Ph.D. Dissertation in Economics:

Endogenous Growth and Underdevelopment Traps:
a theoretical and empirical analysis.

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ABSTRACT

The Thesis investigates issues of growth and development economics from both a theoretical and empirical perspective. The basic stylised fact which motivate the analysis are the existence, documented in the work, of poverty traps and the observation that there is a critical stage in which growth becomes the normal condition of a society. The main objective of the work is to identify economic determinants which explain the growth-stagnation dichotomy, or why countries find more difficulties in activating growth rather than in keeping growth going.

In the first chapter, I construct a model which combines self-sustained growth and 'underdevelopment traps' into a common analytical framework. The model exhibits aggregate non-convexities and thresholds which separate a region in which the equilibrium growth path converges to a stationary steady-state from a region in which growth is self-sustained. The core of the chapter is a set of original formal propositions about non-linear economic dynamics of which I make use in both this and the following chapter. The outcome of some simulations are also discussed. The findings are used to interpret some historical episodes like the take-off experience of different countries during the Industrial Revolution.

The second chapter presents a model built on a similar analytical framework, based on the parable of an economy of many island which grow different fruits from specific trees, whose fertility increases when fertilisers taken from different islands are employed. The parable aims at explaining how the cost of 'market activity' and intermediation affect growth and, possibly, cause underdevelopment traps. The following chapter tests some implications of the model.

The fourth chapter introduces into the analysis foreign direct investments as a potential source of growth, stressing how enforcement problems may limit their flow towards poor countries. Overlapping generations of heterogeneous agents choose the level of individual investment in human capital, whose effects are transmitted between dynasties, and elect the government. The political equilibrium is determined by the distribution of income across generations. Simulations show that it is possible for structurally identical countries to select alternative equilibria at some period and converge to either a constant positive growth rate or a low income stationary state.

The last chapter is an empirical investigation of the sources of macroeconomic fluctuations using the methodology of structural vector autoregression analysis. It extends to a multicountry framework the decomposition analysis of the GNP into permanent and temporary components and present the results of an application to the United States and United Kingdom using a sample of about one century lenghth. The results confirm other authors' findings about the large relative importance of temporary disturbances in explaining business cycle fluctuations.

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INTRODUCTION

In a recent survey article, Lucas (1993) addresses the question of what explanations growth theory can provide to those episodes of very rapid income growth, that he calls 'economic miracles'. He notes that in 1960 the Philippines and South Korea had about the same per capita GDP, and the main structural economic indicators were rather similar (apart from a difference in the higher education schooling rates that was clearly in favour of the Philippines). These preconditions notwithstanding, from 1960 to 1988, GDP per-capita in the Philippines grew at about 1.8 per cent per year, without any evidence of 'convergence' to the GDP levels of the rich countries. Korea, on the other hand, 'grew at 6.2 per cent per year, a rate consistent with the doubling of living standards every 11 years' (p. 251).

Traditional growth literature fails to provide satisfactory explanations to these contrasting performances. On the one hand, neoclassical models following the Solow-Cass-Koopmans tradition predict that backwardness itself is one main reason for rapid growth. When we control for technological and taste parameters, convex growth models predict that countries with a lower initial capital stock will grow faster than rich countries towards the steady-state long-run equilibrium. This may help explain the Korean miracle, but does not say why the Philippines did not do the same, and is even more sharply contradicted by the Sub-Saharan Africa experience of stagnation throughout the last decades. On the other hand, generations of trade and development theorists of more or less radical inclination (Baran, 1957; Frank, 1967; Lewis, 1977; Krugman, 1981) argue, to the opposite, that due to the nature of the North-South relations growth and development are impossible for poor country. According to these authors, relative backwardness, far from being a source of growth, is a cause of stagnation. This claim, consistent with diverse example of poverty traps, seems contradicted by the Korean miracle.

Lucas' viewpoint is different from either school. His requirements for a satisfactory theory of growth and 'miracles' are that:

- 'since it is a fact that the poor are either not gaining on the rich, or are gaining only very slowly, one wants a theory that does not predicts otherwise' (p. 269);
- 'a successful theory of economic miracles should, ..., offer the *possibility* of rapid growth episodes, but should not imply their occurrence as a simple consequence of relative backwardness. It should be as consistent with the Philippine experience as with the Korean' (p.269).

The main objective of this thesis is to develop a theory of economic development which is consistent with these guidelines and with a number of other stylised facts which will be presented throughout the work. Our initial objective is an analytical foundation for the existence of underdevelopment traps. We observe that both historical and cross-country evidence (discussed in the beginning of chapter 1) suggest that it is more difficult for countries to activate a development process than to keep growth going. This leads to a natural emphasis on non-convexities and increasing returns to scale. On the other hand, motivated by the stylised facts identified by Prof. Rostow's monumental work, we aim at explaining why single once-over episodes may activate a self-sustained growth process, with the take-off periods being characterised by particularly high growth rates.

This does not fulfil completely Lucas' guidelines yet. The other point raised by Lucas (that we also document with further examples in the final section of chapter 1) is that countries that seem to be structurally identical do sometimes react differently when hit by the same innovation: some of them activate the engine of growth, whereas others remain locked into stagnation. To address this point, we think it useful to develop models containing economically meaningful multiple equilibria, of the type recently emphasised by Krugman (1991), related to the presence of externalities. Krugman observes that 'in the presence of some kind of externality,..., future returns depend on the factor allocation decisions of other people - which also depend on their expectations of future earnings. Thus, there is at least potentially a possibility of self-fulfilling prophecies' (p.654). This idea stretches back to the traditional 'Big Push' doctrine of Rosenstein Rodan (1943), recently revisited and formalised by Murphy, Shleifer and Vishny (1989). More precisely, Pareto-rankable multiple equilibria can be obtained in macroeconomic models when both externalities and strategic complementarity (Cooper and John, 1988) are present[1].

In the first, second and fourth chapter, we develop a number of analytical models which try to explain why structurally identical country at some stage of their history may take some divergent routes. Our approach abstracts from important idiosyncratic factors (like cultural, anthropological or social differences) and, following Lucas (1988), tries to identify 'a mechanics of economic development - the construction of a mechanical, artificial world, populated by the interacting robots that economics typically studies, that is capable of exhibiting behaviour the gross features of which resemble the actual world ...' (p. 5).

The first chapter of the thesis constructs an endogenous growth model whose most original feature is a particular type of multiplicity of equilibrium solution trajectories. These trajectories all but one converges to a stationary steady-state, whereas one exhibits perpetual growth. This is, in our opinion, the most sensible representation of the growth-stagnation dichotomy. Related literature on underdevelopment trap issues has identified a number of economic mechanisms which generate multiple equilibrium trajectories converging to alternative steady-states. Other papers have shown the possibility of zero-growth corner solutions in endogenous growth models. In the former group of papers, we might mention models of sectoral allocation (Krugman, 1991; Matsuyama,

[1]The former refer to the interactions between agents' decisions at the level of their payoffs, the latter to the interactions at the level of optimal strategy. For example, there are strategic complementarities in investment decisions if not only the representative agent's payoff but also the optimal individual investments is increasing with the aggregate level of investments.

1991, Rodriguez, 1993), models with pecuniary externalities coming from technological complementarities (Young, 1993; Ciccone and Matsuyama, 1992), models with search and trade externalities (Diamond and Fudenberg, 1989), or other types of external effects (Benhabib and Farmer, 1991; Boldrin, 1993). In the second group we find mainly some models which assume non-convexities in the technology of human capital accumulation (Azariadis and Drazen, 1990; Becker, Murphy and Tamura, 1990), but also models with stochastic endogenous innovation (Aghion and Howitt, 1992). To our knowledge, however, our model is the first to demonstrate the possibility of saddle-path (interior) equilibrium solutions a la Cass (1965) co-existing with self-sustained growth equilibria a la Romer (1986). The analytical intuition is that such dynamics emerge whenever both decreasing and increasing returns occurs, such that the productivity of reproducible assets is larger than the social discount rate in the 'large' economy, but falls below it for some range of interior values of the state variable. We rationalise this in a model with externalities of the learning-by-doing type. The main analytical issue is that standard arguments of existence and determinacy[2] of the solutions do not apply to this framework. The core of the contribution is contained in a technical section which provides analytical conditions for such properties to be satisfied. We interpret the model as a formalisation of the Rostovian idea of economic take-off. Multiple equilibria are a possible feature of this model.

Results obtained by manipulating the aggregate production function such as assuming various types of externalities can always be criticised on the grounds of the *ad hoc* nature of the assumptions. In the second chapter, we develop a two-sector model which exhibits the same dynamics as those discussed in the first chapter without any special assumption about technology. The model has the nature of a parable about an economy of islands in which

[2]By determinacy of the solutions we mean the existence of a finite number of equilibrium values of the choice variable(s) for each state of the system. Our equilibria may be non-unique but are always determinate (cfr. Benhabib and Farmer, 1991, for a model with opposite results).

there are transaction costs and specialised operators whose activity consists of trading heterogeneous productive resources owned by agents who live in different island. The main intuition is that if there are imperfections in the 'intermediation sector', and the degree of imperfection is decreasing with the size of the market, it is possible to obtain the alternance of increasing and decreasing returns which makes this model observationally equivalent with that characterised by technological externalities discussed in the first chapter.

Some problems of economic interpretation are worth mentioning. In both the models of the first and second chapter, due to the presence of non-convexities, a critical issue may seem to be the size of the economy. A natural objection (particularly to the second model) is that in an open economy environment all the intermediate inputs that it is expensive to produce locally due to the limited size of the market could be imported. This appears to destroy the argument for the existence of traps. However, one should notice that a large part of intermediate goods and services has a non-traded nature; the addition to the model of (totally or partially) traded goods which can be purchased in the international market would not alter the qualitative results. This seems to be a weak condition. Consider for instance financial services, whose nature of local goods may appear particularly controversial. In many underdeveloped countries the access to international credit markets is highly limited; furthermore, important sector of activities (e.g. credit to small rural enterprises) are characterised by serious information asymmetries which make the credit-customer relation essential for the existence of credit activity. In these cases, financial intermediation turns out to be a quasi-local good. A further issue is that in both models the size of the market or of the externality network seems to be the crucial variable, but there is no evidence that 'large' countries perform better than small countries. We should stress, however, that when we refer to the market or the capital stock size, this does not need to be determined by the political borders of independent states. Many countries lack even significant markets of nationwide scale, whereas other countries are perfectly integrated in international market networks which make their political borders of low importance. The theoretical predictions of the model should be taken as conditional on these dimensions.

Chapter 3 tests some implications of the model of section 2. The model predicts that intermediation costs[3] should be negatively correlated with growth rates. We assume for simplicity that the market power of the banking sector is representative of that of the whole intermediation sector. Then, we take the wedge between the borrowing and lending interest rate as a measure of such markup, which, according to our theory, should be negatively correlated with the growth rates across countries. The results are clearly supportive of the hypothesis, after controlling for several typical explanatory variables in cross-country growth regressions[4]. Though we take this empirical analysis as just preliminary, the results look encouraging. The main limitation, which we plan to cope with in future investigation, is that the test does not consider explicitly the trap issue, which plays a main role in the theoretical analysis.

In the models of the first two chapters, we do not take any explicit view on the nature of the resources whose accumulation determines growth. Traditional growth theorists would call it 'physical capital'; Romer (1986), on the other hand, referred to it as 'knowledge'; in the first chapter we simply define it as the vector of 'reproducible resources'. There exists a class of endogenous growth models, however, which entails a clear characterisation of the driving force of the growth process as the accumulation of human capital (Lucas, 1988; Santos and Caballe, 1993; Barro and Sala-i-Martin, 1993). From an analytical viewpoint, there is a clear distinction between these models and the those discussed earlier. Models with human capital accumulation do not need any

[3]By intermediation we mean here all economic activities which, without being directly productive, allow economic agents to perform exchanges activity which improve the allocation of productive resources. Financial intermediation is the main economic activity of this type; other examples are mentioned in the introduction to chapter 2.

[4]Here, we are not faithful to our previous discussion, and maintain the country as the unit of analysis. We leave to future research to check the robustness of results when control is made for the degree of openness of countries.

force which sustains the productivity of physical capital (or analogue concepts) to keep growth going as the stock of capital increases. Since the productivity of labour is continuously augmented by the investments in education, these models do not have any genuinely fixed factor which enters the production function. The stock of capital can increase for ever without losing productivity at the margin, because the complementary factor - labour in efficiency units - also grows over time and constant returns to scale are sufficient to generate endogenous growth. If the critical assumption of non-decreasing returns to the accumulation of human capital is satisfied, there is an engine of growth. This is the framework of our model of chapter 4.

A theory of underdevelopment traps within this framework needs to explain why agents in poor countries find it lowly profitable to devote time to increase their future labour productivity and wage. The key issue is to identify an economic mechanism such that people in very poor countries expect to receive too low a benefit from augmenting their productivity to justify the effort and the opportunity cost of learning and not invest in education. Earlier models simply assume that there are increasing social returns to human capital accumulation (Azariadis and Drazen, 1990). In our model, low investment in education is caused by the lack of some complementary inputs to labour in the economy. An isolated electronic engineer cannot expect to take great advantage from his qualification in a primitive rural economy. The model identifies a vicious circle that two-way links low human capital accumulation to the absence of development of a modern industrial sector of activity. Recent experience suggests that foreign investments are an important vehicle of industrialization and development, particularly in those activities (manufacturing sector) that entail a significant technological transfer towards poor countries. This suggests a relation of complementarity between the capability of a country to attract foreign investors, and that of inducing the accumulation of local human resources.

The question to be answered becomes then why many poor countries fail to attract foreign investment. The explanation proposed by our model is that investors feel unsafe about property rights. Political changes, revolutions or changes in the attitude of the local governments can undermine the control of the investor over the resources invested. The issue of property right enforcement has been recently discussed within endogenous growth models, but usually without explicit reference to foreign investment. Benhabib and Rustichini (1991) assume an heterogeneous two-group population and show that the poor definition of property rights (each group may lobby to redistribute the existing resources in favour of the consumption of its members) may negatively affect the investment and growth rate of the economy. Persson and Tabellini (1991a) relate this issue to that of income distribution. They assume that heterogeneous agents vote about a linear tax schedule on capital to finance lump-sum redistribution. The higher the degree of inequality, the higher the petition for redistribution and the equilibrium tax rate, and the lower the growth rate. Saint-Paul and Verdier (1991) argue, on the other hand, that high inequality leads to higher growth, since it causes more pressure for redistribution in the form of public education. Other papers include Perotti (1990), Alesina and Rodrik (1991) and Bertola (1991). In all these papers, the conflict is about redistributing a stock of resources owned by members of the social community in a way that could be either favourable or unfavourable to growth.

The issue becomes even more serious when part of the resources which are used in production are owned by foreigners, whose interests are not represented, in principle, by local governments. There is always an incentive for the government to prevent these agents from appropriating the reward perceived from their participation to the productive process. When both local agents and governments are finitely lived and non-altruistic, there is a problem of time-consistency inherent to foreign investment decisions. We show that the issue can be solved by allowing the host country to pre-commit itself to property right enforcement by electing a government consisting of a 'committee of foreign capitalists'. However, the condition for this political solution to be viable through the support of the majority of voters is that the benefit from seizing foreign resources be not too large. This implies that in equilibrium the amount of foreign investments is rationed below the first-best level by the investor themselves. By the previous argument, this feeds back to reducing the rate of human capital accumulation and the growth rate of the economy.

The model allows for heterogeneous agents, with different human capital endowments and income, and contains predictions about the effects of distribution on growth. In particular, it is consistent with the Kuznets curve and predicts a relation between growth and distribution similar to that found by Aghion and Bolton (1992) and Piketty (1993) assuming information issues in capital markets.

The model of chapter 4 shares some features with those presented in the previous chapters. Multiple equilibria are possible due to the nature of the interactions between political equilibrium and individual choices of human capital investments. The selection between alternative outcomes only depends on the expectations prevailing within the members of a particular generation. In one type of equilibrium there is higher investment in education in the host country and higher foreign investment, whereas the other type of equilibrium is characterised by a lower level of both variables. We show by simulation that a single episode, namely the choice of one or another equilibrium may have dramatic long-run consequences. The succeeding generations may no longer face the multiplicity of equilibria, and we can construct cases in which given two initially identical countries, that which selected the good equilibrium takes off into sustained growth, whereas the other declines irreversibly.

The last chapter of the thesis is somewhat self-contained It presents an empirical contribution to the debate about the nature and sources of macroeconomic fluctuations. Some elements of this debate are discussed at the beginning of the chapter. Nelson and Plosser (1982) suggested a new way of looking at the relationship between growth and business cycle in macroeconomics, stressing the unit root properties of the GNP stochastic process. According to these authors, the presence of a stochastic rather than deterministic trend in

GNP is per se sufficient to diminish the role attributed by traditional models to temporary (e.g. monetary) disturbances in explaining output fluctuations. This view was later challenged by work in a multivariate framework (Blanchard and Quah, 1989; King et al., 1991) which rescued the Keynesian argument according to which demand shocks play a main role as a source of business cycle. Recent work using cointegration analysis (Bernard and Durlauf, 1991) established that the data reject both the view that growth is driven by a common trend worldwide and the view that growth has an entirely idiosyncratic nature. This means that output in each country is affected by more than one independent disturbance with a permanent nature and that the share of output variability explained by temporary disturbances might be smaller than that found by Blanchard and Quah assuming that output growth is driven by one stochastic trend only.

To disentangle the effects of multiple trends and measure the importance of short-run interactions between idiosyncratic disturbances in explaining business-cycle fluctuations, we extend to a multicountry framework the analysis of Blanchard and Quah. We then present the result of a simple two-country VAR system. The results show that the qualitative findings of Blanchard and Quah concerning the importance of purely temporary disturbances seem to be confirmed, though quantitatively somewhat diminished. Furthermore, we present results obtained from a sample of data over ninety years which allow us to assess the performance of the model in the face of large scale episodes like the World Wars and the Great Depression. However, some results are puzzling and suggest that more work need to be done before drawing definite conclusions.

CHAPTER 1

A Rostovian Model of Endogenous Growth and Underdevelopment Traps

Sustained growth of per-capita income is a recent event in world economic history. Reynolds (1983) documents a prolonged period in which, though output grows driven by labour force growth ('extensive growth'), no significant trend in per-capita output is observed. He produces historical evidence of this for China from 1368 (the establishment of the Ming dynasty) to 1949, several pre-colonial African countries and India. Europe's similar experience between 1500 and 1800 is documented by Maddison (1982). A large part of the contemporary world has in fact not yet achieved a path of continuing per capita income growth. Recent experience suggests that many poor countries not only fail to exhibit evidence of convergence to the income level of developed countries, but find themselves trapped in stable 'underdevelopment equilibria' with zero growth. The tables of the World Bank Report (1991) for the period 1965-1989, clearly reveal that situations of persistent insignificant per capita income growth are associated with low GNP per head. Forty-one countries out of a hundred have experienced a yearly average growth rate in GNP per head of less than 1%. Among these, twenty-seven had a per capita GNP of less than \$800 (average OECD = \$10,500, figures in 1989 U.S.Dollars) in 1965, eleven had a per capita GNP between \$800 and \$3,500 and only three had a per capita GNP of more than \$3,500. Table 1 reports a list of countries which presents evidence of underdevelopment traps.

Table 1: Evidence of Underdevelopment Traps.

COUNTRY	P.C.GDP 1965	P.C. GDP 1989	AV.GROWTH RATE
Mozambique	..	80	..
Ethiopia	123	120	-0.1
Tanzania	133	130	-0.1
Somalia	158	170	0.3
Bangladesh	164	180	0.4
Lao PDR	..	180	..
Malawi	142	180	1.0
Nepal	156	180	0.6
Chad	253	190	-1.2
Sierra Leone	210	220	0.2
Madagascar	365	230	-1.9
Nigeria	238	250	0.2
Uganda	494	250	-2.8
Zaire	422	260	-2
Niger	519	290	-2.4
Haiti	335	360	0.3
Benin	389	380	-0.1
Central Afr.R.	440	390	-0.5
Ghana	560	390	-1.5
Togo	310	310	0
Zambia	633	390	-2
Mauritania	564	500	-0.5
Bolivia	752	620	-0.8
Senegal	769	650	-0.7
Ivory Coast	652	790	0.8
Papua New G.	848	890	0.2
Honduras	780	900	0.6
Guatemala	734	910	0.9
Peru	1,060	1,010	-0.2
El Salvador	1,178	1,070	-0.2
OECD average	10,554	19,090	2.5

Source: World Bank Report 1991.[1]

[1]Note: according to Summer and Heston (1991) data with adjusted purchasing power parities, we should add Angola, Liberia and Sudan (whose data are not reported by the World Bank), whereas we should not include Tanzania and Guatemala.

In a classic contribution to the economics of development, Rostow (1990) claims that there is a 'decisive interval in the history of a society when growth become its normal condition... The beginning of take-off can usually be traced to a particular sharp stimulus. The stimulus may take the form of a political revolution ..., a technological innovation ..., a newly favourable international environment .... What is essential is ... the fact that the prior development of the society and its economy result in a positive, sustained and self-reinforcing response to it: the result is *not a once-over change* in the production function ..., but a higher proportion of potential innovations accepted in a more or less regular flow, and a higher rate of investment' (pp.36-37).

The idea that underdeveloped economies may be trapped in stable, or 'quasi-stable' equilibria with low per capita income, investment and technical progress has a long tradition in the development literature (Rosenstein-Rodan 1943 and 1957; Nelson, 1956; Leibenstein, 1957; Basu, 1984), stretching back to the Malthusian theory of endogenous population growth. In a recent revival of interest, the poverty traps issue has been analysed in the framework of the coordination failure literature (Cooper and John, 1988). Murphy, Shleifer and Vishny (1989) develop a model with increasing returns to scale and imperfect competition. Other papers include Krugman (1991), Matsuyama (1991); Zilibotti (1993a). The main limitation of this literature is that it only proves the existence of multiple Pareto-rankable steady-state equilibria, rationalising the existence of differences in the long-run income levels between structurally identical countries, but does not explain the stagnation-growth dichotomy described by Rostow.

New growth theories characterise the economy as endowed with a self-sustaining engine of growth. Standard endogenous growth models (Romer, 1986; Lucas, 1988), however, do not answer the question of why starting the engine of growth should be more difficult than keeping it going.

Some papers which focus on the effects of human capital accumulation seem to be closer in spirit to Rostow's point. Azariadis and Drazen (1990) use an overlapping generations model to show that if the private yield on human capital is increasing in the stock of human capital, then there may be bifurcations and multiple equilibria with different growth rates associated with each of them (including a zero-growth solution). A similar result is found by Becker, Murphy and Tamura (1990) in a model where fertility rates are endogenized. A problem with these models is that they characterise stationary traps as zero-income corner solutions. On the other hand, Reynolds (1983) argues convincingly that both contemporary poor countries and pre-industrial European economies actually look quite dynamic and not unresponsive to innovation, though unable to produce sustained per capita income growth (p.948). Also Rostow (1990, ch.3) stresses the importance of those cultural and economic changes taking place during the stages of development at which a society is still unable to produce continuing growth, which represent, in his words, the 'preconditions' for take-off. Models in which the equilibrium dynamics drive any 'trapped' economy to a zero-income solution, however, fail to capture any difference between societies which are in a pre-take-off stage and rule out that any structural change may have permanent effects before the 'big push' occurs. We find it somewhat counterintuitive to think of some centuries of history which preceded the Industrial Revolution in terms of a completely immobile equilibrium.

Our model aims at capturing more closely Rostow's stylised fact. The 'extensive growth' or pre-take-off stage turns out to be characterised by the absence of self-sustained growth, but episodes of short run growth, technical change and permanent increase in per capita income are possible. The Solow-Cass model, with its implication of long-run stationarity, provides the natural framework to describe economies that do not grow at a sustained rate in per capita terms. Once the take-off has occurred, however, the same economies exhibit a Romer-type behaviour with sustained growth. Furthermore, we have a role for both development thresholds and multiple self-fulfilling prophecies.

A secondary objective of the chapter is to capture a stylised fact which emerges from the observation of cross-sectional data, namely the existence of a hump in the pattern of growth. Though very poor countries grow on average less than advanced countries, with many of them exhibiting zero growth, middle-income countries grow on average faster than rich countries. Average per capita growth rates obtained from the Summers and Heston (1988 and 1991) postwar data turn out to be below 1% for low-income countries, 3.2% for middle-income, 2.2% for rich countries. Some Asian newly industrializing countries exhibit growth rates well above 5% per year for prolonged periods. Both traditional and recent growth models, by focusing on balanced growth solutions, are at odd with explaining this evidence.

Section 1 relates our analytical framework to the existing growth literature. Section 2 discusses the assumptions of the model. Section 3 characterizes the dynamic solution of the model through a series of formal propositions. Section 4 presents the results of some numerical simulations based on a specification of the model. Section 5 discusses the economic implications and concludes.

1 The analytical framework. General issues.

The main feature which distinguishes endogenous growth from traditional neoclassical models is the existence of an autonomous engine of growth. In the Solow-Cass framework, long-run growth is not sustainable in the presence of a fixed supply of some non-reproducible factors which enter the production function, because the marginal productivity of the reproducible factor would fall to zero as the accumulation proceeds. The convexity of technology is not sufficient to generate the traditional result of a long-run stationary equilibrium trajectory. A 'convex' economy may still have an autonomous engine of growth if the productivity of the reproducible factor is bounded from below by a sufficiently large positive bound (greater than the social discount rate), as it has been shown by Jones and Manuelli (1990). Traditional growth models rule out this possibility by imposing the Inada conditions (often in the form of a Cobb-Douglas specification), implying that the marginal productivity to capital alone tends to zero in the 'large' economy. Under these circumstances, long-run growth is only possible if an exogenous factor-augmenting technological trend is imposed to the production function. However, a point that seems to have been ignored by the literature is that the analytical solution found by Cass (1965) and Koopmans (1965) carries over to a model in which the productivity of capital falls below the social discount rate only 'locally', rather than globally as in standard neoclassical models. In other words, it is possible to obtain the standard long-run stationary saddle-path as just one of a number of equilibrium solutions of the model, rather than the unique one, when some types of aggregate non-convexities are allowed.

Modern growth theory has examined a number of economic mechanisms which might sustain the marginal productivity of capital, as accumulation proceeds. Among the factors which have been identified are learning-by-doing externalities (Romer, 1986 and 1989), accumulation of human capital (Lucas, 1986), 'intentional' innovation (Romer, 1990; Aghion and Howitt, 1992) and financial development (Greenwood and Smith, 1993; Saint-Paul, 1992; Zilibotti, 1993b). In most cases, reasons of analytical convenience have induced the researchers to focus on models which generate balanced-growth solution trajectories. In other words, the existing literature has accorded a generalised preference for reduced-form solutions which are linear in the reproducible factor (sometimes known as the AK-type).

This chapter seeks to challenge this view, by proposing a model which is asymptotically of the AK-type, but which allows for richer dynamics at lower levels of accumulation. In principle, our basic idea could be built on any of the many engines of growth proposed in the endogenous growth literature. Analytical simplicity, however, leads us to use the simplest mechanism, namely a Jones and Manuelli-type technology which does not require externalities to sustain growth in the long-run. By adding to this model the assumption that technical progress only augments the reproducible factors, we obtain an analytical framework which fits well Rostow's description of take-off and development.

The formal results of this chapter apply to a reduced-form solution which can be generated by a potentially vast set of structural models. The critical feature of these models is that the social productivity of the reproducible factors of production, given the stock of factors in exogenous supply, be decreasing at lower levels of accumulation and increasing at higher level of accumulation. Can we think of economic circumstances which make this picture realistic? In the abstract, reduced-form solutions exhibiting this type of non-linearity are neither more nor less plausible than those displaying the exact linearity of the social productivity of capital in its stock, as in mainstream endogenous growth models. To be concrete, however, we will base the formal discussion on a stylised economic model, close in spirit to Rostow's analysis, which generates the required reduced form.

This is not the unique relevant interpretation for our reduced form, though. Recent models of intermediation and growth (Greenwood and Smith, 1993; Zilibotti, 1993b) also potentially exhibit this type of property. The basic idea is that the productivity of capital is higher when there is (real or financial) intermediation, but there is a cost to opening and operating a market for intermediation. Small economies do not have intermediation markets and are characterised by decreasing returns since the technology of the final sector is assumed to be convex. However, when the intermediation market opens up there is a 'thick market' externality associated with the accumulation process: as the size of the market grows the cost of intermediation falls, and this increases the productivity of capital. If these externalities are strong enough, growth is characterised by increasing returns at higher stages of development.

2 The model.

In this section we discuss the benchmark model. Imagine a world in which two types of innovation take place. The first type, which we will call 'scientific advancements' has the character of large-size episodic events, like changes in the dominant scientific paradigm or development of new machines based on previously unknown principles (e.g. the wheel, the steam engine and so on). The second type is a continuous flow of technical progress resulting from the daily effort to translate the potential warranted by the current state of scientific knowledge into actual improvements to the technology used to produce goods. In other words, we draw a distinction between technical revolution and technical evolution.

We make the simplifying assumption that the former are entirely exogenous; in particular the productive activity does not affect the occurrence of revolutionary changes. One can imagine that these innovations come from the research activity carried on in academic and other institutions, whose accumulated knowledge moves the frontier of possible technical improvements, without being directly applicable to productive activity.[1].

The second type of innovation continuously augments the factor productivity. This technical change is assumed to be the by-product of the investment activity as in standard learning-by-doing models (Arrow, 1962, Romer, 1986)[2]. In other words, technical progress driven by the process of accumulation is the channel through which the state of technical efficiency catches up with the frontier productivity level warranted by the state of knowledge. The central

[1]This assumption is criticizable, since much of the research activity conducted in modern societies receives a determinant impulse from the needs of the productive sector. However, this linkage has not always been so strong as today (for instance, the Copernican revolution was not the result of any stimulus from the productive sector). Furthermore, even in the contemporary world, many countries do not participate in a significant way in the production of great scientific changes, but simply endeavour to catch-up with the technological capabilities of the leading countries, taking as given the frontier of knowledge.

[2]Schmookler, 1967, reports microeconomic evidence about a causal relation going from the rate of investment to the rate of invention.

idea of the model is that when the conditions for sustained investment are not fulfilled, the process of technical change is also arrested and the economy gets locked into an underdevelopment trap. Stagnation and low levels of income and productivity are two sides of the same coin.

We now introduce two strong simplifying assumptions. First, though it would be appealing to use a vintage structure in which innovation is embodied only in newly installed plants, we avoid the formal complications which this would bring about by assuming that the flow of technical progress affects the whole existing capital stock. Second, as in the first generation of endogenous growth models, we treat the learning-by-doing effect as a pure externality, so that any agent cannot exclude others from benefitting from the productivity improvements coming from his investment activity.

Consider a one-sector competitive economy. k (called capital for simplicity) is a composite good consisting of the private component of any reproducible private factor of production like human and physical capital. The technology depends on this composite variable, k_t, a non-reproducible factor in fixed supply, n, and a variable, A_t representing the state of technical efficiency. To fix ideas, the non-reproducible factor could be thought of as labour. The production function $y_t = y(k_t, n, A_t)$ is assumed to be continuous, twice differentiable and to exhibit constant returns to scale in capital and labour and increasing returns to scale in all the three arguments. A_t is a pure public good and does not receive explicit compensation. With reference to the previous discussion, A_t is not just worldwide available knowledge, but is rather that part of it which is applicable to production activity at time t in a certain country.

We characterise the 'learning equation' as a function of two variables:

- the distance between the current state of technical knowledge and the frontier of technical knowledge, determined by past scientific discoveries;

- the current level of aggregate investments.

We define a as the frontier of technical knowledge, which will be kept constant throughout the analysis[3]. We implicitly assume that scientific knowledge is publicly available worldwide at zero cost. The learning function is assumed always to satisfy the following properties:

$$\dot{A}_t = h([a - A_t], \dot{K}_t) \ , \quad h(0, \dot{K}_t) = h([a - A_t], 0) = 0 \ , \quad h_1 > 0, \ h_2 > 0, \quad (1)$$

with subscripts denoting partial derivatives and dots denoting time derivatives. Notice that K_t denotes aggregate capital, whereas k_t is used to indicate capital at the level of the production unit. A convenient specification, originally used for diffusion of techniques among firms in a given market (Mansfield, 1968), but also used in growth models (Ricottilli, 1990) that fits well in our framework is a function of the type:

$$\frac{\dot{A}_t}{A_t} = q\frac{(a - A_t)}{a}\dot{K}_t \quad (2)$$

where $A_t \leq a$ and q is a positive parameter. The rate of technical change is positive only in the presence of a positive rate of accumulation. Furthermore, given the level of investment, it is higher the further a country is from the frontier of technical knowledge. When integrated, this formulation gives a logistic solution of the form:

$$A_t = \frac{a}{1 + \left(\frac{a}{A_o} - 1\right) e^{-qK_t}} \quad (3)$$

where the term inside bracket reflects the initial state of the system. This formulation implies increasing learning effects in the first stage of the accumulation process and decreasing effects at later stages. The core argument of this chapter, summarised by a number of formal propositions in the following section, does not rest, however, on a specific parametrisation. For this reason, when not specified we will only assume some weaker restrictions on the

[3]As we said, we regard here scientific revolutions as secular episodes like changes in the scientific paradigm, rather than continuous advancements on the margin. The model studies the dynamics of the system within each period, emphasising that in each period dominated by a certain scientific paradigm only a subset of countries succeed in activating a sustained growth process. In the last chapter, we will informally consider the effects of revolutions which affects the frontier of knowledge.

function $A(K_t)$, which are satisfied by (3), but could in principle come from an alternative structural model (see the previous section). These restrictions are that $A(K_t)$ be positive, strictly increasing and bounded from above and below. Summarising:

$$A_t = A(K_t) > 0,\ A'(K_t) > 0,\ \lim_{K_t \to \infty} A(K_t) = a,\ \lim_{K_t \to 0} A(K_t) = A_0,\ 0 < A_0 < a. \tag{4}$$

The important assumption here is boundedness, the other restrictions being familiar from other growth models with technological externalities (Romer, 1986 and 1989). If $A(K_t)$ were unbounded above, we could not use an asymptotically linear production function to sustain growth in the long-run, since the technology would become globally unbounded. This restriction notwithstanding, our specification is more robust than those usually employed in Ak-type endogenous growth models with an externality, which rests on non-generic parameter configurations such that the aggregate productivity is exactly linear in capital[4].

The following assumptions characterise the technology, specifying the nature of technical progress and the asymptotic behaviour of the production function.

$$y_t = y(A(K_t)k_t, n), \tag{5}$$

$$y_1(A(K_t)k_t, n) > 0\ ,\ \ y_{11}(A(K_t)k_t, n) < 0, \tag{6}$$

$$\lim_{k_t \to 0} y_1(A(K_t)k_t, n) = \infty\ ,\ \lim_{k_t \to \infty} y_1(A(K_t)k_t, n) = \Omega > 0. \tag{7}$$

Assumption (5) requires that technical progress only affects the productivity of the reproducible factor. In the conventional terminology, technical progress is of the pure 'capital augmenting' type. Notice that if n is interpreted as labour and k as capital, in a literal and statistical sense, this assumption generates some counterfactual implications. Empirical evidence suggests that in most countries the capital-output ratio tends to be constant in the long-run, whereas

[4]A typical example is the technology $Y_t = \overline{K}_t^{\nu} K_t^{\alpha} N^{1-\alpha}$, used for instance by Romer (1989), which generates sustained growth only if $\nu = 1 - \alpha$

labour productivity shows a tendency to grow over time. This is consistent with labour-augmenting rather than capital-augmenting technical progress. In our model, however, the choice of the labels 'capital' and 'labour' as the relevant factors is merely conventional; land or natural resources might also play the role of fixed factor, instead of labour[5]. Assumption (5) means in fact that growth is driven by a vector of reproducible factors, whose productivity is enhanced by endogenous technical progress, and is limited by the presence of a vector of fixed factors whose productivity is not affected by technical progress. Assumption (6) means that the technology is concave when $A(K_t)$ is taken as parametric, as we will assume it to be the case for firms. It is essential to maintain the competitive nature of the model. Assumption (7) requires that the marginal productivity of the reproducible factor be bounded from below by the positive constant Ω, as in Jones and Manuelli (1990).

Throughout the rest of the chapter, we normalize the size of the fixed factor n to unity and ignore it, simply rewriting the production function (with licence of notation) as $y_t = y(A(K_t)k_t)$, where $y' > 0$ and $y'' < 0$.

The set of technological assumptions (5)-(6) and (7) encompasses standard parametrizations like a C.E.S. function with two factors of production (the fixed factor and capital in efficiency units), constant returns to scale and elasticity of substitution between factors strictly greater than one. Since it will be useful at some stage to refer to a fully parametrised version of the model, we specify here the following convenient production function:

$$y_t = DA(K_t)k_t + Z[A(K_t)k_t]^\theta \ , \ 0 < \theta < 1, \tag{8}$$

[5]Still, it is perfectly coherent to interpret n as the non-accumulable component of human activity (purely physical labour) and think, as a matter of simplification, that the productivity of manual activity is not subject to technical progress as opposed to both physical and human capital. Also, notice that assumption (7), when n is taken to be labour, implies that the competitive labour share would tend to zero in the large economy, whereas empirical evidence suggests that the income shares of capital and labour remain approximately constant in the long-run. However, the unconventional definition of k_t and n renders the usual measure of income shares irrelevant. A more correct test of this assumption would involve seeing whether the share of unqualified labour has been declining through time in industrialized countries.

which is a conventional constant returns to scale Cobb-Douglas function (the term n, here omitted because normalised to one, should appear in the second term at the power of $1-\theta$), augmented by a term which increase linearly with the capital stock in efficiency units (D and Z are parameters).

We assume a continuum of atomistic identical profit-maximising firms on the real interval $[0,1]$. Firms act competitively, taking prices as parametric, and atomistically, neglecting the effects of their investment decisions on the aggregate stock of public good A_t[6]. Assumption (6) ensures that the problem of profit maximization facing each firm is concave and well-defined. Under these conditions, the existence of a competitive equilibrium may be proved, using a fixed-point argument (Romer, 1989). In equilibrium, firms' expectations are self-fulfilling.

Finally, we specify the consumers' preferences as parametrised by a standard intertemporally separable isoelastic utility function with a constant time-discount factor $\delta \in]0,1[$. Infinitely-lived identical consumers maximize the utility function:

$$u(c_t) = \int_0^\infty \frac{c_t^{1-\sigma}}{1-\sigma} e^{-\delta t} dt \tag{9}$$

The competitive solution for this economy is equivalent to the solution of a second-best welfare maximization problem, where the planner is constrained to ignore the externalities when he solves the problem (see Romer, 1989, for a proof of the equivalence). This corresponds to treating A_t as parametric. The equilibrium conditions are then substituted into the First Order Condition, obtaining the competitive analogue. The problem is then:

$$\begin{aligned} & \max_{\{c_t;k_t\}} & \int_0^\infty \frac{c_t^{1-\sigma}}{1-\sigma} e^{-\delta t} dt & \qquad (10) \\ s.t. \quad & \dot{k}_t & = \; y(A(K_t)k_t) - c_t & \qquad (11) \end{aligned}$$

with $k_t > 0$; $c_t > 0$; $K_t = \overline{K_t}$; $k_0 = \overline{k}_0$.

[6]Since the total measure of firms is unity, it is clear that $k_t = K_t$. However, it is important to maintain the notational distinction, since firms treat k_t as a choice variable and K_t as parametric, the learning effects having the nature of an externality.

We have assumed for simplicity that capital does not depreciate. The necessary conditions for an optimum can be found from the solution of the current-value Hamiltonian equation:

$$H(c_t, k_t, K_t, \mu_t) = \frac{c_t^{1-\sigma}}{1-\sigma} + \mu_t[y(A(K_t)k_t) - c_t] \tag{12}$$

By the standard procedure, we find the First Order conditions (treating c_t as the control variable, k_t as the state variable, μ_t as the costate variable and K_t as exogenous), then substitute in the equilibrium conditions $k_t = K_t$ and $c_t = C_t$, and finally rearrange the expressions to eliminate μ_t. The necessary conditions for optimality are summarized by a planar autonomous system of differential equations in C_t and K_t (Euler equations).

$$\dot{C}_t = [A(K_t)y'(A(K_t)K_t) - \delta]\frac{C_t}{\sigma} \tag{13}$$

$$\dot{K}_t = y(A(K_t)K_t) - C_t \tag{14}$$

An additional condition that the plan has to satisfy is the transversality condition, which guarantees that the non-negativity constraint on K_t is binding at infinity:

$$\lim_{t\to\infty} e^{-\delta t} C_t^{-\sigma} K_t = 0 \tag{15}$$

A plan that satisfies (13), (14) and (15) plus the non-negativity constraints is a dynamic competitive equilibrium for some set of consistent expectations on the path of K_t.

In order to rule out trivial cases, we will focus on economies whose preferences and technological structure make sustained growth attainable for some set of initial conditions. The problem which we will be interested in is to find conditions that prevent some economies from achieving this outcome. To this aim, we need to assume that the productivity of capital is higher than the social discount rate, when the frontier of technical knowledge is reached and the size of the capital stock is arbitrarily large:

$$\lim_{K_t\to\infty} A(K_t)y'(A(K_t)K_t) = a\Omega \equiv \Phi > \delta \tag{16}$$

where a and Ω have been defined in (1) and (7), respectively.

3 Dynamics.

This section is more technical than the rest of the chapter. We summarize here the main ideas before going through formal propositions.

We are interested in two types of solutions: saddlepaths converging to a stationary state and trajectories characterised by perpetual growth. The assumptions of the model, particularly (7) and (16) guarantee that orbits with sustained growth exist, which satisfy (13) and (14). However, they are not sufficient to ensure that the boundary and the transversality conditions are also satisfied. We provide conditions for the existence of a competitive equilibrium with sustained growth satisfying (13), (14) and (15). We show that such a trajectory asymptotically approaches a 'balanced growth' path where capital, consumption and income grow at a common constant rate.

Stagnation is also a possible outcome when there are trajectories that satisfy the Euler equations and converge to a stationary steady-state. These orbits always satisfy the transversality condition. In fact, they closely resemble the unique solution of the Cass-Koopmans model and are interpreted here as 'extensive growth' trajectories converging to zero long-run growth. Small exogenous shocks may raise the long-run steady-state and produce temporary growth, but it is not self- sustaining. On the other hand, a sequence of cumulated changes (say, subsequent 'revolutions' which shift repeatedly the term a in the learning function) will cause the disappearance of the stationary equilibrium.

Under some additional assumptions we prove that in the take-off stage, along the equilibrium path, capital grows at a faster rate than consumption, with the difference shrinking as accumulation proceeds. Technically, this guarantees the existence of (at least) a competitive equilibrium for all initial conditions.

Finally, we characterise different types of 'transitional' trajectories. We show that, given the state of scientific knowledge, it is possible to find economies in which, for any initial state with low capital, alternative sets of agents' consistent beliefs drive the economy to either a sustained growth or a long-run stationary path. On the other hand, there exist economies that, if the capital stock is below some threshold level, are uniquely destined to be locked into a trap condition.

