

Cleansing by Tight Credit: Rational Cycles and Endogenous Lending Standards*

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Abstract

Endogenous cycles emerge through the two-way interaction between lending standards and production fundamentals. Lax lending standards in booms lead to low interest rates and high output but the deterioration of future loan quality. Low borrower quality in turn precipitates tight standards: the economy enters a recession with high credit spreads and low output but a gradual improvement in the quality of loans. This eventually triggers a shift back to a boom with lax lending, and the cycle continues. The capitalization of expert investors determines the strength of capital reallocation in recessions. Furthermore, although the constrained efficient economy is often cyclical, it features both a static and a dynamic externality in credit supply, hence differing from the decentralized equilibrium.

JEL codes: D82, E32, E44, G01, G10

Keywords: cleansing role of recession; lending standards; endogenous cycles; credit supply; capital reallocation; adverse selection; information choice

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1 Introduction

A growing body of empirical evidence documents economic cycles in which fluctuations in credit supply are strongly correlated with real outcomes. In these cycles booms often correspond to overheated credit markets with low interest rates and increased total quantity of credit, but deteriorating quality of newly issued credit. In the subsequent recessions, credit turns scarce and expensive even for ex-post high-quality investment. However, the quality of newly issued credit improves. The recession finally turns into a boom, and the cycle continues. In this paper, we propose a mechanism to understand this transition between booms and recessions.

We provide a model where the interaction between the credit market and production fundamentals generates cycles. Booms are periods of high output, lax lending standards and abundant credit. Lax lending fosters good investment but also leads to a gradual deterioration of loan quality during the boom, which ultimately triggers lenders to tighten their lending standards and causes a recession.

In the recession output collapses and productive investment slows down: Economic activity is *stifled by tight credit*. However, the reduced credit supply hits the bad firms the hardest and leads to a reallocation of credit in recessions toward higher quality firms: economy is *cleansed by tight credit*. That is, lending standards play a dual role.

In time the economy bounces back to a boom with lax lending. The two-way interaction between the choice of lending standards and the fundamentals of the economy sustains permanent endogenous cycles.

We further show that although the constrained optimal economy tends to be cyclical, it generically differs from the equilibrium cycle. Lenders fail to internalize both the static stifling role, the negative externality, and the dynamic cleansing role, the positive externality, of tight lending standards in recessions. This can culminate in excessively long booms followed by exceedingly deep recessions.

In this economy firms borrow in the credit market to produce, but only some of them pay back. Most investors cannot distinguish between good, creditworthy and bad, not-creditworthy firms, but they have access to an assessment technology that can imperfectly reveal firm type. We call the assessment technology a “test.” A *bold test* approves the credit application of all good firms along with some bad ones. A *cautious test* on the other hand rejects some good applications along all the bad ones. Thus, the bold test implies lax lending standards while the cautious test implies tight lending. As such, investors face a quantity-quality trade-off: Tight lending standards improve the quality but decrease the quantity of credit issued by an investor.

When there are few bad firms among borrowers, investors optimally target a high quantity of lending and choose to be bold: They impose lax lending standards and the credit market exhibits symptoms of overheating. A mixed quality of credit is issued at a low interest rate which induces high credit growth and high output. At the same time, availability of credit to bad firms enables them to expand and multiply, leading to the deterioration of borrower quality in future periods. When the average borrower quality sufficiently deteriorates, lenders rationally switch to be cautious and tighten the lending standards: They target high quality of lending. Tight lending coincides with high credit spreads which dampens bad investment but also slows down some good firms. Put differently, tight standards not only suppress lending to non-creditworthy firms but also harm some creditworthy ones, leading to a downturn. However, the issued credit is of high quality. Thus the pool of credit applications improves, eventually triggering a shift back to lax lending standards. And the cycle continues.

As such, the information that investors choose to acquire and use as the basis for their lending decision implies that lending standards have a dual role. Tight lending standards stifle good investment today, but cleanse the economy of bad firms in the future. On the other hand, lax lending standards enable good investment to thrive today, but sow the seeds of bad future investment.

This mechanism provides novel insights about the extent of capital misallocation in different phases of the economic cycle. We first show that more skilled capital reduces capital misallocation in every state of the world. This finding is intuitive. More interestingly, the model demonstrates that as the economy transitions between a boom and a recession, more skilled capital strengthens the cleansing effect. As such, capital reallocation accelerates during recessions compared to booms if skilled capital is abundant and decelerate otherwise. This is in line with evidence on weaker cleansing effect following the Great Recession (e.g. Foster et al., 2016).

We then use the model to study the welfare properties of the equilibrium cycle. Recessions exhibit the static welfare loss of low output and slowdown of productive investment. However, they also have the dynamic welfare gain of cleansing by tight credit. Investors fail to internalize the effect of their individual choice of lending standards on the concurrent (good) credit quantity, a negative externality, as well as the effect on the future loan quality, a positive externality. In other words, investors ignore both the stifling and cleansing roles of tight lending standards in recessions. The opposite pattern characterizes booms.

Finally, we show that the predictions of our model are consistent with a wide range of stylized facts regarding the role of credit in economic cycles. First, our proposed mechanism implies that the reduction of credit supply to firms in recessions is concentrated among a

certain group, opaque firms, and is affected by factors other than firm fundamental quality. This is consistent with Iyer et al. (2014) who find that in the European interbank freeze, the credit supply reduction was stronger for firms that are smaller and younger. Second, the emerging empirical evidence in the past two decades hints to both a cleansing and a stifling role for recessions, with different strengths pre and post the Great Recession (Caballero et al., 2008; Foster et al., 2016; Osotimehin and Pappadà, 2017). Finally, our model predicts a deterioration of credit quality during the boom, consistent with the empirical evidence that document worsening of vintage performance for the sub-prime mortgages during the boom leading to the Great Recession (Gerardi et al., 2008; Demyanyk and Van Hemert, 2009; Mayer et al., 2009; Palmer, 2015).

Literature To the best of our knowledge, our paper is the first to formalize the dual role of lending standards across booms and recessions and positive and negative externalities implied by it. In particular, lending standards not only contribute to the transition from booms to recessions, but also to the transition back to the boom. Furthermore, while both the equilibrium and constraint efficient outcomes are cyclical, they do not coincide. We demonstrate that the rich two-sided heterogeneity in firm and investor types is crucial to capture this mechanism.

The paper contributes to a few strands of literature. First, it belongs to the growing body of literature on dynamic lending standards. In this literature, lenders' choice to acquire information about borrowers differs in booms and in recessions (Martin, 2005; Gorton and Ordonez, 2014; Asriyan and Vanasco, 2014; Gorton and Ordonez, 2016; Hu, 2017; Fishman et al., 2019; Asriyan et al., 2021). Gorton and Ordonez (2016) and the contemporaneous paper of Fishman et al. (2019) are the closest to our work. Similar to our model, the mechanism in Fishman et al. (2019) relies on the two-way interaction of lenders' information choice and borrowers' average quality. However, unlike our paper, their economy does not feature endogenous cycles. This is a common feature of most of the papers in this literature, with the exception is Gorton and Ordonez (2016). That paper has two dynamic equilibria: a good steady state and an equilibrium that cycles between multiple periods in the good state and one in the bad one. Unlike our model, in this cyclical equilibrium, recessions and the corresponding tight lending standards have no welfare benefit. As such, the only constraint optimal outcome is a steady state with permanent lax lending standards. In our setup on the other hand, a planner often prefers a cyclical economy to a persistent boom, as tight lending standards during downturns have a dynamic cleansing role. Furthermore, a rich set of cycles with different properties emerge in our framework. Finally, in this literature good firms predominantly benefit from information production in recessions. Alternatively, in our

model a large set of creditworthy firms are stifled by tight credit due to investors being cautious in recessions and foregoing good investment opportunities that they cannot assess.

The idea of the cleansing role of recessions goes back to Schumpeter (1939). A number of papers discuss this effect in the labor market (Caballero and Hammour, 1994, 1996; Haltiwanger et al., 2021). There is also a literature arguing that lax lending in the credit market harms growth during booms as it leads to misallocation. A number of theoretical explanations include lenders' unwillingness to terminate inefficient lending relationships due to existence of sunk costs (Dewatripont and Maskin, 1995), impairment of information production in booms (Asriyan et al., 2021), refinancing inefficient projects due to soft budget constraints close to regulatory minimum (Caballero et al., 2008), and gambling for resurrection. We provide an alternative explanation for how lax lending leads to deterioration of pool of borrowers in booms and prompts recessions with tight lending, which then improves the loan quality, leads back to a boom, and endogenously sustains the cycle. We believe this is a particularly powerful mechanism as it can simultaneously explain the transition between booms and recessions and vice-versa.

Our paper also contributes to the literature on endogenous credit cycles (Azariadis and Smith, 1998; Matsuyama, 2007; Myerson, 2012; Gu et al., 2013). These papers present different mechanism that leads to endogenous fluctuations in granted credit quantity. However, none of them capture the interdependence of investors choice of lending standards and economic activity.¹

Finally, there is a literature connecting overheated credit markets to extrapolative expectations (Bordalo et al., 2018; Greenwood et al., 2019; Gennaioli and Shleifer, 2020). Our work is complementary to this literature as we demonstrate that certain features of overheated credit markets are consistent with the rational choice of lax lending standards.

From a methodological perspective, our structure of the credit market builds on Kurlat (2016) and Farboodi and Kondor (2022). Neither of these papers focus on endogenous economic cycles.

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 and 4 characterize the static and dynamic equilibrium. Section 5 discusses welfare. Section 6 provides empirical evidence supporting the implications of the model. Finally, section 7 concludes.

¹In the search literature, a number of mechanisms have been suggested to explain the emergence of endogenous cycles. For instance, see Burdett and Coles (1998) for equilibrium and Shimer and Smith (2001) for optimal cycles.

2 Model

Time is discrete and infinite. Each day is divided into two parts: morning and evening. There is a single perishable good. It can be consumed, invested, or stored at a rate of return $1 + r_f$ between morning and evening. There are two types of agents, entrepreneurs and investors. Each agent is risk-neutral and endowed with one unit of the good every morning.

Entrepreneurs. There is measure one of entrepreneurs (firms) and each one has a two-dimensional type.² An entrepreneur is either good or bad, $\tau = g, b$, and either opaque or transparent, $\omega = 0, 1$. Let $\mu_{0,t}$, $\mu_{1,t}$, $\nu_{0,t}$ and $\nu_{1,t}$ denote the measure of opaque bad, transparent bad, opaque good and transparent good entrepreneurs at time t , respectively, with $\mu_{0,t} + \mu_{1,t} + \nu_{0,t} + \nu_{1,t} = 1 \forall t$. Entrepreneurs know their own type and maximizes their life-time utility. Each period, an entrepreneur (τ, ω) is endowed with a unit of capital and a project. At time t , he obtains credit $\ell_t(\tau, \omega)$ at interest rate $r_t(\tau, \omega)$ and invests $i_t(\tau, \omega)$ in the morning and consumes in the evening. Each unit of investment in the morning produces $\rho > 1 + r_f$ the same evening.³ The cost of investment has to be covered by the entrepreneur's initial endowment or credit, implying the following budget constraint

$$i_t(\tau, \omega) = 1 + \ell_t(\tau, \omega). \quad (1)$$

Furthermore, each entrepreneur has to pledge his investment as collateral to obtain credit. Seizing the collateral is the only threat to enforce repayment from the entrepreneur who borrows, thus $(1 + r_t(\tau, \omega))\ell_t(\tau, \omega) \leq i_t(\tau, \omega)$. Using (1) this simplifies to

$$\ell_t(\tau, \omega) \leq \frac{1}{r_t(\tau, \omega)}. \quad (2)$$

The key friction of the model is that investors cannot seize the investment undertaken by bad entrepreneurs. That is, bad entrepreneurs do not pay back, i.e. they are not creditworthy. As such, if an investor can observe the type of an entrepreneur, she lends only to good ones as repayment from bad entrepreneurs cannot be enforced. However, investors only have imperfect information about entrepreneur type.

At the end of each period, some entrepreneurs exit the market ('die'). An entrepreneur exits either because he is hit by an exogenous shock with probability δ , or because he has not

²Throughout the paper, we use entrepreneur and firm interchangeably.

³We have also solved the model under the alternative assumption that good (bad) investment returns $\rho_g > 1 + r_f$ ($\rho_b < 1$). The expressions are more complex without providing further intuition. Therefore, we have decided to use $\rho = \rho_b = \rho_g > 1 + r_f$. The more general solution is available in the previous circulated versions of the paper, as well as available upon request.

been able to raise credit. Thus, we assume that credit is essential for survival. Entrepreneurs discount future to reflect their exit rate. When an entrepreneur exit, he is replaced with a newborn so as to keep the population fixed at 1. The new entrants are chosen from a fixed type distribution: λ ($1 - \lambda$) of new entrants are bad (good), and $\frac{1}{2}$ ($\frac{1}{2}$) are transparent (opaque). The two dimensions of the type distribution of entrants are independent.

Investors. There are two types of investors. A small, w_1 , measure of investors are skilled, while a large, w_0 , measure are unskilled. Skill is privately observable. Each investor is endowed with one unit of capital. As such, measure of investors also represents their capital. We use h to index individual investors.

Each investor lives for one period and maximizes her period utility. She makes a portfolio decision in the morning and consumes and dies in the evening. A dead investor is replaced by the same type of investor the next day. A portfolio decision involves extending credit to entrepreneurs and/or storing part of investor's unit endowment until the evening.

Each investor chooses to participate in or stay out of the lending market. Skilled investors observe the type of each entrepreneur. Alternatively, unskilled investors who participate in the lending market only observe imperfect signals of the type of entrepreneurs in the sample of loan applications that they receive. These signals are generated by a test of the investor's choice. Each investor can opt for a *bold test* or a *cautious test*. We call the former a bold investor, and the latter a cautious investor. The fixed utility cost of any test is $c \in (0, 1)$, and each unskilled investor runs exactly one type of test.

The tests differ in the signal they generate for opaque entrepreneurs. The bold test pools all opaque entrepreneurs, good or bad, with transparent good ones (a false positive error). The cautious test pools all opaque entrepreneurs with transparent bad ones (a false negative error).⁴ Intuitively, one can envision the bold test to reject projects of transparent bad entrepreneurs only and pass all other ones, while the cautious test passes only projects of transparent good entrepreneurs. When an investor is indifferent between the two tests, we break the tie by assuming that she chooses the bold test.

The size of the sample that an unskilled investor receives and tests is limited by her unit endowment. She can test only as many applications as she could finance.

Credit Market. The credit market operates in the morning. After each unskilled investor h chooses the type of her test, each participating skilled and unskilled investor advertises an

⁴For simplicity, we restrict investor's choice set to these two tests. In appendix F we enrich the model and allow the investors to choose among the continuum of tests lying between the bold and cautious tests. We show that the dominant strategy is always to choose one of the extreme tests. Thus, this assumption is without loss of generality.

interest rate, $\tilde{r}(h)$, at which she is willing to extend credit. Each entrepreneur chooses the measure of loan applications $\sigma(r; \tau, \omega) \in [0, \frac{1}{r}]$ he wishes to submit at each interest rate r . The credit market clears starting from the lowest interest rate. At each interest rate, the unskilled investors sample first.⁵

We assume that there is no credit history for entrepreneurs. That is, investors cannot learn from the past.⁶ Furthermore, in order to keep the problem analytically tractable we assume there is no saving technology available across periods. Therefore, entrepreneurs consume their wealth at the end of each period and if they survive, they start the new period with the unit endowment received in the morning. Moreover, we make the following assumption about skilled and unskilled investor wealth.

Assumption 1 *Skilled and unskilled investor capital w_1 and w_0 are such that*

- (i) *Skilled investor capital, w_1 , is scarce. In particular, it is not sufficient to cover the credit demand of all opaque good entrepreneurs even at the maximum interest rate that any good entrepreneur is willing to borrow.*
- (ii) *Unskilled investor capital, w_0 , is abundant. In particular, it covers the credit demand of all entrepreneurs even at the minimal interest rate at which any unskilled investor is willing to lend.*

Finally, following Kurlat (2016), we impose the following *robustness* criterion to pin down the size of loan applications at interest rates which are not advertised in equilibrium.

Assumption 2 *Suppose that ε fraction of applications submitted are granted unconditionally. We require that the equilibrium strategy of each entrepreneur is the limit of equilibrium strategies as ε goes to 0.*

The formal optimization problem of investors and entrepreneurs, as well as further detail on collateralization and market clearing protocol are stated in Appendix A.

We next define the equilibrium within each period followed by the full dynamic equilibrium of the economy.

⁵Kurlat (2016) shows that if marketplaces with various clearing algorithms coexist, only those where the least skilled sample first will be active. Our assumption that unskilled investors sample first simplifies the market structure while keeping it consistent with this result.

⁶In Appendix H we relax this assumption by introducing a probabilistic signal of past defaults akin to real-world credit scores. We show that our main mechanism remains intact.

Equilibrium Definition. We focus on stationary equilibria, i.e. equilibria where the state variables are members of a finite ergodic set in the long run. We conjecture that in any stationary equilibrium, at each time t , $(\mu_{0,t}, \mu_{1,t})$ are sufficient statistics for the entrepreneur type distribution and in the stationary distribution, the corresponding measures of opaque and transparent good entrepreneurs are equal to $\nu_0 = \nu_1 = \frac{1-\mu_0-\mu_1}{2}$. We will prove this conjecture later. As such, $(\mu_{0,t}, \mu_{1,t})$ are the state variables in the dynamics equilibrium.

We first define the within period static equilibrium for fixed state variables $(\mu_{0,t}, \mu_{1,t})$.

Definition 1 (Stage Game Equilibrium) *For a fixed (μ_0, μ_1) , the stage game equilibrium consists of entrepreneurs' investment schedule $i(\tau, \omega)$ and credit demand schedule $\sigma(r; \tau, \omega)$, investors' advertised interest rate schedule $\tilde{r}(h)$ and unskilled investors' choice of test, equilibrium interest rate schedule $r(\tau, \omega)$, equilibrium credit allocation schedule to entrepreneurs $\ell(\tau, \omega)$, and equilibrium allocation of applications to investors such that*

- (i) *each agent's choice maximizes the agent's stage game utility given the strategy profile of other agents, equilibrium interest rates, and allocations,*
- (ii) *the implied interest rate schedule $r(\tau, \omega)$, credit allocation schedule for entrepreneurs $\ell(\tau, \omega)$, and allocation of applications to investors are consistent with agents' choices and the market clearing process.*

Next, we define the dynamic equilibrium where state variables $(\mu_{0,t}, \mu_{1,t})$ evolve endogenously over an invariant set.

Definition 2 (Dynamic Equilibrium) *The dynamic equilibrium consists of an infinite sequence of $\{(\mu_{0,t}, \mu_{1,t})\}_{t=0}^{\infty}$, individual entrepreneurs' $i_t(\tau, \omega)$ and $\sigma_t(\tau, \omega, r)$, individual investors' $\tilde{r}_t(h)$ and unskilled investors' choice of test, equilibrium $r_t(\tau, \omega)$, $\ell_t(\tau, \omega)$ and allocation of applications to investors, all within each period, such that*

- (i) *there exists a finite κ and a stable invariant set $\{(m_{0,i}, m_{1,i})\}_{i=1}^{\kappa}$ such that if $(\mu_{0,t}, \mu_{1,t}) = (m_{0,i}, m_{1,i})$ then*

$$(\mu_{0,t+1}, \mu_{1,t+1}) = \begin{cases} (m_{0,i+1}, m_{1,i+1}) & \text{if } i < \kappa \\ (m_{0,1}, m_{1,1}) & \text{if } i = \kappa, \end{cases}$$

$$\text{and } \nu_{0,t} = \nu_{1,t} = \frac{1-\mu_{0,t}-\mu_{1,t}}{2},$$

- (ii) *the dynamics of $(\mu_{0,t}, \mu_{1,t})$ are consistent with the birth-death process of entrepreneurs.*
- (iii) *each agent's choice maximizes the agent's life-time utility given the strategy profile of other agents, equilibrium interest rates and allocations,*

(iv) in each period t , the implied interest rate schedule $r_t(\tau, \omega)$, credit allocation schedule for entrepreneurs $\ell_t(\tau, \omega)$, and allocation of applications to investors are consistent with agents' choices and the market clearing process.

The dynamic equilibrium nests both a steady state and a cycle. If $\kappa = 1$, the dynamic equilibrium reduces to a standard steady state. When $\kappa > 1$, it is a *cyclical dynamic equilibrium* and features a stable cycle of length κ .⁷

The last definition of this section restricts attention to symmetric equilibria.

Definition 3 (Symmetric Dynamic Equilibrium) *A symmetric dynamic equilibrium is a dynamic equilibrium in which within each stage, all unskilled investor choose the same test.*

In the next two sections we first describe the stage game equilibrium and then show that the dynamic equilibrium is a sequence of stage game equilibria.

3 Stage Game Equilibrium

We first analyze the within period stage game equilibrium for fixed state variables (μ_0, μ_1) .

3.1 Credit Market

In order to construct the equilibrium in the credit market we start by expressing the optimal strategies of investors and entrepreneurs, which, in turn, inform us about the properties of supply and demand curves for credit. The following lemma describes the unskilled investors' financing decision.

Lemma 1 *Each unskilled investor who participates in the lending market only extends loans to entrepreneurs who pass her test.*

A critical implication of Lemma 1 is that choice of the test maps to different lending standards. A bold investor extends loans to all opaque entrepreneurs along with the transparent good ones. In other words, she applies *lax lending standards*. In contrast, a cautious investor applies *tight lending standards* as she issues credit to good, transparent applicants only.

The next lemma describes the entrepreneurs' optimal strategy, i.e. the credit demand, as a threshold strategy.

⁷Most of our formal analysis focuses on deterministic cycles to focus on the core mechanism of the paper. We introduce an extension to a more realistic stochastic cycle in Section G.

Lemma 2 *Each entrepreneur (τ, ω) chooses a reservation interest rate $r^{max}(\tau, \omega)$. He submits maximum demand, $\sigma(r; \tau, \omega) = \frac{1}{r}$ to all $r \leq r^{max}(\tau, \omega)$ and zero demand to all $r > r^{max}(\tau, \omega)$. Furthermore, $r^{max}(g, \omega) \leq \bar{r} \equiv \rho - 1 \leq r^{max}(b, \omega')$, $\forall \omega, \omega'$.*

Lemma 2 illustrates that it is sufficient to find the equilibrium reservation interest rate for entrepreneurs, instead of working out a full credit demand schedule. A reservation interest rate exists because all borrowed units are equally productive. Then, the structure of $\sigma(r; \tau, \omega)$ described in the Lemma follows from the robustness criterion of Assumption 2. Moreover, there is a maximum interest rate that any good entrepreneur would accept as he does pay back and he has to break even, while bad entrepreneurs are willing to accept any interest rate.

In order to find the stage game equilibrium, we proceed in two steps. The first step finds the equilibrium fixing investors' choice of test, and then the second step endogenizes this choice.

