Distortions in Financial Markets and Monetary Policy

by James Hansen

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Declaration

I certify that this thesis presented by me in February 2012 for examination for the PhD degree of the London School of Economics and Political Science is solely my own work.

Signature ....

Date ....

Abstract

This thesis investigates distortions in credit and equity markets. It provides insight into sources of volatility in these markets and their implications for monetary policy.

Chapter 2 analyses optimal monetary policy in an economy with a credit friction and capital. A central bank implementing policy optimally will face a trade-off in stabilising inflation, the composition of output, and the net worth of borrowers. The importance of net worth is a new finding in the literature, and reflects the central bank's concern that distortions in credit markets can reduce welfare if ignored. In addition, it is shown that some tolerance of inflation can be optimal in response to shocks that reduce borrowers' net worth.

Chapter 3 considers distortions in equity markets and their implications for economic decision-making. It analyses whether changes in the distribution of technology, coupled with optimal expectations on the part of investment-firm managers, can induce endogenous optimism or pessimism. And whether this optimism or pessimism can in turn lead to equity mispricing, and distorted economic decisions. Using a simple general equilibrium model, it is shown that a favourable change in the distribution of technology can induce endogenous optimism leading to over-valued equity prices and over-investment, when compared with an economy in which rational expectations are used.

Chapter 4 focuses on identifying the effects of mispriced equity. I find that equity mispricing has statistically significant effects on household consumption and portfolio allocation decisions. These effects are estimated to be non-trivial when allowing for episodes of significant mispricing such as an equity price bubble.

Taken together, these chapters suggest that distortions in credit and equity markets can be important, and should be taken into consideration by policymakers to the extent that they affect real economic decisions.

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Chapter 1

Introduction

This thesis explores how distortions in financial markets can affect the economy and monetary policy. It addresses three questions:

1. What is the optimal monetary policy response to real or financial shocks in an economy with a credit friction and endogenous capital?

2. Does the theory of optimal expectations, modelled on the part of firms, provide a useful framework for thinking about equity market bubbles?

3. To what extent does mispricing in equity markets affect consumer and firm decisions?

The underlying theme of the chapters is concerned with the appropriate monetary policy response to financial market distortions in either credit or equity markets. This is an important research area in macroeconomics. Financial market volatility has been a recurrent challenge for macroeconomic policy. It is apparent that policymakers, both over time and across countries, differ in their views of how macroeconomic policy should respond to financial volatility including to distortions in credit markets, or to distorted equity prices. The contribution of this thesis is to provide insight into the sources of such financial volatility, and whether they are relevant for monetary policy.

Chapter two of my research focuses on the implications of a distortion in credit markets. Important early research on the macroeconomic implications of credit market frictions has been undertaken by Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), and Bernanke, Gertler and Gilchrist (1999). More recently, the implications of distortions in credit markets for conventional monetary policy has been studied in economies without capital, see for example De Fiore and Tristani (2009), Cúrdia and Woodford (2010) and

Carlstrom, Fuerst and Paustian (2010). There also has been work on unconventional monetary policy in economies with credit frictions and capital, but not price frictions, see for example Gertler and Kiyotaki (2010). The contribution of chapter two is to study how a central bank would conduct conventional monetary policy optimally in an economy with both price and credit frictions, and with endogenous capital.

The key insight is that when both credit and price frictions exist, the central bank faces a meaningful policy trade-off. In particular, the central bank faces a trade-off in stabilising inflation, to mitigate the distortion associated with the pricing friction, and in stabilising the net worth of borrowers, to the mitigate the distortion in credit markets. The importance of net worth is a new result in the literature that studies optimal monetary policy analytically. Its importance can help to explain policy outcomes in the most recent financial crisis, and highlights that credit spreads are not necessarily a sufficient financial statistic when implementing monetary policy optimally in an economy where credit distortions matter.

Chapter three turns to the question of endogenous optimism or pessimism on the part of firms and households, which can distort equity price signals. The scope for endogenous optimism or pessimism to be in the best interest of economic participants is a relatively new development in microeconomic literature, see for example Brunnermeier and Parker (2005), Gollier (2005) and Van den Steen (2004). I examine whether the concept of optimal expectations, as defined by Brunnermeier and Parker, is a useful tool for thinking about one source of an equity price bubble.

More specifically, I focus on whether changes in the distribution of shocks to the economy can affect firms' incentives to be optimistic or pessimistic. And whether this optimism or pessimism can in turn lead to mispricing in equity markets, and distorted economic decisions. The theoretical framework studied augments a recent literature that attempts to build micro-founded dynamic general equilibrium models of equity market mispricing including for example Farhi and Tirole (2011), Martin and Ventura (2010), and Hong, Scheinkman and Xiong (2008).

Chapter four focuses on identifying the effects of mispricing in equity markets on household and firm decisions empirically. There is a voluminous existing literature that focuses on identifying the existence of equity price bubbles.[1] And a smaller literature that focuses on identifying the effects of mispricing on the economy, including for example Chirinko and Schaller (2001), and Gilchrist, Himmelberg and Huberman (2005). The contribution of this chapter is to provide additional evidence on the effects of mispricing on household consumption and portfolio allocation decisions, and firm dividend decisions.

The main finding is that mispricing in equity markets can have statistically significant effects on household and firm decisions. And that the sign of these effects does match those suggested by qualitative accounts of equity price bubbles. As an example, between March 1997 and December 2000, the boom phase phase of the US dot-com bubble, US consumption growth is estimated to be around 0.6 percentage points per annum faster than in the absence of equity mispricing during this period. There is also evidence to suggest that households adjust their portfolio allocations in response to equity mispricing shocks.

In sum, these chapters demonstrate, both theoretically and empirically, that certain types of financial volatility can have important implications for real economic decisions and potentially monetary policy. Each chapter is now discussed in further detail.

[1] See for example Gürkaynak (2008) and Vissing-Jorgensen (2004), which provide useful reviews of this literature.

Chapter 2

Optimal Monetary Policy with a Credit Friction and Endogenous Capital

2.1. Introduction

> A relevant question is why should we be interested in whether an asset price, or indeed any other variable for that matter, appears in some ad hoc class of feedback rule, even though the coefficients of that rule may have been optimised? It seems more instructive to ask first what an optimal rule looks like, and then consider how asset prices ought to figure in it. One might go on to consider whether particular simple rules represent sufficiently close approximations to the optimal rule to be useful guideposts for policy...
>
> Charles Bean (2003)

This chapter is concerned with the objectives of a central bank when implementing monetary policy optimally in an economy with capital, and a credit friction. The aim is to provide insight into whether volatility in financial variables, such as asset prices, affect the path of interest rates optimally chosen by a central bank. In particular, I consider whether asset prices or other financial variables form part of a central bank's stabilisation goals in their own right; or, alternatively, whether such variables are only important to the extent that they are useful for forecasting the output gap and inflation.

There has been much debate on the objectives of a central bank when faced with asset price and financial volatility. Prior to the recent financial crisis, this debate has typically focused on the question of whether asset price volatility should be incorporated into a policymaker's objective. For example, much of the discussion has centered on whether a central bank should look to smooth or "lean against" large movements in asset prices, in either direction, or whether the central bank should respond asymmetrically, using policy only to 'clean up' or stabilise the economy after an asset price bust. Following the recent financial crisis, this debate has been re-invigorated. There is now a wider discussion of the appropriate objectives for policy when faced with volatility in asset and financial markets, and whether monetary policy ought to be used, in conjunction with other macroeconomic tools, to help promote financial stability.

Notwithstanding extensive discussion in policymaking circles, theoretical literature on a central bank's objectives in an economy with capital and financial frictions is limited. There is analytical work addressing the central bank's objectives in a New Keynesian economy with capital but no credit friction – see for example, Edge (2003), and Takamura, Watanabe and Kudo (2006). There is also work addressing optimal monetary policy in an economy with a credit friction but no capital – see for example, Cúrdia and Woodford (2010), De Fiore and Tristani (2009) and Carlstrom *et al* (2010). However, the central bank's objectives in an economy where both capital and financial frictions play an important role remains an open question. Providing analytical insight that addresses this question is important for determining the objectives of monetary policy, especially if policymakers believe endogenous capital and financial frictions are important factors that influence economic volatility and inflation.

The methodology used in this chapter is the Linear Quadratic or LQ approach emphasised by Woodford (2003). Specifically, I derive a first-order approximation of the solution to an optimal monetary policy problem for a central bank concerned with maximising social welfare.[2] The advantage of the LQ approach is that it provides insight into the objectives of a central bank, and whether there exists a meaningful trade-off in achieving

[2] Optimal monetary policy here refers to "optimal monetary policy from a timeless perspective" as discussed by Woodford (2003).

these objectives. This analysis complements previous analytical work undertaken, such as that of Cúrdia and Woodford, and De Fiore and Tristani, and provides additional insight into the findings of previous numerical work on optimal policy, such as that undertaken by Faia and Monacelli (2007).

To preview the main result, I find that the main financial variable of interest to the central bank in an economy with a credit friction and capital is the net worth of borrowers. A central bank implementing policy optimally will look to mitigate volatility in net worth precisely because it is the net worth of borrowers that affects the extent to which a credit friction distorts the economy over time. The incentive to smooth net worth exists in addition to the traditional objectives for monetary policy, that include stabilising volatility in inflation and volatility in the composition of output. The existence of the credit friction, in addition to the price friction, introduces a trade-off for policy, and thus a strict inflation target is no longer equivalent to the optimal policy plan.

Although a policymaker is concerned with stabilising net worth qualitatively, there remains the question of whether this incentive is important quantitatively. The analytical results established here, derived under the assumption that the steady state credit friction is small, highlight that although there is a trade-off between stabilising inflation and mitigating the credit distortion, quantitatively this trade-off is not found to be large. As a consequence, inflation targeting remains a good approximation of optimal policy when the credit friction is not highly distortionary.

I also examine the case of a more distortionary steady state credit friction numerically. Using a calibration of the model similar to that estimated by Queijo von Heideken (2009), for the US and Euro area economies, I find that, in contrast to the small friction case, monetary policy can find it optimal to tolerate some deviation of inflation from target in response to financial shocks. This result confirms that the extent to which policymakers wish to stabilise net worth in the economy is contingent on the extent to which the credit friction is distortionary.

The next section provides a brief sketch of the New Keynesian model with a flexible rental market for capital and a credit friction. Section 3 discusses an analytical representation of the approximate optimal policy problem, including the objectives of the policymaker and the optimal policy plan. Section 4 considers how sensitive the findings are to the magnitude of the credit friction. Conclusions are drawn in the final section.

2.2. The Credit Friction Model

The economy I consider is a New Keynesian economy with capital and a credit friction. The economic environment is very similar to those studied by Carlstrom and Fuerst (1997; 2001) and Faia and Monacelli (2007). However, some small departures from the standard setup are considered to facilitate simplification of the optimal policy analysis. I include a brief review of the full economic environment, and draw attention to the departures from the standard credit friction model.[3]

Households

There is a continuum of identical individual households uniformly distributed on the interval $[0,1]$. Each household, indexed by h, supplies specialised labour $(H_t(h))$ and consumes $(C_t(h))$. Households are able to save by purchasing investment goods $\left(I_t^h(i)\right)$ from entrepreneurs that deliver physical capital $\left(K_{t+1}^h(i)\right)$ for use in an industry i in the period $t+1$, where $i \in [0,1]$. Alternatively, households may save by purchasing a portfolio of state-contingent securities $E_t\left(M_{t,t+1}A_{t+1}^h\right)$ where M_{t+1} is the price of a security delivering one unit of consumption if state s_{t+1} is realised.[4] Households receive income from their portfolio of previously purchased state-contingent securities, A_t^h, rents from

[3] For the reader already familiar with the Carlstrom and Fuerst model, the main departures are:

(a) I focus on an equilibrium where entrepreneurs save all of their available resources until retirement (see Carlstrom and Fuerst 2001). This is in contrast to the more standard assumption in the literature that entrepreneurs are indifferent between saving and consuming;

(b) I assume that in steady state the credit friction distortion is small (of second-order); and

(c) I assume a social welfare function that ensures that monetary policy abstracts from any incentive to redistribute consumption between households and entrepreneurs.

capital held across industries $\left(P_t \int_0^1 R_t(i) K_t^h(i)\, di\right)$, wages for their specialised labour $(P_t W_t(h))$, and are the residual claimants to any profits from production $P_t \int_0^1 D_t^h(i)\, di$. Formally, a household h chooses a sequence of specialised labour supply, consumption, Arrow-Debreu securities and investment goods $\left\{H_t(h), C_t(h), A_{t+1}^h(s_{t+1}), I_t^h(i)\right\}_{t=t_0}^{\infty}$, for all possible states $s_{t+1} \in \Omega$ and $i \in [0,1]$, that solve

$$\max E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[U(C_t(h), \psi_t) - V(H_t(h))\right]$$

subject to the flow budget constraint

$$\begin{aligned} P_t C_t(h) + \int_0^1 Q_t(i) I_t^h(i)\, di + E_t\left(M_{t,t+1} A_{t+1}^h\right) \leq A_t^h &+ P_t W_t(h) H_t(h) - P_t T_t \\ &+ P_t \int_0^1 R_t(i) K_t^h(i)\, di \\ &+ P_t \int_0^1 D_t^h(i)\, di \end{aligned}$$

and the capital accumulation constraint

$$K_{t+1}^h(i) = (1-\delta) K_t^h(i) + I_t^h(i)$$

where T_t is a real lump-sum tax, $W_t(h)$ is the real wage paid for household h's labour, $R_t(i)$ is the real return to capital held in industry i, $Q_t(i)$ is the nominal price of an investment good in industry i, δ is the depreciation rate of capital that is common across industries, and ψ_t is a taste shock common to all households.

The Euler equations in a symmetric equilibrium where all households are the identical, including in their initial endowments ($A_{t_0}^h = A_{t_0}$ for all h), and are indifferent to their

4 I make the usual assumptions regarding the stochastic structure of the economy; specifically, I assume it exists on a non-degenerate probability space $\{\Omega, \mathcal{F}, P\}$ where a history at time t is denoted by $s^t \in \Omega$,and a state, s_t, is an element of a given history (for example, $s^t \equiv \{s_1, ..., s_t\}$). E_t is the time t conditional expectation. I use the generic notation for a random variable $X_t \equiv X_t(s_t)$.

allocation of investment across industries, are given by

$$U_c(C_t,\psi_t) = \beta E_t\left(U_c(C_{t+1},\psi_{t+1})\left(1+i_t^b\right)\frac{P_t}{P_{t+1}}\right) \tag{2.1}$$

$$q_t(i) = \beta E_t\left(\frac{U_c(C_{t+1},\psi_{t+1})}{U_c(C_t,\psi_t)}\left(R_{t+1}(i)+(1-\delta)\,q_{t+1}(i)\right)\right) \tag{2.2}$$

$$\frac{V_H(H_t(h))}{U_c(C_t,\psi_t)} = W_t(h) \tag{2.3}$$

where $C_t = C_t(h)$ for all $h \in [0,1]$, and $q_t(i) \equiv \frac{Q_t(i)}{P_t}$ is the real price of an investment good. It should be noted that Tobin's $q_t(i)$ reflects the shadow price of a unit of capital installed for use in $t+1$ in industry i (or equivalently, is the share price of a firm that holds capital in industry i on the households' behalf).

Entrepreneurs (Investment Supply)

There is a continuum of entrepreneurs in the economy who lie on the interval $[1,1+\zeta]$. Entrepreneurs are risk neutral and supply investment goods to households through their access to a risky investment technology. An entrepreneur, indexed by j, is able to purchase $I_t(j)$ units of final output and invest it in a risky project that yields $\omega_t(j)\,I_t(j)$ units of final capital, which is then sold to households at price Q_t.[5] The stochastic technology underpinning an entrepreneurs' risky project is idiosyncratic and identically distributed with an exogenous cumulative distribution function $\Phi_\omega(x) \equiv \Pr(\omega_t \leq x)$, that is common for all entrepreneurs, and satisfies the first and second moment conditions $\int_{-\infty}^{\infty} x d\Phi_\omega(x) = 1$ and $\int_{-\infty}^{\infty}(x-1)^2 d\Phi_\omega(x) = \sigma_\omega^2$.

Entrepreneurs have limited net worth, and use financial intermediaries (banks) to leverage their project. The amount they borrow, $L_t(j)$, is defined by the total value of their investment, $P_t I_t(j)$, less their net worth, $NW_t(j)$ so that

$$L_t(j) \equiv P_t I_t(j) - NW_t(j)$$

5 I assume that all investment goods are perfectly substitutable. And so, entrepreneurs will all receive the same price for investment goods they sell.

Entrepreneurs' net worth is determined by their capital income and a small government fiscal subsidy (F^e) financed by the lump-sum tax on households,

$$NW_t(j) = P_t F_t^e + (P_t R_t + Q_t(1-\delta)) K_t^e(j) \tag{2.4}$$

where $K_t^e(j)$ is the stock of physical capital held by entrepreneur j. The government subsidy is a simplification that provides some starting capital for entrepreneurs, and allows me to abstract from their labour supply decision (see for example, De Fiore and Tristani 2009). This is a simplification that assists the optimal policy derivations that follow.

I assume that entrepreneurs borrow from perfectly competitive banks.[6] The default threshold, $\overline{\omega}_t(j)$, for an entrepreneur is defined by

$$\overline{\omega}_t(j) \equiv \frac{\left(1 + R_t^L(j)\right)(P_t I_t(j) - NW_t(j))}{Q_t I_t(j)}$$

where $R_t^L(j)$ is the interest rate charged by banks on a loan to entrepreneur j. Entrepreneurs are only able to repay their loans when their investment return satisfies $\omega_t(j) \geq \overline{\omega}_t(j)$. In the event that $\omega_t(j) < \overline{\omega}_t(j)$, entrepreneurs default and receive a zero payoff.

Financial Intermediaries

To model the credit friction I assume asymmetric information in the form of costly state verification for banks (Carlstrom and Fuerst 1997). Although entrepreneurs can observe $\omega_t(j)$ after their investment return has been realised, for banks to observe $\omega_t(j)$ they must pay a verification or bankruptcy cost, $\nu_t \in [0,1]$, that is proportional to the nominal resale value of an entrepreneurs investment income. I allow for exogenous time variation in the proportion of a project that is lost in the event of bankruptcy. Or, more conveniently in

[6] Provided that the expected return from the investment project is large enough, entrepreneurs will be willing to invest their entire net worth in their project and to borrow from banks.

the optimal policy derivations that follow, I focus on the renormalised exogenous shock $\xi_t \equiv \ln(1-\nu_t)$, which can be considered, approximately, as a shock to the proportion of funds recovered by a bank in the event of default. This recovery rate shock is always negative.

Under the optimal contract, banks only pay the verification cost when an entrepreneur defaults and does not repay their loan (Gale and Hellwig 1985). Following Faia and Monacelli (2007), the banks participation constraint can be written as

$$Q_t I_t(j)\, g(\overline{\omega}_t(j), \nu_t) \geq P_t I_t(j) - NW_t(j)$$

where

$$g(\overline{\omega}_t(j), \nu_t) \equiv 1 - \nu_t \Phi(\overline{\omega}_t(j)) - f(\overline{\omega}_t(j)) \tag{2.5}$$

$$f(\overline{\omega}_t(j)) \equiv \int_{\overline{\omega}_t(j)}^{\infty} (\omega_t(j) - \overline{\omega}_t(j))\, d\Phi(\omega_t(j)) \tag{2.6}$$

$g(\overline{\omega}_t(j), \nu_t)$ can be interpreted as the expected share of investment income accruing to banks that lend to entrepreneur j, and $f(\overline{\omega}_t(j))$ can be interpreted as the expected share of investment income accruing to entrepreneur j.

The Optimal Contract

The optimal contract consists of a default threshold and investment level $\{\overline{\omega}_t(j), I_t(j)\}$, chosen by the entrepreneur, that maximises their return subject to the banks' participation constraint. Entrepreneurs solve

$$\max_{\{\overline{\omega}_t(j), I_t(j)\}} Q_t I_t(j)\, f(\overline{\omega}_t(j))$$

$$\text{subject to: } Q_t I_t(j)\, g(\overline{\omega}_t(j), \nu_t) \geq P_t I_t(j) - NW_t(j)$$

The optimal contracting conditions, after aggregation, are

$$q_t f(\overline{\omega}_t) = \frac{f_\omega(\overline{\omega}_t)}{g_\omega(\overline{\omega}_t, \nu_t)} (q_t . g(\overline{\omega}_t, \nu_t) - 1) \tag{2.7}$$

$$I_t = \frac{nw_t}{1 - q_t . g(\overline{\omega}_t, \nu_t)} \tag{2.8}$$

where $nw_t \equiv \frac{NW_t}{P_t}, f_\omega(\overline{\omega}_t) \equiv \frac{\partial f(\overline{\omega}_t)}{\partial \overline{\omega}_t}$ and $g_\omega(\overline{\omega}_t, \nu_t) \equiv \frac{\partial g(\overline{\omega}_t, \nu_t)}{\partial \overline{\omega}_t}$. It should be noted that in equilibrium all entrepreneurs choose the same default threshold, $\overline{\omega}_t(j) = \overline{\omega}_t$ for all $j \in [1, 1+\zeta]$ from (2.7). Also note that investment in effect becomes proportional to the aggregate net worth of entrepreneurs, see (2.8).

Entrepreneurs (Consumption)

Non-defaulting entrepreneurs consume after the returns from their investment decision have been realised.[7] To ensure a well defined and stable equilibrium, I assume entrepreneurs face an exogenous retirement shock, consistent with Carlstrom and Fuerst (2001). Stochastic retirement ensures that when considering equilibria where entrepreneurs save resources through capital, this saving is not sufficient for entrepreneurs to become self-financing. The consumption decision problem for an individual entrepreneur is

$$\max_{\{C_t^e(j)\}} E_{t_0} \sum_{t=t_0}^{\infty} (\kappa \beta^e)^{t-t_0} C_t^e(j)$$

$$\text{subject to: } Q_t K_{t+1}^e(j) + P_t C_t^e(j) \leq (\omega_t - \overline{\omega}_t) Q_t I_t(j)$$

$$K_{t+1}^e(j) \geq 0$$

$$C_t^e(j) \geq 0$$

where $\beta^e \in (0,1)$ is the discount factor for entrepreneurs, $\kappa \in [0,1]$ is the probability than an entrepreneur is not affected by the retirement shock, and $C_t^e(j)$ denotes consumption

[7] Defaulting entrepreneurs forfeit all their investment proceeds and have no resources to consume. Provided these entrepreneurs are not also hit by the retirement shock, they enter the investment supply market in the next period with only the government fiscal subsidy.

by entrepreneur j. It should be emphasised that entrepreneurs are liquidity constrained in the sense that they cannot borrow against expected future project returns to fund additional consumption or saving. It is straightforward to verify that the entrepreneurs' budget constraint will bind, and thus

$$q_t K_{t+1}^e(j) + C_t^e(j) = (\omega_t - \overline{\omega}_t)\, q_t I_t(j)$$

To complete the description of the entrepreneurs' first-order conditions, there are several possible equilibria associated with this decision problem. The appropriate choice can vary depending on the steady state that one chooses to approximate around. To provide insight on optimal monetary policy, I focus on two possible equilibria. The first is an equilibrium where entrepreneurs consume all of their available resources, and so

$$K_{t+1}^e(j) = 0$$

This optimality condition can be supported in a steady state where $\beta^e < \beta$ and $\kappa = 1$. Thus, stochastic retirement is not required since entrepreneurs are assumed to be sufficiently impatient that they are always willing to consume all of their available resources.

As an alternative, I also consider an equilibrium where entrepreneurs save all of their available resources until they are stochastically forced to retire.[8] That is, prior to retirement,

$$C_t^e(j) = 0$$

[8] One can also consider a third equilibrium where entrepreneurs are indifferent between saving and consuming

$$1 = \beta^e E_t\left(\frac{R_{t+1} + q_{t+1}(1-\delta)}{q_t}\frac{q_{t+1} f(\overline{\omega}_{t+1})}{1 - q_{t+1} g(\overline{\omega}_{t+1}, \nu_{t+1})}\right)$$

However, when approximating equilibria with small monitoring costs, this equilibrium is not stable when solved numerically (i.e. perturbations to the economy result in a switch to either a saving or no saving equilibrium). For the case where entrepreneurs are indifferent, and monitoring costs are large, the equilibrium is stable and results are considered in subsequent numerical comparisons in Section 2.4.

which can be supported in a steady state where $\kappa < \beta$ and $1 > \kappa\beta^{e} > \beta$, given that I will latter assume that steady state monitoring costs are small. In this case, the restriction that $\kappa\beta^{e} > \beta$ ensures that entrepreneurs are sufficiently patient to be willing to save until their retirement. The restriction that $\kappa < \beta$ ensures that entrepreneurs retire frequently enough so that they are unable to become self-financing.[9] The measure of entrepreneurs is assumed to be constant over time so that any entrepreneur forced to retire is replaced by the birth of a new entrepreneur that begins life with capital F_t^e.

Aggregating across the measure of entrepreneurs, the equilibrium conditions are

$$q_t K_{t+1}^e + C_t^e = f(\overline{\omega}_t)\, q_t I_t \tag{2.9}$$

and, for the no-saving and saving equilibria respectively,

$$K_{t+1}^e = 0 \text{ if } \beta^e < \beta \text{ and } \kappa = 1 \tag{2.10}$$

or

$$C_t^e = (1-\kappa)\, f(\overline{\omega}_t)\, q_t I_t \text{ if } 1 > \kappa\beta^e > \beta \text{ and } \kappa < \beta \tag{2.11}$$

Whether entrepreneurs save or not will have a substantial bearing on optimal monetary policy. The reason for this is that if (2.10) is used, the endogenous evolution of net worth is shut down in the model since entrepreneurs do not save. In contrast, if (2.11) is used, the endogenous evolution of net worth is retained and will have important consequences for optimal policy. I now briefly review the supply side of the economy, which is standard.

9 The implied restriction that entrepreneurs are very patient, $\beta^e > 1$, may at first seem counterintuitive. However, it arises because I assume that steady state monitoring costs, and returns to saving, are small. This is in contrast to the usual assumption made for numerical work, where steady state monitoring costs, and returns to saving are high, and so entrepreneurs are assumed to be impatient.

Final Producers

Final good producers combine intermediate goods to produce a final good that can be consumed either by households or entrepreneurs, or used by entrepreneurs in their risky investment project. Assuming perfect competition in the market for final goods, each period final good producers solve

$$\max_{X_t(i)} P_t Y_t - \int_0^1 p_t(i) X_t(i) di$$

subject to a Dixit-Stiglitz transformation technology between intermediate inputs and final goods

$$Y_t = \left[\int_0^1 X_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \tag{2.12}$$

where Y_t is output (the final good), $p_t(i)$ is the price of intermediate good i, $X_t(i)$ denotes the quantity of intermediate good i used in production, and θ is the constant elasticity of substitution between intermediate goods used. The demand for intermediate goods produced by firm i is given by

$$X_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t$$

Intermediate Firms

There is a continuum of monopolistically competitive intermediate good producers (firms) on the unit interval. Each firm produces a single differentiated intermediate good using firm-specific labour, and capital. For tractability, I assume that a fully flexible rental market for capital exists that allows capital to be instantaneously reallocated between firms. Consistent with the standard New Keynesian model, only a fraction of firms γ (randomly selected) are able to choose their price optimally each period (Calvo 1983). Firms not able to choose their prices optimally, retain the price they set in the previous period.

Each period all firms, whether they are able to reset their price or not, choose a cost-minimising bundle of labour and capital subject to a Cobb-Douglas production technology. Firm i solves

$$\min_{K_t(i),H_t(i)} \quad R_t(i)K_t(i) + W_t(i)H_t(i)$$

subject to

$$X_t(i) = K_t(i)^{\alpha} \left(Z_t H_t(i)\right)^{1-\alpha} \tag{2.13}$$

where Z_t can be interpreted as a technology or productivity innovation common to all firms. Firm i will choose a capital to labour ratio that satisfies

$$\frac{K_t(i)}{H_t(i)} = \frac{W_t(i)}{R_t(i)} \frac{\alpha}{1-\alpha} \tag{2.14}$$

and the real marginal cost for firm i, $S_t(i)$, is given by

$$S_t(i) = \frac{1}{Z_t^{1-\alpha}} \left(\frac{W_t(i)}{1-\alpha}\right)^{1-\alpha} \left(\frac{R_t(i)}{\alpha}\right)^{\alpha} \tag{2.15}$$

At this point it should be noted that although I assume households supply specialised labour, the capital rental market is perfectly flexible. This assumption ensures that capital rents across industries are the same in each period $R_t(i) = R_t$, and by implication that there is a unique measure of asset prices across all industries, $Q_t(i) = Q_t$ (see (2.2)).

There is a second decision for those firms able to reset their price optimally. These firms choose the price of their intermediate good to maximise the expected value of profits distributed to (and thus discounted on behalf of) households. The program for those firms able to choose their price is

$$\max_{p_{t_0}(i)} E_t \sum_{t=t_0}^{\infty} \gamma^{t-t_0} M_{t_0,t} \frac{P_t}{P_{t_0}} \left(\left(\frac{p_{t_0}(i)}{P_t} - (1-\chi) S_t(i) \right) X_t(i) \right)$$

subject to the demand for their intermediate good,

$$X_t(i) = \left(\frac{p_{t_0}(i)}{P_t}\right)^{-\theta} Y_t$$

where the nominal stochastic discount factor is defined as,

$$M_{t_0,t} \equiv \beta^{t-t_0} \frac{U_c(C_t, \psi_t)}{U_c(C_{t_0}, \psi_{t_0})} \frac{P_{t_0}}{P_t}$$

real marginal costs are given by (2.15), and χ is a fiscal subsidy on real marginal costs that is also financed by the lump-sum tax on households. This subsidy is used to eliminate the steady state distortion associated with monopolistic competition, and is a useful simplification in the optimal policy analysis that follows.

The optimal price chosen by the subset of intermediate firms who can choose their price freely is given by

$$\frac{p_t(i)}{P_t} = (1-\chi)\frac{\theta}{\theta-1} \frac{E_t \sum_{\tau=t}^{\infty} (\beta\gamma)^{\tau-t} \frac{U_c(C_\tau,\psi_\tau)}{U_c(C_t,\psi_t)} P_\tau^\theta Y_\tau S_\tau(i)}{E_t \sum_{\tau=t}^{\infty} (\beta\gamma)^{\tau-t} \frac{U_c(C_\tau,\psi_\tau)}{U_c(C_t,\psi_t)} P_\tau^\theta Y_\tau} \tag{2.16}$$

where the aggregate price index is a Dixit-Stiglitz aggregator of the form

$$P_t \equiv \left[\int_0^1 p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \tag{2.17}$$

Market Clearing Conditions

The aggregate resource constraint, and market clearing conditions are[10]

$$Y_t = C_t + \zeta C_t^e + \zeta I_t \tag{2.18}$$

$$K_{t+1} = (1-\delta) K_t + \zeta I_t - \zeta \nu_t \Phi(\overline{\omega}_t) I_t \tag{2.19}$$

$$K_t = \int_0^1 K_t^h(i)\, di + \zeta K_t^e \tag{2.20}$$

$$K_t(i) = K_t^h(i) + K_t^e(i) \tag{2.21}$$

$$\zeta C_t^e = \int_1^{1+\zeta} C_t^e(j)\, dj \tag{2.22}$$

$$\zeta K_t^e = \int_1^{1+\zeta} K_t^e(j)\, dj \tag{2.23}$$

$$\zeta I_t = \int_1^{1+\zeta} I_t(j)\, dj \tag{2.24}$$

$$\zeta NW_t = \int_1^{1+\zeta} NW_t(j)\, dj \tag{2.25}$$

Definition 2.1. *A rational expectations (RE) equilibrium is defined as a set of sequences* $\left\{K_t(i), K_t^h(i), H_t(i), W_t(i), X_t(i), S_t(i), p_t(i), R_t(i), Q_t(i)\right\}_{t=t_0}^{\infty}$ *for all* $i \in [0,1]$, $\left\{\overline{\omega}_t(j), NW_t(j), K_t^e(j), C_t^e(j), g(\overline{\omega}_t(j), \nu_t), f(\overline{\omega}_t(j)), I_t(j)\right\}_{t=t_0}^{\infty}$ *for* $j \in [1, 1+\zeta]$, *and* $\left\{C_t, Y_t, I_t, K_t, P_t, C_t^e, K_t^e, NW_t, \overline{\omega}_t, Q_t, R_t, i_t^b\right\}_{t=t_0}^{\infty}$ *given initial endowments* $K_{t_0}^h(i) = K_{t_0}^h, K_{t_0}^e(j) = K_{t_0}^e, A_{t_0}(i) = A_{t_0}$ *and bounded shock processes* $\left\{\psi_t, Z_t, \nu_t, F_t^e\right\}_{t=t_0}^{\infty}$ *such that (2.1) to (2.9), (2.12) to (2.25), and either of (2.10) or (2.11) are satisfied.*

Note that in equilibrium all entrepreneurs choose the same default threshold, $\overline{\omega}_t(j) = \overline{\omega}_t$, and that $R_t(i) = R_t$ and $Q_t(i) = Q_t$ given the assumption of perfectly flexible capital markets. The properties of the RE equilibrium, such as uniqueness and determinacy, will

[10] Alternatively, one could re-specify the credit friction to affect the marginal rate of transformation of output into investment by assuming that monitoring costs are incurred in the form of reduced output rather than reduced capital accumulation. The results that follow are robust to re-writing the model in this way.

in general depend on how monetary policy is implemented by the central bank through its choice of nominal interest rates over time, $\left\{i_t^b\right\}_{t=t_0}^{\infty}$.

2.3. Optimal Monetary Policy in the Credit Friction Model

I now consider the implementation of optimal monetary policy in the economy with a credit friction. In particular, I focus on the analytical LQ solution approach, emphasised by Woodford (2003), to attain insight into the objectives of monetary policy, and what policies will achieve these objectives. Numerically, the LQ approach is equivalent to a log-linear approximation of the solution to the Ramsey-policy problem for the central bank.

A Second-Order Approximation of Welfare

To begin, one must define the appropriate normative objective for the central bank. I assume that the central bank is concerned with maximising a measure of aggregate social welfare, that incorporates both the welfare of households and entrepreneurs. In particular, I assume

$$
\begin{aligned}
SW_{t_0} = {} & E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left(\int_0^1 U\left(C_t(i)\right) di - \int_0^1 V\left(H_t(i)\right) di \right) \\
& + E_{t_0} \sum_{t=t_0}^{\infty} \Lambda_t \left(\kappa \beta^e\right)^{t-t_0} \int_1^{1+\zeta} C_t^e(j)\, dj
\end{aligned}
$$

The above social welfare measure implies that all households receive an equal weighting in social welfare, as do all entrepreneurs. In terms of the relative weighting applied across household and entrepreneurs, Λ_t represents a time-varying weight on the welfare of entrepreneurs (the household weight is normalised to one). I choose a deterministic process for Λ_t, which ensures that monetary policy does not have an incentive to redistribute mean consumption between household and entrepreneurs, as will become clearer below.

Taking a second-order approximation[11] of households' contribution to social welfare (see Appendix 6.1) I have[12]

$$E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(U\left(C_t\right)-\int_0^1 V\left(H_t\left(i\right)\right)di\right)=-\left(U_cY\right)E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t^H$$
$$+t.i.p+O\left(\|\vartheta\|^3\right) \tag{2.26}$$

where

$$\begin{aligned}L_t^H \equiv\ & \omega_y\frac{y_t^2}{2}+\omega_i\frac{i_t^2}{2}+\omega_\pi\frac{\pi_t^2}{2}\\ & +\omega_k\frac{k_t^2}{2}+\widetilde{\omega}_e\frac{\left(c_t^e\right)^2}{2}\\ & +s^e c_t^e-\omega_{yi}y_t i_t-\omega_{yk}y_t k_t-\omega_{ey}c_t^e y_t\\ & -\omega_{ei}c_t^e i_t-\omega_\xi\left(\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right)+\xi_t i_t\right)\end{aligned}$$

and all lower case variables denote their log deviation from steady state. It should be noted that $y,i,\pi,k,c^e,\widehat{\Phi}\left(\overline{\omega}\right),\xi$ map to output, investment, inflation, capital, entrepreneurial consumption, the default rate and the recovery rate shock respectively. s^e denotes the steady state share of entrepreneurial consumption in output and all other welfare coefficients, the ω's, are functions of the structural parameters defined in Appendix 6.1. For brevity, and without loss of generality, I abstract from the effects of productivity and taste shocks.[13]

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[11] With the exception of ξ_t, the approximations here are taken with respect to the natural logarithms of variables.

[12] Note that $C_t\left(i\right)=C_t$ in the symmetric equilibrium I focus on, $t.i.p$ stands for terms that are independent of monetary policy, and that ϑ is a vector consisting of the sequences of all exogenous shocks in the economy. I use the notation $O\left(\|\vartheta\|^3\right)$ to denote the approximation residual which is of third-order or higher in the bound $\|\vartheta\|$ on the amplitude of the exogenous shocks (see Benigno and Woodford (2006) for further discussion).

[13] Results that include these shocks are available on request.

A second-order approximation of entrepreneurial welfare yields

$$E_{t_0}\sum_{t=t_0}^{\infty}\left(\kappa\beta^{e}\right)^{t-t_0}\int_{1}^{1+\zeta}C_t^e(j)\,dj = -(Y)\,E_{t_0}\sum_{t=t_0}^{\infty}\left(\kappa\beta^{e}\right)^{t-t_0}L_t^e$$
$$+t.i.p + O\left(\|\vartheta\|^3\right) \tag{2.27}$$

where

$$L_t^e \equiv -s^e c_t^e - s^e\frac{\left(c_t^e\right)^2}{2}$$

Comparing the first-order terms in the household and entrepreneurial loss functions, the presence of first-order terms in entrepreneur consumption, c_t^e, implies that there can be an incentive for the central bank to use monetary policy to redistribute consumption between entrepreneurs and households. The amount of redistribution considered optimal is a function of the weight on entrepreneurial welfare in social welfare, Λ_t, the effective discount factor for entrepreneurs' $\left(\kappa\beta^e\right)$ relative to that for households (β), and the slope of the household utility function evaluated at the steady state (U_c). To ensure that monetary policy abstracts from any incentive to redistribute consumption between households and entrepreneurs, I assume that the weight on entrepreneurial welfare evolves over time according to

$$\Lambda_t \equiv U_c\left(\frac{\beta}{\kappa\beta^e}\right)^{t-t_0}$$

This assumption ensures that the first-order terms relating to entrepreneurial consumption in both household and entrepreneurial welfare cancel in every time period t, and is similar to the approach used by De Fiore and Tristani (2009). The approximation of social welfare in this case simplifies to

$$SW_t = -E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}U_c Y\left(L_t\right)$$
$$+t.i.p + O\left(\|\vartheta\|^3\right) \tag{2.28}$$

where

$$
\begin{aligned}
L_t \equiv\ & \omega_y \frac{y_t^2}{2} + \omega_i \frac{i_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} \\
& + \omega_k \frac{k_t^2}{2} + \omega_e \frac{\left(c_t^e\right)^2}{2} \\
& - \omega_{yi} y_t i_t - \omega_{yk} y_t k_t - \omega_{ey} c_t^e y_t \\
& - \omega_{ei} c_t^e i_t - \omega_\xi \left(\xi_t \widehat{\Phi}\left(\overline{\omega}\right) + \xi_t i_t \right)
\end{aligned}
\tag{2.29}
$$

and $\omega_e \equiv \widetilde{\omega}_e - s^e$.

A useful property of the above social welfare measure is that it does not contain first-order terms. This stems from the specific choice of the weight on entrepreneurial welfare, Λ_t, and that I choose to approximate around a steady state where the distortions associated with monopolistic competition and bankruptcy are assumed to be small. These assumptions imply that monetary policy will focus on time variation in the costs of these distortions associated with economic shocks, rather than using monetary policy to mitigate steady state distortions. These assumptions also contribute significantly to the simplification of the optimal policy problem, as the absence of first-order terms in (2.29) implies that the central bank's objective can be maximised subject to a first-order approximation of the constraints that describe the economy's decentralised equilibrium. For those readers who prefer analysis with large steady state distortions, this question is addressed numerically in Section 2.4.

A First-Order Approximation of the Constraints

Using a first-order approximation of the constraints in a symmetric decentralised equilibrium, I have[14]

$$k_{t+1} = (1-\delta)\,k_t + \delta i_t + \delta\Phi(\overline{\omega})\,\xi_t \tag{2.30}$$

$$\sigma_c c_t^h = \sigma_c E_t c_{t+1}^h - \left(i_t^b - E_t(\pi_{t+1})\right) + \upsilon\psi_t - \upsilon E_t\psi_{t+1} \tag{2.31}$$

$$\begin{aligned}\sigma_c c_t^h = {} & \sigma_c E_t c_{t+1}^h - (1-\beta(1-\delta))\,E_t r_{t+1} + \widehat{q_t} \\ & - \beta(1-\delta)\,E_t\widehat{q_{t+1}} + \upsilon\psi_t - \upsilon E_t\psi_{t+1}\end{aligned} \tag{2.32}$$

$$\widehat{q_t} = \widehat{\widetilde{f_\omega}(\overline{\omega}_t)} - g_\omega\left(\widehat{\overline{\omega}_t, 1-e^{\xi_t}}\right) + \left(1 - q_t.g\left(\widehat{\overline{\omega}_t, 1-e^{\xi_t}}\right)\right) - \widehat{f(\overline{\omega}_t)} \tag{2.33}$$

$$i_t = \widehat{nw_t} - \left(1 - q_t.g\left(\widehat{\overline{\omega}_t, 1-e^{\xi_t}}\right)\right) \tag{2.34}$$

$$\begin{aligned}r_t = {} & \left(\sigma + \frac{1+\eta}{1-\alpha}\right) y_t - s\sigma i_t - \sigma s^e c_t^e - \frac{1+\alpha\eta}{1-\alpha} k_t \\ & - (1+\eta)\,z_t - \upsilon\psi_t\end{aligned} \tag{2.35}$$

$$\pi_t = \Theta r_t - \Theta y_t + \Theta k_t + \beta E_t \pi_{t+1} \tag{2.36}$$

$$\sigma_c c_t^h = \sigma y_t - \sigma s\delta i_t - \sigma s^e c_t^e \tag{2.37}$$

$$c_t^e = \widehat{f(\overline{\omega}_t)} + \widehat{q_t} + i_t \tag{2.38}$$

where $\widetilde{f_\omega}(\overline{\omega}_t) \equiv -f_\omega(\overline{\omega}_t)\,, s \equiv \frac{K}{Y}, \upsilon \equiv \frac{U_{c\psi}}{U_c}, \eta \equiv \frac{V_{HH}H}{V_H}\sigma_c \equiv -\frac{U_{cc}C}{U_c}, \sigma \equiv \sigma_c\frac{Y}{C}$ and variables with hats denote log deviations of functionals from their steady state value, or where the lower case of a variable has already been used to denote its real value.[15] The first three equations describe capital accumulation, the household consumption Euler equation and the household investment Euler equation respectively. The next two equations are the optimal contracting conditions, that determine the relationships between asset prices, the default threshold, bankruptcy costs, investment and net worth. This is followed by the

---

[14] For brevity, I omit the approximation errors here.

[15] For example, $\widehat{f(\overline{\omega}_t)} \equiv \ln\frac{f(\overline{\omega}_t)}{f(\overline{\omega})}$ where $f(\overline{\omega}_t)$ is a function of the default threshold, or real net worth is defined by $nw_t \equiv \frac{NW_t}{P_t}$ and thus $\widehat{nw_t} \equiv \ln\frac{nw_t}{nw}$.

equation for rental returns to capital, the New Keynesian Phillips curve, the aggregate resource constraint and the equation for entrepreneurial consumption.

Entrepreneurial capital holdings and net worth follow either

$$k^e_{t+1} = 0 \tag{2.39}$$

$$\widehat{nw_t} = \widehat{F^e_t} \tag{2.40}$$

if $\beta^e < \beta$ and $\kappa = 1$ in the equilibrium where entrepreneurs do not save, or

$$k^e_{t+1} = \widehat{f(\overline{\omega}_t)} + i_t \tag{2.41}$$

$$\widehat{nw_t} = \frac{F^e}{nw}\widehat{F^e_t} + \beta^{-1}\frac{K^e}{nw}k^e_t + \left(\beta^{-1} - 1 + \delta\right)\frac{K^e}{nw}r_t + (1-\delta)\frac{K^e}{nw}\widehat{q_t} \tag{2.42}$$

if $1 > \kappa\beta^e > \beta$ and $\kappa < \beta$ in the equilibrium where entrepreneurs save and then consume at retirement.[16]

Optimal Monetary Policy

The LQ approximation of the solution to the optimal policy problem is to maximise (2.28) and (2.29) subject to the above constraint system (2.30 to 2.38 and either of 2.39 and 2.40, or 2.41 and 2.42). To understand optimal policy, I re-write the LQ problem in terms of gap variables, where the latter are defined in terms of log deviations from their zero-inflation equilibrium values.[17] That is, gap variables measure the deviation from the policy that would be considered optimal in the absence of the price and credit frictions.[18] The loss

[16] nw, F^e and K^e denote the steady state values of net worth, the lump-sum fiscal transfer, and capital all with respect to entrepreneurs.

[17] See Appendix 6.2 for the system of first-order constraints in the zero-inflation equilibrium.