The first issue is to analyze under what conditions stationary (steady-state) solutions exist. An immediate implication of equation (13) gives the following preliminary result:

Lemma 1 *Let $C(K_t) = \{C_t \in R^+ \mid \dot{K}_t = 0, \text{ given } K_t\}$. For any K_t, $C(K_t)$ is a univalued, continuous and monotonically increasing function of K_t.*

Proof. Follows immediately from (14), since $y(A(K_t)K_t)$ is non-negative, continuous and increasing with K_t. □

We now identify the conditions under which an economy exhibits zero-growth. Define the function $r(K_t) = A(K_t)y'(A(K_t)K_t)$, where r is the interest rate in the laissez-faire economy.

Proposition 1 *(a) If $r(K_t) > \delta$, $\forall K_t$, then the system of differential equations (13)-(14) has no fixed point, i.e. no steady-state solution exists. If a solution exists, it must be characterised by $\dot{C}_t > 0$, $\dot{K}_t > 0$ and $C_t < C(K_t)$, $\forall t$.*
(b) If $\exists\ K_t$ such that $r(K_t) < \delta$, then there will exist at least two steady-state solutions.

Proof. *(a)* That $\dot{C}_t > 0$ follows directly from (13). Assume that, for some t, $\dot{K}_t \leq 0$. Then the condition $K_t > 0$ will be violated in finite time. To show this, differentiate (14) obtaining: $\ddot{K}_t = [A(K_t) + A'(K_t)K_t]y'(A(K_t)K_t)\dot{K}_t - \dot{C}_t < 0$, as $\dot{K}_t \leq 0$, $\dot{C}_t > 0$, $[A_t + A'K_t]y' > 0$. Thus K_t will reach zero in finite time.

(b) By the assumptions (1), (7) and (16) it follows that $\lim_{K_t \to 0} r(K_t) = \infty$ and $\lim_{K_t \to \infty} r(K_t) = \Phi > \delta$. Since $A(K_t)$ and $y(A(K_t)K_t)$ are continuous, then $r(K_t)$ is continuous. We can then apply the Intermediate Value Theorem

and conclude that $r(K_t) = \delta$ for at least two values of K_t. Let us call these values K^* and K^{**}; by (13), we know that $\dot{C}_t = 0$ at K^* and K^{**}. But, by Lemma 1, there exist $C^* = C(K^*)$ and $C^{**} = C(K^{**})$, such that $\dot{K}_t = 0$. Then (C^*, K^*) and (C^{**}, K^{**}) are fixed points of the dynamic system (see fig. 1). □

FIG. 1 HERE

Part *(a)* establishes that, if the marginal productivity of capital always exceeds δ, then no 'trap' will exist. If the reward to capital does not fall too much in the first stage of the accumulation process, then the economy is of the Jones and Manuelli-type. Part *(b)* establishes that, when a steady-state exists, it cannot be the only one (in generic economies). The behaviour of the function $r(K_t)$ in this second case is represented in Fig.1. The particular way in which the externalities affect the marginal productivity of capital in equilibrium (A_t, increasing with K_t, multiplies y'_t, decreasing with K_t) causes the alternation of decreasing and increasing returns to capital alone. There is a range of values of K_t (precisely $K_t \in [K^*, K^{**}]$) in which consumption is decreasing, whereas outside that range it is everywhere increasing, since the competitive interest rate r_t exceeds the social discount rate. Notice that $r(K_t)$ in Fig.1 decreases monotonically to zero in the traditional neoclassical model (Cass, 1965) as well as it decreases monotonically in the model of Jones and Manuelli (1990), where it is however bounded from below by Φ. In standard Ak-type endogenous growth models, finally, it is a horizontal line and the dynamic system does not exhibit any fixed point (steady-state). In our model, if the function $r(K_t)$ takes on the value δ in correspondence of one value of K_t, clearly it must do so in correspondence of at least another value of K_t. Notice that there exist non-generic knife-edge economies, for which $r(K_t) = \delta$ turns out to be a tangency solution and there is only one interior steady-state. This is a 'bifurcation' that separates economies which exhibit traps from those for which self-sustained growth is warranted for any initial condition. In the rest of the discussion we will ignore knife-edge cases and we will limit attention to 'structurally stable' economies to which linearisation techniques can be employed for the purpose of local stability analysis.

The stability properties of the steady-state solutions are dealt with by the next proposition. None of the fixed points of the dynamic system turns out to be stable, the dimension of the unstable manifold associated with each of them being either one or two. 'Odd' steady-state solutions - namely the first, third and so on starting from the fixed point with the lowest level of capital stock - are shown to be saddlepath stable (Cass-Koopmans type) and are interpreted as asymptotes of growth paths for economies that are in a trap region. 'Even' steady-states are fully unstable.

Proposition 2 *Consider a generic economy, described by the system of differential equations (13)-(14), whose fixed points are all hyperbolic. Then, this economy is characterised by an even number of steady-state solutions. The dimension of the unstable manifold associated with each fixed point is either one or two, according to an alternate pattern. The unstable manifold associated with the fixed point with the lowest K has dimension one.*

Proof. Since all fixed points are hyperbolic, we can apply the Theorem of Hartman and Grobman and linearise of the vector field about each steady-state. Let $(\overline{C}, \overline{K})$ be a fixed point. Then:

$$\begin{bmatrix} \dot{C}_t \\ \dot{K}_t \end{bmatrix} = \begin{bmatrix} 0 & \frac{\overline{C}}{\sigma}[A'y' + (A + A'\overline{K})Ay''] \\ -1 & (A'\overline{K} + A)y' \end{bmatrix} \begin{bmatrix} C_t - \overline{C} \\ K_t - \overline{K} \end{bmatrix} \tag{17}$$

where all functions and derivatives are calculated at $(\overline{C}, \overline{K})$. The trace of the Jacobian is always positive. If $r'(\overline{K}) = A'y' + (A'\overline{K} + A)Ay'' < 0$, then the Jacobian has a negative determinant the dimension of the unstable manifold associated with the fixed point is one (saddle-point). If $r'(\overline{K}) = A'y' + (A'\overline{K} + A)Ay'' > 0$, then the determinant is positive, implying that the dimension of the unstable manifold associated with the fixed point is two. In this case, the fixed point is either a source if the eigenvalues of the linearised system are complex and conjugate, or a node, otherwise[7]. If the

[7]The occurrence of complex eigenvalues is associated, ceteris paribus, with low values of σ.

first-order linearisation is not decisive, one can consider higher-order linearisations.

Using (7), compute $\lim_{K_t \to 0} r(K_t) = \infty$, and $\lim_{K_t \to \infty} r(K_t) = \Phi > \delta$. Define K^* as the lowest and K^{**} as the highest steady-state level of capital. Then, by the Intermediate Value Theorem it is clear that $r'(K^*) < 0$ and $r'(K^{**}) > 0$. So, K^* is a saddle-point, whereas K^{**} is a fully unstable fixed point. The argument can be easily extended to cases in which there n steady-states proving that the second fixed point is fully unstable, whereas the $n-1^{st}$ fixed-point is a saddle-point, that the third fixed point is a saddle-point, whereas the $n-2^{nd}$ is a fully unstable and so on.

That the equilibria are even in number is proved by contradiction. Suppose that they are odd in number. Then, by the previous argument, $r'(K^{**}) < 0$. Furthermore, $r(K_t) < \delta$, $\forall K_t > K^{**}$, contradicting assumption (16). So, the number of equilibria must be even. □

Fig. 2 represents the phase diagram of an economy with two steady-states, (C^*, K^*) and (C^{**}, K^{**}). Notice that $\dot{C}_t < 0$ for $K_t \in [K^*, K^{**}]$, whereas $\dot{C}_t > 0$ for all K_t outside this interval. The locus $\dot{K}_t = 0$ represents the aggregate production function which includes the external effects. There is positive (negative) capital accumulation when the value of consumption is greater (smaller) than the level of production. The non-convex region exhibited by the schedule $\dot{K}_t = 0$ is not in contradiction with the assumed concavity of the production function at the firm level. Furthermore, the production function becomes approximately linear for high values of K_t. The saddle-point nature of (C^*, K^*) is evident from the diagram, whereas the purely geometric analysis is not sufficient to establish the repulsive nature of (C^{**}, K^{**}). Without loss of generality, we will limit attention in the rest of the chapter to systems which, like that represented in Fig. 2, exhibit only two fixed points. This allows us to refer unambiguously to (C^*, K^*) and (C^{**}, K^{**}) as the saddle-point and the source/node, respectively.

FIG. 2 HERE

Until now only the existence and the nature of steady-states have been discussed. The following Proposition characterises the equilibrium trajectories. The first part, establishes that a plan converging to a steady-state equilibrium, as in the traditional Cass-Koopmans model, is a competitive equilibrium. The second part establishes the existence (under some conditions) and the uniqueness of an equilibrium trajectory with perpetual growth of K_t and C_t. This equilibrium trajectory converges to an 'asymptotic' balanced growth condition characterized by the equal constant growth rate of K_t and C_t and is the unique trajectory of the dynamic system with perpetual growth which satisfies the transversality condition. The solution identifies the long-run consumption to capital ratio, and the constant long-run saving rate.

Proposition 3 *(a) A plan $\{C_t, K_t\}_{t\geq 0}$ represented by points belonging to the stable manifold of a saddle point is a competitive equilibrium.*

(b) If either $\sigma \geq 1$ or $\Phi < \frac{\delta}{1-\sigma}$, then for some $z \in R^+$ and any $K_0 > z$ there exists a competitive equilibrium, or a plan $\{C_t, K_t\}_{t\geq 0}$ which satisfies (13), (14), (15) and the boundary conditions. This plan is characterised by the following asymptotic conditions:

$$\lim_{t\to\infty} \frac{\dot{C}_t}{C_t} = \lim_{t\to\infty} \frac{\dot{K}_t}{K_t} = \frac{\Phi - \delta}{\sigma} \tag{18}$$

$$\lim_{t\to\infty} \frac{C_t}{K_t} = \frac{\Phi(\sigma - 1) + \delta}{\sigma}. \tag{19}$$

Furthermore, this plan is unique.

(c) There exists no trajectory such that $\lim_{t\to\infty} \frac{\dot{C}_t}{C_t} > \lim_{t\to\infty} \frac{\dot{K}_t}{K_t} > 0$. Trajectories along which $\lim_{t\to\infty} \frac{C_t}{K_t} = 0$ are not competitive equilibria.

Proof *(a)* Since the plan follows an orbit of the vector field, the necessary conditions are obviously satisfied. The transversality condition is also satisfied, since $\lim_{t\to\infty} e^{-\delta t} K_t C_t^{-\sigma} = \lim_{t\to\infty} e^{-\delta t} K^* C^{*-\sigma} = 0$.

(b) First, it has to be proved that such a candidate satisfies the Euler equations. Define g as the common asymptotic growth rate of capital and

consumption. Then, from the assumptions about the asymptotic behaviour of the technology:

$$g = \lim_{t\to\infty} \frac{\dot{C}_t}{C_t} = \lim_{t\to\infty} \left[\frac{A(K_t)y'(A(K_t)K_t) - \delta}{\sigma}\right] = \frac{\Phi - \delta}{\sigma} \tag{20}$$

$$g = \lim_{t\to\infty} \frac{\dot{K}_t}{K_t} = \lim_{t\to\infty} \frac{y(A(K_t)K_t) - C_t}{K_t} = \Phi - \lim_{t\to\infty} \frac{C_t}{K_t} \tag{21}$$

from which:

$$\lim_{t\to\infty} \frac{C_t}{K_t} = \left[\frac{\Phi(\sigma - 1) + \delta}{\sigma}\right] \tag{22}$$

The parameter restrictions stated in the proposition ensure that $\lim_{t\to\infty} \frac{C_t}{K_t} > 0$. Next, it must be shown that this candidate path satisfies the transversality condition. First, observe that:

$$\lim_{t\to\infty} C_t = \lim_{t\to\infty} C_0 e^{\int_0^t g_c(\tau)d\tau} = \lim_{t\to\infty} \Gamma e^{gt} \tag{23}$$

where $\Gamma \equiv lim_{T\to\infty} C_0^{1-\sigma} e^{\int_0^T \frac{r(K_t) - \Phi}{\sigma} d\tau}$ is a constant[8] $g_c(\tau) = \frac{r(K_\tau) - \delta}{\sigma}$ is the consumption growth rate at time τ. We have used the fact that along the equilibrium trajectory $g_c(\tau)$ tends to the constant g. Finally, rewrite (15) as:

$$\lim_{t\to\infty} e^{-\delta t} \frac{K_t}{C_t} C_t^{1-\sigma} = \left[\frac{\sigma}{\Phi(\sigma - 1) + \delta}\right] \Gamma \lim_{t\to\infty} e^{[-\delta + g(1-\sigma)]t} = 0 \tag{24}$$

since $g(1 - \sigma) - \delta = \frac{\Phi(1-\sigma) - \delta}{\sigma} < 0$ by the assumptions of the proposition.

Imposing the asymptotic boundary condition and solving backwards identifies a set of initial states K_0 such that there exists a plan $\{C_t, K_t\}_{t\geq 0}$ which converges to the asymptotic condition. However, we have to show that such a plan is unique, or that given K_0 there exists only one C_0 such that the path through (C_0, K_0) satisfies the asymptotic terminal condition. In order to prove this, we show that trajectories through two arbitrarily close points in the region where $K_t > K^{**}$ and $C_t < C(K_t)$ diverge one from another as t grows (see

[8]There is a technical issue here about the convergence of the integral. In particular, some difficulties may arise if the integral diverges to plus infinity (in which case we should solve a limit of the type zero times infinity to check whether the transversality condition is satisfied). Though we cannot exclude this possibility for general technologies, we limit attention, without great loss of generality, to technologies and learning functions such that Γ is constant and this simple proof is ufficient.

Fig. 3). More precisely, fix K_0 and call $\tilde{C}_0$ the consumption level which identifies the trajectory converging to the balanced growth solution. Then, choose two alternative consumption levels C_0' and C_0'', such that $C_0' > \tilde{C}_0 > C_0''$, and indicate the three trajectories identified by each point with corresponding superindices. From (13) and (14), we have:

$$\dot{C}_0' > \dot{\tilde{C}}_0 > \dot{C}_0'' ; \quad \dot{K}_0' < \dot{\tilde{K}}_0 < \dot{K}_0''. \tag{25}$$

Then, the trajectories diverge one from each other. Since the divergence occurs at each $t \geq 0$, then there will be only one critical trajectory which converges to the asymptotic condition

(c) The first part of the statement stems from the fact that $\lim_{t\to\infty} \frac{\dot{C}_t}{C_t} > \lim_{t\to\infty} \frac{\dot{K}_t}{K_t} > 0$ implies unbounded growth of $\frac{C_t}{K_t}$ violating feasibility, since the productivity of capital is bounded. The second part is proved by checking that a path along which $\frac{C_t}{K_t} \to 0$ as $t \to \infty$ violates the transversality condition. To show this, observe that in this case we would have $\lim_{t\to\infty} \frac{\dot{K}_t}{K_t} = \Phi$ and $\lim_{t\to\infty} \frac{\dot{C}_t}{C_t} = \frac{\Phi-\delta}{\sigma}$ and the transversality condition is violated, since:

$$\lim_{t\to\infty} C_t^{-\sigma} K_t e^{-\delta t} = \Delta e^{\left(-\sigma\frac{\Phi-\delta}{\sigma}+\Phi-\delta\right)t} = \Delta > 0, \tag{26}$$

where $\Delta \equiv C_0^{-\sigma} K_0 \lim_{T\to\infty} e^{\int_0^T \frac{y(r(A(K_T)K_T)}{K_\tau} - r(K_\tau) - \frac{C_\tau}{K_\tau} d\tau}$. To show that $\Delta > 0$ is equivalent to show that the integral of its expression does not diverge to minus infinity. This is necessarily true, since:

(i) - $\frac{y(K_\tau)}{K_\tau} - r(K_\tau) > 0, \forall \tau$, from the concavity of y(.);

(ii) - $\lim_{T\to\infty} \int_0^T \frac{C_\tau}{K_\tau} d\tau < \infty$. This is proved by observing that:

$$\int_0^T \frac{C_\tau}{K_\tau} d\tau = \frac{C_0}{K_0} e^{\int_0^T \frac{\Phi-\delta}{\sigma} - \Phi + \nu(K_\tau) d\tau} \tag{27}$$

where $\nu(K_\tau)$ is an (uninteresting) function of K such that $\lim_{K_\tau\to\infty} \nu(K_\tau) = 0$. Since, by assumption, $\frac{\Phi-\delta}{\sigma} - \Phi \leq 0$, then one can choose a large enough z such that $\frac{\Phi-\delta}{\sigma} - \Phi + \nu(K_\tau) \leq 0, \forall \tau \geq z$. Then, rewrite:

$$\lim_{T\to\infty} \int_0^T \frac{C_\tau}{K_\tau} d\tau = \int_0^z \frac{C_\tau}{K_\tau} + \lim_{T\to\infty} \int_z^T \frac{C_\tau}{K_\tau} d\tau =$$

$$\int_0^z \frac{C_\tau}{K_\tau} + \frac{C_z}{K_z} \lim_{T \to \infty} e^{\int_z^T \frac{\Phi-\delta}{\sigma} - \Phi + \nu(K_\tau) d\tau} < \infty$$

This shows that $\Delta > 0$ and the trajectory is not an equilibrium. □

FIG. 3 HERE

This proposition requires comment. First, since $\Phi > \delta$, by assumption (16), Proposition 3-b restricts the range of feasibility for values of Φ, when $\sigma < 1$, to the open interval $]\delta, \frac{\delta}{1-\sigma}[$. The intuition is that if δ and/or σ are too small, agents postpone consumption too much and no solution to the programme exists.

Second, there is a class of candidate trap trajectories that we have ignored, i.e. closed orbits. It is possible, in fact, to find conditions under which the occurrence of closed orbits is impossible. Bendixson's criterion (Guckenheimer and Holmes, 1983, p.44, Theorem 1.8.2) ensures that if there is no sign change for the trace of the Jacobian evaluated at all points of a two-dimensional flow, then there exists no closed orbit. The expression of the trace as a function of K is given by:

$$Tr(K_t) = \frac{A(K_t)y'(A(K_t)K_t) - \delta}{\sigma} + [A(K_t) + A'(K_t)K_t]y'(A(K_t)K_t). \quad (28)$$

The second term is always positive, whereas the first term is zero at all fixed points, and is positive in the region of the plane in which $\dot{C}_t > 0$, including in particular all points such that $K_t < K^*$ and all points at which $K_t > K^{**}$. So, for Bendixson's criterion to be satisfied - which, remember, provides only sufficient conditions for the non-existence of closed orbits - it remains to be checked that $[(1+\sigma)A(K_t) + \sigma K_t A'(K_t)]y'(A(K_t)K_t) > \delta$, $\forall K_t \in]K^*, K^{**}[$.Though this condition is certainly violated for arbitrarily small values of σ, it is evident that rather mild restrictions on the admissible parameters (particularly, a lower bound on σ dependent on the technological specification) would ensure that it is satisfied. For this reason, we will ignore the issue in the rest of the chapter.

Third, and most important, the main limitation of the proposition is that it does not ensure that an equilibrium with sustained growth exists for arbitrary initial states of the system. In fact it only establishes existence for large enough K_0's. If we can show that an equilibrium with perpetual growth necessarily exists for all initial states $K_0 > K^{**}$, however, this is sufficient to ensure that at least one equilibrium (either long-run stationary or with sustained growth) exists for any initial state of the system. It is in fact easy to establish (the proof is omitted) that for all states K_t belonging to the region of the plane such that $0 < K_t < K^{**}$ there exists at least one point belonging to the stable manifold of the saddle-point (C^*, K^*), namely an equilibrium saving decision. The next proposition provides these conditions. Define the variable $\phi_t \equiv \frac{C_t}{K_t}$ and its long-run equilibrium value as $\phi^* \equiv \frac{\Phi(\sigma-1)+\delta}{\sigma}$. Also, let g be defined, as before, as the asymptotic growth rate along the equilibrium growth path. Using these definitions, rearrange (13)-(14) to obtain:

$$\dot{\phi}_t = \phi_t\left\{(\phi_t - \phi^*) + \frac{1}{\sigma}\left[A(K_t)y'(A(K_t)K_t) - \sigma\frac{y(A(K_t)K_t)}{K_t} - \Phi(1-\sigma)\right]\right\} \tag{29}$$

$$\dot{K}_t = K_t\left\{g + \left[\frac{y(A(K_t)K_t)}{K_t} - \Phi\right] - (\phi_t - \phi^*)\right\} \tag{30}$$

where we notice that $\dot{\phi}_t = \frac{C_t}{K_t}\left(\frac{\dot{C}_t}{C_t} - \frac{\dot{K}_t}{K_t}\right)$. The unique equilibrium trajectory converges here to the conditions $\{\dot{K}_t = gK_t,\ \dot{\phi}_t = 0\}$, with $\phi_t = \phi^*$, and $A(K_t)y'(A(K_t)K_t) = \frac{y(A(K_t)K_t)}{K_t} = \Phi$. It is clear then that our problem is equivalent to establishing that for any $K_0 > K^{**}$ there exists a strictly positive plan $\{\phi_t, K_t\}_{t\geq 0}$ which satisfies these asymptotic conditions. Before going through the formal analysis of the general case, we study geometrically the simplest case, consistent with Fig.1 and Fig.2, in which the competitive interest rate $(r(K_t) = A(K_t)y'(A(K_t)K_t))$ is monotonically increasing with K_t when $K_t > K^{**}$ and the output-to-capital ratio $(\frac{y(A(K_t)K_t)}{K_t})$ is decreasing with K_t in the same range. In this case, the term in square bracket in (29) turns out to be an increasing function of K_t and the term in square bracket in (30) turns out to be a decreasing function of K_t. Accordingly, in the phase diagram in Fig.4 both the isoclines $\dot{\phi}_t = 0$ and $\dot{K}_t = 0$ exhibit a negative slope. It is clear from the figure that the unique equilibrium orbit with sustained growth is characterized by an always declining ϕ_t for all $K_t > K^{**}$. Notice, in particular, that the equilibrium orbit cannot enter the region of the plane where $\phi < \phi^*$ since no orbit which has points in that region tends asymptotically to ϕ^*. Since the equilibrium value of ϕ_t is bounded from below by $\phi^* > 0$, then an equilibrium always exists[9].

FIG. 4 HERE

The main fact that allows us to provide conditions for existence is stated by the following lemma.

Lemma 2 *Let $\phi^{eq}(K_t)$ be the (unique) value taken on by ϕ_t along the equilibrium trajectory in the region of the plane where $K > K^{**}$ and $\dot{K} > 0$. Define $\xi(K_t) = \frac{y(A(K_t)K_t)}{K_t} + \frac{\delta}{\sigma} - \frac{A(K_t)y'(A(K_t)K_t)}{\sigma}$. Then: $\phi^{eq}(K_t) \geq \inf_{K_t > K^{**}} \xi(K_t)$ for all*

$K_t > K^{**}$.

Proof. By *reductio ad absurdum.* Define $\zeta_t \equiv \log(\phi_t)$. Then, using the definition of ϕ^*, rewrite equation (29) as:

$$\dot{\zeta}_t = e^{\zeta_t} - \xi(K_t) \tag{31}$$

where $sign\left(\dot{\zeta}_t\right) = sign\ \left(\dot{\phi}_t\right)$ Assume that, contradicting the Lemma, $e^{\zeta_T^{eq}} \equiv \phi^{eq}(K_T) < \inf_{K_t > K^{**}} \xi(K_t)$ for some K_T. This implies that $\dot{\zeta}_t < 0$ for all $t \geq T$ (to see this, observe that since $\dot{\zeta}_T < 0$, then $\zeta_{T+\Delta_T} < \zeta_T < \inf_{K_t > K^{**}} \xi(K_t)$. Hence $\dot{\zeta}_{T+\Delta_T} < 0$, and so on). Now, notice that $\lim_{K_t \to \infty} \xi(K_t) = \phi^*$. So, $\inf_{K_t > K^{**}} \xi(K_t) \leq \phi^*$. But, then, $\phi^{eq}(K_T) < \phi^*$ and $\dot{\phi}_t^{eq} < 0$, $\forall\, t \geq T$. This implies that $\lim_{t \to \infty} \phi^{eq}(K_t) < \phi^*$, which is a contradiction, since we know that $\lim_{t \to \infty} \phi^{eq}(K_t) = \phi^*$. □

Then, we can establish what follows.

Proposition 4 *a) If $\inf_{K_t > K^{**}} \xi(K_t) > 0$, then there exists $\phi_t^{eq}(K_t) > 0$, $\forall\, K_0 > K^{**}$, $t \geq 0$, i.e. there exists an equilibrium with sustained growth for any $K_0 > K^{**}$.*

[9] This case is coherent with point *(c)* of the following Proposition.

b) A sufficient condition for (a) to hold is that $\sigma \geq 1$.

c) If $\xi'(K_t) < 0$ for all $K_t > K^{**}$, then, in the region of the plane where $K > K^{**}$ and $\dot{K} > 0$, $\dot{\phi}_t^{eq} < 0$ and (a) necessarily holds.

Proof. Part *(a)* follows immediately from the previous Lemma. In fact, $\phi^{eq}q(K_t) \geq \inf_{K_t > K^{**}} \xi(K_t) > 0$, namely for all $K_t > K^{**}$ there exists a choice of consumption which belongs to an equilibrium plan.

To prove part *(b)*, observe (using the definition of $\xi(K_t)$) that *(a)* is always satisfied if, for all K_t, $\sigma \geq 1$ and $\sigma > \frac{y'(\cdot)A(\cdot)K_t}{y(\cdot)}$. The right hand-side term of the latter inequality is the elasticity of output to capital in efficiency units when A_t is treated as parametric, and is smaller than one since $y(A_tK_t)$ is concave when external effects are ignored. So, existence is guaranteed when $\sigma \geq 1$.

Part *(c)* is proved by *reductio ad absurdum.* Using the definitions introduced in the proof to the previous lemma, time-differentiate expression (31) to obtain:

$$\ddot{\zeta}_t = e^{\zeta_t}\dot{\zeta}_t - \xi'(K_t)\dot{K}_t. \tag{32}$$

Assume that at some t $\dot{\zeta}_t^{eq} \geq 0$, in contradiction with point *(c)*. Then, since $\dot{K}_t^{eq} > 0$, $\ddot{\zeta}_t^{eq} > 0$. Since the vector field is continuous and twice differentiable, then $\lim_{t\to\infty} \dot{\zeta}_t^{eq}$ cannot be zero, contradicting the fact that the trajectory is an equilibrium. That an equilibrium exists for any $K_t > K^{**}$ follows from the fact that $\phi^{eq}(K_t) > \phi^* > 0$. □

As a corollary to the Proposition, we provide an additional result which applies to technological parametrizations like (8).

Corollary 1 *Assume that $y(A(K_t)K_t) = DA(K_t) + [A(K_t)K_t]^\theta$, where $0 < \theta < 1$. Define the elasticity of the learning function as $\lambda(K_t) = A'(K_t)\frac{K_t}{A}$. Assume that $\lambda(K_t) < \frac{1-\theta}{\theta}$, for all $K_t > K^{**}$ and that $\theta \leq \sigma$. Then, a competitive equilibrium exists for any initial conditions. If, in addition, $\sigma \leq 1$, then $\dot{\phi}_t^{eq} < 0$ for $K_t > K^{**}$.*

Proof. When $\sigma \geq 1$ an equilibrium always exists by Proposition 4-*b*. Consider now the case in which $\sigma \leq 1$. Under the specified technology, (29) becomes:

$$\dot{\zeta}_t = DA(K_t)\frac{1-\sigma}{\sigma} + A(K_t)^\theta K_t^{\theta-1}\frac{\theta-\sigma}{\sigma} + e^{\zeta_t} - \phi^*, \tag{33}$$

which, when time-differentiated, gives (after rearrangement):

$$\ddot{\zeta}_t = \left\{\frac{1-\sigma}{\sigma}DA'(K_t) + [\theta(1+\lambda(K_t)) - 1]A(K_t)^\theta K_t^{(\theta-2)}\right\}\dot{K}_t + e^{\zeta_t}\dot{\zeta}_t, \tag{34}$$

Assume, in contradiction with the corollary, that $\dot{\zeta}_t^{eq} > 0$. Then, since $\lambda_t < \frac{1-\theta}{\theta}$, $\ddot{\zeta}_t^{eq} > 0$ and $\lim_{t\to\infty}\dot{\zeta}_t^{eq}$ cannot be zero. The rest of the proof is identical to the proof of Proposition 4-*c*. □

To summarise, in competitive models with aggregate non-convexities and externalities, contrarily to the results of standard concave programmes, the existence of an equilibrium (possibly, non-unique) is not guaranteed for arbitrary initial conditions. However, Proposition 4 shows that in our model existence is guaranteed for all initial states when $\sigma \geq 1$, or, otherwise, when some technological restrictions hold. Corollary 1, additionally, shows that the range of values of σ for which existence is guaranteed is wider when the technology is parametrised according to (8), without the need of fully specifying the learning function. The existence issue does not seem, in conclusion, particularly severe, since only highly non-standard technologies and preferences would not satisfy part *(a)* of the proposition. We will see in the Corollary to Proposition 5 that an alternative criterion to ensure existence may be found. Finally, the proposition and its corollary establish that there is a class of economies in which the consumption to capital ratio unambiguously falls in the stage when 'growth becomes the normal condition' of an economy.

We will now consider economies such that an equilibrium exists for all K_0 and extend the analysis to the region of the plane where $K < K^{**}$. The question which we address is where the equilibrium trajectory 'comes from' before entering the region indicated as *(V)* in Fig.2. In more precise terms, we search for the α-limit of the equilibrium orbit with growth. To this aim, observe that generic orbits of the vector field defined by (13) and (14) which have points in the semiplane $W = \{(C, K) \text{ s.t. } 0 < K < K^{**}\}$ belong to either the basin of repulsion of the origin or to the basin of repulsion of the fixed point (C^{**}, K^{**})[10]. The non-generic exception is represented by the one-dimensional unstable manifold of the fixed point (C^*, K^*), whose α-limit is the saddle-point. The geometric inspection suggests that if the equilibrium trajectory is bounded from above by the unstable manifold throughout the semiplane W, then it must belong to the basin of repulsion of the origin (notice in fact that (C^{**}, K^{**}) is bounded from below by the unstable manifold, and trajectories never cross), whereas if it is bounded from below by the unstable manifold it must belong to the basin of repulsion of (C^{**}, K^{**}) Furthermore, in some knife-edge economies the equilibrium trajectory may coincide with the unstable manifold of the saddle-point (C^*, K^*). These points are stated more formally by the next Proposition. We will discuss later why discriminating between economies according to the α-limit behaviour of the equilibrium orbit has important consequences in terms of the economic interpretation of the model.

Proposition 5 *Assume that Bendixson's criterion is satisfied throughout the vector field - cfr. eq. (28). Also, assume that the conditions of Proposition 4-a are satisfied. Define (C_t^{eq}, K_t^{eq}) as the equilibrium orbit with sustained growth, and (C_t^U, K_t^U) as the unstable manifold associated with the fixed point (C^*, K^*), with $(K_0 > K^*)$. Then, one of the following cases will occur:*

(i) $\lim_{t\to\infty} \frac{C_t^U}{K_t^U} = 0$; $\lim_{t\to-\infty} C_t^{eq} = C^{**}$, $\lim_{t\to-\infty} K_t^{eq} = K^{**}$;

(ii) $\lim_{t\to\infty} K_t^U = 0$; $\lim_{t\to-\infty} C_t^{eq} = \lim_{t\to-\infty} K_t^{eq} = 0$;

(iii) $(C_t^U, K_t^U) = (C_t^{eq}, K_t^{eq})\ \forall\, t$; $\lim_{t\to-\infty} C_t^{eq} = C^*$, $\lim_{t\to-\infty} K_t^{eq} = K^*$;

Proof. First, consider the asymptotic (ω-limit) behaviour of the orbit (C_t^U, K_t^U). Since the Bendixon's criterion rules out closed orbits, this trajectory

[10]The the basin of repulsion of $\mathbf{z}$ is defined as the set of points $\mathbf{x}$ with $\mathbf{x}(t) \to \mathbf{z}$ as $t \to -\infty$.

cannot be homoclinic (i.e., a closed orbit connecting the saddle-point with itself). Nor can it spiral 'inwards' towards (C^{**}, K^{**}), since this would imply, given the repulsive nature of this fixed point, that there exists a limit cycle which surrounds (C^{**}, K^{**}). Then, (C_t^U, K_t^U) may either grow for ever as t grows or hit at some finite t one of the axes. If it hits the vertical axis we are in case *(ii)*. If it hits the horizontal axis we are in case *(i)*. If it grows for ever, it is either one of the infinite trajectories along which capital grows faster than consumption for large t (cfr. Proposition 3) or the unique trajectory which converge to the balanced growth condition[11]. The former case is consistent with *(ii)*, whereas the latter, non-generic case is consistent with *(iii)*.

Second, observe that the unstable manifold associated with the saddle-point splits the plane into two regions, corresponding to the basin of repulsion of the origin and the basin of repulsion of the fixed point (C^{**}, K^{**}), respectively.

To prove *(i)*, observe that the equilibrium trajectory cannot be bounded from above by the orbit (C_t^U, K_t^U), since this trajectory does not satisfy - in case *(i)* - the transversality condition and, for this reason, bounds from above only trajectories which do not satisfy the transversality condition (cfr. Fig.3). Since trajectories never cross, the equilibrium trajectory must be bounded from below by (C_t^U, K_t^U). On the other hand, all points $\{(C, K^U) \mid C > C^U\}$ belong to the basin of repulsion of the unstable fixed point (C^{**}, K^{**}). So, the equilibrium orbit has the α-limit in (C^{**}, K^{**}) (see Fig.5a).

To prove case *(ii)*, observe that the basin of repulsion of (C^{**}, K^{**}) is in this case a bounded set whose boundaries are the unstable manifold (C_t^U, K_t^U) and a subset of the vertical axis (see Fig.5b). Since the equilibrium trajectory with sustained growth is not contained in a bounded region, then its α-limit cannot be (C^{**}, K^{**}). So, it must be the origin. The knife-edge case *(iii)* is

[11]Refer to Fig.2. The unstable manifolds, originating from the saddle-point, enter first the region III. Then, unless it hits the horizontal axis, it enters the region V. If it enters V, it either remains there for ever (sustained growth), or it enters the region VI. If this latter case occurs, it necessarily crosses subsequently the regions IV and II, and ends up hitting the vertical axis in finite time. It is impossible that it enters for a second time III coming from IV, because this would imply the existence of a limit cycle.

self-explanatory. □

If the unstable manifold does not hit the boundaries, point *(i)* can be proved under milder assumptions.

Corollary 2 *If* $\lim_{t\to\infty} C_t^U = \infty$ *and* $\lim_{t\to\infty} \frac{C_t^U}{K_t^U} = 0$*, then* $\lim_{t\to-\infty} C_t^{eq} = C^{**}$, $\lim_{t\to-\infty} K_t^{eq} = K^{**}$*. Furthermore, for all* $K_0 > K^{**}$*, there exists an equilibrium plan* $\{C_t^{eq}, K_t^{eq}\}_{t\geq 0} > 0$.

Proof. The prove of the first part is analogous to that of Proposition 5. However, the fact that $(C_t^U, K_t^U) > 0,\ \forall\ K_t^U\ \in]0, \infty[$ guarantees the existence of a competitive equilibrium with sustained growth for all $(K_t > K^{**})$, since the equilibrium trajectory is bounded from below by (C_t^U, K_t^U), without the need of assuming that Proposition 4 hold assumptions. □

The different cases are described by the following figures. The shadowed areas emphasise the basin of repulsion of (C^{**}, K^{**}) in each case. Also, we have indicated with S the saddle-path equilibrium, with T the equilibrium with sustained growth and with U the unstable manifold associated with the fixed point (C^*, K^*). In Fig.5a, corresponding to case *(i)*, the trajectory U grows for ever, but violates the transversality condition (geometrically, it becomes 'horizontal' for large K). The equilibrium trajectory T lies entirely above it and 'comes from' the point (C^{**}, K^{**}). Notice that, as stated by the Corollary, the existence of an equilibrium trajectory for any initial state is guaranteed in this case. In Fig.5b, we have the case *(ii)*. Notice that all trajectories originating from (C^{**}, K^{**}) revert to negative growth and hit the vertical axis in finite time. Fig.5c, finally, represents the knife-edge case.

FIG. 5a, FIG. 5b and FIG. 5c HERE

The difference between the two main cases (ignoring the knife-edge case) has some interesting economic implications. When case *(i)* occurs, we have a theory of development thresholds. More precisely, there is a set of initial conditions $0 < K_0 < K^{lt}$ such that the only equilibrium generated by self-fulfilling expectations is the long-run stationary one. When case *(ii)* occurs, instead, we have a theory of multiple self-fulfilling prophecies. Krugman (1991) stressed how the presence of external economies introduces the possibility of meaningful multiple equilibria in development models. 'When there are external economies, it will often happen that the return to committing resources to some activity is higher, the greater the resources committed' (p. 651). In our model, for all states such that $0 < K_t < K^{ut}$ there are alternative sets of rational expectations, generating different current saving rates, which drive the economy to either take off into sustained growth or long-run stationarity. If expectations of high aggregate investments and productivity growth prevail, all agents find it optimal to choose high savings, since accumulation gets a higher reward. In the opposite case, if low investments are expected, people find it optimal to choose high consumption today. This is a case of strategic complementarity in individual savings decisions (Cooper and John, 1988). There are no minimum thresholds, here: an arbitrarily small economy has always the chance of taking off along a sustained growth path. To our knowledge, this is the first model which allows for multiple Nash equilibria for a continuum of low levels of the state variable with one equilibrium being long-run stationary and the other equilibrium being characterised by sustained growth. On the other hand, the multiple equilibria issue may arise also in case *(i)*, if the equilibrium trajectory cycles about the fixed point (C^{**}, K^{**}), as in Fig.5a when $K_t \in [K^{lt}, K^{ut}]$. Notice that the multiplicity of equilibria exhibited by our model differs from the equilibrium indeterminacy of the type discussed by Benhabib and Farmer (1991), since for any value of the state variable we always have a finite number of saving choices which are equilibrium consistent.

4 A numerical simulation.

In this section we provide the results of some numerical simulations for a parametrised version of the model which generate the dynamics described by the Figg.5$a - b - c$. Together with confirming the analytical findings, they allow us to analyse the sensitivity of the results to some parameter changes. The main findings are that *(i)* the range of multiple equilibria tends to decrease with σ (the inverse of the elasticity of substitution in consumption) and *(ii)* dynamics of the Fig.5*b*-type seem to be more likely, ceteris paribus, when the economy is 'close' to the bifurcation at which the steady-state equilibria disappear (i.e. when $\min_K\ r(K)$ is not much smaller than the time discount rate). Furthermore, we can study the time evolution of the growth rates of the variables of the model and check whether they capture some typical feature of the data. Particular, like mentioned at the beginning of this chapter, cross-country empirical evidence shows that growth patterns are typically hump-shaped, with medium-size developing countries exhibiting higher growth rates than both poor and mature economies. This evidence is not rationalised, to our knowledge, by any existing theory, including AK-type endogenous growth models. The reason for believing that our model is a good candidate to cope with this dilemma is that it relies on two sources of growth, accumulation of capital and technical change, whose relative contribution to growth changes with the stage of development. If the learning effects are stronger in the take-off period, it is possible that a country experiences the highest growth rates during this stage.

Assume the particular learning function (3) and production function (8). The system (13)-(14) becomes then:

$$\dot{C}_t = \left\{ \frac{a}{1+\left[\frac{a}{A_0}-1\right]e^{-qK_t}} \left[D + Z\beta K_t^{\beta-1}\right] - \delta \right\} \frac{C_t}{\sigma} \tag{35}$$

$$\dot{K}_t = \frac{a}{1+\left[\frac{a}{A_0}-1\right]e^{-qK_t}} DK_t + Z\left\{ \frac{a}{1+\left[\frac{a}{A_0}-1\right]e^{-qK_t}} K_t \right\}^{\beta} - C_t \tag{36}$$

We will hold fixed all parameters with the exception of Z and σ. The parameters are assigned the following values: $a = 0.075, A_0 = 0.015, q = 0.01, D = 1, \beta = 0.2, \delta = 0.05$[12]. The effect of an increase in Z is to shift

[12]The choice of these numbers is somewhat arbitrary. We believe that the model is

upward the marginal productivity of capital (cfr. Fig.1) and to make the technology more concave. The effect of an increase of σ is to make the consumption growth rate less responsive to changes in the interest rate. Notice that σ does not affect the steady-state values, but affects the long-run growth rate, whereas Z affects the steady-state values, but does not affect the long-run growth rate. In particular, we can identify the bifurcating economy for which $\delta = \min_{K_t} r(K_t)$ (tangency solution in Fig.1) as parametrised by a critical value of Z. Then, we consider economies parametrised by values of Z below the critical one, which are those which exhibit underdevelopment traps. We will consider three different values of σ (0.5, 1 and 2) and we will identify the ranges of Z's for which each of the type of equilibria described by Figs.5a-b-c, respectively, arise. Notice that the asymptotic growth rate is 1.25% when $\sigma = 2$, 2.5% when $\sigma = 1$ and 5% when $\sigma = 0.5$.

The methodology adopted to obtain numerical solutions for the equilibrium trajectories is the following. First, find the two fixed points of the system of differential equations (35)-(36). Then, choose two points arbitrarily close to the saddle-point (one geometrically above and one below it) as predetermined conditions and ask the computer to solve the system of differential equations backwards and forward. The backward solution identifies, by close approximation, the stable manifold (equilibrium), whereas the forward solution identifies the unstable manifold associated with the saddle-point. Notice that a trajectory which is very close to the saddle-point at time $t = T$ was necessarily even closer to the equilibrium saddle-orbit at time $t = T - j$ (for positive j) since all trajectories tend to diverge from the stable manifold as t grows. On the other hand the same trajectory will be very close to the unstable manifold at time $t = T + j$, in the region in which consumption is decreasing, since all trajectories are 'attracted' by the unstable manifold in such region. The (approximate) solution for the equilibrium trajectory with self-sustained growth is found by

presently too stylised for a substantive calibration exercise, which we leave to further research. We have assumed a standard 5% time discount rate, and a learning effect which increases up to five times the productivity of the reproducible factor.

choosing as a predetermined condition a point very distant from the origin that satisfies the asymptotic condition (19) and solving the system of differential equations backwards so as to trace the whole equilibrium orbit from its α-limit up to the chosen point. By the same argument used for the saddle-orbits, the backward solution gives a good approximation to the true equilibrium orbit, since we have shown that all orbits 'diverge' from the equilibrium trajectory as t grows in the region to the right of the last steady-state. The choice of the predetermined condition may be corrected by checking whether the forward solution originating from it, remains linear as K grows; if this is not the case we rectify the original predetermined state of the system by fine adjustments until we reach a satisfactory approximation[13].

The bifurcation which separates economies with traps from economies without is identified by $Z = 149.5$ and is independent of σ. The following table summarises the results.

