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Abstract

We analyze a general equlibrium model of strategic arbitraging and intermediation. Arbitrageurs take advantage of mispricings, market frictions and manipulation opportunities in order to maximize profits. We analyze the effects of increased competition among arbitrageurs due to lower entry costs. Typically, markets become more liquid and integrated, and Cournot-Walras equilibria converge to Walrasian equilibria, though not uniformly: mispricings persist longer on shallow markets. We also provide a class of economies where the limiting equilibria are neither integrated nor Walrasian. Furthermore, we show that the asset pricing implications for financial innovations are quite different from standard models.
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1 Introduction

In this paper we investigate issues of noncompetitive arbitraging and intermediation in frictional environments where questions of market integration, liquidity and manipulation can be explicitly defined and analyzed.

There are two main motivations for this exercise, one practical and the other one theoretical (with empirical predictions). First, general equilibrium models with arbitrageurs or with imperfectly competitive intermediaries have not been extensively studied in the literature. We believe that this little attention is incommensurate with the prevalence of arbitraging and with the importance of the services arbitrageurs render. It is well-known that arbitraging represents a nonnegligible fraction of daily trading volume. Neal (1993) reports that 47.5% of observed program trading was index arbitrage, and program trading was itself 11.5% of total NYSE trading volume. Furthermore, many of the OTC trades are in fact arbitrage trades. When Investment Banks design and trade complex derivatives with their private clients, they often make arbitrage profits because their positions are completely hedged. Also, practitioners widely believe that arbitrageurs produce a public good for the benefit of all traders by increasing the informational content of market prices and by removing the inefficiencies of the market allocation. They further argue that arbitrageurs provide liquidity and mitigate market fragmentations that may result from the development of new derivative markets. In this paper we show in a general equilibrium setting with strategic arbitrageurs (or intermediaries) that there is some truth to these assertions. While we typically rationalize the no-arbitrage pricing approach by saying that there can't be any arbitrage, for else someone would take care of it, this model is about actually modelling the "taking care of it."

The second motivation is more theoretical and aims to incorporate general equilibrium considerations, strategic behaviour, market frictions and local market making into the literature on the microstructure of financial markets, and to address the issues of arbitrage, integration, financial intermediation, manipulation and asset creation. The model we present is composed of an exchange economy with a given exogenous asset structure and without private information of either the moral hazard or the adverse selection type. We postulate the existence of several security exchanges. Most general equilibrium models assume that investors are price-taking agents who buy securities in order to hedge risks and to intertemporally substitute consumption. Over and above the empirical observation given earlier that indeed a large fraction of trading volume is not due to such investors, it is instructive to see how they interact with large investment banks and proprietary traders. Investors and arbitrageurs have very dissimilar pay-off functions, trading technologies, market power and access to information and exchanges. Investors are competitive and their access to asset markets is restricted because they cannot directly and instantaneously and simultaneously trade on a number of exchanges. Arbitrageurs (hedge funds, investment banks or brokerage houses, say) can trade on every exchange (and consequently reap arbitrage and intermediation profits) upon purchase of the necessary information and trading technology. Subsequently, the active traders play a Cournot-Walras game. The fact that individual investors cannot and do not (simultaneously) arbitrage across exchanges can be explained by the following observations.

1. In order to arbitrage, traders need instantaneous access to the floors, which is why proprietary arbitraging firms purchase seats on the exchanges as well as the necessary information technology. Equipped with advanced computers and telecommunication equipment, they continuously monitor, identify, and profit by eliminating such opportunities. For the technological aspects of arbitraging, the reader is referred to Wong (1993).

2. Individual investors cannot short securities easily, whereas brokerage firms and arbitrageurs have the full use of the proceeds from a short sale.

3. Arbitrageurs hire knowledgeable advisors.

4. Arbitrageurs typically don't bear brokerage fees and other transaction costs, whereas individual investors do.

Arbitrage is done for instance by proprietary traders in investment banks and foremost in so-called "market neutral" or "relative value" hedge funds. Over-the-counter financial engineering can be viewed as arbitrage as well, and so can many financial innovations.

We show the following theoretical results in this paper. First, we characterize the optimal trading strategies of arbitrageurs and show the extent to which they can generate arbitrage and manipulation (properly defined) profits.

Second, we provide a family of conditions under which we verify that arbitrageurs integrate markets. One of the sufficient conditions that guarantee that increased competition eliminates arbitrage opportunities basically requires that markets are locally complete enough. We argue (under these conditions) that markets mostly do not allow for arbitrage exactly because arbitrageurs remove the opportunities. However, under increasing competition among arbitrageurs, the increasing integration (i.e. the vanishing degree of mispricing) is not uniform across asset markets. Shallow markets tend to exhibit more mispricing. This finding neither requires that transaction costs are identical for all market participants, nor that they are higher on some markets than on others, which seems to be a standard argument. We can apply this result to a number of actual asset pricing puzzles.

Third, an as yet unresolved issue in this literature has been to specify the conditions under which imperfectly competitive intermediaries can replace the omniscient Walrasian auctioneer. Under the family of conditions mentioned above, we prove that the Cournot-Walras equilibria indeed converge to Walrasian equilibria (possibly with restricted participation) when entry costs into the arbitraging sector tend to zero. Profit maximizing arbitrageurs induce in the limit exactly the same reallocation of state-contingent consumption that would have occurred in a Walrasian equilibrium (again possibly with restricted participation). We also construct an economy where Cournot-Walras equilibria do not converge to competitive equilibria.

Fourth and finally, this leads us to investigate the asset pricing implications of such arbitraged economies. It is well known that in standard competitive economies redundant assets can be priced by no-arbitrage considerations. The price of a newly introduced redundant asset simply equals its discounted weighted expected payoff using the equilibrium state-prices as weights. We argue that this need not be true in arbitraged economies, even though in the limit the allocations correspond to competitive ones. The reason is simply that introducing an asset that is redundant from an economy-wide point of view nevertheless generates new arbitrage opportunities on the exchange on which it is introduced (where it is not redundant, of course). This is true even in the limit where the intermediated equilibrium coincides with a competitive equilibrium. We illustrate these assertions with a simple example that replicates and generalizes Detemple and Selden's findings. We conclude that the market microstructure matters.

Related Literature. Very few general equilibrium models of economies with uncertainty and with imperfectly competitive financial intermediaries exist. Bisin (1998) and Yosha (1997) are notable exceptions. In a slightly different spirit, Townsend (1983) derived many deep results about competitive, cooperative and noncooperative intermediated general equilibrium structures with a complete set of contingent commodities. Yanelle (1996) analyzes two-sided competition among intermediaries in an economy without uncertainty. Competitive arbitrageurs have also been studied in general equilibrium models by De Long, Shleifer, Summers, and Waldmann (1990), Fremault ((1991), (1993)) and Dow and Gorton (1994). The abstract properties of the restricted participation setup in a competitive economy have for instance been analyzed by Polemarchakis and Siconolfi (1997). Restricted participation was used in finance by Allen and Gale (1994) and by Holden (1995). Holden constructed, to our knowledge, the first general equilibrium model that combines noncompetitive (index futures) arbitrageurs with price-taking investors whose participation is restricted. Chen and Knez (1995) coined the term "integration" (as we use it) and constructed procedures to test its degree in models such as ours. By analyzing a two period economy, we abstract from issues arising when arbitrageurs have a horizon that is shorter than the horizon of the arbitrage position. Such issues were analyzed by De Long, Shleifer, Summers, and Waldmann (1990) in a noise trader framework with short-lived arbitrageurs, by Dow and Gorton (1994) in a model with private information and short-lived speculators and by Shleifer and Vishny (1997) in a principal-agent model.

Structure of the Paper. This paper is divided into 7 sections and two appendices. Section 2 provides an overview of the economy and defines the basic concepts. Section 3 studies the consumer's optimization problem and the existence of competitive equilibria. Section 4 analyzes the arbitrageur's policies. Section 5 introduces the equilibrium concept and proves the general theorems. It also presents results on financial intermediation and its effects on liquidity. Asset pricing implications are discussed in section 6. And section 7 concludes. Most proofs are relegated to Appendix A and Appendix B contains a table with some of the simulation results.

The Economy

Our economy consists of a finite number I of exchanges or islands, each one of them inhabited by a specific clientèle group. For simplicity, client group $i \in I$ can be modelled as a representative investor. At time zero, investor i trades the assets available on his exchange and consumes. The set of assets (and the cardinality) traded on exchange $i \in I$ is denoted by A^i. At time one (the final period), one of S states is randomly chosen by nature. A state s completely describes endowments, preferences and asset payoffs in that state. Investors consume their endowments and the proceeds of their portfolio. The payoff matrix on exchange i is given by

R^i, a matrix of dimension $S \times A^i$ and of full column rank, where S is the state-space (as well as its cardinality, $A^i \leq S < \infty$). A typical dividend of asset a traded on exchange i in state s consists of $d^i_{a,s}$ units of the single commodity, which we choose to be the numeraire. The overall payoff matrix is $R \equiv [R^1 \cdots R^I]$. All assets are in zero-net supply. We assume that all investors (on all exchanges) have the same probability assessments p and the same trivial initial information set. It is without loss of generality to assume that the state space S does not depend on i.

Each type of investors i can only trade with other investors on that exchange or with intermediaries (also called arbitrageurs). Intermediaries, upon paying the required fixed costs, can trade costlessly across exchanges. They are assumed to be risk-neutral profit maximizers.

Investors of type i submit their demand functions (depending on the price on exchange i, q^i), $f^i(q^i) \in \mathbb{R}^{A^i}$. A typical portfolio is denoted as $\theta^i = f^i(q^i)$. Likewise, arbitrageur t submits his market orders $y^{ti} \in \mathbb{R}^{A^i}$. The local (computerized) auctioneer then determines the equilibrium price vector $q^i \in w^i(y^i)$, where w^i maps aggregate asset supplies $y^i \equiv \sum_t y^{ti}$ to the equilibrium prices that equate supplies by arbitrageurs to net demands by investors (this relation may be multi-valued). Section 3 characterizes w in detail. These equilibrium prices do not allow for arbitrage within an exchange. The equilibrium price mapping for the entire economy is $w(y) \equiv (w^1(y^1), \ldots, w^I(y^I))$, with $y \equiv (y^1, \ldots, y^I)$. Due to the segmentation, asset prices across exchanges may exhibit mispricings which arbitrageurs can exploit. Because arbitrageurs are modelled as Cournot players (who are aware of the fact that their asset supplies affect prices), their market supplies are well-defined (we elaborate upon this assumption in Remark 2 in Section 4).

The absence of arbitrage can be defined as follows:

Definition 1 *Given a supply of assets of $Y \in \mathbb{R}^{\sum_i A^i}$, there is* <u>No Arbitrage</u> *(NA) if there is no (further) trade $y \in \mathbb{R}^{\sum_i A^i}$ such that*

$$\begin{aligned} &y'w(Y) \geq 0 \quad \textit{and} \quad Ry < 0 \qquad [\textit{type I}], \textit{or} \\ &y'w(Y) > 0 \quad \textit{and} \quad Ry \leq 0 \qquad [\textit{type II}] \end{aligned}$$

There is <u>No Strategic Arbitrage</u> *(NSA) if there is no trade y such that*

$$\begin{aligned} &y'w(Y+y) \geq 0 \quad \textit{and} \quad Ry < 0 \qquad [\textit{type I}], \textit{or} \\ &y'w(Y+y) > 0 \quad \textit{and} \quad Ry \leq 0 \qquad [\textit{type II}] \end{aligned}$$

For simplicity, an *arbitrage* will always mean a *price-taking arbitrage*, while a *strategic arbitrage* is a non-price-taking arbitrage. The following characterization of no-arbitrage asset prices is well-known and very convenient.

Lemma 1 *The* set of no–arbitrage prices within exchange i *can be shown to equal the open convex cone*

$$\mathcal{Q}^i \equiv \left\{ q^i : \exists \lambda^i \in \mathbb{R}^S_{++} \ s.t.\ q^i = R^{i\prime} \lambda^i \right\}$$

The set of such state-price vectors λ^i on exchange i is denoted by $\Lambda^i \equiv \Lambda(q^i, R^i)$, a smooth convex manifold of dimension $S - A^i$ (due to the full rank assumption on R^i), or else empty. Lemma 1 states that the existence of a state-price vector is equivalent to (q^i, R^i) not admitting arbitrage. The representative investor assumption helps us furthermore identify the intertemporal marginal rates of substitution (IMRS) of investor i between consumption at date zero and in states $s \geq 1$ as a state-price vector. We have now reviewed the ingredients needed to define what we mean by "integration.[1]"

Definition 2 *The exchanges in a set $I' \subset I$ are* integrated *at a given array of prices and return matrices $\{q^i, R^i\}_{i \in I'}$ if there is a state-price vector common to all of them, $\cap_{i \in I'} \Lambda^i \neq \emptyset$.*

The exchanges are called strongly integrated *at a given array of prices and return matrices if $\Lambda^i = \Lambda^j$, for all i and j in I'.*

The first part of the definition says, by an application of Lemma 1, that there is no arbitrage across exchanges. However, this does not necessarily mean that there is no segmentation any longer. The second definition requires in addition that all risk sharing opportunities across exchanges that are permissible by the payoff matrices are exhausted. Indeed, it says that any state-price vector (including the IMRS) of exchange i can be used to correctly price assets on exchange j, both i and j in I'. Arbitraging improves links between exchanges, but does not affect the span on any exchange. The more integrated markets are, the more investors can share their risks because markets become more liquid and more efficient, but not because the span of

[1]Note that the term "integration" as used here does not necessarily imply that there is a flow of commodities or assets across exchanges. Two identical but separated exchanges are integrated, even though there is no trade between them. Of course, such an economy is not interesting.

marketed payoffs increases. Thus, we would expect strong integration only under special circumstances.

3 Investor Optimization and Local Equilibrium

Arbitrageurs maximize their profits by choosing asset supplies, taking as given the general equilibrium demand function w. In this section we derive some properties of w as a function of the asset supplies submitted by arbitrageurs.

We impose the following standard assumptions on investor behaviour.

H(i) The consumption set is $X^i \equiv \mathbb{R}^{S+1}_{++}$. Endowments are $\omega^i \in \mathbb{R}^{S+1}_{++}$.

H(ii) Utility functions are state-separable (but possibly state-dependent), three times continuously differentiable, differentiably strictly concave, differentiably monotonic, by which we mean that ${u^i_s}'(x) > k > 0$ for all x in X^i and all $s \in S \cup \{0\}$, and they satisfy the Inada condition $\lim_{x \to 0+} {u^i_s}'(x) = +\infty$ for sequences in X^i.

The following result can be easily shown.

Lemma 2 *The asset demand function $\theta^i = f^i(q^i)$ is twice continuously differentiable on the set of asset prices not allowing for arbitrage, $\mathcal{Q}^i$.*

Its Slutsky decomposition is $\partial_{q^i} f^i(q^i) = \gamma^i_0 K^i - v^i f^i(q^i)'$, where $\gamma^i_0 > 0$ is the Lagrange multiplier on the time zero budget constraint, K^i is the negative definite substitution matrix and v^i is the vector of income effects. One can rewrite the Jacobian of the demand function as $\partial f^i(q^i) = {u^i_0}' K^i [I + r^i q^i f^i(q^i)']$, with $r^i \equiv -\frac{{u^i_0}''}{{u^i_0}'}$.

The representative agent structure only guarantees that $\partial f^i(q^i)$ is negative definite on $\{q : q'v^i = 0, \text{ or } q' f^i(q^i) = 0, \text{ or } (q'v^i > 0 \text{ and } q'f^i(q^i) > 0), \text{ or } (q'v^i < 0 \text{ and } q' f^i(q^i) < 0)\}$. The following assumption strengthens this.

