

# **Unemployment History and Frictional Wage Dispersion in Search Models of the Labor Market**

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*To my parents Jaime and Cristina, to my wife Katherine, and to my grandparents Victor and Rose Marie*

## Declaration

I certify that the thesis I have presented for examination for the MPhil/PhD degree of the London School of Economics and Political Science is solely my own work other than where I have clearly indicated that it is the work of others (in which case the extent of any work carried out jointly with me is clearly identified in it). The copyright of this thesis rests with the author. Quotation from it is permitted, provided that full acknowledgement is made. This thesis may not be reproduced without the prior written consent of the author. I warrant that this authorisation does not, to the best of my belief, infringe the rights of any third party.

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# Abstract

This thesis studies the inability of search models to match both observed labor market flows and the empirical wage distribution. I show that a known feature of the labor market, that unemployment hurts workers' wages, has an important effect on workers' search behavior, and explains why we observe that similar workers are paid different wages.

The first chapter reviews the relevant literature. I begin by describing the findings in Hornstein, Krusell and Violante (2011) that baseline search models struggle to generate significant wage dispersion, the so-called frictional wage dispersion puzzle. Further, search models face a trade-off between matching the cross-sectional wage distribution and matching the cyclical volatility of unemployment and vacancies. The chapter reviews the unemployment volatility puzzle and explains this trade-off. Given that the thesis introduces the loss of human capital during unemployment, the chapter ends with a review of the related empirical literature.

Chapter 2 studies wage dispersion among identical workers in a random matching search model in which workers lose human capital during unemployment. Wage dispersion increases, as workers accept lower wages to avoid long unemployment spells. I show that the model is an important improvement over baseline search models. The model with unemployment history explains between a third and half of the observed residual wage dispersion.

In Chapter 3 I add on-the-job search to the model with unemployment history. Workers accept lower wages because they keep the option of searching for better paying jobs. Wage dispersion increases significantly. The model accounts for all of the residual wage dispersion. The model also generates substantial wage dispersion even for high values of non-market time. The chapter thus addresses the trade-off between explaining frictional wage dispersion and the cyclical behavior of unemployment.

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# Chapter 1

## The Frictional Wage Dispersion Puzzle and the Effects of Unemployment on Wages: A Review

Empirical evidence suggests that similar workers are paid different wages. The labor literature has studied in detail how worker characteristics, such as education or tenure, affect wages. However, worker characteristics can only explain a fraction of the observed wage dispersion in the data. In a typical Mincerian wage regression with as many controls as possible the residual still displays a large amount of dispersion. Workers that are observationally similar are paid different wages.

Search models of the labor market can explain why similar workers are paid different wages. The intuition is the following. Firms and workers enter the labor market randomly, because job positions are available when firms have an unfilled position, and workers apply to vacant positions when they search for jobs. Knowing that job offers are only available with a given frequency, workers accept a job offer if the associated wage is above their reservation value —if the wage is high enough. When the wage is above the reservation value they are better off taking

the job than remaining unemployed and continuing with their search. This acceptance rule by workers generates wage dispersion, even among identical workers.<sup>1</sup> However, recent work by Andreas Hornstein, Per Krusell and Giovanni L. Violante (2011) shows that baseline search models fail to generate significant wage dispersion. These findings represent a great challenge to the search literature, so this chapter begins by reviewing the frictional wage dispersion puzzle in search models. This puzzle is related to the inability of baseline search models to account for cyclical fluctuations in unemployment and vacancies, what is known in the literature as the Shimer critique. I review this critique and show why there is a trade-off between addressing it and addressing the frictional wage dispersion puzzle. Given that this thesis studies what happens to wage dispersion among identical workers when they lose some human capital during unemployment, the chapter ends with a review of the empirical literature on the effects of job displacement on workers earnings.

## 1.1 Frictional Wage Dispersion in Search Models

In search models workers look for jobs and firms for job applicants. Knowing that job positions are only available with a given frequency, workers' best strategy is to set a reservation wage and accept a job offer when the wage is above this reservation value. This search strategy explains why two identical workers may be paid different wages. Wages may be different because some workers were luckier in their job search.

To understand the frictional wage dispersion puzzle consider first how workers decide when to accept a job offer in baseline search models. When considering a wage offer, unemployed workers compare the value of accepting the job to the value of staying in unemployment —their outside option. First, the value of accepting the job depends on the wage offered and how long the match is expected to last. As for the value of unemployment, if workers remain unemployed they receive

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<sup>1</sup>The literature uses the term frictional wage dispersion to describe wage dispersion among identical workers arising from search frictions. See for example Dale T. Mortensen (2005). To avoid repetition I use wage dispersion to refer to wage dispersion among ex-ante identical workers.

unemployment benefits and can continue looking for other jobs. How much this search option is worth depends on how easy it is to find jobs and what is the expected gain from accepting a job, as given by the wage distribution. Therefore, unemployment benefits, the wage distribution and labor market flows —the job finding and separation rates— are important for workers' choice of reservation wage.

Because the reservation wage determines how quickly workers find jobs, labor market transitions are the result of workers' choice. Therefore, wage dispersion and labor market flows are closely related in the search framework. However, Hornstein, Krusell and Violante (2011) find that baseline search models are unable to reconcile the empirical wage distribution with the observed labor market flows. Intuitively, the expected gain from looking for jobs is higher if wages are more disperse. If one takes the empirical wage distribution as given, in the baseline search model workers wait too long to leave unemployment compared to empirical unemployment durations. Alternatively, unemployment durations can only be as low as in the data if the wage distribution is very compressed. In this case the expected gain from an additional search is very low, so workers accept jobs quickly. This is the frictional wage dispersion puzzle. The empirical values for labor market flows imply that search models struggle to generate significant wage dispersion. Next I review more formally how the frictional dispersion puzzle arises in search models, as found in Hornstein, Krusell and Violante (2011).

### 1.1.1 The $Mm$ ratio in baseline search models

To measure wage dispersion Hornstein, Krusell and Violante (2011) use the ratio between the mean observed wage and the minimum wage, the so-called mean-min or  $Mm$  ratio. This measure has many advantages. In most search models one can derive an expression for the  $Mm$  ratio that depends only on a few parameters, for which reliable estimates exist. Most importantly, the  $Mm$  ratio is independent of the distribution from which workers draw offers. Because in later chapters I extent the random matching model of Christopher A. Pissarides (1985), I show

how to derive the  $Mm$  ratio in this model. However, the  $Mm$  ratio is exactly the same for a large class of search models.

In Pissarides (1985) workers look for jobs and firms look for applicants. Once a worker and a firm meet, the match has a specific productivity  $p$ . The match-specific productivity is drawn from a known distribution  $F(p)$ . Workers receive job offers at rate  $f$ , and firms receive applicants at rate  $q$ , which are endogenous and given by a matching function. Once employed they stay on the job until separation occurs, which happens at the exogenous rate  $s$ . During unemployment workers receive benefits  $b$ .<sup>2</sup> Because of free entry in vacancies, firms post vacancies until the value of the vacancy is zero. The surplus from the match is split between the firm and the worker according to Nash Bargaining, with workers receiving a share  $\beta$  of the surplus. This sharing rule gives a perfect mapping between match productivities and wages  $w = w(p)$ . In this framework, workers' optimal decision consists of setting a reservation productivity  $p^*$ , and accept a job if the match-specific productivity is above  $p^*$ .

The  $Mm$  ratio is given by  $Mm = \bar{w}/w^*$ , where  $\bar{w} = E(w|p > p^*)$  is the average observed wage, and  $w^*$  the minimum observed wage. The mathematical appendix at the end of this chapter derives the following expression relating  $\bar{w}$  and  $w^*$

$$w^* = b + \frac{f^*}{r+s}(\bar{w} - w^*). \quad (1.1)$$

where  $r$  is the interest rate. I use  $f^* = f(1 - F(p^*))$  to denote the job finding rate, as only matches above  $p^*$  are formed. Assume that  $b = \rho\bar{w}$ , where  $\rho$  is the replacement rate. From the above equation one can derive the following expression for the  $Mm$  ratio

$$Mm = \frac{1 + \frac{f^*}{r+s}}{\rho + \frac{f^*}{r+s}}. \quad (1.2)$$

As the expression (1.2) shows, the biggest advantage of using the  $Mm$  ratio to

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<sup>2</sup>One can also interpret  $b$  as the value of non-market time, which may also include the value of leisure.

measure wage dispersion is that its expression is independent of any assumption about the distribution of matches  $F(p)$ , for which there is very little knowledge. While  $F(p)$  is implicit in the equation through  $f^*$ , this is simply the job finding rate, which is observed empirically and can thus be treated as a parameter. This is one of the great contributions of Hornstein, Krusell and Violante (2011).

The  $Mm$  ratio is uniquely determined by the set of parameters  $r$ ,  $f^*$ ,  $s$  and  $\rho$ . In particular, (1.2) shows that increasing the dispersion of  $F(p)$  does not necessarily deliver higher wage dispersion. Wage dispersion is eventually constrained by labor market flows  $f^*$  and  $s$ , which reflect workers' optimal search behavior. The effects of  $f^*$ ,  $s$  and  $\rho$  on the  $Mm$  ratio are intuitive. With a higher job finding rate  $f^*$  workers find jobs more easily, so their option of searching gains value. Workers become pickier and raise their reservation wage. Because now they reject some of the worse matches and only accept the better ones, wage dispersion decreases. The separation rate  $s$  has the opposite effect. When  $s$  is high workers lose their jobs more quickly, so having a job has a lower value. Workers become less picky, they lower their reservation wage, which increases wage dispersion. Finally, if  $\rho$  is higher, the outside option of workers increases, they become pickier and raise their reservation wage, thus lowering wage dispersion.<sup>3</sup>

While expression (1.2) corresponds to the Pissarides random matching model, Hornstein, Krusell and Violante (2011) show that the  $Mm$  ratio has the exact same value for a large class of search models. As the authors state, all models that satisfy the following properties have the same  $Mm$  ratio: "1) perfect correlation between job values and initial wages 2) risk neutrality 3) random search 4) no on-the-job search". The intuition is that the mechanism behind the offer distribution  $F(\cdot)$  from which workers sample offers is irrelevant. All that matters is workers' choice of reservation value, which determines the mass of jobs accepted  $1 - F^*$  and eventually translates into the job finding rate  $f(1 - F^*)$ . Included in this class are the other baseline search models in John J McCall (1970), Robert Jr. Lucas and Edward C. Prescott (1974); search models with vintage of capital such as in

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<sup>3</sup>Section 1.2 discusses the role of  $\rho$  in more detail.

Philippe Aghion and Peter Howitt (1994), Dale T. Mortensen and Christopher A. Pissarides (1998) or Andreas Hornstein, Per Krusell and Giovanni L. Violante (2007b); and search models with efficiency wages.

### 1.1.2 Quantifying the $Mm$ ratio

Expression (1.2) gives the  $Mm$  ratio in baseline search models as a function of  $r$ ,  $f^*$ ,  $s$  and  $\rho$ . Calibrating these parameters to the US economy provides the amount of wage dispersion consistent with data. The interest rate  $r$  is chosen to be consistent with an annual value of 5%, which gives a monthly value of  $r = 0.0041$ . Robert Shimer (2007) provides the values for the finding rate and the job separation rate, which are the reference in the literature.<sup>4</sup> For the period 1994:Q1-2007:Q1, the job finding rate is  $f^* = 0.43$ , and the job separation rate  $s = 0.03$ .<sup>5</sup> Finally, the replacement ratio  $\rho$  is assumed to be 40%, which is the value used in Robert Shimer (2005). This value is based on average unemployment insurance benefits. While the search literature sometimes uses higher values for  $\rho$ , this would only decrease the  $Mm$  ratio and make the frictional wage dispersion puzzle worse, so for now I use this lower value. I discuss the role of  $\rho$  and the implications of using higher values in more detail in section 1.2 and in the final chapter.

With this calibration the  $Mm$  ratio is around 1.05 in baseline search models. This implies that the average observed wage is 5% above the minimum observed wage. However, the 50-10 percentile ratio of the residual in a Mincerian regressions with as many controls as possible is between 1.7 and 1.9. Given that this is a reasonable empirical counterpart to the theoretical  $Mm$  ratio, the large difference between the two values —1.05 vs 1.7 to 1.9— formally illustrates the frictional wage dispersion puzzle. Search models simply cannot generate significant wage dispersion.

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<sup>4</sup>The advantage with Robert Shimer (2007) estimates is that they use information on whether workers found and lost jobs between two employment status observations, something that is missing in previous studies.

<sup>5</sup>Hornstein, Krusell and Violante (2011) choose this time period because it also provides estimates of job-to-job transitions, which are required later when studying on-the-job search.

This inability to generate significant wage dispersion is not due to the use of US data. A European calibration delivers a very similar  $Mm$  ratio, even without changing the replacement ratio to reflect the larger unemployment benefits in Europe. Further, including three employment states —employed, unemployed and inactive— delivers almost the same  $Mm$  ratio.

### 1.1.3 Extensions of baseline search models

In baseline search models workers' labor market outcomes are different because some workers are luckier in their search than others. However, many known features of the labor market are still missing. Hornstein, Krusell and Violante (2011) evaluate wage dispersion when the baseline search model is extended to include some of them. I review their findings in this section. Table 1.1 summarizes the  $Mm$  ratios and their calibrated values.

**Endogenous search costs.** If staying in unemployment is more costly, workers are willing to leave unemployment more quickly. They reduce their reservation wage, and as a result wage dispersion increases. Introducing search costs is one way of inducing this. With search costs the  $Mm$  ratio is given by

$$Mm = \frac{1 + \frac{f^*}{r+s} \frac{1}{1+\sigma}}{\rho + \frac{f^*}{r+s} \frac{1}{1+\sigma}}, \quad (1.3)$$

where  $1/\sigma$  is the elasticity of marginal return to search with respect to search effort. Hornstein, Krusell and Violante (2011) use the findings in Bent Jesper Christensen, Rasmus Lentz, Dale T. Mortensen, George R. Neumann and Axel Werwatz (2005) that  $\sigma \simeq 1$ , which gives an  $Mm$  ratio of 1.088.

**Compensating differentials.** Jobs have many attributes besides the wage they pay. When considering a job, workers take into account the wage, but also other aspects of the job such as location, benefits, etc. If these wage attributes are negatively correlated with the wage, the model reconciles the observed unemployment durations and wage dispersion better. In such scenario workers accept lower wages because they carry positive attributes that compensate for the low

pay. This leads to higher wage dispersion compared to baseline models. Unfortunately, this is not empirically valid. Empirical studies find that the correlation is small and usually positive.

**Stochastic wages.** In baseline search models the quality of the match, or its productivity, remains constant until the job is destroyed. If instead the productivity of the job changes, workers accept lower paying jobs because wages are not low forever. Assume now that at rate  $\lambda$  the productivity of the job changes, which is captured as a draw from the original distribution of wages  $F(\cdot)$ . If the new productivity level is below the reservation value, the match is destroyed. Therefore, separations are now endogenous, and happen at rate  $s^* = \lambda F(w^*)$ . The expression for the  $Mm$  ratio is given by

$$Mm = \frac{1 + \frac{f^* - \lambda + s^*}{r + \lambda}}{\rho + \frac{f^* - \lambda + s^*}{r + \lambda}}. \quad (1.4)$$

However, given that wages are extremely persistent, empirically  $\lambda$  is close to zero. The implied  $Mm$  ratio is almost the same as in the baseline model. The difference between this model and that of Dale T. Mortensen and Christopher A. Pissarides (1994) is that job productivity begins with maximum productivity in the Mortensen-Pissarides model. With this assumption wage dispersion is lower than in (1.4).

**Risk aversion.** In the baseline search model workers are risk-neutral. However, given that unemployment is associated with lower consumption, risk-averse workers want to leave unemployment more quickly than risk-neutral workers. The reservation wage is lower with risk-aversion and wage dispersion increases. The extreme case with no storage technology —i.e. no savings— gives the highest  $Mm$  ratio. If  $u(c)$  is the utility of consumption and worker's consumption is  $c = b$  if unemployed, and  $c = w$  if employed, the  $Mm$  ratio is given by

$$Mm = \left( \frac{\frac{f^*}{r+s} (1 + \frac{1}{2}(\theta - 1)\theta cv^2) + \rho^{1-\theta}}{1 + \frac{f^*}{r+s}} \right)^{\frac{1}{\theta-1}}, \quad (1.5)$$

where  $\theta$  is the coefficient of relative risk aversion, and  $cv$  is the coefficient of variation of the wage distribution. With  $\theta = 2$  and  $cv = 0.3$  the  $Mm$  ratio is 1.2. However, if workers can save or borrow the  $Mm$  ratio would be lower, as they can self-insure, thus bringing the economy closer to risk-neutrality. Per Krusell, Toshihiko Mukoyama and Aysegul Sahin (2010) show that in a Pissarides model with uninsurable risk—i.e. with savings but borrowing only during employment—the  $Mm$  ratio is very small, similar to the one in baseline search models.

**Directed search.** Espen R. Moen (1997) first introduced directed search. Firms post wages, and workers can choose where to direct their search. Because firms that post higher wages attract more applicants, workers face a trade-off between higher wages and shorter unemployment spells. If they direct their search to better paying jobs they can earn higher wages, but the queue is longer, so they have to wait longer before getting an offer. If  $f_i$  is the arrival rate of offers if the worker applies to firm  $i$ , and  $\bar{f}$  that of firms paying the average wage, the  $Mm$  ratio is given by the following

$$Mm = \frac{1 + (1 + \frac{f_{min}}{r+s}) \frac{\bar{f}}{f_{min} - \bar{f}}}{\rho + (1 + \frac{f_{min}}{r+s}) \frac{\bar{f}}{f_{min} - \bar{f}}} \leq \frac{1 + \frac{\bar{f}}{r+s}}{\rho + \frac{\bar{f}}{r+s}}. \quad (1.6)$$

Based on evidence by R Wolthoff (2012),  $\bar{f}$  is almost the same than the average job finding rate. Therefore, the upper bound of the  $Mm$  ratio in the above expression is almost the same as the  $Mm$  ratio for the baseline model.

**On-the-job search.** On-the-job search is one of the most promising avenues to generate significant amounts of wage dispersion. In baseline search models workers stay on the same job until separation occurs. If we allow workers to search for jobs while being employed, unemployed workers become less picky and accept lower wages because they keep the option of searching. Hornstein, Krusell and Violante (2011) show that the  $Mm$  ratio in a generalized version of the on-the-job

search model of Kenneth Burdett (1978) is given by

$$Mm \simeq \frac{1 + \frac{f^u - f^e}{r+s+f^e}}{\rho + \frac{f^u - f^e}{r+s+f^e}}, \quad (1.7)$$

where  $f^u$  is the job finding rate for the unemployed, and  $f^e$  is the arrival rate of offers to employed workers. Using job-to-job transition rates gives a calibration for  $f^e$ , and implies an  $Mm$  ratio between 1.16 and 1.27. Chapter 3 discusses this model in more detail.

Table 1.1:  $Mm$  RATIO IN SEARCH MODELS

Model	$Mm$ ratio	Value	Description/Comments
Baseline	$\frac{1 + \frac{f^*}{r+s}}{\rho + \frac{f^*}{r+s}}$	1.05	
Endogenous search costs	$\frac{1 + \frac{f^*}{r+s} \frac{1}{1+\sigma}}{\rho + \frac{f^*}{r+s} \frac{1}{1+\sigma}}$	1.09	$1/\sigma$ elasticity of marginal return to search w.r.t search effort
Stochastic wages	$\frac{1 + \frac{f^* - \lambda + s^*}{r+\lambda}}{\rho + \frac{f^* - \lambda + s^*}{r+\lambda}}$	1.05	$\lambda$ arrival rate of productivity shocks, $s^*$ endogenous job separation
Risk aversion	$\left( \frac{\frac{f^*}{r+s} (1 + \frac{1}{2}(\theta-1)\theta cv^2) + \rho^{1-\theta}}{1 + \frac{f^*}{r+s}} \right)^{\frac{1}{\theta-1}}$	1.2	$\theta$ CRRA, $cv$ wage distribution's coefficient of variation. Problem: No savings; when add savings $Mm$ same as in baseline
Directed search	$\frac{1 + (1 + \frac{f_{min}}{r+s}) \frac{\bar{f}}{f_{min} - \bar{f}}}{\rho + (1 + \frac{f_{min}}{r+s}) \frac{\bar{f}}{f_{min} - \bar{f}}} \leq \frac{1 + \frac{\bar{f}}{r+s}}{\rho + \frac{\bar{f}}{r+s}}$	$\leq 1.05$	$f_i$ arrival rate of offers from firm $i$ , $\bar{f}$ from firms paying the average wage
On-the-job search	$\approx \frac{1 + \frac{f^u - f^e}{r+s+f^e}}{\rho + \frac{f^u - f^e}{r+s+f^e}}$	[1.16; 1.27]	$f^u$ job finding rate, $f^e$ arrival rate of offers on-the-job
Endogenous search costs and on-the-job	$\left[ \frac{1 + \frac{f_u - f_{w^*}}{r+s} \frac{1}{1+\gamma}}{\rho + \frac{f_u - f_{w^*}}{r+s} \frac{1}{1+\gamma}}, \frac{1 + \frac{f_u - f_{w^*}}{r+s+f^{w^*}} \frac{1}{1+\gamma}}{\rho + \frac{f_u - f_{w^*}}{r+s+f^{w^*}} \frac{1}{1+\gamma}} \right]$	[1.22; 1.90]	Problem: Counterfactual result, implies $< 0$ value of non-market time net of search costs

Notes.-  $f^*$  and  $s$  are the job finding and separation rates;  $r$  the interest rate;  $\rho$  the replacement rate,  $b/\bar{w}$ . See text for details of the calibrations.

Two models that embed on-the-job search generate substantial wage dispersion, although their assumptions are somewhat controversial.

