Essays on credit risk

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Declaration

I certify that the thesis I have presented for examination for the PhD degree of the London School of Economics and Political Science is solely my own work.

I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

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Executive summary

The thesis presents my work on the modelling, explanation and prediction of credit risk through three channels: (binary) default indicator, (ordinal) credit ratings and (continuous) CDS spreads.

Chapter 1

The first chapter aims to investigate the factors useful for the prediction of firm bankruptcy.

The prediction of firm bankruptcy is an important research topic, both in empirical and theoretical research. More importantly, because it reveals the default probability of a firm, this topic has attracted considerable attention from creditors, current and potential investors and policy makers. To discover and model the mechanism of bankruptcy better, it is crucial to find the determinants of the mechanism.

Bankruptcy forecasting can be carried out within either the framework of statistical models or the framework of credit risk models. In the framework of credit risk models, structural models and reduced-form models have been developed. In the framework of statistical models, classification models with accounting information have been explored since 1960's. They are referred to static models. After then, hazard models using yearly or higher frequency data have also been developed. The comparison of empirical estimates obtained by using the hazard models shows that they are more appropriate than the static models for bankruptcy forecasting. A merit of the hazard models is that it does not require an assumption of the joint distribution of the predictor variables. The time dynamics of the predictor variables can further be incorporated into the hazard models to build even more sophisticated models.

In the framework of statistical models for bankruptcy forecasting, we usually need to fit a model to many cross-sectional data. Unfortunately such data often contain missing values. For example, the data of a financially-distressed firm are more likely to have missing values than those of a healthy firm. This leads to a problem of self-selection bias of the data. The problem due to missing values greatly hinders statistical inference of the determinants that we are interested in. Consequently, methods to cope with the missing values and thus correct the self-selection bias play a vital role in forecasting bankruptcy.

The simplest method to tackle missing values is to list-wisely delete the missing values, i.e. to delete all the observations with any missing values. However, this method is not appropriate if the missing values count nontrivial portion of the dataset or play an important role in the analysis, because the important information, which is implicitly conveyed by the pattern of the missing values themselves, is lost. The inference based on this method also suffers from the selection bias due to the drop of observations. Another simple method is to impute the missing values by the closest non-missing values. However, it is still not able to sufficiently recover the information of the missing values, for example, when changes in values at crucial time points are missing. Alternatively, we can use the method of multiple imputation to infer the missing values. In multiple imputation, the uncertainty about the right values to impute can be taken into consideration.

In this context, I construct accounting, market and macro-economic variables as predictors, and investigate the three methods above to tackle the problem of missing values, for the use of the hazard models to forecast bankruptcy.

The contribution of this chapter is that it demonstrates that, compared with the methods of list-wise deleting and closest-value imputation, the method of multiple imputation performs the best and leads to a forecasting model with economically reasonable predictors and estimates. These predictors and estimates reflect firm-specific features of profitability, leverage and stock market information and their impact on the bankruptcy, and thus can be regarded as the determinants for bankruptcy forecasting.

Chapter 2

The purpose of this chapter is to predict the probabilities of credit rating transitions of issuers.

Credit ratings are usually assigned on ordinal scales, expressing the relative likelihood of default from the strongest to the weakest. Credit ratings can be applied to both firms and governments. Meanwhile, the rating transition of an issuer can reflect the change of its default probability. As the rating transition is a signal of a worsening or improving credit quality, an upward move of rating can be viewed as a decrease in the probability of default, while a downward move can be regarded as an increase in the probability of default.

The transition probabilities form a rating-transition matrix. To estimate a rating-transition matrix, one method is to simply adopt the estimates from rating agencies' publications. However, the credit rating agencies have long been under fire for not spotting corporate disasters in time, while rating and rating transitions are expected to capture and respond to a changing economy and business environment. Moreover, the estimates from the agencies' publications are obtained by using a cohort method. The cohort method assumes that the rating-transition process is a discrete-time homogeneous Markov chain. The rating-transition matrix for the next period is estimated by relative frequencies. Although it is easy to carry out and commonly used in the industry, the cohort method suffers from two main weaknesses in its methodology. The first weakness is that it is a discrete-time model and considers ratings only at the two endpoints of the estimation interval, causing it to ignore any transition within the estimation interval. The second weakness is that there are non-Markov behaviours evidently observed in the patterns of rating transitions. Hence some researchers utilise a continuous-time probabilistic method to model the rating transitions. However, both the continuous-time probabilistic method and the cohort method only consider the transition history of the ratings. They do not explicitly exploit other available information, such as the firms' accounting information. Because of this, they cannot capture the factors that may significantly impact rating transitions and thus cannot model how these factors impact rating transitions.

In this context and also because rating is an ordinal categorical variable, a natural choice for modelling ratings transitions is to use a generalised linear regression model, for example the proportional odds logistic regression (POLR) model. However, in the literature the existing methods only use a single POLR model, based on their assumption that the effects of a predictor variable are the same for different current ratings. However, I believe that, for different current ratings, the effects of a variable on their rating transitions should be different in practice. Therefore, instead of using a single model, I develop several level-wise POLR models so as to allow for distinct effects of a predictor variable on the transitions for different current rating levels.

In particular, I develop level-wise POLR models to consider the issuers' initial rating status and construct firm-specific, macro-economic and credit-market variables as predictors. My models demonstrate that the macro-economic predictors have no significant effect on the rating change. The proposed models also outperform the existing models in prediction measured by the Frobenius norm.

Chapter 3

In this chapter, I investigate the effects of the accounting-based and market-based information on the explanation and prediction of credit risk. In particular I examine the difference in their effects between two distinct sample periods, the pre-crisis period and the post-crisis period within 2004-2011.

There are three main types of models for the explanation and prediction of credit risk: accounting-based scoring models, market-based structural models and reduced-form models. Reduced-form models have merit of computational tractability and have proved very useful in the relative pricing of redundant assets. However, the lack of easy interpretation of the latent variables and the difficulty in identifying a stable process to characterise their time-series behaviour make these model not widely viewed as a solid basis for credit risk prediction. There is a long history of the use of accounting-based models to explain and predict credit risk, but such models are often criticised as lacking a solid theoretical underpinning. Market-based structural models are popular in banks and financial institutions because of their theoretical grounding and their use of up-to-date market information.

Because both accounting-based variables and market-based variables can be regarded as salient indicators of financial distress, I am interested in finding whether these two sets of variables have the similar effects on the explanation and prediction of credit risk, and if their effects differ, I am interested in figuring out which of them will be the most significant in volatile periods of heightened systemic instability or at turning points of credit cycles. In particular, I am interested in knowing, between accounting-based models and market-based models, which would have been more reliable in the recent financial crisis period. Such a period is likely to reveal structural instability of the models as manifested, for example, by significant changes in sensitivities to explanatory variables.

To achieve these research objectives, I use the CDS spreads to examine the performance of the accounting-based models, the market-based models and a comprehensive model which combines both accounting-based and market-based information, in terms of the explanatory and prediction abilities for credit risk. In particular, I investigate their performance over the transition from the pre-crisis period to the post-crisis period, using Lehman Brothers' failure in the third quarter of 2008 as the turning-point to separate the pre-crisis and post-crisis periods.

From our studies the following two patterns can be observed. First, compared with the accounting-based models and the comprehensive model, the market-based models perform the best in the explanation of the CDS spreads, in the sense of having a comparable explanatory power and being more parsimonious. Second, if we only look for an optimal prediction of the CDS spreads, an AR time-series model of the CDS spreads would outperform the cross-sectional models.

A contribution of this chapter is that I first divide our sample period into a pre-crisis period and a post-crisis period, then examine the difference in explanatory and predictive abilities of credit risk models between the pre-crisis and post-crisis periods. This examination is undertaken for each of the accounting-based models, market-based models and their combined comprehensive models. That is, our investigation lays emphasis on major cyclical turning points and crises. To my best knowledge, such an investigation has not been found in the literature.

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Chapter 1

Forecasting bankruptcy

1.1 Introduction

In this chapter we aim to investigate the factors useful for the prediction of firm bankruptcy, by considering both the firm-specific accounting and market information and the macro-economic information. Although many investigations have been performed, this problem remains open in the empirical research.

The contribution of this chapter is as follows: our empirical studies demonstrate that, compared with the methods of list-wise deleting and closest-value imputation to tackle missing values, the method of multiple imputation performs the best and leads to a forecasting model with economically reasonable predictors and estimates. These predictors and estimates reflect firm-specific features of profitability, leverage and stock market information and their impact on the bankruptcy.

The problem of missing values often hinders statistical inference for panel data, such as the data collected in clinical trials, biostatistics and credit risk management. In the context of credit risk management, the data of a financially-distressed firm are more likely to have missing values than those of a healthy firm; this leads to a self-selection bias of the data. For example, a distressed firm is generally more reluctant to provide the accounting information such as its net income. Consequently, methods to cope with the missing values and thus correct the self-selection bias play a vital role in forecasting bankruptcy. As observed from our empirical studies, the results of parameter estimation are indeed sensitive to the method chosen to deal with the missing values, at least in terms of bias and efficiency of the estimates.

The simplest method to tackle missing values is to list-wisely delete the missing values, i.e. to delete all the observations with any missing values. However, this method is not appropriate if the missing values count nontrivial portion of the dataset or play an important role in the analysis, because the important information, which is implicitly conveyed by the pattern of the missing values themselves, is lost. The inference based on this method also suffers from the selection bias due to the drop of observations.

Another simple method is to simply impute the missing values by the closest non-missing values. However, it is still not able to sufficiently recover the information of the missing values, for example, when changes in values at crucial time points are missing.

Alternatively, we can use the method of multiple imputation to impute the missing values where the uncertainty about the right values to impute can be taken into account.

Our empirical studies with these three methods are detailed in Section 1.4. In the literature of bankruptcy forecasting, missing values are either substituted with past observations (e.g. Shumway (2001)), or list-wisely deleted or substituted by cross-sectional means or medians (e.g. Campbell et al. (2008)).

After the processing of missing values, bankruptcy forecasting can be carried out within either the framework of statistical models or the framework of credit risk models.

Within the framework of credit risk models, structural models and reduced-form models are widely used. Merton (1974) pioneers in using the structural models for forecasting default: a default occurs when the firm's value falls below the face value of the firm's bond at maturity. Black and Cox (1976) extend the models of Merton (1974) to first-passage models, which allow the occurrence of a default at any time. Leland (1994), Anderson and Sundaresan (1996) and Longstaff and Schwartz (1995), among others, are subsequent extensions. Reduced-form models, as used by Jarrow and Turnbull (1995) and Duffie and Singleton (1999), define the default as the first arrival time of a Poisson event at a mean arrival rate.

Within the framework of statistical models, Shumway (2001) develops a hazard model to forecast bankruptcy using yearly frequency data. Altman (1968) pioneers in using classification models for forecasting bankruptcy, which are referred to as static models by Shumway (2001). Shumway (2001) compares the empirical estimates obtained from the hazard model with those obtained from the static models, and concludes that the hazard model is more appropriate than the static models for forecasting bankruptcy. Chava and Jarrow (2004) confirm the superior forecasting performance of the hazard model of Shumway (2001) to that of the models of Altman (1968) and Zmijewski (1984), using both yearly and monthly frequency data. Campbell et al. (2008) use a similar model to predict the firm bankruptcy at short and long time periods, and claim that their best model has a greater explanatory power than those of Shumway (2001) and Chava and Jarrow (2004). Duffie et al. (2007) incorporate the time dynamics of the predictor variables into their model. Our work to be presented in this chapter falls within this framework.

We apply the hazard model to a sample over the period of 1995-2005. Our empirical studies show that: if we use list-wise deleting or closest-value imputation for the missing values, the results are not fully in lines with the literature (e.g. Shumway (2001) and Campbell et al. (2008)) in terms of statistical significance of the estimates of the predictor variables; however, if we use multiple imputation, the estimation results conformed to those in the literature, in terms of not only statistical significance but also expected signs.

In Section 1.2, the specification of the model is presented. Section 1.3 describes the data and the construction of the predictor variables; the empirical results are shown in Section 1.4 with the three methods to cope with the missing values. Section 1.5 further compares the three methods using an additional model. Section 1.6 presents our conclusions and discussion.

1.2 The model

The hazard model is used to describe the physical default intensity with the merit that a joint distribution for the predictor variables does not have to be assumed. Shumway (2001) shows that a multi-period logit model is equivalent to a discrete-time hazard model with a hazard function $\phi(\tau, X; \alpha, \beta)$. The hazard function is defined as

$$\phi(\tau, X; \alpha, \beta) = \frac{f(\tau, X; \alpha, \beta)}{1 - \sum_{j<\tau} f(j, X; \alpha, \beta)} = \phi_0(\tau) e^{\alpha + X'\beta} \,, \tag{1.1}$$

where $f(\tau, X; \alpha, \beta)$ is the probability mass function of failure, and $\phi_0(\tau)$ is the baseline hazard function at the baseline levels of covariates X. The hazard function $\phi(\tau, X; \alpha, \beta)$ provides the conditional probability of failure at time τ conditional on survival to τ. That is, if we assume that the failure time is the time when the firm files for bankruptcy, then the conditional probability of the firm i filing for bankruptcy at time t, given the information to time $t-1$, is given by

$$Pr(y_{i,t} = 1 | X_{i,t-1}, y_{i,t-1} = 0) = \frac{1}{1 + e^{-\alpha - X'_{i,t-1}\beta}} \,, \tag{1.2}$$

where $y_{i,t}$ is the indicator, which equals one when the firm i filed for bankruptcy at time t, X is the vector of predictor variables, α is the scalar intercept term and β is the vector of the coefficients for the predictor variables. The α and β can be estimated by maximum likelihood estimation via the likelihood function

$$\mathcal{L}(\alpha, \beta) = \prod_{i=1}^{n} \left(\phi(t_i, X_i; \alpha, \beta)^{y_{i,t_i}} \prod_{k_i < t_i} [1 - \phi(k_i, X_i; \alpha, \beta)]^{1-y_{i,k_i}} \right) , \tag{1.3}$$

where t_i is the failure time of the ith firm and $i = 1, \dots, n$.

If the data are collected quarter by quarter, then, in order to forecast the bankruptcy in one quarter ($j = 1$), six months ($j = 2$) or one year ($j = 4$), a logit specification can be rewritten, for the probability of the firm filing for bankruptcy in j quarters, as (Campbell et al., 2008)

$$Pr(y_{i,t-1+j} = 1 | X_{i,t-1}, y_{i,t-2+j} = 0) = \frac{1}{1 + e^{-\alpha_j - X'_{i,t-1}\beta_j}} , \tag{1.4}$$

which degenerates to (1.2) when $j = 1$. If we further assume that the probability of the firm filing for bankruptcy does not change with the prediction horizon, i.e., $\alpha_j = \alpha$ and $\beta_j = \beta$, then the cumulative probability of the firm filing for bankruptcy over j quarters is

$$1 - \prod_{l=1}^{j} Pr(y_{i,t-1+l} = 0 | X_{i,t-1}, y_{i,t-2+l} = 0) = 1 - \left(\frac{e^{-\alpha - X'_{i,t-1}\beta}}{1 + e^{-\alpha - X'_{i,t-1}\beta}} \right)^j , \tag{1.5}$$

which can be approximated by $\frac{1}{1+e^{-j(\alpha + X'_{t-1}\beta)}}$ through Taylor expansion. Hence, the physical default intensity, $\lambda_t^P(j)$, over j quarters at time t, can then be estimated as

$$\lambda_t^P(j) = e^{j(\hat{\alpha} + X'_{t-1}\hat{\beta})} , \tag{1.6}$$

by using the estimated parameters $\hat{\alpha}$ and $\hat{\beta}$.

1.3 The data

1.3.1 Raw variables

Our sample consists of the firms selected from the Mergent Fixed Income Securities Database (FISD), over the period from the beginning of 1995 to the end of 2005. These firms are listed in the US markets including the American Stock Exchange, Boston Stock Exchange, Pacific Stock Exchange, Chicago Board Options Exchange, New York Stock Exchange, Nasdaq Stock Exchange, National Market System, Philadelphia Stock Exchange, Portal Market and Santiago Stock Exchange. Financial firms are excluded from our sample.

Our raw variables contain ten firm-specific variables and nine macro-economic variables for the US market.

The ten firm-specific variables include the indicator of the timing of firm's filing for bankruptcy, the accounting variables, and the quarterly and daily stock prices for non-financial firms. The timings of firms' filing for bankruptcy are collected from FISD. The accounting variables and the quarterly stock price are collected from Compustat North America. The daily stock prices are collected from CRSP.

The nine macro-economic variables include the VIX (Volatility Index), the 3-month, 1-year and 10-year Treasury bill/note rates, three Fama-French factors, and the level and market capitalisation of S&P 500. The daily observations of the VIX are obtained from the website of Chicago Board Options Exchange; the monthly observations of the Treasury bill/note rates are obtained from the website of the Federal Reserve Board; the monthly Fama-French factors are obtained from Ken French's website; and the monthly data on S&P 500 are obtained from CRSP.

The firm-specific variables are first matched to the quarterly frequency dataset by using the common identifier CUSIP (Committee on Uniform Securities Identification Procedures) code amongst Compustat, FISD and CRSP data resources. The macro-economic variables are then added into the dataset by matching the year and the quarter with the firm-specific variables.

In more detail, for each firm, we have 44 quarterly observations (rows); for each observations (rows), we have 16 variables (columns). In this quarterly frequency dataset, there are in total $89,276$ observations representing $2,029$ firms, in which 79 firms filed for bankruptcy.

Variable	Label	N^*	N^* Missing	Mean	Std. Dev.	Min	Max	Med
DATA14	Price Close 3rd Month of Quarter ($)	60151	29125	28	32	0	1100	23
DATA36	Cash and Short Term Investments (MM$)	66809	22467	349	1549	-23	64415	51
DATA44	Assets Total (MM$)	67077	22199	5439	19642	0	752223	1397
DATA49	Current Liabilities Total (MM$)	63974	25302	1089	3174	0	141579	245
DATA51	Long Term Debt Total (MM$)	66615	22661	1493	6860	0	289385	362
DATA54	Liabilities Total (MM$)	67068	22208	3712	16313	0	639686	824
DATA59	Common Equity Total (MM$)	66522	22754	1694	5184	-22295	224234	468
DATA61	Common Shares Outstanding (MM$)	63963	25313	155	466	0	10880	46
DATA69	Net Income (Loss) (MM$)	68863	20413	46.30	451.32	-43029	26615	10

Table 1.1: Simple statistics of the raw firm-specific data (N^*: the number of observations)

Obs	CNUM	y	DATA14	DATA36	DATA44	DATA49	..	VIX	SRRATE
1	000361	0	16.625	28.557	411.362	59.484	..	13.58	0.0591
2	000361	0	18.375	22.960	421.450	67.828	..	12.88	0.0564
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
44	000361	0	24.080	NaN	NaN	NaN	..	11.77	0.0397
45	00081T	0	NaN	NaN	NaN	NaN	..	13.58	0.0591
46	00081T	0	NaN	NaN	NaN	NaN	..	12.88	0.0564
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
80	00081T	0	NaN	60.500	886.70	265.800	..	17.51	0.0091
81	00081T	0	NaN	NaN	NaN	NaN	..	16.73	0.0095
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
84	00081T	0	NaN	79.800	984.50	324.8	..	13.58	0.0222
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
88	00081T	0	24.500	91.100	1929.50	453.000	..	11.77	0.0397
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮

Table 1.2: Illustration of a sample in the raw quarterly dataset

The simple statistics of the raw firm-specific variables are shown in Table 1.1; the dataset is illustrated in Table 1.2. It is observed that the dataset has a severe problem of missing values and some possible occurrence of extreme values of the variables.

1.3.2 The dependent variable

The dependent variable is the binary indicator y_t, such that y_t equals one if the timing of firm's filing for bankruptcy falls at t, and zero otherwise. The timing is described in FISD as the date on which the bankruptcy petition was filed under Chapter 7 (Liquidation) or Chapter 11 (Reorganisation) of the US bankruptcy laws.

1.3.3 Predictor variables

From the raw data and the matched quarterly frequency dataset, we construct 15 predictor variables: nine firm-specific predictor variables to capture a firm's profitability, leverage, liquidity and stock price variation, and six macro-economic predictor variables to capture the macro-economic status. The description for the raw firm-specific and macro-economic predictor variables is presented in Table 1.3.

Predictor variables	Description
Firm-specific predictor variables	
NITA	net income / book value of total asset
TLTA	liability / book value of total asset
CASHTA	cash / book value of total asset
MB	market value / book value of total asset
PRICE	log(minimum (share price, $15))
SIGMA	volatility of equity return of the firm
RSIZE	log(market capitalisation of the firm / that of S&P 500 index)
EXRET	excess log-return
DtD	distance to default
Macro-economic predictor variables	
VIX	implied volatility option index
SRRATE	three-month T-bill rate
LRRATE	ten-year T-note rate
MKTRF	excess return on the market, Fama-French factor
SMB	small minus big, Fama-French factor
HML	high minus low, Fama-French factor

Table 1.3: Description of the predictor variables

Amongst the nine firm-specific predictor variables, the net income over total asset (NITA), the total liability over total asset (TLTA), the cash to total asset (CASHTA), the market over book ratio (MB) are calculated directly from the raw data. The PRICE, an indicator of financial distress as reverse stock splits are relatively rare, is calculated by the natural logarithm of the minimum between the firm's share price and \$15. The explanatory variable PRICE is winsorised above \$15 prior to logarithm transformation: If the share price is smaller than \$15, PRICE will be equal to $\log$(share price), otherwise PRICE will be $\log(15)$. This process is performed for all firms. The reason for winsorisation is that this variable is expected to be relevant for lower price per share (Campbell et al., 2008), as firms in distress tend to trade at low share price. We choose \$15 as the threshold mainly for the convenience of comparison with the work of Campbell et al. (2008). In their paper, the value is obtained from exploratory analysis with no technical details being given. For our data, \$15 is about the first tertile for the share price. The distance to default (DtD) is constructed based on the existing literature (see Section 1.3.4 for its construction). The firm's relative size (RSIZE) and excess return (EXRET) are calculated on the basis of the firm's market capitalisation and stock price, and on the market capitalisation and the level of S&P500. An annualised three-month sample standard deviation of firm's daily return is calculated as a proxy of the firm's equity volatility (SIGMA), i.e.

$$\mathrm{SIGMA}_t = \left(252 \times \frac{1}{N-1} \sum_{j \in t} r_j^2\right)^{\frac{1}{2}},$$

where r is the firm's daily stock return, j is the daily time index, t is the quarterly time index, and N is the daily observation numbers within the quarter t.

