The London School of Economics and Political Science

Essays on Financial Intermediation

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Declaration

I certify that the thesis I have presented for examination for the MPhil/PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly by me and any other person is clearly identified in it).

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Abstract

This thesis investigates the effects of financial frictions such as symmetric information on aspects of financial intermedation process, in particular banks and the securitisation industry. In the first paper, “Contingent capital structure”, I study the optimal financing arrangement of a bank with risk-shifting incentives and private information, in an environment with macroeconomic uncertainty. Leverage mitigates adverse selection problems owing to debt information insensitivity, but leads to excessive risk-taking. I show that the optimal leverage is procyclical, and contingent convertible (CoCo) bonds emerge as part of the implementation of the optimal contingent capital structure. However, the laissez-faire equilibrium entails excessive leverage and risk-taking, due to a bank’s private incentives to minimise arket mispricing of its securities. It is socially optimal to impose countercyclical capital requirements. In the second paper, “Counter-cyclical foreclosure for securitisation”, John Chi-fong Kuong and I investigate the optimal forefclosure policy of delinquent mortgages in a model of mortgage-backed securitisation under asymmetric information. We show that it is optimal for a securitiser to commit to an ex-post value-destroying foreclosure policy to reduce the signalling cost. The optimal foreclosure policy, which can be implemented by contracting with a third-party mortgage-servicer, features a excessive foreclosure rate for a mortgage pools of poor quality, implying a counter-cyclical aggregate foreclosure rate and pro-cyclical repossessed property prices. Finally, the third paper, “Bankruptcy-remote securitisation with implicit guarantee”, explores the role of securitisation in the funding of banks under asymmetric information. In a two-period model, I argue that securitisation as an optimal funding source rely on both features. While implicit guarantee mitigates the asymmetric information problem, bankruptcy-remoteness allows a bank to shield its unsecuritised cash flows in a bad state, thereby relaxing its future financing contraint.

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CONTENTS

CHAPTER 1

CONTINGENT CAPITAL STRUCTURE

1.1 Introduction

The recent financial crisis has brought the prudential regulation of financial institutions to the fore as an issue of critical importance. A new type of "contingent capital" – contingent convertible (CoCo) bonds – a form of debt that automatically converts into additional common shareholders' equity when a bank's original capital is depleted, has received much attention for its potential to restore the incentives for banks to practice prudent risk management and to prevent the disruptive insolvency of large financial institutions.[1] In this paper, I employ an agency-theoretic approach to show that CoCo bonds emerge as part of the optimal bank capital structure to mitigate risk-shifting problems when banks with private information face economic uncertainty.

CoCo bonds were first proposed as an alternative capital instrument by Flannery (2005), followed by modifications put forward by various scholars in the pursuit of a

[1] Straight debt can be interpreted as a special case of a CoCo bond. However, unlike straight debt, which "converts" into equity in the event of a default, CoCo bonds allow flexibility in choosing the conversion trigger of the bonds. The proposed CoCo bonds are typically converted into equity well before a bank enters into distress.

prudential capital structure of banks.[2] CoCo bonds have been positively embraced by regulators including the Swiss banking supervisor, Finma. For example, Lloyds Banking Group announced the first issue of £7bn CoCo bonds (Enhanced Capital Notes) through a bond exchange as early as 2009. The conversion will be triggered if the bank's core capital falls to less than 5% under Basel II rules.[3] CoCo bonds have been proven popular among banks and investors ever since, with issuance in 2012 and 2013 exceeding $20bn, and oversubscription being the norm.[4]

The literature on CoCo bonds, as surveyed below, has expanded in a short period of time, with the focus of discussion on the issues of trigger design and the pricing of the instrument. However, little has been done to address the following fundamental questions. Why does contingent capital improve efficiency as part of the capital structure of financial institutions, if at all? What is the role of financial regulation when contingent capital is available? This paper is the first to formally address these issues in a model of optimal contracting with endogenous risk choice under macroeconomic uncertainty.

The model builds upon two agency problems that are direct consequences of the intermediating functions performed by banks, in an environment with macroeconomic uncertainty. First, banks as informed lenders typically have better information about their investment opportunities than outside investors. Hence there is an inherent asymmetric information problem when banks raise capital. Second, banks as delegated monitors can influence borrowers' behaviour. Without modelling the borrowers explicitly, this paper assumes that banks can affect the riskiness of the businesses they lend to, and charge higher yields on loans to riskier businesses. This creates scope for *ex post* risk-shifting, i.e. the shareholders of levered banks may prefer a portfolio of excessively risky loans at the expense of the debt holders' interests. Moreover, the model takes into account that the general returns on banks' investments fluctuate with macroeconomic conditions, to study the implications of the two agency problems for banks' risk-taking incentives across different economic conditions and the role of a pre-committed contingent capital structure.

The analysis proceeds as follows. I start with showing that, in the *laissez-faire*

[2] The idea of Flannery (2005) first appeared in a 2002 working paper.

[3] Source: Bloomberg (2009).

[4] The most recent issue of CoCo bonds is a $3bn offering of Tier 2 Capital Notes by Credit Suisse. The Credit Suisse CoCo bonds are wiped out if the bank breaches its 5% tier one capital ratio or if the national regulator deems it is near default. Source: Financial Times (2013).

equilibrium, it is optimal for a bank to raise capital *ex ante* with a contingent capital structure employing procyclical leverage that depends on the subsequent realisation of the macroeconomic conditions. In this baseline model, the asymmetric information and risk-shifting problems uniquely determine the equilibrium leverage because of the trade-off effect of leverage: leverage reduces the signalling cost because debt is an information-insensitive funding instrument, but leads to excessive risk-taking *ex post*. Moreover, the optimal contingent capital structure entails higher leverage in booms when the information asymmetry is relatively more severe. Higher leverage must be employed in booms because it is more difficult to differentiate a good issuer from a bad one when asset values are generally higher and the bank's private information becomes relatively less significant. The model implies procyclical leverage ratios for banks, consistent with the empirical evidence documented by Adrian and Shin (2008a,b). In the resulting equilibrium, the bank's equity value is higher in booms and the default probability is lower in booms.

The optimal procyclicality of the equilibrium contingent leverage can be implemented using CoCo bonds, in addition to straight debt and equity, so that the bank has less leverage entering into an economic downturn. The model yields implications regarding the practical design of the CoCo bonds. First, two types of conversion features that are seen in existing CoCo bonds can arise in equilibrium. CoCo bonds issued by better capitalised banks should specify a debt write-down when triggered, because the bank is not in need of much outside capital; CoCo bonds issued by poorly capitalised banks should specify a conversion into equity. For example, Credit Suisse has issued CoCo bond with a contingent convertible feature in 2011 and 2012, followed by CoCo bonds with a write-down feature in 2013, as the capital position of the bank improved.[5] Second, while the model assumes a verifiable macroeconomic state as the trigger of the CoCo bonds, which is analogous to a regulatory declaration proposed by the Squam Lake Working Group (2009), the optimal CoCo bonds can also be implemented with triggers based on the prices of equity (e.g. Flannery 2009; Pennacchi 2011; McDonald 2011) and the CDS spreads of the bank (Hart and Zingales 2011). A concern raised by Sundaresan and Wang (2013) is that the market price of equity may not be unique unless the conversion of the

[5]According to Credit Suisse Regulatory Disclosures (2012), the core tier 1 capital ratio of Credit Suisse under Basel II.5 increased from 10.4% to 14.4% from 2011 to 2012, and the tier 1 capital ratio increased from 15.1% to 18.4%.

CoCo bonds is designed to not transfer value between the existing shareholders and the bond holders, which precludes penalising the existing shareholders, defeating the purpose of CoCo bonds. This paper endogenises the effect of capital structure on banks' value due to risk-shifting incentives and shows that the existing shareholders can be diluted upon the optimally designed CoCo conversion.

The *laissez-faire* equilibrium warrants regulation as it involves excessive leverage and risk-taking driven by a bank's private incentives to minimise the mispricing in the securities it issues under asymmetric information. In this model, the bank can signal its type, either by increasing the amount of leverage in its capital structure, or by deliberately underpricing the securities it issues. While leverage is preferred by the bank as it minimises mispricing, it incurs a social cost through the risk-shifting incentives of the bank by reducing the value of the businesses the bank lends to. I study the optimal regulation, defined as a set of constraints on banks' leverage designed to maximise bank value, which captures social welfare in this baseline model, when the bank privately optimises its capital structure subject to the regulatory constraints. The extent to which the regulator can cap leverage is thus limited by banks' private incentives. The result confirms the optimality of countercyclical capital requirements as proposed by the Basel Committee on Bank Supervision (2010) and advocated by scholars such as Brunnermeier et al. (2009). Faced with binding capital requirements, banks find it (privately) "costly" to issue the excess equity relative to the *laissez-faire* equilibrium because of the mispricing in the market. Banks meet the countercyclical capital requirements by issuing CoCo bonds, in a manner similar to the implementation of the *laissez-faire* equilibrium.

The baseline model thus far considers the role of leverage in a bank's capital structure in signalling its private information at the cost of inducing risk-shifting, which negatively impacts the value of the businesses the bank lends to. However, risk-taking by banks has systemic effects on the broader economy, as highlighted by recurrent financial crises. Large and correlated bank failures tend to pose large negative externalities.[6] Consequently, policy-makers are unwilling or unable to allow major financial institutions to fail.[7] I

[6]For example, Ivashina and Scharfstein (2010) document a run by short-term bank creditors following the failure of Lehman Brothers, contributing to a 46% reduction in the extension of new loans to large borrowers in the fourth quarter of 2008 relative to the previous quarter.

[7]For example, the run on Northern Rock, Britain's fifth-largest mortgage lender, did not stop until a taxpayer-backed guarantee of all existing deposits was announced in September 2007. The US Treasury and the Federal Reserve System bailed out 282 publicly traded banks and insurance companies under the

consider two types of state guarantees, implicit bailouts and explicit deposit insurance, in two extensions of the baseline model, to examine moral hazard problem induced by state guarantees and the implications for the optimal capital regulation.

The first extension assumes the systemic importance of the bank so that the regulator has the incentive to bail out a failed bank *ex post* by repaying the creditors on the bank's behalf. The second extension includes risk averse, unsophisticated depositors as part of the bank's funding base, who are protected by deposit insurance. Both forms of state guarantees create moral hazard problems because they provide an implicit subsidy when the bank issues debt. I show that the optimal capital regulation remains countercyclical and can be implemented using CoCo bonds with the same face value as in the baseline model, since the face value of the CoCo bonds is determined by the relative severity of the asymmetric information problems across different economic states. However, in the presence of state guarantees, the optimal capital regulation permits higher leverage in the form of straight debt or demand deposit. This follows the previous intuition that the extent to which the regulator can cap leverage is limited by the bank's private incentives. Since the moral hazard problem effectively reduces the private cost of leverage to the bank, higher leverage must be permitted to alleviate the asymmetric information problem. The *ex post* bailout of a systemically important bank and explicit deposit insurance therefore hinder the *ex ante* efficient capital regulation of the bank. Bail-in capital, a form of contingent debt that is wiped out in case the bank fails, has been considered by regulators such as the Basel Committee and the Bank of England as part of the resolution regimes for banks, in order to shield taxpayers from the need to bail out a bank that is "too big to fail". In light of the results in this extension, if a regulator can credibly commit to bailing in the debt *ex post*, the capital market would correctly price in the risk of default when the bank raises financing *ex ante*, removing the moral hazard problem of bailouts.

Literature review

This paper relates to a growing literature on contingent capital. In the first strand of the literature, Albul et al. (2010) and Barucci and Viva (2011) endogenise contingent capital in banks' capital structures. These papers extend the Leland (1994) model of tax benefit and bankruptcy cost to consider a firm's choice of capital structure among equity, straight debt and contingent convertible bonds. Unlike this paper, their approach does

Troubled Asset Relief Program (TARP) in 2008–9.

not take into consideration adverse selection or moral hazard. Since risk-shifting problems are perhaps the most important motivation for capital regulation, their settings do not provide implications for regulating the risk of the banking system. The second strand of the literature studies the implications of exogenously imposed CoCo bonds in banks' capital structures. Martynova and Perotti (2012) consider the effect of CoCo bonds on banks' risk-taking incentives. Others focus on the practical issues associated with CoCo bonds using different trigger mechanisms. For example, see Flannery (2005, 2009), Raviv (2004), Squam Lake Working Group (2009), McDonald (2011), Hart and Zingales (2011), Pennacchi et al. (2010), Pennacchi (2011), Bolton and Samama (2012), Calomiris and Herring (2013) and Sundaresan and Wang (2013). This paper is the first to provide a unified analysis of the optimality of CoCo bonds and the subsequent risk-taking behaviour of a bank. The framework also provides economic intuition for the design of CoCo bonds.

This paper is also related to the discussion of countercyclical capital regulation. The point that optimal bank capital regulations should depend on the state of the business cycle is made by Kashyap and Stein (2004) in a model of (exogenously) expensive equity capital and systemic cost of default. Later works by Hanson et al. (2011), Repullo and Suarez (2013) and Shleifer and Vishny (2010) also discuss time-varying capital requirements. A recent paper by Gersbach and Rochet (2013) studies a model of financial frictions with complete markets in which inefficient credit fluctuations arise and can be corrected by countercyclical capital regulation. Similar to this paper, Dewatripont and Tirole (2012) show the optimality of countercyclical capital and self-insurance mechanisms such as CoCos, considering the risk-shifting incentives induced by leverage. The trade-off in their model is generated by the creditor control right in default as disciplining device, which implies that agents should be rewarded only for the part of performance that is under managerial control. In contrast, this paper considers the benefit of mitigating adverse selection using debt financing; generating CoCo bonds that are contingent on exogenous macroeconomic conditions.

More generally, this paper relates to the literature on optimal corporate financing structures. While a significant portion of the theory of corporate finance under frictions can be categorised into two distinct paradigms, agency models (e.g. Jensen and Meckling 1976; Myers 1977; Grossman and Hart 1984; Green 1984) and financial signalling models (e.g. Ross 1977; Leland and Pyle 1977), efforts have been made to explore the implications when the two problems are both present. In the presence of private information, John and

Kalay (1982) study the agency costs of underinvestment, while Darrough and Stoughton (1986) study the agency problems of effort provision. Similar to this paper, John (1987) considers the problems of risk-shifting and asymmetric information in determining the capital structure and the investment policies of a widely held firm. John (1987) emphasises that the risk-shifting problem increases the signalling cost in the equilibrium, relative to the case in which the firm can commit to an investment policy. This paper differs from John (1987) in two aspects. First, I consider the impact of macroeconomic conditions on the trade-off between risk-shifting and asymmetric information problems, generating a role for contingent capital structure. Second, I recognise that the equilibrium is constrained inefficient, and characterise the optimal regulation, which restores constrained efficiency.

Although this paper shows the optimality of CoCo bonds, which are "reverse" convertible bonds, a number of papers have shown that conventional convertible bonds help to mitigate the asymmetric information problem by allowing the security to be contracted on additional signals. Stein (1992) show that callable convertible bonds are used by firms with medium quality as "back-door equity" financing to prevent bad firms from mimicking, in a setting in which the initial asymmetry of information is completely resolved by the time the security is called. Chakraborty and Yilmaz (2011) recognise that the resolution of information asymmetry is likely to be imperfect, and conversion only occurs if good news arrives. The "back-door equity" value of the securities is correlated with the manager's private information, thereby allowing an optimally designed callable convertible bond to resolve the asymmetric information problem without dissipation. In contrast, this paper considers a contingent security contracted upon macroeconomic states, which are uninformative of the private information of the issuer. The optimal conversion of security is therefore determined by the relative severity of the asymmetric information and risk-shifting problems considered in this model across different macroeconomic states, which gives rise to optimal procyclical leverage, implemented by CoCo bonds.

The remainder of the paper is structured as follows. Section 2.2 outlines the baseline model. Section 1.3 analyses the *laissez-faire* equilibrium to show the optimality of a contingent capital structure with procyclical leverage. Section 1.4 illustrates how CoCo bonds are part of the implementation of the optimal contingent capital structure and discusses the optimal design of the CoCo bonds. Noticing that the *laissez-faire* equilibrium entails excessive leverage, Section 1.5 characterises the optimal capital requirement, which is countercyclical. Section 1.6 studies two extensions of the model to incorporate the moral hazard of state guarantees. Section 1.7 concludes.

1.2 Model

There are four dates: 0, $\frac{1}{2}$, 1 and 2. The model's participants consist of a bank and a set of outside investors.[8] All agents are risk neutral and there is no discounting.

At $t = 0$, the bank has an opportunity to extend a total of 1 unit credit to form a loan portfolio that pays off at $t = 2$. The bank is endowed with private information regarding the payoffs of the portfolio at $t = 0$. However, at $t = 1$ after the loans are made, the bank can influence the riskiness of the borrower, and charge a higher yield on loans to riskier businesses. In order to focus on the capital structure of the bank, I abstract from modelling the borrowers explicitly. Instead, in Section 1.2.1 I make assumptions on the cash flows of the bank's loan portfolio directly to capture this intuition.

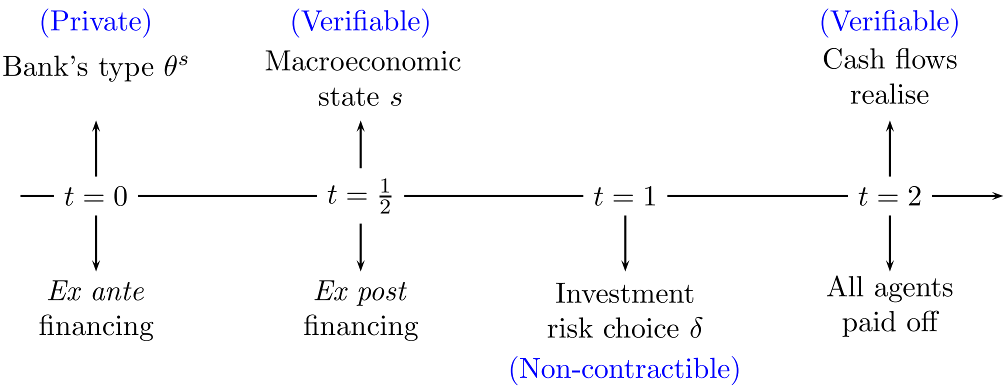
At $t = \frac{1}{2}$, a verifiable macroeconomic state realises, which affects the payoffs of the loan portfolio.[9] In order to fund the lending, the bank chooses its financing arrangement and raises capital either at $t = 0$ (*ex ante* financing), or at $t = \frac{1}{2}$ after the realisation of the macroeconomic state (*ex post* financing). I detail the financing arrangements in both the *ex ante* and the *ex post* cases in Section 1.2.1, and analyse both cases in Section 1.3 for comparison.

The timing of the events is summarised in Fig. 1.1, where the nature of the private information θ^i, the macroeconomic state s and the risk choice δ of the investment are detailed in the following section.

[8] In Section 1.6 I consider as an extension the case in which some outside investors are risk averse and the bank raises funds partially through deposits issued to the risk averse investors.

[9] As there is only one bank in this simple framework, the state s is interpreted as a macroeconomic state for intuitive purposes. Whether the shock to the state s is macroeconomic or idiosyncratic depends on factors outside of this model, such as the correlation of the shock across banks. I discuss other interpretations of the shock in Section 1.4.2 in relation to the design of the CoCo bonds that implement the optimal contingent capital structure.

Figure 1.1: Timeline



1.2.1 Assumptions and discussion

This section presents the assumptions on the distribution of the bank's portfolio cash flows to incorporate both the asymmetric information and the risk-shifting problems, and discusses how the bank can structure itself in order to finance its lending.

The bank's loan portfolio

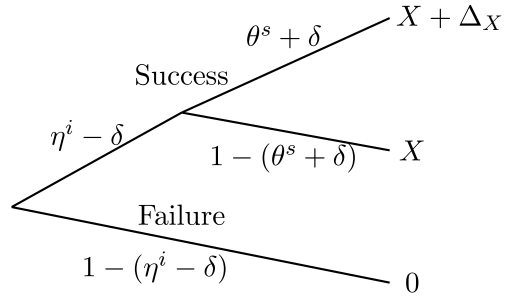
At $t = 0$, the bank has the opportunity to extend 1 unit credit and form a portfolio of risky loans that pays off at $t = 2$. The final payoff of the portfolio can be 0, X or $X + \Delta_X$. I will refer to the case where the portfolio returns 0 a failure, and a positive cash flow X or $X + \Delta_X$ a success.[10] The distribution of the portfolio cash flow over $\{0, X, X + \Delta_X\}$ depends on the type of the bank i, the state of the economy s and the bank's risk choice δ as illustrated in Figure 1.2 and as specified below.

The type of the bank $i \in \{G, B\}$ is characterised by its success likelihood η^i. A *Bad* bank (B) has a loan portfolio with a higher failure probability than a *Good* bank (G): $\eta^G > \eta^B$. The bank privately observes its type at $t = 0$. Outside investors do not observe the type of the bank, but they have a prior belief that the bank is good with probability γ.

The state of the economy $s \in \boldsymbol{S} = \{1, \ldots, S\}$ also affects investment opportunities,

[10] The assumption that the loan portfolio returns 0 in case of a "failure" is without loss of generality. If the bank's loan portfolio produces a positive minimum cash flow, the portfolio contains a portion of cash flow which is safe and therefore does not impose any financing problems on the bank. Backed by this safe part of the cash flow, a bank can issue deposits or safe debt. However, in practice banks typically take on additional risky debt. The model therefore sheds light on understanding the additional leverage taken by banks in the form of risky debt.

Figure 1.2: Distribution of the portfolio cash flow at $t = 2$



and is realised at $t = \frac{1}{2}$. Specifically, the state of the economy affects the expected return of the portfolio upon success. Conditional on success, the likelihood of realising a high cash flow is θ^s. I shall interpret the states with relatively higher θ^s as booms, and those with lower θ^s as recessions.

The bank can then privately choose the risk profile δ of the loan portfolio at $t = 1$. The bank can increase the riskiness of the businesses it lends to, but charge a higher yield on the loans. Specifically, the bank can decrease the success probability by δ, but increase the expected payoff of the portfolio upon success by $\delta\Delta_X$, through an increase in the conditional probability of receiving a high cash flow by δ. This setup loosely captures the trade-off between risk and return in financial investments.[11]

To summarise, for a given bank of type i in a given economic state s, given the risk choice δ of the bank, the investment succeeds with probability $\eta^i - \delta$. If successful, the loan portfolio returns a high cash flow $X + \Delta_X$ with probability $\theta^s + \delta$, or a low cash flow X with probability $1 - (\theta^s + \delta)$.

Bank value

The value of the bank is determined by the risk choice made at $t = 1$ and the type of the bank, in any given state s. For a type i bank in a given state s, the first best risk choice

[11]The specific assumption that a decrease in the success probability brings an increase of the same magnitude in the probability of receiving a high cash flow, conditional on success, is made to simplify the expressions. The results do not change qualitatively, as long as there is a trade-off between the success probability and the conditional probability of a high cash flow.

$\delta_{FB}^{i,s}$ maximises the value of the loan portfolio.[12]

$$\delta_{FB}^{i,s} \equiv \arg\max_{\delta}(\eta^i - \delta)[X + (\theta^s + \delta)\Delta_X] \tag{1.1}$$

As a result, the probability of success when the risk choice is first best is given by

$$q_{FB}^{i,s} \equiv \eta^i - \delta_{FB}^{i,s} = \frac{1}{2}(\eta^i + \theta^s + \frac{X}{\Delta_X}) \tag{1.2}$$

Therefore the model suggests that, when operated at the first best risk level, the bank has a higher probability of success in a higher state than in a lower state, and if it is a Good bank than if it is a Bad bank, other things equal.

Denote the portfolio value when operated at the first best risk level by $V_{FB}^{i,s}$. Assume that $V_{FB}^{G,s} > 1 > V_{FB}^{B,s}$ and $\gamma V_{FB}^{G,s} + (1-\gamma)V_{FB}^{B,s} > 1 \ \forall s$. That is, a Good bank has a positive NPV investment if operated at an appropriate risk level, whereas a Bad bank always has a negative NPV investment. However, at the first best risk level, an average bank has a positive NPV investment. This implies that pooling equilibria are feasible in this model.

As only a Good bank managed without risk-shifting produces positive NPV, the first best outcome in this economy can be produced if the bank (i) raises financing and invests if and only if it is a Good type at $t = 0$ or $t = \frac{1}{2}$, and (ii) chooses the first best level of risk at $t = 1$. However, because the lending can be value-enhancing on average, a pooling equilibrium with financing is feasible in which the bank invests regardless of its type.

Ex post **and** ***ex ante*** **financing**

The main results of the model are derived from the bank's choice of capital structure to finance the lending in equilibrium. I detail the financing game in this section.

After observing its type i, the bank can raise financing either at $t = \frac{1}{2}$ after the resolution of the economic uncertainty (*ex post* financing), or at $t = 0$ prior to the resolution of the economic uncertainty (*ex ante* financing).

I assume that the bank is endowed with internal capital $\bar{e} < 1$. It is therefore unable to self-finance the loans. At $t = 1$ the bank can finance the loan portfolio partly with

[12]Assume that $\frac{X}{\Delta_X} < \eta^i + \theta^s < 2$, so that the probability of success $q^{i,s}$ and the conditional probability of the loan portfolio paying off a high cash flow $(\theta^s + \delta^{i,s})$ can both lie within the range of $(0, 1)$ in the first best case and in all equilibria derived in this paper. The derivation of this assumption is given in Appendix A.1.

its internal capital $e \leq \bar{e}$ and partly from outside investors. This endowment can be interpreted as the sum of the internal capital available within the bank and the maximum amount of funds that can be provided by incumbent shareholders.[13]

This paper takes a security design approach and solves for the equilibrium financing contract. Without loss of generality, I express the overall contract given to the outside investors as a combination of debt with state-contingent face value F^s maturing at $t = 2$ and a state-contingent fraction α^s of the residual equity.[14] The model therefore allows financing via debt and equity, which are the forms of financing used in practice. This framework thus also allows the study of hybrid instruments, as most of the commonly adopted hybrid instruments can be thought of as a combination of debt and equity. As will be shown in Section 1.3, the optimal state-contingent capital structure can be implemented via CoCo bonds, a kind of such hybrid instruments.

In the case of *ex post* financing at $t = \frac{1}{2}$, the state of the economy s is common knowledge. In state s, the bank raises capital by promising the outside investors a combination of debt with face value F^s and a fraction α^s of the residual equity.[15]

In the case of *ex ante* financing at $t = 0$, the bank raises capital prior to the resolution of the economic uncertainty. I consider a general contingent capital structure specification that is given by a set of face values of the debt in each state of the economy $\boldsymbol{F}_C \equiv \{F_C^s\}_{s=1}^S$ and a set of fractions of the equity issued to outside investors $\boldsymbol{\alpha}_C \equiv \{\alpha_C^s\}_{s=1}^S$, so that the *ex post* capital structure of the bank depends on the realisation of the economic state s.[16]

[13]The model assumes that the bank's asset is solely comprised of the loan portfolio. One can also interpret the portfolio as a marginal investment whose payoff can be contracted upon, which would be the case for securitisation. Alternatively, the portfolio can be a part of the on-going operations of a bank, as long as that at $t = 0$ the bank does not have outstanding risky debts. If the bank has existing risk debt, the bank's incentives for financing and investment are distorted by the debt overhang problem. For an analysis on the efficient recapitalisation of banks under debt overhang, see Philippon and Schnabl (2013).

[14]Because of the three point cash flow space $\{0, X, X + \Delta_X\}$, in a given state s, debt with face value $F^s \leq X$ and residual equity replicate any contract that satisfies the usual assumption of monotonicity.

[15]In order to allow the difference between debt and equity as funding instruments, I restrict parameter values such that the cost of risk-shifting is sufficiently high relative to the NPV of the bank's investment, so that the bank cannot be purely debt financed.

[16]To highlight the economic intuition for the properties of the conversion, I assume that the macroeconomic state is verifiable and therefore contractable. In Section 1.4 I discuss potential implementation of the optimal capital structure contracted upon alternative variables including equity prices and CDS spreads.

For each case of the model, the financing game is played as follows. Firstly the bank decides whether to raise financing and invest given its type and its knowledge regarding the state of the economy. Assume that the bank chooses not to invest if it is indifferent between participating or not. Practically, this is the case if there is a small but non-zero cost to participate in the capital market.

If the bank decides to raise financing and invest, it announces in the capital market debt issue with face value F and equity issue of fraction α, where (F, α) are either $(\boldsymbol{F}_C, \boldsymbol{\alpha}_C)$ or (F^s, α^s) as specified above in each case of the model. The bank also puts up $e \leq \bar{e}$ of its own capital and retains the remaining fraction $(1 - \alpha)$ of the equity. After observing the financing plan (e, F, α), capital market investors form a belief regarding the type of the bank, and decide whether or not to accept the terms and provide capital $1 - e$.[17]

1.2.2 Definition of equilibrium

A PBE with financing is a set of financing parameters (e, F, α) representing the amount of internal capital invested by the incumbent bank shareholders, the face value of the debt issued to outside investors and the fraction of the equity issued to outside investors; and a consistent belief assigned by the capital market regarding the type of bank, such that (i) the market valuation is fair given the belief and that the investors at least break even at the issuing price, (ii) the bank optimally makes the financing decision at $t = 0$, and (iii) the bank optimally makes the risk decision at $t = 1$.

Consistent with the existing literature on signalling games (e.g. Spence, 1973) there exists a continuum of PBE. I invoke the Intuitive Criterion of Cho and Kreps (1987) in order to focus on equilibria with reasonable out-of-equilibrium beliefs. Intuitively, given the resulting equilibrium, there cannot exist an off-equilibrium-action such that (i) one type (Bad) is strictly worse off deviating to it, and (ii) if the market indeed believes that the deviation can only come from the other type (Good), this type strictly prefers to defect.

[17] I only consider the case in which the bank raises just enough to finance its lending. This is without loss of generality. A bank can technically raise more than $1 - e$ in terms of outside capital, in which case the excess can only be stored as cash. Since this part of the asset yields zero and poses no information problems to investors, it does not affect the residual payoff structure of the model. In other words, every equilibrium in which a bank raises more than necessary corresponds to an equilibrium in which it raises exactly one unit of capital.

In some cases of the analysis in Section 1.5, the Intuitive Criterion still leaves equilibria with substantially different characteristics. In these cases, I invoke the concept of undefeated equilibrium proposed by Mailath et al. (1993). Consider a proposed equilibrium and an action that is not taken in the equilibrium. Suppose there is an alternative equilibrium in which some types of the player prefer the alternative equilibrium. The criterion then requires that the beliefs at that action in the original equilibrium to be consistent with this set of types. Otherwise, the second equilibrium defeats the proposed equilibrium. In this model, if the Intuitive Criterion leaves both a pooling and a separating equilibrium, the pooling equilibrium defeats the separating equilibrium if both types are better off in the pooling equilibrium.

1.3 *Laissez-faire* equilibria

I derive the *laissez-faire* equilibria in this section and discuss the procyclicality of the equilibrium leverage. The cases of *ex post* and *ex ante* financing are analysed separately. Whereas the case of *ex ante* financing is of primary interest in this paper, the case of *ex post* financing is useful for highlighting the trade-off effects of the asymmetric information and the risk-shifting problem in determining the equilibrium bank capital structure.

1.3.1 Equilibrium with *ex post* financing

In this section I consider the case of *ex post* financing. In the absence of any macroeconomic uncertainty at the time of financing, this version highlights the interaction between the two main frictions considered in this model.

In this case, at $t = \frac{1}{2}$ after the macroeconomic state s becomes common knowledge, the bank announces its financing plan (e, F^s, α^s). Following a backward induction process, I firstly inspect the risk choice of the bank at $t = 1$, and then solve for the optimal financing plan at $t = \frac{1}{2}$.

At $t = 1$, for a bank of type i in state s, given a financing plan (e, F^s, α^s), the risk level δ is chosen to maximise the expected value of the retained cash flow by the bank $(1-\alpha^s)(\eta^i - \delta)[X + (\theta^s + \delta)\Delta_X - F^s]$. Alternatively, the optimal risk choice $\delta^{i,s}(F^s)$ maximises the equity value of the bank

$$\delta^{i,s}(F^s) \quad = \quad \arg\max_{\delta}(\eta^i - \delta)[X + (\theta^s + \delta)\Delta_X - F^s] \tag{1.3}$$

The face value of the outstanding debt F fully determines the bank's risk choice, because for a given capital structure, equity value is independent from the ownership structure α^s. In turn F^s also completely determines the success probability, equity value and the total bank value in equilibrium. Denote the equity value and the total value of a type i bank in state s given the optimal risk choice as $E^{i,s}(F^s)$ and $V^{i,s}(F^s)$ respectively. In particular, denote the equilibrium success probability given leverage F^s by $q^{i,s}(F^s)$, given by

$$q^{i,s}(F^s) \equiv \eta^i - \delta^{i,s}(F^s) = \frac{1}{2}(\eta^i + \theta^s + \frac{X - F^s}{\Delta_X}) \tag{1.4}$$

This highlights the risk-shifting incentives induced by leverage, which decreases the success probability. Therefore the first best risk choice can only be implemented if and only if the bank has an unlevered capital structure, i.e. $F^s = 0$.

I now turn to the security design problem at $t = 0$. Applying the Intuitive Criterion allows the Good firm to select the "least-cost separating equilibria", as stated in Proposition 1. A separating equilibrium (e, F^s, α^s) is characterised by the following constraints

$$(PC^B) \quad : \quad (1 - \alpha^s)E^{B,s}(F^s) \leq e \tag{1.5}$$

$$(PC^G) \quad : \quad (1 - \alpha^s)E^{G,s}(F^s) \geq e \tag{1.6}$$

$$(IR) \quad : \quad V^{G,s}(F^s) - (1 - \alpha^s)E^{G,s}(F^s) \geq 1 - e \tag{1.7}$$

The participation constraints for the Bad bank and the Good bank, (PC^B) and (PC^G) respectively, dictates that only the Good bank raises financing and invests. The investors' rationality constraint (IR) takes into account that the outside investors, regardless of the kind of securities they hold, claim the total value of the bank less the equity retained by the insiders. I will henceforth refer to an equilibrium in which the (IR) holds in equality as a fair-price equilibrium.

Proposition 1. *Any equilibrium under ex post financing that satisfies the Intuitive Criterion is a fair-price separating equilibrium in which only a Good bank raises financing and invests. The equilibrium capital structure is given by*

$$\arg \max_{e,F^s,\alpha^s} (1 - \alpha^s)E^{G,s}(F^s) \qquad s.t.\ e \leq \bar{e}, (PC^B)\ and\ (IR) \tag{1.8}$$

Proof. This and all other proofs are provided in Appendix A.2. ☐

The above optimisation programme allows us to characterise the equilibrium capital structure that satisfies the Intuitive Criterion in further detail. Firstly, notice that in equilibrium, the bank should always prefer internal financing to outside financing. That is, the equilibrium financing plan involves putting in all the internal capital $\bar{e}$, whenever outside leverage is used. This is because internal financing is free from either the risk-shifting problem or the asymmetric information problem in this model.

I proceed to consider the optimal mix of debt and equity when outside debt financing is required, and the unique equilibrium leverage is summarised in the following proposition.

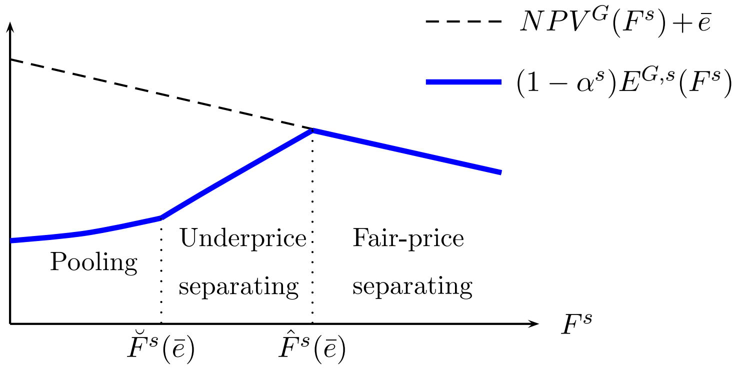
Corollary 1 (to Proposition 1). *There exists a threshold $\breve{e}^s$ such that, in any equilibrium under ex post financing that satisfies the Intuitive Criterion,*

- *If $\bar{e} \geq \breve{e}^s$, the bank issues only equity and no debt.*
- *If $\bar{e} < \breve{e}^s$, the unique equilibrium capital structure is $(\bar{e}, \hat{F}^s(\bar{e}), \hat{\alpha}^s(\bar{e}))$, where $\hat{F}^s(\bar{e}) > 0$ and $\hat{\alpha}^s(\bar{e})$ are characterised by the binding participation constraint of the Bad bank (PC^B) and the investors' rationality constraint (IR).*

The intuition for this result is illustrated in Fig. 1.3, for a Good bank with a given level of internal capital $\bar{e}$. The figure plots the value of the Good bank $NPV^G + \bar{e}$ (dashed line), and the maximum payoff to the inside shareholders of the Good bank $(1-\alpha^s)E^{G,s}(F^s)$ (solid line), in any equilibrium with a given leverage F^s. It can be shown that there exist thresholds $\breve{F}^s(\bar{e})$ and $\hat{F}^s$, such that the maximum payoff to the inside shareholders of the Good bank is obtained in a pooling equilibrium amongst equilibria with face value $F^s \leq \breve{F}^s(\bar{e})$, it is obtained in a separating equilibrium with underpricing amongst equilibria with face value $F^s \in (\breve{F}^s[\bar{e}), \hat{F}^s(\bar{e})]$, and it is obtained in a fair-price separating equilibrium amongst equilibria with face value $F^s \geq \hat{F}^s(\bar{e})$. The leverage in an equilibrium that satisfies the Intuitive Criterion $\hat{F}^s$ (if $\hat{F}^s \geq 0$) maximises the retained equity payoff in equilibrium to the Good bank, as given by Proposition 1.

Fig. 1.3 indicates that the unique leverage level in any equilibrium that satisfies the Intuitive Criterion is the lowest level of leverage that achieves separation at fair-price. This is because, firstly, in any fair-price separating equilibrium, the bank retains the entire NPV created by the bank's lending (the dashed line in Fig. 1.3, which coincides with the solid line for $F^s \geq \hat{F}^s(\bar{e})$), which is decreasing in the amount of leverage due to risk-shifting problems. It therefore does not have the incentive to increase leverage any further than $\hat{F}^s(\bar{e})$. Secondly, if the bank chooses a leverage level $F^s < \hat{F}^s(\bar{e})$,

Figure 1.3: Equilibrium selection using the Intuitive Criterion



it has to either underprice the securities issued to signal its type, or pool with a Bad bank. In either case, the Good bank receives less than the full NPV from the lending. By increasing leverage, the Good bank enjoys a private benefit greater than the cost of risk-shifting because of reduced mispricing. The Good bank therefore prefers to separate by using leverage $\hat{F}^s(\bar{e})$.