	FIG.5a	FIG.5b	FIG.5c
$\sigma = 0.5$	$0 < Z < 53.49$	$53.49 < Z < 149.5$	$Z = 53.49$
$\sigma = 1$	$0 < Z < 112.06$	$112.06 < Z < 149.5$	$Z = 112.06$
$\sigma = 2$	$0 < Z < 149.5$	never	never

In the case with low intertemporal substitution in consumption ($\sigma = 2$) the economy follows a behaviour of the type described by Fig.5a for any Z, namely, the equilibrium orbit is always bounded from below by the unstable manifold. In cases with $\sigma = 1$ and $\sigma = 0.5$, however, economies with relatively large Z behave like in Fig.5b, namely exhibit multiple equilibria for any low level of the state variable. Fig.5a-type behaviour keeps occurring for low values of Z, when the two steady-state are relatively far one from each other. We also observe that cycles about the unstable fixed point are increasingly pronounced the larger the elasticity of substitution in consumption chosen. Also, Fig.5b-type behaviour tends to become predominant when the intertemporal elasticity of

[13]The predetermined conditions used for the different simulations are: $(K_T = 10,000, C_T = 260)$ for the case with $\sigma = 0.5$, $(K_T = 10,000, C_T = 538)$ for the case with $\sigma = 1$, $(K_T = 10,000, C_T = 670)$ for the case with $\sigma = 2$. In all cases $\frac{C_t}{K_t}$ is decreasing along the equilibrium trajectory.

substitution in consumption is high. The intuitive reason is that when people are more willing to substitute present for future consumption, a trajectory with very low consumption at low stages of development is more likely to be viable in equilibrium. As a consequence not only do thrifty economies exhibit higher long-run growth rates (like in standard endogenous growth model) but also they face sooner the opportunity to get out of a stationary state.

The following figures report the result of three simulations with logarithmic preferences ($\sigma = 1$), showing the different cases. In all figures only the equilibrium trajectories (S, the stable manifold of the saddle-point and T, the equilibrium trajectory with sustained growth), plus the unstable manifold, U, are represented. Fig.6a is obtained by taking $D = 105$. It shows that the equilibrium trajectory is bounded from above by the unstable manifold. Notice the pronounced cycle described by the equilibrium trajectory, which causes the occurrence of multiple equilibria for all $K_t \in (44, 132)$. Fig.6b is obtained by taking $D = 120$. Though this is not evident from the scale of the figure, solving forward from a point belonging to the unstable manifold shows that the trajectory reverts to negative growth at $K_t = 1450$ and $C_t = 210$ and is not an equilibrium. Fig.6c, finally, shows the knife-edge case obtained from taking $D = 112.06265$.

FIG. 6a, FIG.6b and FIG. 6c HERE.

We now examine the issue of the time evolution of the growth rates along the equilibrium path with sustained growth. We use the same parametrisations just considered (dropping the knife-edge case) together with two cases in which there is a higher elasticity of substitution ($\sigma = 0.5$). In each figure, the upper pictures offer a comparison of the growth rates of capital (thinner continuous line), income (thicker continuous line) and consumption (dashed line), the time being graphed on the horizontal axis. Notice that the final points in the graphs are such that the different variable grow approximately at the common asymptotic rate. The second picture shows the evolution of the growth rate of output at different log-income levels.

We summarise here the parameter specifications chosen:

	σ	Z	Equilibrium type
$Fig.7$	1	100	Fig. 5a
$Fig.8$	1	120	Fig. 5b
$Fig.9$	0.5	40	Fig. 5a
$Fig.10$	0.5	120	Fig. 5b

FIG. 7, FIG. 8, FIG. 9 and FIG. 10 HERE

Consider Figures 7 and 9 first, corresponding to Fig. 5a-type equilibria. It is evident, in both cases, the existence of fluctuations before the final take-off. The take-off stage is characterised by investments booming long before consumption also starts growing. It is evident that agents 'smooth' consumption, coherently with the rational expectations nature of the model. The sharp increase of the rate of capital accumulation during the take-off stage can be interpreted as the counterpart of consumption smoothness. During this critical period, the marginal productivity of capital is extraordinarily high and output tends to grow fast. However, agents anticipate that in the long-run productivity will settle down at a lower level and do not increase consumption too much, so as to grant themselves higher sustained consumption in the future. Notice that in both cases the growth rate of capital peaks at above 10% about sixteen periods after that output growth has started being positive. Though the quantitative features of the simulations results should not be overemphasised, it seems clear that this specification of the model predicts that the growth rate of output remains for some prolonged period significantly above its long-run value (2.5% for the case of Fig. 7 and 5% for the case of Fig.9). The hump-shaped relation between GDP levels and growth rates is even more evident in the Fig. 5b-type equilibria represented in Figures 8 and 10. The most remarkable difference with respect to the previous figures is that capital here is always increasing over time. Notice, finally, that in all cases the growth rate of consumption is bounded from above by that of capital.

5 Economic implications and conclusions

In what measure has the model succeeded in providing an analytical interpretation to Rostow's stylised fact? First, the model captures the idea that activating growth is more difficult than keeping it going. Second, it is coherent with the fact that once-off shocks may have long lasting effects, namely a single stimulus gives rise to a 'sustained and self-reinforcing' response to it. Third, it suggests that small structural differences between countries that in a stationary world only cause (possibly marginal) differences in levels may give rise to large consequences if some exogenous change enables better positioned countries to take-off, leaving the others in the stationary state. Finally, it accounts for the possibility that countries which are exactly identical from the viewpoint of economic structure are driven by different expectations (one might perhaps relate them to cultural or institutional differences) to alternative destinies of take-off or stagnation. The first three points seem to match Rostow's analysis, whereas the fourth introduces a degree of flexibility which avoids an excess of historical determinism.

Let us clarify these points, with reference to some historical experiences documented by Rostow (1978). The first episode of take-off into sustained growth is Great Britain during the First Industrial Revolution. Rostow dates the British take-off at the last two decades of the eighteenth century. According to his data, the yearly growth rate of industrial production, which had been about 1% between 1700 and 1783, became 3.4% between 1783 and 1802, remaining high throughout all the Nineteenth century. The stimulus which created the take-off opportunity was given, according to Rostow, by a wave of major technical improvements which affected the cotton textile manufacture, iron manufacture with coke as fuel, and the efficiency of steam engine, plus technical progress in agricultural which permitted urbanisation, and an environment of expanding international commerce (Rostow, 1978, p.373). The capacity of countries to absorb these innovations was not uniform, and depended on the degree of industrial development which each country had inherited from earlier experience. Consider two other countries, Sweden and France, whose response to the new technological opportunities was fairly moderate compared to Great Britain, and not such as to be identified as 'turning points'. Sweden in the eighteen century did not enjoy a comparable economic, commercial and political power to that of Great Britain. After the Battle of Poltava in 1709, it had lost the leadership role exerted in the latter part of the seventeenth century, and though it had maintained a vigorous mercantile economy, it had a weaker industrial base than Britain. The case of France looks different. Under Colbert's government France had undertaken a remarkable process of industrial and commercial development between 1665 and 1685 which, though temporarily arrested by a period of war, was resumed between 1715 and 1783. The standard explanations to the absence of take-off in France proposed by economic historians (e.g. the oppressive role of the monarchy in France as opposed to the liberal attitudes emerging in Britain after the second english revolution) do not attribute a major role to differences in the economic structure. An analogous case can be made for Netherlands.

Consider our model. Suppose that before the occurrence of the shock Britain and France were structurally identical, whereas Sweden was lagging behind (say, the total factor productivity parameter Z was lower in Sweden than in the other two countries). Fig.11 (the analogue of Fig.1) represents the situation before and after the 'stimulus'. Imagine that all countries had reached the ex-shock long-run equilibrium in the mid-eighteenth century. This means that the capital level in France and Great Britain was at the level $K_0^{f/gb}$, whereas that of Sweden was at the level K_0^s. As the figure shows, we assume that no country, before the shock, had the possibility to achieve self-sustained growth. We assume that the shock was large enough to make all the economies 'bifurcate', namely a second steady-state and an equilibrium trajectory with sustained growth appear. Also, the lower steady-state capital level shifts to the right ($K^{*,(f/gb)}$, $K^{*,(s)}$). The post-shock marginal productivity schedules are drawn as dashed lines. Consider now our interpretation of the take-off hypothesis. We imagine that Fig.5a represent the post-shock phase diagram for Sweden. Given its initial condition (K_0^s), Sweden only faces an equilibrium path which converges to a higher steady-state level. Though an equilibrium trajectory with perpetual growth now exists, the Swedish 'capital' was below the threshold level which would make take-off an equilibrium. So, Sweden simply converges to a new steady-state[14]. On the other hand, according to our hypothesis, Britain and France can be represented by the same phase diagram and initial condition ($K_0^{f/gb}$). Since the two steady-states are closer than in the case of Sweden (a consequence of the higher value of Z), we interpret Fig.5b as representing the structural condition of these two country after the shock. France and Great Britain were facing the same opportunities, but for reasons that are left out of our analysis, they ended up selecting different equilibria. France was driven to the new steady-state represented by K^* (higher than the Swedish one) moving along the trajectory S[15]. Britain, on the other hand, selected the trajectory T characterised by a higher accumulation rate and went through that great irreversible change represented by the Industrial Revolution which marks the beginning of the stage in which growth became the normal condition for the British economy.

FIG. 11 HERE

The historical accuracy of the Rostovian example just considered is debatable. A detailed consideration of the issue goes beyond the scope of this chapter. Critics of the take-off hypothesis have noted that there is not clear-cut evidence from the data that a substantial wedge opened up between the British and the French investment rate after 1783, as our theory (together with that of

[14]Systematic data for Sweden are only available since 1810 for iron production and 1860 for per capita GDP. Looking at iron production (Rostow, p.412), one observes a moderate constant growth rate of 1% per year between 1810-1855, followed by a sharp increase in the following two decades, at which Rostow dates the Swedish take-off.

[15]The hypothesis that France reached a higher steady- state is not contradicted by a rough analysis of the existing data. Rostow (1978) remarks that 'in the decade after 1783, both nations (England and France) progressed, Britain more rapidly; but the gap widened during the war years and down to the beginning of the French take-off in the 1830's' (p.395).

Prof. Rostow) would predict. On the other hand, there are other examples of countries which, after having been structurally similar for some period of their history, seem to have then taken divergent paths at some stage. The example of Korea and Philippines, drawn from Lucas (1993), has been discussed in the introduction to this Thesis. Argentina and Australia, whose economies was pretty similar at the beginning of this century but followed rather divergent paths since the late 30's, provide another example.

Our model also provides an explanation to the fact that countries seem to have taken off in bunches during some critical periods. According to the model, new growth opportunities (what we called 'scientific revolutions') come up at discrete time intervals and give the chance to better positioned countries to activate sustained growth, whereas worse positioned countries benefit, at most, from level effects. Rostow argues that it was between 1830-1850 when France, Belgium, the United States and Germany underwent the great transformation and activated a sustained growth process (Sweden followed with some delay). The same happened to Japan, Russia, Canada, Italy and Australia between in the last two decades of the century. Large Latin American countries like Argentina, Brazil and Mexico started their take-off stage in the 1930's and 40's. Finally, the late 50's and the 60's have been the years of take-off for many Asian countries like South Korea, China, Taiwan, Thailandia and, perhaps more recently, Malaysia and Indonesia. But as noted in the Introduction, many countries seem to be still locked into underdevelopment traps, particularly the majority of Sub-Saharan African countries, some small Latin American countries, and few Asian countries (Burma, Bangla Desh, Nepal among the others).

From a normative viewpoint, the model provides the rationale for active government development policies, though the highly stylised nature of the model suggests caution in identifying the exact nature of these policies. When a country is in a Fig.5a-type situation, government intervention could force the pace of accumulation, by imposing on the economy an allocation mechanism different from the market. A planned system may impose some higher rate of investment than the decentralized economy would deliver, driving the country out of the 'trap'. This may be thought as the way followed by some socialist countries, such as China or the Soviet Russia, though the problems and the inefficiencies created by these systems are well-known.

A more market-oriented strategy could rely, instead, on a set of incentives (e.g. capital subsidisation) and other forms of intervention that rule out the bad equilibrium, by correcting the externality and increasing the private productivity to the accumulation in the reproducible factor. Furthermore, one could extend the model to allow for the possibility for the government to affect the generation of externalities (by assuming $A = A(K_t, \beta)$, where β is a vector of policy variables). This may be the case with public investment in education or infrastructure. This strategy implies a certain degree of state intervention, but is compatible with a market regime and does not imply the administrative costs of a planned economy. Korea seems to be a classic example of this strategy (see Chenery, Syrquin and Robinson, 1986). Finally, some countries may need just a therapy to enhance economic confidence of consumers and investors, without the necessity of major structural changes.

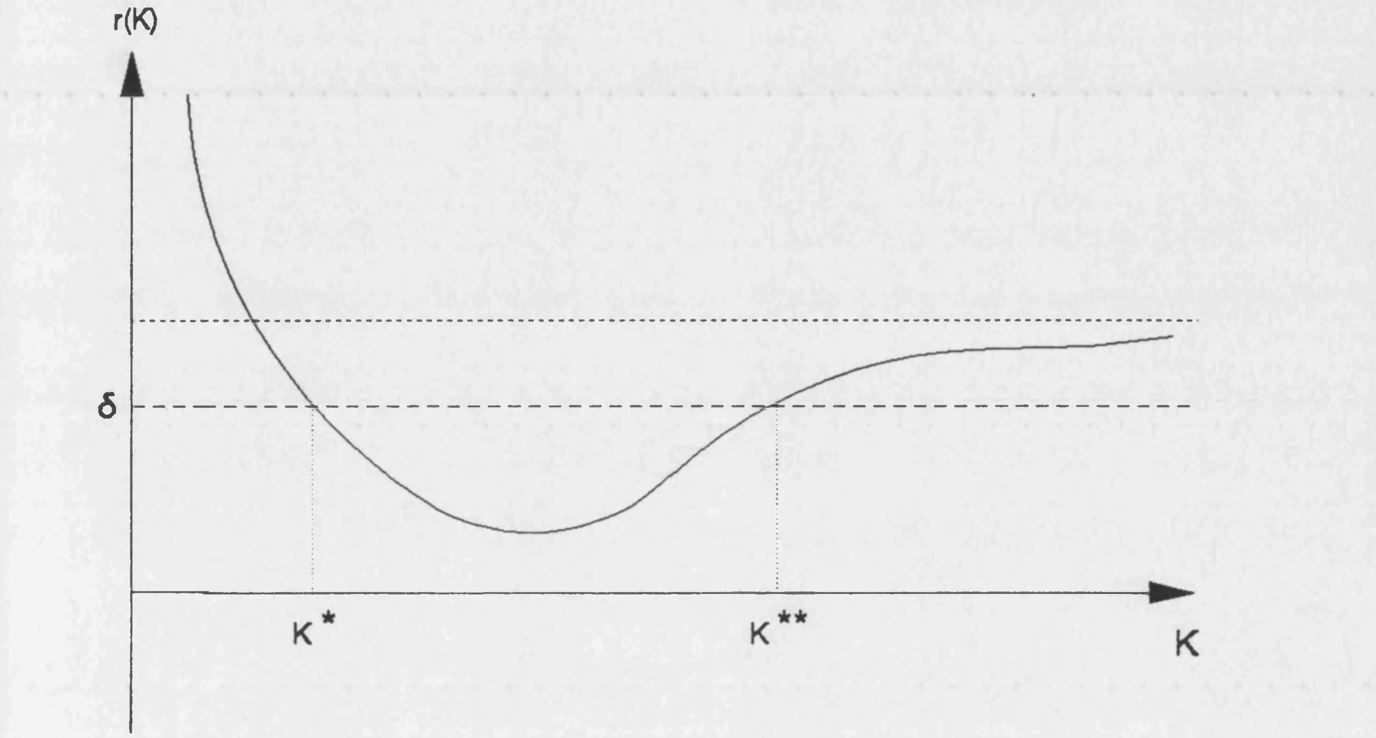


Figure 1

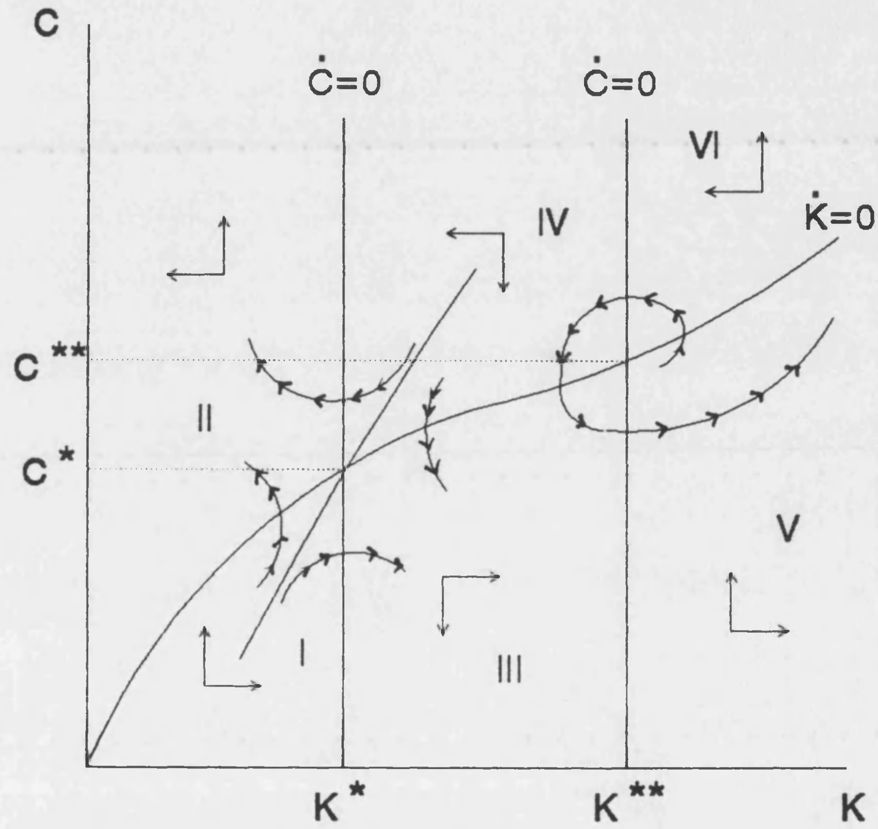


Figure 2

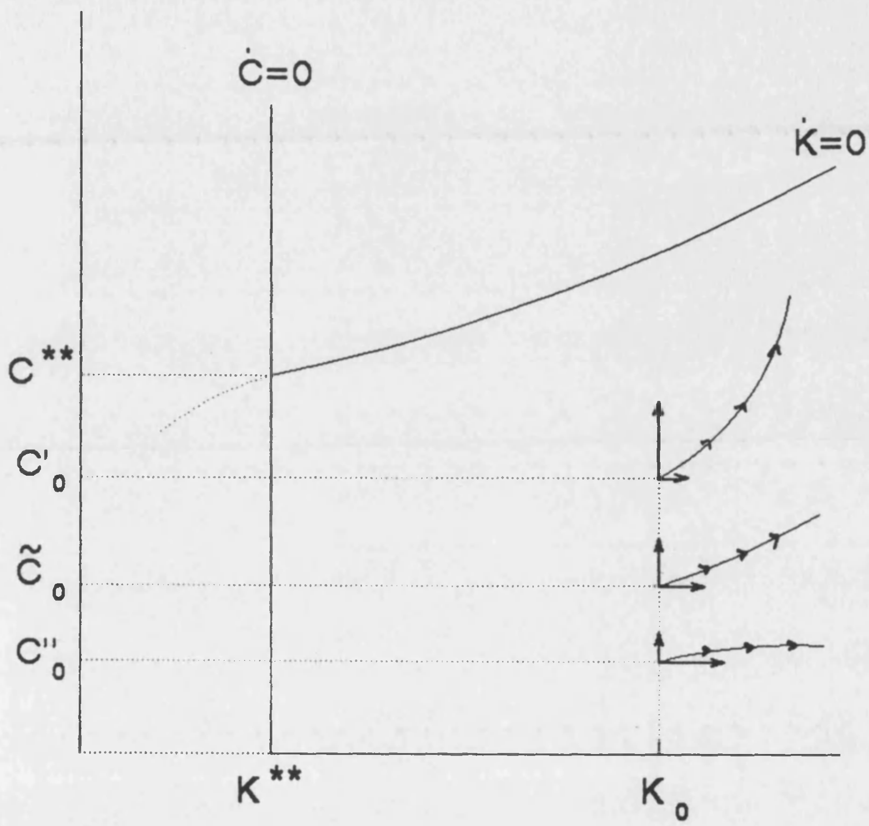


Figure 3

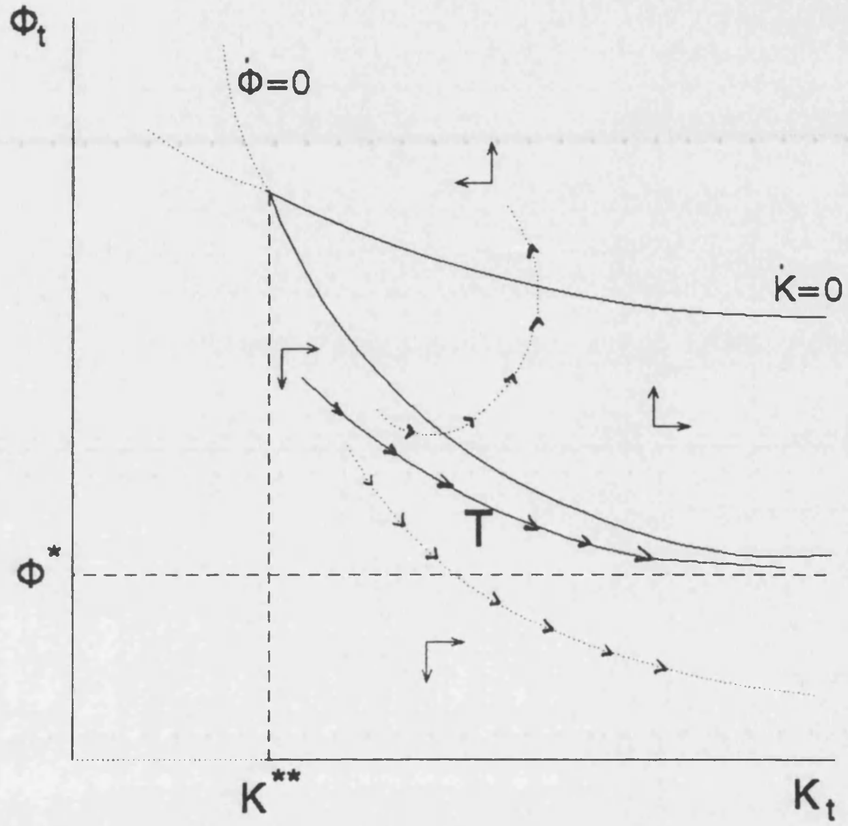


Figure 4

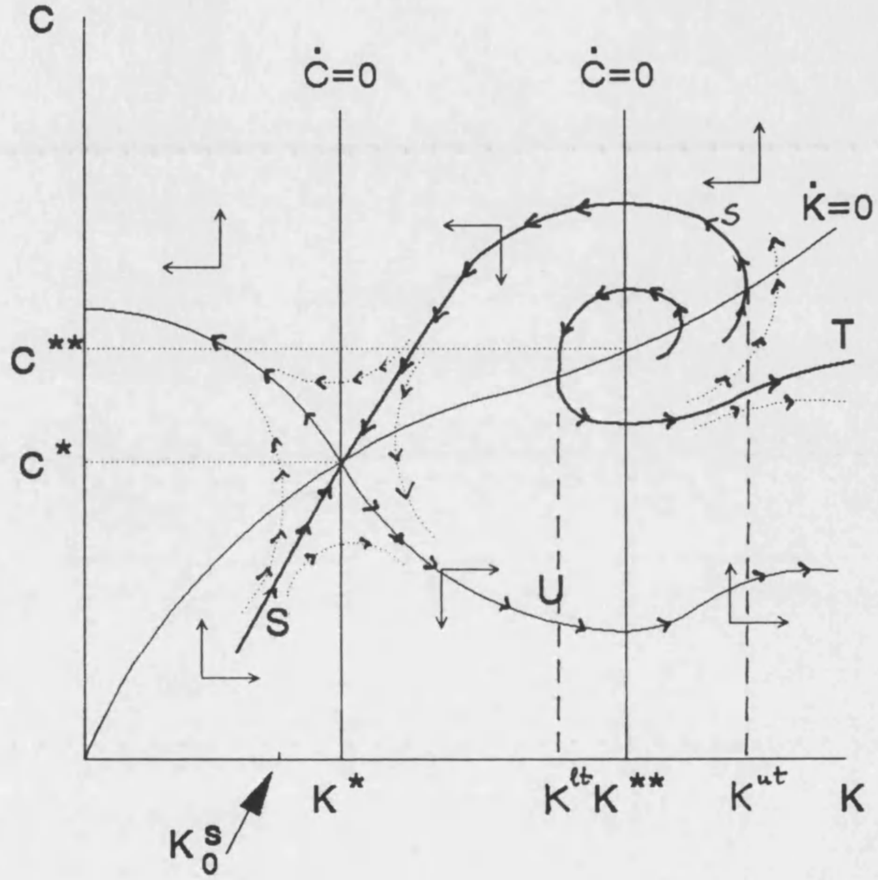


Figure 5a

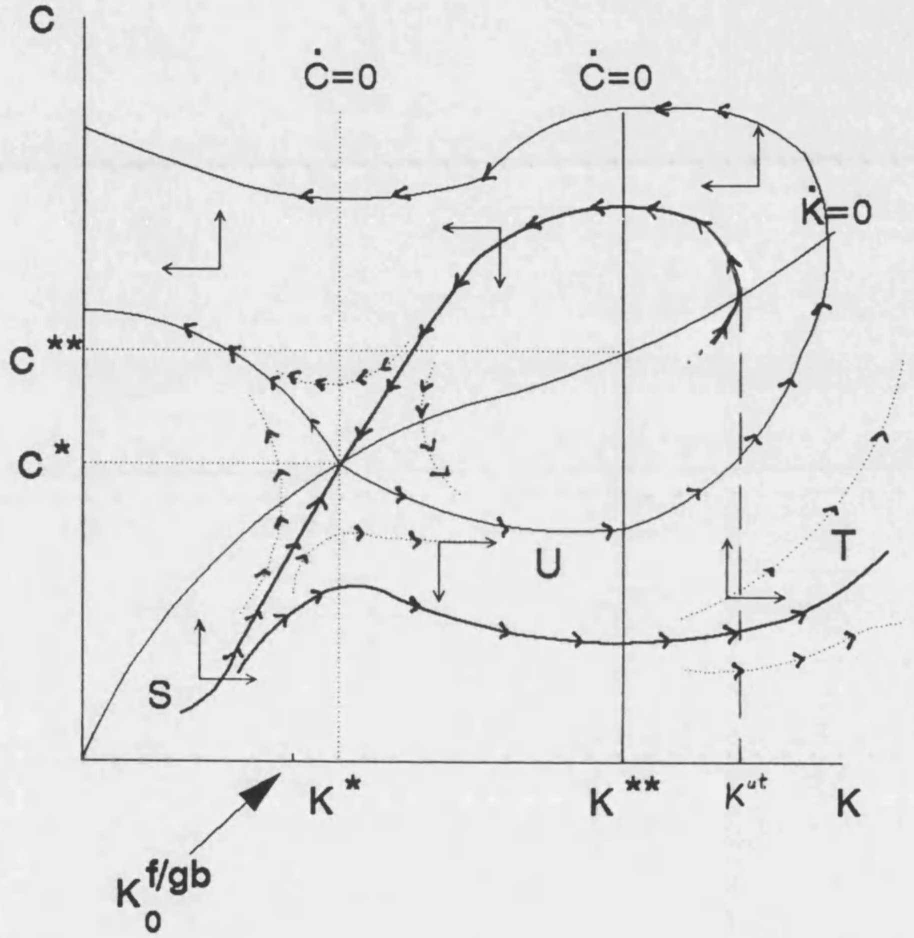


Figure 5b

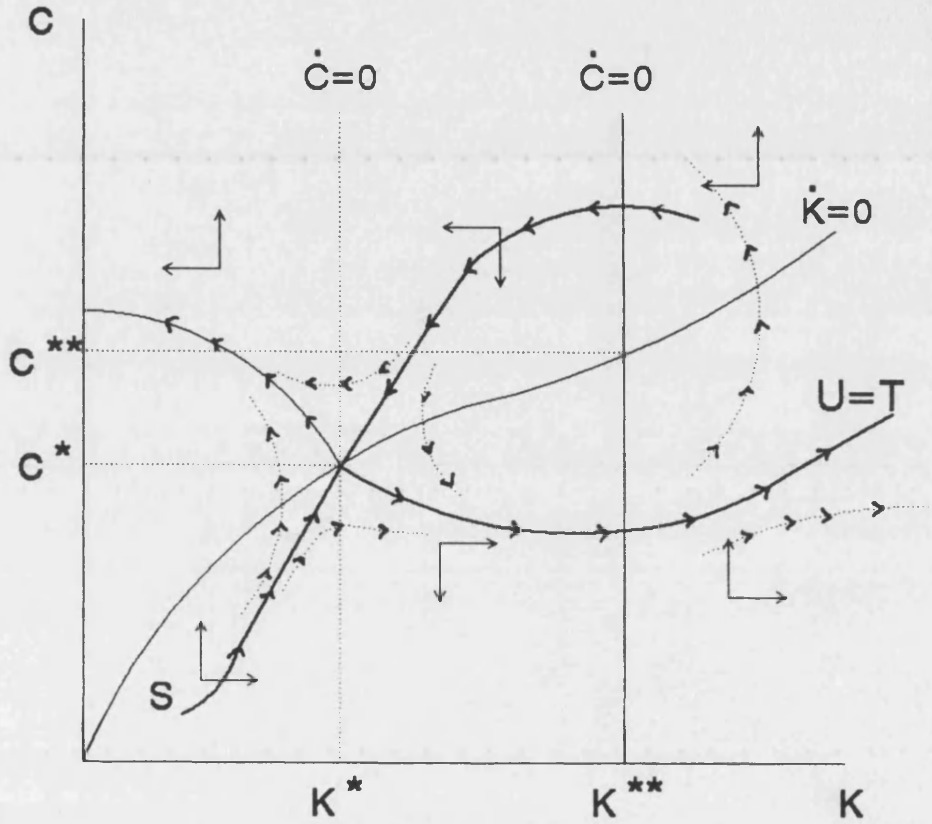


Figure 5c

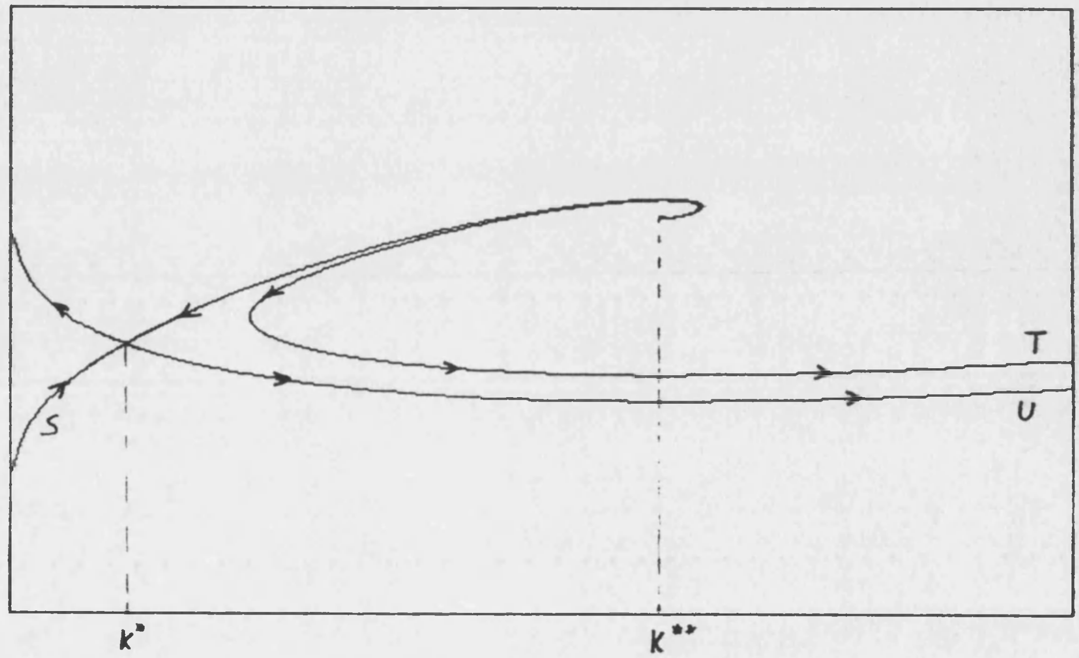


Figure 6a

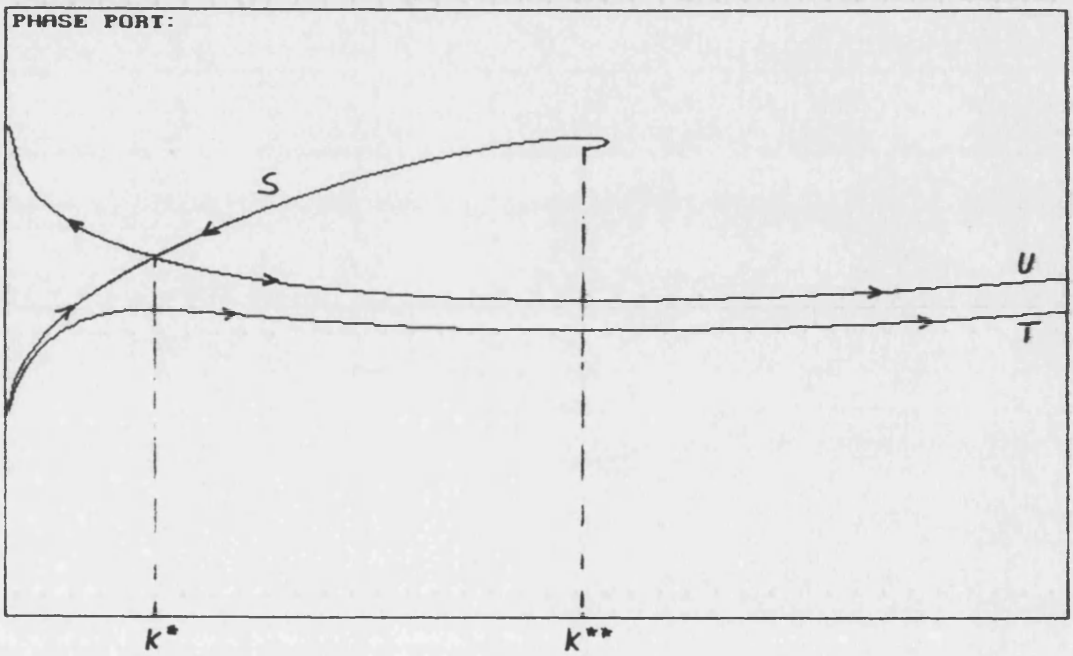


Figure 6b

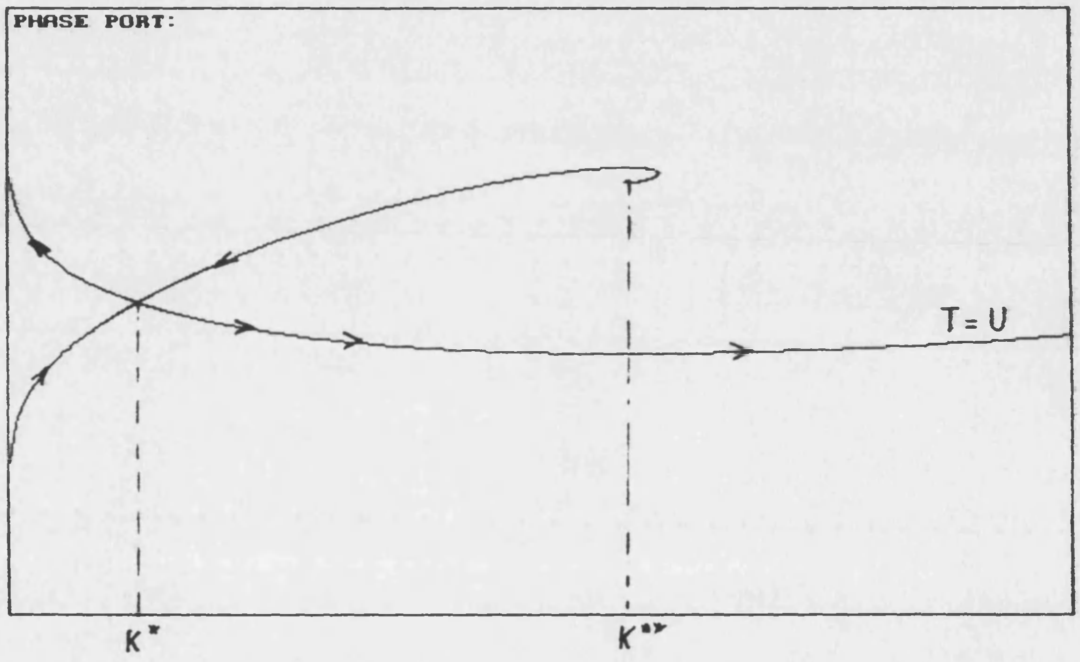
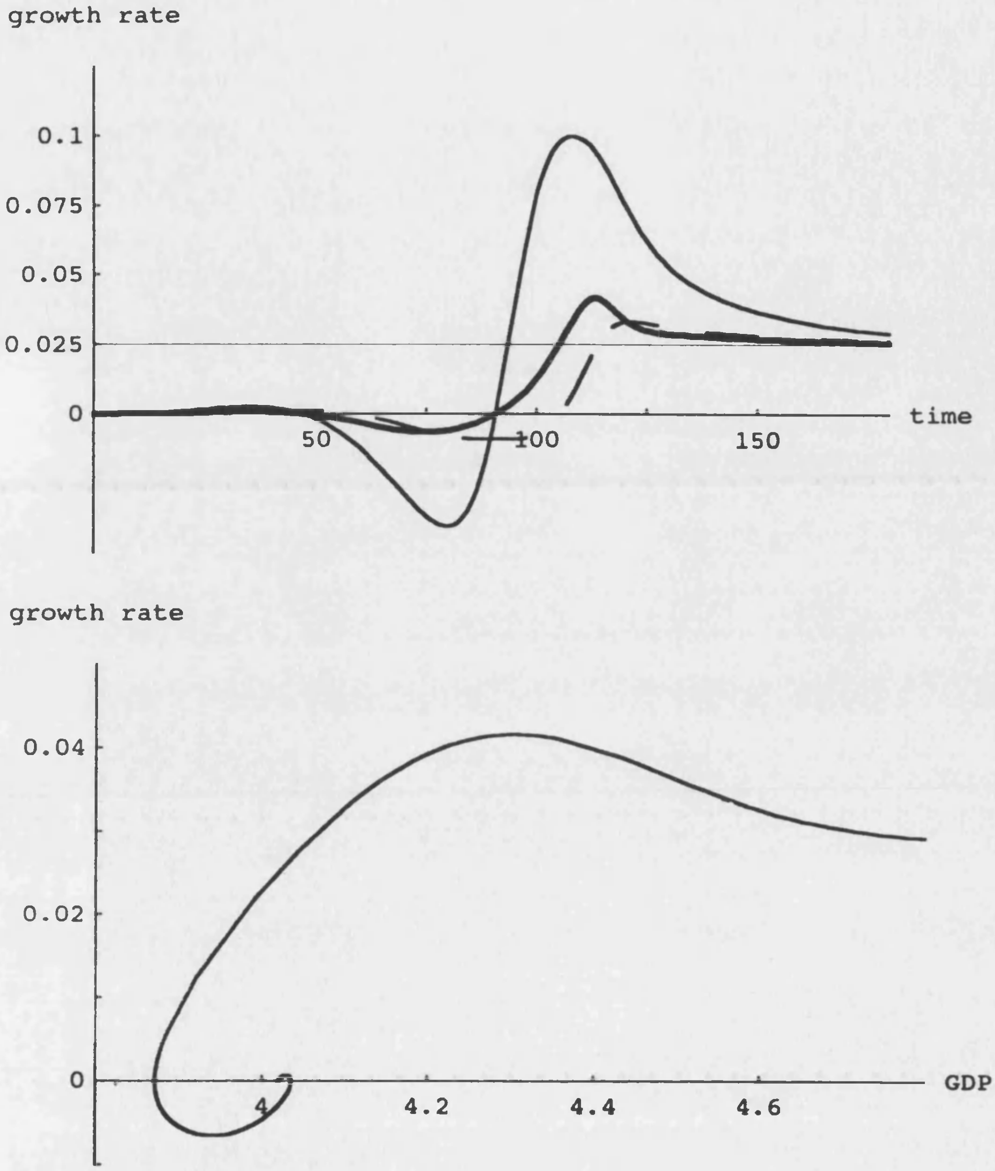


Figure 6c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time
growth rate
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4.2
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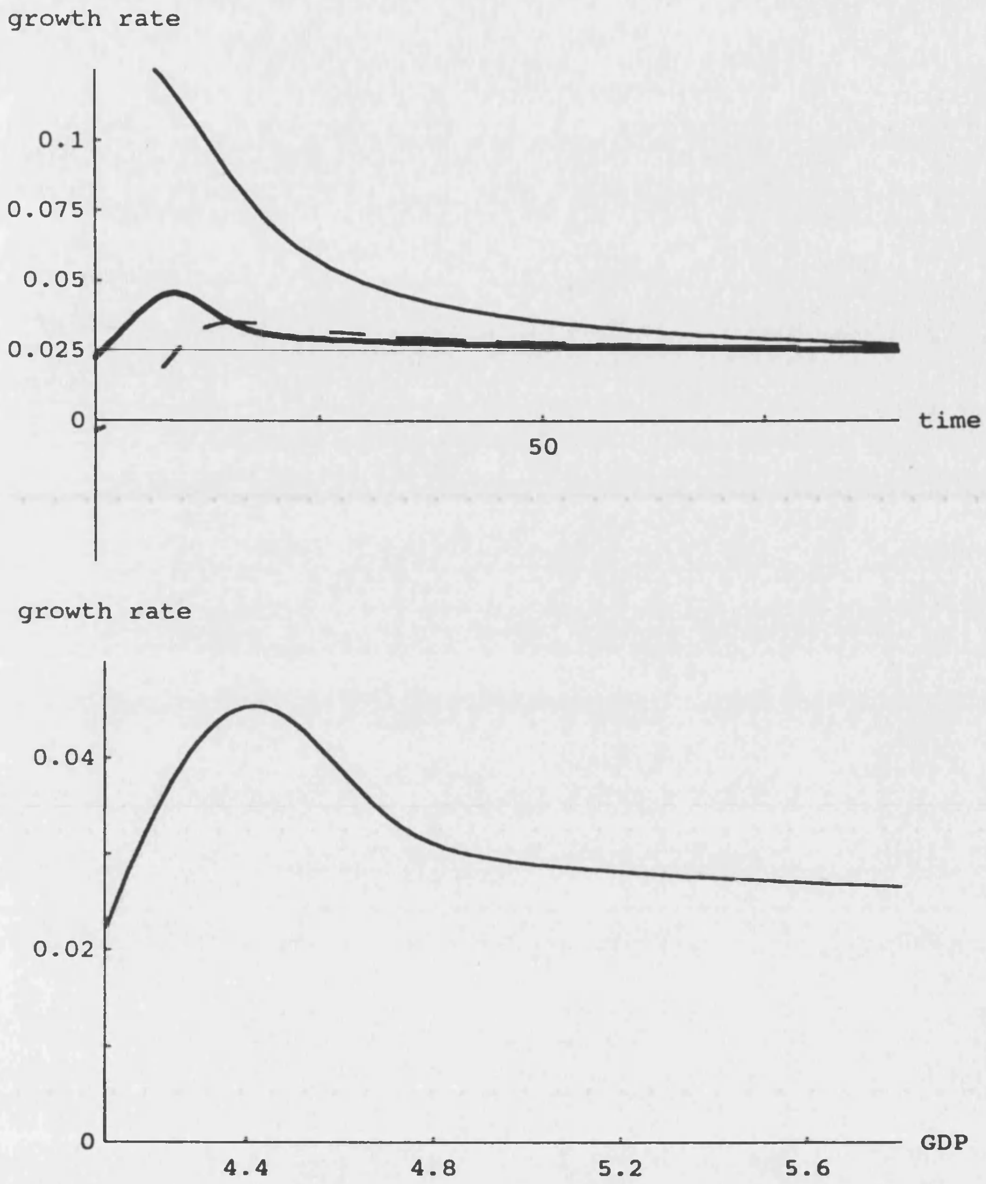
4.6
GDP