To avoid the effect of extreme values and thus obtain an accurate and robust estimation, we winsorise all the firm-specific predictor variables at the 5th and 95th percentiles after processing the missing values.

1.3.4 Construction of distance to default

To construct the distance to default, we need the firm's market asset value and asset volatility. As both the market asset value and the asset volatility are not observable, we use a call option formula to work them out.

According to the Black-Scholes and Merton model, the market asset value A_t follows the Geometric Brownian motion, $\frac{dA_t}{A_t} = \mu_A dt + \sigma_A dW_t$, and the equity value of the firm, E_t, can be viewed as a call option on A_t with the strike price as the face value of debt L_t. The face value of debt is conventionally obtained by a proxy of the short-term debt plus half of the long-term debt. Hence, the call option formula is

$$E_t = A_t N(d_1) - L_t e^{-rT} N(d_2) \ , \tag{1.7}$$

where $N(.)$ is the cumulative distribution function of the standard normal distribution, r is the risk-free return, T is the time to maturity which is assumed to be 1 year, and

$$d_1 = \frac{\ln(\frac{A_t}{L_t}) + (r + \frac{1}{2}\sigma_A^2)T}{\sigma_A \sqrt{T}} \ , \tag{1.8}$$

$$d_2 = d_1 - \sigma_A \sqrt{T} \ . \tag{1.9}$$

Using Eqns (1.7)-(1.9), we can back out the market asset value A_t and asset volatility σ_A from the market equity and accounting information in two ways.

One way (denoted by Method-1 hereafter) to back out A_t and σ_A is through an iterative algorithm including the following five steps (Vassalou and Xing, 2004).

1. Set the initial value of A_t to be the sum of equity E_t and the firm's short-term liability and the long-term liability; set the initial value of σ_A to be the standard deviation of daily initial asset value from the past 12 months; and use the one-year Treasury bill rate as the risk free return r.

2. For each trading day of the past 12 months, use Eqns (1.7)-(1.9) to get the daily value of A_t; compute the standard deviation of A_t over the past 12 months; take this standard deviation as σ_A for the next iteration.

3. Continue the procedure until the values of σ_A from two consecutive iterations converge at a tolerance level, say, 10^{-4}. Once the converged value of σ_A is obtained, Eqns (1.7)-(1.9) are used to back out A_t.

4. μ_A is obtained by taking the mean of the daily value of log return, $\ln A_t - \ln A_{t-1}$.

5. If in Steps 1-3 the quarterly data are processed and the size of the time window is kept as 4 quarters, then we can obtain the estimate of the quarterly value of σ_A and back out the quarterly asset value of A_t.

It follows that the distance to default can be obtained as

$$\mathrm{DtD}_t = \frac{\ln(\frac{A_t}{L_t}) + (\mu_A - \frac{1}{2}\sigma_A^2)T}{\sigma_A\sqrt{T}} \; . \tag{1.10}$$

An alternative way (denoted by Method-2 hereafter) to back out A_t and σ_A is through simultaneously solving two equations for these two unknowns: one equation is Eqn (1.7), and the other equation is the optimal hedge equation (Campbell et al., 2008),

$$\mathrm{SIGMA}_t = \sigma_A N(d_1)\frac{A_t}{E_t} \; , \tag{1.11}$$

where SIGMA_t is the firm's equity volatility.

Then the distance to default can be obtained as

$$\mathrm{DtD}_t = \frac{\ln(\frac{A_t}{L_t}) + (0.06 + r - \frac{1}{2}\sigma_A^2)T}{\sigma_A\sqrt{T}} \; , \tag{1.12}$$

where the equity premium directly takes the value of 0.06 instead of being estimated by the average firms' daily returns as with the Method-1, which might be a noisy estimate.

The Method-2 avoids keeping a rolling window of the previous observations, hence it works for incomplete datasets. Moreover, Method-2 does not require the daily stock price, hence it facilitates the preparation of the data. In this chapter, we use Method-2 to calculate the distance to default.

For convenience, we hereafter refer to a row in the quarterly frequency dataset as a firm-quarter, a column as a variable, a cross intersection of the row and the column as an entry, and the quarter in which the firm filed for bankruptcy as an event-quarter.

Before processing the data for the missing values, we first take the following steps to help to clean the data.

1. Take a one-quarter lag for all the predictor variables to ensure that the predictor information is available before the quarter over which the probability of bankruptcy is to be estimated; and hence the firms with only the first firm-quarter data are removed, giving rise to a decrease in the total number of firms to 1,713 and the number of firms filing for bankruptcy to 65. It should be noted that, as the proportion of firms filing for bankruptcy is very small (around 3.8%), this unbalanced dataset makes forecasting difficult.

2. When any accounting variable at the 4th quarter of a year Y for the firm i is missing, fill in the value with its corresponding annual value of the year Y if the firm's annual data are not missing.

3. Replace any occurrence of zero values in the firm-specific accounting variables by an indicator of missing values. The reasons for this replacement are that the zero values are apparently misrepresented for our accounting and stock price variables, and the data resources do not provide explanation for the occurrence of such zero values.

1.4 Empirical studies

In this section, we shall apply three methods, list-wise deleting, closest-value imputation and multiple imputation, to our sample to handle the missing values. We shall investigate the impact of these methods on the parameter-estimation results, with and without variable selection.

1.4.1 Empirical studies (ES-1) with list-wise deleting

The simplest method to process the missing values is to list-wisely delete the firm-quarters which have missing entries. For our sample, the method of list-wise deleting is performed through the following steps.

1. We delete any firm-quarters with missing entries.

2. For a firm filing for bankruptcy, if its event-quarter has missing entries and thus has been deleted in the last step, we remove such a firm from our sample.

3. For a firm filing for bankruptcy, we delete any of its firm-quarters after its even-quarter.

We observe that, in our dataset, some of the empty firm-quarters are generated from automatically spanning the data to cover the whole sample period while being downloaded from the data resources. Hence, for a firm-quarter, even if its entries are all missing, it still appears in the sample. In addition, for some firms filing for bankruptcy, non-missing entries may reappear several quarters after their event-quarters, as the firms are re-listed in the market. For such firms, we only remove those reappearing observations. The intuition behind our action is that the firm is not expected to have any information in our sample after its event-quarter, and we are to forecast the bankruptcy from data before the event, rather than backing out the bankruptcy from the data after the event.

In a nutshell, after such data processing, we keep in total 45,460 firm-quarters representing 1,637 firms. We call the empirical studies of this dataset “ES-1”.

Parameter	Estimate	Std Error	Wald χ^2	$Pr > \chi^2$
Intercept	-16.2758	4.3434	14.0418	0.0002
NITA	-8.7861	8.2405	1.1368	0.2863
TLTA	8.1357	2.4747	10.8082	0.0010
CASHTA	-1.7476	1.8793	0.8648	0.3524
PRICE	-1.1990	0.9888	1.4704	0.2253
MB	-0.3045	0.2038	2.2328	0.1351
RSIZE	-0.4215	0.3226	1.7068	0.1914
EXRET	-0.4195	0.2350	3.1878	0.0742
SIGMA	5.2340	1.6358	10.2377	0.0014
DtD	0.5294	0.1883	7.9037	0.0049
VIX	-0.0165	0.0360	0.2103	0.6465
SRRATE	-10.6599	21.3551	0.2492	0.6177
LRRATE	8.2882	43.3919	0.0365	0.8485
MKTRF	-0.0402	0.0683	0.3461	0.5563
SMB	0.0733	0.0724	1.0255	0.3112
HML	-0.0297	0.0881	0.1134	0.7363

Table 1.4: Parameter estimates for the full model for ES-1

The estimation results for the full model with all the predictor variables are shown in Table 1.4, from which we can observe the following. Three predictor variables, TLTA, SIGMA and DtD are statistically significant at the 5% significance level. TLTA and SIGMA enter the model with expected signs, reflecting the firm's leverage and stock price volatility. However, DtD, the volatility-adjusted measure of leverage, shows an unexpected positive sign.

Parameter	Estimate	Std Error	Wald χ^2	$Pr > \chi^2$
Intercept	-14.3934	3.4603	17.3017	$<$ .0001
TLTA	9.0198	2.4608	13.4348	0.0002
PRICE	-2.4598	0.8487	8.4000	0.0038
EXRET	-0.4769	0.2293	4.3246	0.0376
SIGMA	5.7652	1.5954	13.0586	0.0003
DtD	0.5894	0.1965	8.9985	0.0027

Table 1.5: Parameter estimates for ES-1 with the predictor variables selected by stepwise model selection

Furthermore, from the 15 predictor variables, a subset of five predictor variables, TLTA, PRICE, EXRET, SIGMA and DtD, are selected by stepwise regression.

The stepwise regression is a simple automatic procedure for model selection. Although it is biased due to the use of the same data for model selection and parameter estimation, this procedure is commonly used in statistics for model selection. The stepwise model selection of the SAS logistic procedure is a modified version of forward selection. It starts from the null model and adds independent variables one by one into the model based on certain criteria. In the SAS logistic procedure, the selection of a variable is based on chi-square statistics. Each forward selection step can be followed by one or more backward elimination steps; that is, the variables already selected in the model may not be necessarily retained. Alternatively, one can use the General-to-Specific methodology to do the selection. The General-to-Specific methodology is a sophisticated model-selection strategy, which "seeks to mimic reduction by commencing from a general congruent specification that is simplified to a minimal representation consistent with the desired criteria and the data evidence (essentially represented by the local DGP)" ("Econometric Modelling", David F. Hendry, Oxford University, 2000, www.folk.uio.no/rnymoen/imfpcg.pdf).

The estimation results for the selected model are shown in Table 1.5. All estimates of the predictor variables have the expected signs, except for that of DtD.

1.4.2 Empirical studies (ES-2) with closest-value imputation

Another simple way to process the missing values is to do a simple imputation of the missing entries with the closest non-missing entries. For our sample, such a closest-value imputation is performed through the following steps.

1. Code all the missing entries as NaN.

2. For each firm, if a missing entry is between any two non-missing entries with regard to an accounting variable, then this missing entry (NaN) is replaced with -99999.

3. Remove the firm-quarters whose missing entries are still shown as NaN. In fact, these missing entries are either before the first non-missing entries or later than the last non-missing entries.

4. Replace -99999 as NaN.

5. For each firm, first replace NaN with the closest non-missing entries later than them, and then replace the remaining NaN with the closest non-missing entries before them to ensure that all NaN are imputed.

In a nutshell, after the above processing, we keep in total 59,378 firm-quarters representing 1,667 firms. The corresponding empirical studies are denoted by ES-2.

Parameter	Estimate	Std Error	Wald χ^2	$Pr > \chi^2$
Intercept	-11.2265	2.6090	18.5153	$<$ .0001
NITA	-10.3210	4.9988	4.2631	0.0389
TLTA	5.8879	1.4365	16.7998	$<$ .0001
CASHTA	-2.5708	1.4465	3.1586	0.0755
PRICE	-1.0319	0.5456	3.5776	0.0586
MB	-0.1412	0.0926	2.3249	0.1273
RSIZE	-0.1708	0.2053	0.6919	0.4055
EXRET	-0.4129	0.1508	7.5026	0.0062
SIGMA	3.3053	0.8421	15.4063	$<$ .0001
DtD	-0.0312	0.0871	0.1282	0.7203
VIX	-0.0182	0.0262	0.4840	0.4866
SRRATE	-11.4882	15.4431	0.5534	0.4569
LRRATE	14.1017	30.5004	0.2138	0.6438
MKTRF	-0.0233	0.0479	0.2354	0.6275
SMB	0.0375	0.0483	0.6036	0.4372
HML	-0.0265	0.0615	0.1850	0.6671

Table 1.6: Parameter estimates for the full model for ES-2

Using all predictor variables, we estimate the full model for ES-2. The estimation results are shown in Table 1.6. Compared with the estimates of the full model for ES-1 (in Table 1.4), we can observe that, for ES-2, NITA and EXRET become statistically significant from being nonsignificant for ES-1, and they have the expected signs. The changes in statistical significance are in line with the economical significance. However, DtD becomes nonsignificant. Meanwhile, all estimates of the predictor variables remain the same signs as those for ES-1, except for that of DtD, the sign of which changes from the unexpected positive to the expected negative.

Furthermore, out of the 15 predictor variables, a subset of four predictor variables, TLTA, PRICE, EXRET and SIGMA, are selected by stepwise model selection. The estimation results

Parameter	Estimate	Std Error	Wald χ^2	$Pr > \chi^2$
Intercept	-11.1471	1.8742	35.3761	$<$ .0001
TLTA	7.1606	1.4145	25.6274	$<$ .0001
PRICE	-1.6014	0.4281	13.9899	0.0002
EXRET	-0.4692	0.1511	9.6376	0.0019
SIGMA	3.6498	0.7593	23.1079	$<$ .0001

Table 1.7: Parameter estimates for ES-2 with the predictor variables selected by stepwise model selection

for the new 4-predictor model are shown in Table 1.7.

Compared with the corresponding estimates in Table 1.5 for ES-1, DtD is no longer selected while other four predictor variables remain being selected. In addition, TLTA and SIGMA are more statistically significant and the magnitudes of their estimates are increased. A reason for the drop of DtD is that the information about the firm's volatility and leverage has already been reflected by TLTA and SIGMA in our model, and TLTA and SIGMA could be better measures than DtD for the firm's leverage and volatility.

1.4.3 Empirical studies (ES-3) with multiple imputation – our best model

The third way to process the missing values is to impute the missing entries by using multiple imputation. Multiple imputation has been widely used for incomplete data analysis in biostatistics. The basic idea is first to obtain m complete datasets through imputing the missing entries m times, then to obtain m estimation results for the m complete datasets, and finally to obtain the estimation results by combining the m estimates. The merit of multiple imputation is that it considers the uncertainty about the right value to impute and, with uncertainty caused by the missing entries effectively incorporated, statistical inference becomes more valid for the final estimation results (Rubin, 1987).

The approach to obtaining the m complete datasets depends on the pattern of missing data, which could be either monotonic or arbitrary, with the assumption of missing at random.

Given a dataset with variables $X_1, X_2, \cdots, X_j, \cdots, X_p$ (arranged in this order), if, for an observation, the fact that X_j is missing means the values of the following variables from X_{j+1} to X_p are all missing, then this dataset has a monotonic missing pattern. For a monotonic missing pattern, either a regression model or a nonparametric method can be used to impute the missing entries (Rubin, 1987).

An arbitrary missing pattern is all other missing patterns rather than the monotonic missing pattern. For a dataset with the arbitrary missing pattern, a Markov Chain Monte Carlo method with an assumption of multivariate normality can be used to impute the missing entries (Schafer, 1997).

An approach to combining the m estimates of the m complete datasets is as the following (SAS Institute Inc., 1999). Given the point estimate $\hat{Q}_i$ and its variance estimate $\hat{U}_i$ for a parameter Q from the ith imputed dataset, $i = 1, \cdots, m$, the combined point estimate $\bar{Q}$ is the average of the $\hat{Q}_1, \cdots, \hat{Q}_m$ such that

$$\bar{Q} = \frac{1}{m}\sum_{i=1}^{m}\hat{Q}_i \ . \tag{1.13}$$

Its total variance estimate U_T is calculated as the weighted sum of the so-called within-imputation variance $\bar{U}$ and between-imputation variance B as

$$U_T = \bar{U} + (1 + \frac{1}{m})B \ , \tag{1.14}$$

where the within-imputation variance $\bar{U}$ is the average of the $\hat{U}_1, \cdots, \hat{U}_m$ such that

$$\bar{U} = \frac{1}{m}\sum_{i=1}^{m}\hat{U}_i \ , \tag{1.15}$$

and the between-imputation variance B is given by

$$B = \frac{1}{m-1}\sum_{i=1}^{m}(\hat{Q}_i - \bar{Q})^2 \ . \tag{1.16}$$

For a simple imputation as with ES-2, the inference of estimates for the variables are solely based on the $\hat{U}_i$. For multiple imputation, besides the within-imputation variance $\bar{U}$, we are able to exploit the between-imputation variance B. With multiple imputation, the confidence intervals of the estimates are narrowed down.

For our sample, multiple imputation is performed through the following six steps, with the first four steps the same as those for ES-2.

1. Code all the missing entries as NaN.

2. For each firm, if a missing entry is between any two non-missing entries with regard to an accounting variable, then this missing entry (NaN) is replaced with -99999.

3. Remove the firm-quarters whose missing entries are still shown as NaN. In fact, these missing entries are either before the first non-missing entries or later than the last non-missing entries.

4. Replace -99999 as NaN.

5. Test for the normality of each predictor variable and take logarithmic transform for the non-normality predictor variables.

6. Obtain $m = 10$ datasets using the `MI` procedure of SAS for the missing entries coded as NaN and then inverse the log-transformed predictor variables.

In a nutshell, after such processing, we have in total 59,716 firm-quarters representing 1,673 firms for our empirical studies (ES-3).

Parameter	Estimate	Std Error	LCL	UCL	t	$Pr > \|t\|$
Intercept	-10.2310	2.3315	-14.8070	-5.6550	-4.39	$<$ .0001
NITA	-11.8802	4.4913	-20.7504	-3.0100	-2.65	0.0090
TLTA	4.7511	1.2512	2.2688	7.2335	3.80	0.0003
CASHTA	-1.3481	1.1466	-3.6022	0.9059	-1.18	0.2404
PRICE	-0.9110	0.4180	-1.7357	-0.0863	-2.18	0.0306
MB	-0.0418	0.0392	-0.1207	0.0370	-1.07	0.2908
RSIZE	-0.2616	0.1787	-0.6128	0.090	-1.46	0.1441
EXRET	-0.2394	0.1186	-0.4721	-0.0068	-2.02	0.0437
SIGMA	1.6400	0.5890	0.4756	2.8043	2.78	0.0061
DtD	-0.0208	0.2014	-0.4163	0.3748	-0.10	0.9178
VIX	-0.0041	0.0248	-0.0527	0.0445	-0.17	0.8687
SRRATE	-12.4174	15.2311	-42.2705	17.4358	-0.82	0.4149
LRRATE	1.7469	29.7873	-56.6352	60.1289	0.06	0.9532
MKTRF	-0.0363	0.0460	-0.1265	0.0538	-0.79	0.4290
SMB	0.0716	0.0484	-0.0233	0.1665	1.48	0.1392
HML	-0.0026	0.0600	-0.1201	0.1149	-0.04	0.9649

Table 1.8: Parameter estimates for the full model for ES-3

The estimation results of the full model for ES-3 using all the predictor variables are shown in Table 1.8. The 95% confidence intervals, of the coefficients of the predictors in the full model for ES-3, are calculated, with the lower confidence limit (LCL) and the upper confidence limit (UCL) also shown in Table 1.8.

Compared with the estimates of the full model for ES-2 (in Table 1.6), we observe the following. Firstly, there are five predictor variables, NITA, TLTA, EXRET, SIGMA and PRICE, statistically significant at the 5% level for ES-3. Previously nonsignificant for ES-2, the variable PRICE becomes significant and keeps the expected negative sign for ES-3. Secondly, DtD is again nonsignificant. Thirdly, all estimates of the predictor variables remain holding the same signs as those in ES-2. In addition, the macro-economic variables are all nonsignificant for ES-3, the same as with ES-1 and ES-2.

Variables	NITA	TLTA	PRICE	MB	EXRET	SIGMA	SRRATE
Frequency	10	10	10	4	7	10	1

Table 1.9: Frequencies of the predictor variables being significant by stepwise model selections for the 10 imputed datasets

Parameter	Estimate	Std Error	LCL	UCL	t	$Pr > \|t\|$
Intercept	-9.3022	1.3226	-11.8993	-6.7051	-7.03	$<$ .0001
NITA	-10.3148	4.1458	-18.4963	-2.1332	-2.49	0.0138
TLTA	4.8065	1.0734	2.6910	6.9220	4.48	$<$ .0001
PRICE	-1.3812	0.3448	-2.0588	-0.7036	-4.01	$<$ .0001
EXRET	-0.2514	0.1150	-0.4770	-0.0258	-2.18	0.0290
SIGMA	1.8190	0.4387	0.9545	2.6835	4.15	$<$ .00014

Table 1.10: Parameter estimates for ES-3 with the predictor variables selected by stepwise model selection

We perform stepwise model selection for the ten imputed datasets, respectively. As the ten stepwise model selections give us distinct subsets of the predictor variables, we choose NITA, TLTA, PRICE, EXRET and SIGMA as the most-frequent significant predictor variables to analyse the ten imputed datasets. The frequencies of the predictor variables being significant amongst these ten model selections are shown in Table 1.9. The estimation results are reported in Table 1.10.

Compared with the corresponding estimates by stepwise model selection in Table 1.5 for ES-1 and in Table 1.7 for ES-2, the predictor variable NITA enters into the model for the first time with statistical significance at the 5% level, while the other four variables TLTA, PRICE, EXRET and SIGMA remain in the model. With the expected negative sign and the high magnitude of the estimate, NITA, as a profitability measure, becomes the most influence predictor variable in forecasting firms' filing for bankruptcy instead of the leverage measure of TLTA. In this sense, ES-3 re-establishes the role of NITA in forecasting firms' filing for bankruptcy. We argue that the model with these five predictor variables, NITA, TLTA, PRICE, EXRET and SIGMA, is the best model for our dataset, based on the above empirical studies.

When controlling for the other explanatory variables, we examine the proportional impact for 0.1 unit increase in each explanatory variables from the current values. Such an increase in NITA (profitability) reduces the odds for bankruptcy by $64\% = 1 - \exp(-1.031)$. Correspondingly, the effects are a 162% increase in the odds for bankruptcy for TLTA (leverage), a 13% reduction for PRICE (price per share), a 2% reduction for EXRET and a 120% increase for SIGMA (volatility).

We note that for ES-1 the variable DtD is significant but with an unexpected sign, while for ES-2 and ES-3 it becomes nonsignificant but with the expected sign. The unstable pattern could be explained by the multicollinearity between DtD and SIGMA, which can be observed from the three correlation matrices of independent variables, as shown in Tables 1.11, 1.12 and 1.13.