Corollary 1 shows that in the unique Intuitive equilibrium, only a Good bank raises funds in the capital market, by issuing fairly priced securities. As in the literature on asymmetric information, the constrained efficient outcome can only be achieved when the bank has sufficient internal capital. For $\bar{e} < \breve{e}^s$, the bank employs additional leverage and subsequently chooses a higher risk profile.

The framework also demonstrates the intuition of Myers and Majluf (1984) that in the presence of asymmetric information, there is a tendency to rely on internal sources of funds, and to prefer debt over equity if external financing is required. If the bank has sufficient amount of internal capital, the first best result can be achieved.

The pecking order theory, based on asymmetric information alone, is silent about the determinants of the debt capacity.[18] The interaction between the risk-shifting incentive and the adverse selection problem in this model, similar to that studied by John (1987), endogenously determines the unique equilibrium level of leverage and hence the capital structure. Specifically, leverage mitigates the adverse selection problem, but incurs a cost due to excessive risk-shifting incentives. The debt capacity in this model is thus provided by the extent of the risk-shifting problem.[19]

[18]The model of Myers and Majluf (1984) shows that the a firm with private information always issues debt and never issues equity.

[19]Other models of capital structure with frictions that predict an interior solution for leverage can also

1.3.2 Equilibrium with *ex ante* financing – Contingent capital structure

I now turn to consider the equilibrium capital structures if the bank raises financing at $t = 0$ prior to knowing the realisation of the underlying economic state. I solve for the equilibrium capital structure within a general class of contingent capital structure given by a set of face values of the debt $\boldsymbol{F}_C$ and a set of fractions of equity issued to outsiders $\boldsymbol{\alpha}_C$ specified for each state s.

A separating equilibrium $(e, \boldsymbol{F}_C, \boldsymbol{\alpha}_C)$ is characterised by the following constraints,

$$(PC_C^B) \quad : \quad \mathbb{E}\left[(1-\alpha_C^s)E^{B,s}(F_C^s)\right] \leq e \tag{1.9}$$

$$(PC_C^G) \quad : \quad \mathbb{E}\left[(1-\alpha_C^s)E^{G,s}(F_C^s)\right] \geq e \tag{1.10}$$

$$(IR_C) \quad : \quad \mathbb{E}\left[V^{G,s}(F_C^s) - (1-\alpha_C^s)E^{G,s}(F_C^s)\right] \geq 1 - e \tag{1.11}$$

This set of conditions is similar to the set of conditions 1.5–1.7 for the case of *ex post* financing which was analysed in the previous section. The difference is that in this section, financing is obtained prior to the realisation of the underlying economic state. Since the economic uncertainty only resolves at $t = \frac{1}{2}$, the risk choice at $t = 1$ takes into account the macro state s, while the financing terms at $t = 0$ only relies on the prior distribution of the economic states. The equilibrium conditions are therefore given in expectation, and are weaker than those in the case of *ex post* financing.

Following similar intuition as in the case with *ex post* financing, a bank prefers internal financing to outside financing, and chooses leverage levels to maximise its retained equity payoff, trading off between the asymmetric information and the risk-shifting problems. The resulting equilibria are given as follows.

Proposition 2. *Any contingent capital equilibrium under ex ante financing that satisfies the Intuitive Criterion is a fair-price separating equilibrium in which only a Good bank raises financing and invests. The set of equilibrium contingent capital structures is given by*

$$\arg\max_{e, \boldsymbol{F}_C, \boldsymbol{\alpha}_C} \mathbb{E}\left[(1-\alpha_C^s)E^{G,s}(F_C^s)\right] \qquad s.t. \quad e \leq \bar{e}, (PC_C^B) \text{ and } (IR_C) \tag{1.12}$$

There exists a threshold $\breve{e}^C$ such that the bank issues debt if and only if $\bar{e} < \breve{e}^C$.

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be interpreted as providing a debt capacity, such as Darrough and Stoughton (1986), Leland (1994) among others.

Similar to the case with *ex post* finance, in any equilibrium with *ex ante* financing that satisfies the Intuitive Criterion, the Good bank chooses the leverage level that allows it to separate from the Bad at the least cost of risk-shifting.

1.3.3 Properties of the *laissez-faire* equilibria

Procyclical equilibrium leverage

This section highlights the procyclicality of the equilibrium leverage in the cases of *ex post* and *ex ante* financing, which creates scope for contingent convertible bonds as discussed in Section 1.4. In this section I examine the implications of procyclical leverage on the bank's risk-taking incentives and the resulting default probabilities.

Proposition 3. *The face values of debt in an equilibrium that satisfies the Intuitive Criterion are procyclical in both the case with ex post financing and the case with ex ante financing. That is,*

$$\hat{F}^s(\cdot) \geq \hat{F}^z(\cdot), \qquad \forall\, \theta^s > \theta^z \tag{1.13}$$

where the inequality is strict if and only if $\bar{e} < \breve{e}^s$*; and*

$$\hat{F}^s_C(\cdot) \geq \hat{F}^z_C(\cdot) \tag{1.14}$$

for any $s, z \in \{s \in \boldsymbol{S} : \hat{\alpha}^C_s < 1\}$ *s.t.* $\theta^s > \theta^z$*, where the inequality is strict if and only if* $\bar{e} < \breve{e}^C$.

The procyclicality result is due to the fact that the information asymmetry problem is relatively more severe in a good state when the returns on the loan portfolio are high in general. This is because in this model, the marginal impact on the value of the bank's claims brought by an increase in the economic fundamentals is greater for a Bad bank than for a Good bank.

$$1 < \frac{E^{G,s}(F)}{E^{B,s}(F)} < \frac{E^{G,z}(F)}{E^{B,z}(F)} \quad \forall\, \theta^s > \theta^z \tag{1.15}$$

This is consistent with the view of diminishing marginal return, or when the complementarity between the economic fundamentals and the bank's type is not too high. As a result, the difference between the Good and the Bad banks' (retained) equity value becomes relatively insignificant in booms. Therefore in equilibrium, relatively higher leverage is required during economic booms in order to resolve the information asymmetry.

The result that the equilibrium leverage of banks is procyclical is supported by Adrian and Shin (2008a), who document that changes in total assets are positively correlated with changes in leverage of financial institutions. This model suggests that banks employ procyclical leverage to minimise the cost of asymmetric information and the cost of risk-shifting incentives.[20]

Although either case produces procyclical leverage in equilibrium, the contingent capital structure in the *ex ante* financing case is less procyclical than the equilibrium in the case with *ex post* financing. This is driven by the fact that the cost of risk-shifting is convex in the amount of leverage in this model, i.e. $\frac{\partial V^{i,s}(F^s)}{\partial F^s} = \frac{F}{2\Delta_X} > 0$.[21] Financing *ex ante* with a contingent capital structure therefore allows the bank to reduce the cyclicality in its leverage to minimise the expected cost of risk-shifting. This is reflected in the cyclicality of the resulting equilibrium default probabilities (Proposition 4). Intuitively, on the one hand, the intrinsic success probabilities are higher in booms, while on the other hand, procyclical equilibrium leverage implies higher risk-taking in booms which tends to increase the default probabilities of the bank.

Proposition 4. *The equilibrium default probabilities are procyclical in the case of ex post financing. For* $\bar{e} \leq \breve{e}^z$,

$$1 - q^{G,s}(\hat{F}^s) > 1 - q^{G,z}(\hat{F}^z) \qquad \forall\, \theta^s > \theta^z \tag{1.16}$$

The equilibrium default probabilities are countercyclical in the case of ex ante financing. For $\bar{e} \geq \breve{e}^C$ *or for any* $s, z \in \{s \in \boldsymbol{S} : \hat{\alpha}^s_C < 1\}$,

$$1 - q^{G,s}(\hat{F}^s_C) \leq 1 - q^{G,z}(\hat{F}^z_C) \qquad \forall\, \theta^s > \theta^z \tag{1.17}$$

With *ex post* financing, the resulting default probabilities are procyclical. In equilibrium, the bank's choice of leverage overcompensates for the better economic prospects, because they do not fully internalise the cost of leverage in the presence of information asymmetry. To see this, notice that for any leverage level less than $\hat{F}^s$, a

[20] It is widely acknowledged that the leverage ratios of financial institutions are procyclical, contrary to non-financial firms, who tend to exhibit negative correlation between asset returns and leverage ratio, known as the "leverage effect". For example, Adrian and Shin (2008b,a) attribute such observation to banks targeting a leverage ratio given by the value-at-risk, while Gromb and Vayanos (2002) and Geanakoplos (2010) show that collateral constraints generate leverage that tends to be procyclical.

[21] The convexity of the cost of risk-shifting in leverage is also what ensures an interior solution for the choice of leverage.

Good bank must issue its securities at a discount because of information asymmetry. The existing shareholders of the bank therefore do not retain the full value created by its loan portfolio. In particular, they share with the outside investors the value destruction brought by an increase in leverage. This conflict of interests leads to excessive risk-taking in booms, sowing the seeds of a bust. Taking the view that securitisation is an important source of funding for banks, this implication is consistent with the findings of Griffin and Tang (2012) that AAA-rated CDO tranches issued between 2003 and 2007, when asset values were high, were of increasingly deteriorating quality leading up to the crisis.

With *ex ante* financing, however, the equilibrium default probabilities given a contingent capital structure are countercyclical. An *ex ante* financing decision allows the bank to take advantage of the relative severity of the information asymmetry problem in different macroeconomic states, which helps the bank to internalise the cost of leverage, alleviating the risk-shifting problem. In particular, to maximise the benefit of leverage in mitigating the information asymmetry problem at the *ex ante* stage, a bank prefers to employ more leverage in the state in which the information asymmetry problem is more severe. This is the state that is less risky as measured by a lower default probability, because the information content in the equity is less pronounced when it is less risky, i.e.

$$1 < \frac{E^{G,s}(F^s)}{E^{B,s}(F^s)} < \frac{E^{G,z}(F^z)}{E^{B,z}(F^z)} \quad \text{iff} \quad q^{i,s}(F^s) > q^{i,z}(F^z) \tag{1.18}$$

Therefore a bank would never take on so much more leverage in a high state such that it results in a default probability higher than that in a low state, because the excessive leverage employed in the high state is inefficient in resolving either of the two frictions. Specifically, a bank in such a situation would benefit from reducing the excessive leverage employed in the high state and increasing leverage in the low state. In doing so, the bank incurs less cost of risk-shifting in expectation due to the convexity of the risk-shifting problem in leverage, as well as improves the efficiency in mitigating the information asymmetry problem, as leverage is shifted to the state in which the information asymmetry problem is more severe.

Efficiency of the contingent capital structure

In this section I compare the *ex ante* efficiency of the equilibria in the case of *ex post* financing and the case of *ex ante* financing. *Ex ante* efficiency is measured by the sum of all agents' expected payoffs, or the expected value of the bank, in equilibrium.

Proposition 5. *Raising capital ex ante using a contingent capital structure is (weakly) preferred to raising capital ex post.*

$$\mathbb{E}\left[V^{G,s}(\hat{F}^s_C(\bar{e}))\right] \quad \geq \quad \mathbb{E}\left[V^{G,s}(\hat{F}^s(\bar{e}))\right] \qquad \forall\, \bar{e} \tag{1.19}$$

where the above inequality is strict for $\bar{e} < \breve{e}^C$.

Since the *ex ante* financing problem nests the *ex post* financing problem, it is clear that it is at least as efficient as the *ex post* problem, because the set of constraints for the *ex ante* financing problem is weaker. Moreover, the *ex ante* financing equilibrium is strictly preferred to the *ex post* financing equilibrium whenever the two are not identical. This is for $\bar{e} < \breve{e}^C$, as evident from the earlier discussion regarding the cyclicality of the equilibrium default probabilities.

This section asserts that the optimal capital structure is contingent on the realisation of the economic state in the *laissez-faire* equilibrium. The contingent capital structure equilibrium minimises the expected cost of risk-shifting necessary to signal the bank's private information, by employing procyclical leverage to balance the information-sensitivity of the residual equity in different states. The bank would therefore voluntarily issue contingent capital securities to implement the procyclical equilibrium leverage, without restriction.[22]

1.4 Implementation using contingent convertible bonds

The key result of the baseline model is that the optimal contingent capital structures feature procyclical leverage that is higher in booms and lower in busts. Contingent convertible bonds are a natural addition to debt and equity in order to implement the optimal capital structures of the bank, as it contractually specifies a reduction of the face value of the debt in an economic downturn. This section presents an example of an economy with the possibility of a tail event to illustrate the key features of the CoCo bonds that implement the optimal contingent capital structure. I then discuss the issues surrounding the practical design of CoCo bonds.

In this section, I consider the special case of an economy with two possible states $\boldsymbol{S} = \{L, H\}$. The High state θ^H occurs with probability β, and the Low state θ^L occurs

[22] In practice, if the banks are subject to a leverage constraint that binds in all states, they would not employ contingent capital structure, because their capital structure is determined by the leverage constraints.

with probability $1-\beta$, where β is large. We can interpret the High state as the normal state, and the Low state as an unlikely adverse state – the "tail event".

1.4.1 CoCo bonds and procyclical leverage

I characterise the two types of the equilibrium that satisfy the Intuitive Criteria in the tail event economy, and then present an implementation of the equilibrium capital structure using contingent convertible bonds contracted upon the verifiable macroeconomic states in this model.

By Proposition 2–4, the equilibrium of contingent capital structure that satisfies the Intuitive Criterion is a fair-price separating equilibrium in which only a Good bank raises financing and invests. The equilibrium capital structure $(\boldsymbol{F}_C, \boldsymbol{\alpha}_C)$ are such that the equilibrium leverage and equity values are procyclical, i.e. $\hat{F}_C^H \geq \hat{F}_C^L$ and $E^{G,H}(\hat{F}_C^H) \geq E^{G,L}(\hat{F}_C^L)$.[23]

The optimality of procyclical leverage naturally points towards contingent convertible bonds as instruments for the implementation of the optimal capital structure. The reverse convertible feature of CoCo bonds reduces the face value of the debt in the bank's capital structure in the Low state. By contrast, conventional convertible bonds, which have been shown to play a role in mitigating adverse selection problems (e.g. Brennan and Kraus, 1987; Constantinides and Grundy, 1989) and moral hazard problems (e.g. Green, 1984; Mayers, 1998), do not implement the required contingency in this model.

The following proposition summarises the equilibrium contingent capital structure and proposes an implementation of the equilibrium contingent capital structure that involves CoCo bonds. Two scenarios of contingent capital structures can arise in this tail event economy depending on the value of $\bar{e}$.

Proposition 6. *There exist thresholds $\breve{e}^C$ and $\breve{e}^T$ such that any contingent capital equilibrium that satisfies the Intuitive Criterion is a fair-price separating equilibrium in which only a Good bank raises financing and invests.*

- *If $\bar{e} \geq \breve{e}^C$, the bank issues only equity and no debt.*
- *If $\bar{e} \in [\breve{e}^T, \breve{e}^C)$, the equilibrium contingent capital structure is $(\bar{e}, \{\hat{F}_C^H(\bar{e}), \hat{F}_C^L(\bar{e})\}, \{\hat{\alpha}_C^H(\bar{e}), \hat{\alpha}_C^L(\bar{e})\})$, where $\hat{F}_C^L(\cdot) = \hat{\alpha}_C^L(\cdot) = 0$, and $\hat{F}_C^H(\cdot) > 0$, $\hat{\alpha}_C^H(\cdot) \geq 0$ are given by*

[23] That the equity value is procyclical follows the fact that the default probability is countercyclical, because $E^{G,s}(F) = \left[q^{G,s}(F)\right]^2 \Delta_X$.

the binding (PC_C^B) *and* (IR_C).

The bank implements the equilibrium contingent capital structure by issuing

- *CoCo bonds with face value* $\hat{F}_C^H(\bar{e})$ *that is written down to zero contingent on the Low state; and,*
- *Warrants of fraction* $\hat{\alpha}_C^H$ *designed such that they are only exercised in the High state.*

- *If* $\bar{e} < \breve{e}^T$, *the equilibrium contingent capital structure is* $(\bar{e}, \{\hat{F}_C^H(\bar{e}), \hat{F}_C^L(\bar{e})\}, \{\hat{\alpha}_C^H(\bar{e}), \hat{\alpha}_C^L(\bar{e})\})$, *where* $\hat{F}_C^H(\cdot) = \hat{F}_C^L(\cdot) + \frac{\theta_H - \theta_L}{\Delta_X}$ *so that* $q^{G,H}(\hat{F}_C^H(\cdot)) = q^{G,L}(\hat{F}_C^L(\cdot))$, *and* $\hat{F}_C^L(\cdot)$, $\hat{\alpha}_C^H(\cdot)$, $\hat{\alpha}_C^L(\cdot)$ *are given by the binding* (PC_C^B) *and* (IR_C).

  The bank implements the equilibrium contingent capital structure by issuing

  - *Straight bonds with face value* $\hat{F}_C^L(\bar{e})$*;*
  - *Equity of fraction* $\hat{\alpha}_C^H(\bar{e})$*; and,*
  - *CoCo bonds with face value* $\frac{\theta^H - \theta^L}{\Delta_X}$ *that convert into equity in the Low state so that the fraction of outside equity becomes* $\hat{\alpha}_C^L(\bar{e}) \geq \hat{\alpha}_C^H(\bar{e})$.

The first scenario (corner solution) is for a bank with an intermediate level of internal capital $\bar{e} \in [\breve{e}^T, \breve{e}_C)$, that issues CoCo bonds with a write-down feature. As the bank is relatively well capitalised, the amount of leverage required to achieve separation is small. Since the debt issued in the High state is relatively less information-sensitive, leverage is only used in the High state, i.e. $\hat{F}_C^H(\bar{e}) > \hat{F}_C^L(\bar{e}) = 0$. This is implemented with CoCo bonds that write down to zero in the Low states. Moreover, given the equilibrium leverage, the residual equity is still less information-sensitive in the High state when the default probability is lower. The equilibrium equity issuance is therefore only in the High state but not in the Low state, implemented with warrant, in order to minimise the "cost of capital".[24] The insiders thus retain full ownership in the Low state to align their incentives, i.e. $\hat{\alpha}_C^H(\bar{e}) \geq \hat{\alpha}_C^L(\bar{e}) = 0$.

[24]This is in line with the results of Chemmanur and Fulghieri (1997) that warrants can be part of the equilibrium signalling device employed a good issuer with private information. In their model, firms issue warrants because the risk-averse inside shareholders of the good firms, which are also riskier, find it less costly to issue warrants than those of the safer firms that also have lower expected values. The warrants in this model, by contrast, are driven by the fact that in equilibrium, equity is less information-sensitive in the High state than in the Low state.

The second scenario (interior solution) is for a bank with a low level of internal capital $\bar{e} < \check{e}^T$, that issues CoCo bonds with a contingent convertible feature, in addition to straight debt and equity. In this case, a high amount of leverage is required to achieve separation, and debt must be issued in the low state as well as in the high state. In equilibrium the default probability $1 - q^{G,s}(\hat{F}_C^s(\cdot))$ is equalised between both states in order to minimise the costs of risk-shifting associated with leverage. Moreover, since in this case the equity value is equal in both states, there is one degree of freedom in determining the equity allocation between the High and the Low states. The optimal contingent capital structure in this scenario is therefore implemented using CoCo bonds with face value $\frac{\theta^H - \theta^L}{\Delta_X}$ that converts into equity in the Low state, in addition to straight bonds and equity.[25] This scenario highlights the efficiency gain provided by using CoCo bonds to implement the optimal contingent capital structures, which take advantage of the relative severity of the asymmetric information and risk-shifting problems across different states. This is evident from the fact that the face value of the CoCo bonds required $\frac{\theta^H - \theta^L}{\Delta_X}$ is determined by information regarding the verifiable state θ^s but not the private information of the bank η^i.[26].

This simple model of agency frictions endogenously gives rise to two types of CoCo bonds that are seen in the market. The model predicts that well capitalised banks issue CoCo bonds with a write-down feature, whereas banks in need of much capital issue CoCo bonds that convert into equity. For example, the first issues of CoCo bonds were by Lloyds Banking Group with a contingent convertible feature in November 2009 and by Rabobank with a write-down feature in March 2010. The tier 1 capital ratios of the two banks were 8.6% and 13.8% respectively prior to the issuances.[27] Credit Suisse issued CoCo bond swith a contingent convertible feature in 2011 and 2012, followed by CoCo bonds with a write-down feature in 2013, as the capital position of the bank improved.[28]

[25]Because there is one degree of freedom in determining the fractions of equity held by outsiders across states, implementation using CoCo bonds is feasible for $\hat{\alpha}_C^H(\bar{e}) \geq \hat{\alpha}_C^L(\bar{e})$. The write-down feature is therefore also a special case of the conversion of the CoCo bonds in this scenario, if the fractions of the equity issued to outsiders are equal in both states.

[26]In this model, the state s is verifiable and thus serves as the trigger for the conversion of the CoCo bonds. Although for the ease of interpretation I have referred to it as a macroeconomic state, the model is also consistent with the interpretation that the state s is driven by idiosyncratic events, provided that they are contractible. I discuss CoCo bonds with market triggers in the following Section

[27]Sources: Lloyds Banking Group (2009) and Rabobank Group (2009).

[28]According to Credit Suisse Regulatory Disclosures (2012), the core tier 1 capital ratio of Credit Suisse

1.4.2 Interpreting the contingent convertible bonds

The role of commitment and CoCo bonds

It is worth noting that the optimal contingent capital structure can only be implemented with *ex ante* contingent contract. As shown in Section 1.3.1, a bank raising capital *ex post* after the realisation of the macroeconomic state employs leverage that is excessively procyclical.

After financing is arranged at $t = 0$, it is crucial for the implementation of the optimal contingent capital structure that the bank commits to its pre-specified contingent capital structure until the payoff of the loan portfolio is realised. The model derives the optimal contingent capital structure assuming that the bank commits to the capital structure it chooses at $t = 0$. In fact, this commitment is necessary, as the equilibrium outcome may not be supported in a sequential PBE if the bank is able to alter its capital structure subsequent to the initial offering.

Specifically, leverage in this model sends a credible signal regarding the type of the issuer because the substitution of debt for equity financing is more costly for the Bad bank than for the Good. After the initial offering, however, a levered bank would be better off buying back the debt by issuing equity in order to remove the risk-shifting incentives whenever the market believes that it is of a Good type. Anticipating that the eventual capital structure would be unlevered, at $t = 0$ the investors would not be able to form the separating belief.

In practice, such commitment is likely to be enforced for a number of reasons. Firstly, it is likely to be costly to issue equity subsequently because of new asymmetric information problems that can arise. Under the current tax regime, there is also a tax disadvantage to issuing equity. Furthermore, a levered bank would be unwilling to issue additional equity or buy back existing debt because of the "debt-overhang" problem.

Current outstanding CoCo bonds are mostly long term. This helps to create a "debt-overhang" problem that reinforces the commitment by the bank to its optimal contingent capital structure. For example, the contingent convertible bonds issued by Credit Suisse have a 30-year maturity, and the Enhanced Capital Note issued by LLoyds Banking Group has fixed maturities ranging between 10–22 years.[29]

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under Basel II.5 increased from 10.4% to 14.4% from 2011 to 2012, and its tier 1 capital ratio increased from 15.1% to 18.4%.

[29]This model provides a supply-side rationale for the long-term nature of CoCo bonds. Bolton and

Trigger design

While so far in this model I refer to the verifiable state s as a macroeconomic state, the model is consistent with a trigger based on idiosyncratic events, provided that they are contractible. Examples of proposals for CoCo bond design based on idiosyncratic events include Flannery (2009), Hart and Zingales (2010, 2011), Pennacchi et al. (2010) and Pennacchi (2011). Others have proposed a dual trigger structure based on both an idiosyncratic event and a measure of macroeconomic downturn, such as Squam Lake Working Group (2009), McDonald (2011) and Kyle (2013).

Alternatives for a trigger that is placed on the idiosyncratic events of the issuing bank include regulatory capital ratios, or market prices of claims on the bank. Although a market price is a "criteri[on] that is informative, objective, timely, difficult to manipulate, and independent of regulators' intervention, avoiding the problems associated with other types of triggers", according to Sundaresan and Wang (2013), all the CoCo bonds that are issued thus far have triggers based on regulatory capital ratios. This is because a conversion trigger based on the market price of equity can suffer from the multiplicity or the absence of equilibria in the price of the equity of the issuing bank. Sundaresan and Wang (2013) show that a unique competitive equilibrium exists only if the conversion is designed to leave the equity price equal before and after the conversion. Moreover, the authors claim that this condition precludes penalising the existing shareholders, defeating the purpose of the CoCo bonds.

This model can be modified to allow a trigger based on the equity price of the bank. In this model, there exists a conversion ratio of the CoCo bonds that gives rise to a unique pricing equilibrium, while diluting the existing shareholders. This contrasts with the result of Sundaresan and Wang (2013), because unlike their model, this model endogenises the risk taking incentives of the bank and hence the bank value. The conversion of the CoCo bonds creates value since it lowers the leverage of the bank and reduces the bank's incentive to take excessive risk. The condition that the equity price remains constant therefore implies that the fraction of equity held by existing shareholders is diluted upon conversion.

Formally, assume that the state s is observable but not contractible, and that there

Samama (2012) and Hart and Zingales (2010) present a demand-side argument that CoCo bonds are likely to be purchased by long-term investors seeking to enhance yield in good times by risking losses in bad times.

is no frictions in the secondary market so that the price of the equity reflects perfectly the value of the equity in state s, given the capital structure of the bank and the market belief.[30] Suppose the bank has outstanding straight debt, CoCo bonds and common equity according the optimal contingent capital structure, and the number of common shares outstanding is N. The equity price in the high state is thus $p^H_{w/o} \equiv \frac{E^{G,H}(\hat{F}^H_C(\bar{e})}{N}$ without conversion. Suppose that the trigger price is set at $p^H_{w/o}$. That is, conversion occurs if the price falls below $p^H_{w/o}$, and the CoCo bonds convert into n shares of common equity, which correspond to a fraction $\frac{n}{N+n} \equiv \hat{\alpha}^L_C(\bar{e}) - \hat{\alpha}^H_C(\bar{e})$ of the equity. In the Low state, the equity prices with and without conversion are given by $p^L_w \equiv \frac{E^{G,L}(\hat{F}^L_C(\bar{e}))}{N+n} < p^H_{w/o}$ and $p^L_{w/o} \equiv \frac{E^{G,L}(\hat{F}^H_C(\bar{e}))}{N}$ respectively.[31] Consider a conversion ratio characterised by n such that the equity prices with and without conversion in the Low state are equal, i.e. $p^L_w = p^L_{w/o}$. It then follows that such a conversion ratio is indeed dilutive, i.e. $\hat{\alpha}^L_C(\bar{e}) - \hat{\alpha}^H_C(\bar{e}) = 1 - \frac{E^{G,L}(\hat{F}^H_C(\bar{e})}{E^{G,L}(\hat{F}^L_C(\bar{e})} > 0$.[32]

Hart and Zingales (2010) propose an alternative trigger mechanism based on the price of the CDS spreads, which reflects the default risk of the issuing bank. In this model, for a bank with a low level of internal capital $\bar{e} < \check{e}^T$, the optimal contingent capital structure specifies a conversion of the CoCo bonds in the Low state to keep the default probability constant. This property suggests that the CDS spread can also act as a conversion trigger that is free from the equilibrium problem, following similar arguments as those given above for a trigger based on equity prices.

[30] I abstract from the discussion regarding the informativeness of the equity price. Martynova and Perotti (2012) discuss the efficiency of an exogenously given CoCo bond triggered by equity prices that are noisy signals of the equity value. The authors find that a mandatory conversion based on such equity prices leads to more frequent conversions, and a regulatory trigger produces fewer conversions.

[31] That $p^L_w = \frac{E^{G,L}(\hat{F}^L_C(\bar{e}))}{N+n} < p^H_{w/o} = \frac{E^{G,H}(\hat{F}^H_C(\bar{e}))}{N}$ is due to the fact that the equity values are equal in both states given the optimal contingent capital structure, i.e. $E^{G,H}(\hat{F}^H_C(\bar{e})) = E^{G,L}(\hat{F}^L_C(\bar{e})$.

[32] It is straightforward to check that given this conversion ratio, the unique equilibrium is indeed for the CoCo bonds to convert only in the Low state. Firstly, since the equity prices in the Low state with and without conversion are both below the trigger price, $p^L_w = p^L_{w/o} < p^H_{w/o}$, the unique equilibrium in the Low state is for the CoCo bonds to convert. Secondly, the unique equilibrium in the High state is for the CoCo bonds not to convert, and for the equity price of be equal to $p^H_{w/o}$. In the High state, the equity price, if the CoCo bonds convert, is given by $p^H_w \equiv \frac{E^{G,H}(\hat{F}^L_C(\bar{e})}{N+n}$. However, since the conversion ratio results in less dilution in the High state than in the Low state when the equity value is higher for any given amount of leverage, a conversion would result in a higher equity price $p^H_w > p^H_{w/o}$, which contradicts with the conversion trigger of the CoCo bonds.

1.5 Optimal countercyclical capital regulation

Although the equilibrium capital structure trades off the cost and benefit of leverage, the bank takes on excessive leverage when its level of internal capital is low, due to its private incentive to minimise the mispricing in the securities it issues. The *laissez-faire* equilibrium thus warrants regulation, since excessive risk-shifting by the bank decreases the value of the businesses the bank lends to, incurring a social cost. In this section I characterise the optimal capital requirements, which are countercyclical, and discuss how CoCo bonds can be used to implement the regulated equilibrium.

In any given state s, a minimum capital requirement is defined as a cap on the face value of the debt, $\bar{F}^s$. Given a capital requirement, there exist Intuitive equilibria in which the bank issues debt with face value no higher than $\bar{F}^s$ in state s. The optimal *ex ante* capital requirement is a set of state caps $\bar{\boldsymbol{F}} \equiv \{\bar{F}^s\}_{s=1}^{S}$ such that it maximises the expected value of the bank in the resulting equilibria that satisfy the Intuitive Criterion.

Imposing a cap on the face value of the debt $\bar{F}^s$ is equivalent to requiring a minimum capital ratio $c^{i,s}(\bar{F}^s) \equiv \frac{E^{G,s}(\bar{F}^s(\cdot))}{V^{G,s}(\bar{F}^s(\cdot))}$ in a given state s, because the capital ratio of the bank is monotonically decreasing in $\bar{F}^s$. In this section I refer to a minimum capital requirement as a cap on the face value of the debt, as opposed to a capital ratio, to simplify exposition and to allow direct comparison with the previous section. The optimal capital regulation $\bar{\boldsymbol{F}}$ derived below can be implemented with a state-contingent minimum capital ratio $\boldsymbol{c} \equiv \{c^s\}_{s \in S}$.

1.5.1 Optimal capital regulation

The following proposition summarises the optimal capital regulation.

Proposition 7. *The optimal minimum capital requirement can improve the efficiency of the laissez-faire (contingent) capital structure equilibrium and produce the constrained efficient outcome. There exists $\breve{e}^C_P$ and $\breve{e}^C_{LP}$ such that*

- *For $\bar{e} \geq \breve{e}^C$, the capital requirement never binds. The bank issues only equity in the capital market and invests if and only if it is of the Good type.*

- *For $\bar{e} \in [\breve{e}^C_P, \breve{e}^C)$, the capital requirement of maximum leverage $\bar{\boldsymbol{F}}(\bar{e}) = \mathbf{0}$ binds. The bank issues only equity in the capital market and invests if and only if it is of the Good type.*

- *For $\bar{e} \in [\breve{e}^C_{LP}, \breve{e}^C_P]$, the capital requirement of maximum leverage $\bar{\boldsymbol{F}}(\bar{e}) = \breve{\boldsymbol{F}}(\bar{e})$ binds. The bank issues both debt and equity in the capital market and invests if and only if it is of the Good type. $\breve{\boldsymbol{F}}(\bar{e})$ is the least-cost leverage cap a regulator can impose that implements a separating equilibrium, which is given by*

$$\arg\max_{\boldsymbol{F}_C} \mathbb{E}\left[V^{G,s}(F^s_C)\right] \quad s.t. \quad v^G(\boldsymbol{F}_C;\bar{e}) \geq v^G_P(\boldsymbol{F}_C;\bar{e}) \tag{1.20}$$

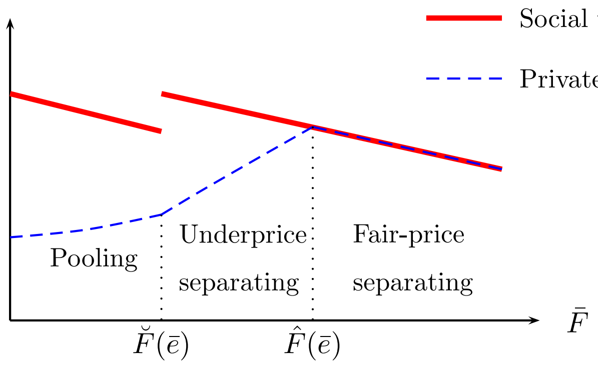
  where $v^G(\boldsymbol{F}_C;\bar{e})$ and $v^G_P(\boldsymbol{F}_C;\bar{e})$ are the expected retained equity value of the Good bank in the least-cost separating equilibrium that in the least-cost pooling equilibrium respectively, given the capital regulation.

- *For $\bar{e} < \breve{e}^C_{LP}$, the capital requirement of maximum leverage $\bar{\boldsymbol{F}} = \boldsymbol{0}$ binds. The bank issues only equity in the capital market and invests regardless of its type.*

I discuss the intuition for Proposition 7 using, as an example, the simplest case where there is only one macroeconomic state, i.e. no macroeconomic uncertainty. In this case the capital regulation is a single cap on leverage $\bar{F}$. I will comment on the cyclicality of the optimal capital regulation in relation to macroeconomic uncertainty in the next section (Section 1.5.2).

Fig. 1.4 highlights the inefficiency in the *laissez-faire* equilibrium and hence the rationale for capital regulation. The figure plots the social value produced by the bank (solid line) and the payoff to the inside shareholders of the Good bank $(1-\alpha^s)E^{G,s}(F^s)$, in the equilibrium that maximises the payoff to the inside shareholders of the Good bank for a given leverage F^s. The social value is measured as the value of the Good bank, $NPV^G(F^s) + \bar{e}$, if the equilibrium is separating, and as the expected value of the bank, $\gamma NPV^G(F^s) + (1-\gamma)NPV^B(F^s) + \bar{e}$, if the equilibrium is pooling.

Figure 1.4: Inefficiency in the *laissez-faire* equilibrium



Section 1.3 has shown that in the *laissez-faire* equilibrium, a bank with $\bar{e} < \breve{e}^C$ chooses leverage level $\hat{F}(\bar{e})$ when raising capital, which maximises its retained payoff. Figure 1.4 illustrates that the *laissez-faire* equilibrium leverage $\hat{F}$ is too high and induces excessive risk-taking by the bank. Although a lower leverage level increases the value of the bank and the social value produced by the bank's lending, the bank is unwilling to reduce its leverage because of the underpricing in the securities it issues due to asymmetric information.

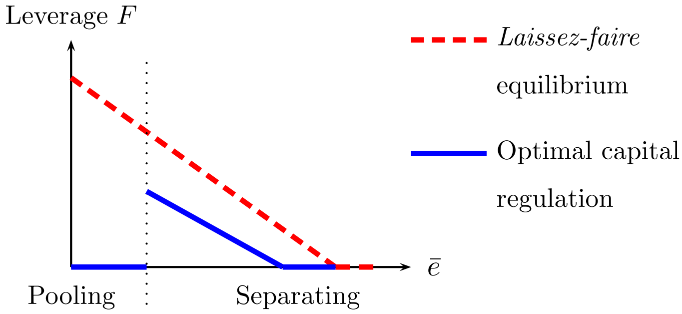
A capital regulation that reduces the equilibrium leverage improves the social value of a Good bank with $\bar{e} < \breve{e}^C$, as represented by the solid line in Fig. 1.4. However, the extent to which the regulator can cap leverage is limited by the bank's private incentives to maximise its retained payoff. As the bank's leverage is capped to be less than $\hat{F}(\bar{e})$, the bank finds it privately costly to signal its type through underpricing. Faced with a stringent capital requirement $\bar{F} < \breve{F}(\bar{e})$, the Good bank would prefer to pool with the Bad bank, instead of incurring the heavy cost of signalling (Eq. 1.20 is violated). This results in another social loss, as it enables the Bad bank to raise capital and make value-destroying investments.

Constrained by the private incentives of a bank with little internal capital $\bar{e} < \breve{e}^C_P$, the optimal capital regulation is either $\bar{F}(\bar{e}) = \breve{F}(\cdot)$ to implement a separating equilibrium, or $\bar{F}(\bar{e}) = 0$ to implement a pooling equilibrium, depending on whether it is less costly to resolve the asymmetric information problem or to curb the risk-shifting incentives. This result highlights the inherent tension between how the information asymmetry and the risk-shifting problems can be solved in this model. That is, higher leverage mitigates the information asymmetry problem as it reduces the information-sensitiveness of the securities issued and hence the potential mispricing, but it also incurs social losses due to the excessive risk-taking incentives it creates. Fig. 1.5 illustrates how this trade-off varies with the bank's internal capital $\bar{e}$.

It is worth noting that the optimal capital requirement $\bar{F}^s(\bar{e})$ depends on the level of the bank's internal capital $\bar{e}$. This raises an important distinction between "inside" and "outside" capital. Regardless of the type of the security issued to obtain outside financing, the equilibrium outcome depends on the bank's "skin in the game" $\bar{e}$, rather than the total debt to equity ratio.

Depending on the bank's internal capital $\bar{e}$, the optimal capital regulation falls into one of the two regions (Fig 1.5). For a relatively better capitalised bank, the optimal

Figure 1.5: Optimal capital regulation



capital regulation limits the amount of leverage the bank can employ, in order to reduce the cost of risk-shifting while allowing the bank to signal through underpricing in addition to leverage (the separating region). For poorly capitalised banks, however, the amount of leverage required to achieve separation is high, and the optimal capital regulation imposes zero leverage and implements a pooling equilibrium (the pooling region).

The optimal capital requirement, while improving social value by curbing excessive risk-taking induced by leverage, nevertheless makes financing costly for the Good bank in terms of mispricing. The amount of mispricing is illustrated by the wedge between the private value received by the bank and the social value (Fig. 1.4). This is consistent with the observation that equity capital is generally perceived to be expensive. For example, Elliott (2009, p.12) states that "the problem with capital is that it is expensive. If capital were cheap, banks would be extremely safe because they would hold high levels of capital." In this model capital is costly in the regulated equilibrium because of the two frictions, in the absence of which the bank would indeed choose no capital market debt and the first best risk level. As argued by Admati et al. (2010), however, it should be noted that capital is not necessarily socially costly, which creates scope for capital regulation.

1.5.2 Countercyclical capital regulation and CoCo bonds

Having characterised the optimal capital requirements, this section explores the countercyclical property of the optimal capital regulation using again the example of the tail event economy, and discusses the role of CoCo bonds in the regulated equilibrium.

