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The art of brevity[☆]

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ABSTRACT

We analyze a class of sender-receiver games with quadratic payoffs, which includes the communication games in Alonso et al. (2008) and Rantakari (2008) as special cases, for which the sender's or the receiver's maximum expected payoff when players have access to arbitrary, mediated communication protocols is attained in one round of face-to-face, unmediated cheap talk. This result is based on the existence for these games of a communication equilibrium with an infinite number of partitions of the state space. We provide explicit expressions for the maximum expected payoff of the sender and the receiver, and illustrate its use by deriving new comparative statics of the quality of optimal communication. For instance, a shift in the underlying uncertainty that reduces expected conflict can worsen the quality of communication.

1. Introduction

Conflicting interests often hinder communication between informed experts and uninformed decision makers. This is certainly the case in Crawford and Sobel's classic contribution where an informed sender strategically sends costless messages to an uninformed receiver–i.e., the sender engages in cheap talk (Crawford and Sobel, 1982). Crawford and Sobel, 1982 considers one-shot communication: the sender makes a single recommendation and the receiver immediately makes a decision based on that recommendation. The fact that the sender can foresee the effect of his influence, by anticipating how each recommendation will be interpreted and which decision it will induce, implies that perfect information transmission is not credible.

The literature has since studied several communication protocols that improve the efficiency of the one-shot equilibria in CS; for instance, by engaging in repeated rounds of communication (Krishna and Morgan, 2004), by using a noisy channel (Blume et al., 2007), by appealing to a correlation device on which to base the encoding and decoding of messages (Blume, 2012) or, more generally, by relying on a trustworthy–or even strategic–mediator (Goltsman et al., 2009; Ivanov, 2010).[1] A driving force behind these efficiency-enhancing communication protocols is that they introduce noise in

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[1] The role of mediation with finite messages is considered in Ganguly and Ray (2012). Ambrus et al. (2013) examine hierarchical cheap talk where the messages get passed through a sequence of agents and show that mixed-strategy equilibria can exist that dominate the direct communication game.

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the sender's message: a communication protocol that induces noisier recommendations for decisions that on average favor the sender can discipline a risk-averse sender and thus enhance information transmission.

In this vein, a natural question is when general communication protocols, including mediated communication, can improve upon one-shot communication and to understand when these gains from mediation are likely to be either large or small. A limitation of this literature to address this question is that efficiency bounds of communication equilibria exist only for the leading example in CS; the characterization of optimal mediation in Goltsman et al. (2009) is done for the case of quadratic preferences, constant bias between sender and receiver and a uniformly distributed state. We extend the analysis of GHPS to a broader class of sender-receiver games and study the structure of the payoff set supported by communication equilibria.

Our main result is a characterization of a class of sender-receiver games for which there exists a one-shot, unmediated communication equilibrium for which the receiver's expected payoff cannot be improved upon by using arbitrary mediation rules (Propositions 2 and 3). That is, for these games the receiver obtains no efficiency gains from prolonged conversations, using noisy channels, employing correlation devices or arbitrary mediation; brief conversations are optimal. In particular, the cheap talk equilibrium that achieves the maximum of the receiver's payoff supports an infinite number of different decisions. That is, brief conversations that are optimal are also very detailed. A particular class of sender-receiver games for which this is true includes the pairwise communication games in Alonso et al. (2008) and Rantakari (2008), where the bias between the sender and the receiver is linear and increasing and vanishes at some point of the state space.[2]

One lesson from GHPS is that the gains from mediation with respect to the most efficient equilibrium in CS are greatest for intermediate levels of conflict: as the conflict vanishes, one-shot communication approaches full revelation, while intensifying the conflict leads to no information transmission, even in the presence of a mediator. Our results show, however, that it is not only the magnitude of the conflict that determines the gains from mediation but also its shape. Indeed, Proposition 2 imposes no constraints on the average conflict (as measured by the expected bias between the sender and the receiver). Therefore, for any level of expected conflict one can find a sender-receiver game in our setup where the receiver does not gain from mediation.

Apart from the insights on the gains from mediation, this result has also a more practical appeal. The cheap talk model in CS, and specifically their leading example, has been a workhorse model in applications featuring costless communication, where the restriction to one-shot communication has often been defended on tractability grounds. A concern with this approach, however, is that the insights derived in such settings may not be robust to the parties agreeing to switch to welfare-improving communication protocols. However, for the class of games and the one-shot infinite equilibrium in those games identified in Propositions 2 and 3, the receiver cannot gain from having access to a trustworthy mediator. This implies, for instance, that the comparative statics in Alonso et al. (2008) and Rantakari (2008) are derived under optimal pairwise communication.

Finally, we explicitly compute the receiver's maximum expected payoff for a sub-class of games satisfying Proposition 2 (Proposition 4). This result is of separate applied interest, as it provides the loss due to strategic communication without explicitly solving for the equilibrium itself and for distributions other than the uniform distribution. We illustrate the use of this result by deriving new comparative statics on optimal communication. For instance, we show that a shift in the underlying distribution that reduces the expected conflict can actually worsen communication (Corollary 2).

The basic logic of the analysis is as follows. First, the revelation principle allows us to restrict attention to games where the sender privately and truthfully discloses the state to a mediator, who in turn issues a recommendation to the receiver which the latter is willing to follow. The need to ensure the sender's honesty and the receiver's obedience implies that, for games with quadratic payoffs, interim and ex-ante payoffs can be expressed as linear functionals of state-contingent average decisions (Lemma 1). From this representation we identify a class of games for which a tight relationship exists between local and global properties of communication equilibria: the ex-ante payoff of the sender or of the receiver depends only on the interim payoff of the sender at one of the extreme types (Proposition 1). Therefore globally optimal communication equilibria must also be locally optimal for either the highest or the lowest type of sender. We then show that if the sender and the receiver are perfectly aligned for either the highest or the lowest state, then there exists a one-shot infinite communication equilibrium such that communication is very detailed around the state of perfect alignment. In particular, an extreme type of sender is able to fully reveal his type, and thus to obtain his preferred decision with certainty. Since local properties of extreme types translate to global properties for these games, perfect communication for an extreme type implies that this communication equilibrium is also ex-ante optimal for the receiver. This logic then naturally extends to settings where the point of alignment is interior, because we can split the communication game around this point of alignment

Our paper follows the literature that analyzes the gains from adopting more sophisticated communication protocols in sender-receiver games. The paper most related to ours is GHPS, who studies different conflict resolution procedures, among them optimal mediation. While we expand their methodology to a broader class of games, our focus is on characterizing the class of games where one-shot communication is optimal. Ivanov (2014) provides sufficient conditions for mediation to be valuable, that is, for the receiver to benefit at all from communicating with the sender. Finally, Blume (2012) is also

[2] See also Alonso (2007) and Gordon (2010). A single round of unmediated communication obtains the upper bound on the receiver's payoff also in Rantakari (2016).

concerned with optimal mediation involving one-shot communication. In particular, Blume (2012) shows that the efficiency bound reported by GHPS for the leading example in CS can be achieved in one-shot communication if parties can rely on a correlation device that sends private signals before the sender becomes informed.[3] In contrast, in our paper optimal communication is face-to-face and does not rely on the use of a correlation device.

2. The model

There are two players, the informed sender (he) and the uninformed receiver (she). The payoffs to both players depend on the realized state of nature $\theta \in \Theta = [0, 1]$ and the chosen action (or decision) $y \in Y \subset \mathbb{R}$. The state of nature is distributed according to the distribution $F(\theta)$ which admits a continuous density $f(\theta)$ with $f(\theta) > 0$ for $\theta \in (0, 1)$. The payoffs of the receiver and the sender are

$$\begin{aligned} u_R(y, \theta) &= -(y - \theta)^2, \\ u_S(y, \theta) &= -(y - y_S(\theta))^2, \end{aligned} \tag{1}$$

where $y_S(\theta)$ is a differentiable function of the state. In our specification, the receiver's preferred action matches the realized state θ while the sender's preferred action is $y_S(\theta)$, where the bias $b(\theta) = y_S(\theta) - \theta$ measures the distance between the sender's and the receiver's preferred choices. Apart from the differentiability requirement, we don't impose any additional assumptions on the shape of $y_S(\theta)$. In particular, for our representation in Lemma 1 we don't require $y_S(\theta)$ to be non-decreasing.

While the sender observes θ, the receiver has authority over the action y. Prior to the receiver selecting an action, the players exchange messages according to a fixed communication protocol. While communication protocols may involve complex communication procedures with multiple rounds in which messages are exchanged, possibly with the help of a trustworthy mediator, the revelation principle applies: any equilibrium outcome of any sender-receiver game with communication can be replicated by a (canonical) communication equilibrium (Forges, 1986; Myerson, 1986). A communication equilibrium involves the use of a trustworthy mediator to which the informed party sends a single, private, costless message, which is a report of the state of nature, after which the mediator issues a recommendation to the receiver. Moreover, in a communication equilibrium the sender is sincere and the receiver obedient: reporting the true state is optimal for the sender, and abiding by the mediator's recommendation is optimal for the receiver. We can then restrict our attention to communication equilibria where the message space is the type space Θ and the space of mediator recommendations is the action space Y. Moreover, the receiver's preferred choice for any belief she may have about θ cannot fall outside $[0, 1]$. Therefore, we can set $Y = [0, 1]$.