DSD (Downward Sloping Demand) ∂f^i is negative quasidefinite for all $i \in I$, i.e. $q' \partial f^i q < 0$ for any nonzero q, without imposing symmetry on the Jacobian.[2]

A sufficient condition for $\partial f^i(q^i)$ to satisfy DSD at a point is that the

[2] The term "downward sloping demand" follows from the observation that $(dq^i)' \partial f^i dq^i < 0$ for $dq^i \neq 0$, i.e. $(dq^i)' d\theta^i < 0$: prices and quantities of portfolios move in opposite directions.

coefficient r^i is "small enough" at that point. In particular, the Jacobian of f^i is (symmetric) negative definite if utility functions are quasi-linear (i.e. if $u_0^{i\,\prime\prime} = 0$, which violates H(ii)). This follows from Lemma 2.

Having characterized f^i, we also know the behaviour of the *equilibrium* price correspondence, defined as $w^i(y^i) \equiv \{q^i : f^i(q^i) = y^i\} = {f^i}^{-1}(y^i)$. The price system $w(0)$ is called the no-intervention price vector. A competitive equilibrium on exchange i, given y^i, satisfies $f^i(q^i) = y^i$. We are looking for the relevant domain of w^i. The set of aggregate supplies y^i that can be supported by a competitive equilibrium on exchange i is called the investment feasible set, and is denoted by $\mathcal{J}^i \equiv \{y^i : w^i(y^i) \neq \emptyset\}$. The set of asset holdings on exchange i that leave investors with enough resources to survive is denoted by $\mathcal{Y}^i \equiv \left\{y^i : \omega_s^i + \sum_a y_a^i d_{as}^i > 0, \quad (\forall s)\right\}$, an open convex subset of $\mathbb{R}^{A^i}$. An element $y^i \in \mathcal{Y}^i$ is called attainable. Both turn out to coincide. We regroup our findings about w^i, the general equilibrium analogue of the inverse demand function on exchange i, in the following Lemma.

Lemma 3 (Characterization of the Price Correspondence)
(i) $\mathcal{Y}^i = \mathcal{J}^i$. In particular, each $y^i \in \mathcal{Y}^i$ can be sustained as an equilibrium on exchange i.
(ii) Not all $y^i \in \mathcal{Y}^i$ may be optimal supplies by the arbitrageurs, however. Given an investment feasible y^{-ti}, arbitrageur t never chooses an action that results in an aggregate supply belonging to the boundary of $\mathcal{Y}^i$ (the proof of this assertion is Lemma A.1 in the appendix).
(iii) $w^i(y^i)$ is typically locally ^{2}.
(iv) DSD implies that $w^i : \mathcal{Y}^i \to \mathcal{Q}^i$ is a diffeomorphism and that $w^i \circ f^i = id(\mathcal{Q}^i)$ and $f^i \circ w^i = id(\mathcal{Y}^i)$. $id(X)$ denotes the identity map on X.

Results (i) to (iii) do not rely on DSD. The merits of the DSD assumption are well-known. First, it guarantees that competitive equilibria are unique for it eliminates the complications linked to singular economies and to discontinuous selections, as shown in (iv). Second, DSD guarantees that Walras tâtonnement is locally asymptotically stable. This is of particular importance in asset markets where equilibrium prices have to be computed in real time. Lastly, DSD also prevents arbitrageurs from manipulating markets by perturbing an integrated equilibrium and thus creating profitable arbitrage opportunities:[3]

[3]This observation is related to Novshek and Sonnenschein's claim that competitive equilibria that fail to satisfy DSD at the competitive equilibrium are artifacts of the Walrasian assumptions. The reason is that such a Walrasian equilibrium is not a limiting

Proposition 1 (anipulation and Nonequivalence of NA with NSA)

(i) There are economies where arbitrageurs can create arbitrage opportunities even when markets do not initially exhibit any (competitive) arbitrage opportunities. This is true if ∂w is not negative quasi-semidefinite at the initial competitive equilibrium.
(ii) Assume that DSD holds. Then NA implies NSA, so that arbitrageurs cannot create arbitrage opportunities.
(iii) If there is an arbitrage of type 2, then there is a strategic arbitrage as well.

In this section we established that investors' demands are well-behaved. This facilitates our analysis in the next subsection where arbitrageurs maximize their profits, anticipating the optimal (re-)actions of investors.

Remark 1 (On DSD and Asymmetric Information) *This section provided assumptions on the fundamental parameters of the economy that give rise to the result that ∂w^i is negative quasidefinite and satisfies some boundary conditions. Any other sets of assumptions would be equally good. In particular, we assumed that information is symmetric across investors and between investors and arbitrageurs. By no means do we claim that asymmetric information is not important. Assume for instance that investor i is not informed about q^{-i}. If the arbitrageur/intermediary submits his market order supply $y_a^i > 0$*

$$\max_{y^t} \left\{ \Pi^t \equiv x_0^t + \beta^t \sum_{s\geq 1} p_s x_s^t - c \right\} \quad \text{s.t.} \quad \begin{cases} x_0^t = \sum_i \sum_{a^i} y_{a^i}^{ti} w_{a^i}^i \geq 0 & & [\phi^t] \\ x_s^t = -\sum_i \sum_{a^i} d_{a^i,s} y_{a^i}^{ti} \geq 0 & (\forall s \in S) & [\gamma_s^t] \\ y^t \in \prod_i \left(\mathcal{Y}^i - \{y^{-t}\}\right) & & \end{cases}$$

The Lagrangian multipliers on the inequality constraints are given in brackets. Notice that budget constraints are imposed with equality due to the assumed monotonicity of the payoff function. These $S+1$ constraints insure that consumption is nonnegative and that the arbitrageur does not default, which is why we sometimes refer to them as self-financing, or no-default constraints. The last constraint restricts the arbitrageur's actions to be investment feasible. It turns out, however, that the boundary behaviour of the inverse demand mapping guarantees that the investment feasibility constraint never binds for any arbitrageur. For a proof refer to Lemma A.1 in the Appendix. For future reference, the set of actions y^t satisfying all the constraints is denoted by $\mathcal{A}^t(y^{-t})$.

Notice that arbitrageurs have no endowments. In particular, this assumption guarantees that profits do not stem from speculation (or risk-arbitrage), but from arbitraging and manipulating only.

We denote $\sum_s \frac{\beta^t p_s + \gamma_s^t}{1+\phi^t} d_{a^i,s}$ by $\sigma_{a^i}^t$, and $\sigma^{ti} \equiv R^{i\prime}(\beta^t p + \gamma^t)\frac{1}{1+\phi^t}$. We also use the convention that $\sigma^{ti} \equiv \{\sigma_a^t\}_{a\in A^i}$. $\sigma^t \equiv (\sigma^{t1}, \ldots, \sigma^{tI})$ is a shadow-price vector which has a state-price representation (using state-prices $(\beta^t p + \gamma^t)\frac{1}{1+\phi^t}$) that does not depend on any exchange in particular. Presumably, this shadow value incorporates all the (potentially different) local valuations into some form of an average. Indeed, assume that the same asset is traded on 3 echanges and $q^1 > q^2 > q^3$. The arbitrageurs buy this asset on exchange 3 and sell it on exchange 1, but it is not obvious whether they buy or sell it on 2: this will depend on the threshold price σ. This will be established rigorously in Proposition 2 and illustrated in length in the discussion following Proposition 2.

The first-order conditions then set marginal cost to marginal revenue,

$$\sum_{a^i=1}^{A^i} y_{a^i}^{ti} \frac{\partial w_{a^i}^i}{\partial y_{b^i}^{ti}} = \sigma_{b^i}^t - w_{b^i}^i \qquad (\forall b^i \in A^i)$$

Suppose that the shadow price of asset b^i is larger than the price on exchange i. The marginal revenue of buying a unit of asset b^i is the difference between what it is worth and what it costs, i.e. $\sigma_{b^i}^t - w_{b^i}^i$. The marginal cost of that operation is the change in the price t has to pay on all his infra-marginal units plus the costs linked to the perturbed prices of all the other assets in his portfolio. Define $\partial w \equiv \operatorname{diag}\{\partial w^i\}_{i=1}^I$, $w \equiv (w^i, \ldots, w^I)$. DSD lets us express asset demands implicitly as

$$y^t = (\partial w')^{-1}(\sigma^t - w)$$

Intuitively, the optimal arbitraging portfolio depends upon both the degree of mispricing in the economy, $\sigma^t - w$, and upon the depth of the different exchanges, ∂w. The arbitrageur holds larger positions the larger the mispricing and the deeper the markets (as summarized by $(\partial w')^{-1}$).

Finally, as second-order sufficient conditions for a local optimum (using the Kronecker product $\otimes$) we require that the Hessian of the Lagrangian

$$\partial^2_{y^{ti}} \mathcal{L}^t \equiv (\partial w^i)' + \partial w^i + \left(y^{ti\prime} \otimes I_{A^i}\right)\left(\partial^2 w^i\right) \tag{1}$$

be negative definite for all $i \in I$ (a block-diagonal matrix is negative definite iff every block is). $\left(\partial^2 w^i\right)$ consists of the vertically stacked Hessians and is of dimension $(A^i)^2 \times A^i$.

Lemma A.2 in Appendix A explicitly solves for the Lagrange multipliers of the arbitrageurs problem. Using this characterization, we can easily derive an expression for the shadow values. Let $\hat{S}$ denote the minimal set of states in which the arbitrageur's self-financing constraint is binding. We define the singular matrix[4] $\Omega \equiv I - R'_{\hat{S}}\left[R_{\hat{S}}(\partial w')^{-1}R'_{\hat{S}}\right]^{-1} R_{\hat{S}}(\partial w')^{-1}$. $R_{\hat{S}}$ consists of the matrix R with all the rows not in $\hat{S}$ deleted. Also, $\zeta^t \equiv \frac{\beta^t}{1+\phi^t}$, where $\zeta^t = \beta^t$ if the arbitrageur consumes at time zero, and $\zeta^t = \frac{w'(\partial w')^{-1}w}{w'(\partial w')^{-1}\Omega R' p} \leq \beta^t$ if the arbitrageur does not consume at time zero.

Proposition 2 *The shadow values are given by the vector* $\sigma^t \in \mathbb{R}^{\sum_i A^i}$,

$$\sigma^t = \zeta^t \Omega R' p + (I - \Omega) w \tag{2}$$

Shadow prices can be interpreted as an combination of the risk neutral prices $\zeta^t R' p$ and of the local valuations w. The first part reflects the impact of the arbitrageurs' risk-neutral valuation on shadow prices.

We can derive some special cases. If the arbitrageur has a short horizon ($\beta^t = 0$) then $\zeta^t = 0$ and only the second part in the expression (2) mat-

[4] Ω is singular if $\hat{S} \neq 0$. If $\hat{S} = 0$, we define $\Omega = I$.

ters: $\sigma^t = R'_{\hat{S}} \left[R_{\hat{S}} (\partial w')^{-1} R'_{\hat{S}} \right]^{-1} R_{\hat{S}} (\partial w')^{-1} w$. In this case the arbitrageur's valuations should not matter at all since the arbitrageur doesn't care for consumption tomorrow, and any payoff pattern is valued by the market only, i.e. using some investor's state-prices, rather than by the arbitrageur's own. On the other end of the spectrum, if it so happens that the arbitrageur consumes in every state tomorrow, then $\Omega = I$ and $\sigma^{ti} = \zeta^t R^{i\prime} p$. Finally, in an economy where $R^i = R^*$, all $i \in I$, if the arbitrageur chooses to hold zero net positions, $\sum_i y^{ti} = 0$, then $\sigma^t = \sum_i \left[\sum_j (\partial w^{j\prime})^{-1} \right]^{-1} (\partial w^{i\prime})^{-1} w^i$ (with an abuse of notation for here σ^t lies in $\mathbb{R}^{A^*}$ rather than in $\mathbb{R}^{\sum_i A^i}$). If arbitrageurs restrict their trading to a single asset across exchanges (say a), define the relative depth of exchange i by $\varpi^i_a \equiv \frac{\delta^i_a}{\sum_j \delta^j_a}$ where $\delta^i_a \equiv -\left(\frac{\partial w^i_a}{\partial y^{ti}_a} \right)^{-1}$ is the depth of the market for asset a on exchange i. The expression then simplifies to[5] $\sigma^t_a = \sum_{i \in I_a} \varpi^i_a w^i_a$. Under these conditions shadow values are given by a weighted arithmetic average (using relative depths) of local prices.

The following proposition decomposes the best-reply function of trader t, denoted by $B^t(y^{-t})$, into a price effect and a depth effect.

Proposition 3 (Best-Reply unction) *Assume that the second–order sufficient conditions hold. Then the best-reply function $B^t(y^{-t})$ is ^{1} except on an interiorless and null set. It is nevertheless always Lipschitzian. Where defined, the Jacobian equals*

$$\partial_y B^t = (\partial^2 \mathcal{L}^t)^{-1} \left[\partial_y \sigma^t - \partial w \right] - (\partial^2 \mathcal{L}^t)^{-1} diag \left\{ (B^{ti\prime} \otimes I_{A^i})(\partial^2 w^i) \right\}_{i=1}^{I}$$

All its eigenvalues have nonpositive real parts.

Given a small change in y^{-t}, arbitrageur t will adjust his supply according to the effect of dy^{-t} on the mispricing $\sigma^t - w$ and on the depth ∂w. The latter effect on t's supplies is small if y^t is small (and is, of course, identically equal to zero if demand functions are linear).

Remark 2 (Cournot vs. Bertrand) *There are two justifications for using a Cournot approach rather than a Bertrand approach. First, arbitrageurs do typically take quantity decisions rather than price decisions. Second, we*

[5] It is easy to see that the condition $\sum_i y^{ti}_a = 0$ holds when either $\beta^t = 0$ or when there is a state s suc

show that the Cournot-Walras paradigm with costly entry (and thus with endogenous participation) leads to more appealing predictions. Indeed, Yanelle (1996) shows that in environments with two-sided competition (i.e. where players compete both upstream and downstream) the outcome of Bertrand competition in general is not Walrasian and exhibits cornering phenomena.

Remark 3 (On Competitive Arbitraging) *Besides casual observations that arbitrage is done exclusively by proprietary traders (like hedge funds, investment banks and major brokerage houses), setup costs and local market making taken together are incompatible with the existence of an equilibrium when arbitrageurs are price-takers.*

Remark 4 (On Setup Costs) *A one-time exogenous cost c is consistent with the stylized fact that the arbitraging industry, as mentioned in the introduction, requires high levels of technological and conceptual sophistication. Indeed, most arbitrage opportunities are not readily exploitable by off-the-exchange or non-dealer traders. The assumption that active arbitrageurs simply start out with $-c$ units of present discounted value of profits allows us to neglect the redistribution and price effects of payments in units of consumption, as well as the induced portfolio effects (due to borrowing) that would be necessary to come up with c units of consumption at time zero.*

Remark 5 (On Risk Arbitrage) *We can capture what is referred to as* risk arbitrage *by endowing arbitrageurs with resources. As we shall insist in Remark 8, with obvious changes all of our analysis goes through when arbitrageurs do start out with endowments.*

Even though the arbitrage business has developed beyond the "riskless" (in the sense of no risk of loss) arbitrages (one reason being the increasing number of arbitrage participants), riskless and low-risk arbitrage is still very much part of arbitrage operations in investment banks (c.f. OTC trades) and hedge funds. We refer the interested reader to Goldman Sachs and Financial Risk Management Ltd. (1998) for data on the performance of market neutral hedge funds.

5 Cournot–Walras Equilibria

In this section we investigate the existence and the characteristics of an equilibrium for the Cournot-Walras game, and in particular the behaviour of equilibria as entry costs for arbitrageurs tend to zero.

5.1 Definition of Equilibrium

As mentioned in the previous section, the players in our model engage in a Cournot-Walras game (Gabszewicz and Vial (1972)). We assume that traders may enter the arbitraging market upon paying a fixed cost c. A Cournot-Walras Equilibrium (CWE) can be an equilibrium with entry only if at the given CWE no inactive arbitrageur has an incentive to enter and no active arbitrageur has an incentive to leave. The former requirement is captured by the no-entry condition (3) below which, at a symmetric equilibrium with n arbitrageurs, requires that no entrant e is able to cover the fixed costs. It is understood that if an entrant is indifferent between staying out and entering, then he stays out. This definition allows active arbitrageurs to make profits at equilibrium.

$$c \geq \max_{\left\{x^{ei}_{a^i}\right\}_{i,a^i}} \sum_i \sum_{a^i} (1+\phi^e) \left[w^i_{a^i} \left(\left\{ n y^{ti}_{b^i}(n) + x^{ei}_{b^i} \right\}_{b^i=1}^{A^i} \right) - \sigma^e_{a^i} \right] x^{ei}_{a^i} \tag{3}$$

We summarize this discussion in the following definition. The reader will have noticed that the game we analyze here is a *generalized game* in the sense that the action space of t depends itself on y^{-t}.