A minimum capital requirement is defined as countercyclical, if in the resulting equilibrium, the bank has a countercyclical capital ratio.

Proposition 8. *In the tail event economy, the optimal minimum capital requirement is* *countercyclical. That is, given the procyclical leverage caps $\bar{F}^H(\bar{e}) \geq \bar{F}^L(\bar{e})$, the bank has a countercyclical capital ratio $c^H \leq c^L$, where $c^s \equiv \frac{E^{G,s}(\bar{F}^s)}{V^{G,s}(\bar{F}^s)}$. The inequalities are strict for $\bar{e} \in [\breve{e}^C_{LP}, \breve{e}^C_P]$.*

In particular, there exists a threshold $\breve{e}^T_{LP}$ such that for $\bar{e} \in [\breve{e}^C_{LP}, \breve{e}^T_{LP}]$, the bank implements the equilibrium contingent capital structure subject to capital regulation by issuing CoCo bonds with the same face value as in the laissez-faire equilibrium $\frac{\theta^H - \theta^L}{\Delta_X}$ that convert into equity in the Low state, in addition to straight debt with a lower face value than in the laissez-faire equilibrium $\bar{F}^L(\bar{e}) < \hat{F}^L_C(\bar{e})$ and equity.

That the capital regulation imposes procyclical leverage follows the same reasoning to those of Proposition 3. As the capital regulation in this case intends to achieve separation (following Proposition 7), higher leverage is required in the High state, when asset values are high and the asymmetric information problem is more severe. The amount of CoCo bonds in the bank's capital structure is therefore the same as in the *laissez-faire* equilibrium, capturing the relative severity of the asymmetric information problem across the High and the Low states (for parameters that give rise to an interior solution). Nevertheless, the optimal capital regulation lowers the amount of leverage in the resulting equilibrium, improving the social value of the bank.

The proposition suggests that the optimal capital regulation is countercyclical. The capital ratio in the regulated equilibrium is countercyclical because the optimal procyclicality of the leverage equalises the equity values across the High and the Low states, in order to maximise the efficiency of the leverage in mitigating the asymmetric information problem, following similar reasoning to those of Proposition 4. The capital ratio is therefore higher in the Low state, as the bank value is lower. This result confirms the optimality of countercyclical capital proposed by the Bank for International Settlement (Basel Committee on Bank Supervision, 2010), in which the regulators call for a capital buffer that takes account of the macro-financial environment in which banks operate. The proposal suggests that the buffer be "deployed ... when excess aggregate credit growth is judged to be associated with a build-up of system-wide risk to ensure the banking system has a buffer of capital to protect it against future potential losses." This idea, advocated by scholars such as Brunnermeier et al. (2009) and Griffith-Jones and Ocampo (2011), is shown to be optimal in this model of risk-shifting incentives and adverse selection.

CoCo bonds emerge as part of the implementation of the optimal countercyclical capital regulation. Currently Basel III developed by the Basel Committee on Bank Supervision (2011, p. 58) requires that the countercyclical capital buffer is deployed by national jurisdictions with a pre-announcement by up to 12 months before the system-wide risk materialises, to allow time for banks to adjust to a buffer level. In practice, it is not clear that the build-up of the buffer can always be achieved in a timely fashion. In light of the results in this model, the optimal countercyclical capital buffer can be implemented using CoCo bonds, which are voluntarily issued by banks well before the build-up of the system-wide risk to meet the countercyclical capital requirement. The CoCo bonds, subject to a mandatory conversion triggered by a regulatory declaration of a state of systemic risk, implements an immediate recaptalisation of the banks.

1.6 Extensions

The baseline model thus far considers the role of leverage in a bank's capital structure in mitigating the asymmetric information problem at the cost of inducing risk-shifting by the bank. The optimal contingent capital structure entails procyclical leverage implemented with CoCo bonds, because the asymmetric information problem is relatively more severe in booms. Due to the bank's private incentive to reduce any mispricing in the securities it issues, however, the bank employs excessive leverage, and incurs a social cost as its subsequent risk-shifting behaviour negatively impacts the value of the businesses the bank lends to. The *laissez-faire* equilibrium thus warrants countercyclical capital regulation, which curbs excessive leverage and risk-taking, while preserving the procyclicality of the bank's leverage.

Recurrent financial crises have highlighted the systemic effect of large and correlated bank failures, including a direct impact on the real economy through credit contraction, and a network effect on the financial health of other financial institutions.[33] Consequently, regulators are unwilling or unable to allow major financial institutions to fail.[34] While

[33]For example, Ivashina and Scharfstein (2010) document a run by short-term bank creditors following the failure of Lehman Brothers, which contributed to a 47% reduction in the extension of new loans to large borrowers in the fourth quarter of 2008 relative to the previous quarter.

[34]For example, the run on Northern Rock, Britain's fifth-largest mortgage lender, did not stop until a taxpayer-backed guarantee of all existing deposits was announced in September 2007, and the US Treasury and the Federal Reserve System bailed out 282 publicly traded banks and insurance companies under the Troubled Asset Relief Program (TARP) in 2008–9. Sources: Economist (2007) and Wilson and Wu (2012).

extending state guarantee can be *ex post* optimal for a regulator to alleviate the systemic impact of a bank failure, it creates a moral hazard problem that incentivises *ex ante* risk-taking by the bank.

This section provides two extensions of the model to consider two types of government guarantees: implicit bailouts and explicit deposit insurance. I analyse the moral hazard problem created by the government guarantees and examine the implications for the optimal capital regulation in relation to the frictions considered in the baseline model, using again the example of a tail event economy.

1.6.1 Bailout and bail-in

This section modifies the baseline model to incorporate the systemic importance of the bank. I assume that the regulator has the incentive to bail out a failed bank *ex post* by repaying the creditors on the bank's behalf, to reduce the systemic externalities posed by the failure of the bank. I analyse the moral hazard problem created by the bailouts and study the optimal *ex ante* capital regulation. Noticing that the *ex post* bailout incentives hinder the efficiency of the *ex ante* capital regulation, I discuss the role of "bail-in" capital in alleviating the moral hazard problem created by bailouts.

Assume that a systematically important bank failing to meet its promised repayment to creditors poses a large cost to the rest of the economy $\bar{\xi}$. Although not modelled explicitly, this large social cost intends to capture the systemic effects of a bank failure and the costs to the real economy. Assume also that an *ex post* bailout by the regulator can reduce the cost to $\underline{\xi} < \bar{\xi}$, which accounts for any remaining externalities that cannot be resolved by a bailout and the deadweight loss associated with financing the bailout.

Bailouts and moral hazard

Suppose the regulator maximises social welfare but cannot commit to any time-inconsistent policies. *Ex post* at $t = 2$, the government has the incentive to bail out a failed bank, by repaying the creditors on the bank's behalf to reduce the social cost from $\bar{\xi}$ to $\underline{\xi}$.

The incentives for a bailout at $t = 2$ creates moral hazard problems for the bank. At $t = 2$, the bank receives a bailout whenever its cash flow is less than its promised repayment F^s. Anticipating a bailout, the bank issues effectively risk-free debt in the capital market at $t = 0$ and receives an implicit subsidy of the amount $\mathbb{E}[(1 - q^{i,s}(F^s))F^s]$

for any given debt structure $\boldsymbol{F}$. Since the value of the implicit subsidy is increasing in F^s, the bank without restriction takes on unlimited leverage and destroys the value in the loans through subsequent risk-taking. This simple extension of the model captures the moral hazard problem of government bailouts, such as those documented by Duchin and Sosyura (2013).[35]

Recognising the moral hazard problem as well as the inefficiency discussed in Section 1.5, the regulator at $t = 0$ designs the optimal *ex ante* capital regulation $\bar{\boldsymbol{F}}$ as defined in 1.5, with the expectation of a bailout at $t = 2$. The *ex ante* objective of the regulator is to maximise the expected social value of the bank less the social loss associated with bank failures, i.e. $\mathbb{E}[V^G, s(\bar{F}^s) - (1 - q^{G,s}(\bar{F}^s))\underline{\xi}]$. The following proposition characterises the optimal *ex ante* capital regulation of a systemically important bank, given the expectation of a bailout *ex post*.

Proposition 9. *In the tail event economy, with the expectation of a bailout ex post, the optimal minimum capital requirement is countercyclical.*

In particular, there exist thresholds $\breve{e}^{BO}_{LP}$ and $\breve{e}^{BO}_{P}$ such that, for $\bar{e} \in [\breve{e}^{BO}_{LP}, \breve{e}^{BO}_{P}]$, the optimal capital requirement for a systemically important bank implements a contingent capital structure equilibrium in which the bank issues CoCo bonds with the same face value as in the baseline model $\frac{\theta^H - \theta_L}{\Delta_X}$ that convert into equity in the Low state, in addition to straight debt with a higher face value than in the baseline model under optimal capital regulation $\bar{F}^L_{BO}(\bar{e}) > \bar{F}^L_T(\bar{e})$, and equity.

The optimal capital regulation of a systemically important bank remains countercyclical, and can be implemented using CoCo bonds with the same face value as in the baseline model (for parameters that give rise to an interior solution). This is consistent with earlier results that the amount of CoCo bonds in a bank's capital structure is determined by the relative severity of the asymmetric information problem across the High and the Low states, in order to minimise the expected cost of risk-shifting induced by the leverage required to signal the private information of the bank.

Relative to the optimal capital regulation derived in Section 1.5, the optimal capital regulation of a systemically important bank permits higher leverage in the form of straight debt. This follows from the earlier intuition that, the extent to which the regulator can

[35] Duchin and Sosyura (2013) show that banks make riskier loans and shift investment portfolios towards riskier securities after being approved for government assistance.

impose leverage caps is limited by the bank's private incentives. Since an *ex post* bailout creates a moral hazard problem which reduces the private cost of risk-shifting internalised by the bank, higher leverage must be allowed to alleviate the asymmetric information problem. The *ex post* bailout of a failed bank to mitigate the *ex post* social loss therefore hinders the efficient capital regulation of the bank *ex ante*.

Bailout and bail-in

Bail-in capital, a form of contingent debt that is automatically wiped out in case the bank fails, has been considered by regulators such as the Basel Committee on Bank Supervision (2011) and the Bank of England, in order to shield taxpayers from the need to bail out a bank that is "too big to fail".[36] In light of the results in this extension, if a regulator can credibly commit to bailing in the debt *ex post*, strictly capital regulation, as characterised in Section 1.5, can be imposed, resulting in lower leverage and less risk-taking by banks in equilibrium.

With all capital market debts "bail-inable", the bank writes down its liabilities at $t = 2$ in case it cannot meet the promised repayments. As the bank now fully internalises the cost of risk-taking when it issues risky debt in the capital market *ex ante*, the equilibrium and the optimal capital regulation are identical to those characterised in Section 1.3–1.5 and can be implemented using CoCo bonds. That is, the bank issues the optimal amount of equity, straight debt and CoCo bonds at $t = 0$, where both the straight debt and CoCo bonds have a bail-in feature. At the interim, if a low microeconomic state realises, the CoCo bond conversion is triggered, subsequently reducing the incentives for risk-shifting. Finally at $t = 2$ when the loan portfolio pays off and the securities mature, the debt holders and CoCo bond holders (if they remain unconverted) are paid off if the bank produces sufficient cash flows. Otherwise, all bonds are written off according to the bail-in arrangement, and the bank enters into an orderly resolution.

This extension highlights the differential roles played by the two types of contingent capital instruments. While CoCo bonds provide early conversion well before bankruptcy to implement the optimal procyclical leverage, bail-in capital features a write-down of the debt only in the event of a default. Bail-in capital is effective in avoiding socially costly bank failures and forcing the bank to internalise the cost of risk-taking *ex ante*, thereby resolving the moral hazard problem created by the government's incentive to bail

[36] Source: Federal Deposit Insurance Corporateion and Bank of England (2012).

out failed banks *ex post*. Given the *ex post* incentives to bail out a failed bank, not only does bail-in capital improve the efficiency of the bank in managing its loan portfolio, it also leads to fewer incidences of bank failures *ex post*, reducing the cost associated with the failure of a systemically important bank.

1.6.2 Depositors and deposit insurance

Banks perform the important functions of maturity transformation and risk transformation by offering deposit contracts. The model thus far has abstracted from this aspect by assuming risk neutral investors, to focus on the trade-off effect of leverage across different macroeconomic states. This section modifies the model to extend the funding base of the bank to include a set of risk averse, unsophisticated depositors with liquidity needs.

I provide conditions for when deposit insurance is essential, and show that deposit insurance creates a moral hazard problem similar to that of a bailout, in that it allows the bank access to cheap funding via insured deposits, without internalising the cost of its risk-taking decision. As a result, the optimal *ex ante* capital regulation in this extension remains countercyclical and can be implemented with CoCo bonds, but permits high levels of leverage using a combination of demand deposits and straight debt, because the extent to which the bank can cap leverage is limited by the bank's private incentives.

Depositors

In this extension I assume that the risk neutral capital market investors have limited capital $k < 1-\bar{e}$, but there is an unlimited supply of funds from a continuum of depositors. The bank thus must raise financing from the depositors in order to finance its lending.

Each depositor has 1 unit of endowment and faces idiosyncratic liquidity shocks following Diamond and Dybvig (1983). I assume the unsophisticated depositors can only observe whether or not their bank runs out of funds. Therefore they would only accept a debt-like contract.

Each depositor demands early consumption at $t = 1$ with probability λ or late consumption at $t = 2$ otherwise. The utility of each depositor is given as

$$U(c_1, c_2) = \begin{cases} u(c_1) & \text{if Early, with prob. } \lambda \\ u(c_2) & \text{if Late, with prob. } 1-\lambda \end{cases} \tag{1.21}$$

At $t = 0$, a depositor is willing to deposit with the bank if the bank offers expected utility of at least $u(1)$. I normalise $u(0) = 0$, $u(1) = 1$. The liquidity shock is i.i.d. across depositors and unobservable while the probability λ is common knowledge.

Deposit insurance and moral hazard

As the bank must raise financing at least partially via demand deposits, this section provides conditions for when deposit insurance is essential for the depositors to be willing to provide capital. I then discuss the moral hazard problem associated with deposit insurance and the implications for the optimal capital regulation.

Deposit insurance is a commitment from the government to inject up to $\bar{D}$ to the bank in case the bank runs out of funds to repay the depositors. I assume that the deposit insurance is financed by charging an *ex ante* fair insurance premium so there is no deadweight loss associated with proving the insurance *ex post*.

Suppose the bank takes $\frac{D}{1-\lambda}$ amount of deposits from a mass $\frac{D}{1-\lambda}$ of depositors, promising each depositor a consumption plan (c_1, c_2) upon request, where $c_1 \leq c_2$ so that it is incentive compatible for a late depositor to wait. *Ex post* exactly a mass $\frac{\lambda}{1-\lambda}D$ of the depositors demand early repayment at $t = 1$, and the remaining mass D of depositors wait until $t = 2$.[37] In order to prevent illiquidity at $t = 1$, the bank holds $\frac{\lambda}{1-\lambda}D$ in liquid reserve (storage) to meet the withdrawal at $t = 1$.[38] The bank's $t = 2$ liability to the depositors is therefore Dc_2. Denote the total liability of the bank in state s as F^s.

At $t = 2$, there is a positive probability that the bank returns 0. Assuming sequential service of deposits, in case the bank runs out of funds, a depositor who joins the queue late receives nothing, whereas one who is at the front of the queue receives full promised repayment. With any given deposit insurance $\bar{D} \leq Dc_2$, in a symmetric equilibrium in which all late depositors wait untill $t = 2$, a late depositor receives the full repayment with probability $\frac{\bar{D}}{Dc_2}$, and 0 otherwise. A depositor is willing to deposit with the bank only if his expected utility provided by the deposit contract is at least that if he enjoyed

[37] Assuming sequential service of deposits and given the government guarantee at $t = 2$, the Late depositors prefer waiting until $t = 2$ to joining the queue at $t = 1$.

[38] The bank optimally chooses to hold reserve to prevent premature bankruptcy given its own capital input $e > 0$. If there is a new generation of depositors at $t = 1$, the bank can also raise fresh capital via deposits at $t = 1$ to meet the withdrawal by the first cohort of depositors. This extension of the model refrains from the coordination problem at $t = 1$ to focus on the moral hazard problem associated with possible bank failures at $t = 2$.

his endowment with certainty, given the capital structure of the bank. Having normalised $u(0) = 0$ and $u(1) = 1$, the break-even condition of a depositor is

$$\lambda u(c_1) + (1-\lambda)\,\mathbb{E}\left[q^{i,s}(F^s) + [1-q^{i,s}(F^s)]\frac{\bar{D}}{Dc_2}\right]u(c_2) \geq 1 \tag{1.22}$$

The following proposition provides a sufficient condition for full deposit insurance to be necessary to fund the bank.

Proposition 10. *If the depositors are sufficiently risk averse such that* $u'(1) \leq \frac{(1-\lambda)(1-\bar{e}-k)}{2\Delta_X}$*, the bank cannot raise deposits without a deposit insurance. The cost to the government for providing the insurance is minimised at full coverage up to* $\bar{D} = 1-\bar{e}-k$*.*

Proof. Appendix A.2.9 □

The intuition is as follows. For a given amount of deposit insurance $\bar{D} < Dc_2$, a risk averse depositor would demand a payoff $c_2 > 1$ at $t = 2$, knowing that the bank defaults with positive probability. In turn, a higher liability induces the bank to increase its portfolio risk in an attempt to maximise shareholders' value, resulting in higher default probability. If the depositors are sufficiently risk averse, they require a high yield which creates a risk-shifting problem so severe that the depositors would not deposit with the bank whenever the deposit is risky. Therefore the government must provide full deposit insurance $\bar{D} = D$. Given full coverage, the demand deposit is risk free and the bank chooses to promise $c_1 = c_2 = 1$ to allow the depositor to break even. This part of the result suggests that the risk aversion of the outside investors exacerbates the risk-taking incentives of the bank.

A full coverage deposit insurance creates a moral hazard problem similar to that of a government bailout. Given the total $t = 2$ liability $\boldsymbol{F}$ and a full coverage guarantee on the deposit $\bar{D}$, the bank has the incentive to maximise its deposit base up to the coverage of the deposit insurance programme, as it enjoys an implicit subsidy of the value $\mathbb{E}[1-q^{i,s}(F^s)]$ per unit deposit taken. The total value of the bank, therefore, consists of the value of the loan portfolio $\mathbb{E}[V^{i,s}(\cdot)]$ and the implicit subsidy of value $\mathbb{E}[1-q^{i,s}(F^s)]\bar{D}$, which is increasing in the deposit coverage $\bar{D}$. As the bank cannot be financed without deposits, which induce risk-shifting in equilibrium, the first best outcome is not attainable. For constrained efficiency, the government should provide the minimum amount of deposit insurance to enable financing $\bar{D} = 1-\bar{e}-k$ as long as the value of the loan portfolio $V^{G,s}(\bar{D}) > 1$. I will assume this is true for comparability with the baseline model.

With the optimal deposit insurance $\bar{D}$ in place, the financing plan of a bank is now given by $(e, D, \boldsymbol{F}, \boldsymbol{\alpha})$, where $F^s \geq D$ is the total book leverage of the bank at $t = 2$, of which $(F^s - D)$ is capital market debt. The bank, when making the capital structure choice, considers the value of the bank as the sum of the value of the loan portfolio and the implicit subsidy provided by the deposit insurance. That is, the bank of type i in state s given the leverage level is $B^{i,s}(F^s) \equiv V^{i,s}(F^s) + [1 - q^{i,s}(F^s)]\bar{D}$. Assuming that this is the case, the following proposition summarises the consequence of the moral hazard problem associated with deposit insurance, which is similar to that of bailouts.

Proposition 11. *Lower availability of capital market funds increases the reliance of banks on fully insured deposits. This induces greater leverage and risk-taking by the banks. The procyclicality of the leverage in the optimal contingent capital structure and the countercyclicality of the optimal capital requirements derived in Section 1.3–1.5 go through. In particular, the face value of CoCo bonds remain the same at* $\frac{\theta^H - \theta^L}{\Delta_X}$ *for banks with little internal capital.*

This result contrasts with the baseline case with only risk neutral investors. Deposit insurance provides the bank with an implicit subsidy, similar to the bailout of capital market debts analysed in Section 1.6.1. However, while capital market debts can be made "bail-inable" to mitigate the moral hazard problem associated with *ex post* bailouts, depositors must be protected in all circumstances, leaving an implicit subsidy of the amount $\mathbb{E}[1 - q^{i,s}(F^s)]\bar{D}$, which lowers the effective cost of leverage for the bank. The equilibrium capital structure chosen therefore entails higher leverage, trading off the benefit of mitigating asymmetric information problems and the cost of inducing risk-shifting. However, it remains optimal for a bank to issue CoCo bonds, since CoCo bonds improve the efficiency of the bank by allowing the bank to employ higher leverage in booms when the asymmetric information problem is relatively severe. In particular, the face value of the CoCo bonds remain the same as in the baseline model, and is only determined by information regarding the microeconomic states θ^s. Since the extent to which the regulator can limit leverage is constrained by the bank's private incentives, the optimal countercyclical capital regulation permits higher leverage than in the baseline case.

1.7 Concluding remarks

Capital regulation of financial institutions has long been at the centre of policy discussions. The recent financial crisis led to the realisation that a single risk-weight capital ratio fails to address macro-prudential concerns. Since then scholars and regulators have called for higher capital requirements and contingent capital to address the procyclical problems of bank leverage. However, a theory is required that incorporates the problems under consideration in order to assess the plausibility of any capital regulation as a solution.

This paper presents a model of the financial structure of a bank that is in need of outside capital for investment. The bank has private information regarding the returns of the investment opportunities, and has a risk-shifting incentive *ex post* to increase the shareholders' value at the expense of the debt holders. These two agency frictions endogenously determine the equilibrium capital structures of the bank, trading off the benefit of leverage as an information-insensitive security and the cost of risk-shifting induced by leverage. Moreover, since the asymmetric information problem is relatively more severe in booms, it is optimal for the bank to raise capital *ex ante* specifying *ex post* procyclical capital structures.

The optimal contingent capital structure can be implemented using contingent convertible (CoCo) bonds in addition to straight debt and equity. The model generates both the write-down feature and the contingent convertible feature seen in the CoCo bonds issued by banks.

Although the model predicts that banks have the incentives to voluntarily issue CoCo bonds in a *laissez-faire* equilibrium, banks are subject to financial regulation in practice. The model notes that the privately chosen leverage levels are generally excessive due to the bank's incentives to minimise market misplacing of its securities. The optimally designed capital requirements, which are countercyclical, can improve the efficiency of the banks. Subject to the optimal capital regulation, the bank maximises its shareholder value by issuing CoCo bonds to meet the capital requirements.

Banks' excessive risk-taking not only harms shareholders' value, but also brings significant instability and associated costs as highlighted by the recent financial crisis. I introduce state guarantees in the form of government bailouts and deposit insurance. The moral hazard problems associated with state guarantees reduces the cost of risk-shifting internalised by a bank. This limits the extent to which a regulator can restrict bank leverage *ex ante*. Nevertheless, countercyclical capital regulation remains optimal, because the optimality of the procyclical leverage implemented by CoCo bonds is determined by the relative severity of the asymmetric information problem across different macroeconomic states.

CHAPTER 2

COUNTERCYCLICAL FORECLOSURE FOR SECURITISATION

2.1 Introduction

The epidemic of mortgage foreclosures in the US which started in 2008 has raised concerns from the general public and policy makers.[1] The number of foreclosures started to surge in 2007 and continued to rise into 2010. It has been argued that foreclosures create significant losses for both the lenders and the borrowers, and have major negative externalities to the broader society.[2] In response, the Federal Reserve has set up a series of programs in an attempt to reduce mortgage foreclosure, such as the Home Affordable Modification Program. The scale and significant economic implication of foreclosure deserves attention in order to achieve an understanding of its driving force and the underlying mechanism.

Recent studies and reports have suggested that securitisation and the biased incentives of mortgage servicers could have contributed to the wave of foreclosure. For instance, Piskorski et al. (2010) show that, during the recent crisis, mortgages in a securitised pool are more likely to be foreclosed than otherwise similar mortgages on bank portfolios. In addition, an analysis of the complex compensation structure of the servicers by Thompson

[1]For example, the Huffington Post has a designated section for news on the foreclosure crisis.

[2]See for example Pennington-Cross (2006) for a survey on the deadweight loss on foreclosure.

(2009) concludes that the servicers' legal and financial incentives bias servicers towards foreclosure instead of modifying delinquent mortgages, even when investors would profit more from modification than foreclosure.

This paper investigates the optimal foreclosure policy of securitisers in a framework of mortgage-backed securitisation under asymmetric information. This framework allows us to answer the following questions. How does information asymmetry in the securitisation process give rise to a foreclosure policy that is *ex post* inefficient? Why does foreclosure appear countercyclical? What is the role of a third-party servicer in the securitisation process? In aggregate, how can a "foreclosure crisis" arise?

We explicitly model the foreclosure decision in the mortgage-backed securitisation and the market for repossessed property. A securitiser has a mortgage pool which returns risky cash flows, and securitisation is motivated by the liquidity needs of the securitiser a là DeMarzo and Duffie (1999). However, some mortgages subsequently become delinquent and the securitiser must decide whether to modify or foreclose the delinquent mortgages. If a mortgage is modified (forbearance), the full repayment is recovered with some probability. If a mortgage is foreclosed, the underlying property is repossessed and sold in a designated market for repossessed properties. Investors in the market for repossessed properties post an aggregate downward sloping demand curve, generating market clearing prices of repossessed properties that are decreasing in the amount of property foreclosed.[3]

The securitiser has private information regarding the probability of recovery on the delinquent mortgages. This may be because the securitiser has access to the specific borrower information. The securitiser then designs and sells a mortgage-backed security to the outside investors. Consistent with existing literature on security design (e.g. Myers and Majluf (1984); Nachman and Noe (1994)), we establish that the securitiser chooses to issue a senior security, or debt, to the outside investors. As in DeMarzo and Duffie (1999), securitisers with high quality mortgage pools signal their type by retaining the residual junior tranche, which entails a liquidity cost.

The main result of the paper is that information asymmetry in the securitisation process leads to countercyclical foreclosure. In the baseline model, we consider the optimal foreclosure policy if the securitiser can choose a set of type-contingent foreclosure rates

[3] This is micro-founded in the model by modelling a mass of investors with heterogeneous renovation costs when investing in repossessed properties.

prior to obtaining any private information on the mortgage pool and commit to the foreclosure policy *ex post*. Compared to the full information benchmark, the optimal foreclosure rates under asymmetric information are more negatively related to the quality of the mortgage pool *ex post*. In other words, signalling concerns lead to more foreclosure when the mortgage pool is of poor quality, and vice versa. This implies that the aggregate foreclosure is negatively related to the overall quality of the mortgage pools in the economy.

The intuition of the above result is as follows. The optimal foreclosure policy under asymmetric information maximises the securitiser's expected payoff by trading off the costs of signalling against the *ex post* inefficiency in the foreclosure decision. In order to reduce the signalling costs, a securitiser designs a policy that discourages the low type from mimicking the high type. The low type's payoff from mimicking comprises the proceeds from selling the debt claim at the high type's price and the value of the retained cash flow. The foreclosure policy of the securitiser has two effects on the incentive for the low type to mimic. Firstly, an inefficiently low foreclosure rate for the high type reduces the incentive to mimic by decreasing the value of the debt security issued by the higher type. Secondly, an excessively high foreclosure rate for the low type discourages mimicking by decreasing the value of the retained junior claim (levered equity), since foreclosure reduces the risk in the overall cash flow from the mortgage pool. This is because, *ex post* under any given market condition, foreclosing a mortgage brings an immediate cash flow equal to the market price of the property, while forbearance entails risk in the potential recovery of the delinquent mortgage. Therefore the equilibrium optimal foreclosure policy is excessively countercyclical.

The ability of the securitisers to commit to the *ex ante* chosen optimal foreclosure policy is crucial in the above mechanism. In an environment where the commitment power is not naturally available, we show that, in equilibrium, third-party servicers play a role in enforcing such commitment. First of all, if the securitiser cannot commit to the *ex ante* chosen optimal foreclosure policy, she would tend to foreclose less *ex post*. Moreover, *ex post* foreclosure policy is positively related to the quality of the mortgage pool. This is because, given that the high quality securitiser retains the junior tranche, which is convex in the cash flows, she benefits from riskier cash flows *ex post*. However, the lack of commitment power hurts the *ex ante* securitisation process, leading to a lower expected payoff in equilibrium for the securitisers.

We then propose a mechanism that resembles the industry practice to enforce such commitment power, which involves mortgage servicers. This is inline with the view of Thompson (2009), who argues that the rise of the servicing industry is a by-product of securitisation.[4] An important function performed by mortgage servicers is the decision of forbearance versus foreclosure. A third-party servicer allows the separation of this decision from the securitiser, potentially enabling the securitiser to commit to a set of *ex ante* chosen foreclosure policies.

In this mechanism, a mortgage originator with a pool of mortgages can choose to (i) securitise the pool himself with the *ex post* servicing done in-house, in which case the foreclosure decision will be made *ex post* as illustrated previously; or (ii) sell the mortgage pool to a securitiser but remain as the servicer of the mortgages. In the latter case, the securitiser offers a compensation contract that includes a payment transfer and fees dependent on the *ex post* cash flow of the mortgage pool. The securitiser then proceeds to issue the optimal mortgage-backed securities, while the servicer makes the *ex post* foreclosure decision according to the incentives given by his compensation.

We show that there exist contracts that implement the optimal foreclosure policy. Because selling the mortgage pool to a securitiser enables commitment and reduces the costs associated with asymmetric information, it is more efficient than in-house servicing. Moreover, for mortgage pools of low quality, the compensation to the servicer is designed to lean towards foreclosure. This implements the optimal foreclosure policy which would appear to be excessive *ex post*, namely, the *ex post* foreclosure may result in a loss to the investors. This is evident in the past financial crisis. For example, Levitin (2009) estimates that lenders lose approximately 50% of their investment in a foreclosure situation.

Finally, we extend the model to consider an economy with multiple securitisers who compete in the market for repossessed property when mortgages are foreclosed. The foreclosure policy is still higher for a lower quality mortgage pool, and lower for a high quality one. This leads to countercyclical foreclosure in equilibrium, that is, the overall foreclosure is higher in an economic downturn in which many mortgage pools are of low quality. The prices in the repossessed property market are hence procyclical.

We also examine the two additional sources of friction brought by the competitive

[4]The servicer performs duties including collecting the payments, forwarding the interest and principle to the lenders, and negotiating new terms if the debt is not being paid back, or supervising the foreclosure process.

environment considered in the market for repossessed property. On the one hand, the fire-sale externality arises with competition, which tends to result in excessive foreclosure in equilibrium. This is because a securitiser does not internalise the negative externality of her decision to foreclose a delinquent mortgage on the other securitisers' payoff due to its price impact. On the other hand, the market power of each securitiser decreases with competition. This tends to increase foreclosure as it reduces the inefficiency associated with oligopoly in terms of insufficient foreclosure. The overall effect suggests that under strong competition, prominent fire-sale externality exacerbates the countercyclical effect of asymmetric information and leads to significant excessive foreclosure in bad economic times, which can be interpreted as a foreclosure crisis.

Related Literature

This paper belongs to the growing body of literature on the incentive problems associated with mortgage securitisation. Various studies argue that securitisation relaxes the *ex ante* lending standards. Keys et al. (2010, 2012), using evidence from securitised subprime loans, show that the ease of securitisation reduces lenders' incentives to carefully screen the mortgage borrowers and that mortgages with higher likelihood to be securitised have higher default rates. Mian and Sufi (2009) find that securitisation of subprime loans is associated with credit expansion and, as a result, counties with a high proportion of subprime mortgages face a larger number of defaults. Elul (2011) also finds securitised prime loans have a higher default rates than otherwise comparable portfolio loans. Our work adds a different dimension to this literature by studying the decision of *ex post* mortgage foreclosure in relation to securitisation.

Our paper also relates to the study of optimal loan modification and foreclosure policy. Wang et al. (2002) show that when a lender (bank) has a high screening cost to ascertain whether a borrower is in distress, it could be optimal for the bank to randomly reject loan workout requests to deter the non-distressed borrower from opportunistically applying for a loan modification. Riddiough and Wyatt (1994) study the case in which the lender's foreclosure cost is private information and the borrowers will infer this cost from past loan foreclosure decisions and consequently decide their default decision and concession request. The lender thus may costly foreclose many loans today to reduce future expected default and loan modification costs. Gertner and Scharfstein (1991) focus on the free-riding problem among multiple creditors and show that when the cost of debt concessions is private but the benefit is shared, a creditor's incentive to grant concessions to a distressed firm is reduced. While the literature typically finds that the frictions lead to excessive foreclosure, this model predicts procyclical foreclosure policy based on the asymmetric information problem which is present in the mortgage-backed securitisation process.

Finally, while this paper is the first to formalise the role played by foreclosure in mortgage-backed securitisation in a model of asymmetric information, several empirical studies identify securitisation as being an important impediment for efficient renegotiation following delinquency, e.g. Agarwal et al. (2011); Piskorski et al. (2010); Zhang (2013). Particularly related to our model are the empirical findings of Agarwal et al. (2011) The authors find that the incentives of servicers present an impediment to loss mitigation of delinquent mortgages and attribute this to the holdup problem posed by dispersed investors of the senior tranche when the servicers hold the junior tranche. Our model provides a theoretical argument for distortions in the foreclosure decision of securitised mortgages.

The remainder of this paper is organised as follows. Section 2.2 presents our model of mortgage-backed securitisation with foreclosure policy under asymmetric information. Section 2.3 solves the model and formulates the optimal pre-committed foreclosure policy. In Section 2.4 discusses the role of third-party servicer in the MBS industry. Section 2.5 extends the baseline model to show that fire-sale externality can generate "foreclosure crisis". Finally Section 2.6 summarises the empirical implications produced by the model, and Section 2.7 concludes.

2.2 Model setup

This section sets up the baseline model and comments on the assumptions which are central to the model.

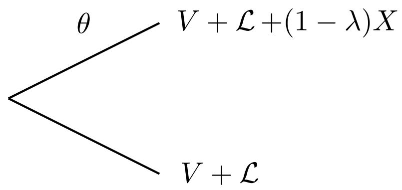
There are four dates: 0, 1, 2 and 3. The baseline model's participants consist of a securitiser and a continuum of outside investors each with one unit of cash. All agents are risk neutral. The securitiser is impatient and has a discount factor $\delta < 1$ between $t = 1$ and $t = 3$. This follows the assumption of DeMarzo and Duffie (1999) and can be interpreted as the securitiser's incentive to raise capital by securitising part of her long term assets as he has access to some positive return investment opportunities. There is no discounting for the outside investors.

Securitiser and mortgage pool

At $t = 0$, the securitiser has a pool of a continuum of identical mortgages that pays off at $t = 3$. We henceforth refer to the securitiser as "she". All mortgages have independent probability to become delinquent at $t = 2$. Therefore a fixed portion of the mortgages becomes delinquent. We normalise the measure of the delinquent mortgages in the pool to 1.The remaining mortgages continue to repay and have an exogenous value of V. The delinquent mortgages can be foreclosed or granted forbearance. In case of foreclosure, the collateral property is repossessed and sold for a liquidation proceed $\mathcal{L}$ to outside investors. In case of forbearance, the fixed mortgage repayment of value X is resumed with probability θ; otherwise the loans are worthless. For simplicity, we assume that the repayments of all delinquent mortgages are perfectly correlated. It can be interpreted to capture the systematic variations in the risk of the mortgages.

Denote λ the fraction of delinquent mortgages foreclosed. The overall cash flow from mortgage pool at $t = 3$ is then $V + \mathcal{L} + (1 - \lambda)X$ with probability θ, and $V + \mathcal{L}$ with probability $(1 - \theta)$, as illustrated in Fig 2.1.

Figure 2.1: Mortgage pool cash flow



At the beginning of $t = 1$, the securitiser receives a private signal regarding the recovering rate of the delinquent mortgages $\theta \in \{\theta_H, \theta_L\}$, where $\theta = \theta_H$ with probability γ. The assumption that the private information only concerns the credit risk of the delinquent mortgages is to simplify analysis and is not central to the model. Nevertheless, one interpretation could be that there is generally less data on delinquent loans, making it more difficult to assess the recovery rate of such borrowers.

After receiving the private information at $t = 1$, the securitiser designs a security that depends on the cash flow of the mortgage pool at $t = 3$, and sells it to outside investors. The securitiser retains the residual cash flow from the mortgage pool after paying off the investors. We will henceforth refer to it as the mortgage-backed securities (MBS). The MBS market is detailed below.

Investors and markets

There are two markets in this model. There is a market of MBS issue at $t = 1$, and a market for distressed property at $t = 2$. The investors are risk neutral, and the discount rate is 0. Since the MBS only pays off at $t = 3$, the investors can participate either in the MBS market, or the distressed property market, or neither.

At $t = 2$ the market for distressed properties opens. Each property is valued at X by the outside investors. However, the investors need to incur a heterogeneous cost $r \geq 0$ per unit capital invested in the distressed properties. This cost r can reflect the significant renovation and repair costs associated with distressed properties, as well as other liens such as unpaid fees and taxes. The heterogeneity in the costs can be driven by the time, skill and experience of the investors to conduct such renovations.

In Section 2.3 and 2.4, we assume that the private cost r of each investor is observable by the securitiser. This allows the monopoly securitiser to implement perfect price discrimination and extract all social surplus. As a baseline model, this setup has the benefit of removing any inefficiency induced by the market structure. This therefore allows a clean representation of the welfare implication of the optimal foreclosure policy under asymmetric information, as presented in Section 2.3.[5] Denote with $I(R)$ the measure of investors with $1 + r \leq R$, with $I(1) = 0$, $I'(R) > 0$ and $I''(R) < 0$.

Given that r is observable to the securitiser, at $t = 2$ the securitiser makes a take it or leave it offer to each investor with a price $b(r)$. The investor then chooses whether or not to accept the offer. An investor is able to accept the offer only if the investor has not invested in the MBS security at $t = 1$. As a tie break convention, we assume that an investor prefers to wait and invest in the distressed property market if the investor expects to be made an offer that will be accepted at $t = 2$, when the investor is indifferent between investing in the MBS security at $t = 1$ and in the distressed property market at $t = 2$. If an investor accepts the offer $b(r)$, the investor purchases a measure $1/b(r)$ of the distressed properties. The cost r is incurred at the end of the periods when the payoffs are realised.

[5]The assumption that the private cost r is observable by the securitiser will be relaxed in Section 2.5 to study the effect of competition amongst securitisers in the market for distressed properties.

