

## SUPPLEMENTARY MATERIALS

### MICRO AND MACRO PARAMETRIC UNCERTAINTY IN CLIMATE CHANGE PREDICTION: A LARGE ENSEMBLE PERSPECTIVE\*

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**NOTE: THIS SUPPLEMENT ACCOMPANIES THE HOMONYM MANUSCRIPT SUBMITTED TO THE BULLETIN OF THE AMERICAN METEOROLOGICAL SOCIETY (BAMS)**

This PDF file includes:

- Supplementary text
- Figures S.1 to S.23

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# 1 Experiments

Here we describe in detail the experimental design of each simulation. The following holds for all experiments, unless otherwise noted:

- **Length of simulation:** 200 years, except experiments 37 and 38.
- **Time step:** 0.01 LTUs (1.2 hours).
- **Output frequency:** 0.2 LTUs (1 day).
- **Number of ensemble members:** 1,001 (1,000 plus central IC), except experiments 37 and 38.
- **Standard deviation in micro ICES:**  $\sigma_{\mathbf{X}_0} = (0.02, 0.02, 0.02, 0.002, 0.000001)$
- **Standard deviation in micro PPEs:** if the order of magnitude of a parameter  $P_l$  is  $10^n$ , then  $\sigma_{P_l} = 2 \cdot 10^{(n-2)}$ .
- **Rate of change:**  $H = 0.01$  (1 unit every 100 years), except experiments 37 and 38.
- **Year when climate change starts:**  $(t_{\text{start}}/73) = 0$ , except experiments 37 and 38.
- **Year when climate change ends:**  $(t_{\text{end}}/73) = 100$ , except experiments 37 and 38.

The ICs for macro PPEs vary across experiments, and are presented in detail below. All figures in the main manuscript and in this supplement are generated from 1-year averages, except Figures S.15 and S.16 which show daily averages.

**Spinup.** The periodically-forced model ( $H = 0$ ) was spun up for 3,000 years starting from  $\mathbf{X}_0 = (1, 1, 1, 5.5, 1.3 \times 10^{-3})$ , with the time step given by 0.01 LTUs as above.

## 1.1 Experiments 1 and 2: Micro ICES

- Initial condition:  $\mathbf{X}_0 = (1.012586, 1.030767, -2.622185 \times 10^{-1}, 4.5, 1.60 \times 10^{-3})$
- Use in the main manuscript: used in Figure 2(a,e), Figure 3(a,e), Figure 4(a,b), Figure 5(a,b)

## 1.2 Experiments 3 to 14: Micro PPEs

- Experiment 3: Micro PPE in  $(F_1, G_1)$ 
  - Initial condition: same as micro ICE
  - Parameters perturbed:  $(F_1, G_1)$
  - Use in the main manuscript: used in Figure 2(b,e), Figure 3(b,e), Figure 4(a,b), Figure 5(a,b)
- Experiment 4: Micro PPE in  $(a, b)$ 
  - Initial condition: same as micro ICE
  - Parameters perturbed:  $(a, b)$
  - Use in the main manuscript: used in Figure 2(e), Figure 3(e), Figure 4(a,b), Figure 5(a,b)
- Experiment 5: Micro PPE in  $M$ 
  - Initial condition: same as micro ICE
  - Parameters perturbed:  $M$
  - Use in the main manuscript: used in Figure 2(e), Figure 3(e), Figure 4(a,b), Figure 5(a,b)
- Experiment 6: Micro PPE in  $(F_1, G_1, a, b, M)$  (5 parameters)
  - Initial condition: same as micro ICE
  - Parameters perturbed:  $(F_1, G_1, a, b, M)$
  - Use in the main manuscript: used in Figure 2(c,e), Figure 3(c,e), Figure 4(a,b), Figure 5(a,b)

- Experiment 7: Micro PPE in  $G_0$

- Initial condition: same as micro ICE
- Parameters perturbed:  $G_0$
- Use in the main manuscript: used in Figure 2(e), Figure 3(e), Figure 4(a-c), Figure 5(a-c)

- Experiment 8: Micro PPE in all parameters minus  $G_0$  (except  $F_m$  and  $H$ )

- Initial condition: same as micro ICE
- Parameters perturbed:  $(F_1, G_1, a, b, M, T_{av}, \gamma, k_w, k_a, \omega, \epsilon, \delta_0, \delta_1)$
- Use in the main manuscript: used in Figure 2(e), Figure 3(e), Figure 4(a-c), Figure 5(a-c)

- Experiment 9: Micro PPE in all parameters (except  $F_m$  and  $H$ )

- Initial condition: same as micro ICE
- Parameters perturbed:  $(F_1, G_1, a, b, M, G_0, T_{av}, \gamma, k_w, k_a, \omega, \epsilon, \delta_0, \delta_1)$
- Use in the main manuscript: used in Figure 2(d,e), Figure 3(d,e), Figure 4(a-c), Figure 5(a-c)

- Experiment 10: Micro PPE in  $\omega, \epsilon$

- Initial condition: same as micro ICE
- Parameters perturbed:  $\omega, \epsilon$
- Used only in Figures S.4 to S.8 of this supplement

- Experiment 11: Micro PPE in  $\delta_0, \delta_1$

- Initial condition: same as micro ICE
- Parameters perturbed:  $\delta_0, \delta_1$
- Used only in Figures S.4 to S.8 of this supplement

- Experiment 12: Micro PPE in  $k_a, \gamma$

- Initial condition: same as micro ICE
- Parameters perturbed:  $k_a, \gamma$
- Used only in Figures S.4 to S.8 of this supplement

- Experiment 13: Micro PPE in  $k_w$

- Initial condition: same as micro ICE
- Parameters perturbed:  $k_w$
- Used only in Figures S.4 to S.8 of this supplement

- Experiment 14: Micro PPE in  $T_{av}$

- Initial condition: same as micro ICE
- Parameters perturbed:  $T_{av}$
- Used only in Figures S.4 to S.8 of this supplement

### 1.3 Experiments 15 to 25: Macro PPEs of Type I

- Experiments 15 to 17: Macro PPEs in  $F_1$

- Initial condition:  $\mathbf{X}_0 = (1.012586, 1.030767, -2.622185 \times 10^{-1}, 4.5, 1.60 \times 10^{-3})$
- Reference run:  $F_1 = 0.02$  (experiment 15)
- Macro perturbations:  $F_1 = 0.01$  (experiment 16),  $F_1 = 0.03$  (experiment 17)
- Use in the main manuscript: used in Figure 6(a), Figure 7(a)

- Experiments 18 to 20: Macro PPEs in  $b$

- Initial condition:  $\mathbf{X}_0 = (1.012586, 1.030767, -2.622185 \times 10^{-1}, 4.5, 1.55 \times 10^{-3})$
- Reference run:  $b = 4$  (experiment 18)
- Macro perturbations:  $b = 3.5$  (experiment 19),  $b = 4.5$  (experiment 20)
- Use in the main manuscript: used in Figure 6(c), Figure 7(c)

- Experiments 21 to 25: Macro PPEs in  $M$

- Initial condition:  $\mathbf{X}_0 = (1.012586, 1.030767, -2.622185 \times 10^{-1}, 4.5, 1.57 \times 10^{-3})$
- Reference run:  $M = 1$  (experiment 21)
- Macro perturbations:  $M = 1.1$  (experiment 22),  $M = 1.3$  (experiment 23),  $M = 0.9$  (experiment 24),  $M = 0.7$  (experiment 25)
- Use in the main manuscript: used in Figure 6(e,g), Figure 7(e,g)

### 1.4 Experiments 26 to 36: Macro PPEs of Type II

- Experiments 26 to 28: Macro PPEs in  $F_1$

- Initial condition and parameter values: Same as experiments 15 to 17.
- Use in the main manuscript: used in Figure 6(b), Figure 7(b)

- Experiments 29 to 31: Macro PPEs in  $b$

- Initial condition and parameter values: Same as experiments 18 to 20.
- Use in the main manuscript: used in Figure 6(d), Figure 7(d)

- Experiments 32 to 36: Macro PPEs in  $M$

- Initial condition and parameter values: Same as experiments 21 to 25.
- Use in the main manuscript: used in Figure 6(f,h), Figure 7(f,h)

### 1.5 Experiments 37 and 38: Attractor for $b = 4$ and $b = 4.5$

- Initial condition:  $\mathbf{X}_0 = (1.012586, 1.030767, -2.622185 \times 10^{-1}, 4.080627, 1.658932 \times 10^{-3})$
- Length of simulation: 5,000 years
- Number of ensemble members: 1 (central IC only)
- Rate of change:  $H = 0$
- Parameter values:  $b = 4$  (experiment 37) and  $b = 4.5$  (experiment 38)
- Use in the manuscript: appears only in Figures S.15 and S.16 of this supplement

## 1.6 Experiments 39 and 40: Micro ICEs with different variances

- Experiment 39: micro ICE with standard deviation reduced by one order of magnitude
  - Initial condition: same as micro ICE
  - Standard deviation:  $\sigma_{\mathbf{x}_0} = (0.02, 0.02, 0.02, 0.002, 0.000001) \times 10^{-1}$
  - Use in the main manuscript: appears only in Figures S.17 and S.18 of this supplement
- Experiment 40: micro ICE with standard deviation increased by one order of magnitude
  - Initial condition: same as micro ICE
  - Standard deviation:  $\sigma_{\mathbf{x}_0} = (0.02, 0.02, 0.02, 0.002, 0.000001) \times 10^1$
  - Use in the main manuscript: appears only in Figures S.19 and S.20 of this supplement

## 1.7 Experiments 41 to 45: Micro PPEs with standard deviation reduced by one order of magnitude

- Experiment 41: Micro PPE in  $(F_1, G_1)$ 
  - Initial condition: same as micro ICE
  - Parameters perturbed:  $(F_1, G_1)$
  - Standard deviation:  $\sigma_{P_l} \times 10^{-1}$
  - Use in the main manuscript: appears only in Figures S.17 and S.18 of this supplement
- Experiment 42: Micro PPE in  $(a, b)$ 
  - Initial condition: same as micro ICE
  - Parameters perturbed:  $(a, b)$
  - Standard deviation:  $\sigma_{P_l} \times 10^{-1}$
  - Use in the main manuscript: appears only in Figures S.17 and S.18 of this supplement
- Experiment 43: Micro PPE in  $(F_1, G_1, a, b, M)$ 
  - Initial condition: same as micro ICE
  - Parameters perturbed:  $(F_1, G_1, a, b, M)$
  - Standard deviation:  $\sigma_{P_l} \times 10^{-1}$
  - Use in the main manuscript: appears only in Figures S.17 and S.18 of this supplement
- Experiment 44: Micro PPE in  $G_0$ 
  - Initial condition: same as micro ICE
  - Parameters perturbed:  $G_0$
  - Standard deviation:  $\sigma_{P_l} \times 10^{-1}$
  - Use in the main manuscript: appears only in Figures S.17 and S.18 of this supplement
- Experiment 45: Micro PPE in all parameters (except  $F_m$  and  $H$ )
  - Initial condition: same as micro ICE
  - Parameters perturbed:  $(F_1, G_1, a, b, M, G_0, T_{av}, \gamma, k_w, k_a, \omega, \epsilon, \delta_0, \delta_1)$
  - Standard deviation:  $\sigma_{P_l} \times 10^{-1}$
  - Use in the main manuscript: appears only in Figures S.17 and S.18 of this supplement

## 1.8 Experiments 46 to 50: Micro PPEs with standard deviation increased by one order of magnitude

- Experiment 46: Micro PPE in  $(F_1, G_1)$

- Initial condition: same as micro ICE
- Parameters perturbed:  $(F_1, G_1)$
- Standard deviation:  $\sigma_{P_t} \times 10^1$
- Use in the main manuscript: appears only in Figures S.19 and S.20 of this supplement

- Experiment 47: Micro PPE in  $(a, b)$

- Initial condition: same as micro ICE
- Parameters perturbed:  $(a, b)$
- Standard deviation:  $\sigma_{P_t} \times 10^1$
- Use in the main manuscript: appears only in Figures S.19 and S.20 of this supplement

- Experiment 48: Micro PPE in  $(F_1, G_1, a, b, M)$

- Initial condition: same as micro ICE
- Parameters perturbed:  $(F_1, G_1, a, b, M)$
- Standard deviation:  $\sigma_{P_t} \times 10^1$
- Use in the main manuscript: appears only in Figures S.19 and S.20 of this supplement

- Experiment 49: Micro PPE in  $G_0$

- Initial condition: same as micro ICE
- Parameters perturbed:  $G_0$
- Standard deviation:  $\sigma_{P_t} \times 10^1$
- Use in the main manuscript: appears only in Figures S.19 and S.20 of this supplement

- Experiment 50: Micro PPE in all parameters (except  $F_m$  and  $H$ )

- Initial condition: same as micro ICE
- Parameters perturbed:  $(F_1, G_1, a, b, M, G_0, T_{av}, \gamma, k_w, k_a, \omega, \epsilon, \delta_0, \delta_1)$
- Standard deviation:  $\sigma_{P_t} \times 10^1$
- Use in the main manuscript: appears only in Figures S.19 and S.20 of this supplement

## 1.9 Experiments 51 to 56: Micro PPEs with different initial conditions

- Experiment 51: Micro PPE in  $(F_1, G_1)$  (reproduction of experiment 3)

- Initial condition: same as micro ICE
- Parameters perturbed:  $(F_1, G_1)$
- Use in the main manuscript: appears only in Figures S.21 and S.22 of this supplement

- Experiment 52: Micro PPE in all parameters (except  $F_m$  and  $H$ ) (reproduction of experiment 9)