[18] This definition is consistent with the fact that I abstract from the effects of cost-push shocks.

function (2.29) can be re-written as (see Appendix 6.3)

$$\begin{aligned}L_t = &\ \omega_y \frac{x_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} \\ &+ \omega_i \frac{g_t^2}{2} + \omega_k \frac{j_t^2}{2} - \omega_{yi} x_t g_t - \omega_{yk} x_t j_t \\ &+ \omega_e \frac{\left(n_t - n_t^*\right)^2}{2} - \omega_{ey} n_t x_t + \omega_{ei} n_t g_t \end{aligned} \tag{2.43}$$

where x_t, g_t, j_t and n_t are the output, investment, capital and net worth gaps respectively,

$$\begin{aligned} x_t &= y_t - y_t^n \\ g_t &\equiv i_t - i_t^n \\ j_t &= k_t - k_t^n \\ n_t &\equiv \widehat{nw}_t - \widehat{nw}_t^n \end{aligned}$$

n_t^* is a target for the net worth gap that is zero when entrepreneurs do not save, and is non-zero when entrepreneurs save,

$$n_t^* = \left\{ \begin{array}{cc} 0 & \text{if } \beta^e < \beta \text{ and } \kappa = 1 \\ \frac{-\left(\frac{s\delta}{s^e}\frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})} f(\overline{\omega}) v_t + \sigma_c c_t^n\right)}{\sigma s^e} & \text{if } 1 > \kappa\beta^e > \beta \text{ and } \kappa < \beta \end{array} \right\}$$

and where c_t^n is the equilibrium value for household consumption that would be chosen in the absence of price and credit frictions.

Comparing (2.43) with the loss function in the economy with capital and exogenous capital adjustment costs, but no credit friction,[19]

$$\begin{aligned}L_t^{No\ CF} = &\ \omega_y \frac{x_t^2}{2} + \omega_\pi \frac{\pi_t^2}{2} \\ &+ \omega_i \frac{g_t^2}{2} + \omega_k \frac{j_t^2}{2} - \omega_{yi} x_t g_t - \omega_{yk} x_t j_t \end{aligned} \tag{2.44}$$

[19] See Hansen (2010) for the derivation of the loss function, $L_t^{No\ CF}$, in the New Keynesian economy with capital and capital adjustment costs, but no credit friction.

it is clear that only difference is in the third line of (2.43), and thus the new variable of interest to the policymaker in the economy with a credit friction is net worth.

More specifically, a central bank will have an incentive to smooth movements in net worth in the equilibrium where entrepreneurs save. This is because the central bank has a non-trivial policy trade-off in this equilibrium. At the margin, the central bank can choose between stabilising inflation, reducing price dispersion that is socially costly, or following an alternative policy that mitigates time variation in the credit friction, that is also socially costly. Importantly, both of these goals cannot be addressed simultaneously, and so there is a trade-off for monetary policy.[20]

The reason that net worth volatility is the key additional variable of interest to the policy maker, is that it is net worth that determines how socially costly the credit friction is. In particular, when net worth is high entrepreneurs are able to choose investment allocations that are similar to those that would be chosen in the absence of a credit friction, and so the distortionary effects of this friction are small. In contrast, when net worth is low, entrepreneurs are constrained in their ability to obtain external finance. This implies that entrepreneurs' investment allocations depart more significantly from those that would be chosen if the credit friction were not present.

Another way to see this point is to analyse the equilibrium where entrepreneurs do not save, and thus the amplification mechanism associated with net worth is shut down. In this case it can be verified that net worth becomes a term that is independent of policy when entrepreneurs do not save,[21] $n_t = t.i.p$ and $n_t^* = 0$, and so the social loss function for the central bank becomes equivalent to that derived in an economy with capital but no credit friction ($L_t = L_t^{No\ CF}$). In this case, as emphasised by Hansen (2010), a zero-inflation target is optimal and the policymaker is only concerned with stabilising dispersion in

[20] This can be seen clearly from analysis of the first-order conditions of the optimal policy problem, and noting that, in general, the target around which net worth volatility is stabilised is non-zero $\left(n_t^* \neq 0\right)$. See Appendix 6.4 for further detail.

[21] See Appendix 6.3.

prices. This makes sense, as although the credit friction still exists when entrepreneurs do not save, the central bank is in fact unable to use monetary policy to mitigate time variation in this distortion.

To understand the determinants of net worth volatility when it is responsive to changes in monetary policy, the net worth gap can be written as

$$n_t = \frac{f_{\overline{\omega}_t}(\overline{\omega})\,\overline{\omega}}{f(\overline{\omega})}\left(\widehat{\overline{\omega}}_t - \widehat{\overline{\omega}^n}_t\right) + \left(\widehat{q}_t - \widehat{q}_t^{\,n}\right) + g_t + O\left(\|\vartheta\|^2\right)$$

in the equilibrium where entrepreneurs save. Thus, when net worth responds to policy, the incentive to smooth movements in net worth is equivalent to an incentive to smooth movements in default rates, the asset price gap, and the investment gap. Interestingly, only investment gap volatility, g_t, and default rate volatility, $\widehat{\overline{\omega}}_t - \widehat{\overline{\omega}^n}_t$, are in fact variables that are endogenous to policy and respond to changes in interest rates. Asset prices volatility, or strictly speaking volatility in the asset prices gap, can be shown to be a term that is in fact independent of policy.[22] Although the policymaker has an incentive to smooth volatility in asset prices, a topic that has received much attention in previous literature, the policymaker in this economy is in fact unable to address this incentive, when the steady state credit friction is small.

Comparing these results with recent literature, it should be noted that the incentive to smooth volatility in the default rate of entrepreneurs is equivalent to an incentive to smooth the credit spread, when the spread is appropriately defined in this economy.[23]

[22] Appendix 6.3, Lemma 6.2, verifies that $\widehat{q}_t = t.i.p$ and so $\widehat{q}_t - \widehat{q}_t^n = 0$.

[23] The credit spread (external finance premium) is defined as

$$\begin{aligned}\widetilde{sp}_t &\equiv \frac{1+R_t^L}{P_t} - 1 \\ &= \frac{\overline{\omega}_t}{g\left(\overline{\omega}_t, 1-e^{\xi_t}\right)} - 1\end{aligned}$$

Focusing on the normalised spread, $sp_t = 1 + \widetilde{sp}_t$, it is straightforward to verify that deviations in the spread gap, $sp_t - sp_t^n$, are proportional to deviations in the default rate gap, $\widehat{\overline{\omega}}_t - \widehat{\overline{\omega}^n}_t$ up to a first-order approximation.

The importance of the credit spread is a finding that is similar to work of Cúrdia and Woodford (2010), De Fiore and Tristani (2009) and Carlstrom *et al* (2010). These authors also show that the credit spread features in the objective of the policymaker in economies with a credit friction but no capital. However, a key difference, between their results and those emphasised here, is that the spread is not the primitive variable in the policymakers' objective in this economy. The spread only matters to the extent that it influences net worth, or more fundamentally the social cost of the credit friction. Although this distinction may appear subtle, a central bank that is concerned with smoothing the credit spread, as opposed to net worth, would not in fact be implementing monetary policy optimally in this economy.

Qualitatively, these results are broadly consistent with the set of monetary policies used across countries in the most recent financial crisis. Arguably, one interpretation is that a number of countries implemented monetary policies, including conventional monetary policy, with the objective of stabilising borrowers' net worth playing some part in policymakers' overall approach. Possible examples include the sharp reduction in interest rates used in the US, UK and Europe at the onset of the crisis, as well as the additional macroeconomic tools used, such as the purchase of financial securities and lending programs, that assisted in stabilising the net worth of borrowers and lenders, and in reducing the incidence of default.[24]

2.4. Quantitative Analysis

I now examine the extent to which these incentives matter quantitatively. For a baseline calibration, I calibrate a similar default rate (3 percent quarterly) and external finance premium (150 basis points) as that used by Faia and Monacelli (2007). All other parameters are calibrated at values that are similar to those used by Faia and Monacelli, or in line with previous empirical literature that estimates the New Keynesian model analysed here (see Table 2.1).

[24] See for example Bernanke (2009), Kohn (2009) and Stark (2009).

Table 2.1: Calibration for the Benchmark Model		
Parameter	Steady State Value[(a)]	Interpretation
β	0.99	Discount factor
δ	0.025	Depreciation rate
α	0.33	Share of factor payments to capital
σ_c	1.00	Coefficient of relative risk aversion
η	2	Inverse of Frisch elasticity of labour supply
γ	0.65	Proportion of firms that cannot change their price each quarter
θ	8	Elasticity of substitution between intermediate goods
κ	0.90	Entrepreneurs' probability of not retiring
$\frac{\zeta F^e}{Y}$	0.02	Aggregagate entrepreneur subsidy relative to income[(b)]
$\phi(\overline{\varpi})$	0.5	Uniform PDF of investment projects[(c)]

Notes: (a) All parameters are chosen to be consistent with a Balanced Growth Path in non-detrended variables, and to match empirical estimates using non-detrended data.
(b) Consistent with a steady state government subsidy to entrepreneurs of 9 per cent (as a share of steady state net worth).
(c) On the $[0,2]$ interval.

Given the recent financial crisis, it is topical to consider two shocks that are financial in nature.[25] The first shock is a negative one percentage point shock to the proportion of funds recovered when an entrepreneur declares default.[26] This results in a decline in the willingness of banks to lend to entrepreneurs (i.e. an increase in the cost of asymmetric information). The second shock is a negative shock to the lump-sum government subsidy to entrepreneurs, which reduces their net worth directly by one percent. This acts as an exogenous reduction in entrepreneurs' pledgable collateral, increasing their external financing requirement and reducing their ability to borrow funds for an investment project.

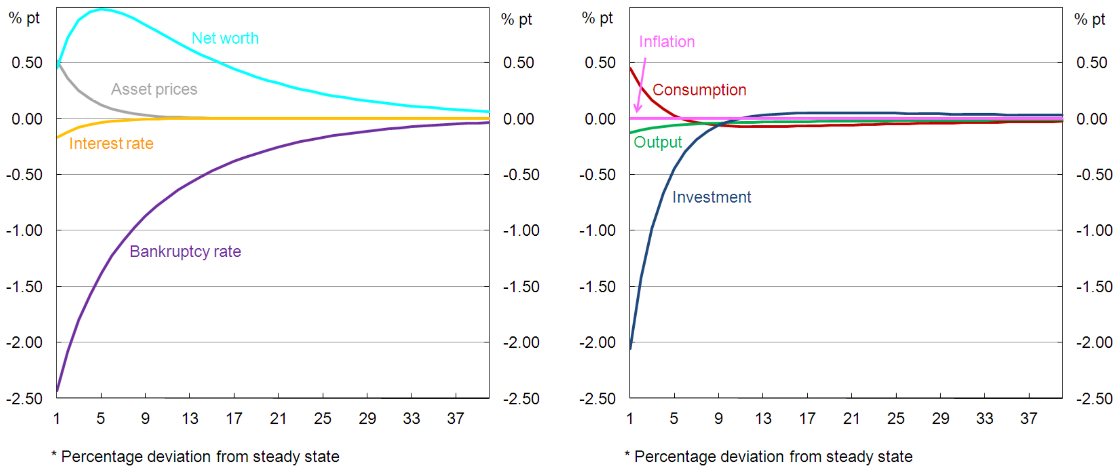
As a useful benchmark for comparison, Figure 2.1 reports the impulse response functions if monetary policy follows a zero-inflation target following a one percentage point decline in the recovery rate. With a small credit friction, it can be observed that a decrease in the recovery rate results in an expansionary monetary policy that stabilises inflation, increases asset prices and the net worth of entrepreneurs, and results in a fall in the default rate.[27]

[25] The implications of the following results are unchanged if productivity shocks or taste shocks are considered.

[26] Recall that, by assumption, the steady state recovery rate is close to one, when monitoring costs are small in steady state

Interestingly, notwithstanding the increase in net worth, investment still falls because although entrepreneurs are now more able to finance projects from their internal funds, the contraction in external finance provided by banks is larger. The net effect is a decline in credit and investment. Banks are less willing to lend precisely because they recover less when an entrepreneur defaults.

Figure 2.1: Decline in the Recovery Rate – Zero Inflation Target



I now consider the extent to which optimal policy deviates from a zero-inflation target. Figure 2.2 reports the deviation of variables from their zero-inflation values. The results highlight that although it is optimal to run a more expansionary monetary policy than that of a zero-inflation target, the order of magnitude of this deviation is small. For example, with a one percent fall in the recovery rate, optimal monetary policy would call for stimulating quarterly inflation in the order of 0.005 per cent.

To be clear, the very small deviations from a strict inflation target are not specific to the type of shock to fundamentals considered. Figure 2.3 reports the deviations from a zero-inflation target policy in response to a negative net worth shock, and a positive productivity shock. Again, though a more expansionary policy is called for in response to both shocks, the optimal deviation of inflation from zero is very small.

[27] In the economy I describe, the external finance premium always moves in the same direction as the default rate.

Figure 2.2: Decline in the Recovery Rate – Optimal Monetary Policy

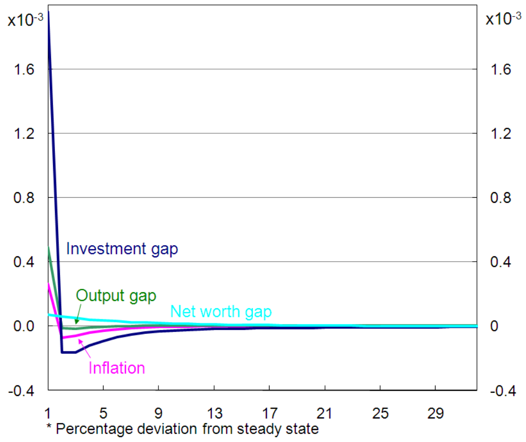
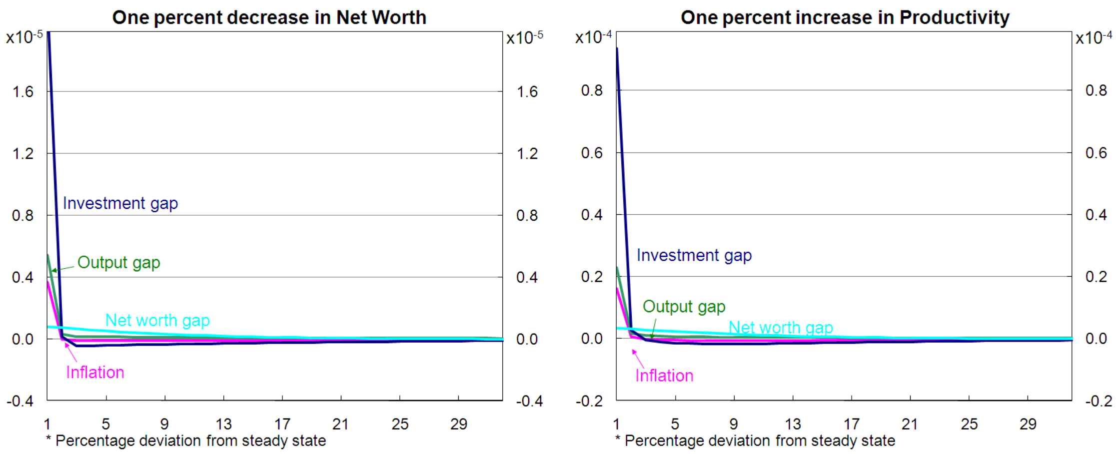


Figure 2.3: Net Worth and Productivity Shocks – Optimal Monetary Policy



The LQ framework provides further insight into these results, as it allows the relative weights of variables in the loss function to be examined. Table 2.2 highlights that inflation is by far the most important variable for social welfare. Of secondary importance is the output gap, the capital and investment gaps, and then the net worth gap. Clearly, the credit friction objective for policy is noticeably sub-ordinate to the objective of stabilising inflation.

Robustness: Does the size of the friction matter?

A natural question is whether the previous quantitative results rely on the assumption of a small steady state credit friction. To address this, I also consider the optimal inflation response to shocks in models with a larger steady state credit friction. Specifically, I

Table 2.2: Loss Function Weights	
Variable	Weight on Variance Term in L_t [(a)]
Inflation (ω_π)	1
Output gap (ω_y)	1.55×10^{-2}
Capital gap (ω_k)	2.63×10^{-3}
Investment gap (ω_i)	2.39×10^{-4}
Net worth gap (ω_e)	2.12×10^{-6}

Notes: (a) Normalised by the weight on inflation.

compare the benchmark economy previously described, with economies that have steady state credit frictions that are similar in size to those estimated for the US and Euro area economies by Queijo von Heideken (2009).

To be clear there are three main distinctions between the large friction economies I now consider, and the benchmark economy previously discussed. These are:

(A) The large friction economies now have monitoring costs that distort the economy to the first-order;

(B) Since steady state returns are higher in the credit friction economies, I focus on an equilibrium where entrepreneurs are impatient, and indifferent to saving and consuming; and

(C) I assume that all firms are able to optimise their price each period, but must pay a quadratic adjustment cost when doing so.

(A) is the distinction of interest I wish to investigate. That is, does the magnitude of the credit friction affect the extent to which a central bank wishes to deviate from a zero-inflation target in response to either productivity, net-worth or bankruptcy rate shocks. (B) and (C) are technical assumptions required when solving for optimal (Ramsey) policy numerically. (B) ensures that entrepreneurs do not become self-financing in an equilibrium where monitoring costs are significant, and steady state investment returns are high. (C) is included as an alternative assumption to Calvo pricing that facilitates a numerical solution.[28]Concerning the measure of social welfare in the larger friction economies, I continue to assume that the social welfare measure maximised by the central bank incorporates both the welfare of households and entrepreneurs, and that monetary policy is not concerned with first-order or mean consumption redistribution between these groups.[29]

Table 2.3 summarises the alternative models I compare. Specifically, I compare the benchmark economy with an economy that assumes steady state bankruptcy costs in the order of 25 per cent, which is comparable to that level of bankruptcy costs estimated for the Euro area (see Queijo von Heideken 2009). The second model I consider assumes steady state bankruptcy costs at 16 per cent, which is similar to estimates for the US, but is otherwise identical to the Euro area model. The third model, US Alt, is identical to the US model with the exception that entrepreneurs are assumed to be more patient. This latter assumption ensures a more realistic steady state rate of default for the US.

Table 2.3: Alternative Credit Friction Models

	Benchmark	Euro Area	US	US Alt
Entrepreneurs	Save[(a)]	Indifferent[(b)]	Indifferent[(b)]	Indifferent[(b)]
SS Recovery Rate	1.00	0.75	0.84	0.84
SS Default Rate	0.03	0.03	0.38	0.03
Entrepreneur Discount Factor	1.1	0.86	0.86	0.91

Notes: (a) Entrepreneurs save until stochastic retirement.
(b) Entrepreneurs are indifferent between consuming and saving.
(c) Firms pay a quadratic adjustment cost when changing their price.
(d) Social welfare includes household and entrepreneur welfare.

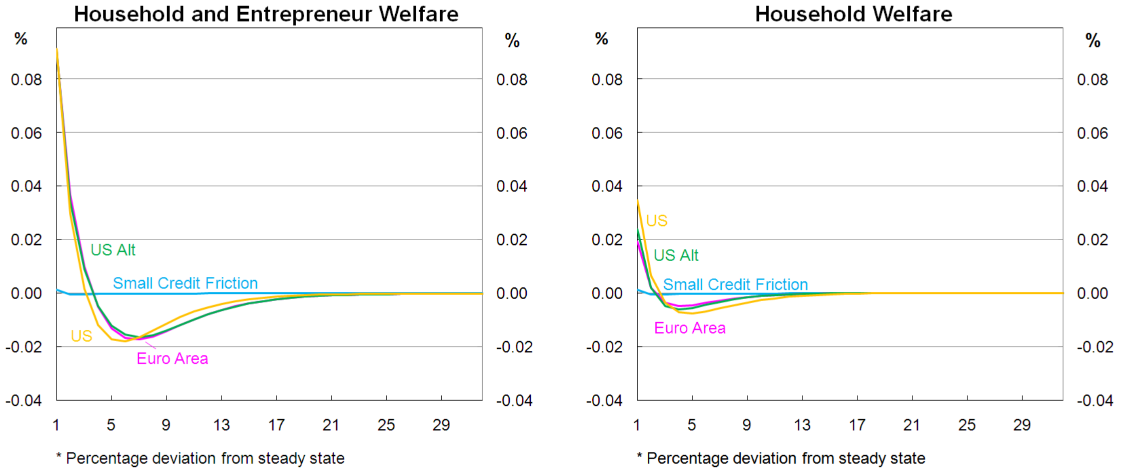
The left hand panel of Figure 2.4 reports results from comparing the response of the inflation rate to a one percentage point decline in the recovery rate, assuming that the central bank implements Ramsey optimal policy. The results highlight that the size of the steady state credit friction does affect the extent to which the central bank deviates from a zero-inflation target. In particular, in the larger friction models a concern with stabilising

28 It overcomes the technical difficulty that, in principle, the whole distribution of prices matters for welfare when solving for numerical Ramsey policy with Calvo pricing. See Faia and Monacelli (2007) for further discussion.

29 In particular, I normalise the weight on households as a group to one, and set $\Lambda_t = U_c\left(\frac{\beta}{\kappa\beta^e}\right)^t$, which is identical in form to the weighting used in the benchmark model.

movements in net worth implies that the policymaker now chooses an inflation rate that is approximately 0.1 percentage points higher than steady state inflation. This is a non-trivial deviation from a zero-inflation target, given that this response is measured at a quarterly frequency, and is in response to a one percentage point decline in the recovery rate.

Figure 2.4: Optimal (Ramsey) Inflation Paths Across Models



The right hand panel of Figure 2.4 recomputes the same responses assuming that the central bank only maximises household welfare. A common assumption made in previous numerical literature on optimal policy, the results highlight that the extent of deviation from the strict inflation target is roughly halved. These results are more in line with the findings of Faia and Monacelli (2007), for example, and emphasise that the results are sensitive to the weights attached to households and borrowers (entrepreneurs) in the policymakers social welfare function. If policymakers, are concerned only with the welfare of households, and not entrepreneurs, then a strict inflation target becomes a better approximation of optimal monetary policy in an economy with a first-order distortionary credit friction.

Overall, these results confirm that credit frictions are potentially important for optimal policy both qualitatively and quantitatively when considering economies that have non-trivial credit distortions. Although inflation remains the primary objective of importance, the quantitative results in this chapter suggest that some deviation of inflation from target may be appropriate if credit frictions are thought to be having a highly distortionary effect on economic activity.

It is informative to compare these results with those obtained in previous work by Faia and Monacelli (2007). Faia and Monacelli consider numerical welfare comparisons of different monetary policy rules for a similar New Keynesian economy with a credit friction and productivity and government spending shocks. These authors find that a high weight on inflation in an interest-rate rule is socially optimal. One assumption that appears important in their findings is that the central bank is only concerned with the welfare of households. The findings here suggest that if the policymaker also takes into consideration the welfare of entrepreneurs (borrowers), motivated by the idea that policymakers abstract from redistribution, then a greater tolerance for non-zero inflation can be induced in response to financial shocks such as increases in the costs of bankruptcy.

It is also informative to compare these results with the findings of Vleighe (2010). In Vlieghe's analysis, the credit friction is assumed to affect the ability of low productivity agents to transfer resources to high productivity agents, a setup that builds on the work of Kiyotaki and Moore (1997). Interestingly, although Vlieghe models quite a different financial friction, he also finds that optimal monetary policy implies small, though not trivial, departures from inflation targeting when solving for optimal policy numerically.

2.5. Conclusion

There has been much debate in the literature regarding optimal monetary policy in response to fluctuations in asset prices and financial volatility. However, much of this debate has been based on a numerical approach, which has left the objectives of monetary policy somewhat unclear. This chapter aims to provide additional insight into this question by taking an analytical approach to the question of optimal monetary policy in a New Keynesian economy with endogenous capital and a credit friction.

The results highlight that policymakers can have an incentive to stabilise volatility in the net worth of borrowers to help mitigate time variation in the distortionary costs of a credit friction in the economy. This incentive exists in addition to the more traditional objectives of stabilising the composition of output and inflation. Interestingly, for the type of credit friction modelled here, credit spreads and asset prices are only variables of interest to policymakers to the extent that they influence the net worth of borrowers in the economy.

Quantitatively, I find that inflation targeting remains a good approximation of optimal monetary policy when the credit friction is not highly distortionary. However, for a more distortionary friction, such as that which can be observed during episodes of financial stress, these results imply that a more expansionary monetary policy with some tolerance of inflation is optimal.

Chapter 3

Optimal Expectations, Investment and Equity Prices

3.1. Introduction

> The sequence goes as follows. Some genuine improvement in economic conditions leads to more optimism. It may be a resource discovery (including the opening up of new productive land), or a technological change, or a rise in the terms of trade, or even just greater confidence in economic policy's capacity to solve problems. Human nature being what it is, people (or governments) are inclined to project into the future with undue confidence and insufficient assessment of risk. They often decide to invest more in ventures that are marginal, or even speculative, borrowing to do so. Because their assessment of permanent income is that it has increased, they also decide to consume more now (either privately or in the form of public services). Financial markets and institutions – which are populated by human beings after all – help them do both these by making capital available. Then, at some point, an event causes people suddenly to realise they have been too optimistic. Maybe the 'new paradigm' disappoints in some way or the terms of trade decline again. The cycle then goes into reverse, usually painfully.
>
> Glenn Stevens, 17 August 2010

The distortionary effects of equity price bubbles have received renewed attention in recent years. The recent global financial crisis, the US dot-com boom, and Japan's experience in the late 1980s, have all stimulated much discussion on the nature of equity price bubbles, and their policy implications. Although there has been a good deal of empirical work on the existence of bubbles, theoretical models of their origins and their welfare implications remains very much an open question. This chapter provides insight in this respect. It focuses on one potential source of a bubble – optimism or pessimism on the part of firms. In particular, I explore whether firm-led optimism, or pessimism, can explain general equilibrium movements in investment, equity prices, consumption, and output that are consistent with the typical equity price bubble.[30]

Being explicit about the source of an equity price bubble is important. Without a micro-founded theory of how bubbles can form, it is difficult to consider normative questions such as the optimal monetary policy response to a bubble or its distortionary effects. A micro-founded theory of bubbles is also useful for empirical work aimed at identifying the existence of equity price bubbles, and for comparing alternative theories that can be used to explain them.

I focus on firm-led optimism because there appears to be a common perception in the literature on bubbles that firm or shareholder optimism can be an important explanation in how they form. Since many observers often refer to optimism on the part of shareholders or firms, or some combination of both, modelling optimism on the production side of the economy is a natural starting point. Many of the results obtained in the simplest model considered in this chapter are shown to be robust to an alternative reduced-form model, where optimism or pessimism stems from shareholders (households).

The modelling device used to capture optimism or pessimism is derived from the work of Brunnermeier and Parker (2005), Gollier (2005) and Brunnermeier, Gollier and Parker (2007). These papers study the effects of optimism or pessimism on the part of households for consumption and portfolio allocation decisions in simple endowment

[30] In the context of this chapter I am usually referring to equity price bubbles unless otherwise specified. Nonetheless, the general methodology used can be applied to alternative assets such as housing.

economies. This chapter extends this work in an alternative direction, by focusing on optimism or pessimism on the production side, and studying how changes in technology can interact with optimism or pessimism to generate an equity price bubble. Specifically, I use optimal expectations on the part of managers of firms making physical investment decisions.[31]

The mechanism I study is whether a change to fundamentals, here the distribution of technology, can provide incentives for firm managers to become optimistic or pessimistic. It is found that this is indeed the case. Following a change in the distribution of technology, allowing for optimal expectations (OE) can amplify the effects of this change on equity prices and investment. Thus, from a rational expectations perspective, equity prices can appear overly sensitive to fundamental shocks, and appear overpriced (underpriced) in response to favourable (unfavourable) technology shocks. I also show that, in a very simple general equilibrium model, predictions for other macroeconomic quantities and prices, such as output and consumption, are qualitatively consistent with historical equity price bubble episodes, provided that households share the optimism or pessimism of firm managers.

The key assumptions delivering these results are that investment managers current felicity is a function of both current payoffs and anticipated future payoffs. And that managers are able to choose optimally the subjective beliefs that affect their anticipated payoffs. After exploring these assumptions in a very simple two-period two-state economy, I consider whether the qualitative predictions obtained still go through if optimism or pessimism in the economy originates from households rather than firms. I also confirm whether the results can be generalised to an economy with many states, and potentially with many periods.

[31] OE is a useful modelling device for optimism or pessimism because it models these beliefs as endogenous outcomes that are in the best interest of decision-makers. It also has the interpretation of a decision maker choosing an optimal prior for his or her beliefs, and then updating these beliefs over time in a Bayesian manner.

The next section provides a review of related literature that motivates the link between optimism, pessimism and equity price bubbles. Section 3 describes a very simple economic environment to demonstrate the key mechanism of interest – a two-period, two-state, general-equilibrium economy with production. Section 4 discusses the main insights. Section 5 confirms that the results generalise to endogenous optimism or pessimism on the part of households, and considers some simple welfare implications. Section 6 confirms that the same mechanism exists and can be studied in an economy with many periods and many states. Conclusions are drawn in the final section.

3.2. Related Literature and Stylised Observations

Optimism followed by subsequent pessimism has long been linked to equity price bubbles. In some of the earliest examples of equity price bubbles, including the the South Sea Bubble (1720) and the British Railway Mania (1840s), optimism has been proposed as an integral part of the upswing observed in equity prices during these episodes. More recently, bubbles such the US Roaring Twenties (1920s), the Japanese equity price bubble in the 1980s, and the US dot-com boom (1990s) have also linked optimism to significant increases in both equity prices and physical investment. In addition to the presence of optimism, a common observation in each of these episodes is the link between the bubble and some fundamental innovation in the economy. For example, the opening of new trade markets, the development of railroads, automobiles, radios, advances in computing, and the internet have all been proposed as fundamental shocks that are linked to equity price bubbles.[32]

Although qualitative descriptions of bubbles have emphasised the importance of optimism, and often subsequent pessimism, a model with a clear micro-foundation of why decision-makers would find it in their interest to be optimistic (or pessimistic) has proved more difficult to formalise. Nonetheless, research by Van den Steen (2004), Brunnermeier and Parker (2005), and Brunnermeier *et al* (2007), has demonstrated

[32] See for example Hong *et al* (2008) and the references cited therein.

economic environments in which optimism or pessimism can indeed be in the decision-makers' interest and consistent with maximising behaviour. These authors motivation for modelling optimism or pessimism is supported by findings in psychology and behavioural economics, which have identified situations in which optimism or pessimism appear to be important in economic decision-making.[33] From a macroeconomic perspective, whether optimism or pessimism is important is a question that should be addressed on both theoretical and empirical grounds. A useful property of the analysis that follows is that it is tractable and empirically testable.

There are two main strands of literature that address the idea that optimism or pessimism can affect firm investment decisions directly.[34] The first strand considers the possibility that managers can be overly optimistic about their firm's future prospects. This optimism can lead to a firm investing in more projects than it should, and reporting forecasts for its future activities and/or profits that are above an objective evaluation of the firm's prospects. Recent discussions of this idea can be found in Heaton (2002), and Baker *et al* (2007). Related empirical work has been undertaken by Polk and Sapienza (2009), Malmendier and Tate (2005), Chirinko and Schaller (2001), and Statman and Tybejee (1985).

A separate literature considers how shareholder optimism can be a potential source of a bubble, which can in turn affect investment decisions. Shareholders who are optimistic about the value of the firm can pay prices for shares that appear unjustified when compared with a rational or objective assessment of the firms' future discounted profit stream. Jensen (2005) has emphasised how shareholder optimism could affect firm-level investment decisions, and especially so in circumstances where liquidity constraints would otherwise prevent investment being undertaken. Empirical work that is related to shareholder optimism, and its ability to affect equity prices and investment, has been

[33] See Van den Steen (2004)and Barberis and Thaler (2003) amongst others for surveys of this literature.

[34] See for example Baker, Ruback and Wurgler (2007).

undertaken by Daniel, Hirshleifer and Teoh (2002), Eames, Glover and Kennedy (2002), Hong and Kubik (2003), and Gilchrist *et al* (2005) amongst others.

More generally, there is a broader literature linking behavioural incentives to outcomes in financial markets. For example, a substantial literature on equity mispricing has emphasised behavioural or psychological explanations for the existence of mispricing, including the role for optimism as one explanation. Recent surveys of this literature include, for example, Shiller (2000a), Shleifer (2000), Hirshleifer (2001), and Barberis and Thaler (2003). Much of this literature focuses on institutional, information or market constraints that prevent mispricing or a bubble from being eliminated, but does not provide a clear microfoundation as to the origin of a bubble. For this reason, the normative implications of bubbles, and how bubbles respond to changing fundamentals, have received less attention in this literature.

In terms of general equilibrium, micro-founded models of bubbles that can be used for welfare and policy analysis, this literature is still very new. Recent theoretical models of bubbles that are heading in this direction include Farhi and Tirole (2011), Martin and Ventura (2010) and Hong *et al* (2008). This chapter should be viewed as complementary to existing literature, with a different focus on the origin of a bubble.

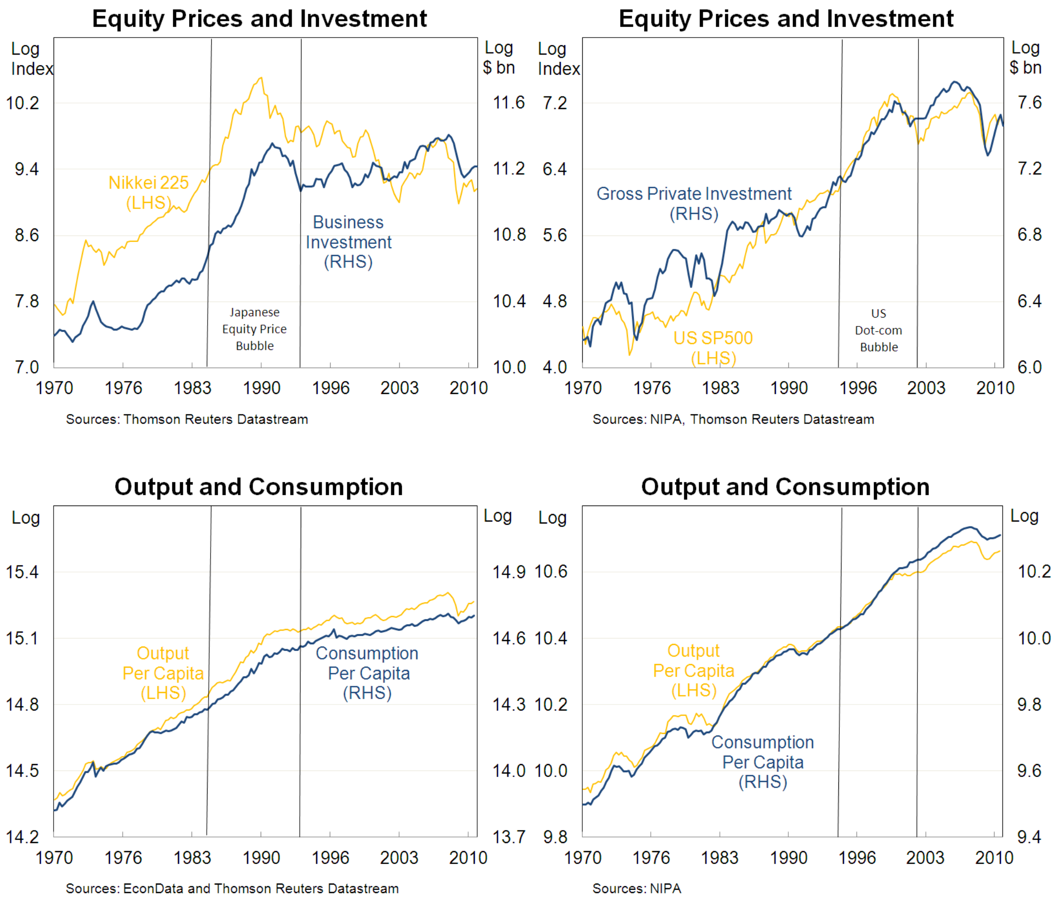
Stylised Observations

Figure 3.1 highlights some interesting time series observed during recent bubble episodes in the US and Japan. It highlights that during perceived bubbles there appears to be a sharp acceleration in equity prices and investment during the “boom” phase. This is matched with more rapid consumption and output growth. At the end of the boom, and during the subsequent downturn, equity prices, investment, consumption and output growth all appear to exhibit lower or even negative growth.

More generally, empirical literature has documented the following stylised facts concerning equity market bubbles:[35]

Figure 3.1: Bubble Episodes in Japan and the US

Japan US



1. Equity prices and investment growth appear to be more highly correlated correlated during bubble episodes (see for example Gilchrist *et al* 2005; Detken and Smets 2004; Helbling and Terrones 2003; Chirinko and Schaller 2001) ;

2. The sharp upswing in equity prices typically observed during a boom, followed by the subsequent sharp decline during a bust, is difficult to explain using either structural or atheoretical models of investment (Chirinko and Schaller 2001); and

[35] There is an extensive literature that has discussed optimism as an explanation for equity price bubbles and the above stylised facts. Some recent examples include Akerlof and Shiller (2009), Stevens (2010), Shiller (2000a), Kindleberger (2000), Camerer (1989) and the references cited therein.

3. Consumption and output growth appear to increase during a boom and decrease during a bust. Some papers find this effect to be statistically significant, while for others it remains an open question (Detken and Smets 2004).

I now consider, in a very simple economy, whether these stylised observations can be replicated qualitatively using endogenous firm-led optimism or pessimism, in response to changes in the distribution of technology.

3.3. Firm-led Optimal Expectations in a 2-Period, 2-State Model

The economy has two periods. Uncertainty is confined to the realisation of a common (aggregate) shock to technology in the second period, where technology can either be high (Z_H) or low (Z_L). There are three types of decision-makers in the economy – households, investment firms, and producers. There is a continuum of each type on the unit interval. Households consume and supply labour. Investment firms undertake physical investment on behalf of their shareholders (households). Perfectly competitive producers rent capital from investment firms and labour from households to produce output that can be either consumed or invested.

The non-standard feature of the economy I describe is in the formulation of beliefs. Rather than assuming that all agents are endowed with rational beliefs, knowing the true probability distribution over technology, I assume that only the managers of investment firms know the true distribution of technology. Furthermore, I assume that these managers optimise their beliefs using OE (see Brunnermeier and Parker 2005). For households, I assume that households do not know the probabilities with which alternative states of technology are realised, and that households accept (or use) the beliefs optimally chosen by firms when making their own consumption and labour supply decisions.[36] Thus, I consider an economy where the beliefs of investment firms and households are symmetric and firm determined. These assumptions are important for retaining tractability.

[36] An extension of this work would be to study the conditions under which households optimally accept firm beliefs. For example, this could be explored by extending the analysis used in Alonso and Matouschek (2008).

Uncertainty

It is useful to be precise about the nature of uncertainty. Uncertainty is confined to the realisation of one unknown, technology, in period two. The true or rational probability space is a triple $\{\Omega, \mathcal{F}, \mathbf{P}\}$, where Ω is the sample space, $\mathcal{F}$ is a σ-field, and $\mathbf{P} : \mathcal{F} \to [0,1]$ is the true or objective probability measure with which events occur.

To fix ideas, and keep the analysis intuitive, I assume that technology is a random variable $Z_2 : \Omega \to R$, that is discretely distributed with two possible states, low technology, Z_L, and high technology, Z_H and with distribution function

$$F(z) = \begin{cases} 0 \text{ if } z < Z_L \\ p_L \text{ if } Z_L \leq z < Z_H \\ 1 \text{ if } Z_H \leq z \end{cases}$$

$$\text{with: } Z_L < Z_H \text{ and } p_L \equiv 1 - p_H$$

Managers know that the low technology state occurs with probability p_L and the high technology state with probability p_H. Although firm managers know the true distribution of technology, I allow managers to use a set of subjective beliefs $\{\widehat{p}_L, \widehat{p}_H\}$ that are consistent with standard probability laws and are absolutely continuous with respect the rational probability measure $(\widehat{p}_s = 0 \text{ if } p_s = 0 \text{ for } s = L \text{ or } H)$. These subjective beliefs may or may not coincide with the objective probabilities known by managers.

Investment-Firm Managers

Allowing for endogenous beliefs, there are two decisions for the investment manager. The first is the choice of subjective beliefs that will be used to make the investment decision. The second is the optimal choice of investment, conditional on the subjective beliefs chosen. I first describe the mechanics underpinning these two decisions, and then the economic rationale for modelling decisions in this way.

To solve the investment managers' two decision problems, one can think of these decision problems as being solved sequentially. In the second-stage, managers choose their optimal level of investment taking their beliefs as given[37]

$$\max_{I_1} R_1 K_1 - I_1 - \psi \frac{I_1^2}{2} + \beta I_1 \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} R_s \widehat{p}_s \tag{3.1}$$

Notice that this problem is quite conventional for a two period model. Managers begin with some starting capital that they rent, $R_1 K_1$, and undertake investment subject to quadratic adjustment costs, $-I_1 - \psi \frac{I_1^2}{2}$, yielding a net first period payoff, $R_1 K_1 - I_1 - \psi \frac{I_1^2}{2}$. In the second period the return from investment is given by $\beta I_1 \frac{U'(C_s)}{U'(C_1)} R_s$, which includes an adjustment for the stochastic discount factor. The stochastic discount factor reflects shareholder (household) preferences over returns earned (dividends paid) in the second period.[38]

Taking subjective beliefs as given, the optimal investment choice is given by the condition

$$1 + \psi I_1 = \beta \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} R_s \widehat{p}_s \tag{3.2}$$

which equates the marginal costs of investment, with the subjective forecast of the expected return. Also notice that if $\widehat{p}_s = p_s$ for $s = L, H$, then the optimal choice of investment will coincide with that selected under rational expectations, and this investment program is standard.

As implied previously, however, managers get to choose their beliefs $\{\widehat{p}_s\}_{s=L,H}$ (the first-stage problem). Following Brunnermeier and Parker (2005), I assume that the manager

[37] To conserve on notation and keep arguments clear, I do not use an index to denote the position of the firm manager on $[0,1]$. Readers who prefer a more detailed exposition of the economy can refer to Appendix 7.2.

[38] This stochastic discount factor is consistent with households receiving the profits from investment firm decisions. Investment firm managers can be thought of as being risk neutral, and paid an arbitrarily small fraction of the dividends paid to households.

chooses their beliefs optimally, knowing that these beliefs will then be used to solve the investment program just described. That is, Equation (3.2) acts as a constraint on the managers optimal choice of beliefs. Concerning the incentives of the manager in his or her selection of beliefs, I assume that the manager chooses their optimal beliefs (optimal expectation) by solving the following program:

$$\max_{\widehat{p}_L, \widehat{p}_H} E\left(\pi_1 + \beta \pi_2\right)$$

subject to:

$$\pi_1 = \left(R_1 K_1 - I_1 - \psi \frac{I_1^2}{2}\right) + \beta I_1 \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} R_s \widehat{p}_s$$

$$\pi_2 = \varepsilon \left(R_1 K_1 - I_1 - \psi \frac{I_1^2}{2}\right) + I_1 \frac{U'(C_2)}{U'(C_1)} R_2$$

$$1 + \psi I_1 = \beta \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} R_s \widehat{p}_s$$

$$1 = \sum_{s=L,H} \widehat{p}_s$$

$$\widehat{p}_s \geq 0 \text{ for } s = L, H$$

This decision problem requires some unpacking. First note, the control variables here are the subjective beliefs $\{\widehat{p}_L, \widehat{p}_H\}$. The objective of the manager is captured in the payoff functions π_1 and π_2.[39] The payoff in period 1 is a function both of profits realised in period 1, $R_1 K_1 - I_1 - \psi \frac{I_1^2}{2}$, and anticipated profits, $\beta I_1 \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} R_s \widehat{p}_s$. Notice that anticipated profits depend on the subjective beliefs of the manager, and so this provides an incentive for a manager to tilt beliefs towards the state that has a higher discounted payoff. This is the key mechanism through which managers have the incentive to distort their beliefs. By attaching a higher subjective probability to the high payoff state, the manager in effect gains a direct utility benefit (enjoyed in period 1) from being optimistic about the future.

[39] Note that the rational probability measure is used to evaluate the objective. This makes sense given that managers know the objective probability space when choosing their optimal subjective beliefs. See Brunnermeier and Parker (2005) for further discussion.

Period 2 payoffs are a function of profits actually realised in period two, $I_1 R_2 \frac{U'(C_2)}{U'(C_1)}$, and the profits previously earned in period one $\varepsilon \left(R_1 K_1 - I_1^* - \psi \frac{I_1^{*2}}{2} \right)$. This latter effect of memory, or persistence in the managers' payoff function, acts a natural counter-balance to the incentive for a manager to become optimistic.[40] By caring about past payoffs, the manager will reduce their welfare if they are too optimistic since over-investment will occur and period 1 profits will be lower than is optimally desired.

Finally, one must unpack the constraints. The first constraint, $1 + \psi I_1 = \beta \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} R_s \widehat{p}_s$, highlights that the manager internalises the effect of their choice of beliefs on their investment decision. That is, a manager knows that once his or her beliefs are chosen they must be used in solving the second-stage investment program (3.1). The final two constraints, $1 = \sum_{s=L,H} \widehat{p}_s$ and $\widehat{p}_s \geq 0$ for $s = L, H$ ensure that the subjective beliefs chosen are standard probabilities.