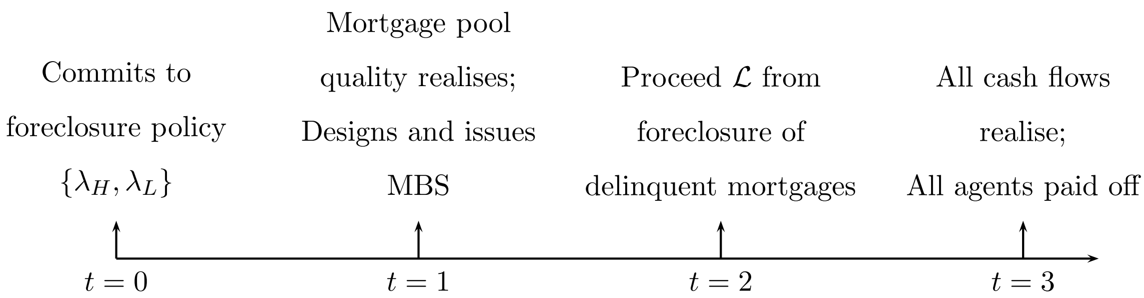
At $t = 1$, the securitiser designs an MBS and issues it to the market. Observing the choice of security on offer, the investors form a belief $\hat{\theta}$ regarding the private information of the issuer and decide whether to subscribe to the issue. The investors strictly prefer to subscribe if the issue is priced below the market valuation, and vice versa. Therefore the market clearing price of the security p is equal to the market value of the security given the investors' belief.

Foreclosure policy

In Section 2.3, we assumed that at $t = 0$, the securitiser commits to a set of foreclosure policies $\{\lambda_H, \lambda_L\}$ contingent on her type when it realises at $t = 1$. We then solve for the optimal foreclosure policy. We relax this assumption in Section 2.4 and provide a mechanism that involves a third party, the mortgage servicer, to implement the optimal foreclosure policy.

The timeline of the model is summarised in Figure 2.2.

Figure 2.2: Baseline model timeline



2.3 Pre-committed foreclosure policy

This section firstly presents the full information (first best) benchmark of the model. We then solve for the optimal foreclosure policy and compare it to the full information benchmark to assess the welfare implications.

2.3.1 First best benchmark

We follow a backward induction process to compute the first best benchmark. First, we solve for the distressed property market pricing equilibrium for a given foreclosure policy.

This then allows the characterisation of the securitiser's problem regarding security design and foreclosure policy choice.

Distressed property market

In order to obtain closed-form results, we use in Section 2.3 and 2.4 the following function form for $I(R)$, the measure of investors with renovation cost such that $1+r \leq R$,

$$I(R) = aX\ln(R), \quad \text{for some } a \in [\underline{a}, \bar{a}]^{6} \tag{2.1}$$

At $t=2$, a fraction λ_i, $i \in \{H, L\}$ of the delinquent mortgages are foreclosed, and the underlying properties offered to the market. In order to maximise the liquidation proceeds, the securitiser prefers to make offers to investors with the lowest r at the highest prices that will be accepted. An investor will only accept an offer if it allows the investor to at least break even. That is, the payoff to the investor after incurring the renovation cost, $\frac{X}{1+r}$, is (weakly) higher than the price $b(r)$ the investor pays for the property.

$$\frac{X}{1+r} \geq b(r) \tag{2.2}$$

In equilibrium, the securitiser sells to the investors with the highest valuation. In this situation, there exists a threshold $\hat{r}(\lambda_i)$ in equilibrium such that the securitiser makes the following offer $\hat{b}(r)$. We will later solve for the threshold $\hat{r}(\lambda_i)$ by market clearing.

$$\hat{b}(r) = \frac{X}{1+r}, \quad \forall\, r \leq \hat{r}(\lambda_i) \tag{2.3}$$

In equilibrium, the strategies of the investors thus depends on their cost r. All investors with $r \leq \hat{r}(\lambda_i)$ will wait until $t=2$. They accept the offer $b(r)$ made by the securitiser and purchase measure $\frac{1+r}{X}$ of the distressed properties, if the offer $b(r)$ allows the investor to break even, i.e. satisfies Eq 2.2. All the investors with $r > \hat{r}(\lambda_i)$ compete in the $t=1$ MBS market.

Finally, the equilibrium threshold $\hat{r}(\lambda_i)$ is given by the clearing condition that the total demand for the properties is equal to the supply,

$$\int_{R=1}^{1+\hat{r}(\lambda_i)} \frac{1}{\hat{b}(r)} dI(R) = \int_{R=1}^{1+\hat{r}(\lambda_i)} \frac{R}{X} dI(R) = \lambda_i \tag{2.4}$$

[6] The bounds for the parameter $\bar{a}$ is imposed to guarantee interior solutions in all the relevant sections, where $\underline{a} =$ and $\bar{a} \equiv \frac{\theta_L}{1-\theta_L} \frac{\gamma(1-\delta)\theta_H + (1-\gamma)(\theta_H - \delta\theta_L)}{\gamma(1-\delta)(\theta_H - \delta\theta_H) + (1-\gamma)(\theta_H - \delta\theta_L)}$.

Using the function form given by Eq. 2.1, the equilibrium threshold $\hat{r}(\lambda_i)$ and liquidation proceed $\mathcal{L}_i(\lambda_i)$ is

$$\hat{r}(\lambda_i) = \frac{\lambda_i}{a} \tag{2.5}$$

$$\mathcal{L}_i(\lambda_i) = aX \ln\left(1 + \frac{\lambda_i}{a}\right) \tag{2.6}$$

The liquidation proceeds are increasing and concave in the foreclosure policy of the mortgage pool. That is,

$$\frac{\partial\, \mathcal{L}_i(\lambda_i)}{\partial \lambda_i} = \frac{a^2 X}{\lambda_i + a} > 0 \tag{2.7}$$

$$\frac{\partial^2\, \mathcal{L}_i(\lambda_i)}{\partial \lambda_i^2} = -\frac{a^2 X}{(\lambda_i + a)^2} < 0 \tag{2.8}$$

First best securitisation and foreclosure

At $t = 1$, a securitiser of type i chooses a security (F_i, f_i) that correspond to the payoffs to outside investors when the delinquent mortgages resume repayments or not respectively. The securitiser's expected payoff at $t = 1$ is comprised of two parts (Eq. 2.9).

$$p(F_i, f_i) + \delta\left(\theta_i\left[V + \mathcal{L}_i(\lambda_i) + (1 - \lambda_i)X - F_i\right] + (1 - \theta_i)\left[V + \mathcal{L}_i(\lambda_i) - f_i\right]\right) \tag{2.9}$$

The first is the proceeds from issuing an MBS at $t = 1$ backed by the mortgage pool, and the second is the residual cash flow from the mortgage pool at $t = 2$. The proceeds from security issuance is given by the market clearing condition (MC) under full information,

$$(MC) \quad p(F_i, f_i) = \theta_i F_i + (1 - \theta_i) f_i \tag{2.10}$$

We can also rewrite Eq. 2.9 as Eq. 2.11 below. This offers an alternative interpretation comprising of a first part that represents the saving of retention cost due on proceeds $p(F_i, f_i)$ from the security issuance, and a second part that is the intrinsic value of the mortgage pool's cash flows to the securitiser (Eq. 2.11).

$$(1 - \delta)p(F_i, f_i) + \delta\left[V + \mathcal{L}_i(\lambda_i) + (1 - \lambda_i)\theta_i X\right] \tag{2.11}$$

In the first best benchmark, the securitiser chooses the security to maximise its expected payoff subject to the limited liability constraints (LL) on the MBS (Eq. 2.13–2.14) and the market clearing constraint (MC) under full information (Eq. 2.10). From Eq. 2.11 it is clear that the securitisation process does not alter the intrinsic payoff of the mortgage pool, the securitiser simply maximises her proceeds from the MBS issue.

$$
\begin{aligned}
(F_i^{FB}, f_i^{FB}) \quad &\equiv \arg \max_{(F_i, f_i)} p(F_i, f_i) && (2.12)\\
s.t. \quad (MC) \quad &\text{and} &&\\
(LL) \quad &F_i \leq V + \mathcal{L}_i(\lambda_i) + (1-\lambda_i)X && (2.13)\\
&f_i \leq V + \mathcal{L}_i(\lambda_i) && (2.14)
\end{aligned}
$$

Since any security issued would be priced correctly under full information, the securitiser chooses to issue a security backed by the entire cash flow of the mortgage pool to minimise her retention cost. The payoff to a securitiser of type i under full information is $V + \mathcal{L}_i(\lambda_i) + (1-\lambda_i)\theta_i X$.

We can now formulate the first best foreclosure policy. Anticipating the securitisation process, at $t = 0$ the securitiser chooses a foreclosure policy $(\lambda_H^{FB}, \lambda_L^{FB})$ to commit to, in order to maximise its expected payoff.

$$
\begin{aligned}
(\lambda_H^{FB}, \lambda_L^{FB}) \quad \equiv \quad \arg \max_{(\lambda_H, \lambda_L)} & \gamma[V + \mathcal{L}_H(\lambda_H) + (1-\lambda_H)\theta_H X] \\
& +(1-\gamma)[V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L)\theta_L X] \quad (2.15)
\end{aligned}
$$

The solutions are characterised by the first order conditions (FOC) because the second order conditions are satisfied. That is, at the first best level of foreclosure, the marginal gain from the property sale in the market is equalised to the expected value of mortgage forbearance (henceforth the forbearance value).

$$
(FOC^{FB}): \qquad \frac{\partial \mathcal{L}_i(\lambda_i^{FB})}{\partial \lambda_i} - \theta_i X = 0 \qquad \forall\, i \in \{H, L\} \tag{2.16}
$$

The following proposition then summarises the first best benchmark results.

Proposition 12. *In the full information equilibrium, the securitiser commits to a foreclosure policy* $(\lambda_H^{FB}, \lambda_L^{FB})$ *at* $t = 0$*, where*

$$
\lambda_i^{FB} = \frac{1-\theta_i}{\theta_i} a \qquad \forall\, i \in \{H, L\} \tag{2.17}
$$

She then securitises all of its mortgage pool cash flow at $t = 1$*, which is fairly priced in the market. At* $t = 2$ *the securitiser forecloses fraction* λ_i *if she is of type* i*, and obtains liquidation proceeds* $\mathcal{L}_i^{FB} \equiv \mathcal{L}_i(\lambda_i^{FB}) = aX \ln\left(\frac{1}{\theta_i}\right)$ *from selling the distressed properties in the market.*

In the full information equilibrium, a high type securitiser forecloses a smaller fraction of delinquent mortgages and obtains less liquidation proceed than a low type, $\lambda_H^{FB} < \lambda_L^{FB}$ and $\mathcal{L}_H^{FB} < \mathcal{L}_L^{FB}$. This is because the good type has a higher forbearance value and is therefore less inclined towards foreclosure.

2.3.2 Foreclosure policy under asymmetric information

We now solve for the optimal foreclosure policy given that securitisation occurs under asymmetric information, following a similar backward induction procedure. Notice that given a foreclosure policy, the liquidation proceeds from distressed property sales at $t = 2$ are the same as before. This section therefore focuses on the optimal security to be issued, and presents the equilibrium foreclosure policy.

Securitisation with signalling

At $t = 1$, the securitiser with private information θ_i designs and issues an MBS security backed by the cash flow of the mortgage pool. In this section we restrict our attention to only consider the least cost separating equilibrium.

First, notice that in a separating equilibrium, the low type securitiser always receives the fair price on the security she issues. Therefore she maximises her payoff by selling the entire cash flow from the mortgage pool to outside investors. There is no distortion in the form of inefficient retention for the low type. Given the pre-committed foreclosure policy, denote $U_i(\lambda_i)$ as the equilibrium payoff to a securitiser of type i. Therefore the equilibrium payoff to the low type securitiser is

$$U_L(\lambda_L) = V + \mathcal{L}_L(\lambda_L) + (1 - \lambda_L)\theta_L X \tag{2.18}$$

Next, we solve for the equilibrium security of the high type securitiser in the least cost separating equilibrium. Consider a general security that specifies a set of payoffs $\mathcal{F} \equiv (F_H, f_H, F_L, f_L)$ for each of the two possible cash flow realisations of the mortgage pool respectively for each type of the securitiser. Table 2.1 details the mapping from the realisation of the cash flow to the payoff of the security to the investors.

Specifically, F_i is the payoff of the security if the cash flow of a type i securitiser realises with the delinquent mortgages recovered, and f_i is the payoff of the security if the cash flow of a type i securitiser realises without any recovery. This is because the final cash flow of the mortgage pool will reveal the true type of the securitiser. We restrict our

Table 2.1: Payoffs of the security issued by the high type

Type	Realisation of cash flow	Security payoff $\mathcal{F}$
High	$V + \mathcal{L}_H(\lambda_H) + (1-\lambda_H)X$	F_H
	$V + \mathcal{L}_H(\lambda_H)$	f_H
low	$V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L)X$	F_L
	$V + \mathcal{L}_L(\lambda_L)$	f_L

attention to only monotonic security payoffs. That is, a higher realisation of the mortgage pool cash flow should leave both the outside investors and the securitiser a (weakly) higher payoff.[7] In the least-cost separating equilibrium, the optimal security for the high type securitiser is given by

$$\hat{\mathcal{F}} = \arg \max_{(F_H, f_H, F_L, f_L)} p(\mathcal{F}) \tag{2.19}$$

$$s.t. \quad (MC) \quad p(\mathcal{F}) = \theta_H F_H + (1-\theta_H) f_H \tag{2.20}$$

$$(LL) \quad \forall\, i \in \{H, L\} \quad \text{and} \tag{2.21}$$

$$\begin{aligned}(IC) \quad U_L(\lambda_L) \geq\ & p(\mathcal{F}) + \delta\theta_L \left[V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L)X - F_L\right] \\ & + \delta(1-\theta_L)\left[V + \mathcal{L}_L(\lambda_L) - f_L\right]\end{aligned} \tag{2.22}$$

where Eq. 2.20 is the market clearing condition (MC) when the market believes that the issuer of the security $\mathcal{F}$ is of the high type, and Eq. 2.22 is the incentive compatibility constraint (IC) for the low type to not mimic the security issued by the high type.

Since the monotonicity of the security payoffs depends on the ranking of the cash flow realisations, which depends on the foreclosure decisions of the securitisers. This significantly complicates the analysis as the foreclosure decisions are endogenously determined in equilibrium. For the rest of the paper, we present the results for the relevant case where (λ_H, λ_L) are such that

$$\mathcal{L}_H(\lambda_H) \leq \mathcal{L}_L(\lambda_L) \quad \text{and} \tag{2.23}$$

$$\mathcal{L}_H(\lambda_H) + (1-\lambda_H)X \geq \mathcal{L}_L(\lambda_L) + (1-\lambda_L)X \tag{2.24}$$

[7] Although this implies some loss of generality, it is not uncommon in the security design literature, e.g. Innes (1990) and Nachman and Noe (1994). One potential justification provided by DeMarzo and Duffie (1999) is that, the issuer has the incentive to contribute additional funds to the assets if the security payoff is not increasing in the cash flow. Similarly, the issuers has the incentive to abscond from the mortgage pool if the security leaves the issuer a payoff that is not increasing in the cash flow. If such actions cannot be observed, the monotonicity assumption is without loss of generality.

It will become clear in Section 2.3.2 that this scenario indeed arises in equilibrium, and we show in Appendix that this is the only equilibrium outcome.

The following proposition summarises the optimal securities in this assuming that Eq. 2.23–2.24 holds.

Proposition 13. *In the least-cost separating equilibrium, the optimal security issued by the low type securitiser is all the equity, whereas that issued by the high type securitiser is a debt with face vaue* $\hat{F}(\lambda_H, \lambda_L)$*, where*

$$\hat{F}(\lambda_H, \lambda_L) = \begin{cases} V + \frac{(1-\delta\theta_L)\,\mathcal{L}_L(\lambda_L)+(1-\delta)(1-\lambda_L)\theta_L X-(1-\theta_H)\,\mathcal{L}_H(\lambda_H)}{\theta_H-\delta\theta_L}, & \\ \qquad\qquad\qquad\qquad \textit{if } \frac{1-\theta_H}{\theta_H-\theta_L}[\mathcal{L}_L(\lambda_L) - \mathcal{L}_H(\lambda_H)] \leq (1-\lambda_L)X & \\ V + \frac{\mathcal{L}_L(\lambda_L)+(1-\lambda_L)\theta_L X-(1-\theta_H)\,\mathcal{L}_H(\lambda_H)}{\theta_H}, & \textit{otherwise} \end{cases} \tag{2.25}$$

Proof. See Appendix. ☐

The result presented in Proposition 13 is two-fold. First, the high type issues a debt security to the outside investors and retains the residual cash flow. The retained cash flow incurs a deadweight loss of $(1 - \delta)$. Such costly retention of the mortgage pool cash flow allows the high type securitiser to signal her type and receive a fair market price for the security it issues. This result is in line with DeMarzo and Duffie (1999). Second, in the presence of asymmetric information, the optimal securities issued by the securitiser are debt contracts, with equity being a special case of extremely high face value. This is because debt contract minimises the information sensitivity, a well established intuition in e.g. Myers and Majluf (1984).

The two cases presented in Proposition 13 correspond to when $\hat{F}$ is greater than or smaller than the low type's good realisation of cash flows, $V + \mathcal{L}_L(\lambda_L) + (1 - \lambda_L)X$, respectively. That is, the cases correspond to whether the low type would have to default even when the delinquent mortgages resume payments, should she mimic the high type. In the first case, information asymmetry measured by $\frac{1-\theta_H}{\theta_H-\theta_L}$ is large. The high type issues a debt with low face value in order to separate from the low type, because a high face value increases the market price of the security, increasing the incentive for the low type to mimic. This, however, incurs a high retention cost on the high type. In the second case, information asymmetry is less severe, and the high type can separate at a relatively high face value of debt with minimal retention cost. In what follows we assume that information asymmetry is so severe that the first case is true. This allows the asymmetric information to have a material effect and generate interesting implications for the optimal foreclosure policy.

In this case, the high type securitiser enjoys a total payoff of $U_H(\cdot)$ (Eq. 2.26) in equilibrium which is comprised of two parts – the saving of retention cost on the proceeds $p(\hat{F}(\cdot), \hat{f}(\cdot))$ from the security issuance, and the intrinsic value of the mortgage pool's cash flows to the securitiser.

$$U_H(\lambda_H, \lambda_L) = (1-\delta)\hat{p}(\lambda_H, \lambda_L) + \delta\left[V + \mathcal{L}_H(\lambda_H) + (1-\lambda_H)\theta_H X\right] \tag{2.26}$$

$$\begin{aligned} \text{where} \qquad \hat{p}(\lambda_H, \lambda_L) &\equiv p(\hat{F}_H(\lambda_H, \lambda_L), \hat{f}_H(\lambda_H, \lambda_L)) & (2.27) \\ \hat{F}_H(\lambda_H, \lambda_L) &= \hat{F}(\lambda_H, \lambda_L) & (2.28) \\ \hat{f}_H(\lambda_H, \lambda_L) &= V + \mathcal{L}_H(\lambda_H) & (2.29) \end{aligned}$$

Optimal ex-ante foreclosure policy

We can now solve for the ex-ante optimal foreclosure policy, given the securitisation game at $t = 1$. The securitiser chooses a foreclosure policy $(\hat{\lambda}_H, \hat{\lambda}_L)$ at $t = 0$ prior to the realisation of her private information, to maximise her expected payoff.

$$(\hat{\lambda}_H, \hat{\lambda}_L) \equiv \arg \max_{(\lambda_H, \lambda_L)} \quad \gamma U_H(\lambda_H, \lambda_L) + (1-\gamma) U_L(\lambda_L) \tag{2.30}$$

The first order conditions that characterise the solutions are

$$(FOC_H): \quad \gamma\left((1-\delta)\frac{\partial \hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial \lambda_H} + \delta\left[\frac{\partial\, \mathcal{L}_H(\hat{\lambda}_H)}{\partial \lambda_H} - \theta_H X\right]\right) = 0 \tag{2.31}$$

$$(FOC_L): \quad \gamma(1-\delta)\frac{\partial \hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial \lambda_L} + (1-\gamma)\left[\frac{\partial\, \mathcal{L}_L(\hat{\lambda}_L)}{\partial \lambda_L} - \theta_L X\right] = 0 \tag{2.32}$$

(FOC_H) is comprised of two components. The first part is the total impact of a change in the foreclosure policy of the high type issuer on her proceeds from security issuance $\frac{\partial \hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial \lambda_H}$, and the second is the impact on the total value of the mortgage pool of the high type. (FOC_L) comprises of two components too. The first part is the impact of a change in the foreclosure policy of the low type issuer on the proceeds from security issuance by the high type, $\frac{\partial \hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial \lambda_L}$, and the second is the impact on the total value of the mortgage pool of the low type. The first component comes from the fact that, when the foreclosure policy is chosen *ex ante*, the securitiser takes into account the effect of the low type's foreclosure policy on her signalling cost if she is of the high type. The second component encompasses the effect on the low type's proceeds from security issuance since the low type securitises all of her cash flow from the mortgage pool in equilibrium.

Proposition 14. *The optimal foreclosure policy* $(\hat{\lambda}_H, \hat{\lambda}_L)$ *under asymmetric information is more countercyclical than under full information. The equilibrium property prices in equilibrium under asymmetric information are more procyclical.*

$$\hat{\lambda}_H < \lambda_H^{FB} < \lambda_L^{FB} < \hat{\lambda}_L, \tag{2.33}$$

$$\hat{b}(\hat{r}(\hat{\lambda}_H) > \hat{b}(\hat{r}(\lambda_H^{FB}) > \hat{b}(\hat{r}(\hat{\lambda}_L) > \hat{b}(\hat{r}(\lambda_H^{FB}) \tag{2.34}$$

That is, there is insufficient foreclosure if the mortgage pool is of high quality, and excessive foreclosure if the mortgage pool is of low quality.

Proof. We express the high type issuer's proceeds and the total impacts of the foreclosure policy on the high type issuer's proceeds as follows.

$$\begin{aligned} \hat{p}(\lambda_H, \lambda_L) &= \theta_H \hat{F}(\lambda_H, \lambda_L) + (1-\theta_H)\left[V + \mathcal{L}(\lambda_H)\right] &(2.35)\\ &= V + \frac{\theta_H(1-\delta\theta_L)}{\theta_H - \delta\theta_L}\mathcal{L}_L(\lambda_L) + \frac{\theta_H - \delta\theta_H}{\theta_H - \delta\theta_L}(1-\lambda_L)\theta_L X \\ &\qquad - \frac{\delta\theta_L(1-\theta_H)}{\theta_H - \delta\theta_L}\mathcal{L}_H(\lambda_H) &(2.36) \end{aligned}$$

$$\begin{aligned} \frac{\partial\hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial\lambda_H} &= -\frac{\delta\theta_L(1-\theta_H)}{\theta_H - \delta\theta_L}\frac{\partial\mathcal{L}_H(\hat{\lambda}_H)}{\partial\lambda_H} &(2.37)\\ \frac{\partial\hat{p}(\hat{\lambda}_H, \hat{\lambda}_L)}{\partial\lambda_L} &= \frac{\delta\theta_H(1-\theta_L)}{\theta_H - \delta\theta_L}\frac{\partial\mathcal{L}_L(\hat{\lambda}_L)}{\partial\lambda_L} + \frac{\theta_H(1-\delta)}{\theta_H - \delta\theta_L}\left[\frac{\partial\mathcal{L}_L(\hat{\lambda}_L)}{\partial\lambda_L} - \theta_L X\right] &(2.38) \end{aligned}$$

It is thus apparent that at the first best level of foreclosure $(\lambda_H^{FB}, \lambda_L^{FB})$, the left hand side of the (FOC_H) (Eq. 2.31) is strictly negative and that of the (FOC_L) (Eq. 2.32) is strictly positive. Therefore the equilibrium is such that there is insufficient foreclosure in the high quality mortgage pool, i.e. the marginal value of foreclosure is greater than the forbearance value of the mortgage $\theta_H X$, and there is excessive foreclosure in the low quality mortgage pool, i.e. the marginal value of foreclosure is lower than the forbearance value of the mortgage $\theta_L X$.

The second part of the proposition regarding property prices follows immediately from the fact that $\hat{b}(\hat{r}(\lambda)) = \frac{aX}{\lambda+1}$ in equilibrium. □

The distortion in the equilibrium foreclosure policy is driven by the signalling concern of the issuer under asymmetric information. Given the equilibrium payoff to the low type issuer, consider the payoff to her is she mimics the high type issuer and issues a debt security. This mimicking payoff is comprised of two parts – the cash proceeds she gets from the security issuance, and the value of the retained cash flow. For a given security issued by the high type, when the high type issuer chooses a less than first best level of foreclosure, the payoff to the debt holders is reduced and hence the value of the security, decreasing the incentive for the low type to mimic. On the other hand, when the low type issuer chooses a higher than first best level of foreclosure, the value of the levered equity she retains decreases, again reducing her incentive to mimic.

The above intuition can be confirmed by the fact that Eq. 2.23 and 2.24 are implied by Proposition 14. This is because the high type is intrinsically riskier than the low type. The results imply that the cash flow from the mortgage pool of the low type is safer in equilibrium than in the first best scenario, and that of the high type is riskier. Therefore in equilibrium the two types become similar, mitigating the asymmetric information problem.

It is also worth noting that the equilibrium foreclosure policy is not time-consistent. Specifically, consider a high type issuer at $t = 2$. Having issued a debt security, the securitiser retains a levered equity stake. This gives her an incentive to prefer the risky cash flow, i.e. that from forbearance, to a safe cash flow, i.e. that from distressed property sales. For a given face value F, the problem of the high type issuer at $t = 2$ is

$$\max_{\lambda_H} \qquad \theta \left[V + \mathcal{L}_H(\lambda_H) + (1-\lambda_H)X - F\right] \tag{2.39}$$

The solution to the above problem is given by $\frac{\partial \mathcal{L}_H(\hat{\lambda}_H)}{\partial \lambda_H} = X$. Using the functional form of Eq. 2.1, this implies zero foreclosure in the high quality mortgage pool if the foreclosure decision was made *ex post* at $t = 2$. This is even lower that the equilibrium foreclosure policy $\hat{\lambda}_H = \frac{\theta_H - \theta_L}{\theta_H - \delta\theta_L} \frac{1-\theta_H}{\theta_H} a$.

2.4 Servicer and the optimal foreclosure policy

Having established the importance of commitment power to a set of foreclosure policies *ex ante* in the previous section, we now propose a mechanism that resembles the real world and involves a mortgage servicer to implement the desired commitment power.

We now assume that at $t = 0$, the loan originator ("he" henceforth) has the pool of mortgages. The loan originator is risk neutral and is subject to the same liquidity constraint δ as the securitiser. Neither the originator nor the securitiser has information about the quality of the mortgage pool until $t = 1$. At $t = 0$, the loan originator is approached by the securitiser intending to acquire the beneficial rights to the mortgage cash flows. After the sale, the initial originator does not retain any claim to the cash flows, but remains the servicer of the mortgages for a fee to be paid by the securitiser.[8] The servicer performs duties including collecting the payments, forwarding the interest and principle to the lender(s), and negotiating new terms if the debt is not being paid back, or supervising the foreclosure process.

This mechanism rests on the fact that the decision of forbearance versus foreclosure is made by the servicer as opposed to the securitiser. Therefore the institutional distinction between the originator and the securitiser does not play a role. In some cases mortgage servicing is done in-house, meaning that an institution is both the lender and the administrator of the loan. In this section we consider also this case when the originator decides whether or not to sell the mortgage pool to the securitiser. If he does not, he is free to securitiser the loan himself, with the servicing done in-house.

Because the incentives required to implement the optimal foreclosure policy can be different depending on the quality of the mortgage pool, the securitiser offers a menu of compensation contracts to the servicer. For simplicity, assume that all cash flows from the mortgage pool are passed on to the securitiser, and that the securitiser has sufficient funds to pay the fees specified by the contract.

The rest of the game is played in a similar way as before. To summarise, at $t = 0$, the originator is offered a menu of contracts by the securitiser to acquire the mortgage pool. If the originator declines the offer, he is free to continue securitising the mortgage pool at $t = 1$ and becomes an originator-securitiser with in-house servicing. If the originator accepts the offer, he remains as a third-party servicer and the menu of contracts is verifiable. At $t = 1$, the securitiser with private information regarding the quality of the mortgage pool designs a security and sells it to the investors. The originator with private information chooses a contract from the menu and agrees to receive compensation according to the chosen contract. At $t = 2$, the servicer makes the foreclosure decision given the incentives provided by his compensation contract.

[8]Practically, the servicer need not be the originator but they are often the party as the skill set required to perform both functions are similar. There is, however, a secondary market for the transfer of servicing rights through a security called Mortgage Servicing Rights (MSR).

In this section, we first solve for the equilibrium payoff to the originator if he securitises the mortgage pool with in-house servicing. We then solve for the incentive contracts that induce the originator to sell the mortgage pool and implement the optimal foreclosure policy. This finally allows us to comment on the implications of the separation of servicing on securitisation.

2.4.1 Securitisation with in-house servicing

At $t = 2$, the originator-securitiser makes the foreclosure decision to maximise his retained cash flow given his type i and the security issued (F_i, f_i) at $t = 1$,[9]

$$\lambda_i^o \equiv \arg\max_{\lambda_i} \theta_i \left[V + \mathcal{L}_i(\lambda_i) + (1-\lambda_i)X - F_i\right] + (1-\theta_i)\left[V + \mathcal{L}_i(\lambda_i) - f_i\right] \quad (2.40)$$

The foreclosure decision therefore depends on the riskiness of the retained cash flow. If the entire cash flow has been sold to the investors, i.e. $F_i = V + \mathcal{L}_i(\lambda_i) + (1-\lambda_i)X$ and $f_i = V + \mathcal{L}_i(\lambda_i)$, assume that there is no conflict of interest and that the first best foreclosure decision λ_i^{FB} is made. For $F_i < V + \mathcal{L}_i(\lambda_i) + (1-\lambda_i)X$, the originator-securitiser chooses the first best level of foreclosure λ_i^{FB} if he retains some cash flow in the downside, i.e. $f_i < V + \mathcal{L}_i(\lambda_i^{FB})$. Otherwise, he chooses zero foreclosure due to the risk-shifting incentive induced by risky retained cash flow, as shown in the last part of Section 2.3.2.

We now turn to the security design problem at $t = 1$, anticipating the consequential foreclosure decisions. The following proposition characterises the optimal security and the equilibrium foreclosure.

Proposition 15. *With in-house servicing, the security issued by the originator-securitiser in the least-cost separating equilibrium is a standard debt with face value $F^o \equiv \hat{F}(0, \lambda_L^{FB})$ if he is of the high type, and an equity contract if he is of the low type. The ex post chosen foreclosure policy is $(\lambda_H^o, \lambda_L^o) = (0, \lambda_L^{FB})$. That is, there is excessive forbearance if the mortgage pool is of high quality.*

Proof. For the low type issuer, he should optimally securitise the entire cash flow and choose the first best foreclosure policy. This is the first best outcome. For the high type issuer, however, he faces the problem as given by Equation 2.19–2.22.

The proof of the high type's security consists of two parts. First we show that the above securities are indeed the optimal security, if the securities issued are such that the

[9]We maintain the assumption that the security issued must satisfy the monotonicity assumption.

equilibrium foreclosure policy is $(\lambda_H^o, \lambda_L^o) = (0, \lambda_L^{FB})$. Given the equilibrium foreclosure policy, the cash flows of the two types satisfy the conditions in Eq. 2.23 and 2.24. That is,

$$\mathcal{L}_H(0) = 0 \quad < \quad \mathcal{L}_L(\lambda_L^{FB}) = aX \ln\left(\frac{1}{\theta_L}\right) \quad \text{and} \tag{2.41}$$

$$\begin{aligned} \mathcal{L}_H(0) + (1-0)X = X \quad > \quad & \mathcal{L}_L(\lambda_L^{FB}) + (1-\lambda_L^{FB})X \\ & = X - aX\left[\frac{1-\theta_L}{\theta_L} - \ln\left(\frac{1}{\theta_L}\right)\right] \end{aligned} \tag{2.42}$$

Therefore Proposition 13 applies and the equilibrium security is as described above. As the high type issues a risky debt, it indeed chooses zero foreclosure *ex post*. The low type issues equity, it then chooses the first best level of foreclosure.

Secondly we show by contradiction that there does not exist an equilibrium in which another foreclosure policy is chosen. Suppose there is an equilibrium in which the high type originator-securitiser chooses the first best foreclosure level. The resulting cash flows of the mortgage pool still satisfy Eq. 2.23 and 2.24 and the high type would issue a risky debt contract. However, given an outstanding risk debt, the originator-securitiser would not choose the first best level of foreclosure. □

Therefore, if the originator does not sell the mortgage pool, his payoff ω_0 from the subsequent equilibrium is given by

$$\omega_H^o \equiv \gamma U_H(0, \lambda_L^{FB}) \tag{2.43}$$

$$\omega_L^o \equiv (1-\gamma) U_L(\lambda_L^{FB}) \tag{2.44}$$

The expected payoff $\omega^o \equiv \gamma\omega_H^o + (1-\gamma)\omega_L^o$ is lower than the expected payoff obtained if the securitiser can commit to the optimal foreclosure policy with commitment $(\hat{\lambda}_H, \hat{\lambda}_L)$. This therefore creates the incentive to trade between the originator and a securitiser, if the securitiser has the commitment power.

2.4.2 Mortgage servicing contract

We now consider the securitiser's problem at $t = 0$. She would like to acquire the mortgage pool from the originator and provide the originator-servicer with incentive to implement the optimal foreclosure policy $(\hat{\lambda}_H, \hat{\lambda}_L)$ at $t = 2$.

Conjecture an affine contract $(\alpha, \beta, \tau) \in \mathbb{R}^{3+}$ that includes a percentage α of the forbearance cash flow to be paid at $t = 3$, a percentage $\alpha\beta$ of the foreclosure cash flow to be paid at $t = 3$ and a flat transfer τ to be paid at $t = 1$, if the mortgage is of high quality. Similarly for the tie break convention, assume that the originator prefers to accept an offer if he is indifferent between accepting or not. Given a contract, the originator's expected payoff $\omega(\alpha, \beta, \tau)$ given his private information θ is given by

$$\omega_i(\alpha, \beta, \tau) \quad \equiv \quad \max_{\lambda} \; \tau + \delta\alpha \left[\beta\, \mathcal{L}_i(\lambda) + (1-\lambda)\theta X\right] \tag{2.45}$$

By construction, the choice of foreclosure policy only depends on β. Specifically, the mortgage servicer chooses λ according to the following first-order condition

$$\beta \frac{\partial\, \mathcal{L}_i(\lambda)}{\partial \lambda} - \theta_i X = 0 \tag{2.46}$$

A comparison between Eq. 2.46 and (FOC_H) and (FOC_L) (Eq. 2.31 and 2.32) which characterise the optimal foreclosure policy suggests that the securitiser must offer different contracts (β_i) to the servicer depending on the type of the mortgage pool in order to implement $(\hat{\lambda}_H, \hat{\lambda}_L)$ respectively. Specifically, the contract to a servicer with type i mortgage pool must be such

$$\hat{\beta}_H \quad = \quad 1 - (1-\delta)\frac{\theta_L(1-\theta_H)}{\theta_H - \delta\theta_L} \tag{2.47}$$

$$\hat{\beta}_L \quad = \quad \frac{\gamma(1-\delta)\theta_H(1-\delta\theta_L) + (1-\gamma)(\theta_H - \delta\theta_L)}{\gamma(1-\delta)\theta_H(1-\theta_L) + (1-\gamma)(\theta_H - \delta\theta_L)} \tag{2.48}$$

The required incentive contracts are such that $\hat{\beta}_H < 1 < \hat{\beta}_L$. This is because $\beta = 1$ should implement the first best level of foreclosure. Therefore the compensation to the servicer must lean towards forbearance if the mortgage pool is of high quality, and towards foreclosure if the mortgage pool is of low quality, in order to implement the optimal foreclosure policy $(\hat{\lambda}_H, \hat{\lambda}_L)$. Contracts that satisfy Eq. 2.47 and 2.48 implement the optimal foreclosure policy regardless of the specific functional form of $I(R)$.

Since the type of the mortgage pool is not contractible, however, the contracts cannot be type-contingent. Instead, at $t = 0$ when both parties are uninformed, the securitiser can offer a menu of incentive-compatible contracts $\{(\alpha_i, \beta_i, \tau_i)\}_{i\in\{H,L\}}$ to the servicer, who chooses a contract from the menu according to the type of his mortgage pool at $t = 1$. Therefore, the securitiser's problem is to design this menu of contracts to implement a set of foreclosure policies that maximises her expected payoff from the mortgage pool less the fees paid to the servicer. Formally, her maximisation problem is

$$\max_{\{(\alpha_i,\beta_i,\tau_i)\}_{i\in\{H,L\}}} \quad \gamma\left[U_H(\lambda_H,\lambda_L)-\omega_H(\alpha_H,\beta_H,\tau_H)\right] + (1-\gamma)\left[U_L(\lambda_L)-\omega_L(\alpha_L,\beta_L,\tau_L)\right] \quad (2.49)$$

$$\text{s.t.} \quad (PC) \; : \; \gamma\omega_H(\alpha_H,\beta_H,\tau_H)+(1-\gamma)\omega_L(\alpha_L,\beta_L,\tau_L) \geq \omega^o \quad (2.50)$$

$$(IC_H) \; : \; \omega_H(\alpha_H,\beta_H,\tau_H) \geq \omega_H(\alpha_L,\beta_L,\tau_L) \quad (2.51)$$

$$(IC_L) \; : \; \omega_L(\alpha_L,\beta_L,\tau_L) \geq \omega_L(\alpha_H,\beta_H,\tau_H) \quad (2.52)$$

$$(IC_\lambda) \; : \; \lambda_i(\beta_j) \equiv \arg\max_{\lambda} \; [\beta_j\,\mathcal{L}_i(\lambda)+(1-\lambda)\theta_i X] \quad (2.53)$$

where (PC) is the participation constraint for the servicer to prefer loan sales to securitisation with in-house servicing at $t = 0$, (IC_i) are the incentive compatibility constraints for a type i servicer to choose the corresponding contract at $t = 1$, and (IC_λ) is the *ex post* incentive compatibility constraint for a type i servicer to choose the foreclosure policy at $t = 2$, after he has picked the contract j at $t = 1$. Thus his potential deviation at $t = 1$ is considered and the *ex post* foreclosure choice $\lambda_i(\beta_j)$ is implicitly embedded in $\omega_i(\alpha_j, \beta_j, \tau_j)$.

Proposition 16. *At* $t = 0$ *the securitiser offers an optimal menu of contracts* $\{(\hat{\alpha}_i, \hat{\beta}_i, \hat{\tau}_i)\}_{i\in\{H,L\}}$ *to the servicer who at* $t = 1$ *implements the optimal foreclosure policy* $(\hat{\lambda}_H, \hat{\lambda}_L)$.