In the first step, Lemma 3 describes the equilibrium when all participating unskilled investors choose the bold test and when all of them choose the cautious test, as in the main text we focus on the symmetric equilibria.⁸

Lemma 3

- (i) *Bold stage: When every participating unskilled investor chooses the bold test the economy is in a bold stage. Every good entrepreneur chooses the reservation interest rate $r^{max} = r_B$ where*

$$r_B(\mu_0, \mu_1, c, r_f) \equiv \frac{\mu_0 + (1 - \mu_1)r_f + c}{1 - \mu_1 - \mu_0}. \quad (3)$$

Bad entrepreneurs choose an arbitrarily high rate. Skilled and unskilled investors both advertise r_B . The credit market is integrated as all good entrepreneurs and opaque bad ones obtain credit at common interest rate r_B .

- (ii) *Cautious stage: When every participating unskilled investor chooses the cautious test the economy is in a cautious stage. Each transparent (opaque) good entrepreneur chooses the reservation interest rate $r^{max} = r_C$ ($r^{max} = \bar{r}$) where*

$$r_C(\mu_0, \mu_1, c, r_f) \equiv r_f + \frac{2c}{1 - \mu_1 - \mu_0}. \quad (4)$$

⁸Lemma C.1 in Appendix C extends Lemma 3 to asymmetric stage game equilibrium in the credit market.

Bad entrepreneurs choose an arbitrarily high rate. The credit market is fragmented. Transparent and opaque good entrepreneurs obtain credit at rates r_C and $\bar{r} > r_C$, respectively. Bad entrepreneurs do not obtain any credit.

The interest rate $r_B(\mu_0, \mu_1, c, r_f)$ defined in Equation (3) is the interest rate at which an unskilled investor is indifferent whether to enter the credit market, pay the cost and choose a bold test, or stay out and earn the risk free rate. Indeed, r_B is the solution to the indifference condition

$$(1 - \mu_1 - \mu_0)(1 + r_B) + \mu_1(1 + r_f) - c = 1 + r_f. \quad (5)$$

The left hand side is the expected utility of an unskilled investor from running the bold test on a representative sample of applications, accepting all good and all opaque applicants out of which only the good ones pay back, investing the remainder of her capital (corresponding to rejected share of applications) in the risk-free asset, and finally paying the cost of the test. The right hand side is the return of not entering the credit market and investing in risk-free asset only.

Similarly, interest rate $r_C(\mu_0, \mu_1, c, r_f)$ defined in Equation (4) is the solution to the analogous indifference condition for an unskilled investor who chooses the cautious test.

$$\frac{(1 - \mu_1 - \mu_0)}{2}(1 + r_C) + \left(\frac{(1 - \mu_1 - \mu_0)}{2} + (\mu_1 + \mu_0) \right)(1 + r_f) - c = 1 + r_f. \quad (6)$$

Note that both (5) and (6) build on the condition that all good and bad entrepreneurs submit the maximum loan applications at each rate where they raise credit, which is implied by Lemma 2 and Lemma 3.

The dashed purple and dotted blue curves on the left panel of Figure 1 illustrate the interest rate that investors receive in bold and cautious stage, r_B and r_C , as a function of the measure of opaque bad entrepreneurs, μ_0 . Each curve is increasing as a larger μ_0 implies increased adverse selection. However, the marginal opaque bad entrepreneur has a differential effect on investors' return when they run different tests. Bold investors lend to the bad opaque entrepreneurs and lose the corresponding unit of capital, while cautious investors reject the applications of this type and earn the risk-free rate on that unit. Thus, bold investors have to be compensated more and r_B curve increases more steeply than r_C as the measure of bad opaque entrepreneurs increases.

Consider the bold stage where all the unskilled investors who enter the credit market choose the bold test and offer an interest rate r_B . The size of this group has to be sufficiently large that, with the help of skilled capital, the credit demand of all good entrepreneurs is

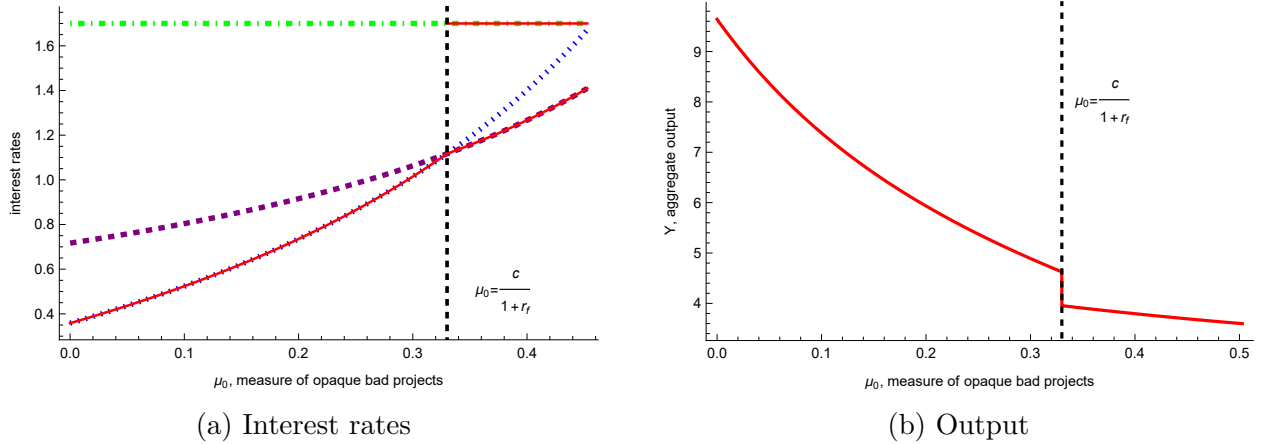


Figure 1: Interest rates and output as a function of μ_0 , for a fixed μ_1 . The left panel displays the break-even interest rates r_B (dashed blue), r_C (dashed purple), the maximum feasible rate $\bar{r}$ (dot-dashed green, horizontal), and the equilibrium interest rates (solid curves). The right panel displays the output. In the left region the stage game equilibrium is bold and in the right region it is cautious. The parameters are: $\rho = 2.7, c = 0.33, r_f = 0, w_0 = 3.3, w_1 = 0.15, \mu_1 = 0.11$.

satisfied at this rate. Furthermore, abundance of unskilled capital implies that all investors have to break even as any positive profits will be driven down to zero by entry of new unskilled investors. Now, consider the skilled investors. They are not willing to deviate to an interest rate lower than r_B as they can lend out all of their capital at this higher rate. Alternatively, they cannot offer a higher rate and still lend to good firms.⁹ As such, r_B is the only prevailing equilibrium interest rate. All good entrepreneurs and some bad opaque entrepreneurs raise financing at this rate.

Alternatively, consider the cautious stage where all the unskilled investors who enter the credit market choose the cautious test and offer interest rate r_C . The cautious unskilled investors only lend to transparent good entrepreneurs. Thus, the size of this group has to be sufficiently large that the credit demand of good transparent entrepreneurs is satisfied at that rate. Alternatively, the skilled investors choose to serve only opaque good entrepreneurs who are not served by cautious unskilled investors. As skilled capital is scarce, they can lend out all of their capital at the maximum interest rate $\bar{r}$ at which good entrepreneurs are willing to borrow. Clearly, skilled investors cannot do better. Each type of entrepreneur chooses the lowest reservation interest rate at which they can still borrow and that constitutes the

⁹Recall that we assume that lower interest rates clear first and, at a given interest rate unskilled investors sample first. Unskilled, by definition, are also indifferent whether to enter at r_B or stay out of the credit market. Therefore, to construct an equilibrium it must be that enough unskilled investors enter at r_B that their combined capital together with all skilled capital is just sufficient to satisfy all good entrepreneurs' credit demand at r_B . If too many unskilled investors would enter at r_B , skilled investors who could not lend would undercut them, deteriorating the applicant pool at r_B , which would violate r_B being an equilibrium.

equilibrium.

For expositional simplicity, in the rest of the paper we focus on the set of parameters that guarantee that all participating unskilled investors choose the same test in each stage. The following assumption provides a sufficient condition.

Assumption 3

$$\mu_1 > 1 - \frac{4c}{\rho + c - (r_f + 1)} - \frac{c}{r_f + 1}. \quad (7)$$

We can now move to the second step and characterize the full stage game equilibrium by endogenizing investors' optimal choice of test. The next proposition presents the first main result of the paper.

Proposition 1 *When $\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r}$,*

(i) If $\mu_0 \in [0, \frac{c}{1+r_f}]$, the economy is in a bold stage.

(ii) If $\mu_0 \in (\frac{c}{1+r_f}, 1]$, the economy is in a cautious stage.

When $\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} \geq \bar{r}$ the economy is in autarky where unskilled investors do not lend, bad entrepreneurs do not borrow, and good ones obtain credit at interest rate $\bar{r}$ from skilled investors only.

Proposition 1 divides the state space into regions where the economy is in the bold stage versus the cautious stage when Assumption 3 holds. In Appendix C we extend the results to the region where Assumption 3 does not hold and the equilibrium can be asymmetric.

In Proposition 1, the critical observation is that abundant unskilled capital implies that the unskilled investors choose the test that allows them to offer the lowest rate to entrepreneurs, as otherwise they are priced out of the market. Indeed, the proof of the proposition shows that $r_B \leq r_C$ if and only if $\mu_0 \in [0, \frac{c}{1+r_f}]$. The upward sloping, solid, red curve in Figure 1a depicts the prevailing interest rate at which unskilled investors lend for each level of μ_0 . The left is the region where the economy is in a bold stage and the right is when the economy is in a cautious stage. Coupled with Lemma 3, this implies that in each stage an unskilled investor cannot offer a lower rate or run a different test and participate.¹⁰

¹⁰Note that in a cautious stage, cautious investors are not lending to opaque good firms. Therefore, in principle, it is possible that given the chance to pick a different test, a group of unskilled investors advertise a higher rate, choose to run the bold test, and lend to opaque good firms. We rule out this possibility in the main text using Assumption 3 for expositional simplicity. We fully characterize such equilibria in Appendices C, D and E.

Investors' choice of the test follows a quality-quantity trade-off. Many applicants pass the bold test, however, the resulting loan portfolio involves some defaults since projects of opaque bad entrepreneurs pass the bold test as well. Therefore, μ_0 is the *quality cost* of a bold test and r_B has to compensate investors for adverse selection. On the other hand, the cautious test is low quantity but high quality. Only projects of good entrepreneurs pass the cautious test. This leads to a high quality loan portfolio which always pays back. However, the rejection rate is high since even some good entrepreneurs fail the test. As the cost of testing a unit measure of applications is fixed at c regardless of the scale of lending and a dollar not lent out earns the risk free return $1 + r_f$, $\frac{c}{1+r_f}$ is the *quantity cost* of a cautious test and r_C has to compensate the investor for the excess rejections at the same cost of the test.

As such, when the quality of the pool of loan applications is high, $\mu_0 \leq \frac{c}{1+r_f}$, the quality cost of a bold test is low, $r_B < r_C$, and a bold stage is realized. In this region investors are more concerned about losing out on good investment opportunities. Thus, lending standards are lax, and many entrepreneurs, including some bad ones, are able to raise financing at the same relatively low rate. On the other hand, if there are many bad entrepreneurs, the adverse selection problem is more severe. Investors are concerned about extending loans to the bad entrepreneurs who will default, thus they require a high r_B . It follows that in this region the quality cost of a bold test is higher than the quantity cost of a cautious test, $\mu_0 > \frac{c}{1+r_f}$, thus $r_C < r_B$ and a cautious stage realizes. Lending standards are tightened and credit market becomes segmented. Not only are bad entrepreneurs unable to raise financing, but even some good ones are able to do so only at extremely high rates.

It is worth mentioning that the bold stages exhibits several features of an overheated credit market. Interest rates are uniformly low and many entrepreneurs, including some bad ones, are financed. As such, the overall quality of initiated credit is low with a sizable share of loans eventually defaulting. This is in contrast to the tight credit market in cautious stages. Most importantly, the latter market is fragmented. Some good entrepreneurs (transparent ones) enjoy ample funding at relatively low interest rates while some other good entrepreneurs (opaque ones) can get only limited funding at very high rates. Furthermore, bad entrepreneurs are not funded at all. Therefore, although the total loan quantity is relatively low, its quality is high, which leads to high subsequent realized returns.

3.2 Investment and Output

In this section, we conclude the characterization of the stage game equilibrium by deriving the implied quantity of credit, investment and output in each stage. Notice that the information

friction influences the quantity of credit through two distinct channels: First, by increasing the prevailing interest rate and thus directly tightening the collateral constraint of borrowers, and second by limiting the supply of credit which leads to rationing the borrower demand even further.

All good entrepreneurs in a bold stage and transparent good ones in the cautious stage are limited by the collateral constraint $\frac{1}{r(\tau, \omega)}$, while opaque bad entrepreneurs in a bold stage and opaque good ones in a cautious stage are restricted by the limited supply of capital and are rationed. The investment of entrepreneur (τ, ω) is given by $i(\tau, \omega) = 1 + \ell(\tau, \omega)$ and his output is $y(\tau, \omega) \equiv \rho i(\tau, \omega)$. Therefore, aggregate output is determined as follows.

Aggregate Output. Aggregate output in state (μ_0, μ_1) is given by

$$\begin{aligned} Y(\mu_0, \mu_1) &\equiv \frac{1 - \mu_0 - \mu_1}{2} (y(g, 1) + y(g, 0)) + \mu_1 y(b, 1) + \mu_0 y(b, 0) \\ &= \rho \left(1 + \frac{1 - \mu_0 - \mu_1}{2} (\ell(g, 1) + \ell(g, 0)) + \mu_0 \ell(b, 0) \right). \end{aligned} \quad (8)$$

In Equation (8), the terms in parenthesis correspond to different sources of investment. 1 is the endowment of all the entrepreneurs. The second term is the outside financing raised by all the good entrepreneurs, and the last term is the outside financing raised by opaque bad entrepreneurs. Transparent bad entrepreneurs do not raise any outside financing.

The main result of this section describes the equilibrium credit allocation in each stage, which in turn determines the investment and output.

Proposition 2

- (i) *In any equilibrium transparent bad entrepreneurs are not financed by any investors, $\ell(b, 1) = 0$.*
- (ii) *In a bold stage, all entrepreneurs face interest rate r_B . All good entrepreneurs borrow $\ell(g, \omega) = \frac{1}{r_B}$. Opaque bad entrepreneurs are limited by unskilled investors' mistakes at interest rate r_B , implying $\ell(b, 0) = \frac{1}{r_B} - \frac{w_1}{1 - \mu_0 - \mu_1}$.*
- (iii) *In a cautious stage, all transparent good entrepreneurs face interest rate r_C and borrow $\ell(g, 1) = \frac{1}{r_C}$. Opaque good ones face $\bar{r}$ and are limited by the short supply of skilled capital, implying $\ell(g, 0) = \frac{2w_1}{1 - \mu_0 - \mu_1}$. Opaque bad entrepreneurs are not financed, $\ell(b, 0) = 0$.*
- (iv) *Aggregate output, $Y(\mu_0, \mu_1)$, is decreasing in μ_0 and discontinuously jumps downward as μ_0 crosses the bold-cautious threshold, $\frac{c}{1 + r_f}$, from below.*

In a bold stage, all good entrepreneurs are fully financed at low interest rate r_B . Transparent bad entrepreneurs are excluded from the credit market. However, opaque bad ones obtain some credit since the bold test does not distinguish them from good entrepreneurs. Yet, their credit is limited by the false positives mistakes of participating unskilled investors. Since all good entrepreneurs and even some bad ones raise credit at a low rate and invest, investment and output are high. Thus, bold stages tend to correspond to a “boom.”

In a cautious stage transparent good entrepreneurs are financed by cautious unskilled investors at the relatively lower interest rate r_C . However, opaque good entrepreneurs can only obtain credit from skilled investors, limited by the restricted capital supply of these investors. We call this the *stifling role of tight credit*: good investment is dampened in a cautious stage. Furthermore, no bad entrepreneur can raise financing and bad investment contracts as well. Due to the low output, cautious stages tend to correspond to a “downturn.”

Figure 1b illustrates aggregate output as a function of μ_0 , for a fixed μ_1 . As part (v) of the proposition states, the aggregate output is continuous and monotonically decreasing in the measure of opaque bad entrepreneurs in any given stage. The output decline is intuitive: In either stage, an increase in μ_0 (weakly) increases the equilibrium interest rates as the adverse selection problem worsens. This, in turn increases the cost of borrowing and suppresses investment and output. When an increase in the fraction of opaque bad entrepreneurs turns a bold stage to a cautious stage, the supply of credit drops as well, which leads to a discontinuously drop in the output.

4 Dynamic Endogenous Cycles

In this section we examine the dynamic behavior of the economy. We describe the defining features of the deterministic cycles that emerge in both the credit market and the real economy.¹¹ Throughout, we use a *boom* or an *upturn* to refer to the times when output is high, which is accompanied by low yields in the credit market. Alternatively, a *bust*, *downturn*, or *recession* happens when output is low. This is accompanied by a fragmented credit market.

The next lemma establishes that the dynamic equilibrium reduces to a sequence of stage game equilibria.

Lemma 4 *In any dynamic equilibrium, the economy is in a stage game equilibrium in each period. The measures of opaque and transparent bad entrepreneurs at time t , $(\mu_{0,t}, \mu_{1,t})$, are sufficient state variables for the stage game equilibrium in the corresponding period.*

¹¹Appendix G presents an extension with aggregate shocks that leads to stochastic cycles.

Lemma 4 demonstrates that maximizing life-time utility leads to the same outcome as maximizing stage game utility for each entrepreneur. That is, it is never optimal for an entrepreneur to take a loss in the stage game in a given period in order to increase his chances to obtain credit and survive to the next period. The lemma also allows us to focus our discussion on the evolution of the state variables $(\mu_{0,t}, \mu_{1,t})$, determined by the prevalent lending standards in the current stage game.

To ease the notation, we omit the time-subscript whenever it does not cause any confusion.

Law of Motion for State Variables. Let (μ_0, μ_1) and (μ'_0, μ'_1) denote the state variables today and tomorrow, respectively. When at least some investors are bold, only transparent bad entrepreneurs cannot raise financing. However, when all investors are cautious, opaque bad entrepreneurs are not financed either. Any entrepreneur who cannot raise financing exits and is replaced by a newborn. The next proposition summarizes the law of motion for measure of opaque and transparent bad entrepreneurs.

Proposition 3 *Assume $\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r} = \rho - 1$ so the economy is not in autarky.*

(i) *Consider a period when a positive measure of participating investors are bold. Then the laws of motion for μ_0 and μ_1 are given by*

$$\mu'_{0B}(\delta, \lambda, \mu_0, \mu_1) = (1 - \delta)\mu_0 + (\delta + (1 - \delta)\mu_1)\frac{\lambda}{2}, \quad (9)$$

$$\mu'_{1B}(\delta, \lambda, \mu_0, \mu_1) = (\delta + (1 - \delta)\mu_1)\frac{\lambda}{2}. \quad (10)$$

(ii) *Consider a period when all participating investors, except possibly a zero measure, are cautious. Then the laws of motion for μ_0 and μ_1 are given by¹²*

$$\mu'_{0C}(\delta, \lambda, \mu_0, \mu_1) = (\delta + (1 - \delta)(\mu_0 + \mu_1))\frac{\lambda}{2}, \quad (11)$$

$$\mu'_{1C}(\delta, \lambda, \mu_0, \mu_1) = (\delta + (1 - \delta)(\mu_0 + \mu_1))\frac{\lambda}{2}. \quad (12)$$

The laws of motion are intuitive. Consider the measure of opaque bad entrepreneurs, μ_0 . When some investors are bold, $\mu'_{0B}(\delta, \lambda, \mu_0, \mu_1)$ describes the evolution of μ_0 . It consists of survivals from the current period, plus the newborns. From the existing opaque

¹²Equations (11) and (12) govern the law of motion of the state variables if the economy is in autarky as well.

bad entrepreneurs, fraction $(1 - \delta)$ survive. The newborns replace two groups of exiting entrepreneurs: First, δ fraction of all entrepreneurs die exogenously and are replaced. Second, the $(1 - \delta)$ remaining fraction transparent bad entrepreneurs cannot raise funding and are replaced as well. A fraction $\frac{\lambda}{2}$ of all newborns are opaque bad entrepreneurs, which implies Equation (9) as the law of motion for opaque bad entrepreneurs. Alternatively, when all investors are cautious, $\mu'_{0C}(\delta, \lambda, \mu_0, \mu_1)$ describes the evolution of μ_0 . In this case no bad entrepreneur survives to the next period so the new opaque bad entrepreneurs are all newborns, who replace all bad entrepreneurs and fraction δ of all good entrepreneurs who die exogenously. The laws of motion for transparent bad entrepreneurs follow a similar intuition in both cases.

The law of motion for opaque and transparent good entrepreneurs are the same in both regimes. The reason is that both groups always raise financing and their measure among the newborns is the same. As such, in the long run both measures are equal to $\frac{1-\mu_0-\mu_1}{2}$. This validates that (μ_0, μ_1) are sufficient state variables for the economy despite four types of entrepreneurs.

The laws of motion for state variables enable us to characterize the dynamic equilibria in this economy, starting with an existence result. Recall that Proposition 1 describes the test chosen by investors in each stage as a function of the state variables and Proposition 3 demonstrates the law of motion for state variables (μ_0, μ_1) as a function of the test chosen by investors. Proposition 4 integrates these two results to state the existence of a dynamic equilibrium.

Proposition 4 *Under weak conditions there exists an invariant set of $\{\mathbf{m}_i\}_{i=1}^\kappa \equiv (m_{0,i}, m_{1,i})_{i=1}^\kappa$ which is a dynamic equilibrium. If Assumption 3 holds for $\mu_1 = (m_{1,i})_{i=1}^\kappa$, then it is a symmetric dynamic equilibrium.*

Note that we focus on the symmetric dynamic equilibria in the main text as we maintain Assumption 3 throughout.

If a single set of laws of motion were to govern the dynamic evolution of the state variables throughout, i.e. either equations (9)-(10) or (11)-(12), then (μ_0, μ_1) would converge to a steady state regardless of the initial conditions. We use this observation to establish the sufficient conditions for the economy to converge to a long-run steady state.

Lemma 5 *Consider two pair of constants $(\bar{\mu}_{0B}, \bar{\mu}_{1B})$ and $(\bar{\mu}_{0C}, \bar{\mu}_{1C})$ such that*

$$\begin{aligned} \bar{\mu}_{0B} &\equiv \frac{\lambda}{2 - \lambda(1 - \delta)}, & \bar{\mu}_{1B} &\equiv \frac{\lambda\delta}{2 - \lambda(1 - \delta)} \\ \bar{\mu}_{0C} &\equiv \frac{\lambda\delta}{2 - 2\lambda(1 - \delta)}, & \bar{\mu}_{1C} &\equiv \frac{\lambda\delta}{2 - 2\lambda(1 - \delta)}. \end{aligned}$$

For any λ and δ , $\bar{\mu}_{0B} > \bar{\mu}_{0C}$ and $\bar{\mu}_{1B} < \bar{\mu}_{1C}$.

(i) If $\bar{\mu}_{0B} \leq \frac{c}{1+r_f}$, then $(\bar{\mu}_{0B}, \bar{\mu}_{1B})$ is a bold steady state equilibrium.

(ii) If $\bar{\mu}_{0C} \geq \frac{c}{1+r_f}$, then $(\bar{\mu}_{0C}, \bar{\mu}_{1C})$ is a cautious steady state equilibrium.