The first-order conditions for the investment firm at an interior solution are [41]

$$1 + \psi I_1 = \beta \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} R_s \widehat{p}_s \tag{3.3}$$

$$1 = \sum_{s=L,H} \widehat{p}_s \tag{3.4}$$

$$\beta \varepsilon (1 + \psi I_1) = \beta \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} R_s p_s + \psi I_1 \tag{3.5}$$

Notwithstanding the introduction of anticipatory and memory welfare, and the ability to optimise on beliefs, the effects on the optimal investment decision are quite intuitive. Comparing (3.5) with the more standard investment decision that holds under rational expectations,

$$1 + \psi I_1 = \beta \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} R_s p_s \tag{3.6}$$

[40] Memory can be thought of as being analogous to habits in a consumption context.

[41] It is straightforward to verify that the above first-order conditions are sufficient when $\varepsilon > \beta^{-1}$.

there are two effects from optimising on beliefs. On the one hand, investment-firm managers obtain an additional perceived return from investing, captured by the term ψI_1. Intuitively, this is the effect of anticipated profits providing managers with the incentive to be optimistic and invest more because it raises their anticipation of future profits earned. On the other hand, managers also care about actual payoffs and their memory welfare. Optimism is constrained by the fact that higher investment reduces actual profits earned in period one, which also has a persistent effect on felicity that is carried forward into period two. With these costs of distorted decision-making, it can be observed that the marginal cost of investment is in effect scaled by the weight on memory welfare, $\beta\varepsilon$.

The key assumptions in the above analysis are that managers can choose their beliefs, and that beliefs enter directly into the objective function of a manager. One can think of the above problem as either a reduced form for capturing incentives for a manager to be optimistic or pessimistic about the future, or more literally as a model of optimal self-deception. Another interpretation, that becomes clearer in a model with many periods (as discussed further in Section 3.6), is that one can think of the manager as selecting an optimal prior before updating in a Bayesian manner.

The assumption that managers' payoffs depend on anticipated future profit is an important component of this framework. One justification is introspection. It is natural to think that people do, at least to some extent, care today about what might happen to them in the future. In particular, decision-makers are likely to have higher welfare if they think the future is going to promising (the investment manager believes future profits are likely to be higher), and lower welfare if the future is likely to be unfavourable (future profits are likely to be low). Brunnermeier and Parker (2005) and Van den Steen (2004) and the references cited therein document psychological and behavioural evidence to suggest that a person's current felicity is affected by what they anticipate is going to happen to them in the future.

There is also empirical evidence to suggest that such preferences may be influencing economic decisions. Recent examples include Landier and Thesmar (2009), Ben-David, Graham and Harvey (2007; 2010), and Puri and Robinson (2007). From a macroeconomic perspective, this preference assumption is one modelling device for incorporating a framework in which agents find it in their best interests to use optimistic, or potentially pessimistic, expectations. The appeal of using this device lie in its clear microfoundations, tractability, and that it can be empirically tested.

Households

I assume a continuum of households on the unit interval. To keep the effects of firm optimism (or pessimism) as transparent as possible, I assume that the continuum of households share the beliefs optimally chosen by investment firms. That is, the continuum of households beliefs are such that $\left\{\widehat{p}_s^h(i) = \widehat{p}_s(i)\right\}_{s=L,H}$ for all $i \in [0,1]$, where $\left\{\widehat{p}_s^h(i)\right\}_{s=L,H}$ are the subjective beliefs used by household type i, and $\{\widehat{p}_s(i)\}_{s=L,H}$ are the subjective beliefs chosen by investment-firm type i. One can think of this as a model where at time $t = 0$ (prior to the opening of financial and goods markets), there is a continuum of investment-firm types reporting their optimal beliefs, and that there is a matching continuum of household types each willing to accept the beliefs reported by the investment-firm manager that matches their type.[42]

After beliefs are matched, households consume, choose the shares of the investment firms that they wish to purchase, and a portfolio of Arrow-Debreu securities in period 1. After the resolution of uncertainty in period 2, households again choose consumption and their desired labour supply (households have no incentive to re-trade in financial markets). The existence of Arrow-Debreu securities ensures that households are able to completely

[42] Although not formally considered here, there are approaches, such as that proposed by Alonso and Matouschek (2008), that can be used to verify the conditions under which households (the principal) will be willing to delegate the investment decision to a more informed firm manager (the agent), even when the manager is potentially biased.

insure with each other against uncertainty, but households still require investment firms to undertake investment in physical capital on their behalf.[43]

I first focus on the intra-temporal allocation made in each period. Each household[44] consumes a basket of goods solving in each period and state $t \in 1, L, H$

$$\min_{C_t^i} \int_0^1 P_t^i C_t^i di$$

subject to

$$C_t \geq \left(\int_0^1 \left(C_t^i \right)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}$$

taking C_t as given, where P_t^i is the price of good i, C_t is a Dixit-Stiglitz basket, C_t^i is a households' consumption of good type i, θ is a common elasticity of substitution across different types of goods and where $i \in [0,1]$. The optimal consumption demand for good i can be express as

$$C_t^i = \left(\frac{P_t^i}{P_t} \right)^{-\theta} C_t$$

where P_t is a Dixit-Stiglitz price index that measure the shadow price of consuming the basket of goods C_t,

$$P_t \equiv \left(\int_0^1 \left(P_t^i \right)^{1-\theta} di \right)^{\frac{1}{1-\theta}}$$

---

[43] Since households cannot purchase investment goods directly, markets are incomplete. This will not affect outcomes in the symmetric equilibrium I focus on provided households use the same probability measure as that optimally chosen by investment firms.

[44] To conserve on notation I do not identify the position of the household on $[0,1]$. See Appendix 7.2 for a more detailed representation of this problem.

As well as making intra-temporal consumption allocations in each period, households also solve the following inter-temporal optimisation problem

$$\max_{C_1, C_s, N_s, \theta_i, a_s} U(C_1) + \beta \sum_{s=L,H} (U(C_s) - V(N_s))\, \widehat{p}_s$$

subject to:

$$\begin{gathered}
P_1 C_1 + P_1 \sum_{s=L,H} \omega_s a_s + P_1 \int_0^1 \left(q_i - D_{1,i}\right) \theta_i di \\
\leq P_1 W_1 N_1 + P_1 \int_0^1 q_i \phi_i di + P_1 D_1^P \\
P_s C_s \leq P_s a_s + P_s W_s N_s + P_s D_s^P + P_s \int_0^1 \theta_i D_{s,i} di \quad \text{for } s = L, H \\
\theta_i \geq 0
\end{gathered} \tag{3.7}$$

where ω_s is the price of an Arrow-Debreu security that delivers one unit of consumption in state s, W is the real wage, D^P is the sum of all real dividends paid by production firms,[45] ϕ_i, D_i, θ_i and q_i are the initial endowment, real dividend paid, equity share purchased and price of equity, all with respect to investment firm i, a_s is the quantity of Arrow-Debreu securities purchased with regard to state s, and N is labour supply. For simplicity, I assume that the first period labour supply and real wages are fixed so that $W_1 N_1$ is exogenous from the household's perspective. A redundant real risk-free equity can be priced using

$$\sum_{s=L,H} \omega_s = \left(1 + r_f\right)^{-1}$$

where r_f is the risk-free rate of return.

[45] For simplicity, I abstract from the ownership of production firms. This has no material effect on any of the analysis that follows.

The interior Euler, labour supply and equity pricing conditions for an individual household are given by[46]

$$U'(C_1) = \sum_{s=L,H} U'(C_s)\left(1+r_f\right)\widehat{p}_s \tag{3.8}$$

$$W_s = \frac{V'(N_s)}{U'(C_s)} \text{ for } s = L, H \tag{3.9}$$

$$q_1 = D_1 + \beta \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} D_s \widehat{p}_s \tag{3.10}$$

Equation (3.10) verifies that the stochastic discount factor used by investment firms, as previously assumed, is consistent with these firms maximising shareholder value in equilibrium.

Producers

I assume perfect competition in the supply of each type of good on the interval $[0,1]$. Producer i has access to a Cobb-Douglas production technology

$$Y_t^i = Z_t \left(K_t^i\right)^{\alpha} \left({N_t}^i\right)^{1-\alpha} \text{ for } t = 1, L, H \tag{3.11}$$

for $i \in [0,1]$, $\alpha \in (0,1)$ and where the supply of labour and capital (and technology) in the first period are fixed exogenously for each good type. It should be noted that Z_t reflects technology that is common to all firms, and thus I abstract from idiosyncratic technology shocks. With perfect competition (zero profits) capital and labour supplied in the production of each good type receive their marginal products

$$R_t^i = \alpha \frac{Y_t^i}{K_t} \text{ for } t = 1, L, H \tag{3.12}$$

$$W_t^i = (1-\alpha) \frac{Y_t^i}{N_t^i} \text{ for } t = 1, L, H \tag{3.13}$$

46 These are sufficient under standard concavity assumptions.

where I assume R_1^i, W_1^i,K_1^i and N_1^i are all exogenous.[47] Furthermore, with perfectly competitive supply in each good type on $[0,1]$ prices reflect nominal marginal cost

$$P_t^i = P_t V_t^i \ \text{ for } t = 1, L, H \tag{3.14}$$

and real marginal costs are given by

$$V_t^i = Z_t^{-1} \left(\frac{R_t^i}{\alpha} \right)^{\alpha} \left(\frac{W_t^i}{1-\alpha} \right)^{1-\alpha} \quad \text{for } t = 1, L, H \tag{3.15}$$

Market Clearing

The market clearing conditions and law of motion for capital for each type of good are given by:

$$Y_1^i = C_1^i + I_1^i + \frac{\psi}{2} \left(I_1^i \right)^2 \tag{3.16}$$

$$Y_s^i = C_s^i \ \text{ for } s = L, H \tag{3.17}$$

$$K_2^i = I_1^i \tag{3.18}$$

where it is straightforward to show that firms will never optimally invest in the second period $\left(I_s^i = 0 \right)$ since the economy terminates at the end of this period.

Equilibrium Concepts

I focus on an equilibrium where all firms are identical, all households are identical, and all households share the same set of beliefs chosen by firms $\{\widehat{p}_s\}_{s=L,H}$.

Definition 3.1. *A symmetric interior OE equilibrium is defined as vector of prices* $\left\{ r_f^{OE}, R_t^{OE}, W_t^{OE}, q_1^{OE}, P_t^{OE}, V_t^{OE} \right\}_{t=1,L,H}$ *and quantities* $\left\{ K_2^{OE}, I_1^{OE}, C_1^{OE}, C_s^{OE}, Y_s^{OE}, N_s^{OE} \right\}$

[47] This assumption simplifies the comparative static analysis by not allowing for an endogenous choice of first period labour supply. Relaxing this assumption did not affect the qualitative results obtained when studying the problem numerically.

for $s = L, H$ and optimal subjective probabilities $\{\widehat{p}_L, \widehat{p}_H\}$ that satisfy (3.3) to (3.5), and (3.8) to (3.18) taking $\{Y_1, R_1, K_1, W_1, N_1, Z_L, Z_H, \varepsilon, p_L, p_H\}$ as given.[48]

To understand how introducing optimism or pessimism on the part of investment firms affects the economy, I compare the economy just described with a standard rational expectations (RE) economy, where investment firms and households use the true probability measure assigned to technology rather than the distorted measure optimally chosen by managers. Specifically, under RE the investment managers' problem (3.1) is replaced by

$$\max_{I_1} \left(R_1 K_1 - I_1 - \psi \frac{I_1^2}{2} \right) + \beta I_1 \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} R_s p_s$$

yielding the more standard Euler equation (3.6). And, the household objective in an RE economy (3.7) is now replaced by

$$U(C_1) + \beta \sum_{s=L,H} \left(U(C_s) - V(N_s)\right) p_s$$

yielding the rational bond and equity pricing conditions[49]

$$U'(C_1) = \sum_{s=L,H} U'(C_s) \left(1 + r_f\right) p_s \tag{3.19}$$

$$q_1 = D_1 + \beta \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} D_s p_s \tag{3.20}$$

Definition 3.2. *A symmetric RE equilibrium is a vector of exogenous beliefs $\{p_L, p_H\}$, endogenous prices $\left\{r_f^{RE}, R_t^{RE}, W_t^{RE}, q_1^{RE}, P_t^{RE}, V_t^{RE}\right\}_{t=1,L,H}$ and endogenous quantities $\left\{K_2^{RE}, I_1^{RE}, C_1^{RE}, C_s^{RE}, Y_s^{RE}, N_s^{RE}\right\}_{s=L,H}$ that satisfy (3.6), (3.9), and (3.11) to (3.20) taking $\{Y_1, R_1, K_1, W_1, N_1, Z_L, Z_H, p_L, p_H\}$ as given.*

48 Interior here also refers to an equilibrium where $1 > \widehat{p}_s > 0$ for $s = L, H$.

49 Note the intratemporal conditions remain the same as those previously discussed.

3.4. Comparative Static Analysis

I now compare the OE and RE economies. I initially assume that, for a given true distribution of technology, OE firms optimally choose rational beliefs and so the OE and RE economies are initially identical in terms of beliefs, prices and quantities.[50] I then study how each economy changes in response to a small change in the objective or true distribution of technology. Specifically, I consider how beliefs, equity prices, investment, output, and consumption growth all change in the OE economy, and in the RE economy, when there is a small perturbation either to the supports of the distribution of technology (Z_L or Z_H) or the probability with which the high state occurs, p_H. Notice that these perturbations all raise the objective mean of technology in the second period, and can be considered as favourable changes to its distribution.

To obtain clear qualitative predictions, I assume that households have a power (or constant relative risk aversion) utility function for consumption, and a power utility function for the disutility of labour

$$\begin{aligned} U(C) &= \frac{C^{1-\sigma}-1}{1-\sigma} \\ V(H) &= \frac{H^{1+\eta}}{1+\eta} \end{aligned} \tag{3.21}$$

I further assume that risk aversion is low ($\sigma < 1$, where σ is the coefficient of relative risk aversion), which ensures that investment increases in response to a technology shock that increases productivity if the high state is realised, under either OE or RE $\left(\frac{\partial I_1^{OE}}{\partial Z_H} > 0, \frac{\partial I_1^{RE}}{\partial Z_H} > 0\right)$, but otherwise does not affect the main qualitative results obtained. In particular, with high risk aversion, $\sigma > 1$, the effects of a shock to the distribution of technology work in the same direction in terms of the effects on optimal subjective probabilities, but reverse the sign of the effects on equilibrium prices and quantities such as investment and equity prices.

[50] The existence of an OE equilibrium that is identical to a given RE equilibrium is verified in Appendix 7.3, Proposition 7.1).

To conserve notation, it should be noted that all of the partial derivatives that follow are evaluated around the point at which optimal and rational expectations initially coincide, and so I use the notation $\frac{\partial y}{\partial x} \equiv \frac{\partial y}{\partial x}\big|_{\varepsilon=\varepsilon^{RE}}$. Notice that ε^{RE} is the value for the memory parameter which ensures that the OE and RE economies are intially identical (see Appendix 7.3, Proposition 7.1), y is the response variable of interest (computed in either the OE or RE economy) and x is the parameter of the technology distribution that is being perturbed (either Z_L, Z_H or p_H).

The first thought experiment considered is whether a positive innovation to the supports of the distribution of technology – that is, to either Z_L or Z_H – leads to higher investment and equity prices under OE than under RE. Intuitively, I examine whether a change in the true distribution of technology can, endogenously, induce beliefs that support over-investment and overvaluation, when compared with a rational expectations economy.

Proposition 3.1 confirms that, starting from a position where OE and RE initially coincide, as the true distribution for technology becomes more favourable (either of the supports are increased), it is in the interest of managers to become optimistic and assign a probability to the high state that is above the objective probability p_H, and a probability to the low state that is below the objective probability p_L. These beliefs induce the manager to over-invest, which in turn leads to equity prices that are above their rational expectations value. It can also be shown that these effects are strongest if the perturbation to technology is through an increase in Z_H, rather than Z_L, provided that the high state realisation for technology is sufficiently likely $\frac{p_H}{p_L} \geq \left(\frac{Z_L}{Z_H}\right)^{\gamma_3 - 1}$, where γ_3 is a positive constant defined in Appendix 7.3.

Proposition 3.1. *Consider (separately) small perturbations to the realisation of technology in each state, Z_L, Z_H, around the equilibrium point at which optimal and rational expectations coincide. It follows that beliefs are optimistic, and investment and equity prices are higher under OE than RE following each of these perturbations. Furthermore, these responses are largest for a perturbation to the realisation of* *technology in the high state – provided that the likelihood of this state is sufficiently large. That is,*

$$\frac{\partial \widehat{p}_H}{\partial x} > 0; \quad \frac{\partial \widehat{p}_L}{\partial x} < 0$$
$$\frac{\partial I_1^{OE}}{\partial x} > \frac{\partial I_1^{RE}}{\partial x} > 0; \quad \frac{\partial q_1^{OE}}{\partial x} > \frac{\partial q_1^{RE}}{\partial x} > 0$$
$$for\, x \in Z_L, Z_H$$

$$and$$

$$\frac{\partial y}{\partial Z_H} \geq \frac{\partial y}{\partial Z_L} \quad for \quad y \in \widehat{p}_H, I_1^{OE}, q_1^{OE}$$
$$if \quad \frac{p_H}{p_L} \geq \left(\frac{Z_L}{Z_H}\right)^{\gamma_{3-1}}$$

Proposition 3.2 confirms that similar results are obtained if the perturbation is to the probability with which the high-state occurs, p_H. Thus, regardless of the source of the perturbation, that is whether the magnitude of realised technology changes across states or whether the objective (true) probability attached to states change such that the high technology state becomes more likely, both can give rise to the incentive for investment-firm managers to optimally distort their expectations and investment decision.

Proposition 3.2. *Consider a perturbation to the true (or rational) probability with which the high state is realised, p_H (with $dp_H = -dp_L$). Firms assign a probability to the high state that is above the true probability, and a probability to the low state that is below the true probability. Investment and equity prices are higher under OE than RE, following this perturbation.*

$$\frac{\partial \widehat{p}_H}{\partial p_H} > 1; \quad \frac{\partial \widehat{p}_L}{\partial p_H} < -1$$
$$\frac{\partial y_1^{OE}}{\partial p_H} > \frac{\partial y_1^{RE}}{\partial p_H} > 0$$
$$for \quad y_1 \in I_1, q_1$$

The next result establishes that output, labour supply and consumption growth are all higher in period two following a perturbation to Z_H, when comparing OE and RE. Similar results are obtained when considering perturbations to either low-state productivity Z_L, or the probability with which the high-state occurs $p_{H.}$

Proposition 3.3. *Output and labour effort in period two, and consumption growth, increase by more under OE than RE in response to a perturbation to* Z_H, Z_L *or* p_H*.*

$$\begin{gathered}
\frac{\partial Y_s^{OE}}{\partial x} > \frac{\partial Y_s^{RE}}{\partial x} > 0 \\
\frac{\partial N_s^{OE}}{\partial x} > \frac{\partial N_s^{RE}}{\partial x} > 0 \\
\frac{\partial \ln \frac{C_s^{OE}}{C_1^{OE}}}{\partial x} > \frac{\partial \ln \frac{C_s^{RE}}{C_1^{RE}}}{\partial x} > 0 \\
for \quad x \in \{Z_L, Z_H, p_H\}
\end{gathered}$$

These results establish that in a very simple general equilibrium model, a favourable change in the objective distribution of technology can induce endogenous optimism on the part of firms, which if also transmitted to households, can lead to over-investment, overvalued equity prices, and higher output when compared with their values determined under rational expectations.

The key mechanism is that for investment managers, whose utility depend on their beliefs, a more favourable objective distribution can increase the benefits from being optimistic, and so agents are willing at the margin to tilt their subjective probabilities towards the more favourable state. Managers do not, however, become excessively optimistic, for example assign probability one to the more favourable state. This is because the incentive to distort beliefs is constrained by the fact that managers are aware that distorted beliefs induce distorted decisions, and this can be costly. It should be noted that this result is not immediate in general equilibrium, since changes in rental returns and the decisions made by consumers also affect the optimal beliefs chosen by the manager.

This mechanism could be one important explanation of an equity price bubble. In qualitative literature on bubbles, it is common for a bubble to be preceded by a positive shock to expected fundamentals, including the development of new technologies such as the railroad, radio, computing and the internet. The mechanism also offers insight into the bust phase after a bubble as well. If, over time, it becomes evident that the objective distribution of outcomes is not as favourable as first thought, then firms who update their beliefs optimally could become pessimistic, reducing investment and the price of their equity. [51]

The previous results consider how beliefs respond endogenously to a change in the objective distribution of technology, and how the change in beliefs in turn influences equity prices, investment and output. I now consider the effects of an exogenous change in preferences that affects beliefs. In particular, I consider how a perturbation to the weight on memory utility, ε, affects equity prices and investment. Proposition (3.4) establishes that an increase in ε induces under-investment and undervalued equity prices. Thus, if managers are sufficiently concerned with the past, they can have an incentive to be pessimistic in this model.

Proposition 3.4. *Agents become pessimistic (over-optimistic) when the weight attached to memory utility increases (decreases)*

$$\frac{\partial I_1^{OE}}{\partial \varepsilon} < 0; \quad \frac{\partial q_1^{OE}}{\partial \varepsilon} < 0$$
$$\frac{\partial \widehat{p}_H^{OE}}{\partial \varepsilon} < 0; \quad \frac{\partial \widehat{p}_L^{OE}}{\partial \varepsilon} > 0$$

3.5. Robustness and Welfare

[51] The previous comparative static results of course reverse when considering negative perturbations to the distribution of technology. With firm-led optimal expectations, reductions in the forecast for technology (a decrease in Z_L, Z_H or p_H) can lead to under investment in capital and under-valuation in equity markets when compared with an economy that has rational expectations.

Household Determined Optimism or Pessimism

An appealing feature of the benchmark model is that the previous results are robust if optimism or pessimism originates from households (in reduced form), rather than from firms. Again, for simplicity, I assume that beliefs across households and firms are symmetric. Proposition (3.5) establishes the main result.

Proposition 3.5. *Re-consider the OE economy described in Definition 3.1. Suppose now that:*

1. *Households optimally choose beliefs rather than investment firms;*
2. *Investment firms maximise shareholder value;*
3. *In the absence of a perturbation to the distribution of technology, households use a rational probability measure; and*
4. *Households become optimistic (pessimistic) in response to favourable (unfavourable) perturbations to the distribution of technology. That is, I assume the reduced-form relationship*

$$\frac{\partial \hat{p}_H^h}{\partial x} > 0 \quad \textit{for} \quad x \in Z_L, Z_H, p_H$$

In such an economy, where beliefs are symmetric and determined by households, equity prices, investment, output, labour supply and consumption growth all increase by more in this reduced-form OE economy than in an RE economy in response to a favourable change in the distribution of technology.

In view of this, the modelling device of firm optimism has the advantage that in a symmetric equilibrium, the qualitative results obtained are isomorphic to an alternative model in which households become optimistic or pessimistic in response to a shock to the distribution of technology, and where this optimism or pessimism is transmitted to investment firms. Thus, irrespective of whether optimism or pessimism arises from the managers of firms, whose beliefs influence those of shareholders, or from the beliefs of shareholders that influence the investment decisions of the firm, the effects on equity prices, investment, consumption growth, and output are qualitatively the same for equilibria with symmetric beliefs.

A model with N-States

I now consider whether the results in the benchmark model, with symmetric beliefs that are firm determined, generalise to a model where there are N possible realisations of productivity in period 2 under the rational (true) probability measure. That is, the model is generalised such that the distribution of technology is given by

$$F(z) = \begin{cases} 0 \text{ if } z < Z_1 \\ \sum_{s=1}^{j-1} p_s \text{ if } Z_{j-1} \leq z < Z_j \text{ for } j = 2, ..., n \\ 1 \text{ if } Z_n \leq z \end{cases}$$

where $\sum_{s=1}^{N} p_s = 1$ and without loss of generality I order the possible realisations of technology such that $Z_1 < Z_2 < ... < Z_N$. Apart from allowing more possible states of nature, the decision-making problems for households and firms are the same as those previously discussed, with the exception that expectations terms, both subjective and objective, are now probability weighted sums over $s = 1, 2, ..., N$ rather than $s = L, H$ (and to avoid any confusion in notation, I redefine consumption in the first period as $\widetilde{C}_1$ which is different to consumption in the second period, when state 1 is realised, now C_1). Definitions of OE and RE equilibriums generalise in a natural way with N states and are reported in Appendix 7.5, as is the confirmation of the existence of these equilibria.

Proposition 3.6 establishes that all of the previous results, and much of the intuition, goes through in a model with N states. In this case, I consider an increase in the realisation of technology in any one given state, and (separately) a marginal shift in the mass of objective probability from a low technology state to some higher technology state.

Consistent with the previous findings, the results highlight that firms do become optimistic when faced with a more favourable objective distribution of technology. Firms under OE choose a unique subjective forecast for discounted investment returns that exceeds the forecast mean that would be used under rational expectations. Again, assuming risk aversion is low ($\sigma < 1$), over-investment and overvaluation in equity markets occur relative to the corresponding RE equilibrium.

Proposition 3.6.

(a) A positive perturbation to realised productivity in one state leads to over-investment and overvaluation in equity markets (assuming risk aversion is low), and an overly optimistic forecast of the return to investment.

$$\frac{\partial I_1^{OE}}{\partial Z_n} > \frac{\partial I_1^{RE}}{\partial Z_n} > 0$$
$$\frac{\partial q_1^{OE}}{\partial Z_n} > \frac{\partial q_1^{RE}}{\partial Z_n} > 0$$
$$\frac{\partial \widehat{E}\left(\frac{U'\left(C_2^{OE}\right)}{U'\left(\widetilde{C}_1^{OE}\right)} R_2^{OE}\right)}{\partial Z_n} > \frac{\partial E\left(\frac{U'\left(C_2^{RE}\right)}{U'\left(\widetilde{C}_1^{RE}\right)} R_2^{RE}\right)}{\partial Z_n} > 0$$
$$\textit{if } \sigma < 1 \textit{ for } n \in \{1,...,N\}$$

(b) Consider a marginal shift in probability mass from some low productivity state (k) to a higher productivity state (j) (i.e. $dp_j = -dp_k$ where $j > k$ and $j,k \in \{1,...,N\}$ and fixing $dp_n = 0$ for all $n \neq j,k$). Investment and equity prices increase beyond their fundamental

values, and again forecast investment returns are overly optimistic.

$$\frac{\partial I_1^{OE}}{\partial p_j} > \frac{\partial I_1^{RE}}{\partial p_j} > 0$$
$$\frac{\partial q_1^{OE}}{\partial p_j} > \frac{\partial q_1^{RE}}{\partial p_j} > 0$$
$$\frac{\partial \widehat{E}\left(\frac{U'\left(C_2^{OE}\right)}{U'\left(\widetilde{C}_1^{OE}\right)} R_2^{OE}\right)}{\partial p_j} > \frac{\partial E\left(\frac{U'\left(C_2^{RE}\right)}{U'\left(\widetilde{C}_1^{RE}\right)} R_2^{RE}\right)}{\partial p_j} > 0$$
$$\textit{if}\ \sigma < 1$$

Welfare Implications

I now study some simple welfare implications associated with the OE economy. Consistent with the assumption that investment-firm managers are risk neutral and paid an arbitrarily small fraction of discounted firm profits, I assume that the social planner is only concerned with maximising the welfare of households. I also assume that the social planner does not optimise on beliefs, but takes their own beliefs as given. Given these assumptions, the social planner chooses a vector of quantities that solves (again assuming N states)

$$\max_{I_1,\widetilde{C}_1,C_s,N_s} U\left(\widetilde{C}_1\right) + \beta \sum_{s=1}^{N} \left(U\left(C_s\right) - V\left(N_s\right)\right) \widehat{p}_s^{SP}$$
$$\text{subject to:}$$
$$Y_1 \geq \widetilde{C}_1 + I_1 + \psi \frac{I_1^2}{2}$$
$$C_s \leq Z_s I_1^{\alpha} N_s^{1-\alpha}$$
$$\text{given:} \quad Y_1, Z_s, \widehat{p}_s^{SP} \tag{3.22}$$

where $\left\{\widehat{p}_s^{SP}\right\}_{s=1}^{N}$ is the exogenous probability measure held by the social planner.[52]

[52] To keep the analysis simple, I also assume that the social planners beliefs are absolutely continuous with respect to the objective probability measure $\left(\widehat{\mathbf{P}}^{SP} \prec \mathbf{P}\right)$.

Since households do not gain directly from optimism or pessimism, it follows naturally that a social planner strictly prefers equilibria where investment firms' beliefs coincide with those of the planner (Proposition 3.7).

Proposition 3.7. *A social planner using probability measure* $\widehat{\mathbf{P}}^{SP} \prec \mathbf{P}$, *prefers investment firms that use **any** probability measure* $\widehat{\mathbf{P}} \prec \mathbf{P}$ *that satisfies*

$$\sum_{s=1}^{N}\left(\frac{U'\left(C_s^{SP}\right)}{U'\left(\widetilde{C}_1^{SP}\right)}R_s^{SP}\right)\widehat{p}_s^{SP} = \sum_{s=1}^{N}\left(\frac{U'\left(C_s^{SP}\right)}{U'\left(\widetilde{C}_1^{SP}\right)}R_s^{SP}\right)\widehat{p}_s$$

where $C_s^{SP}, R_s^{SP} = \alpha\frac{C_s^{SP}}{I_1^{SP}}, \widetilde{C}_1^{SP}$ *are obtained from the solutions to (3.22).*

Two remarks are worth noting on this point. The first is that a social planner that shares the probability measure optimally chosen by investment firms, whether distorted or not, will have no incentive to change the decentralised equilibrium allocation of the OE economy. Conditional on beliefs, all agents are acting in their own best interests. The second is that a social planner that knows the true distribution of technology, and uses this probability measure in evaluating household welfare, will strictly prefer investment firms that have beliefs that are consistent with a rational forecast (expectation) of the discounted investment return. Indeed, any equilibria where firms do not use a rational forecast of the discounted investment return is in fact sub-optimal, from the perspective of households.[53] Although it is in the interest of firm managers to be optimistic or pessimistic, this is not in the direct interest of households.

The policy implications of this model are quite strong. A social planner that believes its own forecast for investment returns are closer to that of the objective or true distribution of technology, will have an incentive, all else constant, to use policy to correct any distortion in investment and consumption decisions due to firm-led optimism or pessimism.

[53] Expected household welfare here refers to the measure $U\left(\widetilde{C}_1\right) + \beta\sum_{s=1}^{N}\left(U\left(C_s\right) - V\left(N_s\right)\right)p_s$.

3.6. Extending the Model Over Time

I now consider how the investment-firm problem can be extended over time, and integrated in a dynamic stochastic general equilibrium model. Again, I define a rational probability space $\{\Omega, \mathcal{F}, \mathbf{P}\}$, that is known by the investment manager, and subjective probability space $\left\{\Omega, \mathcal{F}, \widehat{\mathbf{P}}\right\}$ on which the manager optimises under the restriction $\widehat{\mathbf{P}} \prec \mathbf{P}$. The manager with an infinite horizon chooses an optimal subjective probability measure $\widehat{\mathbf{P}}$, and an investment sequence $\{I_t\}$ solving

$$\begin{aligned}
\max_{\widehat{\mathbf{P}}, \{I_t, K_{t+1}\}_{t=0}^{\infty}} & E\sum_{t=0}^{\infty}\beta^t\left(\sum_{\tau=1}^{t}\varepsilon^{\tau}\phi_{t-\tau}\left(R_{t-\tau}K_{t-\tau}-I_{t-\tau}-\Psi\left(\frac{I_{t-\tau}}{K_{t-\tau}}\right)K_{\tau-\tau}\right)\right)\\
& +E\sum_{t=0}^{\infty}\beta^t\phi_t\left(R_tK_t-I_t-\Psi\left(\frac{I_t}{K_t}\right)K_t\right)\\
+E\sum_{t=0}^{\infty}\beta^t & \left(\sum_{\tau=1}^{\infty}\beta^{\tau}\widehat{E}_t\phi_{t+\tau}\left(R_{t+\tau}K_{t+\tau}-I_{t+\tau}-\Psi\left(\frac{I_{t+\tau}}{K_{t+\tau}}\right)K_{t+\tau}\right)\right) \qquad (3.23)
\end{aligned}$$

subject to:

$$\begin{aligned}
& \widehat{\mathbf{P}} \text{ is a well-defined probability space with } \widehat{\mathbf{P}} \prec \mathbf{P}\\
I_t, K_{t+1} \in \arg & \max_{\widetilde{I}_t, \widetilde{K}_{t+1}:\widetilde{K}_{t+1}=(1-\delta)\widetilde{K}_t+\widetilde{I}_t} \widehat{E}_0\sum_{t=0}^{\infty}\beta^t\phi_t\left(R_t\widetilde{K}_t-\widetilde{I}_t-\Psi\left(\frac{\widetilde{I}_t}{\widetilde{K}_t}\right)\widetilde{K}_t\right)\\
& \phi_t \equiv \frac{U'(C_t)}{U'(C_0)}\\
& K_0 = \widetilde{K}_0\text{: given} \qquad (3.24)
\end{aligned}$$

where the constraint

$$\widehat{\mathbf{P}} \text{ is a well-defined probability space with } \widehat{\mathbf{P}} \prec \mathbf{P}$$

is equivalent to the constraints

$$1 = \sum_{s_{t+1}} \widehat{\Pr}\left(s_{t+1} \mid s^t\right)$$

$$\widehat{\Pr}\left(s_{t+1} \mid s^t\right) \geq 0$$

$$\widehat{\Pr}\left(s^{t+\tau} \mid s^t\right) = \prod_{j=1}^{\tau} \widehat{\Pr}^{(t)}\left(s_{t+j} \mid s^{t+j-1}\right)$$

$$\widehat{\Pr}\left(s_{t+1} \mid s^t\right) = 0 \text{ if } \Pr\left(s_{t+1} \mid s^t\right) = 0$$

$$\text{for all } t, \text{ and for all } \tau \geq 1$$

and where $s_t \in \Omega$ is the state of the economy at time t, and $s^t \in \Omega$ is the history of the economy at time t.

There are two important features to note in the infinite horizon version of the investment-firm problem. The first is that the anticipatory component of felicity in period t is given by $\sum_{\tau=1}^{\infty} \beta^{\tau} \widehat{E}_t \phi_{t+\tau} \left(R_{t+\tau} K_{t+\tau} - I_{t+\tau} - \Psi\left(\frac{I_{t+\tau}}{K_{t+\tau}}\right) K_{t+\tau}\right)$, and so manager welfare is now a function of the discounted sum of the entire stream of anticipated future profits. The memory component of felicity in period t is given by $\sum_{\tau=1}^{t} \varepsilon^{\tau} \phi_{t-\tau} \left(R_{t-\tau} K_{t-\tau} - I_{t-\tau} - \Psi\left(\frac{I_{t-\tau}}{K_{t-\tau}}\right) K_{\tau-\tau}\right)$, and is a discounted sum of weighted profits earned in all previous periods back to time $t = 0$. Apart from the richer dynamics incorporated in the above specification, the intuition behind both anticipatory and memory utility is directly analogous to the intuition discussed previously. An investment managers' felicity in period t is a function of both past and anticipated future payoffs.[54]

Nonetheless, with a richer dynamic structure the manager now must not only optimise about their beliefs over tomorrow, but also beliefs in all future periods. In general this problem is considerably more complex. To obtain insight into this problem I have assumed

[54] Note the parametrisation of the objective used is consistent with that used by Brunnermeier and Parker (2005). It ensures that managers are, in the absence of shocks to preferences, time consistent in their valuation of alternative profit streams. For more detail, see Caplin and Leahy (2004) for discussions of prospective and retrospective time consistency.

in the formulation of (3.24), consistent with Brunnermeier and Parker (2005), that firm managers are able to commit to their optimal beliefs (subjective probability measure, $\widehat{\mathbf{P}}$) at time 0 and do not re-optimise on their beliefs over time. That is, I study the full commitment solution. This ensures that a Law of Iterated Expectations will hold over subjective beliefs, and that the problem can alternatively be interpreted as the selection of an optimal prior with Bayesian updating.

An appealing feature of OE on the part of investment firms under commitment (3.24), is that there exists a recursive solution for investment that is given by the following modified investment Euler equation

Proposition 3.8. *Under full-commitment with $\varepsilon < \beta^{-1}$, and assuming that the optimal investment decision is conceivable (i.e. it is a an investment decision that can be optimally chosen under the restriction $\widehat{\mathbf{P}} \prec \mathbf{P}$), an interior solution to the program (3.24) is given by*

$$\begin{aligned} q_t &= \frac{\beta}{\beta\varepsilon}E_t\left(\frac{U'(C_{t+1})}{U'(C_t)}\left(R_{t+1}+\Psi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}-\Psi\left(\frac{I_{t+1}}{K_{t+1}}\right)+(1-\delta)q_{t+1}\right)\right) \\ &\quad+\frac{(1-\beta\varepsilon)}{\beta\varepsilon}\Psi''\left(\frac{I_t}{K_t}\right)\frac{K_{t+1}}{K_t} \\ &\quad-\frac{(1-\beta\varepsilon)}{\beta\varepsilon}\beta E_t\left(\frac{U'(C_{t+1})}{U'(C_t)}\Psi''\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{K_{t+2}^2}{K_{t+1}^2}\right) \qquad (3.25) \\ q_t &\equiv 1+\Psi'\left(\frac{I_t}{K_t}\right) \\ K_{t+1} &= (1-\delta)K_t+I_t \\ K_0 &: \textit{given} \end{aligned}$$

This will characterise the investment decision made under optimal expectations, provided there exists a set of optimally chosen beliefs, $\widehat{\mathbf{P}}$, satisfying

$$q_t = \beta\widehat{E}_t\left(\frac{U'(C_{t+1})}{U'(C_t)}\left(R_{t+1}+\Psi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}-\Psi\left(\frac{I_{t+1}}{K_{t+1}}\right)+(1-\delta)q_{t+1}\right)\right)$$

and $\widehat{\mathbf{P}}$ is a well-defined probability measure with $\widehat{\mathbf{P}} \prec \mathbf{P}$

To understand the intuition behind (3.25), it is helpful to compare the above investment Euler equation with the Euler equation that would be obtained if a rational probability measure were used when investing

$$q_t = E_t\left(\frac{U'(C_{t+1})}{U'(C_t)}\left(R_{t+1} + \Psi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}} - \Psi\left(\frac{I_{t+1}}{K_{t+1}}\right) + (1-\delta)\, q_{t+1}\right)\right)$$

Intuitively, the effects on the Euler equation look analogous to that identified in the two-period model previously discussed. The presence of anticipation provides an incentive for optimism because the manager attains felicity today from being optimistic about the future. This shows up as an extra perceived fillip to the return from investment captured by the term[55]

$$(1-\beta\varepsilon)\left(\begin{array}{c} U'(C_t)\,\Psi''\left(\frac{I_t}{K_t}\right)\frac{K_{t+1}}{K_t} \\ -\beta E_t\left(U'(C_{t+1})\,\Psi''\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{K_{t+2}^2}{K_{t+1}^2}\right) \end{array}\right)$$

In contrast, memory acts a constraining factor on the incentive to be optimistic. By valuing the past more, agents prefer to have higher profits today because the benefits of doing so will carry forward in to future periods. Thus, ε, again acts as scaling factor on the marginal cost of investment given by

$$\beta\varepsilon U'(C_t)\, q_t$$

At the optimum, the manager optimally trades the gains from optimism with the costs of distorted decision-making. Importantly, it should be noted that as the cost of distorted decision-making is scaled upwards through a higher memory weight, that is as $\varepsilon \to \beta^{-1}$ from below, these costs will the offset gains from optimism, and so in the limit OE converges to RE as the memory weight approaches β^{-1}.

In principle, this dynamic version of investment under optimal expectations could be integrated in certain classes of DSGE models such as an RBC model with investment adjustment costs and consumption habits, which has had success empirically (see

[55] To see this multiply both sides of Equation (3.25) by $\beta\varepsilon U'(C_t)$.

for example Jermann 1998). Or, alternatively, in New Keynesian DSGE models that have become popular in recent literature (see for example Christiano, Eichenbaum and Evans 2005).

In view of the fact that in the simple two-period economy optimal expectations on the part of firms and households amplified the effects of technology shocks on equity prices, investment and output when compared with rational expectations, there is scope for this also to be true in a fully dynamic infinite horizon model. I leave this as an area for further investigation. However, if it is true that OE amplifies investment and equity price responses to shocks quantitatively in a model with an infinite horizon, then this could be useful for resolving puzzles such as the excess volatility observed in equity prices and investment, and the equity premium puzzle.

3.7. Conclusion

This chapter has considered whether optimism or pessimism that is in the interest of economic decision-makers, here the investment firm, could be a plausible explanation for periods in which equity prices appear overvalued or undervalued relative to a model with rational expectations. Qualitatively, optimal expectations on the part of investment firms can amplify the effects of fundamental shocks and can explain periods of overvaluation in equity markets and over-investment in physical capital, when compared with a more standard rational expectations economy. Quantitatively, whether firm driven optimal expectations performs well (or not), appears to be an empirical question that deserves further consideration.

Chapter 4

Does Equity Mispricing Influence Household and Firm Decisions?

4.1. Introduction

The existence of fads or bubbles in equity prices has been a question long debated by economists. Notable historical episodes of mispricing in equity markets that have been identified include the South Sea Bubble, the Great Railway Bubble, and the US Roaring Twenties. More recent examples of apparent bubbles include Japan's equity and property markets in the 1980s, and the US dot-com boom in the late 1990s. In view of these experiences, there has been ongoing interest in the extent to which equity price bubbles distort economic decision-making. However, somewhat surprisingly, there remains only a limited literature that attempts to quantify the effects of mispricing in equity markets on household and firm decisions.[56]

To provide insight into this question, this chapter focuses on identifying the quantitative effects of equity market mispricing on household consumption and portfolio allocation decisions, and firm dividend policies. If mispricing in equity markets exists, the distortion of price signals associated with household wealth held in the form of equity can affect household consumption and portfolio allocation decisions. In addition, mispricing can potentially affect corporate dividend policies. For example, if prices reflect that firms have overly optimistic expectations regarding their future profitability and investment opportunities, this optimism can lead firms to alter their dividend payment decisions. I focus on whether these effects are economically significant. That is, do households

[56] Examples of such literature include Chirinko and Schaller (2001) and Gilchrist *et al* (2005), both of which focus on the effects of bubbles on firms' investment decisions.

consume more or less during a bubble, purchase more or less equity, and do firms change their dividend policies?

There is an extensive literature that seeks to test or identify the existence of equity price bubbles.[57] However, rather than focusing on tests of existence, this chapter takes the view that mispricing shocks exist, and then explores their effect on economic decision-making. It builds on ideas that have previously been used to identify mispricing shocks, including the idea that these shocks should be viewed as transitory[58] and that there exist observable proxies that are correlated with mispricing, but uncorrelated with changes in fundamentals.[59] The contribution is to show how both of these ideas can be incorporated in a simple estimation framework that permits analysis of the effects of equity market mispricing.

Using data for the United States, I estimate a system that allows identification of the effects of mispricing shocks on household consumption and portfolio allocation decisions, firm dividend decisions, and equity prices. The proxies for mispricing that I consider are a measure of analyst forecast dispersion (see Diether *et al* (2002); Gilchrist *et al* (2005)), a survey measure of perceived misvaluation (see Shiller (2000b)), and a measure of expected short-term volatility in equity prices. Importantly, these proxies provide information for identifying equity mispricing that is useful in the case that they are correlated with mispricing, but uncorrelated with shocks to fundamentals.

The identification approach suggested in this chapter is informative for several reasons. The first is that it does not rely on the restriction that mispricing has no effect on economic decisions *a priori*, and is therefore well equipped to identify the effects of bubbles.[60] A second advantage is that the method proposed here does not require

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57 See, for example, Vissing-Jorgensen (2004) and Gürkaynak (2008) for reviews of this literature.

58 For example, this is implicit in the work of Lee (1998). Transitory shocks are defined in this chapter as perturbations that can affect short- but not long-run forecasts. In contrast, permanent shocks are innovations that can influence both short- and long-run forecasts.

59 See, for example, Diether, Malloy and Scherbina (2002) and Gilchrist *et al* (2005).

a unique model of the fundamental structure of the economy. In econometric terms, identifying the effects of the non-fundamental transitory (mispricing) shock does not require identification of the permanent (fundamental) shocks to the system. From this perspective, the methodology proposed can be considered consistent within a class of economic models of fundamentals, rather than requiring a unique model of fundamentals to be identified.

Importantly, the restrictions imposed for identification are made explicit. This is in contrast to statistical approaches used to identify bubbles where the restrictions imposed can be opaque, and the ability to identify the effects of bubbles on economic decisions remains unclear.[61]

The next section outlines related empirical literature. Section 4.3 outlines the estimation methodology and the approach to identification. Sections 4.4 and 4.5 are concerned with the empirical application and the main results. Section 4.6 considers robustness, and some conclusions are drawn in the final section.

4.2. Related Literature and Approaches

One approach to identifying equity price bubbles and their effects is to take a stand on a specific model that describes the evolution of the economy. This includes a specific model for equity prices, and possibly a model of the process underlying an equity price bubble as well. Once the model has been specified, econometric tests for the presence of bubbles can be undertaken. Reviews that summarise this literature include Gürkaynak (2008), concerning rational bubbles, and Vissing-Jorgensen (2004), covering the literature in behavioural finance.