Proof. We rewrite the servicer's payoff as follows

$$\omega_i(\alpha_i,\beta_i,\tau_i) = \tau + \delta\alpha_i K(\theta_i,\beta_i) \quad (2.54)$$

$$\text{where} \quad K(\theta_i,\beta_i) \equiv max_\lambda \; [\beta_i\,\mathcal{L}_i(\lambda)+(1-\lambda)\theta_i X] \quad (2.55)$$

The sufficient condition for the securitiser to prefer to implement the optimal foreclosure policy $(\hat{\lambda}_H, \hat{\lambda}_L)$ in equilibrium, which offers her the maximum expected value from the mortgage pool, is for her to pay the minimum fees to the servicer. That is, (PC) binds.

In particular, consider contracts with $\hat{\beta}_i$ and $\hat{\tau}_i$ such that

$$\hat{\tau}_H = \bar{\omega} - \delta\alpha_H K(\theta_H,\hat{\beta}_H) \quad (2.56)$$

$$\hat{\tau}_L = \frac{\omega^o-\gamma\bar{\omega}}{1-\gamma} - \delta\alpha_L K(\theta_L,\hat{\beta}_L) \quad (2.57)$$

for some $\bar{\omega} > \omega^o$. That is, the servicer receives $\bar{\omega}$ if the mortgage pool is of high quality and $\frac{\omega^o-\gamma\bar{\omega}}{1-\gamma} < \bar{\omega}$ otherwise. As shown in (2.47) and (2.48), $\{\hat{\beta_H}, \hat{\beta_L}\}$ implements $(\hat{\lambda}_H, \hat{\lambda}_L)$

while by construction $\{\hat{\tau}_H, \hat{\tau}_L\}$ binds the (PC). Finally we need to choose $\{\alpha_H, \alpha_L\}$ to ensure (IC_H) and (IC_L) satisfied. That is,

$$\begin{aligned}
\delta\alpha_H \left[K(\theta_H, \hat{\beta}_H) - K(\theta_L, \hat{\beta}_H)\right] &\geq \frac{\bar{\omega} - \omega^o}{1-\gamma} \\
&\geq \delta\alpha_L \left[K(\theta_H, \hat{\beta}_L) - K(\theta_L, \hat{\beta}_L)\right] \geq 0 \qquad (2.58)
\end{aligned}$$

By the Envelope Theorem, it is straight forward that $K(\theta_i, \beta_i)$ is increasing in θ_i. Therefore there exists $\hat{\alpha}_H$ and $\hat{\alpha}_L$ that satisfy the above inequalities. $\square$

2.4.3 In-house versus third-party mortgage servicing

Third-party mortgage servicers frequently come under public criticism for the foreclosure crisis because of their apparent recklessness in foreclosing mortgages. Levitin and Twomey (2011) asserts that the services' compensation structures create a principal-agent conflict and they do not make the decision whether to foreclose or modify a loan based to maximise the net present value of the loan. Indeed, Credit Suisse reports a loss severity rate of 55% on securitised subprime mortgages in the six months ending in May 2008.[10]

We would like to point out with this model that there need not be an agency conflict in equilibrium. The separation of servicing from securitisation allows the securitiser to commit to an ex-ante optimal foreclosure policy, resulting in higher *ex ante* efficiency in securitisation in equilibrium than the in-house servicing case. Through the compensation contract given to the servicer, the optimal foreclosure policy can be implemented.

Nevertheless, the optimal foreclosure policy appears inefficient *ex post*. If the mortgage pool is of low quality, the *ex post* marginal proceeds from foreclosure is lower than the forbearance value as indicated in Proposition 14. In order to implement such foreclosure policy, the compensation given to the servicer is such that $\beta_L > 1$. That is, the servicer receives an incentive tilted towards foreclosure. This is consistent with anecdotal evidence such as that of Goodman (2009).

Moreover, compared to the optimal foreclosure policy with in-house servicing, the foreclosure rate with a third-party servicer is generally higher. Specifically, $\hat{\lambda}_H > \lambda_H^o = 0$ and $\hat{\lambda}_L > \lambda_L^o = \lambda_L^{FB}$. This is an empirically testable implication of our model.

[10]Source: Cordell et al. (2008), page 12. Loss severity measures the total foreclosure costs borne by investors as a proportion of the total unpaid principal on a mortgage.

2.5 Fire-sale externality and foreclosure crisis

We have thus far established that the securitiser under asymmetric information implements inefficiently procyclical foreclosure policy. In the baseline model and the model involving a servicer, we have assumed that the securitiser operates as a monopoly with perfect price discrimination in the distressed property market. In this case, the distressed property market is efficient. In this section, we study an extension of the model in which fire-sale externality in the distressed property market exacerbates the countercyclicality to generate a "foreclosure crisis" when the overall quality of the mortgages in the economy is low.

In this extension we make two changes to the baseline model described in Section 2.2. First, we relax the assumption that the investors' private renovation costs r are contractible. Instead, a investor's cost r is only privately known to the investors and is non-verifiable. Second, we consider an economy in which there are $N \geq 1$ securitisers, each endowed with an i.i.d mortgage pool of the size $1/N$.

We solve for the model assuming that all securitisers can commit to a set of foreclosure policies at $t = 0$.[11] We follow the backward induction process to first consider the distressed property market under competition, then characterise the equilibrium foreclosure policy. This finally allows us to study the welfare implications of competition on the securitisers' foreclosure preference.

2.5.1 Distressed property market under competition

In order to obtain closed-form results, we use the following functional form in this section,

$$I(R) = aX\left(1 - \frac{1}{R}\right), \quad \text{for some } a \in [0, \bar{a}] \tag{2.59}$$

At $t = 2$, the total supply of the distressed properties is given by $\Lambda_s \equiv \frac{1}{N}\Sigma_{n=1}^{N}\lambda_i^n$, where $s \in \mathbb{S} \equiv \{H, L\}^N$ is the overall state of the economy given the realisation of all securitisers' types, λ_i^n is the fraction of delinquent mortgages foreclosed by securitiser n given her type i. Because the investor's renovation costs r are private, the market clears at one price b_s such that the measure of investors who are willing to enter the market at

[11] We have illustrated in the previous section that the exogenous assumption of commitment power is not critical to the implementation of the equilibrium foreclosure policy.

this price clears the market, i.e.

$$I(\frac{X}{b_s}) = b_s \Lambda_s = \mathcal{L}_s \tag{2.60}$$

Using the functional form given by Eq. 2.59, the equilibrium property price $b_s(\Lambda_s)$ and total liquidation proceed $\mathcal{L}_s(\Lambda_s)$ is

$$b_s(\Lambda_s) = \frac{aX}{\Lambda_s + a} \tag{2.61}$$

$$\mathcal{L}_s(\Lambda_s) = aX\frac{\Lambda_s}{\Lambda_s + 1} \tag{2.62}$$

For securitiser n, her liquidation proceed is given by

$$\mathcal{L}_i^n(\lambda_i^n, \Lambda^{-n}) \equiv b_s(\frac{1}{N}\lambda_i^n + \Lambda^{-n})\lambda_i^n = aX\frac{\lambda_i^n}{\lambda_i^n + N(\Lambda^{-n} + a)} \tag{2.63}$$

where $\Lambda^{-n} \equiv \Lambda_s - \frac{1}{N}\lambda_i^n$ is the total supply of properties by all the other securitisers.

The liquidation proceeds of securitiser n are increasing and concave in her foreclosure policy,

$$\frac{\partial\, \mathcal{L}_i^n(\lambda_i^n, \Lambda^{-n})}{\partial \lambda_i^n} = aX\frac{N(\Lambda^{-n} + a)}{[\lambda_i^n + N(\Lambda^{-n} + a)]^2} \tag{2.64}$$

$$\frac{\partial^2\, \mathcal{L}_i^n(\lambda_i^n, \Lambda^{-n})}{\partial {\lambda_i^n}^2} = -2aX\frac{N(\Lambda^{-n} + a)a}{[\lambda_i^n + N(\Lambda^{-n} + a)]^3} \tag{2.65}$$

Moreover, there is a fire-sale externality in the equilibrium. That is, the liquidation proceeds of securitiser n is decreasing in the foreclosure policy of her competitor m of type j,

$$\frac{\partial\, \mathcal{L}_i^n(\lambda_i^n, \Lambda^{-n})}{\partial \lambda_j^m} = -aX\frac{\lambda_i^n}{[\lambda_i^n + N(\Lambda^{-n} + a)]^2} \tag{2.66}$$

This is because an increase in the foreclosure policy of other securitisers leads to a lower market clearing price, which reduces the proceeds obtained by securitiser n for a given foreclosure policy.

2.5.2 Symmetric equilibrium foreclosure policy

At $t = 1$, a securitiser of type i designs and sells an MBS to the investors. Observing all the securities on offer, the investors correctly infer the types of the securitisers in equilibrium. In response, all securitiser are fairly priced as in Section 2.2. Because the securitisers have mortgage pools with independent quality, we restrict our attention to only symmetric equilibria.

Consider for any securitiser n. By the same intuition as in Proposition 13, a low type issuer securitises the entire cash flow from the mortgage pool. For a given set of foreclosure policy $\boldsymbol{\lambda}_i^n \equiv \{\lambda_{i,s}^n\}_{s\in\mathbb{S}}$ for $i \in \{H, L\}$ by securitiser n and a given set of foreclosure policies $\boldsymbol{\Lambda}^{-n} \equiv \{\Lambda_s^{-n}\}_{s\in\mathbb{S}}$ by all other securitisers, the low type securitiser obtains payoff

$$U_L^n(\boldsymbol{\lambda}_L^n; \boldsymbol{\Lambda}^{-n}) = V + \mathbb{E}_L\left[\mathcal{L}_{L,s}^n(\lambda_{L,s}^n, \Lambda_s^{-n}) + (1 - \lambda_{L,s}^n)\theta_L X\right] \tag{2.67}$$

where $\mathbb{E}_i[\cdot]$ is the conditional expectation over all states $s \in \mathbb{S}$ given the securitiser's private information i.

On the other hand, a high type securitiser designs a monotonic security that specifies a set of payoffs $\{(F_{i,s}, f_{i,s})\}_{i\in\{H,L\},s\in\mathbb{S}}$ for the high and low cash flow realisation of the mortgage pool respectively for each type of the securitiser in each state. Since the securitiser does not know the state s, the realisation of the price of the security varies across states. In the least-cost separating equilibrium, the optimal security for the high type securitiser $\{(\hat{F}_{i,s}, \hat{f}_{i,s})\}_{i\in\{H,L\},s\in\mathbb{S}}$ maximises her expected proceed from security issuance subject to the usual limited liability constraints (LL), market clearing constraint (MC) and the incentive compatibility constraint (IC) for the low type not to mimic.

$$\begin{aligned}
&\max_{\{(F_{i,s},f_{i,s})\}_{i\in\{H,L\},s\in\mathbb{S}}} && \mathbb{E}_H\left[p_s^n(F_{H,s}, f_{H,s})\right] && (2.68)\\
s.t.\quad & (LL) && \forall\, i \in \{H, L\}, s \in \mathbb{S} \quad \text{and} && (2.69)\\
& (MC) && p_s^n(F_{H,s}, f_{H,s}) = \theta_H F_{H,s} + (1 - \theta_H) f_{H,s} && (2.70)\\
& (IC) && U_L^n(\boldsymbol{\lambda}_L^n) \le \mathbb{E}_H\left[p_s^n(F_{H,s}, f_{H,s}))\right] + \\
&&& \delta\left(\theta_L\, \mathbb{E}_L\left[V + \mathcal{L}_{L,s}^n(\lambda_{L,s}^n, \Lambda_s^{-n}) + (1 - \lambda_{L,s}^n)X - F_{L,s}\right]\right.\\
&&& \left.+(1 - \theta_L)\, \mathbb{E}_L\left[V + \mathcal{L}_{L,s}^n(\lambda_{L,s}^n, \Lambda_s^{-n}) - f_{L,s}\right]\right) && (2.71)
\end{aligned}$$

We consider foreclosure policy $(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n)$ such that the resulting cash flows satisfy the following conditions in any pair of states s and s' in which all other securitisers are of the same type, except for securitiser n who is of the high type in state s and the low type in state s'. It will become clear that this is indeed the case in equilibrium.

$$\begin{aligned}
\mathcal{L}_{H,s}^n(\lambda_H) &\le \mathcal{L}_{L,s'}^n(\lambda_{L,s'}^n) \quad \text{and} && (2.72)\\
\mathcal{L}_{H,s}^n(\lambda_{H,s}^n) + (1 - \lambda_{H,s}^n)X &\ge \mathcal{L}_{L,s'}^n(\lambda_{L,s'}^n) + (1 - \lambda_{L,s'}^n)X && (2.73)
\end{aligned}$$

Denote the equilibrium face value of the debt $\hat{F}(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n)$. Suppose the face value is such that in all states, a securitiser who issues this security defaults if her cash flow does not recover in case of forbearance of the delinquent mortgages, and never defaults if the cash flow recovers. The conditions for this to be the optimal security is provided in Appendix B.1.2. This simplifies the analysis and allows direct comparison to the baseline case.

Given the optimal security, the high type securitiser enjoys an expected payoff of $U_H^n(\cdot)$ given by

$$\begin{aligned} U_H^n(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n; \boldsymbol{\Lambda}^{-n}) &= (1-\delta)\,\mathbb{E}_H\left[\hat{p}_s^n(\hat{F}(\cdot); \lambda_{H,s}^n, \lambda_{L,s}^n)\right] \\ &\quad + \delta\left(V + \mathbb{E}_H\left[\mathcal{L}_{H,s}^n(\lambda_{H,s}^n, \Lambda_s^{-n}) + (1-\lambda_{H,s}^n)\theta_H X\right]\right) \quad (2.74) \\ \text{where} \quad \hat{p}_s^n(\hat{F}(\cdot); \cdot) &\equiv \theta\hat{F}(\cdot) + (1-\theta)\left[V + \mathcal{L}_{H,s}^n(\lambda_{H,s}^n, \Lambda_s^{-n})\right] \quad (2.75) \end{aligned}$$

The equilibrium foreclosure policy in a symmetric equilibrium is therefore chosen by each securitiser to maximise her expected payoff,

$$(\hat{\boldsymbol{\lambda}}_H^n, \hat{\boldsymbol{\lambda}}_L^n) \equiv \arg\max_{(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n)} \gamma U_H^n(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n; \boldsymbol{\Lambda}^{-n}) + (1-\gamma) U_L^n(\boldsymbol{\lambda}_L^n; \boldsymbol{\Lambda}^{-n}) \quad (2.76)$$

where $\boldsymbol{\Lambda}^{-n}$ is given by all other securitisers choosing the same foreclosure policy. Denote N_L^s the number of securitisers that are of the low type in state s. Therefore the total amount of foreclosure in equilibrium in state s is given by $\hat{\Lambda}_s(\hat{\lambda}_{H,s}^n, \hat{\lambda}_{L,s}^n) \equiv \frac{N-N_L^s}{N}\hat{\lambda}_{H,s}^n + \frac{N_L^s}{N}\hat{\lambda}_{L,s}^n$.

Proposition 17. *(i) The equilibrium foreclosure policy is higher for a low type issuer and lower for a high type issuer. That is, for any pair of states s and s' in which all other securitisers are of the same type, except for securitiser n who is of the high type in state s and the low type in state s',*

$$\hat{\lambda}_{H,s}^n < \hat{\lambda}_{L,s'}^n \quad (2.77)$$

(ii) For $N \geq 2$, the equilibrium foreclosure policy of each securitiser is procyclical. That is, for all states z, z' in which $N_L^z < N_L^{z'}$,

$$\hat{\lambda}_{i,z}^n > \hat{\lambda}_{i,z'}^n \quad \forall\, i \in \{H, L\} \quad (2.78)$$

Proof. See Appendix. □

Proposition 17 highlights the countercyclicality of the equilibrium under fire-sale externality. This result follows similar intuition to those for the baseline model (Proposition 14). This is because there are more securitisers of low quality in a worse state, which are following a high foreclosure policy than the high quality securitisers. This leads to procyclical distressed property prices. In turn, depressed property prices in a worse state discourages the securitisers from foreclosure, leading to procyclical foreclosure policy individually.

Proposition 18. *There exists a menu of affine contracts* $\{(\alpha_i, \beta_i, \tau_i)\}_{i\in\{H,L\}}$ *such that a securitiser can implement the optimal foreclosure policy in equilibrium through a third-party servicer, where* $\beta_i = \hat{\beta}_i$.

Proof. See Appendix. □

Similarly, this equilibrium foreclosure policy can be implemented through a third-party servicer. Because of the independence assumption, there only needs to be two contracts in the menu, one for each type of the mortgage pool. Moreover, the contracts specify the same relative sensitivity towards foreclosure relative to forbearance, $\beta_i = \hat{\beta}_i$, as in the monopoly case, despite a much more complex equilibrium that is being considered in this section. This suggests that it is relatively easy for the securitisers to implement their desired foreclosure policy in practice.

2.5.3 Foreclosure crisis

In this section, we present two benchmarks for comparison in order to understand the equilibrium foreclosure policy characterised above. The equilibrium is affected by three frictions. The first is the fire-sale externality in the distressed property market at $t = 2$, the second is the information asymmetry at $t = 1$, and the last is the market power enjoyed by each securitiser when making the foreclosure policy at $t = 0$.

The first benchmark considered is the full information benchmark (FI) which is absent of the first friction only. Under full information, all securitisers sell their entire cash flows from the mortgage pools to outside investors. The foreclosure policy is thus chosen to maximise the *ex ante* value of each mortgage pool.

$$\begin{aligned}(\boldsymbol{\lambda}_H^{FI}, \boldsymbol{\lambda}_L^{FI}) \quad \equiv \quad & \arg \max_{(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n)} \gamma\, \mathbb{E}_H \left[\mathcal{L}_{H,s}^n(\lambda_{H,s}^n, \Lambda_s^{-n}) + (1-\lambda_{H,s}^n)\theta_H X\right] \\ & + (1-\gamma)\, \mathbb{E}_L \left[\mathcal{L}_{L,s}^n(\lambda_{L,s}^n, \Lambda_s^{-n}) + (1-\lambda_{L,s}^n)\theta_L X\right] \qquad (2.79)\end{aligned}$$

We also present the full information central planner solution – the first best solution (FB). Notice that in the full information benchmark, there are two types of externality ignored by a securitiser. One is the fire-sale externality a securitiser has on other securitisers, and the other is the investor surplus $\frac{X}{b} - (1+r)$ when an investor with private cost r purchases the distressed properties at price b. Therefore in the first best case, a central planner maximises the total surplus in the economy,

$$
\begin{aligned}
(\boldsymbol{\lambda}_H^{FB}, \boldsymbol{\lambda}_L^{FB}) \quad \equiv \quad \arg \max_{(\boldsymbol{\lambda}_H^n, \boldsymbol{\lambda}_L^n)} \sum_{n=1}^{N} \big(& \gamma \, \mathbb{E}_H \left[\mathcal{L}_{H,s}^n(\lambda_{H,s}^n, \Lambda_s^{-n}) + (1-\lambda_{H,s}^n)\theta_H X \right] \\
& + (1-\gamma) \, \mathbb{E}_L \left[\mathcal{L}_{L,s}^n(\lambda_{L,s}^n, \Lambda_s^{-n}) + (1-\lambda_{L,s}^n)\theta_L X \right] \big) \\
& + \mathbb{E} \left[\int_1^{\frac{X}{b_s(\Lambda_s)}} \left(\frac{X}{b_s(\Lambda_s)} - R \right) dI(R) \right] \qquad (2.80)
\end{aligned}
$$

where $\boldsymbol{\Lambda}^{-n}$ is given by all other securitisers choosing the same foreclosure policy.

First of all, we highlight the effect of the market power enjoyed by the securitisers, by comparing the two benchmarks. Consider a state s, in which securitiser n is of type i, and in which the number of low type securitisers among the remaining $N-1$ securitisers is N_L^s. The first order conditions that determine a securitiser's foreclosure policy $\lambda_{i,s}^n$ in the full information case and the first best case respectively are given by

$$
(FOC^{FI}) \quad : \quad \frac{\mathcal{L}_{i,s}^n(\lambda_{i,s}^n, \Lambda_s^{-n})}{\lambda_{i,s}^n} - \theta_i X = 0 \qquad (2.81)
$$

$$
\begin{aligned}
(FOC^{FB}) \quad : \quad & \left(\frac{\mathcal{L}_{i,s}^n(\lambda_{i,s}^n, \Lambda_s^{-n})}{\lambda_{i,s}^n} - \theta_i X \right) \\
& + \left(\frac{N-1-N_L^s}{N} \frac{\mathcal{L}_{H,s}^m(\lambda_{i,s}^m, \Lambda_s^{-m})}{\lambda_{i,s}^n} + \frac{N_L^s}{N} \frac{\mathcal{L}_{L,s}^m(\lambda_{i,s}^m, \Lambda_s^{-m})}{\lambda_{i,s}^n} \right) \\
& + aX \left[I\left(\frac{X}{b_s(\Lambda_s)} \right) \frac{\partial}{\partial b_s} \left(\frac{X}{b_s(\Lambda_s)} \right) \frac{\partial b_s(\Lambda_s)}{\partial \Lambda_s} \frac{1}{N} \right] = 0 \qquad (2.82)
\end{aligned}
$$

It is immediate that the difference between the two first order conditions is in Line 2 and 3 of Eq. 2.82. Specifically, Line 2 of Eq. 2.82 represents the fire-sale externality of securitiser n's foreclosure policy on all other securitisers in a symmetric equilibrium. This effect is negative, suggesting the a securitiser under full information tends to foreclosure excessively relative to the first best case. Line 3 of Eq. 2.82 captures the effect of securitiser n's foreclosure policy on the total investor surplus. This effect is positive, since a higher foreclosure policy tends to lower the market price of the distressed property, benefiting the investors. Failing to account for the investor surplus, a securitiser under full information tends to foreclose insufficiently. The following lemma summarises the important trade-off effects of competition. Two extreme states are of particular interest. Denote $\underline{s}$ the state in which all securitisers have low quality mortgage pools, and $\bar{s}$ the state in which all securitisers have high quality mortgage pools. I will later refer to the former state the "boom" and the latter state the "bust".

Lemma 1. *There exists $\bar{N}, \underline{N}$ such that,*

$$\text{For } N > \bar{N}, \qquad \Lambda_{\bar{s}}^{FI} > \Lambda_{\bar{s}}^{FB} \quad \text{and} \quad \Lambda_{\underline{s}}^{FI} > \Lambda_{\underline{s}}^{FB} \tag{2.83}$$

$$\text{For } N < \underline{N}, \qquad \Lambda_{\bar{s}}^{FI} < \Lambda_{\bar{s}}^{FB} \quad \text{and} \quad \Lambda_{\underline{s}}^{FI} < \Lambda_{\underline{s}}^{FB} \tag{2.84}$$

That is, relative to the first best solution, the full information solution entails excessive foreclosure if the securitisation industry is competitive, but entails insufficient foreclosure if the securitisers enjoy large market power in the extreme states.

Proof. See Appendix. □

This result can be understood by examining the extreme cases. For $N = 1$ when there is only one monopolistic securitiser, there is no fire-sale externality. The market power of the monopoly securitiser therefore leads to insufficient foreclosure in an attempt to maximise her monopoly profit. For $N \to \infty$, the effect of each securitiser's foreclosure policy on the investor surplus diminishes because each securitiser's price impact diminishes. The competitive equilibrium under full information thus lead to excessive foreclosure due to the effect of fire-sale externality.

The following proposition highlights the properties of the equilibrium foreclosure policy, which is affected by asymmetric information as well as the above mentioned trade-off effect of market power.

Proposition 19. *(i) Asymmetric information exacerbates the countercyclicality of the equilibrium foreclosure policy. That is, for any pair of states s, s' in which all other securitisers are of the same type in both states except for securitiser n who is of the high type in state s and of the low type in state s',*

$$\lambda_{H,s}^{\hat{n}} < \lambda_{H,s}^{FI} < \lambda_{L,s'}^{FI} < \hat{\lambda}_{L,s'}^{n} \tag{2.85}$$

In particular,

$$\hat{\Lambda}_{\bar{s}} < \Lambda_{\bar{s}}^{FI} < \Lambda_{\underline{s}}^{FI} < \hat{\Lambda}_{\underline{s}} \tag{2.86}$$

(ii) When competition is strong, the equilibrium entails excessive foreclosure during the bust. That is, for $N > \bar{N}$,

$$\hat{\Lambda}_{\underline{s}} > \Lambda_{\underline{s}}^{FI} > \Lambda_{\underline{s}}^{FB} \tag{2.87}$$

(iii) When competition is weak, the equilibrium entails insufficient foreclosure during the boom. That is, for $N < \underline{N}$,

$$\hat{\Lambda}_{\bar{s}} < \Lambda_{\bar{s}}^{FI} < \Lambda_{\bar{s}}^{FB} \tag{2.88}$$

Proof. See Appendix. □

Part (i) of Proposition 19 follows the same intuitive as the baseline case illustrated in Proposition 14.

Part (ii) can be understood as the "foreclosure crisis" scenario. When competition is strong, fire-sale externality is prominent, leading to excessive foreclosure among low quality mortgage pools. Therefore during the bust, the equilibrium foreclosure is significantly higher than in the first best case. This also leads to depressed property prices. This is consistent with the empirical evidence provided by Piskorski et al. (2010) that the foreclosure rate of delinquent bank-held loans is 3% (13%) to 7% (32%) lower in absolute (relative) terms. The authors also recognise that the primary reason for such findings is whether or not the servicer internalises the costs and benefits from the decision to foreclose a delinquent loan.

Part (iii) is the opposite scenario. If the securitisers have strong market power, they reduce foreclosure to maximise their profits from property sales. During the boom, the securitisers with high quality mortgage pools have the further incentive to avoid foreclosure in order to facilitate their securitisation process under asymmetric information. This results in inefficiently high property prices.

2.6 Empirical implications

This section summarises the empirical implications of our model related to foreclosure policy and characteristics of mortgage servicers' compensation contracts.

1. *In a bad state, securitised mortgages on average have a higher foreclosure rate than comparable bank-held loans.* The main result of our model shows that the asymmetric information friction in the process of mortgage securitisation will exacerbate the countercyclicalilty of foreclosure of delinquent mortgage. In a bad state where most mortgage pools are of low quality, securitised loans on average have a higher foreclosure probability than comparable bank-held loans (no information problem), which is consistent with the empirical finding of Piskorski et al. (2010)

2. *In a bad state, the foreclosure rate of delinquent mortgages in a securitised pool is higher than the ex post efficient level on average.* Specifically we show that when the mortgage pool is of low quality, the proceeds from foreclosing the marginal mortgages are *lower* than their expected recovery value, i.e. its foreclosure will be negative NPV or value-destroying decision from the *ex post* perspective. This is in line with the finding of Levitin (2009).

3. *In a bad state, the third-party servicer's contract on average is biased towards foreclosure.* We show that a securitiser offers an optimal incentive contract to a third-party servicer to implement the optimal foreclosure policy. In a bad state, the incentives for the servicer on average are biased towards foreclosure. This is in line with the anecdotal evidence of Goodman (2009).

4. *Securitised mortgages serviced by third-party servicers are foreclosed more on average than comparable mortgages with in-house servicers.* Our model shows that in-house servicers face a time-inconsistency problem and cannot commit to the ex-ante optimal foreclosure policy. An in-house servicer chooses little foreclosure when he is of the high type because he holds a levered equity claim, but chooses the *ex post* efficient level of foreclosure when he is of the low type. A third-party servicer, on the other hand, implements the optimal foreclosure policy which is insufficient when he is of the high type and excessive when he is of the low type. In either case, the third-party servicer forecloses more than an in-house servicer of the same type. Therefore we expect to observe higher foreclosure rates by third-party servicers on average.

2.7 Conclusion

The recent subprime mortgage crisis has raised concerns regarding the economic and social consequences of mortgage backed securitisation. In particular, the United States experienced a "foreclosure crisis" subsequent to the crisis in 2008 that received much public attention. Recent studies and reports have suggested that securitisation and the biased incentives of mortgage services could have contributed to the foreclosure wave. This paper formally studies the relationship between the foreclosure decision of delinquent loans and the securitisation of mortgages, and examines the role of mortgage servicers in this process.

We investigate the optimal foreclosure decision in a model of mortgage-backed securitisation under asymmetric information. A securitiser with a pool of mortgages has private information regarding the recovery rate of the mortgages that *ex post* become delinquent. The securitiser initially designs and sells a mortgage-backed security, and makes the decision whether to foreclose or modify a mortgage when it becomes delinquent *ex post*.

Relative to the case with full information, we show that the optimal foreclosure policy under asymmetric information involves excessive foreclosure if the mortgage pool is of low quality, and insufficient foreclosure if the mortgage pool is of high quality. This is because the signalling concern at the securitisation stage prompts the securitiser to take procedures at the foreclosure stage to reduce the information sensitivity of the mortgage pool cash flows.

Moreover, we propose a mechanism that involves mortgage servicers that resembles the industry practice, to implement the optimal foreclosure policy. We notice that the optimal foreclosure policy is not time-inconsistent. A securitiser with in-house servicing therefore cannot credibly commit to the *ex ante* optimal policy. In our mechanism, the securitiser designs a contract *ex ante* with a third-party servicer. This rids the securitiser of the commitment problem, and the securitiser can design an incentive contract for the third-party servicer to implement the *ex ante* optimal foreclosure policy.

CHAPTER 3

BANKRUPTCY-REMOTE SECURITISATION WITH IMPLICIT GUARANTEE

3.1 Introduction

Securitisation has become a pervasive means of funding for banks over the last decade.[1] Gorton and Souleles (2007) relate the prevalent use of "special purpose vehicles" (SPVs) in securitisation to two important features of SPVs. The first is bankruptcy-remoteness. An SPV operates as a distinct legal entity so that its bankruptcy has little or no impact on the sponsoring bank. The second is the existence of implicit guarantees on the liabilities of the SPV by the sponsoring bank. During the subprime mortgage crisis of 2007, banks such as Citibank stepped in to rescue its special investment vehicles, or SIVs.[2] This paper show that these two features are important for the implementation of the optimal funding arrangement for banks faced with repeated funding needs under asymmetric information to finance portfolios with a risk-shifting potential.

[1] Calomiris and Mason (2004) state that in the credit card securitisation industry, banks that only around 40% of total outstanding receivables in 2000 were retained on banks' balance sheet.

[2] Higgins and Mason (2004) have also documented 17 discrete recourse events between 1987 and 2001.

This paper seeks to provide a rationale for these two features of securitisation in relation to two specific aspects that have not been studied in the literature. Firstly, the fundamental concept of bankruptcy-remoteness is to allow the sponsoring bank to shield the rest of its assets due to limited liability, even when the SPV defaults.For example, PWC (2011) states that "SPVs can be used ... in particular to isolate the financial risk in the event of bankruptcy or default".[3] Secondly, not only does the provision of implicit guarantee reduce the funding cost of the SPV, an *ex post* realised recourse event also conveys favourable information to investors that helps to reduce the sponsoring bank's future funding costs. The market reaction to a "failure to support the securitisation may impair future access to the capital markets" (FitchIBCA, 1999).

To understand the roles played by these features of securitisation, this paper develops a model of a bank with repeat investment opportunities that needs outside financing, and show that the optimal funding arrangement of the bank can be implemented with bankruptcy-remote securitisation with implicit recourse. In this model, the bank has an opportunity to form a risky portfolio, whose risk increases with portfolio size, eventually decreasing the value of the portfolio. Although not modelled explicitly, this can be interpreted as a portfolio of mortgage, business or personal loans, and the bank has to lower its credit standard in order to extend more loans.

The bank in this model is subject to two financial frictions. On one hand, in each period the bank has an investment opportunity of either good or bad quality, which is private information to the bank. On the other hand, the bank has a moral hazard problem since it cannot commit to a given size of investment. At the beginning of each period, the bank decides how much to raise in the capital market and to invest after acquiring information regarding the quality of its investment portfolio. It may wish to choose a suboptimal portfolio size given its private information. Since these are prevalent frictions in the financial market, the theory may apply to other institutions with repeated funding needs. This model focuses, for concreteness, on the SPVs of banks.

The interaction between these two frictions tends to result in over-investment in equilibrium. This is because a bank with a good portfolio would choose an excessively large and risky portfolio which has a value lower than the first best, as a means to signal

[3]This differs from the view of Gorton and Souleles (2007), who argue that a key source of value to using SPVs is that they help reduce bankruptcy costs, by subjecting less of the banks' assets to bankruptcy costs.

its private information to the capital market. Therefore, there could be deadweight loss in equilibrium in the form of over-investment, as a result of a bank trying to reduce its private funding costs under asymmetric information.

This social cost of over-investment can be avoided in the second period, however, if a good bank is able to signal its private information regarding the quality of its second period investment opportunity by honouring its implicit guarantee. Specifically, suppose the bank finances its first period investment opportunity by structuring a fraction of the portfolio into an SPV and raises financing backed by a senior tranche issued by the SPV. In case the SPV produces insufficient cash flow to repay the debt holders, a bank can allow recourse of the SPV debt onto the remaining fraction of the portfolio on its balance sheet. Given the bankruptcy-remoteness of the SPV, a recourse is a wealth transfer from the bank to its debt holders, and the bank is not willing to do so in the absence of other incentives. In equilibrium, a bank with a good second period portfolio is willing to provide recourse to its first period portfolio to signal its second period portfolio quality, if the cost of providing recourse that deters a bank with a bad second period portfolio from mimicking is lower than the cost of over-investment as a means of signalling. This allows first best level of investment to be achieved in the second period.

The actions by the bank in this equilibrium highlights the two feature of securitisation discussed in this paper. A bank with a bad second period portfolio takes advantage of the bankruptcy-remoteness of the SPV structure, defaults on its first period debt, but retains the cash flows from its on-balance-sheet portfolio. A bank with a good second period portfolio, on the other hand, honours its implicit guarantee and provides recourse, to signal the quality of its second period portfolio and reduce the funding cost in the second period.

While implicit guarantee provided in the first period helps to restore the efficient investment level in the second period, it exacerbates the problem in the first period. Due to the asymmetric information problem between the bank and the outside investors, a debt like security is preferred as it is less information sensitive. In order to provide implicit guarantee, however, the bank must retain a fraction of the portfolio on its balance sheet. This increases the information sensitivity of the security issued to outside investors in exchange for financing. Due to the interaction between the asymmetric information problem and the moral hazard problem, I show that the bank over-invests more in the first period than in an equilibrium without implicit guarantee.

The model yields two empirical implications. Firstly, the extent of securitisation is positively correlated with unobservable asset quality. This is because a bank's on-balance-sheet portfolio provides implicit guarantee for its SPV. Since the assets of a bank with bad portfolio is less valuable, a larger fraction of the portfolio must be retained to provide sufficient guarantee. This is supported by evidence presented by Casu et al. (2011), Vermilyea et al. (2008) and Mandel et al. (2012). Secondly, conditional on recourse becoming necessary, honouring implicit guarantee is positively correlated with future loan performance, due to the signalling effect. Amiram et al. (2011) provides empirical evidence consistent with this prediction.

Related literature

This paper relates to the theoretical literature on securitisation via SPVs. Gorton and Souleles (2007) view a key value brought by SPVs being reducing bankruptcy costs, since transferring assets into an SPV subjects less assets, which remain on a bank's balance sheet, to bankruptcy costs. The authors further argue that commitment to rescue the SPVs in certain states of the world, enforced in an infinite game, helps to mitigate the strategic moral hazard problem that the bank transfers only bad assets into the SPV. Kuncl (2014) shows that creditors signal its quality by reputation-based implicit recourse, and conclude that the information tends to remain private in a boom, further exacerbating a subsequent downturn. This paper emphasises a different role for implicit guarantee to signal quality regarding a sponsoring bank's future investment opportunity, and the bankruptcy-remoteness of securitisation to implement the desired level of implicit guarantee.

Since the recent financial crisis, scholars have empirically investigated the relation between bank performance and securitisation with implicit guarantee. Calomiris and Mason (2004) considers the potential hypothesis that banks use securitisation to engage in regulatory capital arbitrage, but concluded that securitisation is more consistent with optimal contacting view. This paper supports that bankruptcy-remote securitisation with implicit guarantee implements the optimal financing arrangement for a bank under the asymmetric information with repeated funding needs. Gorton and Souleles (2007), using credit card securitisation data, find that riskier firms are more likely to engage in off-balance sheet financing, and that investors in the debt of the SPV incorporate expectations about the risk of the sponsor, who provides implicit guarantee to the SPV debt. Higgins and Mason (2004) study 17 discrete recourse events, and find that recourse events are met with positive stock market reactions and improved long-run operating performance. Using reported fraud losses as opposed to credit losses as a proxy for implicit recourse, Vermilyea et al. (2008) find that the performance of the credit card securitisation portfolio is negatively related to implicit guarantee provision. Casu et al. (2011, 2013) investigate the effect of securitisation on banks' performance in general. Some empirical evidence is discussed further in relation to the model predictions in Section 3.4.2.

The remainder of the paper is organised as follows. Section 3.2 outlines the structure of the model in which the bank chooses its investment and offers a general security to outside investors in exchange for financing. Section 3.3 characterises and analyses the equilibrium in the cases with and without recourses. Section 3.4 discusses the implementation of the equilibrium optimal security through bankruptcy-remote securitisation with implicit guarantee, and provides two empirical implications of the model in relation to existing evidence. Section 3.5 concludes.

3.2 Model setup

There are three dates, 0, 1 and 2 and two periods. The model's participants consist of a long-lived bank (two-period) and a set of short-term outside investors (one-period). All agents are risk neutral and there is no discounting. The bank is endowed with w_0 unit of capital. The outside investors are competitive and deep-pocketed.

At the beginning of each period, i.e. at $t = 0$ and $t = 1$, the bank has an opportunity to form a risky portfolio of size X_t that pays off at the end of the period, i.e. at $t = 1$ and $t = 2$ respectively. A portfolio of size X_t requires an investment of kX_t, and pays off either X_t or $(1 - \delta)X_t$, where $1 > k > 1 - \delta$. The cash flows are verifiable.

At the beginning of each period, the bank receives private information regarding the quality of the portfolio in that period. The quality i, j of the portfolios in the two periods respective can be either good or bad, $i, j \in \{G, B\}$. The quality of the portfolio determines the probability $\theta^i(X_0), \theta^j(X_1) \in [0, 1]$ that a high cash flow is realised at the end of the period, where $\theta^G(X_t) > \theta^B(X_t)$.

At the beginning of each period, the bank chooses the size of its portfolio X_t, $t = 0, 1$.

In order to finance the portfolio, the bank contributes some or all of its own capital w_t, and raises the remaining fund needed through securitisation.

3.2.1 Assumptions and discussion

This section presents the assumptions on the distribution of the bank's portfolio cash flows and discusses how the bank can structure itself in order to finance its portfolio.