	NITA	TLTA	CASHTA	PRICE	MB	RSIZE	EXRET	SIGMA	DtD	VIX	SRRATE	LRRATE	MKTRF	SMB	HML
NITA	1	-0.137	-0.263	0.381	0.120	0.313	0.057	-0.325	0.333	-0.061	0.085	0.079	0.032	-0.014	-0.013
TLTA	-0.137	1	-0.291	-0.155	0.004	-0.081	0.003	-0.053	-0.172	0.019	-0.057	-0.070	-0.011	0.006	-0.002
CASHTA	-0.263	-0.291	1	-0.117	0.258	-0.101	0.024	0.321	-0.141	-0.028	-0.073	-0.073	-0.013	0.003	0.022
PRICE	0.381	-0.155	-0.117	1	0.226	0.553	0.115	-0.447	0.444	-0.073	0.099	0.112	0.068	-0.010	-0.014
MB	0.120	0.004	0.258	0.226	1	0.307	0.125	0.054	0.153	-0.037	0.113	0.101	0.081	-0.011	-0.023
RSIZE	0.313	-0.081	-0.101	0.553	0.307	1	0.039	-0.354	0.476	-0.063	-0.020	-0.002	-0.002	0.001	0.022
EXRET	0.057	0.003	0.024	0.115	0.125	0.039	1	-0.081	0.058	0.010	-0.079	-0.065	-0.082	0.035	0.070
SIGMA	-0.325	-0.053	0.321	-0.447	0.054	-0.354	-0.081	1	-0.798	0.280	0.094	0.026	-0.054	0.001	-0.021
DtD	0.333	-0.172	-0.141	0.444	0.153	0.476	0.058	-0.798	1	-0.324	-0.050	0.017	0.075	-0.006	0.026
VIX	-0.061	0.019	-0.028	-0.073	-0.037	-0.063	0.010	0.280	-0.324	1	0.011	-0.101	-0.358	-0.066	0.042
SRRATE	0.085	-0.057	-0.073	0.099	0.113	-0.020	-0.079	0.094	-0.050	0.011	1	0.822	0.265	-0.111	-0.112
LRRATE	0.079	-0.070	-0.073	0.112	0.101	-0.002	-0.065	0.026	0.017	-0.101	0.822	1	0.247	-0.032	-0.137
MKTRF	0.032	-0.011	-0.013	0.068	0.081	-0.002	-0.082	-0.054	0.075	-0.358	0.265	0.247	1	0.063	-0.480
SMB	-0.014	0.006	0.003	-0.010	-0.011	0.001	0.035	0.001	-0.006	-0.066	-0.111	-0.032	0.063	1	-0.555
HML	-0.013	-0.002	0.022	-0.014	-0.023	0.022	0.070	-0.021	0.026	0.042	-0.112	-0.137	-0.480	-0.555	1

Table 1.11: Correlation of independent variables of the full model for ES-1

	NITA	TLTA	CASHTA	PRICE	MB	RSIZE	EXRET	SIGMA	DtD	VIX	SRRATE	LRRATE	MKTRF	SMB	HML
NITA	1	-0.103	-0.279	0.377	0.089	0.334	0.041	-0.326	0.301	-0.063	0.061	0.058	0.024	-0.011	-0.006
TLTA	-0.103	1	-0.365	-0.130	-0.068	-0.071	-0.003	-0.088	-0.120	0.012	-0.053	-0.063	-0.008	0.007	0.001
CASHTA	-0.279	-0.365	1	-0.113	0.256	-0.120	0.029	0.338	-0.156	-0.012	-0.048	-0.047	-0.007	0.001	0.013
PRICE	0.377	-0.130	-0.113	1	0.211	0.557	0.094	-0.421	0.406	-0.075	0.108	0.115	0.077	-0.013	-0.019
MB	0.089	-0.068	0.256	0.211	1	0.278	0.099	0.065	0.116	-0.033	0.120	0.106	0.080	-0.014	-0.028
RSIZE	0.334	-0.071	-0.120	0.557	0.278	1	0.018	-0.348	0.429	-0.057	-0.010	0.006	0.001	-0.002	0.016
EXRET	0.041	-0.003	0.029	0.094	0.099	0.018	1	-0.055	0.038	0.018	-0.071	-0.057	-0.074	0.031	0.053
SIGMA	-0.326	-0.088	0.338	-0.421	0.065	-0.348	-0.055	1	-0.776	0.256	0.085	0.015	-0.039	-0.001	-0.025
DtD	0.301	-0.120	-0.156	0.406	0.116	0.429	0.038	-0.776	1	-0.296	-0.040	0.028	0.063	-0.004	0.022
VIX	-0.063	0.012	-0.012	-0.075	-0.033	-0.057	0.018	0.256	-0.296	1	-0.039	-0.174	-0.350	-0.059	0.059
SRRATE	0.061	-0.053	-0.048	0.108	0.120	-0.010	-0.071	0.085	-0.040	-0.039	1	0.824	0.276	-0.119	-0.125
LRRATE	0.058	-0.063	-0.047	0.115	0.106	0.006	-0.057	0.015	0.028	-0.174	0.824	1	0.250	-0.045	-0.151
MKTRF	0.024	-0.008	-0.007	0.077	0.080	0.001	-0.074	-0.039	0.063	-0.350	0.276	0.250	1	0.059	-0.486
SMB	-0.011	0.007	0.001	-0.013	-0.014	-0.002	0.031	-0.001	-0.004	-0.059	-0.119	-0.045	0.059	1	-0.550
HML	-0.006	0.001	0.013	-0.019	-0.028	0.016	0.053	-0.025	0.022	0.059	-0.125	-0.151	-0.486	-0.550	1

Table 1.12: Correlation of independent variables of the full model for ES-2

	NITA	TLTA	CASHTA	PRICE	MB	RSIZE	EXRET	SIGMA	DtD	VIX	SRRATE	LRRATE	MKTRF	SMB	HML
NITA	1	-0.121	-0.262	0.368	-0.063	0.312	0.057	-0.274	0.270	-0.057	0.051	0.045	0.018	-0.011	0.000
TLTA	-0.121	1	-0.336	-0.152	0.243	-0.089	-0.006	-0.011	-0.072	0.015	-0.051	-0.061	-0.006	0.007	-0.002
CASHTA	-0.262	-0.336	1	-0.117	0.236	-0.115	0.028	0.248	-0.205	-0.011	-0.041	-0.040	-0.005	0.001	0.010
PRICE	0.368	-0.152	-0.117	1	0.062	0.574	0.114	-0.415	0.394	-0.069	0.075	0.082	0.058	-0.007	-0.006
MB	-0.063	0.243	0.236	0.062	1	0.147	0.114	0.113	-0.023	-0.010	0.061	0.045	0.051	-0.005	-0.021
RSIZE	0.312	-0.089	-0.115	0.574	0.147	1	0.049	-0.320	0.397	-0.054	-0.015	0.000	-0.002	-0.001	0.019
EXRET	0.057	-0.006	0.028	0.114	0.114	0.049	1	-0.087	0.047	0.013	-0.071	-0.060	-0.062	0.030	0.047
SIGMA	-0.274	-0.011	0.248	-0.415	0.113	-0.320	-0.087	1	-0.636	0.188	0.111	0.064	-0.019	-0.005	-0.027
DtD	0.270	-0.072	-0.205	0.394	-0.023	0.397	0.047	-0.636	1	-0.238	-0.083	-0.036	0.042	0.003	0.036
VIX	-0.057	0.015	-0.011	-0.069	-0.010	-0.054	0.013	0.188	-0.238	1	-0.039	-0.174	-0.351	-0.059	0.059
SRRATE	0.051	-0.051	-0.041	0.075	0.061	-0.015	-0.071	0.111	-0.083	-0.039	1	0.824	0.276	-0.119	-0.125
LRRATE	0.045	-0.061	-0.040	0.082	0.045	0.000	-0.060	0.064	-0.036	-0.174	0.824	1	0.250	-0.045	-0.151
MKTRF	0.018	-0.006	-0.005	0.058	0.051	-0.002	-0.062	-0.019	0.042	-0.351	0.276	0.250	1	0.059	-0.486
SMB	-0.011	0.007	0.001	-0.007	-0.005	-0.001	0.030	-0.005	0.003	-0.059	-0.119	-0.045	0.059	1	-0.550
HML	0.000	-0.002	0.010	-0.006	-0.021	0.019	0.047	-0.027	0.036	0.059	-0.125	-0.151	-0.486	-0.550	1

Table 1.13: Correlation of independent variables of the full model for ES-3: Average of the ten datasets from multiple imputation

1.5 Empirical comparison with Campbell et al. (2008)

In the section, we compare the three methods of processing the missing values, by applying an additional model to our datasets. This model, denoted by Campbell-M hereafter, is proposed in the column (2) of Table III of Campbell et al. (2008). The Campbell-M model uses NITA, TLTA, RSIZE, EXRET and SIGMA as predictor variables, slightly different from our models in Tables 1.5, 1.7 and 1.10. Their sample period (1993-1998) is also different from ours (1995-2005). We apply Campbell-M to our datasets generated for ES-1, ES-2 and ES-3, respectively. The results are shown as follows.

Parameter	Estimate	Std Error	Wald χ^2	$Pr > \chi^2$
Intercept	-18.5870	2.5997	51.1198	$< .0001$
	-15.214	–	*39.45**	$**$
NITA	-8.2006	7.6499	1.1491	0.2837
	-14.05	–	*16.03**	$**$
TLTA	9.2075	2.5011	13.5521	0.0002
	5.378	–	*25.91**	$**$
RSIZE	-0.7467	0.2915	6.5607	0.0104
	-0.188	–	*5.56**	$**$
EXRET	-0.4665	0.2315	4.0606	0.0439
	-3.297	–	*12.12**	$**$
SIGMA	3.6198	1.1284	10.2907	0.0013
	2.148	–	*16.40**	$**$

Table 1.14: Parameter estimates for ES-1 with the predictor variables in Campbell et al. (2008). Contents in italic are the results in the column (2) of Table III of Campbell et al. (2008), where the value with * is the absolute value of Z-statistics; $**$ represents statistical significance at the 1% level.

Compared with those in Campbell et al. (2008), our results, as listed in Table 1.14 for ES-1, show the same signs but different magnitudes of estimates, and NITA is nonsignificant here. Note that when constructing the predictor variables, Campbell et al. (2008) adjust the book value of total assets by adding 10% of the difference between market and book equity to them, whereas we do not make such an adjustment.

Compared with those in Table 1.14 for ES-1, our results, as listed in Table 1.15 for ES-2, show that, although still nonsignificant at the 5% level, NITA becomes significant at the 10%

Parameter	Estimate	Std Error	Wald χ^2	$Pr > \chi^2$
Intercept	-16.0864	1.5358	109.7056	$<$ .0001
	-15.214	–	*39.45**	$**$
NITA	-8.3294	4.6490	3.2100	0.0732
	-14.05	–	*16.03**	$**$
TLTA	6.8877	1.4001	24.2014	$<$ .0001
	5.378	–	*25.91**	$**$
RSIZE	-0.5233	0.1691	9.5782	0.0020
	-0.188	–	*5.56**	$**$
EXRET	-0.4715	0.1500	9.8793	0.0017
	-3.297	–	*12.12**	$**$
SIGMA	3.8771	0.7471	126.9305	$<$ .00014
	2.148	–	*16.40**	$**$

Table 1.15: Parameter estimates for ES-2 with the predictor variables in Campbell et al. (2008). Contents in italic are the results in the column (2) of Table III of Campbell et al. (2008), where the value with * is the absolute value of Z-statistics; $**$ represents statistical significance at the 1% level.

level.

In contrast to those in Table 1.14 for ES-1 and Table 1.15 for ES-2, our results, as listed in Table 1.16 for ES-3, show that all the five predictor variables are statistically significant at the 5% level, which is in lines with that of Campbell et al. (2008).

1.6 Conclusions and future work

In this chapter, we have used three different methods, list-wise deleting (ES-1), closest-value imputation (ES-2) and multiple imputation (ES-3), to cope with the severe problem of missing values in our raw dataset. Using the datasets obtained in ES-1 and ES-2, we have estimated the hazard model, and found that estimation results were not fully in lines with the literature in terms of statistical significance of the estimates of the predictor variables. However, when we used the dataset obtained from multiple imputation in ES-3, our estimation results conformed to the literature.

Moreover, using stepwise model selection, we have obtained models and parameter estimations for ES-1, ES-2 and ES-3, respectively, and chosen NITA, TLTA, PRICE, EXRET, and

Parameter	Estimate	Std Error	LCL	UCL	t	$Pr > \|t\|$
Intercept	-13.7896	1.1363	-16.0202	-11.5591	-12.14	$< .0001$
	-15.214	–	–	–	*39.45**	$**$
NITA	-10.9615	4.0802	-19.0098	-2.9132	-2.69	0.0079
	-14.05	–	–	–	*16.03**	$**$
TLTA	4.7678	1.1099	2.5743	6.9613	4.30	$< .0001$
	5.378	–	–	–	*25.91**	$**$
RSIZE	-0.5462	0.1506	-0.8415	-0.2508	-3.63	0.0003
	-0.188	–	–	–	*5.56**	$**$
EXRET	-0.2717	0.1155	-0.4982	-0.0453	-2.35	0.0187
	-3.297	–	–	–	*12.12**	$**$
SIGMA	1.9172	0.4046	1.1224	2.7120	4.74	$< .0001$
	2.148	–	–	–	*16.40**	$**$

Table 1.16: Parameter estimates for ES-3 with the predictor variables in Campbell et al. (2008). Contents in italic are the results in the column (1) of Table 3 of Campbell et al. (2008), where the value with * is the absolute value of Z-statistics; $**$ represents statistical significance at the 1% level.

SIGMA as the predictor variables of our best model.

In order to visualise the prediction performance of the model, here we plot the receiver operating characteristic (ROC) curve in Figure 1.1. The ROC curve is a useful tool to access the accuracy of predictor, where sensitivity is plotted against (1-specificity) for varying thresholds.

In a binary prediction case, the so-called sensitivity represents a statistical measure for the proportion of actual positives which are correctly identified as such, i.e. $P(\hat{y} = 1|y = 1)$. The higher the value of sensitivity the better the predictor. The sensitivity is usually placed on the vertical axis in the ROC space. The so-called specificity defines the proportion of actual negatives which are correctly identified as such, i.e. $P(\hat{y} = 0|y = 0)$. In the ROC space, "1-specificity", usually presented in the horizontal axis, denotes $P(\hat{y} = 1|y = 0)$. The smaller the value of (1-specificity) the better the predictor. Therefore, each point on the ROC curve consists of the pair $(P(\hat{y} = 1|y = 0), P(\hat{y} = 1|y = 1))$ at different thresholds. The perfect predictor is located on the top-left point at $(0, 1)$ with the area under the ROC curve equal to 1.

The ROC curve in Figure 1.1, obtained for ES-3, shows that our model has a quite good prediction performance.

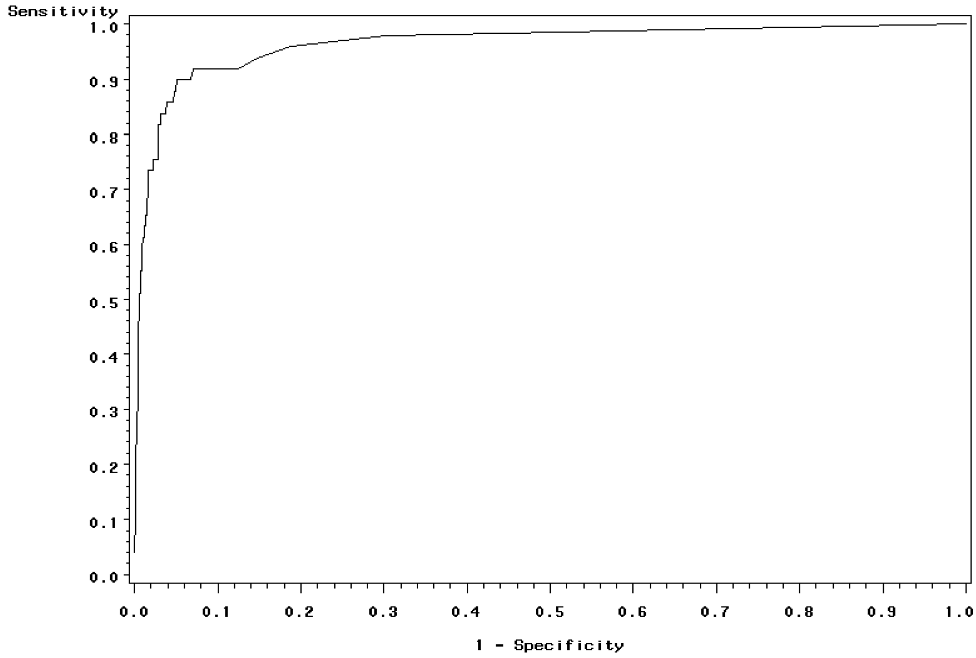


Figure 1.1: ROC plot for the best model

Therefore, from our investigation, we can draw the following two conclusions:

1. Amongst the three methods that we use to process the missing values, we empirically confirm that multiple imputation helps to correct the self-selection bias, and outperforms the methods of list-wise deleting and closest-value imputation, in the sense that its results are more economically reasonable and more consistent to those in the literature.

2. In terms of the determinants of forecasting the probability of bankruptcy, we empirically find that the predictor variables NITA, TLTA, PRICE, EXRET and SIGMA are the most promising ones over our sample period of 1995–2005.

For further investigation, we would like to make three suggestions:

1. Although following the way of utilising a hazard model to forecast the probability of bankruptcy as with the existing literature (Shumway, 2001; Campbell et al., 2008), we think that there are still some aspects worth discussion. These aspects include: each firm-quarter is treated as an independent observation although the data are panel data; the proportion of the firms filing for bankruptcy is very small so that the two classes for logistic regression is very unbalanced, which makes the prediction less reliable; and the

random effects resulted from the discrepancy between individual firms are not considered explicitly in the model.

2. After we obtain the physical default intensity (λ^P), in order to further examine the relationship between the physical bankruptcy risk premium and risk-neutral bankruptcy risk premium, we need to "back out" the risk-neutral default intensity (λ^Q). After we obtain the risk-neutral default intensity, we are able to explore the relationship between the risk-neutral default intensity and the physical default intensity. The simplest way is to regress the physical default intensity with other variables on the risk-neutral default intensity, so that a rough magnitude of the default premium defined as λ^Q/λ^P can be obtained.

   Recently credit default swap (CDS) is traded in a huge volume and a high liquidity, so that it can be regarded as a purer security traded for credit risk than corporate bonds. Researchers have shown strong interest in seeking the credit risk premium through the CDS rate. Duffie et al. (2007) and Berndt et al. (2005) are the papers mostly related to this topic by using the CDS rate, while Driessen (2005) uses corporate bond ratings.

3. Although none of the macro-economics predictor variables is significant in our model, we intend to explore the effect of the macro-economic variables on the default risk premium, because the variables reflecting business cycles have been well documented that they may affect the probability of bankruptcy (Duffie and Kenneth, 2003). In addition, we would like to incorporate industry effect into the determinants of default risk premium.

Chapter 2

Rating-based credit risk modelling

2.1 Introduction

2.1.1 Background

The purpose of this chapter is to predict the probabilities of credit rating transitions of issuers. For this purpose, we shall develop statistical models of credit risk to consider both the issuers' initial ratings and other factors such as firm-specific, macro-economic and credit-market information.

There are mainly three types of credit risk models. The first type is called "structural models", examples of which are the Merton model and the first-passage structural model. In these models, a firm defaults when its asset value falls below a given threshold or its debt value. The second type is called "reduced-form models", which treats the default as a Poisson event involving a sudden loss in the market value. The third type is called "rating-based models", in which the probabilities of upward or downward moves in ratings are estimated by using historical data. Our statistical models belong to the third type, i.e. the rating-based credit risk models.

The rating-based credit risk models address a particular element of credit risk. There are mainly two elements of credit risk often explored by practitioners and regulatory authorities in financial markets using credit risk models. The first element is the probability of default or downgrading. The probability of default is the likelihood that an entity may not meet their financial obligations in full and on time. The second element is the loss given default. The loss given default is to measure the loss of the investors when the default occurs. The rating-based credit risk models address the first element of credit risk, i.e. the default or downgrading probability. This is because credit ratings can reflect the creditworthiness of firms and thus the probability of default or downgrading.

Credit ratings are usually assigned on ordinal scales, expressing the relative likelihood of default from the strongest to the weakest. Credit ratings can be applied to both firms and governments. The ratings reveal a view of rating agencies or financial institutions, and are produced by them through both quantitative and qualitative analyses. Moody's, Standard & Poor's (S&P) and Fitch are three main rating agencies in financial markets. They provide ratings for issuers (e.g. corporations) and for issues (e.g. corporate bonds), and they publish the aggregate historical transition rates. Different rating systems have different emphases. The S&P ratings take relative default risk as a single most important factor; Moody's ratings put more weight on expected loss than on relative default risk. We use the S&P ratings in this study mainly because of the availability of such data to us.

The S&P ratings currently have 21 rating categories. According to Standard & Poor's website, the major rating categories are from "AAA" as the highest rating, through "AA", "A", "BBB", "BB", "B", "CCC", "CC" and "C", to "D" as the lowest rating, denoting the issuer's extremely strong capacity to meet financial commitments ("AAA") to the issuer's payment default on financial commitments ("D"). A plus (+) or minus (-) sign may be used to modify the ratings from "AA" to "CCC" to show relative standing within the major rating categories.

Although rating has a long history of study, it remains very important at present, for at least three reasons. First, Basel 2 links the required measure of bank capital to the credit rating of the bank's obligors. Second, some securities, especially credit derivatives, have their payoff contingent on the ratings. Third, credit ratings are related to equity market liquidity (Odders-White and Ready, 2006).

Not only ratings are important, rating changes are also very important. The rating change of an issuer can reflect the change of its default probability. As a change of ratings is a signal of a worsening or improving credit quality, an upward move of rating can be viewed as a decrease in the probability of default, while a downward move can be regarded as an increase in the probability of default. Hence, given the current rating of an issuer, the prediction of its next rating, i.e. its rating transitions over a certain time period, is always desired. Moreover, regulatory authorities have set requirements contingent on the ratings, and the practitioners, especially institutional investors, have adopted the investment policies that are sensitive to the rating changes. We shall further specify these reasons, among others, for why the prediction of credit rating transitions matters, from the following three aspects.

First, financial institutions are concerned with their rating changes. On the one hand, if their ratings are downgraded to the speculative grade, this signals an increase in the probability of default. Consequently, they will have to increase collateral on their purchases on margins, so more capital is needed. On the other hand, with the decreased share price upon downgrading, the financial institutions need to raise new capital. Adding to the difficulty is the fact that other institutions, such as pension funds and insurance companies, may withdraw their investments in the financial institutions.

Second, the credit ratings are expected to capture the risk regarding the value of an entity's debt. Many institutional investors are obliged by their statutes or regulations to hold investment-grade paper ("BBB-" or higher). In Europe, the Eurosystem, the monetary authority of the Eurozone, has required a minimum rating of "A" for all eligible collateral. Hence, for a portfolio with exposure to credit downgrading risk, the fund managers have to re-balance the portfolio.