$(\bar{\mu}_{0B}, \bar{\mu}_{1B})$ denotes the measure of opaque and transparent bad entrepreneurs in a steady state where investors are always bold. Similarly, $(\bar{\mu}_{0C}, \bar{\mu}_{1C})$ corresponds to the cautious steady state. Observe that $(\bar{\mu}_{0B}, \bar{\mu}_{1B})$ and $(\bar{\mu}_{0C}, \bar{\mu}_{1C})$ are the fixed points of equations (9)-(10) and (11)-(12), respectively. Also, the measure of opaque bad entrepreneurs in the bold steady state is higher than that of the cautious one, $\bar{\mu}_{0,B} > \bar{\mu}_{0,C}$, as the exit rate of opaque bad entrepreneurs is lower when investors are bold. Lemma 5 states that if the cost of the test is so high (low) that unskilled investors always choose the bold (cautious) test, then the economy converges to the corresponding steady state. In these cases, there are no cycles in equilibrium.

It is worth pointing out that the permanent bold steady state is associated with an extended period of low-quality lending, a high fraction of bad entrepreneurs, limited new entry, large share of non-performing loans, and, as a result, low output.¹³ This interpretation matches well the description of the Japanese economy in Caballero et al. (2008), which provide a credit market based explanation for the long Japanese slowdown after the asset price collapse in the early 1990s as a market that involves widespread zombie lending and suffers from depressed restructuring.

Equilibrium Cycles. Throughout the rest of the paper we focus on an intermediate range of cost, $\frac{c}{1+r_f} \in (\bar{\mu}_{0C}, \bar{\mu}_{0B})$, where the economy is not in a steady state and the symmetric dynamic equilibrium is cyclical. The following definition lays out the building blocks of a dynamic cyclical equilibrium

Definition 4 *A bold phase and a cautious phase are a continuous sequence of bold and cautious stages, of length at least one, respectively.*

The economy features different deterministic endogenous cycles, depending on the parameters, as bold and cautious phases alternate. Proposition 5 characterizes the main result of the paper on the emergence of cyclical symmetric dynamic equilibria. This result builds on the observation that the evolution of μ_0 determines the phases of the economy in a symmetric dynamic equilibrium.

¹³While a bold phase corresponds to higher output than a cautious phase for a fixed μ_0 , if all unskilled investors are permanently bold, the economy is stuck with a high fraction of opaque bad entrepreneurs, $\mu_0 = \bar{\mu}_{0B}$. This leads to lower average output relative to an equilibrium cycle with the same parameters.

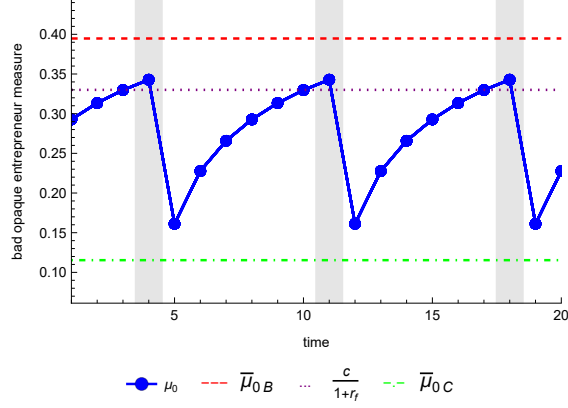
Proposition 5 *In a symmetric dynamic equilibrium λ and δ determine constants $\mu_{0B}^* < \mu_{0C}^* \in (\bar{\mu}_{0C}, \bar{\mu}_{0B})$ such that:*

- (i) *For a 2-period cycle with the two-point support (μ_{0B}^*, μ_{0C}^*) it is necessary that $\frac{c}{1+r_f} \in [\mu_{0B}^*, \mu_{0C}^*)$. In this case, the economy oscillates between a one-period bold phase and a one-period cautious phase.*
- (ii) *For a $\kappa > 2$ period long bold-short cautious cycle it is necessary that $\frac{c}{1+r_f} \in [\mu_{0C}^*, \bar{\mu}_{0B})$. This cycle consists of a long bold phase, $\kappa - 1$ consecutive periods where μ_0 increases, followed by a short cautious phase, a one period decline in μ_0 . A longer bold phase, i.e. a higher κ corresponds to a larger $\frac{c}{1+r_f}$ within this range.*
- (iii) *For a $\kappa > 2$ period short bold-long cautious cycle it is necessary that $\frac{c}{1+r_f} \in (\bar{\mu}_{0C}, \mu_{0B}^*)$. The cycle consists of a long cautious phase, $\kappa - 1$ consecutive periods where μ_0 decreases, followed by a short bold phase, a one period increase in μ_0 . A longer cautious phase, i.e. a higher κ corresponds to a smaller $\frac{c}{1+r_f}$ within this range.*

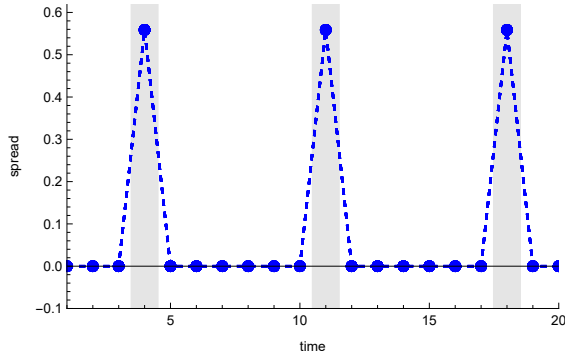
The cycles are the outcome of a two-way interaction between investors' choice of lending standards and the fundamentals of the economy. When the measure of opaque bad entrepreneurs who apply for a loan is relatively low, there are few defaults in the portfolio of a bold investor. As such, the quality cost of the bold test is low and investors use lax lending standards: The interest rate is low, there is a lot of credit, and the economy is in a boom. Nevertheless, as a result of lax lending standards, the quality of the credit pool deteriorates. Once the number of opaque bad applicants has sufficiently risen, the quality cost of the bold test becomes prohibitively high and investors prefer to turn cautious. Being cautious implies tight lending standards, high interest rates, a large credit spread, and little credit to opaque entrepreneurs: A recession hits. However, this also stops opaque bad entrepreneurs from raising funding. We call this *cleansing by tight credit*. The quality of the pool of credit applicants improves in the cautious stage, which prompts the lenders to switch back to lax lending standards in the future. The economy turns into a boom, and the cycle continues.

The output crashes when the investors tighten lending standards by switching from bold to cautious, in line with Proposition 2(iv). The tightening implies that unskilled investors stop lending to opaque entrepreneurs. Opaque good entrepreneurs can only borrow at a higher rate from skilled investors while opaque bad ones lose access to credit all together, which leads to discontinuous drop in credit, investment and output.¹⁴

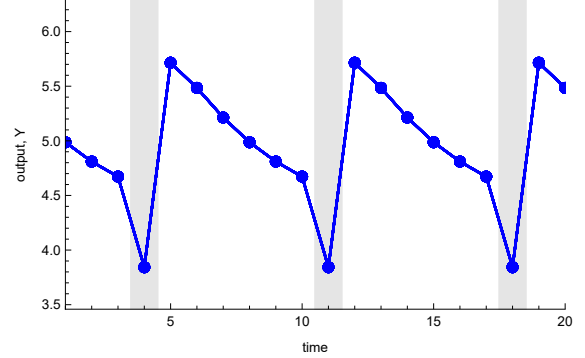
¹⁴Consistent with this interpretation, Leibovici et al. (2019) use Paydex credit score to document that plant shutdown increases during the Great Recession, and the increase is more pronounced among firms with low Paydex score. The Paydex score is a business credit score that is increasing in the likelihood of a business paying its debts on time.



(a) State variable μ_0



(b) Interest rates



(c) Output

Figure 2: This figure plots a cyclical symmetric dynamic equilibrium with a long bold-short cautious cycle. Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates. Panel (c) depicts the total gross output and welfare. The parameters are: $\rho = 2.7, \lambda = 0.6, \delta = 0.2, c = 0.33, r_f = 0, w_0 = 3.3, w_1 = 0.15$.

Proposition 5 illustrates three different classes of cyclical patterns in the long run. Consider the long bold-short cautious cycle of case (ii). Here, the high quantity cost of tight standards implies longer bold and shorter cautious phases, i.e. a long boom is interrupted by a one period recession. A short cautious phase is enough to improve the quality of loan applications sufficiently for investors to be bold again, and not risk losing good investment at the cost of financing too many bad entrepreneurs.

Figure 2 depicts this case. As mentioned earlier, in a symmetric dynamic equilibrium the measure of opaque bad entrepreneurs (μ_0) is sufficient to determine the phase of the economy. Panel 2a shows the evolution of this state variable.¹⁵ Panel 2b plots the interest

¹⁵The indifference threshold $\frac{c}{1+r_f}$ is not a steady state equilibrium. With our tie-breaking assumption, Proposition 3 implies that the bold dynamics apply at the threshold and thus $\mu'_0 > \mu_0$ if $\mu_0 = \frac{c}{1+r_f}$. Any

rates throughout the cycle. As shown in Proposition 1, there is no credit spread in the bold phase, while the credit market is fragmented in the cautious phase and the credit spread spikes. Panel 2c illustrates the cyclical output which crash when lending standards tighten.

Comparison of Panels 2a and 2c shows the negative co-movement of output with the measure of opaque bad entrepreneurs μ_0 . Moreover, the output drop is amplified when there is a switch from lax to tight lending standards, which happens in periods 4, 11 and 18. While μ_0 increases only slightly in these periods, the drop in output is sizable.

Alternatively, a low quantity cost of tight standards implies a short bold-long cautious cycle, where extended downturns are followed by short booms.¹⁶ An intermediate level of $\frac{c}{1+r_f}$ implies a short bold-short cautious cycle, alternating between short booms and short downturns.¹⁷

Lastly, Proposition 5 shows how the length of bold and cautious phases vary with $\frac{c}{1+r_f}$, fixing λ and δ . In particular, it shows that as c declines, a cyclical economy moves from cycles with longer booms, i.e. longer bold phases, to cycles with longer recessions, i.e. longer cautious phases.

4.1 Capital Reallocation

It is instructive to investigate how capital is allocated among good and bad firms in each stage. We define capital allocation quality as the difference between the credit raised by average good and average bad firm in a given stage.¹⁸

$$CAQ(\mu_0, \mu_1) \equiv \frac{\ell(g, 1) + \ell(g, 0)}{2} - \frac{\mu_1 \ell(b, 1) + \mu_0 \ell(b, 0)}{\mu_1 + \mu_0}.$$

The following lemma states how capital allocation is affected by the scarcity of skilled capital.

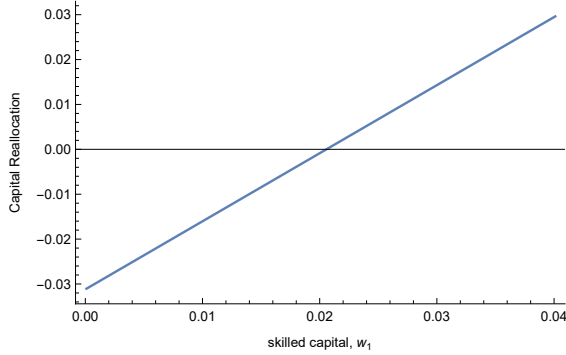
Lemma 6 *For each pair of state variable (μ_0, μ_1) , the capital allocation quality $CAQ(\mu_0, \mu_1)$, is increasing in skilled capital w_1 . Furthermore, $CAQ(\mu_0, \mu_1)$ changes discontinuously as μ_0*

other tie breaking assumption implies a change in μ_0 as well. In particular, if a positive measure of investors chooses to be bold, the bold dynamics apply. If all investors choose to be cautious, then the cautious dynamics apply.

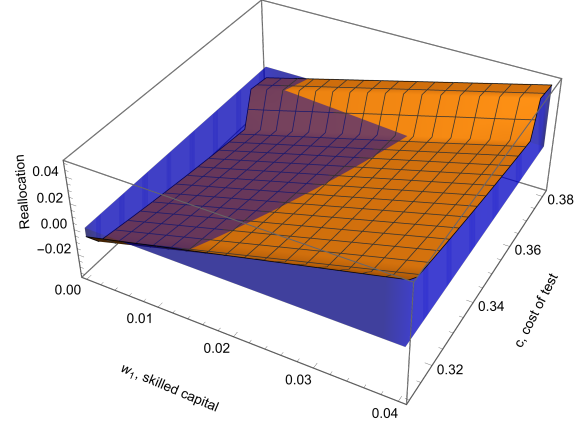
¹⁶Figure D.1 in Appendix D plots the evolution of the measure opaque bad entrepreneurs, output and credit spreads for the case of short bold-long cautious cycles.

¹⁷In Appendices D and E, we show that cycles where both booms and recessions last multiple periods can also arise in our model. Such cycles correspond to parameter sets where Assumption 3 do not always hold, hence the dynamic equilibrium is asymmetric.

¹⁸Note that as $i(\tau, \omega) = 1 + \ell(\tau, \omega)$, defining allocation quality as difference of investment between the average good and bad firm would lead to an identical measure.



(a) Capital reallocation quality for cost of the test $c = 0.35$



(b) Capital reallocation quality for cost of the test $c \in (\bar{\mu}_0^C, \bar{\mu}_0^B)$

Figure 3: Panel (a) plots capital reallocation quality, R , for different wealth levels of skilled investors, w_1 , for the specific value of $c = 0.35$. Panel (b) plots the same quantity but for the complete range of c where the economy has a symmetric dynamic cycle, $c \in (\bar{\mu}_0^C, \bar{\mu}_0^B)$. The orange plane is the equilibrium capital reallocation quality and the blue plain is $R = 0$. Other parameters are: $\rho = 3.5, \lambda = 0.7, \delta = 0.7, r_f = 0, w_0 = 3.2$.

crosses the bold-cautious threshold. Furthermore, there exist a $\bar{w}_1 \geq 0$ such that for all μ_1

$$\lim_{\mu_0 \downarrow \frac{c}{1+r_f}} CAQ(\mu_0, \mu_1) - \lim_{\mu_0 \uparrow \frac{c}{1+r_f}} CAQ(\mu_0, \mu_1) > 0, \quad w_1 > \bar{w}_1$$

and

$$\frac{\partial}{\partial w_1} \left[\lim_{\mu_0 \downarrow \frac{c}{1+r_f}} CAQ(\mu_0, \mu_1) - \lim_{\mu_0 \uparrow \frac{c}{1+r_f}} CAQ(\mu_0, \mu_1) \right] > 0. \quad (13)$$

Lemma 6 states that skilled capital improves capital allocation for any combination of state variables. In the bold phase, skilled capital crowds out unskilled capital and limits the credit allocated to opaque bad firms. In the cautious phase, abundant skilled capital reduces the stifling effect by increasing the capital allocated to opaque good firms.

What is particularly interesting is to study capital reallocation as the economy enters a downturn, i.e. transitions from a bold phase to a cautious phase. Equation (13) provides an approximation of the quality of this reallocation. It shows that at the margin, the capital allocation unambiguously improves when the economy enters a recession from a boom, if there is more skilled capital. In other words, more skilled capital strengthens the cleansing effect. As such, when the skilled capital is abundant, the cleansing effect dominates while if skilled capital is scarce, the stifling effect is stronger.

Lemma 6 is suggestive of when the cleansing effect of recession is dominant and capital

reallocates to good firms in recessions. Yet, it is a partial result. To demonstrate that the corresponding intuition carries through more broadly when the dynamic cycle is determined endogenously in equilibrium, we define *capital reallocation quality* as the difference between the average capital allocation quality between the recession and the boom in a cycle.

$$R \equiv \mathbb{E}_{\mu_0 \leq \frac{c}{1+r_f}}[CAQ(\mu_0, \mu_1)] - \mathbb{E}_{\mu_0 > \frac{c}{1+r_f}}[CAQ(\mu_0, \mu_1)]. \quad (14)$$

Figure 3 plots the capital reallocation quality R as a function of skilled capital w_1 .¹⁹ Panel 3a depicts capital reallocation quality from boom to recession, across different values of skilled wealth, w_1 , for a particular cost of test, $c = 0.36$. Two observations are worth nothing: First, the capital reallocation quality is increasing as function of skilled capital, which is well aligned with the closed form approximation. Second, when skilled wealth is scarce the stifling effect dominates the cleansing effect and the capital misallocation increases during recession. Alternatively, when there is sufficient skilled capital the cleansing role of recession dominates and capital misallocation falls during recessions.

Panel 3b demonstrates that the above two patterns hold more generally, for all values of c , when the economy has a symmetric dynamic cycle. The upward sloping, light, orange plane is the capital reallocation quality for different values of (w_1, c) and the flat, dark, blue plane is $R = 0$. The orange plane is upward sloping in w_1 , and crosses $R = 0$ plane at some intermediate value of w_1 for every level of cost c .

5 Optimal Cycles

Next, we investigate the socially optimal investor information choice and the implied lending standards and allocation of credit, and compare it to equilibrium outcomes. In this economy, a natural measure of welfare in each state is the average aggregate consumption of all entrepreneurs and investors. In order to accommodate the emergence of cycles, we use the notion of *average welfare* defined below.

Definition 5 (Average Welfare) *For any collection of k states characterized by the pair*

¹⁹For any pair of state variables (μ_0, μ_1) and (μ'_0, μ'_1) we have

$$\frac{\partial}{\partial w_1} [CAQ(\mu_0, \mu_1) - CAQ(\mu'_0, \mu'_1)] = \frac{1}{1 - \mu_0 - \mu_1} - \frac{\mu'_0}{(1 - \mu'_0 - \mu'_1)(\mu'_0 + \mu'_1)}.$$

It is possible to compute R as a function of the underlying parameters. However, the calculation is cumbersome and does not provide further intuition. As such, we opted for a numerical illustration of R instead.

of state variables $\{\mu_{0,j}, \mu_{1,j}\}_{j=1}^k$, the average welfare is

$$EW(\{\mu_{0,j}, \mu_{1,j}\}_{j=1}^k) \equiv \frac{1}{k} \sum_{j=1}^k W(\mu_{0,j}, \mu_{1,j}), \quad (15)$$

where the static welfare is given by

$$\begin{aligned} W(\mu_0, \mu_1) \equiv & \rho(1 + \mu_0 \ell(b, 0)) + \frac{1 - \mu_0 - \mu_1}{2} \sum_{\omega=0,1} \ell(g, \omega) [\rho - (1 + r(g, \omega))] \\ & + w_0(1 + r_f) + w_1 \left(1 + \max_{\omega} r(g, \omega)\right). \end{aligned} \quad (16)$$

The first term in static welfare $W(\mu_0, \mu_1)$ is the stage game consumption (or total production) of bad entrepreneurs. The second term is the consumption of transparent and opaque good entrepreneurs, which is their production net of repayment. The third term is the consumption of unskilled investors, noting that they are indifferent between lending and storage at the risk-free rate. The last term is the consumption of the skilled investors.

Note that this measure focuses on the equally weighted aggregate consumption along the cycle without considering whether all agents are weakly better off or only certain groups are. As such, it represents a specific point on the Pareto frontier.

We first show how the static welfare responds to an increase in the measure of opaque bad entrepreneurs.

Lemma 7 *Assume the stage game equilibrium is not autarky. Static welfare is decreasing in the measure of opaque bad entrepreneurs μ_0 , and discontinuously drops in μ_0 when an increase in μ_0 implies a transition change from bold to cautious stage.*

Keeping μ_1 constant, an increase in the measure of opaque bad entrepreneurs decreases static welfare as it exacerbates the borrower adverse selection problem. The cost of capital increases and thus investment and production falls. When investors switch to be cautious the problem is intensified since not only some entrepreneurs lose some (or all) financing, but also some good ones can only borrow at the high rates that skilled investors are willing to lend at. Thus at the level of μ_0 that the equilibrium changes from bold to cautious, static welfare is strictly higher in a bold phase than in a cautious one, re-enforcing our interpretation of these phases as booms and busts. Figure 2c depicts the dynamics of output under our baseline parameters.

Next, consider a social planner who can determine investors' choice of test but cannot directly intervene in the credit market or influence entrepreneurs' investment. Below, we provide the formal definition of the constraint planner problem.

Definition 6 (Constrained Planner Problem) *The constrained planner maximizes average welfare of the ergodic state distribution by choosing a threshold $\hat{\mu}_0^P$ for the fraction of opaque bad entrepreneurs μ_0 and one single test available for investors on each side of the threshold. The resulting allocation corresponds to a dynamic equilibrium, following Definition 2, with the restricted choice of tests.*

The constrained planner has a single tool to influence the economic outcome, the choice of the test, and even that choice is very restricted. The planner can only partition the state space into two segments and in each segment make one test available to investors. For instance, the planner can implement a bold (cautious) steady state by choosing a threshold $\hat{\mu}_0^P > \bar{\mu}_{0,B}$ ($\hat{\mu}_0^P < \bar{\mu}_{0,C}$). Alternatively, the planner can implement various two-phase cycles by choosing different levels of $\hat{\mu}_0^P \in (\bar{\mu}_{0,C}, \bar{\mu}_{0,C})$. He cannot directly intervene in the credit market or choose the investment levels.

The constrained planner has to respect the information friction faced by lenders, hence, he cannot circumvent the adverse selection problem which hinders lending in credit markets. Indeed, once he chooses the test available to investors, optimal equilibrium strategies determine the interest rates, credit quantities and investment levels as described by Lemma 3. An immediate implication is that the planner cannot force lenders to make losses on their extended credit as they can always choose not to participate in the credit market.

The constraint planner faces a trade-off in choosing threshold $\hat{\mu}_0^P$. On the one hand, Lemma 7 implies that the static welfare is low in the cautious stage. Thus, the planner would be inclined to encourage a boom accompanied by lax lending standards. On the other hand, the positive impact of tight lending standards on the future quality of investment suggests that the planner also has an incentive to (periodically) cleanse the economy of opaque bad entrepreneurs by causing a recession. The next proposition formalizes this trade-off by providing sufficient conditions for the constrained planner dynamic allocation to be cyclical.

Proposition 6 *There are $\lambda^{\min}, \lambda^{\max}$ and $\bar{\delta}(\lambda) \in (0, 1)$ such that for any $\lambda \in [\lambda^{\min}, \lambda^{\max}]$ and $\delta < \bar{\delta}(\lambda)$ the dynamic constrained optimal allocation features a cycle.*

Proposition 6 demonstrates that for an intermediate range of parameters, the constrained optimal economy features cycles. Intuitively, choice of investors' test, or equivalently lending standards, is planner's instrument to influence the distribution of states in the long run. Tight lending standards have a cleansing effect: They keep the measure of bad entrepreneurs at bay. However, they also squeeze the entrepreneurs. Therefore, to maximize average welfare the planner wants the investors to be cautious and impose tight lending standards

only periodically, when the measure of entrepreneurs who do not repay is high. It is optimal to have lax lending standards otherwise as it leads to high output.

Externality. It is insightful to consider the inefficiency of the equilibrium cycle through the lens of a static and a dynamic externality.

