\gamma_h}{\gamma_l} \right)^{\frac{1}{\alpha}} N_h + N_l \right] \frac{\phi'}{\phi} \right]^2} < 0$$

by equilibrium stability.

Proof of Proposition 5.7

W^p is defined as the sum of all the incomes in the population, composed of γ_h landowners earning π_h and the wage $\frac{\gamma_h}{\gamma_l}g(L)$, of γ_l landowners earning π_l and the wage $g(L)$, and of workers earning $g(L)$. Hence $W^p = N_h\left[\pi_h + \frac{\gamma_h}{\gamma_l}g(L)\right] + N_l\left[\pi_l + g(L)\right] + g(L)\left(\overline{L} - N_l - N_h\right) = \left[N_h\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}} + N_l\right]\pi_l + \left[\overline{L} + N_h\left(\frac{\gamma_h}{\gamma_l} - 1\right)\right]g(L)$.

Taking the derivative yields:

$$\begin{aligned}
\frac{dW^p}{dN_l} &= \left[N_h\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}} + N_l\right]\frac{d\pi_l}{dN_l} + \pi_l + \left[\overline{L} + N_h\left(\frac{\gamma_h}{\gamma_l} - 1\right)\right]g'(L)\frac{dL}{dN_l} \\
&= \left[N_h\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}} + N_l\right]\frac{\alpha}{1-\alpha}\left[g(L)\frac{dn_l}{dN_l} + (n_l+1)g'(L)\frac{dL}{dN_l}\right] \\
&\quad + \frac{\alpha}{1-\alpha}(n_l+1)g(L) + \left[\overline{L} + N_h\left(\frac{\gamma_h}{\gamma_l} - 1\right)\right]g'(L)\frac{dL}{dN_l} \\
&= \underbrace{\frac{\alpha}{1-\alpha}\underbrace{\left[N_h\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}}\frac{dn_l}{dN_l} + N_l\frac{dn_l}{dN_l} + (n_l+1)\right]}_{=N_h\frac{dn_h}{dN_l}+N_l\frac{dn_l}{dN_l}+(n_l+1)=\frac{d}{dN_l}[N_h+N_hn_h+N_l+N_ln_l]=\frac{dL}{dN_l}} g(L)}_{<0} \\
&\quad + \underbrace{\left[N_h\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}} + N_l\right]\frac{\alpha}{1-\alpha}(n_l+1)g'(L)\frac{dL}{dN_l}}_{<0} + \underbrace{\left[\overline{L} + N_h\left(\frac{\gamma_h}{\gamma_l} - 1\right)\right]g'(L)\frac{dL}{dN_l}}_{<0}
\end{aligned}$$

Hence the result $\frac{dW^p}{dN_l} < 0$ that proves Proposition 5.7.

Note also that $\gamma_h = \gamma_l$ implies $\frac{dL}{dN_l} = 0$ and this implies $\frac{dW^p}{dN_l} = 0$.

Proof of Proposition 5.8

Assuming all the γ_h agents are hired on the privatized resource, $L = N_h + Nn_h$.

Assuming $h > \gamma_h\frac{N_h}{N}$ $\forall N \in [1, N_h]$, we have $h = \gamma_h\frac{N_h}{N} + \gamma_l n_h = \left[\gamma_h\gamma_l(1-\alpha)\frac{\psi}{g}\right]^{\frac{1}{\alpha}}\frac{K}{N}$.

Hence $n_h = \gamma_l^{\frac{1-\alpha}{\alpha}}\left[\gamma_h(1-\alpha)\frac{\psi}{g}\right]^{\frac{1}{\alpha}}\frac{K}{N} - \frac{\gamma_h}{\gamma_l}\frac{N_h}{N}$.

Differentiating with respect to N yields

$$\frac{dn_h}{dN} = -\frac{n_h}{N}$$
$$\frac{dL}{dN} = 0$$

Profits on the privatized resource are given by $\pi = \frac{\alpha}{1-\alpha}\left(n_h + \frac{\gamma_h}{\gamma_l}\frac{N_h}{N}\right)g(L)$ and $\frac{d\pi}{dN} = -\frac{\pi}{N}$.

This easily gives the result that $\frac{dW^p}{dN} = \frac{d}{dN}\left[N\pi + N_h\frac{\gamma_h}{\gamma_l}g(L) + (\bar{L} - N_h)g(L)\right] = 0$.