- Initial condition: same as micro ICE
- Parameters perturbed:  $(F_1, G_1, a, b, M, G_0, T_{av}, \gamma, k_w, k_a, \omega, \epsilon, \delta_0, \delta_1)$
- Use in the main manuscript: appears only in Figures S.21 and S.22 of this supplement

- Experiment 53: Micro PPE in  $(F_1, G_1)$

- Initial condition:  $\mathbf{X}_0 = (1.550668, -1.574188 \times 10^{-1}, 1.380261, 4.166778, 1.656004 \times 10^{-3})$

- Parameters perturbed:  $(F_1, G_1)$
- Use in the main manuscript: appears only in Figures S.21 and S.22 of this supplement

- Experiment 54: Micro PPE in all parameters (except  $F_m$  and  $H$ )

- Initial condition: same as experiment 53
- Parameters perturbed:  $(F_1, G_1, a, b, M, G_0, T_{av}, \gamma, k_w, k_a, \omega, \epsilon, \delta_0, \delta_1)$
- Use in the main manuscript: appears only in Figures S.21 and S.22 of this supplement

- Experiment 55: Micro PPE in  $(F_1, G_1)$

- Initial condition:  $\mathbf{X}_0 = (4.617810 \times 10^{-2}, 1.553979, -5.371011 \times 10^{-2}, 4.084877, 1.645639 \times 10^{-3})$
- Parameters perturbed:  $(F_1, G_1)$
- Use in the main manuscript: appears only in Figures S.21 and S.22 of this supplement

- Experiment 56: Micro PPE in all parameters (except  $F_m$  and  $H$ )

- Initial condition: same as experiment 55
- Parameters perturbed:  $(F_1, G_1, a, b, M, G_0, T_{av}, \gamma, k_w, k_a, \omega, \epsilon, \delta_0, \delta_1)$
- Use in the main manuscript: appears only in Figures S.21 and S.22 of this supplement

## 2 Supplementary figures

Figures S.1 to S.14 supplement those provided in the main manuscript as follows:

- Figures S.1 to S.3 supplements Figures 2 and 3 for variables  $Y, Z$  and  $S$ ;
- Figures S.4 to S.8 supplements Figures 2(a-d) and 3(a-d) for parameters  $a, b, M, G_0, \omega, \epsilon, \delta_0, \delta_1, k_a, \gamma, k_w, T_{av}$  (for all variables).
- Figures S.9 to S.11 supplements Figures 4 and 5 for variables  $Y, Z$  and  $S$ ;
- Figures S.12 to S.14 supplements Figures 6 and 7 for variables  $Y, Z$  and  $S$ .

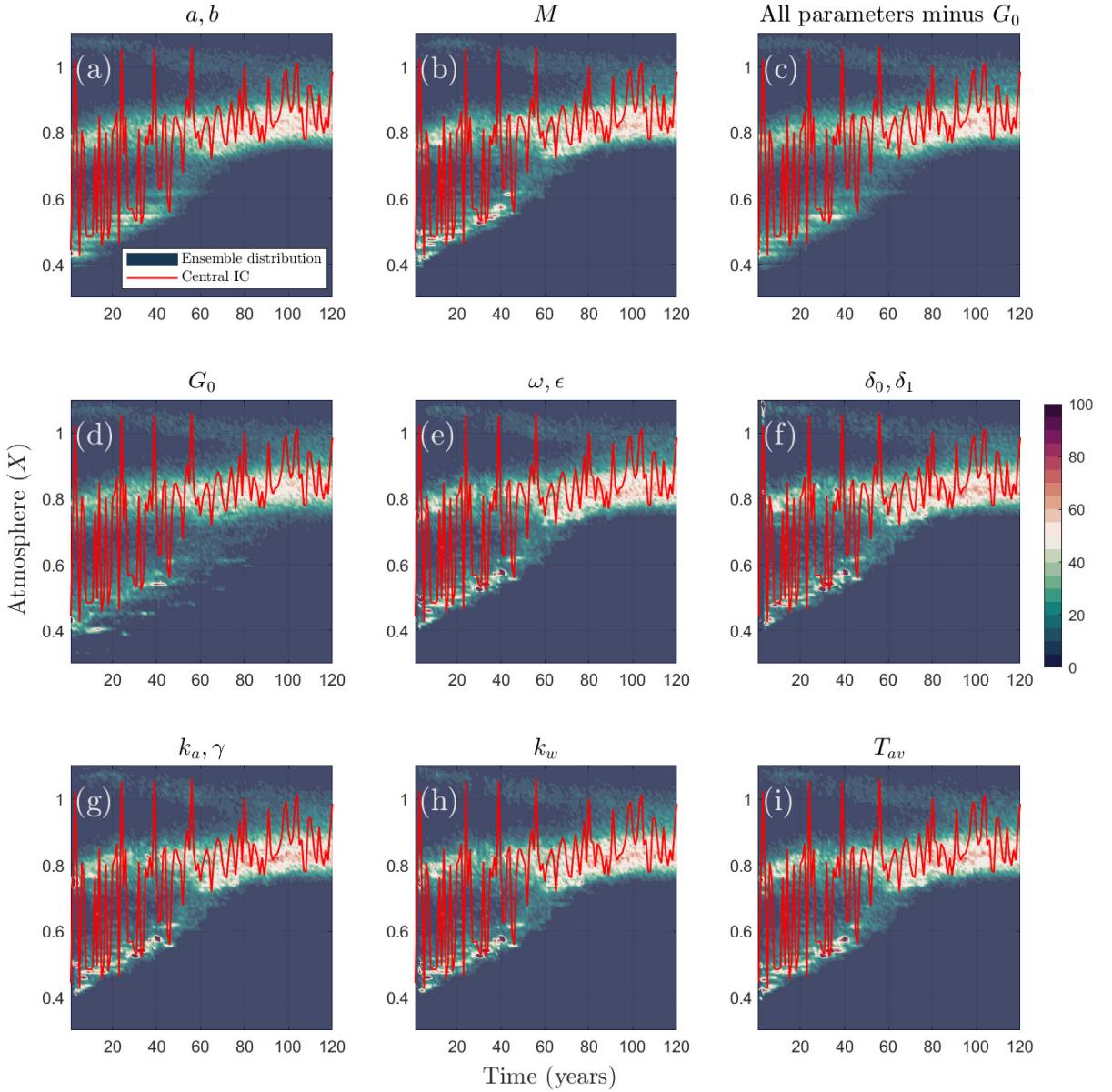


Figure S.1: Micro PPE distributions for different parameter values. The figure shows 120 years of atmosphere variable  $X$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a)  $a, b$  (b)  $M$ ; (c) All parameters minus  $G_0$ ; (d)  $G_0$ ; (e)  $\omega, \epsilon$ ; (f)  $\delta_0, \delta_1$ ; (g)  $k_a, \gamma$ ; (h)  $k_w$ ; (i)  $T_{av}$ .

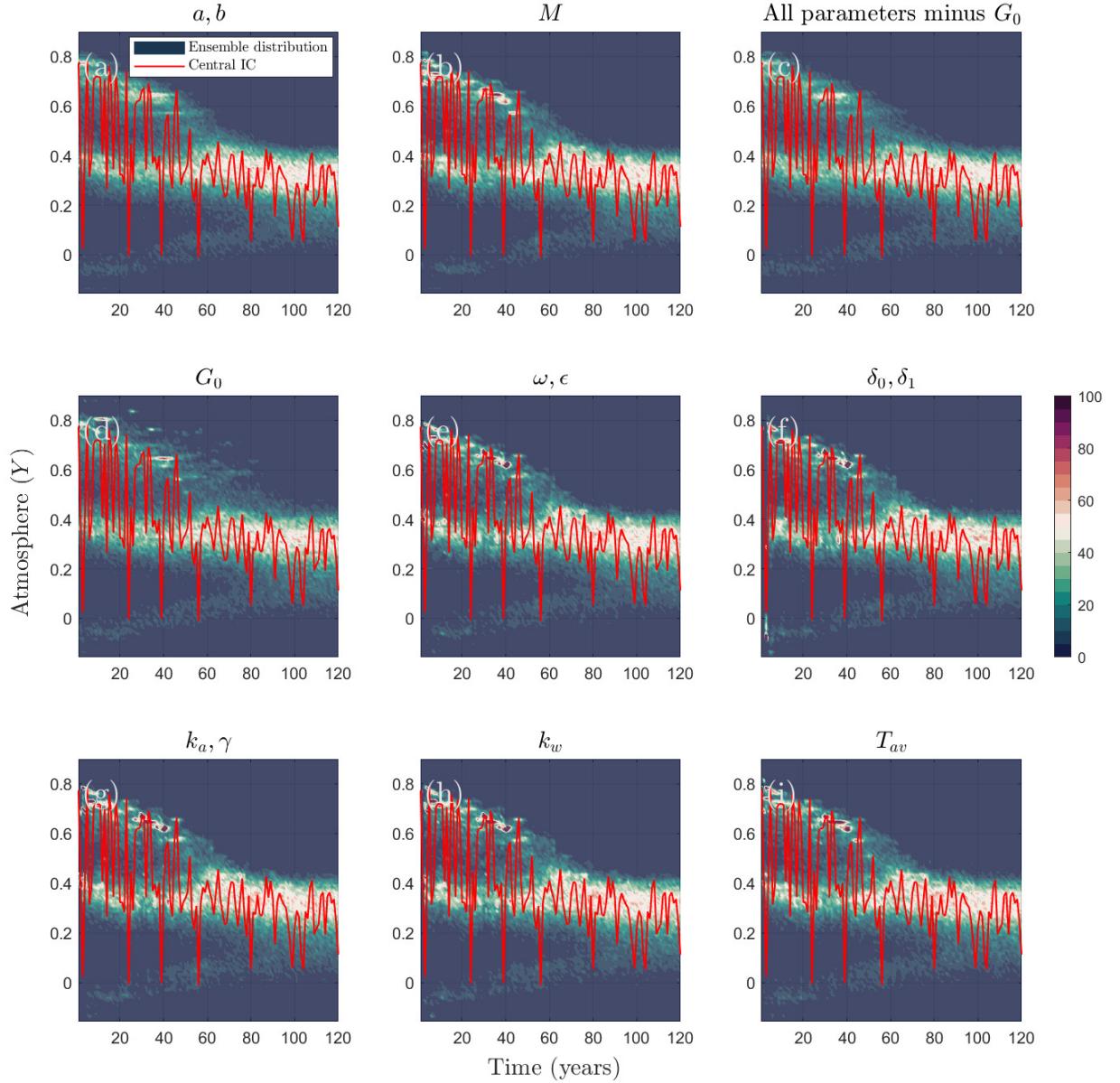


Figure S.2: Micro PPE distributions for different parameter values. The figure shows 120 years of atmosphere variable  $Y$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a)  $a, b$  (b)  $M$ ; (c) All parameters minus  $G_0$ ; (d)  $G_0$ ; (e)  $\omega, \epsilon$ ; (f)  $\delta_0, \delta_1$ ; (g)  $k_a, \gamma$ ; (h)  $k_w$ ; (i)  $T_{av}$ .

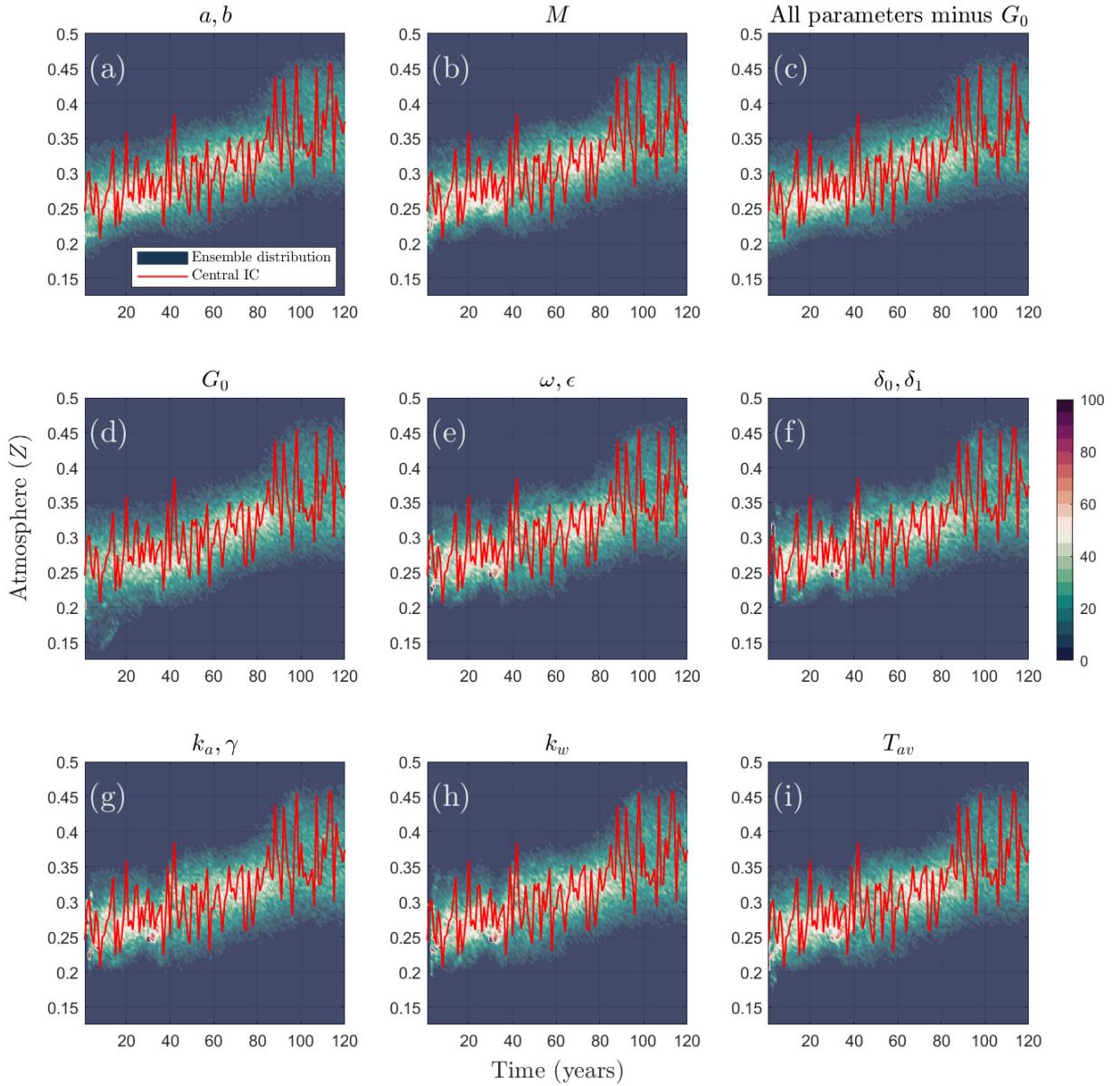


Figure S.3: Micro PPE distributions for different parameter values. The figure shows 120 years of atmosphere variable  $Z$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a)  $a, b$  (b)  $M$ ; (c) All parameters minus  $G_0$ ; (d)  $G_0$ ; (e)  $\omega, \epsilon$ ; (f)  $\delta_0, \delta_1$ ; (g)  $k_a, \gamma$ ; (h)  $k_w$ ; (i)  $T_{av}$ .