Distribution of portfolio cash flows

While the bank can increase the size of its portfolio, I assume that this decreases the probability of the portfolio realising a high cash flow, i.e. $\theta^{i'}(X_0), \theta^{j'}(X_1) < 0$. This captures the intuition that excessive lending by banks can be associated with relaxed credit standard and excessive risk taking, as documented by Dell'Ariccia et al. (2012). As a result, there exists a first best level of investment $X^i_{FB} = X^j_{FB}$ that maximises the NPV of a portfolio of type i, j respectively.[4] Denote the NPV of a portfolio of size X and type i with $V^i(X) \equiv (1-\delta)X + \theta^i(X)\delta X - kX$, and the first best NPV of the portfolio with $V^i_{FB} \equiv V^i(X^i_{FB})$, where $X^i_{FB} \equiv \arg\ \max_X V^i(X)$. I denote $V^j(X)$ and V^j_{FB} similarly for the second period portfolio.

I further assume that an increase in the portfolio size is less detrimental to a good portfolio than a bad one, i.e. $\frac{\partial}{\partial X}\left(\frac{\theta^G(X)}{\theta^B(X)}\right) > 0$. That is, when a bank has access to a pool of high quality loans, the marginal decrease in the credit quality is smaller.

The probability of the first period portfolio being good is γ_0. If the first period portfolio realises a high cash flow, the second period portfolio is good with probability 1. Otherwise, the second period portfolio is good with probability γ_1. The assumption that there is only asymmetric information in the second period following a bad realisation of the cash flow is made to simply the analysis. It can be interpreted as the bad realisation being a possibly transient or permanent shock to the quality of the portfolio. It is also consistent with the case that a high cash flow realisation provided the bank with sufficient retained cash flow to render the asymmetric information problem in the second period immaterial.

[4]Further technical assumptions required to guarantee an interior solution are given in the Appendix C.1.

Financing through securitisation

In this section I take a security design approach to characterise the security to be sold to the outside investors in exchange for financing. I restrict attentions to consider only monotonic securities under limited liability. After describing a general security, I argue that bankruptcy-remote securitisation with implicit guarantee implements the security.

I follow a backward induction process to describe the financing decisions of the bank. At $t = 1$, the bank has w_1 unit of capital from the first period cash flows after paying off the outside investors. The bank also has an investment portfolio of type j. If the bank chooses a portfolio size of X_1, it must design a security to sell to the outside investors in order to raise at least $kX_1 - w_1$. Denote the chosen security (w_2^H, w_2^L), where specifies the amount of cash flows retained by the bank when a high or low cash flow is realised at $t = 2$. Correspondingly, the cash flows promised to the investors are given by $X_1 - w_2^H$ and $(1 - \delta)X_1 - w_2^L$. The competitive investors observes the banks choice of X_1, w_2^H and w_2^L, as well as the history of the actions taken by the bank in the first period, and form a belief regarding the type of the portfolio in the second period. Competitive pressure then determines the price of the security in equilibrium.

The security described above can be implemented with the following financial structure of the bank commonly seen in practice. The bank invests in a portfolio of size X_1, retains a fraction $\alpha_1 = \frac{w_2^L}{(1-\delta)X_1}$ of the portfolio on its balance sheet, while structuring the remaining $(1 - \alpha_1)$ fraction of the portfolio in an SPV for securitisation. The bank sells a senior tranche backed by the SPV with promised cash flow $F_1 = X - w_2^H$ to the investors. Therefore, a bank retains part of its portfolio on balance sheet at $t = 1$ if it wishes to retain a positive cash flow at $t = 2$ in case a low cash is realised. If the cash flows promised to the outside investors are risky, the bankruptcy-remoteness of the securitisation allows the bank to retain the cash flows from the part of the portfolio it keeps on balance while defaulting on the SPV.

At $t = 0$, the bank similarly chooses to invest in a portfolio of size X_0 and designs a security to raise at least $kX_0 - w_0$ from the outside investors. However, at $t = 1$, given a high or low realisation of the first period portfolio cash flow, the bank may wish to specify a different payoff to the investors depending on its private information regarding the type of its second period portfolio j, if it is incentive compatible to do so at $t = 1$. Denote the chosen security $(w_1^H, w_{1,j}^L)_{j \in \{G,B\}}$. I will henceforth refer to an equilibrium in which this is the case an equilibrium with recourse.

This first period security can be implemented in a similar manner as the second period security if it does not depend on the type of the second period portfolio j. Denote the chosen security in this case (w_1^H, w_1^L). I will henceforth refer to an equilibrium in which this is the case an equilibrium without recourse.

The interesting case is when the first period security does depend on the type of the second period portfolio j. In this case, the bank invests in a portfolio of size X_0, retains a fraction $\alpha_0 = \frac{\max_j\{w_{1,j}^L\}}{(1-\delta)X_0}$ of the portfolio on its balance sheet, while structuring the remaining $(1 - \alpha_0)$ fraction of the portfolio in an SPV for securitisation. The bank sells a senior tranche backed by the SPV with promised cash flow $F_0 = X - w_1^H$ to the investors. Consider for example the case where $w_{2,B}^L > w_{2,G}^L$, which will be the case in equilibrium. Following a realisation of a low cash low, a bank with a good second period portfolio provides recourse to a fraction $\frac{w_{2,B}^L - w_{2,G}^L}{(1-\delta)X_0}$ of its on balance sheet. An implicit guarantee of the securitisation vehicle by the bank therefore allows the bank to provide recourse strategically in some of the states of the world but not others. In the following analysis it will become clear that this adds value by allowing the bank to signal its type through a recourse.

3.2.2 Definition of equilibrium

The equilibrium concept I use is subgame-perfect Bayesian equilibrium that satisfies the Intuitive Criterion. An equilibrium consists of choices by the bank of the size of the portfolio and of the security to issue at $t = 0, 1$, and a system of beliefs formed by outside investors that satisfy the following conditions: (i) Choices made by the bank maximises its expected value of retained payoff at each date, given the set of equilibrium beliefs formed by the investors in response to these choices; (ii) Beliefs of the investors are rational, given the equilibrium choices made by the bank, and are formed using Bayes' rule along the equilibrium path while satisfying the Intuitive Criterion off the equilibrium path.

In a separating equilibrium, a bank with a bad portfolio receives the fair price for any security it issues. It therefore always chooses the first best size of the portfolio X_{FB}^B. A bank with a good portfolio behaves differently, since it has to choose a security and a size of portfolio that deters mimicking by a bank with a bad portfolio. I analyse in the following chapter the equilibrium choices of a bank with a good portfolio.

3.3 Equilibrium

This section follows a backward induction process to consider first the second period investment and funding problem, and then the first period problem. In particular, attentions are paid to two types of possible equilibrium, an equilibrium with recourse and one without. After characterising both types of equilibrium, I discuss and compare their properties.

3.3.1 Second period problem

In this section I consider an equilibrium with recourse at the end of the first period and an equilibrium without recourse. This illustrates the signalling value of providing recourse in a model of asymmetric information.

Consider firstly that, following a high realisation of cash flow, it is common knowledge that the second period is good. A bank with a retained cash flow w_1^H after paying off the investors chooses the first best size for the second period portfolio in equilibrium. Because the investors always pay the fair price for any security issued, the bank's objective is equivalent to maximising the NPV of the portfolio. The security choice is irrelevant. That is,

$$\max_{X_1, w_2^H, w_2^L} \quad \theta^G(X_1)w_2^H + [1-\theta^G(X_1)]w_2^L \tag{3.1}$$

$$\text{s.t. } \theta^G(X_1)[X_1 - w_2^H] + [1-\theta^G(X_1)][(1-\delta)X_1 - w_2^L] + w_1^H \geq kX_1 \tag{3.2}$$

$$\iff \quad \max_{X_1} \quad V^G(X_1) + w_1^H \tag{3.3}$$

where the constraint specifies that the sum of the the capital raised from outside investors (the sum of the first two terms) and the bank's own capital (w_1^H) is sufficient to finance the portfolio.

I now turn to analyse the second period problem following a low realisation of cash flow of the first period problem. In this case the bank has private information regarding the type j of the second period portfolio. I consider the different equilibria with and without a recourse for for the bank's first period portfolio separately.

In an equilibrium with recourse

In an equilibrium with recourse, the decision to provide recourse at the end of the first period sends a credible signal regarding the type of the portfolio in the second period.

The second period investment and financing decisions are thus made under complete information.That is, the security design problem is irrelevant, and the portfolio size is at the first best level X^j_{FB}. The value of the bank, with a type j portfolio and w^j_1 capital after providing recourse for its first period portfolio, is given by $V^j_{FB} + w^L_{1,j}$.

In an equilibrium without recourse

In an equilibrium without recourse, the bank makes the financing and investment decisions under asymmetric information. Given a retained capital level w^L_1, a bank with a bad portfolio makes, in a separating equilibrium, chooses the first best portfolio size and realises a value of $V^B_{FB} + w^L_1$. A bank with a good portfolio, on the other hand, chooses its portfolio size and a security (w^H_2, w^L_2) to sell to the outside investors in exchange for financing according to the following optimisation programme

$$\max_{X^G_1, w^H_2, w^L_2} \quad \theta^G(X^G_1)w^H_2 + [1 - \theta^G(X^G_1)]w^L_2 \tag{3.4}$$

$$\text{s.t.} \quad \theta^G(X^G_1)[X^G_1 - w^H_2] + [1 - \theta^G(X^G_1)][(1-\delta)X^G_1 - w^L_2] + w^L_1 = kX_1 \tag{3.5}$$

$$\theta^B(X^G_1)w^H_2 + [1 - \theta^B(X^G_1)]w^L_2 \leq V^B_{FB} + w^L_1 \quad (IC_2) \tag{3.6}$$

where, in addition to the financing constraint, the incentive compatibility constraint (IC_2) specifies that a bank with a bad portfolio would prefer not to mimic a bank with a good portfolio by choosing a portfolio size X^G_1 and issuing a security (w^H_2, w^L_2).

The following proposition characterises the equilibrium in the second period following no recourse at the end of the first period.

Proposition 20. *In an equilibrium with no recourse, for given capital w^L_1, a bank with a bad portfolio chooses at $t = 1$ the first best portfolio size X^B_{FB}, and a bank with a good portfolio chooses $X^{G*}_1(w^L_1) \geq X^G_{FB}$, where*

$$X^{G*}_1(w^L_1) = \begin{cases} X^G_{FB}, & \text{if} \quad w^L_1 \geq w_{1,FB} \\ \hat{X}_1(w^L_1), & \text{otherwise} \end{cases} \tag{3.7}$$

where $\hat{X}_1(w^L_1)$ is such that

$$\frac{\theta^B(\hat{X}_1(\cdot))}{\theta^G(\hat{X}_1(\cdot))}V^G(\hat{X}_1(\cdot)) - V^B_{FB} = \left[1 - \frac{\theta^B(\hat{X}_1(\cdot))}{\theta^B(\hat{X}_1(\cdot))}\right] w^L_1 \tag{3.8}$$

and $w_{1,FB}$ such that $\hat{X}_1(w_{1,FB}) = X^G_{FB}$.

The equilibrium security issued by a bank with a bad portfolio is arbitrary, whereas that issued by a bank with a good portfolio is $\left(w^{H*}_2(w^L_1), w^{L*}_2(w^L_1)\right) = (\frac{V^G(X^{G*}_1(\cdot)) + w^L_1}{\theta^G(X^{G*}_1(\cdot))}, 0)$.

Proof. This and all other proofs can be found in the Appendix. □

Condition 3.8 is given by binding the two constraints Eq. 3.5–3.6 in the maximisation programme, while setting $w_2^L = 0$. The security issued in equilibrium indeed leaves the bank a higher powered stake with $w_2^{L*} = 0$. This confirms the intuition of Myers and Majluf (1984) that issuing a debt-like security to the outside investors mitigates the asymmetric information problem.

The equilibrium portfolio size is equal to the first best level for a bank with a good portfolio if it has sufficient capital $w_1^L \geq w_{FB}$. This is consistent with the literature on asymmetric information that sufficient internal financing eliminates the information problem.

The interesting case is when the bank has insufficient capital enter into the second period $w_1^L \geq w_{FB}$. In this case, a bank with a good portfolio over-invests at $\hat{X}(w_1^L) > X_{FB}^G$. This result is due to two effects that mitigates the asymmetric information problem. Firstly, recall the assumption that the difference between a good and a bad portfolio becomes more significant as the bank increases its portfolio size and lows the credit standard, i.e. $\frac{\partial}{\partial X}\left(\frac{\theta^G(X)}{\theta^B(X)}\right) > 0$. Therefore the retained equity stake in equilibrium becomes more information sensitive, making it less profitable for a bank with a bad portfolio to mimic. Secondly, as a bank over-invests in a good portfolio, the NPV of the portfolio decreases, further reducing the mimicking incentives.

For reasons mentioned above, a bank with a good portfolio in this model essentially signals its private information through over-investment in the presence of the asymmetric information. Since internal financing mitigates the asymmetric information problem, I derive the following corollary.

Corollary 2 (to Proposition 20)**.** *For* $w_1^L \leq w_{FB}$*,* $\frac{\partial X^{G*}(w_1^L)}{\partial w_1^L} < 0$ *and* $\frac{\partial V^G(X^{G*}(w_1^L))}{\partial w_1^L} > 0$. *That is, the extend of over-investment in the second period in equilibrium decreases with the amount of capital the bank has at the beginning of the second period, and the NPV of the bank's portfolio increases.*

3.3.2 The incentives to provide recourse

In this section I first consider the benefits and costs of providing recourse at the end of the first period by comparing the two types of equilibrium in the second period. I then derive the incentive compatibility conditions for recourse to be in equilibrium. I finally discuss the efficiency of an equilibrium with recourse at $t = 1$.

From the analysis in Section 3.3.1, recourse and over-investment are the two means for the bank to signal the type of its second period portfolio, in an equilibrium with and without recourse respectively. In an equilibrium with recourse, a good bank incurs a cost $w^L_{1,B} - w^L_{1,G}$ by providing recourse, and is able to achieve efficient investment in the second period.[5] In an equilibrium without recourse, on the other hand, a good bank over-invests in order to signal its private information, and thus incurs a cost of $V^G_{FB} - V^G(X^{G*}_1(w^L_1))$.

Even in an equilibrium with recourse, a security whose payoff is characterised by $(w^L_{1,G}, w^L_{1,B})$ is not contractable, as mentioned in the model setup. It must therefore be incentive compatible for $(w^L_{1,G}, w^L_{1,B})$ to characterise the equilibrium security issued by the bank in the first period in an equilibrium with recourse.

For a bank with a good portfolio to be willing to provide recourse such that its retained capital becomes $w^L_{1,G}$ at the beginning of the second period, instead of $w^L_{1,B}$. The incentive compatibility constraint (RC^G) thus specifies that the payoff from providing recourse and subsequently investing in an first-best sized portfolio is greater than or equal to the payoff of not providing recourse but subsequently engaging in over-investment, i.e.

$$V^G_{FB} + w^L_{1,G} \geq V^G(\hat{X}(w^L_{1,B})) + w^L_{1,B} \qquad (RC^G) \tag{3.9}$$

This can also be interpreted as when the benefit of efficient second period investment outweighs the cost of providing recourse.

A bank with a bad portfolio, on the other hand, should prefer not to mimic a bank with a good portfolio and provide recourse. The payoff to a bank with a bad portfolio without recourse is $V^B_{FB} + w^L_{1,B}$. Should it choose mimic, it similarly provides recourse, chooses a portfolio size X^G_{FB} and issues a security the same as a bank with a good portfolio. The resulting payoff is

$$\theta^B(X^G_{FB})w^H_2 + [1 - \theta^B(X^G_{FB})]w^L_2 \tag{3.10}$$

$$\text{s.t.} \quad \theta^B(X^G_{FB})w^H_2 + [1 - \theta^B(X^G_{FB})]w^L_2 + w^L_{1,G} \geq kX^G_{FB} \tag{3.11}$$

This payoff is minimised in an equilibrium in which a bank with a good portfolio retains a high powered security $w^L_2 = 0$. Therefore the incentive compatibility constraint (RC^B) therefore specifies that the in this equilibrium, the payoff to a bank with a bad portfolio when it mimics by providing recourse is less than or equal to the payoff when it does not

[5]It will become clear that indeed the cost is positive, i.e. $w^L_{1,B} > w^L_{1,G}$, in equilibrium.

provide recourse and subsequently choosing an efficient portfolio size. That is,

$$\frac{\theta^B(X_{FB}^G)}{\theta^G(X_{FB}^G)}\left[V_{FB}^G + w_{1,G}^L\right] \leq V_{FB}^B + w_{1,B}^L \qquad (RC^B) \tag{3.12}$$

Notice that (RC^B) implies that $w_{1,B}^L > w_{1,G}^L$ whenever $w_{FC} > 0$. That is, whenever the asymmetric information problem is sufficiently severe that a penniless bank cannot achieve the first best outcome, there is signalling value for a bank with a good portfolio to provide costly recourse.

Combining (RC^G) and (RC^B) suggests that, while a larger recourse by a bank with a good portfolio $w_{1,B}^L - w_{1,G}^L$ incurs higher cost, tightening (RC^G), it prevents a bank with a bad portfolio from mimicking, relaxing (RC^B). An equilibrium with recourse is *feasible* if there exists $(w_{1,B}^L, w_{1,G}^L)$ that satisfies (RC^G) and (RC^B).

This leads to the following proposition regarding the efficiency of an equilibrium with recourse.

Proposition 21. *Whenever feasible, an equilibrium with recourse is efficient at $t = 1$ as first best portfolio sizes are achieved in the second period.*

An equilibrium with recourse is efficient at $t = 1$. Although providing recourse incurs a private cost for a bank with a good portfolio, it is not a social cost since a recourse represents a transfer from the bank to outside investors without any dead weight loss. This contrasts with an equilibrium without recourse, in which a bank with a good portfolio engages in over-investment in order to signal its private information, incurring a social cost.

3.3.3 First period problem

I now turn to consider the first period problem. I first consider an equilibrium with recourse. I then consider an equilibrium in which a bank with a good portfolio at $t = 0$ does not provide recourse, and finally discuss whether a bank has the incentives to do so in equilibrium.

In an equilibrium with recourse

In an equilibrium with recourse, a bank at $t = 0$ chooses the size of its first period portfolio, and the security to raise financing characterised by $(w_1^H, w_{1,G}^L, w_{1,B}^L)$. I first derive the equilibrium choices of a bank with a bad portfolio, and then those of a bank with a good portfolio, in a separating equilibrium.

A bank with a bad portfolio chooses its portfolio size and the security to issue to maximise its expected payoff, in an equilibrium with recourse. I write this problem recursively by taking into account that in this equilibrium, the value of the bank at $t = 1$ is the sum of V_{FB}^{j} and its retained cash flow at $t = 1$, i.e.

$$\max_{X_0^B, w_1^{B,H}, w_{1,G}^{B,L}, w_{1,B}^{B,L}} \theta^B(X_0^B)(V_{FB}^G + w_1^{B,H}) + [1-\theta^B(X_0^B)]\left[\gamma(V_{FB}^G + w_{1,G}^{B,L}) + (1-\gamma)(V_{FB}^B + w_{1,B}^{B,L})\right] \quad (3.13)$$

$$\text{s.t.} \quad \theta^B(X_0^B)[X_0^B - w_1^{B,H}] + [1-\theta^B(X_0^B)]\left[(1-\delta)X_0^B - \gamma w_{1,G}^{B,L} - (1-\gamma)w_{1,B}^{B,L}\right] + w_0 \geq kX_0^B \quad (3.14)$$

$$(RC^G) \text{ and } (RC^B) \quad (3.15)$$

Denote the equilibrium expected payoff to a bank with a bad first period portfolio $U^{B*}(w_0)$.

A bank with a good portfolio faces a similar problem, subject an additional incentive compatibility constraint (IC_1) that a bank with a bad portfolio would prefer not to mimic.

$$\max_{X_0^G, w_1^{G,H}, w_{1,G}^{G,L}, w_{1,B}^{G,L}} \theta^G(X_0^G)(V_{FB}^G + w_1^{G,H}) + [1-\theta^G(X_0^G)]\left[\gamma(V_{FB}^G + w_{1,G}^{G,L}) + (1-\gamma)(V_{FB}^B + w_{1,B}^{G,L})\right] \quad (3.16)$$

$$\text{s.t.} \quad \theta^G(X_0^G)[X_0^G - w_1^{G,H}] + [1-\theta^G(X_0^G)]\left[(1-\delta)X_0^G - \gamma w_{1,G}^{G,L} - (1-\gamma)w_{1,B}^{G,L}\right] + w_0 = kX_0^G \quad (3.17)$$

$$\theta^B(X_0^G)(V_{FB}^G + w_1^{G,H}) + [1-\theta^B(X_0^G)]\left[\gamma(V_{FB}^G + w_{1,G}^{G,L}) + (1-\gamma)(V_{FB}^B + w_{1,B}^{G,L})\right] \leq U^{B*}(w_0) \quad (IC_1) \quad (3.18)$$

$$(RC^G) \text{ and } (RC^B) \quad (3.19)$$

Notice that, without the (RC^G) and (RC^B) constraints, the above programme can be optimised by maximising $w_1^{G,H}$ and minimising $\left[\gamma w_{1,G}^{G,L} + (1-\gamma)w_{1,B}^{G,L}\right]$ by setting $w_{1,G}^{G,L} = w_{1,B}^{G,L} = 0$, and $w_1^{G,H}$ to bind the financing constraint. This follows from the simple intuition of financing under asymmetric information, in the absence of concerns for the second period funding problem, as represented by (RC^G) and (RC^B). In this case, the security chosen by a bank with a good portfolio is debt like, leaving the bank a high powered security that only pays off if a high cash flow is realised.

When taking into consideration the need to finance the second period portfolio, a bank with a good portfolio must retain some cash flow when a low cash flow is realised, in order to credibly signal its second period portfolio type through recourse. The equilibrium is summarised in the following proposition.

Proposition 22. *In an equilibrium with recourse, with capital w_0, a bank with a bad portfolio chooses at $t = 0$ the first best portfolio $X_0^{B*}(w_0) = X_{FB}^B$, and a bank with a good portfolio chooses $X_0^{G*}(w_0) \geq X_{FB}^G$, where*

$$X_0^{G*}(w_0) = \begin{cases} X_{FB}^G, & \text{if} \quad w_0 \geq w_{0,FB} \\ \hat{X}_0(w_0), & \text{otherwise} \end{cases} \tag{3.20}$$

where $\hat{X}_0(w_0)$ is such that

$$\begin{aligned} &\frac{\theta^B(\hat{X}_0(\cdot))}{\theta^G(\hat{X}_0(\cdot))} V^G(\hat{X}_0(\cdot)) - V_{FB}^B + \Big(\Big[1 - \theta^B(\hat{X}_0(\cdot))\Big] \\ &\qquad - \frac{\theta^B(\hat{X}_0(\cdot))}{\theta^G(\hat{X}_0(\cdot))}\Big[1 - \theta^G(\hat{X}_0(\cdot))\Big]\Big)(1-\gamma)\hat{w}_1^L = \left[1 - \frac{\theta^B(\hat{X}_0(\cdot))}{\theta^G(\hat{X}_0(\cdot))}\right] w_0 \qquad (3.21) \\ &\hat{w}_1^L = \frac{\theta^B(X_{FB}^G)}{\theta^G(X_{FB}^G)} V_{FB}^G - V_{FB}^B \qquad (3.22) \end{aligned}$$

and $w_{0,FB}$ such that $\hat{X}_0(w_{0,FB}) = X_{FB}^G$.

The equilibrium securities issued by a bank at $t = 0$ with a portfolio of the $i \in \{G, B\}$ is $(w_1^{i,H*}(w_0), w_{1,G}^{i,L*}(w_0), w_{1,B}^{i,L*}(w_0)) = (\hat{w}_1^{i,H}(w_0), 0, \hat{w}_1^L)$, where

$$\hat{w}_1^{i,H}(w_0) = \frac{1}{\theta^i(X^{i*}(\cdot))}\left(V^i(X_0^{i*}(\cdot) + w_0 - [1 - \theta^i(X^{i*}(\cdot))](1-\gamma)\hat{w}_1^L\right) \tag{3.23}$$

It is worth comparing the first period equilibrium to a second period equilibrium without recourse, because both are signalling equilibria in which a bank with a good portfolio signals its private information through over-investment. The corollary below states that, for a given amount of capital w at the beginning of the periods, there is greater distortion in the first period than in the second period.

Corollary 3 (to Proposition 22)**.**

$$X_1^{G*}(w) \geq X_0^{G*}(w) \tag{3.24}$$

where the inequality is strict for $w \leq w_{0,FB}$.

This is because the need to retain capital to facilitate the financing of the second period portfolio hinders the extent to which the bank can use debt financing to mitigate the asymmetric information problem, resulting in greater over-investment in equilibrium.

Moreover, in anticipating of the funding needs in the second period, a bank issues a security with implicit guarantee regardless of the type of its first period portfolio. I discuss the implementation of such security using bankruptcy-remote securitisation with implicit guarantee in Section 3.4.1. This model therefore provides a rationale for such arrangement in the securitisation industry.

In an equilibrium without recourse by a good type

Notice that, in a separating equilibrium, a bank with a bad portfolio at $t = 0$ has similar incentives as one in an equilibrium with recourse. Because the bank raises capital at fair price in a separating equilibrium, it optimally chooses the first best portfolio size and issues a security with recourse so as to resolve the asymmetric information problem in the second period and achieve first best investment decisions.

This section then considers a bank with a good portfolio at $t = 0$ chooses the size of its first period portfolio, and the security to raise financing characterised by (w_1^H, w_1^L), subject to an incentive compatibility constraint (IC_1) to prevent mimicking by a bank with a bad portfolio.

$$\max_{X_0^G, w_1^H, w_1^L} \theta^G(X_0^G)\left[V_{FB}^G + w_1^H\right] \tag{3.25}$$

$$+[1-\theta^G(X_0^G)]\left(\gamma\left[V^G(X_1^{G*}(w_1^L)) + w_1^L\right] + (1-\gamma)\left[V_{FB}^B + w_1^L\right]\right) \tag{3.26}$$

$$\text{s.t.}\quad \theta^G(X_0^G)[X_0^G - w_1^H] + [1-\theta^G(X_0^G)][(1-\delta)X_0^G - w_1^L] + w_0 \geq kX_0^G \tag{3.27}$$

$$\theta^B(X_0^G)\left[V^B(X_1^{G*}(w_1^H)) + w_1^H\right] + [1-\theta^B(X_0^G)]\left(\gamma\left[V^G(X_1^{G*}(w_1^L)) + w_1^L\right]\right.$$

$$\left. +(1-\gamma)\left[V_{FB}^B + w_1^L\right]\right) \leq U^{B*}(w_0) \quad (IC_1) \tag{3.28}$$

The following proposition summarises the equilibrium.

Proposition 23. *In an equilibrium with no recourse provided by a bank with a good portfolio at $t = 0$, a bank with a bad portfolio chooses at $t = 0$ the first best portfolio $X_0^{B*}(w_0) = X_{FB}^B$, and a bank with a good portfolio chooses $X_0^{G**}(w_0) \geq X_{FB}^G$, where*

$$X_0^{G**}(w_0) = \begin{cases} X_{FB}^G, & \textit{if } w_0 \geq w'_{0,FB} \\ \tilde{X}_0(w_0), & \textit{otherwise} \end{cases} \tag{3.29}$$

where $\tilde{X}_0(w_0)$ *is such that*

$$\frac{\theta^B(\tilde{X}_0(\cdot))}{\theta^G(\tilde{X}_0(\cdot))} V^G(\tilde{X}_0(\cdot)) - V^B_{FB} + \left[1 - \theta^B(\tilde{X}_0(\cdot))\right] \gamma \left[V^G(\hat{X}_1(0)) - V^G_{FB}\right]$$
$$= \left[1 - \frac{\theta^B(\tilde{X}_0(\cdot))}{\theta^G(\tilde{X}_0(\cdot))}\right] w_0 \qquad (3.30)$$

and $w'_{0,FB}$ *such that* $\tilde{X}_0(w'_{0,FB}) = X^G_{FB}$.

The equilibrium security issued by a bank with a good portfolio is one without guarantee, $(w_1^{G,H**}(w_0), w_1^{G,L**}(w_0)) = (\frac{V^G(X_0^{G**}(\cdot))+w_0}{\theta^G(X_0^{G**}(\cdot))}, 0)$. *The equilibrium security issued by a bank with a bad portfolio is one with implicit guarantee,* $(w_1^{B,H**}(w_0), w_{1,G}^{B,L**}(w_0), w_{1,B}^{B,L**}(w_0)) = (\hat{w}_1^{B,H}(w_0), 0, \hat{w}_1^L)$.

Notice that in this case, a bank with a good portfolio at $t = 0$ issues a debt like security that leaves a high powered equity stake with $w_1^{G,L**}(w_0) = 0$, in order to mitigate the asymmetric information problem between the bank and outside investors, thereby minimising the amount of over-investment in equilibrium as a signal.

I again compare this first period equilibrium to a second period equilibrium without recourse. In contrast to the previous equilibrium with recourse, for a given amount of capital w, there is less distortion in the investment decision of a bank with a good portfolio in the first period than in the second period $X_1^{G**}(w) < X_0^{G*}(w)$.

Corollary 4 (to Proposition 23)**.**

$$X_1^{G**}(w) \leq X_0^{G*}(w) \qquad (3.31)$$

where the inequality is strict for $w \leq w_{1,FB}$.

This is because, without recourse, the bank over-invests in its second period portfolio if it is of a good type. This reduces the incentive for a bank with a bad portfolio to mimic in the first period, since it otherwise achieves the first best outcome in the second period through providing recourse in equilibrium. Therefore the investment decision of a bank with a good portfolio in the first period involves less over-investment while serving as a signal of its type.

3.3.4 Recourse versus no-recourse equilibrium

This section discusses the economic intuition for the two types of equilibrium for each of the two types of banks.

The trade-off between the two types of equilibrium follows from the above discussion. On the one hand, providing recourse at the end of the first period is a socially efficient way to resolve the asymmetric information problem in the second period, restoring first best investment levels. On the other hand, for recourse to be credible in equilibrium, a bank with a bad project in the second period refrains from providing recourse and retains $w_{1,B}^{B,L*}(w_0) = \hat{w}_1^L > 0$ when the first period portfolio returns a low cash flow.

For a bank with a bad portfolio in the first period, it chooses the first best portfolio size in the first period and is free to design a security with recourse to achieve efficient investment in the second period as well. It enjoys the benefit of providing recourse in equilibrium at no cost, and therefore always chooses a security with recourse in equilibrium.

For a bank with a good portfolio in the first period, however, the need to retain $w_{1,B}^{B,L*}(w_0)$ in order for recourse to be credible limits the bank's ability to issue debt and retain only high powered stake to mitigate the asymmetric information problem in the first period, resulting in greater extent of over-investment. The bank therefore trades off the benefit and cost of providing recourse. The resulting equilibrium is one in which a bank with a good portfolio in the first period issues a security with recourse if and only if the $U^{G*}(w_0) \geq U^{G**}(w_0)$, where $U^{G*}(w_0)$, $U^{G**}(w_0)$ are expected the payoffs to a bank with a good portfolio in an equilibrium with with recourse and one without recourse respectively,

$$\begin{aligned} U^{G*}(w_0) \quad &\equiv \quad \theta^G(X_0^{G*}(\cdot))V_{FB}^G + \left[1 - \theta^G(X_0^{G*}(\cdot))\right]\left[\gamma V_{FB}^G + (1-\gamma)V_{FB}^B\right] \\ &\qquad + V^G(X_0^{G*}(\cdot)) + w_0 \qquad (3.32) \\ U^{G**}(w_0) \quad &\equiv \quad \theta^G(X_0^{G**}(\cdot))V_{FB}^G + \left[1 - \theta^G(X_0^{G**}(\cdot))\right]\left[\gamma V^G(X_1^{G*}(0)) + (1-\gamma)V_{FB}^B\right] \\ &\qquad + V^G(X_0^{G**}(\cdot)) + w_0 \qquad (3.33) \end{aligned}$$

3.4 Bankruptcy-remote securitisation with implicit guarantee

In this section I highlight the two important features of the securitisation process, namely bankruptcy-remoteness and the presence of implicit guarantee, and discuss their roles in implementation the equilibrium with recourse. I then discuss the empirical implications of this model in relation to existing evidence.

3.4.1 Implementation of the equilibrium with recourse

In this section I discuss an implementation of the equilibrium with recourse via bankruptcy-remote securitisation with implicit guarantee.

Proposition 24. *An equilibrium in which a bank of type i at $t = 0$ invests in a portfolio of size $X_0^{i*}(w_0)$ and issues a security characterised by $(w_1^{i,H*}(w_0), w_{1,G}^{i,L*}(w_0), w_{1,B}^{i,L*}(w_0)) = (\hat{w}_1^{i,H}(w_0), 0, \hat{w}_1^L)$ can be implemented according to the following.*

- *Invest in a portfolio of size $X_0^{i*}(w_0)$ at $t = 0$.*
- *Transfer a fraction $(1 - \alpha^{i*}(w_0))$ of the asset into a special purpose vehicle (SPV), and retain the remaining $\alpha^{i*}(\cdot)$ fraction on the book of the bank, where $\alpha^{i*}(\cdot) \equiv \frac{\hat{w}_1^L}{(1-\delta)X_0^{i*}(w_0)}$.*
- *Issue a senior tranche back by the SPV with promised repayment $R^{i*}(w_0) \equiv X_0^{i*}(w_0) - \hat{w}_1^{i,H}(w_0) \in \left[(1 - \delta)X_0^{i*}(\cdot), (1 - \alpha^{i*}(\cdot))X_0^{i*}(\cdot)\right]$, to raise $kX_0^{i*}(\cdot) - w_0$ from the investors.*
- *If a high cash flow $X_0^{i*}(w_0)$ is realised at the end of the period, the investors are paid in full and the bank retains the residual cash flow.*
- *If a low cash flow $(1 - \delta)X_0^{i*}(w_0)$ is realised, the bank decides whether to provide recourse depending on the type of its second period portfolio.*
  - *If the second period portfolio is a good type, the bank provides recourse onto the remaining $\alpha^{i*}(\cdot)$ fraction of its assets and repays $(1 - \delta)X_0^{i*}(w_0) \leq R^{i*}(w_0)$, retaining zero cash flow.*
  - *If the second period portfolio is a bad type, the bank defaults on the SPV, repays the investors with cash flow $(1-\alpha^{i*}(\cdot))(1-\delta)X_0^{i*}(\cdot)$ from the SPV, and retains the payoff $\alpha^{i*}(\cdot)(1 - \delta)X_0^{i*}(\cdot)$ from the assets on its book.*
- *Observing the decision to provide recourse or otherwise, the investors learn the true type of the bank's second period portfolio. The bank forms a portfolio of the first best size and issues a security backed by the portfolio at fair price at $t = 1$.*

The above implementation highlights two important features of the secularisation process, as discussed by, for example, Gorton and Souleles (2007). Firstly, the SPV is structured to be bankruptcy-remote, and secondly, there is often implicit guarantee.

Both features are necessary in this implementation. Since recourse is contingent on the unobservable and uncontractible type of the second period portfolio at $t = 1$, it must be implicit and incentive compatible. Recourse is thus provided only by a bank with a good second period portfolio. Meanwhile, the bankruptcy-remoteness of the SPV enables costless default by a bank with a bad second period portfolio.

3.4.2 Empirical implications

The above implementation gives rise to the following empirical implications.

1. *The extent of securitisation is positively correlated with unobservable asset quality.*

   The discussion in Section 3.3.4 suggests that, while a bank with a bad portfolio at $t = 1$ always provides implicit guarantee in equilibrium, a bank with a good portfolio only does so if providing recourse does not induce too much distortion in its current period investment decision. Further more, in an equilibrium in which a bank with both a good and a bad portfolio provides implicit guarantee, a bank with a bad portfolio retains a larger fraction of its assets on its book, i.e. $\alpha^{B*}(w_0) > \alpha^{G*}(w_0)$. This is because a bad portfolio has a lower expected value, and a larger piece is required to support the level of implicit guarantee provided. This is consistent with the evidence presented by Casu et al. (2011) using US Bank Holding Company data from 2001 to 2007.

   Since in this model the retained fraction of the portfolio provides implicit guarantee, this can also be interpreted as *a negative correlation between implicit recourse and unobservable asset quality*. Vermilyea et al. (2008) show that, reported fraud losses in credit card securitisation, as a proxy for implicit recourse, is negatively related to the performance of the portfolio. Mandel et al. (2012) similarly found a positive relationship between delinquency on securitised assets and enhancements.

2. *Conditional on recourse becoming necessary, honouring implicit guarantee is positively correlated with future loan performance.*

   At the end of the first period, the act of providing recourse serves as a credible signal for the type of the second period portfolio. Indeed Higgins and Mason (2004) show that conditional on being in a situation where recourse is needed, it is met with positive short- and long-term stock market reactions. This is also consistent the point made by Amiram et al. (2011), that an increase in trading volume, equity volatility and spreads when retained interest write downs are announced indicates that the market views such events as providing significant information.

3.5 Conclusion

This paper show that the bankrupt-remoteness and the presence of implicit guarantee are two important feature of securitisation to implement the optimal funding arrangement for banks, in a model of repeated investment with risk-shifting potential and financing under asymmetric information. Consistent with conventional wisdom in the industry, honouring implicit guarantee helps a sponsoring bank reduce its future funding cost. This is because in this model, recourse serves as a signal for the quality of the bank's future investment opportunity. Meanwhile, bankruptcy-remoteness contribute to the credibility of recourse as a signal, be allowing a bank with a bad future investment opportunity to default on its SPV without damaging the rest of the bank asset.

Although providing implicit guarantee mitigates the asymmetric information and risk-shifting problems for the bank in the future, it exacerbates the current period over-investment due to the frictions. A bank therefore would only engage in securitisation if the future benefit outweighs the current cost.

This model has also implications for the regulatory discussion regarding a ban on implicit guarantee. Due to concerns about risks brought to the banks by rescuing failing SPVs, it is argued that implicit recourse should not be allowed. In the context of this model, implicit guarantee as a means to mitigate the asymmetric information and risk-shifting problem is socially efficient, compared to the alternative of over-investment. This paper thus adds to the discussion a potential benefit of allowing implicit guarantee.

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Appendices

APPENDIX A

CHAPTER 1: CONTINGENT CAPITAL STRUCTURE

A.1 Parameter restriction on $\eta^i + \theta^s$

As given in Eq. 1.3, the optimal risk choice $\delta^{i,s}(F^s)$ in state s given a financing plan with debt of face value F^s is $\frac{1}{2}(\eta^i - \theta^s - \frac{X-F^s}{\Delta_X})$. This implies that the success probability is $q^{i,s}(F^s) \equiv \eta^i - \delta^{i,s}(F^s) = \frac{1}{2}(\eta^s + \theta^s + \frac{X-F^s}{\Delta_X})$ and the conditional probability of realising a high cash flow is $\theta^s + \delta^{i,s}(F^s) = \frac{1}{2}(\eta^i + \theta^s - \frac{X-F^s}{\Delta_X})$, where the first best case is produced when $F^s = 0$.