Definition 3 *A (type-symmetric, pure strategy)* CWE with costly entry *(abbreviated as CWECE) is a nonnegative integer n and an attainable array of actions $y^* \equiv \{y^{t*}\}_{t=1}^n \in \prod_t \mathcal{A}^t(y^{-t*})$ (with $y^t = y^1$, all $t = 1, \dots, n$) that satisfy:*
(i) y^* is such that $\Pi^t(y^{*t}, y^{*-t}) \geq \Pi^t(y^t, y^{*-t})$ for all $y^t \in \mathcal{A}^t(y^{-t*})$ and for all $t \in \{1, \dots, n\}$.
(ii) n and y^* satisfy the no-entry inequality (3), and
(iii) profits are non-negative: $\Pi^t(y^*) \geq 0$, $t \in \{1, \dots, n\}$.

It follows from the definition that local asset markets $i \in I$ are in equilibrium at a CWE since $(w^i(y^{*i}), y^{*i})$ is a competitive equilibrium on exchange i if y^{*i} is attainable. Also, by Walras' Law the markets for the consumption commodity clear as well. In states $s > 0$, $\sum_i (x^i_s - \omega^i_s) + \sum_t x^t_s = \sum_i [f^i(w^i(y^{*i})) - y^{*i}]' d^i_s = 0$ for $f^i(w^i(y^{*i})) - y^{*i} = 0$, all $i \in I$, and at time zero, $\sum_i (x^i_0 - \omega^i_0) + \sum_t x^t_0 = \sum_i \sum_a [y^{*i}_a - f^i_a(w^i_a(y^{*i})] w^i_a(y^{*i}) = 0$.

In what follows we limit ourselves to symmetric equilibria, and we drop the superscript t when referring to shadow prices or multipliers.

5.2 Existence and Characterization of Equilibria

It is well known that pure strategy CWE may not exist if profit functions are not quasi-concave (Roberts and Sonnenschein (1977)) or when the price correspondence is not single–valued, even if mixed strategies are allowed (Dierker and Grodal (1986)).

Because we neither replicate the demand side, nor have minimum efficient scales, nor have production frontiers with strictly positive Gaussian curvature, the existence results of Gabszewicz and Vial (1972), Novshek and Sonnenschein (1978) and Roberts (1980) cannot be used. The next proposition shows that equilibria exist under our strong assumptions. It also characterizes the equilibrium relations between a typical arbitrageur's supply y^t and n, denoted by $\xi(n)$, $\xi : \mathbb{R}_+ \to \prod_i \mathcal{J}^i$, and between c and n, denoted by $n = \eta(c)$, $\eta : \mathbb{R}_+ \to \mathbb{N} \cup \{0\}$.

Proposition 4 (Existence and Characterization) *Assume that H(i) - H(iii) and DSD hold (and, to make the problem interesting, that $w(0)$ does admit a strategic arbitrage). Then there is a real number $\bar{c} > 0$ such that pure strategy symmetric CWECE exist for $c \in (0, \bar{c})$. If in addition w is linear, then pure strategy symmetric CWECE exist for $c \in \mathbb{R}_+$. The equilibrium relations can be characterized as follows.*
(i) The mapping η is a weakly decreasing step function between some sets $(0, \bar{c}]$ and $\{n \in \mathbb{N} \cup \{0\} : n \geq \bar{n}\}$, for $\bar{c}$ small enough. η satisfies $\lim_{c \to 0} \eta(c) \to \infty$.
(ii) The supplies of a typical arbitrageur at a Nash equilibrium, $y^t = \xi(n)$, are Lipschitzian functions of n, with $\lim_{n \to \infty} \xi(n) = 0$.
(iii) Asset prices converge to their shadow values, $\lim_{c \to 0} ||w - \sigma|| = 0$.

Of course, when c is high enough to warrant either no entry or the entry of only a single arbitrageur, an equilibrium must exist as well.

5.3 Integration

The first effect of arbitraging that comes to one's mind is that it should tie markets together and guarantee that both the fundamental assets and their derivatives are priced fairly. Proposition 5 below asserts that, given some qualifications, increased competition indeed leads to integrated markets. The qualifications are summarized in the following assumptions, one (and only one) of which will be assumed to hold when we say "under LCM" (by LCM we mean that some markets are locally complete enough relative to the other ones).

LC (a) There is some exchange $k \in I$ such that R^k is of rank S.

LC (b) $\langle R^i \rangle = \langle R^* \rangle$, all $i \in I$ for some matrix R^* of full column rank, and with $A^* \leq S$ ($\langle R \rangle$ denotes the column span of R).

Having shown that limiting equilibria do not allow for arbitrage, Proposition 5 then also shows that this limiting equilibrium coincides with the following candidate Walrasian equilibrium where all agents are price-taking and where a single auctioneer clears markets for state-contingent consumption across all exchanges.

Definition 4 *A* <u>Walrasian Equilibrium with Restricted Participation</u> *of the economy* $\mathcal{E} = \{(u^i)_{i \in I}, (\omega^i)_{i \in I}, (R^i)_{i \in I}\}$ *is an array of excess demand vectors for state-contingent consumption and state-prices* $((z_i)_{i \in I}, \lambda) \in \mathbb{R}^{(S+1)I} \times \mathbb{R}^S_{++}$ *such that, for all* $i \in I$,

$$z^i \in \ \text{argmax}\ u_0^i(\omega_0^i + z_0^i) + \sum_s p_s u_s^i(\omega_s^i + z_s^i) \qquad s.t. \begin{cases} (z_s^i)_{s \in S} \in \langle R^i \rangle \\ z_0^i + \sum_s z_s^i \lambda_s \leq 0 \end{cases}$$

and such that markets clear, $\sum_{i=1}^{I} z^i = 0$.

As before, $\langle R^i \rangle$ denotes the column span of R^i. We now formally state the results aluded to before.

Proposition 5 (Integration) *Assume that H(i)-H(iii), DSD and LCM hold.*
(i) A common strictly positive state-price vector for σ *is given by* $\lambda \equiv \frac{1}{1+\phi}(\beta p + \gamma) \gg 0$, *proving that* (σ, R) *does not allow for arbitrage by invoking Lemma 1.*
(ii) (w, R) *will ultimately not allow for any arbitrage either (since it was shown in Proposition 4 that* $\lim_{c \to 0} ||w - \sigma|| = 0$*).*
(iii) The limiting CWECE coincides with a Walrasian equilibrium with restricted participation. The Walrasian consumption vector ${}^w z^i$ *is supported by the aggregate asset allocation* $\lim_{c \to 0} \eta(c) \xi^i(\eta(c)) = (R^{i\prime} R^i)^{-1} R^{i\prime}\ {}^w z^i$.
(iv) Furthermore, under LCM(b) markets become strongly integrated, and the limiting CWECE corresponds to a competitive equilibrium without restricted participation (but possibly with incomplete markets).
(v) From (ii) we can deduce that the arbitraging sector will not be able to make any gross profits in the limit, $\lim_{c \to 0} \eta(c) \Pi_g^t(c) \to 0$.

(i) shows that LCM guarantees that ϕ remains bounded, which in turn insures that $\lambda \gg 0$ (because $\beta^t > 0$, $p \gg 0$ and $\gamma \geq 0$). Intuitively, an unbounded ϕ arises in situations where the arbitrageur highly values the opportunity to borrow money at time zero but can't. LCM guarantees that the arbitrageur, should he have a need to, can always borrow money today from some investors via a state-contingent repayment scheme that offers enough flexibility to get around possibly binding no-default constraints tomorrow. But even for $\beta^t = 0$ (and $\phi = 0$, hence bounded) it implies that $\lambda = \gamma \gg 0$. The reason is that if such a short-horizon arbitrageur owns an arbitrage portfolio paying off strictly positive amounts tomorrow, he is able to sell this stream to some group of investors, thereby eliminating type one arbitrage as well.

Points (iii) and (iv) provide noncompetitive foundations for Walrasian equilibria in financial economies with restricted participation, and a forteriori for economies without restricted participation. This complements the results of Novshek and Sonnenschein (1978) and Roberts (1980) for perfectly competitive equilibria in atemporal economies with production and without uncertainty.

Point (v) illustrates that under LCM, greater numbers of arbitrageurs drive prices towards NA prices (and hence towards no-strategic arbitrage prices by Proposition 1), thereby shrinking the cake to be shared. Examples and anecdotes illustrating our propositions abound in the literature. Refer for instance to the anecdote of Harry Markowitz as president of 'Arbitrage Management Company' (as reported by Malkiel (1990)), to occurances in emerging markets (where arbitrage opportunities can go unnoticed for quite some time), or to the examples and anecdotes found in Wong (1993) and Hunter (1985) which illustrate why more competition increases the importance of simple trading judgment based on valuations.

Remark 6 (On Alternative Arbitraging Schemes) *We would like to briefly point out that Proposition 5 holds true if arbitrageurs are more specialized or restricted.*

First, assume that arbitrageurs have to pay a cost $c_{i,j}$ to arbitrage between exchanges i and j, and that for each pair (i,j) there is a large number of potential entrants that can only arbitrage across (i,j). Then if technologies to arbitrage across all possible combinations (i,j) are available, it is easy to see that Proposition 5 still holds (since $\cap_{i \in I} \Lambda^i \neq \emptyset$ in the limit as $c_{i,j} \to 0$, all (i,j)).

Second, assume instead that there is a set of linearly independent basis assets, and that each exchange consists of a subset of those. Such an econ-

omy is called a basis economy*. Also assume that each arbitrageur has to specialize in one single asset to arbitrage across all exchanges. The entry cost is c_a for asset a. Then if all assets can be arbitraged, Proposition 5 goes through (if $c_a \to 0$, all $a \in A$).*

We conclude this discussion by saying that even in a more realistic world without global omniscient auctioneers and without investors who can costlessly and simultaneously trade in all the possible assets and markets, we can derive the stylized fact that most markets are typicially rather integrated. In other words, each asset price reflects the deep parameters of seemingly unrelated exchanges and countries. Asset prices can be arbitrarily close to global Walrasian prices even without any global Walrasian auctioneer. It is sufficient that there are enough arbitrageurs who maximize profits and thereby act as auctioneers. They tie all the markets together, even if they are restricted to trade in subsets of the available assets (refer to Remark 6).

Still, we did not claim that all prices converge to their shadow values at the same rate (even if entry costs do). This allows us in Section 6 to derive predictions as to which markets should be expected to be more integrated, and which markets may exhibit asset pricing "puzzles".

5.4 Barriers to Integration

In the present subsection we analyze two major cases where more competition does not lead to integrated markets. In Proposition 6 LCM fails (even though markets may be *globally* complete), and in Proposition 7 below we show that when there are assets that no arbitrageur is allowed to trade, more competition might actually lead to less integration than in the absence of arbitrageurs.

Proposition 6 *If LCM does not hold, CWECE need not converge to integrated equilibria when $c \to 0$ (regardless of whether $\beta^t > 0$ or $\beta^t = 0$). In particular, $\lim_{c\to 0} n\Pi_g^t > 0$ is a possibility. Even though $\lim_{c\to 0} \Pi_g^t = 0$ (i.e. absence of strategic arbitrage), there may still be an arbitrage.*

Furthermore, in that case limiting CWECE do not converge to Walrasian equilibria with restricted participation.

Notice that the reason for not converging to Walrasian equilibria is different from the one in Roberts (1980). There, a critical point in the excess demand function caused the oligopolists to restrain output. Here, we derive a similar result even without such critical points.

The following (robust) example proves the proposition (for the case $\beta^t > 0$, it is trivial to construct an example for the case $\beta^t = 0$).

Example 1 (Endogenous Barrier to Integration) *The example consists of a basis economy with two exchanges and two assets, one traded on each exchange:*

$$R^1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, R^2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Investors on both exchanges have quadratic quasi-linear utility functions, $u_s^i(x) = \alpha^i x - \frac{\beta^i}{2}x^2$*,* $s \geq 1$*, and* $u_0^i(x) = x$*. Equilibrium prices are given by* $w^i(y^i) = \hat{q}^i - (\delta^i)^{-1}y^i$*, where* $\hat{q}^i \equiv \alpha^i E[d_s^i] - \beta^i E[\omega_s^i d_s^i] = w^i(0)$*, the standard CAPM relation and where the depth* δ^i *equals* $(\beta^i E[(d_{a,s})^2])^{-1}$*. The discount factor is* $\beta^t = 1$*, and the optimal supply of assets of an arbitrageur is* $y^{ti} = \frac{\delta^i}{1+n}(\hat{q}^i - \sigma^{ti})$*. As usual, we assume that all arbitrageurs are identical, and we drop the superscript "t" on the multipliers and on the shadow prices.*

We design the preferences and endowments such that $w^1(0) > w^2(0)$*, an arbitrage.*[6] *We then let c tend to zero and we record the equilibrium implications. As c diminishes enough, more and more arbitrageurs enter the business and prices converge to a common value (refer to Figure 1). In the simulations this happens for* $n = 9$*. At that n the multiplier configuration switches from case* 1*, which corresponds to a situation where* $\phi = 0$*,* $\gamma_1 > 0$ *and* $\gamma_2 = 0$*, to case* 2*, which corresponds to* $\phi > 0$*,* $\gamma_1 > 0$ *and* $\gamma_2 = 0$*. Cases other than* 1 *and* 2 *do not occur in this simulation. This can be glanced on Figure 2 where we depict the aggregate consumption evolution of the arbitraging sector (recall that if* $\gamma_s > 0$ *then* $x_s^t = 0$*).*

If c falls further, n rises again, but the aggregate supply of assets is unaffected. Indeed, the larger number of arbitrageurs would like to buy and sell more in aggregate, but given that both prices coincide and that the market-impact is nonnegligible, buying more of security 2 and selling more of security 1 leads to a situation that violates the time-zero self-financing constraint because it would induce $w_2 > w_1$*. They would have to sell more of asset 2 than they buy of asset 1 to satisfy the time 0 self-financing constraint, which in turn violates the self-financing constraint in state 1.*

Due to the fact that no exchange has a complete (enough) set of assets, arbitrageurs find no way around the time zero self-financing constraint. The

[6] *Parameters are such that* $\hat{q}^1 = 4.72$ *and* $\hat{q}^2 = 3$*, and endowments are* $\omega^1 = (2, .1, 2)$ *and* $\omega^2 = (2, 1, 1)$ *Consumptions are always below their respective bliss points,* 2.0667 *for* $i = 1$ *and* 4 *for* $i = 2$.

existing assets do not allow arbitrageurs to short just a little of some asset in order to get around this constraint, while having enough degrees of freedom to repay this loan in the future. This is reflected in the fact that indeed ϕ blows off as the number of players grows. This in turn implies that our limiting candidate for a state-price vector, λ, is not strictly positive.

The aggregate supply of assets is constant after $n = 9$ and independent of c and n. This implies that investors' consumptions are independent of c and n as well. The larger number of competitors simply split the same pie, and each one gets a smaller portion. It follows that (competitive) arbitrage opportunities do not vanish, not even in the limit (refer to Figure 2).

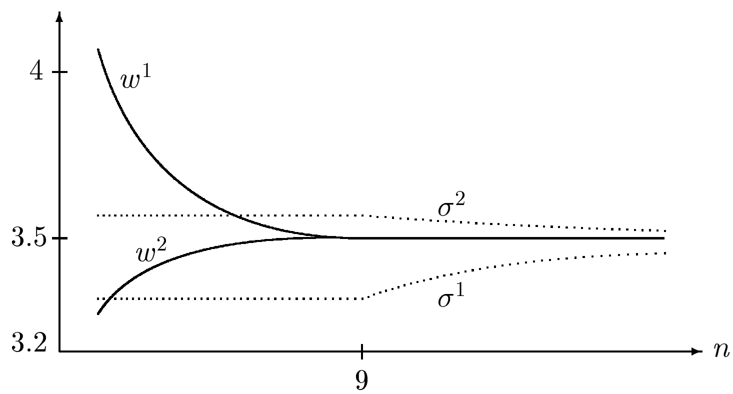

Figure 1: Prices in an Economy Violating LCM

We gather the relevant simulation results in table 1 in Appendix B.