Third, some securities, particularly credit derivatives have indebted the rating in the contract, linking the payoff to the changes in rating. Moreover, the downgrade of credit rating may put pressure on the liquidity of credit derivatives, e.g. the secondary market liquidity for the CDO securities.

2.1.2 Related work

This chapter aims to develop statistical models to predict the probabilities of the credit rating transitions of issuers. The transition probabilities form a rating-transition matrix. The validation of our empirical estimation shows that the models that consider the issuers' initial ratings outperform the models that are otherwise similar. Before presenting our models, we shall discuss some of closely related work.

To estimate a rating-transition matrix, one method is to simply adopt the estimates from rating agencies' publications. However, the credit rating agencies have long been under fire for not spotting corporate disasters in time, while rating and rating transitions are expected to capture and respond to a changing economy and business environment. From the Enron scandal in 2002 to the Lehman Brothers bankruptcy in 2008, credit rating agencies are criticised for having been too slow to lower the corresponding ratings. Hilscher and Wilson (2012) suggest that the reason for the sluggish response by the credit rating agencies is that, because the rating is a single summary measure that relates to two different aspects of credit risk, the firm-level default probability and the systematic default risk, they are not particularly accurate at forecasting default.

Therefore, instead of relying on rating changes made by the credit rating agencies, investors can make investment decision based on home-made models that consider various sources of information, so long as these models are reliable and economical in the prediction of rating transitions. In addition, besides these existing ratings, a model for credit rating is often needed when portfolio risk managers have to predict credit ratings for unrated issuers, which is often the case. Moreover, issuers may seek a preliminary estimate of what their ratings might be prior to entering the capital markets (see Metz and Cantor (2006)).

The estimates from the agencies' publications are obtained by using a cohort method. The cohort method assumes that the rating-transition process is a discrete-time homogeneous Markov chain. The rating-transition matrix for the next period is estimated by relative frequencies. For a population in which there are a total number of N_i firms in rating i at the beginning of the year, the probability of their rating moving to rating j at the end of the year is estimated as $P_{ij} = N_{ij}/N_i$, where N_{ij} is the total number of the firms moved from rating i to rating j. So a rating-transition probability P_{ij} is the portion of the number of the corporations in the population that have moved to rating j from rating i. Pooling the transitions over the years, one can attain a historical transition matrix.

Although it is easy to carry out and commonly used in the industry, the cohort method suffers two main weaknesses in its methodology.

The first weakness is that it is a discrete-time model and considers ratings only at the two endpoints of the estimation interval, causing it to ignore any transition within the estimation interval. Hence, among others, Lando and Skodeberg (2002) criticise that the discrete-time setting cannot obtain efficient estimates of transition rates. Instead they proposed an alternative method, using a continues-time setting to capture the chance of defaulting within a year after successive downgrades (from different firms). They provide estimators in both homogeneous and non-homogeneous chains. Further, Frydman and Schuermann (2008) model the rating-transition process by a mixture of two independent continuous-time homogeneous Markov chains with two different migration speeds.

The second weakness is that there are non-Markov behaviours evidently observed in the patterns of rating transitions. Researchers discern that the history of ratings beyond the estimation interval also carries information about the rating transitions. To overcome this weakness, since the beginning of the 1990s, the patterns of credit rating transitions have been studied, and the non-Markov behaviours have been documented as rating drift (momentum effect), industry and country heterogeneity, duration effect and time heterogeneity (e.g. dependence on the business cycle), etc. in the literature.

Altman (1998) compares three sets of such studies. One set of studies is the series of articles by Altman and Kao (e.g. Altman and Kao (1991) and Altman and Kao (1992)); the second set of studies is done by researchers with Moody's, and the third set of studies is performed by researchers with S&P. Altman (1998) documents the effects of the ages of the bond, the transition horizons and the withdrawn ratings on the rating-transition matrices in the three sets of studies, using both the S&P and Moody's data. Using Moody's data, Nickell et al. (2000) discuss the impact of the industry and domicile of the issuers and the stage of the business cycle on the distribution of rating transitions. They find that the impact exists and varies with the ratings of the issuers. Using the S&P data, Bangia et al. (2002) study the impact of the business cycle on the credit rating migrations. They partition the economy into expansion and contraction, and allow the Markovian credit migrations to switch between the states of the economy. By doing so, they find that the Markovian rating dynamic is a reasonable approximation. Using the S&P data, Lando and Skodeberg (2002) test rating drift and duration effects. Du (2003) explores the duration effects on the credit rating changes.

Up to now, we have mentioned two approaches to obtaining the rating-transition matrices: one is to simple adopt the estimates from the agencies' publications, and the other is to utilise a continuous-time probabilistic method to model the rating transitions. However, these two approaches only consider the transition history of the ratings. They do not explicitly exploit other available information, such as the firms' accounting information. Because of this, they cannot capture the factors that may significantly impact rating transitions and thus cannot model how these factors impact rating transitions. To explain the relationship between rating transitions and potential factors, we shall utilise regression models with the factors as covariates.

Because rating is an ordinal categorical variable, a natural choice in regression models is a generalised linear model. Nickell et al. (2000) use an ordered probit model to quantify the non-Markov effects. Alsakka and Gwilym (2010) add random effects to an ordered probit model in sovereign credit-rating migrations. Altman and Rijken (2004) link credit scores to credit ratings using an ordered logit model with some US data. Kim and Sohn (2008) use a random-effects multinomial regression model to estimate transitions for some Korean data. The methods employed by Altman and Rijken (2004) and Kim and Sohn (2008) belong to proportional odds logistic regression (POLR). Their methods use a single proportional odds logistic regression model: they assume that the effects of a covariate are the same for different current ratings. However, we believe that, for different current ratings, the effects of a covariate on their rating transitions should be different in practice. Therefore, instead of using a single model, we shall develop several level-wise POLR models so as to allow for distinct effects of a covariate on the transitions.

In addition, the models used in Altman and Rijken (2004) and Kim and Sohn (2008) ignore the initial rating status of the issuers, leading to a model actually predicting the *rating* of a firm-year observation rather than predicting the *transition of the rating* of that observation, although their resulting rating can be viewed as a "proxy" of the "next rating" of a firm-year observation. Hence, the issuers' initial rating status will be considered in building our POLR models.

In summary, to predict the probabilities of rating transitions, we shall develop several level-wise POLR models, in which both the firm-specific, macro-economic and credit-market information and the issuers' initial ratings are considered. In this way, a more accurate prediction of the rating transitions can be obtained.

This chapter is organised as follows. Section 2.2 introduces the models. Section 2.3 describes our data. Section 2.4 presents our empirical results, which are obtained by using three methods (i.e. historical matrix, single POLR and level-wise POLRs) to estimate the transition matrices. Section 2.5 draws conclusions and discusses some future work.

2.2 Models

We utilise the so-called "proportional odds logistic regression" (POLR) to set up the logit of the cumulative probability that a rating falls at or below a particular rating level. The details of the POLR model can be found in Agresti (2007).

Suppose we have a variable Y with R categorical levels ordered as $(1, 2, \ldots, r, \ldots, R)$. Each categorical level has a probability, denoted by $p_1, p_2, \ldots, p_r, \ldots, p_R$. The cumulative probability $P(Y \leq r)$ is the sum of the probabilities of the occurrence of Y falling at or below a particular level r, that is to say, $P(Y \leq r) = p_1 + p_2 + ... + p_r$. The probability of the occurrence of Y being at r can be calculated as $P(Y = r) = P(Y \leq r) - P(Y \leq r-1)$. The logit of the cumulative probability is then given by

$$\text{logit}[P(Y \leq r)] = \log \left\{ \frac{P(Y \leq r)}{1 - P(Y \leq r)} \right\} . \tag{2.1}$$

By using the ordinal random variable Y as our response variable, and using a set of covariates X as our explanatory/predictor variables, a POLR model can be established as

$$\text{logit}[P(Y \leq r)|X] = \alpha_r - \beta^T X \ , \quad \text{for } r = 1, 2, \ldots, R-1 \ , \tag{2.2}$$

where using $-\beta$ instead of β is for an interpretation convention, such that a positive β corresponds to Y being more likely to fall at the high end of its ordinal level as X increases (Agresti, 2007). This convention is used by software packages such as R (www.r-project.org) and SPSS.

Each element of the parameter vector β (or more precisely $-\beta$) represents the coefficient that reflect the effect of the increase in X (for a quantitative covariate) on the logit of cumulative probability $P(Y \leq r)$, which is the expected number of units change in the log cumulative odds per unit increase in X. We can observe from Model (2.2) that β is constant and thus the same for each cumulative probability, i.e., β does not depend on r.

There are $R-1$ intercepts α_r, which are the log cumulative odds for the rth category when all the explanatory variables are zero.

In this study, we collected data of Y and X for each quarter in our sample period.

The first model we aim to build is a statistical model for $P(Y_t|X_{t-1})$ for year t, which is fitted to the data of all the firms in our dataset. The model can be written as

$$\text{logit}\{P(Y_t \leq r|X_{t-1})\} = \log \left\{ \frac{P(Y_t \leq r|X_{t-1})}{1 - P(Y_t \leq r|X_{t-1})} \right\} = \alpha_r - \beta^T X_{t-1} \ , \tag{2.3}$$

where $r = 1, \ldots, R-1$ and, as mentioned above, the intercepts α_r are dependent on the rating level r while the coefficients in β are not. We next use the fitted model to predict probabilities $P(Y_{t+1}|X_t)$, supposing that current year's covariates X_t are known but next year's rating Y_{t+1} is not. Finally, we can obtain a matrix of predicted transition probabilities $P(Y_{t+1} = j|Y_t = i)$, by counting the proportion of firms with current year's rating at i and predicted rating at j for next year, for $i, j = 1, \ldots, R$.

The POLR Model (2.3) has been explored in the literature by Altman and Rijken (2004) and Kim and Sohn (2008). However, they use only a single POLR model regardless of original rating levels Y_{t-1}, which implies that the coefficient vector β does not depend on the original rating level Y_{t-1}. This implication is unreasonable in practice. Hence we propose a simple modification to Model (2.3): For different original levels, we develop different POLR models, which we call “level-wise POLR models”.

In detail, our proposed model contains R POLR models, one for each initial rating level. The ith POLR model can be written as

$$\text{logit}\{P(Y_t \leq r | X_{t-1}, Y_{t-1} = i)\} = \alpha_{i,r} - \beta_i^T X_{t-1} \,, \tag{2.4}$$

where $r = 1, \ldots, R-1$ and $i = 1, \ldots, R$. In other words, we separate the dataset (Y_t, X_{t-1}) into R subsets, with each subset corresponding to a current rating level $Y_{t-1} = i$. We then use the ith subset of data to fit the ith POLR Model (2.4).

We note that, based on the models, the probability of rating changes for *each* firm can be predicted without any technical difficulty. However, this study focuses on the aggregate rating-transition matrix, which is the average of the predicted probabilities of rating changes over *all* firms. In order to validate our prediction achieved by using Model (2.4), we shall compare its rating-transition matrix with the historical rating-transition matrix and the transition matrix obtained by using a single POLR Model (2.3). The historical rating-transition matrix is obtained by calculating frequencies of rating changes. Before showing the comparative results in section 2.4 , we first describe our data in section 2.3.

2.3 Empirical data

For a sample period from the year 1999 to the year 2008, we collect, for all firms in the sample period, quarterly accounting data and the S&P domestic long-term issuer credit ratings from Compustat North American Quarterly Updates. We then match the ratings with the accounting data using the identifier Committee on Uniform Security Identification Procedures (CUSIP). Although we can use the sample to predict the transitions matrix in 2007-2008, we choose to predict 2006-2007 transition matrix instead. The reason behind this is that, as pointed out by one of the participants in the LSE seminar talk given by the author in March 2010, the years of 2007-2008 suffered the beginning of a credit crunch, which is highly likely to possess quite different characteristics from those of the rating-transition matrix. In this chapter, we are not particularly interested in exposing and investigating such different characteristics.

In addition to the accounting data, we collect data on the macro-economic variables from the Federal Reserve System and on credit-market variables from the Federal Reserve Economic Data - St. Louis Fed (Table 2.1).

We choose quarterly frequency data, although annual ratings are more standard in the credit risk context. Our choice has at least two advantages. One is that we are open to having higher-frequency transition matrices than a commonly-estimated annual transition matrix; that is, a finer prediction horizon can be achieved. The other is that, if we want to estimate the annual transition matrix, we can readily capture the transition probabilities within one year by aggregating the quarterly transition probabilities.

As we have mentioned in section 2.1, we use the S&P ratings in this study.

In order to facilitate estimation (to ensure a reasonable sample in each rating subset) and to compare with the results reported in the literature, we group the issuers' ratings into six categories, "above AA", "A", "BBB", "BB", "B" and "under CCC", and label correspondingly the new categories from "6" to "1". Moving from a category in higher order (e.g. the rating category "6") to a category in lower order (e.g. the rating category "5") indicates the deterioration in the credit quality of an issuer. Details of grouping can be found in Table 2.2.

After pre-processing the combined dataset, we get a total of 25523 firm-quarters, and the firm-quarters are distributed in the six rating categories as shown in Table 2.3.

Many studies have been carried out on the determinants of ratings and rating transitions. Studies show that models doing a good job of explaining ratings may not necessarily do a good job of predicting rating change (Cantor, 2004). In our modeling we will include three sets of explanatory variables, details of which can be found in Table 2.1. The first set is firm-specific accounting variables (firm-specific variables), which represent the issuer's profitability and credibility. All variables except for WCTA are log-transformed in order to increase the effectiveness in the model estimation. These variables are commonly used in the literature, as in Altman and Rijken (2004) and Kim and Sohn (2008), for example. The second set is macro-economic variables related to business cycles: Discount rate, GDP and Unemployment rate. We use these three macro-economic variables to capture the variation of macroeconomy and the effects of business cycles on the rating transitions. The third set is credit-market variables reflecting credit condition information, as exploited by Anderson (2009): NPCMCM2, NPCMCM5 and NPTLTL.

Firm-specific Variables	
	Description
WCTA	Working Capital / Total Assets; short-term liquidity of a firm
RETA	Retained Earnings / Total Assets; historic profitabilities
EBITA	Earnings Before Interest and Taxes / Total Assets; current profitabilities
MEBL	Market Value of Equity / Total Liabilities; market leverage
SIZE	Total Liabilities / Total Value of US Equity Market; a “too-big-to-fail” default protection
Marco-economical Variables	
	Description
Discount rate	Federal Reserve System Discount Rate
GDP	Real GDP Growth Rate
Unempl	Unemployment Rate
Credit-market Variables	
	Description
NPCMCM2	Nonperforming commercial loans for banks with assets from \$300M to \$1B
NPCMCM5	Nonperforming commercial loans for banks with assets over \$20B
NPTLTL	Nonperforming total loans

Table 2.1: Explanatory variables X

Y	**Standard and Poors' long-term issuer rating**
6	AAA, AA+, AA, and AA-
5	A+, A and A-
4	BBB+, BBB and BBB-
3	BB+, BB and BB-
2	B+, B and B
1	CCC+, CCC, CCC-, CC, C and D

Table 2.2: Dependent variable

Rating category	1	2	3	4	5	6
Observations	886	5692	7564	7068	3569	744

Table 2.3: Property of rating categories

2.4 Results

2.4.1 Significance of the predictors

The p-values for our predictors in our proposed Model (2.4) are shown in Table 2.4. The numbers in the first row denote the models for initial (i.e. current) ratings i, where $i = 1, \ldots, 6$, and $**$ denotes the significance at the 1% level. A graphic presentation of the p-values can be seen in Figure 2.1, where we plot the p-values for all the predictors in our models.

From Table 2.4 and the plots in Figure 2.1, we can observe the following: Firstly, EBITTA and MEBL are always highly significant for all six of the level-wise POLR models. Secondly, SIZE is highly significant for the models with the current rating below "5". Thirdly, all macro-economic variables are not significant at the 1% significance level, and only in three cases the macro-economic variables are significant at the 5% level. Fourthly, the credit-market variables are rarely significant. We also tried the models with the firm-specific variables, the marco-economical variables or the credit-market variables only, and learned that the results do not vary significantly.

This result shows that the firm-specific variables, in particular the variables reflecting current profitabilities and market leverage of the firm, explain the major part of the rating transi-

	1	2	3	4	5	6
WCTA	0.237	0.851	0.009**	0.000**	0.012	0.470
RETA	0.546	0.103	0.000**	0.175	0.398	0.003**
EBITTA	0.000**	0.000**	0.000**	0.000**	0.000**	0.005**
MEBL	0.000**	0.000**	0.000**	0.000**	0.000**	0.000**
SIZE	0.000**	0.000**	0.000**	0.000**	0.279	0.633
Discount	0.110	0.089	0.050	0.698	0.913	0.525
GDP	0.086	0.273	0.032	0.035	0.588	0.823
Unemp	0.065	0.929	0.457	0.828	0.640	0.735
NPTLTL	0.950	0.605	0.323	0.833	0.012	0.430
NPCMCM2	0.261	0.231	0.002**	0.127	0.946	0.840
NPCMCM5	0.696	0.710	0.420	0.774	0.002**	0.983

Table 2.4: p-values for the predictors in the level-wise POLR Models (2.4)

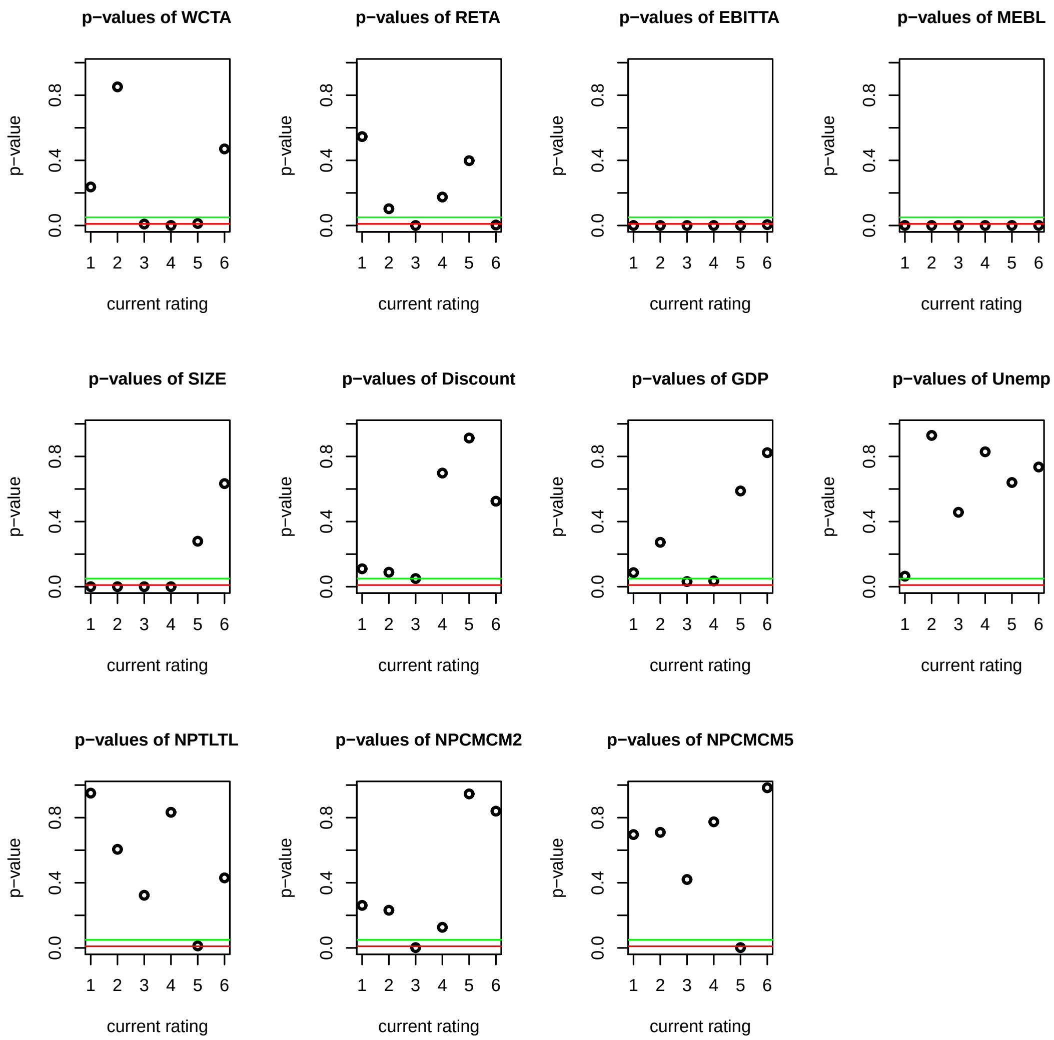
tions. Macro-economic and credit-market variables do not show strong impact on rating transitions in our study. From the literature, it is documented that ratings look through the cycle, that is to say, ratings are intended to measure the default risk over long investment horizons (Cantor, 2004). In this sense, the macro-economic variables should not have significant effects on the rating change. Our results empirically confirm such previous findings in the literature.

2.4.2 Estimates of coefficients of the predictors

Table 2.5 presents the estimates of the coefficients in our models. As we have observed from the p-values of the coefficients in Table 2.4, only EBITTA and MEBL are significant in all the models, so we focus on the estimates for EBITTA and MEBL in Table 2.5. We can find that the coefficients of EBITTA are positive; this means that the response Y is more likely to fall at the high end of the rating levels as EBITTA increases. That is, as the current profitability increases, the firm is more likely to move to a higher rating level. The same holds for MEBL. The signs of the estimates of EBITTA and MEBL are consistent with those reported in Altman and Rijken (2004).

Let us illustrate the interpretation of the estimated coefficients by using an example. From Table 2.5 we see the coefficient on EBITTA is 18.1 for the current rating “BBB” (i.e. “4”).

Figure 2.1: A graphic presentation of p-values of the coefficients of the predictors



This means: For the current rating "BBB", when the other predictors are held constant, for each 0.01 unit increase in EBITTA, the log odds for moving into higher rating levels compared with moving into lower rating levels will increase by an average of 0.181. That is, on average the new odds that a firm upgrades rather than downgrading equal $\exp(0.181) = 1.2$ times the original odds.