Let $\mathcal{F} = \mathcal{B}(Y)$ be the Borel σ-algebra in Y. Formally, a mediation rule M is a family of probability measures on $(Y, \mathcal{F})$ indexed by Θ, $\{p(.|\theta)\}_{\theta\in\Theta}$, that completely describe the mediator's behavior: the mediator's recommendation is distributed according to the measure $p(y|\theta)$ following a report θ. Moreover, to ensure the sender's sincerity and the receiver's obedience, the family $\{p(.|\theta)\}_{\theta\in\Theta}$ must satisfy

$$\int_Y -(y - y_S(\theta))^2 \left(dp(y|\theta) - dp\left(y|\theta'\right)\right) \geq 0, \quad \forall \theta, \theta' \in \Theta, \quad \text{(IC-S)}$$
$$y = \mathbb{E}[\theta|y], \quad \forall y \in Y. \quad \text{(IC-R)}$$

The constraint (IC-S) is the sender's truthtelling constraint: the sender has no incentive to misrepresent the state when the mediator commits to randomizing its recommendation according to $p(y|\tilde{\theta})$ following a report $\tilde{\theta}$. The constraint (IC-R) ensures the receiver's obedience: given the mediation rule $\{p(.|\theta)\}_{\theta\in\Theta}$, and given the sender's truthtelling behavior, the mediator's recommendation refines the receiver's belief about the realized state. Then (IC-R) simply states that whenever the mediator recommends action y, the receiver's optimal action given her updated beliefs over the state is indeed y. The particular form of (IC-R) follows from the fact that, with quadratic payoffs, the receiver's optimal action must equal the expected state given her beliefs. A mediation rule that simultaneously satisfies (IC-R) and (IC-S) is called incentive compatible. Denote by $\mathscr{M}$ the set of incentive compatible mediation rules.

We are interested in mediation rules that maximize the ex-ante welfare of the receiver. We refer to a mediation rule as *unimprovable* for a player, if there is no incentive compatible mediation rule that yields a higher ex-ante expected payoff for that player. An optimal mediation rule is thus an incentive compatible mediation rule that is unimprovable for the receiver. Our quadratic setup allows also a simple informational interpretation of optimal mediation rules. Indeed, given the receiver's behavior (IC-R) and the functional form of the receiver's preferences (1), the receiver's payoff coincides with (the negative of) her residual variance after listening to the mediator's recommendation. Therefore optimal mediation rules also maximize the amount of information that is transmitted, when the informativeness of a signal is measured in terms of its expected residual variance.

We are particularly interested in a class of sender-receiver games where optimal mediation can be achieved through *brief conversations*. To be specific, we define a brief conversation as a Bayesian-Nash equilibrium of a game in which the sender sends a single, costless message observed by the receiver who then takes an action. That is, a brief conversation is the Nash equilibrium of a one-shot, face-to-face, unmediated cheap talk game where players cannot use a correlation

[3] For implementation in correlated equilibria, see also Vida and Forges (2013).

device. Formally, a brief conversation is characterized by a pair of Borel-measurable strategies μ and y and a belief function g defined as follows: (i) the sender's communication rule $\mu(\theta) : \Theta \to \Delta M$ specifies the probability of sending message $m \in M$ conditional on observing state θ, (ii) the receiver's response $y(m) : M \to Y$ maps messages into actions and (iii) the receiver's belief function $g(\theta \mid m) : M \to \Delta\Theta$ states the posterior probability of θ after observing message m. In a Bayesian-Nash Equilibrium, μ is optimal for the sender given the receiver's response, y is optimal for the receiver given the belief function and g is a regular conditional probability derived from μ according to Bayes rule, whenever possible.

3. Analysis

We start by deriving an alternative representation of the equilibrium payoffs induced by a communication equilibrium. To this end, for a given communication equilibrium let $U_S(\theta)$ be the sender's interim payoff when the state is θ, and let V_S and V_R be the ex-ante expected payoffs of the sender and the receiver. As in GHPS, for any incentive compatible M, defined by $\{p(.|\theta)\}_{\theta\in\Theta}$, let $\bar{y}(\theta) = \int_Y y dp(y|\theta)$ and $\bar{\sigma}^2(\theta) = \int_Y (y - \bar{y}(\theta))^2 dp(y|\theta)$ be the equilibrium expected decision and variance of the decision. Quadratic payoffs are convenient as knowledge of $\bar{y}(\theta)$ and $\bar{\sigma}^2(\theta)$ suffices to obtain the state-dependent payoffs for any mediation rule. Indeed, one immediately has

$$\begin{aligned} U_S(\theta) &= -(\bar{y}(\theta) - y_S(\theta))^2 - \bar{\sigma}^2(\theta), \\ V_S &= -\mathbb{E}\left[(\bar{y}(\theta) - y_S(\theta))^2\right] - \mathbb{E}\left[\bar{\sigma}^2(\theta)\right], \\ V_R &= -\mathbb{E}\left[(\bar{y}(\theta) - \theta)^2\right] - \mathbb{E}\left[\bar{\sigma}^2(\theta)\right]. \end{aligned}$$

In principle, knowledge of both values $\bar{y}(\theta)$ and $\bar{\sigma}^2(\theta)$ would be required to obtain $U_S(\theta),\ \theta \in [0,1]$, and knowledge of the functions $\bar{y}$ and $\bar{\sigma}^2$ would be necessary to deduce V_S and V_R. However, the restrictions imposed on the set of equilibrium payoffs by (IC-S) and (IC-R) imply that, for games with quadratic payoffs, interim and ex-ante payoffs can be obtained solely on the basis of the state-contingent average decision $\bar{y}(\theta),\ \theta \in [0,1]$.

Lemma 1. *Let M be an incentive compatible mediation rule that induces in equilibrium $\bar{y}(\theta)$ and $\bar{\sigma}^2(\theta),\ \theta\in[0,1]$. Then*

$$U_S(\hat{\theta}) = \mathbb{E}[\bar{y}(\theta)K_{S(\hat{\theta})}(\theta)] - y_S^2(\hat{\theta}),\ \ \hat{\theta} \in [0,1], \tag{2}$$

$$V_S = \mathbb{E}[\bar{y}(\theta)K_S(\theta)] - \mathbb{E}[y_S^2(\theta)], \tag{3}$$

$$V_R = \mathbb{E}[\bar{y}(\theta)K_R(\theta)] - \mathbb{E}[\theta^2], \tag{4}$$

where, letting $\mathbb{I}_{[0,\hat{\theta}]}$ be the characteristic function of the set $[0,\hat{\theta}]$, we have[4]

$$\begin{aligned} K_{S(\hat{\theta})}(\theta) &= 2y_S(\theta) - \theta - 2\frac{y_S'(\theta)}{f(\theta)}(1 - F(\theta) - \mathbb{I}_{[0,\hat{\theta}]}(\theta)), \\ K_S(\theta) &= 2y_S(\theta) - \theta, \\ K_R(\theta) &= \theta. \end{aligned} \tag{5}$$

Lemma 1 provides expressions for $U_S(\theta)$, V_S and V_R as affine functionals of the average decision $\bar{y}$ without explicit recourse to either $\bar{\sigma}^2$ or to any additional information of the mechanism M.[5] Lemma 1 is analogous to well known results in mechanism design with quasilinear utility and convex type spaces: if $\bar{y}(\theta)$ plays the role of an "allocation" and $\bar{\sigma}^2(\theta)$–which enters additively in $U_S(\theta)$–plays the role of a type-dependent transfer, then an application of the envelope theorem to (IC-S) implies that $U_S(\theta)$ can be obtained from knowledge of the interim payoff of one type and the entire allocation $\bar{y}(\theta), \theta \in [0,1]$ (see, Milgrom and Segal, 2002). Under mediation, however, we must also ensure that the receiver is obedient, i.e. (IC-R) must also hold. Lemma 1 shows that this additional constraint eliminates the degree of freedom in specifying the interim payoff to a fixed sender's type. That is, knowledge of the "allocation" $\bar{y}$ suffices to compute both interim and ex-ante expected payoffs. Note, however, that Lemma 1 remains silent on the set of implementable $\bar{y}$. For example, the conflict of interest may be so severe that only a babbling equilibrium can be sustained (and thus the set of implementable $\bar{y}$ is a singleton), which nevertheless would still satisfy (2), (3) and (4).

3.1. Interim and ex-ante payoffs under mediation.

In principle, if the space of implementable $\bar{y}$ is sufficiently rich, one may conjecture that knowledge of $U_S(\theta)$ for some type θ is not enough to derive $U_S(\theta')$ for some other type θ', and would also be insufficient to infer the ex-ante welfare of the sender and the receiver. In other words, if the set of mediation rules is sufficiently rich one would expect that mediation rules that exhibit the same local behavior, by inducing the same $U_S(\theta)$ for some type θ, may have widely different global

[4] The characteristic function of the set A, $\mathbb{I}_A$, is defined by $\mathbb{I}_A(x) \equiv 1$ if $x \in A$ and $\mathbb{I}_A(x) \equiv 0$ otherwise.

[5] GHPS obtain these expressions for the constant bias case–i.e., when $y_S(\theta) = \theta + b$.

properties and thus generate different ex-ante payoffs. By contrast, the following proposition characterizes a class of sender-receiver games in which local behavior unequivocally determines the global welfare properties of any incentive compatible mediation rule. In fact, for these games, the ex-ante payoff of the sender or of the receiver depends only on the interim payoff of the sender at one of the extreme types.

For the remainder of this paper let $h(\theta) = f(\theta)/(1-F(\theta))$ and $r(\theta) = f(\theta)/F(\theta)$ be the hazard rate and reversed hazard rate of the distribution $F(\theta)$. For clarity of exposition, below we indicate with superscript M the interim and ex-ante payoffs corresponding to the mediation rule M.