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[60] This is in contrast to previous literature, such as Lee (1998), which assumes that mispricing has no real effects.

[61] Helbling and Terrones (2003) and Detken and Smets (2004) are examples of purely statistical approaches that measure reduced-form correlations between bubbles and economic variables of interest.

One advantage of such a structural approach to identification is that the restrictions used are made explicit and can often be tested. However, as noted by Gürkaynak, a common criticism of these hypothesis tests is that they are unable to distinguish between a test for a bubble, and a test of the model assumed as part of the maintained hypothesis. Accordingly, the validity of any results obtained are contingent on the reader accepting the economic model proposed as the correct one. If there was a strong consensus concerning the 'correct' or 'true' model for the economy and equity pricing this would not be too problematic. However, given a lack of consensus over these issues, it has been difficult for any one structural approach to remain convincing in its ability to detect a bubble.

An alternative approach to identifying bubbles is to use a purely statistical or atheoretical approach. Examples of this approach include Helbling and Terrones (2003), Detken and Smets (2004), and Machado and Sousa (2006). The advantage of atheoretical approaches is that they may be less subject to model misspecification, since they do not rely on any particular assumed model, and can be useful for summarising correlations in the data. However, there remains much scepticism of their ability to identify the effects of bubbles on economic decisions more precisely. This stems from the property that these procedures do not appear well-equipped to distinguish between different sources of movements in equity prices, and thus a boom or bust in equity prices that is identified as a bubble could just as likely reflect improved or worsening fundamentals.

A third approach in the literature, which is closest to this chapter, is to use some mix of the structural and statistical approaches. Rather than specifying a tight or unique economic model for fundamentals or bubbles, a weaker set of economic restrictions, consistent with economic theory, is used. These restrictions still provide sufficiently rich information to enable the researcher to get closer to identifying the effects of an equity price bubble, but help to avoid criticisms associated with model specificity. Previous literature in this vein, though not always concerned with identifying the effects of equity market mispricing, includes Cochrane (1994), Lee (1995, 1998), Gallagher (1999), Gallagher and Taylor (2000), Chirinko and Schaller (2001) and Gilchrist *et al* (2005). This chapter contributes to this literature by providing an informative alternative approach to identifying episodes of mispricing in equity markets, and by providing additional evidence on the effects of this mispricing on economic decisions.

4.3. Estimation Methodology

I use two ideas to identify the effects of equity mispricing. The first is that equity mispricing should only have transitory economic effects (see, for example, Lee (1998)). Such an assumption appears reasonable from a theoretical standpoint, given that many economists have the prior that equity prices are not entirely disconnected from the fundamental processes underpinning the economy. If the converse were true, and mispricing shocks had permanent effects, then equity prices would effectively be indeterminate and have no relationship with the underlying value of the dividend streams they pay.

The second idea is that there is observable information that can be used to distinguish between fundamental and non-fundamental transitory shocks. This reasoning follows a recent literature which argues that there are observables that are correlated with equity market mispricing, and that are uncorrelated with measures of economic fundamentals, see, for example, Diether *et al* (2002) and Gilchrist *et al* (2005).

I use these two ideas, in conjunction with a cointegration framework implied by economic theory, to identify the effects of equity mispricing shocks. More specifically, I use five economic relationships to motivate the empirical work in this chapter. The first is an accumulation equation for aggregate household wealth

$$W_{t+1} = \left(1 + R^w_{t+1}\right)\left(W_t - C_t\right)$$

where W_t is beginning of period wealth, C_t is total flow consumption in the period, and R^w_{t+1} is the return to total wealth. This formulation assumes that the market value of human capital is tradeable and included in aggregate wealth. This assumption simplifies exposition, but is an assumption that can be relaxed without substantively affecting any of the analysis that follows (see Lettau and Ludvigson (2004)). The second relationship used is that household wealth can be decomposed into its respective equity, non-equity, and human capital components

$$W_t = E_t + N_t + H_t$$

where E_t is total equity wealth held by households, N_t is total non-equity wealth (such as housing, consumer durables, and other forms of financial non-equity wealth), and H_t is human capital. The third and fourth relationships used are an accumulation equation for tradeable human capital, and the definition of equity wealth

$$\begin{aligned} H_{t+1} &= \left(1 + R^h_{t+1}\right)(H_t - Y_t) \\ E_t &= Q_t (P_t + D_t) \end{aligned}$$

where R^h_{t+1} is the return to human capital, Y_t is labour income, Q_t is the quantity of equity held, P_t is the ex-dividend price of equity, and D_t is the dividend paid on equity held in period t. The final relationship used is the definition of the return to equity, R^e_{t+1}, where

$$1 + R^e_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

Using arguments that are similar to those used by Campbell and Mankiw (1989), Lettau and Ludvigson (2004, 2005) and Kishor (2007), I log-linearise these relationships, assuming a balanced growth path, and obtain the following economic system[62]

[62] In taking these approximations, I assume that each variable in the system can be normalised by an appropriate trend (for example, the level of productivity or another variable that captures the long-run growth rate of the economy), and that limit terms associated with iterating these relationships forwards are small (of second-order). I omit linearisation constants and growth rates in unobserved trends in the above approximations.

$$c_t - w_t \approx \sum_{i=1}^{\infty} \rho_w^i \left(r_{t+i}^w - \Delta c_{t+i} \right) \tag{4.1}$$

$$w_t \approx \omega_e e_t + \omega_n n_t + \omega_h h_t \tag{4.2}$$

$$y_t - h_t \approx \sum_{i=1}^{\infty} \rho_h^i \left(r_{t+i}^h - \Delta y_{t+i} \right) \tag{4.3}$$

$$e_t \approx q_t + \rho_d p_t + (1 - \rho_d) d_t \tag{4.4}$$

$$d_t - p_t \approx \sum_{i=1}^{\infty} \rho_d^{i-1} \left(r_{t+i}^e - \Delta d_{t+i} \right) \tag{4.5}$$

where lower case variables denote natural logarithms,[63] ω_e, ω_n and ω_h are the steady state shares of equity, non-equity and human capital wealth in total wealth respectively, ρ_w is the steady state share of savings in total wealth, ρ_h is one minus the share of labour income in steady state human capital, and ρ_d is the steady state ratio of the ex-dividend equity price to the equity price that includes dividends. It should be noted that the system defined by Equations (4.1) to (4.5) contains two variables that are not directly observable, human capital wealth and total household wealth. To account for this, I substitute human capital and total wealth out of the above system to obtain[64]

$$c_t - \omega_e e_t - \omega_n n_t - \omega_h y_t \approx \sum_{i=1}^{\infty} \rho_w^i \left(r_{t+i}^w - \omega_h r_{t+i}^h - \Delta c_{t+i} + \omega_h \Delta y_{t+i} \right) \tag{4.6}$$

$$e_t \approx q_t + \rho_d p_t + (1 - \rho_d) d_t \tag{4.7}$$

$$d_t - p_t \approx \sum_{i=1}^{\infty} \rho_d^{i-1} \left(r_{t+i}^e - \Delta d_{t+i} \right) \tag{4.8}$$

Assuming that consumption, the quantity of equity held, non-equity wealth, equity prices, labour income, and dividends are integrated of order one, and that returns to total financial

63 Note for any of the return variables I use the notation $r_{t+i} \equiv \ln(1 + r_{t+i})$.

64 Without loss of generality, I assume when making these substitutions that $\rho_w = \rho_h$.

wealth, human capital wealth and equity wealth are stationary, it follows that Equations (4.6) to (4.8) make up a cointegrated system with two cointegrating vectors.

It should be made clear that Equations (4.6) to (4.8) make up a partially specified economic system. Additional model structure, for example including an Euler equation for consumption or an equity pricing equation, could potentially imply more restrictions or additional cointegrating relationships in this system. I choose not to include such structure given existing disagreement over the 'correct' model for either consumption or equity prices. Instead, I use the above framework as a motivation for modelling a system consistent with Equations (4.6) to (4.8), and use empirical analysis to determine the number of cointegrating relationships. I do not impose any model-specific restrictions that could otherwise be incorporated.

A general econometric representation that is consistent with Equations (4.6) to (4.8) is the structural vector error correction model (SVECM)

$$\mathbf{A}_0 \Delta \mathbf{y}_t = -\alpha^* \beta' \mathbf{y}_{t-1} - \mathbf{A}(L) \Delta \mathbf{y}_t + \varepsilon_t \tag{4.9}$$

where $\mathbf{y}_t$ is an $n \times 1$ vector of observables, $\mathbf{y}_t = \begin{bmatrix} c_t & d_t & n_t & y_t & q_t & p_t \end{bmatrix}'$, $\mathbf{A}(L)$ is a lag polynomial of order l, β' is the matrix of cointegrating vectors, and α^* the matrix of loading coefficients on the cointegrating vectors.[65] I assume $\mathbf{A}_0$ is non-singular and that $\alpha^* \beta'$ has rank $r < n$ so that at least one cointegrating vector exists. The ε_t are the primitive structural shocks. I assume these shocks are independently identically distributed, with $E(\varepsilon_t) = 0$ and

$$E\left(\varepsilon_t \varepsilon_\tau'\right) = \begin{cases} \Omega \text{ if } \tau = t \\ 0 \text{ otherwise} \end{cases} \tag{4.10}$$

[65] Note I substitute e_t out of the system in Equations (4.6) to (4.8) in the analysis that follows.

where Ω is a diagonal matrix (with elements that are not necessarily equal).[66] The ε_t are the underlying structural shocks that I am in interested in identifying. Specifically, I wish to identify the elements of ε_t that only have transitory effects, and in particular, non-fundamental transitory effects.

It should be noted that the structural shocks being serially uncorrelated is not necessarily a restrictive assumption in the current context. In particular, Equation (4.9) can be viewed as a finite-order approximation of a model in which the structural shocks are serially correlated (see, for example, Lütkepohl (2006)). That is, Equation (4.9) can be viewed as an approximation of a SVECM with moving average errors,

$$\begin{aligned} \mathbf{A}_0 \Delta \mathbf{y}_t &= -\alpha^* \beta' \mathbf{y}_{t-1} + \nu_t \\ \nu_t &= \Psi(L)\, \nu_{t-1} + \varepsilon_t \end{aligned} \tag{4.11}$$

where $\Psi(L)$ is an infinite-order lag polynomial. In this model, transitory mispricing disturbances in ν_t can be serially correlated with permanent shocks to fundamentals, an assumption that is consistent with the idea that permanent shocks to fundamentals, such as permanent changes in technology, can precede mispricing in the equity market. In the analysis that follows, I focus on estimating Equation (4.9), which can be interpreted as a finite-order approximation of Equation (4.11).[67]

Identification of Reduced-form Shocks

To distinguish between the reduced-form transitory and permanent shocks in Equation (4.9), I follow a re-parameterisation of the approach to identification suggested by Pagan and Pesaran (2008). Without loss of generality, I order the permanent and transitory

[66] Rather than assuming $E\left(\varepsilon_t \varepsilon_t'\right) = \mathbf{I}$, I impose normalisation (unity) restrictions on the main diagonal of $\mathbf{A}_0$.

[67] Lütkepohl (2006) provides a review of the regularity conditions under which such an approximation will be valid.

shocks according to

$$\varepsilon_t = \begin{bmatrix} \varepsilon_t^P \\ \varepsilon_t^T \end{bmatrix}$$

where ε_t^P is a $(n-r) \times 1$ vector of shocks with permanent effects $\left(\lim_{k\to\infty} \frac{\partial E_t(\mathbf{y}_{t+k})}{\partial\left(\varepsilon_t^P\right)'} \neq \mathbf{0}_{n\times n-r}\right)$, and ε_t^T is a $r \times 1$ vector of shocks that have transitory effects $\left(\lim_{k\to\infty} \frac{\partial E_t(\mathbf{y}_{t+k})}{\partial\left(\varepsilon_t^T\right)'} = \mathbf{0}_{n\times r}\right)$.[68] Since I assume that mispricing shocks have only transitory effects on the system, a mispricing shock must be an element of ε_t^T.

I proceed by estimating Equation (4.9) using limited information methods. The first step is to obtain a consistent estimate of the cointegrating matrix, β (or use the known cointegration matrix in the case that β is known). Importantly, as emphasised by Pagan and Pesaran, only a consistent estimate of the cointegration space – the column space of β, $\{\beta Q : Q'\beta'\mathbf{y}_t \sim I(0); \text{given } \mathbf{y}_t \sim I(1) \text{ and } Q \text{ non-singular}\}$ – is required since the instrumental variable (IV) methods described below are invariant to non-singular transformations. A consistent estimate of this space can be obtained, for example, from the Johansen full information maximum likelihood (FIML) estimates of the reduced form of Equation (4.9) or using alternative system methods discussed by Lütkepohl (2006).

Assuming a consistent estimate of β is available, I partition Equation (4.9) into a system of $n-r$ equations with permanent shocks, ε_t^P, and r remaining equations with transitory shocks ε_t^T

$$\begin{bmatrix} A_{11}^0 & A_{12}^0 \\ A_{21}^0 & A_{22}^0 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{y}_{1t} \\ \Delta\mathbf{y}_{2t} \end{bmatrix} = -\alpha^*\beta' \begin{bmatrix} \mathbf{y}_{1t-1} \\ \mathbf{y}_{2t-1} \end{bmatrix} - \begin{bmatrix} A_{11}^2 & A_{12}^2 \\ A_{21}^2 & A_{22}^2 \end{bmatrix} \begin{bmatrix} \Delta\mathbf{y}_{1t-1} \\ \Delta\mathbf{y}_{2t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^P \\ \varepsilon_t^T \end{bmatrix} \tag{4.12}$$

and I assume, without loss of generality, that $\mathbf{A}(L) = A_2 L$. This assumption abstracts from lag dynamics that do not affect the generality of the identification approach proposed.

68 The fact that the number of transitory shocks is equal to the number of cointegrating vectors is an implication of the Granger representation theorem. Lütkepohl (2006) provides a useful review.

Since I previously assumed $E\left(\varepsilon_t \varepsilon_t'\right) = \Omega$, where Ω is a diagonal matrix, I impose n normalisation restrictions on the main diagonals of A_{11}^0 and A_{22}^0. I further assume that A_{11}^0 is non-singular. With these assumptions, a simple matrix premultiplication yields

$$\begin{bmatrix} I & \widetilde{A}_{12}^0 \\ A_{21}^0 & A_{22}^0 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_{1t} \\ \Delta \mathbf{y}_{2t} \end{bmatrix} = - \begin{bmatrix} \widetilde{\alpha}_1^* \\ \alpha_2^* \end{bmatrix} \beta' \begin{bmatrix} \mathbf{y}_{1t-1} \\ \mathbf{y}_{2t-1} \end{bmatrix} - \begin{bmatrix} \widetilde{A}_{11}^2 & \widetilde{A}_{12}^2 \\ A_{21}^2 & A_{22}^2 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_{1t-1} \\ \Delta \mathbf{y}_{2t-1} \end{bmatrix} + \begin{bmatrix} u_t^P \\ \varepsilon_t^T \end{bmatrix} \tag{4.13}$$

where

$$u_t^P = \left(A_{11}^0\right)^{-1} \varepsilon_t^P, \qquad \widetilde{A}_{12}^0 = \left(A_{11}^0\right)^{-1} A_{12}^0$$
$$\widetilde{\alpha}_1^* = \left(A_{11}^0\right)^{-1} \alpha_1^*, \qquad \widetilde{A}_{1j}^2 = \left(A_{11}^0\right)^{-1} A_{1j}^2 \quad \text{for } j = 1, 2$$

The above premultiplication is useful because it allows identification of transitory shocks, without requiring identification of the permanent shocks to the system. That is, I only identify linear combinations of the permanent shocks, u_t^P, and not the underlying permanent structural shocks, ε_t^P.

Using the result that lagged error correction terms should not be present in the structural permanent equations, $\alpha_1^* = 0$,[69] one can use these restrictions to estimate the first $n-r$ permanent equations in Equation (4.13). Specifically, this set of restrictions implies that the $r \times 1$ vector $\xi_{t-1} = \beta' \mathbf{y}_{t-1}$ can be used as instruments for the vector $\Delta \mathbf{y}_{2t}$. And so, the first $n-r$ permanent equations

$$\Delta \mathbf{y}_{1t} = -\widetilde{A}_{12}^0 \Delta \mathbf{y}_{2t} - \widetilde{A}_{11}^2 \Delta \mathbf{y}_{1t-1} - \widetilde{A}_{12}^2 \Delta \mathbf{y}_{2t-1} + u_t^P \tag{4.14}$$

[69] This result is derived in Pagan and Pesaran (2008) with respect to Equation (4.12), and is consistent with the ordering of permanent and transitory shocks.

can be estimated using standard IV methods. This provides consistent estimates of the reduced-form matrices $\widetilde{A}_{12}^{0}, \widetilde{A}_{11}^{2}$ and $\widetilde{A}_{12}^{2}$, and the reduced-form permanent shocks, u_t^P.

To estimate the remaining r transitory equations

$$A_{22}^{0}\Delta\mathbf{y}_{2t} = -A_{21}^{0}\Delta\mathbf{y}_{1t} - \alpha_2^{*}\xi_{t-1} - A_{21}^{2}\Delta\mathbf{y}_{1t-1} - A_{22}^{2}\Delta\mathbf{y}_{2t-1} + \varepsilon_t^T \tag{4.15}$$

I can now use the consistent estimates, $\widehat{u}_t^P$, as instruments for the endogenous variables in $\Delta\mathbf{y}_{1t}$ (see Pagan and Pesaran (2008)). This would enable identification of the reduced-form transitory shocks, $\left(A_{22}^{0}\right)^{-1}\varepsilon_t^T$, but to identify the structural transitory shocks, ε_t^T, it is clear that additional restrictions are required.

Identification of Structural Transitory Shocks

Focusing on the transitory equations in Equation (4.15), recall that I have already imposed r normalisation (unity) restrictions on the main diagonal of A_{22}^{0}. Since I have previously assumed that transitory shocks are uncorrelated (see Equation 4.10), this implies that an additional $r(r-1)/2$ additional restrictions are required to be able to identify the structural shocks, ε_t^T. Although one could proceed by imposing additional restrictions on the elements of A_{22}^{0}, or using restrictions on any of A_{21}^{0}, α_2^{*}, A_{21}^{2} or A_{22}^{2},[70] in some applications such restrictions may not be appealing on theoretical grounds. This is the case, for example, when attempting to distinguish between fundamental and non-fundamental transitory shocks as considered in the empirical application below.

Instead, I assume there exists additional observable information available to the researcher that allows identification of ε_t^T, or at least some of the elements in this transitory shock vector. Specifically, I assume that Equation (4.15) can be partitioned in a form that is

[70] This is the approach followed by Pagan and Pesaran (2008) after fully identifying the effects of permanent shocks.

consistent with the presence of an $(r-1)\times 1$ vector of fundamental transitory shocks, $\varepsilon_t^{f,T}$, and a single non-fundamental transitory shock, $\varepsilon_t^{b,T}$,[71]

$$\begin{bmatrix} A_{11}^{0,22} & a_{12}^{0,22} \\ a_{21}^{0,22} & 1 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_{21t} \\ \Delta \mathbf{y}_{22t} \end{bmatrix} = - \begin{bmatrix} \left[A_{21}^{0}\right]_1 \\ \left[A_{21}^{0}\right]_2 \end{bmatrix} \Delta \mathbf{y}_{1t} - \begin{bmatrix} \left[A_{22}^{2}\right]_1 \\ \left[A_{22}^{2}\right]_2 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_{21t-1} \\ \Delta \mathbf{y}_{22t-1} \end{bmatrix}$$
$$- \begin{bmatrix} \left[A_{21}^{2}\right]_1 \\ \left[A_{21}^{2}\right]_2 \end{bmatrix} \Delta \mathbf{y}_{1t-1} - \begin{bmatrix} \left[\alpha_2^{*}\right]_1 \\ \left[\alpha_2^{*}\right]_2 \end{bmatrix} \xi_{t-1} + \begin{bmatrix} \varepsilon_t^{f,T} \\ \varepsilon_t^{b,T} \end{bmatrix} \tag{4.16}$$

I further assume there exists an observable instrument (or set of instruments) $\mathbf{Z}_t = \left[z_{1t}', ..., z_{kt}'\right]'$, a $k\times 1$ vector $(k \geq 1)$, with the properties that,

$$\begin{aligned} E\left(z_{it}\varepsilon_t^{b,T}\right) &\neq 0 \\ E\left(z_{it}\varepsilon_t^{f,T}\right) &= 0 \\ E\left(z_{it}\varepsilon_t^{P}\right) &= \mathbf{0}_{n-r\times 1} \\ \text{for } i &= 1,...,k \end{aligned} \tag{4.17}$$

That is, there exists one or more instruments for equity prices growth that are correlated with mispricing shocks, and contemporaneously uncorrelated with either fundamental transitory or permanent shocks.[72] Assuming $A_{11}^{0,22}$ is non-singular, again using a simple premultiplication of Equation (4.16) yields

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[71] I use the notation that $\underset{r\times r}{A_{ij}^{x}} = \begin{bmatrix} \underset{r-1\times r}{\left[A_{ij}^{x}\right]_1} \\ \underset{1\times r}{\left[A_{ij}^{x}\right]_2} \end{bmatrix}$. A similar partition is used with respect to α^*.

[72] To be clear, only the first two conditions are required for identification. I use the stronger requirement $E\left(z_{it}\varepsilon_t^{P}\right) = \mathbf{0}_{n-r\times 1}$ since the proxies for mispricing have desirable properties when used as instruments in estimating the permanent equations.

$$\begin{bmatrix} \mathbf{I}_{r-1} & \widetilde{a}_{12}^{0,22} \\ a_{21}^{0,22} & 1 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_{21t} \\ \Delta \mathbf{y}_{22t} \end{bmatrix} = - \begin{bmatrix} \left[\widetilde{A}_{21}^{0}\right]_1 \\ \left[A_{21}^{0}\right]_2 \end{bmatrix} \Delta \mathbf{y}_{1t} - \begin{bmatrix} \left[\widetilde{A}_{22}^{2}\right]_1 \\ \left[A_{22}^{2}\right]_2 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_{21t-1} \\ \Delta \mathbf{y}_{22t-1} \end{bmatrix}$$
$$- \begin{bmatrix} \left[\widetilde{A}_{21}^{2}\right]_1 \\ \left[A_{21}^{2}\right]_2 \end{bmatrix} \Delta \mathbf{y}_{1t-1} - \begin{bmatrix} \left[\widetilde{\alpha}_{2}^{*}\right]_1 \\ \left[\alpha_{2}^{*}\right]_2 \end{bmatrix} \xi_{t-1} + \begin{bmatrix} u_t^{f,T} \\ \varepsilon_t^{b,T} \end{bmatrix} \tag{4.18}$$

where

$$\begin{aligned} \left[\widetilde{A}_{21}^{0}\right]_1 &= \left(A_{11}^{0,22}\right)^{-1} \left[A_{21}^{0}\right]_1 & \left[\widetilde{\alpha}_{2}^{*}\right]_1 &= \left(A_{11}^{0,22}\right)^{-1} \left[\alpha_{2}^{*}\right]_1 \\ \left[\widetilde{A}_{21}^{2}\right]_1 &= \left(A_{11}^{0,22}\right)^{-1} \left[A_{21}^{2}\right]_1 & \left[\widetilde{A}_{22}^{2}\right]_1 &= \left(A_{11}^{0,22}\right)^{-1} \left[A_{22}^{2}\right]_1 \\ \widetilde{a}_{12}^{0,22} &= \left(A_{11}^{0,22}\right)^{-1} a_{12}^{0,22} & u_t^{f,T} &= \left(A_{11}^{0,22}\right)^{-1} \varepsilon_t^{f,T} \end{aligned}$$

This system can be estimated using a method analogous to that used for the permanent equations. Specifically, one can estimate the first $r-1$ transitory equations using $\widehat{u}_t^P$ and $\mathbf{Z}_t$ as instruments for $\Delta \mathbf{y}_{1t}$ and $\Delta \mathbf{y}_{22t}$ respectively. The residuals $\widehat{u}_t^{f,T}$ and $\widehat{u}_t^P$ can then be used as instruments for $\Delta \mathbf{y}_{21t}$ and $\Delta \mathbf{y}_{1t}$ when estimating the final transitory equation.

In sum, this procedure enables identification of the structural mispricing shock, $\varepsilon_t^{b,T}$, including associated impulse response functions and forecast error variance decompositions that identify the effects of this shock. If alternative instruments, or valid restrictions can be imposed to identify the effects of fundamental transitory shocks, then these too can be used. However, such restrictions are not required to identity the effects of the mispricing shock.

In the empirical application that follows I eliminate Equation (4.7) from Equations (4.6) to (4.8) and order the vector of observables such that $\mathbf{y}_t = \begin{bmatrix} c_t & d_t & n_t & y_t & q_t & p_t \end{bmatrix}'$, and so $\mathbf{y}_{1t} = \begin{bmatrix} c_t & d_t & n_t & y_t \end{bmatrix}'$ and $\mathbf{y}_{2t} = \begin{bmatrix} q_t & p_t \end{bmatrix}'$. That is, there are two cointegrating vectors (transitory shocks) in the system $(n = 6, r = 2)$,[73] and I assume that these shocks have direct effects on the quantity and price of equity held by households, and indirect effects on consumption, dividends, non-equity worth and labour income. The latter variables are also those directly perturbed by permanent fundamental shocks.

4.4. Empirical Application

Data and Preliminary Analysis

For estimation I use quarterly data for the United States, covering the sample period from June 1986 to December 2006 when either forecast dispersion or option-implied equity volatility are used as instruments for mispricing (the $\mathbf{Z}_t$ in the methodology described above), or from December 1988 to June 2010 when using the direct survey measure of overvaluation as an instrument. The starting points of these samples reflect data availability on the instruments used for mispricing. The different end points of these samples allow for a comparison of the results with and without the effects of the financial crisis that emerged in 2007.

Following Lettau and Ludvigson (2004), I use household flow consumption of non-durables (excluding clothing and footwear) as a measure of household consumption, and real household after-tax labour income as the measure of income obtained from human capital.[74] Although it would be ideal to use a measure of the total flow services from consumption, excluding durable expenditures, this measure is not directly observed.[75] To measure the cost of purchasing a unit of US equity (equity prices) I use the share price of Vanguard's S&P 500 ETF measured at the end of the quarter.[76] This measure provides a good proxy for the cost of purchasing a diversified equity portfolio that replicates the

[73] This is confirmed by cointegration matrix rank tests (see Appendix 8.2).

[74] For a more detailed description of the data, see Appendix 8.1.

[75] See Lettau and Ludvigson (2004) for further discussion.

[76] The results are very similar if the US S&P 500 index is used.

US S&P 500. I use a seasonally adjusted quarterly dividend measure, also measured with respect to the US S&P 500.

US Flow of Funds Accounts data are used to separate US household net financial wealth into its domestic equity and non-domestic-equity components. I focus on domestic equity because I am interested in studying the effects of mispricing in the US equity market.[77] To construct a measure of household wealth held in US equity, I multiply total household equity holdings (which includes both domestic and foreign components) by the proportion of equity held by all sectors (households and corporate) in US equity. This assumes that the household portfolio share allocations to domestic and foreign equity are similar to the allocations held across the total US private sector.[78] Household non-US-equity net wealth (hereafter non-equity net wealth) is a residual defined as total household net financial wealth less holdings of domestic equity. To construct an internally consistent measure of the quantity of equity held by US households (equity quantities), I divide the domestic equity holdings measure by the share price measure defined above.[79]

In terms of observable information used to distinguish between fundamental and non-fundamental transitory shocks, I consider three measures. The first is a measure of analyst forecast dispersion with respect to the US S&P 500, obtained from the Institutional Brokers' Estimate System (I/B/E/S). Specifically, I use the weighted average standard deviation of analysts long-term average growth in earning per share forecasts for the US S&P 500.[80] The second instrument considered is a measure of option-implied equity volatility. In particular, I use implied 30-day volatility for the US S&P 100 as traded

[77] The preceding theoretical discussion can be appropriately modified to account for the fact that US households own both domestic and foreign equity.

[78] This assumption is required since data on domestic and foreign equity portfolio allocations are only reported for all US sectors, and not specifically for households.

[79] Both the non-equity net wealth and equity quantities measures are lagged one quarter to be consistent with their beginning of period values used in the theory previously discussed.

[80] This index is constructed by weighting the standard deviation of analyst forecasts (of long-term average growth in earnings per share) for each firm in the US S&P 500. The weights used reflect the market capitalisation of each firm in the total index.

on the Chicago Board Options Exchange (with ticker VXO). I use this measure because longer time series are available than for implied 30-day volatility for the US S&P 500 (the VIX), but both measures are highly correlated and the results obtained are not sensitive to this choice over a common sample. The third measure I use is a direct survey measure of perceived over-valuation in the US equity market obtained from surveys of US institutional investors and sourced from the Yale School of Management (see also Shiller (2000b)).[81] The rationale for using each of these measures as instruments for equity prices growth is discussed further below.

Consumption, non-equity net wealth, equity quantities and real after-tax labour income are all in per capita log terms, and data on equity prices and dividends are in log terms. All data, with the exception of equity quantities, consumption and the instruments for mispricing, are deflated by the US personal consumption expenditure deflator.[82] All data used in estimation are measured at a quarterly frequency.

Before proceeding with the proposed identification methodology, it is important to establish that a cointegration framework is in fact a suitable representation of the data. For pre-testing I use all available data on the endogenous variables $(\mathbf{y}_t)$ from March 1953 to June 2010, to ensure accurate inference. Unit root tests are consistent with each of the data series being $I(1)$, and standard information criteria are consistent with two lags in a levels VAR (a VECM with a single lag). Tests of whether the data are cointegrated (the rank of the cointegration matrix) suggest two cointegrating vectors in the data.[83] All pre-testing results are reported in Appendix 8.2.

Turning to estimation of the cointegration matrix, $\beta' = [\beta_1' : \beta_2']$, I restrict attention to the main samples used for estimation, from June 1986 to December 2006 and December

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[81] I am most grateful to Robert Shiller and the Yale School of Management for making these data available.

[82] Consumption of non-durables (excluding clothing and footwear) is deflated by its own implicit price deflator. See Lettau and Ludvigson (2004) for further detail.

[83] Rank tests yield similar results if performed on the main estimation sample, from June 1986 to December 2006. All tests allow for an unrestricted constant in the cointegration model.

1988 to June 2010. Since it is well known that cointegration estimates are more precise if all known information is used by the researcher in estimation, I restrict the second cointegrating vector to have one and minus one coefficients on dividends and equity prices respectively, $\beta_2' = [\begin{array}{cccccc} 0 & 1 & 0 & 0 & 0 & -1 \end{array}]$. This implies that the log dividend to equity price ratio is stationary, which is consistent with the theory described above and is a common assumption that has been used in previous empirical research.[84]

Estimating the cointegration matrix, subject to the above restriction, the first cointegrating vector has coefficients $\widehat{\beta}_1'^{1986-2006} = [\begin{array}{cccccc} 1 & -0.03 & -0.08 & -0.51 & -0.12 & -0.08 \end{array}]$ in the sample from June 1986 to December 2006, and $\widehat{\beta}_1'^{1988-2010} = [\begin{array}{cccccc} 1 & -0.03 & -0.04 & -0.54 & -0.13 & -0.07 \end{array}]$ in the sample from December 1988 to June 2010.[85] These coefficients are consistent with economic theory, with human capital estimated to be the largest share of wealth, and the sum of the coefficients on equity prices and dividends being almost identical to the coefficient on equity quantities (as implied by the previous theoretical motivation). These estimates are also comparable to estimates of the same cointegrating relationship – that do not distinguish between the US equity and non-US equity components of wealth – using single-equation methods, see, for example, Lettau and Ludvigson (2004). In the analysis that follows, I use $\widehat{\beta}'^x = \left[\begin{array}{cc} \widehat{\beta}_1'^x & \beta_2' \end{array}\right]$ for $x = \{1986-2006, 1988-2010\}$ as a consistent estimate of the true cointegration space of the data in each sample, identified up to a non-singular transformation.

Identification

In view of the fact that the estimation methodology in Section 4.3 relies on IV techniques, it is important to establish that the instruments used are both relevant and valid.[86] This is especially so for the instruments used for equity prices growth, that enable identification

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84 See, for example, Campbell and Shiller (1987), Cochrane (1994) and Lee (1998) amongst others.

85 Specifically, I use FIML (Johansen's approach), subject to the restrictions that the rank of β is two and that β_2' is known. The coefficients in β_1 are identified up to a linear scaling factor.

86 Sarte (1997) provides a useful discussion in the context of structural vector autoregressions.

of fundamental and non-fundamental transitory shocks. I first address the question of instrument relevance, before turning to the issue of validity, for each of the instruments in turn.

The rationale for forecast dispersion being correlated with bubbles is that greater heterogeneity in analysts expectations could be consistent with mispricing in equity markets if some analysts are unable (or unwilling) to execute trades that reflect this greater divergence of opinion. For example, a constraint on short-selling is one frequently cited market or institutional constraint that would be consistent with greater heterogeneity implying equity mispricing (see, for example, Diether *et al* (2002) and the references cited therein). However, other explanations such as the existence of heterogenous investors, including rational and non-rational investors, and the inability of rational investors to co-ordinate their actions could also imply a correlation between forecast dispersion and mispricing (see, for example, Shleifer and Vishny (1997); Abreu and Brunnermeier (2003)). Furthermore, heterogenous optimism (Brunnermeier and Parker 2005), and the incentive for informed advisors to inflate their forecasts of fundamentals (Hong *et al* 2008), are also economic environments that can support a correlation between forecast dispersion and mispricing in the equity market.

In terms of exogeneity, Diether *et al* (2002) and Gilchrist *et al* (2005) argue that forecast dispersion is unlikely to be correlated with the fundamental investment opportunities available to firms. The underlying assumption is that shocks that affect mean forecasts for earnings and equity prices are not systematically correlated with shocks to the variance of these forecasts. For example, Diether *et al* (2002) provide evidence supporting the view that forecast dispersion in earnings per share is a useful instrument for equity prices growth in the US context. These authors highlight that on average companies with higher forecast dispersion for their earnings tend to have low future returns. According to the authors, this pattern is consistent with an interpretation where over-confidence or over-optimism on the part of some investors can lead to overpricing when combined with market, institutional or information constraints on non-optimistic investors. Alternatively, such a correlation is inconsistent with an interpretation where fundamental shocks to uncertainty are driving the correlation between equity prices growth and forecast dispersion.

Gilchrist *et al* (2005) also use forecast dispersion as an instrument for mispricing in the US equity market. They argue that forecast dispersion is a better measure of mispricing than other proxies for bubbles that have been suggested in previous literature, such as lagged prices or market to book valuations. The latter measures are thought more likely to be affected by the investment opportunities available to firms, and thus are more likely to be correlated with fundamental shocks.[87]

Turning to the survey measure of valuation confidence compiled by the Yale School of Management, and discussed in Shiller (2000b), this measure is also likely to be correlated with mispricing in the US equity market. This measure reflects a survey of US institutional investors undertaken biannually until July 2001, and at a monthly frequency thereafter.[88] Institutional investors are asked the following question:

> 'Stock prices in the United States, when compared with measures of true fundamental value or sensible investment value, are: [CIRCLE ONE NUMBER]
>
> 1. Too low. 2. Too high. 3. About right. 4. Do not know.'

The responses are designed to provide a direct gauge on whether institutional investors perceive US equity markets as being priced correctly, or whether they are undervalued or overvalued.[89] The exogeneity of this measure is largely assured due to survey design.

87 For additional surety, I use a measure of forecast dispersion with regard to average long-term growth in earnings per share. This should help to ensure that dispersion is not being driven by fundamental shocks relating to near-term uncertainty.

88 When used in estimation, quarterly data are interpolated from the biannual data prior to July 2001.

89 The Yale School of Management measure reports the Valuation Confidence Index as the number of respondents who choose 1 or 3 as a percentage of those who chose 1, 2 or 3. I use one minus this percentage in subsequent empirical analysis.

Shiller (2000b) argues that the wording of this question is informative about potential mispricing in the market, because it explicitly asks survey respondents on their views of valuation controlling for their own knowledge or assessment of market fundamentals.[90] Rather than asking institutional investors whether they expected prices to rise or fall, as some other survey measures that would be correlated with fundamentals do, the survey asks respondents directly about valuation in relation to fundamentals, and whether they perceive the current market as being 'too low' (undervalued), 'too high' (overvalued) or 'about right' (fair value).

The third instrument considered is option-implied equity volatility. Conceptually, this derivative is a measure of market expectations concerning future short-term volatility in a share market index. One might expect that during bubble episodes mispricing in equity markets could be correlated with expected volatility in the index. This could occur if some investors use trading strategies that are based on volatility to profit from bubbles. For example, if informed investors consider the market to be over-valued, but are unable to time exactly when a price correction is likely, then a trading strategy that pays high when markets are expected to move strongly in either direction may be a more profitable risk-adjusted strategy than taking short or long positions on a bubble directly.

Nonetheless, whether short-term option-implied volatility is uncorrelated with fundamental shocks is less clear on theoretical grounds. Although implied volatility may be uncorrelated with conventional fundamental shocks, such as shocks to firm productivity or household preferences, it could be argued that movements in short-term volatility could be correlated with short-term uncertainty that is fundamental in nature. For example, shocks such as terrorism attacks, wars, or uncertainty about major policy changes that affect US corporate profitability could be regarded as fundamental short-term volatility shocks that affect option-implied volatility, and potentially other economic

[90] There is an extensive literature in behavioural finance documenting potential market, institutional or information impediments that can sustain such mispricing, even when certain classes of investors feel confident that current market conditions are consistent with a bubble. See, for example, Shiller (2000a).

variables of interest (see, for example, Bloom (2009)). This suggests that using this third instrument, in conjunction with the identification strategy proposed, may result in inference that is not able to distinguish between the effects of fundamental uncertainty shocks that have transitory effects, and mispricing.

With this caveat in mind, I test whether this third instrument is valid, conditional on either forecast dispersion, or valuation confidence being valid instruments. If it is true that fundamental uncertainty shocks are important in the sample under consideration, and these are correlated with the other permanent or transitory shocks in this system, one would expect option-implied volatility to fail instrument orthogonality tests. Table 4.1 reports the results of Hausman tests that are robust to the presence of weak instruments.[91] The null hypotheses considered are that each of the permanent shocks are individually uncorrelated with option volatility, and that a linear combination of the permanent shocks and the fundamental transitory shock is also uncorrelated with option volatility.[92] The results in Table 4.1 highlight that these null hypotheses cannot be rejected at standard significance levels, and so they are consistent with option volatility being a valid instrument for equity prices growth.[93]

An additional reason to think that option volatility is a valid instrument in the current context is due to the sample under consideration. Although fundamental uncertainty shocks are likely to be relevant in the period following the financial crisis that began in 2007–2008, the importance of these shocks is less clear in the sample from June 1986 to December 2006. In particular, one would have to justify why option-implied volatility drifted upwards from June 1995 to March 2000 (see Figure 4.1), at the same time that equity prices in the United States grew substantially. Such a result is inconsistent with

91 See Hahn, Ham and Moon (2011).

92 Refer to Appendix 8.4 for the appropriate regression specification in the latter case.

93 Additional tests for conditional validity, for example of forecast dispersion being valid conditional on valuation confidence being valid, also fail to reject the hypothesis that both instruments are valid. Results are available on request.

Table 4.1: Instrument Validity Tests for Option-implied Equity Volatility		
Equation	Hausman test statistic	Hausman test statistic
Consumption[(a)]	0.05	0.04
Dividends[(a)]	0.04	0.01
Non-equity net wealth[(a)]	1.62	0.85
Labour income[(a)]	0.01	0.06
Equity quantities[(b)]	0.04	0.40
Critical value[(c)]	3.84	3.84
Exogenous instruments	$\beta_1 \mathbf{y}_{t-1}$ [(a)] Forecast dispersion[(a), (b)]	$\beta_1 \mathbf{y}_{t-1}$ [(a)] Valuation confidence[(a), (b)]
Sample	Jun 1986 to Dec 2006	Dec 1989 to Jun 2010

Notes: (a) The null hypothesis is $H_0 : E(z_t u_{i,t}^P) = 0$ vs the alternative $H_1 : E(z_t u_{i,t}^P) \neq 0$
(b) The null hypothesis is $H_0 : E(z_t \tilde{\varepsilon}_t^{P,T}) = 0$ vs the alternative $H_1 : E(z_t \tilde{\varepsilon}_t^{P,T}) \neq 0$
(c) Obtained from a Chi-squared distribution with one degree of freedom, and at the 5 per cent level of significance

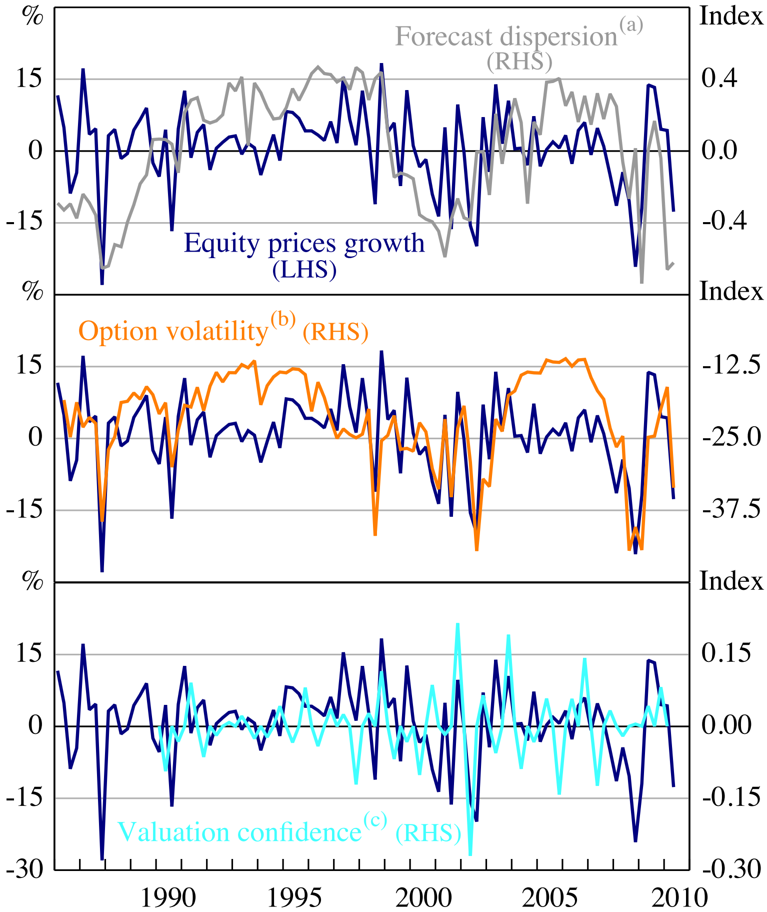
typical fundamental explanations of volatility, which usually suggest that higher volatility should be associated with greater fundamental uncertainty, lower investment and lower equity prices.

I now address whether these instruments are relevant from an empirical perspective. Figure 4.1 reports a graph of equity prices growth compared with each of the three candidate instruments – the second lag of detrended forecast dispersion, contemporaneous option-implied equity volatility, and the first lead of the second difference of the valuation confidence index. These instruments are selected because they have the highest reduced-form correlations with equity prices growth. I use detrended forecast dispersion to account for the upwards drift in earnings per share over time. The second difference, or change in momentum, of valuation confidence is used because it ensures that only information at a biannual frequency is actually used in estimation.[94]

Figure 4.1 highlights that all three variables appear to exhibit some correlation with equity prices growth, a result that is investigated more formally below. Forecast dispersion and

94 Recall that valuation confidence prior to July 2001 is only measured at a biannual frequency. Using the second difference, on a quarterly linear interpolation, in effect implies that only the change in momentum measured at six-monthly intervals is used. Under relatively weak assumptions, this measure will provide consistent IV estimates.

Figure 4.1: Equity Prices Growth and the Instruments for Mispricing



Notes: (a) Negative of the second lag of detrended forecast dispersion
(b) Negative of VXO stock market volatility index
(c) First lead of the second difference in the Valuation Confidence Index

Sources: Bloomberg; Thomson Reuters; Yale School of Management

option volatility appear to be most highly correlated with equity prices growth, although all three measures are consistent with an increase in forecast dispersion, uncertainty and concerns of overvaluation in the late 1990s. This preceded the sharp deceleration in prices growth observed in 2000.

Table 4.2 reports the results from first-stage regressions, and formal tests for instrument relevance, with respect to the permanent equations in Equation (4.14). To be clear, the two relevant first-stage regressions are of the form

$$\Delta y_{2j,t} = \phi_{1j}' \Delta \mathbf{y}_{1t-1} + \phi_{2j}' \xi_{1t-1} + \phi_{3j}' z_t + \varphi_{2j,t} \tag{4.19}$$

for $j = 1,2$, where $\xi_{1t-1} = \beta_1 \mathbf{y}_{t-1}$, and z_t is one of the candidate instruments.

The results are highlighted when using each candidate instrument in turn. The first test for relevance considered is that the instruments are under-identified. Essentially, it is a test of whether the excluded instruments, ξ_{1t-1} and z_t, are sufficiently correlated with the endogenous regressors, $\Delta y_{21,t}$ and $\Delta y_{22,t}$, for meaningful IV inference to be undertaken. Using the Cragg-Donaldson Wald statistic (rows 5 and 6), the null that the instruments are under-identified can be rejected at conventional significance levels.[95]

Although tests for the null of under-identification can be rejected, tests of the null that the instruments are only weakly correlated with the endogenous regressors cannot be rejected at conventional significance levels. In particular, using the test for weak instruments proposed by Stock and Yogo (2005), the critical values published by Stock and Yogo are greater than the relevant Cragg-Donaldson Wald F-statistics (rows 7 to 9). Moreover, F-statistics associated with the first-stage regressions for each of the instruments suggest that weak instruments could be a concern, especially with respect to the equity quantities measure (rows 1 to 2).