Proposition 7 *The equilibrium cycle exhibits two externalities*

- (i) *Static externality: There exists μ_0^{ext} such that for $\mu_0 \in (\frac{c}{1+r_f}, \mu_0^{ext})$ the equilibrium outcome in the stage game is strictly dominated by an alternative outcome in which strictly more investors are bold. In contrast, there is no (μ_0, μ_1) for which the equilibrium outcome in the stage game is weakly dominated by an alternative outcome in which strictly more investors are cautious.*
- (ii) *Dynamic externality: Consider two consecutive periods and let (μ_0, μ_1) and (μ'_0, μ'_1) denote the corresponding state variables. There exist $\bar{\rho}$ and $\bar{w}_1$ such that if $\rho < \bar{\rho}$ and $w_1 < \bar{w}_1$, for any (μ_0, μ_1)*

$$W(\mu'_{0C}(\mu_0, \mu_1), \mu'_{1C}(\mu_0, \mu_1)) > W(\mu'_{0B}(\mu_0, \mu_1), \mu'_{1B}(\mu_0, \mu_1)).$$

This proposition highlights a static and a dynamic externality going in opposite directions. The static externality concerns the welfare effect of lending standards for fixed state variables (μ_0, μ_1) .

The first part of the proposition shows that when μ_0 is just above the threshold $\frac{c}{1+r_f}$, an intervention which leads to the bold test being chosen by unskilled investors increases (equally weighted) welfare in the stage game. In the cautious decentralized equilibrium unskilled investors only lend to transparent good entrepreneurs at $r_C(\cdot)$. Opaque good entrepreneurs can only borrow from skilled investors at the high rate $\bar{r}$. Alternatively, in the bold equilibrium under the intervention unskilled investors lend to all good entrepreneurs at $r_B(\cdot)$.

The key to the welfare gain of the intervention is to notice that the unskilled investors always break even: in the decentralized equilibrium where they choose the cautious test, under the intervention where they choose the bold test, or if they do not participate in the credit market. The reason is that the interest rate adjusts to ensure that unskilled lenders are indifferent between lending or staying out. Thus, they are equally well off with and without the intervention. The main distributive effect of the intervention, however, is that in a bold stage, skilled investors earn a lower information rent while opaque good entrepreneurs borrow at a lower rate and thus are strictly better off.

The proof of Proposition 7 illustrates that the total consumption of these latter two groups is larger under the intervention than in the decentralized equilibrium. Intuitively, the intervention reduces the market power of skilled investors over opaque good entrepreneurs as now they can also borrow from the bold unskilled investors: they borrow at a lower interest rate, $r_B(\cdot)$ instead of $\bar{r}$, from a larger pool of investors. As such, the intervention enhances the opaque good entrepreneurs' access to credit which in turn increases the total investment and thus consumption. Put differently, the source of the static externality is that there is no market to equate the marginal benefit and cost of investors' choice of the test, noting that the cost and benefit are born by different market participants.²⁰

While Proposition 7 states that in some states, choice of bold test by unskilled investors improves welfare, there are no states where tightening lending standards would improve static welfare. That is, the static welfare loss is due to the stifling role of tight credit standards. Thus from a static perspective, investors choose to be cautious too often compared to what is constrained optimal.

The decentralized equilibrium features a dynamic externality as well. The law of motion for state variables directly implies that the measure of opaque bad entrepreneurs is higher following lax lending standards compared to tight ones. For a wide range of parameters, this implies a lower welfare in the period that follows, and all entrepreneurs face a higher interest rate due to the more extreme adverse selection problem. However, lenders ignore that tight lending standards cleanse the economy from opaque bad types, leading to higher welfare in the subsequent period.²¹ Thus, from a dynamic perspective, investors choose to be bold too often compared to what is constrained optimal, exactly opposite the static externality. Put differently, more frequent cautious phases in the optimal cycle keeps the measure of bad entrepreneurs in the pool of applicants lower on average, which in turn makes the following bold phase more productive. Indeed, in our numerical exercises we find that the dynamic externality often dominates: The constrained planner prefers to shorten the length of the boom and have more frequent, albeit milder and less deep recessions.

The dynamic externality arises when use of cautious instead of bold test by the unskilled investors in the current period increases welfare in the future period. A useful way to differentiate between the static and dynamic externality is to consider the margin that they influence. The static externality is relevant for a fixed distribution of entrepreneurs: it corresponds to a reduction in the within-stage welfare due to the "wrong" test being chosen, without any change in the entrepreneur distribution. Alternatively, the dynamic externality

²⁰It is possible to design a market to internalize the static externality and achieve a Pareto improvement in the stage game. The proof is available upon request.

²¹Since markets are incomplete and investors are on their outside option, the dynamic externality persists even if investors were infinitely lived or in a perpetual youth model.

corresponds to a reduction in welfare due to the future distribution of entrepreneurs being adversely selected.

6 Model and Facts

The idea that recessions have a cleansing role goes back to Schumpeter (1939). Caballero and Hammour (1994), Mortensen and Pissarides (1994) and Davis and Haltiwanger (1990) provide formal analyses and early empirical evidence. At the same time, alternative hypotheses highlights that potential distortions, such as credit constraints, might hinder such productivity enhancing reallocation in recessions (e.g. Barlevy, 2003). This debate is especially relevant in relation to the Great Recession and the Eurozone crisis when financial frictions played a central role.

As a contribution to this debate our model provides a rationale for tightening of the lending standards during downturns and, consequently, for the reallocation of credit toward better quality firms. At the same time, we show that this cleansing effect can be substantially mitigated if skilled capital is scarce. Put differently, we connect the extent of cleansing role of recessions to the capital supply. We introduce the notion of “capital reallocation quality” in Section 4.1 to further relate this mechanism to the changing empirical evidence about the strength of cleansing in the recession.

In particular, recent empirical evidence documents both a general cleansing role for recessions and a counteracting stifling role in the Great Recession. Foster et al. (2016) and Osotimehin and Pappadà (2017) find that downturns prior to the Great Recession are periods of accelerated reallocation and even more productivity enhancing than reallocation in normal times. Following the Great Recession in 2008, they find that the reallocation that did occur was less productivity enhancing than in prior recessions, the stifling role of the recession.

A more nuanced prediction of our framework is that tightening of lending standards does not affect the credit supply to firms uniformly even after controlling for credit quality. In particular, deterioration of access to credit is concentrated among a certain group of firms, the more opaque ones. Note that this prediction concerns the differential change in credit access, as opposed to a difference in levels, across different groups of firms.

This prediction is consistent with the recent empirical evidence that credit supply to firms is affected by factors other than firm fundamental quality. Iyer et al. (2014) identify credit supply effects of the unexpected freeze of the European interbank market using Portuguese loan-level data. They find that the credit supply reduction is stronger for firms that are smaller and younger. They particularly suggest “opacity” as one of the explanations for the

disproportional reduction in supply of credit for smaller and younger firms.²²

Alternatively, Almeida et al. (2017) use sovereign credit ceilings to identify an asymmetric change in financial outcomes for corporations following a sovereign downgrade. They show that after a sovereign downgrade firms that are sovereign bound, i.e. those who have higher credit rating than their country before the downgrade, are significantly more likely to be downgraded compared to nonbound firms. In particular, cost of debt financing rises asymmetrically for the sovereign bound firms although they have higher credit quality than nonbound firms. Considering a sovereign downgrade as a downturn accompanied with a credit crunch, the bound firms closely resemble the good opaque firms in our model. When investors become cautious and credit conditions tighten, they face an asymmetric increase in the rate that they face despite being of high credit quality.

This mechanism also underlies the novel prediction of the model suggesting that the strength of cleansing by tight credit in recessions depends on the capitalization of skilled investors.

A third implication of the model is the deterioration of credit quality during booms. This prediction, shared with the literature on extrapolative expectations, relates tightly to the empirical evidence documenting that the sub-prime mortgages originated in 2005-2007, the later years of the boom leading to the Great Recession, performed significantly worse than those originated earlier, in 2003-2004 (Gerardi et al., 2008; Mayer et al., 2009; Palmer, 2015). Demyanyk and Van Hemert (2009) find that the bulk of the deterioration in vintage quality was due to unobservables, suggesting an increasing adverse selection among borrowers as the boom continued.

Our mechanism gives rise to a number of further predictions about financial outcomes which we divide into loan side and investor side predictions. These testable predictions can help identify the boom-bust episodes when our mechanism is likely to have played a first-order role.

On the loan side, our model allows us to compare the dispersion in ex-post performance versus ex-ante interest rate for loans issued in booms and recessions. We predict that loans issued in booms have a higher ex-post default rate and a higher dispersion in ex-post performance despite having a lower dispersion in ex-ante interest rates, compared to the ones issued in the bust.

Furthermore, the model implies that the share of opaque issuance in a given credit rating category inversely predicts the realized return on the opaque loans in subsequent periods.

²²A similar pattern of concentration of exit among the younger firms have been documented for general recessions as well. Figure 1 in Ouyang (2009) plots the exit rate of firms during the period of 1972 – 1989 and illustrates that the increase in exit rate during the recessions is very much skewed towards the younger firms.

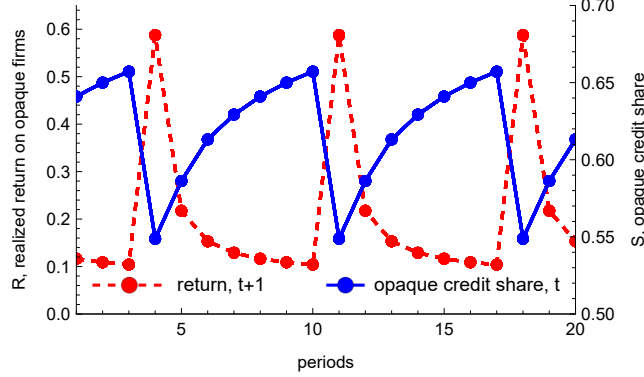


Figure 4: Model generated negative correlation between opaque credit share and its future realized excess return. The solid blue line is the share of issued credit to opaque entrepreneurs relative to all credit in a given period, on the right scale. The dashed red line depicts the realized excess return on opaque credit, one period later, on the left scale.

Figure 4 plots the opaque credit share at time t (solid blue line) along with the one period ahead return (dashed red line) for the two-phase economy simulated in Figure 2.²³ Heightened levels of the solid curve correspond to overheated periods with low subsequent returns. Low levels instead correspond to recessions and tight lending standards accompanied with high subsequent returns.²⁴

On the investor side, we predict that investor's portfolio concentration is countercyclical as investors with different skills focus on specific non-overlapping subsets of assets in downturns. Moreover, investors who perform better derive their excess return from different sources in booms versus downturns. Namely, in booms the main source of over-performance for the highly skilled investors is a portfolio with a low default rate, while in downturns it is investing in higher-yield assets.

²³Formally, let $S(\mu_0, \mu_1)$ denote the share of credit to opaque firms, and $ER(\mu_0, \mu_1)$ denote the net excess realized return on a portfolio of these loans. We have:

$$S(\mu_0, \mu_1) \equiv \frac{\mu_0 \ell(b, 0) + \frac{1-\mu_0-\mu_1}{2} \ell(g, 0)}{\mu_0 \ell(b, 0) + \frac{1-\mu_0-\mu_1}{2} (\ell(g, 0) + \ell(g, 1))}, \quad ER(\mu_0, \mu_1) \equiv \frac{\frac{1-\mu_0-\mu_1}{2} \ell(g, 0) (1 + r(g, 0))}{\mu_0 \ell(b, 0) + \frac{1-\mu_0-\mu_1}{2} \ell(g, 0)} - (1 + r_f).$$

²⁴One might compare this prediction to the growing body of evidence that periods of overheating in credit markets forecast low excess bond returns. Greenwood and Hanson (2013) show that the share of junk bond issuance out of total issuance inversely predicts the excess return on these bonds. While our prediction has a similar flavor, junk bonds are identified by their credit rating, while our prediction refers to the opaque group, say, the young and small firms, within a given credit rating category.

7 Conclusion

The idea that economic fluctuations can be captured by models with endogenous cycles is not new. In fact, the earliest business cycle models by John Hicks and Nicolas Kaldor followed this approach. However, as Boldrin and Woodford (1990) explain, these models fell out of favor by the late 1950's because actual business cycles were found not to show regular cycling behavior.²⁵

In this paper, we argue that despite real world cycles being complex and difficult to forecast, simple models with endogenous cycles are a useful apparatus for macroeconomic theory as indispensable analytical tools to understand the underlying mechanisms that turns booms into recessions and vice-versa.

In this paper, we focus on the credit supply channel. We propose a model where endogenous cycles are generated by the interaction between lenders' choice of lending standards in the credit market and the economic fundamentals. In a downturn, tight credit standards stifle concurrent good investment but also screen out low quality entrepreneurs and thus improve the future quality of credit applications. Once the improvement is sufficiently significant, it triggers a switch to lax lending standards. This in turn leads to the deterioration of fundamentals, which prompts tight credit conditions and a downturn again.

We also characterize two externalities in this framework functioning in opposite directions. Presence of these externalities suggest that one can use this framework to explore the impact of different policy instruments, macro-prudential policy as well as prudential monetary policy introduced by Caballero and Simsek (2019), to control the cyclicity of the economy. We leave a detailed exploration of policy instruments for future research.

References

- Almeida, Heitor, Igor Cunha, Miguel A Ferreira, and Felipe Restrepo, "The real effects of credit ratings: The sovereign ceiling channel," *The Journal of Finance*, 2017, 72 (1), 249–290.
- Asriyan, Vladimir and Victoria Vanasco, "Informed Intermediation over the Cycle," 2014. CREI.
- , Luc Laeven, and Alberto Martin, "Collateral Booms and Information Depletion," *The Review of Economic Studies*, 09 2021.

²⁵See the recent work of Beaudry et al. (2020) for the argument that modern statistical techniques might refute this statement.

- Azariadis, Costas and Bruce Smith**, “Financial intermediation and regime switching in business cycles,” *American economic review*, 1998, pp. 516–536.
- Barlevy, Gadi**, “Credit market frictions and the allocation of resources over the business cycle,” *Journal of monetary Economics*, 2003, *50* (8), 1795–1818.
- Beaudry, Paul, Dana Galizia, and Franck Portier**, “Putting the cycle back into business cycle analysis,” *American Economic Review*, 2020, *110* (1), 1–47.
- Boldrin, Michele and Michael Woodford**, “Equilibrium models displaying endogenous fluctuations and chaos: a survey,” *Journal of Monetary Economics*, 1990, *25* (2), 189–222.
- Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer**, “Diagnostic expectations and credit cycles,” *The Journal of Finance*, 2018, *73* (1), 199–227.
- Burdett, Kenneth and Melvyn G Coles**, “Separation cycles,” *Journal of Economic Dynamics and Control*, 1998, *22* (7), 1069–1090.
- Caballero, Ricardo J and Alp Simsek**, “Prudential monetary policy,” Technical Report, National Bureau of Economic Research 2019.
- **and Mohamad L Hammour**, “The cleansing effect of recessions,” *American Economic Review*, 1994, *84*, 1350–1368.
- **and —**, “On the timing and efficiency of creative destruction,” *The Quarterly Journal of Economics*, 1996, *111* (3), 805–852.
- **, Takeo Hoshi, and Anil K Kashyap**, “Zombie lending and depressed restructuring in Japan,” *American economic review*, 2008, *98* (5), 1943–77.
- Davis, Steven J and John Haltiwanger**, “Gross job creation and destruction: Microeconomic evidence and macroeconomic implications,” *NBER macroeconomics annual*, 1990, *5*, 123–168.
- Demyanyk, Yuliya and Otto Van Hemert**, “Understanding the subprime mortgage crisis,” *The Review of Financial Studies*, 2009, *24* (6), 1848–1880.
- Dewatripont, Mathias and Eric Maskin**, “Credit and efficiency in centralized and decentralized economies,” *The Review of Economic Studies*, 1995, *62* (4), 541–555.
- Farboodi, Maryam and Péter Kondor**, “Heterogeneous global booms and busts,” *American Economic Review*, 2022, *112* (7), 2178–2212.

- Fishman, Michael J., Johanthan A. Parker, and Ludwig Straub**, “A Dynamic Theory of Lending Standards,” 2019. MIT.
- Foster, Lucia, Cheryl Grim, and John Haltiwanger**, “Reallocation in the great recession: cleansing or not?,” *Journal of Labor Economics*, 2016, *34* (S1), S293–S331.
- Gennaioli, Nicola and Andrei Shleifer**, *A Crisis of Beliefs*, Princeton University Press, 2020.
- Gerardi, Kristopher, Andreas Lehnert, Shane M Sherlund, and Paul Willen**, “Making sense of the subprime crisis,” *Brookings Papers on Economic Activity*, 2008, pp. 69–145.
- Gorton, Gary and Guillermo Ordonez**, “Collateral crises,” *The American Economic Review*, 2014, *104* (2), 343–378.
- and —, “Good booms, bad booms,” *Journal of the European Economic Association*, 2016.
- Greenwood, Robin and Samuel G Hanson**, “Issuer quality and corporate bond returns,” *The Review of Financial Studies*, 2013, *26* (6), 1483–1525.
- Greenwood, Robin Marc, Samuel Gregory Hanson, and Lawrence J Jin**, “Reflexivity in Credit Markets,” 2019. Harvard University.
- Gu, Chao, Fabrizio Mattesini, Cyril Monnet, and Randall Wright**, “Endogenous credit cycles,” *Journal of Political Economy*, 2013, *121* (5), 940–965.
- Haltiwanger, John C, Henry R Hyatt, Erika McEntarfer, and Matthew Staiger**, “Cyclical Worker Flows: Cleansing vs. Sullyng,” Technical Report, National Bureau of Economic Research 2021.
- Hu, Yunzhi**, “A Dynamic Theory of Bank Lending, Firm Entry, and Investment Fluctuations,” *Firm Entry, and Investment Fluctuations (August 11, 2017)*, 2017.
- Iyer, Rajkamal, José-Luis Peydró, Samuel da Rocha-Lopes, and Antoinette Schoar**, “Interbank liquidity crunch and the firm credit crunch: Evidence from the 2007–2009 crisis,” *The Review of Financial Studies*, 2014, *27* (1), 347–372.
- Kurlat, Pablo**, “Asset markets with heterogeneous information,” *Econometrica*, 2016, *84* (1), 33–85.

- Leibovici, Fernando, David Wiczer et al.**, “Firm-level credit ratings and default in the Great Recession: theory and evidence,” 2019. Stony Brook University.
- Martin, Alberto**, “Endogenous credit cycles,” *Available at SSRN 1001861*, 2005.
- Matsuyama, Kiminori**, “Credit traps and credit cycles,” *American Economic Review*, 2007, *97* (1), 503–516.
- Mayer, Christopher, Karen Pence, and Shane M Sherlund**, “The rise in mortgage defaults,” *Journal of Economic perspectives*, 2009, *23* (1), 27–50.
- Mortensen, Dale T and Christopher A Pissarides**, “Job creation and job destruction in the theory of unemployment,” *The review of economic studies*, 1994, *61* (3), 397–415.
- Myerson, Roger B**, “A model of moral-hazard credit cycles,” *Journal of Political Economy*, 2012, *120* (5), 847–878.
- Osotimehin, Sophie and Francesco Pappadà**, “Credit frictions and the cleansing effect of recessions,” *The Economic Journal*, 2017, *127* (602), 1153–1187.
- Ouyang, Min**, “The scarring effect of recessions,” *Journal of Monetary Economics*, 2009, *56* (2), 184–199.
- Palmer, Christopher**, “Why did so many subprime borrowers default during the crisis: Loose credit or plummeting prices?,” 2015. MIT.
- Schumpeter, Joseph Alois**, *Business cycles*, Vol. 1, McGraw-Hill New York, 1939.
- Shimer, Robert and Lones Smith**, “Matching, search, and heterogeneity,” *The BE Journal of Macroeconomics*, 2001, *1* (1).

Appendix

A Agent Optimization Problem and Market Clearing Protocol

In this Appendix we formally define the problem of each agent, the market clearing protocol, and a robustness criterion. We also show how the agents' problem reduce to the ones set up in the main text. The structure of our credit market is a modified version of Kurlat (2016). The entrepreneur and investor problems are simplified versions of those in Farboodi and Kondor (2022).

A.1 Stage Game: Entrepreneur and Investor Problem

Let R denote the a set of trading posts, each of which identified by an interest rate r . The problem for an entrepreneur (τ, ω) is

$$\max_{\{\sigma(r; \tau, \omega)\}_{r \in R}} \rho i(\tau, \omega) - \mathbf{1}_{\tau=g} \ell(\tau, \omega) (1 + r(\tau, \omega)) \quad (\text{A.1})$$

s.t.

$$0 \leq \sigma(r; \tau, \omega) \leq \frac{1}{r} \quad \forall r \in R$$

$$\ell(\tau, \omega) = \int_R \sigma(r; \tau, \omega) d\eta(r; \tau, \omega) \quad (\text{A.2})$$

$$r(\tau, \omega) = \frac{\int_R r \sigma(r; \tau, \omega) d\eta(r; \tau, \omega)}{\ell(\tau, \omega)} \quad (\text{A.3})$$

$$\ell(\tau, \omega) \leq \frac{1}{r(\tau, \omega)} \quad (\text{A.4})$$

$$i(\tau, \omega) = \ell(\tau, \omega) + 1.$$

$\sigma(r; \tau, \omega)$ denotes the number of credit units entrepreneur (τ, ω) demands at interest rate r . $\ell(\tau, \omega)$ and $i(\tau, \omega)$ denote the total amount of credit and the investment level for entrepreneur (τ, ω) , respectively.

η is the *rationing function* that assigns $\eta(R_0; \tau, \omega)$ measure of credit, per unit of application, to entrepreneur (τ, ω) who has submitted applications to the subset of trading posts $R_0 \in R$. η is an equilibrium object, determined by the choices of the agents and the market clearing protocol as explained below. The entrepreneur takes η as given.

Let $\bar{\ell}$ denote the maximum available credit for a given entrepreneur,

$$\bar{\ell}(\tau, \omega) \equiv \int_R \frac{1}{r} d\eta(r; \tau, \omega).$$

We are interested in showing that an equilibrium exists. As such, we conjecture and then verify that there exist an equilibrium in which each entrepreneur only raises credit at one single interest rate. From equations (A.3) and (A.2), $r(\tau, \omega)$ denotes the *average* interest

rate that the entrepreneur raises credit at. Under the conjecture that he raises credit at a single interest rate, with some abuse of notation let $r(\tau, \omega)$ denote that unique interest rate. In particular, $r(\tau, \omega)$ does not depend on $\sigma(\cdot)$.

Under this conjecture, the entrepreneur's problem can be rewritten as

$$\begin{aligned} \max_{\ell(\tau, \omega), r(\tau, \omega)} & \rho + \ell(\tau, \omega)(\rho - \mathbf{1}_{\tau=g}(1 + r(\tau, \omega))) \\ \text{s.t.} \quad & \ell(\tau, \omega) \leq \min \left(\bar{\ell}(\tau, \omega), \frac{1}{r(\tau, \omega)} \right). \end{aligned} \quad (\text{A.5})$$

This form suppresses the choice over credit applications, $\sigma(\cdot)$, and focuses on the total obtained credit $\ell(\cdot)$. For any obtained credit $\ell(\tau, \omega)$ along with equilibrium $\eta(r; \tau, \omega)$ schedule, equation (A.2) determines $\sigma(r; \tau, \omega)$.