Proposition 1. *For any two incentive compatible mediation rules M and M' we have the following implications whenever $y_S(\theta)$ takes one of the following forms for some $\alpha, \beta \in \mathbb{R}$:*

$$y_S(\theta) = \alpha\mathbb{E}\left[\tilde{\theta}|\tilde{\theta} \geq \theta\right] + \beta \Rightarrow U_S^{M'}(0) - U_S^M(0) = (2\alpha - 1)\left(V_R^{M'} - V_R^M\right), \tag{6}$$

$$y_S(\theta) = \alpha\mathbb{E}\left[\tilde{\theta}|\tilde{\theta} \leq \theta\right] + \beta \Rightarrow U_S^{M'}(1) - U_S^M(1) = (2\alpha - 1)\left(V_R^{M'} - V_R^M\right), \tag{7}$$

$$y_S(\theta) = \alpha \int_0^\theta h(\theta')d\theta' + \beta \Rightarrow U_S^{M'}(0) - U_S^M(0) = V_S^{M'} - V_S^M, \tag{8}$$

$$y_S(\theta) = \alpha \int_0^\theta r(\theta')d\theta' + \beta \Rightarrow U_S^{M'}(1) - U_S^M(1) = V_S^{M'} - V_S^M. \tag{9}$$

We can interpret Proposition 1 as a refinement of Lemma 1. Lemma 1 shows that knowledge of the function $\bar{y}$ is sufficient to compute ex-ante and interim payoffs for any incentive compatible M. Nevertheless, the functional difference in the linear functionals defining (2), (3) and (4) implies that two average decisions $\bar{y}$ and $\bar{y}'$ that induce the same $U_S(\theta)$ for some type θ may very well yield different ex-ante payoffs. Adding more structure to our model, however, can link local interim payoffs and global ex-ante payoffs. For instance, if $y_S(\theta)$ can be written as (6) or (7), then a linear relation exists between the expected payoff to the receiver and the payoff to the sender at an extreme type for any incentive compatible mediation rule, while if either (8) or (9) holds then the change in the ex-ante payoff to the sender when switching from M to M' equals the change in the payoff to an extreme type. We define the family of *LG (Local-Global) functions* as any $y_S(\theta)$ that can be expressed as in (6), (7), (8), or (9)

The intuition behind Proposition 1 is based on the representations (2), (3) and (4) in Lemma 1 coupled with the obedience constraint by the receiver (IC-R). For example, suppose that (6) holds. Then, the function $y_S(\theta) - y_S'(\theta)/h(\theta)$ is linear in the state, implying that $K_{S(0)}$, obtained by setting $\hat{\theta} = 0$ in (5), can be expressed as an affine function of K_R, which defines the expected utility of the receiver. The final step is to observe that the law of iterated expectations applied to (IC-R) implies that $\mathbb{E}[\bar{y}(\theta)]$ is constant across all incentive compatible mediation mechanisms (and equal to $\mathbb{E}[\theta]$). Then (6) follows immediately from Lemma 1 as $\mathbb{E}\left[\bar{y}(\theta)K_{S(0)}(\theta)\right]$ is a linear transformation of $\mathbb{E}[\bar{y}(\theta)K_R(\theta)]$ in the set of implementable $\bar{y}$. Similar reasoning shows that $\mathbb{E}\left[\bar{y}(\theta)K_{S(1)}(\theta)\right]$ and $\mathbb{E}[\bar{y}(\theta)K_R(\theta)]$ are linearly related if (7) holds. Conversely, if (8) holds then $K_{S(0)}(\theta) - K_S(\theta)$ is a constant, while $K_{S(1)}(\theta) - K_S(\theta)$ is constant if (9) holds. Again, as $\mathbb{E}[\bar{y}(\theta)]$ is constant over the space of mediation rules then we obtain (8) and (9).

We note that both (6) and (7) hold if the sender's preferred decision is an affine function of the state and the state is uniformly distributed.[6] This includes the leading example in CS, which has been the workhorse model in applications involving cheap talk communication. Indeed, the main studies of optimal mediation in CS-type of games all consider this "uniform-quadratic" example (see, Krishna and Morgan, 2004; Blume et al., 2007; GHPS; Ivanov, 2010; Blume, 2012). In fact, for this case we have the following corollary.

Corollary 1. *Suppose that $y_S(\theta) = a\theta + b$ with $a > 1/2$. Then, all incentive compatible mediation rules are Pareto ranked. Letting $\overline{U}(\theta) = \sup_{M' \in \mathscr{M}} U_S^{M'}(\theta)$, if θ is uniformly distributed then M is an optimal mediation rule if, and only if, $U_S^M(\hat{\theta}) = \overline{U}(\hat{\theta}), \hat{\theta} \in \{0, 1\}$.*

It follows that, as long as $a > 1/2$, a mediation rule that is unimprovable for the receiver must also be unimprovable for the sender. Moreover, in the linear-uniform case, an optimal mediation rule must lead to the maximum payoff for the sender at the extremes of the type space. The observation that optimal mediation maximizes $U_S(0)$ is exploited by GHPS to characterize the optimal mediation rule for the constant bias case (i.e., when $a = 1$, $b \neq 0$).

We can use Proposition 1 to enquire, instead, whether the receiver can benefit from mediation in sender-receiver games (see, e.g., Ivanov, 2014).[7] For instance, if $y_S(\theta) = \mathbb{E}\left[\tilde{\theta}|\tilde{\theta} \geq \theta\right]$, so that (6) holds with $\alpha = 1$ and $\beta = 0$, then we must have that any optimal mediation rule must achieve the maximum of the sender's utility at $\theta = 0$. As the babbling equilibrium implements $y = \mathbb{E}\left[\tilde{\theta}\right]$ –i.e., the preferred action of type $\theta = 0$–it follows that babbling is unimprovable for the receiver, in spite of players perfect alignment at $\theta = 1$. The same argument would apply to $y_S(\theta) = \mathbb{E}\left[\tilde{\theta}|\tilde{\theta} \leq \theta\right]$, which satisfies (7) with $\alpha = 1$ and $\beta = 0$. This shows that mediation may not be beneficial if the conflict is on average too large, even if there is locally a point of perfect alignment between sender and receiver.

[6] We leave the analysis of other cases that satisfy (6) to Section 4.1.

[7] We thank a referee for drawing this connection and suggesting these examples.

We end this section with two caveats regarding Proposition 1. First, the implications in 6–(7) and 8–(9) establish a bijection in terms of payoffs, not actions or even average actions. In fact, if either (6) or (7) holds, then different mediation rules that yield the same expected utility for the receiver (and thus for either the lowest or highest type of sender) may induce totally different actions. This is simple to see by noting that in the constant bias-uniform specification with $b < 1/12$, one can construct a mediation rule that induces either the babbling equilibrium or the three partition equilibrium with fixed probabilities and such that the expected payoff to the sender is the same at the extreme types as the two partition cheap talk equilibrium.[8] However, average actions cannot coincide under both communication rules. Second, either (6) or (7) are only sufficient for the existence of a bijection between $U_S(0)$ or $U_S(1)$ and V_R, as we don't incorporate information about the set of implementable $\bar{y}$. For instance, a bijection would trivially follow when the conflict of preferences between sender and receiver is so severe that any incentive compatible mediation rule implements a single action.

4. Art of brevity

We now turn our attention to unimprovable mediation rules, and study when they involve brief conversations. For the cases that satisfy (6) with $\alpha > 1/2$, Proposition 1 establishes that M is optimal if and only if the sender' s payoff in state $\theta = 0$ cannot be improved by any other mediation rule. Similar statements can be made when $y_S(\theta)$ satisfies (7), (8) and (9). Note that $U_S(\hat{\theta}) \leq 0$ and $U_S(\hat{\theta}) = 0$ if and only if the receiver selects the sender's preferred action when his type is $\hat{\theta}$. The next proposition uses this remark to describe a class of games that admit a brief conversation where the receiver selects the sender's preferred action at some type $\hat{\theta}$, and this brief conversation must then be unimprovable for one of the players. For this, we restrict attention to LG functions defined in Proposition 1 that satisfy two additional conditions on the bias $b(\theta)$: (i) it is non-zero in $(0, 1)$, and (ii) it vanishes at one of the extreme types. Thus, these functions, which we label *LG functions with alignment at an extreme type*, take one of the following forms:

$$y_S(\theta) = \alpha(\mathbb{E}[\tilde{\theta}|\tilde{\theta} \geq \theta] - \mathbb{E}[\tilde{\theta}]), \tag{10}$$

$$y_S(\theta) = 1 + \alpha(\mathbb{E}[\tilde{\theta}|\tilde{\theta} \leq \theta] - \mathbb{E}[\tilde{\theta}]), \tag{11}$$

$$y_S(\theta) = \alpha \int_0^{\theta} h(\theta')d\theta', \tag{12}$$

$$y_S(\theta) = 1 - \alpha \int_{\theta}^{1} r(\theta')d\theta', \tag{13}$$

with α such that $y_S(\theta) > \theta$ for $\theta \in (0, 1]$ for the cases (10) and (12), and such that $y_S(\theta) < \theta$ for $\theta \in [0, 1)$ for the cases (11) and (13).

Proposition 2. *Suppose that the sender's preferred action is given by an LG function with alignment at an extreme type.*[9] *Then, there exists a brief conversation that is unimprovable for the receiver. This brief conversation induces an infinite number of different actions, with $y = 0$ an accumulation point for (10) and (12) and $y = 1$ an accumulation point for (11) and (13).*

As (10–13) require full alignment between sender and receiver at some extreme type, Proposition 2 rules out the sender-receiver games studied in CS where the preferred actions of the sender and the receiver never coincide. These types of games, with an infinite type- and action-space and where the bias $b(\theta)$ may vanish or even change sign, have been studied by Gordon (2010) who characterizes communication equilibria and provides conditions for the existence of infinite equilibria. Proposition 2 also shows that the state $\hat{\theta}$ at which preferences are aligned is "communicable" (see Dilmé, 2020) in an unimprovable brief conversation: the sender perfectly reveals his private information if $\theta = \hat{\theta}$.