Table 4.2: Instrument Relevance Statistics – Permanent Equations

	Forecast dispersion	Option volatility	Valuation confidence
Equity quantities F-stat	2.76	2.77	1.71
	3.32	*3.26*	*2.09*
Equity prices F-stat	5.41	11.09	2.10
	3.60	*7.66*	*2.28*
CD Wald stat[(a)(b)]	*4.67*	*5.50*	*3.28*
	(*0.03*)	(*0.02*)	(*0.07*)
CD Wald F-stat[(a)]	2.11	2.37	1.47
	2.08	*2.45*	*1.46*
Critical value[(c)]	3.63	3.63	3.63

Notes: Statistics in italics are computed with robust standard errors
(a) Cragg-Donaldson test statistic
(b) P-values are in parentheses
(c) Based on 25 per cent maximal LIML size and assuming homoskedastic errors

[95] Although the lagged dividend to equity price ratio, $\beta_2' \mathbf{y}_{t-1}$, is also a valid instrument in the first stage, this instrument was found only to be weakly correlated with equity prices growth and is not therefore used when applying IV.

In view of the concern associated with weak instruments for these first-stage regressions, I use two strategies in the analysis that follows. The first is to use just-identified IV estimators, using each of the candidate instruments in turn. There is research suggesting that just-identified estimators can be viewed as approximately median-value unbiased with weak, though identified, instruments.[96] The second strategy, followed in Section 4.6, is to consider an alternative approach to estimation of the system in Equation (4.12) that requires fewer instruments in the procedure used to identify the mispricing shock. By reducing the number of endogenous variables, specifically eliminating the need to instrument for the measure of equity quantities, tests of the null that the instruments are weak can be rejected at conventional significance levels. As discussed in Section 4.6, the results using either strategy are comparable at short- to medium-term horizons.

Proceeding using the just-identified IV estimators, first-stage tests for instrument relevance in the transitory equation for equity quantities are obtained from the following first-stage regressions

$$\Delta \mathbf{y}_{1t} = \Theta_{11}\Delta \mathbf{y}_{t-1} + \theta_{12}\xi_{t-1} + \Theta_{13}\widehat{u}_t^{P,T} + \theta_{14}z_t + \eta_{1t} \tag{4.20}$$

$$\Delta y_{22t} = \Theta_{21}\Delta \mathbf{y}_{t-1} + \theta_{22}\xi_{t-1} + \Theta_{23}\widehat{u}_t^{P,T} + \theta_{24}z_t + \eta_{22t} \tag{4.21}$$

where $\widehat{u}_t^{P,T}$ are the residuals obtained from IV estimates of the permanent equations. Table 4.3 highlights that when using these residuals and each of the proxies for mispricing shocks as instruments (in turn), first-stage F-statistics are large and the null that these instruments are under-identified can be rejected at conventional significance levels.[97] Nonetheless, it should be noted that the F-statistics that provide a measure of the correlation between the excluded instruments and equity prices growth, rows 9 and 10, remain in a range where weak instruments could be a concern. I next present point estimates based on the just-identified IV estimators, before turning to the question of

[96] See, for example, Angrist, Imbens and Krueger (1999) and Angrist and Pischke (2008).

[97] Results are qualitatively similar for the transitory equation for equity prices, and are available on request.

whether concerns associated with weak instruments are likely to be biasing these point estimates.

Table 4.3: Instrument Relevance Statistics – Transitory Equation for Equity Quantities

	Forecast dispersion	Option volatility	Valuation confidence
Consumption F-stat	17.85	13.23	16.45
	11.40	*12.98*	*15.11*
Dividends F-stat	55.77	51.82	231.32
	54.24	*47.59*	*209.17*
Non-equity net worth F-stat	108.70	7.5×10^4	33.54
	60.00	1.6×10^5	*36.09*
Labour income F-stat	142.88	145.73	210.13
	160.69	*168.64*	*295.70*
Equity prices F-stat	7.14	7.48	4.89
	4.54	*4.88*	*3.86*
CD Wald stat[(a)(b)]	*8.90*	*16.90*	*4.62*
	(0.00)	*(0.00)*	*(0.03)*

Notes: Statistics in italics are computed with robust standard errors
(a) Cragg-Donaldson test statistic
(b) P-values are in parentheses

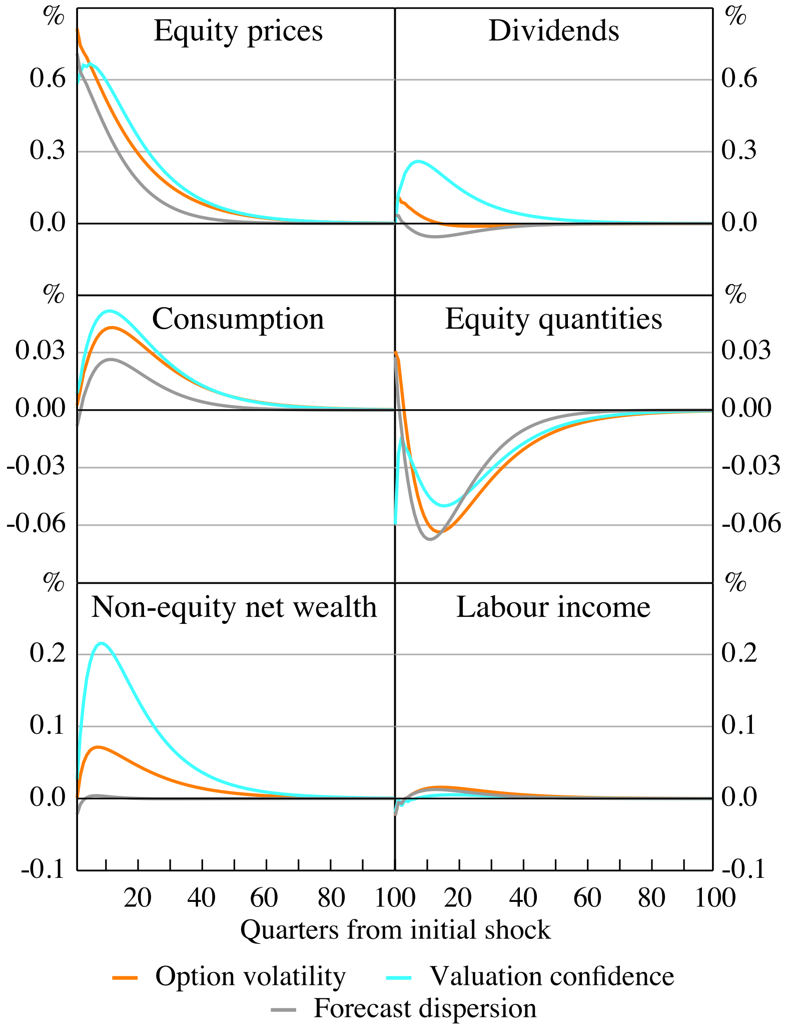
4.5. Results

Response Functions and Variance Decompositions

Figure 4.2 reports the impulse response functions associated with an exogenous 1 per cent increase in mispricing that is transitory, when using forecast dispersion, option volatility, and valuation confidence in turn as instruments.[98] The results highlight that the mispricing shock has a persistent effect on equity prices, with more than one quarter of its initial effect still being observed after five years. In addition, prices do not appear to over-correct in response to a positive mispricing shock, suggesting that both positive and negative mispricing shocks are required to generate the boom and bust patterns often referred to in qualitative accounts of equity market bubbles.

[98] Confidence intervals for these estimates are analysed in Section 4.6.

Figure 4.2: Impulse Response Functions to a Positive 1 Per Cent Mispricing Shock



In terms of the effects on other economic variables in the system, there is a persistent increase in consumption in response to a positive innovation in mispricing. Consumption exhibits a hump-shaped response with the maximum effect being in the order of 0.05 percentage points, which occurs around three years after the initial shock. This effect increases to around 0.5 percentage points in response to a one standard deviation mispricing shock.[99]

An interesting pattern can be observed in the impulse response function for the quantity of equity held by households (equity quantities). Initially, households increase their equity

99 A one standard deviation mispricing shock increases equity prices in the order of 9 to 10 per cent depending on the instrument used.

holdings, when using either forecast dispersion or option volatility as instruments, and then subsequently reduce these holdings as the effects of the mispricing shock begin to dissipate. One interpretation consistent with this result is that households are able to perceive misvaluation in equity markets following a mispricing shock. This could help to explain why households reduce their equity holdings before prices have fully reverted to their fundamental value. The reduction in equity holdings is also consistent with households using the proceeds of equity sales to increase their level of consumption.

Nevertheless, it should be noted that the reduction in equity quantities is smaller than the increase in prices, and so the value of households' equity holdings increases in response to the mispricing shock. In this light, an alternative interpretation of the reduction in equity holdings is that it represents portfolio rebalancing given US households' increased exposure to domestic equities.

Turning to corporate dividend policies, the estimated results are less precise with the magnitude of the impulse response functions sensitive to the choice of instrument used. However, there is evidence to suggest that firms may bring forward the timing of their dividend payments in response to a positive mispricing shock. This could reflect firms using dividends as a signal of their more favourable expectations concerning their future profitability. Interestingly, only when using forecast dispersion as an instrument are dividends subsequently underpaid relative to their value without mispricing.

For the non-equity net wealth measure, the impulse response function is close to zero when using forecast dispersion as an instrument, but positive when using either option volatility of valuation confidence. Thus, whether positive mispricing in equity markets affects other components of household net worth remains an open questions based on these estimates. With regard to labour income, all three measures suggest that positive mispricing shocks have little effect on the after-tax income earned by households.

Table 4.4 reports a forecast error variance decomposition of equity prices and quantities, with the contributions of fundamental shocks and mispricing shocks separately identified at short- to medium-term forecast horizons.[100] The results highlight that transitory mispricing shocks explain the majority of variation in equity prices growth at these forecast horizons. For example, when using forecast dispersion as an instrument, around two-thirds of the forecast error variance in prices growth can be explained by mispricing. In contrast, fundamental shocks are only able to explain between 16 and 40 per cent of the variation in equity prices growth at short to medium forecast horizons, depending on the instrument used. These results suggest that fundamental shocks to two of the most important variables emphasised by economic theory for equity pricing, consumption and dividends, explain some of the variation in equity prices. However, a larger part of this variation remains unexplained.

For equity quantities the proportion of the forecast error that can be explained by permanent and transitory fundamental shocks is much larger, in excess of 90 per cent according to these estimates. This suggests that changing fundamentals, as measured here, are much more able to explain variation in the quantity of domestic equity held by households, rather than the price of domestic equity.

Table 4.5 reports a similar forecast error variance decomposition for consumption and dividends. In this case fundamental shocks explain much of the short term variation in consumption. However, there is evidence to suggest that at medium horizons, from one to four years, a non-trivial fraction of the variation in consumption can be explained by mispricing shocks. These estimates suggest that although fundamental shocks are most important, non-fundamental transitory shocks do not necessarily have trivial effects on the consumption decisions of households.

[100] Note that fundamental shocks include both the reduced-form permanent and transitory fundamental shocks identified.

Table 4.4: Forecast Error Variance Decomposition				
	Equity prices		Equity quantities	
	Fundamental[(a)]	Mispricing	Fundamental[(a)]	Mispricing
Forecast dispersion				
2-quarter	0.33	0.67	0.99	0.01
1-year	0.34	0.66	0.99	0.01
4-year	0.40	0.60	0.95	0.05
Option volatility				
2-quarter	0.17	0.83	0.99	0.01
1-year	0.16	0.84	0.99	0.01
4-year	0.16	0.84	0.97	0.03
Valuation confidence				
2-quarter	0.40	0.60	0.95	0.05
1-year	0.16	0.84	0.97	0.03
4-year	0.17	0.83	0.96	0.04

Notes: (a) Fundamental includes both permanent and transitory fundamental shocks

Table 4.5: Forecast Error Variance Decomposition				
	Consumption		Dividends	
	Fundamental[(a)]	Mispricing	Fundamental[(a)]	Mispricing
Forecast dispersion				
2-quarter	0.96	0.04	0.98	0.02
1-year	0.95	0.05	0.99	0.01
4-year	0.86	0.14	0.98	0.02
Option volatility				
2-quarter	0.94	0.06	0.86	0.14
1-year	0.80	0.20	0.88	0.12
4-year	0.64	0.36	0.96	0.04
Valuation confidence				
2-quarter	0.78	0.22	0.88	0.12
1-year	0.66	0.34	0.74	0.26
4-year	0.64	0.36	0.71	0.29

Notes: (a) Fundamental includes both permanent and transitory fundamental shocks

For dividends, fundamental permanent and transitory shocks again explain the majority of the forecast error variation at all horizons. However, in line with the estimated impulse responses, there is mixed evidence on the importance of mispricing shocks for variation in dividends, with estimates ranging from close to zero to as much as 29 per cent of the variation in dividends at a medium-term horizon, depending on the instrument used.

Analysis of Historical Episodes

The previous forecast error variance decompositions compute the relative importance of alternative shocks over the full estimation sample. To provide additional insight into the importance of mispricing shocks in various historical episodes, Figure 4.3 compares each observed variable with a counterfactual estimate that assumes that all mispricing shocks are zero in the estimation sample from June 1986 to December 2006, and when using forecast dispersion as the relevant instrument. Estimates of the counterfactuals are conditioned taking the December 1985 and March 1986 observations as initial values.[101]

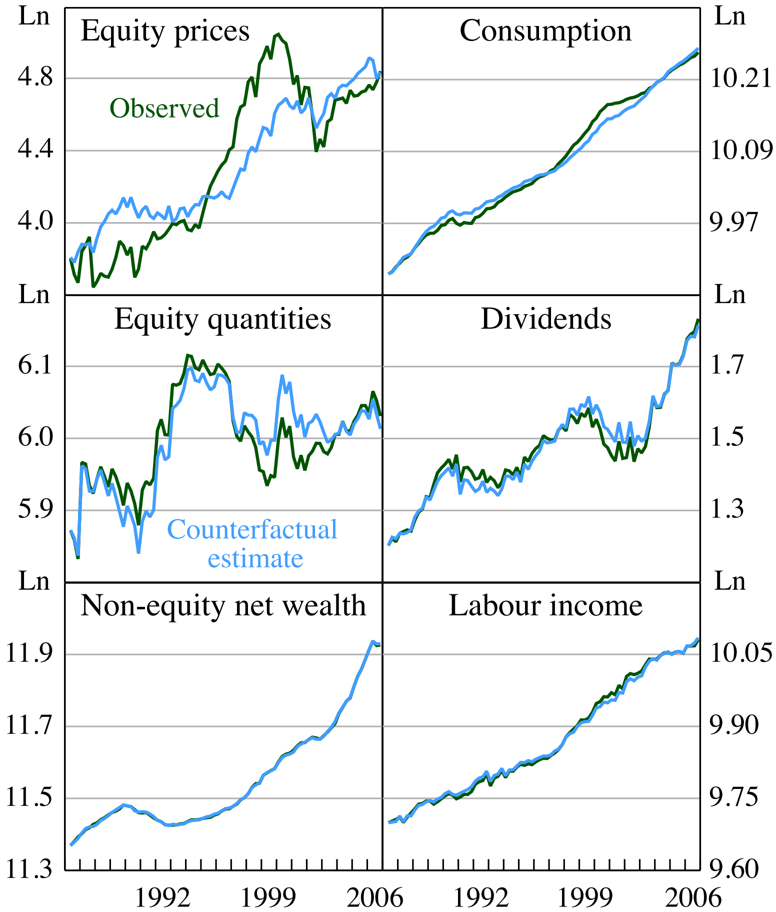
The equity prices panel in Figure 4.3 identifies two notable episodes of mispricing in the data. The first is under-valuation of US equity from 1987 to around 1995, which appears to be at least partly associated with the October 1987 stock market crash. After this time the equity market remains undervalued during the 1990–1991 recession. Prices then start to correct as the US economy recovers from this recession with prices being closer to their fundamental value from around 1992.

The second notable episode of mispricing is the familiar US dot-com bubble. From March 1995 to the height of the bubble in March 2000, the US equity market appears substantially overvalued. In March 2000, these estimates suggest that the US S&P 500 equity index was overvalued by around 45 per cent, before the subsequent collapse in equity prices was observed. Interestingly, equity was only slightly undervalued following the bursting of the dot-com bubble, and there was no clear evidence that equity markets were substantially mispriced in the lead-up to the financial crisis that began to emerge in September 2007.

Turning to consumption, it is clear that the level of consumption and the sign of mispricing shocks are positively correlated. In particular, observed per capita consumption appears

[101] To ensure that these results are not sensitive to initial conditions, I also compute the observed and counterfactual paths for longer time series of historical data. Under the assumption that the structural model is stable over a period prior to the sample used in estimation, the results are qualitatively similar.

Figure 4.3: Observed and Counterfactual Comparison



lower than its counterfactual estimate during the late 1980s and early 1990s, and is above its counterfactual estimate during the dot-com boom. Between March 1997 and December 2000, consumption grew on average 0.6 per cent per annum faster than in the absence of mispricing associated with the dot-com bubble. This suggests that mispricing did have an effect on the consumption decisions made by households.

Again, an interesting pattern emerges with respect to household equity holdings. During the early stages of the dot-com boom, between 1995 and 1997, the counterfactual estimate of the quantity of equity held lies below its corresponding observed value. However, during the latter stages of the bubble, from 1997 onwards, households actually reduce their exposure to US equity. This finding is somewhat surprising given that many qualitative accounts of bubbles do not contend that households reduce their equity holdings when concerns surrounding a bubble are raised. Nonetheless, these results suggest that households may either be selling equity, in part to fund higher consumption, or are consciously reducing their equity exposure due to concerns about mispricing.

For dividends, labour income, and non-equity net worth, the observed and counterfactual estimates are much more closely aligned. This suggests that mispricing shocks had much less effect on the observed variation in these variables, when using forecast dispersion as an instrument.

Another approach for obtaining insight into the relative contributions of alternative shocks is to use a historical forecast error decomposition. Figures 4.4 and 4.5 report the relative contributions to the two-year-ahead forecast errors of transitory mispricing shocks, transitory fundamental shocks, and the reduced-form permanent shocks, again when using forecast dispersion as an instrument. Consistent with the counterfactual analysis discussed previously, it is clear that transitory mispricing shocks explain a non-trivial fraction of the forecast errors in equity prices and consumption. But they explain only a relatively small fraction of the errors for equity quantities, dividends, labour income and non-equity net worth. For these latter variables, reduced-form permanent shocks provide the largest contribution to the forecast errors observed.

Figure 4.4: Two-year Horizon Forecast Error Contributions

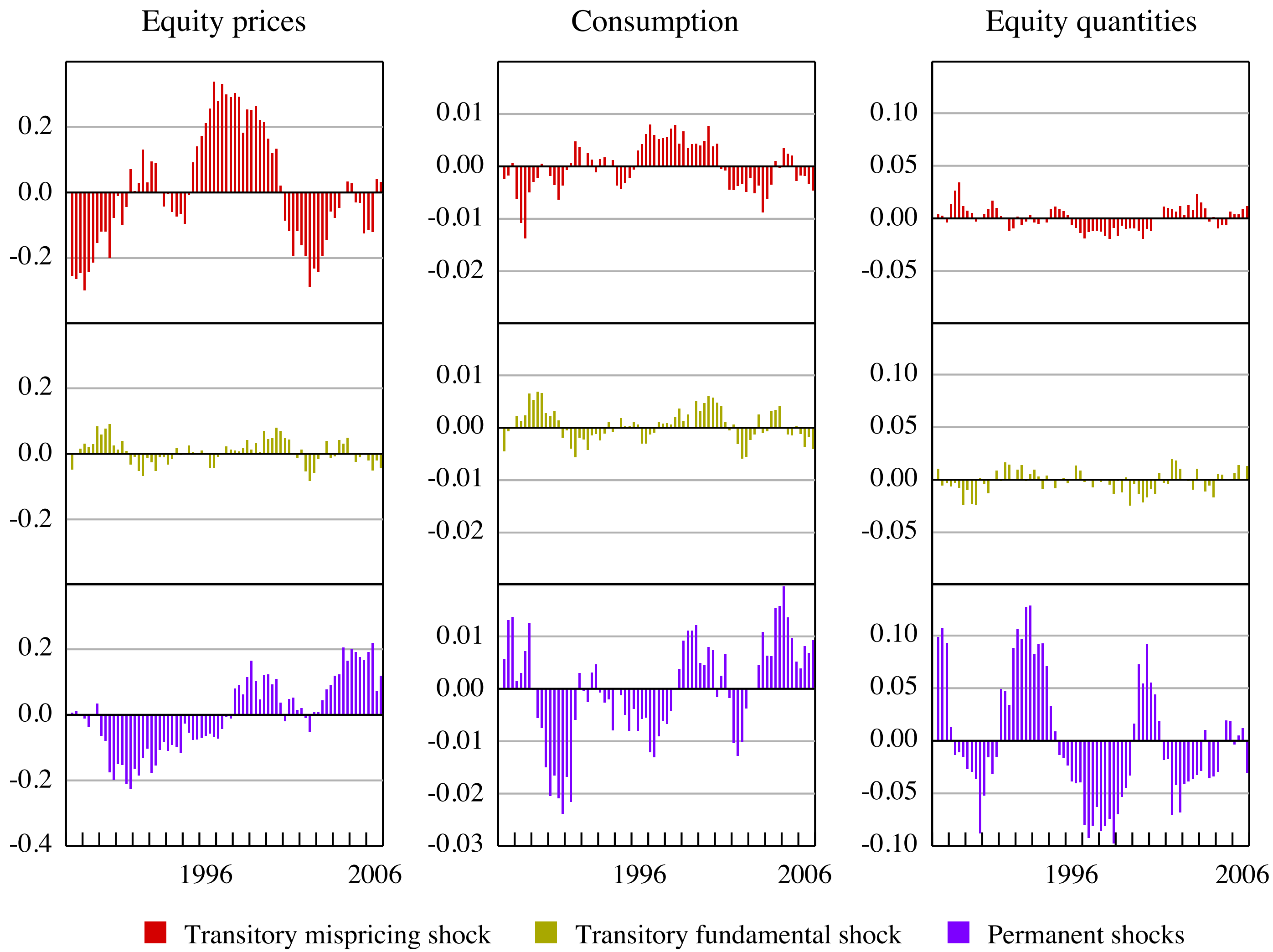
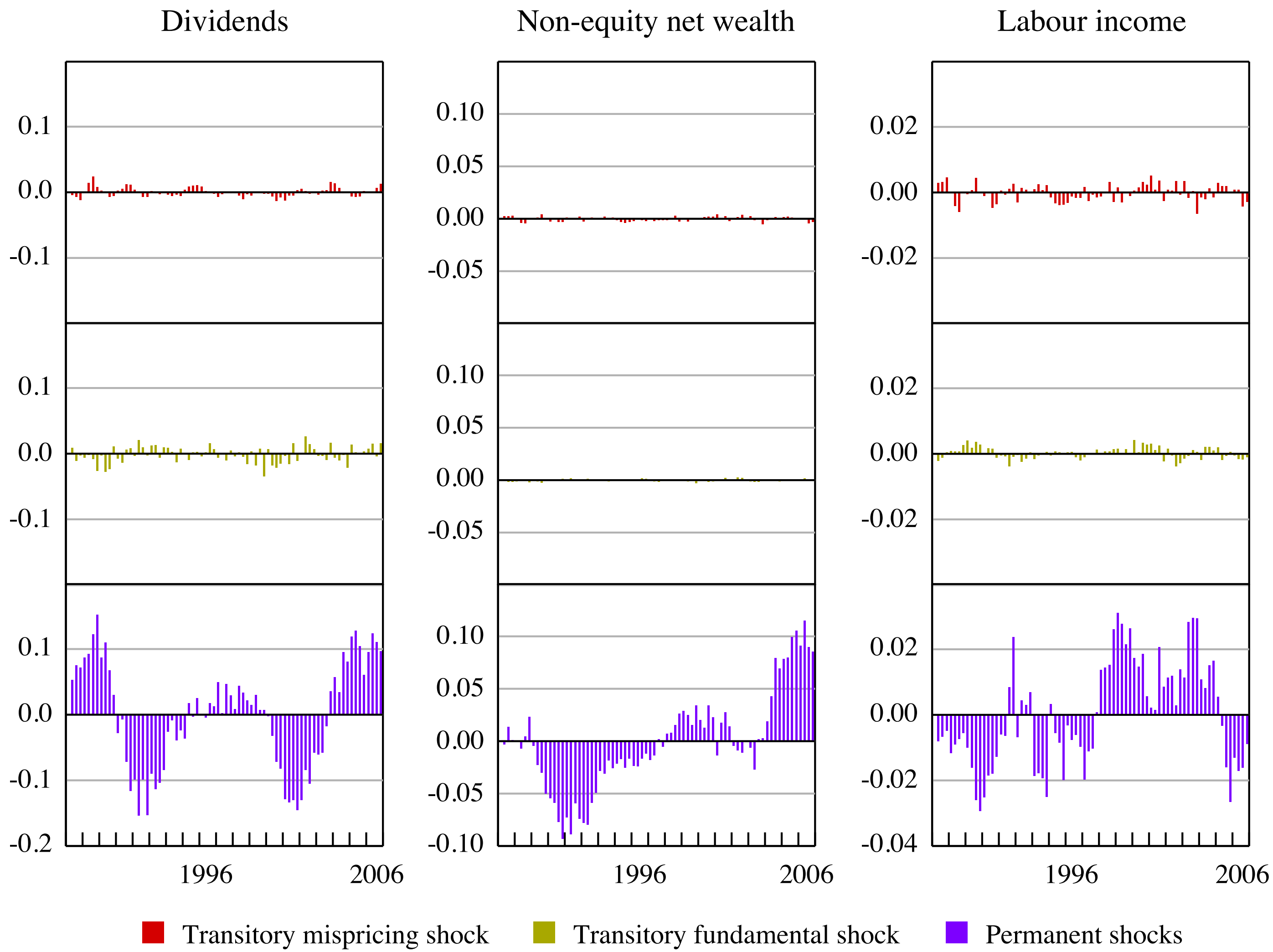


Figure 4.5: Two-year Horizon Forecast Error Contributions



Comparison with Existing Literature

The findings reported here are broadly similar to those obtained from related empirical literature. Focusing first on the effects of mispricing on consumption, Helbling and Terrones (2003) and Detken and Smets (2004) find a higher reduced-form correlation between consumption growth and equity prices growth during periods identified as equity market bubbles, and a weaker correlation during non-bubble periods, when using atheoretical procedures. The semi-structural approach to identification taken in this chapter confirms these findings. A rule of thumb obtained from the estimates presented here would suggest that a one time over-valuation shock in equity markets, in the order of 10 per cent, results in the level of consumption being about 0.5 percentage points higher around three years after the initial shock, all else constant.

Turning to equity prices, the response of equity prices to a non-fundamental transitory shock estimated here is similar to estimates obtained from Lee (1998), who focuses solely on the identification of non-transitory shocks by assuming that such shocks have no real effects (on either dividends or corporate earnings). According to Lee's estimates, a one standard deviation mispricing shock increases equity prices in the order of 4–6 per cent at the end of the first year, with the full effect of the shock dissipating after about 12 years. The estimates presented in this chapter are similar, with a one standard deviation mispricing shock increasing prices in the order of 5–7 per cent at the end of the first year, with the full effects dissipating in about 10 to 12 years. The estimated effects of mispricing on equity prices are very similar, notwithstanding different samples and data frequencies for estimation, and that Lee uses a different identification methodology for identifying non-fundamental transitory shocks.

One interesting difference with the previous literature on bubbles concerns the response of household equity holdings. Although this effect has received limited attention in previous empirical literature, it is interesting to note that the decline in household equity holdings in response to a positive mispricing shock is in contrast to qualitative accounts of bubbles.

It is also in contrast to research by Gilchrist *et al* (2005) who find that firms tend to issue a small positive amount of stock in response to a mispricing shock. The results here suggest that households do not appear to be purchasing additional equity during the latter stages of a bubble episode. If correct, this would imply that either foreign residents or US corporates would need to be purchasing any additional equity issued.

A second remaining question is on the effects of mispricing on dividends. Previous literature has assumed *a priori* that bubbles do not affect corporate dividend policies.[102] The results here suggest that bubbles can potentially influence corporate dividend policies, although the magnitude of this response is sensitive to the instrument used in identification.

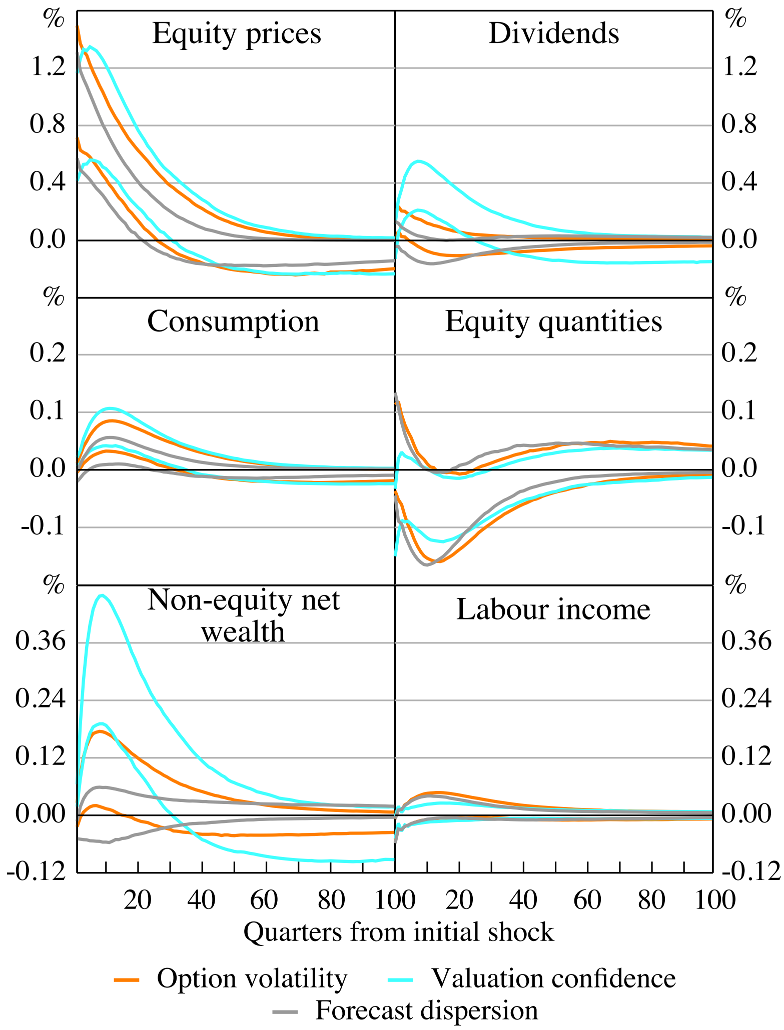
4.6. Robustness

Hall Percentile Confidence Intervals

To provide a gauge of the estimation uncertainty surrounding the previous point estimates, I construct 90 per cent confidence intervals for the estimated impulse response functions using a semi-parametric bootstrap (for further detail see Appendix 8.3). Figure 4.6 reports the bootstrapped confidence intervals using Hall's percentile method, and when each instrument is used in turn as a proxy for mispricing. Two results are noteworthy. First, allowing for estimation uncertainty in this way does not have a substantial effect on the results previously discussed. Mispricing shocks have positive and statistically significant effects on equity prices and consumption, and a negative and significant effect on the quantity of equity held by households irrespective of the instrument used. There is also little evidence of a statistically significant effect of mispricing shocks on household labour income. Again, the results concerning the response of dividends and net worth depend on the instrument used as a proxy for mispricing shocks.

[102] See, for example, Lee (1998).

Figure 4.6: Hall Confidence Intervals
5th and 95th percentiles



The second point to note is how estimation uncertainty changes when an instrument that is more weakly correlated with equity prices growth is used as a proxy for mispricing shocks. Comparing the width of the confidence intervals produced when valuation confidence, the weakest instrument, is used and the alternative instruments, it is clear that the confidence bands associated with valuation confidence are often wider. Thus, as one would expect, using weaker instruments in the identification procedure does result in greater uncertainty about the estimated impulse response functions.

Are Weak Instruments a Problem?

As mentioned previously, weak instruments are potentially of concern given the instrument relevance statistics highlighted in Tables 4.2 and 4.3. To explore whether weak instruments may be resulting in large finite sample biases, I also use an alternative identification procedure that requires fewer instruments when estimating the system in Equation (4.9).[103] In particular, by relaxing the restriction that mispricing shocks have only transitory effects, one can estimate the system in such a way that growth in equity prices is the only endogenous variable requiring a valid instrument. This is in contrast to the previous identification method that required valid instruments for both equity prices and equity quantities growth (the two variables directly perturbed by transitory innovations). I refer to the alternative identification procedure that requires fewer instruments as the ‘alternative’ strategy, and that previously used as the ‘benchmark’ strategy.

The advantage of the alternative strategy is that the concern with weak instruments can be mitigated, since only an instrument that is sufficiently correlated with equity prices growth is required. The disadvantage is that this approach is potentially inefficient, since it no longer uses the restriction that mispricing shocks are assumed to have only transitory effects.

Table 4.6 reports the first-stage results when using the alternative identification strategy. Tests of instrument relevance in this case are able to reject both null hypotheses that the instruments are under-identified and weakly identified when using forecast dispersion and option volatility as instruments, or if both of these measures and valuation confidence are all used collectively. The difference with regard to the previous results, where the null of weak instruments could not be rejected, is that only instruments that are sufficiently correlated with equity prices growth are required when applying the

[103] See Appendix 8.4 for further detail.

alternative strategy.[104] In contrast, the benchmark method required instruments that are sufficiently correlated with both equity prices and equity quantities growth.

Table 4.6: Instrument Relevance Statistics under Alternative Strategy

	Forecast dispersion	Option volatility	Valuation confidence	All instruments
CD Wald stat[(a)(b)]	11.19	23.78	5.40	47.61
	(0.00)	(0.00)	(0.02)	(0.00)
CD Wald F-stat[(a)]	9.84	20.91	4.73	13.07
Critical value[(c)]	8.96	8.96	8.96	4.36

Notes: All statistics are calculated assuming homoskedastic standard errors
(a) Cragg-Donaldson test statistic
(b) P-values are in parentheses
(c) Based on 15 per cent maximal LIML size

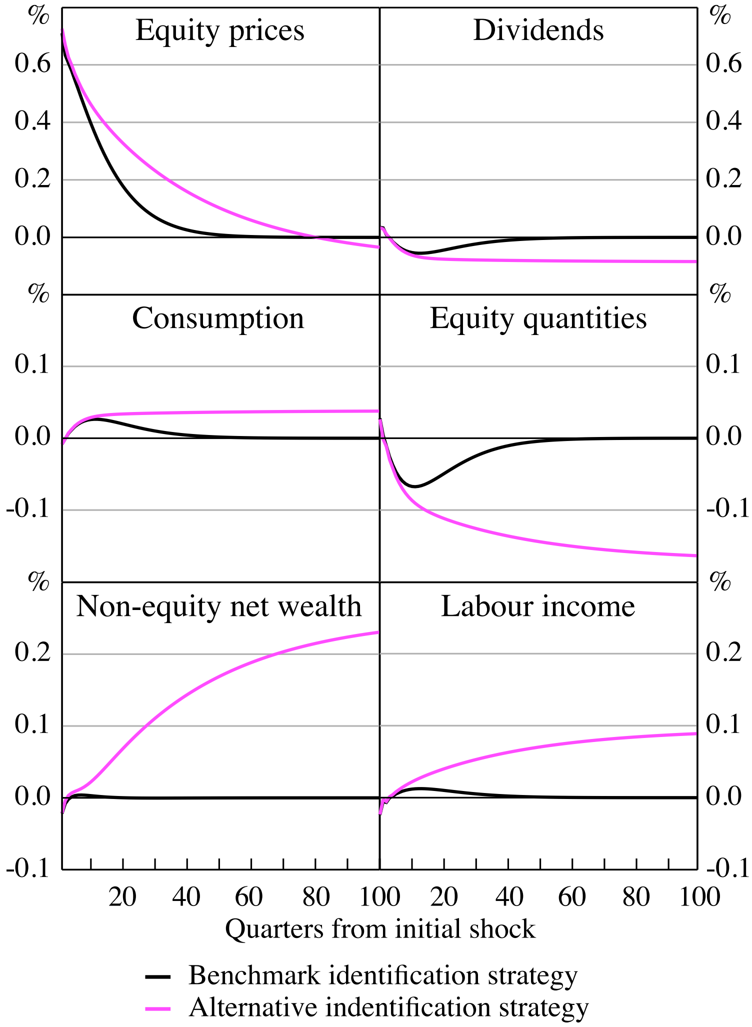
Figure 4.7 compares the estimated impulse response functions using the benchmark and alternative identification strategies when using forecast dispersion as the instrument for equity prices growth.[105] Although it can be observed that estimates using the alternative strategy now permit long-run effects in response to mispricing shocks, it is clear that the sign and magnitude of the responses estimated are very similar to those previously identified, and especially so at horizons of less than four years. This similarity suggests that, at least for short-term horizons, the presence of weak instruments under the benchmark strategy is unlikely to be distorting the sign or magnitude of the responses estimated. At longer horizons, the notable differences in the responses estimated suggests that imposing the restriction that mispricing shocks have only transitory effects is important. Without this restriction, estimates of the response to mispricing shocks under the alternative strategy remain noticeably different from zero.

Overall, the similarity of the results obtained when using just-identified IV estimators with different instruments, the relative widths of the bootstrapped confidence intervals, and the similarity in the short-term point estimates under the benchmark and alternative

[104] More specifically, lags of the cointegrating vectors, $\beta \mathbf{y}_t$, are no longer required as instruments.

[105] Results are similar if the other instruments are used.

Figure 4.7: **Impulse Response Functions under Benchmark and Alternative Identification Strategies**



identification strategies, all suggest that weak instruments are unlikely to be resulting in highly misleading inference. Although valid instruments that are more highly correlated with both equity prices and quantities growth would of course be desirable, there does appear to be sufficient information in the instruments proposed for identifying the sign and magnitude of the effects of transitory mispricing shocks.

4.7. Conclusion

This chapter proposes a new method for identifying the effects of equity market mispricing on household and firm decisions. The key assumptions used are that mispricing shocks only have transitory effects on the economy, and that there exist observable data that are correlated with these shocks, but are not correlated with perturbations to fundamentals.

The results highlighted in this chapter are qualitatively consistent with the idea that equity price bubbles have the potential to distort household and firm decisions. Consumption does appear to increase with a lag in response to a positive mispricing shock, and there is evidence to suggest that firms may change the timing of dividend payments as a signal of their optimism concerning future investment opportunities. However, quantitatively, the results estimated here are not consistent with the idea that mispricing has highly distortionary effects on household and firm decisions. Overall, the effects estimated are statistically significant and modest. There is also evidence to suggest that households reduce their exposure to equity in the latter stages of an equity price bubble.

Taken together, these results suggest that the effects of equity market mispricing are neither trivial, nor as large as has sometimes been claimed in qualitative accounts of bubbles. On balance, they suggest that periods identified as bubbles should be taken into consideration by policy-makers to the extent that variables such as consumption may be growing at a rate which differs to that justified by fundamentals. However, they do not imply that policy-makers should necessarily seek to address distorted equity price signals. This is a broader question which requires consideration of the various costs and benefits associated with using different policy tools to address particular episodes of equity market mispricing.

Chapter 5

Conclusion

This thesis provides insight into how financial volatility can matter for macroeconomic outcomes, and monetary policy. It highlights how distortions in credit and equity markets can affect price signals, which in turn can affect real economic decisions. It also highlights, for the case of a distorted credit market, how monetary policy should optimally respond when faced with financial shocks.

Chapter 2 confirmed that in an economy where both capital and credit matter, financial variables – in particular, net worth – can become relevant in a central bank's optimal policy calculus. Chapter 3 demonstrated how one form of endogenous expectations, optimal expectations, can amplify the effects of technology shocks and be consistent with mispriced equity and over-investment in the economy. Chapter 4 confirmed that empirical estimates of the effects of mispriced equity on household and firm decisions can be both economically and statistically significant.

This work raises additional questions for future research. For example, a natural extension to Chapter 2 would be to allow for micro-founded distortions in both credit and equity markets. One could then explore whether equity market distortions, or the interaction between equity and credit market distortions, are also relevant for optimal monetary policy.

For Chapter 3, a useful extension is to consider whether a dynamic stochastic general equilibrium model, augmented with optimal expectations, is able to better explain empirical volatility in investment and asset prices when compared with a standard rational expectations model. This could provide an alternative explanation to empirical puzzles such as the equity-premium puzzle, and insight into whether optimal expectations are a useful modelling device from an empirical perspective. For Chapter 4, it would be informative to investigate whether the effects of equity mispricing on investment and output are larger than the effects estimated for consumption and dividends. This would provide further insight into whether the different components of economic activity are affected asymmetrically by equity mispricing.

There is much research still to be done on sources of financial market volatility and how these matter for macroeconomic policy. Nonetheless, this thesis has provided important theoretical and empirical contributions to analysing the effects of equity and credit market distortions in the context of monetary policy.

Chapter 6

Appendix to Chapter 2

6.1. Approximation of Household Welfare

Approximation of Household Utility from Consumption

Taking a quadratic approximation of household utility from consumption, which is identical across all households, and using quadratic and linear approximations of the aggregate resource constraint (2.18) to eliminate the linear and quadratic terms in c_t respectively, I have[106]

$$U(C_t) = U_c Y \left(\begin{array}{c} y_t - s\delta i_t - s^e c_t^e \\ \frac{(1-\sigma)y_t^2}{2} - \left(s\delta + \sigma s^2 \delta^2\right) \frac{i_t^2}{2} - s^e \left(1 + \sigma s^e\right) \frac{\left(c_t^e\right)^2}{2} \\ + s\sigma\delta y_t i_t + \sigma s^e c_t^e y_t - s\sigma\delta s^e c_t^e i_t \end{array} \right)$$
$$+ t.i.p + O\left(\|\vartheta\|^3\right)$$

where $s \equiv \frac{K}{Y}$, $s^e \equiv \frac{\zeta C^e}{Y}$, $\sigma \equiv \sigma_c \frac{Y}{C}$, and $\sigma_c \equiv -\frac{U_{cc}(C)C}{U_c(C)}$.

Using a second-order approximation of capital accumulation (2.19), where I use $\xi_t \equiv \ln(1 - \nu_t)$ as a convenient renormalisation of the bankruptcy cost shock, I eliminate the

[106] For brevity, and without loss of generality, I abstract from taste and productivity shocks. All approximations are taken with respect to the natural logarithms of variables unless otherwise specified.

first-order term in investment in the above approximation to obtain

$$U\left(C_t\right)=U_cY\left(\begin{array}{c} y_t+\frac{(1-\sigma)y_t^2}{2}-s\left(k_{t+1}+\frac{k_{t+1}^2}{2}\right) \\ +s\left(1-\delta\right)\left(k_t+\frac{k_t^2}{2}\right)-s^ec_t^e \\ -\sigma s^2\delta^2\frac{i_t^2}{2}-s^e\left(1+\sigma s^e\right)\frac{\left(c_t^e\right)^2}{2} \\ +s\sigma\delta y_ti_t+\sigma s^ec_t^ey_t-s\sigma\delta s^ec_t^ei_t \\ +s\delta\Phi\left(\overline{\omega}\right)\left(\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right)+\xi_ti_t\right) \end{array}\right)$$
$$+t.i.p+O\left(\|\vartheta\|^3\right) \tag{6.1}$$

where I approximate around a stead state such that $K_t=K,\overline{\omega}_t=\overline{\omega},\xi=0,\frac{I}{K}=\frac{\delta}{\zeta}$, treat the recovery share parameter ξ_t as an expansion parameter that can be perturbed, and collect terms that are independent of policy in $t.i.p$.