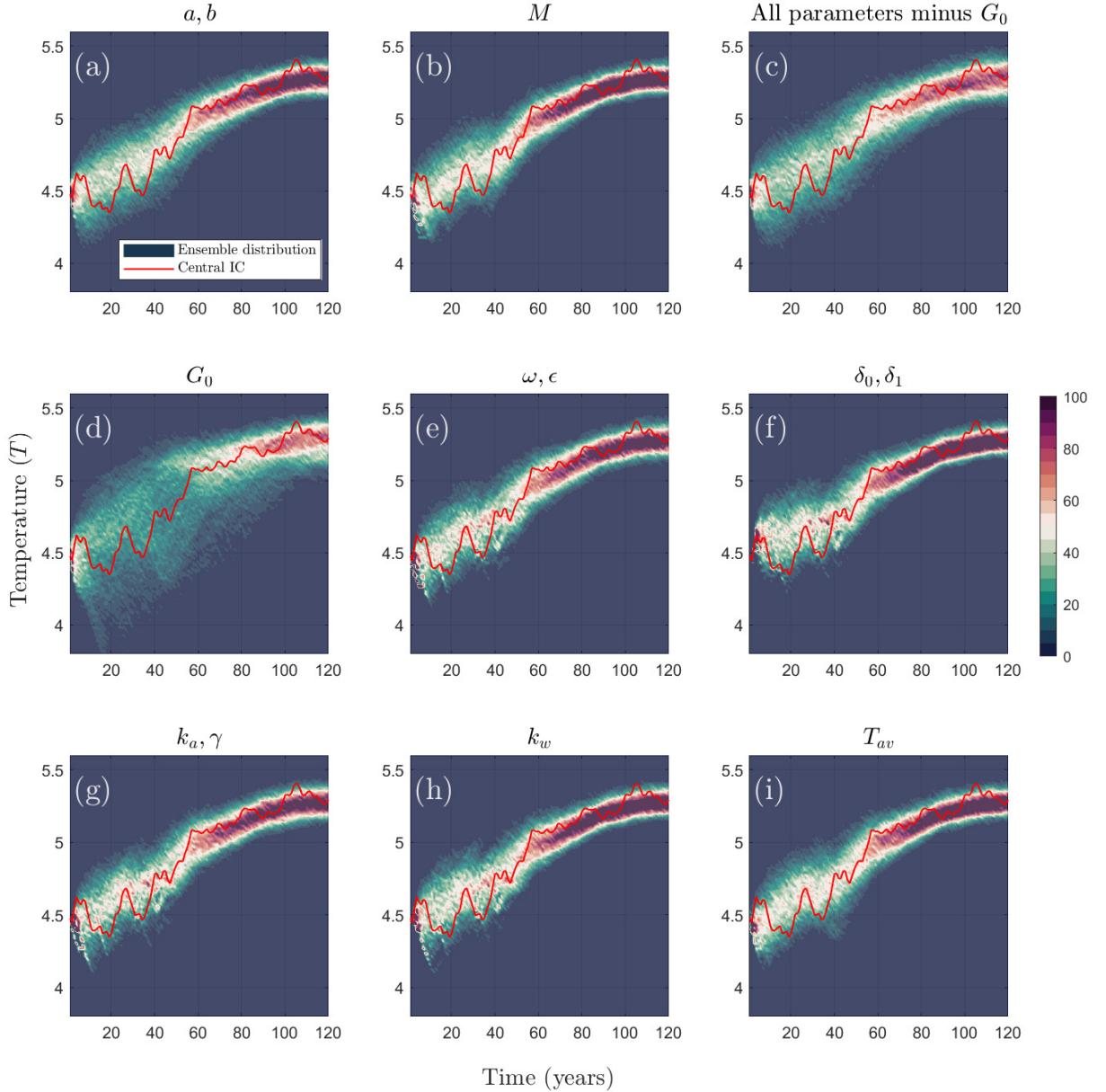


Figure S.4: Micro PPE distributions for different parameter values. The figure shows 120 years of ocean temperature  $T$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a)  $a, b$  (b)  $M$ ; (c) All parameters minus  $G_0$ ; (d)  $G_0$ ; (e)  $\omega, \epsilon$ ; (f)  $\delta_0, \delta_1$ ; (g)  $k_a, \gamma$ ; (h)  $k_w$ ; (i)  $T_{av}$ .

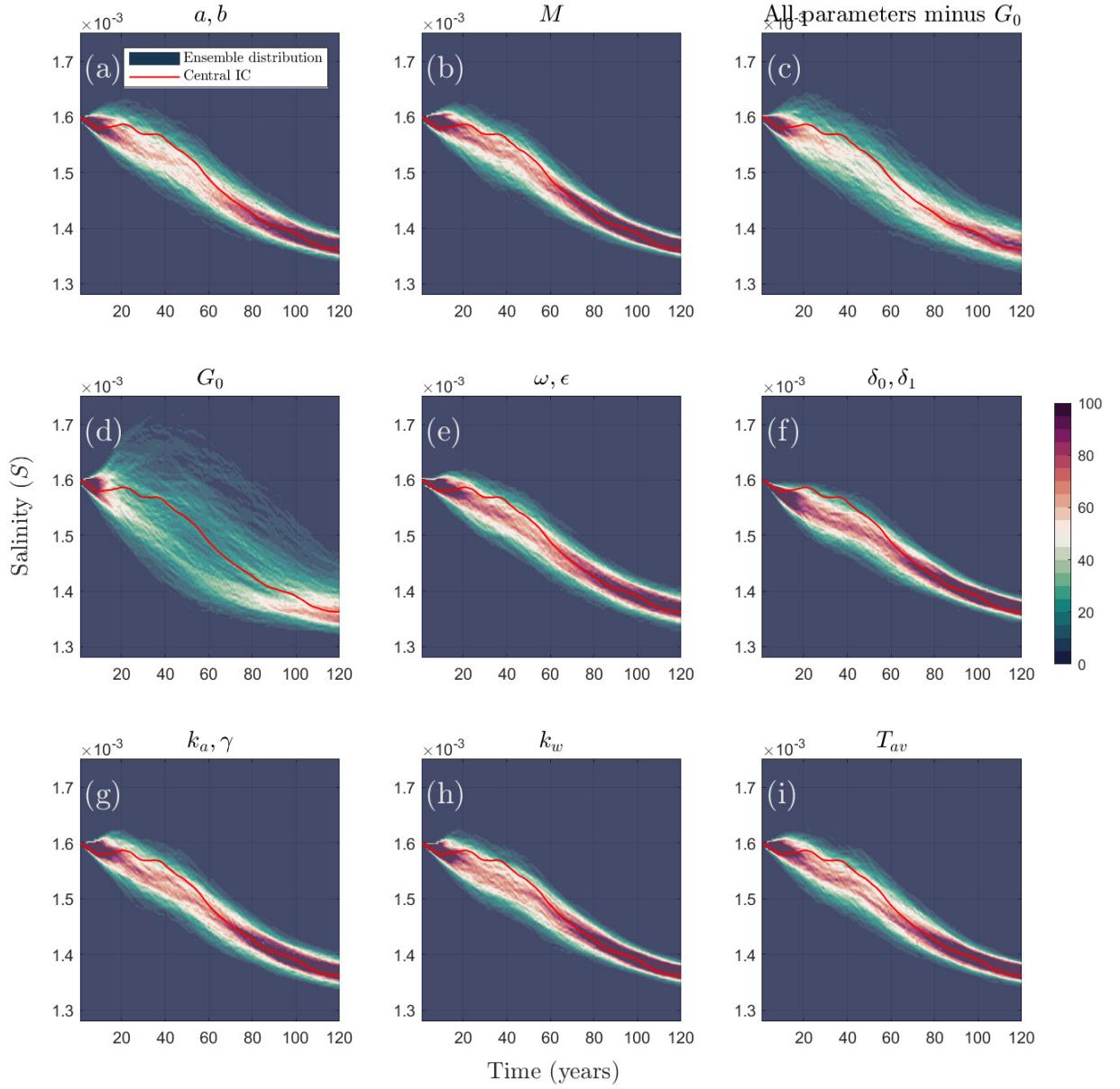


Figure S.5: Micro PPE distributions for different parameter values. The figure shows 120 years of ocean salinity  $S$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a)  $a, b$  (b)  $M$ ; (c) All parameters minus  $G_0$ ; (d)  $G_0$ ; (e)  $\omega, \epsilon$ ; (f)  $\delta_0, \delta_1$ ; (g)  $k_a, \gamma$ ; (h)  $k_w$ ; (i)  $T_{av}$ .

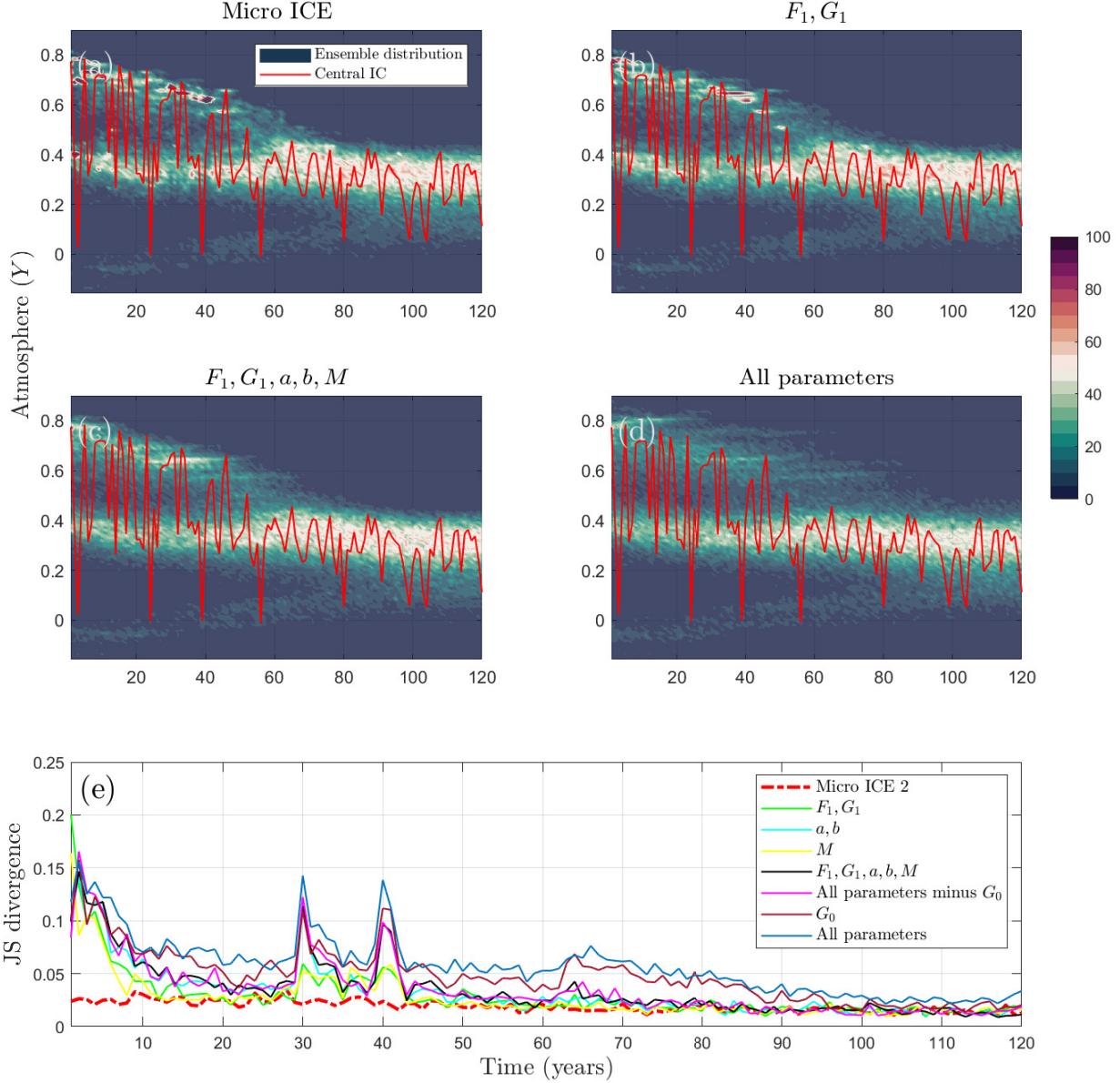


Figure S.6: Micro ICE distribution and micro PPE distributions for different parameter values. The figure shows 120 years of atmosphere variable  $Y$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a) Micro IC; (b)  $(F_1, G_1)$ ; (c)  $(F_1, G_1, a, b, M)$ ; (d) All 14 parameters. Panel (e) shows the Jensen-Shannon divergence comparing the micro ICE with a second micro ICE (red dash-dot line) and several micro PPEs (solid lines).