The logic behind Proposition 2 can be seen in two steps. First, the existence of a state of full alignment implies in our case that an infinite equilibrium exists. This is not immediate as Alonso (2007) and Gordon (2010) show that even if the bias vanishes at some state only finite equilibria maybe possible. However, (10–13) imply that the range of preferred actions of the sender contains the range of preferred actions of the receiver (i.e., the sender is *reactive*, or he has an *outward bias* in the terminology of Gordon (2010)) and thus Theorem 2 in Gordon (2010) guarantees the existence of an infinite equilibrium. Second, an infinite equilibrium on a bounded state space must necessarily have an accumulation point at a state at which the bias disappears. This implies that an infinite equilibrium guarantees that the receiver selects the sender's (and the receiver's) preferred action at some point of alignment. Then (10–13) guarantees that there is a unique point of alignment, which occurs at an extreme type. For instance, the requirement in (10) that $y_S(\theta) > \theta$ for $\theta > \hat{\theta} = 0$ (and in

[8] Formally, for the leading example in CS with bias $b > 0$, let $\{\theta_i\}_{i=0}^{i=N}, \theta_i = 0, \theta_N = 1$, be the cutoff points of an N-partition equilibrium with $\Delta_i^N = \theta_i - \theta_{i-1}, i \geq 1$, the size of each partition. Then, (i) $\Delta_1^N = (1/N) - 2(N-1)b$, (ii) $V_R = -\sum_{i=1}^{i=N} \left(\Delta_i^N\right)^3$, and (iii) $\bar{y}(0) = \Delta_1^N/2$–see CS. For instance, if $b < 1/12$, equilibria with 2 and 3 partitions are possible. For $b = 0.01$, let $p = 0.147$ and consider mediation rule $M_{1,2}$ that, given the sender's message, recommends the action corresponding to the three-partition equilibrium with probability p, and with probability $1 - p$ it recommends the babbling action $y = 1/2$. The expected utility of the receiver is the same under the 2-partition equilibrium and the mediation rule $M_{1,2}$. However, the average action under $M_{1,2}$ for type $\theta = 0$ is $\bar{y}(0) \approx 0.2 < 0.24 = \bar{y}'(0)$ with $\bar{y}'(0)$ induced under the 2-partition equilibrium.

[9] In the proof of the proposition we show that the set of functions satisfying (10–13) is non-empty given our assumption of a positive density $f(\theta) > 0$ in (0,1).

(11) that $y_S(\theta) < \theta$ when $\theta < \hat{\theta} = 1$) implies that the unique point of congruence is $\theta = \hat{\theta}$ so that the infinite equilibrium must necessarily have the sender of type $\theta = \hat{\theta}$ inducing action $y = \hat{\theta}$. As this equilibrium maximizes the sender' payoff at $\theta = \hat{\theta}$, it must also be optimal for the receiver. The same argument applies to (12) and (13). Therefore, when any of (10–13) hold, brief conversations that are unimprovable are also very detailed around the point of full alignment.

An important implication of Proposition 2 is that the one-shot communication equilibrium characterized in some recent applied papers cannot be improved upon by having more rounds of communication, communicating through a noisy channel, deploying a correlation device, or, more generally, by employing a trustworthy mediator. The relevance of this observation is that it addresses a typical concern regarding applied models with cheap talk communication, namely, that the results and insights may not be robust to the receiver adopting a more informative communication protocol. For example, Melumad and Shibano (1991) and Stein (1989) study communication games which are equivalent to sender-receiver games with preferences over actions given by (1) with $y_S(\theta) = a\theta,\ a > 1,$ and $\theta \sim U[0,1]$. Note that this case satisfies (10) with $\alpha = 2a$. Proposition 2 then establishes that the infinite equilibrium studied in those papers is necessarily the optimal communication protocol for the receiver.

Some applied papers (e.g., Alonso et al., 2008 and Rantakari, 2008)[10] consider communication games where the point of congruence between sender and receiver does not correspond to an extreme type. Nevertheless, we can extend Proposition 2 to these cases.

Proposition 3. *Let $y_S(\theta) = a\theta + b,\ a > 1,$ with θ uniformly distributed and $\widetilde{\theta} = b/(1-a) \in (0,1)$. Then, there is a brief conversation that is unimprovable for both sender and receiver. This brief conversation induces an infinite number of different actions, where $y = \widetilde{\theta}$ is the accumulation point.*

The setting analyzed by Alonso et al. (2008) and Rantakari (2008)–with independent communication flows between managers–satisfies Proposition 3 and thus their findings regarding the impact of internal communication on organization are obtained under a pairwise-optimal communication protocol. This result arises from the observation that even if their framework is formally a two-sender game, the quadratic payoff structure implies that the two dimensions are additively separable so that, focusing on independent communication flows, each pairwise communication game satisfies the conditions of Proposition 3.

The conclusions of Propositions 2 and 3 stand in contrast to the result obtained by GHPS (and others) that, in the leading example of CS, it is generically possible to improve upon one round of unmediated communication, as long as informative communication is possible in the one-round case ($b < 1/2$).[11] Thus, while the proof behind Propositions 1, 2 and 3 is primarily technical, it is instructive to consider also a more descriptive intuition of its logic which also establishes a link between the two settings.

Consider the leading example of CS with a positive bias $b > 0$, and a mediation mechanism that induces the smallest action $\underline{y}$ and an upward-binding incentive compatibility constraint. Now, suppose that $y_S(0) = b < \underline{y}$. Then, the mediator is able to separate all $y_S(\theta) < \underline{y}$ at no cost by simply including a lower minimum action in the mechanism. Indeed, lowering the minimum action relaxes the incentive-compatibility constraint of the receiver, which in turn allows a higher maximum action $\bar{y}$. Finally, increasing the maximum action lowers the need for mutually harmful value dissipation through $\overline{\sigma}^2(\theta)$, which is otherwise needed to satisfy the incentive-compatibility constraint of the sender. This benefit is available until $\underline{y} = b$. Conversely, inducing a smallest action below b carries a strictly positive cost, as it reduces the payoff for the lowest type and thus necessitates additional rent dissipation to maintain truthful revelation across the entire type space. Thus, the optimal mechanism maximizes the sender's payoff at the extreme type.

In the one-round CS equilibrium, the equivalent question reduces to asking how the action induced by the lowest message, $y = \mathbb{E}(\theta|m_1)$, compares to the action desired by the lowest type of sender, $y_S(0) = b$. If $\mathbb{E}(\theta|m_1) < b < \mathbb{E}(\theta) = \frac{1}{2}$, we can introduce noise to the communication channel, which will increase $\mathbb{E}(\theta|m_1)$ and thus bring the equilibrium action of the receiver closer to the sender's preferred action. This, in turn, lowers the sender's incentives to exaggerate and the resulting need for rent dissipation. In other words, in the unmediated case, the initial communication is too precise. Introducing noise to the first message increases the alignment between the sender and the receiver for low-enough sender types, which then benefits the receiver globally due to the lower need for value dissipation through $\overline{\sigma}^2(\theta)$. Indeed, as shown by GHPS, when the constant bias b satisfies $b = 1/(2N^2)$ for some integer N, the most informative C-S equilibrium (which involves N different actions) is unimprovable through mediation. The reason is that for this bias, $\mathbb{E}(\theta|m_1) = b$, and so the lowest type gets his desired outcome. As a result, the expected payoff cannot be improved any further. That is, even the constant-bias case admits a (non-generic) set of cases where brief conversations are optimal.

Now, in contrast to the leading example of CS, all the papers referred to above have a multiplicative bias structure with a point of preference alignment. The existence of such a point implies that the sender at that state would have no incentive to misrepresent his type if believed by the receiver, while the conditions of Proposition 2 guarantee that the incentive-compatibility constraints are binding away from this point of alignment ("reactive" agent). Then, it is impossible to improve upon the infinite equilibrium of one round of unmediated communication, as that achieves the maximum payoff for the sender at the point of alignment.

[10] See also Alonso et al. (2015).

[11] See Ivanov (2014) for more general conditions guaranteeing that the receiver benefits from mediation.

Moving away from the fixed bias example also highlights that the benefits of mediation do not depend only on the magnitude of the bias but also on the shape of the bias. In particular, the analysis of GHPS suggests that employing a mediator is most valuable when the conflict of interests is intermediate. Further, one-shot communication can, generically, be improved upon either through long conversations or mediation.[12] We find however that for a state-dependent conflict of interest, the gains from mediation depend not only on the magnitude of the conflict of interest (as given by the expected bias), but also on the shape of the bias $b(\theta)$. Indeed, (10–13) impose no upper limit on the value of α while the expected conflict increases without bound as α increases. The key difference with the constant bias case is that in spite of an increased average conflict, full alignment at an extreme type persists and influential communication remains feasible.

Finally, while the analysis of this paper is focused on sender-receiver games with quadratic preferences, the basic logic of the analysis is clearly broader. For example, Rantakari (2016) establishes that a single round of unmediated communication attains the upper bound on the receiver's payoff in a qualitatively different setting where the receiver is making a choice between two competing alternatives, but which also features preference alignment for an extreme type. It is unclear, however, to what extent the existence of such a point will be sufficient in more general settings where knowledge of the set of implementable decision rules becomes important to characterizing the set of attainable payoffs.

4.1. Brief conversations, uncertainty and the quality of communication

While the cheap talk setting of CS has found wide acceptance as a model of communication under conflicting preferences, applications have generally restricted attention to the "uniform-quadratic" example as expressions for the payoffs in models beyond that case have proven difficult to come by. An additional contribution of our analysis is the result that for the class of sender-receiver games characterized in Proposition 2 we can explicitly compute the ex-ante payoff to the receiver under an optimal communication equilibrium, without needing to explicitly solve for the communication equilibrium itself.

Proposition 4. *For every $y_{i-1}, y_i \in Y$, $y_{i-1} < y_i$, define $\gamma(y_{i-1}, y_i)$ by*

$$\gamma(y_{i-1}, y_i) = \mathbb{E}[\theta | \theta \in [y_{i-1}, y_i]]. \tag{14}$$

If $y_S(\theta)$ satisfies (10), then the maximum expected payoff of the receiver is[13]

$$V_R^* = -\mathbb{E}\left[\theta^2\right] + (1 - F(y^*))\gamma^2(0,1)\frac{\gamma^2(y^*,1) - \gamma^2(0,y^*)}{\gamma^2(0,1) - \gamma^2(0,y^*)} \tag{15}$$

with $y^* \in (0,1)$ the unique solution to

$$2\alpha(\gamma(y^*,1) - \gamma(0,1)) = \gamma(0,y^*) + \gamma(y^*,1). \tag{16}$$

To derive (15), the proof of the proposition constructs two incentive compatible mechanisms that give the sender the same interim utility at $\theta = 0$. The first mechanism is equivalent to a two partition equilibrium where the sender only reports whether his type exceeds a threshold y^*. Truthtelling by the sender and obedience by the receiver requires this threshold to satisfy the "arbitrage" condition (16). To define the second mechanism, M^λ, let M^* be the mechanism that implements the infinite equilibrium described in Proposition 2-i, and let $M^\varnothing$ be the totally uninformative mediation rule (i.e. the babbling equilibrium). Then after the sender's report, with probability λ M^λ issues a recommendation according to $M^\varnothing$, while with probability $1 - \lambda$ it follows M^*. The probability λ is chosen such that the two partition equilibrium and M^λ yield the same $U_S(0)$. Then the linear relation (6) for games where $y_S(\theta)$ satisfies (6) implies that these two mechanisms must generate the same ex-ante payoffs for the receiver, from which we deduce that V_R^* satisfies (15).