Approximation of the Disutility of labour

Approximating $\int_0^1 V(H_t(i))di$, I have

$$\int_0^1 V(H_t(i))di=V_HH\left(E_ih_t\left(i\right)+\frac{1}{2}\left(1+\eta\right)E_ih_t\left(i\right)^2\right)+t.i.p+O\left(\|\vartheta\|^3\right) \tag{6.2}$$

where $\eta\equiv\frac{V_{HH}H}{V_H}$, $E_ih_t\left(i\right)\equiv\int_0^1 h_t\left(i\right)di$ and $E_ih_t\left(i\right)^2\equiv\int_0^1 h_t\left(i\right)^2di.$

Using the steady state values for the intratemporal household condition (2.3), the production technology (2.13), the optimal choice of the labour to capital ratio (2.14), real marginal cost (2.15), and the optimal pricing decision (2.16), it follows that

$$V_HH=\mu^{-1}\left(1-\chi\right)^{-1}\left(1-\alpha\right)YU_c$$

where $\mu^{-1}\equiv(1-\theta^{-1})$ is the inverse of the mark-up. Thus, (6.2) can be written as

$$\int_0^1 V(H_t(i))di=\frac{U_cY}{\mu\left(1-\chi\right)}\left(\left(1-\alpha\right)E_ih_t(i)+\frac{1}{2}\left(1-\alpha\right)\left(1+\eta\right)E_ih_t(i)^2\right) \tag{6.3}$$

Using the production function (2.13), and a second-order approximation of the final producer's demand function for intermediate goods (2.12), I have

$$(1-\alpha)E_i h_t(i) = y_t - \alpha E_i k_t(i) - (1-\alpha) z_t - \frac{1}{2}\mu^{-1} var_i x_t(i) + O\left(\|\vartheta\|^3\right) \tag{6.4}$$

Substituting (6.4) into (6.3) I obtain

$$\int_0^1 V(h_t(i))di = \frac{U_c Y}{\mu(1-\chi)} \left(\begin{array}{c} y_t - \alpha E_i k_t(i) - \frac{1}{2}\mu^{-1} var_i x_t(i) \\ +\frac{1}{2}(1-\alpha)(1+\eta) E_i h_t(i)^2 \end{array} \right) \tag{6.5}$$
$$+ t.i.p + O\left(\|\vartheta\|^3\right)$$

Using a second-order approximation of the capital aggregation condition across industries (2.21), (6.5) can be re-written as

$$\int_0^1 V(H_t(i))di = \frac{U_c Y}{\mu(1-\chi)} \left(\begin{array}{c} y_t - \alpha k_t - \frac{1}{2}\mu^{-1} var_i x_t(i) \\ +\frac{\alpha}{2} var_i k_t(i) + \frac{(1-\alpha)(1+\eta)}{2} var_i(h_t(i)) \\ +\frac{(1-\alpha)(1+\eta)}{2} (E_i h_t(i))^2 \end{array} \right)$$
$$+ t.i.p + O\left(\|\vartheta\|^3\right) \tag{6.6}$$

It is straightforward to show that $var_i(k_t(i))$ and $var_i(h_t(i))$ are proportional to $var_i(x_t(i))$ using first-order approximations of (2.3), (2.13) and (2.14),

$$var_i(h_t(i)) = \frac{1}{(1+\alpha\eta)^2} var_i(x_t(i)) + O\left(\|\vartheta\|^3\right)$$
$$var_i(k_t(i)) = \left(\frac{1+\eta}{1+\alpha\eta}\right)^2 var_i(x_t(i)) + O\left(\|\vartheta\|^3\right)$$

Substituting these relationships into (6.6) yields

$$\int_0^1 V(H_t(i))di = \frac{U_c Y}{\mu(1-\chi)} \begin{pmatrix} y_t - \alpha k_t + \frac{1}{2}\omega_x var_i x_t(i) \\ + \frac{(1-\alpha)(1+\eta)}{2}(E_i h_t(i))^2 \end{pmatrix} + t.i.p + O\left(\|\vartheta\|^3\right) \tag{6.7}$$

where $\omega_x \equiv \frac{\eta+1}{\alpha\eta+1} - \mu^{-1}$. Finally using first-order approximations of (2.12) and (2.13) to substitute out $E_i h_t(i)$, (6.7) can be written as

$$\int_0^1 V(H_t(i))di = \frac{U_c Y}{\mu(1-\chi)} \begin{pmatrix} y_t - \alpha k_t + \frac{\omega_x}{2} var_i x_t(i) \\ + \frac{(1-\alpha)(1+\eta)}{2}\left(\frac{y_t - \alpha k_t}{1-\alpha} - z_t\right)^2 \end{pmatrix} + t.i.p + O\left(\|\vartheta\|^3\right) \tag{6.8}$$

Combining (6.1) and (6.8) it follows that the quadratic approximation of average household felicity is given by

$$U(C_t) - \int_0^1 V(H_t(i))di = U_c Y \begin{pmatrix} \frac{(1-\sigma)y_t^2}{2} - s^2\sigma\delta^2\frac{i_t^2}{2} - \alpha\frac{k_t^2}{2} - s^e(1+\sigma s^e)\frac{(c_t^e)^2}{2} \\ -s^e c_t^e + s\sigma\delta y_t i_t + \sigma s^e c_t^e y_t - s\sigma\delta s^e c_t^e i_t \\ + s\delta\Phi(\overline{\omega})\left(\xi_t\widehat{\Phi}(\overline{\omega}) + \xi_t i_t\right) \\ -\frac{(1-\alpha)(1+\eta)}{2}\left(\frac{y_t-\alpha k_t}{1-\alpha} - z_t\right)^2 - \frac{\omega_x}{2} var_i x_t(i) \end{pmatrix} + t.i.p + O\left(\|\vartheta\|^3\right)$$

To obtain this result I have assumed that the subsidy on intermediate production ensures that the distortion associated with monopolistic competition is eliminated $(1-\chi)\mu = 1$, and that the steady state recovery rate is one ($e^{\xi} = 1$). I have also iterated out linear and quadratic terms involving capital using the result

$$
\begin{aligned}
E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(-sk_{t+1}+s\left(1-\delta\right)k_t+\alpha k_t\right) &= -sE_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(k_{t+1}-\beta^{-1}k_t\right) \\
&= s\beta^{-1}k_{t_0} \\
&= t.i.p \qquad (6.9)
\end{aligned}
$$

which also applies to second-order terms in capital, and is consistent with Takamura *et al* (2006).

For brevity, and without loss of generality, I abstract from productivity shocks (as well as taste shocks) assuming $z_t = 0$. I can then write the approximation of household welfare as

$$
\begin{aligned}
E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(U\left(C_t\right)-\int_0^1 V\left(H_t\left(i\right)\right)di\right) &= E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}U_cY\left(L_t^h\right) \\
&\quad + t.i.p + O\left(\|\vartheta\|^3\right)
\end{aligned}
$$

where

$$
\begin{aligned}
L_t^h \equiv &\left(\sigma+\left(\frac{\alpha+\eta}{1-\alpha}\right)\right)\frac{y_t^2}{2}+s^2\sigma\delta^2\frac{i_t^2}{2}+\omega_\pi\frac{\pi_t^2}{2} \\
&+\left(\alpha+\frac{\alpha^2\left(1+\eta\right)}{1-\alpha}\right)\frac{k_t^2}{2}+s^e\left(1+\sigma s^e\right)\frac{\left(c_t^e\right)^2}{2} \\
&+s^ec_t^e-s\sigma\delta y_ti_t-\frac{\alpha\left(1+\eta\right)}{1-\alpha}y_tk_t-\sigma s^ec_t^ey_t \\
&+s\sigma\delta s^ec_t^ei_t-s\delta\Phi\left(\overline{\omega}\right)\left(\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right)+\xi_ti_t\right)
\end{aligned}
$$

which corresponds to (2.26) in the main text and where $\omega_\pi \equiv \frac{\theta^2\gamma\omega_x}{(1-\gamma)(1-\gamma\beta)}$. In obtaining this last result I follow the approach discussed in Woodford (2003), Chapter 6, where it can be verified in the context of this chapter that $var_i\left(x_t\left(i\right)\right) = \theta^2 var_i\left(p_t\left(i\right)\right)$ and that

$$
\sum_{t=t_0}^{\infty}\beta^{t-t_0}var_i\left(p_t\left(i\right)\right) = \frac{\gamma}{\left(1-\gamma\right)\left(1-\gamma\beta\right)}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\pi_t^2+t.i.p+O\left(\|\vartheta\|^3\right)
$$

6.2. The Constraints in the Zero-Inflation Equilibrium

For the model with the credit friction and endogenous variation in net worth, the zero-inflation equilibrium values must satisfy the system (allowing for taste and productivity shocks):

$$
\begin{aligned}
k_{t+1}^{n} &= (1-\delta)\,k_t^n + \delta i_t^n + \delta \Phi(\overline{\omega})\,\xi_t \\
\sigma_c c_t^n &= \sigma_c E_t c_{t+1}^n + \upsilon \psi_t - \upsilon E_t \psi_{t+1} - r_t^{bn} \\
\sigma_c c_t^n &= \sigma_c E_t c_{t+1}^n - (1-\beta(1-\delta))\,E_t r_{t+1}^n + \widehat{q}_t^{\,n} - \beta(1-\delta)\,E_t \widehat{q_{t+1}}^{\,n} \\
&\quad + \upsilon \psi_t - \upsilon E_t \psi_{t+1} \\
\widehat{q}_t^{\,n} &= -\frac{\phi(\overline{\omega})\,(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_t - \Phi(\overline{\omega})\,\xi_t \\
0 &= \widehat{nw}_t^n - i_t^n - \frac{\phi(\overline{\omega})\,g(\overline{\omega},0)}{1-\Phi(\overline{\omega})}\xi_t + \Phi(\overline{\omega})\,\xi_t - \frac{f_{\overline{\omega}}(\overline{\omega})\,\overline{\omega}}{f(\overline{\omega})}\widehat{\overline{\omega}}_t^{\,n} \\
\widehat{nw_t} &= \varphi_F \widehat{F_t^e} + \varphi_k k_t^{en} + \varphi_r r_t^n + \varphi_q \widehat{q}_t^{\,n} \\
r_t^n &= y_t^n - k_t^n \\
0 &= \left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right) y_t^n - s\sigma\delta i_t^n - \sigma s^e c_t^{en} - \frac{\alpha(1+\eta)}{1-\alpha}k_t^n - (1+\eta)\,z_t - \upsilon \psi_t \\
\sigma_c c_t^n &= \sigma y_t^n - \sigma s \delta i_t^n - \sigma s^e c_t^{en} \\
c_t^{en} &= \frac{f_{\overline{\omega}}(\overline{\omega})\,\overline{\omega}}{f(\overline{\omega})}\widehat{\overline{\omega}}_t^{\,n} + \widehat{q}_t^{\,n} + i_t^n \\
k_{t+1}^{en} &= \frac{f_{\overline{\omega}}(\overline{\omega})\,\overline{\omega}}{f(\overline{\omega})}\widehat{\overline{\omega}}_t^{\,n} + i_t^n
\end{aligned}
$$

This is a first-order difference system with 11 equations in 11 unknowns that can be used to solve for the real rate of interest, r_t^{bn}, consistent with the absence of price and credit frictions.

6.3. Re-writing the Loss Function

I begin with the loss function (2.29)[107]

$$
\begin{aligned}
L_t \equiv & \left(\sigma + \frac{\alpha+\eta}{1-\alpha}\right)\frac{y_t^2}{2} + s^2\sigma\delta^2\frac{i_t^2}{2} + \omega_\pi\frac{\pi_t^2}{2} \\
& + \left(\alpha + \frac{\alpha^2(1+\eta)}{1-\alpha}\right)\frac{k_t^2}{2} + \sigma\left(s^e\right)^2\frac{\left(c_t^e\right)^2}{2} \\
& - s\sigma\delta y_t i_t - \frac{\alpha(1+\eta)}{1-\alpha} y_t k_t - \sigma s^e c_t^e y_t \\
& + s\sigma\delta s^e c_t^e i_t - s\delta\Phi(\overline{\omega})\left(\xi_t\widehat{\Phi}(\overline{\omega}_t) + \xi_t i_t\right)
\end{aligned}
$$

As noted in the main text, it is useful to re-write variables in terms of the log deviation from their respective zero-inflation equilibrium values. Abstracting from the effects of productivity shocks (z_t) and taste shocks (ψ_t) without loss of generality, conditional expectations of zero-inflation (hereafter natural) variables must satisfy the following real marginal cost condition in a zero-inflation equilibrium[108]

$$
0 = \left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right) y_{t|t_0}^n - s\sigma\delta i_{t|t_0}^n - \sigma s^e c_{t|t_0}^{en} - \frac{\alpha(1+\eta)}{1-\alpha} k_{t|t_0}^n \tag{6.10}
$$

where I use the notation $E_{t_0}\left(y_t^n\right) = y_{t|t_0}^n$. Using this condition, it follows that the loss function (2.29), for any given period t, can be written as

$$
\begin{aligned}
L_t \equiv & \left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right)\frac{\left(y_t - y_t^n\right)^2}{2} + s^2\sigma\delta^2\frac{\left(i_t - i_t^n\right)^2}{2} + \omega_\pi\frac{\pi_t^2}{2} \\
& + \alpha\frac{\left(k_t - k_t^n\right)^2}{2} + \left(\frac{\alpha^2(1+\eta)}{1-\alpha}\right)\frac{\left(k_t - k_t^n\right)^2}{2} + \sigma\left(s^e\right)^2\frac{\left(c_t^e - c_t^{en}\right)^2}{2} \\
& - s\sigma\delta\left(y_t - y_t^n\right)\left(i_t - i_t^n\right) - \frac{\alpha(1+\eta)}{1-\alpha}\left(y_t - y_t^n\right)\left(k_t - k_t^n\right) \\
& - \sigma s^e\left(c_t^e - c_t^{en}\right)\left(y_t - y_t^n\right) + s\sigma\delta s^e\left(c_t^e - c_t^{en}\right)\left(i_t - i_t^n\right) + \widetilde{R}_t
\end{aligned} \tag{6.11}
$$

[107] For brevity, I omit approximation errors in the analysis that follows.

[108] This result can be derived from analysis of the first-order approximation of the constraints in the zero-inflation equilibrium (see Appendix 6.2).

and where the remainder, $\widetilde{R}_t$ is given by

$$\begin{aligned}
\widetilde{R}_t \equiv\, & \frac{\alpha(1+\eta)}{1-\alpha}\alpha k_t^n k_t - \frac{\alpha(1+\eta)}{1-\alpha} y_t^n k_t - s\sigma\delta y_t^n i_t \\
& + s^2\sigma\delta^2 i_t^n i_t + \sigma\left(s^e\right)^2 c_t^{en} c_t^e - \sigma s^e y_t^n c_t^e \\
& + s\sigma\delta s^e c_t^{en} i_t + s\sigma\delta s^e i_t^n c_t^e + \alpha k_t^n k_t \\
& - s\delta\Phi(\overline{\omega})\left(\xi_t \widehat{\Phi}(\overline{\omega}_t) + \xi_t i_t\right)
\end{aligned} \tag{6.12}$$

The next two propositions establish that (6.11) can be rewritten as

$$\begin{aligned}
L_t \equiv\, & \left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right)\frac{\left(y_t - y_t^n\right)^2}{2} + s^2\sigma\delta^2\frac{\left(i_t - i_t^n\right)^2}{2} + \omega_\pi\frac{\pi_t^2}{2} \\
& + \alpha\frac{\left(k_t - k_t^n\right)^2}{2} + \left(\frac{\alpha^2(1+\eta)}{1-\alpha}\right)\frac{\left(k_t - k_t^n\right)^2}{2} + \sigma\left(s^e\right)^2\frac{\left(nw_t - nw_t^n - n_t^*\right)^2}{2} \\
& - s\sigma\delta\left(y_t - y_t^n\right)\left(i_t - i_t^n\right) - \frac{\alpha(1+\eta)}{1-\alpha}\left(y_t - y_t^n\right)\left(k_t - k_t^n\right) \\
& - \sigma s^e\left(nw_t - nw_t^n\right)\left(y_t - y_t^n\right) + s\sigma\delta s^e\left(nw_t - nw_t^n\right)\left(i_t - i_t^n\right) + t.i.p
\end{aligned}$$

where

$$n_t^* = \left\{\begin{array}{cc} 0 & \text{if } \beta^e < \beta \text{ and } \kappa = 1 \\ \frac{-\left(\frac{s\delta}{s^e}\frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})} f(\overline{\omega})\nu_t + \sigma_c c_t^n\right)}{\sigma s^e} & \text{if } 1 > \kappa\beta^e > \beta \text{ and } \kappa < \beta \end{array}\right\}$$

and that $n_t = 0$ if $\beta^e < \beta$ and $\kappa = 1$, and $n_t \neq 0$ if $1 > \kappa\beta^e > \beta$ and $\kappa < \beta$, which is consistent with (2.43) and the surrounding discussion in the main text.

I first consider the case where entrepreneurs do not save and so $\beta^e < \beta$ and $\kappa = 1$.

Proposition 6.1. *In the case that entrepreneurs do not save ($\beta^e < \beta$ and $\kappa = 1$) and entrepreneur consumption behaviour is described by (2.9) and (2.10) in the main text, to* *the first-order*

$$\widetilde{R}_t = t.i.p$$

$$c_t^e = \widehat{nw_t} = t.i.p$$

$$nw_t - nw_t^n = 0$$

Proof. It is sufficient to show that $\widetilde{R}_t$ is made up of terms that are independent of policy and $c_t^e = \widehat{nw_t} = t.i.p$. To begin, I substitute out $-\frac{\alpha(1+\eta)}{1-\alpha}\alpha k_t^n k_t$ from the definition of $\widetilde{R}_t$ (6.12) using the property that natural value of real marginal costs is zero (see 6.10)

$$\begin{aligned}
\widetilde{R}_t = & \left(s\sigma\delta i_t^n - \left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right) y_t^n + \sigma s c_t^{en}\right)\alpha k_t \\
& - \frac{\alpha\left(1+\eta\right)}{1-\alpha} y_t^n k_t - s\sigma\delta y_t^n i_t \\
& + s^2\sigma\delta^2 i_t^n i_t + \sigma\left(s^e\right)^2 c_t^{en} c_t^e - \sigma s^e y_t^n c_t^e \\
& + s\sigma\delta s^e c_t^{en} i_t + s\sigma\delta s^e i_t^n c_t^e + \alpha k_t^n k_t \\
& - s\delta\Phi\left(\overline{\omega}\right)\left(\xi_t \widehat{\Phi}\left(\overline{\omega}_t\right) + \xi_t i_t\right)
\end{aligned}$$

Next I substitute for δi_t using the first-order approximation of capital accumulation and drop terms that are independent of policy to obtain

$$\begin{aligned}
\widetilde{R}_t = & \left(\left(\sigma + \left(\frac{\alpha+\eta}{1-\alpha}\right)\right) y_t^n - s\sigma\delta i_t^n - \sigma s^e c_t^{en}\right)\alpha k_t \\
& - \frac{\alpha\left(1+\eta\right)}{1-\alpha} y_t^n k_t - \sigma y_t^n s\left(k_{t+1} - \left(1-\delta\right)k_t\right) \\
& + s\sigma\delta i_t^n s\left(k_{t+1} - \left(1-\delta\right)k_t\right) + \sigma\left(s^e\right)^2 c_t^{en} c_t^e - \sigma s^e y_t^n c_t^e \\
& + \sigma s^e c_t^{en} s\left(k_{t+1} - \left(1-\delta\right)k_t\right) + s\sigma\delta s^e i_t^n c_t^e + \alpha k_t^n k_t \\
& - s\delta\Phi\left(\overline{\omega}\right)\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right) - \Phi\left(\overline{\omega}\right)\xi_t s\left(k_{t+1} - \left(1-\delta\right)k_t\right)
\end{aligned}$$

Now I iterate out terms that are multiplicative in capital using

$$\begin{aligned}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\begin{pmatrix} s\left(m_{t+1}k_{t+1}\right) \\ -s\left(1-\delta\right)m_tk_t-\alpha m_tk_t \end{pmatrix} &= sE_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}\left(m_{t+1}k_{t+1}-\beta^{-1}m_tk_t\right) \\ &= -s\beta^{-1}m_{t_0}k_{t_0}+s\lim_{j\to\infty}\beta^jE_{t_0}\left(m_{t_0+j}k_{t_0+j}\right) \\ &= t.i.p\end{aligned}$$

for *any* given endogenous variable m_t in the system under consideration. This follows from the property that I approximate around a non-distorted steady state where $\beta^{-1} = 1-\delta+\frac{\alpha}{s}$ (noting $s\equiv\frac{K}{Y}$), that I focus on stationary policies such that $\lim_{j\to\infty}\beta^jE_{t_0}\left(m_{t_0+j}k_{t_0+j}\right)=0$, and that I restrict attention to policies that are optimal from a timeless perspective ($m_{t_0}k_{t_0}=t.i.p$). Thus, for the purposes of dealing with the remainder in period t, I can use the expression

$$m_t\left(sk_{t+1}-s\left(1-\delta\right)k_t\right)=\alpha m_tk_t+s\left(m_t-m_{t+1}\right)k_{t+1}$$

for *any* given endogenous variable m_t. Using this result, the remainder can be written as

$$\begin{aligned}\widetilde{R}_t = &-\alpha k_ty_t^n+\alpha k_t^nk_t \\ &+\sigma\left(y_{t+1}^n-y_t^n\right)sk_{t+1}-s\sigma\delta\left(i_{t+1}^n-i_t^n\right)sk_{t+1} \\ &-\sigma s^e\left(c_{t+1}^{en}-c_t^{en}\right)sk_{t+1} \\ &+s^ec_t^e\left(\sigma s^ec_t^{en}-\sigma y_t^n+s\sigma\delta i_t^n\right) \\ &-s\delta\Phi\left(\overline{\omega}\right)\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right)-\Phi\left(\overline{\omega}\right)\xi_t\alpha k_t \\ &-\Phi\left(\overline{\omega}\right)\left(\xi_t-\xi_{t+1}\right)sk_{t+1}\end{aligned} \tag{6.13}$$

I now use the first-order approximations of the household Euler equation and the aggregate resource constraint (in the zero-inflation equilibrium) to obtain

$$\begin{aligned}\widehat{q^n_{t|t_0}} &= \sigma\left(y^n_{t|t_0} - y^n_{t|t_0+1}\right) - s\sigma\delta\left(i^n_{t|t_0} - i^n_{t|t_0+1}\right) - \sigma s^e\left(c^{en}_{t|t_0} - c^{en}_{t|t_0+1}\right) \\ &\quad + \left(1-\beta\left(1-\delta\right)\right)\left(y^n_{t|t_0+1} - k^n_{t|t_0+1}\right) + \beta\left(1-\delta\right)\widehat{q^n_{t|t_0+1}}\end{aligned}$$

which I can then use to eliminate the following term in $\widetilde{R}_t$

$$\left(\sigma\left(y^n_t - y^n_{t+1}\right) - s\sigma\delta\left(i^n_t - i^n_{t+1}\right) - \sigma s^e\left(c^{en}_t - c^{en}_{t+1}\right)\right)sk_{t+1}$$

The remainder (6.13) becomes

$$\begin{aligned}\widetilde{R}_t &= -\alpha k_t y^n_t + \alpha k^n_t k_t \\ &\quad + \left(\left(1-\beta\left(1-\delta\right)\right)\left(y^n_{t+1} - k^n_{t+1}\right) - \widehat{q}_t^{\,n} + \beta\left(1-\delta\right)\widehat{q_{t+1}}^{\,n}\right)sk_{t+1} \\ &\quad + s^e c^e_t\left(\sigma s^e c^{en}_t - \sigma y^n_t + s\sigma\delta i^n_t\right) \\ &\quad - s\delta\Phi\left(\overline{\omega}\right)\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right) - \Phi\left(\overline{\omega}\right)\xi_t\alpha k_t - \Phi\left(\overline{\omega}\right)\left(\xi_t - \xi_{t+1}\right)sk_{t+1}\end{aligned}$$

Iterating out $\left(1-\beta\left(1-\delta\right)\right)\left(y^n_{t+1} - k^n_{t+1}\right)sk_{t+1} - \alpha\left(y^n_t - k^n_t\right)$
(noting $1-\beta\left(1-\delta\right) = \alpha\beta$ in steady state) I have

$$\begin{aligned}\widetilde{R}_t &= \left(\beta\left(1-\delta\right)\widehat{q_{t+1}}^{\,n} - \widehat{q}_t^{\,n}\right)sk_{t+1} \\ &\quad + s^e c^e_t\left(\sigma s^e c^{en}_t - \sigma y^n_t + s\sigma\delta i^n_t\right) \\ &\quad - s\delta\Phi\left(\overline{\omega}\right)\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right) - \Phi\left(\overline{\omega}\right)\xi_t\alpha k_t - \Phi\left(\overline{\omega}\right)\left(\xi_t - \xi_{t+1}\right)sk_{t+1}\end{aligned} \tag{6.14}$$

Using Lemma (6.1) (see the end of Proposition 6.2) it follows that

$$\begin{aligned}c^e_t &= \widehat{nw}_t \\ &= t.i.p\end{aligned}$$

with respect to the loss function. This follows directly from the fact that I focus on an equilibrium where non-defaulting entrepreneurs are assumed to be sufficiently impatient that they consume all of their available resources once their investment return has been realised. Using this property of the equilibrium where entrepreneurs do not save, the remainder can be written as

$$\widetilde{R}_t = \left(\beta\left(1-\delta\right)\widehat{q_{t+1}}^n - \widehat{q}_t^n - \Phi\left(\overline{\omega}\right)\left(\xi_t - \xi_{t+1}\right)\right)sk_{t+1} \\ - s\delta\Phi\left(\overline{\omega}\right)\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right) - \Phi\left(\overline{\omega}\right)\xi_t\alpha k_t \tag{6.15}$$

Using Lemma (6.2) it follows that in the flexible price equilibrium, asset prices are determined by shocks in the recovery rate

$$\widehat{q}_t^n = -\frac{\phi\left(\overline{\omega}\right)\left(1-g\left(\overline{\omega},0\right)\right)}{1-\Phi\left(\overline{\omega}\right)}\xi_t - \Phi\left(\overline{\omega}\right)\xi_t$$

Substituting this expression into (6.15) and iterating out the term $\left(1-\beta\left(1-\delta\right)\right)\Phi\left(\overline{\omega}\right)\left(\xi_{t+1}\right)sk_{t+1} - \alpha\Phi\left(\overline{\omega}\right)\xi_t k_t$ I obtain

$$\widetilde{R}_t = \left(\beta\left(1-\delta\right)\left(-\frac{\phi\left(\overline{\omega}\right)\left(1-g\left(\overline{\omega},0\right)\right)}{1-\Phi\left(\overline{\omega}\right)}\xi_{t+1}\right) + \frac{\phi\left(\overline{\omega}\right)\left(1-g\left(\overline{\omega},0\right)\right)}{1-\Phi\left(\overline{\omega}\right)}\xi_t\right)sk_{t+1} \\ - s\delta\Phi\left(\overline{\omega}\right)\xi_t\widehat{\Phi}\left(\overline{\omega}_t\right)$$

Using $\Phi\left(\overline{\omega}\right)\widehat{\Phi\left(\overline{\omega}_t\right)} = \phi\left(\overline{\omega}\right)\overline{\omega}\widehat{\overline{\omega}}_t$ and Lemma (6.3), it follows that the above expression can be re-written as

$$\widetilde{R}_t = \left(\beta\left(1-\delta\right)\left(-\frac{\phi\left(\overline{\omega}\right)\left(1-g\left(\overline{\omega},0\right)\right)}{1-\Phi\left(\overline{\omega}\right)}\xi_{t+1}\right) + \frac{\phi\left(\overline{\omega}\right)\left(1-g\left(\overline{\omega},0\right)\right)}{1-\Phi\left(\overline{\omega}\right)}\xi_t\right)sk_{t+1} \\ - s\delta\xi_t\phi\left(\overline{\omega}\right)\frac{f\left(\overline{\omega}\right)}{f_{\overline{\omega}}\left(\overline{\omega}\right)}\left(\widehat{nw_t} - i_t - \frac{\phi\left(\overline{\omega}\right)g\left(\overline{\omega},0\right)}{1-\Phi\left(\overline{\omega}\right)}\xi_t + \Phi\left(\overline{\omega}\right)\xi_t\right)$$

Again noting that $\widehat{nw_t} = c_t^e = t.i.p$ it follows that

$$\widetilde{R}_t = \left(\beta\left(1-\delta\right)\frac{\phi\left(\overline{\omega}\right)f\left(\overline{\omega}\right)}{f_{\overline{\omega}}\left(\overline{\omega}\right)}\xi_{t+1} - \frac{\phi\left(\overline{\omega}\right)f\left(\overline{\omega}\right)}{f_{\overline{\omega}}\left(\overline{\omega}\right)}\xi_t\right)sk_{t+1}$$
$$+s\delta\phi\left(\overline{\omega}\right)\frac{f\left(\overline{\omega}\right)}{f_{\overline{\omega}}\left(\overline{\omega}\right)}i_t\xi_t + t.i.p$$

where I have used that in the steady state without monitoring costs

$$f\left(\overline{\omega}\right) = 1 - g\left(\overline{\omega},0\right)$$
$$f_{\overline{\omega}}\left(\overline{\omega}\right) = \Phi\left(\overline{\omega}\right) - 1$$

Again substituting for investment using a linear approximation of capital accumulation and then iterating out the remaining terms that are multiplicative in capital I have

$$\widetilde{R}_t = \beta s\left(1-\delta\right)\frac{\phi\left(\overline{\omega}\right)f\left(\overline{\omega}\right)}{f_{\overline{\omega}}\left(\overline{\omega}\right)}\left(\xi_{t+1}k_{t+1} - \beta^{-1}\xi_t k_t\right)$$
$$= t.i.p$$

which verifies the desired result. □

I now consider the case where entrepreneurs save ($1 > \kappa\beta^e > \beta$ and $\kappa < \beta$).

Proposition 6.2. *In the case that entrepreneurs do save ($1 > \kappa\beta^e > \beta$ and $\kappa < \beta$) and their consumption behaviour is described by (2.9) and (2.11),*

$$\widetilde{R}_t = -\sigma\left(s^e\right)^2\left(n_t^*\right)n_t + t.i.p$$
$$\widehat{nw}_t - \widehat{nw}_t^n = c_t^e - c_t^{en} + O\left(\left\|\vartheta\right\|^2\right)$$
$$n_t^* = \frac{s\delta\frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})}f\left(\overline{\omega}\right)\nu_t + s^e\sigma_c c_t^n}{\sigma\left(s^e\right)^2}$$

Proof. The steps in this proof are almost identical to those used in the proof of Proposition (6.1). The only exception is that the policymaker must now keep track of entrepreneurial consumption and net worth, given that these variables are no longer independent of policy. Using Lemma (6.1), it is straightforward to establish (irrespective of whether entrepreneurs save or not)

$$c_t^e = \frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})}\xi_t + \widehat{nw}_t + O\left(\|\vartheta\|^2\right) \tag{6.16}$$

and so

$$\widehat{nw}_t - \widehat{nw}_t^n = c_t^e - c_t^{en} + O\left(\|\vartheta\|^2\right)$$

I now re-evaluate the remainder for the equilibrium in which entrepreneurs save all of their available resources before consuming at retirement. Applying the same reasoning that is used in Proposition (6.1), but retaining terms in c_t^e and $\widehat{nw}_t$, that are no longer independent of policy, it can be observed that

$$\widetilde{R}_t = s^e c_t^e \left(\sigma s^e c_t^{en} - \sigma y_t^n + s\sigma\delta i_t^n\right) - s\delta\phi(\overline{\omega})\frac{f(\overline{\omega})}{f_{\overline{\omega}}(\overline{\omega})}\widehat{nw_t}\xi_t + t.i.p$$

Using (6.16) and a first-order approximation of the resource constraint (in the zero-inflation equilibrium),

$$-\sigma_c c_t^n = \sigma s^e c_t^{en} - \sigma y_t^n + s\sigma\delta i_t^n$$

I have

$$\widetilde{R}_t = \left(-s^e\sigma_c c_t^n - s\delta\phi(\overline{\omega})\frac{f(\overline{\omega})}{f_{\overline{\omega}}(\overline{\omega})}\xi_t\right)\widehat{nw_t} + t.i.p$$

where

$$\widehat{nw_t} = \varphi_F \widehat{F_t^e} + \varphi_k k_t^e + \varphi_r r_t + \varphi_q q_t$$

From a first-order approximation of the definition of the recovery rate shock $e^{\xi_t} \equiv 1 - \nu_t$

$$\xi_t = -\nu_t + O\left(\|\vartheta\|^2\right)$$

where ν_t is the proportion of funds lost in the case of bankruptcy (with $\nu = 0$).[109] Using this result, and that in steady state, $f_\omega(\overline{\omega}) = \Phi(\overline{\omega}) - 1$, the desired result

$$\widetilde{R}_t = \left(-s\delta\frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})}f(\overline{\omega})\nu_t - s^e\sigma_c c_t^n\right)n_t + t.i.p$$

follows. □

The rest of this appendix discusses the lemmas used in Propositions (6.1) and (6.2).

Lemma 6.1. *In either of the saving or no saving equilibriums,* $c_t^e = \frac{\phi(\overline{\omega})}{1-\Phi(\overline{\omega})}\xi_t + \widehat{nw}_t$. *Moreover, specific to the equilibrium where entrepreneurs do not save (*$\beta^e < \beta$ *and* $\kappa = 1$*),* $\widehat{nw}_t = t.i.p$ *and so* $c_t^e = t.i.p$ *.*

Proof. To establish the first result, I begin by taking a first-order approximation of the derivative of the expected share of investment returns accruing to financial intermediaries with respect to the default threshold, (2.5),

$$g_{\overline{\omega}_t}\left(\widehat{\overline{\omega}_t, 1-e^{\xi_t}}\right) = \frac{\phi(\overline{\omega})}{\widetilde{f_{\overline{\omega}_t}}(\overline{\omega})}\xi_t + \widetilde{\widehat{f_{\overline{\omega}_t}(\overline{\omega}_t)}} \tag{6.17}$$

Combining (6.17) with the optimal choice of default threshold, (2.33), I have

$$\widehat{q}_t + \widehat{f(\overline{\omega}_t)} = \frac{-\phi(\overline{\omega})}{\widetilde{f}_{\overline{\omega}_t}(\overline{\omega})}\xi_t + \left(1 - q_t . g\left(\widehat{\overline{\omega}_t, 1-e^{\xi_t}}\right)\right) \tag{6.18}$$

Using the investment supply equation (2.34) and (6.18)

$$\widehat{q}_t + \widehat{f(\overline{\omega}_t)} = \frac{-\phi(\overline{\omega})}{\widetilde{f}_{\overline{\omega}_t}(\overline{\omega})}\xi_t + \widehat{nw}_t - i_t \tag{6.19}$$

[109] ν_t is in levels and not in logarithmic form.

Combining (6.19) with (2.38), I obtain

$$c_t^e = \widehat{nw_t} - \frac{\phi\left(\overline{\omega}\right)}{\widetilde{f}_{\overline{\omega}_t}\left(\overline{\omega}\right)}\xi_t \tag{6.20}$$

which establishes the first-result. Note that this result applies in both the equilibrium where entrepreneurs do not save, and in the equilibrium where entrepreneurs save.

To establish the second result, that is specific to the equilibrium where entrepreneurs do not save, recall that entrepreneurial net worth is given by

$$nw_t = F_t^e + \left(R_t + q_t\left(1-\delta\right)\right)K_t^e$$

Combining the initial condition that entrepreneurs have no starting capital,[110] with the optimality condition that $K_{t+1}^e = 0$ for all $t \geq t_0$ *in the equilibrium where entrepreneurs do not save*, it follows that

$$nw_t = F_t^e \quad \text{for all } t \geq t_0$$

and thus

$$\begin{aligned} \widehat{nw_t} &= \widehat{F_t^e} \\ &= t.i.p \end{aligned} \tag{6.21}$$

since the lump-sum transfer to entrepreneurs is not a control variable for the central bank. Using (6.21) and (6.20), it also follows that

$$c_t^e = t.i.p$$

which establishes the second result. □

[110]Note that this is consistent with the economy beginning at its deterministic steady state where $K_{t_0}^e = 0$, when entrepreneurs do not save.

Lemma 6.2. $\widehat{q_t} = -\frac{\phi(\overline{\omega})(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_t - \Phi(\overline{\omega})\xi_t$

Proof. I begin with the definition

$$\left(1-q_t.g\widehat{\left(\overline{\omega}_t, 1-e^{\xi_t}\right)}\right) \equiv \ln\left(\frac{1-q_t.g\left(\overline{\omega}_t, 1-e^{\xi_t}\right)}{1-q.g(\overline{\omega},0)}\right)$$

Taking a first approximation of $\ln\left(1-q_t.g\left(\overline{\omega}_t, 1-e^{\xi_t}\right)\right)$ around the deterministic steady state without monitoring costs where $q=1$ and $\xi=0$ it follows that

$$\left(1-q_t.g\widehat{\left(\overline{\omega}_t, 1-e^{\xi_t}\right)}\right) = -\frac{g(\overline{\omega},0)}{1-g(\overline{\omega},0)}\widehat{q_t} - \frac{g(\overline{\omega},0)}{1-g(\overline{\omega},0)}g\widehat{\left(\overline{\omega}_t, 1-e^{\xi_t}\right)} \tag{6.22}$$

Using a first-order approximation of the expected share of investment returns that accrues to banks (2.5),

$$g\widehat{\left(\overline{\omega}_t, 1-e^{\xi_t}\right)} = \frac{-f(\overline{\omega})}{1-f(\overline{\omega})}\widehat{f(\overline{\omega}_t)} + \frac{\Phi(\overline{\omega})}{1-f(\overline{\omega})}\xi_t \tag{6.23}$$

Substituting (6.23) into (6.22) implies

$$\left(1-q_t.g\widehat{\left(\overline{\omega}_t, 1-e^{\xi_t}\right)}\right) = -\frac{g(\overline{\omega},0)}{1-g(\overline{\omega},0)}\widehat{q_t} + \widehat{f(\overline{\omega}_t)} - \frac{\Phi(\overline{\omega})}{1-g(\overline{\omega},0)}\xi_t \tag{6.24}$$

where $f(\overline{\omega}) = 1-g(\overline{\omega},0)$. Combining (6.18) with (6.24) it follows that

$$\widehat{q_t} = -\frac{\phi(\overline{\omega})(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_t - \Phi(\overline{\omega})\xi_t \tag{6.25}$$

where I have used the property that $\widetilde{f_{\overline{\omega}_t}}(\overline{\omega}) = 1-\Phi(\overline{\omega})$ in steady state. $\square$

Lemma 6.3. $(\overline{\omega})\widehat{\overline{\omega}}_t = \frac{f(\overline{\omega})}{f_{\overline{\omega}}(\overline{\omega})}\left(\widehat{nw_t} - i_t - \frac{\phi(\overline{\omega})g(\overline{\omega},0)}{1-\Phi(\overline{\omega})}\xi_t + \Phi(\overline{\omega})\xi_t\right)$

Proof. I begin by rearranging (6.24)

$$\widehat{f(\overline{\omega}_t)} = \left(1-q_t.g\widehat{\left(\overline{\omega}_t, 1-e^{\xi_t}\right)}\right) + \frac{g(\overline{\omega},0)}{1-g(\overline{\omega},0)}\widehat{q_t} + \frac{\Phi(\overline{\omega})}{1-g(\overline{\omega},0)}\xi_t$$

Using the investment supply condition (2.34) and (6.25), the above expression becomes

$$\begin{aligned}\widehat{f(\overline{\omega}_t)} &= \widehat{nw_t} - i_t + \frac{g(\overline{\omega},0)}{1-g(\overline{\omega},0)}\left(-\frac{\phi(\overline{\omega})(1-g(\overline{\omega},0))}{1-\Phi(\overline{\omega})}\xi_t - \Phi(\overline{\omega})\xi_t\right) \\ &+ \frac{\Phi(\overline{\omega})}{1-g(\overline{\omega},0)}\xi_t \end{aligned} \tag{6.26}$$

Using $\widehat{f(\overline{\omega}_t)} = \frac{f_\omega(\overline{\omega})\overline{\omega}}{f(\overline{\omega})}\widehat{\overline{\omega}}_t$ in (6.26) I have

$$(\overline{\omega})\widehat{\overline{\omega}}_t = \frac{f(\overline{\omega})}{f_{\overline{\omega}}(\overline{\omega})}\left(\widehat{nw_t} - i_t - \frac{\phi(\overline{\omega})g(\overline{\omega},0)}{1-\Phi(\overline{\omega})}\xi_t + \Phi(\overline{\omega})\xi_t\right)$$

as required. □

6.4. The Optimal Policy Problem

Using a first-order approximation (for brevity the steps are omitted here, but are available from the author on request), the constraints describing the decentralised equilibrium of the economy (measured in terms of deviations from zero-inflation target values) are given by

$$\begin{aligned} 0 &= \sigma x_t - \left(\beta(1-\delta)\sigma - (1-\beta(1-\delta))\frac{1+\eta}{1-\alpha}\right)E_t x_{t+1} \\ &- \left(s\sigma\left(1+\beta(1-\delta)^2\right) + (1-\beta(1-\delta))\frac{1+\alpha\eta}{1-\alpha}\right)j_{t+1} \\ &+ s\sigma(1-\delta)j_t + \beta(1-\delta)s\sigma E_t j_{t+2} \\ &- \sigma s^e n_t + \beta(1-\delta)\sigma s^e E_t n_{t+1} \\ 0 &= \varphi_k n_{t-1} + \varphi_r\left(\sigma + \frac{1+\eta}{1-\alpha}\right)x_t - \varphi_r s\sigma j_{t+1} \\ &+ \varphi_r\left(s\sigma(1-\delta) - \frac{1+\alpha\eta}{1-\alpha}\right)j_t - \left(1+\varphi_r\sigma s^e\right)n_t \end{aligned}$$

and

$$
\begin{aligned}
0 &= \Theta\left(\sigma + \frac{\alpha+\eta}{1-\alpha}\right)x_t - \Theta s\sigma j_{t+1} + \Theta\left(s\sigma(1-\delta) - \frac{\alpha(1+\eta)}{1-\alpha}\right)j_t \\
&\quad - \Theta\sigma s^e n_t + \beta E_t \pi_{t+1} - \pi_t \\
0 &= \sigma(x_t - E_t x_{t+1}) + s\sigma(1-\delta)j_t - s\sigma(1-\delta)E_t j_{t+1} \\
&\quad - \sigma s^e(n_t - E_t n_{t+1}) + i_t^b - r_t^{bn} - E_t(\pi_{t+1})
\end{aligned}
$$

where

$$
\begin{aligned}
\varphi_k &\equiv \beta^{-1}\frac{K^e}{nw} \\
\varphi_r &\equiv \left(\beta^{-1} - 1 + \delta\right)\frac{K^e}{nw}
\end{aligned}
$$

The optimal policy problem is to minimise $E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} L_t$ where L_t is given by (2.43), subject to the above four constraints, and choosing sequences $\{i_t^b, g_t, j_t, n_t, \pi_t\}_{t=t_0}^{\infty}$ and where r_t^{bn} is given by the system defined in Appendix 6.2. It is straightforward to show from the first-order conditions of this problem (again results omitted for brevity but are available on request), that a zero gap solution for all variables does not satisfy these conditions in the general case that $n_t^* \neq 0$, and given an economy that is initially at its efficient steady state. Therefore, in general, optimal policy will deviate from a zero-inflation policy.

Chapter 7

Appendix to Chapter 3

7.1. Information and Timing

We have three decision makers – households, investment firms and producers. Investment firms know the true distribution of the random variable Z_2 described in Section 3.3. That is, investment firm managers information set at $t = 1$ contains $\mathcal{I}_1^{IF}(Z_L, Z_H, p_L, p_H, \mathbf{x}_1)$ where $\mathbf{x}_1$ is a vector of all observed prices and quantities in period 1 (information is complete for the investment firm manager).

In contrast, I assume that households know the possible realisations of Z_2 (Z_L and Z_H), but not the probabilities with which these states are realised. Thus, the information set for households at $t = 1$ is given by $\mathcal{I}_1^{HH}(Z_L, Z_H, \mathbf{x}_1)$. Since households do not know p_L, p_H, an additional assumption on how households form their beliefs is required. For simplicity, I assume that the continuum of households have beliefs such that $\left\{\widehat{p}_L^{HH}(i), \widehat{p}_H^{HH}(i)\right\} = \{\widehat{p}_L(i), \widehat{p}_H(i)\}$ for all $i \in [0,1]$. That is, there is an atom of households sharing the beliefs of each atom of investment firms across the unit interval.

In period 2, after uncertainty has been resolved and the state of technology realised, the information sets for firm managers and households are identical $\mathcal{I}_2^{IF}(\mathbf{x}_2) = \mathcal{I}_2^{HH}(\mathbf{x}_2)$, where $\mathbf{x}_2$ is a vector of all observed prices and quantities in period 2. Notice that since producers are only concerned with intra-temporal decisions, they have no interest in forecasting Z_2.

The timing of decisions is as follows:

$t = 0$

1. A continuum of investment firms optimise on their subjective probabilities $\{\widehat{p}_L(i), \widehat{p}_H(i)\}$, and report these to a continuum of households. Beliefs across

the household continuum are assumed to match the beliefs across the continuum of firms, so that for each type of firm ($\varepsilon(i)$), there is an atom of households willing to accept the firm-reported probabilities $\{\widehat{p}_L(i), \widehat{p}_H(i)\}$, and to use these probabilities when making their optimal portfolio allocation and consumption decisions.

$t = 1$

1. Given their beliefs, investment firms, households and producers make their optimal consumption, portfolio allocation, investment and production plans.

2. Arrow-Debreu securities, share markets and goods markets open.

3. The Walrasian auctioneer quotes a market-clearing price vector consistent with a decentralised equilibrium.

4. Consumption, portfolio selection, investment and production decisions take place at market-clearing prices.

$t = 2$

1. The state of technology is realised in period 2 ($Z_2 \in \{Z_L, Z_H\}$), and is observed by all agents.

2. Households, investment firms and producers make their optimal consumption, labour effort, investment and production plans.