In the Christensen et al. (2005) model of endogenous search effort workers can search on-the-job. During unemployment they choose the arrival rate of job offers at a cost. Hornstein, Krusell and Violante (2011) find bounds for the  $Mm$  ratio

in this model, which are given by

$$\frac{1 + \frac{f_u - f_{w^*}}{r+s} \frac{1}{1+\gamma}}{\rho + \frac{f_u - f_{w^*}}{r+s} \frac{1}{1+\gamma}} \leq Mm \leq \frac{1 + \frac{f_u - f_{w^*}}{r+s+f^{w^*}} \frac{1}{1+\gamma}}{\rho + \frac{f_u - f_{w^*}}{r+s+f^{w^*}} \frac{1}{1+\gamma}}. \quad (1.8)$$

As in the case of endogenous search efforts,  $\gamma$  is the inverse of the elasticity of marginal return to search with respect to search effort. Using Danish data, Christensen et al. (2005) estimate  $\gamma$  to be around 1. Parameters  $f_u$  and  $f_{w^*}$  refer to the job finding rate of the unemployed and of workers who earn the reservation wage. Michael Rosholt and Michael Svarer (2004) find that the ratio between these two finding rates is around 64% for Danish data. Left with no other choice but to assume the same ratio for US data, Hornstein, Krusell and Violante (2011) calibrate  $f_{w^*}$  to be 0.27. This calibration implies that the  $Mm$  ratio is between 1.22 and 1.90. However, this model has controversial results. While it can potentially generate substantial wage dispersion, the bounds of the  $Mm$  ratio imply that non-market time net of search costs is between zero and a large negative number.<sup>6</sup>

A number of papers introduce sequential auctions between employers in a model of on-the-job search. These include Fabien Postel-Vinay and Jean-Marc Robin (2002a), Fabien Postel-Vinay and Jean-Marc Robin (2002b) Matthew S. Dey and Christopher J. Flinn (2005) and Pierre Cahuc, Fabien Postel-Vinay and Jean-Marc Robin (2006). When an employed worker receives an offer from an outside employer, the current employer can make a counteroffer to keep the worker, and so both firms enter in an auction. Wage dispersion is larger in this setting because wages grow even if workers stay on the same job. In its most extreme case, which delivers the largest wage dispersion, workers extract a share  $\beta$  of the surplus when they start at a job, and get the marginal product  $p$  from their employer once they are contacted by an outside firm. If  $\beta$  is low enough, the  $Mm$  ratio is larger than  $1/\rho$ .<sup>7</sup> However, these models are controversial. True, these type of auctions

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<sup>6</sup>The baseline search model also generates substantial wage dispersion if the value of non-market time is negative and large enough.

<sup>7</sup>Hornstein, Krusell and Violante (2011) show that models with tenure contracts, such as

may be common in the academic labor market. But as both Dale T. Mortensen (2005) and Robert E. Hall and Paul R. Milgrom (2008) point out, these auctions are absent in almost all occupations in the labor market, probably because of asymmetric information problems. Once the worker holds an outside offer, it is difficult to credibly communicate its terms to her employer, for the outside firm has little incentive to corroborate the existence and the terms of the offer. And if firms were to communicate, nothing would stop them from colluding. Further, because reacting to outside offers may create incentives for workers to look for jobs, firms' optimal strategy may be to commit to no counter-offers in response to this moral hazard problem.<sup>8</sup>

While these explanations may be appealing and interesting, lack of data and some controversial assumptions and results make them insufficient, suggesting more work needs to be done.

## 1.2 Trade-off with Cyclical Unemployment Fluctuations

The frictional wage dispersion puzzle has further implications for search models' ability to account for empirical observations. Looking at the intuition behind the puzzle —that given the empirical wage distribution, workers in baseline search models wait too long before accepting an offer— shows that making unemployment less attractive helps to solve this problem. The value of non-market time, as captured by the replacement ratio  $\rho$ , is one of the determinants of the value of unemployment. So when the value of non-market time is low, unemployment is less attractive and workers accept worse offers. In other words, a lower value of non-market time reduces workers' reservation wage. Therefore, search models require low values of non-market time to account for wage dispersion.

However, Hornstein, Krusell and Violante (2011) observe that this relationship

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Stevens (2004) and Burdett and Coles (2003) are equivalent to this model with a  $\beta$  equal to 0. The problem is that if  $\beta$  is 0, the reservation wage is negative.

<sup>8</sup>Competition between job applicants would also slow down wage growth.

between non-market time and wage dispersion leads to a trade-off between solving the frictional wage dispersion puzzle and accounting for cyclical fluctuations in unemployment and vacancies. Shimer (2005) finds that search and matching models are unable to generate enough volatility in unemployment and vacancies compared to the data, a problem often referred to as the Shimer critique. The trade-off arises because low values of non-market time increase wage dispersion but reduce unemployment volatility during the cycle. And vice-versa, with high values of non-market time baseline search models can account for the cyclical volatility of unemployment and vacancies, but then they struggle even more to generate significant wage dispersion.

Next, I briefly review the literature on the Shimer critique and formally show the trade-off with the frictional wage dispersion problem.

### 1.2.1 Labor market fluctuations: The Shimer critique

The intuition for why search models are unable to account for the cyclical fluctuations in unemployment and vacancies is the following. When labor productivity increases relative to vacancy costs and the value of non-market time, firms respond by posting more vacancies, which drives the vacancy rate up. At the same time, as firms post more vacancies workers find jobs more quickly, which reduces unemployment. Overall, labor market tightness —the ratio of the vacancy and unemployment rates— increases with productivity. This is the prediction of the canonical search model, as in Pissarides (1985) or Christopher A. Pissarides (2000). Labor market tightness is procyclical, and unemployment and vacancies are negatively correlated —this negative correlation is known as the Beveridge Curve.

However, Shimer (2005) finds that fluctuations in labor market tightness are much larger in the data than what the model predicts. At the source of the problem lies the fact that the job finding rate increases with labor productivity. When productivity increases, this relationship between the job finding rate and labor productivity raises the value of unemployment. Because wage payments have

to cover workers' outside option, the increase in productivity pushes wages up. The increase in wages shrinks profits, which reduces the incentive to post vacancies. Overall, the value of unemployment plays such a strong role in the determination of wages that workers keep most of the gain from the rise in productivity, leaving profits almost unaffected. As vacancies barely move, labor market tightness is only mildly procyclical, and the economy experiences only a small movement along the Beveridge Curve.

Using data from the Bureau of Labor Statistics (BLS), Shimer (2005) reports that the standard deviation of market tightness and labor productivity —around their trend— is 0.38 and 0.02, so labor productivity is 20 times more volatile than productivity. By contrast, using estimates on job finding and separation rates for the US economy the elasticity of labor market tightness with respect to productivity is close to one.<sup>9</sup> This value implies that the empirical fluctuations and the model's prediction can only be reconciled if the value of non-market time is close to labor productivity. In the model the standard deviation of labor market tightness around its trend is 0.037, only 10% of its empirical counterpart.<sup>10</sup>

Shimer (2005) and some of the papers that followed argued that the inability to account for these cyclical fluctuations is not really a failure of the search model, but rather the result of assuming that wages are determined by Nash Bargaining. A productive match between the firm and the worker creates a surplus. To close the model, one needs to make an assumption as to how these rents are shared between both parties. With Nash Bargaining wages respond almost one to one to productivity, thus unfolding the mechanism described earlier of small profits and little incentives to post vacancies.

I follow Dale T. Mortensen and Eva Nagypal (2007) and Christopher A. Pis-

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<sup>9</sup>This is another of the great contributions of the work in Shimer (2005). Estimates of the job finding and separation rates based only on employment status fail to account for the fact that workers may find and lose jobs between employment status observations. By using information on short term unemployment, Shimer (2005) is able to account for this. The estimates have become the standard in the literature.

<sup>10</sup>Shimer (2005) also finds that in the model shocks to the separation rate lead to a counterfactual upward sloping Beveridge Curve.

sarides (2009) and formally describe the Shimer critique using a continuous time version of the Pissarides (2000) model because it provides a more intuitive exposition. Shimer (2005) solves its stochastic version. In his model one equation determines labor market tightness for each state of the economy, which can be easily solved numerically. Once labor market tightness is known, the rest of the variables are determined as in the Pissarides model. As labor productivity is very persistent and labor market flows are large, unemployment and vacancies adjust very quickly compared to productivity. Labor market tightness is the key driver in the model, so comparing steady states at different levels of productivity, as Mortensen and Nagypal (2007) and Pissarides (2009) do, is an accurate description of cyclical fluctuations.<sup>11</sup>

The model is the same as the one in section 1.1.1, only that labor productivity  $p$  is no longer drawn from a distribution  $F(p)$ . Instead all matches have the same productivity, but  $p$  changes with the cycle. As before, the job finding rate  $f$  is a function of labor market tightness  $\theta$ , and satisfies  $f(\theta) = \theta q(\theta)$ .<sup>12</sup> Separations are exogenous.<sup>13</sup> As before, unemployed workers receive  $b$ , and firms post vacancies at a cost  $k$ . Wages are determined by Nash Bargaining, with workers receiving a share  $\beta$  of the surplus.

Let  $U$  and  $W$  be the value of being unemployed and employed, and  $V$  and  $J$  of a vacancy and a filled position. I show here the equations and results that are relevant for the exposition, but the derivations are included in the mathematical appendix at the end of the chapter. The model gives the following expression for

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<sup>11</sup>Shimer (2005) made the same point. He shows that the steady state version of the model is an accurate description of the stochastic model.

<sup>12</sup>I can drop  $\theta$  from the notation when I studied frictional wage dispersion because it focuses on the steady state.

<sup>13</sup>Pissarides (2009) discusses in detail why endogenous separations barely affect job creation and shows that the results are the same if separations are exogenous. Intuitively, jobs at the reservation productivity yield zero profits. Therefore, changes in the reservation productivity leaves the expected profits from an average job almost unchanged, which is what drives job creation.

wages

$$w = rU + \beta(p - rU - (r + s)V). \quad (1.9)$$

Expression (1.9) illustrates more formally the point made by Shimer. The value of unemployment  $U$  enters the wage expression because it is workers' outside option, and thus their threat point in the bargaining process. However,  $U$  responds to current labor conditions. When productivity increases, firms post more vacancies and drive the job finding rate up. This increases  $U$  and thus wages. Solving further gives the following expression for wages

$$w = (1 - \beta)b + \beta(p + \theta k), \quad (1.10)$$

which is the same as equation (7) in Shimer (2005) and (10) in Pissarides (2009).

I use the notation  $\varepsilon_\theta$  and  $\varepsilon_w$  for the elasticities of  $\theta$  and  $w$  with respect to  $p$ , and  $\eta$  for the elasticity of  $q(\theta)$  with respect to  $\theta$ .<sup>14</sup> Using the free entry condition for vacancies and the asset equations, in the appendix I find  $\varepsilon_\theta$  as a function of  $\varepsilon_w$

$$\varepsilon_\theta = \frac{1}{\eta} \frac{p - \varepsilon_w w}{p - w}. \quad (1.11)$$

What is the target for the elasticity  $\varepsilon_\theta$ ? As mentioned earlier, Shimer (2005) finds that the standard deviation of labor market tightness  $\theta$  is around 19.1 times that of labor productivity  $p$ . However, as work by Mortensen and Nagypal (2007) points out, empirically labor productivity is not the only variable affecting labor market tightness. So the model should only try to explain the volatility of labor market tightness that is correlated with productivity. In other words, the residual in a regression of labor market tightness on labor productivity is very large. As this residual captures fluctuations in labor market tightness that are generated by movements in other variables, we should not expect the model to account for it. Instead, the coefficient in a regression of  $\theta$  on  $p$  should be the target. It is obtained

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<sup>14</sup>i.e.  $\eta = -q'(\theta)\theta/q(\theta)$ , with a minus sign because  $q'(\theta) < 0$ .

by multiplying the ratio of standard deviations with value 19.1 by the correlation coefficient between  $\theta$  and  $p$ , which gives a target of 7.56.

When the canonical search model is calibrated to the US economy, wages are almost as cyclical as productivity. To illustrate Shimer's point, if  $\varepsilon_w$  is close to 1, equation (1.11) shows that  $\varepsilon_\theta = 1/\eta$ , which is around 2 using a standard calibration for  $\eta$  of 0.5. As the target for this elasticity is 7.56, the canonical model is unable to generate the observed cyclical volatility. This is in a nutshell the Shimer critique. More formally, the elasticity  $\varepsilon_\theta$  is given by

$$\varepsilon_\theta = \frac{r + s + \beta f(\theta)}{\eta(r + s) + \beta f(\theta)} \cdot \frac{p}{p - b}. \quad (1.12)$$

Pissarides (2009) and Mortensen and Nagypal (2007) calibrate this model, and find that  $\varepsilon_\theta$  is 1.71 when Shimer's value of the replacement ratio is used. Although with the new target of 7.56 the canonical model's performance is not as terrible, the generated fluctuations still fall short. It generates only around a fourth of the empirical value.<sup>15</sup>

What drives the model's inability to generate enough fluctuations in its key variable  $\theta$  is the following. Because the wage closely follows productivity, most of the productivity shock translates into wages. This leaves firms' profits unaffected, so firms have no incentives to post more or less vacancies. More formally, equilibrium  $\theta$  is the solution to

$$\frac{(1 - \beta)(p - b)}{k} = \frac{r + s}{q(\theta)} + \beta\theta. \quad (1.13)$$

This gives the following elasticity  $\tilde{\varepsilon}_\theta$  of market tightness  $\theta$  with respect to *net* productivity  $p - b$

$$\tilde{\varepsilon}_\theta = \frac{r + s + \beta f(\theta)}{\eta(r + s) + \beta f(\theta)}. \quad (1.14)$$

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<sup>15</sup>Hall and Milgrom (2008) also run regressions on lagged productivity and find similar results. The canonical model with Shimer's calibration explains around one fourth of the observed volatility.

With Shimer's calibration, (1.14) implies that  $\tilde{\varepsilon}_\theta$  is around 1.03.<sup>16</sup> Using this value and wage equation (1.10) shows that a one percent increase in net labor productivity  $p - b$  leads to a one percent increase in  $\theta$ , thus increasing  $w - b$  by one percent. Profits are unaffected, so firms do not post more vacancies in response to this one percent rise in productivity.

Finally, to provide an indication of what changes in a stochastic setting, Shimer (2005) solves a stochastic version of this model in which productivity  $p$  and the separation rate  $s$  change with a frequency  $\lambda$ . Labor market tightness is a function of the state  $(p, s)$ . All equations are similar, only that labor market tightness  $\theta_{p,s}$  is found by solving numerically the following equation, which is equation (6) in his paper

$$\frac{r + s + \lambda}{q(\theta_{p,s})} + \beta\theta_{p,s} = (1 - \beta)\frac{p - b}{k} + \lambda\mathbb{E}_{p,s}\left(\frac{1}{\theta_{p',s'}}\right), \quad (1.15)$$

where  $\mathbb{E}_{p,s}(\cdot)$  is the expectation over  $p$  and  $s$ . His simulations confirm that the predictions of this model are a good approximation of the earlier static model — which corresponds to  $\lambda = 0$ . This comes from the high persistence of productivity compared to labor market flows.

### 1.2.2 Attempts to reconcile model and data

The response that followed Shimer's critique initially focused on blaming the Nash bargaining process. With this wage rule, wages respond just too much to labor productivity shocks. If wages are instead rigid, the argument goes, wages respond less to productivity shocks and firms' profits fluctuate more over the cycle. This generates more fluctuations in vacancies.

However, as Mortensen and Nagypal (2007) point out, unless an unrealistic wage rigidity is assumed, wage rigidity is not enough.<sup>17</sup> As they argue, the low

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<sup>16</sup>Note that  $\tilde{\varepsilon}_\theta$  in Shimer (2005) has  $1 - \eta$  instead of  $\eta$ . But this is because  $\eta$  here is the elasticity of  $q(\cdot)$ , not of  $f(\cdot)$ . The result is thus the same.

<sup>17</sup>For example, the wage as a social norm in Hall (2005) can only exist if the only source of fluctuations are very small aggregate shocks, which is inconsistent with the large flows in the labor market. Otherwise it violates rationality.

fluctuations found in Shimer (2005) come from three sources. First, a low value of the elasticity of the job finding rate,  $1 - \eta$  in the model of the previous section. Second, a low value of non-market time. Finally, too much feedback from the job finding rate  $f(\theta)$  to wages, which is exactly what happens if wages are determined by Nash bargaining.

I discuss in what follows how addressing the three sources of low fluctuations improves the performance of the canonical model. Two results become clear. First, one cannot generate significant fluctuations by addressing only one of these sources. Second, in all cases higher values of non-market time are needed. This is crucial for frictional wage dispersion, as with these higher values search models can barely generate any wage dispersion.

### Early successful attempts

Shimer (2005) already discusses the role of the elasticity of the job finding rate  $1 - \eta$ . He uses a value of 0.28, which delivers an elasticity of labor market tightness with respect to *net* labor productivity  $\tilde{\varepsilon}_\theta$  of 1.03. However, when the value is close to zero,  $\tilde{\varepsilon}_\theta$  is still only 1.39, and 1.15 when the value is 0.1. Mortensen and Nagypal (2007) find a similar result. The elasticity  $\varepsilon_\theta$ , this time with respect to labor productivity, goes from 1.71 to only 1.82 when one uses the alternative calibration  $1 - \eta = 0.544$ . So the elasticity  $1 - \eta$ , by itself, is not the reason why the elasticities  $\varepsilon_\theta$  or  $\tilde{\varepsilon}_\theta$  are so low in the model.

Turning to the value of non-market time, Hall and Milgrom (2008) use evidence from consumption and hours data, and calibrate the value of non-market time  $b$  to match the Frisch elasticities of consumption demand and labor supply. This procedure gives them a value of 0.71 for non-market time. With this value the elasticity  $\varepsilon_\theta$  is 3.67, around half the target, which is quite an improvement. To further amplify fluctuations and match the empirical values, Hall and Milgrom (2008) drop the Nash Bargaining assumption. Instead, they introduce the sequential bargaining in Binmore, Rubinstein and Wolinsky (1986). Workers and firms make alternating offers and incur some delay costs during bargaining. There is

now a difference between the outside option and the disagreement payoffs. This is the sequential bargaining's key innovation. With Nash Bargaining, if workers get a poor offer they break negotiations and get the outside option. However, the outside option is not a credible threat unless its value is very high. In sequential bargaining, when an offer is low workers reject the offer but continue to bargain by making a counteroffer.<sup>18</sup> The resulting wage responds less to current labor market conditions, as it breaks the link between wages and the value of unemployment  $U$ . Thus firms' hiring fluctuates more. With both these alternative assumptions, Hall and Milgrom (2008) match the volatility in unemployment and vacancies without unrealistic wage stickiness.<sup>19</sup>

Hagedorn and Manovskii (2008) argue differently. The problem is not the wage rule, but the calibration used. They propose a new calibration for the value of non-market time  $b$  and for the bargaining power  $\beta$ . The values  $b$  and  $\beta$  are chosen to match the elasticities of labor market tightness and wages  $\varepsilon_\theta$  and  $\varepsilon_w$ .<sup>20</sup> This calibration strategy gives them a very high value for non-market time of 0.955, and a very low value for  $\beta$  of 0.052. Intuitively, the extreme values for  $b$  and  $\beta$  imply very small firm profits.<sup>21</sup> Because of this small size, small changes in profits drive big relative changes. This generates large fluctuations in vacancies. Mechanically, such a high  $b$  implies a large value for  $p/(p - b)$  in (1.12), and thus in  $\varepsilon_\theta$ . However, their target is to generate the large fluctuations in the elasticity of labor market tightness  $\varepsilon_\theta$  as found in Shimer (2005). The elasticity  $\varepsilon_\theta$  is 26.83, which as Mortensen and Nagypal (2007) mentions, is too high of a target.

These findings show that generating significant labor market fluctuations requires high values of non-market time. With such values, search models struggle

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<sup>18</sup>Similarly when the firm receives an offer.

<sup>19</sup>In this literature wage rigidity or stickiness refers loosely to how much wages respond to productivity.

<sup>20</sup>As in Hall and Milgrom (2008), they need a value for the cost of vacancies. They calibrate the capital and labor costs of vacancies, which provides them with the total cost of vacancies. In their calibration capital costs are 47% of labor productivity, and labor costs 11%.

<sup>21</sup>The high vacancy costs also imply lower profits.  $\beta$  has a very small impact on fluctuations in  $\theta$ , but allows the wage rule to match an empirical wage rigidity of 0.449. Otherwise, with a standard  $\beta$  the wage elasticity is 0.964, exactly Shimer's point.

even more to account for frictional wage dispersion.

## Criticisms

Despite the success of Hall and Milgrom (2008) and Hagedorn and Manovskii (2008), there has been some criticism of their findings, especially of the work of Hagedorn and Manovskii (2008).

I begin with Hall and Milgrom (2008). First, the authors close their model by calibrating two key parameters in their model —the cost of delay in negotiations  $\gamma$  and the probability that negotiations break  $\delta$ — to match the average unemployment and the elasticity  $\varepsilon_\theta$ . Although their calibrated values are reasonable, there is little information about their empirical value.<sup>22</sup> In particular, if delay costs depend on wages or productivity the mechanism would generate less fluctuations. Further, Pissarides (2009) argues that wage rigidity can not be the answer to Shimer’s critique. He proves that if new matches are determined by Nash Bargaining, then the job creation condition is the same no matter how the continuation wages are determined.<sup>23</sup> Since he argues that empirically wages in new matches follow productivity very closely, rigidity cannot be the answer. However, as Hall and Milgrom (2008) argue, the standard errors in the estimate for new matches includes their value of wage rigidity, so this seems less controversial.

Hagedorn and Manovskii (2008) has received more criticism. First, with such a high value of non-market time the response of unemployment to unemployment insurance is too high, as research by Costain and Reiter (2008) shows. Hall and Milgrom (2008) find a similar result. The value of non-market time in Hagedorn and Manovskii (2008) implies that the Frisch labor supply elasticity is too high, around 2.6. The second source of criticism argues that with the low value for the share of surplus  $\beta$ , workers only receive 2.3% of the surplus. While including taxes lowers the value of non-market time to around 0.67, it still implies the same low share of surplus. Finally, a milder source of criticism takes issue with their

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<sup>22</sup>If we consider delay costs to include the costs of idle capital, their value for  $\gamma$  of 0.23 is well below that found in Hagedorn and Manovskii (2008), which is 0.43. But, it is particularly difficult to disentangle delay costs from vacancy costs.

<sup>23</sup>Intuitively, the timing of wages does not matter for job creation.

calibration of the capital costs of a vacancy. It assumes that a vacancy is caused by a short fall in employment and that the capital that was being used becomes idle.

In their defense, Hagedorn and Manovskii (2008) argue theirs is not the only model with such shortcomings. Strong responses to policy are common in models that increase the amplification of shocks —such as some of the extensions of the RBC model. Further, they argue that responses to policy such as unemployment insurance are hard to measure in data, and suffer from endogeneity problems. Finally, their focus is not on the source of the shock, but rather the response of the model’s key variables. If policy is instead the focus, one needs to specify the source —whether it is technology, capital or employment—, as it has important implications for the endogenous response of the model to the policy.

### Further attempts

In his work, Shimer (2005) already suggests some of the modifications that could address his critique. These have been developed in some of the papers that followed.<sup>24</sup>

Pissarides (2009) shows that if there are some fixed costs involved in creating a vacancy, the model generates more fluctuations because costs are less cyclical —fixed costs act in a similar way as delay costs in Hall and Milgrom (2008). However, as Mortensen and Nagypal (2007) note, we do not have enough evidence on the nature of fixed costs. Do they depend on  $w$  or  $p$ ? This is crucial for the argument to work. In a recent paper, Silva and Toledo (2012) show that if fixed costs can be partially passed on to new hired workers in the form of lower wages, then amplification is reduced. They also find a similar Costain and Reiter (2008) critique. The response to changes in unemployment benefits is too high if the fixed costs are set to match the fluctuations in labor market tightness  $\theta$ .

