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# Agnostic Structural Disturbances (ASDs): Detecting and reducing misspecification in empirical macroeconomic models

Wouter J. Den Haan a,b,1,\*, Thomas Drechsel c,1

- <sup>a</sup> Centre for Macroeconomics, London School of Economics and Political Science, Houghton Street, London WC2A 2AE, UK
- <sup>b</sup> CEPR, London, UK
- <sup>c</sup> Department of Economics, University of Maryland, College Park, MD, USA

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#### ABSTRACT

Constructing empirical specifications for structural economic models is difficult, if not impossible. As shown in this paper, even minor misspecifications may lead to large distortions for parameter estimates and implied model properties. We propose a novel concept, namely an agnostic structural disturbance (ASD), that can be used to both detect and correct for misspecification of structural disturbances and is easy to implement. While agnostic in nature, the estimated coefficients and associated impulse response functions of the ASDs allow us to give them an economic interpretation. We adopt the methodology to the Smets-Wouters model and formulate an improved risk-premium and an improved investment-specific productivity disturbance.

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#### 1. Introduction

Exogenous random shocks are the lifeblood of modern macroeconomic business cycle models. They enter the model as innovations to structural disturbances that affect key aspects of the model. Recent generations of business cycle models include a multitude of structural disturbances. Structural disturbances impose restrictions on model equations and, thus, on the model's solutions. Therefore, each structural disturbance has to enter *each* model equation correctly. This is a concern, since we often do not have independent evidence on how structural disturbances should affect the system and there is substantial disagreement in the profession on the specific shocks needed to bridge the gap between the model and the data. For example, should a risk-premium disturbance enter all Euler equations or only some? Is it correct to assume that structural disturbances are uncorrelated as is commonly done? Chari et al. (2007) propose "wedges" as alternatives to standard structural disturbances, but wedges also impose restrictions. As shown in this paper, misspecification of the structural disturbances will distort the empirical analysis of the underlying model itself such as the degree of price stickiness or the importance of adjustment costs. We propose a procedure that allows us to systematically search for better specifications of structural disturbances. This will not only be useful in understanding what the ultimate driving processes are, but by having better empirical models will lead to improved model fit and less distortion due to misspecification.

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<sup>\*</sup> Corresponding author.

E-mail addresses: w.denhaan@lse.ac.uk (W.J. Den Haan), drechsel@umd.edu (T. Drechsel).

<sup>&</sup>lt;sup>1</sup> Both authors contributed equally to the paper.

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Specifically, the contributions of this paper are threefold. First, we propose the agnostic structural disturbance (ASD) as an alternative type of *structural* disturbance. The procedure simply involves adding structural disturbances with associated reduced-form coefficients to *each* model equation or alternatively to each policy rule. In contrast to regular structural disturbances and wedges, ASDs impose *no* additional restrictions on policy rules. Nevertheless, they are different from measurement error, because they are structural and propagate through the system like regular structural disturbances.

Our ASD procedure can be used to test whether regular structural disturbances are correctly specified and to enrich an empirical specification by adding ASDs as additional structural disturbances. Using Monte Carlo experiments, we document that the ASD procedure is capable of detecting and correcting for misspecification in samples of typical size.

The second contribution of our paper is to test whether the structural disturbances of the model in Smets and Wouters (2007) (SW) are correctly specified using the same US postwar data set. We find that the risk-premium and the investment-specific productivity disturbance are not. We use our procedure to improve on the SW empirical specification. Our preferred specification (based on marginal likelihood considerations) has three ASDs and excludes the SW risk-premium and the SW investment-specific disturbance.

Although the ASD procedure itself does not rely on any economic reasoning, the estimation results – both the associated coefficients and their impulse response functions (IRFs) – may reveal a lot about the type of structural disturbance the data has identified. For example, we interpret one of the ASDs in our adjusted empirical specification of the SW model as an "investment-modernization" disturbance, because it stimulates investment, but at the same time leads to faster depreciation of the existing capital stock. The second ASD of our empirical model has features in common with both a risk-premium and a preference disturbance but is also different from both. Finally, the third ASD captures increases in the wage mark-up disturbance that are associated with an increase in the utilized capital stock.

The third contribution of our paper consists of showing that *minor* misspecifications of the empirical model regarding structural disturbances can easily lead to *large* distortions for parameter estimates and model properties, such as business cycle statistics and IRFs. We document that ASDs can alleviate these problems.

The next section explains the ASD procedure. Section 3 documents the ability of ASDs to detect and correct for misspecification using Monte Carlo experiments for a typical application. Section 4 discusses the results when our procedure is applied to the SW model on US data. Section 5 concludes.