We note that the coefficients on EBITTA are not monotone in rating level. Meanwhile, we can observe that the coefficient for category "4" (corresponding to "BBB") is the smallest one among all six coefficients on EBITTA. This indicates that, compared with the firms currently at other ratings, the firms currently at the rating "BBB" are least likely to move into a higher or lower rating level given an increase or decrease in the current profitability. In other words,

	Model						Altman
	1	2	3	4	5	6	
WCTA	-0.72	0.05	-0.77	-1.60	-1.21	0.91	-
RETA	0.27	0.21	0.95	-0.30	-0.14	-0.83	+
EBITTA	20.57	28.90	25.54	18.10	18.87	26.69	+
MEBL	0.46	0.74	0.74	0.98	0.72	1.03	+
SIZE	0.51	0.40	0.50	0.35	0.06	0.04	+
Discount	54.64	-19.01	-19.29	4.73	1.73	-21.10	
GDP	-169.88	24.07	44.46	51.38	16.48	12.34	
Unemp	169.16	2.41	-17.82	6.36	-17.62	27.32	
NPTLTL	0.26	0.66	-1.14	-0.29	-4.32	-3.00	
NPCMCM2	3.19	-0.98	2.33	1.34	0.08	0.45	
NPCMCM5	-0.49	-0.13	-0.26	-0.10	1.38	-0.02	

Table 2.5: Estimates of coefficients of the predictors in Model (2.4)

the rating “BBB” is the most “sticky” rating, insensitive to a change in the current profitability. Furthermore, in the junk grade group (“BB” and below), the trend of upgrading of the current level “3” (corresponding to “BB”) is weaker than that of “2” (corresponding to “B”), implying that the current rating “BB” is also more sticky than “B”. In summary, these results imply that the ratings are sticky around the barrier between the junk grade and the investment grade.

2.4.3 Predicted rating-transition matrices

For illustrative purposes, we show for the year 2007 the credit-rating-transition matrix $T_{2007} = [T_{ij}]$, where $i, j = 1, \ldots, R$ are indices of the R rating levels, and $T_{ij} = P(Y_{2007} = j|Y_{2006} = i)$, in which Y_{2007} is assumed not yet been observed.

For comparison, we present the true (observed) transition matrix of Y_{2007} in Table 2.6. This table shows the true (observed) probabilities for firms’ changing ratings from rating i (in the row categories) in the year 2006 to rating j (in the column categories) in the year 2007. For example, the entry $(1, 2)$ of the table is read that the firms being rated “1” in 2006 have a probability of 0.327 to move up to the rating “2” in 2007.

Let $t = 2006$. Here we present three methods to predict the transition matrix T_{t+1}.

	1	2	3	4	5	6
1	0.673	0.327	0.000	0.000	0.000	0.000
2	0.011	0.862	0.127	0.000	0.000	0.000
3	0.000	0.066	0.896	0.038	0.000	0.000
4	0.000	0.006	0.050	0.924	0.021	0.000
5	0.000	0.000	0.021	0.051	0.922	0.006
6	0.000	0.000	0.000	0.019	0.038	0.943

Table 2.6: Observed transition matrix of 2006→2007

A simple method to predict T_{t+1} is to simply adopt the up-to-date transition matrix $T_{1:t}$ calculated from all historical data, i.e.

$$T_{t+1} = T_{1:t} \ . \tag{2.5}$$

This method uses the relative frequencies in the same way as that used by credit rating agencies in the publications. Table 2.7 shows a transition matrix obtained by applying this method to our empirical data.

	1	2	3	4	5	6
1	0.858	0.134	0.007	0.000	0.000	0.000
2	0.082	0.838	0.079	0.000	0.000	0.000
3	0.010	0.082	0.874	0.034	0.000	0.000
4	0.001	0.006	0.063	0.917	0.014	0.000
5	0.001	0.000	0.006	0.088	0.896	0.008
6	0.000	0.000	0.000	0.011	0.137	0.851

Table 2.7: Predicted transition matrix of 2006→2007 by simply using that of 1999→2006

A transition matrix obtained by applying the single POLR Model (2.3) to our empirical data is shown in Table 2.8, over which the nonzero transition probabilities are more spread out than over Table 2.7.

Table 2.9 presents a transition matrix obtained from applying our level-wise POLR Models (2.4) to the same empirical data as those for Tables 2.7 and 2.8.

	1	2	3	4	5	6
1	0.316	0.477	0.175	0.030	0.003	0.000
2	0.075	0.423	0.353	0.130	0.018	0.001
3	0.015	0.191	0.415	0.309	0.066	0.004
4	0.002	0.051	0.256	0.447	0.224	0.021
5	0.002	0.030	0.108	0.347	0.405	0.108
6	0.001	0.025	0.086	0.142	0.378	0.369

Table 2.8: Predicted transition matrix of 2006→2007 by using the single POLR Model (2.3)

	1	2	3	4	5	6
1	0.812	0.179	0.009	0.000	0.000	0.000
2	0.048	0.869	0.083	0.000	0.000	0.000
3	0.009	0.074	0.884	0.034	0.000	0.000
4	0.000	0.004	0.046	0.936	0.014	0.000
5	0.001	0.000	0.005	0.078	0.908	0.008
6	0.000	0.000	0.000	0.008	0.102	0.890

Table 2.9: Predicted transition matrix of 2006→2007 by using the level-wise POLR Models (2.4)

2.4.4 Prediction performance

The prediction performance of a model for rating transitions can be measured by comparing the distance between a predicted matrix T_{pred} and the true (observed) matrix T_{true}. There are many approaches to measuring the distance between two matrices; a natural choice is to calculate a matrix norm of $T_{pred} - T_{true}$. Here we use one type of matrix norm: the entry-wise Frobenius norm for its simplicity and easy understandability.

The Frobenius norm of $T_{pred} - T_{true}$ is an entry-wise norm, defined as

$$\|T_{pred} - T_{true}\|_F = \sqrt{\sum_i^R \sum_j^R \{[T_{pred} - T_{true}]_{ij}\}^2}\ . \tag{2.6}$$

Using our empirical data, we can 'predict' yearly-transition matrices and validate the predicted matrices by comparing with the true (observed) matrices in terms of the Frobenius norm. The results are plotted in Figure 2.2.

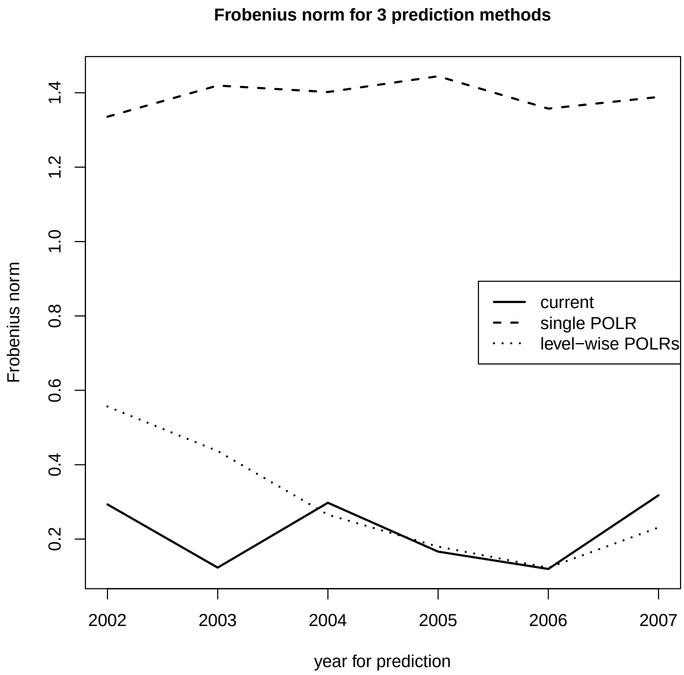


Figure 2.2: Prediction performance for yearly-transition matrices, in terms of the Frobenius norm of $T_{pred} - T_{true}$

From Figure 2.2, we can make the following observations: Firstly, the method using the single POLR Model (2.3) always performs the worst. Secondly, our method using the level-wise POLR Models (2.4) and the simple method in (2.5) demonstrate similar performance. Nevertheless, our method has an ability to explain the transitions by linking them to firm-specific, macro-economic and credit-market variables that the simple method does not possess.

2.4.5 Momentum effect (rating drift)

Momentum effect is also called rating drift. It refers to the dependence between rating transitions and the rating history. More specifically, the firms that have been downgraded tend to be downgraded further, while the firms that have been upgraded are less likely to be downgraded subsequently. This effect has been documented in the literature (e.g. Xing et al. (2012)), as we have mentioned in section 2.1, where we discussed the violation of the properties of Markov chains. We now explore whether the momentum effect exists in our data.

An illustration of our results for detecting the momentum effect can be found from Table 2.10 to Table 2.13. The left-hand panel of Table 2.10 is the pattern that we expect to find for the model with current rating category "2", if a momentum effect does exist; the right-hand panel is the empirical pattern obtained from our results.

In more detail, the left-hand panel of Table 2.10 reads that the current year is 2006 and the current rating level is "2". The current rating can be upgraded from rating level "1", come from the same rating level "2", or be downgraded from rating level "3". We want to predict the rating transition probability in next year, 2007. Here the predicted transition probability is denoted by P_{xyz} where x, y and z are the rating levels of the last, current and next years, respectively. For instance, P_{121} reads the probability of downgrading to level "1" in year 2007 given the current level "2" in year 2006 that was upgraded from level "1" in year 2005. The momentum effect occurs when $P_{121} < P_{221} < P_{321}$, indicating that the firm having been downgraded from rating level "3" to level "2" is more likely to downgrade further to level "1".

Similarly, Tables 2.11, 2.12 and 2.13 are for current rating levels "3", "4" and "5", respectively.

From these tables, we can observe that: For the model for current rating category "2", the momentum effect is found; for the models for current rating categories "3" and "4", only half of the pattern can be found; and for the model for current rating category "5", we can even find an inverse pattern of the momentum effect. In short, there is no clear pattern of momentum effect that can be observed for the years 2005-2006-2007.

2005	2006	2007					
		1	2	3	4	5	6
1	2	P_{121}					
		$\wedge$					
2	2	P_{221}					
		$\wedge$					
3	2	P_{321}					
4	2						
5	2						
6	2						

2005	2006	2007					
		1	2	3	4	5	6
1	2	$.026$					
		$\wedge$					
2	2	$.041$					
		$\wedge$					
3	2	$.043$					
4	2						
5	2						
6	2						

Table 2.10: Momentum effect for current rating category “2”

2005	2006	2007					
		1	2	3	4	5	6
1	3						
2	3		P_{232}				
			$\wedge$				
3	3		P_{332}				
			$\wedge$				
4	3		P_{432}				
5	3						
6	3						

2005	2006	2007					
		1	2	3	4	5	6
1	3						
2	3		$.121$				
			$\vee$				
3	3		$.068$				
			$\wedge$				
4	3		$.105$				
5	3						
6	3						

Table 2.11: Momentum effect for current rating category “3”

2005	2006	2007					
		1	2	3	4	5	6
1	4						
2	4						
3	4			P_{343}			
				$\wedge$			
4	4			P_{443}			
				$\wedge$			
5	4			P_{543}			
6	4						

2005	2006	2007					
		1	2	3	4	5	6
1	4						
2	4						
3	4			.034			
				$\wedge$			
4	4			.046			
				$\vee$			
5	4			.039			
6	4						

Table 2.12: Momentum effect for current rating category “4”

2005	2006	2007					
		1	2	3	4	5	6
1	5						
2	5						
3	5						
4	5					P_{454}	
						$\wedge$	
5	5					P_{554}	
						$\wedge$	
6	5					P_{654}	

2005	2006	2007					
		1	2	3	4	5	6
1	5						
2	5						
3	5						
4	5					.119	
						$\vee$	
5	5					.084	
						$\vee$	
6	5					.070	

Table 2.13: Momentum effect for current rating category “5”

We further examine whether the pattern occurs in years 2004-2005-2006 and 2003-2004-2005. The results are summarised in Table 2.14, which indicate that there is no strong evidence to support the existence of the momentum effect. Nevertheless, we note that this conclusion may not be generic beyond the tests performed in our study.

last year-current year-next year	Current rating level			
	2	3	4	5
2005-2006-2007	yes	no	no	no
2004-2005-2006	yes	no	yes	no
2003-2004-2005	yes	no	no	no

Table 2.14: Summary of whether the momentum effect has been observed

2.4.6 Computational time complexity

Our level-wise POLR Models (2.4) need to estimate R POLR models; the model in (2.3) only needs to estimate a single POLR model. However, each POLR in our models only needs to fit much less data that the single POLR model does; our experiments show that, for the yearly prediction for our empirical data, Model (2.3) and Models (2.4) have similar computational time complexity, each running less than one second by using the package `MASS` of the software R via computer.

2.5 Conclusions and future work

In this chapter, we have developed level-wise POLR models to predict rating transitions probabilities of issuers, incorporating firms' accounting information, macro-economic variable and credit-market variables as explanatory variables in the model.

Compared with the use of a single POLR model to predict the rating transitions, where the effects of explanatory variables on the logit of the cumulative probabilities do not change with the initial rating levels, our use of the level-wise POLR models allows these effects to differ with the rating levels, and thus more accurate prediction is obtained. Our comparison of the prediction performance of the models has demonstrated that our level-wise POLR models outperformed the single POLR model.

Moreover, the parameter-estimation results obtained by applying our models to empirical data have indicated that the firm-specific variables, in particular the variables containing current operational profitabilities and market leverage of the firm, explained the bulk of the rating transitions. Macro-economic and credit market variables did not show strong impact on the rating transitions in this study.

Finally, we have examined the momentum effect in the rating transitions (i.e. downgrading is more likely followed by another downgrading). The results did not show strong evidence in our study to support the existence of such an effect.

For future extensions of our models, it may be helpful to investigate a more sophisticated version of the models, such as adding random effects into the current models. Kim and Sohn (2008) develop a random-effects model by assigning Dirichlet prior distributions to the cumulative probabilities $P(Y_t \le r|X_{t-1})$ on the left-hand side of Model (2.3) and thus using Bayesian methods to estimate the transition matrix.

In our case, a random-effects model can be obtained by adding firm-dependent random effects γ_c (for firm c) to the right-hand side of Model (2.4), leading to

$$\text{logit}\{P(Y_{t,c} \le r|X_{t-1,c}, Y_{t-1,c} = i)\} = \alpha_{i,r} - \beta_i^T X_{t-1,c} - \gamma_c \ . \tag{2.7}$$

Furthermore, more effects such as industry-dependent random effects and year-dependent random effects can also be added to the right-hand side of Model (2.7). The random effects can be assumed to be mutually independent. Let Y_{t,g,c_g} be the rating of firm c in industry g at time t with the industry-dependent random effects ϕ_g and the year-dependent random effect η_t. The model (2.7) becomes

$$\text{logit}\{P(Y_{t,g,c_g} \le r|X_{t-1,g,c_g}, Y_{t-1,g,c_g} = i)\} = \alpha_{i,r} - \beta_i^T X_{t-1,g,c_g} - \gamma_c - \phi_g - \eta_t \ , \tag{2.8}$$

where $g = 1, \ldots, G$, the number of firms in industry group g is n_g, and $c_g = 1, \ldots, n_g$. We can assume $\phi_g \sim N(0, \sigma_g^2)$, $\gamma_c \sim N(0, \sigma_c^2)$ and $\eta_t \sim N(0, \sigma_t^2)$, and assume that the random effects are mutually independent.

However, according to Professor Brian Ripley, the Professor of Applied Statistics at the University of Oxford, there is no reliable way to fit such a model and even no package in R available for this undertaking. He suggested using Markov chain Monte Carlo (MCMC) methods. A candidate software for this is WinBUGS.

Chapter 3

Accounting-based and market-based models for credit risk

3.1 Introduction

3.1.1 Background

In this chapter we compare the performance of the two main categories of models for the explanation and prediction of credit risk. These two categories are called accounting-based models and market-based models, respectively.

There is a long tradition of the use of accounting variables to explain and predict credit risk. The main early contribution in this area is Altman's Z-score that was originated in the 1960s, and since then methodology has been developed and refined in subsequent generations of accounting-based scoring models (Altman et al., 1977; West, 1985; Platt and Platt, 1991; Altman and Saunders, 1998). However, this type of models is often criticised as lacking a solid theoretical underpinning (Agarwal and Taffler, 2008).

In the 1970s, Black, Scholes, Merton and others developed the contingent claims approach to modelling the liabilities of the firm. Merton (1974) is the seminal contribution in the analysis of defaultable bonds by means of a "structural model", driven by assumptions about the stochastic process of the firm's assets as well as information about the terms and conditions of the firm's liabilities (e.g. coupon, leverage and term). In contrast to the accounting-based scoring models, the implementation of structural models typically makes use of market information, most notably stock market prices. The market-based structural approach is the cornerstone of the KMV model, which becomes popular in banks and financial institutions because of its theoretical grounding and its use of up-to-date market information. As this approach considers the capital structure of the firm, i.e. the assets value and debts value, it is called "structural model".

Since the 1990s, another approach to tackling credit risk has appeared. This approach assumes that the firm's default time is driven by the default intensity based on the market prices of credit securities. As this approach purely reduces all information to latent states variables, it is called "reduced-form model". Reduced-form models have merit of computational tractability and have proved very useful in the relative pricing of redundant assets. However, the lack of easy interpretation of the latent variables and the difficulty in identifying a stable process to characterise their time-series behaviour have meant that they are not widely viewed as a solid basis for credit risk prediction. For this reason in our research we focus on the models that relate credit risk to observable variables, which are easier to interpret.

Market-based structural models are based on a clear interpretation of the credit event, namely the bankruptcy process. The variables in market-based models can be regarded as the main indicators of financial distress. Meanwhile, the variables that typically are used in accounting-based credit models arguably are also salient indicators of distress. Accounting-based models make use of variables derived from firm's financial reports: balance sheets, income statements and cash-flow statements. Since information from these financial reports reflects the recently-past performance of the firm, the accounting-based models may be regarded as a backward-looking vision of the creditworthiness for the firm. In contrast, the concept of the market-based models is based on the evolution of the market value of assets. The market value of assets, through a channel of the market value of equity, usually reflects the view of market participants on the future performance of the firm. Therefore the information used in the market-based models is normally regarded as a forward-looking indicator of the creditworthiness for the firm.

In this context, we are interested in finding whether these two sets of information (models) have the same performance in the explanation and prediction of credit risk. If their performances differ, we are then interested in figuring out which of them will be the most useful in volatile periods of heightened systemic instability or at turning points of credit cycles. In particular, we are interested in knowing which would have proved to be more reliable in the recent financial crisis period. Such a period is likely to reveal structural instability of models as manifested, for example, by significant changes in sensitivities to explanatory variables.

A new contribution of this chapter is as follows: We first divide our sample period into a pre-crisis period and a post-crisis period, then examine the difference in explanatory and predictive abilities of credit risk models between the pre-crisis and post-crisis periods. This examination is undertaken for each of the accounting-based models, market-based models and their combined comprehensive models. That is, our investigation lays emphasis on major cyclical turning points and crises. To our best knowledge, this has not been found in the literature.

3.1.2 Related work and executive summary

The literature on performance comparison between the market-based models and the accounting-based models is limited, as noted in Agarwal and Taffler (2008). Motivated by this, Agarwal and Taffler (2008) find that the two models capture different aspects of bankruptcy risk. Based on the UK data they find that there is little difference in their predictive ability. Das et al. (2009) provide evidence on the US market. They find that the accounting-based model performs comparably to the market-based model, and the combination of the two sources of information performs better than either of the two models.

In this chapter we build on the methodology of Das et al. (2009) to study the relative performance of the accounting-based and market-based credit risk models as well as a comprehensive model which includes both types of information. We apply this to an unbalanced panel of observations on the credit default swap (CDS) spreads for North American *non-financial* firms between 2004 to 2011. The CDS spreads are a good proxy for credit risk, in terms of the continuity of the CDS data in contrast to the dichotomy of the default data, and their market perception in contrast to that of the rating data (Das et al., 2009). Because the CDS markets are standardised and are typically more liquid than the underlying bonds and notes, and because they are less affected by tax considerations, they are widely regarded as a relatively pure indicator of a firm's financial distress.

We separate our sample into two sub-samples: pre-crisis and post-crisis. In September 2008, Lehman Brothers collapsed. This bankruptcy is widely viewed as the watershed event beyond, which the entire financial system entered a period of crisis thus provoking wide-spread bankruptcies, financial distress and a large global recession. Therefore, we choose the third quarter of 2008 as a break-point for the crisis.

We investigate the performance of an accounting-based model, a market-based model and a comprehensive model, for each of the sub-samples representing for the market conditions before the Lehman Brothers' failure and after its failure, respectively.

We find that, in the pre-crisis sample, there is little difference in fit of the two basic models, i.e. the accounting-based and the market-based. The accounting-based variables are able to explain 74% of the variation of the CDS rates, while the market-based variables can explain 72% of the variation of the CDS rates. These results are consistent with the findings in Agarwal and Taffler (2008). We also find that the combination of both accounting-based and market-based variables is able to explain 77% of the variation of the CDS rates. Furthermore, most of the variables entering the two basic models remain significant in the comprehensive model. Hence the accounting information and the market information are complementary. When we apply the same methodology to the post-crisis period we find similar results: the explanatory powers of the two basic models are comparable, and the performance is improved when the two sources of information are combined.

When comparing the results for the pre-crisis sample versus the post-crisis sample, we find that the explanatory power of the variables decreased. For example, for the comprehensive models, from the before-crisis period to the after-crisis periods, the adjusted R-squared falls from 77% to 62%. This may reflect the increased volatility in latent factors that has not been captured by our accounting and market variables. One such factor could be the liquidity risk predominating in the financial sector during the crisis period. Focusing on the banking sectors, Gefang et al. (2011) suggest the importance of the liquidity risk relative to the credit risk to the financial crisis in explaining the LIBOR-OIS (overnight index swap) rate. They find that, particularly at the 1 month and 3 month terms, the role of the liquidity risk is much more important than that of the credit risk. Alternatively, the decline in the model fit may reflect an increase in the sensitivity of the CDS pricing to the perceived counter-party risk in these OTC derivatives contracts.

While the explanatory powers of the accounting-based and market-based models are comparable in both the pre-crisis and post-crisis periods, some of the results suggest that the accounting-based model is susceptible to structural instability. Our market-based model is more parsimonious. It uses three explanatory variables versus thirteen variables for our accounting-based model. Furthermore, there is considerable change between the pre-crisis and post-crisis subsamples in the patterns of sign and significance of the estimated coefficients in the accounting-based model.

We further establish one-quarter predictive models based on the pre-crisis subsample, and use these models to predict the one-quarter ahead CDS spreads for the post-crisis period. We calculate the mean squared prediction errors (MSE) across firms for each quarter in the post-crisis period. A large MSE would indicate a greater change of economy from the pre-crisis period.

We find that the MSE in the second quarter 2010 is the highest. This indicates that in the first quarter of 2010 the economic situation is significantly different from that in the pre-crisis period. Moreover, we find that overall the comprehensive model performs the best in cross-sectional models for the prediction of distress. In addition, we find that, compared with the accounting-base model, the market-based model does not always perform better in prediction.