Figure 7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GDP
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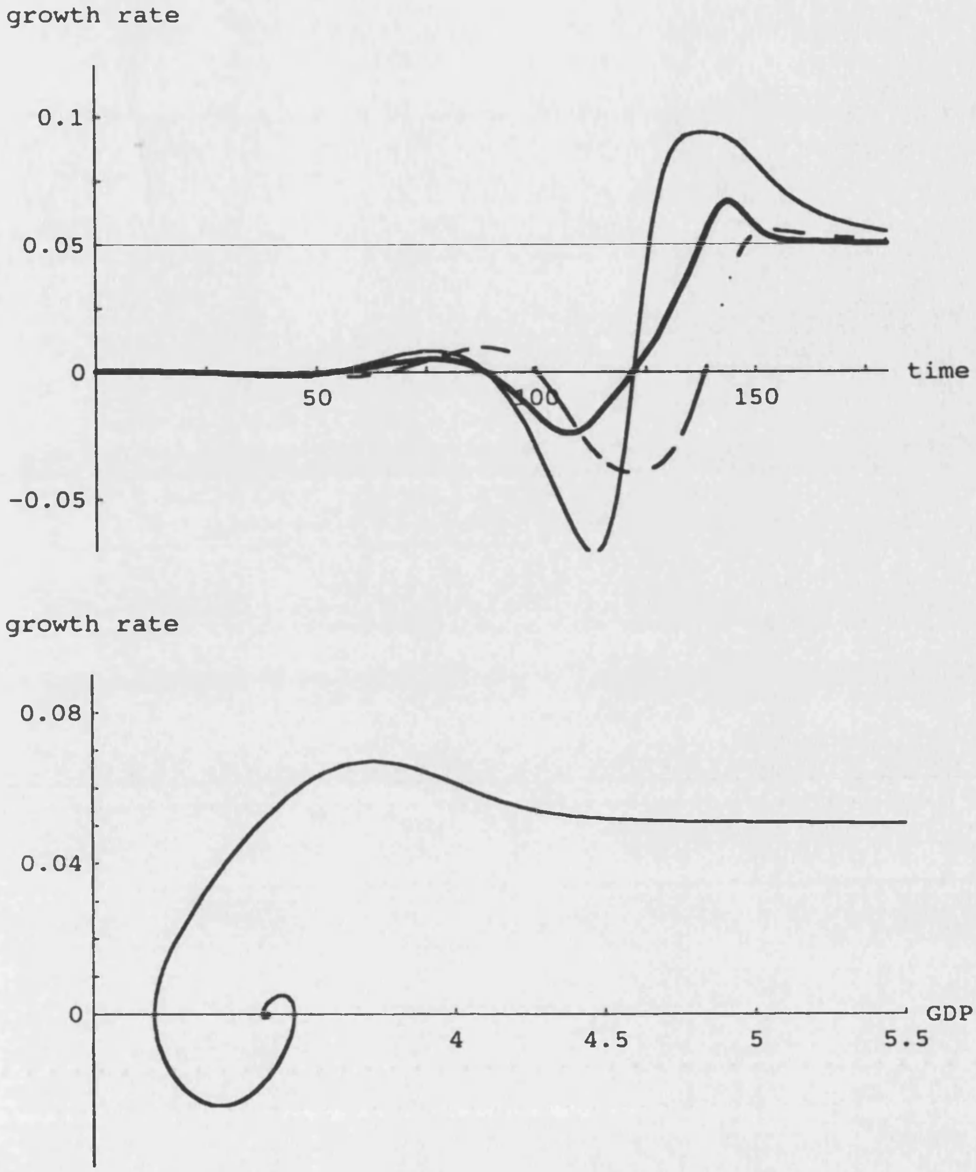
5.2
5.6

Figure 8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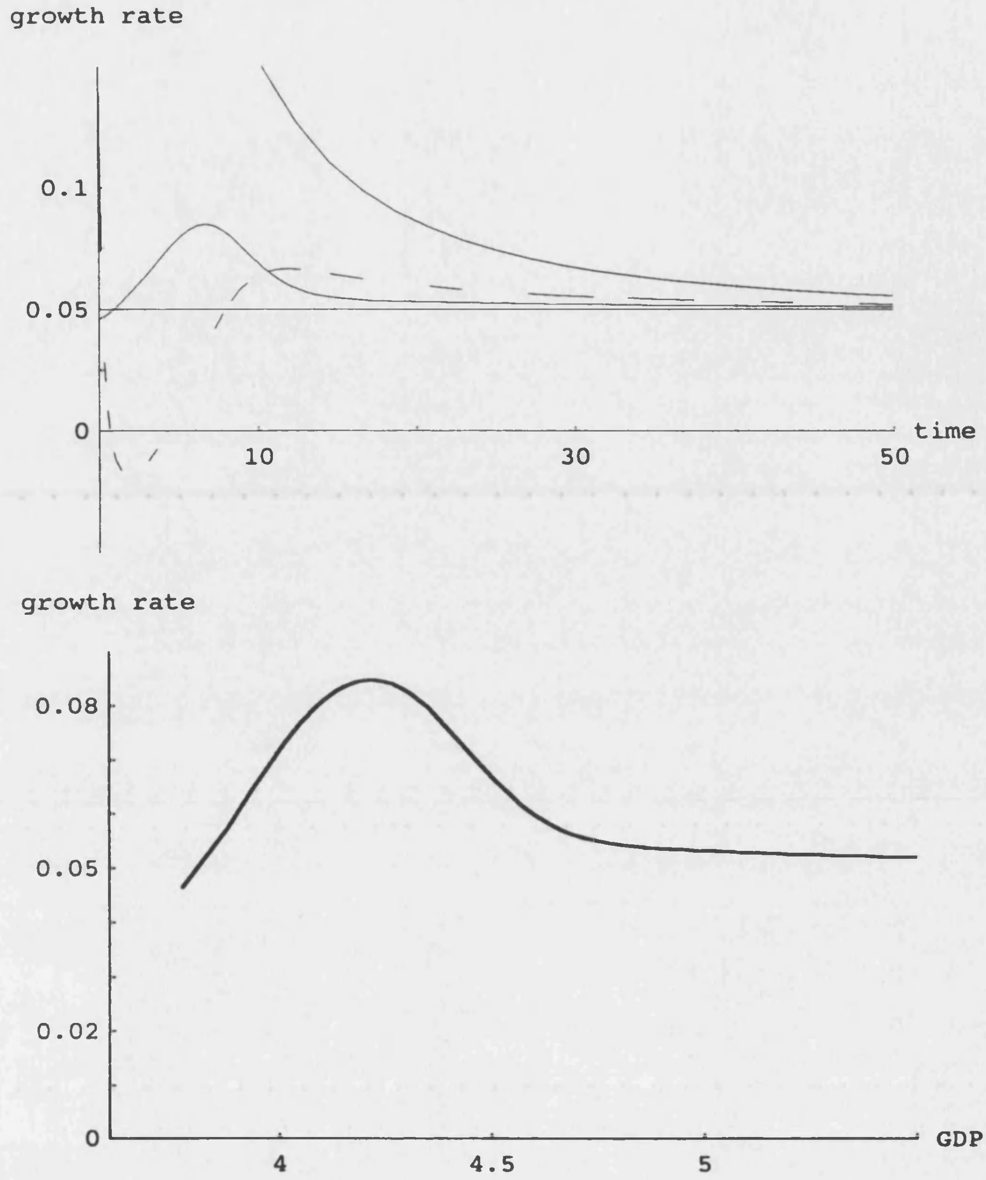
GDP

Figure 9



growth rate
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Figure 10

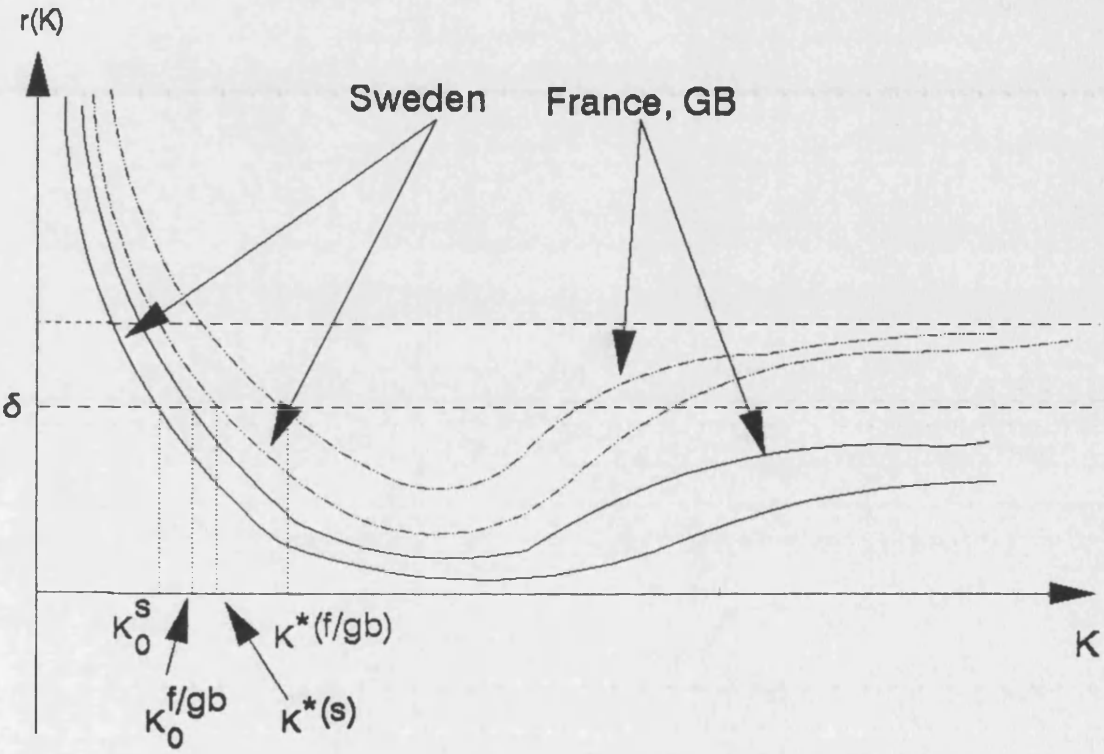


Figure 11

CHAPTER 2

Endogenous Growth and Intermediation in an 'Archipelago' Economy

This chapter investigates the relationship between growth and intermediation, emphasising the impact of market imperfections on the process of economic development. Traditional growth models assume that market activity is frictionless and costless. However, modern economies devote a large amount of resources to activities that are not directly productive. Diverse specialised institutions like financial intermediaries, estate agencies, job placement agencies, wholesalers, transport companies and so on take part in a process through which the economic system transfers resources (including physical and human capital) from the place where they are accumulated to the place where they are most productive. The degree of efficiency achieved by this 'intermediation sector' affects the productivity of investments and this, according to modern growth theory, gives rise to growth effects.

The low development of intermediation and trade activity may then be an explanation to the poor growth performances of a number of less developed countries. Recent models in this vein emphasise the role of fixed costs to opening new markets (particularly financial markets),and include Greenwood and Jovanovic (1990), Saint-Paul (1992), Greenwood and Smith (1993) and Blackburn and Hung (1993). This chapter examines a related, but somewhat different issue. 'Thick market' externalities originating in market imperfections in the trade sector cause the cost of intermediation to be higher in poor than in rich countries. This induces firms to choose techniques intensive in internal rather than in traded resources, and reduces the efficiency of accumulation. We show that economies that would attain self-sustained growth under the first-best allocation might get locked into a stationary equilibrium under a laissez-faire regime if the initial endowment of capital is below some threshold level. Multiple dynamic equilibria may exist, since the investment demands of final producers are 'strategically complementary' to each other in the presence of imperfect intermediation markets (Cooper and John, 1988; Murphy, Shleifer and Vishny, 1989; Benhabib and Farmer, 1991).

The idea of introducing market imperfections in endogenous growth models is not new. Some papers which studied the relation between growth and both horizontal (Romer, 1990; Helpman and Grossman, 1991) and vertical (Aghion and Howitt, 1992) innovation assumed an imperfectly competitive sector for the production of intermediate goods. However, these models typically introduce assumptions which guarantee constant markups in the non-competitive sector. Accordingly, growth is sustained by the increasing productivity warranted by the introduction of new differentiated (horizontal innovation) or higher quality (vertical innovation) inputs, but it is never affected by changes in the degree of market imperfections (i.e. the markup) which remains in fact constant. In our model, on the other hand, the price charged by the monopolistically competitive intermediation sector decreases with the size of the economy and tends asymptotically to the marginal cost. This implies that wedge between the reward to savings and the marginal productivity of capital which is opened by the existence of a costly intermediation activity is higher in the small than in the large economy, providing an offsetting effects to the fall of the marginal productivity of capital throughout the accumulation process due to the convexity of technology. A similar mechanism in which changes in the aggregate productivity are caused by endogenously falling markups has been independently shown in an exogenous growth framework by Gali (1993a).

1 The Archipelago economy

Consider an economy consisting of a number of islands. Each island is endowed with the same number of trees. Each tree yields a specific fruit and each island grows only one kind of tree. Each tree bears fruits in some amount which increases less than proportionally with the quantity of fertiliser applied to it in the previous period. The fruits that are not consumed can be used to produce fertiliser. Different types of fertilisers made with different fruits, however, are only imperfect substitutes. The inhabitants of each island own shares of the respective forest and their income consists of dividends on such shares plus a reward to foregone consumption depending on the interest rate. As consumers, they share identical tastes and endowments and are indifferent between consuming different types of fruit. Each island has the same number of inhabitants.

Fertiliser is non-storable. The fertiliser purchased by firms at time t is paid to the sellers after the completion of the following harvest, at time $t+1$. Now consider a representative island in the archipelago. When a local firm employs a unit of home-produced fertiliser, it pays directly to the savers a reward per unit of foregone consumption equal to the marginal contribution to production of the fertiliser (since firms are owned by consumers, we can imagine that part of the fruits are retained by the firm to make fertiliser). Consumers may alternatively give the fruits which they do not consume to outsider intermediaries which pay, in the following period, a yield which must equal, in equilibrium, the return paid by local firms. The business of a middleman consists of taking fruits from savers in each island and delivering them at time t to investing firms located in other islands, receiving from producers a revenue at time $t+1$, part of which is used to pay the original owners of the fertiliser. The cost of transferring fertilisers without intermediation from one island to another is assumed to be prohibitive for single producers, so firms wishing to employ external fertiliser have to pay the services of the middlemen. The existence of intermediation costs raises a wedge between the effective price paid by firms and the reward perceived by savers for fertiliser-capital. Householders' gross unit reward for savings will be denoted as r, whereas producers' effective cost will be a weighted average between r, the price of home fertiliser, and $(r+p)$, the price of foreign fertiliser, p being the unit fee charged by intermediaries.

Since different fruits are perfect substitutes in consumption, we can treat them as a unique final good, y, whose production requires a non-reproducible factor, z (trees), and a reproducible factor, k (fertiliser). This is a composite good, consisting of variable proportions of different types of fertilisers. We assume the following technology, identical across islands:

A1-Production function: $y = y(k, z);\ k = \Pi_{i=1}^{m} k_i^{\alpha};\ \alpha = \frac{1}{m},$

where $y(k, z)$ is linearly homogeneous and strictly concave. k_i are units of foregone consumption of each type of products, m being the number of islands.[1] Since in this section we do not deal with dynamic issues, time subscripts are ignored.

Firms behave competitively in both factor and product markets. We solve the profit maximisation problem in two stages, by first minimising the cost of fertilisers for given k and then choosing the optimal $\frac{k}{z}$. Define k_h as the units of home fertiliser and k_f as the units of a composite good consisting of equal proportions of each fertiliser produced in external islands. By symmetry the demand for fertilisers produced in any island, other than that where the firm is located, is simply $\frac{k_f}{m-1}$. Also define q as the relative price of foreign fertilisers to home fertilisers, $q \equiv \frac{r+p}{r}$. The problem is then:

$$\textbf{P1:} \quad \min_{k_h, k_f} r(k_h + qk_f), \quad \text{s.t. } k = k_h^{\alpha} k_f^{1-\alpha},$$

whose solution is:

$$k_h = \alpha\Phi q^{1-\alpha} k, \quad k_f = (1-\alpha)\Phi q^{-\alpha} k, \quad \Phi \equiv \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} (1-\alpha)^{-1}. \tag{1}$$

Given the symmetric Cobb-Douglas nature of the aggregator function for k, each firm spends a constant uniform amount on each type of fertiliser. The

[1]The technology $A1$ is somewhat restrictive, especially by implying that every input is an essential factor. Our results are however independent of this specification, which we maintain throughout this section for analytical convenience. Section II discusses a case in which there exists an alternative technology which uses only internal capital, and intermediation is not strictly necessary to production.

elasticity of the demand for external capital with respect to the relative price q is constant and equal to α. We can also derive the unit 'effective cost' of composite capital, p_k, defined as the minimised capital costs (i.e. the solution to P1) per unit of k:

$$p_k = r\left[\frac{\alpha\Phi q^{1-\alpha}k + q(1-\alpha)\Phi q^{-\alpha}k}{k}\right] = \Phi r q^{1-\alpha}. \tag{2}$$

Equation (2) shows that the effective price of capital responds with a constant elasticity $(1-\alpha)$ to changes in the intermediation costs. In equilibrium, firms invest until the point at which the marginal productivity of capital, $y_1(k,z)$, equals p_k, taking r and q as parametric.

Specialised companies provide inter-island shipment services. To avoid complications concerning the location of companies, we assume that each intermediary necessarily makes a complete tour of the archipelago at each period, regardless of where its headquarters are situated. The provision of services is subject to the payment of a fixed cost F plus a cost which is proportional to the amount of fertiliser traded. For reasons of notational convenience, we define the marginal cost as $(A-1)$. All costs are in terms of units of capital (fertiliser of any type is used as 'fuel'). The total cost function that each intermediary faces is then:

A2-Cost function of intermediaries $\quad TC(x) = r[F + (A-1)x]$,

where x represents the number of units of output (services) supplied by a representative intermediary firm. We assume that intermediaries act in a Cournot-Nash fashion in an imperfectly competitive market, taking their competitors' outputs $(\overline{x})$ as given.[2] The total demand for services in the archipelago is

[2]We assume, for simplicity, that individual shipment companies neglect the effects of their decision on the average price p_k and, through this, on the aggregate demand of K. Cournot competition is not essential for our argument. The main results, for instance, carry over to a regime of Bertrand competition cum fixed costs (monopoly). What is essential is that prices are linear (e.g., we rule out two-part tariffs). Middlemen are price-takers in the collection of resources (they take r as parametric), since savings have an alternative use in local firms.

given by $X = K_f = mk_f$ (capital letters denote aggregate variables at the archipelago level), whose price-elasticity with respect to q is, as we saw, α. Each shipping company, then, faces a constant individual demand elasticity equal to $n\alpha$, where n is the number of companies that are active, and maximises the following profit function:

$$\textbf{P2:}\quad \max_{\{x\}} p(X)x - r(A-1)x - rf = r[q(X)x - ax - F],$$

where $X = x + (n-1)\overline{x}$ and $p(X)$, $q(X)$ denote inverse demand functions. The condition for profit maximisation for each shipment company is given, as usual, by equalising the marginal revenue to marginal cost, or by the following mark-up rule:

$$\left(1 - \frac{1}{\alpha n}\right) q = A. \tag{3}$$

We assume free entry in the intermediation market. This implies that every shipment company breaks even:

$$(q - A)x = F. \tag{4}$$

The last condition for the industry equilibrium is that the market clears:

$$nx = K_f = [(1-\alpha)\Phi]\, Kq^{-\alpha}. \tag{5}$$

Equations (3), (4) and (5) provide a solution for the endogenous variables n, x and q, given K and the technological parameters. We focus here on the solution for q, leaving to the reader the derivation of the results for the other variables (in particular, it turns out that both x and n grow unboundedly as the market size grows, and n tends to $1/\alpha$ for arbitrarily small K). We obtain:

$$\Gamma(q) \equiv \frac{q^{1+\alpha}}{(q-A)^2} = \frac{\alpha(1-\alpha)\Phi}{F} K\ ,\quad q \in\,]A, \infty[\ ,\ \Gamma'(q) < 0,\ \Gamma''(q) > 0. \tag{6}$$

The following proposition summarises the key results.

Proposition 1 *Consider the solution* $q = q(K)$ *implicitly defined by (6). Then: (i)* $q'(K) < 0,\ q''(K) < 0$; *(ii)* $\lim_{K\to\infty} q(K) = A,\ \lim_{K\to 0} q(K) = \infty$; *(iii)* $q(K) \sim J\ K^{1/(\alpha-1)},\ K \to 0$, *for some constant* J.[3]

[3] We say that $f(x) \sim$ (is equivalent to) $g(x)$, $x \to \overline{x}$ if $\lim_{x\to\overline{x}}\{f(x)/g(x)\} = 1$.

Proof. Parts *(i)* and *(ii)* follow immediately from (6) by the application of the implicit function theorem. Part *(iii)* follows from observing that $q^{\alpha-1} \sim \Gamma(q) = \frac{\alpha(1-\alpha)\Phi}{F}K, q \to \infty$ and from *(ii)*. □

Remember that $q(K)$ is the relative price of external to internal capital faced by firms. Then , point *(i)* of the proposition implies that as K grows final producers find it relatively cheaper to buy other islands' fertiliser and have an incentive to improve productive efficiency by adopting a technology more intensive in external capital. Point *(ii)* shows that the degree of market imperfection vanishes in the 'large' economy. Notice that $q = A \Leftrightarrow p = r(A-1)$. On the other hand, it establishes that in the 'small' economy the intermediation costs become arbitrarily large. Point *(iii)*, finally, shows that $q(K)$ behaves like the function $K^{1/(\alpha-1)}$ when K is small. The relevance of this technical result will become apparent in the next section.

We assume that the final sector of each island consists of a continuum of identical atomistic firms on the real interval [0,1]. Firms' atomistic behaviour implies that they neglect the effects of their decisions on aggregate variables, particularly on the total demand for intermediation services and, through this, on the unit cost of services paid by each of them. So, the equilibrium investment for the individual firm is given by the choice of k which satisfies the condition: $p_k(r, q(K)) = y_1(k, z)$, where K is taken as parametric. Given the symmetric nature of the archipelago and the constant returns to scale of the technology, we can use this equilibrium condition together with (2) to obtain an expression for r as a function of aggregate variables only,

$$r(K) = \frac{y_1(K, Z)}{q(K)^{1-\alpha}}\Phi, \tag{7}$$

which is identical to (2) apart from that here q is the function $q = q(K)$ characterised by Proposition 1. K, until here treated as exogenous, is in fact the state variable of the dynamic model. Condition (7) shows that the reward to savings is in general a non-monotonic function of K, unlike in convex one-sector growth models in which it typically decreases with the total accumulation. Since the economy is subject to higher costs to the intermediation process at a lower than at a higher stage of development, this provides a countervailing effect to the decreasing returns to investments induced by the final sector technology. If intermediation costs are relatively large, we can expect that the productivity of investments, ceteris paribus, be higher in large than in small economies. This will be a central issue in the next section.

For future reference, we also write the expression for the absolute price per unit of shipment, p, as an implicit function of K. Since, by definition, $p(K) = r(K)(q(K) - 1)$, then

$$p(K) = y_1(K, Z)\left[\frac{q(K) - 1}{q(K)}q(K)^{\alpha}\right]\Phi, \;\; p'(K) < 0, \tag{8}$$

where we note that the term in square brackets is increasing with q and decreasing with K.

The laissez-faire economy suffers from two sources of inefficiency. The first is the existence of a non-competitive sector. The second is a demand or 'thick market' externality. Each firm, when investing, adds to the total demand of intermediation services and causes a fall in the wedge between the price of internal and external capital. In a second-best world, firms internalise such external effects and coordinate their demands. This means that they invest until the point where $y_1(k, z) = p_k(K) + p_k'(K)K$, whose second term on the right hand side is negative. Then, (7) is replaced by

$$r^{sb}(K) = \frac{y_1(K, Z)\Phi}{q(K)^{1-\alpha} + (1-\alpha)q(K)^{-\alpha}q'(K)K} > r(k, K). \tag{9}$$

For any K, the reward to saving is higher in a world of second best than in a laissez-faire economy with atomistic behaviour. In a world of first-best, finally, trade services are offered at their marginal cost and the interest rate is

$$r^{fb}(K) = \frac{y_1(K, Z)\Phi}{A^{1-\alpha}} > r^{sb}(K) > r(K), \tag{10}$$

which is monotonically decreasing with capital, as in standard neoclassical growth models.

2 Dynamic general equilibrium

We focus on economies whose growth is self-sustained in the absence of intermediation costs. This is obtained by assuming, following Jones and Manuelli (1990), that the marginal productivity of capital, uniformly decreasing with K_t, is bounded away from a sufficiently large positive lower bound. For simplicity, we limit attention to a well-known class of production functions which exhibits this feature, namely CES functions with elasticity of substitution greater than one. We assume that:

$$\textbf{A3-Jones-Manuelli} \qquad y_t = y(k_{t-1}, z) = [(Bk_{t-1})^\rho + z^\rho]^{\frac{1}{\rho}},$$

where: $0 < \rho < 1$, $y_1(k_{t-1}, z) = [(Bk_{t-1})^\rho + z^\rho]^{\frac{1-\rho}{\rho}} B^\rho k_{t-1}^{\rho-1}$,

$\lim_{k_{t-1}\to\infty} y_1(k_{t-1}, z) = B$, $\lim_{k_{t-1}\to 0} y_1(k_{t-1}, z) = \infty$.

A representative householder maximises a standard utility function, taking the paths of dividends on shares and rates of reward on savings as given. Formally:

$$\textbf{P3} \qquad \max_{\{c_t, k_t;\ t=1,\dots,\infty\}} U(c_1, c_2, \dots) = \sum_{t=1}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \qquad s.t.$$

$$c_t + k_t \leq r_t k_{t-1} + d_t z, \tag{11}$$

with $k_t > 0$, k_0 given. Here, k_{t-1} denotes the fertiliser accumulated at $t-1$ and used in the production of the fruits which are gathered and distributed at t; d_t is the dividend per-tree; r_t is the interest perceived at time t per unit of savings made at time $t-1$; β $(0 < \beta < 1)$ is the time discount rate. The solution to this standard programme is given by:

$$c_t = (\beta r_t)^{\frac{1}{\sigma}} c_{t-1}, \tag{12}$$

together with (11) - with equality - and the transversality condition. The laissez-faire general equilibrium solution is found by substituting the equilibrium values of r_t and d_t into (11) and (12), and using the symmetric equilibrium conditions $K_t = m\ k_t$, $C_t = m\ c_t$ and $Z = m\ z$. The equilibrium value of r_t is given by (7). Instead of computing directly the expression for d_t, we use the fact that the net product is entirely distributed to consumers, so the aggregate budget constraint implies that $r_t K_{t-1} + d_t Z = y(K_{t-1}, Z) - p(K_{t-1})K_f(K_{t-1})$, where $p(K_{t-1})K_f(K_{t-1})$ is the total revenue of the intermediation sector, K_f being the total amount of 'external' fertiliser used in the archipelago. Since intermediaries break even, this revenue is equal to the total cost of the intermediation process, or the amount of output socially dissipated in moving fertiliser between islands. It is convenient to use (8) to reformulate this cost as follows:

$$p(K_{t-1})K_f(K_{t-1}) = \left(y_1(K_{t-1}, Z)\frac{q(K_{t-1}) - 1}{q(K_{t-1})}\right) K_{t-1} \equiv w(K_{t-1})K_{t-1} \quad (13)$$

where $w'(K_{t-1}) < 0$. By substituting (13) into (11) and rearranging, we express the equilibrium conditions in terms of an autonomous system of first-order difference equations plus the initial and the transversality conditions:

$$C_t = \left(\beta\, \frac{y_1(K_{t-1}, Z)\Phi}{q(K_{t-1})^{1-\alpha}}\right)^{\frac{1}{\sigma}} C_{t-1}, \quad (14)$$

$$K_t = y(K_{t-1}, Z) - w(K_{t-1})K_{t-1} - \left(\beta\, \frac{y_1(K_{t-1}, Z)\Phi}{q(K_{t-1})^{1-\alpha}}\right)^{\frac{1}{\sigma}} C_{t-1}, \quad (15)$$

$$K_0 = \overline{K}_o, \quad \lim_{t\to\infty} \beta^t C_t^{-\sigma} K_t = 0, \quad C_t > 0, \quad K_t > 0.$$

Consider the asymptotic behaviour of the system, in correspondence of arbitrarily large values of the state variable K. By Assumption $(A3)$, Proposition 1 and (13) we have:

$$\lim_{K_{t-1}\to\infty} \frac{C_t}{C_{t-1}} = \left(\beta\, \frac{B\Phi}{A^{1-\alpha}}\right)^{\frac{1}{\sigma}}, \quad (16)$$

$$\lim_{K_{t-1}\to\infty} \frac{K_t}{K_{t-1}} = B - B\frac{(A-1)}{A} - \left(\beta\, \frac{B\Phi}{A^{1-\alpha}}\right)^{\frac{1}{\sigma}} \lim_{K_{t-1}\to\infty} \frac{C_{t-1}}{K_{t-1}}. \quad (17)$$

It has been proved in the first chapter that in (possibly non-convex) models with bounded asymptotic productivity there exists *at most one* equilibrium path with self-sustained growth. This path converges to a balanced growth condition according to which consumption, income and capital grow at a common rate. Here, we only provide the conditions for the existence of this path, referring to the first chapter for the argument of the proof of uniqueness. Let g be defined as the asymptotic growth rate.

Proposition 2 *A self-sustained growth path converging to a balanced growth path is characterised by the following conditions:*

(i) $g = \left(\frac{\beta B \Phi}{A^{1-\alpha}}\right)^{1/\sigma} - 1$ *(ii)* $\lim_{t\to\infty} \frac{C_t}{K_t} = \frac{B-A(1+g)}{A(1+g)}$.

Such path exists if and only if:

$$(a)\ 1 < \left(\frac{\beta B \Phi}{A^{1-\alpha}}\right)^{1/\sigma} < \left(\frac{B}{A}\right);\ (b)\ \frac{1}{\beta} > \left(\frac{B\Phi}{A^{1-\alpha}}\right)^{1-\sigma}. \tag{18}$$

Proof (sketch). *(i)* and *(ii)* are found as the solution to (14), (15), after imposing the asymptotic balanced growth condition $\lim_{t\to\infty}\left(\frac{C_t}{C_{t-1}}\right) = \lim_{t\to\infty}\left(\frac{K_t}{K_{t-1}}\right) = 1+g$. The condition (18)-a ensures that $g > 0$ and $\lim_{t\to\infty}\left(\frac{C_t}{K_t}\right) > 0$ along the asymptotic equilibrium path. The condition (18)-b ensures that the transversality condition $\lim_{t\to\infty} \beta^t C_t^{-\sigma} K_t = 0$ be satisfied. □

We will limit attention to economies which satisfy (18). Paths with perpetual growth are not the only candidate laissez-faire equilibria. If there exist saddlepaths converging to stationary points of the system (14)-(15), they necessarily satisfy the transversality condition and are equilibria. An important feature of the model that ensures the existence of 'traps' of this kind is that, due to the intermediation costs, the accumulation is more productive in 'very large' than in 'very small' economies, and $r(K)$ is increasing with K in some interval. We formally state and prove this property.

Proposition 3 *Consider* $r_{t+1} = r(K_t)$, $K_t \in R^+$, *as implicitly defined by (7), (6) and A3. Then: (i)* $\lim_{K_t\to\infty} r(K_t) = 0$; *(ii)* $\exists I \subseteq R^+$ *such that* $r'(K_t) > 0$ *for* $K_t \in I$; *(iii)* $r(K_t) = 1/\beta$ *for some* $K_t \in I$.

Proof. We know that $\lim_{K_t \to 0} r(K_t) = \lim_{K_t \to 0} \frac{y_1(K_t,Z)}{q(K_t)^{1-\alpha}} \Phi$. To prove *(i)* amounts to show that $y_1(K_t, Z)$ is an infinity of lower order than $q(K_t)^{1-\alpha}$ as $K_t \to 0$. From *A3*: $\lim_{K_t \to 0} y_1(K_t, Z) = Z^{1-\rho} K_t^{\rho-1} = \infty$. From Proposition 1, point *(iii)*: $\lim_{K_t \to 0} q(K_t)^{1-\alpha} = J^{1-\alpha} \lim_{K_t \to 0} K^{-1} = \infty$. Then, it is evident that the denominator is an infinity of higher order than the numerator. As to *(ii)* and *(iii)*, notice that (18) and $A3$ imply that $\lim_{K_t \to \infty} r(K_t) = \frac{B\Phi}{A^{1-\alpha}} > \frac{1}{\beta}$. Then, *(ii)* follows from *(i)* and the continuity of $r(K_t)$. *(iii)* follows from *(i)* and *(ii)*, by the application of the intermediate value theorem. □

Proposition 3 and equation (14) ensure that when the initial capital stock is below some threshold level, consumption declines through time. A phase diagram shows informally the existence of a 'bad' equilibrium path. Fig. 1 represents a case in which $r(K_t)$ is monotonically increasing and takes on the value $r(K_t) = 1/\beta$ for one value of K_t only.

FIG. 1 HERE

The locus $C_t = C_{t-1}$ corresponds to the condition $r(K_{t-1}) = 1/\beta$ - cfr. (14) - and necessarily exists by Proposition 3. When $K_{t-1} < K^*$, $r_t < 1/\beta$, since low productivity techniques are adopted, and consumption declines over time. The locus $K_t = K_{t-1}$ is derived from (15). The fixed point S is completely unstable (linearisation of (14) and (15) in the neighbourhood of S shows that both roots are larger than one in absolute value, when $r'(K^*) > 0$) and is not an interesting equilibrium. On the other hand, there exists a saddle-path (T) converging to the origin which is also a dynamic laissez-faire equilibrium for a non-trivial set of states of the system. The equilibrium path with sustained growth is indicated by T'. The uniqueness of the growing equilibrium is ensured by the fact (see ch. 1) that all paths which remain below T' are characterised by perpetual growth, but violate the transversality condition (capital grows too fast, and the ratio $\frac{C}{K}$ declines towards zero), whereas all paths which lie above it are destined to revert to negative growth. In the case represented in Fig.1, the path T' has no points in the region of the space with $K < K_L$. So K_L may be interpreted as a development threshold. If a laissez-faire economy has an initial capital endowment inferior to the threshold level, it is condemned to stagnation and underdevelopment. The equilibrium interest rate, because of the high intermediation costs, is too low to warrant sustained accumulation; people consume more now than in the future and the economy converges to a zero-income equilibrium.

It seems desirable to abandon the straightjacket assumption that the fertilisers of all islands are 'essential' factors in the production of each fruit ($A1$). Let us consider a case in which there exists an additional technology, less productive, whose adoption makes trees fertile even if no external fertiliser at all is used. This is formalised by replacing $A1$ by the following technology:

A1 bis: $y_t = y(k_{t-1}, z), \quad k = \max\{\theta k_h, \Pi_i k_i^{\alpha}\}, \quad \alpha = \frac{1}{m}.$

Clearly, the 'autarkic' technology will be adopted when shipment costs are prohibitive. $A3$ ensures now that consumption grows in an arbitrarily small economy when the marginal productivity of capital tends to infinity.[4] If θ is large enough, no threshold will be observed and we have the standard solution of an endogenous growth model. We consider instead the case in which θ is relatively small and the productivity of capital falls below the social discount rate for an interior set of states of the system, before that the technology with intermediation becomes productive enough to warrant self-sustained growth. Fig. 2 shows the behaviour of the function $r(K)$. Notice that the no-intermediation technology is more productive at low levels of K and there is an interval in which neither of the technologies is productive enough to sustain a positive growth rate of consumption ($r(K) < 1/\beta$).

FIG. 2 HERE

Fig. 3 represents the corresponding vector field. Observe that as far as $K_t < K^*$ consumption grows, due to the high productivity of capital warranted by the autarkic technology at low levels of accumulation. However, as the size of the economy grows, the productivity of capital declines to a level which is

[4]We ignore the formal issues introduced by the technological discontinuity, since they do not affect the qualitative analysis.

not sufficient to sustain consumption growth. At some critical level between K^* and K^{**}, however, the size of the market becomes large enough to justify the development of the market for intermediation. For values of K_t which are larger than such threshold, we observe that $r'(K_t) > 0$ (see Fig.2), namely the economy switches from a regime of decreasing to one of increasing returns. Eventually, as $K_t \geq K^{**}$ the productivity of capital returns to a high enough level to sustain consumption growth.

As before, there are two equilibrium paths: T and T', but there exist now two interior steady-states (a saddle-point and a fully unstable node, respectively). The saddlepath T is a 'trap' solution which converges to a stationary interior long-run equilibrium. Notice its resemblance to the solution of traditional neoclassical convex models, like in Cass (1965). T can be thought as a no-intermediation equilibrium path, characteristic of an economy that will never create the opportunity for an intermediation market to be profitably open. T', on the other hand, describes the unique self-sustained growth path with an active intermediation market converging to a balanced growth condition with constant saving rate.

Fig. 3 displays an interesting possibility of this model. If K_o lies between K_L and K_H, there are alternative sets of consistent beliefs which may drive the economy to either take-off into sustained growth or irreversible economic decline.[5] In the language of Krugman (1991), for either very low or very high values of the state variable K_t only 'history' matters, the future path of the economy being entirely determined by inherited conditions. However, there is a set of states of the system in which 'expectations' matter, the destiny of the economy being open to multiple self-fulfilling prophecies.

FIG. 3 HERE

[5]This possibility is related to the occurrence of explosive cycles about the unstable fixed point. Simulations show that the lower σ the more pronounced, ceteris paribus, the cyclical pattern. Another theoretical possibility is that the α-limit of the equilibrium path with sustained growth be the origin rather than the fixed point S'. In this case the region with multiple equilibria is larger.

Note, finally, that dynamics of the type described by Fig. 3 may also be generated by the system (14)-(15) (i.e., without the autarkic technology), since $r(K)$ is in general non-monotonic and can generate multiple interior steady-states.

Clearly, traps are less likely in second-best economies, since firms' investment demand is higher and $r^{sb}(K) > r(K)$ for all K. In a first-best world, no trap exists. Since the 'wedge' is constant ($q = A$), consumption grows in a smaller economy at a faster rate than in a large economy. If the intermediation costs are covered through lump sum payments which do not affect the incentive to accumulate, the growth rate will be bounded from below by the asymptotic rate and the economy will never get locked into a stationary equilibrium.

3 Conclusions

We have constructed a theoretical model with an imperfectly competitive transportation-type intermediation sector, which shows in what sense the cost of trading productive resources affects growth and can lead to stationary (possibly interior) underdevelopment equilibria. We have also shown that a model with convex technology in the final goods sector can produce strong non-convergence results, with structurally identical economies moving either along a self-sustained growth path, or towards a long-run stationary trajectory. The policy implications are clear-cut, though the highly stylised nature of the model suggests caution. Countries should aim at reducing imperfections in the markets for inputs and services, either real or financial, which are complements of productive capital, so as to lower the opportunity-cost to accumulation. Fiscal policies or public investments that stimulate the process of accumulation (of both physical and human capital) are also welfare-improving, given the presence of a 'thick market externality'.