Some more refined extensions include Menzio (2005) and Kennan (2010) ,

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<sup>24</sup>Among others he mentions fixed costs (he shows that when  $q(\theta) = \mu$  is constant, which is equivalent to fixed costs, fluctuations can be very large), asymmetric information, on-the-job search, wage rigidity or specific capital.

which create some endogenous wage rigidity by adding some asymmetric information about match's productivity, and Eyigungor (2010), which combines specific capital and embodied technology to generate the observed fluctuations.<sup>25</sup>

Finally, Mortensen and Nagypal (2007) shows that shocks to  $s$  amplify fluctuations only if the feedback from  $f(\theta)$  to the wage is weak. Otherwise the Beveridge Curve could even slope upwards, as Shimer (2005) shows. When using Hall and Milgrom's calibration and their wage determination mechanism the ratio  $\sigma_\theta/\sigma_p$  is 12.56 with shocks to labor productivity and separations. This is around two thirds of its empirical counterpart —here one needs to use as a target the empirical  $\sigma_\theta/\sigma_p$ , which is 19.1, because both  $s$  and  $p$  shocks are the source of fluctuations.

A more detailed exposition of these extensions of the search models is unnecessary for the study of frictional wage dispersion. What matters is that all these attempts to generate reasonable fluctuations require at least the value of non-market time in Hall and Milgrom (2008). And the implications for the frictional wage dispersion puzzle are significant.

### Search and matching in RBC models

Despite the failure of the search and matching framework to match the cyclical fluctuations in unemployment and vacancies, it is worth noting that search and matching frictions are important for business cycle fluctuations. Merz (1995), Andolfatto (1996) and den Haan, Ramey and Watson (2000) show that search and matching frictions improve the performance of RBC models. Among other successes, they show that with this extension RBC models accurately replicate movements in employment and wage fluctuations —in particular output and hours fluctuate more than wages—, predict that productivity leads hours and deliver a Beveridge Curve. Therefore, RBC models with search and matching frictions explain business cycle fluctuations better than the competitive market model. However, the problem is that these extensions of the RBC model are still unable to

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<sup>25</sup>Vintage capital clearly increases fluctuations in vacancies and unemployment, but has counterfactual implications for job destruction —when a technology shock arrives firms have incentives to sell the old capital to acquire the new vintage. By adding specific capital the model corrects for this and matches well the behavior of unemployment and vacancies.

match fluctuations in  $\theta$ , as wages are still too flexible because of Nash Bargaining.

### 1.2.3 Taking Stock: Implications for the Frictional Wage Dispersion Puzzle

In the above discussion of what drives cyclical fluctuations in unemployment and vacancies, the one element that has implications for frictional wage dispersion is the value of non-market time. Wage dispersion in search models depends on labor market flows, the interest rate and the value of non-market time —as captured by the replacement ratio. Except for the value of non-market time, the rest of the determinants of wage dispersion remain the same in all the different modifications of the baseline search model that account for cyclical fluctuations.

One conclusion is clear from the discussion of the Shimer critique. To account for fluctuations in unemployment and vacancies one needs high values of non-market time. No matter what the extension or modification of the baseline search model is, it needs to assume at the very least the value in Hall and Milgrom (2008). However, higher values of the non-market time deliver even lower amounts of wage dispersion. Therefore, there is a trade-off between accounting for labor market cyclical fluctuations and generating significant wage dispersion.

Table 1.2:  $Mm$  RATIO WITH DIFFERENT REPLACEMENT RATIOS

Source	$\rho$	$Mm$ ratio
Shimer (2005)	0.40	1.046
Hall & Milgrom (2008)	0.71	1.022
Hagedorn & Manovskii (2008)	0.95	1.004

More formally, consider the expression for the  $Mm$  ratio in (1.2). Clearly, the  $Mm$  ratio decreases with the replacement ratio  $\rho$ . Table 1.2 shows the value of the  $Mm$  ratio with replacement ratio  $\rho$  as in Hall and Milgrom (2008) and Hagedorn and Manovskii (2008). The  $Mm$  ratio is 1.05 with the original value for  $\rho$  of 0.4. With  $\rho$  equal to 0.71 —the value in Hall and Milgrom (2008)— it goes down to

1.02. Finally, the replacement ratio in Hagedorn and Manovskii (2008), which has a value of 0.955, delivers an  $Mm$  ratio of only 1.004. This latter value means that the average wage is only 0.4% above the minimum observed wage, an extremely low value. Figure 1.1 shows the  $Mm$  ratio as a function of the replacement ratio  $\rho$ . It illustrates one of Hornstein, Krusell and Violante (2011) points. Unless the value of non-market time is extremely negative, the baseline search model cannot generate amounts of wage dispersion similar to those observed empirically. Even with a value for  $\rho$  of -1 the  $Mm$  ratio is less than 1.2. By requiring high values of non-market time, accounting for cyclical fluctuations makes the frictional wage dispersion problem worse.

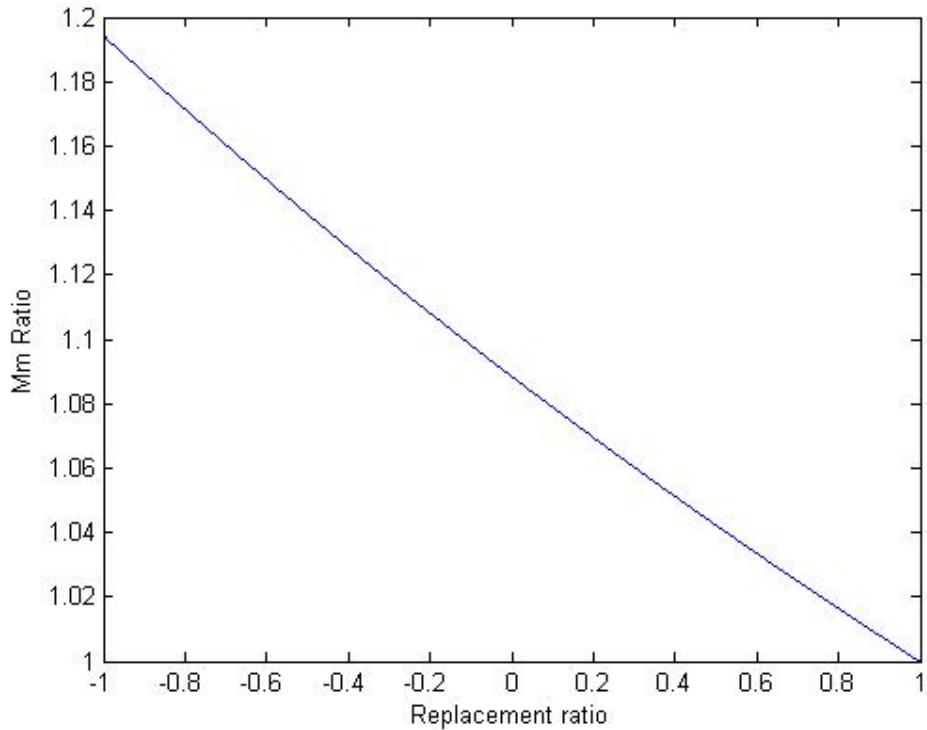


Figure 1.1:  $Mm$  RATIO AND REPLACEMENT RATIOS

## 1.3 The Effects of Unemployment on Wages

Looking at how workers decide when to accept a job offer provides an immediate solution to the frictional wage dispersion puzzle. Find a way of making unemployment more costly and wage dispersion will increase. Intuitively, with more costly unemployment workers' value of being unemployed decreases. Workers are willing to accept lower offers as the value of their outside option —unemployment— is lower. A lower reservation wage raises wage dispersion.

A way to achieve this is by introducing a loss of skills or human capital associated with unemployment. This mechanism delivers higher wage dispersion. Workers lower their reservation wage to give themselves a better chance of leaving unemployment and avoid the human capital losses incurred during unemployment. In view of this, this thesis explores in detail how much wage dispersion can be generated through this mechanism.

There is empirical evidence that unemployment hurts the workers who suffer it. Most of this evidence comes from the job displacement literature, which studies the effects of job displacement —a particular case of unemployment— on wages and other labor market outcomes. This literature finds that job displacement causes large and very persistent earning losses to displaced workers. The focus is slightly different than the one I take in later chapters, but provides one of the sources of evidence on the effects of unemployment on wages. Therefore, I briefly summarize the results in what follows.

### 1.3.1 Who are the displaced workers?

At the heart of the job displacement literature lies the concern of how structural changes affect workers' prospects. Structural changes may be brought about by technological change, increasing competition from international trade, changes in the composition of demand or changes in government regulation —such as environmental protection. Why is the literature concerned with the fortunes of workers suffering losses from structural changes? If these changes benefit society

as a whole, then the optimal policy should promote structural change while compensating the workers who suffer losses in the process. Even to this day the US Congress debates and approves legislation that compensates displaced workers, usually through extended unemployment insurance (UI) protection. To determine the magnitude of such public policy, we need to know who suffers losses and the size of such losses.<sup>26</sup>

The job displacement literature focuses on workers with a high attachment to their industry, firm or occupation. Displaced workers are thus a subset of all unemployed workers. They have accumulated large amounts of specific human capital. They are involuntarily separated from their jobs, and have little chance of being recalled by their employer or finding a similar job within a reasonable span of time. Because the job loss must be involuntary, quits, temporary layoffs and firings for cause are not job displacements. As opposed to the empirical work included in Chapter 2, this literature is therefore not as concerned with the effects of unemployment on workers' wages brought about by cyclical fluctuations.

To select workers who are attached to their job, the job displacement literature usually focuses on workers with a minimum tenure on a job —usually 6 years, although in some cases the minimum tenure is 3 years. The Bureau of Labor Statistics (BLS) provides the following definition of a displaced worker. A displaced worker is at least 20 years old, with more than 3 years of tenure with their current employer,<sup>27</sup> who has lost their job without a recall either due to “slack work, abolition of position or shift, or plant closing or relocation”. However, most studies in the literature use an even stricter definition to make sure that they select only displaced workers. For example, in most cases only plant closures and mass-layoffs are considered.

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<sup>26</sup>Fallick (1996) and Kletzer (1998) are excellent reviews of the early findings of the job displacement literature. Couch and Placzek (2010) and von Wachter, Song and Manchester (2009) provide more recent results.

<sup>27</sup>To give some idea of how many unemployed workers fall in that category, 58% of workers being separated have less than 3 years of tenure.

### 1.3.2 Data sources

Although the estimates vary with the data source, location and period of the study, the job displacement literature finds unambiguous evidence that job displacement causes large and persistent losses to displaced workers. Next, I discuss the advantages and disadvantages of each data source. In the following section I present the results by data source, as this allows for a better comparison between the different studies and helps understand where the differences come from.

Each of the data sources comes with advantages and disadvantages. To help understand job displacement, starting in 1984 the Bureau of Labor Statistics (BLS) began preparing a survey of displaced workers, the Displaced Workers Survey (DWS). The information is drawn from another data collected by the BLS, the Current Population Survey (CPS).

The main advantages of the DWS are its size and representativeness. Because it draws from the CPS, it has access to information from 60,000 households. Further, the CPS sample is representative of the US population, so the DWS is a national sample. However, the DWS also has some important shortcomings. First, it lacks a control or comparison group. One can only measure earnings losses as the difference between pre-displacement and post-displacement earnings. But doing so fails to account for the fact that earnings may have increased had displacement not occurred. So to estimate the costs of displacement one needs to compare post-displacement earnings to the level of earnings had displacement not occurred—essentially the earnings of the non-displaced. Farber (1997) tries to control for this by constructing a “synthetic” sample of non-displaced workers using the CPS. Another important shortcoming of using the DWS can not be so easily corrected. Because the DWS does not follow workers, it can not control for average earnings. Presumably displaced workers may have lower average earnings than non-displaced workers. If less productive workers are more likely to be displaced, then estimates of the effects of job displacement may be biased.<sup>28</sup>

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<sup>28</sup>Some relatively less important disadvantages of using the DWS include the following. The DWS asks workers about displacements that have occurred in the past. As with any survey that

To overcome the limitations of the DWS, some studies use the Panel Study of Income Dynamics (PSID). It consists of a panel of US workers, and covers 30 years of data —1968 to 1997.<sup>29</sup> Using the longitudinal dimension of the PSID, these studies can control for fixed-effects, which controls for the different average earnings of displaced and non-displaced.<sup>30</sup> It is also a national sample, so the results also apply to the whole of the US. The only shortcomings of the PSID is its relatively small size and, as with any survey, measurement error.

To overcome the measurement error in DWS and PSID, many studies use administrative records from US states used to calculate UI benefits.<sup>31</sup> Further, the sample sizes are very large, usually 5% of the state. Although both administrative data and the PSID follow workers through time and allow for a control group, administrative data provides some advantages over the PSID. It is a much bigger sample, its income records are not top-coded and it includes information about the employer. In particular, it provides the exact quarter of separation. However, there are some disadvantages with using administrative data. Administrative data is only available at the state level and for a given period of time, so there may be concerns that the results apply only to certain states and not nationwide, and to the time period of the study. Further, administrative data lacks demographic information such as occupation, education or marital status, and lacks information on hours worked.<sup>32</sup> Finally, the reason for separation is not available in admin-

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ask respondents to recall past events, it contains some mistakes. As von Wachter, Handwerker and Hildreth (2007) show, the DWS tends to underestimate the number of job displacements. Another problem is that in the DWS the post-displacement employment is identified as the job at the time of the survey. The DWS is unable to identify if the worker had a different job between displacement and the job at the time of the survey.

<sup>29</sup>Chapter 2 contains a more detailed description of the PSID.

<sup>30</sup>To see how important controlling for fixed-effects is, consider the following case. Using the DWS, Gibbons and Katz (1991) find that earnings losses are lower for workers displaced because of a plant closure or due to mass layoffs. They argue that this provides evidence that employers take unemployment spells as a signal of low performance. When Stevens (1997) uses the PSID the results are completely reverse. Mostly, the DWS fails to take into account that workers employed at firms with plant closure and mass layoffs already start losing earnings before they are displaced, which is what drives the result.

<sup>31</sup>The state's administrative records used to construct the data are used to calculate firms' tax liabilities, so they can be assumed free of measurement error.

<sup>32</sup>The only demographic information available is age and sex.

istrative data. For this reason, studies using administrative data focus on mass layoffs or plant closure.<sup>33</sup>

### 1.3.3 Empirical evidence from the job displacement literature

To give a general picture of the magnitude of the estimated losses, the DWS studies find the lowest estimate for the earnings losses due to displacement — not surprisingly given that they can not use fixed-effects. Estimates from the PSID are larger but relatively closer to estimates from the DWS. Studies based on administrative records tend to produce the largest estimated losses.

#### Administrative data.

The reference in the study of job displacement is the work by Jacobson, LaLonde and Sullivan (1993) (JLS). Not only was their work the first to use administrative data to study job displacement, their methodology has become the standard in such studies.

The data in JLS comes from administrative records from the state of Pennsylvania which are used to calculate UI benefits, providing quarterly information on earnings from 1974 through 1986. JLS focus on workers with least 6 years of tenure with their employer. They further restrict to workers between 20 and 49 years old at the time of separation. Thus, the selected workers are likely to possess large amounts of specific human capital. Now, their data does not provide the reason for job separation, so they are unable to distinguish between voluntary and involuntary quits. To overcome this shortcoming, JLS focus on mass-layoffs and plant closure to make sure that the sample selected includes only involuntary separations. Using information about workers' firms, workers are considered separated from mass-layoff if their firm has experienced a 30% drop in employment around the time of the separation. As many firms in the manufacturing and mining sector suffered plant closures and mass-layoffs in Pennsylvania during the

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<sup>33</sup> As von Wachter, Handwerker and Hildreth (2007) show, administrative data tends to overstate the number of job losses.

period of the study, the problem with this selection is that it is likely picking up workers that suffer larger earnings losses.

The statistical methodology in JLS represents another great contribution to the job displacement literature. They apply program evaluation techniques to study the effects of job displacement on workers earnings, which in a nutshell consists of using both fixed-effects and time trends. The statistical model used follows this specification

$$\ln Y_{it} = \alpha_i + \gamma_t + X_{it}\beta + D_{it}\delta + \varepsilon_{it}. \quad (1.16)$$

In the above equation  $Y_{it}$  represents earnings,  $\alpha_i$  is worker fixed-effects, quarterly dummies  $\gamma_t$  capture time trends and  $X_{it}$  includes demographic variables, although the only information available is sex and age —and their interactions. The  $D_{it}$  dummies indicate when the displacement occurs, so the vector  $\delta$  captures the effects of displacement before and after displacement.

Using this statistical model, JLS find that displaced workers suffer 40% earning losses after job displacement, and after 6 years earnings losses are still around 25%. Earnings losses are large and persistent for all industries and for workers who return to same industry, although losses are larger in manufacturing and mining and for those who change sectors.

Because JLS use data from Pennsylvania during the 70s and 80s, they cover a period of high unemployment in a state with a big manufacturing sector. Their sample includes high seniority workers laid-off at a time with many plant closures in the manufacturing and mining sector. This raises concerns about the limitation of their results. Are the results representative of the US as a whole? How much can we generalize them? Given these concerns, Couch and Placzek (2010) use administrative data, similar to the one used in JLS, but from the state of Connecticut during the much calmer period from 1993 to 2004. They use the same estimation techniques in JLS, and find large and persistent losses, but more in line with evidence from the DWS or the PSID. When they only focus on workers

separated in mass-layoffs, earnings losses are around 33% the year after separation, and 13 to 15% after 6 years from the year of separation. If all separators are included, losses are still 33% after separation and 7 to 9% 6 years after separation. As opposed to JLS, Couch and Placzek (2010) find that workers in finance, insurance and real estate have the largest earnings losses 5 years after job loss, followed by workers in business and professional services and workers in manufacturing. Workers in education and health services suffer the smallest losses, with earnings fully recovered in the long-run.

Although the study in Couch and Placzek (2010) provides evidence on the costs of job displacement in more tranquil economic times, one still wonders how general the results are. Further, both studies share a common problem, that some workers leave the state and it is impossible to distinguish between workers who have left the state and workers who simply have no earnings —both are registered with zero earnings.<sup>34</sup> It would also be interesting to find evidence of the earnings losses beyond 6 years after job separations. In an ambitious project, von Wachter, Song and Manchester (2009) use administrative data from the Social Security to draw a large random sample of the *entire* US population. The data, with similar characteristics to the previous two studies, contains employment and earnings information covering 30 years of longitudinal data. Therefore, they follow workers' earnings losses for up to 20 years after job separations in the early 80s. The long span of the study and the fact that it covers the entire US labor market are major advantages of this study.

Although their results are preliminary, they also find very large and persistent earnings losses from job displacement. Losses last over 20 years. The JLS is the benchmark in their study. They use the same statistical methodology and also focus on mass-layoffs to make sure they isolate job displacements. They consider workers in their middle age who are separated between 1980 and 1986. There are two differences with the JLS study. First, they only look at male workers, to avoid

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<sup>34</sup>However, JLS find that workers who move within Pennsylvania suffer larger losses, so most probably those who move out of state have larger losses too, so the estimates may not be biased.

having to model labor force participation. Second, they also estimate losses for a sample of workers with shorter tenure at the time of job loss —3 years instead of 6 years.

They find that displaced workers suffer 30% immediate earnings losses, 20% losses 10 years after separation, 10% 15 years after, and that losses are not faded after 20 years. The results are very robust. They hold if one controls for firm or industry fixed effects, and show that losses are large for all age and industry groups. Although, as the authors mention, these results are preliminary.

### **The Panel Study of Income Dynamics (PSID).**

The reference in the study of job displacement using the PSID is Stevens (1997). The advantage with the PSID is that it is longitudinal, so Stevens (1997) can use the same estimation technique as JLS of fixed effects combined with time-trends. However, it is not possible to identify if a worker is laid off or fired due to performance. So displacement is identified as involuntary termination of a position, and includes workers who lose their jobs due plant or business closing, but also those who are laid off and fired. Temporary layoffs and end of temporary jobs are not considered displacements. As the PSID does not provide information on firms, it is impossible to focus on mass-layoffs and plant closures.

As with results from administrative data, Stevens (1997) finds large and persistent losses. Immediate earnings losses are around 25% at the time of displacement, 15% a year after and between 7% and 12% 7 years or more later —the last period is 10 years after displacement. A similar pattern emerges from wages, falling 12% at time of displacement, around 8% a year after, 9% 7 years after separation and remain roughly the same up until 10 years after displacement.

Also using the PSID, Ruhm (1991) finds between 14 and 18% earnings losses at the time displacement, and between 10 and 13% after 4 years of separation.

### **Displaced Workers Survey (DWS).**

Studies based on the DWS include Carrington (1993), Farber (1997) and Topel (1990). These studies do not include a control group, except for Farber (1997), who

constructs a ‘synthetic’ control group of non-displaced workers based on the CPS. These DWS studies estimate earnings losses to be between 9% and 16% losses from displacement. In Farber (1997), when a control group is used estimated losses are larger, as losses go from 9% to 12%.

### **National Longitudinal of Youth (NLSY).**

Using similar techniques to JLS, work by Kletzer and Fairlie (2003) use the National Longitudinal of Youth (NLSY), a panel of young workers, to look at the effects of displacement on young workers’ earnings. Presumably these workers should have little specific human capital. However, the authors still find big losses. Earnings losses are around 18% a year after displacement, and 9% 5 years after. When looking at wages, losses are 10% a year after separation, and 21.2% 5 years after separation. Interestingly, losses are bigger for college graduates, with 25% earnings losses in the fifth year after separation. However, in the case of young workers Kletzer and Fairlie (2003) show that these losses mostly reflect missed opportunities to accumulate skills early in the career, as young workers out of work miss on steep earnings growth.