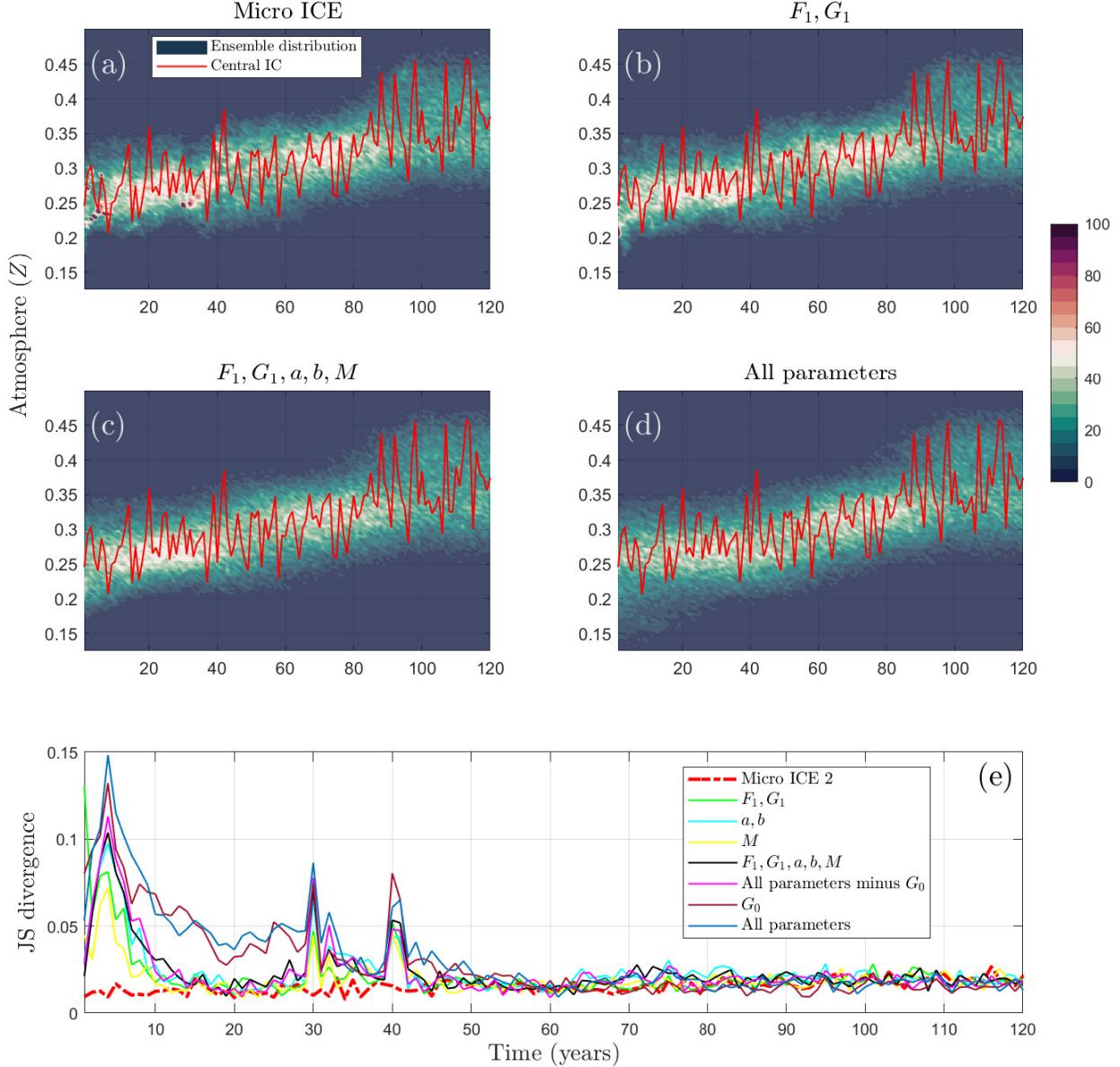


Figure S.7: Micro ICE distribution and micro PPE distributions for different parameter values. The figure shows 120 years of atmosphere variable  $Z$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a) Micro IC; (b)  $(F_1, G_1)$ ; (c)  $(F_1, G_1, a, b, M)$ ; (d) All 14 parameters. Panel (e) shows the Jensen-Shannon divergence comparing the micro ICE with a second micro ICE (red dash-dot line) and several micro PPEs (solid lines).

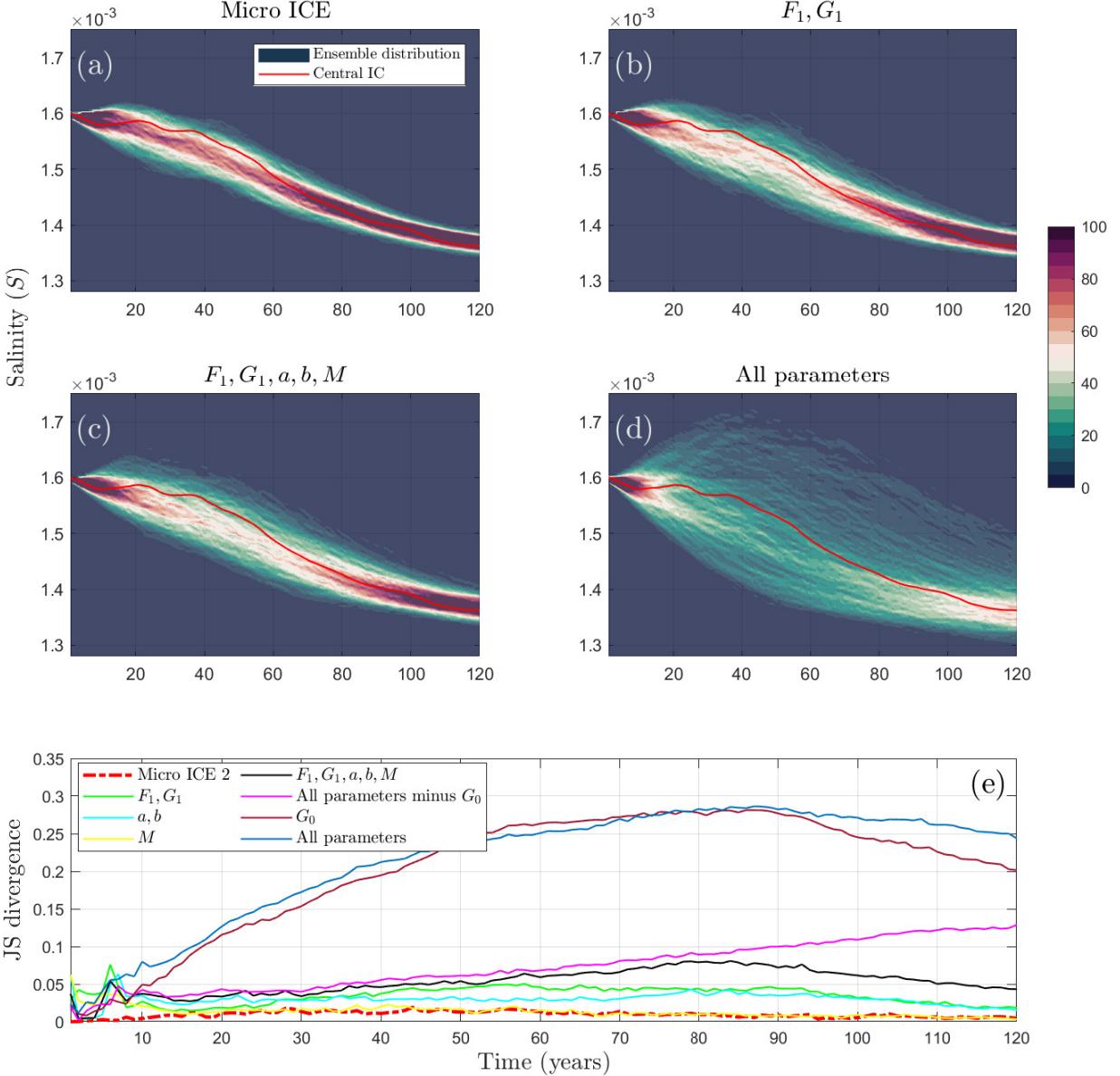


Figure S.8: Micro ICE distribution and micro PPE distributions for different parameter values. The figure shows 120 years of ocean salinity  $S$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a) Micro IC; (b)  $(F_1, G_1)$ ; (c)  $(F_1, G_1, a, b, M)$ ; (d) All 14 parameters. Panel (e) shows the Jensen-Shannon divergence comparing the micro ICE with a second micro ICE (red dash-dot line) and several micro PPEs (solid lines).

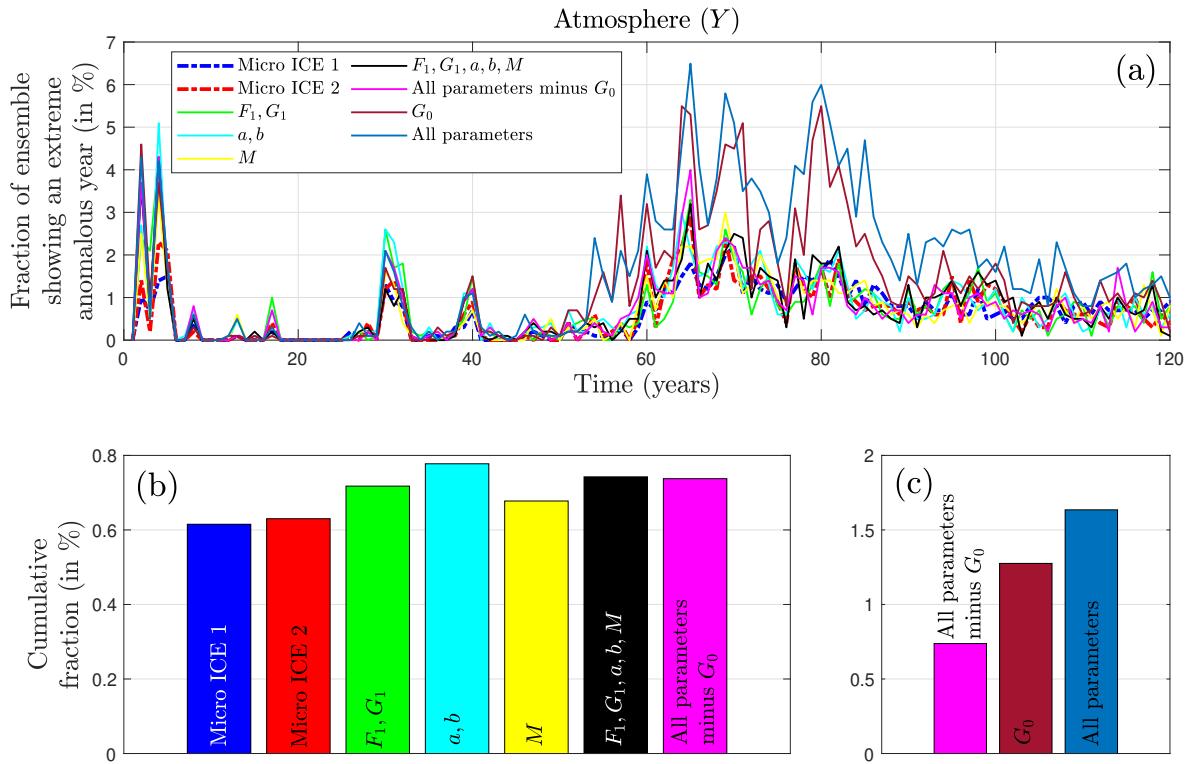


Figure S.9: Comparing a micro ICE with micro PPEs for the atmosphere variable  $Y$ . Top panel (a) shows the fraction of ensemble members projecting an extreme anomalous year (here considered as states over 3 standard deviations from the micro ICE mean) over time. Bottom panels (b,c) show the cumulative fraction from (a) for the entire 120-year period simulated.