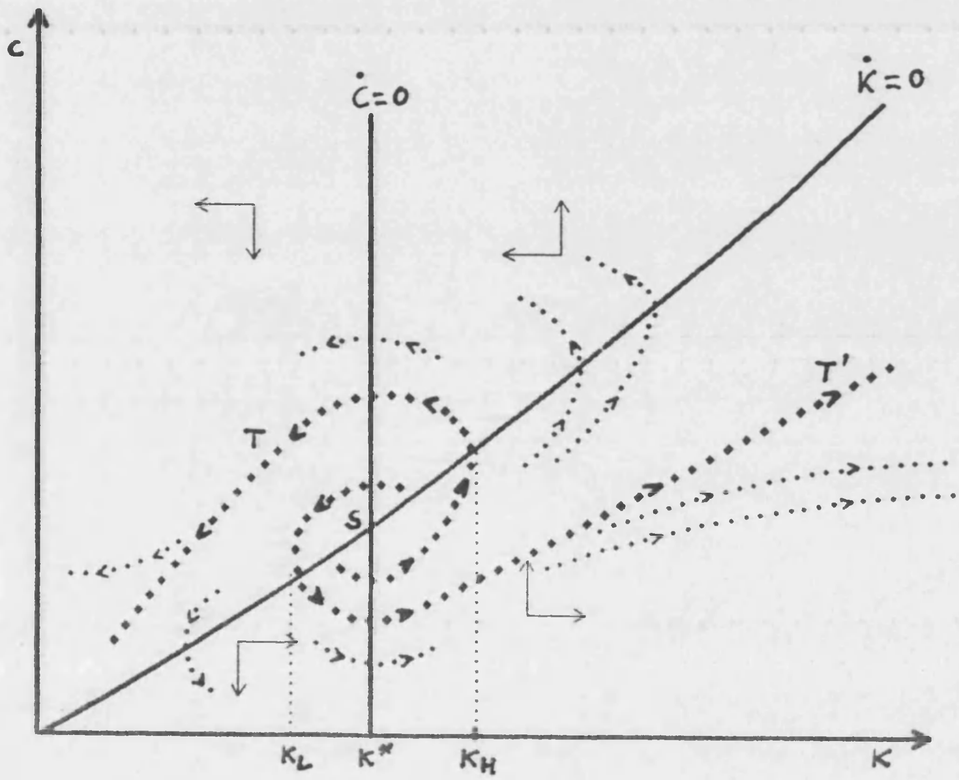


Figure 1

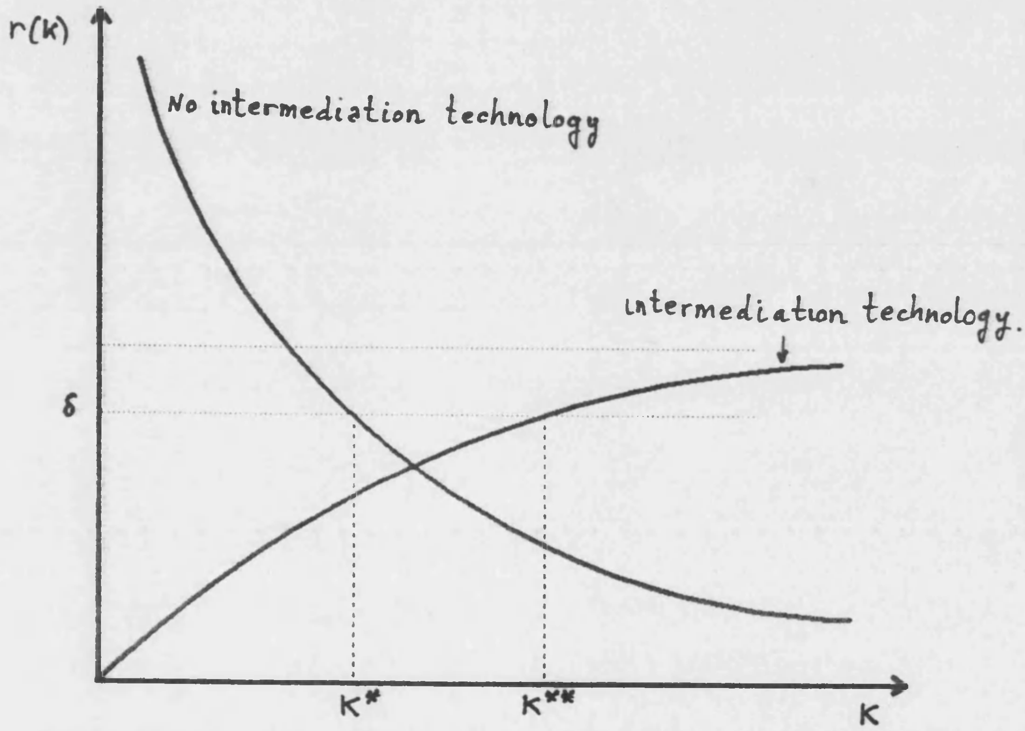

r(k)
No intermediation technology
intermediation technology.
δ
K*
K**
K

Figure 2

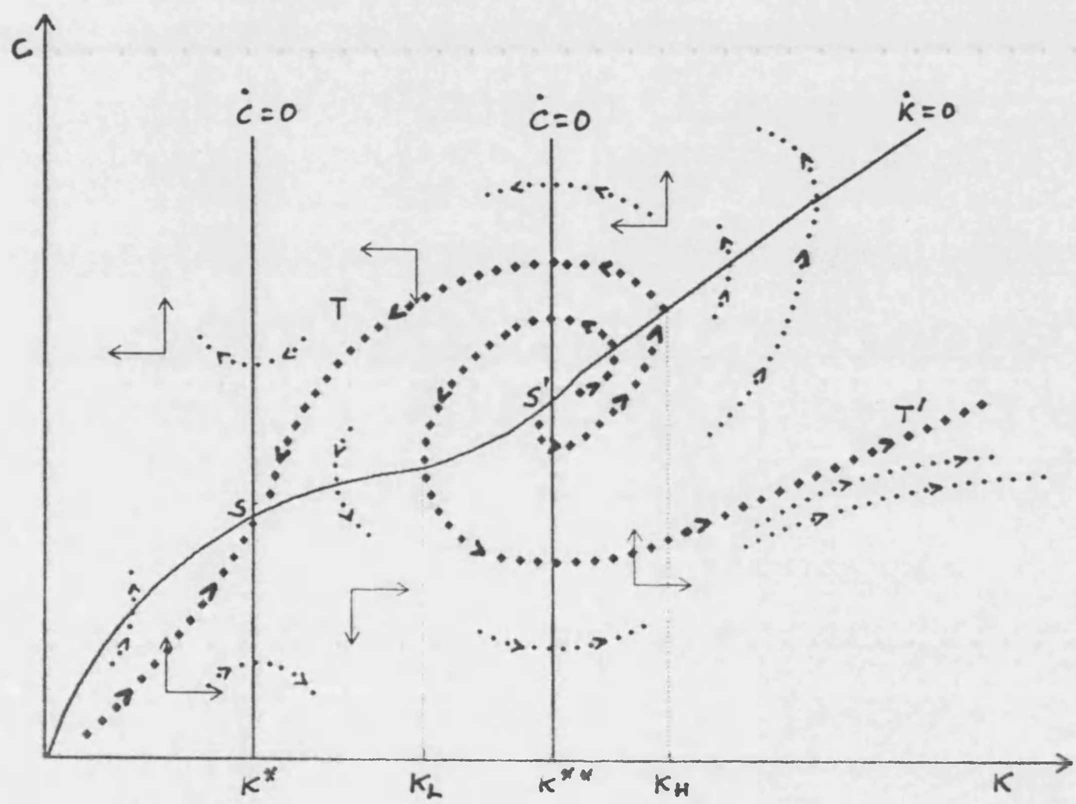


Figure 3

CHAPTER 3

Growth and Intermediation: an Empirical Analysis

This chapter investigates from an empirical perspective the relationship between growth and intermediation. In the previous chapter we emphasised the effects of market imperfections in the intermediation activity on the process of economic development, including the possible emergence of poverty traps. The most evident phenomenon in the contemporary world to which this analysis can be applied is financial intermediation.

The link between growth and financial development has attracted increasing attention in the recent theoretical literature, though the empirical research is still limited. Among the first authors who ascribed a positive growth-inducing role to intermediating institutions, we can mention Gerschenkron (1962), Cameron (1967), Patrick (1967) and McKinnon (1973) who supported their argument with casual historical evidence. Goldsmith (1969) first attempted to provide a rigorous empirical support to such conjecture. Using the value of all financial instruments outstanding divided by the value of national wealth as an indicator of the development of the intermediation system, Goldsmith found some evidence that a progressive deepening of financial activity accompanies the process of growth and development. Taking homogeneous samples of countries, he found that this ratio increases over time in the period 1880-1963 in both a group of developed and a group of underdeveloped countries considered separately. Furthermore he found that the index takes on lower values in underdeveloped than in developed countries (p.208). However, though both of these findings point in favour of the conjecture, Goldsmith himself recognised that evidence as 'loose and irregular' and the formal cross-sectional tests as essentially inconclusive (p.374-5).

Atje and Jovanovic (1992) provide some renewed evidence of a positive association between financial development and growth. They run a cross-sectional regression in which the growth rate of GDP per capita is the dependent variable and the ratio of the cumulative credit extended by banks plus the value of stock outstanding to GDP is included in the list of regressors, and find that remarkable cross-country differences in growth performances may be attributed to different degrees of financial development. The reliability of this result is however weakened by the lack of control in their regression for other potentially important determinants of growth like human capital, indices of political instability, etc.. Furthermore, they obtain less encouraging results in time-series analysis for a selected group of developed countries.

From a theoretical standpoint, financial intermediation introduces a separation between savers' and investors' decisions in aggregate growth models. We have seen in the previous chapter that in models with self-sustained growth and costly intermediation activity, the technology and market structure of the intermediation sector can alter the rate of capital accumulation and growth of the economy. In particular, intermediation costs affect both the propensity to saving of agents and the productivity of investments allowing for a more efficient allocation of resources. Though our model is highly stylised and abstracts from many specific features of financial intermediation, we find it plausible to think of the financial sector as performing the economic activity of 'transporting' resources from the places where they are saved and accumulated to that where they are invested and, possibly, get the highest return. This is close in spirit to the traditional analysis of Gurley and Shaw (1960), who motivated the existence of intermediaries on the grounds of transaction costs. Like has been recently emphasised by Hellwig (1991), this line of analysis sees intermediation as a kind of transportation activity. "(J)ust as the transporter takes oranges from Spain and transforms them into oranges in Germany, so the intermediary takes bonds issued by firms and transforms them into demand deposits or savings deposits held by consumers" (p.42). The function of specialised middlemen is that of overcoming the frictions from transaction costs, particularly when non-convexities in the transaction technologies occur.

The interpretation of intermediation in terms of 'transportation activity' is certainly restrictive. We believe, however, that possible extensions of the model designed to introduce explicitly the dimensions of risk and asymmetric information would not alter the essence of the argument and the testable implications which are delivered by our simple parable.

There are other papers which provides a more specific description of the financial intermediation process, resulting in similar predictions to those generated by our model. Greenwood and Jovanovic (1990) construct a competitive model with heterogeneous agents which deals with growth, intermediation and income distribution issues. Intermediaries collect and analyze information, channelling resources to the place where they earn the highest social returns. Entry into the intermediation activity requires paying a lump-sum cost proportional to the number of agents being intermediated. In exchange for a once-for-all payment (a sort of initial 'subscription' fee), each intermediary commits himself to paying the savers an agreed return on the capital invested. Intermediated savings turn out to be more efficiently allocated and to give a higher reward than those not intermediated. Agents with low-income, however, still find it optimal to choose the less efficient investment technology in order to avoid the payment of the lump-sum fee. As growth proceeds, also poorer people gradually switch to develop a link with the exchange network and foster an accelerating growth. In the long run, all savings are intermediated and efficiently allocated and the growth rate becomes constant at the maximum sustainable rate.

Bencivenga and Smith (1991) identify the growth enhancing role of financial intermediation in the encouragement which it provides to agents to switch their savings from lowly productive liquid assets to highly productive illiquid ones. Saint-Paul (1992) constructs a model in which the development of capital markets, by allowing the spreading of risk through financial diversification makes possible the choice of more specialized and risky techniques, characterised by a higher degree of division of labour and productivity. Multiple equilibrium growth paths (one "financial" and another 'nonfinancial') are a possible outcome of his model.

In the archipelago model, like Greenwood and Jovanovic, we made the critical assumption that the provision of intermediation services is subject to some fixed costs. However, in our model these do not take the form of a once-for-all entry cost, but of a continuous flow of capital costs independent of the output of services produced viz a fixed cost of operation rather than entry. The main difference between our model and that of Greenwood and Jovanovic is that the latter is entirely competitive. This is made consistent with the existence of fixed costs by the assumption that the lump-sum cost (subscription) paid by the investors just covers the lump-sum cost suffered by the intermediary for establishing its activity. This guarantees Pareto-efficiency in their model. In our model, instead, we allow for market imperfections in the intermediation sector and the laissez-faire outcome is then suboptimal.

When interpreted as referred to financial intermediation activity, our parable identifies the wedge between borrowing and lending interest rates as an important variable which can be used for empirical tests. This prediction is model-specific, since the existence of such wedge makes sense only in an imperfectly competitive framework. The crucial point of our theory is that the higher the price of intermediation services, the higher the effective cost of investment. Though firms can substitute external capital with own capital, there is an increasing cost in terms of productive efficiency in doing so, since the marginal rate of substitution across different types of capital is decreasing. The lower the intermediation costs, the less intensive in own capital and the more productive the technology chosen by firms. Since the productivity of reproducible assets is higher, so is the incentive to make new investments and the growth rate in the economy.

This chapter proposes some empirical tests related to the predictions of the model, based on cross-country data. According to the model the price of intermediation services should affect the growth performances of countries. To test this prediction, we first add the wedge between borrowing and lending interest rates to the list of regressors proposed by Barro (1990) and find that it has a highly significant coefficient with the expected (negative) sign, provided that countries subject to hyperinflation are excluded from the samples. Differences up to 2.75-3.5(depending on the sample considered) in the average yearly growth rate of GDP per-capita are explained by different interest rate wedges. The result is consistent with the findings of Atje and Jovanovic, who estimated quantitative effects slightly smaller but also significant. Furthermore, we find that the effects of intermediation costs are particularly strong for African countries.

1 Framework of the analysis

We reconsider here the existing results from the cross-country growth regression literature (Barro, 1991). The core prediction that we want to test is that per-capita growth rates[1] should depend on the efficiency of the intermediation process, which should be reflected by the wedges between borrowing and lending interest rates.

Our analysis is based on a simplified version of the archipelago model. Consider the following Euler equations derived from an AK-type model in continuous time with intermediation costs, which can be taken as the simplified analogue of the model of chapter 2. Apart from the discrete vs. continuous time difference, in fact, the model is identical to the asymptotic version of that discussed in the previous chapter. First, we write down the equilibrium balanced growth conditions, summarised by the following differential equations:

$$\dot{C}_t = \left[\frac{\Omega}{a^{1-\alpha}} - \delta\right]\frac{C_t}{\sigma} \tag{1}$$

[1] We assume throughout this and the following sections that the results of the theoretical model are normalized in per- capita terms

$$\dot{K}_t = [(\Omega - a)K_t - C_t]. \tag{2}$$

The first equation is the usual condition saying that consumption growth rate is equal to the elasticity of substitution times the difference between the marginal productivity of capital and the time discount rate, modified to account for intermediation costs (the notation is analogous to the previous chapter). The productivity of capital (Ω) is reduced by an amount that depends on the size of intermediation charges. The second equation expresses the usual feasibility condition. The term 'a' will be assumed to be country-specific, and captures the effect of intermediation costs along the balanced growth trajectory. Observe that, departing from the discussion of chapter 2, we treat here for simplicity intermediation costs as exogenous, determined by depending on institutional country-specific features, like policies towards the banking and financial sector and so on. Though this exogeneity assumption can be questioned, the nature of the problem is not different from that concerning other standard explanatory variables of cross-country growth regressions, like human capital or index of political instability which are likely to be two-way related to growth rates. From the Euler equation we can derive the following balanced growth conditions:

$$g = \frac{\Omega}{\sigma a^{1-\alpha}} - \frac{\delta}{\sigma} \tag{3}$$

$$\frac{I}{Y} = \frac{a\sigma + a^{\alpha-1}\Omega - \delta}{\sigma\Omega} \tag{4}$$

where $\frac{I}{Y}$ is the investment to GDP ratio. To obtain (4), we have used the expressions (1), (2), the identity $\frac{I}{Y} = 1 - \frac{C}{Y} = 1 - \frac{C}{K}\frac{1}{\Omega}$, and the balanced growth condition.

Throughout the empirical analysis, we will control in the growth regressions for a list of variables. First, we add the the 'initial state' of the system, namely the GDP per-capita at the initial year, which previous work has found to be highly significant. This variable is normally interpreted as capturing 'convergence effect' towards the long-run path when a country has not yet reached such path. Second, we control for the set of explanatory variables identified by Barro's work as being significant in explaining cross-country growth rates.

Equation (3) predicts that growth rates should be negatively correlated with intermediation costs. On the other hand, equation (4) shows that the sign of the correlation between $\frac{I}{Y}$ and a is ambiguous. The reason is the traditional conflict between substitution and income effect. On the one hand, lower intermediation costs make the rewards to saving higher and induce people to be more parsimonious. On the other hand, they increase the permanent income and, since current consumption is a normal good, induce agents to enjoy higher consumption in current time. Ceteris paribus, low values of σ (high intertemporal elasticity of substitution in consumption), tend to make $\frac{I}{Y}$ decreasing with a.

The previous analysis suggests another implication of the model. Regardless of whether intermediation costs decrease or enhance the savings rates, they affect positively growth rates when we control for savings rates. This descends from the fact that resources are better allocated in economies with lower intermediation costs. In the archipelago parable, firms choose a technology which is relatively more intensive in external fertilizer and, thus, more efficient. This is shown explicit by using equations (2) and (4) to obtain:

$$g = \frac{\dot{K}_t}{K_t} = \frac{I}{Y} - \frac{a}{\Omega} \tag{5}$$

Equation (5) confirms that growth depends negatively on intermediation costs after controlling for investment to GDP ratio.

2 Cross-country regressions reconsidered

The data set from which all but one of the variables are drawn is that constructed by Barro and Wolf and used by Barro (1991). Only 79 countries from that data set, however, enter our sample for diverse reasons. First, no data are available from the International Financial Statistics for the wedge between borrowing and lending interest rates for Algeria, Ethiopia, Madagascar, Sudan, Zaire, Hongkong, Iraq, Jordan, Pakistan, Taiwan, Austria, Turkey, Guyana, Haiti, Nicaragua, Panama and Paraguay. Second, some countries, particularly in South America, were affected throughout the 80s by extraordinary fluctuations characterised by high inflation rates and violent adjustment policies. This makes unrealistic to take the interest rates of that decade as representative of 'structural' conditions. In particular Argentina, Bolivia, Brazil, Colombia, Ecuador, Peru, Uruguay and Venezuela, plus Israel, have been excluded because they exhibit anomalous (negative in some cases negative, very high in others) or highly volatile wedges that clearly reflect the particular contingencies of the 80's rather than long run stable conditions of the credit market.

For the reader's convenience, we resume here the list of the variables drawn from Barro's data set, referring to his work for further details. GR6085 (GR7085) is the annual growth rate of per-capita GDP over the indicated period (e.g. GR6085 is the annual growth rate in the period 1960-85); GDP60 (GDP70) is the log of per-capita GDP level in the referred year (all data are in Kravis Dollars); PRIM60 (PRIM70) and SEC60 (SEC70) are the of school-enrolment rates at the primary and secondary levels, and so proxy human capital; FERT60 (FERT70) is the fertility rate; GOV6085 (GOV7085) is the real government consumption expenditure to real GDP ratio (average for the indicated period); INV6085 (INV7085) is the average investment to GDP ratio; REVOL is the number of revolutions and coups per year (1960-85), and is a measure of political instability; AFRICA is a dummy for African countries; JAMAICA is a dummy for this caribbean country.

The new variable WEDGE (defined as the difference between the credit market and deposit rates) is instead obtained from the International Financial Statistics Yearbook, 1992. The main problem is that observations for a large enough number of countries are only available for recent years. So, we have constructed WEDGE by taking the average for reported observations from

1981 to 1990 (when available) for each country[2]. This data constraint raises serious questions. In principle, it is always preferable to use begin-of-period data as explanatory variables of our regression, in order to weaken the issue of reciprocal causation between dependent and independent variables. Unfortunately, this is not possible for the variable WEDGE. With some 'suspension of disbelief', we will treat our variable as an estimate of the wedge over a longer period[3].

To start with, we test the significance of adding the variable WEDGE to linear Barro-type regressions whose dependent variable is the average yearly growth rate in the period 1960-85 (GR6085) and 1970-85 (GR7085), respectively. The list of control variables include REVOL, PRIM60 (PRIM70), SEC60 (SEC70), GOV60 (GOV70), and FER60 (FERT70) defined earlier. The dummy variables AFRICA (sub- Saharan countries) and JAMAICA are also sometimes added. Particularly, Jamaica seems to be a constant 'outlier'. In this set of regressions we do not add the investment to GDP ratio to the list of right hand-side variables and present the result of strictly reduced form regressions. This means that the measured effects on growth of the variables introduced can work either directly, namely by affecting the productivity of investments, or indirectly through their effects on the investments. The scope of this section is mainly to provide results directly comparable with the existence mainstream literature.

Several experiments were performed starting from a larger set of control variables drawn from Barro's data set and then eliminating those with non-significant coefficients through subsequent tests. We present the following results[4]:

[2]In cases with more than two missing observations, we have used also observations for 1979 and 1980.

[3]The usual choice of taking averages over different annual observations has the scope of purging the data from short-run country-specific effects, though it is also questioned by some authors. (Quah, 1992).

[4]We are more agnostic than Barro about the role of the variable fertility, which he treats as endogenous, like in Becker et al. (1990). In such model, fertility is a choice variable of the agents, but it can alternatively be conceived, more traditionally, as an exogenous variables

Table 1.

	depvar: gr1960-85			depvar: gr1970-85	
ind.var.	regrl	regr2	regr3	ind.var.	regr4
const.	0.026 (0.006)	0.012 (0.006)	0.024 (0.006)	const.	0.032 (0.01)
revol	-0.055 (0.011)	-0.054 (0.012)	-0.053^{**} (0.011)	revol	-0.064 (0.014)
prim60	0.027 (0.005)	0.038 (0.005)	0.027 (0.005)	prim70	0.021 (0.005)
sec60	0.018 (0.006)	0.023 (0.009)	0.019 (0.006)	sec70	0.033 (0.005)
gov6085	-0.047 (0.023)		-0.051 (0.023)	gov7085	-0.090 (0.029)
gdp60	-0.014 (0.002)	-0.018 (0.003)	-0.013 (0.002)	gdp70	-0.017 (0.003)
africa	-0.004^{**} (0.004)	-0.009 (0.0035)		africa	
jamaica	-0.033 (0.003)	-0.033 (0.003)	-0.032 (0.002)	jamaica	-0.053 (0.003)
fert60	-0.005 (0.0014)		-0.006 (0.0012)	fert70	-0.007 (0.002)
wedge	-0.0018 (0.00045)	-0.0018 (0.00047)	-0.0018 (0.00045)	wedge	-0.0023 (0.0006)
R^2	0.71	0.64	0.71	R^2	0.62
adj-R^2	0.67	0.60	0.67	adj-R^2	0.57
F	18.25	17.25	20.36	F	13.84

Dependent variable: per capita GDP growth rate. Number of observations: 78 (regrl, regr2, regr3); 79 (regr4). Standard Errors in brackets. x^* = significance level of the coefficient of x inferior to 0.95. x^{**} = significance level of the coefficient of x inferior to 0.90. Method of estimate: OLS (standard errors corrected for heteroscedasticity)

Regressions (1), (2) and (3) in Table 1 consider average growth rates in the period 1960-85. We have also provided the results when the fertility index is omitted in order to facilitate the comparison with Barro's result. In this case government consumption also becomes non-significant. Also we have omitted the non-significant dummy for Africa. The coefficient of WEDGE is in all cases

reflecting cultural and religious attitudes of a population rather than a process of economic maximization negative, highly significant (more than 95 per cent significance level) and stable across experiments. The sign is in accordance with theoretical expectations, showing that a reduction in the wedge has positive effects on growth.

We have also repeated the regression for the period 1980-85 (not reported). In this case, the fit of the regression is smaller ($R^2 = 0.48$) and some coefficients become non-significant. This is no surprise, since the average growth rates become highly sensitive to short-run fluctuations when the period considered is so short. Noticeably, however, the coefficient of WEDGE remains stable (its point estimate being -0.0025, s.e. 0.0009) and statistically significant.

The suspect can arise that the variable WEDGE captures in fact the effect of inflation rates on growth. If wedges are positively correlated with inflation rates and inflation rates are negatively correlated with growth rates, then we would have a case of spurious correlation. However, the introduction of average yearly inflation rates into the list of regressors does not affect the estimated parameters and turns out to be lowly significant. We have also controlled separately for inflation rates in the period 1980-90 (the period in which the variable WEDGE has been computed) and verified that the results are never significantly different.

We can draw some quantitative inferences about the extent to which cross-country differences in growth rates can be attributed to differences in the costs of intermediation in a typical country. A one percent fall in the wedge between the borrowing and lending rates brings about a 0.18 percent increase in the annual growth rate, raised to 0.23 percent when the restricted sample is considered. Since the variable WEDGE ranges between 0.0006 to 0.1587, differences in intermediation costs account for differences up to 2.75 percent (regressions 1,2,3) or 3.5 percent (regression 4) of the observed distribution of annual average growth rates across countries. In conclusion, the effect of intermediation costs on growth can be regarded as fairly large.

To check of the robustness of the results, we have tested the structural stability of the sample. Some interesting insight comes from splitting the sample into African and non-African countries. We report the results of regressions (1) and (4) only. The estimated coefficients of WEDGE is -0.0026 (s.e. 0.0011) in (1) and -0.0036(s.e. 0.0013) in (4) for African countries, and -0.0014 (s.e.0.0005) in (1) and -0.0020 (s.e. 0.0008) in (4) for the rest of the sample. This suggests that the effect of differences in intermediation costs are particularly remarkable in the African continent. On the other hand, such differences are not statistically significant. An added shift dummy (WEDGE times AFRICA) is never statistically significant. The F-test of structural stability also fails to reject the null hypothesis.

A potential issue which has been considered is the measurement error in the variable WEDGE. To correct for the potential bias, we have run the same regressions by using the Instrumental Variable method, using a rank indicator as an instrument for WEDGE. This indicator classifies countries in five countries according to their wedge. Provided that the measurement error is not too serious, this variable should be uncorrelated with the error term, being so a valid instrument. The results (not reported) are almost unchanged, apart from the fact that the point estimate of WEDGE is slightly larger in absolute value (-0.0019 in regr1, -0.0025 in regr4, with the same standard errors as before).

After finding that the wedge between borrowing and lending rate has a positive explanatory power in growth regressions, we can test the prediction of the model (equation 5) that such effect remains positive when we introduce the saving rates into the list of regressors. The results obtained are reported in the following table.

Table 2.

depvar: gr1960-85		*depvar: gr1970-85*	
ind.var.	regr1	ind.var.	regr2
const.	0.018 (0.006)	const.	0.018 (0.008)
revol	-0.047 (0.009)	revol	-0.053 (0.012)
prim60	0.023 (0.005)	prim70	0.020 (0.005)
sec60	0.015* (0.0078)	sec70	0.028 (0.01)
gov6085	-0.061 (0.022)	gov7085	-0.088 (0.031)
gdp60	-0.016 (0.002)	gdp70	-0.022 (0.003)
africa	-0.006 (0.003)	africa	−0.011** (0.0044)
jamaica	-0.032 (0.002)	jamaica	-0.049 (0.003)
fert60	-0.004 (0.0014)	fert70	-0.004 (0.0017)
wedge	-0.0016 (0.0004)	wedge	-0.0020 (0.0005)
inv6085	0.07 (0.03)	inv7085	0.102 (0.033)
R^2	0.75	R^2	0.70
adj-R^2	0.71	adj-R^2	0.65
F	20.04	F	15.28

Dependent variable: per capita GDP growth rate. Number of observations: 78 (regr1); 79 (regr2). Standard Errors in brackets. x^* = significance level of the coefficient of x inferior to 0.95. x^{**} = significance level of the coefficient of x inferior to 0.90. Method of estimate: OLS (standard errors corrected for heteroscedasticity)

There is clear evidence that in both periods the effect of WEDGE on growth works mainly through the productivity of capital, rather than through the saving rates. On the other hand, the coefficients of WEDGE are lower in absolute value than those in Table 1, showing that lower intermediation costs also affect growth through savings rates.

The highly stylised nature of the model suggests caution in interpreting these results. Though these simple regressions cannot be taken as a direct test of the model of the previous chapter, it seems remarkable that the prediction generated by the theory about the relation between economic development and cost of intermediation are clearly supported from our empirical analysis. This looks a robust result, the limitations of the methodology notwithstanding. Many important issues have not been considered, like the existence of credit rationing, financial repression, segmentation of local markets, access to international financial markets etc.. Altogether, these issues make extremely problematic to assess the 'efficiency' of the intermediation process by simply lookin at the figures of borrowing and lending interest rates. Finally, the 'trap issue' has been ignored throughout this empirical analysis. Still, such figures seem to contain some interesting preliminary support to our theoretical work, and add to the findings of other authors in cross-country growth regressions.

CHAPTER 4

Enforcement Constraints in a Model of Growth Driven by Human Capital Accumulation

A critical issue about theories of development thresholds and underdevelopment traps is their robustness to an open economy environment. The explanations proposed by the models of the first and second chapter are, respectively, that (*a*) accumulation is characterised by locally increasing returns, causing the low marginal productivity of capital at some low stages of development, and (*b*) increasing returns and market imperfections in some intermediate input markets make these inputs highly expensive when the market size is small, causing the low 'effective' productivity of capital and accumulation rate. A natural objection (particularly to type (*b*) models) is that thresholds should vanish when the productive factors and resources, particularly capital, are allowed to move freely across countries. A theory of underdevelopment traps should then explain not only what prevents the activation of an internal engine of growth in a closed economy, but also why foreign investments do not flow into poor countries and rule out the trap equilibria. There is good deal of empirical evidence that the amount of foreign direct investments (fdi's) which flows into poor stationary countries is pretty moderate.

Two explanations may be proposed to this puzzle. The first, already mentioned in the introduction, is that these economies lack some non-tradable productive resources, e.g skills embodied in labour force, network services, etc., which are complements of tradable factors in production. For this reason, the productivity of fdi's and the power of poor countries to attract them remains low. This argument could be used to defend the robustness of the models of the first two chapters. There is a second point, however, that might be also important, and is that the fdi's inflow is low because property rights are perceived as unsafe by foreign investors. Political instability and incentive problems make them fear to loose the control over the resources invested, due to nationalisation, taxation etc.. The relationship between incentive constraints, financing opportunities and growth, already discussed by the 'dept repudiation' literature (Atkeson, 1991), have been recently reconsidered within the general framework of a stochastic competitive equilibrium model with infinite horizon by Marcet and Marimon (1992). From an empirical viewpoint, Barro (1991) finds that an index of political instability has a very strong predictive power in cross-country growth regressions. Some more direct evidence which supports the hypothesis of a negative effect of enforcement constraints on growth with reference to some African countries is found by Giovannetti, Marcet and Marimon (1993).

This chapter considers the two issues (accumulation of non-tradable factors and property rights) jointly, and identifies a vicious circle in which enforcement problems reduce foreign investments, and this affects negatively the local people's propensity to invest in human capital. The slow growth (if any) of human capital feeds back into a low levels foreign investments. The main features of our model are the following:

a) the government of the host country can 'expropriate' the resources invested by the foreigners and redistribute them among the local population. However, fdi's have a component of specific knowledge that cannot be seized, and the host country cannot run the productive process combining the nationalised and the local resources at the same level of efficiency as the investor would do. The productivity fall is biased against the local qualified labour; in particular, we make the (unnecessarily) extreme assumption that qualified labour becomes unproductive at all when non-seizable resources are withdrawn.

b) fdi's are an essential factor for the development of a 'modern' sector. We assume a dual economy in which productivity growth takes place only in the sector which is fdi-intensive (the 'modern' sector). On aggregate, growth in per-capita term is entirely driven by the development of the modern sector;

c) the average and marginal productivity of fdi's is increasing in the stock of human capital available in the host country. Foreign capital and human capital are assumed to be the only factors which enter a constant returns to scale production function of final goods in the modern sector;

d) the host country is populated by overlapping generations of two-period lived agents. They can save in two forms: either by investing in education when young and using their expertise in the modern sector when old, or by storing the resources earned in the first period when employed in the traditional sector. Accumulated knowledge can be transmitted from one generation to its offspring.

e) to accumulate human capital is costly to the indigenous inhabitants. The opportunity-cost is given by the time spent in education rather than in production during the youth. The aggregate investment in education in the host country is a non-decreasing function of the wage rate in the modern sector;

f) the enforcement problem reduces the amount of foreign capital per efficiency unit of human capital which enters the country. This causes a lower wage in the modern sector than in a first-best world and affects negatively the accumulation of human capital and the growth rate of the economy.

The solution of the model is simple and unsurprising when agents are assumed to be identical. Given the structure of preferences and technology, the economy will converge to either a stationary or a steady-growth solution depending on whether the initial human capital endowment is above or below a critical level. The enforcement problem affects the threshold, and leads a set of economies that would converge to the good equilibrium in a first-best world to converge to the 'bad' equilibrium.

The model produces richer dynamics when agents are heterogeneous in their individual human capital endowments. The intergenerational transmission of human capital is assumed to take place partly inside each dynasty (family education), partly as a social process (school education). The stock of productive knowledge held by each agent is a function of the human capital of his parents, of the total human capital accumulated in the society when he is young and of his personal investment in education. People belonging to different dynasties choose, in general, different levels of investment in human capital. Furthermore, more educated people gain more from selling their labour services in the 'modern' sector. So, rich people are less inclined to 'seize' the foreign investments than poor, less educated people. The main 'results' are the following:

a) the extent to which the enforcement problem binds depends on the distribution of income. Particularly, it is more severe when the 'median voter' has low education compared with the average human capital. A very unequal distribution of human capital in which a majority of uneducated people live together with an elite of highly educated people is unfavourable to growth. A highly egalitarian distribution is not the most favourable situation, though;

b) growth increases inequality in the beginning, when the inflow of foreign investment induces people from better educated families to undertake high investment in education, whereas poor people find it optimal not to invest in education. At a later stage, however, growth is equalising, since people progressively switch into investing time in education and those dynasties which have less human capital than average benefit more from the process of social transmission of knowledge produces. The result is a 'convergence' of individual productivities to a uniform growing level. This evolution is coherent with the traditional Kuznet's curve argument also captured by recent models about growth and distribution (Aghion and Bolton, 1992);

c) multiple equilibria are possible in the model, for a full-dimensional set of initial distributions of knowledge. In one type of equilibrium, the majority of local workers anticipate low wages in the modern sector and do not invest in education. These expectations are confirmed by the behaviour of the foreign investors who anticipate unfavourable political conditions and enter with a low amount of foreign investments per efficiency unit of human capital. This implies low wages in the modern sector. In the other type of equilibrium the opposite happens: a high level of foreign investments flow into the country, and the majority of workers invest in education. The expectations which generate this behaviour are again self-fulfilling. It is possible for the initial selection of the equilibrium to have long-lasting consequences, namely to determine whether an economy is to converge to a long-run stationary equilibrium or to a self-sustained growth path. This is illustrated by a simple example by simulations.

The chapter is structured as follows. We first describe the basic model with identical agents and no enforcement constraints (section 1). Then, we introduce the enforcement constraint and its relation to the political equilibrium (section 2). In the following two sections we generalize the model to the more interesting case in which workers are heterogeneous. Policy implications are discussed in section 5. An example is then presented to discuss the relation between short-run and long-run equilibria (section 6).

1 The basic model

An economic system is populated by overlapping generations of two period-lived identical agents belonging to a continuum of dynasties. The successive generations have a constant size, whose measure is assumed to be unity. In the first period of their life agents choose to allocate their time between working in a 'traditional' household activity and receiving formal education (leisure is worthless). The goods produced by a young person can be stored for one period and consumed by the same agents when old. Alternatively, we can imagine that first-period savings can be invested in a non-productive asset, like foreign currency, which plays the role of a store of value. In the second period, agents sell their labour force in the modern sector. Here they earn a wage which is proportional to their productivity (h). This income, together with the savings of the previous period, is entirely consumed in the second period. For convenience, we assume that a young worker is entirely unskilled, and is worthless in the modern sector, as well as an old worker is unsuitable for working in the traditional sector.

Agents have standard intertemporally separable logarithmic preferences and do not care about their offspring. There is no uncertainty. A representative member of the generation which is young at time t solves the following programme:

$$\max_{s_t, u_t} V_t = \log c_t + \beta \log c_{t+1} \tag{1}$$

$$\begin{aligned} s.t. \qquad c_t &\leq w(1-s_t)(1-u_t) \\ c_{t+1} &\leq s_t R w(1-u_t) + w_{t+1}^M h_{t+1} \\ h_{t+1} &= (\delta + u_t^{1-b}) h_t \\ 0 \leq s_t \leq 1&; \quad 0 \leq u_t \leq 1. \end{aligned}$$

where w is the wage rate in the traditional activity, w_{t+1}^M is the wage rate per efficiency unit in the modern sector, β $(0 < \beta < 1)$ is the time-discount factor, s_t is the saving rate out of the income earned in the first period, u_t is the share of time spent in education, R is the gross rate of return paid by the storage technology, δ $(0 < \delta < 1)$ is one minus the depreciation rate of the human capital inherited from the former generations without any personal investment. The parameter b is such that $0 \leq b < 1$, meaning that there are non-increasing returns to the individual investment in education. The representative agent takes as given the stock of human capital of the previous generation, h_t, and the wage rate in each activity.

The First Order Conditions ($V_{u_t} \leq 0$, $V_{s_t} \leq 0$) of this problem can be expressed - after simple manipulations- as follows:

$$w_{t+1}^M h_t[\beta(1-b) - u_t(1+\beta(1-b)) - \delta u_t^b] \leq s_t \left[u_t^b R(1-u_t)(1+\beta)w\right] \tag{2}$$

$$w\beta R u^b(1-u_t) - w_{t+1}^M h_t(u_t + \delta u_t^b) \leq s_t \left[u_t^b R(1-u_t)(1+\beta)w\right] \tag{3}$$

where strict inequality in (2) and (3) implies, respectively, $u_t = 0$ and $s_t = 0$ (slackness conditions). Notice that no corner solutions at which either $u_t = 1$ or $s_t = 1$ (or both) can be a maximum, since this would imply consumption at time t to be zero and the utility to be minus infinity. So, the gradient of the value function evaluated in the optimum is always non-positive.

When $b > 0$ (strictly decreasing returns to the individual investment in human capital) and $h_t > 0$, the optimum u_t is always positive. This is evident from the fact that (2) never holds for $u_t = 0$. Let us consider, first, strictly interior solutions. By using (2) and (3) to eliminate s_t, we obtain:

$$u_t = \left[\frac{(1-b)w^M_{t+1}}{wR} h_t\right]^{\frac{1}{b}} \tag{4}$$

The second condition which needs to hold is that $s_t > 0$. From the inspection of (2), it results that this condition is satisfied if and only if:

$$(1-b)\beta - [1 + (1-b)\beta]u_t \geq \delta u_t^b \tag{5}$$

Call u^* the value of u_t for which (5) holds with equality (figure 1). Then $u_t = \left[\frac{(1-b)w^M_{t+1}}{wR} h_t\right] < u^* \Rightarrow s_t > 0$. One can check that this condition is both necessary and sufficient for the solution to be strictly interior.

FIGURE 1 here

It can also be verified that any corner solution at which $s_t = 0$[1] will be characterised by $u_t = u^*$. This solution is independent of the state variable h_t.

Finally, it is useful to obtain the limit behaviour of the policy function when $h_t \to 0$. In this case, the logarithmic preferences generate the simple solution:

$\lim_{h_t \to 0} s_t = \frac{\beta}{1+\beta}$; $\lim_{h_t \to 0} u_t = 0$

The characterization of the equilibrium dynamics requires us to obtain the value of w^M_t. To this end, consider the technology of the productive sectors in

[1]Agents cannot issue debts when young.

the economy. In the 'traditional' activity only physical labour of young people is used, and there is no technical progress. Formally:

$$y_t^T = w(1 - u_t) \tag{6}$$

having normalized the measure of the labour force to one, as already said. In the modern sector, output is produced by using human capital and imported resources. For analytical convenience, we assume that each vintage of fdi's is productive for only one period. In the basic case described in this section, each mature agent's belonging to a particular generation is endowed with the same amount of human capital. The production function is assumed to be Cobb-Douglas, of the form:

$$y_t^M = k_t^{1-\alpha} h_t^{\alpha} \tag{7}$$

Human capital is supplied inelastically by mature people. Foreign investors decide at time $t-1$ the amount k_t which is invested in the host country and which becomes productive at time t. They are assumed to act competitively, taking as given the opportunity-cost of resources given by the international one-period interest rate, r, and paying wages to workers at the value of their marginal productivity. This means that the level of foreign investment will be set at the following level[2]:

$$k_t = \left[\frac{(1-\alpha)}{r}\right]^{\frac{1}{\alpha}} h_t \tag{9}$$

and the wage per efficiency unit paid to workers will be:

$$w_t^M = a\left[\frac{(1-a)}{r}\right]^{\frac{1-\alpha}{a}} \tag{10}$$

[2]One could assume, alternatively, that foreign investors' decisions are taken according to a joint profit maximization principle. In this case, less fdi's per efficiency unit of human capital would enter. Precisely, we would have:

$$k_t = \left[\frac{(1-\alpha)^2}{r}\right]^{\frac{1}{\alpha}} h_t \tag{8}$$

The change is qualitatively unessential for our argument.

So, in a world with no enforcement problems the wage per efficiency unit paid in the modern sector is constant. Notice that in an economy characterised by a growing stock of human capital, this means that per-capita wages are growing over time.

Since we have established that w^M is constant over time, the only state variable which determines the solution to (2) and (3) will be h_t. In particular, we have the following cases:

$$\begin{aligned} h_t = 0 \quad &\Rightarrow \quad u_t = 0; \quad s_t = \frac{\beta}{1+\beta} \\ 0 < h_t < \frac{wRu^*}{(1-b)w^M} \quad &\Rightarrow \quad u_t = u(h_t); s_t = s(h_t) \\ h_t \geq \frac{wRu^*}{(1-b)w^M} \quad &\Rightarrow \quad u_t = u^*; s_t = 0 \end{aligned}$$

The function $u(h_t)$ is defined in (4). Clearly, $u'(h_t) > 0$. The behaviour of $s(h_t)$ can be derived by totally differentiating (2) (the details are in the appendix). This establishes that $s'(h_t) < 0$ in all the relevant range.

We can now establish the main result for the basic case dealt with in this section:

Proposition 1 *Let the parameters be such that $\delta + u^* > 1$. Define $h^{thr} \equiv \frac{wR}{(1-b)w^M}(1-\delta)^{\frac{b}{1-b}}$ and g_t as the growth rate of human capital from t to $t+1$. Then:*

- *if $h_0 > h^{thr}$, then the equilibrium path will be such that:*

  $u_{t+1} \geq u_t; s_{t+1} \leq s_t; h_{t+1} > h_t$

  and will converge to a steady-growth path in which

  $s = 0; u = u^*; g = (\delta + u^* - 1)$.

- *if $h_0 < h^{thr}$, then the equilibrium path will be such that:*

  $u_{t+1} \leq u_t; s_{t+1} \geq s_t; h_{t+1} < h_t$

  and will converge to a steady-state in which

  $s = \frac{\beta}{1+\beta}; u = 0; g = 0; y = y^T$.

- *if* $h_0 = h^{thr}$, *then the equilibrium path will be stationary and such that, for all t:*
  $u = \left[\frac{(1-b)w^M h^{thr}}{wR}\right]^{\frac{1}{b}}$; $s = \frac{w^M h_t[\beta(1-b)-u_t(1+\beta(1-b))-\delta u_t^b]}{u_t^b R(1-u_t)(1+\beta)w}$; $h = h^{thr}$

PROOF - Since we know that $u'(h_t) > 0$ and $s'(h_t) < 0$, it is then sufficient to prove that $h_t > (<) h^{thr} \Leftrightarrow h_{t+1} > (<) h_t$ in order to ensure that the Proposition holds true. By substituting (4) into the human capital accumulation equation, we obtain:

$$h_{t+1} = \left[\delta + \frac{(1-b)w^M}{wR} h_t^{\;(1-b)}\right] h_t. \tag{11}$$

One can check that the term inside square brackets takes on the unit value when $h_t = h^{thr}$ and is larger (smaller) than one when $h_t > (<) h^{thr}$. $\square$

The following pictures give an intuitive representation of the dynamics described by the Proposition. When $h_0 > h^{thr}$, h_t grows over time (figure 2), since $\delta + u_t > 1$. Then, u_t grows and s_t falls over time (figure 3). The opposite occurs when $h_0 < h^{thr}$ (figure 4).

FIGURES 2, 3, 4 here

2 The enforcement constraint.

In this section we allow for the possibility that the host country's government seizes the foreign resources before the productive process is set. Given the Cobb-Douglas technology, a constant share of the production in the modern sector is appropriated by the foreign investors as a reward to the productive resources invested. The complement of this share represents the income earned by the inhabitants of the host country from the activity of the modern sector. This can be compared with the income which can be obtained by nationalising the foreign resources.

An important assumption is that the country cannot nationalise the capital and manage the productive activity efficiently without the cooperation of foreign investors. We imagine, to motivate this assumption, that some non-seizable factor,like specific knowledge, managerial capabilities etc. be owned by foreign investors. The government can, however, seize the tangible assets and obtain an income which is assumed to be proportional to the amount of capital planted in the previous period. We will refer to this income as 'scrap value of capital' and indicate it by ψk_t[3]. We will structure the sequence of the relevant decisions as follows. At the beginning of period t foreign investments are planted in the host country; such vintage of investments becomes productive only at time $t+1$. Meanwhile, also at time t, the young workers who contemplate being employed in the modern sector in the following period decide the time to invest in education. At the beginning of time $t+1$ elections are called and this generation, just become mature (the newborn do not have right to vote at this stage), chooses the government in a two-party system. Of the parties which run the competition, the party A, 'liberal', is credibly committed to enforce property rights over the entire mandate. The party B, to the opposite, is believed (regardless of its explicit programme) to nationalise the foreign capital and to redistribute in equal shares to the members of the voting generations of the income obtained. We will show that it is indifferent to assume, alternatively, that party B is expected, when in power, to tax at full rate the profits of foreign capitalists.