3. The Walrasian auctioneer quotes a market clearing price vector consistent with a decentralised equilibrium.

4. Consumption, labour supply, investment and production decisions take place at market-clearing prices.

7.2. More Detailed Description of the OE Economy

To clarify the problem for an individual investment manger. The second stage problem (i.e. the optimal choice of investment taking beliefs as given) for investment firm i is

$$\max_{I_1(i)} R_1(i) K_1(i) - I_1(i) - \psi \frac{I_1(i)^2}{2} + \beta I_1(i) \sum_{s=L,H} \frac{U'(C_s(i))}{U'(C_1(i))} R_s(i) \widehat{p}_s(i)$$

where $i \in [0,1]$. At an interior, the optimal investment choice is given by

$$1 + \psi I_1(i) = \beta \sum_{s=L,H} \frac{U'(C_s(i))}{U'(C_1(i))} R_s(i) \widehat{p}_s(i)$$

The first stage problem (the optimal choice of beliefs) is given by

$$\max_{\widehat{p}_L(i), \widehat{p}_H(i)} E(\pi_1(i) + \beta \pi_2(i))$$

subject to:

$$\pi_1(i) = \left(R_1(i) K_1(i) - I_1(i) - \psi \frac{I_1(i)^2}{2} \right) + \beta I_1(i) \sum_{s=L,H} \frac{U'(C_s(i))}{U'(C_1(i))} R_s(i) \widehat{p}_s(i)$$

$$\pi_2(i) = \varepsilon(i) \left(R_1(i) K_1(i) - I_1(i) - \psi \frac{I_1(i)^2}{2} \right) + I_1(i) \frac{U'(C_2(i))}{U'(C_1(i))} R_2(i)$$

$$1 + \psi I_1(i) = \beta \sum_{s=L,H} \frac{U'(C_s(i))}{U'(C_1(i))} R_s(i) \widehat{p}_s(i)$$

$$1 = \sum_{s=L,H} \widehat{p}_s(i)$$

$$\widehat{p}_s(i) \geq 0 \text{ for } s = L, H$$

and has interior optimality conditions

$$1+\psi I_1(i)=\beta\sum_{s=L,H}\frac{U'(C_s(i))}{U'(C_1(i))}R_s(i)\,\widehat{p}_s(i)$$

$$1=\sum_{s=L,H}\widehat{p}_s(i)$$

$$\beta\varepsilon(i)(1+\psi I_1(i))=\beta\sum_{s=L,H}\frac{U'(C_s(i))}{U'(C_1(i))}R_s(i)\,p_s+\psi I_1(i)$$

where $C_s(i)$ is the consumption basket purchased by the shareholders (households) that own firm i.

To clarify the problem for households, individual household $j\in[0,1]$ solves the following intratemporal consumption allocations for $t=1,L,H$

$$\min_{C_t^i(j)}\int_0^1 P_t^iC_t^i(j)\,di$$

subject to

$$C_t(j)\geq\left(\int_0^1\left(C_t^i(j)\right)^{\frac{\theta-1}{\theta}}di\right)^{\frac{\theta}{\theta-1}}$$

taking $C_t(j)$ as given, where P_t^i is the price of consumption good i, $C_t(j)$ is a Dixit-Stiglitz basket of goods consumed by household j, and θ is a common elasticity of substitution across goods. The optimal demand for consumption good i by household j can be expressed as

$$C_t^i(j)=\left(\frac{P_t^i}{P_t}\right)^{-\theta}C_t(j)$$

where P_t is a Dixit-Stiglitz price index that measures the shadow price of consuming the basket of goods $C_t(j)$,

$$P_t\equiv\left(\int_0^1\left(P_t^i\right)^{1-\theta}di\right)^{\frac{1}{1-\theta}}$$

The intertemporal and labour supply decisions made by households can be obtained from the solution to:

$$\max_{\substack{C_1(j),\theta_i(j),a_s(j)\\ C_s(j),N_s(j)}} U\left(C_1\left(j\right)\right)+\beta\sum_{s=L,H}\left(U\left(C_s\left(j\right)\right)-V\left(N_s\left(j\right)\right)\right)\widehat{p}_s\left(j\right)$$

subject to:

$$P_1C_1\left(j\right)+P_1\sum_{s=L,H}\omega_s a_s\left(j\right)+P_1\int_0^1\left(q_i-D_{1,i}\right)\theta_i\left(j\right)di$$
$$\leq P_1W_1\left(j\right)N_1\left(j\right)+P_1\int_0^1 q_i\phi_i\left(j\right)di+P_1D_1^P\left(j\right)$$
$$P_sC_s\left(j\right)\leq P_sa_s\left(j\right)+P_sW_s\left(j\right)N_s\left(j\right)+P_sD_s^P\left(j\right)+P_s\int_0^1\theta_i\left(j\right)D_{s,i}di$$
$$\theta_i\left(j\right)\geq 0$$
$$\text{for } s=L,H$$

The household optimality conditions (at an interior solution) satisfy

$$U'\left(C_1\left(j\right)\right)=\sum_{s=L,H}U'\left(C_s\left(j\right)\right)\left(1+r_f\right)\widehat{p}_s\left(j\right)$$
$$W_s\left(j\right)=\frac{V'\left(N_s\left(j\right)\right)}{U'\left(C_s\left(j\right)\right)}\text{ for } s=L,H$$
$$q_{1,i}=D_{1,i}+\beta\sum_{s=L,H}\frac{U'\left(C_s\left(j\right)\right)}{U'\left(C_1\left(j\right)\right)}D_{s,i}\widehat{p}_s\left(j\right)$$

7.3. Existence and Comparative Statics

I first show that an OE equilibrium exists where all firms optimally choose rational expectations.

Proposition 7.1. *A symmetric OE equilibrium where all firms and households optimally choose rational expectations exists. This equilibrium will be chosen when $\varepsilon=\varepsilon^{RE}$, where*

$$\varepsilon^{RE}\equiv\beta^{-1}\frac{1+2\psi I_1^{RE}}{1+\psi I_1^{RE}}$$

and I_1^{RE} is the strictly positive solution to any valid RE equilibrium satisfying Definition 3.2.

Proof. Fix values for Y_1, Z_s and p_s where $Y_1 > 0, Z_s > 0$ and $\sum_s p_s = 1$. I first establish existence and uniqueness of an interior RE equilibrium. An interior RE equilibrium can be represented as a (strictly positive) vector $\left\{I_1^{RE}, C_1^{RE}, C_s^{RE}, N_s^{RE}\right\}_{s=L,H}$ that solves

$$
\begin{aligned}
1+\psi I_1 &= \beta \sum_{s=L,H} \frac{U'(C_s)}{U'(C_1)} \alpha Z_s (I_1)^{\alpha-1} (N_s)^{1-\alpha} p_s \\
C_s &= Z_s I_1^{\alpha} N_s^{1-\alpha} \\
C_1 &= Y_1 - I_1 - \psi \frac{I_1^2}{2} \\
V'(N_s) &= U'(C_s)(1-\alpha) Z_s (I_1)^{\alpha} (N_s)^{-\alpha}
\end{aligned}
$$

taking Y_1, Z_s, p_s as given and for $s = L, H$. Using the Second Welfare theorem note that the decentralised equilibrium can be obtained by solving the following social planner problem:

$$
\max_{I_1, C_1, C_s, N_s} U(C_1) + \beta \sum_{s=L,H} \left(U(C_s) - V(N_s)\right) p_s
$$

$$
\begin{gathered}
\text{subject to:} \\
Y_1 \geq C_1 + I_1 + \psi \frac{I_1^2}{2} \\
C_s \leq Z_s I_1^{\alpha} N_s^{1-\alpha} \\
I_1 \geq 0, C_1 \geq 0, C_s \geq 0 \\
\text{given:} \quad Y_1, Z_s, p_s \\
\text{for } s = L, H
\end{gathered}
$$

Under standard assumptions on the objective (U is strictly increasing and concave, V is strictly increasing and convex), and given that the constraints are quasi-convex, it follows that a solution to the above problem exists and the first-order conditions are both necessary and sufficient for a unique global maximum. Since the decentralised and social planner optimality conditions are identical, it follows that the interior RE equilibrium exists, and it is unique.

Now note that an OE equilibrium can be represented by a vector $\left\{I_1^{OE}, C_1^{OE}, C_s^{OE}, N_s^{OE}, \widehat{p}_s\right\}_{s=L,H}$ solving

$$\beta\varepsilon\left(1+\psi\left(1-\beta^{-1}\varepsilon^{-1}\right)I_1\right) = \beta\sum_{s=L,H}\frac{U'(C_s)}{U'(C_1)}\alpha Z_s(I_1)^{\alpha-1}(N_s)^{1-\alpha}p_s$$

$$C_s = Z_s I_1^{\alpha} N_s^{1-\alpha}$$

$$C_1 = Y_1 - I_1 - \psi\frac{I_1^2}{2}$$

$$V'(N_s) = U'(C_s)(1-\alpha)Z_s(I_1)^{\alpha}(N_s)^{-\alpha}$$

$$1 = \sum_{s=L,H}\widehat{p}_s$$

$$1+\psi I_1 = \beta\sum_{s=L,H}\frac{U'(C_s)}{U'(C_1)}\alpha Z_s\left(\frac{N_s}{I_1}\right)^{1-\alpha}\widehat{p}_s$$

taking $\{Y_1, Z_s, p_s\}_{s=L,H}$ and $\varepsilon > \beta^{-1}$ as given.

Our goal is to show there exists an ε such that $\left\{I_1^{OE} = I_1^{RE}, C_1^{OE} = C_1^{RE}, C_s^{OE} = C_s^{RE}, N_s^{OE} = N_s^{RE}, \widehat{p}_s = p_s\right\}_{s=L,H}$. To see this, choose $\varepsilon = \varepsilon^{RE} > \beta^{-1}$ where

$$\varepsilon^{RE} \equiv \beta^{-1}\frac{1+2\psi I_1^{RE}}{1+\psi I_1^{RE}}$$

and $I_1^{RE} > 0$ is the solution to a given RE equilibrium above (i.e for given $\{Y_1, Z_s, p_s\}_{s=L,H}$).

Plugging the above value for ε into the conditions characterising an OE equilibrium it follows that $\left\{I_1^{OE}, C_1^{OE}, C_s^{OE}, N_s^{OE}, \widehat{p}_s\right\}_{s=L,H}$ is now a vector that solves

$$\frac{1+2\psi I_1^{RE}}{1+\psi I_1^{RE}}\left(1+\psi\left(1-\frac{1+\psi I_1^{RE}}{1+2\psi I_1^{RE}}\right)I_1\right)=\beta\sum_{s=L,H}\frac{U'(C_s)}{U'(C_1)}\alpha Z_s(I_1)^{\alpha-1}(N_s)^{1-\alpha}p_s$$

$$C_s = Z_s I_1^{\alpha} N_s^{1-\alpha}$$

$$C_1 = Y_1 - I_1 - \psi\frac{I_1^2}{2}$$

$$V'(N_s) = U'(C_s)(1-\alpha)Z_s(I_1)^{\alpha}(N_s)^{-\alpha}$$

$$1 = \sum_{s=L,H}\widehat{p}_s$$

$$1+\psi I_1 = \beta\sum_{s=L,H}\frac{U'(C_s)}{U'(C_1)}\alpha Z_s\left(\frac{N_s}{I_1}\right)^{1-\alpha}\widehat{p}_s$$

One can verify that $\left\{I_1^{OE}=I_1^{RE}, C_1^{OE}=C_1^{RE}, C_s^{OE}=C_s^{RE}, N_s^{OE}=N_s^{RE}, \widehat{p}_s=p_s\right\}$ for $s=L,H$ is a solution to the above system, given that $\left\{I_1^{RE}, C_1^{RE}, C_s^{RE}, N_s^{RE}\right\}_{s=L,H}$ solves the RE equilibrium previously defined. □

I now study the main comparative static results of interest, recalling that I use the notation $\frac{\partial y}{\partial x}\equiv\frac{\partial y}{\partial x}\big|_{\varepsilon=\varepsilon^{RE}}$. That is, I study the effects of small perturbations around the equilibrium point where optimal and rational expectations coincide.

Proposition 3.1. *Consider (separately) small perturbations to the realisation of technology in each state, Z_L, Z_H, around the equilibrium point at which optimal and rational expectations coincide. It follows that beliefs are optimistic, and investment and equity prices are higher under OE than RE following each of these perturbations. Furthermore, these responses are largest for a perturbation to the realisation of technology in the high state – provided that the likelihood of this state is sufficiently large.*

That is,

$$\frac{\partial \widehat{p}_H}{\partial x} > 0; \quad \frac{\partial \widehat{p}_L}{\partial x} < 0$$
$$\frac{\partial I_1^{OE}}{\partial x} > \frac{\partial I_1^{RE}}{\partial x} > 0; \quad \frac{\partial q_1^{OE}}{\partial x} > \frac{\partial q_1^{RE}}{\partial x} > 0$$
$$for\ x \in Z_L, Z_H$$

and

$$\frac{\partial y}{\partial Z_H} \geq \frac{\partial y}{\partial Z_L} \quad for \quad y \in \widehat{p}_H, I_1^{OE}, q_1^{OE}$$
$$if \quad \frac{p_H}{p_L} \geq \left(\frac{Z_L}{Z_H}\right)^{\gamma_{3-1}}$$

Before starting the proof, I begin with a lemma that characterises one minimal representation of the equilibrium conditions for the OE and RE equilibria respectively. These minimal representations are useful for deriving the comparative statics that follow.

Lemma 7.1. *A minimal representation of the OE equilibrium is a solution* $\left\{I_1^{OE}, \widehat{p}_s^{OE}\right\}_{s=L,H}$ *that satisfies*

$$\beta\varepsilon\left(1+\psi I_1 - \frac{\psi}{\beta\varepsilon} I_1\right) = I_1^{\gamma_1}\gamma_2 \sum_{s=L,H} Z_s^{\gamma_3} p_s \left(Y_1 - I_1 - \psi\frac{(I_1)^2}{2}\right)^{\sigma}$$
$$(1+\psi I_1) = I_1^{\gamma_1}\gamma_2 \sum_{s=L,H} Z_s^{\gamma_3} \widehat{p}_s \left(Y_1 - I_1 - \psi\frac{(I_1)^2}{2}\right)^{\sigma}$$
$$1 = \sum_{s=L,H} \widehat{p}_s$$
$$where\ \gamma_1 \equiv -\frac{\sigma+\eta-\alpha\eta+\alpha\sigma\eta}{\alpha+\sigma+\eta-\alpha\sigma} < 0$$
$$\gamma_2 \equiv \alpha\beta\,(1-\alpha)^{\frac{(1-\alpha)(1-\sigma)}{\eta+\sigma(1-\alpha)+\alpha}} > 0$$
$$\gamma_3 \equiv -(\sigma-1)\,\frac{\eta+1}{\alpha+\sigma+\eta-\alpha\sigma} > 0$$
$$since\ \alpha, \sigma \in (0,1) \tag{7.1}$$

taking $\{Y_1, Z_s, p_s\}_{s=L,H}$ as given. By comparison, a minimal representation of the RE equilibrium is a solution $\left\{I_1^{RE}\right\}$ to

$$\left(Y_1 - I_1 - \psi\frac{(I_1)^2}{2}\right)^{-\sigma}(1+\psi I_1) = I_1^{\gamma_1}\gamma_2\sum_{s=L,H} Z_s^{\gamma_3}p_s$$

again taking $\{Y_1, Z_s, p_s\}_{s=L,H}$ as given.

Proof. This can be verified by solving the OE and RE equilibriums, given the assumed power functions for household preferences (see 3.21). Notice that once the above solutions are obtained, one can solve for all prices and quantities of interest given the OE and RE equilibria previously defined. □

I now proceed with the proof of Proposition 3.1.

Proof. I first establish the results with respect to investment and equity prices, and then turn to the optimal subjective probabilities. Using Lemma 7.1, taking logs, and totally differentiating, holding $\{p_s\}_{s=L,H}$ and Z_L fixed, it follows that

$$\frac{\partial I_1^{OE}}{\partial Z_H} = \gamma_3\frac{Z_H^{\gamma_3-1}p_H}{\sum_s Z_s^{\gamma_3}p_s}\left(\sigma\frac{1+\psi I_1^{RE}}{C_1^{RE}} + \frac{\psi - \frac{\psi}{\beta\varepsilon^{RE}}}{1+\psi I_1^{RE} - \frac{\psi}{\beta\varepsilon^{RE}}I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}}\right)^{-1}$$
$$> 0$$

where $\gamma_2 > 0, \gamma_3 > 0$ (assuming $\sigma < 1$), and $\gamma_1 < 0$.

Again, using Lemma 7.1, an analogous exercise for the RE economy implies

$$\frac{\partial I_1^{RE}}{\partial Z_H} = \gamma_3\frac{Z_H^{\gamma_3-1}p_H}{\sum_s Z_s^{\gamma_3}p_s}\left(\sigma\frac{1+\psi I_1^{RE}}{C_1^{RE}} + \frac{\psi}{1+\psi I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}}\right)^{-1}$$
$$> 0$$

Comparing the derivatives $\frac{\partial I_1^{OE}}{\partial Z_H}$ and $\frac{\partial I_1^{RE}}{\partial Z_H}$ and noting that $\frac{\psi}{1+\psi I_1^{RE}} > \frac{\psi - \frac{\psi}{\beta \varepsilon^{RE}}}{1+\psi I_1^{RE} - \frac{\psi}{\beta \varepsilon^{RE}} I_1^{RE}}$ since $\beta \varepsilon^{RE} > 1$, it follows that

$$\frac{\partial I_1^{OE}}{\partial Z_H} > \frac{\partial I_1^{RE}}{\partial Z_H} > 0$$

An analogous argument can be used to establish

$$\frac{\partial I_1^{OE}}{\partial Z_L} > \frac{\partial I_1^{RE}}{\partial Z_L} > 0$$

Also note that

$$\frac{\partial I_1^{OE}}{\partial Z_L} = \gamma_3 \frac{Z_L^{\gamma_3 - 1} p_L}{\sum_s Z_s^{\gamma_3} p_s} \left(\sigma \frac{1+\psi I_1^{RE}}{C_1^{RE}} + \frac{\psi - \frac{\psi}{\beta \varepsilon^{RE}}}{1+\psi I_1^{RE} - \frac{\psi}{\beta \varepsilon^{RE}} I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}} \right)^{-1}$$

and thus

$$\frac{\partial I_1^{OE}}{\partial Z_H} \geq \frac{\partial I_1^{OE}}{\partial Z_L}$$

if

$$Z_H^{\gamma_3 - 1} p_H \geq Z_L^{\gamma_3 - 1} p_L$$

The intuition behind this last inequality is that if the high state payoff is sufficient likely and/or large, then a perturbation to realised technology in the high state will have a greater effect on investment than a perturbation to realised technology in the low state. That is,

$$\frac{\partial I_1^{e}}{\partial Z_H} \geq \frac{\partial I_1^{e}}{\partial Z_L} \quad \text{if} \quad Z_H^{\gamma_3 - 1} p_H \geq Z_L^{\gamma_3 - 1} p_L$$
$$\text{for} \quad e \in \{OE, RE\}$$

To establish the results for equity pricing, it can be observed from the equilibrium conditions, either under OE or RE, that the measure of equity prices of interest (households' valuation of the dividend stream paid by investment firms) is given by

$$q_1^e = R_1 K_1 - I_1^e - \frac{\psi \left(I_1^e\right)^2}{2} + \beta I_1^e E^e \left(\frac{U'\left(C_2^e\right)}{U'\left(C_1^e\right)} R_2^e \right)$$

$$\text{where } e \in \{OE, RE\}$$

Using the equilibrium investment condition under the optimal subjective probability measure for an OE equilibrium, and under the rational probability measure for an RE equilibrium,

$$q_1^e = R_1 K_1 + \frac{\psi \left(I_1^e\right)^2}{2}$$

and thus

$$\frac{\partial q_1^e}{\partial x} = \psi I_1^e \frac{\partial I_1^e}{\partial x} \quad \text{for} \quad x \in \{Z_L, Z_H, p_H\}$$

$$\text{and} \quad e \in \{OE, RE\}$$

since $R_1 K_1$ is fixed exogenously. From this condition, and that I am considering perturbations around the equilibrium where initially $I_1^{OE} = I_1^{RE}$, it follows that the above results derived with respect to investment, equally apply to equity prices.

Turning to the optimal subjective probabilities, again using Lemma 7.1, the OE equilibrium conditions can be re-written as

$$\sum_{s=L,H} Z_s^{\gamma_3} \widehat{p}_s = \frac{\sum_s Z_s^{\gamma_3} p_s}{\beta \varepsilon} \frac{1 + \psi I_1}{1 + \psi I_1 - \frac{\psi}{\beta \varepsilon} I_1}$$

$$I_1^{\gamma_1} \gamma_2 \sum_{s=L,H} Z_s^{\gamma_3} p_s = \beta \varepsilon \left(Y_1 - I_1 - \psi \frac{\left(I_1\right)^2}{2} \right)^{-\sigma} \left(1 + \psi I_1 - \frac{\psi}{\beta \varepsilon} I_1 \right) \tag{7.2}$$

Taking logs and totally differentiating the first condition (holding $\{p_s\}_{s=L,H}$ and Z_L fixed), and noting $\frac{d\widehat{p}_L}{d\widehat{p}_H} = -1$, I have

$$\begin{aligned}\frac{\partial \widehat{p}_H}{\partial Z_H} &= \frac{\sum_s Z_s^{\gamma_3} p_s}{Z_H^{\gamma_3} - Z_L^{\gamma_3}} \left(\frac{\psi}{1+\psi I_1^{RE}} - \frac{\psi - \frac{\psi}{\beta\varepsilon^{RE}}}{1+\psi I_1^{RE} - \frac{\psi}{\beta\varepsilon^{RE}} I_1^{RE}} \right) \frac{\partial I_1^{OE}}{\partial Z_H} \\ &> 0\end{aligned}$$

since $Z_H > Z_L$, $\frac{\psi}{1+\psi I_1^{RE}} - \frac{\psi - \frac{\psi}{\beta\varepsilon}}{1+\psi I_1^{RE} - \frac{\psi}{\beta\varepsilon} I_1^{RE}} > 0$, $\gamma_3 > 0$ and $\frac{\partial I_1^{OE}}{\partial Z_H} > 0$ (see above). Using the restriction that subjective probabilities must sum to one immediately immediately implies $\frac{d\widehat{p}_L}{dZ_H} < 0$.

By symmetry

$$\begin{aligned}\frac{\partial \widehat{p}_H}{\partial Z_L} &= \frac{\sum_s Z_s^{\gamma_3} p_s}{Z_H^{\gamma_3} - Z_L^{\gamma_3}} \left(\frac{\psi}{1+\psi I_1^{RE}} - \frac{\psi - \frac{\psi}{\beta\varepsilon^{RE}}}{1+\psi I_1^{RE} - \frac{\psi}{\beta\varepsilon^{RE}} I_1^{RE}} \right) \frac{\partial I_1^{OE}}{\partial Z_L} \\ &> 0\end{aligned}$$

and $\frac{\partial \widehat{p}_L}{\partial Z_L} < 0$. Finally, since $\frac{\partial I_1^{RE}}{\partial Z_H} \geq \frac{\partial I_1^{RE}}{\partial Z_L}$ if $Z_H^{\gamma_3 - 1} p_H \geq Z_L^{\gamma_3 - 1} p_L$, it follows that

$$\frac{\partial \widehat{p}_H}{\partial Z_H} > \frac{\partial \widehat{p}_H}{\partial Z_L} \quad \text{if } Z_H^{\gamma_3 - 1} p_H \geq Z_L^{\gamma_3 - 1} p_L$$

which completes the proof. □

Proposition 3.2. *Consider a perturbation to the true (or rational) probability with which the high state is realised, p_H (with $dp_H = -dp_L$). Firms assign a probability to the high state that is above the true probability, and a probability to the low state that is below the true probability. Investment and equity prices are higher under OE than RE, following* *this perturbation.*

$$\frac{\partial \widehat{p}_H}{\partial p_H} > 1; \quad \frac{\partial \widehat{p}_L}{\partial p_H} < -1$$
$$\frac{\partial y_1^{OE}}{\partial p_H} > \frac{\partial y_1^{RE}}{\partial p_H} > 0$$
$$\mathit{for} \quad y_1 \in I_1, q_1$$

Proof. As in the previous proposition I start with the equilibrium conditions in the system (7.1). Fixing Z_L and Z_H, taking logs and totally differentiating both conditions I obtain

$$\begin{aligned}\frac{\partial \widehat{p}_H}{\partial p_H} &= 1 + \frac{\frac{\psi}{1+\psi I_1^{RE}} - \frac{\psi - \frac{\psi}{\beta \varepsilon^{RE}}}{1+\psi I_1^{RE} - \frac{\psi}{\beta \varepsilon^{RE}} I_1^{RE}}}{\sigma \frac{1+\psi I_1^{RE}}{C_1^{RE}} + \frac{\psi - \frac{\psi}{\beta \varepsilon^{RE}}}{1+\psi I_1^{RE} - \frac{\psi}{\beta \varepsilon^{RE}} I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}}} \\ &> 1\end{aligned}$$

since $\gamma_1 < 0$ and $\beta \varepsilon^{RE} > 1$. Note $\frac{\partial \widehat{p}_L}{\partial p_H} < 1$ follows immediately from $1 = \widehat{p}_L + \widehat{p}_H$. In addition, it is straightforward to verify

$$\begin{aligned}\frac{\partial I_1^{OE}}{\partial p_H} &= \frac{Z_H^{\gamma_3} - Z_L^{\gamma_3}}{\sum_s Z_s^{\gamma_3} p_s} \left(\sigma \frac{1+\psi I_1^{RE}}{C_1^{RE}} + \frac{\psi - \frac{\psi}{\beta \varepsilon^{RE}}}{1+\psi I_1^{RE} - \frac{\psi}{\beta \varepsilon^{RE}} I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}} \right)^{-1} \\ &> 0\end{aligned}$$

The second result follows directly by comparing the above partial derivative with

$$\begin{aligned}\frac{\partial I_1^{RE}}{\partial p_H} &= \frac{Z_H^{\gamma_3} - Z_L^{\gamma_3}}{\sum_s Z_s^{\gamma_3} p_s} \left(\sigma \frac{1+\psi I_1^{RE}}{C_1^{RE}} + \frac{\psi}{1+\psi I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}} \right)^{-1} \\ &> 0\end{aligned}$$

The result for equity prices follows directly from $\frac{\partial q_1^e}{\partial p_h} = \psi I_1^e \frac{\partial I_1^e}{\partial p_h}$ for $e \in \{OE, RE\}$ (see the proof of Proposition 3.1). $\square$

Proposition 3.3. *Output and labour effort in period two, and consumption growth, increase by more under OE than RE in response to a perturbation to Z_H, Z_L or p_H.*

$$
\begin{gathered}
\frac{\partial Y_s^{OE}}{\partial x} > \frac{\partial Y_s^{RE}}{\partial x} > 0 \\
\frac{\partial N_s^{OE}}{\partial x} > \frac{\partial N_s^{RE}}{\partial x} > 0 \\
\frac{\partial \ln \frac{C_s^{OE}}{C_1^{OE}}}{\partial x} > \frac{\partial \ln \frac{C_s^{RE}}{C_1^{RE}}}{\partial x} > 0 \\
for \quad x \in \{Z_L, Z_H, p_H\}
\end{gathered}
$$

Proof. The first two results follow immediately by noting that in general (under either an OE or RE equilibrium)

$$
\begin{aligned}
Y_s^e &= Z_s \left(I_1^e\right)^{\alpha} \left(N_s^e\right)^{1-\alpha} \\
N_s^e &= \left((1-\alpha) Z_s^{1-\sigma} \left(I_1^e\right)^{\alpha(1-\sigma)}\right)^{\frac{1}{\eta+\sigma(1-\alpha)+\alpha}}
\end{aligned}
$$

where $e \in \{OE, RE\}$

and so (taking logs and totally differentiating)

$$
\begin{aligned}
\frac{dY_s^e}{Y_s} &= \gamma_4 \frac{dZ_s}{Z_s} + \alpha\gamma_4 \frac{dI_1^e}{I_1^e} \\
\frac{dN_s}{N_s} &= \gamma_5 \frac{dZ_s}{Z_s} + \alpha\gamma_5 \frac{dI_1^h}{I_1^h} \\
\text{where } \gamma_4 &\equiv \frac{\eta+1}{\alpha+\sigma+\eta-\alpha\sigma} > 0 \\
\gamma_5 &\equiv \frac{(1-\sigma)}{\eta+\sigma(1-\alpha)+\alpha} > 0
\end{aligned}
$$

since $\sigma, \alpha \in (0,1)$

Noting that these results can be treated locally as identities in the neighbourhood of the equilibrium where RE and OE coincide, it follows that

$$\frac{\partial Y_s^e}{\partial x} = \frac{Y_s^{RE}}{Z_s}\gamma_4\frac{\partial Z_s}{\partial x} + \frac{Y_s^{RE}}{I_1^{RE}}\alpha\gamma_4\frac{\partial I_s^e}{\partial x}$$
$$\frac{\partial N_s^e}{\partial x} = \frac{N_s^{RE}}{Z_s}\gamma_5\frac{\partial Z_s}{\partial x} + \frac{N_s^{RE}}{I_1^{RE}}\alpha\gamma_5\frac{\partial I_s^e}{\partial x}$$
$$\text{for } x \in \{Z_L, Z_H, p_H\} \text{ and } e \in \{OE, RE\}$$

which in conjunction with the results in Propositions 3.1 and 3.2 can be used to verify that $\frac{\partial Y_s^{OE}}{\partial x} > \frac{\partial Y_s^{RE}}{\partial x} > 0$ and $\frac{\partial N_s^{OE}}{\partial x} > \frac{\partial N_s^{RE}}{\partial x} > 0$ for $x \in \{Z_L, Z_H, p_H\}$. The third result follows by noting that

$$C_1^e = Y_1 - I_1^e - \frac{\psi}{2}I_1^{e2}$$
$$Y_s^e = C_s^e$$
$$\text{for } s = L, H \text{ and } e \in \{OE, RE\}$$

and again using the results derived in Propositions 3.1 and 3.2 with respect to investment. □

Proposition 3.4. *Agents become pessimistic (over-optimistic) when the weight attached to memory utility increases (decreases)*

$$\frac{\partial I_1^{OE}}{\partial \varepsilon} < 0; \quad \frac{\partial q_1^{OE}}{\partial \varepsilon} < 0$$
$$\frac{\partial \widehat{p}_H^{OE}}{\partial \varepsilon} < 0; \quad \frac{\partial \widehat{p}_L^{OE}}{\partial \varepsilon} > 0$$

Proof. Using Lemma 7.1 one can verify

$$\frac{\partial I_1^{OE}}{\partial \varepsilon} = -\left(\frac{1}{\varepsilon^{RE}} + \frac{\psi I_1^{RE}}{\beta\left(\varepsilon^{RE}\right)^2\left(1+\psi\left(1-\beta^{-1}\left(\varepsilon^{RE}\right)^{-1}\right)I_1^{RE}\right)}\right)$$
$$\times\left(\sigma\frac{1+\psi I_1^{RE}}{C_1^{RE}} + \frac{\psi - \frac{\psi}{\beta\varepsilon^{RE}}}{1+\psi I_1^{RE} - \frac{\psi}{\beta\varepsilon^{RE}}I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}}\right)^{-1}$$
$$< 0$$

Using the result derived in Proposition 3.1

$$\frac{\partial q_1^{OE}}{\partial \varepsilon} = \psi I_1^{OE}\frac{\partial I_1^{OE}}{\partial \varepsilon}$$

verifies the analogous result for equity prices.

Again using Lemma 7.1,

$$\frac{\partial \widehat{p}_H^{OE}}{\partial \varepsilon} = \frac{\sum_s Z_s^{\gamma_3} p_s}{Z_H^{\gamma_3} - Z_L^{\gamma_3}}\left(\sigma\frac{1+\psi I_1^{RE}}{C_1^{RE}} + \frac{\psi}{1+\psi I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}}\right)\frac{\partial I_1^{OE}}{\partial \varepsilon}$$
$$< 0$$

since $\frac{\partial I_1^{OE}}{\partial \varepsilon} < 0$ (above). The last result follows from $1 = \widehat{p}_H^{OE} + \widehat{p}_L^{OE}$. □

7.4. Household Optimism

Proposition 3.5. *Re-consider the OE economy described in Definition 3.1. Suppose now that:*

1. *Households optimally choose beliefs rather than investment firms;*
2. *Investment firms maximise shareholder value;*
3. *In the absence of a perturbation to the distribution of technology, households use a rational probability measure; and*

4. *Households become optimistic (pessimistic) in response to favourable (unfavourable) perturbations to the distribution of technology. That is, I assume the reduced-form relationship*

$$\frac{\partial \widehat{p}_H^h}{\partial x} > 0 \quad for \quad x \in Z_L, Z_H, p_H$$

In such an economy, where beliefs are symmetric and determined by households, equity prices, investment, output, labour supply and consumption growth all increase by more in this reduced-form OE economy than in an RE economy in response to a favourable change in the distribution of technology.

Proof. I assume investment firm managers now solve

$$\max_{I_1(i)} \left(R_1(i) K_1(i) - I_1(i) - \psi \frac{I_1(i)^2}{2} \right) + \beta I_1(i) \sum_{s=L,H} \frac{U'(C_s(i))}{U'(C_1(i))} R_s(i) \widehat{p}_s^h(i)$$

taking $\widehat{p}_s^h(i)$ as beliefs that are now household determined. It is straightforward to verify that managers are investing to maximise the value of the firm, given shareholder (household) beliefs.

Now suppose households choose their expectations optimally, as well as their consumption, portfolio choice and labour supply decisions. That is, households solve[111]

$$\max_{\{\widehat{p}_s^h(j)\}_{s=L,H}} \widehat{F}\left(C_1^*(j), C_s^*(j), N_s^*(j), \theta_i^*(j), a_s^*(j), \widehat{p}_s^h(j); \xi\right)$$

subject to:

$$\widehat{p}_s^h(j) \geq 0 \text{ and } \widehat{p}_s^h(j) = 0 \text{ if } p_s = 0$$

$$\sum_{s=L,H} \widehat{p}_s^h(j) = 1$$

$$\text{for } s = L, H$$

[111]The intratemporal allocation of consumption purchases remains identical to that described in the main text.

and the functions $\{C_1^*(j), C_s^*(j), N_s^*(j), \theta_i^*(j), a_s^*(j)\}_{s=L,H}$ are derived from the solutions to the constrained optimisation program

$$\max_{\substack{C_1(j), C_s(j), \\ N_s(j), \theta_i(j), a_s(j)}} U(C_1(j)) + \beta \sum_{s=L,H} (U(C_s(j)) - V(N_s(j))) \widehat{p}_s^h(j)$$

subject to:

$$P_1 C_1(j) + P_1 \sum_{s=L,H} \omega_s a_s(j) + P_1 \int_0^1 \left(q_i - D_{1,i}\right) \theta_i(j) di$$
$$\leq P_1 W_1(j) N_1(j) + P_1 \int_0^1 q_i \phi_i(j) di + P_1 D_1^P(j)$$
$$P_s C_s(j) \leq P_s a_s(j) + P_s W_s(j) N_s(j) + P_s D_s^P(j) + P_s \int_0^1 \theta_i(j) D_{s,i} di$$
$$\theta_i(j) \geq 0$$
$$\text{for } s = L, H$$

where $\widehat{F}$ is a twice differentiable strictly concave function, and ξ is parameter vector (that includes $\beta, \varepsilon^h, \eta, \sigma$). I assume there exists, at least locally, a continuously differentiable solution for optimal household beliefs given by the functions

$$\widehat{p}_H^h = \widehat{f}(Z_L, Z_H, p_L, p_H, Y_1; \xi)$$
$$\widehat{p}_L^h = 1 - \widehat{p}_H^h$$

with the property that

$$\frac{\partial \widehat{p}_H^h}{\partial x} > 0 \text{ for } x \in Z_L, Z_H, p_H$$

and that $\widehat{p}_H^h = p_H$, $\widehat{p}_L^h = p_L$ in the absence of any perturbations. I need to show that all of the comparative statics previously derived also hold for this alternative household OE economy.

To see this, first note that given the above assumptions a household-led OE equilibrium can be defined minimally as a set $\left\{I_1^h, \widehat{p}_L^h, \widehat{p}_H^h\right\}$ solving

$$\begin{aligned}\left(Y_1 - I_1^h - \psi\frac{\left(I_1^h\right)^2}{2}\right)^{-\sigma}\left(1+\psi I_1^h\right)\left(I_1^h\right)^{-\gamma_1} &= \gamma_2 \sum_{s=L,H} Z_s^{\gamma_3}\widehat{p}_s^h \\ \widehat{p}_H^h &= f\left(Z_L, Z_H, p_L, p_H, Y_1; \xi\right) \\ \widehat{p}_L^h &= 1 - \widehat{p}_H^h \\ \text{where } \gamma_1 < 0, \gamma_2 > 0, \gamma_3 < 0 &\end{aligned}$$

taking $\{Z_L, Z_H, p_L, p_H, Y_1; \xi\}$ as given.

Comparing this with the RE equilibrium condition for investment

$$\left(Y_1 - I_1^h - \psi\frac{\left(I_1^h\right)^2}{2}\right)^{-\sigma}\left(1+\psi I_1^h\right)\left(I_1^h\right)^{-\gamma_1} = \gamma_2 \sum_{s=L,H} Z_s^{\gamma_3} p_s$$

it is straightforward to show that

$$\begin{aligned}\frac{\partial I_1^h}{\partial x} > \frac{\partial I_1^{RE}}{\partial x} > 0 \text{ if } \frac{\partial \widehat{p}_H^h}{\partial x} > 0 \\ \text{for } x \in Z_L, Z_H, p_H\end{aligned}$$

Given that all other prices and quantities in the household-led OE economy are identical functions of investment when compared with the benchmark OE economy previously discussed, and that the RE economy is unchanged, it follows that all of the previous comparative statics derived also hold in the household-led OE economy. □

7.5. A Model with N-states

The definition on an OE equilibrium with N states generalises to:

Definition 7.1. *A symmetric interior OE equilibrium in an economy with N states is defined as vector of prices* $\left\{r_f^{OE}, q_1^{OE}, R_s^{OE}\right\}_{s=1}^{N}$*, quantities* $\left\{\widetilde{C}_1^{OE}, I_1^{OE}, C_s^{OE}, Y_s^{OE}, N_s^{OE}\right\}_{s=1}^{N}$*, and optimal subjective probabilities* $\{\widehat{p}_s\}_{s=1}^{N}$ *that satisfy*

$$\begin{aligned}
1+\psi I_1 &= \beta \widehat{E}\left(\frac{U'(C_2)}{U'\left(\widetilde{C}_1\right)} R_2\right) \\
U'\left(\widetilde{C}_1\right) &= \beta\left(1+r_f\right)\widehat{E}\left(U'(C_2)\right) \\
\beta\varepsilon(1+\psi I_1) &= \beta E\left(\frac{U'(C_2)}{U'\left(\widetilde{C}_1\right)} R_2\right)+\psi I_1 \\
q_1 &= R_1 K_1 - I_1 - \frac{\psi}{2}(I_1)^2 + \beta I_1 \widehat{E}\left(\frac{U'(C_2)}{U'\left(\widetilde{C}_1\right)} R_2\right) \\
(1-\alpha)\frac{C_s}{N_s} &= \frac{V'(N_s)}{U'(C_s)} \\
Y_1 &= \widetilde{C}_1 + I_1 + \frac{\psi}{2}(I_1)^2 \\
R_s &= \alpha\frac{Y_s}{I_1} \\
C_s &= Z_s (I_1)^{\alpha}(N_s)^{1-\alpha} \\
Y_s &= C_s \\
1 &= \sum_{s=1}^{N}\widehat{p}_s \quad \textit{and} \quad \widehat{\mathbf{P}} \prec \mathbf{P} \\
\textit{for} \quad & s = 1,\ldots,N
\end{aligned}$$

taking Y_1, R_1, K_1 *and the rational probability space* $\{\Omega, \mathcal{F}, \mathbf{P}\}$ *as given, and where* $\widehat{E}(X) \equiv \sum_{s=1}^{N} X_s \widehat{p}_s$ *and* $E(X) \equiv \sum_{s=1}^{N} X_s p_s$ *for the generic random variable* $X:\Omega \to \mathbb{R}$*.*

For comparative purposes, with N states the RE equilibrium is defined by:

Definition 7.2. *An RE equilibrium is a vector of exogenous beliefs* $\{p_s\}_{s=1}^{N}$*, endogenous prices* $\left\{r_f^{RE}, q_1^{RE}, R_s^{RE}\right\}_{s=1}^{N}$ *and endogenous quantities* $\left\{\widetilde{C}_1^{RE}, I_1^{RE}, C_s^{RE}, Y_s^{RE}, N_s^{RE}\right\}_{s=1}^{N}$

that satisfy

$$
\begin{aligned}
1+\psi I_1 &= \beta E\left(\frac{U'(C_2)}{U'\left(\widetilde{C}_1\right)}R_2\right) \\
U'\left(\widetilde{C}_1\right) &= \beta\left(1+r_f\right)E\left(U'(C_2)\right) \\
q_1 &= R_1K_1 - I_1 - \frac{\psi}{2}(I_1)^2 + \beta I_1 E\left(\frac{U'(C_2)}{U'\left(\widetilde{C}_1\right)}R_2\right) \\
(1-\alpha)\frac{C_s}{N_s} &= \frac{V'(N_s)}{U'(C_s)} \\
Y_1 &= \widetilde{C}_1 + I_1 + \frac{\psi}{2}(I_1)^2 \\
R_s &= \alpha\frac{Y_s}{I_1} \textit{ for } s = L,H \\
Y_s &= Z_s(I_1)^{\alpha}(N_s)^{1-\alpha} \\
Y_s &= C_s \\
\textit{for } s &= 1,\ldots,N
\end{aligned}
$$

taking Y_1, R_1, K_1 *and the rational probability space* $\{\Omega,\mathcal{F},\mathbf{P}\}$ *as given.*

Importantly, one can show, in the case of N states, there exists an OE economy that coincides with a RE economy, in terms of prices and quantities, and is uniquely defined up to the optimal expectation, $\widehat{E}\left(\frac{U'\left(C_2^{OE}\right)}{U'\left(\widetilde{C}_1\right)}R_2^{OE}\right) = E\left(\frac{U'\left(C_2^{RE}\right)}{U'\left(\widetilde{C}_1^{RE}\right)}R_2^{OE}\right)$.[112] However, it should be note that uniqueness of beliefs is no longer assured with $N > 2$. This is because there are many sets of subjective beliefs that are consistent with the same optimal expectation (or indeed, rational expectation).

[112] I omit this proof for brevity, though it is available on request.

Remark 7.1. *Note that with N-states the optimality conditions that characterise prices and quantities in an OE equilibrium are given by*

$$
\begin{aligned}
\beta\varepsilon\left(Y_1 - I_1^{OE} - \psi\frac{I_1^2}{2}\right)^{-\sigma} &= \left(1 + \psi I_1^{OE} - \frac{\psi}{\beta\varepsilon} I_1^{OE}\right)^{-1} \\
&\times \left(I_1^{OE}\right)^{\alpha - 1 - \alpha\sigma} \alpha\beta \\
&\times \sum_{s=1}^{N} Z_s^{1-\sigma} N_s^{(1-\alpha)(1-\sigma)} p_s \\
N_s &= \left((1-\alpha) Z_s^{1-\sigma} \left(I_1^{OE}\right)^{\alpha(1-\sigma)}\right)^{\frac{1}{\eta + \sigma(1-\alpha) + \alpha}}
\end{aligned}
$$

and that the unique expectation is given by

$$
\widehat{E}\left(\frac{U'\left(C_2\right)}{U'\left(\widetilde{C}_1\right)} R_2\right) = \frac{1 + \psi I_1^{OE}}{\beta}
$$

I now turn to the key results of interest.

Proposition 3.6.