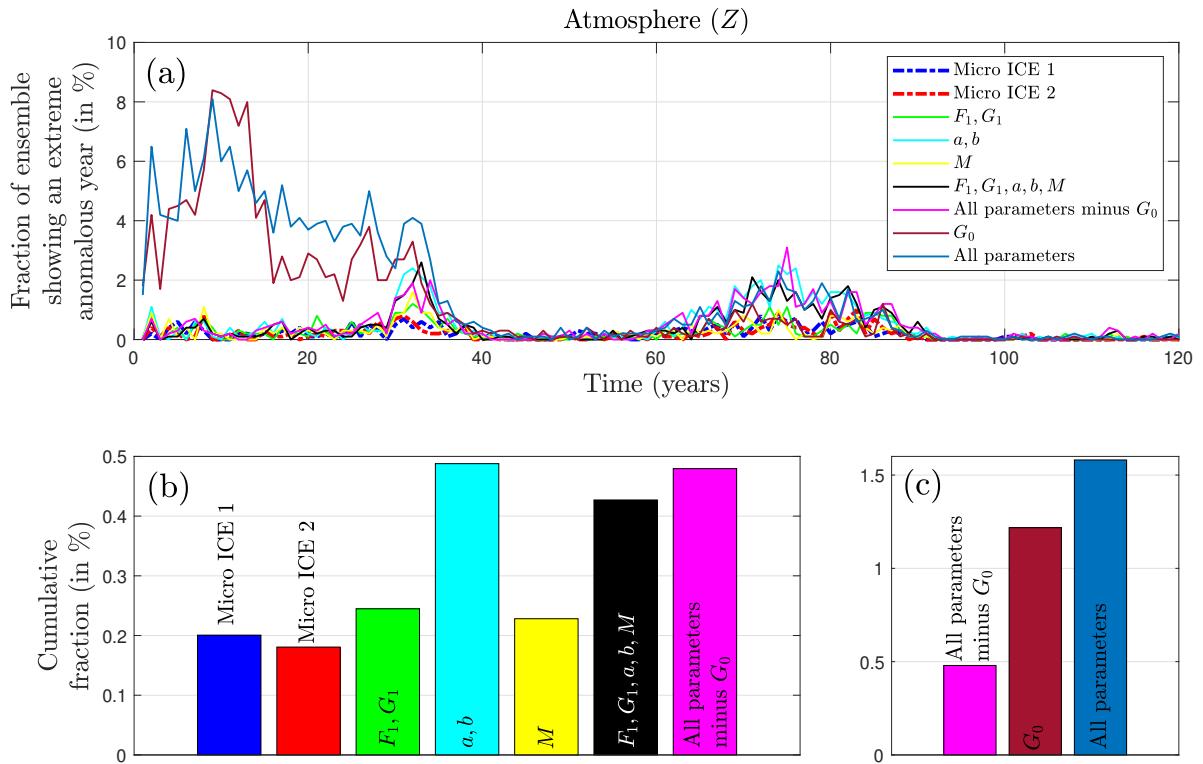


Figure S.10: Comparing a micro ICE with micro PPEs for the atmosphere variable  $Z$ . Top panel (a) shows the fraction of ensemble members projecting an extreme anomalous year (here considered as states over 3 standard deviations from the micro ICE mean) over time. Bottom panels (b,c) show the cumulative fraction from (a) for the entire 120-year period simulated.

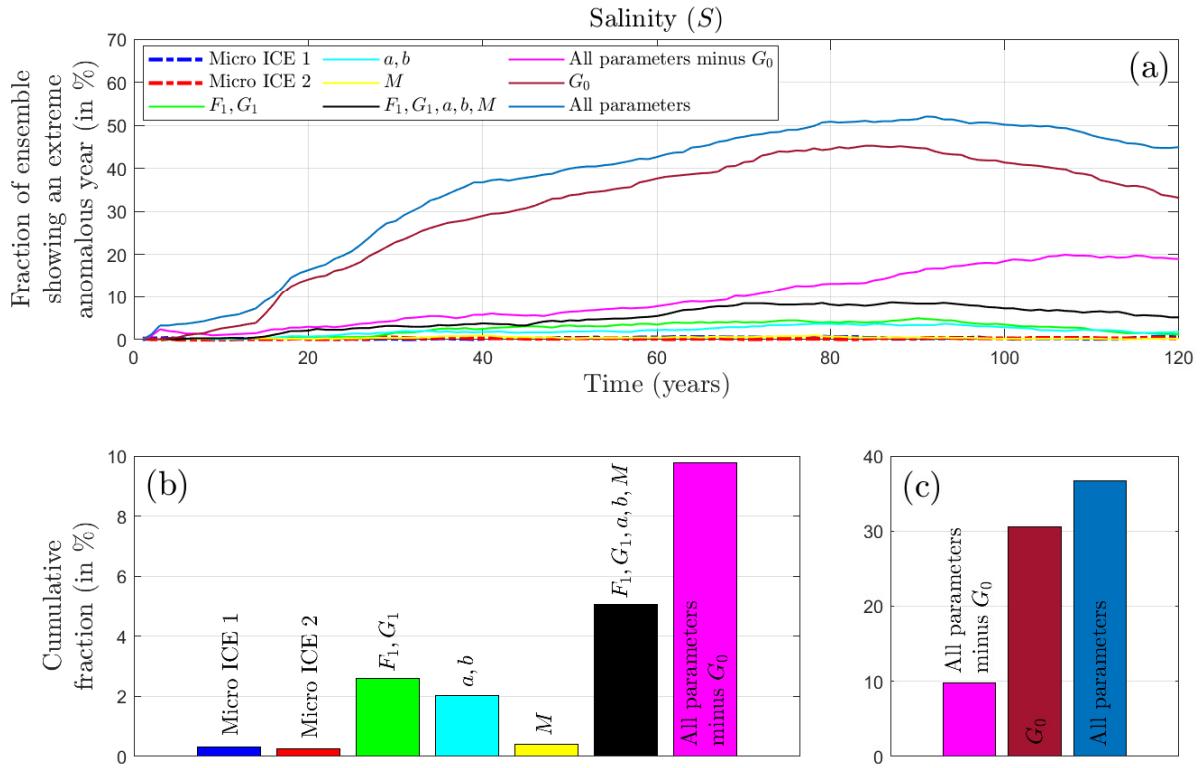


Figure S.11: Comparing a micro ICE with micro PPEs for the ocean salinity  $S$ . Top panel (a) shows the fraction of ensemble members projecting an extreme anomalous year (here considered as states over 3 standard deviations from the micro ICE mean) over time. Bottom panels (b,c) show the cumulative fraction from (a) for the entire 120-year period simulated.

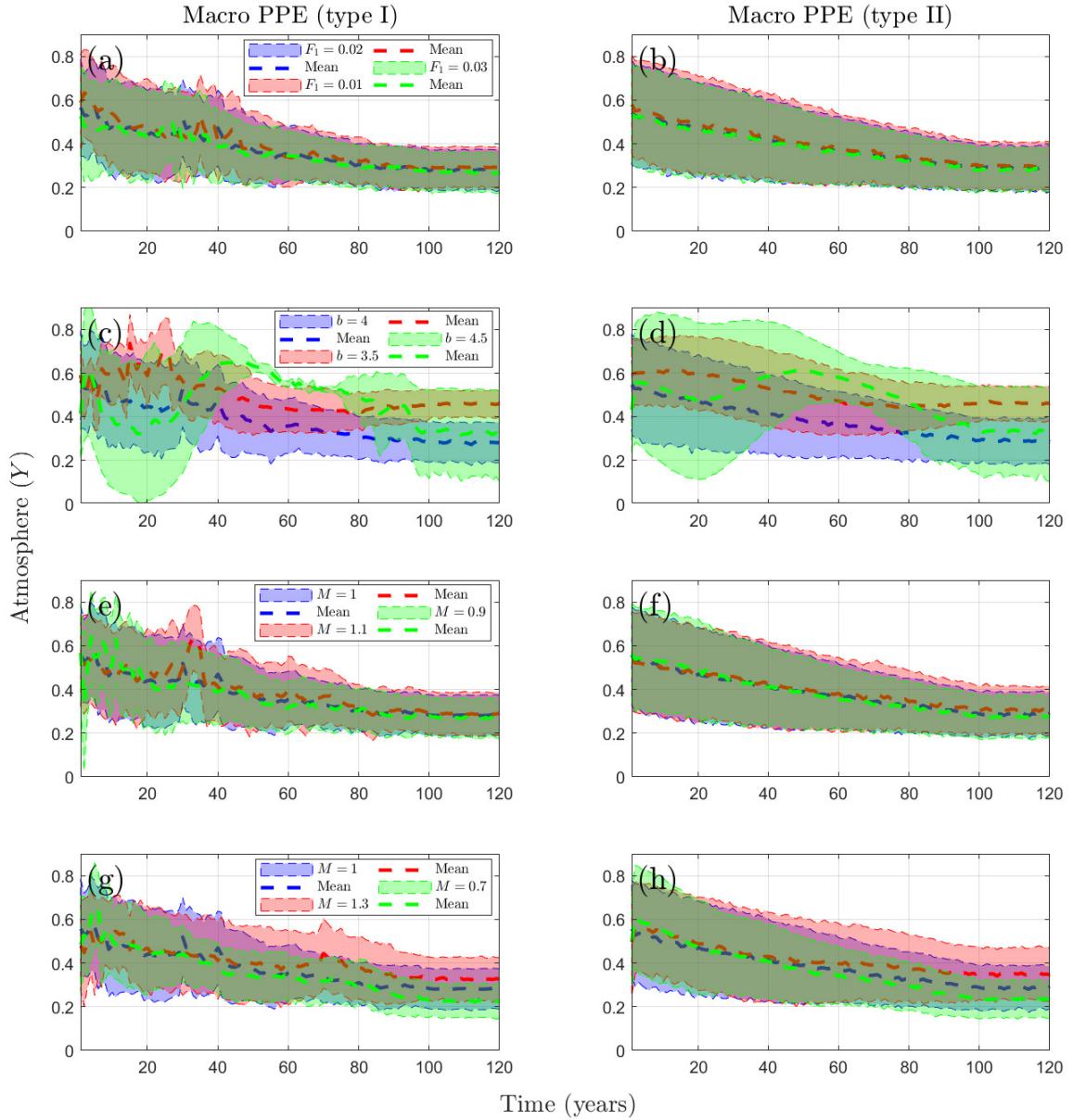


Figure S.12: Macro PPE of types I (left column) and II (right column), for different parameters. The figure shows 120 years of atmosphere variable  $Y$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a,b)  $F_1$ ; (c,d)  $b$ ; (e-h)  $M$ .

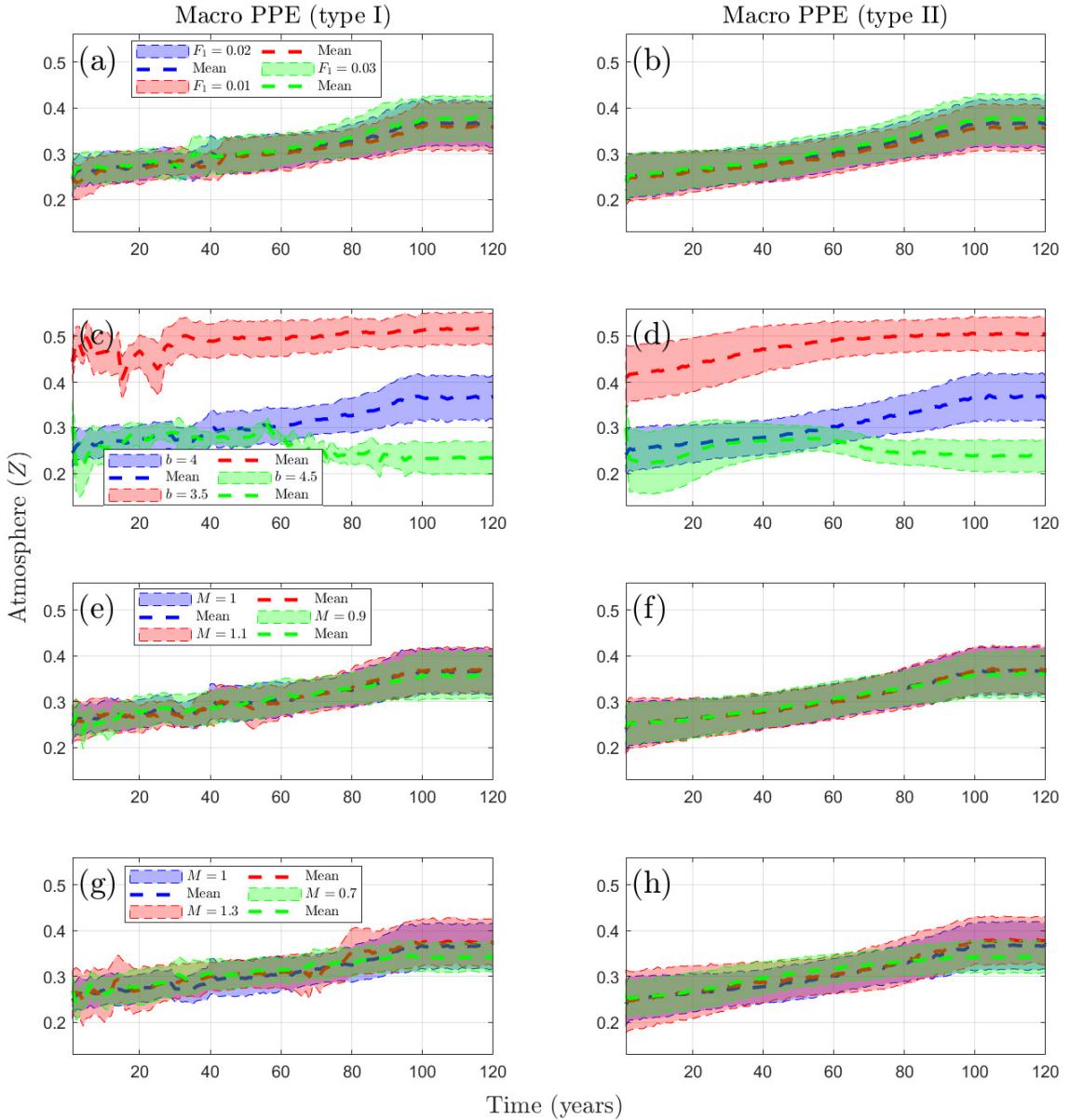


Figure S.13: Macro PPE of types I (left column) and II (right column), for different parameters. The figure shows 120 years of atmosphere variable  $Z$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a,b)  $F_1$ ; (c,d)  $b$ ; (e-h)  $M$ .