## 1.4 Mathematical Derivations

### 1.4.1 $Mm$ ratio in the Pissarides random-matching model

Let  $U$  and  $W$  be the asset values of being unemployed and employed, and  $J$  and  $V$  those of a filled position and a vacancy. Workers get wages  $w(p)$ . The following asset equations hold

$$rU = b + f \int_{p^*}^{p^{max}} (W(p) - U) dF(p) \quad (1.17)$$

$$rW(p) = w(p) - s(W(p) - U) \quad (1.18)$$

$$rJ(p) = p - w(p) - sJ(p) \quad (1.19)$$

$$rV = -k + q \int_{p^*}^{p^{max}} J(p) dF(p), \quad (1.20)$$

where  $k$  is the cost of posting a vacancy. Assuming Nash Bargaining (NB) of the match surplus gives that

$$\beta J(p) = (1 - \beta)(W(p) - U). \quad (1.21)$$

Using the asset equations and the NB rule I get the following expression for wages

$$w(p) = (1 - \beta)rU + \beta p. \quad (1.22)$$

The minimum observed wage corresponds to the wage evaluated at  $p^*$ . I denote it by  $w^*$ , i.e.  $w^* = w(p^*)$ . Since  $W(p^*) = U$ , and  $J(p^*) = 0$ , the following properties hold

$$p^* = rU \quad (1.23)$$

$$w^* = p^*. \quad (1.24)$$

In particular, combining wages in (1.22) and (1.23) gives that

$$w(p) = p^* + \beta(p - p^*). \quad (1.25)$$

Using the asset equations for  $U$  and  $W$  gives that

$$w^* = b + f \int_{p^*}^{p^{max}} (W(p) - U) dF(p). \quad (1.26)$$

Substituting (1.22) into the asset equation for  $W(p)$  gives that

$$W(p) - U = \frac{\beta(p - p^*)}{r + s}. \quad (1.27)$$

Therefore, combining (1.27) and (1.26) gives

$$w^* = b + \frac{f}{r + s} \int_{p^*}^{p^{max}} \beta(p - p^*) dF(p), \quad (1.28)$$

which after adding and subtracting  $p^*$  and dividing and multiplying by  $1 - F(p^*)$  in the integrand on the right-hand side gives

$$w^* \left( 1 + \frac{f^*}{r + s} \right) = b + \frac{f^*}{r + s} \int_{p^*}^{p^{max}} w(p) \frac{dF(p)}{1 - F(p^*)}. \quad (1.29)$$

As  $b = \rho \bar{w}$  and

$$\bar{w} = \int_{p^*}^{p^{max}} w(p) \frac{dF(p)}{(1 - F(p^*))}, \quad (1.30)$$

I get that

$$w^* \left( 1 + \frac{f^*}{r + s} \right) = \bar{w} \left( \rho + \frac{f^*}{r + s} \right). \quad (1.31)$$

The above equation gives the expression for the  $Mm$  ratio in the text

$$Mm = \frac{\bar{w}}{w^*} = \frac{1 + \frac{f^*}{r+s}}{\rho + \frac{f^*}{r+s}}. \quad (1.32)$$

### 1.4.2 Unemployment fluctuations in the canonical search model

The asset equations for  $U$ ,  $W$ ,  $J$  and  $V$  are given by

$$rU = b + f(\theta)(W - U) \quad (1.33)$$

$$rW = w - s(W - U) \quad (1.34)$$

$$rJ = p - w - s(J - V) \quad (1.35)$$

$$rV = -k + q(\theta)(J - V) \quad (1.36)$$

There is free entry in the market for vacancies, so  $V = 0$  yields

$$J = \frac{k}{q(\theta)} \quad (1.37)$$

Using the asset equation for  $J$  gives to substitute  $J$  in the above equation gives

$$\frac{p - w}{r + s} = \frac{k}{q(\theta)}. \quad (1.38)$$

Nash Bargaining implies that

$$(1 - \beta)(W - U) = \beta J. \quad (1.39)$$

Using the asset equations for  $W$  and  $J$  and the above Nash sharing rule gives

$$w = rU + \beta(p - rU - (r + s)V). \quad (1.40)$$

Rearranging and substituting (1.38) gives the following expression for wages

$$w = (1 - \beta)b + \beta(p + \theta k). \quad (1.41)$$

Equilibrium  $\theta$  is determined by free entry condition (1.38) and wages (1.41). Substituting the wage (1.41) into (1.38) and rearranging gives that  $\theta$  is the solution to the following equation

$$\frac{(1 - \beta)(p - b)}{k} = \frac{r + s}{q(\theta)} + \beta\theta \quad (1.42)$$

which is equation (1.13) in the text, and is the same equation as in page 36 of Shimer (2005) with no stochastic shocks ( $\lambda = 0$ ). I use  $\varepsilon_\theta$  and  $\varepsilon_w$  to denote the elasticities of  $\theta$  and  $w$  with respect to  $p$ , and  $\eta$  to denote the elasticity of  $q(\theta)$  with respect to  $\theta$ , i.e.  $\eta = -q'(\theta)\theta/q(\theta)$ , as  $q'(\cdot) < 0$ . Taking logs of the free entry condition (1.38) and differentiating with respect to  $p$  gives equation  $\varepsilon_\theta$  as a function of  $\varepsilon_w$  as given by (1.11) in the text

$$\frac{1 - \frac{dw}{dp}}{p - w} = \frac{\eta}{\theta} \frac{d\theta}{dp} \quad (1.43)$$

$$\varepsilon_\theta = \frac{1}{\eta} \frac{p - \varepsilon_w w}{p - w}. \quad (1.44)$$

This corresponds to expression , which is the same as equation (20) in Pissarides (2009).

I use now the notation  $\tilde{\varepsilon}_\theta$  to denote the elasticity of  $\theta$  with respect to *net* labor productivity  $p - b$ . Taking log and derivative of (1.42) and rearranging gives the expression (1.14) for  $\tilde{\varepsilon}_\theta$

$$\tilde{\varepsilon}_\theta = \frac{r + s + \beta f(\theta)}{\eta(r + s) + \beta f(\theta)}. \quad (1.45)$$

The above is the same value as in Shimer (2005) with no shocks, only with  $\eta$  instead of  $1 - \eta$ , as in Shimer (2005)  $\eta$  is the elasticity of  $f(\theta)$  and not  $q(\theta)$ . Following the

same procedure gives the elasticity with respect to labor productivity

$$\varepsilon_\theta = \frac{r+s+\beta f(\theta)}{\eta(r+s)+\beta f(\theta)} \cdot \frac{p}{p-b}. \quad (1.46)$$

Unemployment evolves according to the following transition equation

$$\dot{u} = f(\theta)u - s(1-u), \quad (1.47)$$

which gives the following steady-state unemployment

$$u = \frac{s}{s+f(\theta)}. \quad (1.48)$$

The above equation defines what is known as the Beveridge Curve, and describes a negative relationship between unemployment and vacancies.

Shimer (2005) solves a stochastic version of this model. All equations are similar, only that labor market tightness  $\theta_{p,s}$  equation (6) gives

$$\frac{r+s+\lambda}{q(\theta_{p,s})} + \beta\theta_{p,s} = (1-\beta)\frac{p-z}{k} + \lambda\mathbb{E}_{p,s}\left(\frac{1}{\theta_{p',s'}}\right). \quad (1.49)$$

Solving the above equation numerically gives the values for labor market tightness  $\theta_{p,s}$ . The rest of the equations are the same.

## Chapter 2

# Unemployment History and Frictional Wage Dispersion

A large number of papers in labor economics explore the determinants of wages. The effects on wages of worker characteristics, such as education or tenure, are well documented. However, worker characteristics can only explain a fraction of the observed wage dispersion in the data. Once one controls for these characteristics, the residual still displays a large amount of dispersion. Therefore, observationally similar workers are paid different wages.

Search models of the labor market can explain why apparently similar workers are paid different wages. In these models, workers adopt a reservation wage strategy when looking for jobs. Job offers are only available with a given frequency, so workers accept a job offer if the associated wage is above their reservation value. This acceptance rule by workers generates wage dispersion, even among identical workers.<sup>1</sup> However, recent work by Hornstein, Krusell and Violante (2011) shows that baseline search models fail to generate significant wage dispersion. The authors use the ratio between the mean and minimum wage observation, the mean-min or  $Mm$  ratio, to measure wage dispersion. In search models the  $Mm$  ratio is a function of labor-market flows and preference parameters, for which re-

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<sup>1</sup>The literature uses the term frictional wage dispersion to describe the wage dispersion among identical workers that arises from search frictions. For example, see Mortensen (2005).

liable estimates exist. These estimates imply an  $Mm$  ratio in search models of around 1.05, implying that the mean wage is 5% higher than the minimum observed wage. By contrast, the residual in a Mincerian regression, with as many controls as possible, gives a 50-10 percentile ratio between 1.7 and 1.9. Given that this 50-10 percentile ratio is a reasonable empirical counterpart to the  $Mm$  ratio, the gap between the two values is remarkable.<sup>2</sup>

This paper introduces a search model in which workers lose some human capital or skills during unemployment. Workers become less productive while they remain unemployed, so wages depend on workers' unemployment histories—their cumulative time spent in unemployment. I use this model to address the following question: What happens to wage dispersion among identical workers if they lose human capital during unemployment? The model generates further wage dispersion compared to baseline search models because workers adjust their search behavior. The intuition is the following. Unemployment “hurts” workers. They lose human capital during unemployment, which depreciates their wages. Since workers are aware that longer unemployment spells lead to larger wage losses, they are willing to accept lower wages to leave unemployment more quickly. With a lower reservation wage, wage dispersion increases among identical workers.<sup>3</sup>

The paper shows that wage dispersion increases significantly if workers lose some human capital during unemployment. I derive an expression of the  $Mm$  ratio in the model that does not rely on any assumption about the underlying distribution of match productivities. The  $Mm$  ratio is uniquely determined by a set of parameters, for which reliable estimates exist.<sup>4</sup> To illustrate the amount of wage dispersion generated by the model, I compare its implied  $Mm$  ratio to the one in the baseline search model and the data. Using estimates from the Panel Study of Income Dynamics (PSID), the  $Mm$  ratio in the model with loss of human

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<sup>2</sup>Using the 10th percentile reduces some of the measurement error associated with the minimum observation.

<sup>3</sup>To avoid repetition I use wage dispersion to refer to wage dispersion among ex-ante identical workers, the focus of the paper.

<sup>4</sup>In their work Hornstein, Krusell and Violante (2011) find that in most search models one can express the  $Mm$  ratio as a function of a few parameters.

capital has a value of 1.15. By contrast, the  $Mm$  ratio in the baseline search model is 1.04. In the PSID the 50-10 percentile ratio is 1.34. A similar picture emerges if one uses estimates from the Current Population Survey (CPS) for the labor market flows, as given by Shimer (2005). The  $Mm$  ratio has a value of 1.22 in the model with loss of human capital, whereas for the baseline search model the value is 1.05. Empirically, the 50-10 percentile ratio is between 1.7 and 1.9. These results imply that, while the baseline model explains around 11% of the residual wage dispersion in the PSID, and 6% of that in the CPS, the mechanism of the model accounts for around 45% of the wage dispersion in the PSID, and 28% of that in the CPS.

The model incorporates workers' loss of human capital during unemployment in the following way. Workers' human capital depreciates at a constant rate while they stay in unemployment.<sup>5</sup> This feature is introduced in an otherwise typical search model, the Pissarides (1985) random matching model. Each match between the firm and the worker has a match-specific productivity. In contrast to the standard model, the productivity of the match further depends on the worker's human capital, which is uniquely determined by workers' unemployment history. When the worker and the firm meet, if the match-specific productivity is above a reservation productivity value they start to produce.

I assume that unemployment benefits are proportional to workers' human capital. As a result, benefits gradually decrease while workers stay unemployed. There is no reason to believe that benefits should satisfy this property, but assuming it greatly simplifies the solution.<sup>6</sup> However, in the paper I also solve the model with constant unemployment benefits, using numerical methods, and show that it generates very similar amounts of wage dispersion. With proportional benefits, a closed form solution exists. The  $Mm$  ratio is independent of any distributional assumption for match productivities. Evaluating the  $Mm$  ratio only

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<sup>5</sup>This human capital should not be confused with the human capital given, for example, by education, which is observable and thus controlled for in Mincerian regressions.

<sup>6</sup>A number of papers use similar assumptions to simplify the derivations. For example, see Mortensen and Pissarides (1998), Postel-Vinay and Robin (2002b), or Kenneth Burdett, Carlos Carrillo-Tudela and Melvyn G. Coles (2011).

requires knowledge of a few parameters, namely the depreciation rate of human capital during unemployment, the labor market flow rates, the interest rate, and the replacement ratio.

The paper contains some empirical work to quantify the amount of wage dispersion consistent with the data. I use the PSID, one of the large panels of US workers, to construct workers' unemployment history and estimate the rate at which they lose human capital during unemployment. The regressions results indicate that an additional month of unemployment history is associated with around 1.2% wage loss. The  $Mm$  ratio in the model is compared to the 50-10 percentile ratio of the residual in the Mincerian regression.

This paper is motivated by the findings in Hornstein, Krusell and Violante (2011) that baseline search models fail to generate significant wage dispersion. The paper is also related to two literatures.

First, a large empirical literature explores the effects of job displacement on workers' earnings. Chapter 1 summarized the findings of this literature. The magnitude of the earnings losses are much larger than those of this paper. The difference in their estimates comes from their focus on displaced workers, a smaller set of unemployed workers who usually suffer larger losses.<sup>7</sup> Further, because these studies are concerned about the effects of displacement, they do not control for occupation or industry sector. If both displaced and non-displaced workers suffer losses because of a declining industry the effect on both displaced and non-displaced workers is included in their estimates, as they argue this matters for policy.<sup>8</sup> Using the estimates from the job displacement literature would only increase wage dispersion, so this difference is not problematic. However, the em-

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<sup>7</sup>Displaced workers are a subset of all unemployed workers. The formal definition says that displaced workers are fairly attached to their job and are involuntarily separated from it, with little chance of being recalled by their employer or finding a similar job within a reasonable span of time. To select workers who are attached to their job, the job displacement literature usually focuses on workers with a minimum tenure on a job —6 years of tenure for most studies, and at least 3 years. The job loss must also be involuntary, so quits, temporary layoffs and firings for cause are not job displacements.

<sup>8</sup>That is, if for example opening to trade hurts both displaced and non-displaced workers in a particular sector, estimated losses should include these. Nevertheless, the studies report how earnings losses vary per sector.

pirical work in this paper is better suited for the model for the following reasons. First, only some unemployed workers are displaced. Wage losses coming from cyclical fluctuations are not included in the job displacement literature. Second, because my empirical work focuses on how the wage loss depends on workers' unemployment history, it provides a better mapping between the empirical estimates and the corresponding variable in the model.<sup>9</sup> Finally, I do want to control for industry and occupation, because the focus of the paper is wage dispersion among identical workers.

The second literature introduces the loss of human capital during unemployment into search models. Aside from modeling differences, these papers answer different questions. Lars Ljungqvist and Thomas J. Sargent (1998) offer an explanation for the high unemployment in Europe compared to the US.<sup>10</sup> Christopher A. Pissarides (1992) finds that unemployment becomes more persistent when unemployed workers lose skills and studies the implications for long term unemployment. Robert Shimer and Ivan Werning (2006) and Nicola Pavoni (2011) study unemployment insurance. In Kenneth Burdett, Carlos Carrillo-Tudela and Melvyn G. Coles (2011) workers accumulate human capital when they are employed, but there is no loss during unemployment. Workers can search on-the-job, and employed and unemployed workers receive job offers at the same rate.<sup>11</sup> Because they focus on explaining why younger workers move jobs more frequently and are more likely to experience wage gains, they calibrate the parameters taking the empirical  $Mm$  ratio as a target.<sup>12</sup> In Melvyn Coles and Adrian Masters (2000) long-term unemployment arises endogenously. They also introduce train-

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<sup>9</sup>von Wachter, Song and Manchester (2009) find that losses are smaller for workers displaced at the peak of the recovery of the late 80s. This further suggests that losses could depend on the length of the unemployment spell, as in a recovery workers find jobs quicker and unemployment durations are lower.

<sup>10</sup>See also the related papers by Wouter J. den Haan, Christian Haefke and Garey Ramey (2005), and Lars Ljungqvist and Thomas J. Sargent (2007) and (2008).

<sup>11</sup>This assumption of same job offer rates implies very large  $Mm$  ratios, but it is at odds with the data.

<sup>12</sup>Whereas I proceed differently. The  $Mm$  is not a target in a calibration exercise. The model shows a direct link between the parameters in the model and the dispersion of wages. I estimate the parameters from the data, and use them to quantify the dispersion of wages consistent with those estimates.

ing, so workers can recover some of the lost human capital when they start a job. In equilibrium, firms train workers until they regain all of their lost human capital.<sup>13</sup> Their framework suggests that job creation subsidies are a more efficient policy than training for the unemployed. The question in this paper is different. I implement the effects of unemployment history on wages to explore the capacity of search models to generate wage dispersion.

The paper starts with the model, and continues with the empirical work using the PSID. Next, using numerical methods, I derive the solution for the case of constant benefits during unemployment, which to my knowledge is not a widely used approach.<sup>14</sup> I show that both models generate very similar amounts of wage dispersion. Finally, I assess the amount of wage dispersion consistent with the CPS.

## 2.1 The Labor Market

The model builds on the random matching model of Pissarides (1985). I introduce the assumption that workers gradually lose human capital during unemployment at a constant rate  $\delta$ . The loss depends on the time the worker spends in unemployment. Throughout the paper the term unemployment history refers to the cumulative duration of unemployment spells. I use  $\gamma$  to denote unemployment history.

Given the focus on residual wage dispersion, human capital in the model is net of other controls such as education. Thus, human capital depends only on unemployment history. I denote workers' human capital by  $h(\gamma)$ . Normalizing  $h(0) = 1$ , the constant depreciation rate during unemployment implies human capital is given by  $h(\gamma) = e^{-\delta\gamma}$ . Although workers do not accumulate human capital when they are employed, the model is isomorphic to a model in which workers

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<sup>13</sup>Although the empirical evidence shows important wage losses after becoming unemployed. Louis S. Jacobson, Robert J. LaLonde and Daniel G. Sullivan (2005) find that displaced workers regain some lost earnings if they receive training, but they remain far from recovering all lost earnings.

<sup>14</sup>Coles and Masters (2000) do consider constant benefits during unemployment, but their solution is simplified by the retraining assumption.

accumulate human capital during employment and lose it during unemployment, because what matters for workers' search decision is that the human capital gap widens between employment and unemployment.<sup>15</sup>

Workers search for jobs, and firms for job applicants. I assume that the worker and the firm draw a productivity parameter  $p$  from a known distribution  $F(p)$  when they meet. The productivity of the match is determined by the product of match-specific productivity  $p$  and the worker's human capital  $h(\gamma)$ , i.e. by  $h(\gamma)p$ .

Following the approach in Pissarides (2000), labor market flows are determined by a matching function  $m(V, U)$ , where  $V$  denotes vacancies and  $U$  unemployed workers. Market tightness  $\theta$  is defined as the ratio of vacancies to unemployed workers,  $\theta = V/U$ . I assume the usual conditions for the matching function, that it is increasing in both its arguments and concave, and that it displays constant returns. Workers find jobs at a rate  $f(\theta) = m(V, U)/U$ , and firms receive applicants at a rate  $q(\theta) = m(V, U)/V$ . The properties of the matching function imply that  $f(\theta) = \theta q(\theta)$ . If the labor market is tight ( $\theta$  high, many vacancies for a given number of unemployed workers) workers find jobs more easily, and firms have more difficulty finding applicants. In other words,  $\partial f/\partial \theta \geq 0$ , and  $\partial q/\partial \theta \leq 0$ . Separations occur at an exogenous rate  $s$ . The paper only considers the steady-state, so for simplicity I drop  $\theta$  from the notation.

Workers are identical when they first join the labor market. However, they find and lose jobs, so in equilibrium they have different unemployment histories. I assume that workers leave the labor force at a rate  $\mu$ , and are replaced by new workers with zero unemployment history. This allows for a stationary distribution of unemployment histories. I denote the distribution of unemployment histories among unemployed workers by  $G^U(\gamma)$ , and among employed workers by  $G^E(\gamma)$ . These distributions are endogenous.

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<sup>15</sup>Returns to work experience are implicit. If an unemployed worker were employed she would be accumulating human capital. The parameter  $\delta$  captures both this foregone human capital accumulation and the loss of human capital from being unemployed. More formally, the two models are isomorphic because in a model in which workers accumulate human capital during employment and lose it during unemployment, the equation that determines the reservation productivity is the same as in the model of the paper.

I assume that unemployed workers receive payments  $bh(\gamma)$ . With this assumption, payments during unemployment are proportional to workers' human capital level  $h(\gamma)$ , and decrease at the rate  $\delta$  while they remain unemployed. This assumption greatly simplifies the analysis, and allows for a closed-form solution. In section 2.3, I solve the model with constant  $b$  numerically, and assess how this assumption changes the results.

### 2.1.1 Asset equations for workers and firms

Unemployed workers accept a job if the match-specific productivity is above their reservation productivity. Given that human capital decreases while the worker stays unemployed, the reservation productivity may depend on unemployment history  $\gamma$ . In section 2.1.2 I show that Nash Bargaining implies that the reservation productivity is the same for workers and firms. To avoid unnecessary complications in notation, I denote this reservation productivity by  $p_\gamma^*$ .

Let  $U(\gamma)$  be the value function of an unemployed worker with unemployment history  $\gamma$ , and  $W(\gamma, p)$  the value function of an employed worker in a job with match-specific productivity  $p$ . If  $r$  is the interest rate, the asset equation for the unemployed worker is

$$(r + \mu)U(\gamma) = bh(\gamma) + f \int_{p_\gamma^*}^{p^{max}} (W(\gamma, y) - U(\gamma))dF(y) + \frac{\partial U(\gamma)}{\partial \gamma}. \quad (2.1)$$

The left-hand side of (2.1) represents the returns to being unemployed with unemployment history  $\gamma$ , taking into account that workers leave the labor force at rate  $\mu$  (so  $r + \mu$  is the effective discount rate). Consider now the right-hand side. The first term corresponds to the payments workers receive while unemployed. The second term captures the option value of being unemployed, namely that at rate  $f$  the worker receives a job offer with expected gain  $\int_{p_\gamma^*}^{p^{max}} (W(\gamma, y) - U(\gamma))dF(y)$ . The last term captures the capital depreciation of the value of unemployment  $U(\gamma)$ , caused by the depreciation of human capital while the worker stays unemployed.

Wages depend on the human capital of the worker and the match-specific

productivity. I use  $w(\gamma, p)$  to denote the wage of a worker with unemployment history  $\gamma$ , and employed in a job with match-specific productivity  $p$ . The asset equation for the employed worker is

$$(r + \mu)W(\gamma, p) = w(\gamma, p) - s(W(\gamma, p) - U(\gamma)). \quad (2.2)$$

The intuition behind this equation is similar. The worker receives a wage  $w(\gamma, p)$ , and at a rate  $s$  the worker loses the job, which carries a net loss of size  $W(\gamma, p) - U(\gamma)$ .

To find successful candidates, firms post vacancies at cost  $k$ . Remember that later I prove that the reservation productivity  $p_\gamma^*$  is the same for firms and workers. So the firm hires a worker with unemployment history  $\gamma$  if  $p \geq p_\gamma^*$ . The firm receives applications from unemployed workers with  $\gamma$  given by the endogenous distribution  $G^U(\gamma)$ . The asset equation for vacancies is

$$rV = -k + q \int_0^\infty \left( \int_{p_\gamma^*}^{p^{max}} (J(\gamma, y) - V) dF(y) \right) dG^U(\gamma). \quad (2.3)$$

I assume free entry in the market for vacancies, meaning that firms post vacancies until  $V = 0$ .

When production begins, a worker produces  $h(\gamma)p$ . Having to pay wages, the firm receives  $h(\gamma)p - w(\gamma, p)$ . The asset equation for a filled job position is

$$(r + \mu)J(\gamma, p) = h(\gamma)p - w(\gamma, p) - s(J(\gamma, p) - V). \quad (2.4)$$

The intuition is similar. The left-hand side represents the returns to a filled position. The right-hand side captures that the filled position produces a flow  $h(\gamma)p - w(\gamma, p)$ , and that at rate  $s$  the job is destroyed, with a net loss of  $J - V$ .

### 2.1.2 Reservation productivity and wages

In the next few paragraphs I find two results about reservation productivities. First, given the assumption that unemployment benefits and the productivity of

matches are proportional to human capital, I prove that the reservation productivity  $p_\gamma^*$  is independent of  $\gamma$ . This simplifies greatly the analysis. Second, I find an expression that links wages and the reservation productivity.