Each investor h advertises a single rate $r(h)$. Unskilled investor h solves

$$\begin{aligned} \max_{\chi(h), \tilde{r}(h)} & (1 + \tilde{r}(h)) (S_u(r; g, 1) + \mathbf{1}_{\chi(h)=B} S_u(r; g, 0)) \\ & + (1 + r_f) (S_u(r; b, 1) + \mathbf{1}_{\chi(h)=C} (S_u(r; b, 0) + S_u(r; g, 0))), \end{aligned}$$

while skilled investor h solves

$$\max_{\tilde{r}(h)} (1 + \tilde{r}(h)) (S_s(r; g, 1) + S_s(r; g, 0)).$$

$\chi(h)$ is the unskilled agent's choice of test. S_u and S_s are the sampling functions for unskilled and skilled investors.

An unskilled investors has one unit of wealth, thus she samples total one unit of applications at the interest rate she advertises. $S_u(r; \tau, \omega)$ denotes the measure of applications submitted by (τ, ω) entrepreneurs that the unskilled investor who has advertised interest rate r receives. Importantly, this measure is independent of unskilled investor's choice of test. $S_s(r; \tau, \omega)$ is the analogous object for skilled investors. The sampling functions are aggregate equilibrium objects determined by the market clearing protocol and the choices of agents, and are taken as given by investors.

Market Clearing Protocol. Let r' denote the lowest interest rate which is both advertised by some investor and some entrepreneurs have submitted demand at this rate. If there is no such interest rate, then no applications are financed.

First, each entrepreneur who submits an application at that rate posts r' *down-payment* per unit of application from her endowment. Applications without a down-payment are automatically discarded. Then, each unskilled investor who has advertised rate r' obtains a sample of the (non-discarded) applications submitted at that rate with the underlying distribution. As such, $S_u(r'; \tau, \omega)$ is equal to the fraction of non-discarded (τ, ω) application submitted at interest rate r' .

If there are not enough applications to fill up every unskilled present investor's capacity limit, then all applications have been sampled and the sampling process stops. Otherwise, all unskilled investors sample a measure (of value) one of applications and provide financing

to all applications in their sample that passes their chosen test. Their remaining endowment is invested in the risk-free asset.

If all unskilled investors reach their sampling capacity and there are remaining applications from good entrepreneurs, then they are distributed pro rata across skilled investors up to their capacity given by their one unit of endowment. As such, $S_s(r'; g, \omega)$ is the ratio of remaining non-discarded (g, ω) applications at interest rate r' relative to sum of remaining non-discarded $(g, \omega) + (g, \omega')$ applications after unskilled investors make their financing decision at rate r' . Skilled investors grant credit to these entrepreneurs.

Entrepreneurs who receive financing invest the credit they obtain along with the down-payment, and the invested units are posted as collateral for the loan. These invested units enter into a public registry, so they cannot serve as collateral to other loan applications. Applications that are submitted but do not receive financing are discarded, and the down-payment is returned to the entrepreneur who can only invest it in the risk-free asset.

Then, the process is repeated at the next lowest advertised interest rate at which there are applications. The process stops once there is no such rate any more. $\eta(r; \tau, \omega)$ is computed by aggregating over all investors who grant credit to entrepreneur (τ, ω) at interest rate r .

A.2 Dynamic Economy: Entrepreneur and Investor Problem

Since each investor lives for a single period, she solves the identical utility maximization problem in the stage game and the full game.

For entrepreneurs the only change is that they maximize the expected sum of their future utility while alive. This consists of entrepreneur's period utility, as well as his expected continuation value. That is, instead of (A.5), the value function of the entrepreneur can be written as

$$\begin{aligned}
 V(\tau, \omega; \mu_{0,t}, \mu_{1,t}) = & \\
 \max_{\ell_t(\tau, \omega), r_t(\tau, \omega)} & \rho + \ell_t(\tau, \omega) \left(\rho - \mathbf{1}_{\tau=g}(1 + r_t(\tau, \omega)) \right) + (1 - \delta) \mathbf{1}_{\ell_t(\tau, \omega) > 0} V(\tau, \omega; \mu_{0,t+1}, \mu_{1,t+1}) \quad (\text{A.6}) \\
 \text{s.t.} \quad & \ell_t(\tau, \omega) \leq \min \left(\bar{\ell}_t(\tau, \omega), \frac{1}{r_t(\tau, \omega)} \right),
 \end{aligned}$$

where the entrepreneur takes the equilibrium dynamics of $(\mu_{0,t}, \mu_{1,t})$ as given.

B Proofs

Proof of Lemma 1

As we explained in the text, in any equilibrium unskilled investors have to lend at the break-even interest rate which makes them indifferent whether to participate or not. Also, they never extend credit to entrepreneurs not passing their test. This is so, because tests are informative. Therefore extending credit to those entrepreneurs who do not pass an investors' test increases her break-even interest rate. Therefore, if there were such a group of investors in equilibrium, non-participating investors would deviate by entering at a slightly lower

interest rate, extending credit only to those who pass their test, and stealing the business of the first group.

Proof of Lemma 2

The market clearing mechanism and Assumption 2 implies that in the stage game if any agent would like to raise credit at an interest rate $r^{\max}$, she would want to submit a maximum measure of applications, $\sigma(r; \tau, \omega) = \frac{1}{r}$ at every interest rate smaller than $r^{\max}$ too. This makes it possible to receive a fraction of their credit at a lower rate (as markets clear from the lowest interest rate), and potentially even without the requirement to invest the received amount (Assumption 2). This latter possibility is attractive for bad entrepreneurs. Because applications with no down-payment are discarded, there is no possibility of having more credit granted as intended. Agents also want to submit the maximum measure of applications at $r^{\max}$. Given the linear structure, if, at a given interest rate an agent would like borrow to invest, she also would like to borrow up to the limit $\frac{1}{r}$ and invest at that rate. This concludes the first part of the Lemma.

For the second part, observe that the objective function (A.1) implies that a good entrepreneur does not apply for credit at any interest rate $r(g, \omega) > \rho - 1$ as that would imply negative return on her investment. As we noted before, Assumption 2 and objective (A.1) imply that bad entrepreneurs instead apply for maximum credit at any interest rate as they do not plan to pay back.

Proof of Lemma 3 and Proposition 1

Consider the interest rate functions (3), (4) and

$$r_M(\mu_0, \mu_1, c, r_f) \equiv \frac{2\mu_0 + (1 + \mu_0 - \mu_1)r_f + (1 - \mu_1 - \mu_0)c}{1 - \mu_1 - \mu_0}. \quad (\text{A.7})$$

In what follows, we show that these functions are indeed break-even interest rates under the stated conditions and that they support the equilibrium in Proposition 1.

Bold stage We conjecture and verify that under the conditions of statement (i) of Proposition 1, all unskilled investors run a bold test, all investors advertise rate r_B , all entrepreneurs submit maximum demand at that rate and all good entrepreneurs choose $r^{\max} = r_B$. Note that r_B must satisfy the indifference condition (5), which in turn implies (3). The entering measure of unskilled investors is determined by the market clearing condition spelled out in the proof of Proposition 2. This condition ensures that the fraction of bold unskilled investors' capital that finances good entrepreneurs, together with the capital of skilled investors (who only finance projects of good entrepreneurs), satisfies the credit demand of all good entrepreneurs regardless of their opacity.

In order to prove that this is an equilibrium, we show that none of the agents have a profitable deviation. The left hand side of (5) is the expected utility of running the bold test on a representative sample of applications, accepting all good and all opaque applications out

of which only the good ones pay back (the first term), and investing the capital corresponding to rejected share of applications in the risk-free asset (second term) minus the cost of the test (last term). The right hand side is the return of not entering the market and investing in risk-free asset only. That is, bold unskilled break even at r_B . (Note that (5) is using the assumption that unskilled investors sample first.) Therefore, unskilled cannot offer a lower rate and participate.

If the unskilled investor was to run a cautious test, she would break even at a rate r_C given by the condition (6), which implies (4). As long as $r_B \leq r_C < \bar{r}$, switching to a cautious test leads to a loss. This condition is equivalent to $\mu_0 \in [0, \frac{c}{1+r_f}]$. As such, unskilled investors do not have a profitable deviation to a different rate or a different test in this region.

Finally, skilled investors could offer a lower rate profitably but they lend out all their capital in equilibrium, so there is no advantage of offering a lower rate. Offering a higher rate for any investor would lead to a zero measure of applications from good entrepreneurs. Good entrepreneurs raise all the capital they need, hence they would not deviate to a higher r^{max} . None of the entrepreneurs would raise any capital at a lower r^{max} given strategies.

Cautious stage We conjecture and verify that under the conditions of statement (ii) of Proposition 1, all unskilled investors run a cautious test and advertise the rate r_C , all skilled advertise $\bar{r}$, transparent good entrepreneurs choose $r^{max} = r_C$, while opaque good entrepreneurs choose $\bar{r}$. Bad entrepreneurs choose r^{max} weakly higher than $\bar{r}$. r_C must satisfies the indifference condition (6). Analogous to the bold stage, the entering measure of unskilled investors is determined by the market clearing condition ensuring that all transparent good entrepreneurs can obtain the capital they demand at r_C .

Cautious unskilled investors would make losses at a smaller rate, and would not pass any applications submitted at a higher rate. However, they could consider running a bold test, advertising a higher rate, and finance the opaque good entrepreneurs (along some bad ones). Under this strategy, they would break even at the rate r_M determined by the indifference condition

$$\frac{\frac{(1-\mu_1-\mu_0)}{2}}{\frac{(1-\mu_1-\mu_0)}{2} + (\mu_1 + \mu_0)} (1 + r_M) + \frac{\frac{\mu_1}{2}}{\frac{(1-\mu_1-\mu_0)}{2} + (\mu_1 + \mu_0)} (1 + r_f) - c = (1 + r_f). \quad (\text{A.8})$$

which is equivalent to (A.7). This deviation is not possible if $\bar{r} < r_M$ whenever $r_C < r_B$. This is equivalent to the condition of Assumption 3. As such, unskilled investors do not have a profitable deviation to a different rate or a different test.

Skilled investors do not deviate either as they are offering the highest rates any good entrepreneur is willing to pay and lend out all their capital. Transparent good entrepreneurs raise the capital they demand at the lowest rate available. Opaque good entrepreneurs are just indifferent to raise capital at $\bar{r}$ by definition. If they pick a lower r^{max} they could not raise any capital, thus they do not deviate from their current strategy.

Proof of Proposition 2

We described in the main text how entrepreneurs' decide on investment i and borrowing ℓ taking the interest rate $r(\tau, \omega)$ and the borrowing limit $\bar{\ell}(\tau, \omega)$ as given. Then, expressions

in Proposition 2 follow from the determination of $r(\tau, \omega)$ in Proposition 1 and the borrowing limits $\bar{\ell}(\tau, \omega)$ which we derive here. We also derive here $k(\mu_0, \mu_1)$, the equilibrium fraction of unskilled investors who decide to not to enter the credit market in a given state. Consider the bold stage first. The market clearing condition for credit to transparent good and opaque entrepreneurs is

$$w_1 + (1 - k_B) w_0 (1 - \mu_0 - \mu_1) = (1 - \mu_0 - \mu_1) \frac{1}{r_B}$$

where $k(\mu_0, \mu_1) = k_B$ in a bold stage. Then, $\bar{\ell}(b, 0)$ is determined by the endowment of unskilled investors which is allocated to bad, opaque credit by the false positives of the bold test:

$$\mu_0 \bar{\ell}(b, 0) = (1 - k_B) w_0 \mu_0$$

implying

$$\bar{\ell}(b, 0) = \frac{1}{r_B} - \frac{w_1}{(1 - \mu_0 - \mu_1)} \quad (\text{A.9})$$

and

$$i(b, 0) = \bar{\ell}(b, 0) (1 + r_B) = \frac{(1 + r_B)}{r_B} - \frac{(1 + r_B) w_1}{(1 - \mu_0 - \mu_1)}.$$

Assumption 1 requires $\frac{w_1}{(1 - \mu_0 - \mu_1)} < \frac{1}{r_B}$, thus the bad entrepreneurs are constrained in a bold stage.

In the cautious stage market clearing for opaque good firms gives

$$\frac{(1 - \mu_0 - \mu_1)}{2} \bar{\ell}(g, 0) = w_1$$

implying

$$\bar{\ell}(g, 0) = \frac{2w_1}{(1 - \mu_0 - \mu_1)} \quad (\text{A.10})$$

and investment

$$i(g, 0) = 1 + \frac{2w_1}{(1 - \mu_0 - \mu_1)}.$$

Assumption 1 requires $\frac{w_1}{(1 - \mu_0 - \mu_1)} < \frac{1}{2\bar{r}}$ implying that opaque good entrepreneurs are indeed constrained in this stage. The fraction of entering unskilled investors in a cautious stage, $(1 - k_C)$, is determined by the market clearing condition for the low interest rate market,

$$\frac{(1 - \mu_0 - \mu_1)}{2} \frac{1}{r_C} = (1 - k_C) w_0 \frac{(1 - \mu_0 - \mu_1)}{2}.$$

Also, w_0 has to be sufficiently large that $k_B, k_C \in [0, 1]$. We can summarize the requirements on w_1 for later use as:

$$\frac{w_1}{(1 - \mu_0 - \mu_1)} < \min \left(\frac{1}{2\bar{r}}, \frac{1}{r_B} \right) = \frac{1}{2\bar{r}}. \quad (\text{A.11})$$

For part (iv), recall that $Y(\mu_0, \mu_1)$ is the population weighted sum of the outputs $\rho(1 + \ell(\tau, \omega))$ for each group of entrepreneurs (τ, ω) . The statement follows from the observation that that $\ell(g, 1) = \frac{1}{r(\tau, \omega)}$ is continuous, while $\ell(g, 0)$ and $\ell(b, 0)$ discontinuously decreases in μ_0 as it crosses the threshold from below between a bold and a cautious stage as implied by (A.9)-(A.10) and (A.11).

Proof of Lemma 4

Comparing (A.5) and (A.6) and using the equilibrium definitions, it is sufficient to show that maximizing life-time utility leads to the same outcome as maximizing stage game utility within each period. That is, introducing endogenous continuation does not change equilibrium strategy profiles.

First, consider a sequence of stage game equilibria consistent with the law of motion for state variables. We will show that in every period, there is no individual deviation from the optimal strategy in the stage game equilibrium which would increase the life-time utility of an entrepreneur who lives more than one period. That is, any sequence of stage game equilibria consistent with the equilibrium law of motion of the state variables (μ_0, μ_1) is a dynamic equilibrium. Proposition 1 implies that in any stage game equilibrium all good entrepreneurs obtain positive credit. That is, they hit the upper limit of their probability of survival, $1 - \delta$. As such, they cannot increase the interest rate that they accept, compared to the stage game $\bar{r}$, in order to improve their survival probability. On the other hand, more credit always increases bad entrepreneurs' stage game utility. Furthermore, as long as they are able to raise credit they are indifferent about the corresponding interest rate. Hence, they have no incentive to reduce their reservation interest rate below $\bar{r}$. For them there is no trade-off between stage game utility and increasing the chance of survival by obtaining more credit.

Second, we show that there is no dynamic equilibrium where the economy is not in a stage game equilibrium in each period. By contradiction, assume that such dynamic equilibrium exist. This implies that there is at least one period in which some good entrepreneur obtains credit at rate $r > \bar{r}$. First note that any good entrepreneur can obtain some credit if he demands a positive amount at an interest rate which a skilled investor advertises. Furthermore, by assumption, any amount of credit is sufficient for an entrepreneur to survive, i.e. maximizes the survival probability at $1 - \delta$. Thus, a necessary condition for such an equilibrium is that all skilled investors advertise an interest rate which is larger than $\bar{r}$.

In such an equilibrium, a good entrepreneur might be willing to borrow at interest rate above $\bar{r}$, lose in the short-term but in return survive with positive probability. Let $r' \equiv \bar{r} + \Delta$ denote the lowest advertised rate by any skilled investor. Note that since continuation value of an entrepreneur is finite, Δ cannot be arbitrarily large. Furthermore, all good entrepreneurs financed at $r > \bar{r}$ would submit only a diminishingly small demand at r' because that leads to minimal current loss and guarantees maximum survival probability. They submit 0 at every higher interest rate. Moreover, assumption 2 implies that they demand maximum credit at all rates equal or lower than $\bar{r}$, where they make positive current profit and guarantees maximum survival probability. The first consequence is that all skilled investors must advertise the same rate r' as by advertising a higher rate would not lend anything. Second, each skilled investors can only lend out a diminishingly small fraction

of her endowment and thus obtains a diminishingly small return on her capital. Hence, a skilled investor can deviate to $r \leq \bar{r}$ and lend a positive measure of her endowment, which is a contradiction. Thus, such an equilibrium does not exist.

Finally, since there is no profitable deviation from the strategies described by the stage game equilibrium, and investors' optimal strategies in the stage game depend only on the concurrent state variables (μ_0, μ_1) (determining the phases of Proposition A.6), there is no dynamic equilibrium where investors would condition on past values of (μ_0, μ_1) . That verifies our conjecture that the contemporaneous values of (μ_0, μ_1) are the only state variable of the economy.

Proof of Lemma 5

See appendix E.1 for the proof.

Proof of Propositions 3

The proposition directly follows from birth-death process for entrepreneurs, the equilibrium information choice and lending choice of investors.

Proof of Proposition 4

See appendix E for the proof.

Proof of Proposition 5

See appendix E for the proof.

Proof of Lemma 6

By definition and substituting for $\ell(\cdot)$ from Proposition 2, we have

$$\frac{\partial CAQ(\mu_0, \mu_1)}{\partial w_1} \Big|_{\mu_0 < \frac{c}{1+r_f}} = \frac{\mu_0}{(1 - \mu_0 - \mu_1)(\mu_0 + \mu_1)} > 0$$

and

$$\frac{\partial CAQ(\mu_0, \mu_1)}{\partial w_1} \Big|_{\mu_0 < \frac{c}{1+r_f}} = \frac{1}{(1 - \mu_0 - \mu_1)} > 0.$$

Also,

$$\begin{aligned} & CAQ(\mu_0, \mu_1) \Big|_{\mu_0 > \frac{c}{1+r_f}} - CAQ(\mu_0, \mu_1) \Big|_{\mu_0 < \frac{c}{1+r_f}} = \\ & -\mu_1 \frac{1 - \mu_0 - \mu_1}{(\mu_0 + \mu_1)(c + \mu_0 + r_f(1 - \mu_1))} + \frac{1}{2 \left(r_f + \frac{2c}{1 - \mu_0 - \mu_1} \right)} + \mu_1 \frac{w_1}{(1 - \mu_0 - \mu_1)(\mu_0 + \mu_1)}. \end{aligned}$$

Hence, the statements hold with

$$\bar{w}_1 = \min \left(0, (1 - \mu_0 - \mu_1) \left(\frac{1 - \mu_0 - \mu_1}{c + \mu_0 + r_f(1 - \mu_1)} - \frac{\mu_0 + \mu_1}{\mu_1 2 \left(r_f + \frac{2c}{1 - \mu_0 - \mu_1} \right)} \right) \right).$$

Proof of Lemma 7

The lemma follows from the following three Lemmas.

Lemma B.1 *Within the bold phase, welfare is decreasing in μ_0 .*

Proof. Welfare in the bold phase is

$$\begin{aligned} W_B = & (1 - \mu_0 - \mu_1) (\rho - 1) \left(1 + \frac{1}{r_B} \right) + \mu_0 \rho \left(1 + \frac{1}{r_B} - \frac{w_1}{(1 - \mu_0 - \mu_1)} \right) + \mu_1 \rho \\ & + w_0(1 + r_f) + w_1(1 + r_B) \end{aligned}$$

which we rewrite as

$$\begin{aligned} W_B = & \rho + w_0(1 + r_f) + w_1 \rho \\ & + (\rho(1 - \mu_1) - (1 + r_B)(1 - \mu_0 - \mu_1)) \left(\frac{1}{r_B} - \frac{w_1}{(1 - \mu_0 - \mu_1)} \right) \end{aligned}$$

Note that

$$d \left(\frac{1}{r_B} - \frac{w_1}{1 - \mu_0 - \mu_1} \right) / d\mu_0 = \left(-\frac{1}{r_B^2} \frac{dr_B}{d\mu_0} - \frac{w_1}{(1 - \mu_0 - \mu_1)^2} \right) < 0$$

also

$$(1 - \mu_1)(\rho - (1 + r_f)) - c = (1 - \mu_1)(\bar{r} - r_B) + \mu_0(1 + r_B) > 0,$$

implying the result. ■

Lemma B.2 *Within the cautious phase, welfare is decreasing in μ_0 .*

Proof. Welfare in the cautious phase is

$$\begin{aligned} W_C = & \frac{1 - \mu_0 - \mu_1}{2} \left(\rho \left(1 + \frac{1}{r_C} \right) - \frac{1}{r_C} (1 + r_C) + \rho \left(1 + \frac{2w_1}{1 - \mu_0 - \mu_1} \right) - \frac{2w_1}{1 - \mu_0 - \mu_1} \rho \right) + \mu_0 \rho + \mu_1 \rho \\ & + w_0(1 + r_f) + w_1(1 + \bar{r}) \end{aligned}$$

which we rewrite as

$$W_C = \rho + \frac{1 - \mu_0 - \mu_1}{2} \frac{(\rho - 1 - r_C)}{r_C} + w_0(1 + r_f) + w_1 \rho$$

Then

$$\frac{\partial \left(\frac{1 - \mu_0 - \mu_1}{2} \frac{(\rho - 1 - r_C)}{r_C} \right)}{\partial \mu_0} = \frac{1 - \mu_0 - \mu_1}{2} \left(-\frac{\rho - 1}{r_C^2} \right) \frac{\partial r_C}{\partial \mu_0} - \frac{1}{2} \frac{(\rho - 1 - r_C)}{r_C} < 0$$

where we used $\frac{\partial r_C}{\partial \mu_0} > 0$. This implies the Lemma. ■

Lemma B.3 Fix μ_1 and μ_0 at any level $\mu_0 \leq \frac{c}{1+r_f}$. Welfare is strictly larger in a bold stage than it would be in a – counterfactual – cautious stage, $W_B(\mu_0, \mu_1) > W_C(\mu_0, \mu_1)$, as long as $\mu_0 \leq \frac{c}{1+r_f}$.