Remark 7 (On Welfare) *In this example, welfare is strictly increasing for each clientèle group up to $n = 9$, and constant thereafter. The intuition is simple: the fact that some states or some assets are overpriced on some exchange reflects the fact that the local investors would like to trade these assets, but can't. Arbitrageurs then act as intermediaries and allow the welfare improving trades. In order to induce investors to trade away from the autarcy point with them, arbitrageurs have to bid prices in favour of the investors (by assumption DSD). Even in cases where arbitrageurs manipulate markets, we find that welfare is improved (these results are briefly mentioned in the conclusion). However, a general theorem is not available.*

One reason not to push welfare results is the dependency on the representative investor assumption within each clientèle class. If investors' pref-

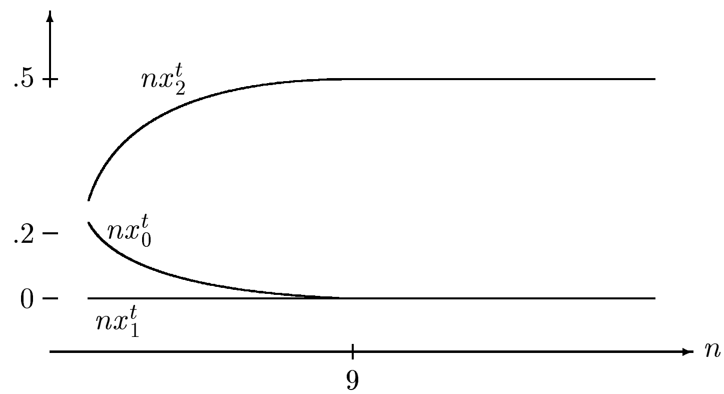

Figure 2: Aggregate Arbitrageur Consumption in an Economy Violating LCM

erences and endowments are sufficiently diverse on exchange i, say, then it is likely that the arbitrageurs compete with some investors on that exchange who thereby lose out on the opening of markets.

Remark 8 (On Arbitrageur Capital) *Let us contrast these results with those arising in an economy where arbitrageurs own endowments at time zero (i.e. capital). We thus add a further parameter to the economy representing the arbitrageur's time zero endowment in the consumption commodity, denoted by $\omega_0^t(n)$, and raise the question whether the barrier to integration vanishes in case $\omega_0^t(n) > 0$. We denote aggregate arbitrage capital by $\omega_0(n) \equiv n\omega_0^t(n)$. The results depend on the magnitude of $\omega_0(n)$ for n large.*

If $\omega_0(n)$ is very small (at least for large n), then the limiting results are again quantitatively very similar to the ones where $\omega_0^t(n) \equiv 0$, all n. For instance, say that $\omega_0 = .00001$, all n. It turns out that $\phi \to 10117 < \infty$, so that in effect there is no arbitrage in the limit any longer and the economy becomes integrated. Indeed, the economy starts out again in the multiplier configuration 1 and switches to 2 at $n = 9$ (as before), but $w^1(9) = 3.505868 < 3.505888 = w^2(9)$. If $\lim_{n\to\infty} \omega_0(n)$ is large, say equal to 1, then the switch never occurs. Each arbitrageur has enough endowments to always consume at zero and $\phi = 0$, no matter how high n is.

As a general rule, the smaller ω_0, the smaller the n at which the multipliers switch and the higher ϕ. As $\omega_0(n)$ becomes small, at least for all large n, we recover the results from the case without endowments.

As an illustration, Figure 3 depict the situation where $\omega_0 = .01$, all n.

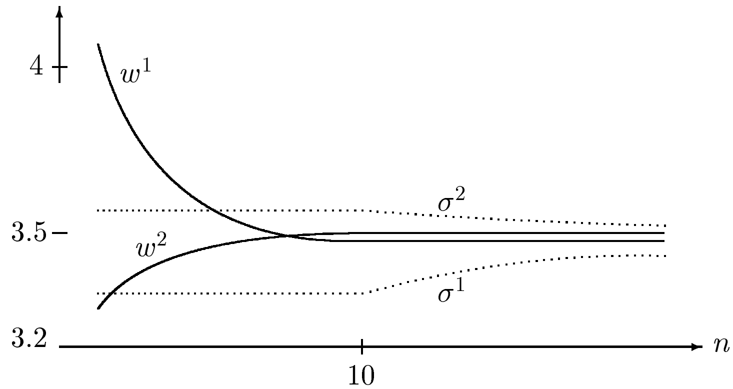

Figure 3: Prices in an Economy where Individual Arbitrageurs have Small Endowments but where Aggregate Endowments are Nonnegligible

We want to insist again that the vast majority of the results in this article hold if arbitrageurs have (nonzero) endowments, only the interpretations of these propositions are less clear. The reader can then simply replace the qualification LCM in the statements by the assumption that $\lim_n n\omega_0^t(n) > 0$.

We now turn to a basis economy where each arbitrageur is specialized in a single asset. This problematic is inspired from the observation that when analyzing international financial mark *imal in the limit* $\left(\frac{\partial m}{\partial y_a^{t1}}=0\right)$ *as* $|c|\to 0$ *since the Law of One Price will eventually hold.*

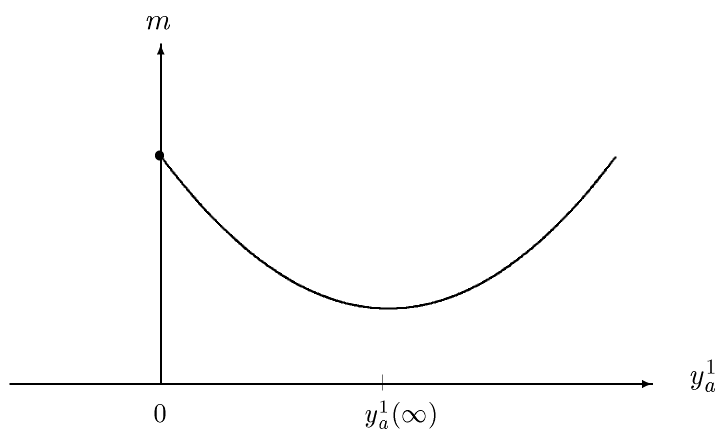

Figure 4: Integration when Arbitraging is Restricted to a Subset of the Available Assets

We can guarantee eventual complete integration if $d_{a,s}=\pi_s^1(0)-\pi_s^2(0)$. *The interpretation is that it basically allows each exchange to get its most-valued asset in exchange of giving the other exchange their favourite one in return. This trading of autarky state-price deflators brings the actual state–price deflators in line. This fact crucially relies on quadratic preferences.*

That arbitrage in one asset always suffices to increase integration is clearly wrong, as the following example suggests.

Example 3 *Assume that markets are complete, that preferences are given by* $u_s^i(x)=-\delta_s^i e^{-x}$ *and that asset a is a bond. Then* $\frac{\partial m(y_a^1)}{\partial y_a^1}=-2\sum_s p_s\left[(u_s^{1\prime})^2 -(u_s^{2\prime})^2\right]$. *This term is strictly positive if* $u_s^{2\prime}$ *has a higher second moment than* $u_s^{1\prime}$. *Indeed, assume endowments and preferences are such time 1 marginal utilities satisfy* $u^{1\prime}=(2,2)$, $u^{2\prime}=(3,1)$ *and* $p=(2/5,3/5)$. *Then* $w^1-w^2=\frac{1}{5}>0$, *but* $\frac{\partial m(y_a^1)}{\partial y_a^1}=\frac{2}{5}>0$.

In general the relationship between arbitraging in a small subset of assets and the level of integration is ambiguous.

6 Asset Pricing Implications

The first two subsections analyze some of the implications of our model as to the pricing of *new* and redundant assets. Due to the restricted participation structure, there is a demand for matched-book derivatives which rationalizes the observation that innovators are indeed ready to invest up to $5M for such an innovation.

We then move on to answer the question of whether options and futures exchanges are truly derivative of the stock exchanges, or whether it is the other way round. Finally we investigate the claim that deeper markets are typically more integrated than relatively shallow but otherwise identical ones.

6.1 No-Arbitrage Pricing Versus Equilibrium Pricing

Barring private information economies or economies with sunspots, a redundant asset (defined here as an asset for which there is a replicating portfolio in the economy) can in frictionless economies be priced by no-arbitrage (i.e. by replicating portfolio considerations), leaving the state-prices unaffected. This need not be the case in intermediated economies, basically because introducing assets that are redundant from an economy-wide viewpoint may create new arbitrage opportunities when they are traded locally.

Proposition 8 (Derivative Pricing) *Assume that LCM holds and that $c = 0$. Assume the intermediaries introduce on exchange i an asset a that is redundant at the economy-wide level, but that is not redundant on i. Then generically the price at which a is traded on i is not equal to the no-innovation price of the replicating portfolio.*

Arbitrageurs will take advantage of this additional mispricing, thereby affecting both λ and λ^i, which in turn affects w and (typically) σ. We showed that we cannot reason as if the prices of the assets in the replicating portfolio were given, because these very prices (generically) get perturbed by the financial innovation. This concludes our claim that we cannot price redundant assets precisely without recomputing the general equilibrium. Of course, at the new equilibrium the price of the innovated asset equals the new price of the replicating portfolio (since we assumed free entry).

On one hand, this removes much of the simplicity of traditional no-arbitrage pricing. On the other hand, it allows us to rationalize the huge market of matched-book intermediation. Indeed, if the price of such a derivative equalled the value of the respective replicating portfolio, these companies would in general (absent off-balance sheet considerations and the like) have little incentive to issue them. The following subsection illustrates this idea.

6.2 Derivative Pricing and Detemple and Selden's Result

The contribution of Detemple and Selden (1991) was to show that, in economies with incomplete markets, the introduction of nonredundant securities typically affects equilibrium prices and allocations. In particular, they showed that the stock is more valuable in the presence of an option. They point out that this violates the partial equilibrium intuition that investors reduce their demand for the stock and instead purchase some of the new option, thereby lowering the price of the stock. Their predictions have been corroborated in the empirical literature.

Here we replicate this result for any derivative security, even when globally redundant (but not locally). A financial innovator, really an arbitrageur, introduces the asset and sells it on the exchange where it captures the highest price, replicating it cheaply on some other exchange, thereby raising the price of the replicating assets.

For the sake of concreteness, let there be two exchanges, labelled 1 and 2. Two assets are traded on the first exchange, a and b. We can think of security a as being a share. Now introduce a derivative asset on the second exchange, and assume that it can be replicated using the two assets on exchange 1: there is $(\varphi_a, \varphi_b) \in \mathbb{R}^2$ such that $d^2_{c,s} = \varphi_a d^1_{a,s} + \varphi_b d^1_{b,s}$, all $s \in S$. Assume, as in Detemple and Selden, that $R^i \geq 0$, $i = 1, 2$ (limited liability).

Buying one unit of the synthetic asset c on exchange 1 amounts to holdings of $y^1_a = \varphi_a y^1_c$ and $y^1_b = \varphi_b y^1_c$. Typically, the innovators (members of the exchange) are the writers of the option, so that we assume that $w^1_c(0) < w^2_c(0)$, prompting supplies of $y^1_c < 0$ and $y^2_c > 0$. The desired result follows if it was true that $\frac{\partial w^1_a}{\partial y^1_c} < 0$.

For simplicity, assume that preferences are quasi-linear. State-prices are then $\lambda^1_s(y^1_c) = p_s \ u^{i\prime}_s(\omega^1_s + y^1_c d^2_{c,s})$. Since $\frac{\partial \lambda^1_s}{\partial y^1_c} = p_s u^{1\prime\prime}_s d^2_{c,s} \leq 0$ and strictly negative in some states, it follows that $\frac{\partial w^1_a}{\partial y^1_c} = \sum_s \frac{\partial \lambda^1_s}{\partial y^1_c} d^1_{a,s} < 0$. The intuition is that the innovator induces investors of type 1 to sell more securities promising positive payoffs tomorrow. This causes type 1's marginal utilities tomorrow to increase, which gets reflected in higher share prices today.

This result is coherent with a partial equilibrium intuition if the derivative is a positive delta asset ($\varphi_a > 0$), for then we buy asset a in order to replicate asset c and the price of asset a ought to rise. However, interestingly even if $\varphi_a < 0$, q_a^1 will rise as long as $R^i \geq 0$.

6.3 Fundamental Values

Both in partial equilibrium and in arbitrage analysis, the price of the replicating portfolio consisting of the underlying assets is taken as the "fundamental" value of the payoff. This is of course at best an approximation. As has already been pointed out by Miller (1997),

> *One can make a case these days for saying that the stock market is really derivative of the index futures and options exchanges. (...) Derivative markets have won the role of pricing Equity away from the traditional stock exchanges.*

For sake of concreteness assume that there are only two exchanges. We also assume that the arbitrageur trades the same payoff across different exchanges, and that β^t is low enough. The fundamental value of the payoff is given by $\sigma_a = \sum_i \varpi_a^i w_a^i$. The derivative is traded on the first exchange while the underlying is traded on the second exchange. If it is also true that the market where the derivative is traded is shallow, then $|\sigma_a - w_a^2|$ is small (by the very definition of σ_a). Thus, we can approximate the shadow value by the value of the replicating portfolio on exchange two. However, if it is true that the derivatives market is more "liquid" (deeper) than the underlying market, then in the limit for a (relatively) very shallow cash asset market, the shadow value of the replicating portfolio is actually best approximated by the price of the derivative. This argument would be reinforced by the asymmetries of information between traders.

6.4 Depth, Depth-Related Puzzles and Integration

Most asset combinations hardly admit any arbitrage profits, but examples abound where some assets momentarily exhibit puzzling mispricings. The depth of markets plays a major role in such puzzles. In this subsection we analyze the claim that the depth of a market determines the extent of integration. We argue that deeper markets lead to more integration because arbitrage profits are higher and attract more arbitrageurs.

For simplicity arbitrageurs can only arbitrage a single asset a across two exchanges. Furthermore, we assume that arbitrageurs are either of the short-horizon type ($\beta^t = 0$) or that asset payoffs are such that there are states (s, s') with $d_{a,s} > 0$ and $d_{a,s'} < 0$. Either of these assumptions will insure at a CWECE that $\sum_i y_a^t = 0$. Preferences allow the representation $u_s^i(x) = \alpha^i x - \frac{\beta^i}{}$

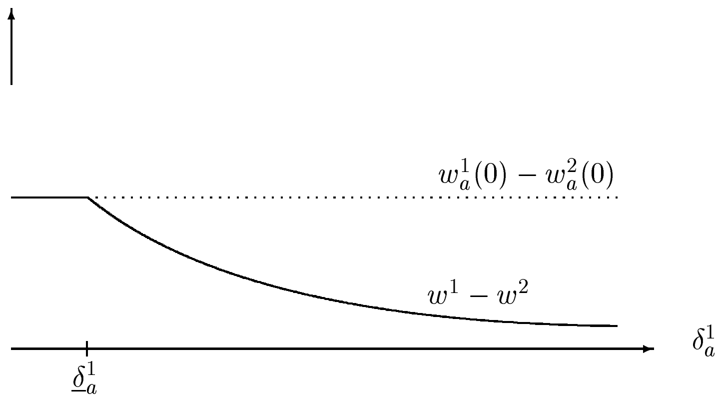

Figure 5: The Effects of Depth on Mispricings

both effects exactly cancel each other out. However, when we do allow for entry, more arbitrageurs choose to enter for the same magnitude of the mispricing. The extensive margin thus induces the quantity effect to dominate, leading to smaller mispricings at equilibrium. It is also easy to verify that the hyperbola shifts down if either c is diminished or if δ_a^2 is raised.