When we compare the predictive performance of the cross-sectional models with that of the autoregressive time-series (AR) models of the CDS spreads, we find that the latter outperforms the former. This could be due to the fact that: 1) although the cross-sectional models are comprehensive, they still miss certain variables affecting the variation of the CDS spreads; and 2) the error terms in the cross-sectional models might be correlated. Therefore, we investigate the autocorrelation in the error terms and find that indeed the autocorrelation presents there. In order to alleviate the impact of the autocorrelation on the model inference, we incorporate lagged dependent variables (LDV), i.e. the lagged log CDS spreads, into the cross-sectional models. The estimation results show that the addition of the LDV improves the model greatly in terms of explanatory power (higher the adjusted R^2) and predictive power (lower MSE) for all the models in both pre-crisis and post-crisis periods.

In short, from our studies we observe the following two patterns. First, compared to the accounting-based model and the comprehensive model, the market-based model performs the best in the explanation of the CDS spreads, in the sense of having a comparable explanatory power and being more parsimonious. Second, if we purely look for an optimal prediction of the CDS spreads, we find that an AR time-series model of the CDS spreads outperforms the cross-sectional models.

The remaining sections are organised as follows. Firstly, we introduce credit default swap and its pricing formula. Secondly, we describe the data and the construction of variables. Thirdly, we establish models and present the estimation results for each of the sample periods. Fourthly, we carry out comparison of the performance of the predictive models. Fifthly, we check for autocorrelation of the error terms and present the results for the models including the

LDV, and finally we conclude on the findings.

3.2 Credit default swap and its pricing model

Although the recent growth of the credit derivatives market has been concentrated on more sophistic structured products, such as the collateralised debt obligations (CDOs), the credit default swap (CDS) is still standing the largest position among other credit derivatives products. The CDS market provides a relatively good platform to study the credit default risk, because a credit default swap is seen as a purer credit indicator than a corporate bond.

A credit default swap is a bilateral swap contract between a protection buyer and a protection seller, against the credit default risk of financial securities of a reference entity. It can also be regarded as a far out-of-the-money put option with the reference event of default: the protection buyer has the right to sell the securities of the reference entity to the protection seller in the event of default.

According to the contract, the protection buyer pays a periodic fee to the seller either at the time of default or at the expiration time of the contract, whichever is the first; and the protection seller promises to make a payment in the event of default of the reference entity. The periodic fee, also called default risk price, with a fixed rate, is often referred to as credit default swap spread. Following Berndt et al. (2005), the pricing formula for the CDS spreads is derived as the following.

The risk-neutral probability of the firm surviving to T conditional on survival to t, under the doubly stochastic assumption, is given as

$$p^*(t,T) = \mathbb{E}^Q[e^{-\int_t^T \lambda^Q(u)du}|F_t] \ . \tag{3.1}$$

The CDS provides an insurance against potential loss due to the risk of default of a reference entity, hence the market value of the payment by the protection seller if default occurs before the payment date t_n is given by

$$h(t,s) = \mathbb{E}^Q[\delta(t,\tau)W^s_\tau 1_{\{\tau \leq t_n)\}}|F_t] \ , \tag{3.2}$$

where the default-free market discount factor $\delta(t,\tau)$ is given as $\mathbb{E}^Q_t[e^{-\int_t^T r(u)du}]$ and is assumed to be independent of the default time under the probability measure. If default occurs at time τ, the payment W^s_τ at the default time adjusted by accrued premium since last payment date is

$$W^s_\tau = L^*_\tau - s(\tau - \frac{\lfloor 4\tau \rfloor}{4}) \; , \tag{3.3}$$

where $\lfloor 4\tau \rfloor$ denotes the largest integer less than 4τ, L^*_τ denotes the risk neutral expected loss as a fraction of notional at the default time, and s is the annualised CDS rate.

In return, the protection buyer pays quarterly premiums at the annualised rate of s to the seller till the maturity date of the CDS contract or when a credit default occurs, whichever is first. The present value of the payments by the buyer of unit notional size is $sg(t)$, for the quarterly payment dates $t_i, ..., t_n$, where $g(t)$ is given by

$$g(t) = \frac{1}{4}\sum_{i=1}^{n} \delta(t, t_i) p^*(t, t_i) \; . \tag{3.4}$$

The two present values are equal when the CDS contract is originated such that the CDS price is fair, hence s solves

$$sg(t) = h(t, s) \; . \tag{3.5}$$

By assuming that, if default occurs between payment dates t_{i-1} and t_i and the protection seller make a payment at the middle of the payment dates, a numerical approximation to $h(t, s)$ is

$$h(t, s) \approx \sum_{i=1}^{n} \delta\left(t, \frac{t_i + t_{i-1}}{2}\right) \{p^*(t, t_{i-1}) - p^*(t, t_i)\} \left(L^* - \frac{s}{8}\right) \; . \tag{3.6}$$

From Eqns 3.4-3.6, the CDS rate s is derived as

$$s = \frac{8\sum_{i=1}^{n} \delta\left(t, \frac{t_i + t_{i-1}}{2}\right) \{p^*(t, t_{i-1}) - p^*(t, t_i)\} L^*}{2\sum_{i=1}^{n} \delta(t, t_i) p^*(t, t_i) + \sum_{i=1}^{n} \delta\left(t, \frac{t_i + t_{i-1}}{2}\right) \{p^*(t, t_{i-1}) - p^*(t, t_i)\}} \; . \tag{3.7}$$

If we assume a flat term structure, then s can be simplified as

$$s = \frac{8\sum_{i=1}^{n} \{p^*(t, t_{i-1}) - p^*(t, t_i)\} L^*}{2\sum_{i=1}^{n} p^*(t, t_i) + \sum_{i=1}^{n} \{p^*(t, t_{i-1}) - p^*(t, t_i)\}} \; . \tag{3.8}$$

From the pricing model for the CDS, one can observe that the level of the market price of the CDS would incorporate such information as default probability of issuers and macroeconomic factors, and the variation of the market price of the CDS would reflect the overall functioning of the credit market.

This observation motives us to set up two type of models. One is to use accounting-based variables plus macro-economic variables, and the other is to use market-based variables plus macro-economic variables. Including macro-economic variables in both types of the models allows us to distinguish the performance of the accounting-based variables and the market-based variables in indicating the default probability of issues and in explaining the CDS rate.

3.3 Data

3.3.1 Collection and merger of the data

Our firm-specific data consist of four types of data.

- Firstly, the CDS data are collected on Datastream from January 2003 to Dec 2011. In detail, daily 1-, 2-, 3-, 5- and 10-year constant maturity spreads whose notional values are dollar-denominated are selected. The daily data are transformed into quarterly data by taking the spreads at the end of each quarter.

- Secondly, the accounting data are collected on Compustat (North America Fundamentals Quarterly) from the first quarter of 2003 to the fourth quarter of 2011, in which the financial firms have been excluded.

- Thirdly, the daily share prices are collected from CRSP.

- Fourthly, the S&P domestic long-term issuer credit rating is collected from Compustat.

The CDS sample and the accounting data are merged by Ticker names. The merged sample is further refined by requiring availability of at least 50 trading days equity returns prior to the end of each quarter. After data processing, the firm-quarter data in the sample ranges from the first quarter of 2004 to the fourth quarter of 2011.

Note that the CDS data on Datastream contain spreads from two data providers: Credit Market Analysis (CMA) and Thomas Reuters (TR). CMA provides the CDS data starting from January 2003 until September 2010, after then the CMA CDS data are restricted with an academic license. Hence since October 2010, TR has become the unique data source available with the academic license. We combine the data from the two sources by integrating the CMA entity mnemonics with the TR CDS mnemonics. Often the single CMA entity mnemonics can be mapped to several TR CDS mnemonics with different restructuring type. In such cases, we choose the restructuring type XR (No restructuring) from the TR data because of its common use in the US region[1].

Our macro-economic data include the following.

- 3-month constant maturity US Treasury bill rate from the website of U.S. Department of the Treasury
- Monthly S&P 500 index level from the website of Standard and Poor's
- Monthly average value-weighted returns for 17-industry portfolios from Fama-French's Data Library

Other related data, used to assist the construction of the explanatory variables, are listed as follows.

- The industry definitions for 17-industry portfolios from Fama-French's Data Library. According to the definitions, we assign the industry to each firm in the sample by using their Compustat SIC codes. (This is consistent to the industry definitions, where the Compustat SIC code is applied; only when it is missing, the CRSP SIC code is used instead. Das et al. (2009) use the CRSP SIC code.)
- The Consumer Price Index on all urban consumers (all items with the period 1982-1984 as a base), from the website of the Bureau of Labor Statistics of U.S. Department of Labor.
- One year Treasury constant maturity rate from Board of Governors of the Federal Reserve System.

3.3.2 Construction of the explanatory variables

A summary of all the explanatory variables and their expected relationship with CDS (sign displayed as a proxy) are listed in Table 3.1.

3.3.2.1 Accounting-based variables

From the collected accounting data, we construct 10 variables: size, roa, incgrowth, interest coverage (split into $c1$–$c4$), quick, cash, trade, salesgrowth, booklev and retained.

[1]See Thomas Reuters CDS–FAQ from data sources on Datastream

Explanatory variable	Description	Sign
	Accounting-based variables	
size	Deflated assets	-
- Profitability		
roa	Return on assets	-
incgrowth	Income growth	-
$c1$	Interest coverage $\in$ [0,5)	-
$c2$	Interest coverage $\in$ [5,10)	-
$c3$	Interest coverage $\in$ [10,20)	-
$c4$	Interest coverage $\in$ [20, 100]	-
- Financial liquidity		
quick	Quick ratio	-
cash	Cash to asset ratio	-
- Trading account activity		
trade	Inventories to COGS ratio	+
salesgrowth	Sales growth	-
- Capital structure		
booklev	Leverage ratio	+
retained	Retained earnings to assets ratio	-
	Market-based variables	
DTD	Distance to default	-
ret	Annualised prior 65-trading day equity returns	-
sdret	Annualised stdev of prior 65-trading day equity returns	+
	Macro-economic variables	
r	3-month constant maturity T-bill rates	-
snp	Prior year returns of S&P500	-
indret	Prior year returns in the same Fama-Fench industry	-
	Contract-specific variables	
invgrade	Equal to 1 for firm in investment grade, 0 otherwise	-
seniority	Equal to 1 for senior underlying debt, 0 otherwise	-
maturity	Maturity of CDS contract (1, 2, 3, 5, 10 years)	+

Table 3.1: List of the explanatory variables

These variables represent the characteristics of firms such as profitability, financial liquidity, trades, size and leverage. They are usually regarded reflecting creditworthiness of the firms, and are broadly used in academic research, such as Campbell et al. (2008) and Das et al. (2009).

Size Size is the deflated total value of assets. It is constructed as Total Asset (Compustat item ATQ) divided by the Consumer Price Index.

Profitability

- Return on assets (roa). It is constructed as Net Income (item NIQ) divided by Total Asset.
- Net income growth. It is equal to the ratio of quarterly increase in Net Income over Total Asset.
- Interest coverage. It is calculated as the sum of Pretax Income (PIQ) and Interest Expense (XINTQ) divided by Interest Expense.

Financial liquidity

- Quick ratio. It is constructed as the difference of Current Assets (ACTQ) and Inventories (INVTQ) divided by Current Liabilities (LCTQ).
- Cash to asset ratio. It is constructed as Cash and Equivalents (CHEQ) divided by Total Assets.

Trade account activities Trade is calculated as Inventories divided by Cost of Good Sold (COGSQ).

Sales growth Sales growth is obtained by using quarterly increase in Sales (SALEQ) divided by last quarter Sales.

Leverage

- Booklev is calculated as the ratio of Total Liabilities (LTQ) to Total Assets.
- Retained is the Retained Earnings (REQ) over Total Assets.

As in Das et al. (2009), we adjust roa, sales growth, interest coverage and trades for seasonal effects. That is, we use the trailing 1-year average of these variables in the models.

In particularly, before taking the trailing 1-year average for interest coverage ratio, we set any negative ratio to zero and censor the ratio at 100 if they are greater than 100. This takes into account the conjecture that: 1) a negative interest ratio should not last long and firm must find a way to meet the interest expense; and 2) when the pre-tax income is much larger than the interest expense, the magnitude of the difference would convey no additional information on the firm's creditworthiness.

Furthermore, we change the specification of the interest coverage ratio so as to consider the shape of the non-linearity. The ratio is split into 4 variables $c1_{it} - c4_{it}$ (Table 3.2) and the coefficients will be determined in the models with other variables. The non-linearity would be reflected in the estimated coefficients, i.e. the slope of interest ratio intervals.

	$c1_{it}$	$c2_{it}$	$c3_{it}$	$c4_{it}$
$IC_{it} \in$ [0, 5)	IC_{it}	0	0	0
$IC_{it} \in$ [5, 10)	5	IC_{it} -5	0	0
$IC_{it} \in$ [10, 20)	5	5	IC_{it} -10	0
$IC_{it} \in$ [20, 100]	5	5	10	IC_{it} -20

Table 3.2: Transformation of interest coverage ratio

3.3.2.2 Market-based variables

We construct 3 market-based variables: DTD, ret and sdret.

The distance-to-default (DTD) concept is originated from Merton's model and applied as a cornerstone in Moodys' KMV model. The DTD has advantages that it combines the market value of assets, business risk and financial leverage into a single credit risk measure. The evidence that the DTD outperforms accounting variables has been documented in several research papers (Hillegeist et al., 2004).

The DTD can be calculated by solving for the asset value of firm V and its standard deviation σ_V from a system of non-linear equations:

$$\begin{aligned} E &= VN(d_1) - e^{-rT}FN(d_2) \ , \\ \sigma_E &= \frac{V}{E}N(d_1)\sigma_V \ , \end{aligned}$$

where E is the market value of the firm's equity, F is the face value of the firm's debt, r is the instantaneous risk-free interest rate, $N(\cdot)$ is the cumulative distribution function for standard normal distributed random variables, and

$$\begin{aligned} d_1 &= \frac{\ln(\frac{V}{F}) + (r + 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}}, \\ d_2 &= d_1 - \sigma_V\sqrt{T} \ . \end{aligned}$$

In the calculation, the market value of the firm's equity is estimated as the multiplication of the number of shares outstanding (Compustat item CSHOQ) and the end-of-quarter closing stock prices (item PRCCQ). The face value of the firm's debt is calculated by current liabilities (item LCTQ) plus half of the long-term debt (item DLTTQ). The rate r is proxyed by the one-year Treasury constant maturity rate at the end of each quarter; T is set to be 1 year as in convention; σ_E is estimated by the standard deviation of the firm's daily stock price returns for trailing 65 trading days ($\sim$ 1 quarter). As in Das et al. (2009) and other papers, we calculate the DTD only for the quarter when at least prior 50 trading-day returns are available.

After obtaining the values of V and σ_V, we calculate the DTD as follows:

$$DTD = \frac{\ln(\frac{V}{F}) + (\mu_V - 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}},$$

where μ_V is estimated by the annualised mean equity returns on the prior 65 trading days.

3.3.2.3 Macro-economic and contract-specific variables

The 3-month constant maturity US Treasury bill (T-bill) rates are used as the measure of macroeconomic conditions. If the T-bill rate is low, the credit default spread would be expected high. We use also the prior year returns in the same Fama-French industry where the firm belongs to, in order to reflect the firm's industry risk. Prior year returns on the S&P 500 index are also included.

For contract-specific variables, we set a dummy variable for seniority of the CDS contract. The dummy variable is equal to 1 if the underlying debt of the contract is in seniority, and to 0 otherwise. In addition, we include the maturity of the CDS contract as a variable which takes values of 1, 2, 3, 5 and 10. In normal cases and ceteris paribus, the longer the maturity, the higher the CDS rate. This is due to the greater uncertainty in the credit risk for the longer-term CDS contract. We also have a dummy, equal to 1 if the underlying debt of the contract carry an investment grade and to 0 otherwise.

3.4 Empirical studies

We are interested in finding whether the accounting-based information/models and the market-based information/models have the same performance in the explanation and prediction credit risk. If their performances differ, which will be the most useful in volatile periods of heightened systemic instability or at turning points of credit cycles? In particular, we are interested in the relative reliability of these two sets of information/models for the recent crisis period.

To answer these questions, we investigate various models with the accounting-based variables, with the market-based variables and with both the accounting-based and market-based variables. Furthermore, for each of the models, we will fit the model to two sets of data: one dataset represents a relatively quiet market, and the other represents a relatively volatile market.

We partition samples into the pre-crisis sample and the post-crisis sample. We choose the third quarter of 2008 as the cut-off quarters, taking into account the turmoil in the US when Lehman Brothers failed in September 2008. That is, the data in a quarter prior the second quarter of 2008 belong to the pre-crisis sample, while the data in a quarter after the third quarter of 2008 are used for estimating the post-crisis models. The term of pre-crisis is interchangeably used as before-crisis in the chapter; similarly, the post-crisis is interchangeably used with after-crisis.

In total, our pre-crisis sample contains the data for 18 quarters from 2004Q1 to 2008Q2. The post-crisis sample contains 14 quarters in total from 2008Q3 to 2011Q4. The descriptive statistics for each sample are listed in Table 3.3.

3.4.1 Explanatory models

We set up three types of models:

- model 1: accounting-based models;

Variable	Mean		Standard Deviation	
	Pre-crisis	Post-crisis	Pre-crisis	post-crisis
size	89.63	87.30	131.98	135.99
roa	0.02	0.01	0.02	0.02
incgrowth	0.21	0.55	45.22	48.09
c1	4.12	3.77	1.35	1.54
c2	2.22	1.75	2.26	2.15
c3	2.11	1.39	3.70	3.09
c4	3.72	2.36	13.45	11.38
quick	1.11	1.21	0.61	0.58
cash	0.08	0.08	0.08	0.07
trade	0.61	0.64	0.58	0.62
salesgrowth	0.04	0.01	0.07	0.14
booklev	0.63	0.65	0.16	0.19
retained	0.24	0.18	0.46	0.72
DTD	7.91	5.75	6.49	5.61
ret	0.00	0.01	0.73	1.09
sdret	0.30	0.46	0.24	0.35
r	0.03	0.00	0.01	0.00
snp	0.07	0.00	0.10	0.27
indret	0.13	0.05	0.13	0.30
invgrade	0.74	0.68	0.44	0.47
maturity	4.24	4.20	3.22	3.19
seniority	0.96	0.95	0.20	0.21

Table 3.3: Simple statistics of the explanatory variables

- model 2: market-based models;

- model 3: comprehensive models.

For each type of models, we fit the models to both the pre-crisis and post-crisis samples. It can be observed from Table 3.3 that the standard deviations are large for some variables. Therefore, in order to avoid undesirable effects of extreme values on the model estimation, we winsorise some variables by their 5% and 95% quantiles. That is, the data value greater than the 95% quantile is set to be the 95% quantile and the data value smaller than the 5% quantile is set to be the 5% quantile. The variables winsorised include CS (denoting the CDS spreads), size, roa, incgrowth, quick, cash, trade, salesgrowth, booklev, retained, DTD, ret, sdret, r, snp and indret. We also take logarithm for size considering its relatively large magnitude to other variables.

3.4.1.1 Accounting-based models (M1)

The specification for the accounting-based model follows Eqn (3.9). The estimation results of the model can be found in the second column (Column 2) and the third column (Column 3) of Table 3.4, for the before-crisis and after-crisis periods, respectively.

$$
\begin{aligned}
\ln(\mathrm{CS}_{it}) =& \alpha + \beta_1 \mathrm{size}_{it} + \beta_2 \mathrm{roa}_{it} + \beta_3 \mathrm{incgrowth}_{it} + \beta_4 c1_{it} + \beta_5 c2_{it} + \beta_6 c3_{it} \\
&+ \beta_7 c4_{it} + \beta_8 \mathrm{quick}_{it} + \beta_9 \mathrm{cash}_{it} + \beta_{10} \mathrm{trade}_{it} + + \beta_{11} \mathrm{salesgrowth}_{it} \\
&+ \beta_{12} \mathrm{booklev}_{it} + \beta_{13} \mathrm{retained}_{it} + \beta_{14} r_{it} + \beta_{15} \mathrm{snp}_{it} \\
&+ \beta_{16} \mathrm{indret}_{it} + \beta_{17} \mathrm{invgrade}_{it} + \beta_{18} \mathrm{maturity}_{it} + \beta_{19} \mathrm{seniority}_{it} + \epsilon_{it} \ .
\end{aligned} \tag{3.9}
$$

Column 2 in Table 3.4 provides the results for the pre-crisis sample. Some patterns can be observed as follows.

- Firstly, all variables have significant effects except for quick, cash, salesgrowth and booklev. For these significant variables, the signs of estimated coefficients are all consistent to our expectation except for trade. That is, amongst the accounting-based variables, the size of firms (Log of assets), roa, income growth rate, interest coverage and retained earnings have a negative relationship to the CDS spreads.