Proof of Proposition 5.9

From (5.18), and differentiating L with respect to γ_l.

$$\frac{dL}{d\gamma_l} = \frac{\left[\frac{1-\alpha}{\alpha}\frac{1}{\gamma_l}N_h\left(n_h + \frac{\gamma_h}{\gamma_l}\right) + \frac{2-\alpha}{\alpha}\frac{1}{\gamma_l}N_l\left(n_l+1\right) + N_h\frac{\gamma_h}{\gamma_l^2}\right]}{1 - \frac{1}{\alpha}\left[N_h\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}} + N_l\right]\left(n_l+1\right)\frac{\phi'}{\phi}}$$

Differentiating with respect to γ_h yields

$$\frac{dL}{d\gamma_h} = \frac{N_h\frac{1}{\alpha}\frac{1}{\gamma_h}\left[n_h + \frac{\gamma_h}{\gamma_l}(1-\alpha)\right]}{1 - \frac{1}{\alpha}\left[N_h\left(\frac{\gamma_h}{\gamma_l}\right)^{\frac{1}{\alpha}} + N_l\right]\left(n_l+1\right)\frac{\phi'}{\phi}}$$

$\frac{dL}{d\gamma_l}$ and $\frac{dL}{d\gamma_h}$ are positive by using the stability condition (5.15).

Chapter 6

Conclusion

This thesis has presented various aspects of how institutions shaped behaviors, and how in return these shaped institutions. Chapter 2 argued that institutions created an environment that defeated their primary purpose through cultural transmission. There is a feeling in this result that individuals free ride (though unwillingly in the model) on the institutions, and trust them to take care of behaviors. This is a mistake, and good institutions may result in a society plagued by bad behavior, with good but inefficient institutions. Chapter 3 went a step further, and let institutions be chosen by individuals. Because of the same effect, now not only do institutions crowd out good behavior, but they also crowd themselves out. They initiate themselves the mechanism that undermines their own existence. However it is possible for cultural groups to control this process and create other institutions to affect cultural transmission. Education is done publicly because it helps to support a set of rules, and avoid crowding out. The idea that cultural groups could affect socialization is the starting point for Chapter 4. What are the mechanisms of cultural transmission, and how is it affected by group characteristics? The focus in this chapter shifts from institutions as rules for the society, to institutions as rules internal to each cultural group. Cultural groups could be tempted to minimize contacts with other cultures to avoid being "contaminated". By recognizing that children meet friends from other cultural groups, and that parents try to influence them in this choice, it appeared that intolerance is not always a good strategy for groups. In order to survive cultural transmission they should try to have a very strong internal structure, and be open to other cultures. As in the preceding chapters, institutional design has to be finely tuned in order to promote a cultural trait. Chapter 5 builds on this idea and uses the prominent example of commons as an institutional failure. The right institution requires clear property rights, and privatization is a good reform to achieve the first best outcome. However it may be more complex in a second best world, where labor flows from one resource to the other. Privatization may have unfavorable consequences.

These chapters suggest several ways for future research. Social interactions play an important role in crowding out, particularly if one wants to study the formation of norms in a society. Chapter 2 provided a framework for doing so, but it might be valuable to further elaborate on the shape of these interactions. The inclusion of interactions in the political equilibrium of Chapter 3 would require further modifications, but would achieve the extension of the crowding out result. Cultural groups find it profitable to set up institutions that strategically affect socialization. I considered only the case where one group is allowed to do so, but clearly both groups can. The competition that it creates between groups has been left aside but it is surely of interest to understand how different cultural groups choose to affect socialization. Chapter 4 offers to look at the internal organization of groups. Parents try to influence their children but they could also collectively decide about the level of intolerance, or how to reach children external to the group, etc. There is a broad research agenda to be set in order to understand how cultures try to spread their ideas in a world with many different traits. The last ambitious extension of Chapter 4 is to use network theory to derive strong properties about equilibrium network properties. Cultural transmission has recently prompted some empirical research to check the predictions of the theory. The introduction of interactions, with the econometric challenges it raises, would complement the existing literature and provide further evidence on the consequences of sorting, either geographically, or by characteristics. Finally Chapter 5 makes predictions about the impact of privatization on the remaining commons and the returns to labor. An empirical investigation that looks at the level of resource depletion after privatization would validate or reject the theory.

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