I assume that wages are determined by Nash Bargaining. When the worker and the firm meet, if  $p \geq p_\gamma^*$  production begins and they split the surplus. Given a bargaining strength  $\beta$ , the wage is the solution to

$$w(\gamma, p) = \arg \max_{w(\gamma, p)} (W(\gamma, p) - U(\gamma))^\beta (J(\gamma, p) - V)^{1-\beta}. \quad (2.5)$$

The surplus of the match is given by  $J(\gamma, p) - V + W(\gamma, p) - U(\gamma)$ . Nash Bargaining implies that the worker gets a share  $\beta$  of the surplus, and the firm a share  $1 - \beta$ . Combining (2.5) and the asset equations for the worker and the firm (2.2) and (2.4) gives the following result:

$$\beta J(\gamma, p) = (1 - \beta)(W(\gamma, p) - U(\gamma)). \quad (2.6)$$

Two properties about  $p_\gamma^*$  are useful. Consider a firm and a worker that meet and draw a productivity parameter  $p$ . Accepting the offer has a value  $W(\gamma, p)$  to the worker. If he rejects the offer the worker walks away with  $U(\gamma)$ . It follows that  $p_\gamma^*$  satisfies

$$W(\gamma, p_\gamma^*) = U(\gamma). \quad (2.7)$$

Similarly, if production starts the match has a value  $J(\gamma, p)$  to the firm. If the firm does not hire the worker it gets the value of the vacancy  $V = 0$ . Thus, the reservation productivity  $p_\gamma^*$  satisfies

$$J(\gamma, p_\gamma^*) = 0. \quad (2.8)$$

The asset equations (2.2) and (2.4), and the sharing rule (2.6) give the first

expression for  $w(\gamma, p)$ :

$$w(\gamma, p) = \beta h(\gamma)p + (1 - \beta)(r + \mu)U(\gamma) \quad (2.9)$$

Evaluating the asset equations for the employed worker and the filled job position (2.2) and (2.4) at  $p = p_\gamma^*$ , and using (2.7) and (2.8) gives

$$(r + \mu)U(\gamma) = w(\gamma, p_\gamma^*), \quad (2.10)$$

$$h(\gamma)p_\gamma^* = w(\gamma, p_\gamma^*). \quad (2.11)$$

Combining these two properties gives

$$(r + \mu)U(\gamma) = h(\gamma)p_\gamma^*. \quad (2.12)$$

Finally, (2.9) and (2.12) give the first expression linking wages  $w(\gamma, p)$  and  $p_\gamma^*$ :

$$w(\gamma, p) = h(\gamma)(\beta p + (1 - \beta)p_\gamma^*). \quad (2.13)$$

The following result simplifies the model.

**Proposition 1.** *The reservation productivity  $p_\gamma^*$  is independent of  $\gamma$ , i.e.  $p_\gamma^* = p^*$ .*

The proof is included in appendix 2.5. The assumption of proportional benefits  $bh(\gamma)$  is crucial for this result. Intuitively, in the wage bargaining process the worker expects to be compensated for giving up  $U(\gamma)$ . While the value of output  $h(\gamma)p$  decreases with unemployment, the value of benefits  $bh(\gamma)$  does too. The first process raises the reservation productivity, as better matches are required if  $h(\gamma)$  is low, and the second lowers it. Given that all quantities are proportional to  $h(\gamma)$  these two opposing effects cancel out, and the reservation productivity stays constant. I formally prove this by guessing a solution and proving that the guess is correct. By contrast, as I show in section 2.3, when benefits are constant this result disappears. With constant benefits  $b$ , the worker expects to be compensated for giving up  $b$ , but the value of expected output  $h(\gamma)p$  decreases with unemployment.

Workers and firms then require better matches for longer unemployment histories. Proposition 1 gives the following wage expression

$$w(\gamma, p) = h(\gamma)(\beta p + (1 - \beta)p^*). \quad (2.14)$$

Next, I derive the  $Mm$  ratio to assess the amount of wage dispersion generated by the model.

### 2.1.3 The $Mm$ ratio

As work by Hornstein, Krusell and Violante (2011) shows, in most search models one can derive the  $Mm$  ratio without assuming any distribution for match-specific productivities  $F(p)$ . This property represents a major advantage over other measures of wage dispersion, such as the variance. I show that the  $Mm$  ratio in the model displays the same property, and is independent of the distributional assumption for  $F(p)$ .<sup>16</sup>

Similar to Hornstein, Krusell and Violante (2011), I define the replacement ratio as the ratio between unemployment benefits and the average wage, and denote it by  $\rho$ . Workers receive different unemployment benefits depending on their human capital. As a result, to find the replacement ratio in the labor market, I compare average benefits with average wages. More specifically,  $\rho$  is given by  $\rho = b\bar{h}(\gamma)/\bar{w}$ , where  $\bar{h}(\gamma) = E(h(\gamma))$  is the average value of  $h(\gamma)$ , and  $\bar{w}$  is the average wage, which is given by

$$\bar{w} = E(w(\gamma, p) | p \geq p^*) = \int_0^\infty \left( \int_{p^*}^{p^{max}} w(\gamma, p) \frac{dF(p)}{1 - F(p^*)} \right) dG^E(\gamma). \quad (2.15)$$

Taking expectations of wage expression (2.14), I find that  $\bar{w} = \bar{h}(\gamma)(\beta\bar{p} + (1 - \beta)p^*)$ ,

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<sup>16</sup>One of the disadvantages of using the  $Mm$  ratio is its reliance on the minimum observation, which suffers from measurement error. However, as suggested by Hornstein, Krusell and Violante (2011), one can use the 10th percentile observation instead to minimize measurement error.

where  $\bar{p} = E(p|p \geq p^*)$  is defined in a similar way

$$\bar{p} = \int_{p^*}^{p^{max}} p \frac{dF(p)}{1 - F(p^*)}. \quad (2.16)$$

This implies that

$$b = \rho(\beta\bar{p} + (1 - \beta)p^*). \quad (2.17)$$

Equation (2.14) shows that wages are proportional to the human capital level  $h(\gamma)$ . Taking the logarithm of (2.14), and using that  $\log(h(\gamma)) = -\delta\gamma$ , shows that log wages are linear in unemployment history  $\gamma$ , with a coefficient  $\delta$ . Therefore,  $h(\gamma)$  can be removed from wages by controlling for unemployment history in a Mincerian wage regression —this is done in the next section, which contains the empirical part of the paper. Given the focus of the paper on residual wage dispersion, I focus on the dispersion in  $\beta p + (1 - \beta)p^*$ . Therefore, the  $Mm$  ratio is given by

$$Mm = (\beta\bar{p} + (1 - \beta)p^*)/p^*. \quad (2.18)$$

To derive the  $Mm$  ratio, use asset equation (2.4), and wage expression (2.14) to find

$$(r + \mu + s)J(\gamma, p) = (1 - \beta)h(\gamma)(p - p^*). \quad (2.19)$$

Substituting equation (2.19), the Nash Bargaining sharing rule (2.6), and (2.12) into the equation for  $U(\gamma)$  given by (2.1) implies the following expression

$$\frac{r + \mu + \delta}{r + \mu}p^* = b + \beta f \int_{p^*}^{p^{max}} \frac{p - p^*}{r + \mu + s} dF(p). \quad (2.20)$$

The above equation allows for a simple expression of the  $Mm$  ratio

$$Mm = \frac{\frac{r+\mu+\delta}{r+\mu} + \frac{f^*}{r+\mu+s}}{\rho + \frac{f^*}{r+\mu+s}}, \quad (2.21)$$

where  $f^* = f(1 - F(p^*))$  is the job finding probability.<sup>17</sup> Appendix 2.5 shows how to derive the  $Mm$  ratio from (2.20) in more detail.

Expression (2.21) shows that the  $Mm$  ratio measures wage dispersion without relying on any distributional assumption for  $F(p)$ . While it depends on the job finding rate  $f^* = f(1 - F(p^*))$ , this rate is eventually determined by the data, and no functional assumption for  $F(p)$  is required. Further, substituting  $\delta = 0$  yields the same  $Mm$  ratio as in the Pissarides (1985) model.<sup>18</sup>

The relationship between the job finding and separation rates and the  $Mm$  ratio is intuitive. With higher  $f^*$ , workers find jobs more quickly, and the value of workers' outside option increases. Workers respond to the higher outside option by increasing their reservation productivities, which lowers wage dispersion and the  $Mm$  ratio. The  $Mm$  ratio is thus decreasing in  $f^*$ . The separation rate has the opposite effect, so higher separation rates increase the  $Mm$  ratio. Finally, a higher  $\delta$  makes unemployment more costly for workers. Workers are willing to lower their reservation productivity to leave unemployment more rapidly, which increases the  $Mm$  ratio.

By using the  $Mm$  ratio, one only requires knowledge of  $r$ ,  $\rho$ ,  $s$ ,  $f^*$ ,  $\mu$  and  $\delta$  to assess the amount of wage dispersion in the model. Reliable estimates for these parameters can be found from data. In the next section, I use micro data to estimate them and quantify the size of wage dispersion in the model.

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<sup>17</sup>While job offers arrive at rate  $f$ , the worker accepts them if  $p \geq p^*$ , which happens with probability  $1 - F(p^*)$ . Therefore, the job finding rate is given by  $f^* = f(1 - F(p^*))$ .

<sup>18</sup>With  $\delta = 0$  the  $Mm$  ratio is given by  $Mm = (1 + \frac{f^*}{r+\mu+s}) / (\rho + \frac{f^*}{r+\mu+s})$ , which is the expression Hornstein, Krusell and Violante (2011) find for baseline search models. Further, if  $\delta = 0$  one can see that the model corresponds to the Pissarides (1985) model.

## 2.2 Empirical work

I use data from the 1968-1997 waves of the PSID.<sup>19</sup> Appendix 2.6 describes the sample selection. The main motivation for using the PSID is that it follows workers over time. In the model wages depend on workers' unemployment history. Given the panel structure of the PSID, I construct workers' unemployment history to estimate  $\delta$ . The panel structure has the further advantage of allowing for fixed effects estimation. There may be some unobserved characteristics that make some workers more productive than others. If less productive workers are more likely to be unemployed, the estimation may be biased. By controlling for workers' constant unobserved characteristics, fixed effects estimation solves this problem. Finally, one concern may be that when a worker joins the sample, previous unemployment history is unknown. Fixed-effects again solves this problem. When a worker joins the sample, prior unemployment history remains constant in later observations, so worker fixed effects controls for it. This is another reason for preferring fixed-effects regression over cross-sectional.

### 2.2.1 Estimating $\delta$

In the model,  $\delta$  captures the percentage wage loss caused by unemployment history, which consists of the accumulated unemployment spells of the worker. The PSID asks workers how many weeks they were unemployed in the previous year.<sup>20</sup> I use the answers to this question to construct unemployment history. I include this information into the variable *Unhis*, which contains unemployment history in months. To estimate  $\delta$ , I regress the log of wages on *Unhis* and other covariates

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<sup>19</sup>The PSID collected data biennially after 1997 for funding reasons. Therefore, after 1997 information on the number of weeks in unemployment is unavailable for the years without interviews. See Appendix 2.6 for more details.

<sup>20</sup>Most PSID questions are retrospective, and ask the household head about the year prior to the interview. For example, in the 1981 wave the PSID asks about the income or hours worked during 1980.

$X$ , as in the following expression

$$\log W = -\delta \text{ Unhis} + \beta X + \varepsilon. \quad (2.22)$$

In the regression above,  $\delta$  gives the percentage wage loss for an additional month of unemployment history. Taking the logarithm of the equation for wages in (2.14) shows that log wages are linear in unemployment history  $\gamma$ , with a coefficient  $\delta$ . Therefore, wage equation (2.14) is the theoretical equivalent to equation (2.22). As the estimated  $\delta$  captures the exact same effect on wages that  $\delta$  has in the model, this empirical strategy provides an estimate that can be consistently entered in the model.

The results are included in Table 2.1. Column (1) of Table 2.1 corresponds to the regression with worker fixed-effects. Fixed effects regression controls for all constant characteristics, so in column (1)  $X$  also includes potential experience (cubic), regional dummies, and one-digit occupational dummies.<sup>21</sup> The regression gives an estimate for  $\delta$  of  $-.0122$ . I use this value in the rest of the paper.

To check the robustness of the estimate for  $\delta$ , I employ a variety of alternative specifications for the regression in (2.22). The results, shown in Table 2.1, are very robust. Column (2) gives  $\delta$  in the cross-sectional regression, without fixed-effects. Not having fixed-effects, I add several time-invariant regressors to  $X$ . Covariates  $X$  in column (2) include the covariates in column (1), plus race dummies, educational dummies and year-dummies. Column (3) corresponds to the same regression as in column (1), but with two-digit occupation. The regression has fewer observations because it covers only the years 1975-1997, as the PSID did not record two-digit occupations before 1975.<sup>22</sup> However, running column (1) for the same years as in

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<sup>21</sup>One can also add educational dummies. The results do not change.

<sup>22</sup>The PSID recontacted heads of households to get the three-digit occupation for 1968-1974, and included the updated data in a supplement. However, while having some advantages, using this supplement also has some disadvantages. Some people could not be recontacted, so one needs to drop some individuals to use this supplement. If recontact was possible, the PSID asks about events that happened many years before. In any case, as the text points out, one or two digit regressions give very similar results for  $\delta$  over the period for which both are available, so this is not an issue.

Table 2.1: THE EFFECTS OF UNEMPLOYMENT HISTORY ON WAGES

	(1)	(2)	(3)	(4)	(5)
<i>Unhis</i>	-.0122 (.00043)	-.0107 (.00098)	-.0143 (.00057)	-.0164 (.00082)	-.0101 (.00034)
<i>Unhis</i> <sup>2</sup>				.0001 (.00002)	
Occ 1-Digit	yes	yes		yes	yes
Occ 2-Digit			yes		
FE	yes		yes	yes	
Heckman					yes
<i>N</i>	34,542	42,203	25,178	34,542	43,260
LR test, $\chi^2(1)$					157.28

Notes.- *Unhis* in months. Robust standard errors (clustered by worker). Numbers in brackets are standard errors. The  $\chi^2(1)$  is the likelihood ratio test for selection bias. Section 2.2 describes the different columns.

column (3) gives very similar results. Column (4) includes the quadratic  $Unhis^2$ . Including industry does not change the results.

### 2.2.2 Estimating labor-market transition rates

I now estimate the labor market transition rates. Why not use existing estimates from the literature, such as the values from the CPS in Shimer (2005)? To test the explanatory power of the model, one should compare the  $Mm$  ratio in the model with its empirical counterpart in the PSID. The model shows a direct link between labor market flows and the  $Mm$  ratio. If flows are different in the PSID (as they are), using the flows from the CPS would provide a wrong test of the model. Although the evaluation of wage dispersion in this section is very specific to the PSID, later in the paper I explore the amount of wage dispersion in the CPS, and use the values in Shimer (2005).

The estimation strategy is as follows. The PSID gives the worker's employment state at the time of the interview. Using the date of the interview I calculate the time elapsed between interviews. With these two pieces of information I esti-

mate the transition rates as continuous time Markov chains.<sup>23</sup> This probabilistic model arises when workers can find and lose jobs (i.e. change state) between two employment status observations, and they find and lose jobs (change state) with a frequency given by Poisson processes. The last property differentiates the continuous time Markov chain from a simple Markov chain. The assumptions from the probabilistic model are the same as those implicit in the model, so the estimates correspond exactly to the variables in the model. I use  $E$  and  $U$  to denote employed and unemployed workers. The probabilities  $P_{EE}(t)$  and  $P_{UE}(t)$  of transitions  $EE$  and  $UE$  when the time between interviews is  $t$ , taking into account that workers can find and lose jobs between employment status observations, are

$$P_{EE}(t) = \frac{f^*}{s + f^*} e^{-(f^* + s)t} + \frac{s}{s + f^*}, \quad (2.23)$$

$$P_{UE}(t) = \frac{s}{s + f^*} - \frac{s}{s + f^*} e^{-(f^* + s)t}. \quad (2.24)$$

I apply Maximum Likelihood Estimation to the above expressions. The estimation gives the Poisson rates for the separation and job finding rates  $s$  and  $f^*$ .

### 2.2.3 Comparing empirical and model's $Mm$ ratio

To estimate the empirical  $Mm$  ratio, I use the 50-10 percentile ratio of the residual of log-wages in a Mincerian wage regression. Although the  $Mm$  ratio is the ratio of the average and the minimum wage, using the 10th percentile reduces the measurement error associated with the minimum observation. I consider the residual of wage regression (2.22), with fixed effects. Given that the dependent variable is log-wages, one must take the exponential of the residual before extracting the 50th and 10th percentiles. The resulting 50-10 percentile ratio has a value of 1.34.<sup>24</sup>

I compare the empirical  $Mm$  ratio with the one in the model by using expression (2.21). The  $Mm$  ratio is uniquely determined by  $r$ ,  $\rho$ ,  $s$ ,  $f^*$ ,  $\mu$  and  $\delta$ . The earlier estimations give  $\delta$ ,  $f^*$ , and  $s$ . Table 2.2 presents these values, where

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<sup>23</sup>See Sheldon M. Ross (2007) for an exposition of this type of processes.

<sup>24</sup>Andreas Hornstein, Per Krusell and Giovanni L. Violante (2007a) follow a similar approach. They find an  $Mm$  ratio of 1.33 for the PSID, after controlling for fixed effects.

the flow rates correspond to monthly rates, and the value for  $\delta$  to the effect of one month of unemployment history on wages. The value for  $\mu$  is consistent with a working life of 40 years on average. I choose the interest rate  $r$  to be 5% on average, which implies a monthly value for  $r$  of 0.0041. Finally, I consider the same assumption as in Hornstein, Krusell and Violante (2011), and use the value in Shimer (2005) for  $\rho$  of 0.4.<sup>25</sup> With these values, the model generates an  $Mm$  ratio of 1.15.

Table 2.2: ESTIMATES FROM PSID

Parameter	Value	Description
$\delta$	0.0122	Depreciation rate of wage during unemployment
$f^*$	0.1591	Job finding probability
$s$	0.0035	Separation rate

To provide further evidence of the model's contribution, I compare its  $Mm$  ratio to the one corresponding to  $\delta = 0$ . With  $\delta = 0$ , the model corresponds to the Pissarides (1985) model, which delivers an  $Mm$  ratio close to 1.04. The baseline search model struggles to generate significant wage dispersion, precisely the point made by Hornstein, Krusell and Violante (2011).

While the mechanism of the paper accounts only for some of the wage dispersion observed in the data, comparing the above values for the  $Mm$  ratio shows that the improvement brought by the model is important.

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<sup>25</sup>Some papers use higher values for  $\rho$ . Hall and Milgrom (2008) choose  $\rho = 0.71$  and Hagedorn and Manovskii (2008)  $\rho \simeq 0.955$ . As I discussed in Chapter 1, Hornstein, Krusell and Violante (2011) point out that with higher replacement ratios search models match the volatility of unemployment, but generate less wage dispersion, making the frictional wage dispersion problem worse. The highest value used by Hagedorn and Manovskii (2008) also implies that labor supply becomes very responsive to unemployment benefits, as work by Costain and Reiter (2008) shows, which is at odds with data. Hall and Milgrom (2008) make a similar point, with the value from Hagedorn and Manovskii (2008) Frisch elasticities are too large. In Chapter 3 of the paper I discuss higher replacement ratios in more detail.

#### 2.2.4 Correcting for selection bias

Attrition in panels may lead to selection bias problems. Individuals drop from the sample, and the reason may not be random. What determines whether we observe an individual may be correlated with wages. I use a version of James J. Heckman (1979) two-steps procedure to correct for selection bias. It consists of using some information that affects the probability of leaving the sample without directly affecting wages. I use the number of children under 18 and marital status as a determinant of whether the individual stays in the sample or not. Intuitively, married workers with young children are less likely to move and leave the sample than non-married workers without children. I exploit the hypothesis that these variables affect the probability of sample selection, but are not directly correlated with wages, to produce Heckman's two-step correction term. More specifically, I run the following probit regression

$$s_i^* = Z_i\gamma + v_i, \quad (2.25)$$

where  $s_i^*$  is the latent variable, such that  $s_i^* > 0$  if the worker is present in the sample. Regressors  $Z_i$  contain the variables in  $X$ , plus number of children under 18, marital status and number of periods present in the sample. The number of children and marital status are highly significant in the first stage. The Likelihood-Ratio test is included in Table 2.1. The probit estimation produces the Heckman correction term  $\hat{\lambda}$ , that is then added as a covariate in (2.22). Column (5) in Table 2.1 displays the results. In column (5)  $X$  contains the covariates of regression column (2), plus the correction term. The estimates of  $\delta$  in columns (2) and (5) are very close, so selection bias does not appear to affect the estimate for  $\delta$ .

### 2.3 Labor market with constant benefits

In the previous sections I assumed that unemployed workers receive benefits proportional to their human capital, i.e. they receive  $bh(\gamma)$ . From this assumption,

reservation productivities become independent of unemployment history, thus simplifying the model and allowing for a closed form solution. To understand how this assumption may affect the results, consider the following relationship between benefits and workers' reservation choice. If unemployment benefits are higher, workers' outside option increases in value. Workers respond to this by increasing their reservation productivity. This relation between benefits and workers' reservation choice may suggest that by assuming decreasing benefits workers become less picky, thus potentially driving the results. To address this concern, I develop the model of the previous section, but now with constant unemployment benefits  $b$ . I analyze how this new assumption affects the  $Mm$  ratio in the model.

### 2.3.1 Search behavior and labor market outcomes

Reservation productivities depend on unemployment history  $\gamma$ , so I can not simplify the notation  $p_\gamma^*$ . Equations (2.1) to (2.13) remain the same, except that  $bh(\gamma)$  should be replaced by  $b$ . I reproduce here expression (2.13) for wages, which is the result of combining the asset equation of an employed worker (2.2), the asset equation of a filled vacancy (2.4), and the surplus sharing rule (2.6):

$$w(\gamma, p) = h(\gamma)(\beta p + (1 - \beta)p_\gamma^*). \quad (2.26)$$

The next proposition characterizes workers' search behavior, and provides some important results about reservation productivities  $p_\gamma^*$ . The following results are independent of any distributional assumption for  $F(p)$ .