- Secondly, the nonlinearity of interest coverage is reflected via the different values of the estimated coefficients and the decreasing significance (as the t-statistics decreases)

Variable	Acc-based model		Market-based model		Comprehensive model	
	Before	After	Before	After	Before	After
(Intercept)	6.41 ***	5.77 ***	5.88 ***	5.06 ***	6.39 ***	5.12 ***
	(110.38)	(93.43)	(154.95)	(126.9)	(106.51)	(77.16)
Log of assets	-0.21 ***	-0.13 ***			-0.15 ***	-0.07 ***
	(-35.11)	(-20.41)			(-25.17)	(-11.2)
roa	-7.05 ***	-9.68 ***			-4.43 ***	-4.71 ***
	(-10.13)	(-14.87)			(-6.76)	(-7.47)
incgrowth	-0.38	-2.62 ***			-1.00 *	-1.56 ***
	(-0.88)	(-6.10)			(-2.48)	(-3.82)
$c1$	-0.09 ***	-0.06 ***			-0.09 ***	-0.05 ***
	(-14.09)	(-10.29)			(-15.33)	(-9.00)
$c2$	-0.02 ***	-0.05 ***			-0.03 ***	-0.05 ***
	(-6.26)	(-10.81)			(-8.07)	(-11.96)
$c3$	-0.01 ***	0.01 ***			-0.01 ***	0.01 *
	(-4.03)	(4.54)			(-4.51)	(2.42)
$c4$	0.00 **	0.00 *			0.00 ∘	0.00 ***
	(-2.94)	(-2.50)			(-1.68)	(-3.29)
quick	0.02	-0.04 *			0.06 ***	-0.02 ∘
	(1.21)	(-2.46)			(5.07)	(-1.79)
cash	0.15	1.02 ***			-0.12	0.77 ***
	(1.57)	(9.36)			(-1.34)	(7.44)
trade	-0.18 ***	-0.07 ***			-0.13 ***	-0.05 ***
	(-14.97)	(-5.75)			(-11.16)	(-3.86)
salesgrowth	-0.22	0.94 ***			-0.45 ***	0.36 **
	(-1.55)	(6.90)			(-3.33)	(2.77)
booklev	-0.03	0.34 ***			-0.07 ∘	0.40 ***
	(-0.72)	(6.98)			(-1.67)	(8.58)
retained	-0.34 ***	-0.35 ***			-0.19 ***	-0.23 ***
	(-13.45)	(-13.02)			(-7.96)	(-8.8)
DTD			-0.10 ***	-0.06 ***	-0.07 ***	-0.04 ***
			(-52.63)	(-26.14)	(-37.51)	(-16.70)
ret			0.43 ***	0.21 ***	0.30 ***	0.09 ***
			(32.39)	(17.88)	(23.93)	(7.94)
sdret			-0.31 ***	1.03 ***	0.06	0.88 ***
			(-6.53)	(23.95)	(1.25)	(21.01)
r	-14.91 ***	-18.72 ***	-13.41 ***	-20.45 ***	-13.09 ***	-15.96 ***
	(-38.64)	(-9.56)	(-33.81)	(-10.49)	(-35.87)	(-8.48)
snp	-1.72 ***	-0.45 ***	-0.66 ***	0.53 ***	-0.96 ***	0.21 **
	(-26.23)	(-6.74)	(-9.75)	(7.77)	(-15.14)	(3.13)
indret	-0.46 ***	-0.57 ***	-0.73 ***	-0.53 ***	-0.59 ***	-0.40 ***
	(-9.82)	(-9.16)	(-15.67)	(-8.52)	(-13.02)	(-6.61)
invgrade	-1.10 ***	-0.75 ***	-1.36 ***	-0.91 ***	-0.96 ***	-0.62 ***
	(-73.49)	(-50.65)	(-106.1)	(-70.57)	(-66.05)	(-42.76)
maturity	0.15 ***	0.08 ***	0.15 ***	0.08 ***	0.15 ***	0.08 ***
	(101.25)	(51.80)	(99.15)	(51.29)	(108.13)	(54.63)
seniority	-0.27 ***	-0.12 ***	-0.29 ***	-0.15 ***	-0.24 ***	-0.11 ***
	(-11.1)	(-4.88)	(-11.89)	(-5.79)	(-10.63)	(-4.48)
R^2	73.53%	58.18%	72.15%	57.34 %	76.77 %	62.46
Adj. R^2	73.50%	58.13 %	72.13%	57.32 %	76.74%	62.41
N	16103	15922	16103	15922	16103	15922

Table 3.4: Estimation results. (The numbers in bracket are t-statistics. The significance codes: $0 *{*}*$; 0.001 $**$; 0.01 $*$; 0.05 $\circ$)

from $c1$ to $c4$, which confirms the conjecture that an increased value of interest coverage provides little additional information to the firm's performance.

- Thirdly, as expected, the macro-economic variables are all statistically significant and are negatively related to the CDS spreads, indicating the sensitivity of the CDS spreads to the macro-economic environment.

- Finally, regarding the contract-specific variables, both maturity and seniority are significant and have the expected signs: a positive sign for maturity and a negative sign for seniority.

Column 3 of Table 3.4 presents the results for the post-crisis period. For comparison between the model for the before-crisis period and the model for the after-crisis period, we identify changes in significance and sign of the estimated coefficients and in the adjusted R^2 of the models, from the before-crisis to the after-crisis. Based on the comparison, some interesting patterns of change and their implications are discussed as follows.

- The nonsignificant variables (quick, cash, sales growth and booklev) in the pre-crisis model become significant. This indicates that the financial liquidity factor (represented by quick and cash ratio) is more relevant to the firm's creditworthiness in a crisis market than in a relative quiet market, and the liquidity variation plays a more important role in affecting the credit spread widening in the financial crisis.

- As for the changes in the sign, the sales growth changes sign, from negative to positive. Although the negative sign is not consistent to the expected univariate relationship, this is also observed in Das et al. (2009). The book leverage changes sign, from negative to positive; the quick ratio changes sign, from positive to negative; both are consistent to our expectation. Cash is significant in the after-crisis period, and in a positive association with the CDS spreads. This is somewhat counterintuitive as one would expect the more cash holding the lower CDS rate. This could be explained by the fact that the firm prefers holding cash to making investment in a crisis period; and the market views such holding-cash behaviour as a temporary decision of the firm. The decision benefits the short-term liability, but is not good in meeting the long-term liability.

- Further comparing the models on the pre-crisis and post-crisis periods, we find that the adjusted coefficient of determination R^2 decreased by 26% from 74% to 58%, indicating that the explanatory power of the model decreased. This may be due to the increased volatility in latent factors that have not been captured by our accounting and market variables. One such factor could be the liquidity risk predominating in the financial sector during the crisis time period. The liquidity risk in financial sectors eventually spreads out to other industries.

3.4.1.2 Market-based models (M2)

The specification of the market-based model (Model 2) follows Eqn (3.10). Besides the market-based variables, here we also include the macro-economic variables as well as the contract-specific variables.

$$\begin{aligned}\ln(\mathrm{CS}_{it}) =& \alpha + \beta_1 \mathrm{DTD}_{it} + \beta_2 \mathrm{ret}_{it} + \beta_3 \mathrm{sdret}_{it} \\ & + \beta_4 r_{it} + \beta_5 \mathrm{snp}_{it} + \beta_6 \mathrm{indret}_{it} + \beta_7 \mathrm{invgrade}_{it} \\ & + \beta_8 \mathrm{maturity}_{it} + \beta_9 \mathrm{seniority}_{it} + \epsilon_{it} \ .\end{aligned} \tag{3.10}$$

Columns 4-5 in Table 3.4 provide the estimation results for the pre-crisis period and the post-crisis period, respectively.

All the market variables are significant in both the post-crisis and pre-crisis periods. The macro-economic and contract-specific variables are also all significant in both periods and display the same pattern of signs as when they appeared in the accounting-based model. The overall fit of the market-based model is very close to that obtained in the accounting-based model in both the pre-crisis and post-crisis periods. Given that the market-based model is more parsimonious, with three market variables compared to thirteen accounting variables, this perhaps is surprising and may be taken as a strength of this model. While the sign of the coefficient on the variable sdret does switch from negative in the pre-crisis sample to positive in the post-crisis period, this does appear more stable than in the case of the accounting-based model where there are four changes of sign.

3.4.1.3 Comprehensive models (M3)

Putting accounting and market variables together as well as the macro-economic and contract-specific variables, we set up a comprehensive model as in Eqn (3.11).

$$\begin{aligned}\ln(\mathrm{CS}_{it}) =& \alpha + \beta_1 \mathrm{size}_{it} + \beta_2 \mathrm{roa}_{it} + \beta_3 \mathrm{incgrowth}_{it} + \beta_4 c1_{it} + \beta_5 c2_{it} + \beta_6 c3_{it} \\ & + \beta_7 c4_{it} + \beta_8 \mathrm{quick}_{it} + \beta_9 \mathrm{cash}_{it} + \beta_{10} \mathrm{trade}_{it} + \beta_{11} \mathrm{salesgrowth}_{it} \\ & + \beta_{12} \mathrm{booklev}_{it} + \beta_{13} \mathrm{retained}_{it} + \beta_{14} \mathrm{DTD}_{it} + \beta_{15} \mathrm{ret}_{it} + \beta_{16} \mathrm{sdret}_{it} \\ & + \beta_{17} r_{it} + \beta_{18} \mathrm{snp}_{it} + \beta_{19} \mathrm{indret}_{it} + \beta_{20} \mathrm{invgrade}_{it} \\ & + \beta_{21} \mathrm{maturity}_{it} + \beta_{22} \mathrm{seniority}_{it} + \epsilon_{it} \ .\end{aligned} \tag{3.11}$$

The estimation results are listed in Columns 6-7 of Table 3.4. Some thought-provoking patterns can be observed from the comparison between the comprehensive models and the two basic models. Examples include the following.

- We find that, most of the variables, that are statistically significant in the two basic models, remain statistically significant in the comprehensive model. This indicates that the two sets of information, the accounting-based and the market-based, are complementary. This pattern is reliable for both periods. We note that variables used in estimating DtD are then used again in a regression to explain the size of the CDS premium (credit spread). We have followed the wide-spread practice of implementing distance to default using the historical distribution of firm value. This can be justified by the empirical nature of our model. In applications of the Merton model for pricing purposes distance to default theoretically should be measured from the risk neutral distribution. In principle, this might give rise to different results in empirical applications such as our own; however, we have not explored this here.

- As for the explanatory power, the comprehensive models get improved in both the pre-crisis and post-crisis periods, compared with the basic models. For example, for the pre-crisis period, the accounting-based variables are able to explain 74% of the variation of the CDS rates, the market-based variables can explain 72%, while the combination of both accounting-based and market-based variables is able to explain 77%. These results are consistent with the findings in Agarwal and Taffler (2008).

- Similarly to that for the two basic models, we observe that the adjusted R^2 falls from 77% to 62% from the pre-crisis period to the post-crisis period. As discussed before, this may reflect the increased volatility in latent factors that has not been captured by our accounting and market variables. One such factor could be the liquidity risk predominating in the financial sector during the crisis period. Concentrating on financial sectors, Gefang et al. (2011) suggest the importance of the liquidity risk relative to the credit risk on the financial crisis. They find that, for short terms especially for the 1 month and 3 month terms, the role of the liquidity risk is much more important. Since our accounting and market variables are designed to capture the credit risk rather than the liquidity risk, we would expect our model is better fit for the long-term CDS spreads than for short-term ones. To confirm this we fit the comprehensive models to the 1-year CDS spreads and to the 5-year CDS spreads respectively, and we find that the adjusted R^2 for the former is 0.58 while for the latter is 0.64. Alternatively, the decline in the model fit may reflect an increase in the sensitivity of the CDS pricing to the perceived counter-party risk in these OTC derivatives contracts.

3.4.2 Predictive models

The focus so far has been on the ability of alternative sets of information to account for the cross-sectional variation of the CDS spreads across different firms. In practical applications we might consider the use of these models in portfolio choice. For example, the models might be used to construct portfolios of over-valued and under-valued contracts which could be used to take convergence type risk-arbitrage trades. Or the models might be used to extrapolate in cross-section, for example, in pricing CDS on a name that is not currently quoted in the market. However, for predictions over time, the models as specified may need to be adapted. The accounting-based, market-based and comprehensive models that we have considered so far all use contemporaneous explanatory variables as in Agarwal and Taffler (2008) and Das et al. (2009). To make time-series predictions one could attempt to forecast the explanatory variables and find the implied forecast. Given the number of explanatory variables involved, however, this seems unlikely to be the most practicable approach in most contexts. Consequently, we develop predictive models based on lagged variables.

Specifically we will use predictive regression models estimated on the data from the pre-crisis period to predict the CDS spreads for the post-crisis period. In general, we regress log(CS) on the one-quarter lagged explanatory/predictor variables. We use various combinations of lagged predictors for the models M1-M3, as illustrated below:

$$
\begin{aligned}
\text{Pred.M1}: \quad & \text{CDS}_t = \text{acc}_{t-1} + \phantom{\text{market}_{t-1}} + \text{macro}_{t-1} + \text{contract}_t\ ; \\
\text{Pred.M2}: \quad & \text{CDS}_t = \phantom{\text{acc}_{t-1} +} \ \text{market}_{t-1} + \text{macro}_{t-1} + \text{contract}_t\ ; \\
\text{Pred.M3}: \quad & \text{CDS}_t = \text{acc}_{t-1} + \text{market}_{t-1} + \text{macro}_{t-1} + \text{contract}_t\ ,
\end{aligned}
$$

where acc_{t-1}, market_{t-1} and macro_{t-1} denote accounting-based, market-based and macro-economic variables with a one-quarter lag, respectively.

In more detail, the predictive model 1 (Pred.M1) accounts for the lagged accounting and macro-economic variables, as specified in Eqn (3.12):

$$
\begin{aligned}
\ln(\text{CS}_{it}) =& \alpha + \beta_1 \text{size}_{i,t-1} + \beta_2 \text{roa}_{i,t-1} + \beta_3 \text{incgrowth}_{i,t-1} + \beta_4 c1_{i,t-1} + \beta_5 c2_{i,t-1} \\
& + \beta_6 c3_{i,t-1} + \beta_7 c4_{i,t-1} + \beta_8 \text{quick}_{i,t-1} + \beta_9 \text{cash}_{i,t-1} + \beta_{10} \text{trade}_{i,t-1} \\
& + \beta_{11} \text{salesgrowth}_{i,t-1} + \beta_{12} \text{booklev}_{i,t-1} + \beta_{13} \text{retained}_{i,t-1} \\
& + \beta_{14} r_{i,t-1} + \beta_{15} \text{snp}_{i,t-1} + \beta_{16} \text{indret}_{i,t-1} + \beta_{17} \text{invgrade}_{i,t-1} \\
& + \beta_{18} \text{maturity}_{it} + \beta_{19} \text{seniority}_{it} + \epsilon_{it}\ .
\end{aligned} \tag{3.12}
$$

In the predictive model 2 (Pred.M2), the market variables and macro-economic variables are lagged by one quarter as in Eqn (3.13):

$$
\begin{aligned}
\ln(\text{CS}_{it}) =& \alpha + \beta_1 \text{DTD}_{i,t-1} + \beta_2 \text{ret}_{i,t-1} + \beta_3 \text{sdret}_{i,t-1} \\
& + \beta_4 r_{i,t-1} + \beta_5 \text{snp}_{i,t-1} + \beta_6 \text{indret}_{i,t-1} + \beta_7 \text{invgrade}_{i,t-1} \\
& + \beta_8 \text{maturity}_{it} + \beta_9 \text{seniority}_{it} + \epsilon_{it}\ .
\end{aligned} \tag{3.13}
$$

The predictive model 3 (Pred.M3) includes the lagged accounting, market and macro-economic variables, as shown in Eqn (3.14):

$$
\begin{aligned}
\ln(\text{CS}_{it}) =& \alpha + \beta_1 \text{size}_{i,t-1} + \beta_2 \text{roa}_{i,t-1} + \beta_3 \text{incgrowth}_{i,t-1} + \beta_4 c1_{i,t-1} + \beta_5 c2_{i,t-1} \\
& + \beta_6 c3_{i,t-1} + \beta_7 c4_{i,t-1} + \beta_8 \text{quick}_{i,t-1} + \beta_9 \text{cash}_{i,t-1} + \beta_{10} \text{trade}_{i,t-1} \\
& + \beta_{11} \text{salesgrowth}_{i,t-1} + \beta_{12} \text{booklev}_{i,t-1} + \beta_{13} \text{retained}_{i,t-1} \\
& + \beta_{14} \text{DTD}_{i,t-1} + \beta_{15} \text{ret}_{i,t-1} + \beta_{16} \text{sdret}_{i,t-1} \\
& + \beta_{17} r_{i,t-1} + \beta_{18} \text{snp}_{i,t-1} + \beta_{19} \text{indret}_{i,t-1} + \beta_{20} \text{invgrade}_{i,t-1} \\
& + \beta_{21} \text{maturity}_{it} + \beta_{22} \text{seniority}_{it} + \epsilon_{it}\ .
\end{aligned} \tag{3.14}
$$

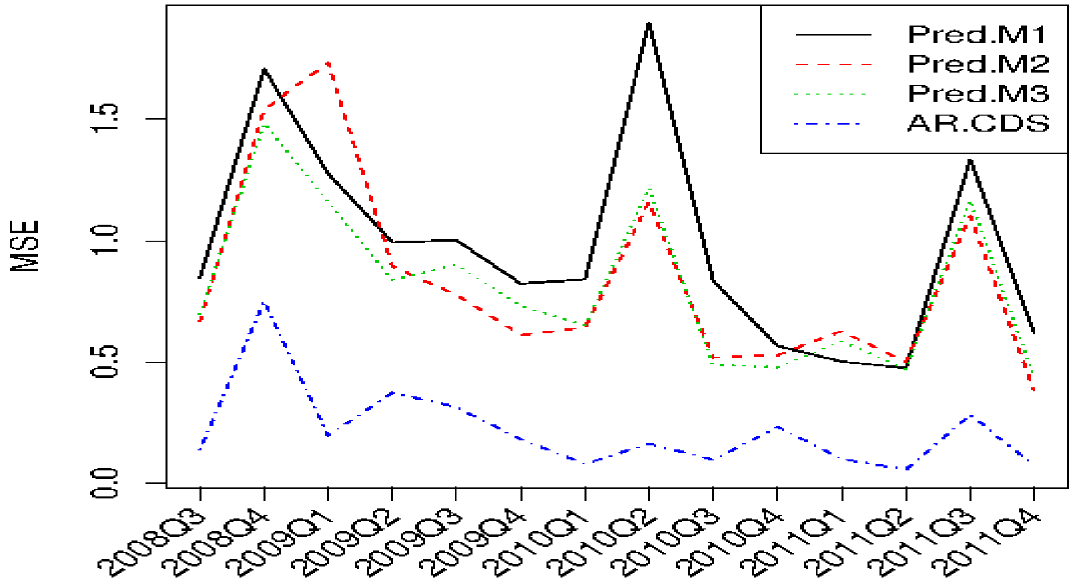


Figure 3.1: Prediction mean squared errors

	2008Q3	2008Q4	2009Q1	2009Q2	2009Q3	2009Q4	2010Q1	2010Q2	2010Q3	2010Q4	2011Q1	2011Q2	2011Q3	2011Q4
Pred.M1	0.85	1.71	1.28	0.99	1.00	0.82	0.84	1.90	0.83	0.56	0.50	0.47	1.33	0.62
Pred.M2	0.67	1.54	1.73	0.89	0.77	0.61	0.64	1.16	0.52	0.53	0.63	0.50	1.10	0.38
Pred.M3	0.70	1.48	1.16	0.84	0.90	0.73	0.65	1.22	0.49	0.48	0.59	0.47	1.17	0.44
AR.CDS	0.14	0.75	0.20	0.37	0.31	0.18	0.08	0.16	0.10	0.23	0.10	0.05	0.28	0.07

Table 3.5: Prediction mean squared errors

For these 3 cross-sectional predictive models, Figure 3.1 and Table 3.5 present the mean squared errors (MSE) over all firms at each quarter for the post-crisis period. Specifically, the prediction error is the logarithm of the ratio of the predicted spread to the observed spread.

If the MSE for quarter Q is large, it can be implied that the properties of the predictor variables at $Q-1$ are quite different from those in the pre-crisis period. That is, the economic situation described by the variable spaces has changed from the pre-crisis period. On the other end, if the MSE is small for quarter Q, then the economic status in quarter $Q-1$ is similar to that in the pre-crisis period.

From Figure 3.1 and Table 3.5 we can observe a number of interesting patterns.

- In all but three quarters the market-based model has a smaller prediction error than the accounting-based model.

- The accounting-based, market-based, and comprehensive models all have very large prediction errors in three periods— the immediate aftermath of Lehman Brothers in 2008Q4 and 2009Q1, in 2010Q2 when the fears of double-dip recession were great, and in 2011Q3 when the euro crisis had reach alarming proportions.

- Some details of timing might suggest different relative strengths of different information sources. As for Pred.M1, where only the accounting information and the macro-economic information are lagged, we can observe that the MSE hits the highest in the second quarter of 2010. Because of the one-quarter lag, the economic status explained by the accounting variables refers to the first quarter of 2010. Hence this indicates that the economic environment (expanded mainly by the accounting and macro-economic information) during the first quarter of 2010 is remarkably different on average from that in the pre-crisis period. Therefore the quarter could be a time point that the economy is enduring a potential transition.

- Throughout all the predictive models, Pred.M1, Pred.M2 and Pred.M3, we can observe a common trend that the MSEs for the models start relatively low, increase in the forth quarter of 2008, decrease back to the level of the third quarter of 2008 and hit a peak in the second quarter of 2010. Since then, the MSEs continuously decline for 3 quarters. The common trend indicates that there exists two stages. The first stage is from 2008Q3 to 2010Q1, where the economy behaviours were quite different from that in the pre-crisis period. The second stage starts from 2010Q2 where the economy moved towards the before-crisis level, a sign of potential recovery.

- Figure 3.1 also shows that overall the comprehensive model performs the best in the cross-sectional prediction.

All these are comparisons of predictive versions of the cross-sectional models considered so far. However, for pure predictions over time, it is interesting to compare these to a simple, pure time-series model. We have estimated such a model as well. Specially, we set up a fourth-order auto-regression model, as displayed in Eqn (3.15) for the CDS spreads over the pre-crisis period, and then use the fitted model to obtain the predictive MSE for the post-crisis period.

$$\ln(\mathrm{CS}_{it}) = \alpha + \beta_1 \ln(\mathrm{CS}_{i,t-1}) + \beta_2 \ln(\mathrm{CS}_{i,t-2}) + \beta_3 \ln(\mathrm{CS}_{i,t-3}) + \beta_4 \ln(\mathrm{CS}_{i,t-4}) + \epsilon_{it} \quad (3.15)$$

The MSE for the AR benchmark model are presented in Figure 3.1 and Table 3.5. It is noted that the AR time-series model outperforms all the cross-sectional models. In our opinion, there may be two reasons for this phenomenon. First, there may be some omitted variables in the cross-sectional models. Second, the variation of CDS rates may be driven by its own supply-demand shocks, but such information is not able to be captured by the cross-sectional models. If these omitted variables or shocks can be represented by persistent latent variables, this may be reflected in autocorrelated residuals in the cross-sectional models, something that is not typically considered in the previous literature. This motivates us to our further investigation in Section 3.4.3.

3.4.3 Time-series models or cross-sectional models?

3.4.3.1 Autocorrelation check for the residuals

We examine the autocorrelation of the residuals that we obtained from the estimation for Model 1, Model 2 and Model 3, presented in sections 3.4.1.1, 3.4.1.2 and 3.4.1.3, respectively. We fit a fourth-order AR model to the residuals, taking into account possible seasonal effects. That is, the model is set up as

$$\mathrm{Resid}_{it} = \alpha + \beta_1 \mathrm{Resid}_{i,t-1} + \beta_2 \mathrm{Resid}_{i,t-2} + \beta_3 \mathrm{Resid}_{i,t-3} + \beta_4 \mathrm{Resid}_{i,t-4} + \epsilon_{it}\ . \quad (3.16)$$

The results are listed in Table 3.6, where resid.lag1-resid.lag4 denote the first-fourth orders AR terms of the residuals. We see that there is a very large positive coefficient at first-order lag for all the models in both the pre-crisis and post-crisis periods. However, the AR terms are significant out to four lags. That is, all t-statistics are highly significant, at least at the 1% significance level. We interpret this to be strong confirmation of our conjecture that there is some persistent latent variable that has been omitted from the cross-sectional models. Therefore, we alter the specification to take this into account.