Proof. As welfare is aggregate consumption, we can decompose $W_B(\mu_0, \mu_1) - W_C(\mu_0, \mu_1)$ as follows. The difference in transparent good entrepreneurs' consumption is

$$\frac{(1 - \mu_0 - \mu_1)}{2} (\rho - 1) \left(\frac{1}{r_B} + 1 \right) - \frac{(1 - \mu_0 - \mu_1)}{2} (\rho - 1) \left(\frac{1}{r_C} + 1 \right)$$

which is non-negative in any point when $r_B \leq r_C$, that is, in the pooling region. The difference in opaque good plus skilled consumption is

$$\left[\frac{(1 - \mu_0 - \mu_1)}{2} (\rho - 1) \left(\frac{1}{r_B} + 1 \right) + w_1 (1 + r_B) \right] - \left[\frac{(1 - \mu_0 - \mu_1)}{2} \rho + w_1 (1 + \bar{r}) \right] \quad (\text{A.12})$$

note that the term in the first squared bracket is decreasing in r_B as

$$\begin{aligned} & \frac{\partial \left(\frac{(1 - \mu_0 - \mu_1)}{2} (\rho - 1) \left(\frac{1}{r_B} + 1 \right) + w_1 (1 + r_B) \right)}{\partial r_B} = \\ & = -\frac{1}{r_B^2} \frac{(1 - \mu_0 - \mu_1)}{2} (\rho - 1) + w_1 \leq -\frac{1}{r_B^2} \frac{(1 - \mu_0 - \mu_1)}{2} (\rho - 1) + \frac{1 - \mu_0 - \mu_1}{r_B} = \\ & = \frac{(1 - \mu_0 - \mu_1)}{r_B} \left(1 - \frac{\rho - 1}{r_B} \right) < 0 \end{aligned}$$

where we used (A.11), and equals to the term in the second left bracket when $r_B = \bar{r}$. That is, (A.12) is non-negative at any point as long as $r_B \leq \bar{r}$. Unskilled consumption is equal under the two regimes, while the difference in bad consumption is equal to

$$\mu_0 \left(\frac{1}{r_B} - \frac{w_1}{1 - \mu_0 - \mu_1} \right) > 0.$$

■

Proof of Proposition 6

Let $\lambda^{\min} \equiv \frac{2c+2r_f}{3c+3r_f+1}$ and $\lambda^{\max} \equiv 2 \frac{\rho-c-r_f-1}{2\rho-c-r_f-1}$. We show that under the conditions of the proposition, there is at least one cyclical economy (the one with short-booms and short recessions) which is preferred by the planner compared to both the always bold and always cautious economies. We will argue that for this conclusion, it is sufficient to show that $\lambda \in [\lambda^{\min}, \lambda^{\max}]$ implies

$$\max(\lim_{\delta \rightarrow 0} W_C(\bar{\mu}_{0C}, \bar{\mu}_{1C}), \lim_{\delta \rightarrow 0} W_B(\bar{\mu}_{0B}, \bar{\mu}_{1B})) < \lim_{\delta \rightarrow 0} \frac{W_B(\mu_{0B}^*, \mu_{1B}^*) + W_C(\mu_{0C}^*, \mu_{1C}^*)}{2}.$$

Note that $\lim_{\delta \rightarrow 0} \bar{\mu}_{0B} = \frac{\lambda}{2-\lambda}$ and

$$\lim_{\delta \rightarrow 0} \bar{\mu}_{1B}, \bar{\mu}_{1C}, \mu_{1C}^*, \mu_{1B}^*, \bar{\mu}_{0C}, \mu_{0C}^*, \mu_{0B}^* = 0.$$

In an economy where investors are always bold or always cautious, welfare converges to $W_B(\mu_{0B}^*, \mu_{1B}^*)$ and $W_C(\bar{\mu}_{0C}, \bar{\mu}_{1C}^*)$ by definition. First, note that

$$\lim_{\delta \rightarrow 0} W_C(\bar{\mu}_{0C}, \bar{\mu}_{1C}) = W_C(0, 0) < \lim_{\delta \rightarrow 0} \frac{W_B(\mu_{0B}^*, \mu_{1B}^*) + W_C(\mu_{0C}^*, \mu_{1C}^*)}{2} = \frac{W_B(0, 0) + W_C(0, 0)}{2}.$$

This is implied by Lemma B.3. Then, we show that $\lambda \in [\lambda^{\min}, \lambda^{\max}]$ is a sufficient condition that

$$\lim_{\delta \rightarrow 0} W_C(\mu_{0C}^*, \mu_{1C}^*) > \lim_{\delta \rightarrow 0} W_B(\bar{\mu}_{0B}, \bar{\mu}_{1B}). \quad (\text{A.13})$$

or

$$W_C(0, 0) > W_B\left(\frac{\lambda}{2-\lambda}, 0\right)$$

which we can rewrite as

$$\begin{aligned} (\rho - 1 - (r_f + c)) \frac{1}{2r_f + c} &> \\ &> (\rho - 1 - (r_f + c)) \left(\frac{1}{r_B(\frac{\lambda}{2-\lambda}, 0, c, r_f)} - \frac{w_1}{(1 - \frac{\lambda}{2-\lambda})} \right). \end{aligned}$$

This holds when $\lambda \in [\lambda^{\min}, \lambda^{\max}]$, because by (3) $\lambda \in [\lambda^{\min}, \lambda^{\max}]$ is the condition for

$$\frac{1}{2r_f + c} > \frac{1}{r_B(\frac{\lambda}{2-\lambda}, 0, c, r_f)}$$

and $r_B(\frac{\lambda}{2-\lambda}, 0, c, r_f) < \bar{r}$ to hold simultaneously. As all inequalities are strict and all relevant functions are continuous from the left in (μ_0, μ_1) , for any $\lambda \in [\lambda^{\min}, \lambda^{\max}]$ we can pick a $\bar{\delta}(\lambda)$ that if $\delta < \bar{\delta}(\lambda)$ then our statement holds. Picking

$$\bar{\delta} = \max_{\lambda \in [\lambda^{\min}, \lambda^{\max}]} \bar{\delta}(\lambda)$$

defines the threshold for δ .

Proof of Proposition 7

(i) Recall that $W(\mu_0, \mu_1) = W_B(\mu_0, \mu_1)$ for $\mu_0 \leq \frac{c}{1+r_f}$ but $W(\mu_0, \mu_1)$ discontinuously drops at $\mu_0 = \frac{c}{1+r_f}$ in μ_0 by Lemma 7. However, $W_B(\mu_0, \mu_1)$ is differentiable in μ_0 around $\mu_0 = \frac{c}{1+r_f}$. This implies that $W_B(\mu_0, \mu_1) > W(\mu_0, \mu_1)$ if $\mu_0 > \frac{c}{1+r_f}$ as long as μ_0 is sufficiently close to $\frac{c}{1+r_f}$. That is, μ_0^{ext} with the given properties exists.

(ii) Consider the following trade-off:

Consider the case where $w_1 \rightarrow 0$, so by continuity, $\exists \bar{w}_1$ such that the result holds for $w_1 < \bar{w}_1$.

Let W_{xy} denote the welfare in the second period if the equilibrium in the first and second periods are x and y respectively (B or C). Also let $\mu'_{i,xy}$ denote μ_i in the second period with equilibrium y following the first period with equilibrium x . Finally, let (μ_0, μ_1) denote the state variables in the first period.

Then $\mu'_{0,By} > \mu'_{0,Cy}$ and $\mu'_{0,By} + \mu'_{1,By} > \mu'_{0,Cy} + \mu'_{1,Cy}$, but $\mu'_{1,By} < \mu'_{1,Cy}$. Now, the second period is either cautious or bold

(i) $y = C$: It is straightforward to show that $W_{BC} < W_{CC}$. in CC there are total fewer bad entrepreneurs in the second period, so more entrepreneurs get credit. Interest rate is also lower. Thus more entrepreneurs get cheaper credit too (more transparent good), and for sufficiently small w_1 the production of opaque good entrepreneurs is negligible.

(ii) $y = B$: This case is more complicated as there is a quantity-price trade-off.

(a) Quantity. More entrepreneurs get credit in the second period in a BB sequence of equilibria compared to a CB sequence (μ'_1 lower).

(b) Price. $r_{BB} > r_{CB}$, so investment per entrepreneur is lower in BB .

In order to resolve the trade-off, note that $\exists \bar{\rho}$ such that if $\rho < \bar{\rho}$, in the second period the production of the opaque bad entrepreneurs who survive the first bold period and also get credit in the second bold period is sufficiently low, so that it does not compensate the lower production of everyone else who faces a higher interest rate in the second period because of the presence of a higher measure of opaque bad entrepreneurs following a first-period B equilibrium, compared to a first-period C equilibrium. As such, the price effect in the quantity-price trade-off is stronger, which implies $W_{BB} < W_{CB}$.

C Stage Game Equilibrium: The Mix Stage

Throughout the main text, we focused on the set of parameters where Assumption 3 holds for all $(\mu_{0,t}, \mu_{1,t})$ along the equilibrium cycle. In this section, we explain how the equilibrium changes when this assumption is violated. For the purposes of this Appendix, we define

$$\tilde{\mu}_0(\mu_1) \equiv \frac{(\bar{r} - r_f)(1 - \mu_1) - 2\mu_1 c}{2 + c + r_f + \bar{r}},$$

hence, Assumption 3 is equivalent to

$$\frac{c}{1 + r_f} < \tilde{\mu}_0(\mu_1).$$

In the stage game, whenever Assumption 3 does not hold, apart from the bold stage and the cautious stage, a third type of equilibrium can also manifest. The following proposition generalizes Lemma 3 and Proposition 1 to accommodate this case.

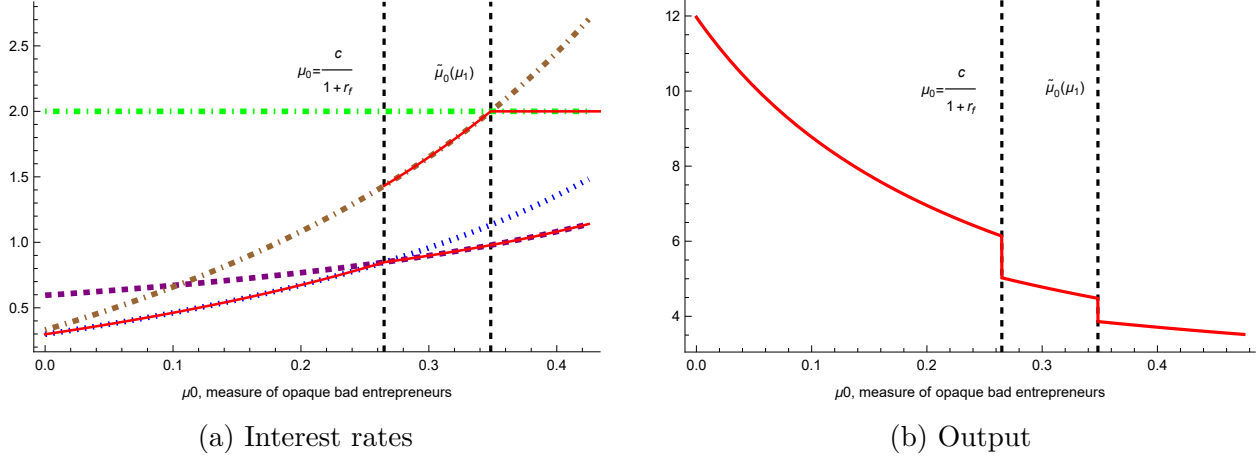


Figure C.1: Interest rates and output as a function of μ_0 , for a fixed μ_1 . All three phases of stage game equilibrium occur for some μ_0 . The left panel displays the break-even interest rates r_B (dotted blue), r_C (dashed purple), r_M (dot-dashed brown), the maximum feasible rate $\bar{r}$ (dot-dashed green, horizontal), and the equilibrium interest rates (solid curves). The right panel displays the output. In the leftmost region the stage game equilibrium is bold, in the middle range it is mix, and in the rightmost region it is cautious. The parameters are: $\rho = 3, c = 0.265, r_f = 0, w_0 = 3.99, w_1 = 0.01, \mu_1 = 0.11$.

Proposition C.1 *Let $r_M(\mu_0, \mu_1, c, r_f)$ be defined by Equation (A.7). If Assumption 3 does not hold and $\min\{r_B(\mu_0, \mu_1, c, r_f), r_C(\mu_0, \mu_1, c, r_f)\} < \bar{r}$, the following equilibria can arise.*

- (i) *If $\mu_0 \in [0, \frac{c}{1+r_f}]$, the economy is in a **bold stage**. Every unskilled investor chooses the bold test. The credit market is integrated. All good entrepreneurs and opaque bad ones obtain credit at common interest rate $r_B(\mu_0, \mu_1, c, r_f)$.*
- (ii) *If $\mu_0 \in (\tilde{\mu}_0(\mu_1), 1]$, the economy is in a **cautious stage**. Every unskilled investor chooses the cautious test. The credit market is fragmented. Transparent and opaque good entrepreneurs obtain credit at rates $r_C(\mu_0, \mu_1, c, r_f)$ and $\bar{r} > r_C$, respectively. Bad entrepreneurs do not obtain any credit.*
- (iii) *If $\mu_0 \in (\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)]$, the economy is in a **mix stage**. Some unskilled investors choose the bold test while others choose the cautious test. The credit market is fragmented. Transparent good entrepreneurs obtain credit at rate $r_C(\mu_0, \mu_1, c, r_f)$, while all opaque entrepreneurs obtain credit at rate $r_M(\mu_0, \mu_1, c, r_f) > r_C$.*

Otherwise the economy is in autarky, where unskilled investors do not lend, bad entrepreneurs do not borrow, and good ones obtain credit at interest rate $\bar{r}$ from skilled investors only.

The equilibrium in the bold and cautious stages are characterized as before. However, when $\frac{c}{1+r_f}$ is at an intermediate range, a mix stage equilibrium arises.

In a mix stage, a group of unskilled investors enter as bold and advertise rate r_M and while another group enter as cautious and advertise r_C . r_M is defined by the break-even condition of an unskilled investor who were to run a bold test when all other unskilled investors run a

cautious test. That is, r_M is determined by the indifference condition defined by Equation (A.8), which is equivalent to (A.7). Skilled advertise r_M . Transparent good entrepreneurs choose $r^{\max} = r_C$, while opaque good entrepreneurs choose r_M . Bad entrepreneurs choose an $r^{\max}$ weakly higher than $\bar{r}$. The entering measures of unskilled investors in each of these markets is determined by the market clearing conditions ensuring that all transparent good entrepreneurs can obtain the capital they demand at r_C , while all opaque good entrepreneurs can obtain the capital they demand at r_M taking into account that all skilled investors should be able to lend out all their capital. (This condition is spelled out in proof of Proposition C.2 below.)

As long as $r_M < \bar{r}$ and $r_B > r_C$, by the definition of r_M and r_C , unskilled are indifferent whether to enter as part of the bold or cautious group or to stay inactive. $r_M < \bar{r}$ implies that Assumption 3 is violated. If these conditions are met, skilled investors cannot offer a higher rate than r_M to opaque good entrepreneurs as they would be undercut by bold unskilled ones. Possible deviations of all other groups can be ruled out analogously to the other cases.

Figure C.1a depicts the prevailing interest rates as a function of μ_0 , the fraction of opaque bad entrepreneurs, for a particular level of μ_1 . For this set of parameters all three stages arise for some μ_0 . The left most region represents the bold region, the intermediate region is the mix region, and the rightmost region is the cautious region.

The next proposition characterizes output in the mix stage.

Proposition C.2

- (i) *In the mix stage, all transparent good entrepreneurs face r_C , while opaque good ones face r_M . Neither are constrained by information frictions, $\ell(g, 1) = \frac{1}{r_C}$ and $\ell(g, 0) = \frac{1}{r_M}$. Opaque bad entrepreneurs are limited by unskilled investors' mistakes at interest rate r_M , $\ell(b, 0) = \frac{1}{2r_M} - \frac{w_1}{1-\mu_0-\mu_1}$.*
- (ii) *Aggregate output, $Y(\mu_0, \mu_1)$, is decreasing in μ_0 and discontinuous at the threshold across any two stages.*

For part (i) recall that in the mix stage $\frac{1-\mu_0-\mu_1}{\mu_0+\mu_1+1}$ fraction of invested unskilled capital finances good, opaque entrepreneurs at the high interest rate market, $2\frac{\mu_0}{\mu_0+\mu_1+1}$ finances opaque bad entrepreneurs and $2\frac{\mu_1}{\mu_0+\mu_1+1}$ ends up at risk-free storage. Then market clearing for opaque good firms then is

$$\frac{(1-\mu_1-\mu_0)}{2}\ell(g, 0) = (1-k_I)w_0\frac{(1-\mu_1-\mu_0)}{1+(\mu_1+\mu_0)} + w_1$$

as opaque good entrepreneurs are not constrained, this implies

$$\frac{1}{2}\frac{1}{r_M} - \frac{w_1}{1-\mu_0-\mu_1} = (1-k_I)w_0\frac{1}{1+(\mu_1+\mu_0)}$$

Then market clearing for bad, opaque entrepreneurs gives

$$\mu_0\bar{\ell}(b, 0) = (1-k_I)w_02\frac{\mu_0}{\mu_0+\mu_1+1}.$$

Substituting back $(1 - k_I)$ implies

$$\bar{\ell}(b, 0) = \left(\frac{1}{2} \frac{1}{r_M} - \frac{w_1}{1 - \mu_0 - \mu_1} \right) \quad (\text{A.14})$$

and

$$i(b, 0) = (1 + r_M) \left(\frac{1}{2} \frac{1}{r_M} - \frac{w_1}{1 - \mu_0 - \mu_1} \right).$$

Assumption 1 requires the additional conditions that w_0 is sufficiently large to imply $k_I \in (0, 1)$ and $\frac{w_1}{(1 - \mu_0 - \mu_1)} < \frac{1}{2r_M}$.

Part (ii) follows from the discontinuity of $\ell(g, 0)$ and $\ell(b, 0)$ at the thresholds between stages, similar to Proposition 2.(iv).

In a mix stage, transparent good entrepreneurs face the same credit conditions as a cautious stage, i.e. interest rate r_C . Opaque entrepreneurs, good and bad, borrow at a higher rate r_M , thus they are able to raise less credit. The credit raised by opaque bad entrepreneurs is further limited by the mistakes made by unskilled bold investors who lend at rate r_M . Thus the output and investment is between that of the bold and cautious stages.

Figure C.1b illustrates aggregate output as a function of μ_0 , for a fixed μ_1 , for a set of parameters that all three stages arise. As Proposition C.2 states, even in the presence of the mix stage, the aggregate output is continuous and monotonically decreasing in the measure of opaque bad entrepreneurs within each stage and it discontinuously drops when an increase in the fraction of opaque bad entrepreneurs leads to a change in stage.

Finally, the next lemma shows that the discontinuity in welfare that we stated in Lemma 7 generalizes for the three-stage economy.

Lemma C.1 *Assume the stage game equilibrium is not autarky. Static welfare is decreasing in the measure of opaque bad entrepreneurs μ_0 , and discontinuously drops in μ_0 when an increase in μ_0 implies a transition change from bold to mix stage or from mix to cautious stage.*

The proof follows from augmenting the proof of Lemma 7 with the following two auxiliary lemmas.

Lemma C.2 *Within the mix phase, welfare is decreasing in μ_0 .*

Proof. Welfare in the mix phase is

$$\begin{aligned} W_M = & \frac{1 - \mu_0 - \mu_1}{2} \left(\rho \left(1 + \frac{1}{r_C} \right) - \frac{1}{r_C} (1 + r_C) + \rho \left(1 + \frac{1}{r_M} \right) - \frac{1}{r_M} (1 + r_M) \right) + \\ & \mu_0 \rho \left(1 + \left(\frac{1}{2} \frac{1}{r_M} - \frac{w_1}{1 - \mu_0 - \mu_1} \right) \right) + \mu_1 \rho \\ & + w_0 (1 + r_f) + w_1 (1 + r_I) \end{aligned}$$

which we rewrite as

$$W_M = \rho + w_1\rho + w_0(1 + r_f) + \frac{1 - \mu_0 - \mu_1}{2} \left((\rho - 1) \frac{1}{r_C} - 1 \right) + \\ (\rho(1 - \mu_1) - (1 - \mu_1 - \mu_0)(1 + r_M)) \left(\frac{1}{2r_M} - \frac{w_1}{1 - \mu_0 - \mu_1} \right)$$

Then, the statement follows from the observations that

$$\frac{1}{r_C}, \frac{1 - \mu_0 - \mu_1}{2}, (\rho(1 - \mu_1) - (1 + \mu_0 - \mu_1)(1 + r_f) - (1 + \mu_1 + \mu_0)c)$$

are decreasing in μ_0 ,

$$(\rho - 1) \frac{1}{r_C} - 1 > 0$$

$$(\rho(1 - \mu_1) - (1 - \mu_1 - \mu_0)(1 + r_M)) = (1 - \mu_1)(\rho - (1 + r_M)) + \mu_0(1 + r_M) > 0$$

as $r_C \leq \bar{r}$, and

$$\frac{1}{2r_M} > \frac{w_1}{1 - \mu_0 - \mu_1}$$

by (A.11), finally

$$\frac{\partial \left(\frac{1}{2r_M} - \frac{w_1}{1 - \mu_0 - \mu_1} \right)}{\partial \mu_0} < 0$$

as

$$\frac{\partial r_M}{\partial \mu_0} < 0.$$

■

Lemma C.3 Fix μ_1 and μ_0 at any level $\mu_0 \leq \frac{c}{1+r_f}$. Welfare is strictly larger in a bold stage (pooling equilibrium) than it would be in a – counterfactual – mix stage $W_B(\mu_0, \mu_1) > W_M(\mu_0, \mu_1)$, as long as $\mu_0 \leq \frac{c}{1+r_f}$.

Proof. As welfare is aggregate consumption, we can decompose $W_B(\mu_0, \mu_1) - W_M(\mu_0, \mu_1)$ analogously to the decomposition of $W_B(\mu_0, \mu_1) - W_C(\mu_0, \mu_1)$ in the proof of Lemma B.3. The difference is that in the second step we use that opaque good plus skilled consumption has the form of

$$\left[\frac{(1 - \mu_0 - \mu_1)}{2} (\rho - 1) \left(\frac{1}{r_x} + 1 \right) + w_1(1 + r_x) \right] \quad (\text{A.15})$$

with interest rates $r_x = r_B, r_M$ in the pooling and mix phases, respectively, which term is decreasing in r_x by (A.11). That is, (A.12) is non-negative at any point as $r_B \leq r_M \leq \bar{r} =$

$\rho - 1$. Finally, the difference in bad consumption is

$$\begin{aligned} \mu_0 \rho \left(\frac{1}{r_B} - \frac{w_1}{(1 - \mu_0 - \mu_1)} \right) - \mu_0 \rho \left(\frac{1}{2 r_M} - \frac{w_1}{1 - \mu_0 - \mu_1} \right) &= \\ &= \mu_0 \rho \left(\frac{1}{r_B} - \frac{1}{2} \frac{1}{r_M} \right) > 0. \end{aligned}$$

■

Lemma C.4 *Welfare jumps downward in μ_0 at the mix-cautious threshold whenever that threshold exists. That is, $W_M(\mu_0, \mu_1) > W_C(\mu_0, \mu_1)$ at $\mu_0 = \tilde{\mu}_0(\mu_1)$.*

Proof. Consider the definition (16) where each element corresponds to the consumption of a group of agent of a given type. Recall that at $\mu_0 = \tilde{\mu}_0(\mu_1)$, $r_M = \bar{r}$ by definition. This the interest at which good investors are indifferent whether to borrow. Therefore, by Propositions 1 and 2, only the consumption of transparent bad types, $\rho(1 + \ell(b, 0))$ is discontinuous at $\mu_0 = \tilde{\mu}_0(\mu_1)$. $\ell(b, 0)$ jumps downward to 0 as opaque bad types cannot borrow when all investors turn to cautious which proves the Lemma. ■

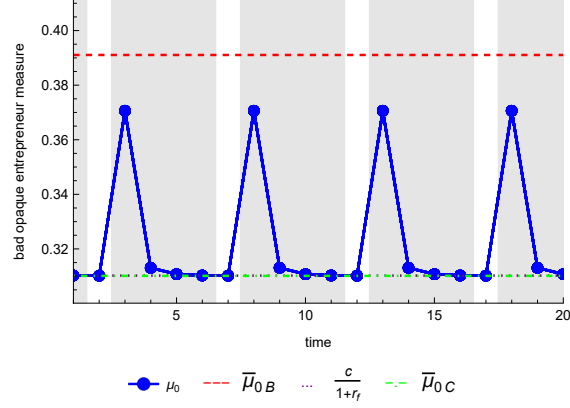
D Other Dynamic Equilibria

Long Recoveries and the Three-Phase Cycle

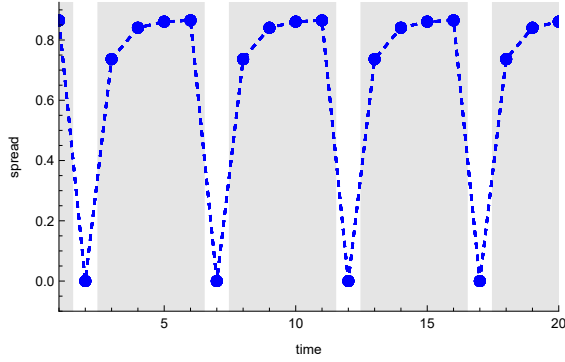
In this section we discuss the cycles that we either only briefly mentioned or we did not include in the main text. It is insightful to distinguish the dynamic cycles that emerge in this economy based on the number of distinct phases that they have. In the main text, we introduced *two-phase* cycles, long bold-short cautious and short bold-long cautious cycle. In Appendix C, we introduced the new stage game equilibrium which arises if Assumption 3 is violated, the mix stage. In this case, a *three-phase* cycle emerges which has a mix phase as well.