There is an abundance of evidence that deep markets exhibit less arbitrage opportunities than shallow markets. Good illustrations of our predictions satisfy two conditions: the arbitrage is unwound fast (because our model ignores dynamic rebalancing) and the *same* market goes through periods of differing depths. Hemler and Miller Jr. (1997) provide such an example. Typically, arbitrage opportunities in box-spreads exist only for agents having the lowest transaction costs (i.e. registered traders) who implement the strategy expeditiously. Their main findings concentrate on the period around the 1987 Crash. The pre-crash sample from 9.1.87 to 10.15.87 contained only few apparent box-spread arbitrage opportunities, and they vanished in less than a minute. However, the post-crash sample from 10.26.87 to 11.30.87 exhibits numerous apparent arbitrage opportunities, even assuming 5 minute lags. This situation lasted for 3 weeks. They provide statistics that show that the frequency and the size of mispricings indeed vary with the shallowness of the markets. In this example, c denotes the opportunity costs of traders during these times.

The literature on international finance also abounds in examples that illustrate this point. For instance, Demirgüç-Kunt and Levine (1995) analyze the empirical relations between extent, liquidity and volatility of domestic stock exchanges and international integration. The integration statistics are taken from Korajczyk (1996). They find that market liquidity is significantly negatively correlated with risk mispricing and volatility.

7 Conclusion

We constructed a simple two-period general equilibrium model where investors need not have the opportunity to trade all of the existing assets. This allows us to capture two related aspects of financial markets. First, often only registered traders have the opportunity to arbitrage. Second, the segmentation assumption can be interpreted as a need for intermediation (say via OTC transactions).

Because the assumed frictions give rise to arbitrage opportunities (maybe in the form of intermediation profits such as bid-ask spreads), they will be seized by profit maximizing arbitrageurs who thereby (partly) undo the frictions. Intermediaries earn profits related to the size of the (possibly discriminatory) bid-ask spreads that are determined endogenously. One of the motivations of this paper is to provide market microfoundations that can explain why markets are typically integrated. The claim is that most markets exhibit few arbitrage opportunities precisely because there is a lot of arbitraging. Indeed, we prove that integration rises with competition. In the limit, arbitrageurs act as intermediaries, thereby replacing the direct market link that was missing. For instance, assume that $A^i = A$, all $i \in I$. Intermediaries than clear markets when investors cannot directly engage in bilateral trades, and the trade flows thereby induced exactly coincide with the ones generated on a common Walrasian auction.

Still, since the level of integration arises endogenously at a general equilibrium, integration need not rise uniformly as entry costs drop. Typically, equilibrium will simultaneously exhibit a lot of integration on deep markets and less integration on shallow ones. We do not need to assume that transaction costs are higher on shallow markets to derive this result as the levels of mispricing are derived endogenously in our model.

Arbitraging also improves market liquidity as well as investors' hedging opportunities. More precisely, we show that if entry costs become small, the arbitraged allocation converges to the Walrasian allocation of an economy with restricted participation (under certain conditions). Besides taking advantage of mispricings, arbitrageurs can also manipulate markets in a certain sense and under some conditions. We discussed a definition of manipulation, which did not rely on private information or on a long horizon. While it was argued that this definition was too strong, we can reinterpret some phenomena as manipulative behaviour. One can indeed easily construct examples where arbitrageurs can *profitably increase existing mispricings* by exploiting strategic complementarities across assets. Assume that asset a gets traded on exchanges 1 and 2, and that $w_a^2(0) \gg w_a^1(0)$. Then one can construct economies where arbitrageurs find it profitable to purchase some asset b traded on exchange 2, in order to increase $w_a^2 - w_a^1$ (and of course the arbitrageurs also get the benefits of consuming the proceeds), even though that same asset b might be traded at a lower cost on some third exchange 3. In other words, the extent of manipulation can be measured by the premium they pay on asset b, $w_b^2 - w_b^3$: this measures both how much more they pay, and the arbitrage profit margin they forego by not arbitraging that mispricing away as well. Results from a simple simulation are available from the author upon request. In those, the welfare of all investors increases with a drop in c. What looks in effect like manipulation by informed intermediaries with market power need however not be detrimental to individual investors because of the gains from diversification.

We argue that modelling the noncompetitive foundations of market microstructure is essential to the understanding of a number of phenomena. First, from an empirical point of view, we showed that depth-related puzzles can very easily be explained in this setup. Second, our model accounts for a number of cross-exchange and cross-country phenomena. For instance, Koch and Koch (1991) analyze the dynamic linkages of stock indices. They find that markets grew more interdependent since 1972, but that this increasing interdependence is concentrated among countries in the same geographical region, whose trading hours overlap.

Third, we showed that the market microstructure matters. Even when the equilibrium allocations of the intermediated economy are observationally equivalent to the ones of a frictionless competitive economy, the asset pricing implications can still be quite different. For instance, it is not true that the price of a redundant asset equals the price of its replicating portfolio if the agents who replicate behave strategically.

To summarize, we provided a natural and tractable framework to study a variety of problems in a unified way. Two problems that have not been addressed here and that warrant further research are the following. First, we would like to model the implications of asymmetric information and interlinked markets on systemic risk and on crises. Secondly, and relatedly, interesting price dynamics may exist in models allowing for dynamic arbitraging and manipulation.

A Proofs

Proof of Lemma 2 $\mathcal{Q}^i$ denotes the set of no–arbitrage prices on exchange i and will be defined in greater detail in the next subsection.

We now decompose the effects of a change in prices into a substitution and an income effect. Assume consumer i finds himself in state s at time 1. His Lagrangian multiplier attached to the budget constraint in state s is γ_s^i. Denote his income (other than his endowments) by $t_s^i \equiv \sum_{a^i} d_{a^i,s}^i \theta^i$. Standard arguments reveal that $\alpha_s^i \equiv -\frac{\partial \gamma_s^i}{\partial t_s^i} > 0$ (due to the concavity of the Von-Neumann–Morgenstern utility functions). Given such a relation in each state s, we turn to the consumer's problem at time zero. The first-order conditions with respect to asset demands θ^i, the budget constraint and the first-order conditions with respect to excess demands of state-contingent consumption z_s^i can be written respectively as

$$\begin{aligned} 0 &= -\gamma_0^i q^i + \sum_{s\geq 1} \gamma_s^i {d_s^i}' \\ t_0^i = 0 &= {q^i}'\theta^i + z_0^i \\ 0 &= {u_0^i}' - \gamma_0^i \end{aligned}$$

Totally differentiating this system and using the definition of α^i and the observation that $dt_s^i = d_s^i d\theta^i$ leads to the system of equations

$$\begin{bmatrix} {u_0^i}'' & -1 & 0 \\ -1 & 0 & -{q^i}' \\ 0 & -q^i & -\sum_s \alpha_s^i {d_s^i}' d_s^i \end{bmatrix} \begin{bmatrix} dz_0^i \\ d\gamma_0^i \\ d\theta^i \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ {\theta^i}' & -1 \\ \gamma_0^i I & 0 \end{bmatrix} \begin{bmatrix} dq^i \\ dt_0^i \end{bmatrix}$$

Defining the inverse matrix

$$\begin{bmatrix} \ell^i & -\nu^i & {\mu^i}' \\ -{\nu^i}' & \alpha_0^i & -{v^i}' \\ \mu^i & -v^i & K^i \end{bmatrix} \begin{bmatrix} {u_0^i}'' & -1 & 0 \\ -1 & 0 & -{q^i}' \\ 0 & -q^i & -\sum_s \alpha_s^i {d_s^i}' d_s^i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & I \end{bmatrix}$$

This definition implies, among others, the relations

$$\mu^i {u_0^i}'' + v^i = 0 \tag{5}$$

$$-\mu^i - K^i q^i = 0 \tag{6}$$

$$v^i {q^i}' - K^i \sum_s \alpha_s^i {d_s^i}' d_s^i = I \tag{7}$$

as well as $v^i = -{u_0^i}'' \mu^i$ and $\mu^i = -K^i q^i$ which imply

$$v^i = {u_0^i}'' K^i q^i \tag{8}$$

First we have to show, however, that an inverse exists. Define the matrix

$$\Theta \equiv \begin{bmatrix} {u_0^i}'' & -1 & 0 \\ 1 & 0 & {q^i}' \\ 0 & -q^i & -\sum_s \alpha_s^i {d_s^i}' d_s^i \end{bmatrix}$$

Assume that there is a $z \equiv (z^1, z^2, z^3)$ such that $\Theta z = 0$. Then $z'\Theta z = 0$ as well, implying that $z_1^2 {u_0^i}'' - z_3' \big(\sum_s \alpha_s^i {d_s^i}' d_s^i\big) z_3 = 0$, so that $z_1 = 0$ and $z_3 = 0$. Since we also have $\Theta z = 0$, we know that $z_1 {u_0^i}'' - z_2 = 0$, i.e. $z_2 = 0$. It follows that Θ is invertible so that, by an application of the implicit function theorem, demand functions are ^{2} if utility functions are ^{3}.

Using the usual notation $\theta^i = f^i(q^i)$, we solve the system for $\partial_{q^i} f^i$,

$$\partial_{q^i} f^i = \gamma_0^i K^i - v^i {\theta^i}'$$

It is apparent that v^i represents the vector of income effects. K^i is evidently symmetric. That it is negative definite as well will be shown next. Notice that the matrix $\sum_s \alpha_s^i {d_s^i}' d_s^i = {R^i}' \mathrm{diag}(\{\alpha_s^i\}_{s \geq 1}) R^i$ is positive definite since R^i is of full column rank and $\alpha_s^i > 0$, $s \in \{1, \ldots, S\}$.

First, assume that $K^i y = \beta y$, $\beta \neq 0$. Then ${q^i}' K^i y = \beta {q^i}' y = -{\mu^i}' y$ by (6). Using (7), we know that $y' v^i {q^i}' y - \beta y' (\sum_s \alpha_s^i {d_s^i}' d_s^i) y = y'y > 0$. The first expression $y' v^i {q^i}' y$ can be rewritten, using (5), as $\beta^{-1} {u_0^i}'' (y'\mu^i)^2$. Gathering all these results,

$$\beta^{-1} \underbrace{\left[{u_0^i}''(y'\mu^i)^2\right]}_{<0} - \beta \underbrace{\left[y' \left(\sum_s \alpha_s^i {d_s^i}' d_s^i\right) y\right]}_{>0} = \underbrace{y'y}_{>0}$$

from which we can deduce that $\beta < 0$.

Second, assume that there is a zero eigenvalue: $K^i y = 0$, $y \neq 0$. From (7) $y' v^i {q^i}' y - y' K^i (\sum_s \alpha_s^i {d_s^i}' d_s^i) y = y' y$ it follows that $y' v^i {q^i}' y = y' y$. Also, (6) tells us that $-\mu^i = K^i q^i$, so $y' \mu^i = 0$ by the assumption of a zero eigenvalue. Inserting the results thus derived into (5), $\underbrace{y' \mu^i {u_0^i}''}_{=0} + y' v^i = 0$ entails that $y' v^i = 0$. But since $y' v^i {q^i}' y = y' y > 0$, this is impossible. This completes the proof for we showed that any eigenvalue of the symmetric matrix K^i is strictly negative.

Finally, we show how the coefficient of absolute risk aversion influences negative quasidefiniteness. From (8) we know that $v^i = {u_0^i}'' K^i q^i$. Using ${u_0^i}' = \gamma_0^i$,

$$\partial f^i(q^i) = {u_0^i}' K^i [I + r^i q^i {\theta^i}']$$

In particular, assume that preferences are quasi-linear, i.e. ${u_0^i}'' \equiv 0$. Then from (5) we can deduce that indeed $v^i = 0$ and $r^i = 0$. ■

Proof of Lemma 3 (Characterization of the Price Correspondence)
(i) For a fixed $y^i \in \mathcal{Y}^i$ the price correspondence is given implicitly by the set $w^i(y^i) \equiv \{q^i : q^i = {R^i}' IMRS^i(y^i, q^i)\}$ so that, writing $q^i = w^i(y^i)$ for some element of $w^i(y^i)$ and $w_b^i(y^i)$ for its b^{th} coordinate,

$$w_b^i(y^i) = \sum_s p_s d_{b,s}^i \frac{{u^i}'_s(\omega_s^i + \sum_a y_a^i d_{a,s}^i)}{{u^i}'_0(\omega_0^i}$$

determines $\kappa(y^i) \in \mathbb{R}$. Notice that the LHS does not depend on the particular choice of y^i_a given m^i. The Inada condition of unbounded marginal utility as consumption at time zero goes to zero together with the monotonicity assumption H(i) imply that the range of the LHS as a function of m^i is $\mathbb{R}$. It follows from the intermediate value theorem that for every $y^i \in \mathcal{Y}^i$ there is at least one $m^i \in \mathbb{R}$ such that the LHS$= \kappa(y^i)$. This in turn implies that for every choice of $y^i \in \mathcal{Y}^i$ by the arbitrageurs there exists an equilibrium asset price vector on i. This shows that indeed $\mathcal{Y}^i \subseteq \mathcal{J}^i$. The relation $\mathcal{J}^i \subseteq \mathcal{Y}^i$ is definitional.
(ii) This result is shown separately in Lemma A.1 below.
(iii) We now show that for each y^i, there is at least one element of $w^i(y^i)$ (which we denote for simplicity $w^i(y^i)$ as well) that is ^{2} in a neighbourhood of y^i.

Totally differentiating $m^i {u^i_0}'(\omega^i_0 - m^i) = \kappa(y^i)$, we get that

$$\partial_{y^i} \mathcal{M}^i(y^i)^i = \frac{1}{{u^i_0}'(\omega^i_0 - m^i) - m^i {u^i_0}''(\omega^i_0 - m^i)} \left(\partial_{y^i} \kappa(y^i)\right)'$$

If the denominator is different from zero (which we know has to be true for at least some element m^i), then $m^i = \mathcal{M}^i(y^i)$ is ^{2}. It follows from equation (9) that $w^i(y^i)$ is itself ^{2} around such points.
(iv) Given that ∂f^i is nonsingular, f^i is a local diffeomorphism. It is also evident that f^i is one-to-one (any ^{1} function with a Jacobian that is negative quasidefinite in a convex set is one-to-one). It follows that f^i is a diffeomorphism. Since $w^i(y^i) = {f^i}^{-1}(y^i)$, so is w^i and $\partial w^i = (\partial f^i)^{-1}$. ■

Proof of Proposition 1 (Nonequivalence of NA and of NSA) (i) Indeed, assume there are two identical exchanges with $\partial w^i(0)$ not negative quasi-semidefinite, $i = 1, 2$. In other words, there is a (small) $\underline{\epsilon}$ such that $\underline{\epsilon}' \partial w^i(0) \underline{\epsilon} > 0$. Of course, there is no (competitive) arbitrage if $y = 0$, and the equilibrium is a Walrasian equilibrium of the integrated economy. Now let $dy^1 = \underline{\epsilon}$ and $dy^2 = -\underline{\epsilon}$. This strategy generates profits of $\Pi^t = \left(w^1 - w^2\right)' \underline{\epsilon} - c$. For small $\underline{\epsilon}$, $w^i(\underline{\epsilon}) \approx w^i(0) + \partial w^i(0)\underline{\epsilon}$, so that $\Pi^t \approx 2\underline{\epsilon}' \partial w^1(0) \underline{\epsilon} - c > 0$, an arbitrage profit (if entry costs are low enough).

(ii) The basic result we need is the following. If DSD holds, then $y'w(Y) \geq y'w(Y + y)$, for all y. Indeed, by the mean-value theorem (applied to the function $R(\mu; y) \equiv y'w(Y + \mu y)$, $\mu \in \mathbb{R}$) there is a $\theta \in (0, 1)$ such that $y'w(Y + \mu y) = y'w(Y) + y'\partial w(Y + \theta \mu y))y < y'w(Y)$ for $y \neq 0$ and for $\mu > 0$, and in particular for $\mu = 1$.