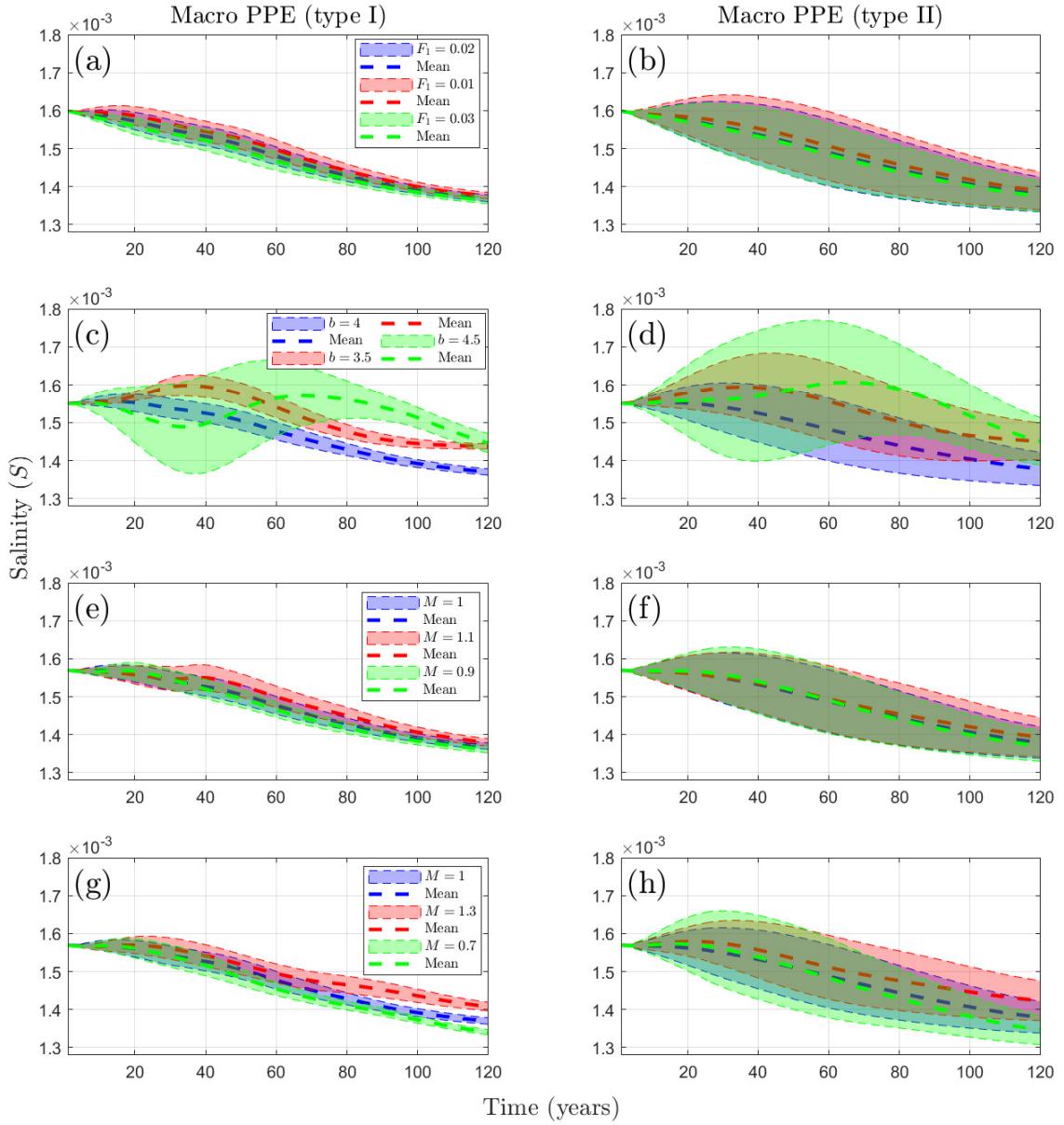


Figure S.14: Macro PPE of types I (left column) and II (right column), for different parameters. The figure shows 120 years of ocean salinity  $S$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a,b)  $F_1$ ; (c,d)  $b$ ; (e-h)  $M$ .

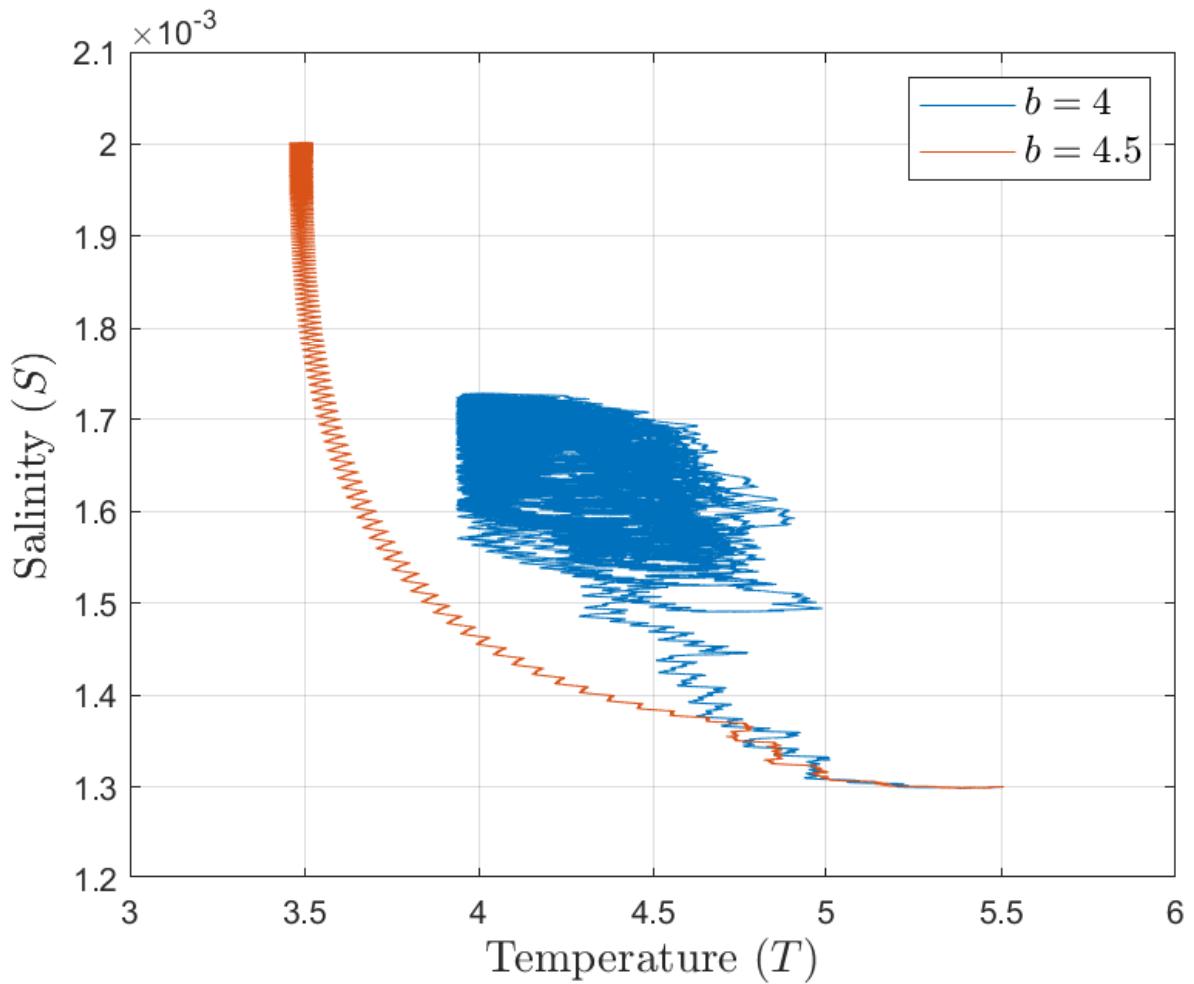


Figure S.15: Projection of the attractor of the stationary ( $H = 0$ ) L84-S61 system onto the ocean variables  $T, S$ . Both attractors are computed as a 5,000-year spinup from the same initial condition. The attractor for  $b = 4$  (reference model version) is shown in blue. The attractor for  $b = 4.5$  (perturbed model version) is shown in red.

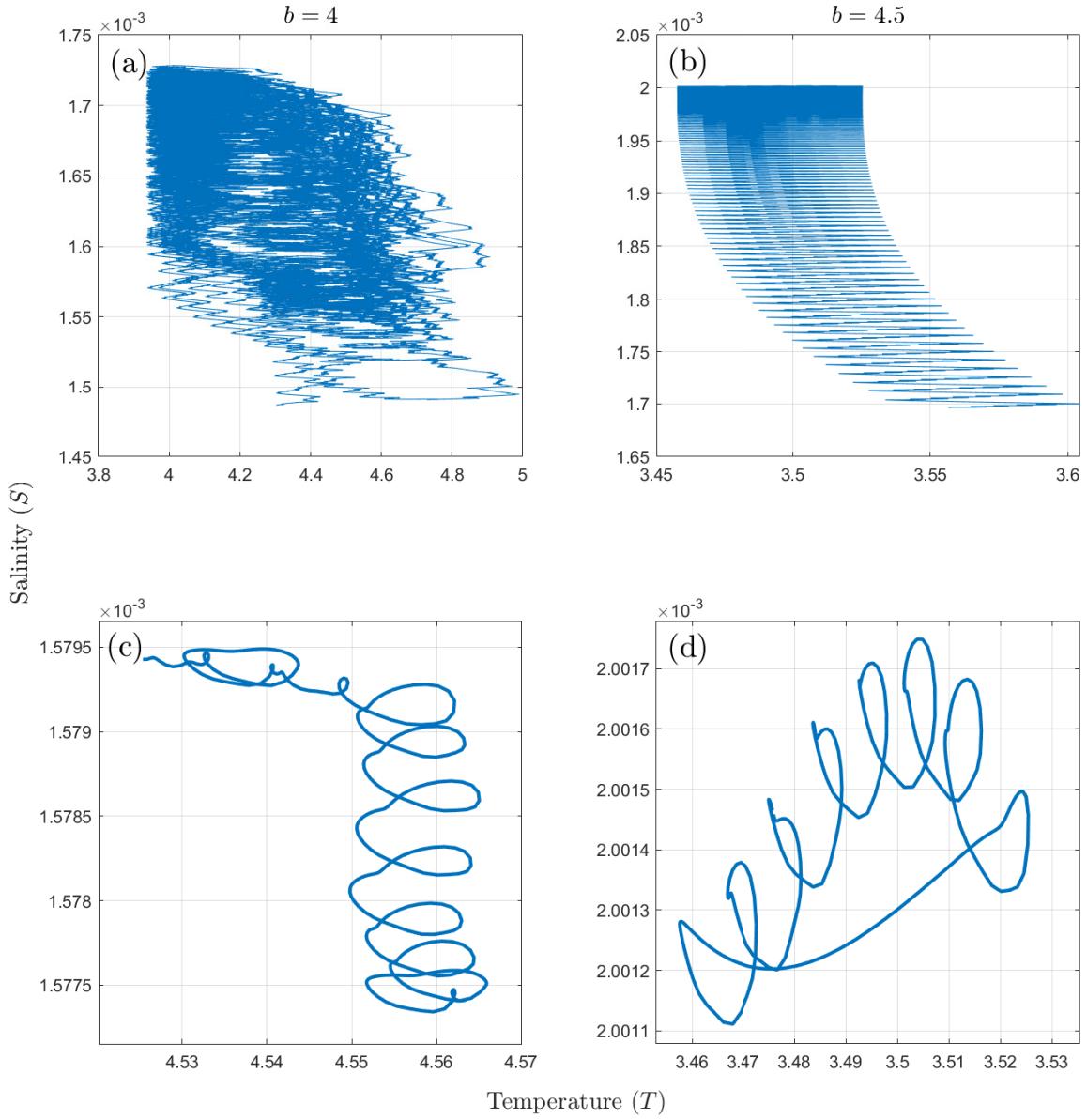


Figure S.16: Projection of the attractor of the stationary ( $H = 0$ ) L84-S61 system onto the ocean variables  $T, S$ . Both attractors are computed as a 5,000-year spinup from the same initial condition. (a) Attractor for  $b = 4$  (reference model version); (b) Attractor for  $b = 4.5$  (perturbed model version); (c) last year of simulation (year 4999) for  $b = 4$ ; (d) last year of simulation (year 4999) for  $b = 4.5$ . Plots (a) and (b) are the same as in Figure S.15, excluding the first 50 years. Note that the scales in the horizontal and vertical axis vary across the panels.