[3]In our notation k_t is the capital which is productive at time t. The formulation proposed in the text can be interpreted as if the country can do no better than to sell the machinery seized on the international market. Alternatively, one could assume that the country can still operate a certain number of efficiency units of foreign capital in the modern sector. However, in this case we should impose an additional assumption, namely that the relative contribution of the local factor to the production process be smaller when the foreigners are expropriated than when they are not. This makes the loss to the indigenous population from not running the process at full efficiency decreasing with the level of foreign investments, a fact which is essential, as it will become clear soon, in order to have interior solutions. Informally speaking, we want the productivity of the local factor to be strongly reduced by the withdrawal of the non-seizable component. The case which we consider is the extreme one in which the local factor looses its productivity entirely.

We need to justify this political structure, in which apparently the two parties only cover the two radical options, whereas a continuum of tax rates on foreign investors' income seem to be viable policies. First, we stress that governments remain in power for only one period, implying that no party may act strategically to build a reputation to the eyes of foreign investors.[4]. Consider the case in which the government can decide to tax profits at the end of the production process. Then, a time-consistent strategy for the local government will be necessarily characterised by the full taxation of the corporate income belonging to the foreigners. The expectations about the behaviour of party B simply come from this time-consistency issue. Notice that if party B's victory were anticipated, no foreign investment at all would be observed in equilibrium. How to justify then the existence of a party, the 'liberals', which supports property right enforcement and never deviates from its electoral manifesto? One might argue that the leaders of this party obey some ideological convictions or have a moral commitment to the future generations (it will become clear soon why). More realistically, we can think that they receive financing and bribes from foreign investors. As marxist literature would say, party A's leaders are 'agents of the foreign capital'. Leaving aside ethical issues, it is possible that the current generation find it optimal to 'tie his (and government's) hands' and choose to delegate political power rather to party A than to the 'honest' politicians of party B. To be concrete, let us review the complete scenario. Party B's leaders are expected to operate under any circumstance in the interest of the electors. This sound, to the eyes of foreign investors, as the threat of facing ex-post taxation at the full rate. Party A's propaganda hides the corrupted nature of its leaders under the banner of an ethical defense of the intangibility of private property rights. This sounds credible to foreign investors, on whose behalf these politicians in fact speak. Consider now the event of party B's electoral victory. The investors,

[4]The lack of consideration for reputational issues is certainly a major limitation of the model which is open to further research. However, we expect the technically issues that this would arise to be non-trivial.

anticipating full taxation, put themselves 'on strike', leaving the country alone with the tangible assets which can be seized but withdrawing all the intangible assets which are necessary to carry on the productive activity (something like this happened in Allende's Chile and in the Sandinist Nicaragua). If party A wins, instead, the modern sector will be active and no seizure will occur.

To summarize, it is unessential how radical the party B's manifesto is. The game is in fact structured in such a way that a party which is loyal to the interests of the local electors will always tax foreign capital income at the full rate. The threats of taxation or nationalisation turn out to be observationally equivalent in our framework. It is instead essential that *(i)* governments remain in power for only one period and are linked to only one (non-altruistic) generation, *(ii)* there is a party which acts, ex-post, on behalf of the investors rather than the electors. This party may gain, ex-ante, the support of the majority of the country, because it represents the actual instruments through which credible precommitment may be taken by the indigenous population, and *(iii)* some assumption is made about the way in which the benefit from the seizure-taxation is redistributed to the local agents. Though the actual form in which the redistribution takes place is unessential, we do not consider the potentially interesting, and certainly non-trivial, issue to endogenize the fiscal policy by allowing for a multi-party system in which any redistribution mechanism may be proposed by a party or a coalition.

Now, we compute what constraint the enforcement problem imposes on the foreign investors' decisions, given that they need to act so as to induce the party A's victory. This is easy to determine when the population is entirely homogeneous The party A will win the elections held at time t if and only if:

$$\psi k_t \leq w^M h_t = \alpha k_t^{1-\alpha} h_t^{\alpha} \tag{12}$$

This condition can be rewritten as:

$$k_t \leq \left(\frac{a}{\psi}\right)^{\frac{1}{\alpha}} h_t \tag{13}$$

We assume that, in the equilibrium, the foreign investors can coordinate their decisions and thus avoid creating the conditions for a victory for party B. This means that they will optimally restrict the inflow of investments in such way that (13) always holds. We will refer to (13) as the 'enforcement constraint' condition. Clearly, such a constraint is not binding (strict inequality) under certain parameter configurations. In these cases the results of the previous section carry over. We will focus, however, on those cases in which such constraint is binding, by assuming, throughout the rest of the chapter, that:

$$\frac{a}{\psi} < \frac{1-\alpha}{r} \tag{14}$$

When (14) holds, the wage rate in the modern sector is lower in the presence of incentive problems than under first-best Pareto efficiency. In particular, we have now:

$$w^M = \alpha^{\frac{1+\alpha}{\alpha}} \psi^{\frac{-1}{\alpha}} \tag{15}$$

This means that the threshold level h^{thr} which guarantees convergence to a sustained growth path is now larger than in the previous section (cfr. Proposition 1).

3 Heterogeneous agents.

In this section we consider a population with heterogeneous levels of education between agents of the same generation. To this end, we define a generic distribution function over human capital level, $F_t(h) : R \to [0,1]$, $F_t'(h) \geq 0$, where $F_t(h_c)$ is the proportion of old people whose human capital is inferior to h_c at time t. We also define the density function $f_t(h) \equiv \frac{dF_t(h)}{dh}$.

Two main changes are introduced here with respect to section 2. The first, merely technical, is that we restrict the analysis to the case in which there are constant returns to scale to the individual investments in education. This means to restrict the parameter b in (1) to be zero. The second, more substantial, concerns the process of transmission of human capital between generations. We assume that knowledge is transferred between generations through both a family and a social channel. As before, individual investment in education increases the power of this process of transmission. We formalize this idea through the following transition equation:

$$h_{t+1}^i = (\delta + u_t^i)(\gamma h_t^{av} + h_t^i) \tag{16}$$

where i ($i \in R$, $0 \leq i \leq 1$) indexes dynasties, $h_t^{av} \equiv \int_0^\infty f_t(h^i)h_t^i di$ is the average human capital at time t, and γ is an arbitrary constant which indicates the importance of the social channel of transmission (e.g. public education) of knowledge between generations.

The programme (1) can be now reformulated, with reference to the member of the dynasty i who is young at time t, as follows:

$$\max_{s_t^i, u_t^i} V_t^i = \log c_t^i + \beta \log \ _{t+1}^i \tag{17}$$

$$\begin{aligned} s.t. \qquad c_t^i &\leq w(1 - s_t^i)(1 - u_t^i) \\ c_{t+1}^i &\leq s_t^i R w(1 - u_t^i) + w^M h_{t+1}^i \\ h_{t+1}^i &= (\delta + u_t^i)(\gamma h_t^{av} + h_t^i) \\ 0 &\leq s_t^i \leq 1; \quad 0 \leq u_t^i \leq 1. \end{aligned}$$

where h_t^i and $f_t(h^i)$ are given. The First Order Conditions can be expressed as:

$$-\frac{1}{1-u_t^i} + \beta \frac{w^M(h_t^i + \gamma h_t^{AV}) - r s_t^i w}{r s_t^i w(1-u_t^i) + w^M(\delta + u_t^i)(h_t^i + \gamma h_t^{AV})} \leq 0 \tag{18}$$

$$-\frac{1}{1-s_t^i} + \beta \frac{w(1-u_t^i)}{r s_t^i w(1-u_t^i) + w^M(\delta + u_t^i)(h_t^i + \gamma h_t^{AV})} \leq 0 \tag{19}$$

It is easy to verify that only corner solutions exist. For technical reasons that will become clear soon, we assume that $\beta > \delta$ throughout the rest of the work. The solution can be summarized as follows. Define the human capital 'endowment' of a young agent at time t as $\hat{h}_t^i \equiv \left(h_t^i + \gamma h_t^{AV}\right)$. Then:

- $\forall \{i,t\}$ such that $\hat{h}_t^i < h^* \equiv \frac{\beta w}{\delta w^M}$:

$$u_t^i = 0; \quad s_t^i = \frac{\beta w - \delta w^M \hat{h}_t^i}{(\beta + r)w} \tag{20}$$

- $\forall \{i,t\}$ such that $\hat{h}_t^i \geq h^* \equiv \frac{\beta w}{\delta w^M}$:

$$u_t^i = u^* = \frac{\beta - \delta}{1 + \delta}; \quad s_t^i = 0 \tag{21}$$

When the population is heterogeneous, it turns out that young agents coming from poor, uneducated families have less incentive to invest in education than those coming from better educated families. We will refer to h^*, the level of human capital endowment that makes workers indifferent between investing or not in education, as the 'critical level of human capital'.

We now determine at which conditions an individual dynasty accumulates human capital at a positive or negative rate in the time interval between one generation and its offspring. Remember that part of the process of transmission of knowledge depends on institutional features of an economy, like the establishment of a system of compulsory primary education (cfr. the parameter γ) and on the size of the aggregate human capital (h_t^{AV}). As a result, the degree of inequality, if we control for differences in the individual investments in education, tends to decrease as the stock of knowledge grows over successive generations. Some relatively poor dynasties may accumulate human capital at a positive rate even when their members do not invest in education. To the opposite, some relatively rich dynasties may decumulate human capital even when their members do invest in education. The precise conditions are stated in the following Lemma.

Lemma 1 *Define* $\underline{h}_t = \frac{\delta\gamma}{1-\delta} h_t^{AV}$, *and* $\overline{h}_t = \frac{1+\delta}{\beta-\delta}\gamma h_t^{AV}$.

Then:

- $\forall j$ *s.t.* $h_t^j < \underline{h}_t$, $\quad h_{t+1}^j > h_t^j$

- $\forall k$ *s.t.* $h_t^k > \overline{h}_t, \quad h_{t+1}^k < h_t^k$

PROOF - Consider an agent from dynasty j who is young at time t and chooses $u_t^j = 0$. From (16):

$$h_{t+1}^j = \delta\left(1 + \gamma\frac{h_t^{AV}}{h_t^j}\right)h_t^j$$

then: $h_{t+1}^j \geq (<) h_t^j \Leftrightarrow \delta\left(1 + \gamma\frac{h_t^{AV}}{h_t}\right) \geq (<) 1$

from which the first part of the Lemma is established.

The second part is established similarly, using the condition (16) for an agent who chooses $u = u^*$. □

To rule out uninteresting cases, we will impose parameter restrictions such that a society whose all members invest in education ($u_t^i = u^*$, $\forall i$) accumulates human capital, whereas a society in which no member invest in education ($u_t^i = 0$, $\forall i$), decumulates human capital. This is guaranteed by the following assumption.

Assumption 1 $\frac{1}{\delta} > (1+\gamma) = \left(\frac{1+\delta}{\beta+\delta^2}\right)\frac{1}{\theta}$
for some $\theta \in (0,1)$

One can check from (16) that this implies that the income of a homogeneous population ($h_t^i = h_t^{AV}$, $\forall i$) which chooses $u_t = u^*$ grows over time, whereas the income of a homogeneous population which chooses $u_t = 0$ falls over time. The role of the constant θ will become clear soon. A corollary which follows from this assumption is that all agents who invest in education and belong to a less educated dynasty than average will add to the stock of human capital of their parents, whereas all agents who do not invest in education and belong to a more educated dynasty than average will remain below the level of education/productivity achieved by their parents.

In a world with heterogeneous population, the relation between short-run and long-run equilibrium is less straightforward than that given by Proposition

1 in the second section. The state of the system is now defined by the entire distribution of human capital across dynasties, and, in general, we will observe at any moment some dynasties accumulating and some others decumulating human capital. The dynamics of average human capital is in general ambiguous unless the entire distribution of wealth is specified.

Still, we can construct some useful intuition. If the initial state of the system ($t = 0$) is such that a richer part of the population invests in education whereas a poorer part does not, we observe the tendency for the density function to accumulate about two levels of human capital which evolve over time. Consider, for example, a case in which agents with an original endowment h_0^* (the 'critical level of human capital') ends up with more human capital than the parents if they invest in education and less human capital than the parents if they do not invest (figure 5). Then, dynasties with an initial human capital endowment just below h_0^* will decline over time in terms of their stock of knowledge. However, very poor dynasties ($h_0^j < \underline{h}_0$) will increase their human capital stock by the effect of the social channel of transmission. Dynasties with an initial human capital endowment just above h_0^* will increase their human capital stock. Finally, very rich dynasties ($h_0^j > \overline{h}_0$) will experience negative accumulation.

FIGURE 5 here

Imagine that the resulting average human capital has grown from period zero to period one and keeps growing for the following periods. This causes a rightward shift over time of both $\underline{h}$ and $\overline{h}$ (figure 6). If the members of the 'poor' dynasties (i.e. those originarily below the critical level h_0^*) kept choosing $u_t = 0$ for all t and the rich dynasties kept choosing $u_t = u^*$ for all t, then the density function would converge to a two-spikes distribution such that the human capital and income of both groups grows at the same rate. The mass of the population, in other words, would concentrate at two growing points, 'close' to $\underline{h}_t$ and $\overline{h}_t$. This cannot be, however, the long-run outcome. At some point in time, there will be a generation whose all members (both 'rich' and 'poor') will find it optimal to invest in education. Since that moment, the income of the poor dynasties will grow faster than that of the rich dynasties, and the economy will converge to a steady-growth equilibrium in which the differences across dynasties tend to vanish.

FIGURE 6 here

According to this sketchy picture, earlier stages of development appear characterised by growing inequality. Later stages of development, however, are characterised by decreasing inequality , since poor dynasties invest the same in education as rich ones but gain more from the process of social transmission of knowledge. Nothing ensures, unfortunately, that this case is general, and one could construct other examples in which the average human capital always decreases over time or follows mixed patterns. It is possible, however, to identify a set of initial conditions which guarantees that the equilibrium dynamics necessarily converge to the 'good' long-run equilibrium. Let us define $F_t(\hat{h}) : R \rightarrow [0,1]$ as a generic distribution function of the young population over (inherited) human capital levels at time t. Consider the following Proposition.

Proposition 2 *If* $F_0(\hat{h}^j = h^*) < 1 - \theta \Rightarrow h_{t+1}^{AV} > h_t^{AV}\ \forall\, t \geq 1$ *and the economy converges to the long-run equilibrium with the maximum growth rate,* $g = (\delta + u^*)(1+\gamma)$, *where* $u^i = u^*$, $\forall i$.

PROOF - Consider (16). By integrating on both sides, over i we get:

$$\begin{aligned}
h_1^{AV} &= \left(\delta + \int_0^\infty u_0^i f_0(h^i)di\right)\left(\int_0^\infty h_0^i f_0(h^i)di + \gamma h_0^{AV}\right) \\
&= \delta(1+\gamma)h_0^{AV} + u^*(1 - F_0(\hat{h}^j = h^*))\gamma h_0^{AV} + u^*\left(\frac{\int_{h^*}^\infty h_0^i f_0(h^i)di}{h_0^{AV}} h_0^{AV}\right) \\
&\geq \delta(1+\gamma)h_0^{AV} + u^*(1 - F_0(\hat{h}^j = h^*))(1+\gamma)h_0^{AV} \\
&> \theta(\delta + u^*)(1+\gamma)h_0^{AV} > h_0^{AV}.
\end{aligned}$$

where the first inequality follows from the fact that $\frac{\int_{h^*}^{\infty} \hat{h}_0^i f_0(h^i)di}{h_0^{AV}} \geq (1 - F_0(\hat{h}^j = h^*))$, since the richest x per cent of the population holds at least x per cent of total wealth. The second inequality follows, instead, from the Assumption 1.

Since $h_1^{AV} > h_0^{AV}$, then $F_1(\hat{h}^j = h^*) \leq F_0(\hat{h}^j = h^*)$. The previous argument applies recursively for all $t \geq 1$, so $h_{t+1}^{AV} > h_t^{AV}, \forall\, t$.

Also, $h_{t+1}^{AV} > h_t^{AV} \Rightarrow \underline{h}_{t+1} > \underline{h}_t$, $\forall\, t$. Then $\exists\, T$ such that $\underline{h}_t > G$, $\forall\, t \geq T$. Since then, $\hat{h}_{t+1}^j > \hat{h}_t^j$, $\forall\, j$. Then, $\exists\, \tilde{T}$ such that $\hat{h}_t^j > G$, $\forall\, j$, $\forall\, t \geq \tilde{T}$. This implies that $u_t^i = u^*$, $\forall\, i$, $\forall\, t \geq \tilde{T}$. That $g \to (\delta + u^*)(1+\gamma)$ as $t \to \infty$ follows from (16). □

4 Heterogeneous population and the enforcement constraint

When the population is heterogeneous, the relation between political equilibrium and foreign investments acquires new dimensions. Agents are no longer unanimous in their political attitude. The 'rich' will find their interests represented by the liberal attitude of the party A, whereas the 'poor' will lobby for nationalisation and redistribution, supporting the party B. The solution of the electoral competition will be determined by the will of a decisive individual, the 'median voter'. As the intuition might suggest, if the median voter has a marginal preference for the party A over the party B, just more than half population, i.e. those who are at least as rich as him, will support the party A, and just less than half population, i.e. those who are less wealthy than him, will support the party B.

The characterisation of the political equilibrium is not trivial, though. The key issue is that agents take their decisions about education before the election, and these decisions affect their political choice. If a voter marginally prefers the party A after having invested in education, for instance, he would have come up with a preference for party B had he chosen, ceteris paribus, not to invest in personal education in the previous period. As in the previous sections, no outcome in which the foreign capital is seized is an equilibrium. In the equilibrium, foreign capitalists are careful enough to restrain the inflow of investments below the level that would trigger a 'bad' political outcome. However, when the population is heterogeneous it is possible to have interesting multiple equilibria in which all agents take time-consistent decisions.

To fix ideas, we construct a particular example in a game-theoretical framework, before moving to the general case. Consider a three-class society with an homogeneous politically decisive 'middle class' which has a positive measure in the total population. A representative middle class agent chooses, at time t, to invest (E) or not to invest (NE) a fixed amount of time in education depending on his expectations about the wages paid in the modern sector. The 'rich' and the 'poor' class find it optimal, respectively, to invest and not to invest time in education, and their decisions can be treated as exogenous and ignored. This allows us to discuss the model in the form of a two-player game between the middle class and the foreign investors. The latter choose high (H) or low (L) investment in tradable resources on the basis of their expectations about the political equilibrium. If they choose H (L), high (low) wages are paid in the modern sector in the second period[5]. With an opportune choice of the pay-offs[6] it is the case that the middle class, if it chooses E in the first period, always finds it optimal to support the party A in the second stage and the property rights are safeguarded. But if it chooses NE in the first period, it only supports the party A in the second stage if L is chosen by foreign investors. Otherwise it supports the party B, since the potential gain from seizing the foreign resources is larger than the benefit from working in the modern sector when H occurs. Figure 7 gives the extensive form representation

[5]In fact, the wages are also affected by the investments in education, being decreasing with the social stock of human capital. In this example we will neglect this second effect, with the motivation that in the general case it is never entirely offsetting (namely the wages given H and E are always larger than the wages given L and NE) when the measure of the middle class is less than that of the entire population.

[6]The numerical payoffs chosen are arbitrary, but are coherent with the assumptions of the model, as it will be shown more precisely in section 7.

of the game. Notice that only the political decision of the local people is taken with perfect information about the opponent's choice, whereas all investment decisions in the first stage are taken without knowledge of the behaviour of the other 'player'. This motivates the dashed oval in the figure (according to usual conventions this means that the representative foreign investor ignores the first-stage choice of the middle class when he takes his decision).

FIGURE 7 here

It can be checked, by using backward induction, that this game has two subgame perfect Nash Equilibria, given by the sequences of actions $((E, H), A)$ and $((NE, L), A)$, respectively. More precisely, the two equilibrium strategy pairs are:

- Foreign investors choose H; middle class agents choose E in the first stage and vote for the party A under any circumstance in the second stage;
- Foreign investors choose L; middle class agents choose NE in the first stage and vote for the party B conditional on H (information set not reached at equilibrium) and for party A conditional on L (information set reached at equilibrium).

The first Nash Equilibrium dominates the second one in welfare terms.

Having built the intuition through the example, we move now to show that the case for multiple equilibria does not depend on the particular distribution of human capital assumed in this example. First, we discuss how the enforcement constraint is modified by the heterogeneity of the population. Before taking their decisions, foreign investors speculate about the political equilibrium which will prevail in the following period, that is to say about the attitude of the median voter in the next election. By maintaining the assumption that if the capital is seized, the income obtained is equally distributed within the local population[7] equation (12) becomes:

$$w^M h_t^{med} \geq \psi k_t \tag{22}$$

When (22) holds, there is a majority in the country which supports the party A and no seizure occurs. In an equilibrium, foreign investors choose a level of k such that (22) holds with equality. The wage rate per efficiency unit in the modern sector keeps being determined by the marginal productivity of human capital, meaning that $w_t^M = \alpha \left(\frac{k_t}{h_t^{AV}}\right)^{(1-\alpha)}$. Using this condition, we can rewrite (22) - taking equality - as:

$$\alpha \left(\frac{k_t}{h_t^{AV}}\right)^{(1-\alpha)} h_t^{med} = \psi k_t \tag{23}$$

It is clear from this expression that we no longer have a time-invariant $\frac{k}{h}$ ratio like in (8), and that neither the wage in the modern sector, w^M, nor the 'critical level of human capital', h^*, are now constant and time-invariant.

As the game-theoretical example suggested, we have two types of candidate equilibria. We will call *equilibrium of type 1* an equilibrium in which the median voter chooses to invest in education, and *equilibrium of type 2* one in which the median voter does not invest in education. In each type of equilibrium all agents adopt optimal, time-consistent rules. This implies, amongst the other things, that under no circumstance does the decisive agent vote for the party B after investing in education at the previous period. Some people (i.e. the 'poor') may be dissatisfied with the political equilibrium which prevails, but all agents are 'realistic' enough to take decisions which are optimal on the basis of the effective political outcome rather than on their political 'desires'. In other words, the political equilibrium is always perfectly anticipated by all agents. Some non-decisive agents might choose to invest in education at time

[7] A natural extension could be to allow party B to device a redistribution schedule which maximises its chances of winning the election. This would imply, informally speaking, to redistribute nothing to the richest, whose support would be very costly to achieve, little to the poorest, whose support is easily bought, and most of the resources seized to the 'marginal voters'. This policy would make tougher the electoral task for party A and more serious the effects of the incentive constraint on investments.

t based on the (correct) expectations about the one period-ahead wage rate in the modern sector, and then, at time $t+1$, support the party B, because the nationalisation would still make them better off. The event of a victory of the party B would be welcome by such agents, though it would make them regret about the choice of youth. This inconsistency will never show up, anyway, since the political equilibrium is always in favour of the party A.

Let us define formally the two types of equilibria.

Definition 1 *An equilibrium is characterised by the following conditions*[8]*:*

$$\alpha \left(\frac{k_{t+1}}{h_{t+1}^{AV}}\right)^{(1-\alpha)} h_{t+1}^{med} = \psi k_{t+1} \tag{24}$$

$$\alpha \left(\frac{k_{t+1}}{h_{t+1}^{AV}}\right)^{(1-\alpha)} h_t^* = \frac{\beta w}{\delta} \tag{25}$$

$$\int_0^{h_t^*} \delta \hat{h}_t^i f(\hat{h}_t^i)\, di + \int_{h_t^*}^{\infty} (\delta + u^*) \hat{h}_t^i f(\hat{h}_t^i)\, di = h_{t+1}^{AV} \tag{26}$$

An equilibrium is said to be of type 1 iff:

$$u_t^{med} = u^* \tag{27}$$

$$h_t^* \leq \hat{h}_t^{med} \tag{28}$$

and is said to be of type 2 iff:

$$u_t^{med} = 0 \tag{29}$$

$$h_t^* \geq \hat{h}_t^{med} \tag{30}$$

The condition (25) is the familiar definition of 'critical level' of human capital endowment. Notice from (26) that h_{t+1}^{AV} is a decreasing function of h_t^*, since the higher h_t^* the lower the proportion of people who invests in education at time t in equilibrium. In the characterisation of each type of equilibrium, the time-consistency requisites are expressed by (28) and (30), respectively. The former says that should the median voter invest in education, the ex-post

[8]We remind that $\hat{h}_t^i \equiv h_t^i + \gamma h_t^{AV}$ is the capital-endowment of an agent who is young at time t, apologizing with the reader for the proliferation of notation

'critical level of human capital' determined by the equilibrium wage rate in the modern sector, cannot be higher than the endowment of the median voter, $\hat{h}_t^{med}$; otherwise, the median voter would regret about his choice of investing in education. The opposite must hold true should he not invest in education when young. Given the distribution of human capital across dynasties at time t, the equilibrium solution(s) is (are) found, by solving (24), (25) and (26) for the endogenous variables h_t^*, h_{t+1}^{AV} and k_{t+1} and checking whether either of the pairs of conditions (27)-(28) and (29)-(30), or both, hold.

The main results of this section are summarized by the following Proposition. It establishes that, for any distribution of human capital endowment, there exists (at least) one equilibrium outcome determined by the choices of a generation of indigenous agents (investment in education and political election) and foreign investors. It also establishes that multiple equilibria exist for a non-trivial set of distributions of human capital.

Proposition 3 *Let H be the space of continuous distribution functions over inherited human capital, $F(\hat{h})$. Then for any $F \in H$ there exists an equilibrium as characterised by Definition 1.*

Let $D(F, F^*) \equiv \mid \int_0^\infty F(\hat{h}) - F^*(\hat{h})d\hat{h} \mid$ be a metric on H. Let $N(F^{**}, \epsilon)$ be an ϵ-neighbourhood of F^{**}, i.e. $N(F^{**}, \epsilon) \equiv \{F \in H \mid D(F, F^{**}) < \epsilon)\}$. Let $A \subset H$ be the subset of distributions such that both a type 1 and a type 2 equilibrium exist, and let A° be the interior of A. Then A is non-empty, and for any distribution $\tilde{F} \in A^\circ$ there exists a $\delta > 0$ such that $N(\tilde{F}, \delta) \subset A^\circ$.

PROOF. Rewrite (26) as $h_{t+1}^{AV} = h(h_t^*, f_t(h))$, where $h_1 < 0$. Then, rearrange (24) and (25) to obtain:

$$k_{t+1} = \left(\frac{\alpha}{\psi}\right)^{\frac{1}{\alpha}} h(h_t^*, .)^{\frac{\alpha-1}{\alpha}} (\delta + u_t^{med})\hat{h}_t^{med} \tag{31}$$

$$k_{t+1} = \left(\frac{\beta w}{\alpha\delta}\right)^{\frac{1}{1-\alpha}} (h_t^*)^{\alpha-1} h(h_t^*, .) \tag{32}$$

and, by eliminating k_{t+1}:

$$\left(\frac{\alpha}{\psi}\right)^{\frac{1}{\alpha}}\left(\frac{\alpha\delta}{\beta w}\right)^{\frac{1}{1-\alpha}}(\delta+u_t^{med})\hat{h}_t^{med}=(h_t^*)^{\alpha-1}h(h_t^*,.)^{\frac{1}{\alpha}} \tag{33}$$

which we rewrite as:

$$\Gamma\;(\delta+u_t^{med})\hat{h}_t^{med}=\zeta\,(h_t^*,f(h_t))\ \ ,\ \ \zeta_1\,(h_t^*,f(h_t))<0 \tag{34}$$

where Γ is a constant term. From (34) it is clear that in equilibrium, given the state variables $f(h_t)$ and $\hat{h}_t^{med}$:

$$(h_t^*\mid u_t^{med}=u^*)<(h_t^*\mid u_t^{med}=0) \tag{35}$$

Now, assume that a type 1 equilibrium does not exist. This implies that

$(h_t^*\mid u_t^{med}=u^*)>\hat{h}_t^{med}$

But then, from the inequality (35):

$(h_t^*\mid u_t^{med}=0)>\hat{h}_t^{med}$.

So, an equilibrium of type 2 exists.

Suppose, to the opposite, that a type 2 equilibrium does not exist. This implies:

$(h_t^*\mid u_t^{med}=0)<\hat{h}_t^{med}$.

But then, again from (35):

$(h_t^*\mid u_{t-1}^{med}=u^*)<(h_t^{med}\mid u_{t-1}^{med}=u^*)$.

So, an equilibrium of type 1 exists. This completes the proof of the first part of the Proposition.

To prove the second part, observe, using (34), that, given $\hat{h}_t^{med}$, it is always possible to choose a density function $\tilde{f}(h_t)$ (and a corresponding distribution function $\tilde{F}(h_t)$) having the given median such that:

$$(\tilde{h}_t^*\mid u_t^{med}=u^*)<\hat{h}_t^{med}<(\tilde{h}_t^*\mid u_t^{med}=0) \tag{36}$$

where $\tilde{h}_t^*$ is the value of h^* originating from the distribution $\tilde{F}$. So, A is non-empty. Also, the continuity of (34) ensures that for any function $F\in N(\tilde{F},\delta)$

and some $\delta > 0$ there exists a condition analogous to (36) which continues to be satisfied. $\square$

Figure 8 gives an intuitive geometric argument. The locus (1) represents the positive association between k_{t+1} and h_t^* described by the enforcement condition (31), whereas the locus (2), negatively sloped, represents the condition (32). Notice that u_t^{med} shifts schedule (1), whereas it does not affect the schedule (2). When $u_t^{med} = u^*$ (type 1 equilibrium), there are more foreign investments and higher wages in the modern sector (lower h_t^*) than when $u_t^{med} = 0$ (type 2 equilibrium). In the case represented, multiple equilibria exist.

FIGURE 8 here

5 Transitional dynamics and long-run equilibrium.

The analysis of the transitional dynamics towards a long-run equilibrium is complicated here by the fact that h^* is not time-invariant. So, we should keep track not only of the dynamic path of h_t^{AV} but also of that of h_t^*. The conditions given by Proposition 2 are no longer sufficient to ensure that convergence to a 'good' long-run equilibrium occurs. It turns out that, in fact, sufficient conditions for the economy to converge to the high growth long-run equilibrium can still be found, but they are more restrictive and involve 'distributional issues'.

It is convenient to write explicitly the relation between the 'critical level of human capital' at time t and the ratio between the median and the average human capital at time $t+1$ which holds in equilibrium. By eliminating k_{t+1} from (24) and (25), and rearranging, we obtain:

$$h_t^* = \left(\frac{\beta w \psi^{\frac{1-\alpha}{\alpha}}}{\delta \alpha^{\frac{1}{\alpha}}} \right) \left(\frac{h_{t+1}^{AV}}{h_{t+1}^{med}} \right)^{\frac{1-\alpha}{\alpha}} \tag{37}$$

The following Lemmas establish important facts for proving the main result.

Lemma 2 *If in the equilibrium $u_t^{med} = u^*$ (type 1 equilibrium), and $h_t^{med} < h_t^{AV}$, then $h_t^* < h_{t-1}^*$.*

If in the equilibrium $u_t^{med} = 0$ (type 2 equilibrium), and $h_t^{med} > h_t^{AV}$, then $h_t^* > h_{t-1}^*$.

PROOF - First part. Under the assumptions of the first statement, the equation (16) implies that $\frac{h_{t+1}^{med}}{h_t^{med}} > (\delta+u^*)(1+\gamma)$, and that $\frac{h_{t+1}^{AV}}{h_t^{AV}} \leq (\delta+u^*)(1+\gamma)$. So, $\frac{h_{t+1}^{AV}}{h_{t+1}^{med}} < \frac{h_t^{AV}}{h_t^{med}}$, which implies, by (37), $h_t^* < h_{t-1}^*$.

Second part. Under the assumptions of the second statement, the equation (16) implies that $\frac{h_{t+1}^{med}}{h_t^{med}} < \delta(1+\gamma)$, and that $\frac{h_{t+1}^{AV}}{h_t^{AV}} \geq \delta(1+\gamma)$. So, $\frac{h_{t+1}^{AV}}{h_{t+1}^{med}} > \frac{h_t^{AV}}{h_t^{med}}$, which implies, by (37), $h_t^* > h_{t-1}^*$. □

Definition 2 *Let G be defined as h_t^* evaluated at an equilibrium in which $h_{t+1}^{med} = h_{t+1}^{AV}$. Then, from (24), (25) and (37):*

$$G \equiv \frac{\beta w \psi^{\frac{1-\alpha}{\alpha}}}{\delta \alpha^{\frac{1}{\alpha}}}.$$

Lemma 3 *If a type 1 equilibrium occurs at time T and $h_T^{med} > h_T^{AV}$, then $h_{T+1}^{med} > h_{T+1}^{AV}$. Hence, if only type 1 equilibria occur for $t > T$, and $h_T^{med} > h_T^{AV}$, then $h_t^* < G$, $\forall\, t \geq T-1$.*

If a type 2 equilibrium occurs at time T and $h_T^{med} < h_T^{AV}$, then $h_{T+1}^{med} < h_{T+1}^{AV}$. Hence, if only type 2 equilibria occur for $t > T$, and $h_T^{med} < h_T^{AV}$, then $h_t^* > G$, $\forall\, t \geq T-1$.

PROOF - For the first part, apply directly (16) and obtain:

$$\begin{aligned} h_{T+1}^{med} &- h_{T+1}^{AV} = (\delta+u^*)(h_T^{med} + \gamma h_T^{AV}) - \\ &- \delta(1+\gamma)h_T^{AV} - u^*\left(\int_{h_T^*}^{\infty} h^i f(h^i)di + (1 - F_T(\hat{h} = h_T^*))\gamma h_T^{AV}\right) = \\ &= F_T(\hat{h} = h_T^*)\gamma h_T^{AV} + \delta(h_T^{med} - h_T^{AV}) + u^*\left(h_T^{med} - \int_{h_T^*}^{\infty} h^i f(h^i)di\right) \end{aligned}$$

which is greater than zero if $h_T^{med} > h_T^{AV}$ (notice that the integral is smaller than h_T^{AV}). This implies from (37) that $h_T^* < G$. The rest of the statement follows from the recursive structure of the result.

For the second part, analogously:

$$\begin{aligned} h_{T+1}^{med} - h_{T+1}^{AV} &= \delta(h_T^{med} + \gamma h_T^{AV}) - \\ &- \delta(1+\gamma)h_T^{AV} - u^* \left(\int_{h_T^*}^{\infty} h^i f(h^i) di + (1 - F_T(\hat{h} = h_T^*))\gamma h_T^{AV} \right) = \\ &= \delta(h_T^{med} - h_T^{AV}) - u^* \left(\int_{h_T^*}^{\infty} h^i f(h^i) di + (1 - F_T(\hat{h} = h_T^*))\gamma h_T^{AV} \right) \end{aligned}$$

which is smaller than zero if $h_T^{med} > h_T^{AV}$. Then, again, apply recursion. □

Figure 9 gives a visualisation of the facts involved in the Lemmas. When only type 1 equilibria occur for $t \geq 0$, we have two possible cases: either $h_0^* < G$ ($h_0^{med} > h_0^{AV}$) - like in the case represented in the figure -, and G bounds from above h_t^* the dynamics of h^* being in general ambiguous, or $h_0^* > G$ ($h_0^{med} < h_0^{AV}$), and $\dot{h^*}_t < 0$, at least as far as $h_t^* \geq G$. The opposite happens when only type 2 equilibria occur.

FIGURE 9 here

The following Proposition uses the Lemmas 2 and 3, and jointly gives conditions for a sequence of type 1 equilibria to exist. The result is a generalisation of the Proposition 2.

Proposition 4 *Let $F_0(\hat{h})$ be the distribution function over initial human capital endowments. If the initial conditions are such that:*

- *either* (i) $h_0^{med} \geq h_0^{AV} \geq \frac{G}{1+\gamma}$, *and* (ii) $F_0(\hat{h} = G) < 1 - \theta$
- *or* (i) $h_0^{AV} < h_0^{med}$, (ii)$\hat{h}_0 > h_0^*$, *and* (iii) $F_0(\hat{h} = h_0^*) < 1 - \theta$

then there exists a path characterised by a sequence of type 1 equilibria such that $h_{t+1}^{AV} > h_t^{AV}$, $\forall\, t \geq 1$, which converges to the long-run equilibrium with the maximum growth rate, where $u^i = u^*$, $\forall\, i$.

PROOF - Consider the first set of conditions. The Lemma 3 ensures that G is an upper bound to h_t^*, if a sequence of type 1 equilibria occurs for $t \geq 0$. The strategy of the proof is similar to that of the Proposition 2, with the upper bound G being treated similarly to the fixed h^*. The condition (i) guarantees that a type 1 equilibrium exists[9] in the first period: since $h_0^{med} > h_0^{AV}$, then $\hat{h}_0^{med} > (1+\gamma)h_0^{AV} > G \geq h_0^*$. The condition (ii) guarantees that $h_1^{AV} > h_0^{AV}$: since $G \geq h_0^*$, then $F_0(\hat{h} = h_0^*) \leq F_0(\hat{h} = G) \leq 1 - \theta$.

Then, verify that the problem has a recursive structure and that *(a)* $h_{t+1}^{AV} > h_t^{AV}$, $\forall\, t \geq 1$, and *(b)* type 1 equilibria exist for all $t \geq 1$. To verify (a), observe that $F_1(\hat{h} = G) \leq F_0(\hat{h} = G) < 1 - \theta$, since the offspring of all agents with a human capital endowment $\hat{h}_0^j > G$ certainly has $\hat{h}_1^j > G$, whereas that of some agents with $\hat{h}_0^j < G$ may have $\hat{h}_1^j > G$. This ensures that $h_1^{AV} \geq h_0^{AV}$. The last inequality, together with (i) implies that (b) necessarily holds true for $t = 1$, so a type 1 equilibrium also exists at $t = 1$. The argument applies recursively for $t > 1$.

The rest of the proof is identical to that provided in the Proposition 2.

Consider the second set of conditions. In this case, the Lemma 2 ensures that, if a sequence of type 1 equilibria occurs, then $h_{t+1}^* < h_t^*$, $\forall\, t \geq 0$. The conditions (ii) and (iii) ensure that a type 1 equilibrium exists in the first period, and that $h_1^{AV} > h_0^{AV}$. The rest of the prove is based on the recursive structure of the problem and is analogous to that given for the first part. □

To give sufficient conditions for convergence to the 'bad equilibrium', in which no activity in the modern sector exists, is less straightforward. The technical difficulty lies in finding simple conditions which guarantee that the average human capital falls over time when a sequence of type 2 equilibria

[9] It is possible to find less restrictive conditions, but at the cost of going through a list of algebraically intriguing and uninteresting different cases. Notice that in this case it is not sufficient to impose $\hat{h}_t^{med} > G$ when $t = 0$, in order to ensure that it holds also true for $t \geq 1$, since there is no restriction which ensures that $\hat{h}_t^{med}$ grows over time. In the second set of conditions this is instead sufficient, since $h_t^{med} < h_t^{AV}$, where h_t^{AV} grows over time along the equilibrium path, and h_t^{med} must grow, in a type 1 equilibrium, faster than the average human capital.

arise. It is intuitive, however, that the occurrence of a type 2 equilibria at the initial time tends to be associated with the future decline of an economy.

6 Policy implications.

Two main points are identified by the previous discussion. On the one hand high initial levels of human capital are a favourable condition to avoid the lock-in into a stationary equilibrium. The existence of thresholds in the accumulation of human capital had been already revealed by the study of the basic case with homogeneous population. On the other hand, the lower the human capital of the median voter compared with the average human capital, the higher the total amount of human capital that the society needs to be endowed with in order to sustain a type 1 equilibrium and to take-off into sustained growth. This point emerges as a non-obvious effect of the relation between political equilibrium, property right enforcement and profitability of individual investments in education.

To study this point more closely, consider some economies which are identical in all but the distribution of human capital. It is clear that a very unequal distribution is not a good pre-condition for growth. Take the extreme case in which more than half the population has no human capital at all, whereas high competence and skills are concentrated in the hands of a minority. In this case, there will be no development of the modern sector, because the political pressure for seizure and redistribution of the foreign resources would be overwhelming should any positive amount of foreign investment enter the country. On the other hand, a perfectly egalitarian society in which all people have the same amount of human capital, though a more favourable environment, is not the best situation for activating the growth process. Imagine that the initial aggregate stock of human capital is too low for promoting sustained growth in an egalitarian society. Still, it is possible that a less egalitarian society, in which the same total human capital is concentrated in the hands of a *majori-*

tarian group, can sustain a take off process. The reason is that in this society there is a firmer support for property rights enforcement from the dominant class which, by squeezing the interests of the 'poor', is consistent with a higher inflow of foreign investments, higher wages in the modern sector and a higher rate of human capital accumulation. The 'ideal' distribution[10] of a given stock of knowledge would be to give (just more than) half the population all the human capital equally distributed among the members and the remnant (just less than) half population nothing at all.

We can try to relate this rather abstract discussion to more realistic scenarios, in order to draw meaningful policy implications. Assume that in a less developed country there exists an elite with a relatively high endowment of knowledge or capabilities and a large majority of uneducated unskilled people. Should the government use its budget for education for the elite to acquire higher human capital, say by paying grants for studying abroad? Or should it give priority, to the opposite, to plans for large-scale basic literation of the mass of uneducated people? According to the model, as far as pure growth objectives are concerned, neither policy is ideal. If the first policy is adopted, the few highly educated people are likely never to find good opportunities at home, because the political environment will remain unfavourable to foreign investments and modernization. Possibly, they will be induced to spend their competence abroad. If the second policy is adopted, the government budget may get too much 'disperse' to produce significant effects. The expenditure should target the formation of a substantial, majoritarian middle class, whose interests are coincident with those of foreign investments and modernization. The change in the social structure puts the basis for credible protection of property rights, increase of salaries in the modern sector and large 'spontaneous' investments in human capital from the current and future generations.

The model is viable to a 'right-wing' interpretation as providing the ratio-

[10]Obviously, we mean 'ideal' for a social planner who aims at maximizing the growth rate of the economy, without any regard for equity or other welfare considerations.

nale to a period of non-democratic rules, with the repression of the political rights of the lower classes at an early stage of development. Even more, it seems to suggest that a government controlled by agents of foreign capital is the most desirable thing for the growth perspectives of a less developed country, because it creates a powerful instrument of pre-commitment which favours foreign investments. To defend this argument from ethically-based criticism, one could argue that a non-democratic 'oligarchic' government would operate in fact on behalf not only of the alive wealthy groups (and foreigners), but also of the future generations whose welfare is affected by the accumulation of human capital and which would be damaged by the 'selfishness' of a democratic government. However, repressive policies are always subject to the uncertainty that political outbreaks occur, and are likely not to be the best option. An opposite political interpretation, more in tune with the views of the author, is that the model indicate to the 'liberal party' (A) the need of being sensitive to social issues and introducing in its electoral programme some redistributive policies. In particular, it would be advisable to tax at a highly progressive rate the incomes earned by the richest groups of the local population in the modern sector and rebate them lump-sum. Tax schedules can be designed in such a way that they hit the most wealthy groups, but not the middle class (i.e. the median voter), and they do not affect the amount of investments in education (the rich finds it optimal to invest in education even if his income is going to be taxed up to some extent). The perspective of income redistribution enlarges on the other hand the share of supporters of property right enforcement at a given level of foreign investments, and makes more foreign investments enter the country in equilibrium. If we insist on the interpretation of the liberal party as an agency controlled by international investors, we could also argue that it might be optimal for them to accept (by introducing a populistic note in the electoral programme of party A) a certain degree of taxation, so as to enlarge the support to the defense of property rights and reduce the extent to which foreign investments need to be 'rationed' due to the enforcement problem.