That the success probability lies in $(0,1)$ is equivalent to $-\frac{X-F^s}{\Delta} < \eta^i + \theta^s < 2 - \frac{X-F^s}{\Delta}$. That the conditional probability of realising a high cash flow lies in $(0,1)$ is equivalent to $\frac{X-F^s}{\Delta} < \eta^i + \theta^s < 2 + \frac{X-F^s}{\Delta}$.

As it will become clear later, the equilibrium face value of the debt $F^s \in [0, X)$. Therefore the necessary condition that guarantees that the equilibrium probabilities lie in $(0,1)$ is that

$$\frac{X - F^s}{\Delta} < \eta^i + \theta^s < 2 \tag{A.1}$$

A.2 Proofs

A.2.1 Proof of Proposition 1 and 2

The problem in the case of *ex post* financing can be interpreted as a special case of the problem in the case of *ex ante* financing, when the set of macroeconomic states $\boldsymbol{S}$ is a singleton. Proposition 1 is therefore implied by Proposition 2. Here I present the proof for the latter proposition, which then also implies the former.

I first show in Claim 1 that the set The set of equilibria that satisfy the Intuitive Criterion is given by the programme given in Proposition 2. I then show in Claim 2 that all equilibria that satisfy the Intuitive Criterion are fair-price separating. Finally I derive the threshold $\breve{e}^C$ such that the bank issues debt if and only if $\bar{e} < \breve{e}^C$.

Claim 1. *The set of equilibria that satisfy the Intuitive Criterion is given by the programme given in Proposition 2.*

Proof of Claim 1. Denote $(\hat{e}, \boldsymbol{F}_C, \boldsymbol{\alpha}_C)$ a maximiser of the programme given in Proposition 2. The proof is consisted of two parts. Firstly I apply the Intuitive Criterion to discard any equilibrium in which the capital structure is not a maximiser of the said programme. Intuitively, the Good bank always has the incentive to deviate to a capital structure that gives it a higher payoff which still allows it to separate from the bad. I then show that a maximise of the programme satisfies the Intuitive Criterion.

The first part of the proof is to show that any equilibrium capital structure that satisfies the Intuitive Criterion a maximiser of the programme given in Proposition 2. I show this in the following two steps: (i) any separating equilibrium that is not a maximiser of the said programme does not satisfy the Intuitive Criterion, and (ii) any pooling equilibrium does not satisfy the Intuitive Criterion.

(i) Consider a separating equilibrium with a contingent capital structure $(e, \boldsymbol{F}, \boldsymbol{\alpha})$, which is not a maximiser of the said programme. A deviation to the capital structure $(\hat{e}, \boldsymbol{F}_C, \boldsymbol{\alpha}_C)$ provides the Good bank with a strictly higher payoff whereas the Bad bank receives a lower payoff than in the equilibrium, while the outside investors at least break even, if the outside investors hold a belief that the deviation can only come from the Good type. The separating equilibrium $(\boldsymbol{F}, \boldsymbol{\alpha})$ thus does not satisfy the Intuitive Criterion. Therefore any contingent capital structure equilibrium that survives the Intuitive Criterion must be a maximiser of the said programme.

(ii) Consider a pooling equilibrium with a contingent capital structure $(e, \boldsymbol{F}, \boldsymbol{\alpha})$. The equilibrium capital structure satisfies the following constraints,

$$
\begin{aligned}
(PC^B_{Pool}) \quad &: \quad \mathbb{E}[(1-\alpha^s)E^{B,s}(F^s)] \geq e && \text{(A.2)}\\
(PC^G_{Pool}) \quad &: \quad \mathbb{E}[(1-\alpha^s)E^{G,s}(F^s)] \geq e && \text{(A.3)}\\
(IR_{Pool}) \quad &: \quad \mathbb{E}[\gamma V^{G,s}(F^s) + (1-\gamma)V^{B,s}(F^s) - (1-\alpha^s)(\gamma E^{G,s}(F^s) + (1-\gamma)E^{B,s}(F^s))] \\
&\qquad \geq 1 - e && \text{(A.4)}
\end{aligned}
$$

In equilibrium, the payoff to the existing shareholders of the bank, if the bank issues securities according to the equilibrium financing plan, is given by

$$\mathbb{E}[(1-\alpha^s)E^{i,s}(F^s)] + (\bar{e} - e) = u \qquad \text{(A.5)}$$

for some constant u. Implicitly different F^s in Eq. A.5 with regard to α^s to find the derivative

$$f^{i,s}_{\alpha^s,F^s}(e, \boldsymbol{F}, \boldsymbol{\alpha}) \equiv \frac{\partial \alpha^s}{\partial F^s} = -\frac{1-\alpha^s}{q^{i,s}(F^s)\Delta_X} \qquad \text{(A.6)}$$

The derivative is negative, and more so for the Bad type than for the Good type. That is, the Bad bank requires the equity issuance to be decreased more, in order to compensate for an increase in the leverage. Therefore there exists a deviation $(e, \boldsymbol{F'}, \boldsymbol{\alpha'})$, such that $F^{s'} > F^s$, $\alpha^{s'} < \alpha^s$, and all other financing parameters are the same as in the equilibrium. This deviation, compared to the original pooling equilibrium, provides the Good bank with a strictly higher payoff whereas the Bad bank receives a lower payoff than in equilibrium, if the outside investors are willing to provide capital.

If the investors believe that such a deviation can only come from a Good bank, they should accept as long as they at least break even. In a separating equilibrium, the investors' rationality constraint is given by Eq. 1.7. Binding the (IR) and implicitly differentiating it yields the derivative

$$f^{IR,s}_{\alpha^s,F^s}(e, \boldsymbol{F}, \boldsymbol{\alpha}) \;\equiv\; \frac{\partial \alpha^s}{\partial F^s} = \frac{F^s}{2\Delta_X} - \frac{1-\alpha^s}{q^{G,s}(F^s)\Delta_X} \qquad \text{(A.7)}$$

Notice that $|f^{G,s}_{\alpha^s,F^s}(\cdot)| > -f^{IR,s}_{\alpha^s,F^s}(\cdot)\ \forall F > 0$. That is, an increase in F and a decrease in α that leaves the marginally lower payoff to a Bad bank would strictly benefit the investors. Therefore there indeed exists such a deviation that the investors would be willing to accept and allow the Good bank to be strictly better off.

(i) and (ii) thus collectively suggest that any contingent capital structure equilibrium that survives the Intuitive Criterion must be a maximiser of the said programme.

The second part establishes that a maximiser of the programme satisfies the Intuitive Criterion. Suppose that there exists a capital structure $(\boldsymbol{F}, \boldsymbol{\alpha})$ which is a maximiser of the programme but it does not satisfy the Intuitive Criterion. That is, there exists a capital structure $(\boldsymbol{F}', \boldsymbol{\alpha}')$ such that the good bank receives a strictly higher payoff and the bad bank receives a lower payoff than at $(\boldsymbol{F}, \boldsymbol{\alpha})$, and that the outside investors at least break even. This suggests that $(\boldsymbol{F}', \boldsymbol{\alpha}')$ satisfies the constraints (PC_C^B) and (IR_C) and gives the good bank a strictly higher payoff than $(\boldsymbol{F}, \boldsymbol{\alpha})$. This contradicts with the assumption that $(\boldsymbol{F}, \boldsymbol{\alpha})$ is a maximiser of the programme.

Therefore, the set of equilibrium contingent capital structures that satisfy the Intuitive Criterion is given by the set of maximisers of the programme given in Proposition 2. This result implies Proposition 1. ☐

Claim 2. *Any equilibrium that survives the Intuitive Criterion is fair-price separating.*

Proof of Claim 2. I prove this claim by showing that otherwise there exists a deviation to eliminate the equilibrium according to the Intuitive Criterion. This part of the proof builds upon the previous claim and only considers separating equilirbia.

Intuitively, if the (IR_C) is not binding given a separating belief, a Good bank can increase its leverage and decrease its equity issue to maintain the same payoff, while still allowing the investors to at least break even, despite the value destruction due to increase leverage and risk-taking. Moreover, substituting debt for equity also hurts the Bad bank more than the Good because it reduces the mispricing. A change that leaves the Good bank indifferent should then make the Bad strictly worse off.

This argument is formally established below. Consider an underpricing separating equilibrium with financing plan $(e, \boldsymbol{F}, \boldsymbol{\alpha})$ such that

$$\mathbb{E}[V^{G,s}(F^s) - (1-\alpha^s)E^{G,s}(F^s)] > 1 - e \tag{A.8}$$

There thus exists a deviation to financing plan $(e, \boldsymbol{F}', \boldsymbol{\alpha}')$, where where $F^{s'} > F^s$, $\alpha^{s'} < \alpha^s$ and all other financing parameters are the same, such that, if accepted by the investors, this financing plan makes a Good bank strictly better off, whereas it makes a Bad bank strictly worse off in comparison to the equilibrium outcome, following similar argument as in Part 1(ii) of the proof of Claim 1.

If the investors believe that such an deviation can only come from a Good bank, they are willing to accept the financing terms if the increase in leverage is small. This is because the investors make a positive payoff in equilibrium. A sufficiently small increase in leverage reduces the payoff left for investors but they can still at least break even.

Therefore in any Intuitive equilibrium, (IR_C) binds. The equilibrium thus must be a fair-price separating equilibrium. $\square$

I now derive the threshold $\breve{e}^C$ in Proposition 2. First consider the fair-price separating equilibria with no capital market debt that survives the Intuitive Criterion. Combining (IR_C) with (PC_C^B) yields that, at $\boldsymbol{F} = \mathbf{0}$, the conditions can be satisfied if

$$e \geq \breve{e}^C \equiv \frac{\mathbb{E}\left[(1-\alpha^s)E_{FB}^{G,s}\right]}{\mathbb{E}\left[(1-\alpha^s)(E_{FB}^{G,s} - E_{FB}^{G,s})\right]} \left(\mathbb{E}\left[V_{FB}^{G,s}\right] - 1\right) \tag{A.9}$$

That is, a Good bank must put in at lease $\breve{e}^C$ of its internal capital in order to separate from the Bad. This implies that no separating equilibrium exists if $\bar{e} < \breve{e}^C$. Moreover, for $\breve{e} \geq \breve{e}^C$, the bank is able to achieve separation without leverage. A separating equilibrium without leverage therefore cannot satisfy the Intuitive Criterion. In summary, the bank issues debt in equilibrium if and only if $\bar{e} < \breve{e}^C$.

A.2.2 Proof of Corollary 1

Because of the complex nature of the problem, I impose parameter restrictions to guarantee an interior solution, that $\max_{F^s} V^{B,s}(F^s) + \frac{\eta^G - \eta^B}{2} F < 1$. Intuitively, this is the case if the Good and the Bad types are not too different, i.e. $\eta^G - \eta^B$ small.

To characterise the unique equilibrium leverage that satisfies the Intuitive Criterion, I start by establishing the following claims. The Intuitive Criterion is applied to discard equilibria in which there exists an out-of-equilibrium action such that (i) a Bad bank is strictly worse off deviating to it, and (ii) if the investors believe that such a deviation can only come from a Good bank, the Good bank is strictly better off deviating to it. This process is equivalent to establishing that there is a unique solution $\hat{F}^s$ to the maximisation programme given by Proposition 1.

Claim 3. *Any equilibrium with capital market debt $F^s > 0$ that survives the Intuitive Criterion has a binding (PC^B).*

Proof of Claim 3. I show this claim by constructing a deviation according to the Intuitive Criterion. Intuitively, if the (PC^B) is slack, a Good bank can reduce its leverage to enjoy the value created by the reduction in risk-shifting incentive, while still achieving separation from the Bad.

Formally, consider a fair-price separating equilibrium with financing plan $(\bar{e}, F^s, \alpha^s)$ such that $(1-\alpha^s)E^{B,s}(F^s) < \bar{e}$ where $F^s > \bar{D}$. There exists a deviation to financing plan $(\bar{e}, F', \alpha')$, where $F' < F^s$ and $\alpha' > \alpha^s$, such that a Good bank strictly benefits if the financing plan is accepted by the investors, and that $(1-\alpha')E^{B,s}(F') < \bar{e}$ so that a Bad bank has no incentive to deviate to this financing plan.

Consider any financing plans (e, F, α) such that the payoff to the insiders of a bank of type i is

$$(1-\alpha)E^{i,s}(F) + (\bar{e} - e) = u \tag{A.10}$$

where u is any constant. The existence of such a deviation thus follows from the properties of the functions $f^{i,s}_{\alpha^s,F^s}(\cdot)$ and $f^{IR,s}_{\alpha^s,F^s}(\cdot)$ given by Eq. A.6–A.7 and discussed in Part 1(ii) of the proof of Claim 1. □

Claims 2–3 thus suggest that both (PC^B) and (IR) bind in an equilibrium that satisfy the Intuitive Criterion. Combining both binding constraints implies that, for a a given F^s, the equilibrium input of internal capital e is given by

$$e = \breve{e}^s(F^s) \equiv \frac{E^{B,s}(F^s)}{E^{G,s}(F^s) - E^{B,s}(F^s)} \left[V^{G,s}(F^s) - 1\right] \tag{A.11}$$

This implies an equilibrium that satisfies the Intuitive Criterion with F^s exists only for $\bar{e} \geq e$. Particularly, for $\bar{e} \geq \breve{e}^s$, fair-price separation can be achieved without leverage. The financing plan in the unique Intuitive equilibrium is thus $(\hat{e}, 0, \hat{\alpha}^s(\bar{e}))$ where $\hat{e} \geq \breve{e}^s$ and $\alpha^s(\cdot)$ is given by a binding (IR). That is, $\hat{\alpha}^s(\cdot) = (1-\bar{e})/V^{G,s}_{FB}$.

For $\bar{e} < \breve{e}^s$, however, the bank must resort to taking leverage in order to achieve fair-price separation.

Claim 4. *In any equilibrium with capital market debt $F^s > 0$ that survives the Intuitive Criterion, the bank puts up all of its internal capital $\bar{e}$.*

Proof of Claim 4. The proof builds upon that of Claims 2–3 and only considers fair-price separating equilibria with a binding (PC^B).

Intuitively, because internal capital is even less information sensitive than debt, substituting internal capital for debt hurts the Bad bank and benefits the Good by reducing the mispricing from debt issuance. Therefore a Good bank can deviate to a lower leverage level and puts up more internal capital. This deviation still allows the outside investors to break even since a reduction in leverage increases the total value of the bank.

Formally, I implicitly differentiate Eq. A.10 to find the derivative

$$f^i_{\alpha,e}(e,F,\alpha) \equiv \frac{\partial \alpha}{\partial e} = -\frac{1}{E^{i,s}(F)} \tag{A.12}$$

The derivative is negative, suggesting that increasing internal capital input by a marginal unit while decreasing the fraction of equity issued by $1/E^{i,s}(F)$ leaves a type i bank with the same amount of payoff, holding the face value of the debt outstanding unchanged.

Notice that $|f^B_{\alpha,e}(\cdot)| > |f^G_{\alpha,e}(\cdot)|$. That is, a Bad bank requires to retain more equity in order to compensate for the additional internal capital input than a Good bank.

Consider a fair-price separating equilibrium (e, F^s, α), where $F^s > 0$. There exists a financing plan (e', F', α), where $e' > e$ and $0 < F' < F$ such that if raises financing successfully , a Good bank is strictly better off, whereas a Bad bank is strictly worse off than in equilibrium.

If the investors believe that such an deviation can only come from a Good bank, they are willing to accept the financing terms. Because the reduction in leverage increases the value of the bank, a Good bank can be better off and still allow the investors to at least break even.

Therefore in any Intuitive equilibrium in which the bank issues capital market debt, it puts up all of its internal capital. $\square$

Therefore for $\bar{e} < \breve{e}^s$, any fair-price separating equilibrium in which the (PC^B) holds with strict inequality does not satisfy the Intuitive Criterion because it entails some leverage $F^s > 0$. Intuitively, because reducing leverage increases the bank's value, the bank can share some of it with the investors and still benefit from the deviation. The unique Intuitive equilibrium is thus characterised by a binding (PC^B) and a binding (IR).

Specifically, the equilibrium face value of debt $\hat{F}^s(\bar{e})$ is given by $\bar{e} = \breve{e}^s(\hat{F}^s(\cdot))$. The equilibrium fraction of outside equity issued is then given by $\hat{\alpha}^s(\bar{e}) = 1 - \bar{e}/E^{B,s}(\hat{F}^s(\cdot))$.

A.2.3 Proof of Proposition 3 and 4

Ex post financing

In the equilibrium with *ex post* financing, the equilibrium leverage is given by binding (PC^B) and (IR), which yields

$$\breve{e}(\hat{F}^s) \equiv \frac{E^{G,s}(\hat{F}^s)}{E^{G,s}(\hat{F}^s) - E^{B,s}(\hat{F}^s)} \left[V^{G,s}(\hat{F}^s) - 1\right] = \bar{e} \tag{A.13}$$

The procyclicality of book leverage $\hat{F}^s(\cdot)$ is derived by implicitly differentiating Eq. A.13 with regard to θ^s. The equity value of a bank given the face value of the debt and the optimal risk choice can be expressed as $E^{i,s}(F^s) \equiv \left[q^{i,s}(F^s)\right]^2 \Delta_X$. I then express Eq. A.13 as

$$\left[q^{G,s}(\hat{F}^s(\cdot))\right]^2 \Delta_X + q^{G,s}(\hat{F}^s(\cdot))\hat{F}^s(\cdot) - \bar{e}\frac{\left[q^{G,s}(\hat{F}^s(\cdot))\right]^2}{\left[q^{B,s}(\hat{F}^s(\cdot))\right]^2} = 1 - \bar{e} \tag{A.14}$$

Denote by A the total derivative of the left hand side of Eq. A.14 with respect to $q^{G,s}(\cdot)$, when expressing $q^{B,s}(\cdot) = q^{G,s}(\cdot) - \frac{\eta^G - \eta^B}{2}$. Implicitly differentiating Eq. A.14 with regard to θ^s yields

$$\begin{aligned}
\frac{\partial \hat{F}^s(\cdot)}{\partial \theta^s} &= \Delta_X \frac{A}{A - 2q^{G,s}(\cdot)\Delta_X} > \Delta_X > 0 && \text{(A.15)}\\
\text{where} \quad A &\equiv 2q^{G,s}(\cdot)\Delta_X + \hat{F}^s(\cdot) + \bar{e}\left[q^{G,s}(\cdot) - q^{B,s}(\cdot)\right]\frac{q^{G,s}(\cdot)}{q^{B,s}(\cdot)} && \text{(A.16)}\\
&> 2q^{G,s}(\cdot)\Delta_X && \text{(A.17)}
\end{aligned}$$

The partial derivative of the success probability $q^{G,s}(\hat{F}^s(\bar{e}))$ with respect to θ^s is therefore

$$\frac{dq^{G,s}(\cdot)}{d\theta^s} = \frac{\partial q^{G,s}(\cdot)}{\partial \theta^s} + \frac{\partial q^{G,s}(\cdot)}{\partial F^s}\frac{\partial \hat{F}^s(\cdot)}{\partial \theta^s} < \frac{1}{2} - \frac{1}{2\Delta_X}\Delta_X = 0 \tag{A.18}$$

Ex ante financing

The intuition for cyclical leverage is as follows. If the default probability is higher in one state than in another, effectively the debt in the higher leverage state is more information sensitive than in the other state. Then Good bank should have incentive to equalise the resulting default probabilities across states to reduce the mispricing. Since a better economic fundamental sustains higher leverage to produce the same default probability, leverage tends to be procyclical. However, since high leverage is required in booms to equalise the resulting default probabilities, it may becomes too costly in terms of the risk shifting incentives. Therefore in equilibrium, the default probabilities may not be equal across the states, but countercyclical.

Formally, I implicitly differentiate Eq. A.6 to find the derivative

$$f^{i}_{F^s,F^z}(e,\boldsymbol{F},\boldsymbol{\alpha}) \equiv \frac{\partial F^s}{\partial F^z} = -\frac{1-\alpha^z}{1-\alpha^s}\frac{q^{i,z}(F^z)}{q^{i,s}(F^s)}\frac{p(z)}{p(s)} \tag{A.19}$$

where $p(s)$ is the probability of state s.

The derivative is negative if $\alpha^z < 1$, suggesting that increasing the leverage in state z by a marginal unit while decreasing leverage in state s by $\frac{1-\alpha^z}{1-\alpha^s}\frac{q^{i,z}(F^z)}{q^{i,s}(F^s)}$ leaves a type i bank with the same amount of payoff, holding all other financing parameters unchanged.

Notice that $|f^{B}_{F^s,F^z}(\cdot)| > |f^{G}_{F^s,F^z}(\cdot)|$ iff $q^{i,z}(F^z) > q^{i,s}(F^s)$. That is, a Bad bank requires to reduce leverage in the highly levered state by more in order to compensate for the increase in leverage in the less levered state than a Good bank.

If the investors believe that a financing plan can only come from a Good bank, they should accept the financing plan if they can at least break even. I implicitly differentiate the binding (IR_C) (Eq. 1.11) to find the derivative

$$f^{IR}_{F^s,F^z}(e,\boldsymbol{F},\boldsymbol{\alpha}) \equiv \frac{\partial F^s}{\partial F^z} = -\frac{(1-\alpha^z)q^{G,z}(F^z)-\frac{F^z}{2\Delta_X}}{(1-\alpha^s)q^{G,s}(F^s)-\frac{F^s}{2\Delta_X}}\frac{p(z)}{p(s)} \tag{A.20}$$

That is, increasing leverage in state z by a marginal unit while decreasing leverage in state s by $\frac{(1-\alpha^z)q^{i,z}(F^z)-\frac{F^z}{2\Delta_X}}{(1-\alpha^s)q^{i,s}(F^s)-\frac{F^s}{2\Delta_X}}\frac{p(z)}{p(s)}$ leaves the investors with the same amount of payoff, holding all other financing parameters unchanged.

I now show that there exists a deviation to eliminate any equilibrium in which the contingent leverage that is countercyclical or in which the default probability is procyclical. Consider any states s, z s.t. $\theta^s > \theta^z$.

Consider a fair-price separating equilibrium financing plan $(\bar{e}, \boldsymbol{F}_C, \boldsymbol{\alpha}_C)$ such that (PC^B_C) binds, where $F^z > F^s$. This implies that $q^{i,z}(F^z) < q^{i,s}(F^s)$. There exists a financing plan $(\bar{e}, \boldsymbol{F}', \boldsymbol{\alpha}')$, where $F^{s'} > F^s$, $F^{z'} < F^z$ and all other financing parameters are the same, such that if raises financing successfully, a Good bank is strictly better off, whereas a Bad bank is strictly worse off than in equilibrium. Thus only a Good bank has the incentives to deviate to this financing plan.

Notice that $|f^{G}_{F^z,F^s}(\cdot)| < -f^{IR}_{F^z,F^s}(\cdot)$ if $F^z > F^s$ and $q^{i,z}(F^z) < q^{i,s}(F^s)$. That is, an increase in leverage in state s and a decrease in leverage in state z that leaves the same payoff to the Good bank would strictly benefit the investors. Therefore there indeed exists such a deviation as described above that the investors would be willing to accept.

Consider now a fair-price separating equilibrium financing plan $(\bar{e}, \boldsymbol{F}, \boldsymbol{\alpha})$ such that (PC_C^B) binds, where $F^z < F^s$ and $q^{i,z}(F^z) > q^{i,s}(F^s)$. By the same reasoning, there again exists a financing plan $(\bar{e}, \boldsymbol{F}', \boldsymbol{\alpha}')$, where $F^{s'} < F^s$, $F^{z'} > F^z$ and all other financing parameters are the same, such that if raises financing successfully, a Good bank is strictly better off, whereas a Bad bank is strictly worse off than in equilibrium; and the investors are willing to accept the financing plan given a belief that it can only come from the Good bank.

Therefore in any equilibrium that satisfies the Intuitive Criterion, leverage is such that $\hat{F}_C^s > \hat{F}_C^z$ and $E^{i,s}(\hat{F}^s) \geq E^{i,z}(\hat{F}^z)\ \forall\ \theta^s > \theta^z,\ s, z \in \{s \in \boldsymbol{S} : \hat{\alpha}_C^s < 1\}$. This also implies that $1 - q^{i,s}(\hat{F}^s) \leq 1 - q^{i,z}(\hat{F}^z)$.

A.2.4 Proof of Proposition 5

First, I notice that the equilibrium capital structures in the case of *ex post* financing also satisfy the equilibrium conditions for the case of *ex ante* financing, since in the former case the conditions are satisfied in each state *ex post*. Proposition 2 then implies that the Intuitive contingent capital structure equilibrium must give the Good bank at least as high an expected payoff as the equilibrium with *ex post* financing. Moreover, given that the outside capital market investors always break even in equilibrium, the bank captures the entire value created by the bank. The contingent capital structure therefore produces an expected bank value that is at least as high as in the other two cases.

Secondly, the contingent capital structure is strictly preferred to the *ex post* capital structure when capital market debt is issued in equilibrium, i.e. $\bar{e} < \breve{e}^C$. This is because by Proposition 3, the default probability in a contingent capital structure equilibrium is countercyclical, while the face value of the debt is procyclical. The *ex post* capital structure equilibrium has procyclical default probability, whereas the non-contingent capital structure equilibrium has constant face value of debt. Therefore neither capital structure belongs to the optimal set of contingent capital structures. The optimal contingent capital structure thus must deliver strictly higher bank value than either of the other two cases.

A.2.5 Proof of Proposition 6

The equilibrium contingent capital structure in the example of a tail event economy is given by Proposition 2. In order to fully characterise the equilibrium, consider the following two scenarios for $\bar{e} < \breve{e}^C$.

Firstly, suppose an interior solution such that $E^{i,H}(\hat{F}_C^H) = E^{i,L}(\hat{F}_C^L)$, i.e. $\hat{F}_C^H = \hat{F}_C^L + \frac{\theta^H - \theta^L}{\Delta_X}$. This also implies that $q^{i,H}(\hat{F}_C^H) = q^{i,L}(\hat{F}_C^L)$. Given this restriction, denote the equity value and the success probability with $E^i(\boldsymbol{F}_C)$ and $q^i(\boldsymbol{F}_C)$ respectively, which are equal in both states. The maximisation programme can then be rewritten as

$$\max_{e,F_C^L,\alpha_C^H,\alpha_C^L} \quad [\beta(1-\alpha_C^H) + (1-\beta)(1-\alpha_C^L)]E^G(\boldsymbol{F}_C) \tag{A.21}$$

$$s.t. \quad e \le \bar{e} \tag{A.22}$$

$$[\beta(1-\alpha_C^H) + (1-\beta)(1-\alpha_C^L)]E^B(\boldsymbol{F}_C) \ge e \tag{A.23}$$

$$\begin{aligned} &\beta V^{G,H}(F_C^H) + (1-\beta)V^{G,L}(F_C^L) \\ &\quad -[\beta(1-\alpha_C^H) + (1-\beta)(1-\alpha_C^L)]E^G(\boldsymbol{F}_C) \ge 1-e \end{aligned} \tag{A.24}$$

Applying similar reasoning as in the proof given in Appendix A.2.2, it is easy to show that objective function is maximised when all three constraints bind, which determines $e = \bar{e}$ and the equilibrium $\hat{F}^H = \hat{F}_C^L + \frac{\theta^H - \theta^L}{\Delta_X}$, where $\hat{\boldsymbol{F}}_C$ is given by

$$\bar{e} = \frac{E^B(\hat{\boldsymbol{F}}_C)}{E^G(\hat{\boldsymbol{F}}_C) - E^B(\hat{\boldsymbol{F}}_C)} \left[\beta V^{G,H}(\hat{F}_C^H) + (1-\beta)V^{G,L}(\hat{F}_C^L) - 1\right] \tag{A.25}$$

as well as $\hat{\boldsymbol{\alpha}}_C$ up to one degree of freedom, i.e.

$$\beta(1-\alpha_C^H) + (1-\beta)(1-\alpha_C^L) = \frac{\bar{e}}{E^B(\hat{\boldsymbol{F}}_C)} \tag{A.26}$$

The restrictions $\hat{F}_C^H = \hat{F}_C^L + \frac{\theta^H - \theta^L}{\Delta_X}$ implies that $\hat{F}_C^H \ge \frac{\theta^H - \theta^L}{\Delta_X}$. Therefore the condition for the first scenario to arise is that in equilibrium, $E^i(\hat{\boldsymbol{F}}_C) \le V_{FB}^{i,L}$. This is the case if

$$\bar{e} \le \breve{e}^T \equiv \frac{V_{FB}^{B,L}}{V_{FB}^{G,L} - V_{FB}^{B,L}} \left[\beta V^{G,H}(\frac{\theta^H - \theta^L}{\Delta_X}) + (1-\beta)V_{FB}^{G,L} - 1\right] \tag{A.27}$$

For $\bar{e} \in [\breve{e}^T, \breve{e}^C)$, the second scenario arises, in which case $\hat{F}_C^H < \frac{\theta^H - \theta^L}{\Delta_X}$ and $\hat{F}_L = 0$. Imposing the restriction that $\hat{F}_L = 0$, the maximisation programme can then be rewritten as

$$\max_{e,F_C^H,\alpha_C^H,\alpha_C^L} \quad \beta(1-\alpha_C^H)E^{G,H}(F_C^H) + (1-\beta)(1-\alpha_C^L)V_{FB}^{G,L} \tag{A.28}$$

$$s.t. \quad e \le \bar{e} \tag{A.29}$$

$$\beta(1-\alpha_C^H)E^{B,H}(F_C^H) + (1-\beta)(1-\alpha_C^L)V_{FB}^{B,L} \ge e \tag{A.30}$$

$$\begin{aligned} &\beta V^{G,H}(F_C^H) + (1-\beta)V_{FB}^{G,L} - \beta(1-\alpha_C^H)E^{G,H}(F_C^H) \\ &\quad -(1-\beta)(1-\alpha_C^L)V_{FB}^{G,L} \ge 1-e \end{aligned} \tag{A.31}$$

It is straightforward that the solution entail $e = \bar{e}$. The restriction $F_C^H < \frac{\theta^H - \theta^L}{\Delta_X}$ implies that $E^{G,H}(F_C^H) > V_{FB}^{G,L}$. Therefore the equity in the high state is relatively less information sensitive in the low state. The solution should therefore entail $\hat{\alpha}_C^L = 0$. The equilibrium leverage $\hat{F}_C^H$ and the equity issuance $\hat{\alpha}_C^H$ in the high state is therefore given by binding the (IR) and (PC^B),

$$\bar{e} = \frac{E^{B,H}(\hat{F}_C^H)}{E^{G,H}(\hat{F}_C^H) - E^{B,H}(\hat{F}_C^H)} \left[\beta V^{G,H}(\hat{F}_C^H) + (1-\beta) V_{FB}^{B,L} \frac{E^{G,H}(\hat{F}_C^H)}{E^{B,H}(\hat{F}_C^H)} - 1 \right] \tag{A.32}$$

$$\hat{\alpha}_C^H = 1 - \frac{\bar{e} + (1-\beta) V_{FB}^{B,L}}{\beta E^{B,H}(\hat{F}_C^H)} \tag{A.33}$$

The implementation using CoCo bonds in addition to straight debt and equity provided in the proposition follows intuitively from the characterisation of the equilibrium contingent capital structure.

A.2.6 Proof of Proposition 7

To derive the optimal capital regulation, I first show in Claim 5 that any regulation that imposes a leverage cap below the equilibrium level will bind in the resulting equilibrium. I then characterise $\breve{\boldsymbol{F}}$, the minimum leverage cap the regulate can impose, while implementing a separating equilibrium. Finally I provide conditions for when this is feasible, and derive the optimal capital regulation for the other cases.

Claim 5. *The capital regulation $\bar{\boldsymbol{F}}$ binds in any Intuitive equilibrium if the bank has insufficient internal capital $\bar{e} < \breve{e}^s(\bar{\boldsymbol{F}})$.*

Proof. This result follows the intuition of Claim 2. At any leverage level $\boldsymbol{F}$ such that $\bar{e} < \breve{e}^s(\boldsymbol{F})$, the Good bank's payoff is increasing in the leverage level, in any separating equilibrium. It therefore has the incentive to increase leverage until the capital regulation binds. This is due to the underpricing of the claims issued by the Good bank in order to separate at low leverage. □

This effectively allows the regulator to set leverage levels in the equilibrium. I now characterise $\breve{\boldsymbol{F}}$, the minimum leverage cap the regulate can impose, while implementing a separating equilibrium.

I invoke the concept of undefeated equilibrium proposed byMailath et al. (1993). Consider a pooling equilibrium and a separating equilibrium. If the pooling equilibrium provides the Good bank with a strictly higher payoff, the pooling equilibrium defeats the separating equilibrium, by restricting that the out-of-equilibrium belief upon observing the pooling equilibrium action in the separating equilibrium to be consistent with the set of types who strictly benefit form such a deviation. $\breve{\boldsymbol{F}}$ is therefore characterised by the optimisation programme given by Eq. 1.20, where the expected retained payoffs to the Good bank in the least-cost separating equilibrium $v^G(\boldsymbol{F};\bar{e})$ and the least-cost pooling equilibrium $v^G_P(\boldsymbol{F};\bar{e})$ are given by, respectively,

$$v^G(\boldsymbol{F}_C;\bar{e}) \equiv \max_{e,\boldsymbol{\alpha}_C} \mathbb{E}\left[(1-\alpha^s_C)E^{G,s}(F^s_C)\right] \; s.t. \; e \leq \bar{e}, (PC^B_C), (IR_C) \tag{A.34}$$

$$v^G_P(\boldsymbol{F}_C;\bar{e}) \equiv \max_{e,\boldsymbol{\alpha}_C} \mathbb{E}[(1-\alpha^s_C)E^{G,s}(F^s_C)] \quad s.t. \; e \leq \bar{e}, \; (PC^B_{Pool}) \text{ and } (IR_{Pool}) \tag{A.35}$$

I now derive the thresholds $\breve{e}^C_P$ and $\breve{e}^C_{LP}$. $\breve{e}^C_P < \breve{e}^C$ is the threshold for a separating equilibrium that satisfy the Intuitive Equilibrium, given a leverage cap of zero $\bar{\boldsymbol{F}} = 0$, to be undefeated by a pooling equilibrium. Because the equilibrium payoff to the Good bank is increasing in its internal capital $\bar{e}$, the threshold $\breve{e}^C_P$ is given by when the Good bank is indifferent between a pooling equilibrium and a separating equilibrium at zero leverage. That is,

$$v^G(\mathbf{0};\breve{e}^C_P) = v^G_P(\mathbf{0};\breve{e}^C_P) \tag{A.36}$$

For $\bar{e} < \breve{e}^C_{LP}$, the leverage required to implement a separating equilibrium is too socially costly because of the risk-shifting problem, and the regulator prefers to implement a pooling equilibrium at zero leverage. Such threshold $\breve{e}^C_{LP}$ is therefore given by

$$\mathbb{E}[V^{G,s}(\breve{F}^s(\breve{e}^C_{LP})] = \mathbb{E}[\gamma V^{G,s}_{FB} + (1-\gamma)V^{B,s}_{FB}] \tag{A.37}$$

A.2.7 Proof of Proposition 8

For $\bar{e} \in [\breve{e}^C_{LP}, \breve{e}^C_P]$, the optimal capital regulation $\bar{\boldsymbol{F}} = \breve{\boldsymbol{F}}$, where $\breve{\boldsymbol{F}}$ is the least-cost leverage cap a regulator can impose that implements a separating equilibrium given by the optimisation programme Eq. 1.20. Within the example of a tail event economy, I first characterise the functions $v^G_P(\boldsymbol{F};\bar{e})$ and $v^G(\boldsymbol{F};\bar{e})$. This allows me to then show that $\breve{\boldsymbol{F}}$ is procyclical and derive the results in comparison to the *laissez-faire* equilibrium. Lastly show that this implies that the capital ratio under the optimal capital regulation is countercyclical for all $\bar{e}$.

Characterisation of $v_P^G(\boldsymbol{F};\bar{e})$

In this part of the proof I characterise the function $v_P^G(\boldsymbol{F};\bar{e})$ given by Eq. A.35. The solution to this maximisation programme must entail the constraints $e \leq \bar{e}$ and (IR_{Pool}) binding. (i) $e = \bar{e}$ following the reasoning for Claim 4. If $e < \bar{e}$, there exists an alternative financing plan that involves more internal capital investment and less equity issuance that produces a higher payoff to the Good bank while satisfying all the other constraints. (ii) Similarly, if the (IR_{Pool}) is slack, there exists an alternative financing plan that involves less equity issuance that produces a higher payoff to the Good bank while satisfying all the constraints.

(iii) Given $\boldsymbol{F}$, equity should be issued primarily in the state in which the equity value is higher. This is shown in the following. I differentiate α^H in Eq. A.5 with regard to α^L to find the derivative

$$f^i_{\alpha^H,\alpha^L}(e,\boldsymbol{F},\boldsymbol{\alpha}) \equiv \frac{\partial \alpha^H}{\partial \alpha^L} = -\frac{E^{i,L}(F^L)}{E^{i,H}(F^H)}\frac{1-\beta}{\beta} \tag{A.38}$$

This is derivative is negative, suggesting that increasing the equity issuance in the Low state by a marginal unit while decreasing leverage in the High state by $\frac{E^{i,L}(F^L)}{E^{i,H}(F^H)}\frac{1-\beta}{\beta}$ leaves a type i bank with the same payoff.

Notice that $|f^B_{\alpha^H,\alpha^L}(\cdot)| > |f^G_{\alpha^H,\alpha^L}(\cdot)|$ iff $E^{i,H}(F^H) < E^{i,L}(F^L)$. That is, a Bad bank requires to equity issuance in the highly levered state by more in order to compensate for an increase in equity issuance in the less levered state than a Good bank.

If the investors believe that a financing plan can only come from a Good bank, they should accept the financing plan if they can at least break even. I implicitly differentiate α^H in the binding (IR_{Pool}) with regard to α^L to find the derivative

$$f^{IR,Pool}_{\alpha^H,\alpha^L}(e,\boldsymbol{F},\boldsymbol{\alpha}) \equiv \frac{\partial \alpha^H}{\partial \alpha^L} = -\frac{\gamma E^{G,L}(F^L) + (1-\gamma)E^{B,L}(F^L)}{\gamma E^{G,H}(F^H) + (1-\gamma)E^{B,H}(F^H)}\frac{1-\beta}{\beta} \tag{A.39}$$

That is, increasing equity issuance in the Low state by a marginal unit while decreasing equity issuance in the High state by $\frac{\gamma E^{G,L}(F^L)+(1-\gamma)E^{B,L}(F^L)}{\gamma E^{G,H}(F^H)=(1-\gamma)E^{B,H}(F^H)}\frac{1-\beta}{\beta}$ leaves the investors with the same amount of payoff, holding all other financing parameters unchanged. In particular, $|f^{IR,Pool}_{\alpha^H,\alpha^L}(\cdot)| \leq |f^B_{\alpha^H,\alpha^L}(\cdot)|$, iff $E^{i,H}(F^H) < E^{i,L}(F^L)$. That is, an increase in equity issuance in the less levered state and a decrease in equity issuance in the highly livered state such that the Bad bank enjoys the same payoff, leaves the investors strictly better off.