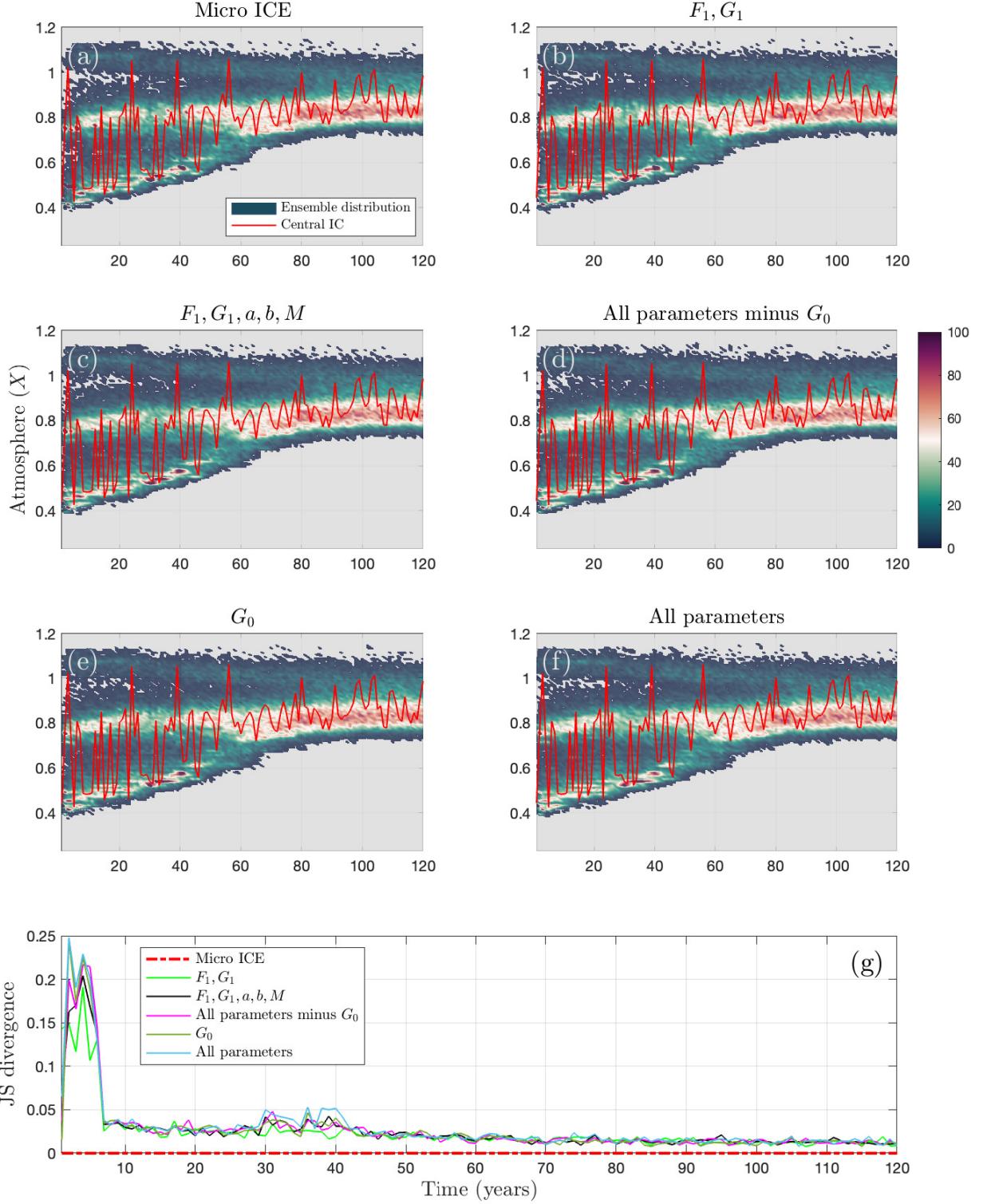


Figure S.17: Micro ICE distribution and micro PPE distributions for different parameter values, for a variance that is one order of magnitude smaller than the reference. The figure shows 120 years of atmosphere variable  $X$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a) Micro IC; (b)  $(F_1, G_1)$ ; (c)  $(F_1, G_1, a, b, M)$ ; (d) All 14 parameters. (e)  $G_0$ . (f) All minus  $G_0$ . Panel (f) shows the Jensen-Shannon divergence comparing the micro ICE with a second micro ICE (red dash-dot line) and several micro PPEs (solid lines).

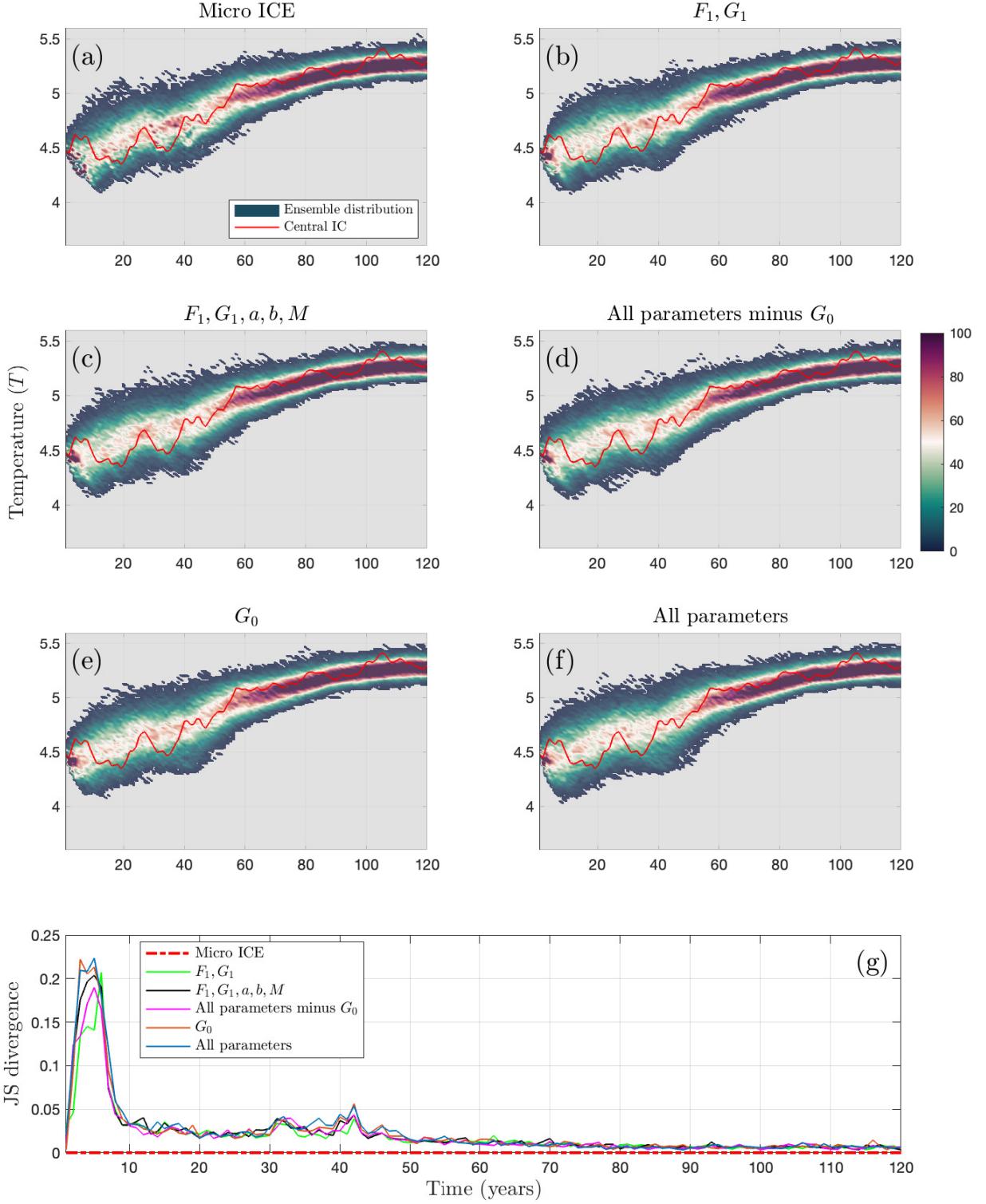


Figure S.18: Micro ICE distribution and micro PPE distributions for different parameter values, for a variance that is one order of magnitude smaller than the reference. The figure shows 120 years of ocean temperature  $T$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a) Micro IC; (b)  $(F_1, G_1)$ ; (c)  $(F_1, G_1, a, b, M)$ ; (d) All 14 parameters. (e)  $G_0$ . (f) All minus  $G_0$ . Panel (f) shows the Jensen-Shannon divergence comparing the micro ICE with a second micro ICE (red dash-dot line) and several micro PPEs (solid lines).

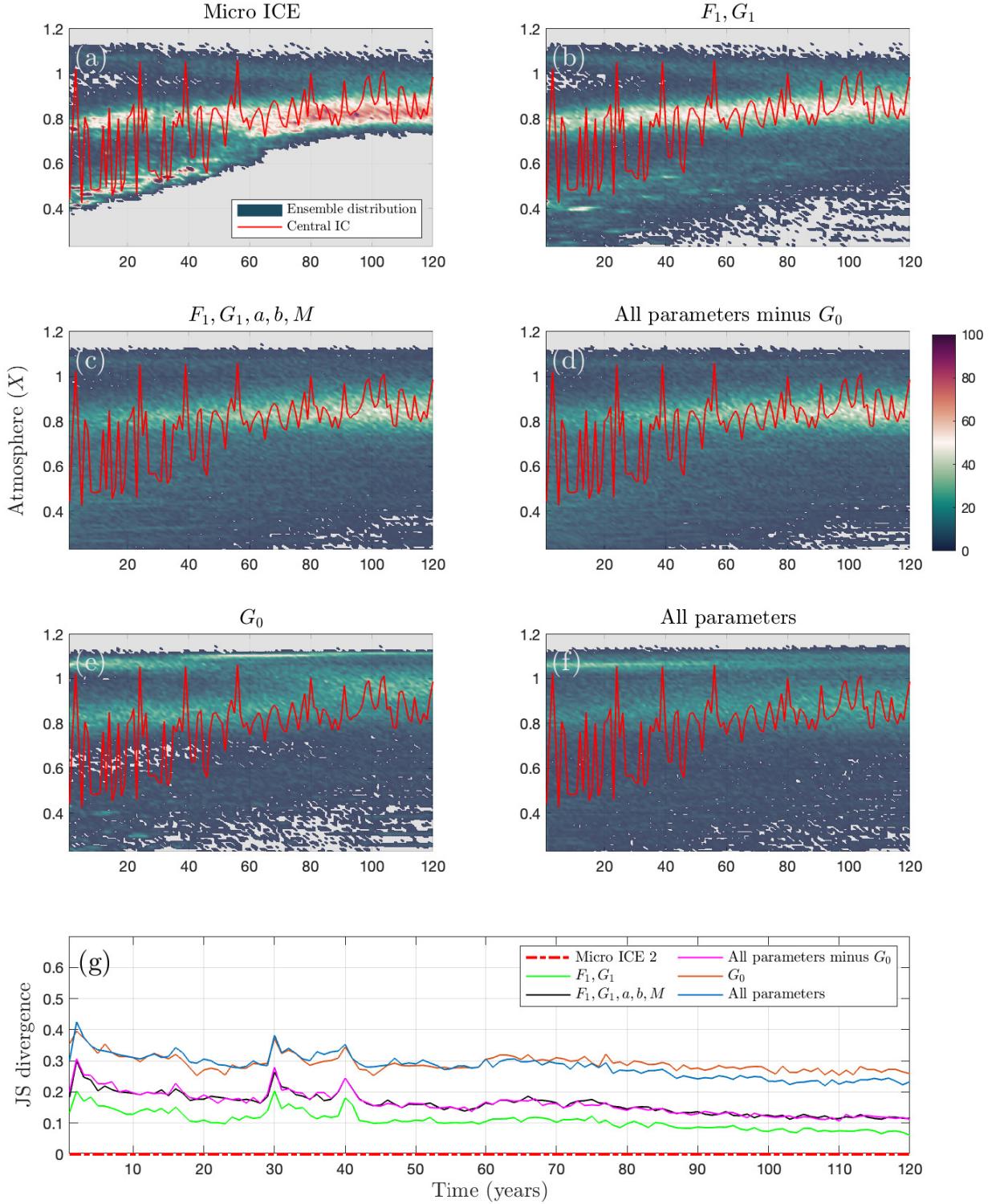


Figure S.19: Micro ICE distribution and micro PPE distributions for different parameter values, for a variance that is one order of magnitude larger than the reference. The figure shows 120 years of atmosphere variable  $X$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a) Micro ICE; (b)  $(F_1, G_1)$ ; (c)  $(F_1, G_1, a, b, M)$ ; (d) All 14 parameters. (e)  $G_0$ . (f) All minus  $G_0$ . Panel (f) shows the Jensen-Shannon divergence comparing the micro ICE with a second micro ICE (red dash-dot line) and several micro PPEs (solid lines).

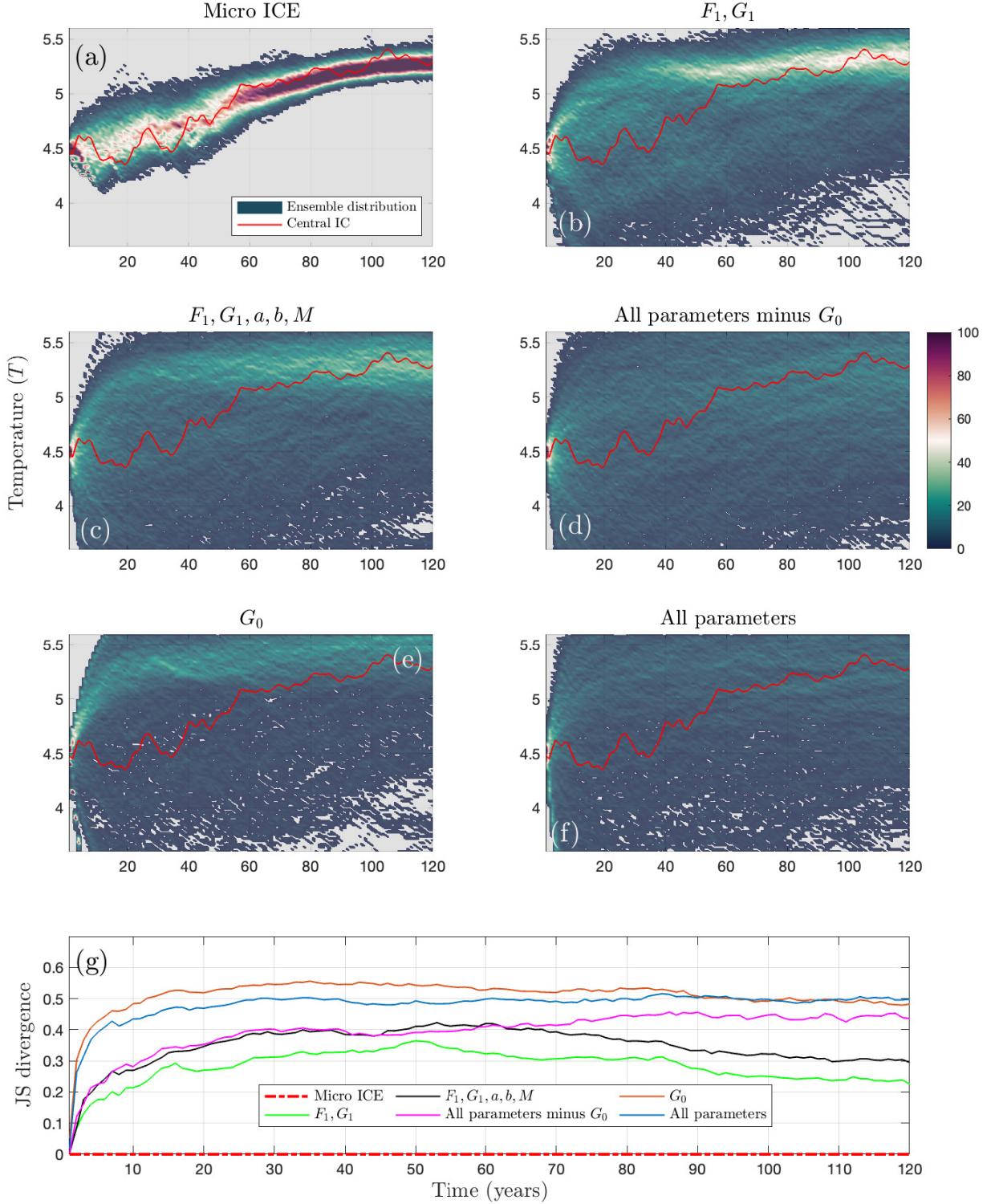


Figure S.20: Micro ICE distribution and micro PPE distributions for different parameter values, for a variance that is one order of magnitude larger than the reference. The figure shows 120 years of ocean temperature  $T$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. (a) Micro ICE; (b)  $(F_1, G_1)$ ; (c)  $(F_1, G_1, a, b, M)$ ; (d) All 14 parameters. (e)  $G_0$ . (f) All minus  $G_0$ . Panel (f) shows the Jensen-Shannon divergence comparing the micro ICE with a second micro ICE (red dash-dot line) and several micro PPEs (solid lines).