7 Long-run equilibria and historical accidents. Pallas-land vs. Lotus-land.

We have shown that multiple equilibria are an intrinsic feature of our model. Should we believe that structurally identical countries which select different equilibria at some stage of their history are destined to converge to alternative long-run equilibria? In other words, do historical accidents have long-lasting consequences? We will show that according to our model two countries that have achieved altogether different levels of economic development may have had, at some past stage, identical conditions and opportunities. Models which contain this prediction have been discussed in the previous chapters (see also Krugman, 1991, and Matsuyama, 1991). The main difference of the model of this chapter is that it does not need any type of technological non-convexity. Rather, it is the intergenerational externality in the accumulation of human capital that plays a central role.

In this section, we specify the initial distribution of knowledge and we give an example of non-uniqueness of the long-run outcome for given initial conditions. We leave to future research a more detailed study of the issue and the derivation of more general analytical results.

Consider the three-class economy in whose framework we have discussed the game of a previous section. A fourth of the population belongs to the 'poor' class, a fourth to the 'rich' class, and half of the population belong to the middle class. Each class consists of identical individuals. Though special the case is, we believe that a large class of distributions (particularly, symmetric and lowly skewed) would generate similar dynamics.

At time $t = 0$, the old middle class agents are two times as productive, and the rich three times as productive as the poor. The productivity of the poor is normalised to one. The other parameters are chosen as follows:

$\beta = 0.875,\ \delta = 0.5,\ w = 1.714,\ \alpha = 0.5,\ \psi = 0.25$

It is easy to check that, with this parameter choice, the following relations hold:

$u^* = 0.25,\ \ w^M_{t+1} = \frac{h^{med}_{t+1}}{h^{AV}_{t+1}},\ \ h^*_t = 3\frac{h^{AV}_{t+1}}{h^{med}_{t+1}}$

where the expression for the 'critical level of human capital' is easily derived from (37). The representative member of the middle class is, obviously, the median voter.

When the first generation, alive at time $t = (0, 1)$, is considered, there are two equilibria. In the type 1 equilibrium, the middle class and the rich invest in education, there is a high level of foreign investments, the wage rate per efficiency unit is $w^M_1 = 1.06$, and the critical level of human capital endowment is $h^*_0 = 2.73 < \hat{h}^{med}_0 = 3$. This implies that it is optimal for both the rich and the middle class (not for the poor) to invest in education in the first stage. As a result, the average productivity grows from 2 at time $t = 0$ to 2.125 at time $t = 1$. In the type 2 equilibrium, instead, the middle class does not invest in education, and the equilibrium is characterised by less foreign investments, a lower wage rate per efficiency unit ($w^M_1 = 0.86$), and a critical level of human capital endowment of $h^*_0 = 3.5 > \hat{h}^{med}_0 = 3$. So, in this case it is optimal to invest in education only for the rich ($3.5 < 4$), and the average productivity falls from 2 to 1.75.

If alternative sets of self-fulling expectations are viable in the first period, the future faced by the following generations is uniquely determined. Imagine that two countries, Pallas-land and Lotus-land[11], shared identical conditions, like those just described, at the beginning of their history, but in Pallas-land the type 1 equilibrium occurred in the first period, whereas in Lotus-land the type 2 equilibrium occurred. In the second period, the Palladiensis face a unique

[11]In Greek mythology, Pallas was the goddess of sciences. Lotus was a legendary plant inducing luxurious languor when eaten. In a well-known episode of the 'Odyssey', Ulysses and his mates land at the island of the Lotus-eaters, and risk loosing the recall of the fatherland Ithaca by eating the flower.

type 1 equilibrium. One can check that neither a type 1 equilibrium in Lotus-land, nor a type 2 equilibrium for Pallas-land exist. Pallas-land is destined to the route of progress and development, Lotus-land to an inexorable decline.

Let us follows the path taken by these two economies (figures 10 and 11). In Pallas-land, the first years are characterised by an irresistible escalate of the middle class, with the productivity and income of both the poor and the rich growing only moderately (figure 10.a). At the sixth generation, the gap between the rich and the poor is almost unchanged (in fact it is slightly wider) with respect to the initial difference, whereas the middle class has almost caught-up with the rich group. Since then, the social redemption of the lower class starts. At the ninth generation, the gap between the richest and the poorest is less than half as much it used to be, and continues to fall in the following periods, as the figure shows. The figure 10.b shows why this happens. At the seventh generation, also the poor start investing in education and their income grows at the highest rate.

In Lotus-land, to the opposite, the society continuously forgets something of what their ancestors could do[12]. As far as the first couple of generations is concerned, rich people do invest in education, and their productivity declines more slowly than that of the other classes. However, since the third generation, all the inhabitants of Lotus-land give up devoting time to education and the decline is generalised (figures 11.a and 11.b)

FIGURES 10.a, 10b, 11.a, 11.b here

[12]If one finds this possibility unrealistic, he can think that some type of knowledge gets in fact obsolete as time goes on, and can no longer be productively used together with foreign capital in a modern industrial sector.

8 Appendix

We prove here that $s'(h_t) < 0$ for interior solutions in the basic model. This fact is used in the proof to Proposition 1. First, substitute (4) into u_t^b in the left hand-side of (2) (taken with equality) and get:

$$w_{t+1}^M h_t[\beta(1-b)-u_t(1+\beta(1-b))-\delta u_t^b] = s_t\left[\frac{(1-b)w_{t+1}^M h_t}{wR}R(1-u_t)(1+\beta)w\right]. \tag{38}$$

Then, rearrange to obtain:

$$s_t = \frac{\beta(1-b)-u_t[1+\beta(1-b)]-\delta u_t^\beta}{(1-b)(1-u_t)}. \tag{39}$$

By differentiating the last expression, it turns out that:

$$\frac{ds_t}{du_t} = -\frac{(1-u_t)\left(1+b\delta u_t^{b-1}\right)+u_t+\delta u_t^b}{(1-u_t)(1+\beta)(1-u_t)^2} < 0. \tag{40}$$

Since from (4) it follows that $u'(h_t) > 0$, then we have proved that in an interior solution $s'(h_t) < 0$.

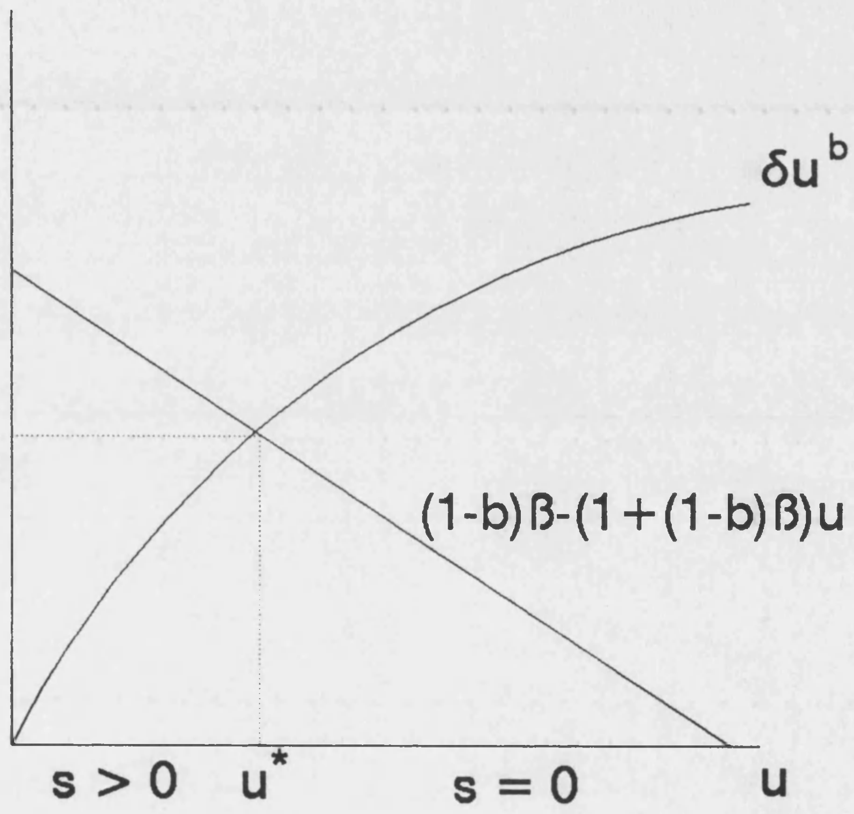


Figure 1

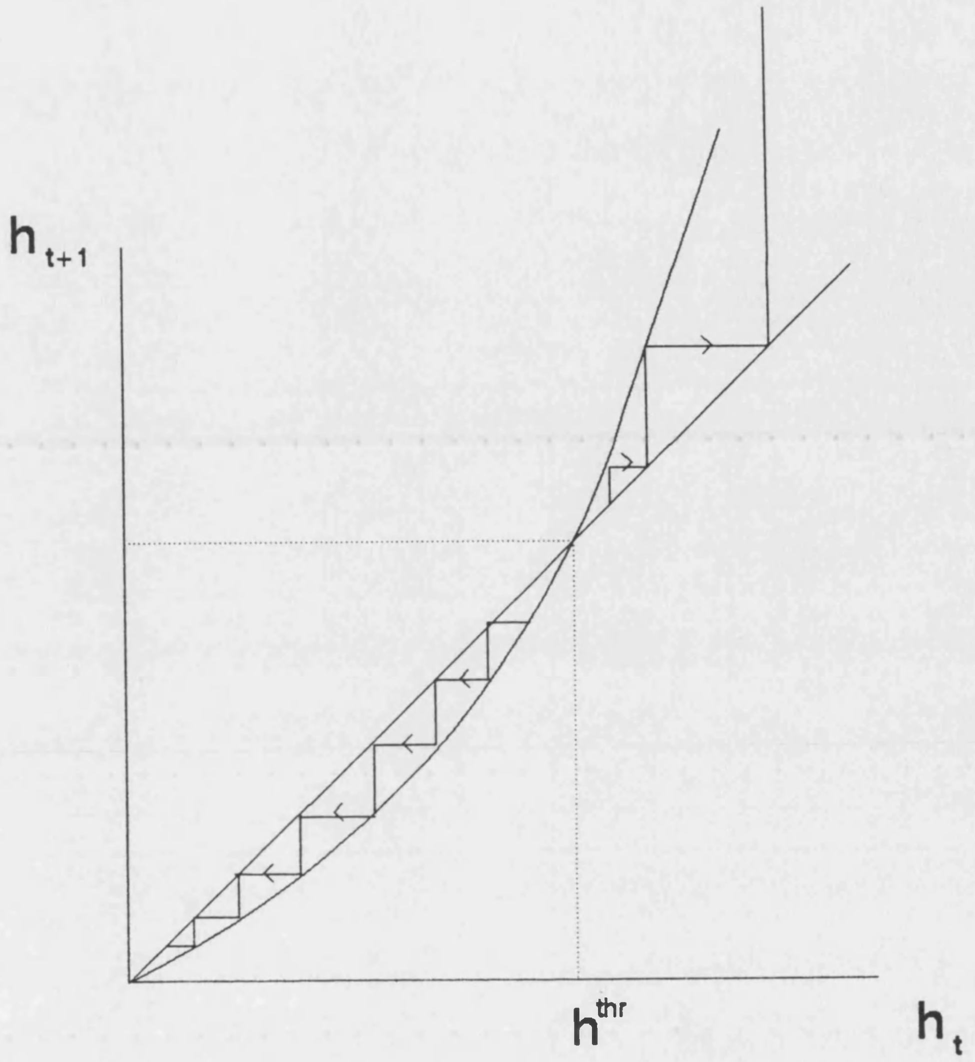


Figure 2

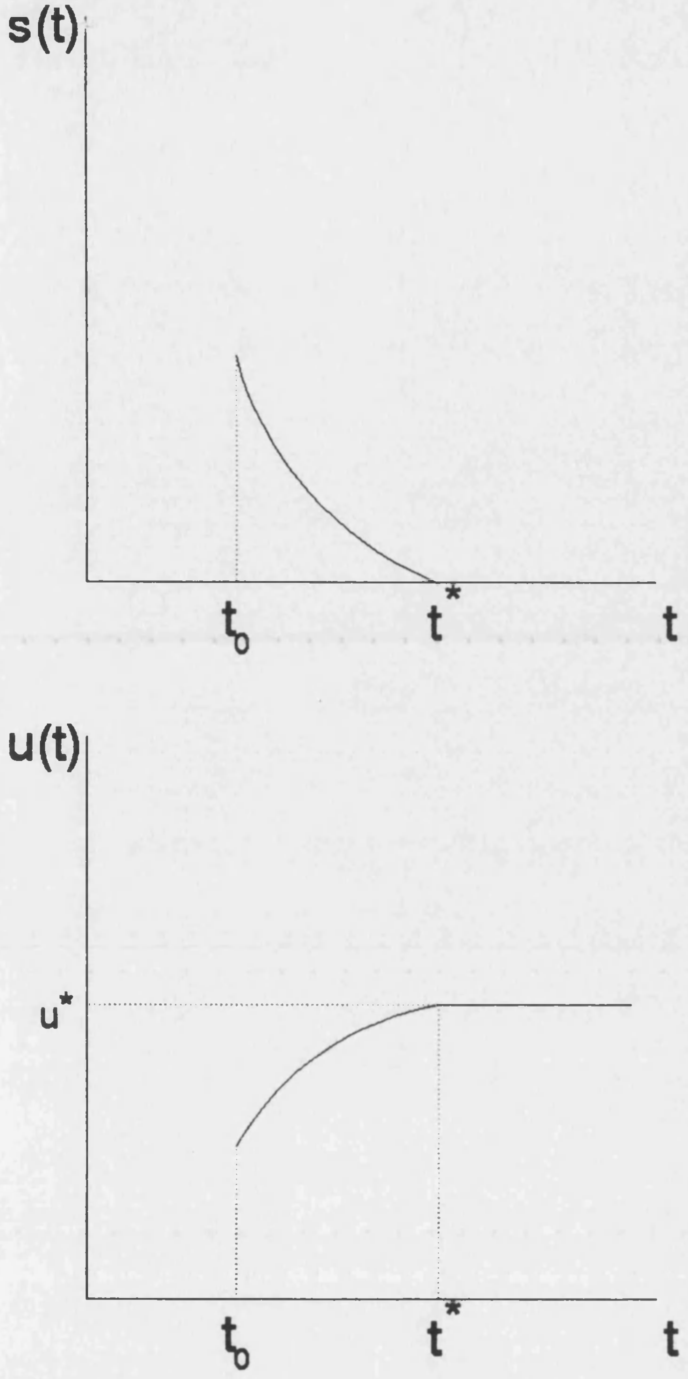


Figure 3

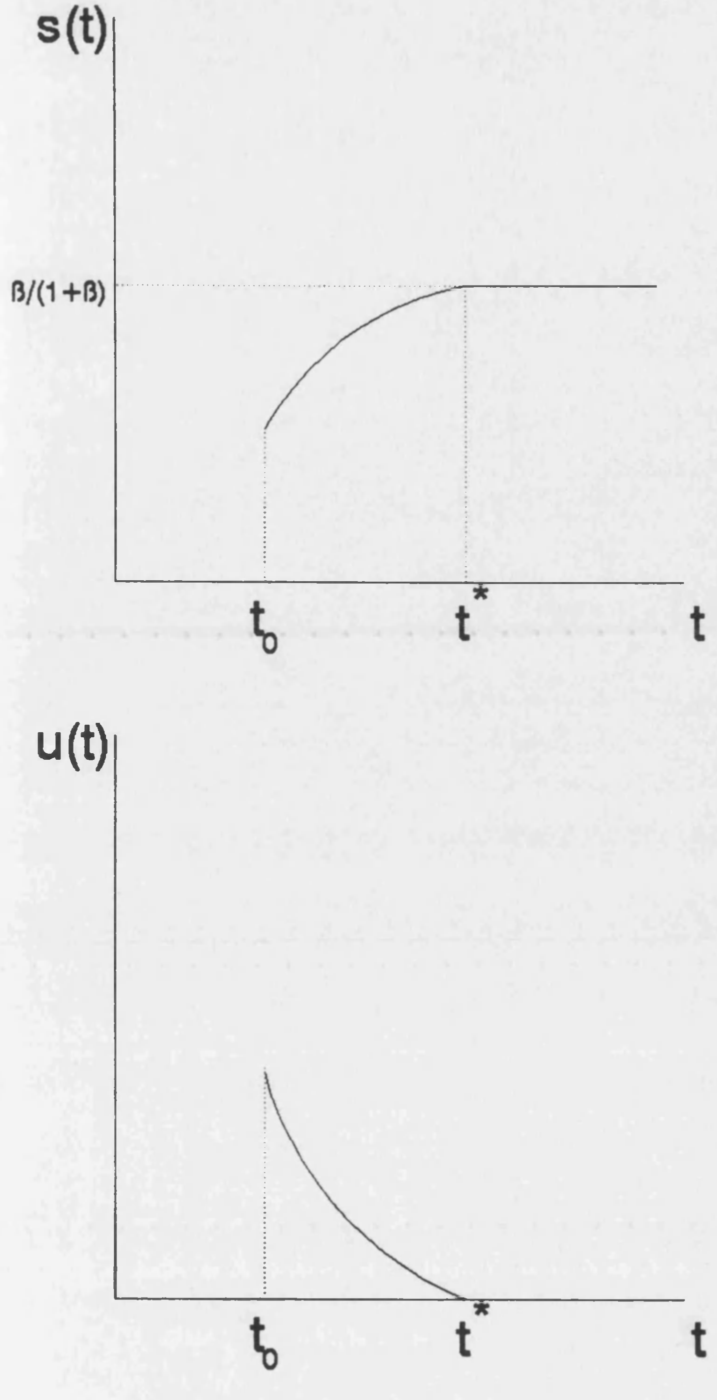


Figure 4

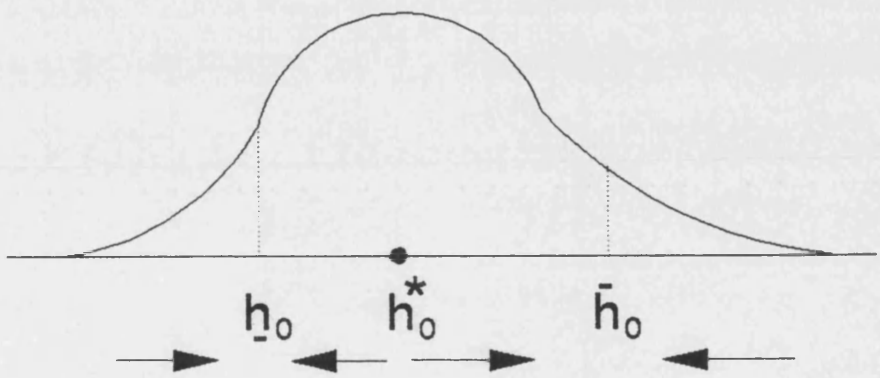


Figure 5

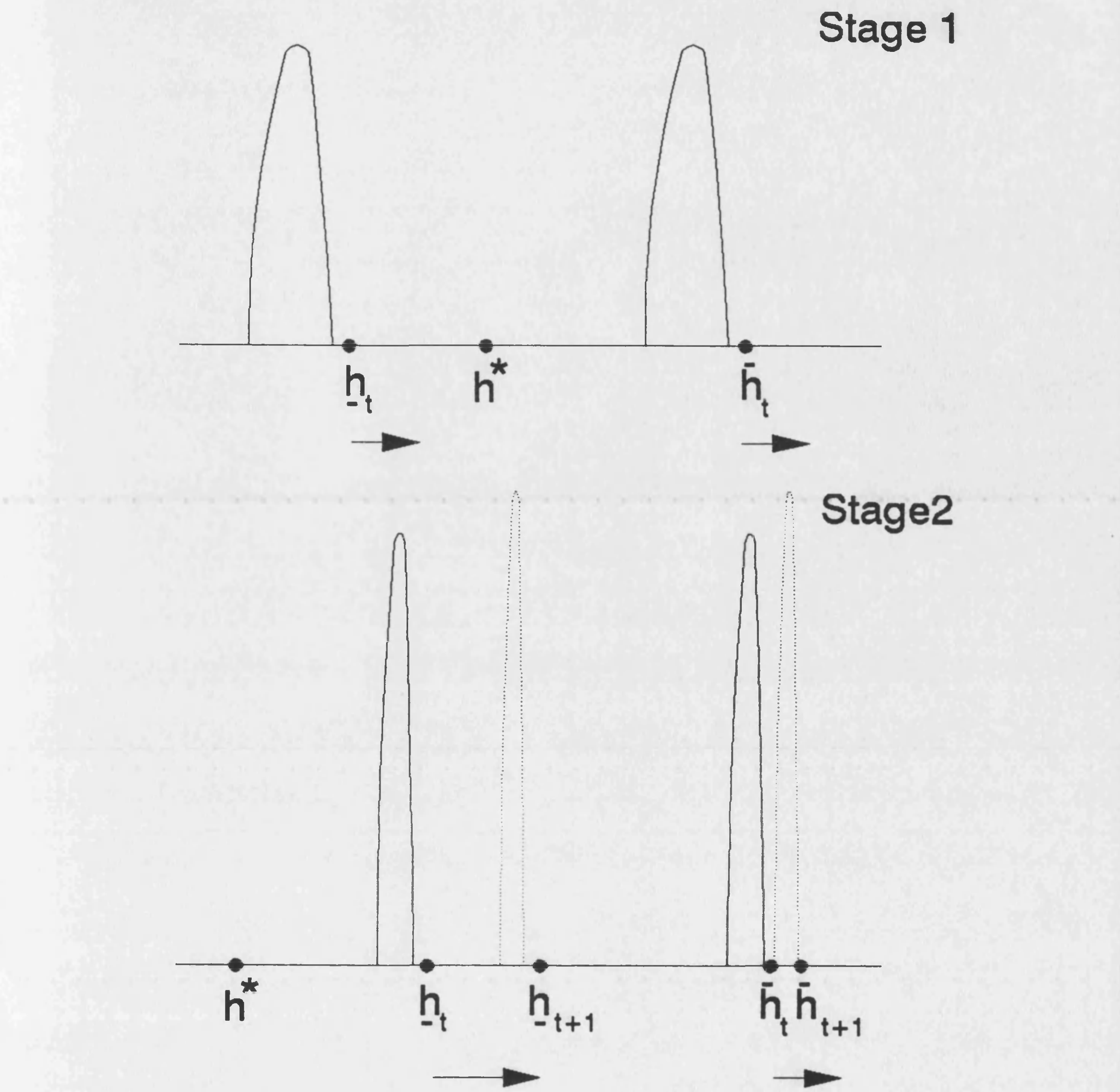

Stage 1
$\underline{h}_t$
h^*
$\bar{h}_t$
Stage2
h^*
$\underline{h}_t$
$\underline{h}_{t+1}$
$\bar{h}_t$
$\bar{h}_{t+1}$

Figure 6

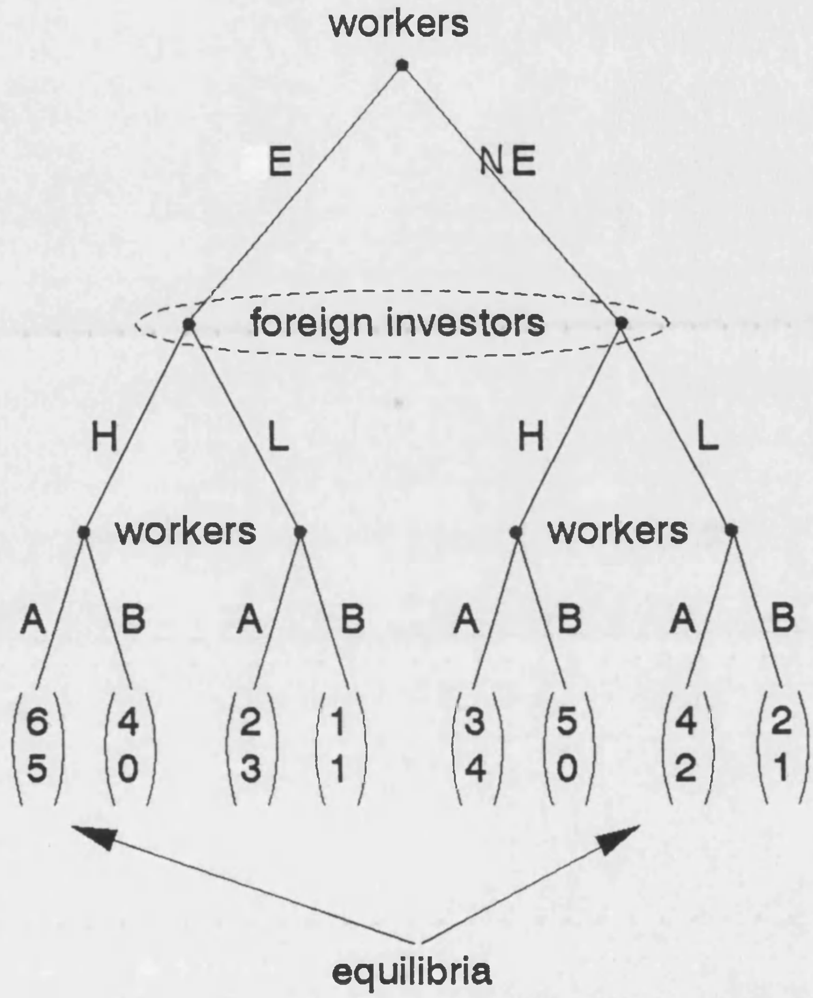

workers
E
NE
foreign investors
H
L
H
L
workers
workers
A
B
A
B
A
B
A
B
6 5
4 0
2 3
1 1
3 4
5 0
4 2
2 1
equilibria

Figure 7

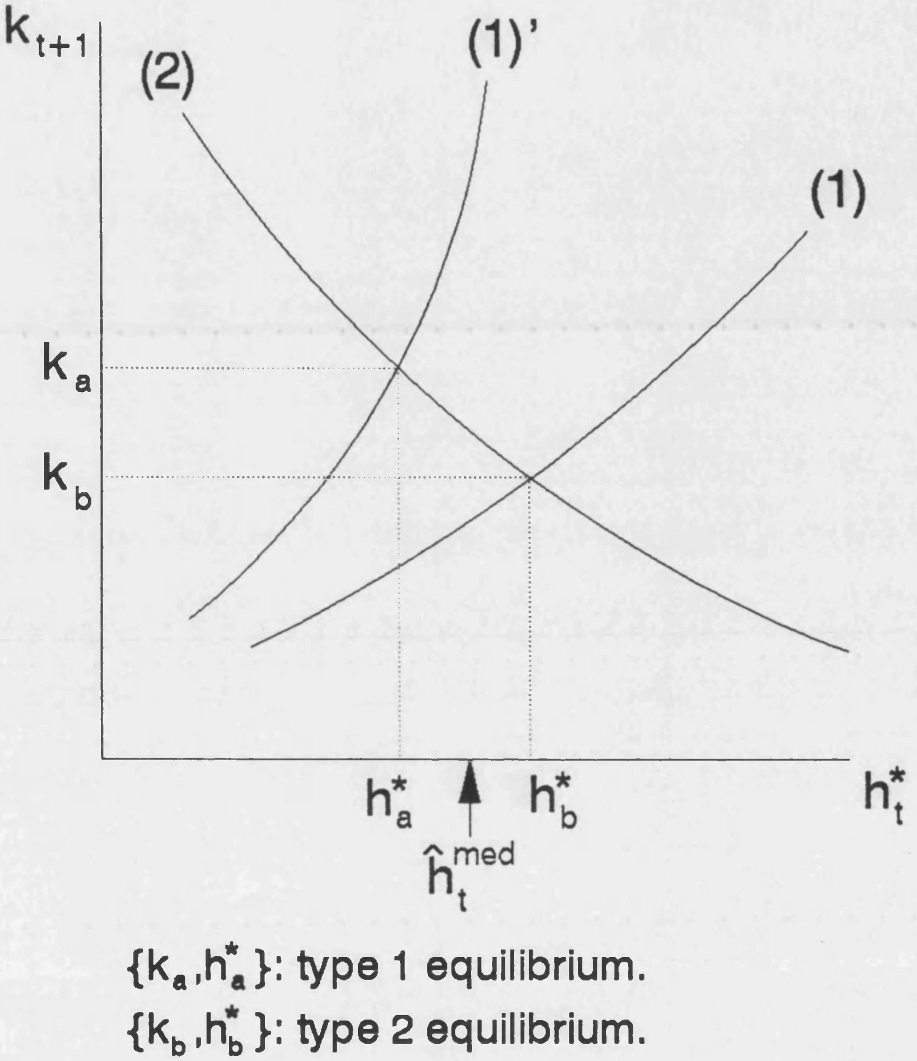

k_{t+1}
(2)
(1)'
(1)
k_a
k_b
h*_a
h*_b
h*_t
ĥ_t^med
{k_a, h*_a}: type 1 equilibrium.
{k_b, h*_b}: type 2 equilibrium.

Figure 8

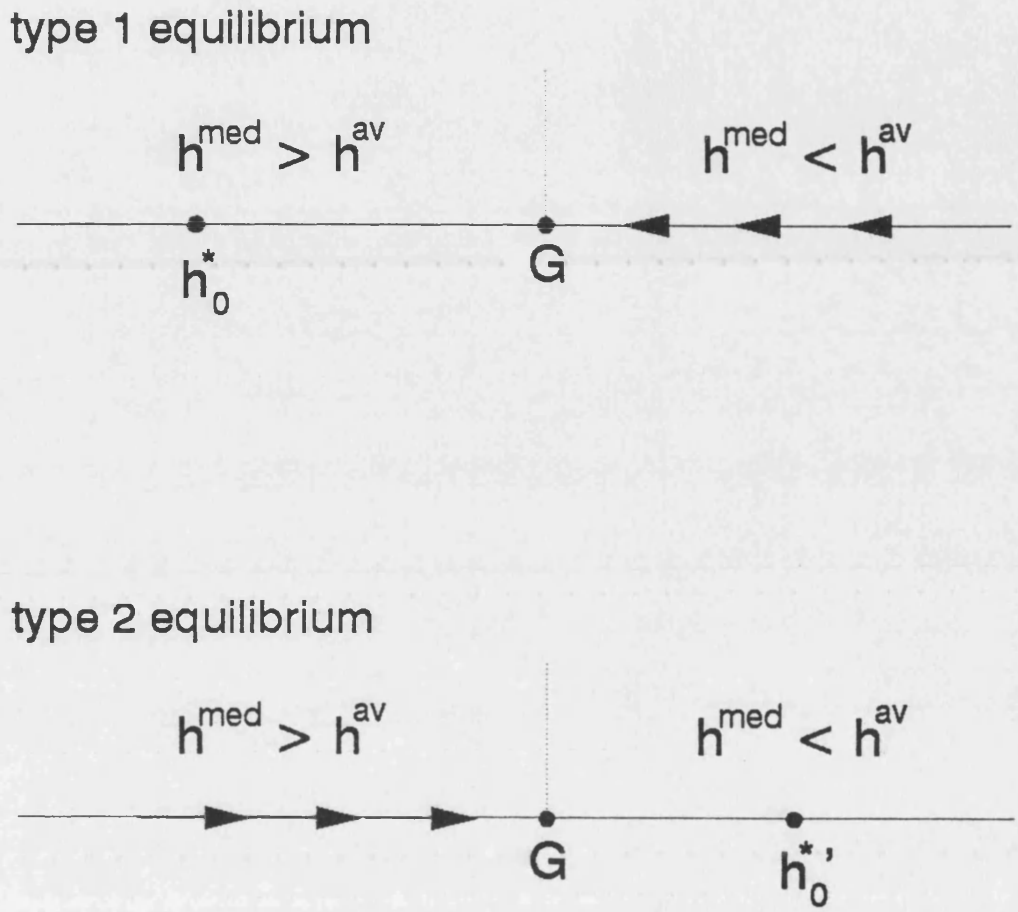
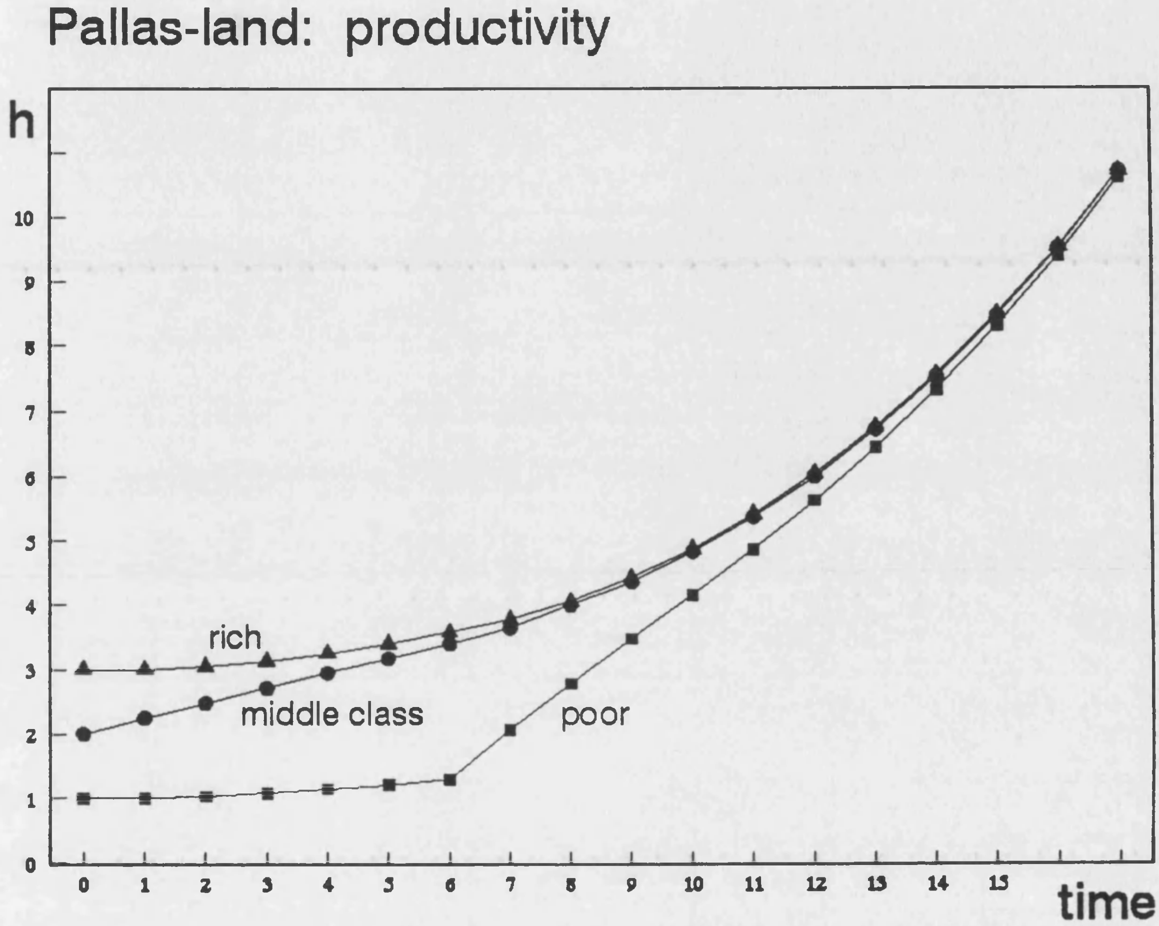

type 1 equilibrium
hmed > hav
hmed < hav
h*0
G
type 2 equilibrium
hmed > hav
hmed < hav
G
h*'0

Figure 9



Pallas-land: productivity
h
10
9
8
7
6
5
4
3
2
1
0
rich
middle class
poor
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
time

Figure 10a

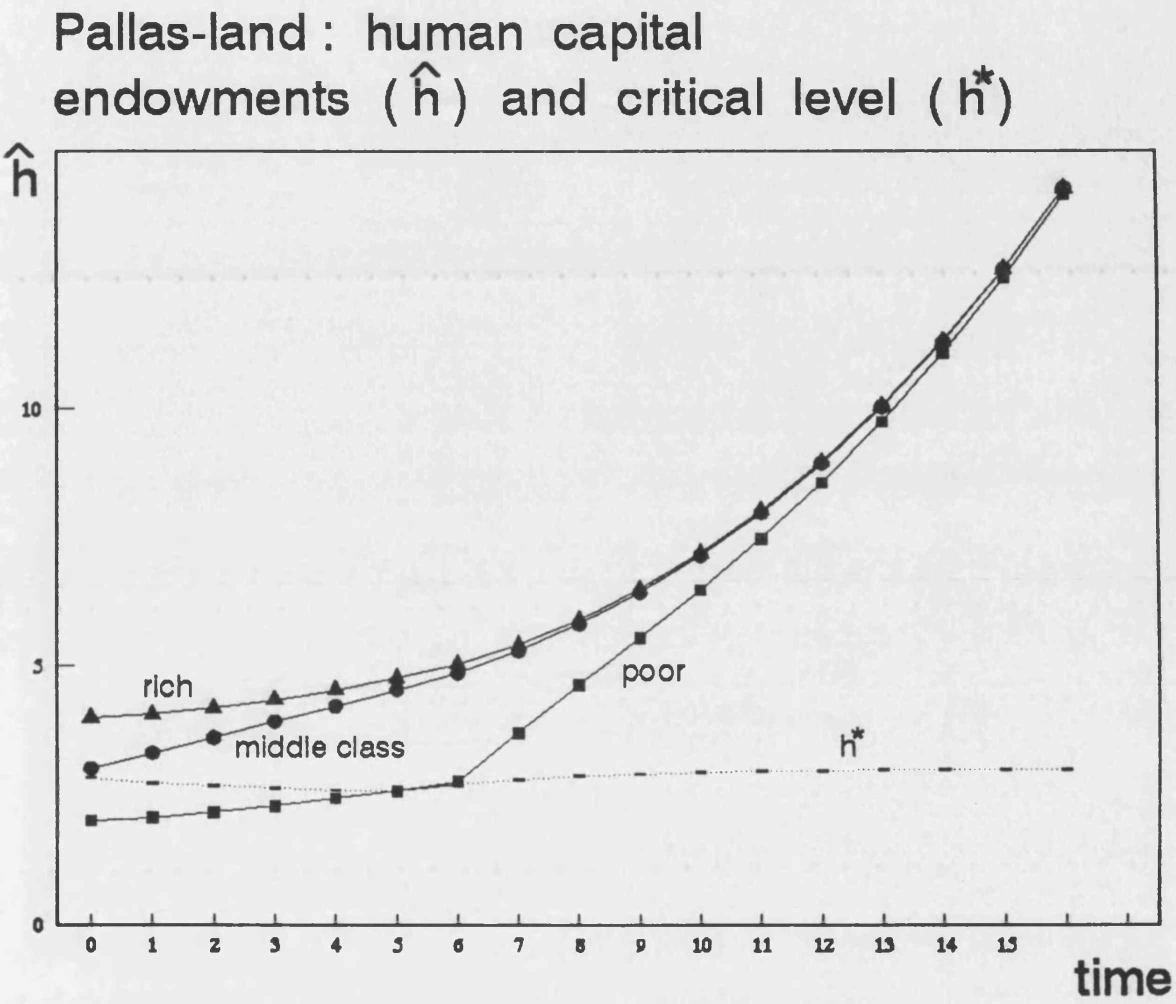

Pallas-land : human capital endowments (ĥ) and critical level (h*)
ĥ
10
5
0
rich
middle class
poor
h*
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
time

Figure 10b

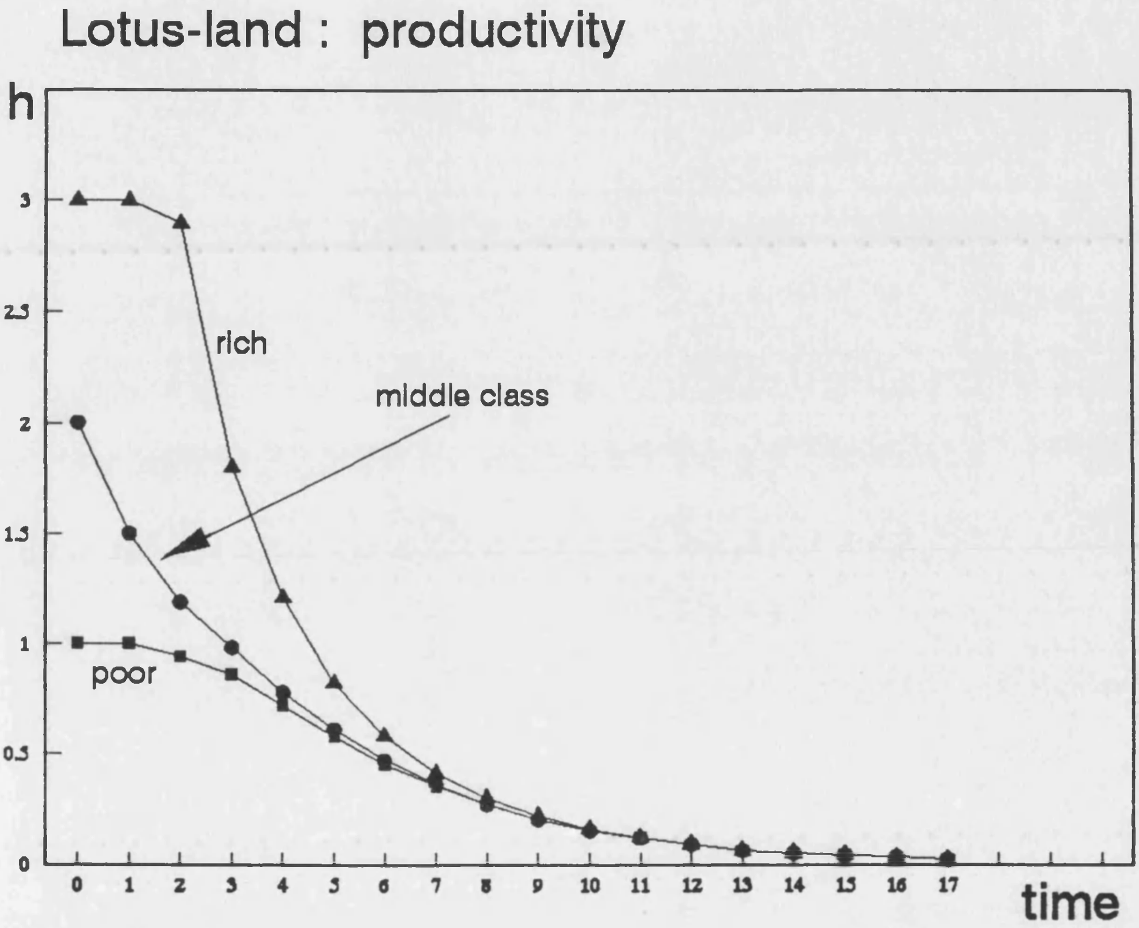

Lotus-land : productivity
h
3
2.5
2
1.5
1
0.5
0
rich
middle class
poor
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
time

Figure 11a

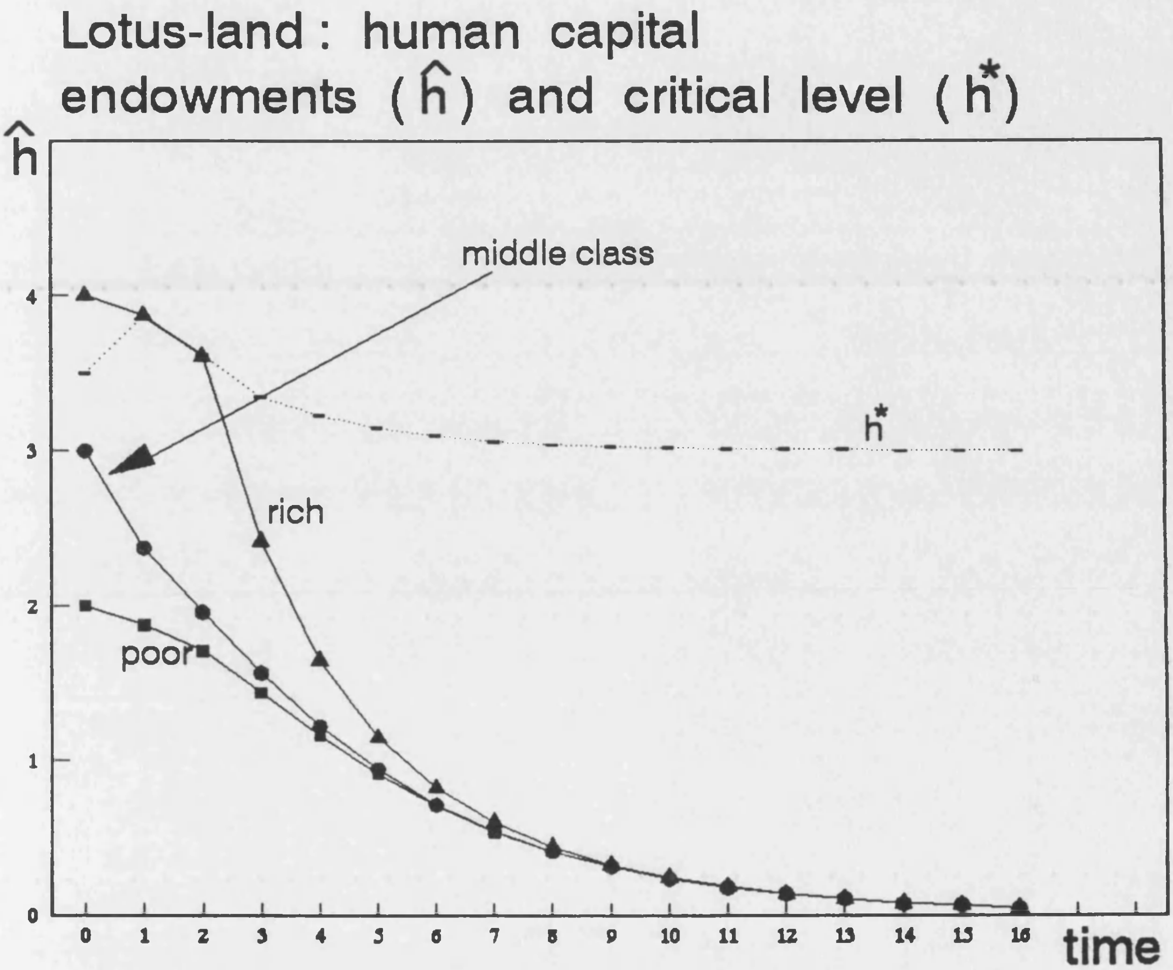

Lotus-land : human capital
endowments ($\hat{h}$) and critical level (h^*)
$\hat{h}$
middle class
h^*
rich
poor
0
1
2
3
4
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
time

Figure 11b

CHAPTER 5

Stochastic Trends and Macroeconomic Fluctuations: a Multicountry Analysis

This chapter is intended as a contribution to the debate opened by the seminal paper of Nelson and Plosser(1982) about the relative importance of permanent and transitory shocks in macroeconomic fluctuations. It is now well-known that GNP in most countries is characterised as a unit root process. The success of the idea that trends in output have a stochastic rather than deterministic nature has caused the breakdown of the traditional clear-cut distinction between business cycle and growth theory, the former being related to the study of fluctuations around a trend and the latter to the study of the persistent features of the economy. This debate has some implications on the recent developments in growth theory. If economic fluctuations are caused by repeated shocks which affect a self-sustained growth path (including policy shocks like distortionary taxation) the stochastic version of the AK-type endogenous growth model can in principle be used to characterise both the long-run and short-run behaviour of the economic system. This diminishes the role traditionally attributed to temporary disturbances (e.g. monetary shocks) as a source of fluctuations in the economy.