First assume that $y \neq 0$ is such that $Ry \leq 0$. By NA we know that

$y'W(Y) \leq 0$, and by the above result that $y'W(Y+y) < 0$. Hence there is NSA of type 2.

Second, assume that y is such that $Ry < 0$. Similarly we then have that $y'w(Y+y) < 0$, eliminating strategic arbitrages of type 1 as well.

(iii) is a trivial result. ■

Lemma A.1 (Boundary Suboptimality) *It is without loss of generality to restrict arbitrageur t's actions (given an investment feasible y^{-t}) to lie in a compact set $\bar{\mathcal{Y}}^i(-t)$. The boundary of that set cannot contain arbitrageur t's optimal actions. At a Nash equilibrium aggregate actions must lie in the interior of a compact set $\bar{\mathcal{Y}}^i$, int $\bar{\mathcal{Y}}^i \subseteq \mathcal{Y}^i$.*

Proof :
Denote $\omega_s^i + y^{-ti} \cdot d_s^i$ by $\omega_s^i(-t)$, which is the total amount of consumption commodity available to i in state s, given y^{-ti}. We prove first that we can impose on each arbitrageur that his actions have to lie in a compact set, given that y^{-t} is investment feasible. Define $K_s^i(-t) \equiv \left[-\omega_s^i(-t), \sum_{j \neq i} \omega_s^j(-t)\right]$.

(a) First pick a sequence of supplies $\{y^t\}$ such that $\lim y^{ti} \cdot d_s^i = -\omega_s^i(-t) < 0$ for some states s. The set of such states is denoted by $\underline{S}^i$. That $\omega_s^i(-t) > 0$ follows from the assumption that y^{-t} be investment feasible. For each such $s \in \underline{S}^i$ the IMRS grows without bound, $IMRS_s^i \equiv \frac{u^{i\prime}_s}{u^{i\prime}_0} \to +\infty$. The reader can readily verify that the arbitrageur's consumption gotten from exchange i satisfies $x_0^{ti}(m) = \sum_{s \in S \setminus \underline{S}^i} (y^{ti}(m) \cdot d_s^i) IMRS_s^i(m) + \sum_{s \in \underline{S}^i} (y^{ti}(m) \cdot d_s^i) IMRS_s^i(m) \to -\infty$. This follows from the fact that the first term is bounded and that y^{-t} is investment feasible, so that $\lim_{m \to \infty} y^{ti}(m) \cdot d_s^i = -\omega_s^i(-t) < 0$. Because $\sum_{j \neq i} x_0^{tj} \leq \sum_j \omega_0^j < \infty$, $x_0^t = \sum_i x_0^{ti} \to -\infty$. This argument provides us with lower bounds on the amount arbitrageur t can transfer to i in s, i.e. $y^{ti} \cdot d_s^i \geq -[\omega_s^i + \sum_{a^i} y_{a^i}^{-ti} d_{a^i,s}^i] \equiv -\omega_s^i(-t)$. Such a bound must hold for all $s \in S$.

(b) Second, assume we pick a sequence $\{y^t\}$ such that $\lim y^{ti} \cdot d_s^i = \sum_{j \neq i} \omega_s^j(-t)$, for some s. The set of such states is denoted by $\bar{S}^i$. Then $\lim y^{tj} \cdot d_s^j = -\omega_s^j(-t)$, all $j \neq i$. By (a) this cannot be optimal.

We can conclude that given an investment feasible y^{-t}, for all $s \in S$ we know that $y^{ti} \cdot d_s^i \in K_s^i(-t)$. In vector notation, $R^i y^{ti} \in \prod_{s \in S} K_s^i(-t) \equiv K^i(-t)$, where $K^i(-t)$ is compact. We need to show that this restricts the set of asset supplies (call this new set $\bar{\mathcal{Y}}^i(-t)$) to be compact as well, with $R^i \bar{\mathcal{Y}}^i(-t) = K^i(-t) \cap (R^i \mathbb{R}^{A^i}) \equiv K_y^i(-t)$, the right-hand side being nonempty (both intersecting sets own zero), closed and bounded.

It follows from the injectivity of the linear function that $\bar{\mathcal{Y}}^i(-t)$ is compact as well. Indeed, assume there is a sequence $\{y^i(m)\}$ such that $||y^i(m)|| \to \infty$. By the compactness of $K^i(-t)$, there is a converging subsequence of $\{R^i y^i(m)\}$, and assume for simplicity that this subsequence is the sequence itself, converging to an element $k^i \in K_y^i(-t)$. That is, we have $\lim_{m\to\infty} R^i y^i(m) = k^i$. By the full rank assumption of R^i, the rank of $R^{i\prime} R^i$ is A^i, and it follows that $y^i(\infty) = \left(R^{i\prime} R^i\right)^{-1} R^i k^i$, which contradicts the assumption that $||y^i(m)|| \to \infty$. We can conclude that the preimage of $K_y^i(-t)$ (i.e. $\bar{\mathcal{Y}}^i(-t)$) is closed (by continuity) and bounded.

We verify that $y^{ti} \in \partial\bar{\mathcal{Y}}^i(-t)$ can never be an *optimal* strategy for t. Given y^{-t}, let there be a sequence $\{y^{ti}(m)\}_{m=1}^{\infty}$ converging towards the boundary of $\bar{\mathcal{Y}}^i(-t)$. Because $R^i(\partial\bar{\mathcal{Y}}^i(-t)) = \partial K_y^i(-t)$, $\{Ry^i(m)\}_m$ converges to an element of $\partial K_y^i(-t)$, which we show *otherwise* $\phi = 0$.

Proof :
Let the set of states where the self-financing constraints are active at time one be $S_b \equiv \{s \in S : \sum_i \sum_{a^i} d_{a^i,s} y^{ti}_{a^i} = 0\}$. The arbitrageur's problem remains unaffected if we replace the original constraint set formed by the time one self-financing constraints by the new constraint set $\sum_i R^i_{S_b} y^{ti} = 0$.

Denote the rank of R_{S_b} by $\rho(R_{S_b})$. R_{S_b} may not be of full row rank. The new constraint set is unaffected if we drop the "right" $S_b - \rho(R_{S_b})$ rows to form a $\rho(R_{S_b}) \times \sum_i A^i$ matrix $R_{\hat{S}}$ of rank $\hat{S} \equiv \rho(R_{S_b}) = \rho(R_{\hat{S}})$. Notice that if there is an exchange k with $\rho(R^k) = S$ (this is assumption LCM(a)), then $\rho(R) = S$, and $\hat{S} = S_b$.

Assuming $\hat{S} > 0$ and using $\sigma^i = \frac{1}{1+\phi} R^{i\prime}(\beta^t p + \gamma)$ we can rewrite $\sum_i R^i_{\hat{S}} y^{ti} = 0$ as

$$\sum_i R^i_{\hat{S}} (\partial w^{i\prime})^{-1} \left[R^{i\prime}(\beta^t p + \gamma) \frac{1}{1+\phi} - w^i \right] = 0$$

The equation above can then be rewritten by denoting the block diagonal matrix $\mathrm{diag}(\partial(w^{i\prime}))^{-1}$ by $(\partial w')^{-1}$:

$$R_{\hat{S}} (\partial w')^{-1} R'_{S_b} \gamma_{S_b} = \sum_i R^i_{\hat{S}} (\partial w^{i\prime})^{-1} \left\{ (1+\phi) w^i - \beta^t R^{i\prime} p \right\} \qquad (11)$$

If the rank of R_{S_b} equals $\hat{S}$, we can solve for $\gamma_{\hat{S}}$ unambiguously. Otherwise, notice that $R'_{S_b} \gamma_{S_b} = R'_{\hat{S}} \gamma_{\hat{S}} + R'_{S_b \setminus \hat{S}} \gamma_{S_b \setminus \hat{S}}$ with $\rho(R'_{\hat{S}}) = \hat{S}$. Solving, we get

$$\gamma_{\hat{S}} = (R_{\hat{S}} (\partial w')^{-1} R'_{\hat{S}})^{-1} R_{\hat{S}} (\partial w')^{-1} \left[(1+\phi^t) w - R' \beta^t p - R'_{S_b \setminus \hat{S}} \gamma_{S_b \setminus \hat{S}} \right]$$

Any choice of $\gamma_{S_b \setminus \hat{S}} \geq 0$ yields a solution (for it leaves the arbitrageur's optimum unaffected), and for the time being we simply put $\gamma_{S_b \setminus \hat{S}} = 0$ to break the indeterminacy. We want to remark that we have $S_b - \hat{S}$ degrees of freedom to pick γ, and this will be used again in the proof of Proposition 5.

$(1+\phi)$ is then the solution to $\sum_i w^{i\prime} y^{ti} = 0$, which can be rewritten as $w'(\partial w')^{-1} \sigma = w'(\partial w')^{-1} w$. Now $\sigma^i = \frac{1}{1+\phi} R^{i\prime}(p\beta + \gamma)$, with γ already computed above. We can then trivially solve for $1 + \phi$. ∎

Proof of Proposition 3 We divide the proof into two steps. In step 1 we need to establish that the best-response function is Lipschitzian. In step 2, we show that the eigenvalues of the Jacobian have nonpositive real parts.

Step 1: Recall that the arbitrageur's optimization problem contains S linear inequality constraints and one nonlinear constraint. The desired result follows from the work of Jittorntrum (1978) (see also Fiacco (1983), Theorem 2.4.5) if we are able to show that the Jacobian of the active constraints at the point in question is of full rank (over and above the assumed satisfaction of the second-order sufficient conditions for a local optimum that $\partial^2\mathcal{L}$ be negative definite). Without loss of generality, assume that the set of binding constraints is $J = \{0, \hat{S}\}$, so that the self-financing constraints at time zero as well as in the states $s \in \hat{S} \subseteq S$ are active (where the set of states $\hat{S}$ can be assumed to be such that $R_{\hat{S}}$ is of full rank, refer to the argument in Lemma A.2).

We now show that the Jacobian $\left[-\sigma^t \; R'_{\hat{S}}\right]'$ is indeed of full rank. Recall that $\sigma^t \equiv \frac{1}{1+\phi^t} R'(\beta^t p + \gamma^t)$. It is easy to see that the Jacobian is of full rank iff there is no $\zeta \in \mathbb{R}^{\hat{S}}$ such that $R'p = R'_{\hat{S}}\zeta$. By definition of $\hat{S}$, $R_{\hat{S}} y^t = 0$ and $R_{S\setminus\hat{S}} y^t < 0$, where y^t is the typical arbitrageur's asset supplies, and where the inequality follows from $0 \in J$. We now argue by contradiction. Assume there is a ζ such that $R'p = R'_{\hat{S}}\zeta$. Then

$$y^{t\prime} R' p \equiv [\underbrace{y^{t\prime} R'_{\hat{S}}}_{=0} \quad y^{t\prime} R'_{S\setminus\hat{S}}] p = y^{t\prime} R'_{\hat{S}} \zeta = 0$$

This is impossible, for the left-hand side is a strictly negative scalar (for $p_{S\setminus\hat{S}} \gg 0$). We can conclude that there cannot exist such a ζ.

Hence we showed that the policy functions are locally Lipschitz. The set of critical points of B^t is determined by $\;_R(B^t) \equiv \{y^{-t} :$ the Jacobian $\partial_y B^t$ is not defined$\}$. $\;_R(B^t)$ can be shown to be interiorless and of measure zero (cf Mas-Colell (1985)). It follows that B^t is differentiable on the open set $(\mathrm{int}\bar{\mathcal{Y}}) \setminus C_R(B^t)$. We now characterize the Jacobian assuming $y^{-t} \notin \;_R(B^t)$. Then the derivative exists in an open neighbourhood of y^{-t}, and we can impose the strict complementary slackness: if $w'y^t = 0$ then $\phi > 0$ and when $\sum_i \sum_{a^i} d_{a^i,s} B^{ti}_{a^i} = 0$ then $\gamma^t_s > 0$. It is then w.l.g. to
(i) delete nonbinding constraint information, since if $\gamma^t_s(y^{-t}) = 0$ and if the payoff is interior, $\sum_i \sum_{a^i} d_{a^i,s} B^{ti}_{a^i}(y^{-t}) < 0$, then $\gamma^t_s(y^{-t} + \epsilon) = 0$ if ϵ is small, and
(ii) replace $\gamma^t_s \sum_i \sum_{a^i} d_{a^i,s} B^{ti}_{a^i} = 0$ by $\sum_i \sum_{a^i} d_{a^i,s} B^{ti}_{a^i} = 0$ if $\gamma^t_s > 0$ by continuity of the multipliers.

It follows then (see Fiacco (1983)) that the unique continuously differentiable function $(B^t(y^{-t}), \phi^t(y^{-t}), \gamma^t_s(y^{-t}))$ is such that $B^t(y^{-t} + \epsilon)$ is the unique argmax at $y^{-t}+\epsilon$, ϵ small.

The $(\sum_i A^i + |J|) \times 1$ system of first-order-conditions is

$$G_{(J)}(B^t, y^{-t}, \phi^t, \gamma^t_{\hat{S}}) \equiv \begin{bmatrix} (1+\phi^t)\left[(\partial w)' B^t + w - \frac{1}{1+\phi^t} R'(\beta^t p + \gamma^t)\right] \\ w' B^t \\ -R_{\hat{S}} B^t \end{bmatrix} = 0$$

where B^t is the best reply of the arbitrageur under scrutiny taking as given y^{-t}.

For simplicity, define $E \equiv [\sigma^t \quad -R'_{\hat{S}}]$. It is easy to verify that $\partial_{(B^t,\phi^t,\gamma^t_{\hat{S}})} G_{(J)} = \left[\begin{smallmatrix} (1+\phi^t)\partial^2\mathcal{L} & E \\ E' & 0 \end{smallmatrix}\right]$. Also, define the inverse matrix by

$$\begin{bmatrix} (1+\phi^t)\partial^2\mathcal{L} & E \\ E' & 0 \end{bmatrix} \begin{bmatrix} \tilde{M} & -N \\ -N' & Q \end{bmatrix} = I$$

from which we can deduce, among others, that

$$\begin{aligned}
\tilde{M} &\equiv \frac{1}{1+\phi^t}(\partial^2\mathcal{L})^{-1} - \frac{1}{1+\phi^t}(\partial^2\mathcal{L})^{-1}E[E'(\partial^2\mathcal{L})^{-1}E]^{-1}E'(\partial^2\mathcal{L})^{-1} \\
M &\equiv (1+\phi^t)\tilde{M} \\
N &\equiv -(\partial^2\mathcal{L})^{-1}E[E'(\partial^2\mathcal{L})^{-1}E]^{-1} \\
Q &\equiv -[E'(\partial^2\mathcal{L})^{-1}E]^{-1}(1+\phi^t) \\
0 &= E'M \\
I &= (\partial^2\mathcal{L})M - EN'
\end{aligned}$$

The Jacobian is indeed easily seen to be invertible. Assume that $Hz = 0$ for some $z \equiv (z_1, z_2)$, with $H \equiv \left[\begin{smallmatrix} (1+\phi^t)(\partial^2\mathcal{L}) & E \\ -E' & 0 \end{smallmatrix}\right]$. Then $z'Hz = 0$, and hence $z_1'(1+\phi^t)(\partial^2\mathcal{L})z_1 = 0$, which is possible only for $z_1 = 0$. Finally, $Hz = 0$ and $z_1 = 0$ then implies that $Ez_2 = 0$, and hence that $z_2 = 0$. Notice that N is of dimension $\sum_i A^i \times (\hat{S}+1)$, and that E has a submatrix of rank $\hat{S}$. E is of rank $\hat{S}+1$ since the only way that E is not of rank $\hat{S}+1$ is the case $\sum_i A^i = \hat{S}$, which contradicts $\phi^t > 0$.