While 10–(11) are still relatively restrictive conditions, they are not uniquely tied to the uniform distribution. As a particular application of Proposition 4, we now use (15) to compare V_R^* for different distributions of the state. We present three examples below, which are constructed so as to have $y_S(\theta) = a\theta, a > 1$, ensuring that comparative statics follow from changes in the distribution rather than the bias. To allow for changes in the distribution, however, we change the (bounded) support of Θ for each example.

Example 1. (exponential) For each truncated exponential $f(\theta, \bar{\theta}, \lambda) = \lambda e^{-\lambda\theta}/(1 - e^{-\lambda\bar{\theta}}), \theta \in [0, \bar{\theta}]$, let $y_S(\theta, \bar{\theta}, \lambda)$ be given by (10). Then $f(\theta, \bar{\theta}, \lambda)$ converges pointwise to $\lambda e^{-\lambda\theta}$ and $y_S(\theta, \bar{\theta}, \lambda)$ converges pointwise to $a\theta$ as $\bar{\theta} \to \infty$, where $a = \alpha$.[14] The limit variance of the truncated exponentials is the variance of an exponential $1/\lambda^2$.

Example 2. (linear) Consider a linear pdf that vanishes at the upper bound of the support, $f(\theta) = \left(2/\bar{\theta}_l\right)(1 - \left(\theta/\bar{\theta}_l\right)), \theta \in [0, \bar{\theta}_l]$, with $Var[\theta] = \bar{\theta}_l^2/18$. Then, applying (10), we obtain $y_S(\theta) = a\theta$ with $a = 2\alpha/3$.

Example 3. (uniform) Finally, applying (10) to a uniform distribution $f(\theta) = 1/\bar{\theta}_u, \theta \in [0, \bar{\theta}_u]$, with $Var[\theta] = \bar{\theta}_u^2/12$, we have $y_S(\theta) = a\theta$ with $a = \alpha/2$.

[12] For the leading example in CS one has that: (i) For $b \geq 1/2$ no information transmission is possible, (ii) for $b < 1/2$, one-shot communication is generically not optimal, and (iii) if $b < 1/8$ multiple rounds of unmediated communication can achieve the maximum payoff (see GHPS for details).

[13] A similar expression can be obtained if instead (11) holds.

[14] See proof of Corollary 2.

For each example, we can use (16) to solve for the threshold y^*, which given $F(\theta)$ uniquely determines the expected payoff of the receiver. This result is summarized in the following corollary:

Corollary 2. *Suppose that $y_S(\theta) = a\theta,\ a \geq 1$.*

(i) The receiver's maximum expected payoff is

$$V_R^* = -\frac{x(a-1)}{xa-1} Var[\theta], \tag{17}$$

where $x = 2$ for the limit of truncated exponentials (example 1), $x = 3$ for the linear case (example 2), and $x = 4$ for the uniform case (example 3).[15]

(ii) Recall that $\bar{\theta}_l$ and $\bar{\theta}_u$ are the upper bound of the support of θ for the linear and uniform cases. If

$$\frac{2}{3} < \frac{\bar{\theta}_u}{\bar{\theta}_l} < \sqrt{\frac{4a-1}{2(3a-1)}} \text{ and } 3 < \lambda\bar{\theta}_l < \sqrt{\frac{12(3a-1)}{2a-1}}, \tag{18}$$

then the expected bias is highest for the uniform case while it is lowest for the limit of truncated exponentials. However, the maximum expected payoff to the receiver is highest for the uniform case but lowest for the exponential case.

As expected, (17) shows that the quality of communication improves when the conflict between the sender and the receiver decreases (i.e., as a decreases). More interestingly, the corollary also shows that a shift in the distribution that lowers the expected bias $\mathbb{E}[y_S(\theta) - \theta]$ can actually worsen communication and lower the receiver's expected payoff under an optimal communication protocol.

To see this result, note first that $dV_R^*/dx > 0$, so that holding the variance of the underlying distribution constant, the receiver's expected payoff improves as we move from the exponential to the linear and finally to the uniform case. But at the same time, if we hold $Var[\theta]$ constant, we have that $\frac{1}{\lambda} < \frac{\bar{\theta}_l}{3} < \frac{\bar{\theta}_u}{2}$, so that the expected conflict between the sender and the receiver is also increasing. In other words, while the expected conflict between the sender and the receiver increases, the flow of information improves and the receiver is better off. Corollary 2-ii then provides the range of parameter values such that the distribution that leads to the highest expected payoff for the receiver is also the one with the highest expected conflict.

The intuition behind this result relies on the two separate roles that uncertainty plays in determining the gains from communication. First, note that all three distributions are weakly decreasing in θ and all communication equilibria are partitional equilibria, where the intervals become smaller as one approaches the point of congruence at $\theta = 0$. Therefore, holding constant the partition of the state space, a shift in the distribution that puts more mass on the larger states where communication is less detailed and the realized conflict is larger can only worsen the receiver's payoff.

The change in the distribution, however, also changes the "arbitrage condition" that determines the equilibrium partition. Intuitively, as higher states become relatively more likely, the receiver becomes more responsive to the recommendations of the (more reactive) sender and, as a result, the flow of information is improved. To see this, suppose that the receiver knows that the state lies in $[y, y+\Delta]$ so that her optimal choice exceeds y by $\psi(y,\Delta) = \gamma(y, y+\Delta) - y$. That is, $\psi(y,\Delta)$ measures the responsiveness of the receiver when she knows that the state lies in an interval of length Δ. Then, $\psi(y,\Delta)$ does not vary in y for the uniform case, while it decreases in y for the linear case. That is, the receiver is less responsive under a linear distribution than a uniform. To preserve incentive compatibility by the sender, the size of the intervals must then grow more rapidly and thus the equilibrium partition will be coarser under the linear distribution. Then Corollary 2-ii indicates that this second effect dominates for the range of parameters in (18) and a lower expected conflict actually leads to a lower expected payoff for the receiver.[16]

5. Conclusion

The literature has emphasized the beneficial role of mediation in sender-receiver games where conflicting preferences hinder information transmission. We have identified a class of games, however, for which neither lengthy conversations nor mediation enhances the amount of information exchanged in equilibrium. In short, brief conversations are optimal in these cases. This implies, for instance, that the pairwise communication equilibria described in Alonso et al. (2008) and Rantakari (2008) cannot be improved upon mediation, and thus comparative statics in those models are derived under optimal communication. Importantly, the optimality of brief conversations persists even if the average conflict between the sender and the receiver is arbitrarily large. This shows that the value of mediation not only depends on the magnitude of the conflict between the sender and the receiver but also on how this conflict varies over the state space. In our case, as long as the conflict vanishes at one of the extreme points of the state space, brief conversations remain optimal.

Our proof of optimality of brief conversations (Proposition 2) relies on the existence of a one-to-one relation between the sender's interim payoffs at extreme types and the ex-ante expected payoffs of the players. This bijection also implies that

[15] This expression for the uniform case already appears in Alonso et al. (2008) and Rantakari (2008).

[16] Note that this same logic should hold in the leading example of CS with additive bias, as the intuition depends simply on the effect that the underlying distribution has on the responsiveness of the receiver, independent of the exact bias structure of the sender. But when Proposition 2 is satisfied, we can explicitly compute the expected payoff of the receiver and compute the effect.

optimal mediation rules are locally optimal for some sender's type. A natural question is the extent to which this assertion holds true in general. In other words, does an optimal mediation rule necessarily maximize the interim utility of some sender's type? Furthermore, our proofs made no use of the characteristics of the set of implementable average actions, as we rely instead on properties of the payoff functions. Better understanding implementability can further our understanding of the benefits of mediation. We leave these two observations for future work.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Proofs

Proof of Lemma 1: Let M be an arbitrary incentive compatible mediation rule, with $H(y)$ the induced distribution over actions and $g(\theta|y)$ an induced conditional distribution over the state given the recommended action; i.e., $H(y) \equiv \int_{[-\infty,y]\times\Theta} dp(y'|\theta)dF(\theta)$ and $g(\theta|y)$ satisfying $\int_A dp(y|\theta)dF(\theta) = \int_A dg(\theta|y)dH(y)$ for any measurable $A \subset Y \times \Theta$. We will derive the relations (2), (3) and (4) in three steps. First, we have that

$$\int_{Y\times\Theta} y\theta dp(y|\theta)dF(\theta) = \int_Y y\Big(\int_\Theta \theta dg(\theta|y)\Big)dH(y) = \int_Y y^2 dH(y), \tag{A.1}$$

where in the first equality we apply the Fubini-Tonelli Theorem and the second equality follows from (IC-R). Therefore, V_R can be written as

$$\begin{aligned} V_R &= -\int_{Y\times\Theta} (y-\theta)^2 dp(y|\theta)dF(\theta) = -\int_{Y\times\Theta} \big(y^2 - 2y\theta + \theta^2\big)dp(y|\theta)dF(\theta) \\ &= \int_{Y\times\Theta} \big(y\theta - \theta^2\big)dp(y|\theta)dF(\theta) = \mathbb{E}[\bar{y}(\theta)\theta] - \mathbb{E}\big[\theta^2\big]. \end{aligned}$$

where we applied (A.1) to the third equality and the law of iterated expectations to the last equality. This establishes (4) with $K_R(\theta) = \theta$.

Second, we can write

$$\begin{aligned} V_S &= -\mathbb{E}\big[(y - y_S(\theta))^2\big] = -\mathbb{E}\big[y^2 - 2yy_S(\theta) + y_S^2(\theta)\big] = \\ &= -\mathbb{E}\big[y\theta - 2yy_S(\theta) + y_S^2(\theta)\big] = -\mathbb{E}\big[\bar{y}(\theta)(\theta - 2y_S(\theta)) + y_S^2(\theta)\big], \end{aligned} \tag{A.2}$$

where we have again applied (A.1) to the third equality. This establishes (3) with $K_S(\theta) = 2y_S(\theta) - \theta$.