The first generation of papers which discussed the issue focused on univariate methods of time-series analysis, which decompose the stochastic process of GNP into the effects of a permanent - typically random walk - and a stationary component. The objective of the analysis was to quantify the relative 'size' of the random walk (Cochrane, 1988), by comparing the variance of the non-stationary component with the total variance exhibited by the time series. A seemingly different approach, though in fact equivalent in results, is to compare the impact effect of a shock with its cumulated (long-run) effect over time, providing a measure of the degree of persistence in the economy (Campbell and

Mankiw, 1986). The finding that movements in permanent components are a quantitatively important part of GNP fluctuations was taken by some authors as the evidence that 'real (non-monetary) disturbances are likely to be a much more important source of fluctuations than monetary disturbances' (Nelson and Plosser, 1982, p. 159).

Both the methodology and economic implications of this research have been later criticised by the influential paper of Blanchard and Quah (1989), who argued that the issue of identifying permanent and transitory components is not satisfactorily solved by restricting the permanent component to be random walk. This is in fact a purely statistical criterion. If we aim at giving some economic interpretation to the different disturbances, however, there are no valid reasons to justify the apriori assumption that the permanent components is a random walk rather than any more general unit root process. Blanchard and Quah (BQ) showed that the identification of the permanent vs. transitory components is possible within more a general class of decompositions - particularly, arbitrary orthogonal decompositions - by using additional information revealed by the behaviour of macroeconomic variables other than the GNP. This multivariate methodology has the advantage of delivering a 'structural' interpretation of the residual based on the effects which they have - coherently with some identifying restrictions - on the endogenous variables of the model. For this reason, this methodology has become known as structural vector auto-regression (VAR) analysis (see Giannini, 1992 for an overview of methods). The crucial identifying assumptions used by BQ is that transitory shocks (e.g. monetary shocks) do not have permanent effects on GNP. Both types of disturbances, on the other hand, affect both GNP and unemployment in the short run. The shocks are also assumed to be orthogonal. This identification scheme has been exploited by a number of authors with diverse stationary 'auxiliary' variables (Fachin, Gavosto and Pellegrini, 1992, for example, use an index of industrial capacity utilization). BQ found that large part of the variance of GNP can be still attributed to 'demand' shocks. The same result is obtained in a higher dimensional multivariate framework by King et al. (1991), who conclude that in VAR systems with nominal variables productivity shocks typically explain less than half of the business-cycle variability. It is temptative to interpret the results of this 'second generation' of papers as evidence in favour of traditional models which predict that demand shocks play a prominent role in explaining business cycle fluctuations. The equivalence between temporary and demand disturbances is certainly controversial, though is coherent with the predictions of some models with a neo-keynesian flavour. BQ, for instance, explicitely relate their findings to a version of the model of Fischer (1977) in which the temporary disturbances are monetary shocks.

The recognition that the decomposition of GNP must take into account the existence of diverse sources of fluctuations in the economy is a remarkable progress. However, the bivariate analysis of BQ can still be inadequate if the economy is in fact subject to multiple independent permanent shocks. The importance of temporary or 'demand' disturbances may be incorrectly measured (possibly, overstated) due to misspecification. In this work, we explore the possibility that an important role is played by the propagation of the international business cycle (see also Canova, 1993). More precisely, we assume that there exist both 'technological' permanent shocks with an idiosyncratic country-specific nature, and a worldwide stochastic trend which is shared by all countries in the sample. The cointegration properties of the time-series of GNP for the different countries are used to disentangle the effects of the different stochastic trends. This relates our contribution to the work of Bernard and Durlauf (1992), who find that the cointegration analysis reject the hypothesis that the GNPs of a set of industrialized countries are driven by a common trend.

In the following section, we show that the multicountry version of BQ which includes a stationary 'conjunctural' variable for each country provides a 'block-identification' scheme which separates the effects of what we call 'supply' shocks with respect to 'demand' shocks. The identifying assumption is that supply shocks have permanent effects on output levels in at least some countries, whereas demand shocks have no permanent effect on output in any country. The long-run properties are then used to fully identify each permanent shock. As far as the demand shocks are concerned, it is more difficult to find economically meaningful restrictions and we do not opt for any particular identifying assumptions. This is in fact not crucial for the scope of the present paper. VAR analysis permits, once the joint effects of these purely temporary ('demand') disturbances are identified, to check 'how the world would have looked' without these components.

1 Identification

We first derive the generalization of the Blanchard and Quah model to a multicountry framework. Let $Y^k(t)$ and $U^k(t)$ denote the logarithm of per capita GNP and the level of the unemployment (or other conjunctural variable) in country k. Let $X(t)$ denote the $2n$-dimensional vector of stationary variables $(\Delta Y, U)'$, where n is the number countries, and e be the vector of $2n$ 'structural' disturbances. As usual in the VAR literature, we assume that such disturbances are orthogonal and that X follows a stationary process given by:

$$X(t) = \Sigma_{j=0}^{\infty} A(j)e(t-j), \qquad Var(e) = I \tag{1}$$

where the diagonal covariance matrix has been normalised to unity. Let us impose the BQ-type restrictions on the long-run matrix. In the multicountry case, this is not sufficient to achieve complete identification. However, we want to show that it allows us to 'block-identify' the system, so that the lack of individual identification of the transitory shocks does not affect the identification of the permanent disturbances. The long-run BQ restrictions can be represented as follows. Let $A(1) = \Sigma_{j=0}^{\infty} A(j)$ be the matrix of long-run effects. Then, A(1) can be written in the form of the following partitioned matrix:

$$A(1) = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix} \tag{2}$$

where the zero block has dimension (nxn). This implies that the temporary disturbances have no effects on the level of Y in any country.

The estimation procedure goes in two steps. First, obtain the Wold moving average representation by estimating and inverting the VAR representation; then transform the Wold representation by imposing that the residual be orthogonal and that the impact matrix $B(0)$ have a lower triangular structure (Cholesky factorization). Let the Cholesky decomposition be represented as follows:

$$X(t) = \Sigma_{j=0}^{\infty} B(j)\epsilon(t-j), \qquad Var(\epsilon) = I \quad B(0)B(0)' = \Omega \tag{3}$$

where Ω is the covariance matrix of the residuals in the Wold representation and B(0) is lower triangular. To obtain the Cholesky decomposition is routine in current econometric packages. The problem of identification amounts to reconstructing the structural representation (1) from (3). The following relationships must hold true:

$$A(0)e(t) = B(0)\epsilon(t), \qquad A(j) = B(j)A(0), \qquad A(1) = B(1)A(0) \tag{4}$$

where $B(1)$ is the long-run matrix according to the Cholesky decomposition, defined analogously to $A(1)$. From $A(0)A(0)' = \Omega$, it follows that:

$$A(1)A(1)' = B(1)\Omega B(1)' \equiv \tilde{\Omega} \tag{5}$$

where $\tilde{\Omega}$ can be estimated. Now, partition $\tilde{\Omega}$ as follows:

$$\tilde{\Omega} = \begin{bmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} \\ \tilde{\Omega}'_{12} & \tilde{\Omega}_{22} \end{bmatrix} \tag{6}$$

where each block has dimension (nxn) and $\tilde{\Omega}_{11}$ and $\tilde{\Omega}_{22}$ are symmetric. Then (5) implies that:

$$A_{11}A'_{11} = \tilde{\Omega}_{11}, \tag{7}$$

$$A_{11}A'_{21} = \tilde{\Omega}_{12} \tag{8}$$

The permanent residuals are (exactly) identified if and only if there exist unique matrices A_{11} and A_{21} which can be recovered from the estimates of $\tilde{\Omega}_{11}$ and $\tilde{\Omega}_{12}$. Since $\tilde{\Omega}_{11}$ is symmetric, a necessary condition for identification (cfr. equation (7)) is that at least n(n-1)/2 restrictions be imposed on A_{11}. If there are no singularities, (8) can then be used to find a unique solution for A_{21}. In particular, this allows the estimation of all the 'structural' parameters corresponding to the first and second column of $A(j)$ for $j = 0, 1, ..., \infty$, using (5), together with the first n disturbances. This generates an estimate of the effects of each of the permanent shocks on all the endogenous variables. It is clear from this argument that whether or not the temporary disturbances are individually identified does not matter for the sake of the identification of the permanent disturbances. Nor does it matter for decomposing the forecast variance into the effects of permanent and temporary disturbances. Then, we need to impose a sufficient number of restrictions on the long-run effects of technological shocks on output. In the next section we will derive these restrictions from the long run properties of the output series.

2 A two-country case

As a simple example of the procedure illustrated, we will consider a two-country case, with the countries chosen being the US and UK. Departing from previous literature, we use a longer sample than that usually considered for decomposition purposes. This allows us to test the robustness of the decomposition results to the addition of a number of historical episodes, such as the two world wars and the Great Depression, though at the cost of using yearly rather than quarterly data. We use the logarithm of GNP per capita for the US and of GDP per capita for the UK and the levels of the unemployment rate for these two countries in the period 1900-1992.

To start with, we consider the univariate properties of the time series. The augmented Dickey-Fuller (ADF) test with different lag structures and no trend (see Engle and Granger, 1991) confirms, as expected, that the output series are I(1) processes. The results of the ADF tests for the unemployment series are more controversial. The data do not reject the null of the presence of a unit root at the 95% confidence level for either of the countries. However, in both cases there is some evidence of reversion to the mean, particularly for the US series where we only marginally fail to reject the non-stationarity null. The relevant coefficients for the ADF test with two lags and a constant exhibit a t-value of -2.81 for the US which is very close to the 5% critical value of -2.86. In the case of the UK unemployment rate the t-value found was -1.99[1]. Since the evidence is not conclusive, we choose to maintain the hypothesis of stationarity of unemployment in the text, and to report the results of the same model with unemployment in first differences in the appendix. As we will see, the results are affected substantially by this choice [2].

Next, we test for cointegration between the two output level series. Different tests for cointegration are available. The most traditional for bivariate analysis (Engle and Granger test) consists of running first a cointegrating regression with an intercept, and then an augmented Dickey-Fuller test on the residuals of this regression. The evidence from this test is again not clear-cut. The ADF test with one lag on the cointegrating regression returns a t-value of the critical coefficient of -2.65, failing to reject the non-cointegration null hypothesis at both 5% and 10% levels (the critical value of the 5 per cent Engle-Granger test reported in MacKinnon, 1991 is -3.34). In conclusion, we do not reject the null of non-cointegration, though one might think that the

[1]This failure to reject the unit-root null in the UK case was noted by Bean, 1992. Some issues about the stationarity of unemployment are also mentioned by Blanchard and Quah. Notice, that they introduce a fitted-linear time-trend regression line in their analysis. This solution does not seem of great help with our data.

[2]Our prejudice in favour of unemployment being stationary comes from the fact that the theoretical interpretation of the model with unemployment in first differences is problematic. In particular, we have no clear explanation to why temporary disturbances should have a permanent effect on unemployment, but not on output. We notice, however, that models with 'hysteresis' effects like Blanchard and Summers (1986) predict precisely that unemployment should be non-stationary and affected in the long-run by both demand and supply disturbances.

results are due to the low power of the test.

The cointegration analysis suggests that the matrix of the long-run effects of the shocks (A_{11}) is full rank. It leaves a degree of freedom, however, about the choice of the basis of such space. The choice of a basis which is economically interesting is used here as the device to identify the permanent disturbances. Unfortunately, any solution is subject to some degree of arbitrariness, as it is often the case in VAR analysis. We can consider three candidate solutions. The first is that growth in the two countries is driven by entirely idiosyncratic shocks which have permanent effects only in the country where they have been generated. This implies to impose that the off-diagonal entries in the matrix A_{11} be zero. This structure implies two zero restrictions in the long-run matrix, and is overidentified since perfect identification requires one restriction only. However, the overidentifying restriction implied by the model with purely idiosyncratic trends is massively rejected by the data, so we ignore this case. Both of the other identification schemes which we consider assume the existence of a shared 'world' stochastic trend. The model which we will refer to as *case a* follows from the assumption that, together with a world technological trend which drives growth in both countries, there exists an idiosyncratic technological trend in one of the two countries (particularly, the UK). Though this asymmetry is somewhat unpalatable, it is rather convenient from an analytical viewpoint since it implies that the long-run matrix is lower triangular. The alternative model (*case b*) eliminates the asymmetry by assuming, alternatively, the existence of a 'relative shock' with a unit root that drives the difference ($Y^{US} - Y^{UK}$) between the output of the two countries. This assumption implies one restriction on the long-run matrix, which takes on the form:

$$A_{11} = \begin{bmatrix} \alpha & \beta \\ \gamma & -\beta \end{bmatrix} \tag{9}$$

Since both the identifying restrictions suggested are controversial, we present the results of both cases. Notice that this specification issue is mildened by the fact that the choice of the identifying restrictions within the block A_{11} does not affect the results of the permanent vs. transitory decomposition, but only the attribution to each permanent disturbance of the total forecast variance explained by permanent components and determined by the Blanchard and Quah-type restrictions.

3 Results.

The observation of the GNP series (see later, Fig. 2) reveals a higher degree of volatility in the first half of the century than in second half, with three main shocks in the forefront: the First World War, the Great Depression and the Second World War. It is also noticeable that the first thirty years of the century are characterised by a very low average growth rate in the UK. It seems interesting to check how the model fits the data in the presence of these large-size fluctuations that are left outside of the analysis which considers only post-war data.

First, we proceed to estimate the impact and long-run matrices. In order to make estimation feasible, we impose a technical 'fictitious' identifying restriction with no economic interpretation, namely that A_{22} is lower triangular. This makes A(1) itself lower triangular. When interpreting the results, however, we will be careful to avoid to disentangle the effects of of the two temporary disturbances, and we only assess their joint effects on each endogenous variable. In all cases, we have estimated a three lags VAR system with a constant (deterministic trend in output). Changes in the lag structure do not affect the results significantly.

Table 1 summarizes the estimates of the impact and long-run matrix obtained from the VARs using model *a* model *b*. Notice that only the first two columns of each matrix bear a meaningful interpretation, according to our previous discussion. In the former cases, the disturbances labelled as 'world technological shock' has a significantly stronger impact on the US than on the UK economy. We also notice that the UK shock has a positive impact effect on the US economy. In the latter case, instead, the impact effect of the world shock is almost identical, whereas the cumulated long-run effect is higher in the UK than in the US. The 'relative shock' has a negative impact effect on both countries, though stronger in the UK, whereas the long-run effect is positive for the US and negative for the UK. We can compare the short-run effects with the long-run effects of permanent disturbances, in order to measure their degree of persistence, similalrly to Campbell and Mankiw (1987). In both cases, the so-called 'world shock' exhibits a high degree of persistence with long run effects being more pronounced than the impact effects. In model *a* the global shock has a long-run effect which is twice as large as the short-run effect in the US and more than three times as large as the short-run effect in the UK. In model *b*, a 0.8% GNP rise in both economies following a world shock has a long-run effect of 0.9% in the US and more than 1.3% in the UK.

Table 1

MODEL *a*

Estimate of A(0) - Impact effects				*Estimate of A(1) - Long-run effects*			
0.00532	0.00620	-0.00813	0.01345	0.01063	0	0	0
0.00260	0.01044	-0.00839	-0.00288	0.00826	0.01194	0	0
0.48103	-1.10866	0.58480	-1.43004	4.21228	-4.20664	11.90462	0
0.58998	-0.31436	1.33584	-0.07751	6.17355	1.49854	12.31294	8.40371

MODEL *b*

Estimate of A(0) - Impact effects				*Estimate of A(1) - Long-run effects*			
0.00781	-0.00240	-0.00813	0.01345	0.00899	0.00568	0	0
0.00778	-0.00744	-0.00839	-0.00288	0.01336	-0.00568	0	0
-0.18562	1.19541	0.58481	-1.43000	1.31372	5.80597	11.90482	0
0.33081	0.58086	1.33586	-0.07751	6.01957	2.03048	12.31310	0.40348

Figure 1 represents the cumulated effects for each of the permanent shocks on output in the two countries. We observe some pronounced oscillatory behaviour of the impulse-response functions whose interpretation is puzzling. Since we use annual data, we find it reasonable to expect that the dynamic effects of a shock should die off after few periods (i.e., we expect to see the impulse-response function settling down about its long run value rather soon).

FIGURE 1 (Impulse response functions)

The next step represents the main point of our exercise. The objective is to estimate the weight attributed to each disturbance in explaining the variability of output by each model. To this aim, we decompose the forecast error variance at various horizons into the part due to each of the innovation processes. Before going through our results, we remind the reader the findings of Blanchard and Quah. They estimated that the percentage of variance due to demand shocks at a one (ten) year horizon, depending on alternative treatments of structural breaks and trend in unemployment, is 97.9% (39.3%), 78.9% (18.7%), 98.6% (50.4%) and 38.9% (5.2%). Though the evidence is not clear-cut in one of the case mentioned, they considered their results as supportive of models which predict an important role for demand disturbances in the short run. We want to check whether the same finding holds true in our two-country framework and with a different sample. The results are summarised in Table 2.

Table 2

MODEL *a*

(N.B.: decomposition without the period 1930-46 in parenthesis)

Decomposition of variance for US output				
Years	Std. Error	World shock	UK shock	Demand
1	0.0177 (0.0116)	9.1 (46.3)	12.4 (1.9)	78.5 (51.8)
5	0.0215 (0.0145)	10.3 (42.0)	22.7 (3.3)	67.0 (54.7)
10	0.0225 (0.0146)	10.2 (42.1)	23.6 (3.4)	66.2 (54.5)

Decomposition of variance for UK output				
Years	Std. Error	World shock	UK shock	Demand
1	0.0139 (0.0118)	3.5 (51.5)	56.1 (35.2)	40.4 (13.3)
5	0.0157 (0.0140)	6.1 (46.0)	48.4 (26.9)	45.5 (27.1)
10	0.0162 (0.0141)	6.2 (46.0)	46.6 (26.9)	41.2 (27.1)

Decomposition of variance for US unemployment				
Years	Std. Error	World shock	UK shock	Demand
1	1.9615 (0.9376)	6.0 (0.1)	31.9 (22.0)	62.1 (77.9)
5	4.2700 (1.4355)	1.6 (9.0)	35.8 (24.0)	62.6 (67.0)
10	4.6380 (1.4963)	2.7 (9.2)	31.4 (24.4)	65.4 (66.4)

Decomposition of variance for UK unemployment				
Years	Std. Error	World shock	UK shock	Demand
1	1.4958 (0.6871)	15.6 (3.0)	4.4 (8.3)	80.0 (88.7)
5	3.1237 (2.2680)	9.9 (14.7)	6.0 (22.6)	84.1 (62.7)
10	3.8489 (2.9287)	11.4 (13.6)	5.6 (24.9)	83.0 (61.5)

MODEL *b*

Decomposition of variance for US output				
Years	Std. Error	World shock	Relative shock	Demand
1	0.0177	19.5	2.0	78.5
5	0.0215	25.4	7.6	67.0
10	0.0225	10.2	23.6	66.2

Decomposition of variance for UK output				
Years	Std. Error	World shock	Relative shock	Demand
1	0.0139	31.1	28.5	40.4
5	0.0157	30.0	24.5	45.5
10	0.0162	29.4	23.4	41.2

Decomposition of variance for US unemployment				
Years	Std. Error	World shock	Relative shock	Demand
1	1.9615	0.9	37.0	62.1
5	4.2700	7.7	29.7	62.6
10	4.6380	7.5	26.6	65.4

Decomposition of variance for UK unemployment				
Years	Std. Error	World shock	Relative shock	Demand
1	1.4958	4.9	15.1	80.0
5	3.1237	3.1	12.8	84.1
10	3.8489	8.5	8.5	83.0

Our estimates seem to confirm the prominent role of demand disturbances found by BQ, particularly for the US economy. Almost 79% of the one-year forecast variance is attributed to temporary disturbances. This figure falls to about 66% at the ten-years horizon (which we still find it a surprisingly high percentage). In model *a*, the only shock which is assumed to have a permanent effects on the US output explains just about 10% of the forecast error at different horizons. We should notice, however, that an important role in these results is played by the highly turbulent period 1930-1946. When this period is omitted from the sample, the decomposition outcome changes different. In this case (estimates within brackets in the table), the forecast variance explained by 'supply' disturbances turns out to be substantially higher, as reported in the table. Still, more than half of the output variability is explained by temporary disturbances in the US. Model *b* estimates attribute a more important role to the world shock than model *a* in explaining output fluctuations, particularly in the US case.

Finally, we can check how our model would fit the output data in the absence of temporary disturbances[3]. This provides similar information to that rendered by the decomposition, but is intuitively appealing and allows us to visualize the role of the different shocks throughout the history of the two countries. Furthermore, it is directly comparable with the picture obtained by BQ for the common period in the sample. Our model, once a deterministic trend is allowed, can be given the following representation:

$$X_t = \Sigma_{j=0}^{t-1} A(j) e_{t-s} + \beta \tag{10}$$

where capital letter terms are (4x4) matrices and lower case terms are (4x1) vectors, with the variable ordered in the conventional way. If we constrain the third and fourth term of each vector e_{t-s} to be zero, and simulate the dynamic path of the economy, we obtain the representation of how the world would have looked in the absence of temporary disturbances. The visualisation of the results of this exercise are reported in the upper part of Figure 2, together with the graph of actual GNP per capita. The lower part reports the difference between actual and simulated GNP per capita absent demand.

FIGURE 2

We find these results interesting. Consider the US, first. According to model *a*, supply disturbances are the main cause of both the boom and the recession which preceded and followed, respectively, the First World War. The drastic fall of output during the Great Depression is a consequence of both demand and supply disturbances, though demand disturbances come first and have a much more violent effect. The recovery experienced by the economy throughout the New Deal is also mainly due to demand effects. After a brief slowing down of growth caused by a demand slump, both demand and supply

[3]For this part of the analysis, the results of models *a* and *b* are, obviously, identical.

move to cause the production boom during the Second World War, as well as the immediate post-war recession. The picture of the post-war period and the respective decomposition look pretty similar to that produced by Blanchard and Quah (cfr. Fig. 7, p. 664). Compared with their results, however, our stochastic trend is characterised by a higher variability, as the graph makes clear. Our estimates, accordingly, confer to demand shocks a quantitatively smaller role than those of BQ. The band of fluctuation of postwar output due to demand effects is $(-4.3\%, +3\%)$ according to our estimates, whereas Blanchard and Quah detect fluctuations between +6% and -9%. This is only in a part due to the inclusion of the effects of propagation of the business cycle, which in fact play a marginal role. Like BQ, we detect demand-determined recessions in 1954 and 1961. Demand policies keep the output at a higher level than warranted by its structural component throughout most of the 60s and 70s, with important episodes of demand-pulled booms in 1964 and 1972. The slump of 1969 is supply-driven according to our model, whereas it was demand-driven in BQ. The recessions of 1974-75 and 1979-80 are instead attributed to the effects of falling demand plus a slowing down in the structural component of growth. The most recent recession, starting in 1989, is characterised as a supply-determined episode, which has been offset in 1992 by a favourable demand shock.

Looking at the United Kingdom, the analysis identifies some remarkable favourable demand shocks in 1910, 1915, 1934-35, 1939-41, and adverse demand shocks in 1908-09, 1918-19, 1921-23 and 1930-31. The contribution of demand to the Great Depression is smaller in the UK than in the US. After the immediate post-war recession, the analysis does not detect any sharp demand-pull fluctuations up to the mid-70's. The main fluctuations in the actual output level are also recorded by the series which is constructed absent demand, including the most recent slump episode. This is coherent with the finding of the decomposition analysis that fluctuations have a more important supply-determined component in the US than in the UK. Notice that the 80's are characterised by a switch of regime such that the actual output ceases to be above its 'structural level' and remains constantly below it.

4 Conclusions.

We have extended the BQ framework in terms of both sample length and number of countries and independent sources of fluctuations considered. If the hypothesis of stationarity of the unemployment rate series is maintained, the results are similar to those of Blanchard and Quah. The finding that temporary shocks play an important role in explaining short-run fluctuations is confirmed by our analysis. Furthermore, the decomposition of the post-war US business cycle into 'demand' and 'supply' effects renders a picture which is, at least qualitatively, consistent with that of Blanchard and Quah. On the other hand, the impulse-response function presents some counterintuitive features that undermine the reliability of the results. The importance of the 'world shock' vs. idiosyncratic or relative trends in explaining fluctuations depends significantly on the assumptions chosen to identify each permanent components. As agenda for future research, we intend to test the structural stability of the sample, checking whether some significant changes occur when some large-size shocks (the Great Depression, the two wars) are better modelled as once-for-all deterministic changes rather than as part of the stochastic process (see Perron, 1991, for a univariate analysis).

5 Appendix

In this appendix we report the estimates obtained by differentiating the unemployment rate. We will refer to this case as *model c*. Table 3 corresponds to Table 1 (estimates of impact and long-run effects) whereas Table 4 corresponds to Table 2 (decomposition). The most remarkable feature of this case is that the role of demand in explaining output fluctutations becomes very small (about 5-6 percent in both countries). We find these results hardly plausible.

Table 3

MODEL c

Estimate of $A(0)$ - Impact effects				*Estimate of $A(1)$ - Long-run effects*			
0.01799	-0.00516	0.00088	0.00211	0.02340	0	0	0
0.00956	0.00997	-0.00296	-0.00024	0.00418	0.01563	0	0
-1.58902	0.62256	0.96829	-0.58282	-2.24788	0.42304	0.85722	0
-0.96889	-0.19432	0.88319	0.81358	-0.77348	-0.56463	0.59841	0.81076

Table 4

MODEL c

Decomposition of variance for US output				
Years	Std. Error	World shock	UK shock	Demand
1	0.0188	91.0	7.5	1.5
5	0.0222	85.4	8.6	6.0
10	0.0225	85.4	8.5	6.1

Decomposition of variance for UK output				
Years	Std. Error	World shock	UK shock	Demand
1	0.0141	45.8	49.7	4.5
5	0.0161	48.7	46.1	5.2
10	0.0163	49.8	44.6	5.6

Decomposition of variance for US unemployment				
Years	Std. Error	World shock	UK shock	Demand
1	2.0469	60.3	9.3	30.4
5	2.4050	56.0	9.3	34.7
10	2.4329	55.9	9.3	34.6

Decomposition of variance for UK unemployment				
Years	Std. Error	World shock	UK shock	Demand
1	1.5551	38.8	1.6	59.6
5	1.6961	38.9	6.6	54.5
10	1.7171	39.2	6.5	54.3

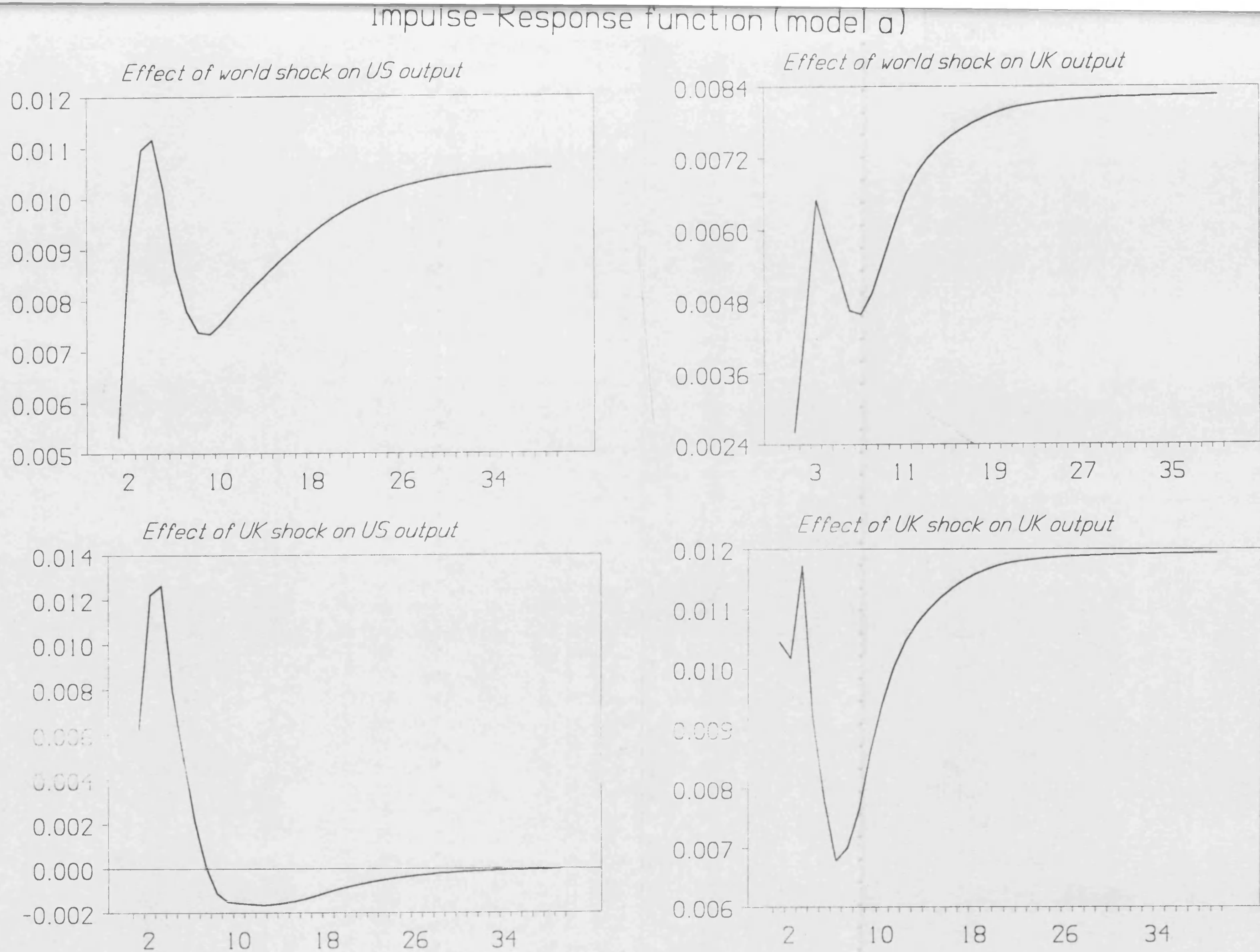

Impulse-Response function (model a)
Effect of world shock on US output
0.012
0.011
0.010
0.009
0.008
0.007
0.006
0.005
2
10
18
26
34
Effect of world shock on UK output
0.0084
0.0072
0.0060
0.0048
0.0036
0.0024
3
11
19
27
35
Effect of UK shock on US output
0.014
0.012
0.010
0.008
0.006
0.004
0.002
0.000
-0.002
2
10
18
26
34
Effect of UK shock on UK output
0.012
0.011
0.010
0.009
0.008
0.007
0.006
2
10
18
26
34

Figure 1

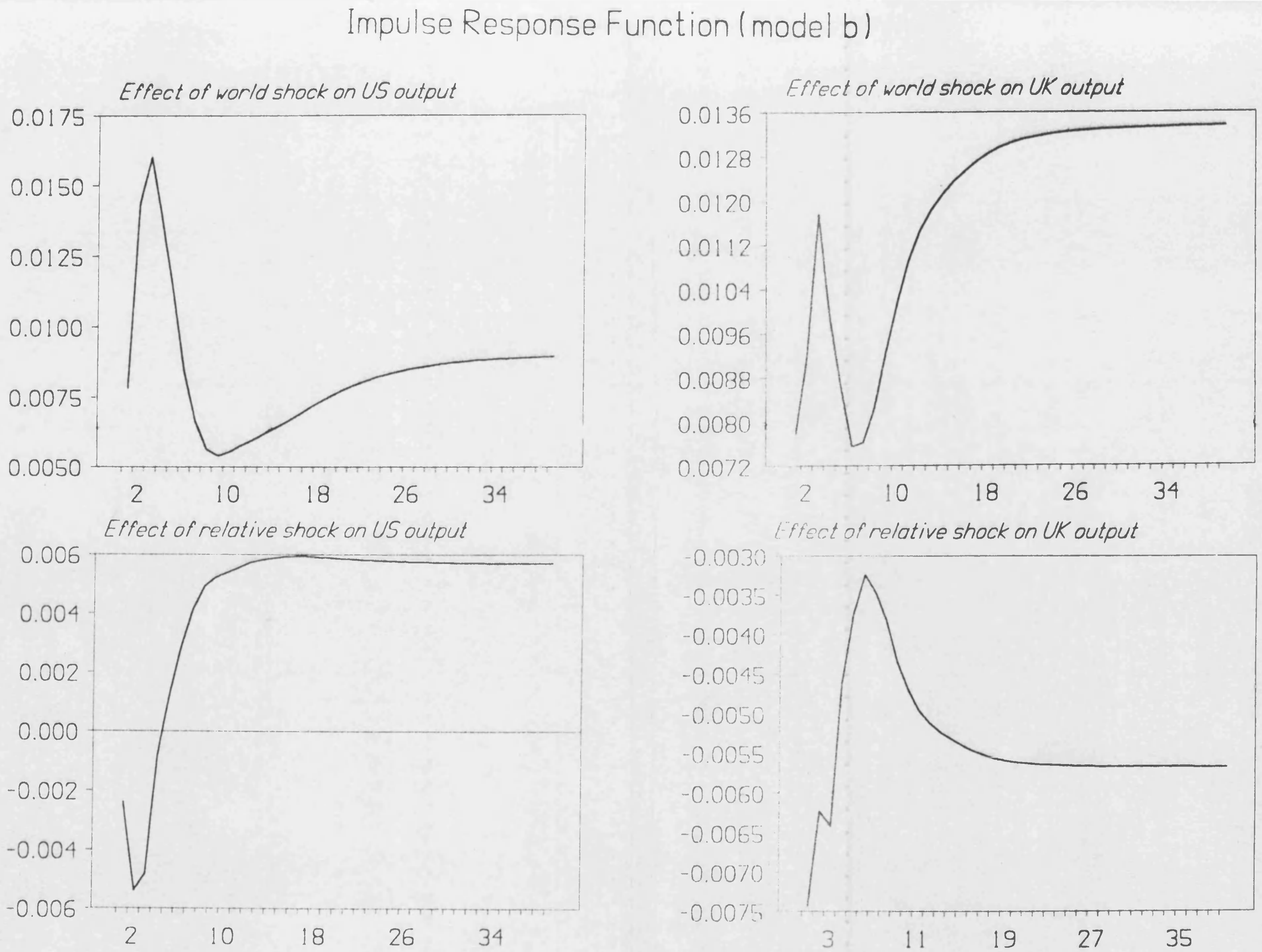

Impulse Response Function (model b)
Effect of world shock on US output
Effect of world shock on UK output
Effect of relative shock on US output
Effect of relative shock on UK output

Figure 1

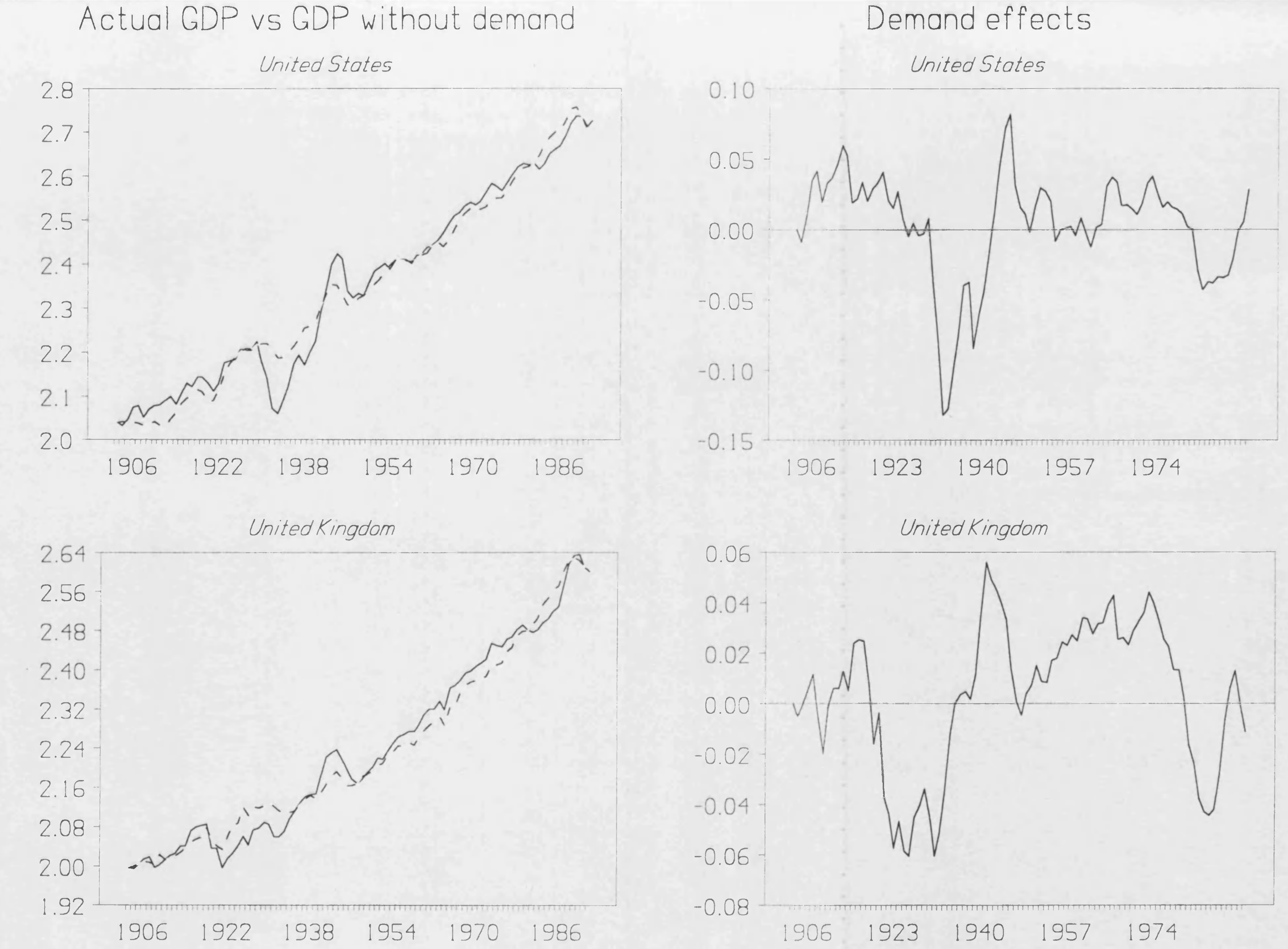

Actual GDP vs GDP without demand
Demand effects
United States
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1906
1922
1938
1954
1970
1986
United States
0.10
0.05
0.00
-0.05
-0.10
-0.15
1906
1923
1940
1957
1974
United Kingdom
2.64
2.56
2.48
2.40
2.32
2.24
2.16
2.08
2.00
1.92
1906
1922
1938
1954
1970
1986
United Kingdom
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
1906
1923
1940
1957
1974

Figure 2

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