The implicit function theorem shows that the best-reply function is [1] and that

$$\partial_y B^t = -M(\partial^2\mathcal{L} - \partial w') + N \begin{bmatrix} B^{t\prime}\partial w \\ 0 \end{bmatrix} \tag{12}$$

$$\partial_y \begin{bmatrix} \phi^t \\ \gamma^t_{\hat{S}} \end{bmatrix} = (1+\phi^t)N'(\partial^2\mathcal{L} - \partial w') - Q \begin{bmatrix} B^{t\prime}\partial w \\ 0 \end{bmatrix}$$

The latter expression can be used to compute

$$\partial_y \sigma^t = \frac{-1}{1+\phi^t} E \partial_y \begin{bmatrix} \phi^t \\ \gamma^t_{\hat{S}} \end{bmatrix}$$

Taken together, we can make use of $N - (\partial^2\mathcal{L})^{-1} EQ\frac{1}{1+\phi^t} = 0$ and deduce the expression given in the statement of the proposition,

$$\partial_y B^t = (\partial^2\mathcal{L})^{-1}\left[\partial_y \sigma^t - \partial w\right] - (\partial^2\mathcal{L})^{-1}\mathrm{diag}\left\{(B^{ti\prime} \otimes I_{A^i})(\partial^2 w^i)\right\}_{i=1}^{I}$$

Step 2: In this step we show that we can express the supplies at the fixed point as Lipschitz functions of the (so far exogenous) number of players, n. We denoted these functions by $\xi(n)$.

In order to accomplish this, it is sufficient to show that every element of the generalized Jacobian of F (recall that $F(y^t, n) \equiv y^t - B^t(\max\{0, n-1\}y^t))$ with respect to y^t, for simplicity also denoted by $\partial_{y^t} F = I - \partial_{y^t} B^t$, is nonsingular.

Definition 5 *Suppose $F : X \to Y$, $X \subseteq \mathbb{R}^n$ and $Y \subseteq \mathbb{R}^m$ is Lipschitzian and the Jacobian $\partial F(x)$ fails to exist on $\ _R(F) \subset X$. Then the* <u>Generalized Jacobian</u> *of F at $x \in X$ is $\mathrm{co}\{\lim_{x_k \to x, x_k \notin \mathcal{C}_R} \partial F(x_k)\}$, where $\mathrm{co}\{\cdot\}$ is the convex hull of the set $\{\cdot\}$.*

We now show that for every noncritical y^{-t}, $\partial_y B^t$ only has eigenvalues with nonpositive real parts for n large enough (the number of zero eigenvalues is no larger than the number of active constraints).

First, notice that $\ker(M) = \langle E \rangle$. Second, also notice that M is negative definite on $\langle E \rangle^\perp$ if n is large enough[7] and that $(\partial^2\mathcal{L} - \partial w')$ is negative quasidefinite for n large enough.

[7]Indeed, assume that b is a non-zero eigenvalue of M: $Mv = bv$. Then $E'Mv = bE'v = 0$, and hence $v'E = 0$, showing that every eigenvector of M to which is attached a nonzero eigenvalue lies in $\langle E \rangle^\perp$. For large n, $\partial^2\mathcal{L}$ is negative definite as it is approximately equal to $\partial w + \partial w'$ in view of the fact that quasi definiteness is preserved with respect to small perturbances (which applies due to the fact that $n\xi(n)$ is bounded as it lies in a compact

We wish to show that (from equation (12))

$$(\partial_y B^t)' z = -(\partial^2\mathcal{L} - \partial w) M z + \big[(\sigma - w)\ 0\big] N' z = \mu z \tag{13}$$

cannot hold for $\mu > 0$ (with $z \in \mathbb{R}^{\sum_i A^i} \setminus \{0\}$). We consider two cases.

First, assume that $z \in \langle E \rangle \setminus \{0\}$ and that $\mu \neq 0$. Then $Mz = 0$ so that $\big[(\sigma - w)\ 0\big] N' z = \mu z$. Since $z \in \langle E \rangle$, we can write it as $z = Ev$ for some vector v, which means (using $N'E = -I$) that $-\big[(\sigma - w)\ 0\big] v = \mu z$ and hence that $-v_1(\sigma - w) = \mu z$ where v_1 is the first element of v. We can premultiply both sides by $B^{t\prime}$ and get $-v_1(\sigma - w)'(\partial w)^{-1}(\sigma - w)= \mu B^{t\prime} z$ $= \mu B^{t\prime} E v = \mu [B^{t\prime}\sigma \quad 0] v = \mu v_1 B^{t\prime}\sigma$. Notice that $v_1 \neq 0$, which is due to $-v_1 z'(\sigma - w) = \mu z' z > 0$, and that $B^{t\prime}\sigma = B^{t\prime}(\partial w')B^t < 0$ (since $(\partial w')B^t = \sigma - w$ so $B^{t\prime}(\partial w)' B^t = B^{t\prime}\sigma - 0$ by the self-financing constraints at time zero). This implies that $-(\sigma - w)'(\partial w')^{-1}(\sigma - w) = \mu B^{t\prime}\sigma$ so that $\mu < 0$. We conclude that if $z \in \langle E \rangle \setminus \{0\}$ then $\mu \leq 0$.

Second, assume that $z \notin \langle E \rangle$. To begin with, analyze the eigenvalue problem for $-AB$ where we define $A \equiv -(\partial^2\mathcal{L} - \partial w)$ and $B = -M$. A is positive quasidefinite for n large, while B is symmetric and positive semidefinite. So $-AB\tilde{z} = \tilde{\mu}\tilde{z}$. We want to show that if $\tilde{z} \notin \langle E \rangle$ then $\tilde{\mu} < 0$. The spectrum of $-AB$ only owns nonpositive elements in view of the result that the spectrum of AB is a subset of the product of the field of values of A and B if B is positive semidefinite: $\sigma(AB) \subset \mathcal{F}(A)\mathcal{F}(B)$, where $\mathcal{F}(A) \equiv \{y^* A y : y \in \mathbb{C}^n, y^* y = 1\}$ (refer to Horn and Johnson (1991)). Since B is a real symmetric matrix, $\mathcal{F}(B) = \{y' B y : y \in \mathbb{R}^n, y' y = 1\}$, so $\mathcal{F}(B) \subset [0, \infty)$. To compute $\mathcal{F}(A)$, notice that $y^* A y = (y_R' - i y_I') A (y_R + i y_I) = [y_R' A y_R + y_I' A y_I] + i[y_r' A y_I - y_I' A y_R]$, i.e. $\mathrm{Re}\mathcal{F}(A) \subset (0, +\infty)$. Also, $\mathcal{F}(A)\mathcal{F}(B) = \{z \in \mathbb{C} : z = x^* y \quad, x \in \mathcal{F}(A), y \in \mathcal{F}(B)\} = \{z \in \mathbb{C} : z = x_R y + i x_I y \quad, x \in \mathcal{F}(A), y \in \mathcal{F}(B)\}$, so that $z_R \in [0, +\infty)$. We can conclude by saying that $\mathrm{Re}\mathcal{F}(A)\mathcal{F}(B) \subset [0, +\infty)$. Hence all the real parts of the eigenvalues of $-AB$ are nonpositive. Finally we show that if $\tilde{\mu} = 0$ then $\tilde{z} \in \langle E \rangle$. Indeed, assume $AB\tilde{z} = 0$. Then $M\tilde{z} = 0$ and $\tilde{z} \in \ker(M) = \langle E \rangle$, a contradiction. We can conclude that if $\tilde{z} \notin \langle E \rangle$ then $\tilde{\mu} < 0$. Now perturb $-AB$ by adding $\Theta \equiv \big[(\sigma - w)\ 0\big] N'$, and the new eigenvector-eigenvalue pair satisfies $-(AM + \Theta) z = \mu z$. From the first case, we know that if $z \in \langle E \rangle \setminus \{0\}$, then $\mu < 0$. So we assume that $z \notin \langle E \rangle$. Since we can choose the perturbation to be as small as we wish by choosing n large enough, there

set (Lemma A.1)), and so $v'\partial^2\mathcal{L} v < 0$, implying that $v'v = v'\big(\partial^2\mathcal{L} M - E N'\big) v = v' \partial^2\mathcal{L} M v$. The first equality stems from the identity $(1 + \phi^t)(\partial^2\mathcal{L})\tilde{M} - EN' \equiv I$ and the second one from $v'E = 0$. We can then conclude that $v'(\partial^2\mathcal{L}) b v > 0$ so that $b < 0$.

is a large n such that we still have $\mu < 0$, contradicting the assumption that $\mu > 0$. The reader will have noticed that the complication in the argument stems from the fact that the time zero self-financing constraint was assumed to be binding. If it doesn't bind, the last argument is superfluous. ■

Proof of Proposition 4 We split this proof into three steps. The first step derives the existence result and characterizes the equilibrium treating n as exogenous. The second step derives some intermediate results used in step 3 by exploiting the Lipschitz properties established in Proposition 3. The last step then endogenizes n as a function of c. In this proof, the expression Π_g^t stands for gross profits, i.e. $\Pi^t + c$.

Step 1: The number of players will be taken as fixed for the moment. We first introduce a truncation by constraining players to choose actions in a small compact cube C around zero. We denote the truncated economy by $\mathcal{E}'$. Second, we also introduce an artificial economy $\mathcal{E}''$ derived from $\mathcal{E}'$ in order to guarantee the continuity of the payoff functions and of the best-reply functions. The problem is that if for some reason t believes that $-t$ supply a quantity that is not investment feasible, it is easy to construct examples where t's policy function need not be defined. At an equilibrium of $\mathcal{E}'$ this cannot happen by Lemma A.1, of course. This is why we introduce the artificial economy $\mathcal{E}''$ that will be shown to have an equilibrium, from which we can deduce that $\mathcal{E}'$ must have an equilibrium as well.

Assume that arbitrageurs in $\mathcal{E}''$ believe that the auctioneers' order books only record supplies up to a certain limit. Feasibility requires that $y \in \mathcal{Y}$, so define K as being a large convex and compact subset of $\mathcal{Y}$ owning 0 in its interior that satisfies the additional requirement that no element of the boundary can contain an optimal action (that such a set always exists was proved in Lemma A.1). K defines the limits of the auctioneers' order books. Orders larger than those in K will be truncated at K's boundary. We now formally construct this truncation mapping. Given y^{-t}, define the linear path with constant velocity $g : [0,1] \times \mathbb{R}^{\sum_i A^i} \to \mathbb{R}^{\sum_i A^i}$ with $g(0; y^{-t}) = 0$ and $g(1; y^{-t}) = y^{-t}$. Also define the set $\mathcal{T}(y^{-t}) \equiv \{\tau \in [0,1] : g(\tau; y^{-t}) \cap \partial K \neq \emptyset\}$ (which is either empty or a singleton) and the element

$$\bar{\tau} \equiv \begin{cases} \tau \in \mathcal{T}(y^{-t}) \text{ if } \mathcal{T}(y^{-t}) \neq \emptyset \\ 1 \text{ otherwise} \end{cases}$$

Finally, the mapping from perceived y^{-t} to supplies appearing in the order book is given by ${}^*y^{-t} = \varphi(y^{-t}) \equiv g(\bar{\tau}; y^{-t})$. This mapping is easily seen to be continuous from the fact that $0 \in \text{int } K$ and from the convexity of K.

Given DSD, the truncation C can be chosen small enough as to make each player's problem strictly quasi-concave, insuring that reaction correspondences (ignoring individual rationality) are continuous functions. Indeed, the Lagrangian to be solved is

$$\max \sum_i \sum_{a^i} y^{ti}_{a^i} \left[w^i_{a^i}(1+\phi) - \sum_s (\beta^t p_s + \gamma_s) d_{a^i,s} - \mu^i_{a^i} + \nu^i_{a^i} \right] + \sum_i \sum_{a^i} y^* (\mu^i_{a^i} + \nu^i_{a^i})$$

where $\mu^i_{a^i}$ is the Lagrangian multiplier on $y^{ti}_{a^i} \leq y^*$ and $\nu^i_{a^i}$ the multiplier on $y^{ti}_{a^i} \geq -y^*$, $y^* > 0$ and "small." The first-order and second-order conditions are the same as in the unconstrained case except that we now have $\sigma_{a^i} \equiv \frac{1}{1+\phi} \sum_s (\beta^t p_s + \gamma_s) d_{a^i,s} + \mu^i_{a^i} - \nu^i_{a^i}$.

Notice that for $|y^*|$ small $(\partial w^i)' + \partial w^i + \left(y^{ti'} \otimes I_{A^i} \right) (\partial^2 w^i)'$ is negative definite by Lemma A.1 and by the fact that a matrix close to a quasi definite matrix is also quasi definite, so that the truncated optimization problem is indeed strictly quasi-concave. Arrow-Hurwicz-Uzawa's theorem together with the Kuhn-Tucker theorem then guarantee the existence of the Lagrangian multipliers. The continuity of the optimal policy functions B^t (as functions of ${}^*y^{-t}$) follows from Berge's Theorem of the Maximum.

Given n, a symmetric equilibrium of $\mathcal{E}''$ is determined by the following fixed point

$$\begin{bmatrix} \vdots \\ y^{ti}_{a^i} \\ \vdots \end{bmatrix} = \begin{bmatrix} \vdots \\ B^{ti}_{a^i} \left(\varphi \left(\max\{0, n-1\} y^t \right) \right) \\ \vdots \end{bmatrix}$$

The function maps the convex compact set C continuously into itself, and admits a fixed point $\xi(n)$ in view of Brouwer's fixed point theorem. By Lemma A.1 this must be an equilibrium of $\mathcal{E}'$ as well, for $y^{-t} = \varphi(y^{-t})$. Indeed, assume that $y^{-t} \neq \varphi(y^{-t})$. Then $y^{-t} \notin K$, and by symmetry $(n-1)y^t \notin K$. Also by symmetry, $ny^t \notin K$, but we know that this cannot happen at a Nash equilibrium.

Denote the fixed point system in $\mathcal{E}'$ by $F(y^t, n) \equiv y^t - B^t(\max\{0, n - 1\}y^t)$. Now assume n is large. By Lemma A.1 aggregate profits are bounded, so by symmetry $n\Pi^t$ is bounded. By raising n we can make Π^t arbitrarily small, i.e. $-(1+\phi^t) \sum_i \xi^{i'} (\partial w^i) \xi^i \to 0$. Given that ∂w^i is continuous and negative quasidefinite, we have $\xi \to 0$ and $||w - \sigma|| \to 0$. In particular, the truncation C doesn't bind for n large enough, and the equilibrium for $\mathcal{E}'$ is an equilibrium for $\mathcal{E}$ as well. This is true for all $c \leq \Pi_g^t(n)$, which we assume to be the case. Remains to verify that the asset supplies given by the first-order conditions are optimal. We showed that we can restrict the set of rational (nontruncated) actions to a compact set, the boundary of which cannot constitute optimal actions. It follows that optimal asset supplies must be an interior point, at which the first-order conditions then have to hold with equality. But the latter can only be satisfied in a neighbourhood of zero for n sufficiently large.

Step 2: We now show that for n large enough, $\partial_{y^t} F = I - \max\{0, n-1\}\partial_y B^t$ is nonsingular on the complement of ${}_R(B^t)$, so ${}_R(F) \equiv \{(n, y^t) : \max\{0, n-1\}y^t \in {}_R(B^t)\}$. This enables us to derive, among others, the result that $\frac{\partial \Pi^t}{\partial n} < 0$ across equilibria. Recall from Proposition 3 that all eigenvalues of $\partial_y B^t$ have nonpositive real parts. It is sufficient to show that $\frac{1}{n-1}$ is not an eigenvalue of $\partial_y B^t$. But we know that all its eigenvalues have nonpositive real parts for n large enough.