**Proposition 2.** *a.* *There exists  $\bar{\gamma}$  such that  $p_{\bar{\gamma}}^* = p^{max}$ , and the reservation wage of workers with  $\gamma = \bar{\gamma}$  is given by  $w(\bar{\gamma}, p_{\bar{\gamma}}^*) = b$ . Furthermore,  $\bar{\gamma} = -\log(b/p^{max})/\delta$ .*  
*b.* *The reservation productivity  $p_\gamma^*$  is increasing in  $\gamma$ .*  
*c.* *The reservation wage  $w(\gamma, p_\gamma^*)$  is decreasing in  $\gamma$ .*

**Corollary.** *The job finding probability  $f(1 - F(p_\gamma^*))$  is decreasing in  $\gamma$ .*

The proofs are included in appendix 2.5, but I provide some intuition here. When bargaining over wages, both workers and firms receive their outside option

plus a share of the surplus of the match. The worker's outside option  $U(\gamma)$  includes the constant benefits  $b$ , so the worker must always get payments  $b$  at the very least. While benefits are constant, the potential output  $h(\gamma)p$  decreases with unemployment history. Eventually, if the worker stays unemployed for too long, no matches yield  $p$  high enough to cover  $b$ . At that point no matches are profitable, and the worker drops from the labor force. The proposition shows in result (a) that if unemployment history goes beyond  $\bar{\gamma}$ , workers leave the labor force.<sup>26</sup> A similar mechanism explains result (b) that  $p_\gamma^*$  is increasing in  $\gamma$ . Benefits  $b$  are constant, but output  $h(\gamma)p$  and the surplus of the match decrease with higher unemployment history  $\gamma$ . The higher  $\gamma$  gets, the better matches are required for the match to be profitable. In the model of previous sections, assuming decreasing benefits  $bh(\gamma)$  pushed down the reservation productivities by lowering the worker's outside option  $U(\gamma)$ .<sup>27</sup> Finally, the result of the corollary, that the job finding probability is decreasing with  $\gamma$ , follows from the increasing reservation productivity.

### 2.3.2 Reservation productivities

Because the reservation productivity  $p_\gamma^*$  depends on  $\gamma$  if benefits are constant, a closed form expression is not straightforward. Further, one needs to assume a distribution for match-specific productivities  $F(p)$  to be able to solve the model. To simplify the calculations I assume that  $F(p)$  follows a uniform with support  $[0, p^{max}]$ .<sup>28</sup> Given this distributional assumption, I use numerical methods to solve the model. Consider (2.1), with constant  $b$  instead of  $bh(\gamma)$ , as a differential

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<sup>26</sup>Coles and Masters (2000) also find that if benefits are constant, and skills depreciate during unemployment, workers leave the labor force if they stay unemployed too long, becoming long-term unemployed.

<sup>27</sup>The additional assumption that benefits are proportional to  $h(\gamma)$  gives the constant reservation productivity in section 2.1.

<sup>28</sup>The parameter  $p^{max}$  plays no role in the results, because independently of its value  $p_\gamma^*$  and  $p^{max}$  keep the same ratio. Its value would matter if one introduces endogenous and exogenous separations, and is interested in matching their empirical values, as in Pissarides (2009).

equation. Integrating it gives the following expression for  $U(\gamma)$

$$U(\gamma) = \frac{b}{r + \mu} + f \int_{\gamma}^{\bar{\gamma}} e^{-(r+\mu)(\Gamma-\gamma)} \left( \int_{p_{\Gamma}^*}^{p^{max}} (W(\Gamma, p) - U(\Gamma)) dF(p) \right) d\Gamma. \quad (2.27)$$

I derive an equation similar to (2.19). Substituting wage expression (2.26) into the asset equation for  $J(\gamma, p)$  gives

$$(r + \mu + s)J(\gamma, p) = (1 - \beta)h(\gamma)(p - p_{\gamma}^*). \quad (2.28)$$

Given the above equation and the Nash Bargaining sharing rule (2.6), the expression for  $U(\gamma)$  as given in (2.27) becomes:

$$p_{\gamma}^*h(\gamma) = b + \int_{\gamma}^{\bar{\gamma}} e^{-(r+\mu)(\Gamma-\gamma)} \left( \alpha \int_{p_{\Gamma}^*}^{p^{max}} h(\Gamma)(p - p_{\Gamma}^*) dF(p) \right) d\Gamma, \quad (2.29)$$

where  $\alpha = \beta f(r + \mu)/(r + \mu + s)$ . Equation (2.29) provides a way of solving for  $p_{\gamma}^*$  numerically. Using numerical integration and iteration methods I find  $p_{\gamma}^*$  for a grid  $\{\gamma_1 = 0, \gamma_2, \dots, \gamma_n = \bar{\gamma}\}$  of the possible unemployment histories  $[0, \bar{\gamma}]$ . Appendix 2.5 presents the details of the computational strategy.

### 2.3.3 Endogenous distributions $G^U(\gamma)$ and $G^E(\gamma)$

With constant benefits, deriving the  $Mm$  ratio requires knowledge of the endogenous distributions of unemployment histories. I use  $G^U(\gamma)$  and  $G^E(\gamma)$  to denote their cumulative density functions for unemployed and employed workers, and  $N$  and  $E$  for the number of unemployed and employed workers.

To find the endogenous distribution of  $\gamma$ , I look at the flows in the labor market. Unemployed workers find jobs at a rate  $f(1 - F(p_{\gamma}^*))$ , employed workers lose their jobs at rate  $s$ , and all workers leave the labor force at rate  $\mu$ . Workers leaving the labor force are replaced by new entrants with zero unemployment history.

First, consider the group of unemployed workers with unemployment history

lower than  $\gamma$ . In steady-state, a stationary distribution requires that the flows in and out of this group be equal. For  $\gamma \leq \bar{\gamma}$ , this condition gives the following flow equation

$$\begin{aligned} g^U(\gamma)N + f \left( \int_0^\gamma (1 - F(p_\gamma^*)) dG^U(\gamma) \right) N + \mu G^U(\gamma)N &= \\ &= sG^E(\gamma)E + \mu(E + N). \end{aligned} \quad (2.30)$$

The left-hand side corresponds to flows out of the group of unemployed workers with unemployment history lower than  $\gamma$ . The first term represents the workers in that group who have exactly  $\gamma$  unemployment history; the second term those who find a job; and the third term those who leave the labor force. The right-hand side of (2.30) captures the flows in. The first term are the employed workers with unemployment history lower than  $\gamma$  who lose their jobs, and the last term are the new entrants (they have zero unemployment history).

Similarly, consider now the group of employed workers with unemployment history lower than  $\gamma$ . In steady-state, for  $\gamma \leq \bar{\gamma}$ , the following flow equation holds

$$(s + \mu)G^E(\gamma)E = f \left( \int_0^\gamma (1 - F(p_\gamma^*)) dG^U(\gamma) \right) N. \quad (2.31)$$

The intuition is similar. The left-hand side of (2.31) captures the flows out of the group of employed workers with unemployment history lower than  $\gamma$ , and the right-hand side are the flows in.

When  $\gamma > \bar{\gamma}$ , only workers with unemployment history lower than  $\bar{\gamma}$  find jobs.<sup>29</sup> Similar equations to (2.30) and (2.31) hold for  $\gamma > \bar{\gamma}$

$$\begin{aligned} g^U(\gamma)N + f \left( \int_0^{\bar{\gamma}} (1 - F(p_\gamma^*)) dG^U(\gamma) \right) N + \mu G^U(\gamma)N &= \\ &= sG^E(\bar{\gamma})E + \mu(E + N), \end{aligned} \quad (2.32)$$

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<sup>29</sup>The results would be the same if workers are replaced after  $\bar{\gamma}$  by new workers with zero unemployment history.

and

$$(s + \mu)G^E(\bar{\gamma})E = f \left( \int_0^{\bar{\gamma}} (1 - F(p_\gamma^*))dG^U(\gamma) \right) N. \quad (2.33)$$

To simplify the exposition, I define  $\Phi(\gamma)$  as

$$\Phi(\gamma) = \mu \left( \frac{f(1 - F(p_\gamma^*)) + s + \mu}{s + \mu} \right). \quad (2.34)$$

The flow equations shown above give a solution for the density  $g^U(\gamma)$ . For  $\gamma \leq \bar{\gamma}$

$$g^U(\gamma) = g^U(\bar{\gamma}) \exp \left( \int_\gamma^{\bar{\gamma}} \Phi(y) dy \right), \quad (2.35)$$

and for  $\gamma > \bar{\gamma}$

$$g^U(\gamma) = g^U(\bar{\gamma}) e^{\mu(\bar{\gamma} - \gamma)}. \quad (2.36)$$

The derivations are included in appendix 2.5. To find the final expression for the densities, I derive  $g^U(\bar{\gamma})$  by using the following identities:

$$G^U(\bar{\gamma}) = g^U(\bar{\gamma}) \int_0^{\bar{\gamma}} \exp \left( \int_\Gamma^{\bar{\gamma}} \Phi(y) dy \right) d\Gamma \quad (2.37)$$

$$1 - G^U(\bar{\gamma}) = \frac{1}{\mu} g^U(\bar{\gamma}). \quad (2.38)$$

The above equations result from integrating (2.35) between 0 and  $\bar{\gamma}$ , and (2.36) between  $\bar{\gamma}$  and infinity.

Given that  $g^U(\gamma)$  is known now, I derive  $G^E(\gamma)$  by using (2.31) and (2.33):

$$G^E(\gamma) = \frac{\int_0^\gamma (1 - F(p_\Gamma^*))dG^U(\Gamma)}{\int_0^{\bar{\gamma}} (1 - F(p_\Gamma^*))dG^U(\Gamma)}. \quad (2.39)$$

The density  $g^E(\gamma)$  follows by taking derivative:

$$g^E(\gamma) = \frac{(1 - F(p_\gamma^*))}{\int_0^{\bar{\gamma}} (1 - F(p_\Gamma^*))dG(\Gamma)} g^U(\gamma). \quad (2.40)$$

### 2.3.4 Calibration

Before evaluating wage dispersion in the model I need to calibrate  $b$  and  $f$ . I choose two targets. In line with the choice for the replacement ratio  $\rho$  in the previous sections, the first target imposes that  $\rho$  be around 0.4, so benefits  $b$  must be around 40% of average wages  $\bar{w}$ . Average wages are given by

$$\bar{w} = \int_0^{\bar{\gamma}} \left( \int_{p_{\Gamma}^*}^{p^{max}} w(\Gamma, p) \frac{dF(p)}{1 - F(p_{\gamma}^*)} \right) dG^E(\Gamma). \quad (2.41)$$

Given this expression for average wages, the target requires  $b/\bar{w} = 0.4$ .

The second target imposes that the job finding probability in the model must match its empirical counterpart. Thus, I choose the value for  $f$  for which the average job finding rate  $f^e = f \int_0^{\bar{\gamma}} (1 - F(p_{\Gamma}^*)) dG^U(\gamma)$  equals the empirical estimate from the PSID, i.e. such that  $f^e = 0.159$ . These two targets, and the rest of equations in the model, give unique values for  $b$  and  $f$ .

### 2.3.5 The $Mm$ ratio with constant benefits

Figure 2.1 depicts the reservation productivities  $p_{\gamma}^*$  as a function of unemployment history  $\gamma$  over its support  $[0, \bar{\gamma}]$ . As proven in proposition 2, the reservation productivity is increasing in  $\gamma$ , and when  $\gamma = \bar{\gamma}$  it reaches the maximum value  $p^{max}$ . Figure 2.2 shows the probability density function  $g^E(\gamma)$  as a function of  $\gamma$ , also over its support  $[0, \bar{\gamma}]$ .

Similar to the model with proportional benefits, wages are proportional to the human capital level  $h(\gamma)$ , with  $w(\gamma, p) = h(\gamma)(\beta p + (1 - \beta)p_{\gamma}^*)$ . The focus of the paper is on residual wage dispersion, and given that one can remove  $h(\gamma)$  in wage regressions, I focus on the wage dispersion generated by  $\beta p + (1 - \beta)p_{\gamma}^*$ . Using that the lowest reservation productivity corresponds to  $\gamma = 0$ , the  $Mm$  ratio is given by

$$Mm = \int_0^{\bar{\gamma}} \left( \int_{p_{\Gamma}^*}^{p^{max}} (\beta p + (1 - \beta)p_{\Gamma}^*) \frac{dF(p)}{1 - F(p_{\Gamma}^*)} \right) dG^E(\Gamma) / p_0^*. \quad (2.42)$$

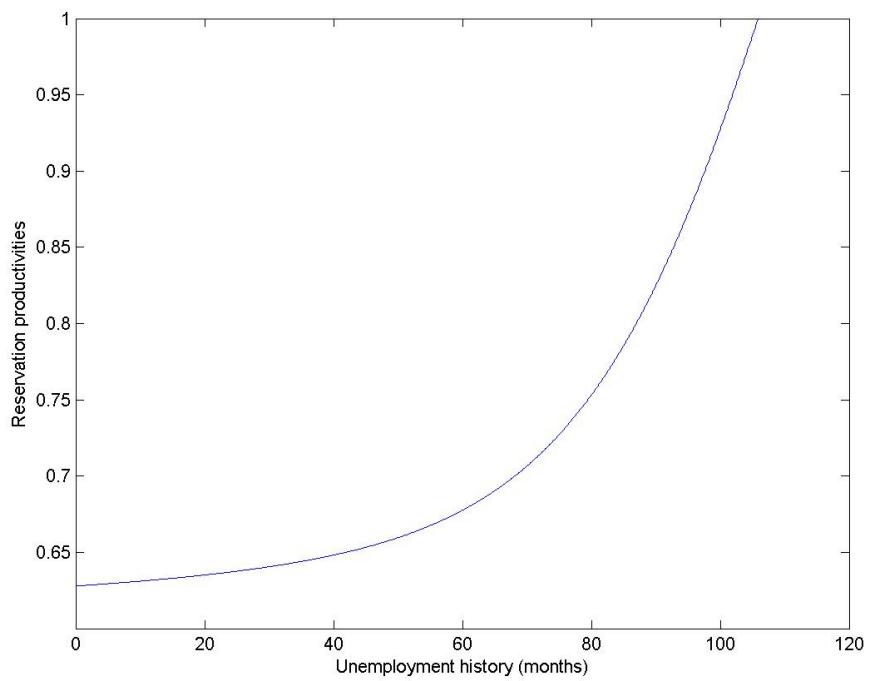


Figure 2.1: RESERVATION PRODUCTIVITIES  $p^*_\gamma$  WITH CONSTANT BENEFITS

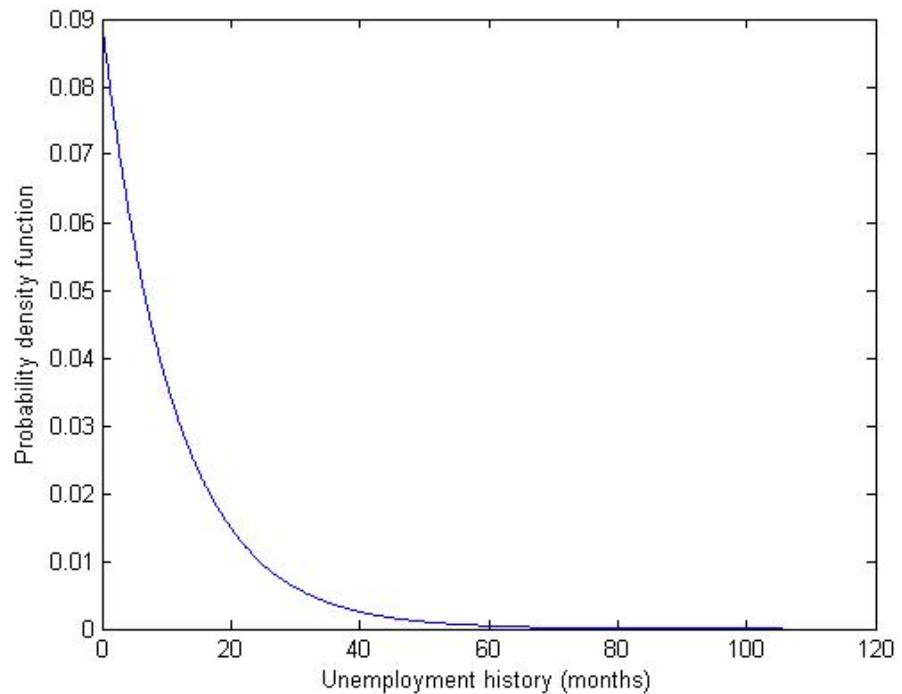


Figure 2.2: DISTRIBUTION OF UNEMPLOYMENT HISTORY  $\gamma$

For the model with constant benefits, the  $Mm$  ratio has a value of around 1.16. Table 2.3 summarizes the several results found for the  $Mm$  ratio.

Table 2.3: THE  $Mm$  RATIO WITH PSID FLOWS

Empirical	1.3367		
Baseline model	1.0357		
$f^* - 1std$	1.0380	$s - 1std$	1.0349
$f^* + 1std$	1.0336	$s + 1std$	1.0364
Proportional benefits $bh(\gamma)$	1.1529		
$f^* - 1std$	1.1628	$s - 1std$	1.1497
$f^* + 1std$	1.1441	$s + 1std$	1.1560
Constant benefits $b$	1.1600		
$f^* - 1std$	1.1686	$s - 1std$	1.1567
$f^* + 1std$	1.1522	$s + 1std$	1.1632

*Notes.*- Table 2.3 shows the  $Mm$  ratio when the job finding and separation rates are taken from PSID estimates. Baseline model refers to the model with  $\delta$  equal 0. The  $bh(\gamma)$  and  $b$  models correspond to the models with proportional and constant benefits. The cell  $f^* - 1std$  gives the  $Mm$  ratio evaluated at the value of  $f^*$  minus one standard deviation of the estimate for  $f^*$  in the PSID.

Although unemployment punishes workers less when benefits are constant, the  $Mm$  ratio is higher than that of the model with proportional benefits. To understand why, consider workers with unemployment history below and above average. Workers with below average unemployment history are less picky, and have lower reservation productivity, whereas the opposite is true for workers with above average unemployment history. Given that the model tries to match a replacement ratio of 40%, to match this target workers with below average unemployment history become less picky than workers in the model with proportional benefits. Because the  $Mm$  ratio relies on the lowest reservation productivity, this mechanism increases the  $Mm$  ratio. However, the resulting increase is mild, because workers with below average unemployment history behave in a very similar way than workers in the model with proportional benefits. Given that the distribution of workers is very concentrated on workers with low unemployment history, who

have very similar reservation productivities compared to the proportional benefits model, this barely affects the average productivity. In particular, if  $b$  is chosen to match the replacement ratio in the model with proportional benefits, the same  $b$  almost matches the replacement ratio when benefits are constant, because a large mass of workers (those with low unemployment history) have almost the same reservation productivity.

## 2.4 Extending the exercise to the economy at large

Up to this point, the quantitative assessment of wage dispersion has been very specific to the PSID. While the PSID was constructed to be representative of the US economy, one wonders how the results extend to other larger data sets, such as the CPS. This question gains interest if one compares the labor market flows in both data sets. The estimates for the job finding and separation rates from the PSID and the CPS show that the PSID is a more stable sample of workers than the CPS. Both the job finding and separation rates are smaller in the PSID, but relative to the separation rate the job finding rate is higher in the PSID. In light of these observations, I explore the amount of wage dispersion consistent with CPS labor market flows.

I focus on the CPS because its size and characteristics make it very representative of the US labor market. Among other things, it provides the estimates for the unemployment rate in the US, and the estimates in Shimer (2005) for the job finding and job separation rates, which are the standard reference in the literature. Hornstein, Krusell and Violante (2011) use the values in Shimer (2005) to assess wage dispersion in search models, so choosing the CPS allows for further comparison between their results and those of this paper.

Similar to Hornstein, Krusell and Violante (2011), I assign a value of 0.43 to the job finding rate, and a value of 0.03 to the separation rate. I use the value for  $\delta$  from the PSID, and the value for  $\mu$  that is consistent with a working life of

40 years. The  $Mm$  ratio increases to around 1.21-1.22, both with constant and proportional benefits. Intuitively, if  $\mu$  and  $\delta$  are constant, what matters for wage dispersion is the relative size of  $f^*$  and  $s$ .<sup>30</sup> As discussed earlier, while both  $f^*$  and  $s$  are higher in the CPS, the ratio  $f^*/s$  is higher in the PSID.<sup>31</sup> Thus the increase in the  $Mm$  ratio.

Table 2.4: THE  $Mm$  RATIO WITH CPS LABOR FLOWS

Source	$Mm$ ratio
Empirical	1.70 - 1.90
Baseline model	1.05
Proportional benefits $bh(\gamma)$	1.2095
Constant benefits $b$	1.2185

*Notes.*- Table 2.4 shows the  $Mm$  ratio when the job finding and separation rates are taken from Robert Shimer (2005). Baseline model refers to the model with  $\delta$  equal 0. The  $bh(\gamma)$  and  $b$  models correspond to the models with proportional and constant benefits.

Table 2.4 summarizes the results. With CPS labor flows, the  $Mm$  ratio is around 1.05 for the baseline random matching model, which corresponds to the model with  $\delta = 0$ . Using the same labor market flows, I find that the model with loss of human capital during unemployment gives an  $Mm$  ratio of up to 1.22. Empirically, the 50-10 percentile ratio in the CPS is between 1.7 and 1.9. Some wage dispersion remains unexplained, but the model explains between 24% and 31% of the observed residual wage dispersion in the CPS. This is an important improvement over the baseline model, as the baseline model can only explain 6% of observed wage dispersion.

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<sup>30</sup>Or the average job finding rate  $f^e$  in the case of constant benefits.

<sup>31</sup>That a labor outcome depends on the ratio of  $f^*$  and  $s$  is not surprising, for example in search models the unemployment rate also depends on the ratio rather than the separate values.

## 2.5 Mathematical Appendix

### 2.5.1 Proof of proposition 1

The proof proceeds by guessing a solution and then confirming the guess. Guessing that  $U(\gamma) = h(\gamma)U$ , with  $U$  independent of  $\gamma$ . Given the guess, I use now that  $(r + \mu)U(\gamma) = h(\gamma)p_\gamma^*$  to get

$$p_\gamma^* = (r + \mu)U, \quad (2.43)$$

which is independent of  $\gamma$ . In the text I use  $p^*$  to denote the reservation productivity. Substituting these results in (2.1), the asset equation for  $U(\gamma)$ , gives a unique solution for  $U$ . Given that the Bellman equation has a unique solution (it is a contraction), this proves the result.

### 2.5.2 Derivation of the $Mm$ ratio

The text shows that taking expectations of (2.14) gives that  $\bar{w} = h(\bar{\gamma})(\beta\bar{p} + (1 - \beta)p^*)$ , where  $\bar{p}$  is defined by  $\bar{p} = E(p|p \geq p^*) = \int_{p^*}^{p^{max}} p \frac{dF(p)}{1 - F(p^*)}$ . Further, because  $\rho = b\bar{h}(\bar{\gamma})/\bar{w}$ , we also have that  $b = \rho(\beta\bar{p} + (1 - \beta)p^*)$ . The  $Mm$  ratio is given by (2.21), which I reproduce here

$$Mm = (\beta\bar{p} + (1 - \beta)p^*)/p^*. \quad (2.44)$$

Denote the job finding rate  $f(1 - F(p^*))$  by  $f^*$ . Using (2.20) one can derive the  $Mm$  ratio in the following way

$$\frac{r + \mu + \delta}{r + \mu}p^* = b + \beta f^* \int_{p^*}^{p^{max}} \frac{p - p^*}{r + \mu + s} \frac{dF(p)}{1 - F(p^*)}, \quad (2.45)$$

$$\left( \frac{r + \mu + \delta}{r + \mu} + \frac{f^*}{r + \mu + s} \right) p^* = \left( \rho + \frac{f^*}{r + \mu + s} \right) (\beta\bar{p} + (1 - \beta)p^*) \quad (2.46)$$

where the second equation is obtained using that  $\beta(p - p^*) = (\beta p + (1 - \beta)p^*) - p^*$ . The expression for  $Mm$  in the text results from rearranging the last equation.