(a) A positive perturbation to realised productivity in one state leads to over-investment and over-valuation in equity markets (assuming risk aversion is low), and an overly optimistic forecast of the return to investment.

$$
\begin{gathered}
\frac{\partial I_1^{OE}}{\partial Z_n} > \frac{\partial I_1^{RE}}{\partial Z_n} > 0 \\
\frac{\partial q_1^{OE}}{\partial Z_n} > \frac{\partial q_1^{RE}}{\partial Z_n} > 0 \\
\frac{\partial \widehat{E}\left(\frac{U'\left(C_2^{OE}\right)}{U'\left(\widetilde{C}_1^{OE}\right)} R_2^{OE}\right)}{\partial Z_n} > \frac{\partial E\left(\frac{U'\left(C_2^{RE}\right)}{U'\left(\widetilde{C}_1^{RE}\right)} R_2^{RE}\right)}{\partial Z_n} > 0 \\
\text{if } \sigma < 1 \text{ for } n \in \{1, \ldots, N\}
\end{gathered}
$$

(b) Consider a marginal shift in probability mass from some low productivity state (k) to a higher productivity state (j) (i.e. $dp_j = -dp_k$ where $j > k$ and $j,k \in \{1,\ldots,N\}$ and fixing

$dp_n = 0$ *for all* $n \neq j,k$*). Investment and equity prices increase beyond their fundamental values, and again forecast investment returns are overly optimistic.*

$$\frac{\partial I_1^{OE}}{\partial p_j} > \frac{\partial I_1^{RE}}{\partial p_j} > 0$$

$$\frac{\partial q_1^{OE}}{\partial p_j} > \frac{\partial q_1^{RE}}{\partial p_j} > 0$$

$$\frac{\partial \widehat{E}\left(\frac{U'\left(C_2^{OE}\right)}{U'\left(\widetilde{C}_1^{OE}\right)} R_2^{OE}\right)}{\partial p_j} > \frac{\partial E\left(\frac{U'\left(C_2^{RE}\right)}{U'\left(\widetilde{C}_1^{RE}\right)} R_2^{RE}\right)}{\partial p_j} > 0$$

$$\text{if } \sigma < 1$$

Proof. Focusing on result (a), and using methods analogous to those used in Proposition 3.1, it is straightforward to show that

$$\frac{\partial I_1^{OE}}{\partial Z_n} = \gamma_3 \frac{Z_n^{\gamma_3 - 1} p_n}{\sum_s Z_s^{1-\sigma} N_s^{(1-\alpha)(1-\sigma)} p_s} \left(\sigma \frac{1 + \psi I_1^{RE}}{\widetilde{C}_1^{RE}} + \frac{\psi - \frac{\psi}{\beta \varepsilon^{RE}}}{1 + \psi I_1^{RE} - \frac{\psi}{\beta \varepsilon^{RE}} I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}}\right)^{-1} > 0$$

$$\frac{\partial I_1^{RE}}{\partial Z_n} = \gamma_3 \frac{Z_n^{\gamma_3 - 1} p_n}{\sum_s Z_s^{1-\sigma} N_s^{(1-\alpha)(1-\sigma)} p_s} \left(\sigma \frac{1 + \psi I_1^{RE}}{\widetilde{C}_1^{RE}} + \frac{\psi}{1 + \psi I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}}\right)^{-1} > 0$$

and so $\frac{\partial I_1^{OE}}{\partial Z_n} > \frac{\partial I_1^{RE}}{\partial Z_n}$. Also, it still follows that with N-states and symmetric household and investment firm beliefs

$$\frac{\partial q_1^e}{\partial Z_n} = \psi I_1^{RE} \frac{\partial I_1^e}{\partial Z_n} \quad \text{for } e \in OE, RE$$

which can be used to confirm the second result. The third result in (a) can be derived by differentiating the equilibrium conditions with respect to Z_n and using $\frac{\partial I_1^{OE}}{\partial Z_n} > \frac{\partial I_1^{RE}}{\partial Z_n} > 0$

$$1 + \psi I_1^{OE} = \beta \widehat{E} \left(\frac{U'\left(C_2^{OE}\right)}{U'\left(\widetilde{C}_1^{OE}\right)} R_2^{OE} \right)$$
$$1 + \psi I_1^{RE} = \beta E \left(\frac{U'\left(C_2^{RE}\right)}{U'\left(\widetilde{C}_1^{RE}\right)} R_2^{RE} \right)$$

Turning to the results in (b), first I concentrate on the effect on investment. Recall that in equilibrium

$$\beta \varepsilon \left(Y_1 - I_1^{OE} - \psi \frac{\left(I_1^{OE}\right)^2}{2} \right)^{-\sigma} \left(1 + \psi I_1^{OE} - \frac{\psi}{\beta \varepsilon} I_1^{OE} \right) = \left(I_1^{OE}\right)^{\gamma_1} \gamma_2 \sum_s Z_s^{\gamma_3} p_s$$

Taking logs, and totally differentiating (fixing Z_s for all s, and fixing p_s for all $s \neq j$ or k)

$$\frac{\partial I_1^{OE}}{\partial p_j} = \left(\sigma \frac{1 + \psi I_1^{RE}}{\widetilde{C}_1^{RE}} + \frac{\psi \left(1 - \left(\beta \varepsilon^{RE} \right)^{-1} \right)}{1 + \psi I_1^{RE} - \frac{\psi}{\beta \varepsilon^{RE}} I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}} \right)^{-1} \frac{Z_j^{\gamma_3} + Z_k^{\gamma_3} \frac{dp_k}{dp_j}}{\sum_s Z_s^{\gamma_3} p_s}$$

Since $1 = -\frac{dp_k}{dp_j}$ by construction it follows that

$$\frac{\partial I_1^{OE}}{\partial p_j} = \left(\sigma \frac{1 + \psi I_1^{RE}}{\widetilde{C}_1^{RE}} + \frac{\psi \left(1 - \left(\beta \varepsilon^{RE} \right)^{-1} \right)}{1 + \psi I_1^{RE} - \frac{\psi}{\beta \varepsilon^{RE}} I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}} \right)^{-1} \frac{Z_j^{\gamma_3} - Z_k^{\gamma_3}}{\sum_{s=1}^N Z_s^{\gamma_3} p_s}$$

Thus, provided that the increment in probability involves a marginal shift in mass from some low state to a higher state, it follows that investment will increase following this marginal shift in mass if $\sigma < 1$ ($\gamma_3 > 0$).

Using a similar argument

$$\frac{\partial I_1^{RE}}{\partial p_j} = \left(\sigma \frac{1+\psi I_1^{RE}}{\widetilde{C}_1^{RE}} + \frac{\psi}{1+\psi I_1^{RE}} - \frac{\gamma_1}{I_1^{RE}}\right)^{-1} \frac{Z_j^{\gamma_3} - Z_k^{\gamma_3}}{\sum_{s=1}^{N} Z_s^{\gamma_3} p_s}$$

thus completing the first result (recall $\beta \varepsilon^{RE} > 1$).

The second result in (b) immediately follows from

$$\frac{\partial q_1^e}{\partial p_j} = \psi I_1^{RE} \frac{\partial I_1^e}{\partial p_j} \quad \text{for } e \in OE, RE$$

The third result in (b) follows from using the equilibrium conditions under OE and RE respectively

$$1 + \psi I_1^{OE} = \beta \widehat{E}\left(\frac{U'\left(C_2^{OE}\right)}{U'\left(\widetilde{C}_1^{OE}\right)} R_2^{OE}\right)$$
$$1 + \psi I_1^{RE} = \beta E\left(\frac{U'\left(C_2^{RE}\right)}{U'\left(\widetilde{C}_1^{RE}\right)} R_2^{RE}\right)$$

and that $\frac{\partial I_1^{OE}}{\partial p_j} > \frac{\partial I_1^{RE}}{\partial p_j} > 0$ (recall these derivatives are evaluated at $\varepsilon = \varepsilon^{RE}$, and so $I_1^{OE} = I_1^{RE}$ in the absence of the perturbation). □

7.6. Welfare

Proposition 3.7. *A social planner using probability measure $\widehat{\mathbf{P}}^{SP} \prec \mathbf{P}$, prefers investment firms that use **any** probability measure $\widehat{\mathbf{P}} \prec \mathbf{P}$ that satisfies*

$$\sum_{s=1}^{N}\left(\frac{U'\left(C_s^{SP}\right)}{U'\left(\widetilde{C}_1^{SP}\right)} R_s^{SP}\right) \widehat{p}_s^{SP} = \sum_{s=1}^{N}\left(\frac{U'\left(C_s^{SP}\right)}{U'\left(\widetilde{C}_1^{SP}\right)} R_s^{SP}\right) \widehat{p}_s$$

where $C_s^{SP}, R_s^{SP} = \alpha \frac{C_s^{SP}}{I_1^{SP}}, \widetilde{C}_1^{SP}$ are obtained from the solutions to (3.22).

Proof. This follows from the fact that $\left\{\widetilde{C}_1^{SP}, C_s^{SP}, I_s^{SP}, N_s^{SP}\right\}_{s=1}^{N}$ is the unique solution to the social planner problem in (3.22), and that for any alternative set of beliefs, $\{\widehat{p}_s\}_{s=1}^{N}$ satisfying Definition 7.1, these beliefs only induce the unique allocation $\left\{\widetilde{C}_1^{SP}, C_s^{SP}, I_s^{SP}, N_s^{SP}\right\}_{s=1}^{N}$, if and only if, $\sum_{s=1}^{N} \frac{U'\left(C_s^{SP}\right)}{U'\left(\widetilde{C}_1^{SP}\right)} R_s^{SP} \widehat{p}_s^{SP} = \sum_{s=1}^{N} \frac{U'\left(C_s^{SP}\right)}{U'\left(\widetilde{C}_1^{SP}\right)} R_s^{SP} \widehat{p}_s$. The latter result can be verified by showing that the FOC of the social planner problem are identical to those associated with OE equilibrium (in terms of quantities $\left\{\widetilde{C}_1, C_s, I_s, N_s\right\}_{s=1}^{N}$) in the case that $\sum_{s=1}^{N} \frac{U'\left(C_s^{SP}\right)}{U'\left(\widetilde{C}_1^{SP}\right)} R_s^{SP} \widehat{p}_s^{SP} = \sum_{s=1}^{N} \frac{U'\left(C_s^{SP}\right)}{U'\left(\widetilde{C}_1^{SP}\right)} R_s^{SP} \widehat{p}_s$ and not otherwise. □

7.7. Optimal Investment Expectations with an Infinite Horizon

Before proceeding to the full commitment solution to this problem, the following lemma will prove useful.

Lemma 7.2. *The constraints on I_t and K_{t+1}*

$$I_t, K_{t+1} \in \arg \max_{\widetilde{I}_t, \widetilde{K}_{t+1} : \widetilde{K}_{t+1} = (1-\delta)\widetilde{K}_t + \widetilde{I}_t} \widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U'(C_t)}{U'(C_0)} \left(R_t \widetilde{K}_t - \widetilde{I}_t - \Psi\left(\frac{\widetilde{I}_t}{\widetilde{K}_t}\right) \widetilde{K}_t \right) \tag{7.3}$$

can be equivalently be represented by

$$U'(C_t)\left(1 + \Psi'\left(\frac{I_t}{K_t}\right)\right) K_{t+1} = \sum_{\tau=1}^{\infty} \beta^{\tau} \widehat{E}_t U'(C_{t+\tau}) \left(R_{t+\tau} K_{t+\tau} - I_{t+\tau} - \Psi\left(\frac{I_{t+\tau}}{K_{t+\tau}}\right) K_{t+\tau} \right)$$

$$K_{t+1} = (1-\delta) K_t + I_t$$

Proof. Consider that

$$I_t, K_{t+1} \in \arg \max_{\widetilde{I}_t, \widetilde{K}_{t+1} : \widetilde{K}_{t+1} = (1-\delta)\widetilde{K}_t + \widetilde{I}_t} \widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U'(C_t)}{U'(C_0)} \left(R_t \widetilde{K}_t - \widetilde{I}_t - \Psi\left(\frac{\widetilde{I}_t}{\widetilde{K}_t}\right) \widetilde{K}_t \right)$$

can be solved using the Lagrangian

$$\Lambda = \widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U'(C_t)}{U'(C_0)} \left(R_t K_t - I_t - \Psi\left(\frac{I_t}{K_t}\right) K_t - q_t \left(K_{t+1} - (1-\delta) K_t - I_t\right) \right)$$

which has first-order conditions

$$U'(C_t)\,q_t = \beta \widehat{E}_t\left(U'(C_{t+1})\left(R_{t+1}+\Psi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}-\Psi\left(\frac{I_{t+1}}{K_{t+1}}\right)+q_{t+1}(1-\delta)\right)\right)$$

$$K_{t+1} = (1-\delta)K_t + I_t$$

where $q_t \equiv 1+\Psi'\left(\frac{I_t}{K_t}\right)$. A useful relationship to note is that

$$q_tK_{t+1} = \beta E_t\left(\frac{U'(C_{t+1})}{U'(C_t)}\begin{pmatrix} R_{t+1}K_{t+1}+\Psi'\left(\frac{I_{t+1}}{K_{t+1}}\right)I_{t+1} \\ -\Psi\left(\frac{I_{t+1}}{K_{t+1}}\right)K_{t+1}+q_{t+1}K_{t+1}(1-\delta)\end{pmatrix}\right)$$

Since

$$K_{t+1}(1-\delta) = K_{t+2} - I_{t+1}$$

the above relationship can be re-written as

$$q_tK_{t+1} = \beta E_t\left(\frac{U'(C_{t+1})}{U'(C_t)}\begin{pmatrix} R_{t+1}K_{t+1}+\Psi'\left(\frac{I_{t+1}}{K_{t+1}}\right)I_{t+1} \\ -\Psi\left(\frac{I_{t+1}}{K_{t+1}}\right)K_{t+1}-q_{t+1}I_{t+1}+q_{t+1}K_{t+2}\end{pmatrix}\right)$$

Solving forward

$$q_tK_{t+1} = \widehat{E}_t\sum_{\tau=1}^{\infty}\beta^{\tau}\frac{U'(C_{t+\tau})}{U'(C_t)}\left(R_{t+\tau}K_{t+\tau}-I_{t+\tau}-\Psi\left(\frac{I_{t+\tau}}{K_{t+\tau}}\right)K_{t+\tau}\right)$$

and using the definition of q_t yields the desired result. □

I now turn to the main result of interest.

Proposition 3.8. *Under full-commitment with $\varepsilon < \beta^{-1}$, and assuming that the optimal investment decision is conceivable (i.e. it is a an investment decision that can be optimally* *chosen under the restriction $\widehat{\mathbf{P}} \prec \mathbf{P}$), an interior solution to the program (3.24) is given by*

$$
\begin{aligned}
q_t &= \frac{\beta}{\beta\varepsilon} E_t \left(\frac{U'(C_{t+1})}{U'(C_t)} \left(R_{t+1} + \Psi'\left(\frac{I_{t+1}}{K_{t+1}}\right) \frac{I_{t+1}}{K_{t+1}} - \Psi\left(\frac{I_{t+1}}{K_{t+1}}\right) + (1-\delta)\, q_{t+1} \right) \right) \\
&\quad + \frac{(1-\beta\varepsilon)}{\beta\varepsilon} \left(\Psi''\left(\frac{I_t}{K_t}\right) \frac{K_{t+1}}{K_t} \right) \\
&\quad - \frac{(1-\beta\varepsilon)}{\beta\varepsilon} \beta E_t \left(\frac{U'(C_{t+1})}{U'(C_t)} \Psi''\left(\frac{I_{t+1}}{K_{t+1}}\right) \frac{K_{t+2}^2}{K_{t+1}^2} \right) \\
q_t &\equiv 1 + \Psi'\left(\frac{I_t}{K_t}\right) \\
K_{t+1} &= (1-\delta) K_t + I_t \\
K_0 &: given
\end{aligned}
\tag{7.4}
$$

This will characterise the investment decision made under optimal expectations, provided there exists a set of optimally chosen beliefs, $\widehat{\mathbf{P}}$, satisfying

$$
q_t = \beta \widehat{E}_t \left(\frac{U'(C_{t+1})}{U'(C_t)} \left(R_{t+1} + \Psi'\left(\frac{I_{t+1}}{K_{t+1}}\right) \frac{I_{t+1}}{K_{t+1}} - \Psi\left(\frac{I_{t+1}}{K_{t+1}}\right) + (1-\delta)\, q_{t+1} \right) \right)
$$

and $\widehat{\mathbf{P}}$ is a well-defined probability measure with $\widehat{\mathbf{P}} \prec \mathbf{P}$

Proof. Following the approach of Brunnermeier and Parker the OE program for firm managers with an infinite horizon is given by:

$$
\begin{aligned}
\max_{\widehat{\mathbf{P}}, \{I_t, K_{t+1}\}_{t=0}^{\infty}} & E \sum_{t=0}^{\infty} \beta^t \left(\sum_{\tau=1}^{t} \varepsilon^{\tau} U'(C_{t-\tau}) \left(R_{t-\tau} K_{t-\tau} - I_{t-\tau} - \Psi\left(\frac{I_{t-\tau}}{K_{t-\tau}}\right) K_{\tau-\tau} \right) \right) \\
& + E \sum_{t=0}^{\infty} \beta^t U'(C_t) \left(R_t K_t - I_t - \Psi\left(\frac{I_t}{K_t}\right) K_t \right) \\
& + E \sum_{t=0}^{\infty} \beta^t \left(\sum_{\tau=1}^{\infty} \beta^{\tau} \widehat{E}_t U'(C_{t+\tau}) \left(R_{t+\tau} K_{t+\tau} - I_{t+\tau} - \Psi\left(\frac{I_{t+\tau}}{K_{t+\tau}}\right) K_{t+\tau} \right) \right)
\end{aligned}
\tag{7.5}
$$

subject to:

$$\widehat{\mathbf{P}} \text{ is a well-defined probability space with } \widehat{\mathbf{P}} \prec \mathbf{P}$$

$$\widetilde{I}_t, \widetilde{K}_{t+1} \in \arg \max_{I_t, K_{t+1}: \widetilde{K}_{t+1} = (1-\delta)\widetilde{K}_t + \widetilde{I}_t} \widehat{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U'(C_t)}{U'(C_0)} \left(R_t \widetilde{K}_t - \widetilde{I}_t - \Psi\left(\frac{\widetilde{I}_t}{\widetilde{K}_t}\right) \widetilde{K}_t \right)$$

where the constraint that

$$\widehat{\mathbf{P}} \text{ is a well-defined probability space with } \widehat{\mathbf{P}} \prec \mathbf{P}$$

is equivalent to the following restrictions on the optimal subjective probabilities

$$\begin{aligned}
1 &= \sum_{s_{t+1}} \widehat{\Pr}\left(s_{t+1} \mid s^t\right) \text{ for all } t, \text{ and for all } \tau \geq 1 \\
\widehat{\Pr}\left(s_{t+1} \mid s^t\right) &\geq 0 \text{ for all } t \\
\widehat{\Pr}\left(s^{t+\tau} \mid s^t\right) &= \prod_{j=1}^{\tau} \widehat{\Pr}^{(t)}\left(s_{t+j} \mid s^{t+j-1}\right) \text{ for all } t, \text{ and for all } \tau \geq 1 \\
\widehat{\Pr}\left(s_{t+1} \mid s^t\right) &= 0 \text{ if } \Pr\left(s_{t+1} \mid s^t\right) = 0 \text{ for all } t
\end{aligned}$$

To solve this program, I begin by defining the following random variables on the probability space $\{\Omega, \mathcal{F}, \mathbf{P}\}$.

$$\begin{aligned}
Y_t &\equiv \beta E_t\left(m_{t+1} X_{t+1}\right) + \beta Y_{t+1} \\
H_t &\equiv \varepsilon X_{t-1} + \varepsilon H_{t-1} \\
X_t &\equiv U'(C_t) \left(R_t K_t - I_t - \Psi\left(\frac{I_t}{K_t}\right) K_t \right) \\
m_{t+1} &\equiv \frac{\widehat{\Pr}\left(s_{t+1} \mid s^t\right)}{\Pr\left(s_{t+1} \mid s^t\right)}
\end{aligned}$$

where I have made use of the property that under absolute continuity, there exists a random variable m_{t+1} satisfying

$$\widehat{E}_t\left(X_{t+1}\right) = E_t\left(m_{t+1}X_{t+1}\right)$$
$$1 = E_t\left(m_{t+1}\right)$$

Using Lemma 7.2, the above definitions, and reformulating the problem under the restriction of absolutely continuity, it follows that the optimal expectations program can be re-written as

$$\max E \sum_{t=0} \beta^t \left(H_t + X_t + Y_t\right)$$

subject to:

$$0 = Y_t - \beta E_t\left(m_{t+1}X_{t+1}\right) - \beta E_t Y_{t+1}$$
$$0 = H_t - \varepsilon H_{t-1} - \varepsilon X_{t-1}$$
$$0 = 1 - E_t\left(m_{t+1}\right)$$
$$0 = X_t - U'\left(C_t\right)\left(R_t K_t - I_t - \Psi\left(\frac{I_t}{K_t}\right)K_t\right)$$
$$0 = K_{t+1} - (1-\delta)K_t - I_t$$
$$0 = U'\left(C_t\right)\left(1 + \Psi'\left(\frac{I_t}{K_t}\right)\right)K_{t+1} - Y_t$$
$$m_{t+1} \geq 0$$
$$H_{-1} = 0, X_{-1} = 0 \text{ and } K_0 \text{ given}$$

The FOC w.r.t $X_t, H_t, Y_t, m_{t+1}, I_t, K_{t+1}$ at an interior are respectively

$$0 = 1 - \lambda_{4,t} + \lambda_{1,t-1} m_t + \beta \varepsilon E_t \lambda_{2,t+1} \tag{7.6}$$

$$0 = 1 - \lambda_{2,t} + \beta \varepsilon E_t \lambda_{2,t+1} \tag{7.7}$$

$$0 = 1 - \lambda_{1,t} + \lambda_{6,t} + \lambda_{1,t-1} \tag{7.8}$$

$$0 = \beta \lambda_{1,t} E_t X_{t+1} + \lambda_{3,t} \tag{7.9}$$

$$\begin{aligned} 0 = {} & -\lambda_{4,t} U'(C_t) \left(1 + \Psi'\left(\frac{I_t}{K_t}\right)\right) + \lambda_{5,t} \\ & - \lambda_{6,t} U'(C_t) \Psi''\left(\frac{I_t}{K_t}\right) \frac{K_{t+1}}{K_t} \end{aligned} \tag{7.10}$$

$$\begin{aligned} 0 = {} & -\lambda_{5,t} - \lambda_{6,t} U'(C_t) \left(1 + \Psi'\left(\frac{I_t}{K_t}\right)\right) \\ & + \beta E_t \lambda_{4,t+1} U'(C_{t+1}) \left(R_{t+1} + \Psi'\left(\frac{I_{t+1}}{K_{t+1}}\right) - \Psi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right) \\ & + \beta (1-\delta) E_t \lambda_{5,t+1} + \beta E_t \lambda_{6,t+1} U'(C_{t+1}) \Psi''\left(\frac{I_{t+1}}{K_{t+1}}\right) \frac{I_{t+1} K_{t+2}}{K_{t+1}^2} \end{aligned} \tag{7.11}$$

with the constraints completing the full set of FOC at an interior solution. I now simplify the above conditions.

Notice there are two possible solutions to (7.9)

$$\lambda_{1,t} = \lambda_{3,t} = 0$$

$$\text{or } \beta \lambda_{1,t} E_t X_{t+1} = -\lambda_{3,t}$$

I choose to focus on an equilibrium where the first solution holds. In this equilibrium the lagrange multiplier on the absolute continuity constraint is valued at 0, implying that at the optimum the restriction of absolute continuity does not constrain the optimal choice of investment and beliefs. For further discussion on this interior equilibrium, see Remark 7.2.

Proceeding under an equilibrium where $\lambda_{1,t} = \lambda_{3,t} = 0$ it follows from (7.8) and (7.7) that

$$\lambda_{6,t} = -1$$
$$\lambda_{2,t} = \frac{1}{1-\beta\varepsilon}$$

Using these results in (7.6)

$$\lambda_{4,t} = \frac{1}{1-\beta\varepsilon}$$

And so (7.10) and (7.11) collapse respectively to

$$\begin{aligned}
0 &= -\frac{1}{1-\beta\varepsilon}U'(C_t)\left(1+\Psi'\left(\frac{I_t}{K_t}\right)\right)+\lambda_{5,t}+U'(C_t)\Psi''\left(\frac{I_t}{K_t}\right)\frac{K_{t+1}}{K_t} \\
0 &= -\lambda_{5,t}+U'(C_t)\left(1+\Psi'\left(\frac{I_t}{K_t}\right)\right) \\
&\quad +\frac{\beta}{1-\beta\varepsilon}E_t\left(U'(C_{t+1})\left(R_{t+1}+\Psi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}-\Psi\left(\frac{I_{t+1}}{K_{t+1}}\right)\right)\right) \\
&\quad +\beta E_t\lambda_{5,t+1}(1-\delta)-E_t\left(U'(C_{t+1})\Psi''\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}K_{t+2}}{K_{t+1}^2}\right)
\end{aligned}$$

or more simply

$$\begin{aligned}
\frac{\beta\varepsilon}{1-\beta\varepsilon}U'(C_t)q_t &= \frac{\beta}{1-\beta\varepsilon}E_t\left(U'(C_{t+1})\begin{pmatrix} R_{t+1}+\Psi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}} \\ -\Psi\left(\frac{I_{t+1}}{K_{t+1}}\right)+(1-\delta)q_{t+1}\end{pmatrix}\right) \\
&\quad +U'(C_t)\Psi''\left(\frac{I_t}{K_t}\right)\frac{K_{t+1}}{K_t} \\
&\quad -\beta E_t\left(U'(C_{t+1})\Psi''\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{K_{t+2}^2}{K_{t+1}^2}\right) \\
q_t &\equiv 1+\Psi'\left(\frac{I_t}{K_t}\right)
\end{aligned}$$

Note that the implicit assumption in this solution for investment is that the investment firm manager would chose the above solution even if he or she was not restricted to choosing optimal subjective probabilities that are absolutely continous with respect to the rational probability measure. That is, this solution assumes there exists a subjective probability measure,$\widehat{\mathbf{P}}$, satisfying

$$q_t = \beta \widehat{E}_t \left(\frac{U'(C_{t+1})}{U'(C_t)} \left(R_{t+1} + \Psi' \left(\frac{I_{t+1}}{K_{t+1}} \right) \frac{I_{t+1}}{K_{t+1}} - \Psi \left(\frac{I_{t+1}}{K_{t+1}} \right) + (1-\delta) q_{t+1} \right) \right)$$

and $\widehat{\mathbf{P}}$ is a well-defined probability measure with $\widehat{\mathbf{P}} \prec \mathbf{P}$

□

Remark 7.2. *To clarify, the interior equilibrium I focus on is equivalent to solving the program in (7.5) using the following algorithm.*

Step 1. Obtain a sequence $\left\{ I_t^* \right\}$ *that solves*

$$\begin{gathered}
\max E \sum_{t=0} \beta^t (H_t + X_t + Y_t) \\
\textit{subject to:} \\
0 = H_t - \varepsilon H_{t-1} - \varepsilon X_{t-1} \\
0 = X_t - U'(C_t) \left(R_t K_t - I_t - \Psi \left(\frac{I_t}{K_t} \right) K_t \right) \\
0 = K_{t+1} - (1-\delta) K_t - I_t \\
0 = U'(C_t) \left(1 + \Psi' \left(\frac{I_t}{K_t} \right) \right) K_{t+1} - Y_t \\
H_{-1} = 0, X_{-1} = 0 \textit{ and } K_0 \textit{ given}
\end{gathered} \tag{7.12}$$

Step 2. Find a set of beliefs $\widehat{\mathbf{P}}$ *that satisfies the constraints*

$$U'(C_t) \left(1 + \Psi' \left(\frac{I_t^*}{K_t^*} \right) \right) = \beta \widehat{E}_t \left(\begin{array}{c} U'\left(C_{t+1}^*\right) \left(R_{t+1}^* + \Psi' \left(\frac{I_{t+1}^*}{K_{t+1}^*} \right) \frac{I_{t+1}^*}{K_{t+1}^*} - \Psi \left(\frac{I_{t+1}^*}{K_{t+1}} \right) \right) \\ + (1-\delta) \left(1 + \Psi' \left(\frac{I_{t+1}^*}{K_{t+1}^*} \right) \right) \end{array} \right)$$

$$K_{t+1}^* = (1-\delta) K_t^* + I_t^*$$

$$K_0^* = K_0 : \textit{given}$$

and $\widehat{\mathbf{P}}$ *is a well-defined probability space with* $\widehat{\mathbf{P}} \prec \mathbf{P}$

Thus, implicitly the equilibrium I focus on assumes there exists a probability measure $\widehat{\mathbf{P}}$ *that satisfies the constraints specified in step 2.*

Conjecture 7.1. *A sufficient condition for establishing that such a* $\widehat{\mathbf{P}}$ *exists satisfying the conditions in step 2 is to verify that*

$$\overline{I}_t\left(s^t\right) \geq I_t^*\left(s^t\right) \geq \underline{I}_t\left(s^t\right) \text{ for } s^t \text{ and } t \geq 0$$

where $\underline{I}_t\left(s^t\right)$ *is the first element of the sequence* $\left\{\underline{I_\tau}\left(s^\tau\right)\right\}_{\tau\geq t}$ *solving*

$$\left\{\underline{I_\tau}\left(s^\tau\right)\right\}_{\tau\geq t} = \arg\max_{\{I_\tau\}_{\tau\geq t}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{U'\left(C_\tau\left(\underline{s}^\tau\right)\right)}{U'\left(C_t\left(s^t\right)\right)} \begin{pmatrix} R_\tau\left(\underline{s}^\tau\right) K_\tau\left(\underline{s}^\tau\right) - I_\tau\left(\underline{s}^\tau\right) \\ -\Psi\left(\frac{I_\tau\left(\underline{s}^\tau\right)}{K_\tau\left(\underline{s}^\tau\right)}\right) K_\tau\left(\underline{s}^\tau\right) \end{pmatrix}$$

$$s.t: K_{\tau+1} = (1-\delta) K_\tau + I_\tau$$

$$\text{with } K_t\left(s^t\right) \text{ given.}$$

and $\left\{\underline{s}^\tau\right\}_{\tau\geq t} \in \Omega$ *is a sequence of histories that results in the smallest possible investment decision at time* t, *given observed history* s^t, *that is conceivable assuming the future sequence* $\left\{\underline{s}^\tau\right\}_{\tau\geq t}$ *is realised almost surely. Similarly,* $\overline{I}_t\left(s^t\right)$ *is the first element of the sequence sequence* $\left\{\overline{I}_t\left(s^\tau\right)\right\}_{\tau\geq t}$ *solving*

$$\left\{\overline{I}_t\left(s^\tau\right)\right\}_{\tau\geq t} = \arg\max_{\{I_\tau\}_{\tau\geq t}} \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{U'\left(C_\tau\left(\overline{s}^\tau\right)\right)}{U'\left(C_t\left(s^t\right)\right)} \begin{pmatrix} R_\tau\left(\overline{s}^\tau\right) K_\tau\left(\overline{s}^\tau\right) - I_\tau\left(\overline{s}^\tau\right) \\ -\Psi\left(\frac{I_\tau\left(\overline{s}^\tau\right)}{K_\tau\left(\overline{s}^\tau\right)}\right) K_\tau\left(\overline{s}^\tau\right) \end{pmatrix}$$

subject to:

$$K_{\tau+1} = (1-\delta) K_\tau + I_\tau$$

$$\text{with } K_t\left(s^t\right) \text{ given.}$$

and $\left\{\overline{s}^\tau\right\}_{\tau\geq t} \in \Omega$ *is a sequence of histories that results in the largest possible investment decision at time* t, *given observed history* s^t, *that is conceivable assuming the future sequence* $\left\{\overline{s}^\tau\right\}_{\tau\geq t}$ *is realised almost surely.*

Proof. A formal proof is beyond the scope of this chapter. As a sketch of the conjectured argument, sufficiency of the proposed condition should follow from the fact that probability spaces are convex. If $\underline{I_t}\left(s^t\right)$ is the smallest possible investment decision that can be implemented given some probability measure $\underline{\mathbf{P}} \prec \mathbf{P}$, and $\overline{I_t}\left(s^t\right)$ is the largest possible investment decision that can be implemented given some probability measure $\overline{\mathbf{P}} \prec \mathbf{P}$, then if

$$\overline{I_t}\left(s^t\right) \geq I_t^*\left(s^t\right) \geq \underline{I_t}\left(s^t\right)$$

the convexity of the space defined by the general set of probability measures $\left\{\Omega, \mathcal{F}, \tilde{\mathbf{P}} \prec \mathbf{P}\right\}$ implies there should exist at least one subjective measure $\widehat{\mathbf{P}} \prec \mathbf{P}$ that implements $I_t^*\left(s^t\right)$, and thus satisfies the constraint specified in Step 2. $\square$

Chapter 8

Appendix to Chapter 4

8.1. Data

Equity prices

I use the Vanguard S&P 500 ETF price measured at the close of trading of each quarter from September 1976 to June 2010. These data are used in all estimation samples and are sourced from Thomson Reuters. For the pre-estimation specification tests that use a sample from March 1952 to June 2010, I backcast (splice) the Vanguard S&P 500 ETF price index using the price changes history for the US S&P 500 (again sourced from Thomson Reuters).

Consumption and labour income

These data are obtained from Martin Lettau's website (available from March 1952 to June 2010 at the time of writing), and are reported in log real per capita terms.[113] For a full description of these data see Lettau and Ludvigson (2004).

Dividends

Dividends per share are measured as the sum of gross dividends paid in the quarter, with respect to the US S&P 500 index (SPX). Non-seasonally adjusted data are sourced from Bloomberg. Seasonally adjusted estimates are calculated by the author using the US Census Bureau X12 method applied at a quarterly frequency.

[113] See http://faculty.haas.berkeley.edu/lettau/data/cay_q_10Q2.txt.

Non-equity net wealth

To construct a measure of non-equity net wealth I use:

$$\begin{aligned}\text{Non-Equity Wealth}_t = \quad & \text{Total Household Net Wealth}_t \\ & - \text{Household US Equity Wealth}_t\end{aligned}$$

where:

$$\begin{aligned}\text{Household US Equity Wealth}_t \quad = \quad & \text{Household Total Equity Wealth}_t \\ & \times \left(1 - \frac{\text{All US sector foreign equity}_t}{\text{All US sector domestic and foreign equity}_t}\right)\end{aligned}$$

Wealth data are obtained from the US Flow of Funds Accounts. Total household net wealth is reported under identifier FL152090005.Q, household total equity wealth under FL153064475.Q, all US sector holdings of foreign equity under FL263164103.Q, and all US sector holdings of domestic and foreign equity under FL893064125.Q. In line with Lettau and Ludvigson (2004), all wealth variables are lagged one quarter to be consistent with their beginning of quarter values.

Equity quantities

I use:

$$\text{Equity Quantities}_t = \frac{\text{Household US Equity Wealth}_t}{\text{Vanguard 500 ETF Price}_t}$$

where the denominator is the equity prices measure previously described. This measure is also lagged one quarter to be consistent with its beginning of quarter value.

Personal consumption expenditure deflator

Equity prices, non-equity wealth, dividends and labour income are deflated using the personal consumption expenditure deflator, Bureau of Economic Analysis NIPA Table 1.1.9 Line 2.

Population

Non-equity wealth and equity quantities are converted to per capita values using US population estimates, Bureau of Economic Analysis NIPA Table 7.1 Line 18.

Details of the instruments used in estimation are now discussed.

Valuation confidence index

These data are obtained from the Yale School of Management website. I use the proportion of respondents who viewed the stock market as overvalued. From October 1989 to April 2001 biannual survey data are available. From September 2001, six-month-ended averages are reported at a monthly frequency. I construct a biannual measure for the full sample, from October 1989 to April 2010, and then linearly interpolate the data to a quarterly frequency.[114] As noted in the main text, the use of a linear interpolation will have no effect on the validity of this instrument once the second difference of this measure is used in estimation.

Option volatility

This measure is 30-day option-implied equity volatility with respect to the US S&P 100. These data are sourced from Bloomberg (with ticker VXO) and are measured at market close on the last trading day of the relevant quarter. For comparison, estimation on a shorter sample was also undertaken using 30-day option-implied equity volatility with respect to the US S&P 500. These data are also sourced from Bloomberg (with ticker VIX).

[114] From September 2001 onwards, the Yale School of Management reports the six-month-ended average percentage responses, $\bar{s}_t = \frac{1}{6}\sum_{k=0}^{5} s_{t-k}$. To adjust for this change in reporting, biannual monthly survey responses are calculated by the author using

$$s_t = 6\left(\bar{s}_t - \bar{s}_{t-1}\right) + s_{t-6}$$

Forecast dispersion

This measures the weighted standard deviation in long-term earnings-per-share-growth forecasts for companies in the US S&P 500, using the Institutional Brokers' Estimate System (I/B/E/S). The weights used reflect market capitalisation, with this series sourced from Thomson Reuters. This series is measured at the beginning of the quarter.

8.2. Specification Tests

Table 8.1 highlights that the null of a unit root cannot be rejected for each of the endogenous regressors using either Augmented Dicky-Fuller or Phillips-Perron tests.

Table 8.1: Unit Root Tests
March 1953–June 2010 sample period

Variable	ADF test statistic[(a)]	PP test statistic[(b)]
Consumption	−1.71	−2.48
Dividends	−1.53	−1.37
Non-US-equity net worth	−0.79	−0.70
Labour income	−1.32	−1.83
Equity quantities	−1.04	−0.93
Equity prices	−1.91	−1.93

Notes: All tests include four lags in their construction; ***,**,* denote test statistics that reject the null of a unit root at the 1, 5 and 10 per cent significance levels
(a) Augmented Dicky-Fuller test statistic
(b) Phillips-Perron test statistic

Table 8.2 reports results from lag-order selection criteria tests, and Table 8.3 reports results from Johansen Trace Tests concerning the rank of the cointegration matrix.

Table 8.2: Lag-order Selection Criteria

March 1953–June 2010 sample period

Lags	LR(a)	FPE(b)	AIC(c)	HQIC(d)	SBIC(e)
0		$3.4 \times e^{-15}$	-33.42	-33.42	-33.42
1	3 651.9	$5.9 \times e^{-22}$	-48.99	-48.77	-48.45
2	197.67	$3.4 \times e^{-22^{*}}$	-49.53^{*}	-49.10^{*}	-48.46^{*}
3	67.15	$3.5 \times e^{-22}$	-49.51	-48.86	-47.90
4	56.61^{*}	$3.8 \times e^{-22}$	-49.45	-48.58	-47.29

Notes: * denotes lag length selected
(a) Likelihood ratio test statistic
(b) Final prediction error
(c) Akaike information criterion
(d) Hannan Quinn information criterion
(e) Schwarz Bayesian information criterion

Table 8.3: Johansen Trace Tests

Maximum rank	**Trace test statistic** Mar 1953–Jun 2010	**Trace test statistic** Jun 1986–Jun 2010	**Critical value(a)**
0	111.95	110.68	94.15
1	73.74	71.33	68.52
2	41.84*	38.75*	47.21
3	22.92	19.71	29.68
4	10.27	9.76	15.41
5	2.69	3.29	3.76

Notes: * Denotes the implied rank of the cointegration matrix
(a) 5 per cent level of signficance

Table 8.4 reports results from Lagrange Multiplier tests for up to third-order serial correlation in the VECM residuals (estimated subject to the restrictions that the cointegration rank $r = 2$ and that $\beta_2^{'} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$). A check on the stability properties of the eigenvalues for this restricted VECM are consistent with estimated model being stable.

Table 8.4: Lagrange Multiplier Tests for Residual Serial Correlation

June 1986–June 2010 sample period

Lags	Test statistic	P-value(a)
1	50.40	0.06
2	40.00	0.30
3	43.45	0.18

Note: (a) Obtained from a Chi-squared distribution with 36 degrees of freedom

8.3. Bootstrap Methodology

90 per cent confidence intervals are constructed using the following semi-parametric bootstrap procedure:

1. Using the procedure outlined in Section 4.3, I obtain estimates of the semi-structural residual vector $\widetilde{\varepsilon}_t = \left[\left(u_t^P\right)', \left(\varepsilon_t^T\right)'\right]'$ conditioning on $\widehat{\beta}$ and the instruments $\widehat{\beta}_1\mathbf{y}_t$ and z_t (recall z_t is the relevant instrument for mispricing shocks, either forecast dispersion, option volatility or valuation confidence).

2. Randomly draw with replacement (by column) from the matrix of estimation residuals and z_t, $\begin{bmatrix} \widetilde{\varepsilon}_1 & & \widetilde{\varepsilon}_T \\ & ,..., & \\ z_1 & & z_T \end{bmatrix}$, so that in effect a form of 'pairs' bootstrap is used that accounts for the joint empirical distribution of the errors and the instrument used in identification. One thousand random samples of length $T = 83$ are drawn.

3. Simulate data to construct the vector $\begin{bmatrix} \mathbf{y}_t^i \\ z_t^i \end{bmatrix}$ using

$$\begin{aligned} z_t^i &= z_t^i \\ \mathbf{y}_t^i &= \left(\mathbf{I} - \widehat{\widetilde{\mathbf{A}}}_0^{-1} \widehat{\widetilde{\alpha}}^* \beta' - \widehat{\widetilde{\mathbf{A}}}_0^{-1} \widehat{\widetilde{\mathbf{A}}}_2\right) \mathbf{y}_{t-1}^i + \widehat{\widetilde{\mathbf{A}}}_0^{-1} \widehat{\widetilde{\mathbf{A}}}_2 \mathbf{y}_{t-2}^i + \widetilde{\varepsilon}_t^i \end{aligned}$$

   for $t = 1,...,T$ and for $i = 1,...,1\ 000$ where i is an index identifying the relevant draw in Step 2, and where $\widehat{\widetilde{\alpha}}^*, \widehat{\widetilde{\mathbf{A}}}_0, \widehat{\widetilde{\mathbf{A}}}_2, \widehat{\beta}$ are the point estimates used to construct the statistics of interest discussed in the main text.[115]

4. For each artificial sample, i, estimate $\widehat{\widetilde{\alpha}}_i^*, \widehat{\widetilde{\mathbf{A}}}_{0,i}, \widehat{\widetilde{\mathbf{A}}}_{2,i}$ and then construct the estimated impulse response function (moving average) matrices $\left\{\widehat{\Psi}_{j,i}\right\}_{j=1}^{100}$ for $i = 1,...,1\ 000$. Note that $\widehat{\beta}$ is treated as known and is not re-estimated with each sample.

[115] For brevity, I abstract from deterministic terms. In implementation I allow for an unrestricted constant in the SVECM.

5. Construct Hall percentile confidence intervals following Lütkepohl (2006). Let $s^*_{j,0.05}$ and $s^*_{j,0.95}$ be the 5 and 95 percentiles of the statistic $s^*_j = \left(\widehat{\Psi}_{j,i} - \widehat{\Psi}_j\right)$ where $\widehat{\Psi}_j$ is the estimated impulse response function based on the observed data, j quarters after the initial shock of interest. The Hall confidence interval is given by

$$CI_H = \left[\widehat{\Psi}_j - s^*_{j,0.95}, \widehat{\Psi}_j - s^*_{j,0.05}\right]$$

8.4. Alternative Identification Strategy

Rather than partitioning the system in Equation (4.9) according to those variables directly influenced by permanent and transitory shocks, I now partition the system into equity prices $(\mathbf{y}_{22t})$ and other variables $\left(\widetilde{\mathbf{y}}_{1t} = [\mathbf{y}'_{1t}, \mathbf{y}_{21t}]'\right)$. That is,

$$\begin{bmatrix} B^0_{11} & B^0_{12} \\ B^0_{21} & 1 \end{bmatrix} \begin{bmatrix} \Delta\widetilde{\mathbf{y}}_{1t} \\ \Delta\mathbf{y}_{22t} \end{bmatrix} = -\alpha^*\beta' \begin{bmatrix} \widetilde{\mathbf{y}}_{1t-1} \\ \mathbf{y}_{22t-1} \end{bmatrix} - \begin{bmatrix} B^2_{11} & B^2_{12} \\ B^2_{21} & B^2_{22} \end{bmatrix} \begin{bmatrix} \Delta\widetilde{\mathbf{y}}_{1t-1} \\ \Delta\mathbf{y}_{22t-1} \end{bmatrix} + \begin{bmatrix} \widetilde{\varepsilon}^{P,T}_t \\ \varepsilon^{T,b}_t \end{bmatrix}$$

where again I use six normalisation restrictions on the main diagonal of $\mathbf{B}_0$. Using the same methodology as that discussed previously, it is straightforward to verify that provided $\left(B^0_{11}\right)^{-1}$ exists, one can proceed estimating

$$\begin{aligned}
\Delta\widetilde{\mathbf{y}}_{1t} = &-\left(B^0_{11}\right)^{-1} B^0_{12}\Delta\mathbf{y}_{22t} - \left(B^0_{11}\right)^{-1} \left[\alpha^*\beta'\right]_{11} \widetilde{\mathbf{y}}_{1t-1} \\
&- \left(B^0_{11}\right)^{-1} \left[\alpha^*\beta'\right]_{12} \mathbf{y}_{22t-1} - \left(B^0_{11}\right)^{-1} B^2_{11}\Delta\widetilde{\mathbf{y}}_{1t-1} \\
&- \left(B^0_{11}\right)^{-1} B^2_{12}\Delta\mathbf{y}_{22t-1} + \left(B^0_{11}\right)^{-1} \widetilde{\varepsilon}^{P,T}_t
\end{aligned}$$

using z_t as an instrument for $\Delta\mathbf{y}_{22t}$. The estimated reduced-form residuals, comprising both permanent and transitory shocks $\left(\widehat{\left(B^0_{11}\right)^{-1} \widetilde{\varepsilon}^{P,T}_t}\right)$, can then be used as instruments for $\Delta\widetilde{\mathbf{y}}_{1t}$ in the estimation of

$$\begin{aligned}\Delta\mathbf{y}_{22t} &= -B_{21}^{0}\Delta\widetilde{\mathbf{y}}_{1t} - \left[\alpha^{*}\beta'\right]_{21}\widetilde{\mathbf{y}}_{1t-1} \\ &\quad - \left[\alpha^{*}\beta'\right]_{22}\mathbf{y}_{22t-1} - B_{21}^{2}\Delta\widetilde{\mathbf{y}}_{1t-1} \\ &\quad - B_{22}^{2}\Delta\mathbf{y}_{22t-1} + \varepsilon_{t}^{T,b}\end{aligned}$$

where I have used the conformable partition

$$\alpha^{*}\beta' = \left[\begin{array}{cc} \left[\alpha^{*}\beta'\right]_{11} & \left[\alpha^{*}\beta'\right]_{12} \\ \left[\alpha^{*}\beta'\right]_{21} & \left[\alpha^{*}\beta'\right]_{22} \end{array}\right]$$

The additional restriction that mispricing shocks have only transitory effects, $\left(\lim_{k\to\infty}\frac{\partial E_t\left(\mathbf{y}_{t+k}\right)}{\partial\left(\varepsilon_t^{T,b}\right)'} = \mathbf{0}_{n\times 1}\right)$, is not imposed.

Chapter 9

References

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