I can now establish that an maximiser of the programme given by Eq. A.35 should entail equity issuance primarily in the state in which the is higher. Otherwise, there exists an alternative financing plan with high equity issuance in the state in which the equity value is higher, and lower equity issuance in the state in which the equity value is lower. Such a financing plan provides the Bad bank with lower payoff while leaving the Good bank and the investors strictly better off.

This discussion allows us to examine the marginal impact of a change in $\boldsymbol{F}$ on $v_P^G(\boldsymbol{F})$. Specially, given that (IR_{Pool}) binds and that the equity issuance is a corner solution, consider the marginal effect of an increase in F_C^s on the equilibrium equity issuance in the state z in which the solution is interior, where z can be either H or L. This is obtained by implicitly differentiating (IR_{Pool}).

$$\frac{\partial \alpha^z}{\partial F^s} = \frac{\frac{F}{\Delta_X} + (1-\alpha^s)\left[\gamma q^{G,s}(F^s) + (1-\gamma)q^{B,s}(F^s)\right]}{\gamma E^{G,z}(F^s) + (1-\gamma)E^{B,z}(F^z)} \frac{p^s}{p^z} \tag{A.40}$$

where p^s is the probability of state s realising, $s \in \{H, L\}$. Substituting this into the total differentiation equation of the payoff of the Good bank yields

$$\begin{aligned} \frac{\partial v_P^G(\boldsymbol{F})}{\partial F^s} &= p^s \frac{F}{\Delta_X} \frac{E^{G,z}(F^z)}{\gamma E^{G,z}(F^z) + (1-\gamma)E^{B,z}(F^z)} \\ &+ p^s(1-\alpha^s)\left(\frac{E^{G,z}(F^z)\left[\gamma q^{G,s}(F^s) + (1-\gamma)q^{B,s}(F^s)\right]}{\gamma E^{G,z}(F^s) + (1-\gamma)E^{B,z}(F^z)} - q^{G,s}(F^s)\right) \end{aligned} \tag{A.41}$$

Characterisation of $v^G(\boldsymbol{F}; \bar{e})$

In this part I characterise the function $v^G(\boldsymbol{F}; \bar{e})$ given by Eq. A.34 by examining the constraints in the maximisation programme. (i) The constraint $e \leq \bar{e}$ binds given the solution. This follows similar reasoning for Claim 4. (ii) (IR_C) is slack, implied by Claim 5, for $\bar{e} < \breve{e}^s(\bar{F})$. (iii) Given that the (IR_C) is slack, (PC_C^B) binds. This is because otherwise there exists an alternative financing plan with lower equity issuance that still satisfies all constraints but leaves the Good bank a strictly higher payoff. (iv) Given $\boldsymbol{F}$, equity is issued primarily in the state in which the equity value is higher. This follows the same reasoning as given in Part (iii) of the previous section when characterising $v_P^G(\boldsymbol{F}; \bar{e})$.

Characterisation of $\breve{\boldsymbol{F}}$

I now turn to characterise $\breve{\boldsymbol{F}}$, the least-cost capital regulation that implements a separating equilibrium.

(i) The constraint that such a separating equilibrium is undefeated by a pooling equilibrium binds given the $\breve{\boldsymbol{F}}$, i.e. $v^G(\breve{\boldsymbol{F}}; \bar{e}) = v_P^G(\breve{\boldsymbol{F}}; \bar{e})$. I show this by contradiction. Suppose $(e, \boldsymbol{F}, \boldsymbol{\alpha})$ is a solution to the maximisation programme, such that $v^G(\boldsymbol{F}; \bar{e}) >$

$v_P^G(\boldsymbol{F};\bar{e})$. Consider a financing plan $(e,\boldsymbol{F}',\boldsymbol{\alpha}')$ such that $F^{s'} < F^s$, $\alpha^{s'} > \alpha^s$ and all other financing parameters are the equal. Because of the property of Eq. A.6 discussed in Part 1(ii) of the proof of Claim 1, there exists a financing plan $(e,\boldsymbol{F}',\boldsymbol{\alpha}')$ such that it provides the Bad bank with the same payoff in the separating equilibrium as the equilibrium financing plan, but reduces the Good bank's payoff relative to the equilibrium financing plan. This also relaxes the (IR_C) since it increases the value of the Good bank in equilibrium. This alternative financing plan produces a higher social value as it entails lower leverage. This contradicts with the supposition that the original financing plan $(e,\boldsymbol{F},\boldsymbol{\alpha})$ is a solution to the maximisation programme, as the constraint $v^G(\boldsymbol{F};\bar{e}) \geq v_P^G(\boldsymbol{F};\bar{e})$ can still be satisfied for a sufficiently small deviation in F^s.

I now examine the procyclical property of $\breve{\boldsymbol{F}}$ in the following. I show in the following that $\breve{\boldsymbol{F}}$ is procyclical and that the resulting default probability is procyclical.

(ii) Consider $F^H < F^L$. This implies that $q^{i,H}(F^H) > q^{i,L}(F^L)$. I eliminate this case by constructing a socially preferred alternative. Consider another set of leverage caps $\boldsymbol{F}'$ such that $F^{H'} > F^H$ and $F^{L'} < F^L$. Notice that in this case equity is only issued in the high state in both the least-cost pooling and separating equilibria. By the reasoning in Appendix A.2.3, there exists $\boldsymbol{F}'$ such that the resulting least-cost separating equilibrium produces a higher payoff for the Good bank than $\boldsymbol{F}$, which implies that the social value is also higher. Such an alternative is therefore socially preferred, if $v^G(\boldsymbol{F}';\bar{e}) \geq v_P^G(\boldsymbol{F}';\bar{e})$ is still satisfied.

Indeed the constraint is satisfied. The payoffs to the Good bank in the least-cost pooling and separating equilibria can be expressed as

$$\begin{aligned} v_P^G(\boldsymbol{F};\bar{e}) &= \underbrace{\frac{E^{G,H}(F^H)}{\gamma E^{G,H}(F^H)+(1-\gamma)E^{G,H}(F^H)}}_{B}\left(\mathbb{E}\left[\gamma V^{G,s}(F^s)+(1-\gamma)V^{B,s}(F^s)\right]-1+\bar{e}\right) \\ &\quad -(1-\beta)\Big(E^{G,L}(F^L) \\ &\quad -\left[\gamma E^{G,L}(F^L)+(1-\gamma)E^{B,L}(F^L)\right]\underbrace{\frac{E^{G,H}(F^H)}{\gamma E^{G,H}(F^H)+(1-\gamma)E^{B,H}(F^H)}}_{B}\Bigg) \end{aligned} \tag{A.42}$$

$$v^G(\boldsymbol{F};\bar{e}) = \underbrace{\frac{E^{G,H}(F^H)}{E^{B,H}(F^H)}}_{C}\bar{e}-(1-\beta)\left[E^{G,L}(F^L)-E^{B,L}(F^L)\underbrace{\frac{E^{G,H}(F^H)}{E^{B,H}(F^H)}}_{C}\right] \tag{A.43}$$

where $C > B$, and

$$\frac{\partial B}{\partial F^H} = (1-\gamma)\frac{q^{G,H}(F^H)q^{B,H}(F^H)\left[q^{G,H}(F^H)-q^{B,H}(F^H)\right]}{[\gamma E^{G,H}(F^H)+(1-\gamma)E^{B,H}(F^H)]^2}\Delta_X > 0 \quad \text{(A.44)}$$

$$\frac{\partial C}{\partial F^H} = (1-\gamma)\frac{q^{G,H}(F^H)q^{B,H}(F^H)\left[q^{G,H}(F^H)-q^{B,H}(F^H)\right]}{[E^{B,H}(F^G)]^s}\Delta_X > \frac{\partial B}{\partial F^H} \quad \text{(A.45)}$$

This suggests, for β sufficiently large, an increase in F^H and a decrease in F^L such that it leaves the social value unchanged would increases $v^G(\cdot)$ more than $v_P^G(\cdot)$. Therefore there exists $\boldsymbol{F}'$ such that the social value is improved while satisfying $v^G(\cdot) \geq v_P^G(\cdot)$.

(iii) The case in which $F^H > F^L$ such that $q^{i,H}(F^H) < q^{i,L}(F^L)$ can also be eliminated as it does not arise in equilibrium. Notice that, in this case, equity is only issued primary in the Low state. Similar reasoning as above rules out this case as the optimal $\breve{\boldsymbol{F}}$.

To conclude, $\breve{\boldsymbol{F}}$ is such that $\breve{F}^H \geq \breve{F}^L$ and $q^{i,H}(\breve{F}^H) \geq q^{i,L}(\breve{F}^L)$.

I now explicitly derive the optimal capital regulation that implements a separating equilibrium $\breve{\boldsymbol{F}}$. This part of the derivation is similar to those in of Proposition 6 given in Appendix A.2.5. There are two scenarios.

Firstly, suppose an interior solution such that $E^{i,H}(\breve{F}^H) = E^{i,L}(\breve{F}^L), i.e. \breve{F}^H = \breve{F}_L + \frac{\theta_H - \theta_L}{\Delta_X}$. This also implies that $q^{i,H}(\breve{F}^H) = q^{i,L}(\breve{F}^L)$. Given this restriction, denote the equity value and the success probability with $E^i(\breve{\boldsymbol{F}})$ and $q^i(\breve{\boldsymbol{F}})$ respectively, which are both equal in both states. The solution is characterised by imposing $v_P^G(\cdot) = v^G(\cdot)$ as given by Eq. A.42–A.43, given the result in Part (i) of this section. The solution is given by

$$\bar{e} = \frac{E^B(\breve{\boldsymbol{F}})}{\gamma E^G(\breve{\boldsymbol{F}}) + (1-\gamma)E^B(\breve{\boldsymbol{F}})}\left(\mathbb{E}\left[\gamma V^{G,s}(\breve{F}^s) + (1-\gamma)V^{B,s}(\breve{F}^s)\right] - 1\right) \quad \text{(A.46)}$$

The condition for the first scenario to arise is that in equilibrium, $E^i(\breve{\boldsymbol{F}}) \leq V_{FB}^{i,L}$. This is the case if

$$\begin{aligned}\bar{e} \leq \breve{e}_{LP}^T \equiv & \frac{E_{FB}^{B,L}}{\gamma E_{FB}^{G,L} + (1-\gamma)E_{FB}^{B,L}}\left(\beta\left[\gamma V^{G,H}(\frac{\theta^H-\theta^L}{\Delta_X}) + (1-\gamma)V^{B,H}(\frac{\theta^H-\theta^L}{\Delta_X})\right]\right. \\ & \left. +(1-\beta)\left[\gamma V_{FB}^{G,L} + (1-\gamma)V_{FB}^{B,L}\right] - 1\right) \quad \text{(A.47)}\end{aligned}$$

For $\bar{e} \in [\breve{e}_{LP}^T, \breve{e}_P^C]$, the second scenario arises, in which case $\breve{F}^H < \frac{\theta^H - \theta_L}{\Delta_X}$ and $\breve{F}^L = 0$. The solution in this case is

$$\begin{aligned}\bar{e} = & \frac{E^{B,H}(\breve{\boldsymbol{F}})}{\gamma E^{G,H}\breve{F}^H) + (1-\gamma)E^{B,H}(\breve{F}^H)}\left(\beta\left[\gamma V^{G,H}(\breve{F}^H) + (1-\gamma)V^{B,H}(\breve{F}^H)\right]\right. \\ & \left. +(1-\beta)\left[\gamma V_{FB}^{G,L} + (1-\gamma)V_{FB}^{B,L}\right] - 1\right) \\ & +(1-\beta)\left(\frac{\gamma E_{FB}^{G,L} + (1-\gamma)E_{FB}^{B,L}}{\gamma E^{G,H}(\breve{F}^H) + (1-\gamma)E^{B,H}(\breve{F}^H)} - \frac{E_{FB}^{B,L}}{E^{B,H}(\breve{F}^H}\right)E^{B,H}(\breve{F}^H) \quad \text{(A.48)}\end{aligned}$$

Countercyclical capital ratio

It is now straightforward to show that the capital ratio under the optimal capital regulation is countercyclical. For all cases in which $\bar{F}^L = \bar{F}^L = 0$, the capital ratio is always 100%. For $\bar{e} \in [\breve{e}^T_{LP}, \breve{e}^C_P]$, the capital ratio is 100% in the Low state and less than 100% in the High state. For $\bar{e} \in [\breve{e}^C_P, \breve{e}^T_{LP}]$, the capital ratio is $c^H < c^L$, because the bank's equity value is equalised in both state, but the bank has more debt in the high state.

A.2.8 Proof of Proposition 9

Given the *ex post* bailout guarantee, the bank receives an implicit subsidy when it issues debt to financing its loan portfolio. Therefore the only difference between this extension and the baseline model is that the (IR_C) is replaced with the following

$$(IR^{BO}_C) \quad : \quad \mathbb{E}\left[V^{G,s}(F^s_C) + \left(1 - q^{G,s}(F^s_C)\right) F^s - (1-\alpha^s_C)E^{G,s}(F^s_C)\right] \geq 1 - e \tag{A.49}$$

All results the baseline model therefore hold qualitatively. I show below that the optimal regulation in this case must permit higher leverage in order to implement a separating equilibrium.

Consider imposing $\breve{\boldsymbol{F}}$, the optimal regulation in the baseline model, in this economy with bailouts. Since (PC^B_C) binds in the regulated separating equilibrium, $v^G(\cdot)$ is equal to that in the baseline model. However, $v^G_P(\cdot)$ is higher than before, because the (IR_{Pool}) is now relaxed, which enables the Good bank to issue less equity in the pooling equilibrium. This violates the constraint $v^G(\cdot) \geq v^G_P(\cdot)$. Therefore leverage must be reduced in order to implement a separating equilibrium.

In the interior solution region, the optimal capital regulation is given by

$$\begin{aligned}\bar{e} \;=\; & \frac{E^B(\breve{\boldsymbol{F}}_{BO})}{\gamma E^G(\breve{\boldsymbol{F}}_{BO}) + (1-\gamma)E^B(\breve{\boldsymbol{F}}_{BO})}\Big(\mathbb{E}\left[\gamma V^{G,s}(\breve{F}^s_{BO}) + (1-\gamma)V^{B,s}(\breve{F}^s_{BO})\right] - 1 \\ & + \left[1 - \gamma q^G(\breve{\boldsymbol{F}}_{BO}) - (1-\gamma)q^B(\breve{\boldsymbol{F}}_{BO})\right]\mathbb{E}[\breve{F}^s_{BO}]\Big)\end{aligned} \tag{A.50}$$

For a given $\bar{e}$, the RHS of Eq. A.50 is strictly higher than the RHS of Eq. A.46 for a given $\breve{\boldsymbol{F}}$ such that $\breve{F}^H = \breve{F}^L + \frac{\theta^H - \theta^L}{\Delta_X}$, the solution to Eq. A.50 is strictly higher than the solution to Eq. A.46. That is, higher leverage must be permitted than in the baseline model to implement a separating equilibrium, when there is the expectation of bailouts.

A.2.9 Proof of Proposition 10

Denote $b \equiv q^{i,s}(F^s) + \left[1 - q^{i,s}(F^s)\right] \frac{\bar{D}}{Dc_2} \leq 1$ the probability of receiving the repayment c_2 at $t = 2$ given the deposit insurance coverage $\bar{D}$. A depositor breaks even at a deposit contract (c_1, c_s) if

$$\lambda u(c_1) + (1 - \lambda) b u(c_2) = u(1) \tag{A.51}$$

This condition is satisfied for a risk-free deposit contract $c_1 = c_2 = 1$ under full deposit insurance $\bar{D} = D$.

I implicitly differentiate Eq. A.51 to find that $\frac{\partial b}{\partial c_2} = -\frac{bu'(c_2)}{u(c_2)}$. That is, if increase the $t = 2$ repayment by a marginal unit, the probability of repayment b must decrease by at most $\frac{bu'(c_2)}{u(c_2)} \leq u'(1)$ to allow the depositor to break even.

Holding other liabilities of the bank constant, a marginal increase in the promised repayment c_2 reduces the success probability of the bank by $\frac{\partial q^{i,s}(F)}{\partial c_2} = -\frac{D}{2\Delta_X}$. Holding the deposit insurance level $\bar{D}$ constant, the probability of repayment to a depositor b decrease by

$$\frac{\partial}{\partial c_2}\left(q^{i,s}(F^s) + \left[1 - q^{i,s}(F^s)\right] \frac{\bar{D}}{Dc_2}\right) \tag{A.52}$$

$$= \quad \frac{D}{2\Delta_X} + \frac{\bar{D}}{Dc_2^s}[1 - q^{i,s}(F^s - Dc_2)] \geq \frac{D}{2\Delta_X} \tag{A.53}$$

If $u'(1) \leq \frac{(1-\bar{e}-k)}{2\Delta_X}$, a depositor will not be willing to deposit with the bank unless he receives a certain repayment $c_1 = c_2 = 1$, given that the bank has to raise at least $\frac{D}{1-\lambda} = \frac{1-\bar{e}-k}{1-\lambda}$ from the depositors. This is because Eq. A.51 is not satisfied for any $c_2 > 1$ for a given level of D and $\bar{D} = D$. Therefore deposits must be raised with a certain promised repayment $c_1 = c_2 = 1$, backed by a full deposit insurance coverage $\bar{D} = D$.

APPENDIX B

CHAPTER 2: COUNTERCYCLICAL FORECLOSURE FOR SECURITISATION

B.1 Proofs

B.1.1 Proof of Proposition 13

The optimal security for the low type is to issue all of the equity to outside investors in order to minimise the retention cost. The optimal security for the high type, however, has to satisfy an additional (IC) to prevent the bad type from mimicking. The remainder of the proof characterises the optimal security issued by the high type.

Central to the characterisation is the monotonicity of the security payoffs. The set of permitted securities (F_H, f_H, F_L, f_L) therefore depends on the ranking of the cash flows from each type of securitisers. Notice also that the functional form of the liquidation proceeds $\mathcal{L}_i(\lambda_i)$ implies that, for λ_i, λ_j such that $\mathcal{L}_i(\lambda_i) \geq \mathcal{L}_j(\lambda_j)$, $\mathcal{L}_i(\lambda_i) + (1-\lambda_i)X \leq \mathcal{L}_j(\lambda_j) + (1-\lambda_j)X$. Therefore two cases discussed below are relevant, as analysed below.

Case 1: $\lambda_H \leq \lambda_L$

This is the most important case as this will be the case in equilibrium. The solution to this case is a debt contract as presented in Proposition 13 and derived below.

In this case, the liquidation proceeds are less for the high type securitiser than for the low type securitiser. But if the delinquent mortgage recover, the total cash flows are higher for the high type securitiser.

$$\begin{aligned} V + \mathcal{L}_H(\lambda_H) + (1 - \lambda_H)X &\geq V + \mathcal{L}_L(\lambda_L) + (1 - \lambda_L)X \\ &\geq V + \mathcal{L}_L(\lambda_L) \geq V + \mathcal{L}_H(\lambda_H) \end{aligned} \tag{B.1}$$

The two monotonicity constraints for the insiders and the outsiders, combined with the limited liability constraints (LL), are

$$F_H \geq F_L \geq f_L \geq f_H \geq 0 \tag{B.2}$$

$$\begin{aligned} V + \mathcal{L}_H(\lambda_H) + (1 - \lambda_H)X - F_H &\geq V + \mathcal{L}_L(\lambda_L) + (1 - \lambda_L)X - F_L \\ &\geq V + \mathcal{L}_L(\lambda_L) - f_L \geq V + \mathcal{L}_H(\lambda_H) - f_H \\ &\geq 0 \end{aligned} \tag{B.3}$$

Firstly, examining the incentive compatibility constraint (IC) Eq. 2.22. The constraint is relaxed by increasing F_L and f_L. That is, decreasing the payoff to the Low type securitiser if she mimics the High type and issues this security. However, this increase will bind either the insider's monotonicity constraint or the outsider's. Depending on which constraints bind, many cases can arise, as discussed below.

(i) Suppose f_L binds the insider's monotonicity constraint, i.e.

$$V + \mathcal{L}_L(\lambda_L) - f_L = V + \mathcal{L}_L(\lambda_L) - f_H \tag{B.4}$$

This implies that $f_L > f_H$. Considering F_L, there are again two scenarios. (a) Suppose F_L binds the insider's monotonicity constraint, i.e.

$$V + \mathcal{L}_L(\lambda_L) + (1 - \lambda_L)X - F_L = V + \mathcal{L}_L(\lambda_L) - f_L \tag{B.5}$$

This implies that $F_L > f_L$ and $F_H \geq V + \mathcal{L}_L(\lambda_L)$. Substituting Eq. B.4 and B.5 into the (IC) yields

$$U_L(\lambda_L) \geq \theta_H F_H + (1 - \theta_H) f_H + \delta[V + \mathcal{L}_H(\lambda_H) - f_H] \tag{B.6}$$

It is now clear that the (IC) must bind in order to maximise $\theta_H F_H + (1-\theta_H) f_H$. Substituting the (IC) into the objective function yields

$$\max_{f_H} \quad [A_1 - (1-\theta_H) f_H + \delta f_H] + (1-\theta_H) f_H \tag{B.7}$$

where $A_1 \equiv U_L(\lambda_L) - \delta[V + \mathcal{L}_H(\lambda_H)]$. The solution is therefore to increase f_H until the (LL) binds. The solution in this scenario is given by

$$\begin{aligned} f_H &= V + \mathcal{L}_H(\lambda_H) && \text{(B.8)} \\ f_L &= V + \mathcal{L}_L(\lambda_L) && \text{(B.9)} \\ F_L &= V + \mathcal{L}_L(\lambda_L) + (1-\theta_L) X && \text{(B.10)} \\ F_H &= V + \frac{\mathcal{L}_L(\lambda_L) + (1-\lambda_L)\theta_L X - (1-\theta_H)\,\mathcal{L}_H(\lambda_H)}{\theta_H} && \text{(B.11)} \end{aligned}$$

This is a solution if F_H is indeed such that $F_H \geq V + \mathcal{L}_L(\lambda_L)$, which is equivalent to

$$\frac{1-\theta_H}{\theta_H - \theta_L}[\mathcal{L}_L(\lambda_L) - \mathcal{L}_H(\lambda_H)] \geq (1-\lambda_L) X \tag{B.12}$$

I now turn to scenario (b) in which I suppose F_L binds the onsider's monotonicity constraint, i.e.

$$F_L = F_H \tag{B.13}$$

Which implies that $F_H \leq V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L) X$. Following similar reasoning as in the previous scenario, we can substitute Eq. B.4, B.13 and the (IC) into the objective function, which again suggests that the solution involves maximising f_H until the (LL) binds. The solution in this scenario is given by

$$\begin{aligned} f_H &= V + \mathcal{L}_H(\lambda_H) && \text{(B.14)} \\ f_L &= V + \mathcal{L}_L(\lambda_L) && \text{(B.15)} \\ F_L &= F_H = V + \frac{(1-\delta\theta_L)\,\mathcal{L}_L(\lambda_L) + (1-\delta)(1-\lambda_L)\theta_L X - (1-\theta_H)\,\mathcal{L}_H(\lambda_H)}{\theta_H - \delta\theta_L} && \text{(B.16)} \end{aligned}$$

This is a solution if F_L satisfy the (LL), which is equivalent to the violation of the condition given in Eq. B.12.

(ii) Suppose now that f_L binds the outsider's monotonicity constraint, i.e. $f_L = F_L$. In this case, the only possibility for this to be the case is if $F_L \leq V + \mathcal{L}_L(\lambda_L)$. Under this constraint, we can increase F_L to bind the outsider's monotonicity constraint $F_L = F_H$ because the previous constraint implies that the insider's monotonicity constraint for F_L

is not binding. Substituting $f_L = F_L = F_H$ and the (IC) into the objective function implies that the objective function maximised when f_H is minimised to bind the (CC) at f_H. This implies that

$$\begin{aligned} f_H &= 0 & \text{(B.17)} \\ f_L &= F_L = F_H = \frac{1-\delta}{\theta_H - \delta}[V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L)\theta_L X] & \text{(B.18)} \end{aligned}$$

However, this is not a solution as this result does not satisfy the (LL) for F_L and f_L.

Therefore for cash flows ranked according to Case 1, the optimal security is a debt contract because the security payoff is always equal to the cash flow apart from the highest payoff, which corresponds to the face value of a debt contract. The optimal face value of the debt is summarised in Proposition 13 corresponding to the two scenarios in part (i) of Case 1.

Case 2: $\lambda_H > \lambda_L$

We show that, in this case, the optimal security issued by a high type issuer is not debt. However, we show that given the optimal security, the securitiser does no choose her optimal foreclosure policy such that $\lambda_H > \lambda_L$. Therefore this case does not arise in equilibrium.

In this case, the cash flows generated by the High securitiser dominates the cash flows generated by the Low securitiser.

$$\begin{aligned} V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L)X &\geq V + \mathcal{L}_H(\lambda_H) + (1-\lambda_H)X \\ &\geq V + \mathcal{L}_H(\lambda_H) \geq V + \mathcal{L}_L(\lambda_L) & \text{(B.19)} \end{aligned}$$

The two monotonicity constraints for the insiders and the outsiders, combined with the limited liability constraints (LL), are

$$F_L \geq F_L \geq f_h \geq f_L \geq 0 \tag{B.20}$$

$$\begin{aligned} & V + \mathcal{L}_L(\lambda_L) + (1-\lambda_L)X - F_L \geq V + \mathcal{L}_H(\lambda_H) + (1-\lambda_H)X - F_H \\ \geq \quad & V + \mathcal{L}_H(\lambda_H) - f_H \geq V + \mathcal{L}_L(\lambda_L) - f_l \geq 0 & \text{(B.21)} \end{aligned}$$

Following a similar procedure, it can be shown that in the solution is as follow. If xxxxxx

B.1.2 The optimal securities under competition

Following similar intuition, the low type securitiser issues equity. We now turn to consider the optimal security for the high type when the conditions Eq. 2.72–2.73 for the ranking of the cash flows in each state are satisfied.

Notice first that, in this case, the good cash flows produced by a securitiser in all states are higher than the bad cash flows in all states, i.e.

$$\mathcal{L}^n_{i,s}(\lambda^n_{i,s}, \Lambda^{-n}_s) + (1-\lambda^n_{i,s})X > \mathcal{L}^n_{i,z}(\lambda^n_{i,z}, \Lambda^{-n}_z) \qquad \forall\, s, z \in S \tag{B.22}$$

This is because the good cash flow is the highest if there is no foreclosure, at $\frac{1}{N}X$, while the bad cash flow is the highest if there is a foreclosure rate of 1, at X. This reflects the intuition that foreclosure reduces the risk in the mortgage pool.

The remainder of the argument proceeds as follows. We first conjecture that the optimal contract a debt contract with face value F such that

$$\max_s\{\mathcal{L}^n_{L,s}(\lambda^n_{L,s}, \Lambda^{-n}_s) \leq F \leq \min_s\{\mathcal{L}^n_{L,s}(\lambda^n_{L,s}, \Lambda^{-n}_s) + (1-\lambda^n_{L,s})X\}] \tag{B.23}$$

The face value of this debt is relatively low so that the bad securitiser does not have to default should she mimic the good type securitiser and issues this security, but sufficiently high so as to leave zero cash flow to the bad securitisers when their cash flows do not recover. This enables direct comparison of this extension with competition with the baseline case. We then discuss conditions for this security to be the optimal security.

Within the class of debt contracts with face value satisfying the above condition, the optimal face value of the debt $\hat{F}(\boldsymbol{\lambda}^n_H, \boldsymbol{\lambda}^n_L)$ is given by the binding (IC),

$$\begin{aligned}\hat{F}(\boldsymbol{\lambda}^n_H, \boldsymbol{\lambda}^n_L) \quad = \quad & V + \frac{1}{\theta_H - \delta\theta_L}\,\mathbb{E}[(1-\delta\theta_L)\,\mathcal{L}^n_{L,s}(\lambda^n_{L,s}, \lambda^{-n}_s) \\ & -(1-\theta_H)\,\mathcal{L}^n_{H,s}(\lambda^n_{H,s}, \lambda^{-n}_s) + (1-\delta)(1-\lambda^n_{L,s})\theta_L X]\end{aligned} \tag{B.24}$$

If the face value characterised above indeed satisfies the condition, it is the optimal contract. This can be shown by considering possible deviations. In this case, $f_{H,s}$ and $f_{L,s}$ bind the (LL) for all s. The only possibly deviations are to increase or decrease $F_{i,s}$ for some s. I show in the following that there do not exist any deviations that can increase the good type issuer's payoff while satisfying the monotonicity constraints, the (IC) and the (LL).

(i) Suppose we decrease $F_{L,s}$ by for some s. This violates the (IC), and therefore must be accompanied by a decrease in $F_{H,z}$ for some z, if possible without violating any constraints. This, however, is not optimal as it reduces the payoff to the good type issuer.

(ii) Suppose we increase $F_{L,s}$ by ϵ for some s. This violates the monotonicity constraint on $F_{H,s}$, where also need to increase by at least ϵ. This, however, not violates the (IC) as the payoff to the low type issuer increases by $(\theta_H - \delta\theta_L)\epsilon$. If more monotonicity constraints are violated, we must also increase either $F_{H,z}$ for some z, or both $F_{L,z}$ and $F_{H,z}$ for some z. Similar intuition holds.

B.1.3 Proof of Proposition 17

This proposition can be proved by examining the relevant first order conditions of the maximisation problem given by Eq. 2.74–2.76.

Consider a pair of states s and s' in which all other securitisers are of the same type, except for securitiser n who is of the high type in state s and the low type in state s'. It follows that the probability of state s conditional on the securitiser n being a high type is equal to the probability of state s' conditional on the securitiser n being a low type. Denote $p^n_{s,s'}$ this conditional probability.

The first order conditions for $\lambda^n_{H,s}(\Lambda^{-n}_s)$ and $\lambda^n_{L,s'}(\Lambda^{-n}_{s'})$, taking the foreclosure policies of other securitisers as given, can be expressed as

$$(FOC_{H,s}) \quad : \quad \gamma\left[(1-\delta)\theta_H \frac{\partial \hat{F}(\cdot)}{\partial \lambda^n_{H,s}} + (1-\delta)(1-\theta_H)\frac{\partial\, \mathcal{L}^n_{H,s}(\cdot)}{\partial \lambda^n_{H,s}} p^n_{s,s'} + \delta\left(\frac{\partial\, \mathcal{L}^n_{H,s}(\cdot)}{\partial \lambda^n_{H,s}} - \lambda^n_{H,s}\theta_H X\right) p^n_{s,s'}\right] = 0 \tag{B.25}$$

$$(FOC_{L,s'}) \quad : \quad \left[\gamma(1-\delta)\theta_H \frac{\partial \hat{F}(\cdot)}{\partial \lambda^n_{L,s'}} + (1-\gamma)\left(\frac{\partial\, \mathcal{L}^n_{L,s'}(\cdot)}{\partial \lambda^n_{L,s'}} - \lambda^n_{L,s'}\theta_L X\right) p^n_{s,s'}\right] = 0 \tag{B.26}$$

where

$$\frac{\partial \hat{F}(\cdot)}{\partial \lambda^n_{H,s}} = -\frac{1-\theta_H}{\theta_H - \delta\theta_L}\frac{\partial\, \mathcal{L}^n_{H,s}(\cdot)}{\partial \lambda^n_{H,s}} p^n_{s,s'} \tag{B.27}$$

$$\frac{\partial \hat{F}(\cdot)}{\partial \lambda^n_{L,s'}} = \left[\frac{\delta(1-\theta_L)}{\theta_H - \delta\theta_L}\frac{\partial\, \mathcal{L}^n_{L,s'}(\cdot)}{\partial \lambda^n_{L,s'}} + \frac{1-\delta}{\theta - \delta\theta_L}\left(\frac{\partial\, \mathcal{L}^n_{L,s'}(\cdot)}{\partial \lambda^n_{L,s'}} - \theta_L X\right)\right] p^n_{s,s'} \tag{B.28}$$

(i) These conditions are very similar to those in the baseline case. Therefore similar reasoning shows that for any foreclosure followed by other securitisers Λ^{-n}, the foreclosure rate of the low type is higher than that of the high type. Therefore this is also true in equilibrium, i.e.

$$\hat{\lambda}^n_{H,s} < \hat{\lambda}^n_{L,s'} \tag{B.29}$$

(ii) In order to prove this part of the proposition, we first need to establish that $\hat{\Lambda}_s^{-n} < \hat{\Lambda}_z^{-n}$ and $\hat{\Lambda}_{s'}^{-n} > \hat{\Lambda}_{z'}^{-n}$. The result then follows.

This part of the proposition concerns equilibrium results. We here impose that the equilibrium is symmetric. The equilibrium foreclosure decision is therefore given by the following fixed point problems.

$$\begin{cases} \hat{\lambda}_{H,s}^n & = \lambda_{H,s}^n(\hat{\Lambda}_s^{-n}), \\ \hat{\lambda}_{L,s'}^n & = \lambda_{L,s'}^n(\hat{\Lambda}_{s'}^{-n}), \end{cases} \tag{B.30}$$

where $\hat{\Lambda}_s^{-n} = \hat{\Lambda}_{s'}^{-n} = \frac{N-N_L^s-1}{N}\hat{\lambda}_{H,s}^n + \frac{N_L^s}{N}\hat{\lambda}_{L,s'}^n$

Notice that the functions $\lambda_{H,s}^n(\cdot)$ and $\lambda_{L,s'}^n(\cdot)$ are decreasing in the argument. This can be easily checked by implicitly differentiating the $(FOC_{H,s})$ and $(FOC_{L,s'})$. The intuition is, an increase the aggregate amount of foreclosure by the other securitisers increases reduces the incentive for the securitiser n to foreclose because of the lower price in the market.

Consider another pair of states z, z' analogous to s, s' such that $N_L^s < N_L^z$, and express the solution to the optimal foreclosure policy in state z as the solution to a system similar to the above. We now prove this part of the proposition by contradiction. Suppose that the equilibrium is such that $\hat{\Lambda}_s^{-n} > \hat{\Lambda}_z^{-n}$. That the functions $\lambda_{H,s}^n(\cdot)$ and $\lambda_{L,s'}^n$ implies that $\hat{\lambda}_{H,s}^n < \hat{\lambda}_{H,z}^n$ and $\hat{\lambda}_{L,s'}^n < \hat{\lambda}_{L,z'}^n$. Combined with the fact that there are fewer low type issuers in state s than in z and the result of part (i), this contradicts with the supposition that the aggregate foreclosure is greater in states s and s' than in states z and z'. Similar arguments show that $\hat{\Lambda}_{s'}^{-n} > \hat{\Lambda}_{z'}^{-n}$ also leads to a contradiction.

Part (ii) of the proposition thus follows immediately. Since $\hat{\Lambda}_s^{-n} < \hat{\Lambda}_z^{-n}$ and $\hat{\Lambda}_{s'}^{-n} > \hat{\Lambda}_{z'}^{-n}$, it must be that $\hat{\lambda}_{H,s}^n > \hat{\lambda}_{H,z}^n$ and $\hat{\lambda}_{L,s'}^n > \hat{\lambda}_{L,z'}^n$.

APPENDIX C

CHAPTER 3: BANKRUPTCY-REMOTE SECURITISATION WITH IMPLICIT GUARANTEE

C.1 The first best portfolio sizes

The first best portfolio size X^i_{FB} maximises the NPV of the portfolio. For tractability, I assume that the second order condition is satisfied to guarantee an interior solution. The first order condition that determines the solution is

$$1 - k - \delta + \theta^i(X^i_{FB})\delta + \theta^{i'}(X^i_{FB})\delta X^i_{FB} = 0 \tag{C.1}$$

It then follows from the previous stated assumptions that $\theta^G(\cdot) > \theta^B(\cdot)$ and $\frac{\partial}{\partial X}\left(\frac{\theta^G(X)}{\theta^B(X)}\right) > 0$ that $X^G_{FB} > X^B_{FB}$, because $\frac{\partial}{\partial X}\left(\frac{\theta^G(X)}{\theta^B(X)}\right) > 0$ implies that $\theta^{G'}(\cdot) > \theta^{B'}(\cdot)$.

C.2 Proofs

C.2.1 Proof of Proposition 20

To derive the equilibrium security in the second period, notice firstly that if (w_2^H, w_2^L), $w_2^L > 0$ is a solution to programme given by Eq. 3.4–3.6, then there exists $w_2^{H'} > w_2^H$ such that $(w_2^{H'}, 0)$ is also a solution. This is because an increase in w_2^H accompanied by a decrease in w_2^L such that the constraint Eq. 3.5 is kept constant keeps the objective function constant while relaxes the (IC_2) constraint.

The binding financing constraint and $w_2^L = 0$ then determines w_2^H in equilibrium for a given investment level X_1^G.

Substituting the binding financing constraint Eq. 3.5 and $w_2^L = 0$ into the objective function and (IC_2) yields the following programme which determines the equilibrium portfolio size X_1^G,

$$\max_{X_1^G} \quad V^G(X_1^G) + w_1^L \tag{C.2}$$

$$\frac{\theta^B(X_1^G}{\theta^G(X_1^G)} V^G(X_1^G) - V_{FB}^B - \left[1 - \frac{\theta^B(X_1^G)}{\theta^B(X_1^G)}\right] w_1^L \leq 0 \tag{C.3}$$

If the above equation is satisfied at $X_1^G = X_{FB}^G$, the first best payoff is received by a good bank. Otherwise, notice that the left hand side of the constraint is increasing in X_1^G. Therefore the programme is maximised at the larger solution to the binding constraint, which is given by $\hat{X}_1(w_1^L)$. Moreover, in equilibrium, $\hat{X}_1(w_1^L) > X_{FB}^G$.

Finally, since all constraints bind in the equilibrium with $w_0^L = 0$, there exists no other equilibrium with $w_0^L > 0$.

In a separating equilibrium, a bank with a bad second period portfolio always issues fairly priced security. Its security choice is therefore arbitrary, and it always chooses the first best level of investment.

C.2.2 Proof of Propositions 23 and 22

The proofs of Propositions 23 and 22 are established immediately following similar reasoning to those in Appendix C.2.1.

C.2.3 Proof of Corollaries 4 and 3

To show Corollary 3, compare Eq. 3.8 and **??**, which determine $X_1^{G*}(\cdot)$ and $X_0^{G*}(\cdot)$ respectively. Since the LHS of Eq. **??** is larger than the LHS of Eq.

Eq:BR:Equilibrium:wFB, the solution to the former must be greater than the latter.

Similarly, to show Corollary 4, compare Eq. 3.8 and Eq. **??**, which determine $X_1^{G*}(\cdot)$ and $X_0^{G**}(\cdot)$ respectively. The LHS of the former equation is smaller than that of the latter equation. Hence the result in the corollary.