Here, we first use Figure D.1 to provide some more explanation about the short bold-long cautious two-phase cycle introduced in Proposition 1(iii), when Assumption 3 holds. Then we proceed to three-phase cycles which can happens when this assumption is violated.

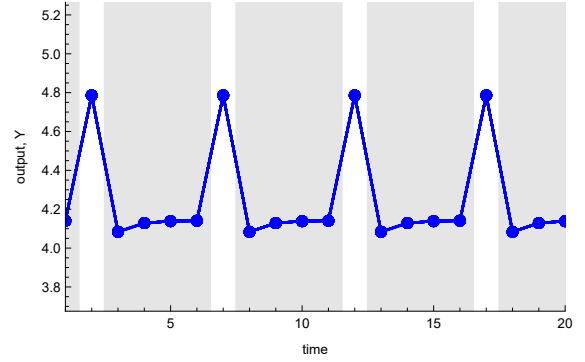
Short Bold-Long Cautious Cycle: Long Recovery. Figure D.1 illustrates the short bold-long cautious two-phase cycle which can emerge when Assumption 3 holds. Panel D.1a shows the evolution of the state variable, the measure of opaque bad entrepreneurs, μ_0 . Panel D.1b plots the interest rates throughout the cycle. As we see, in this cycle the credit spread increases during the long recession. Panel D.1c illustrates the cyclicalilty of output, and its boom when lending standards loosen. Comparison with panel D.1a shows the negative co-movement of output with the measure of opaque bad entrepreneurs μ_0 . Moreover, the output drop is amplified when there is a switch from lax to tight lending standards. Notice that while μ_0 increases only slightly in these periods, the drop in output is sizable, which is a direct consequence of the change in lending standards.



(a) State variable μ_0



(b) Interest rates



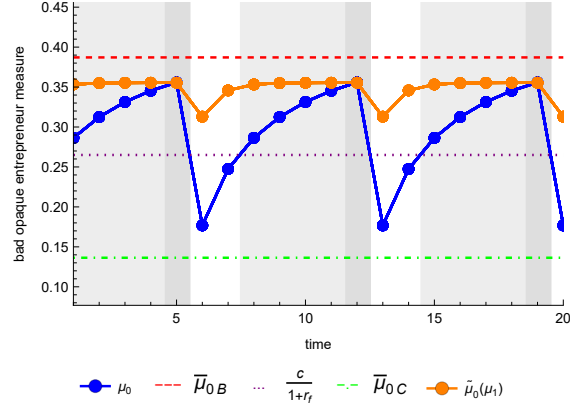
(c) Output

Figure D.1: This figure plots a short bold-long cautious two-phase cycle. Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates. Panel (c) depicts the total gross output. The parameters are: $\rho = 3.5, \lambda = 0.7, \delta = 0.7, c = \bar{\mu}_{0C} + 0.0001, r_f = 0, w_0 = 3.2, w_1 = 0.05$.

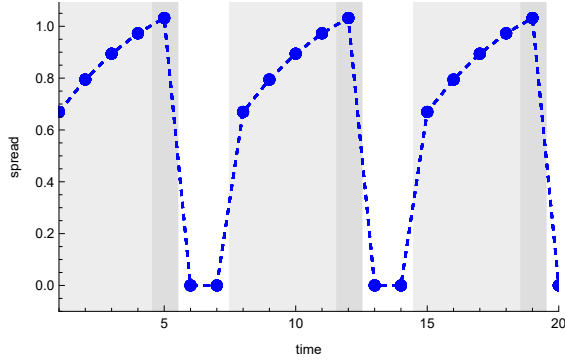
Three-Phase Cycle. Note that the laws of motion stated in (9)-(12) continue to hold in an economy with three potential types of stages. In fact, the economy follows the same dynamics in a mix stage and in a bold stage as in both a positive measure of unskilled investors choosing the bold test.

This also implies that if $\tilde{\mu}_0(\mu_1) > \frac{c}{1+r_f}$ then the dynamic economy might not directly transition from a bold phase into a cautious phase. Instead, it might pass through an intermediate phase in which investors are indifferent between being bold and cautious, the *mix phase*. Here, a fraction of unskilled investors are bold and a fraction are cautious.

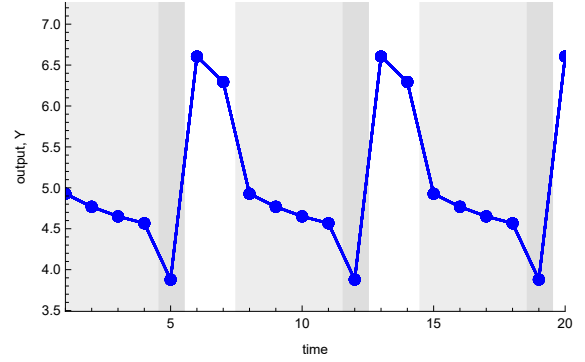
In the mix phase, the credit market is fragmented and interest rates rise relative to a bold phase. As such, the output experiences a first drop. However, the credit market fragmentation is not as extreme as a cautious phase and some investors stay bold and impose lax lending standards, which has two implications. First, the opaque bad entrepreneurs are still able to get some financing, which leads to a further decline of credit quality as the economy transitions through the mix phase. Moreover, although the output declines relative



(a) State variable μ_0



(b) Interest rates



(c) Output

Figure D.2: This figure plots a three-phase cycle. Panel (a) depicts the law of motions of state variables. Panel (b) shows the interest rates. Panel (c) depicts the total gross output. The parameters are: $\rho = 3, \lambda = 0.6, \delta = 0.25, c = 0.265, r_f = 0, w_0 = 3.99, w_1 = 0.01$.

to the bold phase and decreases throughout the mix phase, it is still higher than the cautious phase.

The mix phase ends when the credit quality is sufficiently low that it is not optimal for any investor to be bold any more. All investors switch to being cautious and impose tight standards. The economy enters a cautious phase and the output experiences a second drop. However, this final output drop is accompanied by a dramatic improvement in quality of the credit applicants, to which the investors respond by switching to lax lending standards. The economy switches back to a bold phase, and the cycles continues. Figure D.2 depicts a three-phase economy.

Intuitively, the recession can be exacerbated if the initial decline in credit quality is not sufficiently bad to make all investors adopt a cautious strategy and impose tight lending standards. As such, the cleansing effect in the credit market is delayed. Although the fragmentation of credit market leads to a drop in output, it does not entail an improvement in loan quality. For some time, the credit market is fragmented, but since some investors are still bold, bad entrepreneurs continue securing some financing and thus credit quality

worsens. At some point however, the credit quality has deteriorated so much that every investor chooses to use tight lending standards. The output takes a second hit, but this time it is accompanied by an improvement in the loan quality and leads to a boom. This phenomena is captured in the three-phase economy, as illustrated in Figure D.2.

E Characterization and Existence: Proofs of Lemma 5 and Proposition 4 and 5

In this appendix, we provide detailed characterization for a class of cyclical dynamic equilibria in our economy. All the cases we highlight in the main text and in Appendix D are within this class. Here, we present necessary and sufficient conditions for the existence of each member of this class. We also show uniqueness within this class, that is, at most one equilibrium within this class can exist for a given set of parameters.

At the end of this section, we also explain that while for some parameter values cyclical equilibria might exists outside of our preferred class, they tend to have very similar properties to the ones exposed here.

E.1 Steady States: Proof of Lemma 5

Let

$$\begin{aligned}\boldsymbol{\mu}_t &= \begin{bmatrix} \mu_{0t} \\ \mu_{1t} \end{bmatrix} \\ \mathbf{a} &= \begin{bmatrix} \delta \frac{\lambda}{2} \\ \delta \frac{\lambda}{2} \end{bmatrix} \\ \mathbf{A}_C &= \begin{bmatrix} (1-\delta) \frac{\lambda}{2} & (1-\delta) \frac{\lambda}{2} \\ (1-\delta) \frac{\lambda}{2} & (1-\delta) \frac{\lambda}{2} \end{bmatrix} \\ \mathbf{A}_B &= \begin{bmatrix} (1-\delta) & (1-\delta) \frac{\lambda}{2} \\ 0 & (1-\delta) \frac{\lambda}{2} \end{bmatrix}\end{aligned}$$

By Proposition 3, if $\mu_0 \in \left[0, \max\left\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\right\}\right]$ then

$$\mathbf{a} + \mathbf{A}_B \boldsymbol{\mu}_t = \boldsymbol{\mu}_{t+1} \tag{A.16}$$

and $\bar{\boldsymbol{\mu}}_B$ solves

$$\mathbf{a} + \mathbf{A}_B \bar{\boldsymbol{\mu}}_B = \bar{\boldsymbol{\mu}}_B \tag{A.17}$$

or

$$\bar{\boldsymbol{\mu}}_B = (\mathbf{I} - \mathbf{A}_B)^{-1} \mathbf{a} = \begin{pmatrix} \frac{\lambda}{-\lambda + \lambda\delta + 2} \\ \lambda \frac{\delta}{-\lambda + \lambda\delta + 2} \end{pmatrix},$$

a unique fixed point under the permanent bold regime. Clearly, the stationary steady state $\bar{\boldsymbol{\mu}}_B$ exists if $\bar{\mu}_{0B} \leq \max\left\{\frac{c}{1+r_f}, \tilde{\mu}_0(\bar{\mu}_{1B})\right\}$.

If $\mu_0 \in (\max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\mu_1)\}, 1]$ then

$$\mathbf{a} + \mathbf{A}_C \boldsymbol{\mu}_t = \boldsymbol{\mu}_{t+1} \quad (\text{A.18})$$

and $\bar{\boldsymbol{\mu}}_C$ solves

$$\mathbf{a} + \mathbf{A}_C \bar{\boldsymbol{\mu}}_C = \bar{\boldsymbol{\mu}}_C \quad (\text{A.19})$$

or

$$\bar{\boldsymbol{\mu}}_C = (\mathbf{I} - \mathbf{A}_C)^{-1} \mathbf{a} = \begin{bmatrix} \frac{1}{2} \lambda \frac{\delta}{-\lambda + \lambda \delta + 1} \\ \frac{1}{2} \lambda \frac{\delta}{-\lambda + \lambda \delta + 1} \end{bmatrix} = \begin{bmatrix} \bar{\mu}_{0C} \\ \bar{\mu}_{1C} \end{bmatrix},$$

a unique fixed point under the permanent bold regime. The stationary steady state $\bar{\boldsymbol{\mu}}_C$ exists if $\bar{\mu}_{0C} \geq \max\{\frac{c}{1+r_f}, \tilde{\mu}_0(\bar{\mu}_{1C})\}$. Note that $\bar{\mu}_{0C} \leq \bar{\mu}_{0B}$ but $\bar{\mu}_{1C} \geq \bar{\mu}_{1B}$. Furthermore,

$$0 < \tilde{\mu}_0(\bar{\mu}_{1B}) - \tilde{\mu}_0(\bar{\mu}_{1C}) = (\bar{\mu}_{1C} - \bar{\mu}_{1B}) \frac{\left(\frac{\rho}{(r_f+1)} + \frac{c}{(r_f+1)} - 1 \right)}{1 + \frac{c}{1+r_f} + \frac{\rho_f}{1+r_f}} < (\bar{\mu}_{1C} - \bar{\mu}_{1B}) < (\mu'_{0B} - \mu'_{0C})$$

for any $\frac{\rho}{(r_f+1)} > 1$ and $\delta, \lambda \in (0, 1)$. That is, at most one of the steady states can exist. Furthermore, both systems (A.16) and (A.18) are stable as the all eigenvalues of A_B and A_C are within the unit circle. This concludes Lemma 5.

E.2 B-Cycles and C-Cycles

Next, we construct a large class of finite invariant sets, $\{\mathbf{m}_i\}_{i=1}^\kappa \equiv (m_{0,i}, m_{1,i})_{i=1}^\kappa$ as candidates for the cycles in our dynamic equilibria. For each $\kappa > 2$, we consider two candidates.

- (i) A B -cycle cycles through $(\mathbf{m}_i^{B,\kappa})$ $i = 1, \dots, \kappa$ a sequence of two-dimensional vectors. In this cycle, the first $\kappa - 1$ steps are implied by (A.16) and then a step implied by (A.18) pushes back the economy to its starting point $\mathbf{m}_1^B$. This implies that $\mathbf{m}_1^{B,\kappa}$ has to satisfy

$$\begin{aligned} \mathbf{m}_1^{B,\kappa} &= \mathbf{a} + \mathbf{A}_C \left((\mathbf{m}_{\kappa-1}^{B,\kappa} - \bar{\boldsymbol{\mu}}_B) + \bar{\boldsymbol{\mu}}_B \right) = \mathbf{a} + \mathbf{A}_C \left((\mathbf{A}_B)^{\kappa-1} (\mathbf{m}_1^{B,\kappa} - \bar{\boldsymbol{\mu}}_B) + \bar{\boldsymbol{\mu}}_B \right) = \\ &= \mathbf{a} + \mathbf{A}_C (\mathbf{A}_B)^{\kappa-1} \mathbf{m}_1^{B,\kappa} - \mathbf{A}_C (\mathbf{A}_B)^{\kappa-1} \bar{\boldsymbol{\mu}}_B + \mathbf{A}_C \bar{\boldsymbol{\mu}}_B + \bar{\boldsymbol{\mu}}_B - \bar{\boldsymbol{\mu}}_B = \\ &= \mathbf{A}_C (\mathbf{A}_B)^{\kappa-1} \mathbf{m}_1^{B,\kappa} + \mathbf{a} + (\mathbf{I} - \mathbf{A}_C (\mathbf{A}_B)^{\kappa-1}) \bar{\boldsymbol{\mu}}_B - (\mathbf{I} - \mathbf{A}_C) \bar{\boldsymbol{\mu}}_B \end{aligned}$$

implying

$$\mathbf{m}_1^{B,\kappa} - \bar{\boldsymbol{\mu}}_B = (\mathbf{I} - \mathbf{A}_C (\mathbf{A}_B)^{\kappa-1})^{-1} (\mathbf{a} - (\mathbf{I} - \mathbf{A}_C) \bar{\boldsymbol{\mu}}_B)$$

Clearly, there is a unique such point. Then, for any $i \in [2, \kappa]$ we have

$$\mathbf{m}_i^{B,\kappa} - \bar{\boldsymbol{\mu}}_B = (\mathbf{A}_B)^{i-1} (\mathbf{m}_1^{B,\kappa} - \bar{\boldsymbol{\mu}}_B) = (\mathbf{A}_B)^{i-1} (\mathbf{I} - \mathbf{A}_C (\mathbf{A}_B)^{\kappa-1})^{-1} (\mathbf{a} - (\mathbf{I} - \mathbf{A}_C) \bar{\boldsymbol{\mu}}_B)$$

- (ii) A C -cycle has the support of $(\mathbf{m}_i^C)$ $i = 1, \dots, \kappa$. In this cycle, the first $\kappa - 1$ steps are implied by (A.18) and then a step implied by (A.16) pushes back the economy to its

starting point $\mathbf{m}_1^C$. This implies that $\mathbf{m}_1^C$ has to satisfy

$$\begin{aligned}\mathbf{m}_1^{C,\kappa} &= \mathbf{a} + \mathbf{A}_B \left(\left(\mathbf{m}_{\kappa-1}^{C,\kappa} - \bar{\mu}_C \right) + \bar{\mu}_C \right) = \mathbf{a} + \mathbf{A}_B \left((\mathbf{A}_C)^{\kappa-1} \left(\mathbf{m}_1^{C,\kappa} - \bar{\mu}_C \right) + \bar{\mu}_C \right) = \\ &= \mathbf{a} + \mathbf{A}_B (\mathbf{A}_C)^{\kappa-1} \mathbf{m}_1^{C,\kappa} - \mathbf{A}_B (\mathbf{A}_C)^{\kappa-1} \bar{\mu}_C + \mathbf{A}_C \bar{\mu}_C + \bar{\mu}_C - \bar{\mu}_C = \\ &= \mathbf{A}_B (\mathbf{A}_C)^{\kappa-1} \mathbf{m}_1^{C,\kappa} + \mathbf{a} + (\mathbf{I} - \mathbf{A}_B (\mathbf{A}_C)^{\kappa-1}) \bar{\mu}_C - (\mathbf{I} - \mathbf{A}_B) \bar{\mu}_C\end{aligned}$$

implying

$$\mathbf{m}_1^{C,\kappa} - \bar{\mu}_C = (\mathbf{I} - \mathbf{A}_B (\mathbf{A}_C)^{\kappa-1})^{-1} (\mathbf{a} - (\mathbf{I} - \mathbf{A}_B) \bar{\mu}_C)$$

Clearly, there is a unique such point. Then, for any $i \in [2, \kappa]$ we have

$$\mathbf{m}_i^{C,\kappa} - \bar{\mu}_C = (\mathbf{A}_C)^{i-1} \left(\mathbf{m}_1^{C,\kappa} - \bar{\mu}_C \right) = (\mathbf{A}_C)^{i-1} (\mathbf{I} - \mathbf{A}_B (\mathbf{A}_C)^{\kappa-1})^{-1} (\mathbf{a} - (\mathbf{I} - \mathbf{A}_B) \bar{\mu}_C)$$

For $\kappa = 2$, two algorithms above imply the same $\{\mathbf{m}_i\}_{i=1}^2$ values

$$\begin{aligned}\mathbf{m}_1^2 &= \mathbf{m}_1^{B,2} = \mathbf{m}_2^{C,2} = (\mathbf{I} - (\mathbf{A}_B \mathbf{A}_C)^{-1} (\mathbf{I} + \mathbf{A}_B) \mathbf{a} \\ \mathbf{m}_2^2 &= \mathbf{m}_2^{B,2} = \mathbf{m}_1^{C,2} = (\mathbf{I} - (\mathbf{A}_B \mathbf{A}_C)^{-1} (\mathbf{I} + \mathbf{A}_C) \mathbf{a}.\end{aligned}$$

In the main text, we denote the first element of $\mathbf{m}_1^2$ and $\mathbf{m}_2^2$ as μ_{0C}^* and μ_{0B}^* respectively.

These expressions give the following closed-form solutions

$$\begin{aligned}\mathbf{m}_i^{B,\kappa} - \bar{\mu}_B &= \left[\frac{\lambda \left(-\frac{2^{-i} \lambda (2^\kappa - (2-2\delta)^\kappa) ((2-2\delta)^i - (\lambda - \delta \lambda)^i)}{\lambda (2-2\delta)^\kappa - 2(\lambda-1)(\lambda - \delta \lambda)^\kappa + \lambda 2^\kappa - 2^\kappa + 1} - (1-\delta)^i \right)}{(\delta-1)\lambda + 2} \right. \\ &\quad \left. - \frac{(\lambda-2)\lambda((1-\delta)^\kappa - 1)2^{\kappa-i}(\lambda - \delta \lambda)^i}{((\delta-1)\lambda + 2)(\lambda(2-2\delta)^\kappa - 2(\lambda-1)(\lambda - \delta \lambda)^\kappa + \lambda 2^\kappa - 2^\kappa + 1)} \right] \\ \mathbf{m}_i^{C,\kappa} - \bar{\mu}_C &= \left[\frac{\delta(\lambda-1)\lambda(\lambda - \delta \lambda)^i}{2((\delta-1)\lambda + 1)((\lambda+1)(\lambda - \delta \lambda)^\kappa - 2\lambda)} \right. \\ &\quad \left. \frac{\delta \lambda^2 ((\delta-1)(\lambda - \delta \lambda)^\kappa - 1)(\lambda - \delta \lambda)^i}{2(\lambda-1)((\delta-1)\lambda + 1)} \right]\end{aligned}$$

The next Lemma establishes the basic properties of these invariant sets which we need for the construction of our dynamic equilibrium.

Lemma E.1 *Let $[\mathbf{x}]_1$ stand for the first element of a vector $\mathbf{x}$. Then*

- (i) $\left[\mathbf{m}_i^{B,\kappa} - \bar{\mu}_B \right]_1 < 0$ and $\left[\mathbf{m}_i^{B,\kappa} \right]_1$ is monotonically increasing in i for any κ and λ , and $\frac{\partial}{\partial \delta}$.
- (ii) $\left[\mathbf{m}_i^{C,\kappa} - \bar{\mu}_C \right]_1 > 0$ and $\left[\mathbf{m}_i^{C,\kappa} \right]_1$ is monotonically decreasing in i for any κ and λ , and $\frac{\partial}{\partial \delta}$.
- (iii) $\left[\mathbf{m}_\kappa^{B,\kappa+1} \right]_1 - \left[\mathbf{m}_\kappa^{B,\kappa} \right]_1 > 0$ and $\left[\mathbf{m}_\kappa^{C,\kappa+1} \right]_1 - \left[\mathbf{m}_\kappa^{C,\kappa} \right]_1 < 0$.