Third, fixing a probability measure $p(\cdot|\hat{\theta})$ from the mediation rule M, the function $-\int_Y (y - y_S(\theta))^2 dp(y|\hat{\theta})$ has (at least) the same smoothness properties as $y_S(\theta)$. Our assumption that $y_S(\theta)$ is differentiable and Theorem 2 of Milgrom and Segal (2002) then imply that $U_S(\theta)$ is absolutely continuous, and for any two states θ and θ' satisfies the integral representation

$$\begin{aligned} U_S(\theta') - U_S(\theta) &= \int_\theta^{\theta'} \Big(\int_Y 2(y - y_S(\tau))y_S'(\tau)dp(y|\tau)\Big)d\tau \\ &= 2\int_\theta^{\theta'} (\bar{y}(\tau) - y_S(\tau))y_S'(\tau)d\tau. \end{aligned} \tag{A.3}$$

Fixing a reference state $\tilde{\theta}$, integrating by parts, and rearranging we have

$$\begin{aligned} V_S &= U_S(\tilde{\theta}) - 2\int_0^{\tilde{\theta}} (\bar{y}(\theta) - y_S(\theta))y_S'(\theta)d\theta + 2\int_0^1 (\bar{y}(\theta) - y_S(\theta))y_S'(\theta)(1 - F(\theta))d\theta \\ &= U_S(\tilde{\theta}) + \mathbb{E}\left[\bar{y}(\theta)\left(2y_S'(\theta)\frac{1 - F(\theta)}{f(\theta)} - 2\frac{y_S'(\theta)}{f(\theta)}\mathbb{I}_{[0,\tilde{\theta}]}\right)\right] - \mathbb{E}\big[y_S^2(\theta)\big] + y_S^2(\tilde{\theta}) \end{aligned}$$

where $\mathbb{I}_{[0,\tilde{\theta}]}$ is the characteristic function of the set $[0,\tilde{\theta}]$ (i.e. $\mathbb{I}_{[0,\tilde{\theta}]}(x) \equiv 1$ if $x \in [0,\tilde{\theta}]$ and $\mathbb{I}_{[0,\tilde{\theta}]}(x) \equiv 0$ otherwise). Using the expression for V_S given in (A.2) and substituting above we obtain (2) with $K_{S(\hat{\theta})}$ given by (5).

Proof of Proposition 1: (i) Suppose that (6) holds so that

$$y_S'(\theta) = \alpha h(\theta)\big(\mathbb{E}\big[\theta'|\theta' \geq \theta\big] - \theta\big),$$

implying

$$\begin{aligned} K_{S(0)}(\theta) &= 2y_S(\theta) - \theta - 2y_S'(\theta)\frac{1}{h(\theta)} \\ &= 2\alpha\mathbb{E}\big[\theta'|\theta' \geq \theta\big] + 2\beta - \theta - 2\alpha\big(-\theta + \mathbb{E}\big[\theta'|\theta' \geq \theta\big]\big) \end{aligned}$$

$$= (2\alpha - 1)\theta + 2\beta = (2\alpha - 1)K_R + 2\beta.$$

Conversely, if (7) holds then

$$y_S'(\theta) = \alpha r(\theta)\big(\theta - \mathbb{E}\big[\theta'|\theta' \le \theta\big]\big),$$

so that

$$\begin{aligned} K_{S(1)}(\theta) &= 2y_S(\theta) - \theta + 2y_S'(\theta)\frac{1}{r(\theta)} \\ &= 2\alpha\mathbb{E}\big[\theta'|\theta' \le \theta\big] + 2\beta - \theta + 2\alpha\big(\theta - \mathbb{E}\big[\theta'|\theta' \le \theta\big]\big) \\ &= (2\alpha - 1)\theta + 2\beta = (2\alpha - 1)K_R + 2\beta \end{aligned}$$

Let $\widehat{\theta} = 0$ if (6) holds, and $\widehat{\theta} = 1$ if (7) holds. Then,

$$\begin{aligned} U_S(\widehat{\theta}) &= \mathbb{E}\big[\bar{y}(\theta)K_{S(\widehat{\theta})}(\theta)\big] - y_S^2(\widehat{\theta}) = \mathbb{E}[\bar{y}(\theta)((2\alpha - 1)K_R + 2\beta)] - y_S^2(\widehat{\theta}) = \\ &= (2\alpha - 1)\mathbb{E}[\bar{y}(\theta)K_R] + 2\mathbb{E}[\bar{y}(\theta)\beta] - y_S^2(\widehat{\theta}) \\ &= (2\alpha - 1)V_R + (2\alpha - 1)\mathbb{E}\big[\theta^2\big] + 2\beta\mathbb{E}[\bar{y}(\theta)] - y_S^2(\widehat{\theta}). \end{aligned}$$

Applying the law of iterated expectations to (IC-R) one readily obtains

$$\mathbb{E}[\bar{y}(\theta)] = \mathbb{E}[y] = \mathbb{E}[\mathbb{E}[\theta|y]] = \mathbb{E}[\theta]. \tag{A.4}$$

Therefore, for any incentive compatible mediation rule we have

$$U_S(i) = (2\alpha - 1)V_R + C_R, \tag{A.5}$$

with $C_R = (2\alpha - 1)\mathbb{E}\big[\theta^2\big] + 2\beta\mathbb{E}[\theta] - y_S^2(\widehat{\theta})$ finite and independent of the mediation mechanism. This establishes (6). Finally, if $\alpha > 1/2$ then (A.5) implies that there is a linear and increasing relation between $U_S(\widehat{\theta})$ and the ex-ante payoff to the receiver V_R. Therefore, a mediation rule achieves the maximum $U_S(\widehat{\theta})$ in $\mathscr{M}$ if and only if it maximizes V_R.

(ii) If $y_S'(\theta) = \alpha h(\theta)$, then $K_{S(0)}$ can be written as

$$K_{S(0)}(\theta) = 2y_S(\theta) - \theta - 2\alpha = K_S(\theta) - 2\alpha,$$

while if $y_S'(\theta) = \alpha r(\theta)$, then $K_{S(1)}$ can be written as

$$K_{S(1)}(\theta) = 2y_S(\theta) - \theta + 2\alpha = K_S(\theta) + 2\alpha$$

As average actions must equal the state (as shown in (A.4)), then letting $\widehat{\theta} = 0$ if (8) is satisfied, and $\widehat{\theta} = 1$ if (9) is satisfied, we can write

$$\begin{aligned} U_S(\widehat{\theta}) &= \mathbb{E}\big[\bar{y}(\theta)K_{S(\widehat{\theta})}(\theta)\big] - y_S^2(\widehat{\theta}) = \mathbb{E}[\bar{y}(\theta)K_S(\theta)] - 2(1 - 2\widehat{\theta})\mathbb{E}[\theta] - y_S^2(\widehat{\theta}) = \\ &= V_S + C_S, \end{aligned}$$

with $C_S = -2(1 - 2\widehat{\theta})\mathbb{E}[\theta] - y_S^2(\widehat{\theta})$ finite and independent of the mediation mechanism, from which (8) follows.

Proof of Corollary 1: Applying both (A.1) and (A.4) we have

$$\begin{aligned} V_S &= -\mathbb{E}\big[\big(y - (a\theta + b)^2\big)\big] = -\mathbb{E}\big[y^2 - 2ay\theta - 2by + (a\theta + b)^2\big] = \\ &= (2a - 1)\mathbb{E}[y\theta] - \mathbb{E}\big[-2by + (a\theta + b)^2\big] = (2a - 1)V_R + \tilde{C} \end{aligned}$$

with $\tilde{C} = \mathbb{E}\Big[(2a - 1)\theta^2 + 2b\theta - (a\theta + b)^2\Big]$ independent of the mediation rule. Since $a > 1/2$, we have $V_S^{M'} \ge V_S^M$ if and only if $V_R^{M'} \ge V_R^M$ for any two incentive compatible mediation rules M and M'. If, in addition, θ is uniformly distributed then (6) and (7) are both satisfied with $\alpha = 2a$. Then (6) implies that if $a > 1/4$ a mediation rule is optimal if and only if it maximizes $U_S(\widehat{\theta})$, $\widehat{\theta} \in \{0, 1\}$.

Overall, if $a > 1/2$ we have that optimal mediation rules maximize the interim payoff of extreme types, and also the sender's expected payoff and, conversely, any mechanism that maximizes the interim payoff at an extreme state must necessarily be unimprovable for both the sender and the receiver.

Proof of Proposition 2: We first establish that the set of functions $y_S(\theta)$ satisfying the conditions of Proposition 2-i and the set satisfying Proposition 2-ii are both non-empty if $f(\theta) > 0$, $\theta \in [0, 1]$.