Since the generalized Jacobian at a point is defined as the convex hull of all limiting Jacobians, we also need to know whether each element in the generalized Jacobian $\partial_{y^t} F$ is invertible. We only need to check points in ${}_R(F)$. Let there be two sequences $(n_{k_l}, y_{k_l}^t) \to (n, y^t) \in {}_R(F)$ with $(n_{k_l}, y_{k_l}^t) \notin {}_R(F)$ for any $k_l \in \{0, 1, \dots\}$, $l = 1, 2$. It follows that $\lim_{k_l \to \infty} \partial_{y^t} F(y_{k_l}^t, n_{k_l})$ $= I - \lim_{k_l \to \infty}(n_{k_l} - 1)\partial_{y_{k_l}} B^t = I - (n-1)\lim_{k_l \to \infty} \partial_{y_{k_l}} B^t$, and define $J_l \equiv \lim_{k_l \to \infty} \partial_{y_{k_l}} B^t$, $l = 1, 2$. At such a point, a typical element of the generalized Jacobian of F is $\partial_{y^t} F = I - \alpha(n-1)J_1 - (1-\alpha)(n-1)J_2$, $\alpha \in (0, 1)$. Notice that J_1 and J_2 only differ in as far as the matrices M_1 and M_2 as well as the matrices N_1 and N_2 may be different, namely via $\hat{S}_1$ and $\hat{S}_2$, which follows from the continuity of B^t, ∂w and $\partial^2 w$. By the same argument as before, simply replacing M by $\alpha M_1 + (1-\alpha)M_2$ and N by $\alpha N_1 + (1-\alpha)N_2$, we can show that $\partial_{y^t} F$ is nonsingular because $\frac{1}{n-1} > 0$ is never an eigenvalue. This is true for any choice of α and of J_1 and J_2.

The Lipschitz implicit function theorem then guarantees that ξ is locally Lipschitz, and since the chosen point was arbitrary, ξ is Lipschitzian. Recalling that $\partial_n F = -(\partial_y B^t)B^t$, it follows for $y^{-t} \notin {}_R(B^t)$ and for n large enough that

$$\partial_n \xi = \left[I - (n-1)\partial_y B^t\right]^{-1} (\partial_y B^t) B^t \tag{14}$$

The typical arbitrageur's gross profit function (i.e. excluding the investment of c) across Nash equilibria is $\Pi_g^t : (\bar{n}, \infty) \to (0, \bar{c})$, where $\Pi_g^t : n \mapsto \Pi_g^t(n)$ and where $\bar{n}$ is large enough to guarantee the existence of an equilibrium and $\bar{c} = \Pi_g^t(\bar{n})$. Of course, at a symmetric Nash equilibrium $\Pi_g^t = (1+\phi^t)\sum_i \sum_{a^i}\left[w_{a^i}^i(n\xi^i(n)) - \sigma_{a^i}^t\right]\xi_{a^i}^i(n)$ In view of the Lipschitz property of ξ, Π_g^t is itself Lipschitz in n. Appealing to the envelope theorem (as extended by Jittorntrum (1978)), to the first-order conditions and to the negative quasidefiniteness of ∂w^i,

$$\frac{\partial \Pi^t}{\partial n} = (1+\phi^t)\sum_i \sum_{a^i}\sum_{b^i}\xi_{a^i}^i \xi_{b^i}^i \frac{\partial w_{b^i}^i}{\partial y_{a^i}^i} < 0$$

Step 3 At this stage it remains to be shown that we can replace "for n large enough" by "for c small enough" and that we can restrict n to be a nonnegative integer. Fix an n that is large enough. By definition of no-entry, c then has to satisfy $\Pi^e(n) - c = 0$. We know that at equilibrium, $0 \leq \Pi_g^e(n) < \Pi_g^t(n)$, $\lim_{n\to\infty}\Pi_g^t(n) = 0$ and $\frac{\partial \Pi^t}{\partial n} < 0$. So if $(\Pi^e)^{-1}(c)$ is nonmonotonic and thus multivalued, we choose the smallest element of the correspondence (which exists) and we round it up to the next integer in the preimage of $(0, c)$, $(\Pi^e)^{-1}((0,c))$. That way we build up the weakly decreasing step function $n = \eta(c)$, with $\lim_{c\to 0}\eta(c) = +\infty$ and with $n \in \mathbb{N}\cup\{0\}$.

At an n large enough, any potential entrant can at best break even, and we assume that they choose to stay out in that case. From what has been said above, we know that $\Pi^t(n) > 0$, for any active arbitrageur $t = 1,\ldots,n$. We also know that $\Pi_g^t(n)$ can be made arbitrarily small by choosing an n that is arbitrarily large. Since $\Pi^e(n) < \Pi^t(n)$, Π_b^e can be made arbitrarily small as well (and smaller than c). Due to the monotonicity (for large enough n) of $\Pi_b^e(n)$, we found that $n = \eta(c)$ is a decreasing function and that $\lim_{c\to 0}\eta(c) = +\infty$. ■

Proof of Proposition 5 (Integration)

That we must have $||w^i - \sigma^i|| \to 0$ was shown in Proposition 4. We now prove each item in turn.

(i) The arbitrage properties of limiting prices can be easily verified as well. Indeed, in the limit we have that $w_{a^i}^i = \sigma_{a^i} = \sum \frac{\beta^t p_s + \gamma_s}{1+\phi} d_{a^i,s}$. In the economy with short horizon arbitrageurs, in the limit, $w_{a^i}^i = \sigma_{a^i} = \sum \gamma_s d_{a^i,s}$. Under LCM, common stochastic discount factors in the limit are $\lambda \equiv \frac{1}{1+\phi}(\beta^t p + \gamma) \in \cap_{i\in I}\Lambda^i$.

Indeed, when LCM(a) holds, in the limit we need to have $\sigma^k = w^k$, i.e. $\frac{1}{1+\phi}R^{k\prime}(\beta^t p + \gamma) = R^{k\prime}\lambda^k$ (where $\lambda^k \equiv IMRS^k$) so that $\frac{1}{1+\phi}(\beta^t p + \gamma) = \lambda^k \gg 0$. Interestingly, this is true even for $\beta^t = 0$ as γ is equal to λ^k in the limit.

When LCM(b) holds, in the limit we can replace on each exchange the original matrix R^i by R^* without affecting the consumption allocation or the state-prices. We can then set $\lambda \equiv \frac{1}{1+\phi}(\beta^t p + \gamma) = \lambda^i \gg 0$, some $i \in I$. In particular, if $\beta^t = 0$ we can set $\gamma = \lambda^i \gg 0$, for some $i \in I$. Unless $\rho(R^*) = S$, λ^i and λ^j may differ for i and $j \neq i$ in I as both lie in a manifold of dimension $S - A^*$, in which case we typically cannot simultaneously have $\lambda = \lambda^i$ and $\lambda = \lambda^j$. As mentioned in the proof of Lemma A.2, we indeed have $S - A^*$ degrees of freedom to choose γ, and it is obvious from equation (11) that γ is compatible with any (λ^i, λ^j) that satisfy $R^{*'}\lambda^i = R^{*'}\lambda^j$.

(ii) is self-explanatory.

(iii) Let there be a sequence $\{c_m\}_{m=1}^{\infty}$ such that $c_m \to 0$. Given a large enough m (guaranteeing that equilibria exist), at a CWECE of $\mathcal{E}(m)$

$$z_s^i(\lambda^i(m)) = \sum_{a^i} f_{a^i}^i(q^i(m))d_{a^i,s} = \sum_{a^i} y_{a^i}^i(m)d_{a^i,s}$$
$$z_0^i(\lambda^i(m)) = -\sum_{a^i} f_{a^i}^i(q^i(m))q_{a^i}^i(m) = -\sum_{a^i} y_{a^i}^i(m)q_{a^i}^i(m)$$

from which it follows that

$$\sum_i z_s^i(\lambda^i(m)) = \sum_i \sum_{a^i} y_{a^i}^i(m)d_{a^i,s} = -\sum_t^{n(m)} x_s^t(m) = -n(m)x_s^t(m)$$
$$\sum_i z_0^i(\lambda^i(m)) = -\sum_i \sum_{a^i} y_{a^i}^i(m)q_{a^i}^i(m) = -\sum_t^{n(m)} x_0^t(m) = -n(m)x_0^t(m)$$

Given that $c_m \to 0$, arbitrageurs' consumption vanishes, $\lim_{m\to\infty} n(m)x_s^t(m) \to 0$, $s = 0, \ldots, S$ (refer to Proposition 5 and to the self-financing constraints), from which we can deduce that $\lim_{m\to\infty} \sum_i z_s^i(\lambda^i(m)) = 0$, i.e. $\sum_i z_s^i(\lambda^i(\infty)) = 0$, $s = 0, \ldots, S$ (by continuity). Since we showed in Proposition 5 that both $\lambda^i(\infty)$ and $\lambda \equiv \frac{1}{1+\phi}(\beta^t p + \gamma)$ are in $\Lambda^i(\infty)$, it has to be the case that $\sum_i z_s^i(\lambda) = 0$ as well, verifying that $\big((z^i)_{i\in I}, \lambda\big)$ is a Walrasian equilibrium with restricted participation.

(iv) follows from (ii) and (iii).

(v) Assume $c_m \to 0$. We know from Proposition 5 that in the limit asset prices w^i equal σ^i, which itself allows a state-price representation $\lambda \gg 0$. Hence in the limit, there is no Y^i such that either

(a) $\sum_i \sum_{a^i} Y_{a^i}^i w_{a^i}^i(\infty) = 0$ and $\sum_i \sum_{a^i} Y_{a^i}^i d_{a^i,s} \leq 0$ with strict inequality in at least one state, or

(b) $\sum_i \sum_{a^i} Y^i_{a^i} w^i_{a^i}(\infty) > 0$ and $\sum_i \sum_{a^i} Y^i_{a^i} d_{a^i,s} \leq 0$.

We need to show that $\lim_{m\to\infty} n(m)\Pi^t(m) = 0$. Now using as usual $y^i_{a^i}(m) \equiv$ $n(m)y^{ti}_{a^i}(m)$,

$$n(m)\Pi^t(m) = \sum_i \sum_{a^i} y^i_{a^i}(m) w^i_{a^i}(y^i_{a^i}(m)) + \beta^t \sum_{s\geq 1} p_s(-1) \sum_i \sum_{a^i} d_{a^i,s} y^i_{a^i}(m)$$

If for m large enough $n(m)\Pi^t(m) > 0$, then either *(i)* or *(ii)* has to hold, or both, due to the self-financing constraints. But for m large enough we know that neither *(i)* nor *(ii)* can be satisfied, proving that $\lim_{m\to\infty} n(m)\Pi^t(m) = 0$. ■

Proof of Proposition 8 (Derivative Pricing) Even though we have an economy-wide state-price vector λ, we cannot say that the price at which a will trade on i equals $\lambda \cdot d_a$, the price of the replicating portfolio. The reason is that in order to be able to price by replicating portfolio considerations, we need $q^i_a = \lambda \cdot d_a = \lambda^i \cdot d_a$. We can w.l.g. set $\lambda^i = IMRS^i$, so $\lambda^i \cdot d_a$ is the price at which the derivative gets traded on exchange i when arbitrageurs don't change their supplies. Given that the dimension of the manifold Λ^i is $S - A^i$, there is no compelling reason (other than $S = A^i$, which we excluded when we assumed that a was not redundant on i) for this equality to hold, and indeed generically it doesn't. T be solved for

$$B_a^{ti}(y^{-t}) = \frac{\delta_a^i}{2}(\hat{q}_a^i - \sigma_a^*) - \frac{1}{2}y_a^{-ti} + \frac{1}{2}\varpi_a^i \sum_j y_a^{-t,j}$$

$$\sigma_a^t(y^{-t}) = \sigma_a^* - \frac{1}{\sum_l \delta_a^l} \sum_j y_a^{-t,j}$$

At a symmetric Nash equilibrium in quantities it has to be the case that $y_a^{ti} = y_a^{t'i}$, all t, t', and that $\sum_i y_a^{-ti} = 0$. At such an equilibrium prices and quantities satisfy $\sigma_a(n_a) = \sigma_a^*$, $\xi_a^i(n_a) = \frac{\delta_a^i}{n_a+1}\left[\hat{q}_a^i - \sigma_a^*\right]$ and $w_a^i(n_a) = \frac{1}{1+n_a}\hat{q}_a^i + \frac{n_a}{1+n_a}\sigma_a^*$.

We now relate the actual number of arbitrageurs to the setup costs. The potential "entrant's" optimisation problem is equivalent to insider t's, simply replacing t by e, and at a Nash equilibrium with n_a symmetric active players,

$$y_a^{ei} = \frac{1}{2(1+n_a)}\delta_a^i(\hat{q}_a^i - \sigma_a^*)$$

The entrant's profit can be deduced to equal $\Pi_a^e(n^a) = \frac{1}{4(1+n_a)^2}\sum_i \delta_a^i(\hat{q}_a^i - \sigma_a^*)^2 = \frac{1}{4(1+n_a)^2}\frac{\delta_a^1\delta_a^2}{\delta_a^1+\delta_a^2}(\hat{q}_a^1 - \hat{q}_a^2)^2 = \frac{1}{4}\Pi_a^t$. n_a is then determined by $\Pi_a^e(n^a) = c_a$ so that

$$1 + n_a = \max\left\{1, \sqrt{\frac{\Pi_a^e(0)}{c_a}}\right\}$$

and $\Pi_a^t = 4c_a > c_a$. The no-entry condition still allows for profits.

We now explore the depth-implications of this simple model. At a CWE with n_a specialized arbitrageurs,

$$w_a^i(n_a) - w_a^j(n_a) = \frac{\hat{q}_a^i - \hat{q}_a^j}{1+n_a} \tag{15}$$

Also notice that the condition that $\frac{\Pi_a^e(0)}{c_a} \geq 1$ is equivalent to $\delta_a^1 \geq \underline{\delta}_a^1 \equiv \frac{4\delta_a^2 c_a}{\delta_a^2(\hat{q}_a^1 - \hat{q}_a^2)^2 - 4c_a}$.

Filling in the details yields the expression found in equation (4). It can also be easily verified that the asymptote is given by $\iota\sqrt{\frac{4c_a}{\delta_a^2}}$. ∎

B Simulation Results for Barrier to Integration

n	1	2	5	8	9	100	100000
multipl. config.	1	1	1	1	2	2	2
σ^1	3.3647	3.3647	3.3647	3.3647	3.371	3.4937	3.5059
σ^2	3.5647	3.5647	3.5647	3.5647	3.56 21	3.5109	3.5059
w^1	4.0424	3.8165	3.5906	3.5153	3.5059	3.505 9	3.5059
w^2	3.2824	3.3765	3.4706	3.502	3.5059	3.5059	3.5059
ϕ	0	0	0	0	0.0465	10.6279	1.16E+04
γ_1	2.5647	2.5647	2.5647	2.5647	2.7278	39.8249	4.08E+04
γ_2	0	0	0	0	0	0	0
λ_1	3.3647	3.3647	3.3647	3.3647	3.371	3.4937	3.5059
λ_2	0.2	0.2	0.2	0.2	0.1911	0.0172	0
y_1	0.2824	0.1882	0.0941	0.0627	0.0562	0.0051	5.06E-06
y_2	-0.2824	-0.1882	-0.0941	-0.0627	-0.0562	-0.0051	-5.06E-06
ny_1	0.2824	0.3765	0.4706	0.502	0.5059	0.5059	0.5059
ny_2	-0.2824	-0.3765	-0.4706	-0.502	-0.5059	-0.5059	-0.5059
x_0^t	0.2146	0.0828	0.0113	0.0008	0	0	0
x_1^t	0	0	0	0	0	0	0
x_2^t	0.2824	0.1882	0.0941	0.0627	0.0562	0.0051	5.06E-06
nx_0^t	0.2146	0.1656	0.0565	0.0067	0	0	0
nx_1^t	0	0	0	0	0	0	0
nx_2^t	0.2824	0.3765	0.4706	0.502	0.5059	0.505 9	0.5059
x_0^1	0.8586	0.5632	0.3103	0.2355	0.2264	0.226 4	0.2264
x_1^1	0.3824	0.4765	0.5706	0.602	0.6059	0.605 9	0.6059
x_2^1	2	2	2	2	2	2	2
x_0^2	2.9268	3.2711	3.6332	3.7578	3.7736	3.773 6	3.7736
x_1^2	0.7176	0.6235	0.5294	0.498	0.4941	0.494 1	0.4941
x_2^2	0.7176	0.6235	0.5294	0.498	0.4941	0.494 1	0.4941
U^1	3.8597	3.9341	4.0297	4.0664	4.0711	4.0711	4.0711
U^2	5.5399	5.5709	5.6107	5.626	5.628	5.628	5.628

Table 1: Barrier to Integration

(The second row refers to the configuration of the multipliers.)

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