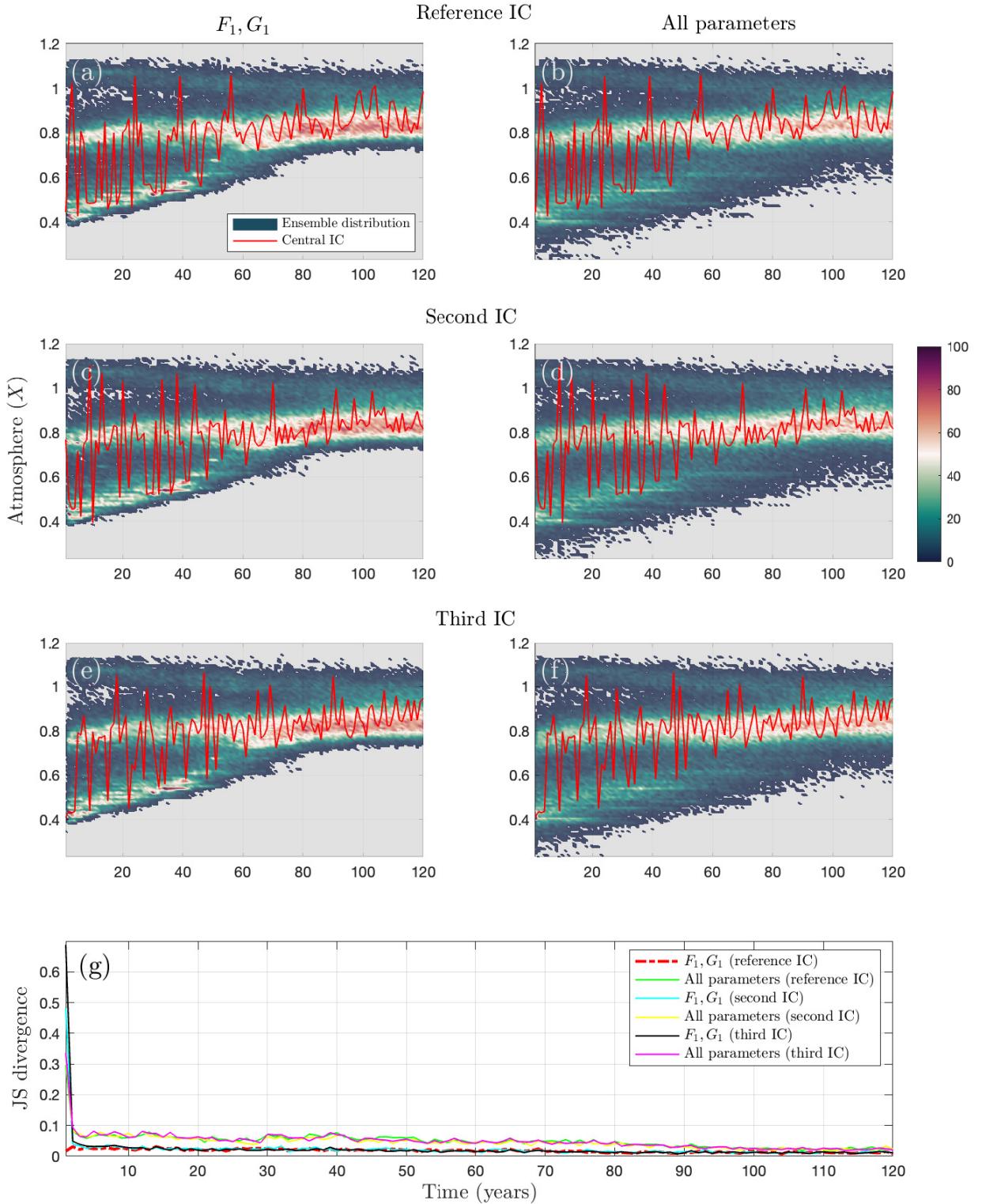


Figure S.21: Micro ICE distribution and micro PPE distributions for different parameter values, for different initial conditions (ICs). The figure shows 120 years of atmosphere variable  $X$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. The left column shows micro PPEs for  $(F_1, G_1)$  and the right column for All 14 parameters. Panels (a,b) shows a second run of the micro PPEs starting from the reference IC. Panels (c,d) and (e,f) show micro PPEs starting from two distinct ICs. Panel (f) shows the Jensen-Shannon divergence comparing the micro PPE for  $(F_1, G_1)$  with a second micro PPE (red dash-dot line) and the other micro PPEs (solid lines).

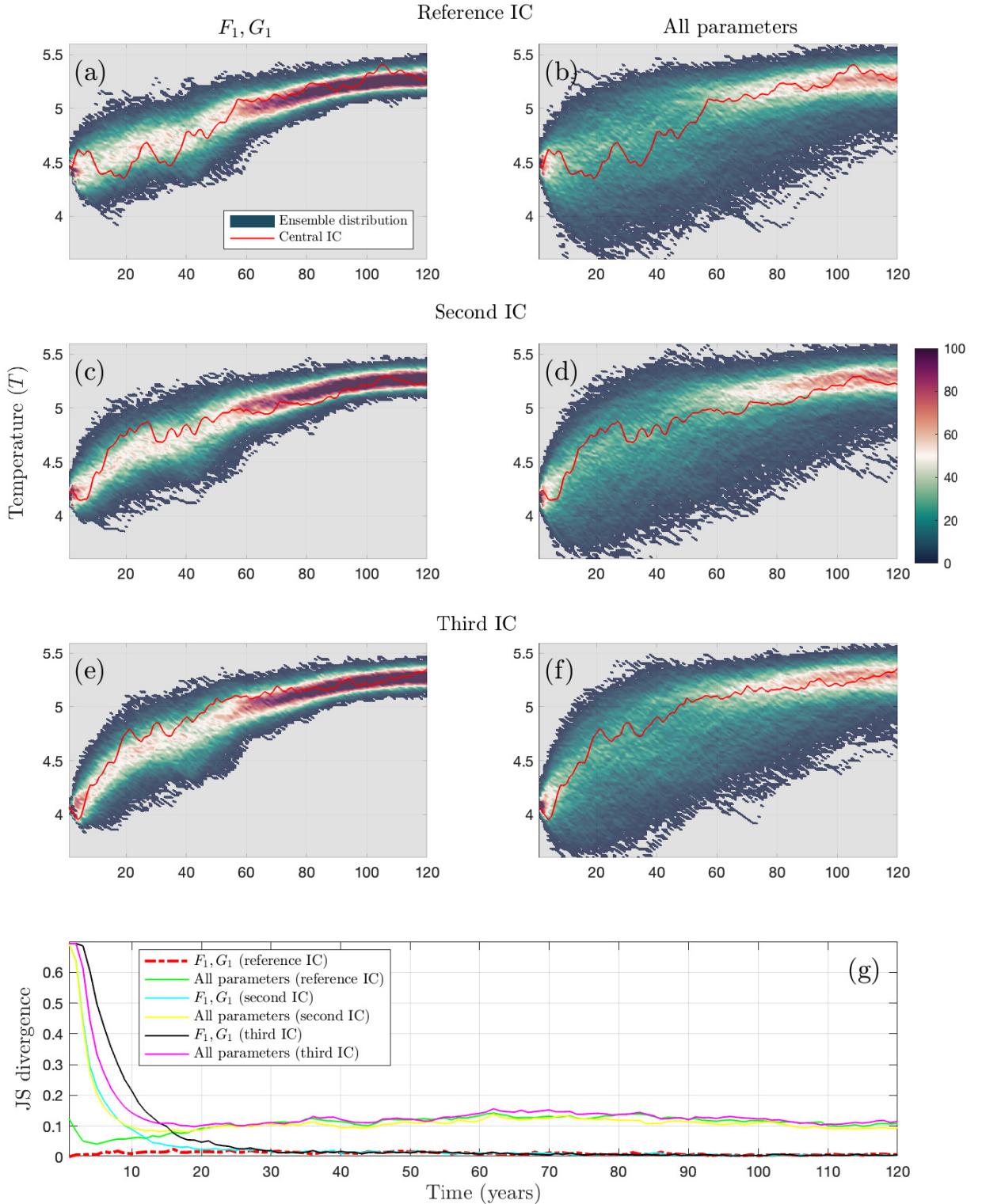


Figure S.22: Micro ICE distribution and micro PPE distributions for different parameter values, for different initial conditions (ICs). The figure shows 120 years of ocean temperature  $T$  distribution, consisting of 100 years of climate change followed by 20 years of stationary climate. The left column shows micro PPEs for  $(F_1, G_1)$  and the right column for All 14 parameters. Panels (a,b) shows a second run of the micro PPEs starting from the reference IC. Panels (c,d) and (e,f) show micro PPEs starting from two distinct ICs. Panel (f) shows the Jensen-Shannon divergence comparing the micro PPE for  $(F_1, G_1)$  with a second micro PPE (red dash-dot line) and the other micro PPEs (solid lines).

JS divergence between  $\mathcal{N}(0, 1)$  and  $\mathcal{N}(0, \sigma)$  as a function of  $\sigma$

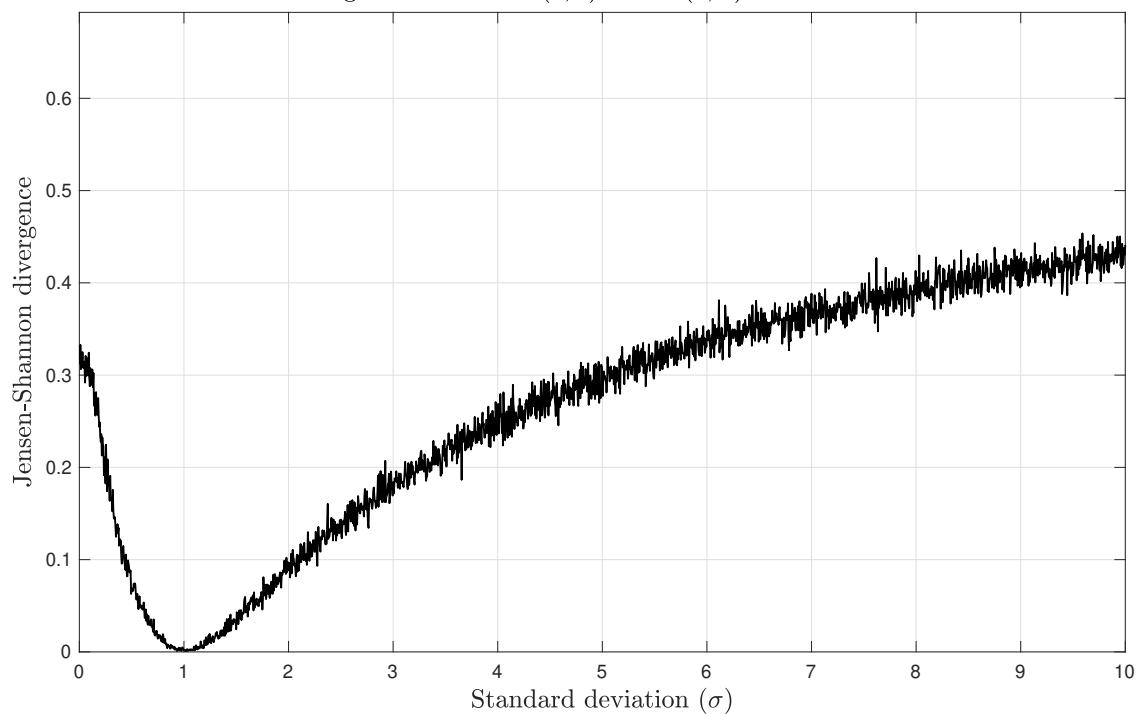


Figure S.23: Jensen-Shannon divergence comparing a normal distribution  $\mathcal{N}(0, 1)$  with a second normal distribution  $\mathcal{N}(0, \sigma)$  as a function of the standard deviation  $\sigma$ .