### 2.5.3 Proof of proposition 2

**Proof of a.** The proof proceeds by contradiction. Function  $h(\gamma)$  is decreasing in  $\gamma$  and tends to zero as  $\gamma$  tends to infinity (remember,  $h(\gamma) = e^{-\delta\gamma}$ ). Therefore, there exists a  $\bar{\gamma}$  such that  $h(\bar{\gamma})p^{max} = b$ , and  $h(\gamma)p^{max} < b$  for all  $\gamma > \bar{\gamma}$ . Suppose now that part (a) of the proposition is not true, and  $p_{\bar{\gamma}}^* < p^{max}$ . Then

$$p_{\bar{\gamma}}^*h(\bar{\gamma}) < h(\bar{\gamma})p^{max} = b. \quad (2.47)$$

From (2.11) we know that  $p_{\bar{\gamma}}^*h(\bar{\gamma}) = w(\gamma, p_{\bar{\gamma}}^*)$ . It is clear from the Nash bargaining sharing rule that the worker is always compensated for his outside option  $U(\gamma)$ , so workers must get at the very least the stream of payments  $b$  (one can see this more formally by solving (2.1) as a differential equation). As a result  $w(\gamma, p_{\bar{\gamma}}^*) \geq b$  for all  $\gamma$ , so combining this with the above gives

$$b \leq p_{\bar{\gamma}}^*h(\bar{\gamma}) < h(\bar{\gamma})p^{max} = b, \quad (2.48)$$

which is a contradiction. Taking  $\bar{\gamma}$  as defined earlier proves the first part of part (a). By definition of  $\bar{\gamma}$ ,  $h(\bar{\gamma})p^{max} = b$ . Combining this with  $p_{\bar{\gamma}}^*h(\bar{\gamma}) = w(\gamma, p_{\bar{\gamma}}^*)$  and  $p_{\bar{\gamma}}^* = p^{max}$  gives the second result in a. Finally, taking logarithm of  $h(\bar{\gamma})p^{max} = b$  and rearranging gives the expression for  $\bar{\gamma}$ .

**Proof of b.** The proof requires a few steps. First, define the total surplus of a match as  $S(\gamma, p) = J(\gamma, p) + W(\gamma, p) - U(\gamma)$ , for  $p \geq p_{\gamma}^*$ . The Nash Bargaining sharing rule implies that workers and firms get their outside option plus their share of the total surplus ( $\beta$  for the worker,  $1 - \beta$  for the firm). As potential output is decreasing in  $\gamma$ , it is clear that the surplus is decreasing in  $\gamma$ . To prove this more formally, using the asset equations one finds that  $S(\gamma, p) = h(\gamma)p - (r + \mu)U(\gamma)$ . When  $p = p_{\gamma}^*$  the surplus equals zero for all  $\gamma$ , so  $dS(\gamma, p)/d\gamma = 0$  for all  $\gamma$ . Taking derivative of  $S(\gamma, p)$  with respect to  $\gamma$  one sees that it is decreasing in  $p$ . This implies that  $dS(\gamma, p)/d\gamma < 0$  if  $p > p_{\gamma}^*$ . Note that all these expressions are well-defined. Given that  $J(\gamma, p) = (1 - \beta)S(\gamma, p)$ , then  $J(\gamma, p)$  is also decreasing in  $\gamma$

if  $p > p_\gamma^*$ .

Now assume that there exist  $\gamma_0$  and  $\gamma_1$  such that  $\gamma_0 < \gamma_1$  and  $p_{\gamma_0}^* > p_{\gamma_1}^*$ . This assumption implies that  $J(\gamma_1, p_{\gamma_0}^*) > 0$ , as  $J(\gamma, p) > 0$  for all  $p > p_\gamma^*$ . But  $J$  is decreasing in  $\gamma$ , and given the assumption  $p_{\gamma_0}^* > p_{\gamma_1}^*$ ,  $J(\gamma_1, p_{\gamma_0}^*)$  is well defined and  $J(\gamma_1, p_{\gamma_0}^*) < J(\gamma_0, p_{\gamma_0}^*) = 0$ . This is a contradiction, thus  $p_\gamma^*$  is non-decreasing in  $\gamma$ . That  $p_\gamma^*$  is not constant can be readily seen from the expression for  $U(\gamma)$ .

**Proof of c.** I prove first that if some function  $\pi(\gamma)$  is non-increasing, i.e.  $\pi'(\gamma) \leq 0$  for all  $\gamma$ , then the function  $\Pi(\gamma) = \int_\gamma^{\bar{\gamma}} e^{-r(\Gamma-\gamma)} \pi(\Gamma) d\Gamma$  is decreasing in  $\gamma$ . Intuitively,  $\Pi(\gamma)$  is the present value of the stream of payments  $\pi$  between  $\gamma$  and the final period  $\bar{\gamma}$ . If these payments are decreasing  $\Pi(\gamma)$  is unambiguously decreasing. I apply the result to the following expression for  $U(\gamma)$ , that is derived by integrating (2.1), with constant  $b$  instead of  $bh(\gamma)$ :

$$U(\gamma) = \frac{b}{r + \mu} + f \int_\gamma^{\bar{\gamma}} e^{-(r+\mu)(\Gamma-\gamma)} \left( \int_{p_\Gamma^*}^{p_{\max}} (W(\Gamma, p) - U(\Gamma)) dF(p) \right) d\Gamma. \quad (2.49)$$

This proves that  $U(\gamma)$  is decreasing. Since  $(r + \mu)U(\gamma) = w(\gamma, p_\gamma^*)$ , this proves part (c) of the proposition.

To prove that  $\Pi(\gamma)$  is decreasing, first observe that  $\Pi(\bar{\gamma}) = 0$ , and  $\Pi(\gamma) > 0$  if  $\gamma < \bar{\gamma}$ . This implies that  $\Pi(\gamma)$  must be decreasing near  $\bar{\gamma}$ . Therefore, if it is not true that  $\Pi'(\gamma) < 0$  for all  $\gamma$ , there must exist  $\gamma_0$  such that  $\Pi'(\gamma_0) = 0$ . I show that this leads to a contradiction.

Take derivative of  $\Pi(\gamma)$ :

$$\Pi'(\gamma) = -\pi(\gamma) + r \int_\gamma^{\bar{\gamma}} e^{-r(\Gamma-\gamma)} \pi(\Gamma) d\Gamma. \quad (2.50)$$

That  $\Pi'(\gamma_0) = 0$  implies that

$$\pi(\gamma_0) = r \int_{\gamma_0}^{\bar{\gamma}} e^{-r(\Gamma-\gamma_0)} \pi(\Gamma) d\Gamma, \quad (2.51)$$

which further implies that

$$\Pi(\gamma_0) = \frac{\pi(\gamma_0)}{r}. \quad (2.52)$$

That  $\pi(\gamma)$  is decreasing implies that  $\pi(\gamma_0) \geq \pi(\gamma)$  for all  $\gamma \geq \gamma_0$ . Therefore

$$\Pi(\gamma_0) \geq \int_{\gamma_0}^{\bar{\gamma}} e^{-r(\Gamma-\gamma_0)} \pi(\gamma_0) d\Gamma = \frac{\pi(\gamma_0)}{r} (1 - e^{-r(\bar{\gamma}-\gamma_0)}). \quad (2.53)$$

The expression above implies that  $\Pi(\gamma_0) < \frac{\pi(\gamma_0)}{r}$ , leading to a contradiction.  $\square$

#### 2.5.4 Endogenous distributions $G^U(\gamma)$ and $G^E(\gamma)$ with constant benefits $b$

Equation (2.31) implies

$$G^E(\gamma)E = \frac{f}{s+\mu} \left( \int_0^{\gamma} 1 - F(p_{\gamma}^*) dG^U(\gamma) \right) N. \quad (2.54)$$

Similarly, using (2.33) and that  $G^E(\bar{\gamma}) = 1$  as no worker is hired if  $\gamma > \bar{\gamma}$

$$E = \frac{f}{s+\mu} \left( \int_0^{\bar{\gamma}} 1 - F(p_{\gamma}^*) dG^U(\gamma) \right) N. \quad (2.55)$$

Now take the derivative of (2.30) with respect to  $\gamma$  to find

$$g^U(\gamma)N + f(1 - F(p_{\gamma}^*))g^U(\gamma)N + \mu g^U(\gamma)N = sg^E(\gamma)E. \quad (2.56)$$

Taking now derivative of (2.31)

$$g^E(\gamma)E = \frac{f}{s+\mu} (1 - F(p_{\gamma}^*))g^U(\gamma)N. \quad (2.57)$$

Combining the two equations above gives

$$\frac{dg^U(\gamma)}{d\gamma}N + (f(1 - F(p^*)) + \mu)g^U(\gamma)N = \frac{s}{s + \mu}f(1 - F(p^*))g^U(\gamma)N, \quad (2.58)$$

which implies the following differential equation

$$\frac{dg^U(\gamma)}{d\gamma} + \Phi(\gamma)g^U(\gamma) = 0, \quad (2.59)$$

where  $\Phi(\gamma)$  is defined in the text as  $\Phi(\gamma) = \mu \left( \frac{f(1 - F(p_\gamma^*)) + s + \mu}{s + \mu} \right)$ . The solution gives the expression in the text:

$$g^U(\gamma) = g^U(\bar{\gamma}) \exp \left( \int_\gamma^{\bar{\gamma}} \Phi(y) dy \right). \quad (2.60)$$

To find  $g^U(\gamma)$  for  $\gamma > \bar{\gamma}$  I proceed similarly. Taking derivative of (2.32)

$$\frac{dg^U(\gamma)}{d\gamma}N + \mu g^U(\gamma)N = 0. \quad (2.61)$$

This implies the result of the text:

$$g^U(\gamma) = \exp(\mu(\bar{\gamma} - \gamma)) g(\bar{\gamma}). \quad (2.62)$$

### 2.5.5 Numerical strategy to find $p_\gamma^*$

Consider equation (2.29). Using the assumption of  $F(p)$  being uniform  $[0, p^{max}]$  gives

$$p_\gamma^* = (h(\gamma))^{-1} \left( b + \beta f \frac{r + \mu}{r + \mu + s} \int_\gamma^{\bar{\gamma}} e^{-(r + \mu)(\Gamma - \gamma)} h(\Gamma) \frac{(p^{max} - p_\Gamma^*)^2}{2p^{max}} d\Gamma \right). \quad (2.63)$$

I use the following strategy to solve for  $p_\gamma^*$ . Consider a grid  $\{\gamma_1 = 0, \gamma_2, \dots, \gamma_n = \bar{\gamma}\}$  of the range of unemployment histories  $[0, \bar{\gamma}]$ . Given an initial guess for the vector

of unemployment histories  $\{p_{\gamma_i}^*\}_i$ , I calculate the integral in the above equation using numerical integration. Entering the result in the above equation gives the new value for  $p_\gamma^*$ . Iterating until convergence provides the solution for  $p_\gamma^*$ .

See Kenneth Judd (1998) for a bound on the size of the error involved in the numerical integration. This error decreases as the grid uses more nodes. However, as the strategy combines numerical integration and iteration methods, the final bound on the error is not straightforward. To assess the error involved in this numerical strategy, I test how well it approximates a known solution. This provides, in the language of Judd (1998), some backward error analysis. Consider the general case of the equation above

$$f(\gamma) = a_1(\gamma) + a_2(\gamma) \int_\gamma^{\bar{\gamma}} \phi(\Gamma, f(\Gamma)) d\Gamma. \quad (2.64)$$

The method described earlier provides a solution for this general case. For the case where the  $a_i$  are constant and  $\phi(\Gamma, f(\Gamma)) = f(\Gamma)$  an exact solution exists. Comparing the exact solution to the one provided by the numerical strategy shows that the numerical solution can get as close as necessary to the exact solution within a reasonable time.

## 2.6 Data Appendix: PSID

The Panel Study of Income Dynamics (PSID) started collecting data in 1968. The original PSID consisted of two samples: the core sample, representative of the US population; and the SEO sample, an oversample of low income families. In 1968 the PSID initially followed 5000 families, 3000 of them from the core sample, and the rest from the SEO sample. Since then the PSID has grown in size, as it followed young family members when they formed their own family.

The PSID collected data annually between 1968 and 1997, but biennially after 1997 for funding reasons. Therefore, after 1997 information on the number of weeks in unemployment is unavailable for the years without interviews.<sup>32</sup> As the empirical work requires knowledge of workers' unemployment history, I only include the 1968-1997 PSID waves, and discard waves after 1997.

Information in the PSID is mostly retrospective. For example, questions about hours worked or income earned refer to the previous calendar year. For example, the year  $t$  wave asks about the income earned during year  $t - 1$ . As is standard in the literature, I construct wages by dividing total labor earnings by the number of hours worked.

I now describe the sample choice, which is standard in the literature. I restrict my attention to male workers between 18 and 65 years old in the core sample. I include only observations with positive earnings. Similar to Andreas Hornstein, Per Krusell and Giovanni L. Violante (2011), I exclude observations with too many hours or too few work hours, more specifically, if work hours are lower than 520 or greater than 5096. Excluding these outliers reduces measurement error. I also ignore observations for which the real wage is lower than half the minimum wage of 1982, or with top-coded earnings. I exclude workers in farming or in the army. For the fixed effects estimation only workers with a minimum of 8 observations are included. Finally, I exclude individuals currently self-employed, in school,

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<sup>32</sup>For example, if the worker is interviewed in 1997 and again in 1999, information on number of weeks unemployed in 1998 is unavailable.

inactive, or living in Alaska or Hawaii. I drop individuals with inconsistent or missing information.<sup>33</sup>

This selection leaves 2,160 workers, with observations spanning the 30 years between 1968-1997. In total the sample includes 34,542 observations, or 1,151 worker/year observations.

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<sup>33</sup>Although very few workers are affected by this once I implement the previous selection criteria.

# Chapter 3

## Unemployment History with On-the-Job Search

Search models struggle to generate significant wage dispersion. As previous chapters discuss, Hornstein, Krusell and Violante (2011) show that in baseline search models the average observed wage is only 5% above the minimum observed wage, whereas empirically it is between 70% and 90%. Or more formally, the  $Mm$  ratio is 1.05 in baseline search models, whereas its empirical counterpart, the 50-10 percentile ratio of the residual in a Mincerian wage regression, is between 1.7 and 1.9 .

In the previous chapter I show that if workers lose some human capital or skills during unemployment, wage dispersion increases significantly among identical workers. Because workers are aware that unemployment hurts their wages, they accept lower wages to give themselves a better chance to leave unemployment and avoid big human capital losses. With a lower reservation wage, wage dispersion increases. I show that wage dispersion is uniquely determined by a set of parameters, namely labor market flows, the interest rate, the value of non-market time and the rate at which workers lose human capital during unemployment. Using evidence from micro data to simulate the model I show that adding unemployment history is an important improvement over baseline search models.

The mechanism accounts for between 28% and 45% of the observed residual wage dispersion. In particular, with CPS labor market flows the  $Mm$  ratio is around 1.22.

Hornstein, Krusell and Violante (2011) show that allowing for on-the-job search improves the performance of search models. Intuitively, the problem with search models is that they predict that workers wait a long time before accepting a job offer. If workers are allowed to search while being employed, they are willing to accept lower paying offers because they do not give up the option to search when they accept a job. Wage dispersion in a search model with on-the-job search is similar to that in the model with unemployment history, with an  $Mm$  ratio between 1.16 and 1.27.

This paper adds on-the-job search to the framework with loss of human capital during unemployment. The  $Mm$  ratio is again uniquely determined by a set of parameters for which reliable estimates exist. Compared to the model with unemployment history, wage dispersion further depends on how frequently job offers arrive on the job. As with the baseline model and its extension with unemployment history, wage dispersion does not rely on any assumption about the distribution of match qualities. I show that adding on-the-job search significantly increases wage dispersion among identical workers. The model with unemployment history and on-the-job search delivers an  $Mm$  ratio of around 2, thus accounting for all of the observed residual wage dispersion in the CPS.

The model also addresses the trade-off found by Hornstein, Krusell and Violante (2011) between explaining frictional wage dispersion and cyclical fluctuations in unemployment and vacancies. Matching the cyclical behavior of unemployment and vacancies requires high values of non-market time, as measured by the replacement ratio —benefits over average wages. However, a high replacement ratio makes the frictional wage dispersion problem significantly worse. I show that even with the highest value in the literature for the replacement ratio, the model with unemployment history and on-the-job search yields a high  $Mm$  ratio of 1.68, thus accounting for almost all of the observed residual wage dispersion.

First I derive wage dispersion in the model with on-the-job search. Next, I find the  $Mm$  ratio for the model with both unemployment history and on-the-job search and quantify the resulting wage dispersion. I then explore how wage dispersion depends on the value of non-market time, and show that wage dispersion is large even for high values of the replacement ratio.

### 3.1 Frictional Wage Dispersion with On-the-Job Search

In baseline search models, workers hold their jobs until separation occurs, so workers must go through unemployment to change jobs. However, if workers can search for jobs when they are employed, they keep the option of searching when they accept a job offer. For example, in the extreme case in which job offers arrive at the same rate to unemployed and employed workers, unemployed workers would take any offer that pays more than unemployment benefits —or non-market time. So with on-the-job search unemployed workers accept lower wages and wage dispersion increases. The size of the increase depends on how frequently employed workers receive job offers relative to unemployed workers.

Hornstein, Krusell and Violante (2011) extend a version of Burdett's job ladder model to quantify wage dispersion with on-the-job search. Assume that workers draw wage offers from a distribution  $F(w)$ . Unemployed workers receive job offers at rate  $f^u$ , and employed workers at rate  $f^e$ . Jobs are destroyed at an exogenous rate  $s$ , and during unemployment workers receive unemployment benefits  $b$ . Workers set a reservation wage  $w^*$  and accept any job above this wage. The asset equations for unemployment and employment  $U$  and  $W(w)$  are

$$rU = b + f^u \int_{w^*}^{w^{max}} (W(y) - U) dF(y), \quad (3.1)$$

$$rW(w) = w + f^e \int_w^{w^{max}} (W(y) - W(w)) dF(y) - s(W(w) - U). \quad (3.2)$$

The above equations capture that at rate  $f^e$  an employed worker takes a new draw from  $F(w)$ , and if the new wage is above the current wage the worker switches to the new job. To simplify the calculations, assume that no firm makes an offer below  $w^*$ , so  $F(w^*) = 0$ .<sup>1</sup> However, it is worth noting that with this assumption  $f^u$  is now also the job finding rate.

As workers move jobs, the distribution of wages becomes endogenous. Denoting this endogenous distribution by  $G(w)$ , one can derive this distribution by looking at labor flow equations. Consider the group of workers with wages below  $w$ . In steady state the flows out of this group must equal the flows in. Therefore, the following equation holds

$$f^u F(w)N = (s + f^e(1 - F(w)))G(w)E, \quad (3.3)$$

where  $N$  and  $E$  are the number of unemployed and employed workers. The left-hand side captures the flows in. They are given by the unemployed workers who find a job that pays a wage lower than  $w$ . The flows out are on the right-hand side, which are due to workers either losing their job or finding a job that pays a wage above  $w$ . Since in steady-state the flows in and out of unemployment must also be equal, we also have that  $E/N = f^u/s$ . Therefore,  $G(w)$  as a function of  $F(w)$  is given by

$$G(w) = \frac{s}{s + f^e(1 - F(w))} F(w). \quad (3.4)$$

The  $Mm$  ratio is given by  $Mm = \bar{w}/w^*$ , where the average observed wage is now given by

$$\bar{w} = \int_{w^*}^{w^{max}} wdG(w). \quad (3.5)$$

Using the asset equation for the unemployed (3.1) and the expression for the

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<sup>1</sup>The results are the same without this assumption, but the derivation is simpler.

distribution of  $G(w)$  in (3.4) gives the expression for the  $Mm$  ratio

$$Mm \simeq \frac{1 + \frac{f^u - f^e}{r+s+f^e}}{\rho + \frac{f^u - f^e}{r+s+f^e}}. \quad (3.6)$$

The details of the derivation are in the appendix. The  $Mm$  ratio shows what the intuition suggested, that the amount of wage dispersion depends on the relative size of  $f^u$  and  $f^w$ . The more frequently job offers arrive to employed workers relative to unemployed workers, the more attractive employment is to workers. Therefore, the reservation wage decreases with the arrival rate of offers on-the-job, and wage dispersion increases. For example, if job offers arrive with the same frequency to unemployed and employed workers, workers accept any wage offer above the benefits  $b$ .

## 3.2 Combining Unemployment History and On-the-Job Search

In this section I extend the model with unemployment history and add on-the-job search. I derive a closed form solution for the  $Mm$  ratio, and use empirical evidence to quantify its magnitude.

### 3.2.1 The labor market

I assume that workers search on-the-job in the model with unemployment history and proportional benefits  $bh(\gamma)$  of section 2.1. Given that I prove in Chapter 2 that the models with proportional and constant benefits behave in almost the same way, this restriction is without a loss of generality. The assumptions are the same, only that now unemployed workers receive job offers at rate  $f^u$ , and employed workers receive job offers at rate  $f^w$ . The asset equations for unemployed workers (2.1) and vacancies (2.3) remain the same. So for unemployed workers the asset

equation is

$$(r + \mu)U(\gamma) = bh(\gamma) + f^u \int_{p_\gamma^*}^{p^{max}} (W(\gamma, y) - U(\gamma))dF(y) + \frac{\partial U(\gamma)}{\partial \gamma}. \quad (3.7)$$

However, the equation for employed workers is now given by

$$(r + \mu)W(\gamma, p) = w(\gamma, p) + f^w \int_p^{p^{max}} (W(\gamma, y) - W(\gamma, p))dF(y) \quad (3.8)$$

$$- s(W(\gamma, p) - U(\gamma)).$$

Compared to (2.2), equation (3.8) captures that workers now also receive job offers at a rate  $f^w$ . If the productivity is above the current level  $p$  they change jobs. Similarly, the asset equation for a filled vacancy is now given by

$$(r + \mu)J(\gamma, p) = h(\gamma)p - w(\gamma, p) - (s + f^w(1 - F(p)))(J(\gamma, p) - V). \quad (3.9)$$

Intuitively, firms get a net flow  $h(\gamma)p - w(\gamma, p)$ . The job is destroyed either because of separation, which happens with frequency  $s$ , or because the worker finds a better job, which happens with frequency  $f^w(1 - F(p))$ .

As in section 2.1, assuming that benefits are proportional implies that reservation productivities are independent of unemployment history  $\gamma$ , so  $p_\gamma^* = p^*$ .

**Proposition 3.** *The reservation productivity  $p_\gamma^*$  is independent of  $\gamma$ , i.e.  $p_\gamma^* = p^*$ .*

This simplifies the model and allows for a closed form expression of wage dispersion. The proof proceeds in the same way as in the proof of proposition 1.<sup>2</sup> The intuition is also similar. All payments, both during employment and unemployment, are proportional to the human capital level  $h(\gamma)$ . As a result, unemployment history  $\gamma$  is irrelevant for workers' choice of reservation productivity.