Define $\overline{\Delta}_R(\theta) = (\mathbb{E}[\tilde{\theta}|\tilde{\theta} \ge \theta] - \mathbb{E}[\tilde{\theta}|\tilde{\theta} \ge \hat{\theta}])/(\theta - \hat{\theta})$ for $\theta > \hat{\theta}$ and $\underline{\Delta}_R(\theta) = (\mathbb{E}[\tilde{\theta}|\tilde{\theta} \le \hat{\theta}] - \mathbb{E}[\tilde{\theta}|\tilde{\theta} \le \theta])/(\hat{\theta} - \theta)$ for $\theta < \hat{\theta}$ and continuously extend $\overline{\Delta}_R(\theta)$ and $\underline{\Delta}_R(\theta)$ to $\theta = \hat{\theta}$ using $\lim_{\theta \to \hat{\theta}^+} \overline{\Delta}_R(\theta) = h(\hat{\theta})\Big(\mathbb{E}[\tilde{\theta}|\tilde{\theta} \ge \hat{\theta}] - \hat{\theta}\Big)$ and $\lim_{\theta \to \hat{\theta}^-} \underline{\Delta}_R(\theta) = r(\hat{\theta})\Big(\hat{\theta} - \mathbb{E}[\tilde{\theta}|\tilde{\theta} \le \hat{\theta}]\Big)$. If $f(\theta) > 0$ for $\theta \in [0, 1]$ then $\overline{\Delta}_R(\hat{\theta}) > 0$ if $\hat{\theta} < 1$ and $\underline{\Delta}_R(\hat{\theta}) > 0$ if $\hat{\theta} > 0$. The function $\overline{\Delta}_R(\theta)$ is then continuous and strictly positive in $[\hat{\theta}, 1]$, and $\underline{\Delta}_R(\theta)$ is continuous and strictly positive in $[0, \hat{\theta}]$, so that $\bar{e}_R \equiv \inf_{\theta \in [\hat{\theta}, 1]} \overline{\Delta}_R(\theta) > 0$ and $\underline{e}_R \equiv \inf_{\theta \in [\hat{\theta}, 1]} \underline{\Delta}_R(\theta) > 0$. Thus, any $\alpha > \max\{1/\bar{e}_R, 1/2\}$ and $\alpha' > \{1/\underline{e}_R, 1/2\}$ satisfy the conditions in

Proposition 2-i. Second, define $\overline{\Delta}_S(\theta) = \int_{\hat{\theta}}^{\theta} h(\theta')d\theta'/(\theta - \hat{\theta})$ for $\theta > \hat{\theta}$ and $\underline{\Delta}_S(\theta) = \int_{\theta}^{\hat{\theta}} r(\theta')d\theta'/(\hat{\theta} - \theta)$ for $\theta < \hat{\theta}$ and continuously extend $\overline{\Delta}_S(\theta)$ and $\underline{\Delta}_S(\theta)$ to $\theta = \hat{\theta}$ using $\lim_{\theta \to \hat{\theta}'^+} \overline{\Delta}_S(\theta) = h(\hat{\theta})$ and $\lim_{\theta \to \hat{\theta}'^-} \underline{\Delta}_S(\theta) = r(\hat{\theta})$. A similar argument as above implies that $\bar{e}_S \equiv \inf_{\theta \in [\hat{\theta},1]} \overline{\Delta}_S(\theta) > 0$ and $\underline{e}_S \equiv \inf_{\theta \in [0,\hat{\theta}]} \underline{\Delta}_S(\theta) > 0$, so that any $\alpha > 1/\bar{e}_S$ and $\alpha' > 1/\underline{e}_S$ satisfy the conditions in Proposition 2-ii.

We now show that there are brief conversations that are unimprovable for some player. Let $Y_S = \{y : y_S(\theta) = y\}$ which is a connected set given the continuity of $y_S(\theta)$ satisfied by all cases (10–13). For all (10–13), we have $[0,1] \subset Y_S$. This implies that the sender has an outward bias and an infinite equilibrium exists (Gordon (2010), Theorem 2). Furthermore, by construction of (10–13) we have $y_S(\theta) \neq \theta$ for $\theta \neq 0$ and $\theta \neq 1$. This means that the unique point of alignment in (10–13) is at an extreme point $\theta \in \{0,1\}$ and the equilibrium with an infinite number of actions must necessarily have an accumulation point at that extreme point so $y = y_S(\theta) = \theta$, and thus $U_S(\theta) = 0$. As (10) satisfies (6), and (11) satisfies (7), then both (6) and (7) hold and the infinite equilibrium must be optimal. Conversely, as (12) satisfies (8), and (13) satisfies (9), then both (8) and (9) hold and the infinite equilibrium must be unimprovable for the sender.

The following Lemma will be used in the proof of Proposition 3. It formalizes the intuition that average actions must be less dispersed than the state under an incentive compatible mediation rule.

Lemma 1. *Let $\bar{y}_M(\theta)$ be the average action induced by some incentive compatible mediation rule. Then, for any $z \in [0,1]$, we must have*

$$\int_0^z (\bar{y}_M(\theta) - \theta)dF(\theta) \geq 0. \tag{A.6}$$

Proof of Lemma 1: Define the random variable $\bar{Y} = \bar{y}_M(\theta)$ and let $\theta_M(y) = \max\{\theta : \bar{y}_M(\theta) \leq y\}$ be an inverse of $\bar{y}_M(\theta)$. Then, $\bar{Y}$ is distributed according to $F_{\bar{Y}}(y) = F(\theta_M(y))$. Note that the random variable θ is a mean-preserving spread of $\bar{Y}$,[17] so θ dominates $\bar{Y}$ in the convex order and we can write (see Shaked and Shanthikumar, 2007-equations 3.A.5–6):

$$\int_0^z (z-\theta)dF(\theta) \geq \int_0^z (z-\bar{y})dF_{\bar{Y}}(\bar{y}) = \int_0^{\theta_M(z)} (z - \bar{y}_M(\theta))dF(\theta),$$
$$\int_z^1 (\theta - z)dF(\theta) \geq \int_z^1 (\bar{y}-z)dF_{\bar{Y}}(\bar{y}) = \int_{\theta_M(z)}^1 (\bar{y}_M(\theta) - z)dF(\theta),$$

where each of the equalities follows from the definition of $\bar{Y} = \bar{y}_M(\theta)$ and $\theta_M(y)$.

Fix $z \in [0,1]$. Since $z - \bar{y}_M(\theta) \geq 0$ for $\theta \leq \theta_M(z)$, if $\theta_M(z) \geq z$ we must have

$$\int_0^z (z-\theta)dF(\theta) \geq \int_0^{\theta_M(z)} (z - \bar{y}_M(\theta))dF(\theta) \geq \int_0^z (z - \bar{y}_M(\theta))dF(\theta),$$

which implies (A.6). If instead $\theta_M(z) < z$, and since $\bar{y}_M(\theta) - z \geq 0$ for $\theta \geq \theta_M(z)$, then we have

$$\int_z^1 (\theta - z)dF(\theta) \geq \int_{\theta_M(z)}^1 (\bar{y}_M(\theta) - z)dF(\theta) \geq \int_z^1 (\bar{y}_M(\theta) - z)dF(\theta)$$

which combined with $\mathbb{E}[\bar{y}_M(\theta) - \theta] = 0$ gives (A.6).

Proof of Proposition 3: We first construct a mediation rule as a concatenation of scaled mediation rules satisfying Proposition 2. We then show that the associated brief conversation is unimprovable for the receiver–and thus for the sender, see Corollary 1.

Let $M_H = \{p_{M_H}(y|\theta)\}_{\theta \in [0,1]}$ be the mediation rule corresponding to the optimal brief conversation when (10) holds, and $M_L = \{p_{M_L}(y|\theta)\}_{\theta \in [0,1]}$ the one when instead (11) holds, both for a uniform distribution and with $\alpha = 2a$. For $y_S(\theta) = a\theta + b$ with $a > 1$ and $\widetilde{\theta} = b/(1-a) \in (0,1)$, so that $\widetilde{\theta}$ is the unique point of congruence between sender and receiver, define the mediation rule $M^* = \{p_{M^*}(y|\theta)\}_{\theta \in [0,1]}$ by

$$p_{M^*}(y|\theta) \equiv \begin{cases} p_{M_L}\left(\frac{y}{\widetilde{\theta}} | \frac{\theta}{\widetilde{\theta}}\right) & \text{if } \theta \leq \widetilde{\theta}, \\ p_{M_H}\left(\frac{y - \widetilde{\theta}}{1-\widetilde{\theta}} | \frac{\theta - \widetilde{\theta}}{1-\widetilde{\theta}}\right) & \text{if } \theta > \widetilde{\theta}. \end{cases} \tag{A.7}$$

Note that for any $\theta \leq \widetilde{\theta}$, p_{M^*} assigns the same probability to action y as the mediation rule M_L assigns to action $y/\widetilde{\theta}$ when the state is $\theta/\widetilde{\theta}$. Conversely, for $\theta > \widetilde{\theta}$, p_{M^*} assigns the same probability to action y as the mediation rule M_H assigns to action $\left(y - \widetilde{\theta}\right)/\left(1 - \widetilde{\theta}\right)$ when the state is $\left(\theta - \widetilde{\theta}\right)/\left(1 - \widetilde{\theta}\right)$. We now show that: (i) M^* is an incentive compatible mediation rule, and (ii) we have

$$\int_0^{\widetilde{\theta}} (\bar{y}_{M^*}(\theta) - \theta)dF(\theta) = \int_{\widetilde{\theta}}^1 (\bar{y}_{M^*}(\theta) - \theta)dF(\theta) = 0. \tag{A.8}$$

[17] Note that y is a mean preserving spread of $\bar{Y}$ and, from IC-R, θ must also be a mean-preserving spread of y.

For $\theta = \widetilde{\theta}$, M^* induces the action $y = \widetilde{\theta}$ with certainty. Consider first a state $\theta < \widetilde{\theta}$ so that $y_S(\theta) < \widetilde{\theta}$. The sender would rather send message $m = \widetilde{\theta}$ than any $m' > \widetilde{\theta}$ as this would lead with probability one to an action weakly larger than $y = \widetilde{\theta}$. Moreover, any deviation to a message $m'' \in [0, \widetilde{\theta}]$ with $m'' \neq \theta$ cannot be profitable. Otherwise, in the game where the sender's payoffs are (10) with $\alpha = 2a$ and the mediation rule is M_L, deviating to the message $m''/\widetilde{\theta}$ when the state is $\theta/\widetilde{\theta}$ would also be profitable for the sender, violating the claim that M_L is incentive compatible. This shows that M^* is incentive compatible for any $\theta < \widetilde{\theta}$. The same argument, *mutatis mutandi*, shows that M^* is also incentive compatible for any $\theta < \widetilde{\theta}$. As the sender induces his preferred decision when $\theta = \widetilde{\theta}$, this completes the proof that M^* is incentive compatible.