Proof. For the first statement, note that

$$\frac{\frac{\partial [\mathbf{m}_i^{B,\kappa} - \bar{\mu}_B]_1}{\partial i}}{[\mathbf{m}_i^{B,\kappa} - \bar{\mu}_B]_1} = \log(1 - \delta) - \frac{2^\tau (1 - (1 - \delta)^\tau) \lambda^{i+1} \log\left(\frac{2}{\lambda}\right)}{2^\tau (1 - (1 - \delta)^\tau) \lambda^{i+1} + 2^{i+1} (1 - \lambda) (2^\tau - (1 - \delta)^\tau \lambda^\tau)} < 0$$

It is also bounded away from 0 for any $0 < i \leq \kappa$ for fixed parameters of λ and δ . Therefore, $\left[\mathbf{m}_i^{B,\kappa} - \bar{\mu}_B \right]_1$ is not changing sign for any $i \leq \kappa$. Thus, by continuity, it is sufficient to show that $\left[\mathbf{m}_i^{B,\kappa} - \bar{\mu}_B \right]_1 < 0$ for some pair of i, κ . For this, observe that

$$\lim_{\kappa \rightarrow \infty} \left[\mathbf{m}_1^{B,\kappa} - \bar{\mu}_B \right]_1 = -\frac{(1-\delta)(2-\lambda)\lambda}{2(2-(1-\delta)\lambda)} < 0.$$

The second statement directly follows from the closed form expression by differentiating $\frac{\delta(\lambda-1)\lambda(\lambda-\delta\lambda)^i}{2((\delta-1)\lambda+1)((\lambda+1)(\lambda-\delta\lambda)^\kappa-2\lambda)}$ with respect to i .

For the first part of the third statement we have to show that

$$\begin{aligned} 0 &< \frac{\partial \left[\mathbf{m}_i^{B,\kappa} \right]_1}{\partial \kappa} \Big|_{i=\kappa} = \\ &= -\frac{2^{1-\kappa}(1-\lambda)\lambda^2(1-\delta)^\kappa(2^\kappa-\lambda^\kappa)(\lambda^\kappa(1-\delta)^\kappa(2^\kappa(((1-\delta)^\kappa-1)\log(\lambda-\delta\lambda)+\log(2))-(2-2\delta)^\kappa\log 2(1-\delta)))+(4-4\delta)^\kappa\log(1-\delta))}{(2-(1-\delta)\lambda)(-\lambda(2-2\delta)^\kappa+2(\lambda-1)(\lambda-\delta\lambda)^\kappa-2^\kappa\lambda+2^{\kappa+1})^2} \end{aligned}$$

Note that this holds if

$$2^\kappa(1-\delta)^\kappa(\lambda^\kappa((1-\delta)^\kappa-1)\log \lambda + (2^\kappa-\lambda^\kappa)\log(1-\delta) + \lambda^\kappa(1-(1-\delta)^\kappa)\log(2)) < 0.$$

As

$$\begin{aligned} \frac{\partial((2^\kappa-\lambda^\kappa)\log(1-\delta) + \lambda^\kappa(1-(1-\delta)^\kappa)\log(2))}{\partial \lambda} = \\ -\kappa\lambda^{\kappa-1}(\ln(1-\delta) + ((1-\delta)^\kappa-1)\ln 2) > 0 \end{aligned}$$

we need only

$$\begin{aligned} ((2^\kappa-\lambda^\kappa)\log(1-\delta) + \lambda^\kappa(1-(1-\delta)^\kappa)\log(2)) < \\ < ((2^\kappa-1)\log(1-\delta) + (1-(1-\delta)^\kappa)\log(2)) < 0. \end{aligned}$$

This must be true for every κ as

$$\begin{aligned} \frac{\partial((2^\kappa-1)\log(1-\delta) + (1-(1-\delta)^\kappa)\log(2))}{\partial \kappa} = \\ = (\ln 2 \ln(1-\delta))(2^\kappa - (1-\delta)^\kappa) < 0 \end{aligned}$$

and even for $\kappa = 1$,

$$\log(1-\delta) + (1-(1-\delta))\log(2) < 0$$

for every $1 > \delta > 0$. The second part also holds, because

$$0 > \frac{\partial \left[\mathbf{m}_i^{C,\kappa} \right]_1}{\partial \kappa} \Big|_{i=\kappa} = \frac{\delta\lambda(1-\lambda^2)\lambda^{2\kappa}(1-\delta)^{2\kappa}\log \lambda(1-\delta)}{2(1-(1-\delta)\lambda)((\lambda+1)(\lambda-\delta\lambda)^\kappa-2\lambda)^2}.$$

■

Given these result, we can fully characterize parameter sets implying the cycles described in Proposition 5 in the main text and in the Appendix D.

Consider B-cycles first. For the invariant set $\left(\mathbf{m}_i^{B,\kappa}\right)_{i=1}^{\kappa}$ to be part of a cyclical dynamic equilibrium, we need that

$$\left[\mathbf{m}_{\kappa}^{B,\kappa}\right]_1 > \max\left\{\frac{c}{1+r_f}, \tilde{\mu}_0\left(\left[\mathbf{m}_{\kappa}^{B,\kappa}\right]_2\right)\right\} \quad (\text{A.20})$$

$$\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1 \leq \max\left\{\frac{c}{1+r_f}, \tilde{\mu}_0\left(\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_2\right)\right\}, \quad (\text{A.21})$$

where $\left[\mathbf{m}_{\kappa}^{B,\kappa}\right]_1$ and $\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1$ denote the largest and second largest implied μ_0 value along this invariant set. Note, that under these conditions, this is a locally stable cycle because all the eigenvalues of $\mathbf{A}_C (\mathbf{A}_B)^{\kappa-1}$ are inside the unit cycle for any κ .

The corresponding equilibrium is a bold-cautious two-phase economy, if

$$\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1 \leq \frac{c}{1+r_f} < \left[\mathbf{m}_{\kappa}^{B,\kappa}\right]_1 \quad (\text{A.22})$$

and

$$\tilde{\mu}_0\left(\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_2\right) < \left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1, \quad (\text{A.23})$$

and a bold-mix-cautious three-phase economy²⁶ if

$$\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1 \leq \tilde{\mu}_0\left(\left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_2\right) < \left[\mathbf{m}_{\kappa}^{B,\kappa}\right]_1 \quad (\text{A.24})$$

and

$$\frac{c}{1+r_f} < \left[\mathbf{m}_{\kappa-1}^{B,\kappa}\right]_1. \quad (\text{A.25})$$

and

$$\left[\mathbf{m}_1^{B,\kappa}\right]_1 < \frac{c}{1+r_f}. \quad (\text{A.26})$$

Then, our result $\left[\mathbf{m}_{\kappa}^{B,\kappa+1}\right]_1 - \left[\mathbf{m}_{\kappa}^{B,\kappa}\right]_1 > 0$, and the monotonicity properties of $\left[\mathbf{m}_i^{B,\kappa}\right]_1$ imply that the relevant intervals for the thresholds to imply a B-cycle of length κ are non-overlapping and longer κ cycles correspond to higher thresholds $\frac{c}{1+r_f}$. Given Proposition 1, the characterization of cycles with long booms in Proposition 5 follows.

Analogously, if $\left(\mathbf{m}_i^{C,\kappa}\right)_{i=1}^{\kappa}$ is part of a cyclical dynamic equilibrium then conditions

$$\begin{aligned} \left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_1 &< \max\left\{\frac{c}{1+r_f}, \tilde{\mu}_0\left(\left[\mathbf{m}_{\kappa}^{C,\kappa}\right]_2\right)\right\} \\ \left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_1 &\geq \max\left\{\frac{c}{1+r_f}, \tilde{\mu}_0\left(\left[\mathbf{m}_{\kappa-1}^{C,\kappa}\right]_2\right)\right\} \end{aligned}$$

²⁶If (A.24)-(A.25) hold, but (A.26) is violated, we have a cautious-mix economy. This case is qualitatively similar to a bold-cautious two-phase economy, hence we do not discuss it in the paper.

must hold, implying a locally stable cycle because all the eigenvalues of $\mathbf{A}_B(\mathbf{A}_C)^{\kappa-1}$ are inside the unit cycle for any κ . Also, $[\mathbf{m}_\kappa^{C,\kappa+1}]_1 - [\mathbf{m}_\kappa^{C,\kappa}]_1 < 0$. That is, the relevant intervals for the thresholds to imply a C-cycle of length κ are decreasing and non-overlapping. If the corresponding cyclical dynamic equilibrium is a bold-cautious two-phase economy²⁷, then

$$[\mathbf{m}_{\kappa-1}^{C,\kappa}]_1 > \frac{c}{1+r_f} \geq [\mathbf{m}_\kappa^{C,\kappa}]_1 \quad (\text{A.27})$$

and

$$\tilde{\mu}_0([\mathbf{m}_{\kappa-1}^{C,\kappa}]_2) \leq [\mathbf{m}_{\kappa-1}^{C,\kappa}]_1 \quad (\text{A.28})$$

must also hold. Case (iii) in Proposition 5 is implied by these conditions. Case (i) corresponds to a cyclical dynamic equilibrium of length $\kappa = 2$. A necessary condition for this case is

$$[\mathbf{m}_1^2]_1 \leq \frac{c}{1+r_f} < [\mathbf{m}_2^2]_1 \quad (\text{A.29})$$

and

$$\tilde{\mu}_0([\mathbf{m}_2^2]_2) < [\mathbf{m}_2^2]_1, \quad (\text{A.30})$$

in line with the statement.

There is one additional condition to make sure that a given invariant set $\{\mathbf{m}_i\}_{i=1}^\kappa$ is part of a cyclical dynamic equilibrium. It is that the economy is not in autarky, or

$$\min \left(\frac{r_B(\mu_0, \mu_1, c, r_f) + 1}{1 + r_f}, \frac{r_C(\mu_0, \mu_1, c, r_f) + 1}{1 + r_f} \right) < \frac{\rho}{1 + r_f}$$

for any $(\mu_0, \mu_1) \in \{\mathbf{m}_i\}_{i=1}^\kappa$. Given our closed form expressions, this condition can be simply evaluated for any given set of parameters.

This concludes the proof of Proposition 4.

E.3 Other Classes of Cyclical Dynamic Equilibria

Suppose, that $\tilde{\mu}_0(\bar{\mu}_{1B}) \leq \frac{c}{1+r_f}$, so we must have a two-phase economy. As we have established, intervals of the form $([\mathbf{m}_{\kappa-1}^{x,\kappa}]_1, [\mathbf{m}_\kappa^{x,\kappa}]_1)$, $x = B, C$ are non-overlapping. That is, there must be a set of parameters that

$$\frac{c}{1+r_f} \in \left([\mathbf{m}_{\kappa-1}^{C,\kappa-1}]_1, [\mathbf{m}_{\kappa-1}^{C,\kappa}]_1 \right).$$

This implies that the necessary conditions established in previous subsection for a cyclical dynamic equilibrium with monotonic $(\mathbf{m}_i^\kappa)_{i=1}^\kappa$ are violated. Is there a cyclical dynamic equilibrium for such set of parameters? Our simulations show that in these sets, our economy

²⁷A mix-cautious 2-phase economy is also possible, if

$$\frac{c}{1+r_f} < [\mathbf{m}_\kappa^{C,\kappa}]_1 \leq \tilde{\mu}_0([\mathbf{m}_{\kappa-1}^{C,\kappa}]_2) < [\mathbf{m}_{\kappa-1}^{C,\kappa}]_1.$$

still converge to a cyclical dynamic equilibrium where $(\mathbf{m}_i^\kappa)_{i=1}^\kappa$ consists of a finite number of subsequent monotonic series. For instance, when $\frac{c}{1+r_f}$ is too high for a $\kappa = 3$ B-cycle, but still too low for a $\kappa = 4$ B-cycle, then the economy converges to a cycle which is in a bold phase for 4 periods, then cautious for a single period, then bold for 3 periods and only then, after an additional cautious period, returns to its starting point. By a trivial modification of our arguments in the previous section, it is possible to establish necessary conditions for these slightly more complex cycles. However, given that the economic properties of these cycles are very similar to the B-cycles and C-cycles we characterize, this would not add anything to the analysis. Hence, we leave it for the interested reader to work out.

F Continuum of Tests

Assume there is a continuum of tests, indexed by $s \in [0, 1]$. Every test s passes all $\frac{1-\mu_0-\mu_1}{2}$ applications of transparent good entrepreneurs and rejects all μ_1 applications of transparent bad entrepreneurs. Furthermore, test s passes s fraction of the applications of opaque entrepreneurs, i.e. $s\frac{1-\mu_0-\mu_1}{2}$ good projects and $s\mu_0$ bad opaque projects. Thus, $s = 0$ corresponds to the cautious test, and $s = 1$ corresponds to the bold test. Tests with $s \in (0, 1)$ cover everything in between. We follow the logic as in proof of Proposition 1 to show that both the bold and the cautious equilibrium are robust to this modification. In particular, investors strictly prefer to choose the bold test when $\mu_0 < \frac{c}{1+r_f}$ and the cautious test when $\mu_0 > \frac{c}{1+r_f}$ even if the intermediate choices are also available.

Recall that the unskilled investors choose a test which allows them to advertise the lowest break-even interest rate under the conjecture that at that interest rate all types will submit an application. If that were not true, unskilled investors not entering in equilibrium could choose a test and advertise an interest rate which leads to higher profit than staying outside. (We rely here on Lemma 2 (i) ensuring that if an entrepreneurs applies for a given rate in equilibrium, he also applies for all lower rates, advertised or not.) The break-even interest rate for any test characterized by s is

$$\begin{aligned} & \left(\frac{1-\mu_0-\mu_1}{2} + s \frac{1-\mu_0-\mu_1}{2} \right) (1+r(s)) \\ & + \left(\mu_1 + (1-s)\mu_0 + (1-s) \frac{1-\mu_0-\mu_1}{2} \right) (1+r_f) - c = 1+r_f, \end{aligned}$$

which in turn implies

$$\frac{(1+r_f) \left(1 - \left(\mu_1 + (1-s)\mu_0 + (1-s) \frac{1-\mu_0-\mu_1}{2} \right) \right) + c}{\left(\frac{1-\mu_0-\mu_1}{2} + s \frac{1-\mu_0-\mu_1}{2} \right)} - 1 = r(s).$$

Note that

$$\frac{\partial r(s)}{\partial s} = -2 \frac{c - \mu_0 - \mu_0 r_f}{(s+1)^2 (1 - \mu_0 - \mu_1)},$$

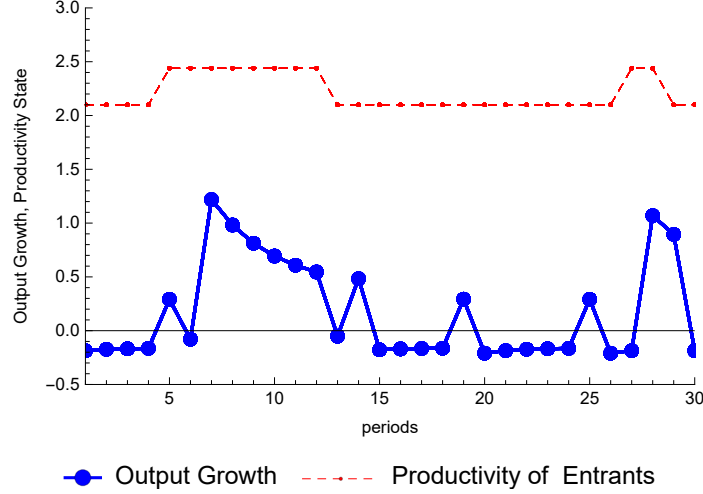


Figure G.3: The figure plots a realization of the path for exogenous productivity of entrants (dashed red) along with the endogenous output growth path (solid blue). The parameters are: $\rho_g = 2.7, \rho_b = 1.7, \lambda_1 = 0.4, \lambda_2 = 0.6, \alpha = 0.8, \delta = 0.2, c = 0.116, r_f = 0, w_0 = 6.5, w_1 = 0.2$

implying that whenever $\mu_0 < \frac{c}{(1+r_f)}$, the smallest interest rate is implied by the test $s = 1$, while in the opposite case it is $s = 0$. Thus, by the same argument as in the main text, if $\mu_0 < \frac{c}{(1+r_f)}$, the equilibrium advertised interest rate by unskilled investors corresponds to the test $s = 1$ (bold test), and in the opposite case they choose $s = 0$ (cautious test). In this sense, the continuum of intermediate tests are always dominated by either the bold or the cautious test, and restricting investor choice to these two tests is without loss of generality.

G Stochastic Cycles

In this section we introduce an aggregate shock to the model. This extension makes the framework more realistic while keeping the analysis tractable, and replaces deterministic cycles with more plausible stochastic ones.

Suppose that the fraction of bad entrepreneurs among the new entrants is stochastic, denoted by λ_t . Assume λ_t follows an S state Markov process with transition matrix $\Sigma \in [0, 1]^S$. Furthermore, relax the assumption that the good and bad firms are equally productive and let $\rho_g > \rho_b > 1 + r_f$. As such, λ_t represents an exogenous productivity shock and the average productivity of entrants in a given period is $(1 - \lambda_t)\rho_g + \lambda_t\rho_b$.

Observe that λ_t enters the equilibrium only through the law of motion for (μ_0, μ_1) described in Proposition 1. Therefore, the only change to the equilibrium characterization is replacement of (9)-(12) in the deterministic cycle with their state-dependent, stochastic counterparts. For instance, (9) is replaced by

$$\mu'_{0B}(\delta, \lambda_s, \mu_0, \mu_1) = (1 - \delta)\mu_0 + (\delta + (1 - \delta)\mu_1) \frac{\lambda_{s'}}{2} \quad \text{w.p. } \Sigma_{s,s'}, \forall s'. \quad (\text{A.31})$$

We study the properties of the extended model through a simple example.

Example 1 *Let λ_t follow a two state Markov process, where $\lambda_1 < \lambda_2$ and $\Sigma_{ss} = \alpha_s > \frac{1}{2}$ for $s = 1, 2$.*

As such, state $s = 1$ corresponds to high productivity of entrants. Figure G.3 illustrates the implied stochastic cycle. The dashed red line is the average exogenous productivity of entrants, while the solid blue line is the endogenous aggregate growth of the economy. Aggregate growth depends on both the exogenous productivity of new entrants and the endogenous credit market outcomes, similar to the baseline economy where state variables (μ_0, μ_1) and lending standards determine which firms are financed and survive. This leads to an intricate relationship between productivity shocks and aggregate growth. In particular, output growth is not solely driven by exogenous productivity shocks. For instance, periods 14-26 correspond to an interval of continuously low exogenous productivity. Still, the economy switches between recessions and booms as lending standards endogenously fluctuate within this interval. Furthermore, this figure makes it clear that the introduction of an aggregate state leads to stochastic cycles in which booms and recessions of varying magnitude and length alternate.

It is also interesting to consider the highest aggregate growth episodes, periods 7 and 28 in this example. These episodes happen when the exogenous high productivity of entrants coincides with the endogenous cleansing effect of a long cautious phase leading to an unusually low fraction of surviving bad firms. As such, the aggregate growth is exacerbated.

H Publicly Observable Credit Score

A simplifying assumption in the baseline model is that no credit score exists. This assumption has two important implications. First, in the period after a bold phase no information is revealed about the surviving bad firms. Thus if the stage game equilibrium remains bold, all of these bad opaque entrepreneurs keep getting financed. Second, in the period following a cautious phase, no opaque good surviving entrepreneur is known to unskilled investors as good. This is relevant when the stage game equilibrium remains cautious, as these entrepreneurs are not identified by unskilled investors and thus they are rationed.

In this section we incorporate credit scores in the baseline framework in a simple fashion. Consider all the entrepreneurs who survive from period t to $t + 1$. We model the credit score as a probability ζ that the true type τ of these surviving entrepreneurs is revealed to unskilled investors. ζ captures the precision of the credit score. When the true type of an entrepreneur is revealed to be τ , he is perceived as a $(\tau, 1)$ entrepreneur to investors, i.e. a transparent entrepreneur of type τ . We further assume $\zeta < \bar{\zeta}$.

Next we adjust the law of motion for the state variables of the benchmark economy,

expressed in equations (9)-(12):

$$\mu_{0B}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = (1 - \zeta)(1 - \delta)\mu_0 + (\delta + (1 - \delta)\mu_1)\frac{\lambda}{2}, \quad (\text{A.32})$$

$$\mu_{1B}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = \zeta(1 - \delta)\mu_0 + (\delta + (1 - \delta)\mu_1)\frac{\lambda}{2}, \quad (\text{A.33})$$

$$\mu_{0C}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = (\delta + (1 - \delta)(\mu_0 + \mu_1))\frac{\lambda}{2}, \quad (\text{A.34})$$

$$\mu_{1C}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = (\delta + (1 - \delta)(\mu_0 + \mu_1))\frac{\lambda}{2}, \quad (\text{A.35})$$

where superscript *cs* denotes the economy with credit scores. Consider equation (A.32). The first term is the measure of opaque bad entrepreneurs from last period who both survive and whose credit score is not revealed. The second term is the newborn additions and is identical to the baseline economy. Equation (A.33) is the complementary equation. It incorporates the measure of surviving opaque bad entrepreneurs whose type is revealed by the credit score to be bad, and are now transparent bad entrepreneurs from the perspective of unskilled investors. The last two equations do not change.

In this economy we need a third state variable, the measure of opaque good entrepreneurs, as the measure of the two groups of good entrepreneurs evolve differently. We have

$$\nu_{0B}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = (1 - \zeta)(1 - \delta)\nu_0 + (\delta + (1 - \delta)\mu_1)\frac{\lambda}{2}, \quad (\text{A.36})$$

$$\nu_{0C}^{cs'}(\delta, \lambda, \zeta, \mu_0, \mu_1, \xi_0) = (1 - \zeta)(1 - \delta)\nu_0 + (\delta + (1 - \delta)(\mu_0 + \mu_1))\frac{\lambda}{2}. \quad (\text{A.37})$$

Both equations incorporate the measure of opaque good entrepreneurs whose type is not revealed by credit scores. The complementary measure is added to the measure of transparent good entrepreneurs.

The most critical difference with the baseline economy is in deriving Lemma 4, i.e. proving that the dynamic equilibrium reduces to a sequence of stage game equilibria. In order to establish this result, the deviation to consider is that of a opaque bad entrepreneur in a market where investors are bold and all entrepreneurs behave as in a stage game. We need to show that a opaque bad entrepreneur does not have an incentive to deviate from his optimal stage game strategy and pay investors back, in order to be able to survive and earn future profits.

The key to the proof is to first observe that any opaque bad entrepreneur who does not pay back investors raises strictly positive profits in a bold equilibrium. The reason is that all projects are positive NPV and unskilled lenders who determine the interest rate only break even. Thus even good entrepreneurs who do pay back make positive profits as they choose to participate. This implies that the opaque bad ones who do not pay back make higher, strictly positive profits. However, if an opaque bad entrepreneur does not pay back he will be excluded from the market with probability ζ . Alternatively, if he pays back, he makes strictly lower profits this period but his participation probability next period increases by $\zeta\%$, where he can make positive profits. Furthermore, depending on his repayment strategy he can make positive profits in the periods farther in the future as well.

As such, $\exists \bar{\zeta}$ such that if $\zeta < \bar{\zeta}$ no deviation exists for a opaque bad entrepreneur: the discounted probability of more likely survival by paying back at time t , $\frac{z}{1+r}$, is sufficiently small for a this entrepreneur that he prefers not to deviate.

In this case, the steady state levels of state variables, interest rates, the switching thresholds for choice of the test, as well as the stage game output adjust according to the new equations governing the evolution of the state variables, but the stage game and dynamic equilibrium logic remains the same. As such, similar to the baseline economy, the equilibrium will be cyclical for an intermediate range of c .