Our approach is to include the lagged CDS spreads into the cross-sectional models. We are expecting that the addition of lagged dependent variables will reduce the autocorrelation in the residuals, and will improve the models' performance in terms of explanatory power and prediction power. Hence, on the basis of the accounting-based, market-based and comprehensive models described in Section 3.4, we develop new models with additional 1-4 lags of the log CDS spreads.

Variable	Acc-based model		Market-based model		Comprehensive model	
	Before	After	Before	After	Before	After
(Intercept)	0.04	0.02	0.03	0.01	0.04	0.01
	11.91	5.63	7.50	2.79	10.22	3.33
resid.lag1	0.71	0.84	0.57	0.65	0.59	0.70
	74.70	86.91	60.63	66.71	62.21	71.70
resid.lag2	0.08	-0.06	0.14	0.15	0.14	0.11
	6.83	-4.66	13.19	12.68	12.94	9.00
resid.lag3	0.04	-0.07	0.05	-0.05	0.06	-0.06
	3.05	-5.61	4.96	-4.01	5.11	-5.26
resid.lag4	0.02	0.14	0.06	0.09	0.03	0.09
	2.10	15.82	6.06	10.12	2.83	10.27
R^2	66.43%	70.02%	56.41%	66.57%	55.34%	65.62%
Adj R^2	66.41%	70.01%	56.39%	66.55%	55.33%	65.60%
N	16103	15922	16103	15922	16103	15922

Table 3.6: Estimation results of the AR(4) models for residuals (The t-statistics are reported below the coefficients)

Variable	Acc-based model		Market-based model		Comprehensive model	
	Before	After	Before	After	Before	After
(Intercept)	1.76***	1.12 ***	1.59 ***	0.57 ***	1.86***	0.68 ***
	(30.57)	(22.37)	(40.92)	(16.22)	(31.77)	(13.71)
size	-0.03***	-0.05 ***			-0.02 ***	-0.02***
	(-7.4)	(-12.27)			(-3.99)	(-5.82)
roa	-2.00 ***	-2.38***			-1.09 *	-0.3
	(-4.25)	(-5.95)			(-2.38)	(-0.8)
incgrowth	-0.4	-3.19 ***			-0.66 *	-2.06 ***
	(-1.36)	(-11.98)			(-2.33)	(-8.4)
c1	-0.02***	0.00			-0.02 ***	0.00
	(-4.9)	(-0.05)			(-5.57)	(1.09)
c2	0.00	-0.01 **			0.00	-0.01 ***
	(1.5)	(-2.97)			(0.18)	(-5.97)
c3	0.00	0.00 **			0.00	0.00
	(-1.29)	(2.68)			(-1.45)	(0.12)
c4	0.00	0.00			0.00	0.00
	(-1.24)	(1.04)			(-0.85)	(0.12)
quick	-0.01	-0.02 *			0.01	-0.02 **
	(-0.87)	(-2.3)			(0.61)	(-2.97)
cash	-0.29***	0.1			-0.3 ***	0.12 $^{\circ}$
	(-4.24)	(1.48)			(-4.56)	(1.87)
trade	-0.03***	-0.01			-0.01	0.00
	(-3.46)	(-1.41)			(-1.34)	(-0.44)
salesgrowth	-0.29 **	1.25***			-0.3 **	0.7 ***
	(-2.86)	(14.8)			(-3.07)	(9.01)
booklev	0.00	0.11 ***			-0.02	0.1 ***
	(0.01)	(3.63)			(-0.77)	(3.4)
retained	0.02	-0.11 ***			0.05 **	-0.05 ***
	(1.19)	(-6.44)			(2.89)	(-3.36)
DTD			-0.03***	0.00	-0.03***	0.00**
			(-20.25)	(1.27)	(-18.23)	(2.79)
ret			0.01	-0.17***	0.02 $^{\circ}$	-0.17***
			(1.49)	(-24.71)	(1.94)	(-24.59)
sdret			-0.15 ***	0.7 ***	-0.07 *	0.69 ***
			(-5.01)	(28.04)	(-2.2)	(26.89)
r	-20.68***	0.71	-17.75***	-1.12	-17.78***	-2.75*
	(-42.78)	(0.51)	(-36.87)	(-0.88)	(-37.14)	(-2.13)
snp	1.12***	-0.14**	1.12***	0.15 ***	1.05 ***	0.12**
	(15.09)	(-3.29)	(15.68)	(3.67)	(14.53)	(2.91)
indret	-0.19***	-0.35 ***	-0.15***	-0.03	-0.13***	-0.05
	(-5.63)	(-9.21)	(-4.85)	(-0.73)	(-3.97)	(-1.5)
invgrade	-0.13 ***	-0.13 ***	-0.13***	-0.1 ***	-0.12 ***	-0.07***
	(-9.83)	(-12.69)	(-10.57)	(-11.05)	(-9.63)	(-7.36)
maturity	0.02 ***	0.02 ***	0.01***	0.02 ***	0.02 ***	0.02 ***
	(10.35)	(19.22)	(10.87)	(20.11)	(13.63)	(21.5)
seniority	-0.03 *	-0.01	-0.02	-0.02	-0.03	-0.01
	(-1.97)	(-0.68)	(-1.5)	(-1.17)	(-1.62)	(-1.01)
ln(CS)$_{i,t-1}$	0.71 ***	0.86 ***	0.70***	0.84 ***	0.68 ***	0.83 ***
	(76.12)	(100.39)	(76.96)	(104.85)	(75.09)	(104.25)
ln(CS)$_{i,t-2}$	0.12 ***	-0.25 ***	0.12***	-0.14***	0.12 ***	-0.13 ***
	(10.27)	(-23.54)	(10.38)	(-13.35)	(9.91)	(-13.27)
ln(CS)$_{i,t-3}$	0.05 ***	0.21 ***	0.07 ***	0.14 ***	0.06 ***	0.13 ***
	(3.78)	(19.97)	(5.6)	(13.9)	(5.09)	(13.28)
ln(CS)$_{i,t-4}$	-0.03***	-0.02 *	-0.04 ***	-0.02 **	-0.04 ***	-0.03 ***
	(-3.34)	(-2.01)	(-3.97)	(-3.04)	(-4.32)	(-3.64)
R^2	91.70%	85.34 %	92.12 %	87.39 %	92.25 %	87.68%
Adj. R^2	91.68 %	85.32 %	92.12 %	87.38 %	92.24 %	87.65%
N	11188	14913	11188	14913	11188	14913

Table 3.7: Estimation results with the lagged dependent variables (The numbers in bracket are t-statistics. The significance codes: 0 $***$; 0.001 $**$; 0.01 $*$; 0.05 $\circ$)

The estimation results for these new models are presented in Table 3.7. Comparing with the results in Table 3.4 where no lagged CDS spreads was added, we can make the following observations.

- Firstly, we find that the adjusted R^2 are greatly increased for all the models. This could be due to that the lagged CDS spreads incorporate variation of some latent variables.
- Secondly, amongst the accounting-based model, the market-based model and the comprehensive model, the comprehensive model remains the best one in the sense of producing the highest R^2, although the margin is not substantial.
- Thirdly, overall, the inclusion of the lagged CDS spreads into the models reduces the number of significant accounting variables. For example, in the comprehensive model for the before-crisis sample, interest coverage ($c2$ and $c3$), trade and book leverage become nonsignificant. This implies that these variables explained some variation between the CDS spreads.
- Fourthly, comparing corresponding models for the before-crisis sample and the after-crisis sample, we can find patterns similar to those discussed before, such as the pattern that quick ratio and booklev change to the expected signs.
- Finally, the inclusion of lagged dependent variables results in a dramatic change in the market-based model. Specifically, the important distance to default variable is now insignificant in the post-crisis period.

Before proceeding to examine the predictive performance of this version of our models we check whether the residuals still poses AR characteristics. Therefore, we fit the AR(4) models for the new residuals as done in the beginning of this section. The new estimation results are shown in Table 3.8. Comparing with the results with no lagged dependent variables (as in Table 3.6), we can find that

- the coefficients of the lagged residuals become less significant and close to zero;
- the adjusted R^2 are decreased from a range of 55%-75% to a range of 0%-6%.

This indicates no evidence that the residuals are still autocorrelated.

Variable	Acc-based model		Market-based model		Comprehensive model	
	Before	After	Before	After	Before	After
(Intercept)	0.04	0.00	0.03	0.00	0.03	0.00
	9.06	-0.35	8.37	0.85	8.35	0.75
resid.lag1	0.02	0.00	-0.01	-0.02	0.00	-0.02
	2.06	-0.35	-0.65	-1.85	-0.27	-2.28
resid.lag2	-0.04	0.11	-0.06	0.10	-0.05	0.10
	-3.57	11.04	-4.58	10.01	-4.19	9.63
resid.lag3	-0.02	-0.15	0.00	-0.07	0.00	-0.06
	-1.40	-16.75	-0.20	-7.16	0.13	-6.77
resid.lag4	0.05	-0.10	0.05	-0.08	0.06	-0.07
	3.62	-11.47	4.15	-9.23	4.52	-7.61
R^2	0.45 %	5.73%	0.56%	2.57%	0.55%	2.16
Adj R^2	0.39%	5.69%	0.5%	2.52%	0.49%	2.12
N	11188	14913	11188	14913	11188	14913

Table 3.8: Estimation results of the AR(4) models for residuals with the lagged dependent variables (The t-statistics are reported below the coefficients)

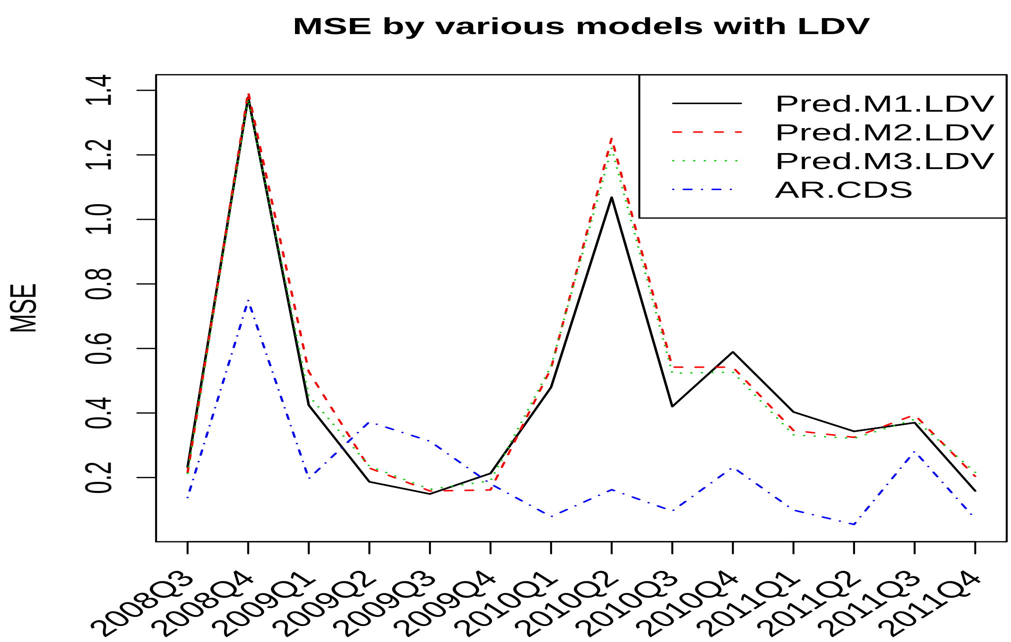


Figure 3.2: Prediction mean squared errors for models with lagged dependent variables

3.4.3.2 Predictive MSE for the models with lagged CDS spreads

We note that the standard errors need to be corrected for cross firm effects and autocorrelation. This correction can be conducted by estimating clustered standard errors. Alternatively, one can use lagged dependent variables to mitigate the autocorrelation.

Now we examine the predictive performance of the new models that have been built by adding the lagged dependent variables into the models Pred.M1 - Pred.M3. The new models are denoted by Pred.M1.LDV - Pred.M3.LDV. As with section 3.4.2, we predict the CDS spreads in the post-crisis period, calculate the MSEs and aggregate the MSEs over firms at each quarter. The aggregated MSEs are listed in Table 3.9 and plotted in Figure 3.2. From these results, we observe the following.

- The predictive performances of the accounting-based, market-based and comprehensive models are quite comparable to each other.
- At 2010Q2 the MSE remains high for the cross-sectional models.
- The AR model of the CDS spreads remains the best predictive model, outperforming the cross-sectional models and producing the lowest MSE in all but three quarters.

	2008Q3	2008Q4	2009Q1	2009Q2	2009Q3	2009Q4	2010Q1	2010Q2	2010Q3	2010Q4	2011Q1	2011Q2	2011Q3	2011Q4
Pred.M1.LDV	0.23	1.38	0.42	0.19	0.15	0.21	0.48	1.07	0.42	0.59	0.40	0.34	0.37	0.16
Pred.M2.LDV	0.21	1.39	0.53	0.23	0.16	0.16	0.54	1.25	0.54	0.54	0.35	0.32	0.39	0.20
Pred.M3.LDV	0.21	1.37	0.45	0.24	0.16	0.19	0.55	1.22	0.52	0.53	0.33	0.32	0.38	0.22
AR.CDS	0.14	0.75	0.20	0.37	0.31	0.18	0.08	0.16	0.10	0.23	0.10	0.05	0.28	0.07

Table 3.9: Prediction mean squared errors of models with lagged dependent variables

3.5 Conclusions and future work

Using the CDS spreads as the credit risk measure, we have examined the performance, in terms of the explanatory and prediction powers, of the accounting-based models, the market-based models and a model combining both accounting-based and market-based information. We have particularly investigated their performance over the transition from the pre-crisis period to the post-crisis period, using Lehman Brothers' failure in the third quarter of 2008 as the turning-point to separate the pre-crisis and post-crisis periods.

Based on our investigation, we have found that the accounting information and the market information are complementary in the explanation of firms' performance and the prediction of firms' distress. This finding confirms the assessment by Das et al. (2009).

We have also found that the explanatory performance of the accounting variables changes with the economic environment, while that for the market variables is more reliable.

We have used the one-quarter predictive models, which was fitted by using the pre-crisis data, to predict the one-quarter ahead CDS spreads for the post-crisis period. Using the mean squared error as a measure of the predictive power, we have found that, amongst these the three cross-sectional models, the comprehensive model performs the best in the prediction of distress. In addition, we have found that in the first quarter of 2010 there could have been a potential structure break of the economic situation, a sort of turning-point of economy starting to recover.

We have found evidence that the pure cross-sectional models may omit some persistent latent variable. We have then added lagged CDS spreads into the models in order to tackle the autocorrelated residuals and potentially-omitted predictive variables. The inclusion of the lagged dependent variables has improved greatly the model fitting and the predictive MSE.

In summary, from our studies we have observed the following two patterns. Firstly, compared with the accounting-based models and the comprehensive model, the market-based models perform the best in the explanation of the CDS spreads, in the sense of having a comparable explanatory power and being more parsimonious. Secondly, if we only look for an optimal prediction of the CDS spreads, an AR time-series model of the CDS spreads would outperform the cross-sectional models.

For further research we would like to make three suggestions. As we have noted, the standard errors of estimates need to be corrected for cross firm effects and autocorrelation. So the first future work would be to conduct the correction by estimating clustered standard errors for our models, and the second future work would be to compare the corrected models to the regression models with lagged dependent variables. The third future work that we are interested in is to establish the link between credit ratings and CDS spreads, that is, to investigate how sensitive the credit rating is to variations of CDS spreads. More specifically, for a fixed rating level, we expect to establish a threshold of the variation in CDS spreads, above which the rating migration will occur with a high probability. If the link can be established, this will greatly facilitate the separation of rating migration risk and default risk from market risk.

Bibliography

Agarwal, V., Taffler, R., 2008. Comparing the performance of market-based and accounting-based bankruptcy prediction models. Journal of Banking & Finance 32 (8), 1541–1551.

Agresti, A., 2007. An Introduction to Categorical Data Analysis, 2nd Edition. John Wiley & Sons, Inc., Hoboken, New Jersey.

Alsakka, R., Gwilym, O. a., 2010. A random effects ordered probit model for rating migrations. Finance Research Letters 7 (3), 140–147.

Altman, E. I., 1968. Financial ratios, discriminant analysis, and the prediction of corporate bankruptcy. The Journal of Finance 23 (4), 589–609.

Altman, E. I., 1998. The importance and subtlety of credit rating migration. Journal of Banking & Finance 22 (10-11), 1231–1247.

Altman, E. I., Haldeman, R., Narayanan, P., 1977. Zeta analysis: A new model to identify bankruptcy risk of corporations. Journal of Banking & Finance 1 (1), 29–54.

Altman, E. I., Kao, D. L., 1991. Corporate Bond Rating Drift: An Examination of Credit Quality Rating Changes Over Time. Research Foundation Publications.

Altman, E. I., Kao, D. L., 1992. The implications of corporate bond ratings drift. Financial Analysts Journal 48 (3), 64–75.

Altman, E. I., Rijken, H. A., 2004. How rating agencies achieve rating stability. Journal of Banking & Finance 28 (11), 2679–2714.

Altman, E. I., Saunders, A., 1998. Credit risk measurement: Developments over the last 20 years. Journal of Banking & Finance 21 (11–12), 1712–1742.

Anderson, R. W., 2009. What accounts for time variation in the price of default risk? London School of Economics, Working Paper.

Anderson, R. W., Sundaresan, S., 1996. Design and valuation of debt contracts. The Review of Financial Studies 9 (1), 37–68.

Bangia, A., Diebold, F. X., Kronimus, A., Schagen, C., Schuermann, T., 2002. Ratings migration and the business cycle, with application to credit portfolio stress testing. Journal of Banking & Finance 26 (2-3), 445–474.

Berndt, A., Douglas, R., Duffie, D., Ferguson, M., Schranz, D., 2005. Measuring default risk premia from default swap rates and EDFs. BIS working paper (173).

Black, F., Cox, J. C., 1976. Valuing corporate securities: Some effects of bond indenture provisions. The Journal of Finance 31 (2), 351–367.

Campbell, J. Y., Hilscher, J., Szilagyi, J., 2008. In search of distress risk. The Journal of Finance 63 (6), 2899–2939.

Cantor, R., 2004. An introduction to recent research on credit ratings. Journal of Banking & Finance 28 (11), 2565–2573.

Chava, S., Jarrow, R. A., 2004. Bankruptcy prediction with industry effects. Review of Finance 8 (4), 537–569.

Das, S. R., Hanouna, P., Sarlin, A., 2009. Accounting-based versus market-baseed cross-selectional models of CDS spreads. Journal of Banking & Finance 33 (4), 719–730.

Driessen, J., 2005. Is default event risk priced in corporate bonds? The Review of Financial Studies 18 (1), 165–195.

Du, Y., 2003. Predicting credit rating and credit rating changes: A new approach. Working paper.

Duffie, D., Kenneth, J. S., 2003. Credit Risk: Pricing, Measurement, and Management. Princeton University Press, Princeton and Oxford.

Duffie, D., Saita, L., Wang, K., 2007. Multi-period corporate default prediction with stochastic covariates. Journal of Financial Economics 83 (3), 635–665.

Duffie, D., Singleton, K. J., 1999. Modeling term structures of defaultable bonds. The Review of Financial Studies 12 (4), 687–720.

Frydman, H., Schuermann, T., 2008. Credit rating dynamics and Markov mixture models. Journal of Banking & Finance 32 (6), 1062–1075.

Gefang, D., Koop, G., Potter, S. M., 2011. Understanding liquidity and credit risks in the fianncial crisis. Journal of Empirical Finance 18 (5), 903–914.

Hillegeist, S. A., Keating, E. K., Cram, D. P., Lundstedt, K. G., 2004. Assessing the probability of bankruptcy. Review of Accounting Studies 9 (1), 5–34.

Hilscher, J., Wilson, M., 2012. Credit ratings and credit risk. SSRN Working Paper.

Jarrow, R. A., Turnbull, S. M., 1995. Pricing derivatives on financial securities subject to credit risk. The Journal of Finance 50 (1), 53–85.

Kim, Y., Sohn, S. Y., 2008. Random effects model for credit rating transitions. European Journal of Operational Research 184 (2), 561–573.

Lando, D., Skodeberg, T. M., 2002. Analyzing rating transitions and rating drift with continuous observations. Journal of Banking & Finance 26 (2-3), 423–444.

Leland, H. E., 1994. Corporate debt value, bond covenants, and optimal capital structure. The Journal of Finance 49 (4), 1213–1252.

Longstaff, F. A., Schwartz, E. S., 1995. A simple approach to valuing risky fixed and floating rate debt. The Journal of Finance 50 (3), 789–819.

Merton, R. C., 1974. On the pricing of corporate debt: The risk structure of interest rates. The Journal of Finance 29 (2), 449–470.

Metz, A., Cantor, R., 2006. Moody's Credit Rating Prediction Model. Moody's Investors Service.

Nickell, P., Perraudin, W., Varotto, S., 2000. Stability of rating transitions. Journal of Banking & Finance 24 (1-2), 203–227.

Odders-White, E. R., Ready, M. J., 2006. Credit ratings and stock liquidity. The Review of Financial Studies 19 (1), 119–157.

Platt, H. D., Platt, M. B., 1991. A note on the use of industry-relative ratios in bankruptcy prediction. Journal of Banking & Finance 15 (6), 1183–1194.

Rubin, D. B., 1987. Multiple Imputation for Nonresponse in Surveys. John Wiley & Sons, Inc, New York.

SAS Institute Inc., 1999. SAS OnlineDoc^{TM} Version 8. Cary, NC, USA.

Schafer, J. L., 1997. Analysis of Incomplete Multivariate Data. Chapman and Hall, New York.

Shumway, T., 2001. Forecasting bankruptcy more accurately: A simple hazard model. The Journal of Business 74 (1), 101–124.

Vassalou, M., Xing, Y., 2004. Default risk in equity returns. The Journal of Finance 59 (2), 831–868.

West, R. C., 1985. A factor-analytic approach to bank condition. Journal of Banking & Finance 9 (2), 253–266.

Xing, H., Sun, N., Chen, Y., 2012. Credit rating dynamics in the presence of unknown structural breaks. Journal of Banking & Finance 36 (1), 78–89.

Zmijewski, M. E., 1984. Methodological issues related to the estimation of financial distress prediction models. Journal of Accounting Research 22, 59–82.