Using (A.7), we can represent the average decision under M^* as $\bar{y}_{M^*}(\theta) = \bar{y}_{M_L}(\theta/\widetilde{\theta})$ for $\theta \le \widetilde{\theta}$ and $\bar{y}_{M^*}(\theta) = \bar{y}_{M_L}(\frac{\theta-\widetilde{\theta}}{1-\widetilde{\theta}})$ for $\theta > \widetilde{\theta}$. Since M_L and M_H are incentive compatible, then $\mathbb{E}\left[\bar{y}_{M_L}(\theta)\right] = \mathbb{E}\left[\bar{y}_{M_H}(\theta)\right] = \mathbb{E}[\theta]$. A change of variable in these expressions then gives (A.8).

We conclude by showing by contradiction that M^* is optimal. To reach a contradiction, we will use Lemma 1 that shows that that the integral between 0 and any z of the difference $\bar{y}_M(\theta) - \theta$ for any incentive compatible M must be non-negative–a reflection that, viewing $\bar{y}_M(\theta)$ as a random variable, θ second order stochastically dominates $\bar{y}_M(\theta)$.

By way of contradiction, suppose that some other incentive compatible mediation rule M' provides the receiver an ex ante higher payoff. Then, we must have that: (i) $U_S^{M'}(0) > U_S^{M^*}(0)$; and (ii) $U_S^{M'}(\widetilde{\theta}) \le U_S^{M^*}(\widetilde{\theta})$. The first observation follows from Corollary 1, while the second holds as, under M^*, the sender obtains his preferred action at $\theta = \widetilde{\theta}$. With these observations and using (A.3) we can write

$$\left(U_S^{M'}(\widetilde{\theta}) - U_S^{M^*}(\widetilde{\theta})\right) - \left(U_S^{M'}(0) - U_S^{M^*}(0)\right) = 2\int_0^{\widetilde{\theta}} (\bar{y}_{M'}(\theta) - \bar{y}_{M^*}(\theta))d\theta < 0$$

But then, (A.8) implies that $\int_0^{\widetilde{\theta}} (\bar{y}_{M'}(\theta) - \theta))d\theta < 0$, violating (A.6) in Lemma 1 and reaching a contradiction. Therefore, M^* must be optimal.

Proof of Proposition 4: To obtain (15) we will use the relation (6) and the fact that the set of implementable $\bar{y}$ is convex. To see this last point note that for any two incentive compatible M' and M'', that induce $\bar{y}'$ and $\bar{y}''$, a mediation rule that with probability λ issues recommendations according to M' and with probability $1-\lambda$ according to M'',where, importantly, the probability λ does not vary with the report of the sender, is incentive compatible and induces an average action $\lambda\bar{y}' + (1-\lambda)\bar{y}''$. Let M^* replicate the infinite equilibrium described in Proposition 2 and let $M^{\varnothing}$ replicate the babbling equilibrium (i.e. under $M^{\varnothing}$ the receiver selects a single action $\mathbb{E}[\theta]$ is induced).

We now construct a two partition equilibrium of the cheap talk game when y_S is given by (10). In such equilibrium the sender only discloses whether the state is above or below y^*, the receiver selects $\gamma(0, y^*) = \mathbb{E}[\theta|\theta \in [0, y^*]]$ if $\theta \le y^*$ and $\gamma(y^*, 1) = \mathbb{E}[\theta|\theta \in [y^*, 1]]$ otherwise, and y^* must satisfy the arbitrage condition

$$y_S(y^*) - \gamma(0, y^*) = \gamma(y^*, 1) - y_S(y^*). \tag{A.9}$$

As $y_S(\theta)$ that satisfies (10) can be expressed as $y_S(\theta) = \alpha(\gamma(\theta, 1) - \gamma(0, 1))$, the existence of a solution $y^* \in [0, 1]$ to (A.9) then requires the existence of a solution to

$$2\alpha = \frac{\gamma(0, y^*) + \gamma(y^*, 1)}{\gamma(y^*, 1) - \gamma(0, 1)} \tag{A.10}$$

The right hand side of (A.10) is positive and decreasing in y^*, unbounded as y^* approaches 0, and achieves a minimum $(1 + \mathbb{E}[\theta])/(1 - \mathbb{E}[\theta])$ when $y^* = 1$. From (10) we have

$$2\alpha > \max \frac{2\theta}{\mathbb{E}\left[\tilde{\theta}|\tilde{\theta} \ge \theta\right] - \mathbb{E}\left[\tilde{\theta}\right]} \ge \frac{2}{1 - \mathbb{E}[\theta]} \ge \frac{1 + \mathbb{E}[\theta]}{1 - \mathbb{E}[\theta]}$$

This implies that we can always find an y^* that solves (A.10) and a two partition equilibrium exists. Denote by M^2 the mediation rule that induces this two partition equilibrium. The receiver's expected utility under M^2 is

$$\begin{aligned} V_R^{M^2} &= -\int_0^{y^*} (\gamma(0, y^*) - \theta)^2 dF(\theta) - \int_{y^*}^1 (\gamma(y^*, 1) - \theta)^2 dF(\theta) = \\ &= -\mathbb{E}\left[\theta^2\right] + F(y^*)\gamma^2(0, y^*) + (1 - F(y^*))\gamma^2(y^*, 1). \end{aligned} \tag{A.11}$$

Next consider the mediation mechanism M^{λ} that is a convex combination of M^* and $M^{\varnothing}$, that is with probability λ the mechanism M^{λ} issues the same recommendation as $M^{\varnothing}$ while with probability $1-\lambda$ it issues the same recommendation as M^*. We then have

$$V_R^{M^{\lambda}} = -\lambda Var\theta + (1 - \lambda)V_R^*. \tag{A.12}$$

From Proposition 2, under the optimal one-shot equilibrium we have $U_S^{M^*}(0) = 0$. Moreover, $U_S^{M^2}(0) = -\gamma^2(0, y^*)$ and $U_S^{M^{\varnothing}}(0) = -\gamma^2(0, 1) < U_S^{M^2}(0)$. Therefore, there exists $\bar{\lambda}$ such that $U_S^{M^{\lambda}}(0) = U_S^{M^2}(0)$, which is then given by

$$-\gamma^2(0, y^*) = -\bar{\lambda}\gamma^2(0, 1) + (1 - \bar{\lambda})U_S^{M^*}(0),$$

$$\bar{\lambda} = \frac{\gamma^2(0, y^*)}{\gamma^2(0, 1)}, \tag{A.13}$$

and (A.12) leads to

$$(1 - \bar{\lambda})V_R^* = V_R^{M^2} + \bar{\lambda}Var\theta$$

substituting the value $\bar{\lambda}$ given by (A.13) and $V_R^{M^2}$ given by (A.11) into this expression one obtains (15). A similar argument can then be used to derive similar expressions if instead $y_S(\theta)$ satisfies (11).

Proof of Corollary 2: First consider a truncated exponential of parameter λ, $f(\theta) = \frac{\lambda e^{-\lambda\theta}}{1-e^{-\lambda\bar{\theta}}}, \theta \in [0, \bar{\theta}]$. Then (10) translates to

$$y_S(\theta) = \frac{\alpha}{1 - e^{-\lambda(\bar{\theta}-\theta)}}\left(\theta - \bar{\theta}e^{-\lambda(\bar{\theta}-\theta)}\frac{\left(1 - e^{-\lambda\theta}\right)}{\left(1 - e^{-\lambda\bar{\theta}}\right)}\right),$$

and pointwise we have $y_S(\theta) \to \alpha\theta$ as $\bar{\theta} \to \infty$. Taking the limit as $\bar{\theta} \to \infty$ to the arbitrage condition (16) gives

$$2(1 - \lambda y^*(a - 1)) = \frac{\lambda y^*}{1 - e^{-\lambda y^*}},$$

and (15) gives

$$\begin{aligned} V_R^* &= -\frac{2}{\lambda^2} + \frac{1}{\lambda^2}\left[1 + \frac{y^*\lambda\left(1 - e^{-\lambda y^*}\right)}{2\left(1 - e^{-\lambda y^*}\right) - y^*\lambda e^{-\lambda y^*}}\right] \\ &= -\frac{2(a-1)}{2a-1}\frac{1}{\lambda^2} = -\frac{2(a-1)}{2a-1}Var[\theta]. \end{aligned}$$

Now consider the linear case where (10) translates to $y_S(\theta) = \frac{2}{3}\alpha\theta$ with $\frac{2}{3}\alpha > 1$, implying that $\alpha = \frac{3}{2}a$ with $a > 1$. The solution to (16) is

$$y^* = \left(1 - \frac{\sqrt{36a^2 - 48a + 17} - 1}{6a - 4}\right)\bar{\theta}_l,$$

which, substituted in (15) leads to

$$\begin{aligned} V_R^* &= -\frac{1}{18}\bar{\theta}_l^2\frac{-3\bar{\theta}_l y^* + \bar{\theta}_l^2 + (y^*)^2}{\bar{\theta}_l y^* + \bar{\theta}_l^2 - (y^*)^2} \\ &= -\frac{(a-1)}{3a-1}\frac{\bar{\theta}_l^2}{6} = -\frac{3(a-1)}{3a-1}Var[\theta]. \end{aligned}$$

Finally, consider the uniform case where (10) translates to $y_S(\theta) = \frac{\alpha}{2}\theta$ with $\frac{\alpha}{2} > 1$, so that $\alpha = 2a$ with $a > 1$. The solution to (16) is

$$y^* = \frac{\bar{\theta}_u}{2(2a-1)},$$

which, substituted in (15) leads to

$$\begin{aligned} V_R^* &= -\frac{\bar{\theta}_u - 2y^*}{12\left(\bar{\theta}_u + y^*\right)}\bar{\theta}_u^2 = \\ &= -\frac{a-1}{3(4a-1)}\bar{\theta}_u^2 = -\frac{4(a-1)}{4a-1}Var[\theta]. \end{aligned}$$

Part ii follows from the fact that expected bias for the exponential, linear and uniform case are $(a-1)/\lambda$, $(a-1)/3\bar{\theta}_l$ and $(a-1)/2\bar{\theta}_u$ while the maximum expected payoff to the receiver in each case is $2(a-1)/\lambda^2(2a-1)$, $(a-1)\bar{\theta}_l^2/(18a-6)$ and $(a-1)\bar{\theta}_u^2/(12a-3)$.

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