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<sup>2</sup>In their version of the Burdett-Mortensen model with human capital accumulation, Burdett, Carrillo-Tudela and Coles (2011) also find that the reservation piece rate is independent of experience.

### 3.2.2 The $Mm$ ratio

Using the asset equations one gets that wages are of the form  $w(\gamma, p) = h(\gamma)\hat{w}(p)$ , where  $\hat{w}(p)$  is independent of  $\gamma$ . This shows that, as in the model with human capital losses during unemployment, log wages are linear in unemployment history with coefficient  $\delta$ . Given the focus of the paper on wage dispersion among identical workers, I control for unemployment history in the empirical exercise and focus on the wage dispersion of  $\hat{w}(p)$ .

For a given  $F(p)$ , workers move jobs once they become employed. I denote  $G(p)$  the endogenous distribution of match productivities. The average of  $\hat{w}(p)$ , which I denote  $\bar{\hat{w}} = E(\hat{w}(p)|p > p^*)$ , is thus given by

$$\bar{\hat{w}} = \int_{p^*}^{p^{max}} \hat{w}(y) dG(y). \quad (3.10)$$

Equation (2.11) gives that  $w(\gamma, p^*) = h(\gamma)p^*$ , so the  $Mm$  ratio is given by

$$Mm = \bar{\hat{w}}/p^*. \quad (3.11)$$

Using the asset equation for unemployed workers (2.1) and the asset equation for employed workers (3.8) evaluated at  $p = p^*$  gives the following equation

$$\begin{aligned} h(\gamma)p^* + f^w \int_{p^*}^{p^{max}} (W(\gamma, y) - U(\gamma)) dF(y) &= \\ &= \frac{r + \mu}{r + \mu + \delta} \left( b h(\gamma) + f^u \int_{p^*}^{p^{max}} (W(\gamma, y) - U(\gamma)) dF(y) \right). \end{aligned} \quad (3.12)$$

Given the paper's focus on the steady state, without loss of generality I assume that  $F(p^*) = 0$  for the derivation of the  $Mm$  ratio. Therefore  $f^u$  is the job finding rate. This assumption makes the exposition simpler, but the results are exactly the same without it. Combining (3.12) equation with the other equations in the

model gives the following expression for the  $Mm$  ratio

$$Mm = \frac{1 + \frac{(\frac{r+\mu}{r+\mu+\delta})f^u - f^w}{r+\mu+s+f^w}}{(\frac{r+\mu}{r+\mu+\delta})\rho + \frac{(\frac{r+\mu}{r+\mu+\delta})f^u - f^w}{r+\mu+s+f^w}}, \quad (3.13)$$

where  $f^u$  is the job finding probability and  $f^w$  is the rate at which job offers arrive on the job. Appendix 2.5 includes the details of the derivation.

By shutting down the appropriate channel, expression (3.13) also provides the  $Mm$  ratio for the baseline search model, the model with unemployment history, and the on-the-job search model. If  $\delta = 0$ , the unemployment history mechanism is shut down and (3.13) gives the expression for the  $Mm$  ratio in the on-the-job search model (3.6). If  $f^w = 0$ , on-the-job search is shut down and (3.13) corresponds to (2.21). Finally, setting both  $\delta = 0$  and  $f^w = 0$  gives the  $Mm$  ratio for the baseline search model (1.2).

### 3.2.3 Quantifying the Model's $Mm$ Ratio

Wage dispersion, as measured by the  $Mm$  ratio in (3.13), depends uniquely on a few parameters,  $r$ ,  $\rho$ ,  $s$ ,  $\mu$ ,  $f^u$ ,  $f^w$  and  $\delta$ . Further, it is independent of any distributional assumption about  $F(p)$ . The relationship between these parameters and wage dispersion is the same as before, only that now it further depends on the arrival rate of offers on the job  $f^w$ . Larger values of  $f^w$  imply that it is easier to switch jobs, making the option value of searching on the job larger. Workers thus accept lower wages and wage dispersion increases.

I use the CPS values for the job finding and separation rates  $f^u$  and  $s$  as in Chapter 2. To quantify wage dispersion in the model, the expression for the  $Mm$  ratio requires a value for the arrival rate  $f^w$ . Similar to Hornstein, Krusell and Violante (2011), I follow Eva Nagypal (2008) and choose the value that is consistent with empirical job-to-job transitions. Using data from the Survey of Income and Program Participation (SIPP), Nagypal (2008) finds monthly job-to-job flows of around 2.2%.<sup>3</sup> These job-to-job flows imply a unique value for  $f^e$ .

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<sup>3</sup>Empirical job-to-job transitions can be found in Bruce C. Fallick and Charles A. Fleischman

Consider the job-to-job flows implied by the model, they are given by

$$\chi = f^e \int_{p^*}^{p^{\max}} (1 - F(p)) dG(p). \quad (3.14)$$

Intuitively, workers move jobs if they receive and offer, which happens at rate  $f^e$ , and if the offer implies a higher wage, which happens with probability  $1 - F(p)$ . The above expression averages these “job transition” rates over the distribution of matches  $G(p)$ . The appendix shows that the transition rate  $\chi$  is given by

$$\chi = \frac{s(f^e + s) \log(\frac{s+f^e}{s})}{f^e} - s. \quad (3.15)$$

Solving this equation numerically implies a value for  $f^w$  of around 0.07.<sup>4</sup> The other parameters are chosen in the same way as in section 2.4.

With on-the-job search only, the magnitude of the  $Mm$  ratio is similar to the one of the model with loss of human capital during unemployment, around 1.16.<sup>5</sup> As discussed in Chapter 2, with unemployment history the model delivers an  $Mm$  ratio of 1.21. When both unemployment history and on-the-job search are included, the model generates an  $Mm$  ratio of around 2.07, and accounts for all of the observed residual wage dispersion.

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(2004), G. Moscarini and F. Vella (2008) and Nagypal (2008). I use the lowest value, which corresponds to Nagypal (2008) to show that even with the lowest estimates the model generates large amounts of wage dispersion.

<sup>4</sup>The expression for  $\chi$  is the same as in Hornstein, Krusell and Violante (2011), thanks to the result that the reservation productivities are constant and independent of workers’ unemployment history  $\gamma$ .

<sup>5</sup>If the highest value for the arrival of offers on-the-job is used instead, the  $Mm$  ratio goes up to 1.27.

### 3.3 Trade-off between Frictional Wage Dispersion and Cyclical Unemployment Fluctuations

As discussed in Chapter 1, Hornstein, Krusell and Violante (2011) show that in search models frictional wage dispersion and unemployment volatility are closely related. The baseline search model usually requires high values of the replacement ratio to match the cyclical volatility of unemployment and vacancies. However, high values of the replacement ratio make the frictional wage dispersion problem worse. Intuitively, if benefits are higher relative to average wages the value of workers' outside option increases, so workers accept only better paying jobs. Workers wait longer to accept a job offer and wage dispersion decreases. On the other hand, higher benefits are needed to generate fluctuations in unemployment and vacancies that are in line with empirical observations. Intuitively, high replacement ratios —or value of non-market time— shrink firms' profits, which implies that small changes in profits imply large percentage changes in profits. As profits is one of the main drivers of job creation, higher replacement ratios amplify the movements in vacancies and unemployment. Therefore, in search models there is a trade-off between matching the unemployment volatility during the cycle and generating sizable wage dispersion.

I calculate the  $Mm$  ratio in the model with both on-the-job search and unemployment history for the different values of the replacement ratio  $\rho$  used in the literature. As discussed in Chapter 1, Shimer (2005) uses a value of 0.40, which is the value Hornstein, Krusell and Violante (2011) and the previous sections use to assess frictional wage dispersion. Hall and Milgrom (2008) choose  $\rho$  to be 0.71 based on data on the elasticity of labor supply. Hagedorn and Manovskii (2008) choose the value of the replacement ratio that matches the elasticity of labor market tightness, which gives the highest value in the literature of around 0.95.<sup>6</sup>

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<sup>6</sup>Chapter 1 discusses the criticisms Hagedorn and Manovskii (2008) received for using this

Table 3.1:  $Mm$  RATIO WITH ON-THE-JOB SEARCH

Calibration source	$\rho$	$Mm$ ratio
Shimer (2005)	0.40	2.07
Hall & Milgrom (2008)	0.71	1.84
Hagedorn & Manovskii (2008)	0.95	1.68

Table 3.1 gives the  $Mm$  ratio for the three values of the replacement ratio. The  $Mm$  ratio is 2.07 with a replacement ratio of 0.40, goes down to 1.84 if the replacement ratio is 0.71, and bottoms at 1.68 for the highest replacement ratio of 0.95. Therefore, even for the highest value of the replacement ratio the amount of wage dispersion is very large. Although some wage dispersion remains unexplained for higher values of the replacement ratio, the model with both on-the-job search and unemployment history accounts for almost all of the observed residual wage dispersion. Figure 3.1 shows the  $Mm$  ratio as a function of the replacement ratio  $\rho$  for the three models: the baseline search model, the model with unemployment history, and the model with unemployment history and on-the-job search.

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high value. In a nutshell, Costain and Reiter (2008) and Hall and Milgrom (2008) point out that with this very high value labor supply becomes very responsive to unemployment benefits. See Chapter 1 for the reply in Hagedorn and Manovskii (2008).

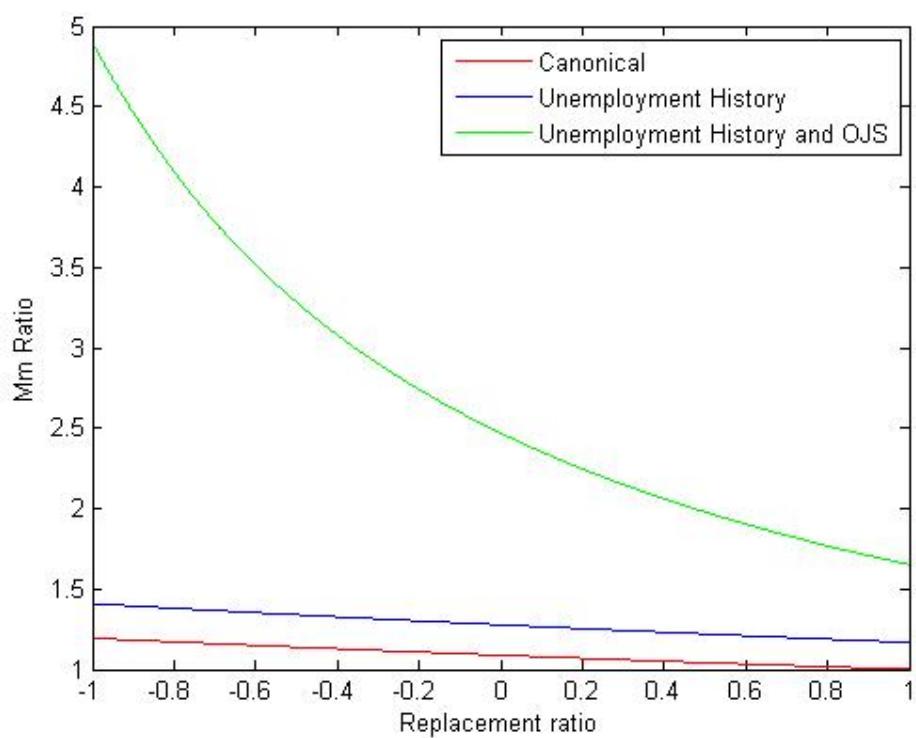


Figure 3.1: *Mm* RATIO AND REPLACEMENT RATES

## 3.4 Mathematical Derivations

### 3.4.1 $Mm$ Ratio with On-the-Job Search

The asset equations are given by

$$rU = b + f^u \int_{w^*}^{w^{max}} (W(y) - U) dF(y), \quad (3.16)$$

$$rW(w) = w + f^e \int_w^{w^{max}} (W(y) - W(w)) dF(y) - s(W(w) - U). \quad (3.17)$$

As shown in the text, the endogenous distribution of wages  $G(w)$  is

$$G(w) = \frac{s}{s + f^e(1 - F(w))} F(w). \quad (3.18)$$

The average observed wage is now given by

$$\bar{w} = \int_{w^*}^{w^{max}} w dG(w). \quad (3.19)$$

Given that  $U$  is workers' outside option,  $W(w^*) = U$ . Combining both asset equations with  $w = w^*$  gives

$$w^* = b + (f^u - f^e) \int_{w^*}^{w^{max}} (W(y) - U) dF(y) \quad (3.20)$$

Now, solving the integral by parts gives that

$$\int_{w^*}^{w^{max}} (W(y) - U) dF(y) = [(W(y) - U)F(y)]_{w^*}^{w^{max}} - \int_{w^*}^{w^{max}} W'(y)F(y) dy \quad (3.21)$$

$$= W(w^{max}) - U - \int_{w^*}^{w^{max}} W'(y)F(y) dy \quad (3.22)$$

$$= \int_{w^*}^{w^{max}} W'(y)(1 - F(y)) dy. \quad (3.23)$$

Differentiating the asset equation (3.17) with respect to  $w$  and substituting in the above equation gives that

$$\int_{w^*}^{w^{max}} (W(y) - U)dF(y) = \int_{w^*}^{w^{max}} \frac{1 - F(y)}{r + s + f^e(1 - F(y))} dy. \quad (3.24)$$

Using the expression for  $G(w)$  gives that

$$1 - G(w) = \frac{s + f^e}{s + f^e(1 - F(w))} (1 - F(w)) \quad (3.25)$$

$$\simeq \frac{r + s + f^e}{r + s + f^e(1 - F(w))} (1 - F(w)), \quad (3.26)$$

where the last approximation holds if  $r$  is small relative to  $s$  and  $f^e$ . Finally, plugging these results into (3.20) one gets that

$$w^* = b + (f^u - f^e) \int_{w^*}^{w^{max}} \frac{1 - G(y)}{r + s + f^e} dy. \quad (3.27)$$

To get the  $Mm$  ratio first note that using integration by parts again gives

$$\bar{w} = \int_{w^*}^{w^{max}} y dG(y) \quad (3.28)$$

$$= w^{max} - \int_{w^*}^{w^{max}} G(y) dy \quad (3.29)$$

$$= w^* + \int_{w^*}^{w^{max}} (1 - G(y)) dy, \quad (3.30)$$

where the last equation is derived by adding and subtracting  $w^*$ . Finally, the  $Mm$  ratio can be solved for by combining all these results to get

$$w^* = b + \frac{f^u - f^e}{r + s + f^e} (\bar{w} - w^*), \quad (3.31)$$

which after noting that  $b = \rho\bar{w}$  gives the expression for the  $Mm$  ratio in the text.

### 3.4.2 $Mm$ Ratio with On-the-Job Search and Unemployment History

Using the asset equations wages are given by  $w(\gamma, p) = h(\gamma)\hat{w}(p)$ , and similarly  $W(\gamma, p) = h(\gamma)\hat{W}(p)$ . The paper investigates wage dispersion among identical workers, so I focus on the dispersion of  $\hat{w}(p)$ , the wage adjusted for human capital. The  $Mm$  ratio is given by

$$Mm = \bar{\hat{w}}/p^*. \quad (3.32)$$

where

$$\bar{\hat{w}} = \int_{p^*}^{p^{max}} \hat{w}(y) dG(y). \quad (3.33)$$

Using integration by parts one gets

$$\bar{\hat{w}} = [\hat{w}(y)G(y)]_{p^*}^{p^{max}} - \int_{p^*}^{p^{max}} \hat{w}'(y)G(y) dy. \quad (3.34)$$

Given that  $G(p)$  is the endogenous distribution of observed productivities,  $G(p^{max})$  is 1 and  $G(p^*)$  is 0. Using that  $\hat{w}(p^*) = p^*$  gives

$$\bar{\hat{w}} = p^* + \int_{p^*}^{p^{max}} \hat{w}'(y)(1 - G(y)) dy \quad (3.35)$$

Using the asset equations gives

$$(r + \mu + s)\hat{W}'(p) = \hat{w}'(p) + f^w \left( 0 + \int_{p^*}^{p^{max}} -\hat{W}'(p)dF(y) \right), \quad (3.36)$$

which implies the following expression

$$(r + \mu + f^w(1 - F(p)))\hat{W}'(p) = \hat{w}'(p). \quad (3.37)$$

Use asset equations  $U(\gamma)$  and  $W(\gamma, p^*)$  to get expression (3.12), which after dividing by  $h(\gamma)$  and rearranging becomes

$$p^* = \frac{r + \mu}{r + \mu + \delta} b + \left( f^u \frac{r + \mu}{r + \mu + \delta} - f^w \right) \int_{p^*}^{p^{max}} \hat{W}'(y)(1 - F(y))dy, \quad (3.38)$$

where the above equation uses that

$$\int_{p^*}^{p^{max}} (\hat{W}(y) - \hat{W}(p^*))dF(y) = \int_{p^*}^{p^{max}} \hat{W}'(y)(1 - F(y))dy. \quad (3.39)$$

To find the average wage  $\bar{w}$  I now derive the endogenous distribution  $G(p)$ . Given the paper's focus on the steady state, without loss of generality I assume that  $F(p^*) = 0$ . The results are exactly the same without this assumption, but the derivation is simpler.<sup>7</sup> The interpretation is that  $f^u$  becomes the job finding rate. Consider the group of employed workers in a job with productivity lower than  $p$ . In steady state the following flow equation holds

$$(1 - u)G(p)(s + f^w(1 - F(p))) = f^u F(p)u. \quad (3.40)$$

The left-hand side of the above equation are the flows out of the group, and consists of job separations and quits to better jobs with productivity still lower than  $p$ . The right-hand side are the flows in, which consist of unemployed workers who find a job with productivity lower than  $p$ . Similarly, in steady state

$$uf^u(1 - F(p^*)) = (1 - u)s. \quad (3.41)$$

These equations imply that

$$G(p) = F(p) \left( \frac{s}{s + f^w(1 - F(p))} \right), \quad (3.42)$$

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<sup>7</sup>Reservation productivity  $p^*$  is constant in steady state. Unless one is interested in business cycle properties, or in how shocks affect the economy, this assumption is irrelevant.

Thus

$$1 - G(p) = \left( \frac{s + f^w}{s + f^w(1 - F(p))} \right) (1 - F(p)) \quad (3.43)$$

$$\simeq \left( \frac{r + \mu + s + f^w}{r + \mu + s + f^w(1 - F(p))} \right) (1 - F(p)), \quad (3.44)$$

where the last approximation is true if  $r$  is small relative to  $s$  or  $f^w$ . Substituting the above expression for  $1 - F(p^*)$  in the equation for  $p^*$  gives

$$p^* = \frac{r + \mu}{r + \mu + \delta} b + \left( \frac{f^u(\frac{r+\mu}{r+\mu+\delta}) - f^w}{r + \mu + s + f^w} \right) (\bar{w} - p^*), \quad (3.45)$$

which gives, noting that  $b = \rho \bar{w}$ , the expression for the  $Mm$  ratio in the text.

$$Mm \simeq \frac{1 + \frac{(\frac{r+\mu}{r+\mu+\delta})f^u - f^w}{r + \mu + s + f^w}}{(\frac{r+\mu}{r+\mu+\delta})\rho + \frac{(\frac{r+\mu}{r+\mu+\delta})f^u - f^w}{r + \mu + s + f^w}}. \quad (3.46)$$

### 3.4.3 Job-to-job flows and arrival rate of job offers on-the-job $f^e$

Similar to Hornstein, Krusell and Violante (2011), I follow Nagypal (2008) to find  $f^e$  from the job-to-job flows  $\chi$ . Begin with the following equation

$$\chi = f^e \int_{p^*}^{p^{\max}} (1 - F(p)) dG(p). \quad (3.47)$$

Integrating by parts gives the following

$$\chi = f^e - f^e \left( [F(p)G(p)]_{p^*}^{p^{\max}} - \int_{p^*}^{p^{\max}} G(p) dF(p) \right) \quad (3.48)$$

$$= f^e \int_{p^*}^{p^{\max}} G(p) dF(p). \quad (3.49)$$

Using (3.42) finally gives

$$\chi = f^e \int_{p^*}^{p^{max}} \frac{sF(p)}{s + f^e(1 - F(p))} dF(p). \quad (3.50)$$

Now, the integrand is simply a function of  $F(p)$  that is integrated over  $dF(p)$ . Using the change of variable  $F(p) = z$ , and noting that at the boundaries of integration  $p^*$  and  $p^{max}$  variable  $z$  is 0 and 1, gives that

$$\chi = f^e \int_0^1 \frac{z}{s + f^e(1 - z)} dz, \quad (3.51)$$

which has a closed form solution, as given in the text

$$\chi = \frac{s(f^e + s) \log(\frac{s+f^e}{s})}{f^e} - s. \quad (3.52)$$

Given  $\chi$  from the data, I back out  $f^e$  by solving the above equation numerically, which gives the value in the text.

# Concluding Remarks

Motivated by the findings of Hornstein, Krusell and Violante (2011) that baseline search models fail to generate significant wage dispersion, this thesis investigates how much wage dispersion arises if workers lose some skills during unemployment. I begin by illustrating the frictional wage dispersion puzzle and how it relates to the Shimer critique, the inability of search models to generate significant fluctuations in unemployment and vacancies. A summary of the empirical literature on the effects of job displacement on workers's labor outcomes provides evidence that unemployment causes large and persistent earnings losses.

I develop a search model in which workers gradually lose some human capital while they stay unemployed. The wage losses caused by unemployment matter for workers' search behavior. Knowing that unemployment hurts their earnings, workers lower their reservation productivity and accept lower wages to leave unemployment more quickly. The paper shows that the model generates significant wage dispersion among identical workers and is an important improvement over baseline search models. Using the measure proposed by Hornstein, Krusell and Violante (2011), the  $Mm$  ratio, I find a closed form expression for wage dispersion that depends only on a few parameters. Combining the theoretical predictions of the model and estimates from micro data, the model explains between 45% and 48% of the observed residual wage dispersion in the PSID, and between 24% and 31% of that in the CPS. By contrast, the baseline model accounts for around 11% for the PSID, and 6% for the CPS.

When workers also search on-the-job, the model with unemployment history

accounts for all of the observed residual wage dispersion. In search models there is a trade-off between accounting for frictional wage dispersion and cyclical fluctuations in unemployment and vacancies. High values of non-market time make it more difficult for search models to generate significant amounts of wage dispersion, but high values of non-market time are required to account for the cyclical volatility of unemployment and vacancies. I show that even for high values of non-market time the search framework with both unemployment history and on-the-job search generates large amounts of wage dispersion, and accounts for almost all of residual wage dispersion. The paper thus addresses the trade-off in search models between matching frictional wage dispersion and the cyclical behavior of unemployment and vacancies.

In recessions jobs become scarce, unemployment rises, and workers take longer to find jobs. The results of this paper suggest that, by accounting for workers' human capital depreciation during unemployment, the welfare losses in recessions may be larger than previously thought. In the context of the recent crisis, the consequences for workers' labor market prospects have been dramatic, suggesting that one should take these costs into account.

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