#### 2. Agnostic Structural Disturbances

We use a simple business cycle model to explain what ASDs are and how they can be used for building theoretical models that one wants to bring to the data. Appendix A provides a general formulation.

#### 2.1. Model

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Agents' choices for consumption,  $C_t$ , investment,  $I_t$ , and capital,  $K_t$  are the outcomes of the following maximization problem:

$$\max_{\{C_{t+j}, I_{t+j}, K_{t+j}\}_{j=0}^{\infty}} \sum_{i=0}^{\infty} \frac{C_{t+j}^{1-\gamma} - 1}{1 - \gamma} \tag{1}$$

s.t. 
$$e^{\varepsilon_{a,t}}K_{t-1}^{\alpha} = C_t + I_t + e^{\varepsilon_{g,t}}\overline{G}$$
, (2)

$$K_t = (1 - \delta)K_{t-1} + I_t e^{\varepsilon_{i,t}}.$$
 (3)

This model contains three exogenous random variables. Using the terminology of Chari et al. (2007), these are an efficiency wedge,  $\varepsilon_{a,t}$ , an investment wedge,  $\varepsilon_{i,t}$ , and a government consumption wedge,  $\varepsilon_{g,t}$ . Consistent with the literature, the variables are generated by the following stochastic process:

$$\varepsilon_{m,t} = \rho_m \varepsilon_{m,t-1} + \sigma_m \eta_{m,t}, m \in \{a, i, g\}, \tag{4}$$

$$\mathbb{E}_{t}[\eta_{m,t+1}] = 0, \mathbb{E}_{t}[\eta_{m,t+1}^{2}] = 1, \text{ and } \mathbb{E}_{t}[\eta_{m,t+1}\eta_{m^{*},t+1}] = 0 \text{ for } m \neq m^{*}.$$
(5)

This economy is represented with the following set of linearized first-order conditions<sup>2</sup>:

$$\mathbb{E}_{t}[c_{t+1} - c_{t}] = \frac{1 - \beta(1 - \delta)}{\gamma} (\rho_{a} \varepsilon_{a,t} + (\alpha - 1)k_{t}) + \frac{1 - \beta(1 - \delta)\rho_{i}}{\gamma} \varepsilon_{i,t}, \tag{6a}$$

$$\overline{Y}(\varepsilon_{a,t} + \alpha k_{t-1}) = \overline{I} i_t + \overline{C} c_t + \overline{G} \varepsilon_{g,t}, \tag{6b}$$

$$k_t = (1 - \delta)k_{t-1} + \frac{\bar{I}}{\bar{K}}i_t + \frac{\bar{I}}{\bar{K}}\varepsilon_{i,t},\tag{6c}$$

<sup>&</sup>lt;sup>2</sup> Throughout this paper, we focus on linearized systems and treat those as the true data generating process. We do this because most structural empirical macroeconomic models are based on such systems. In principle, one could include ASDs in nonlinear systems as well.

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where lower case letters denote variables expressed as a percentage difference from their steady state values and  $\overline{X}$  indicates the steady state value of variable  $X_r$ .

The random disturbances can be interpreted literally as regular exogenous structural disturbances affecting the economy. As illustrated in Chari et al. (2007), however, these wedges can also be seen as manifestations of frictions in more elaborate models or as the part that is not modeled explicitly.<sup>3</sup> Although they are somewhat general, these three wedges *do* impose restrictions on the model and they differ from each other exactly because of these restrictions. First, none of the wedges appear in all equations, which is typical. Second, the model imposes cross-equation restrictions that depend on the structural parameter values of the model.<sup>4</sup>

The three policy functions for this model can be expressed as follows.

$$c_t = A_c(\Psi)k_{t-1} + B_{c,a}(\Psi)\varepsilon_{a,t} + B_{c,i}(\Psi)\varepsilon_{i,t} + B_{c,g}(\Psi)\varepsilon_{g,t}, \tag{7a}$$

$$i_t = A_i(\Psi)k_{t-1} + B_{i,a}(\Psi)\varepsilon_{a,t} + B_{i,i}(\Psi)\varepsilon_{i,t} + B_{i,g}(\Psi)\varepsilon_{g,t}, \tag{7b}$$

$$k_t = A_k(\Psi)k_{t-1} + B_{k,\sigma}(\Psi)\varepsilon_{a,t} + B_{k,i}(\Psi)\varepsilon_{i,t} + B_{k,\sigma}(\Psi)\varepsilon_{g,t}, \tag{7c}$$

where  $\Psi$  is a vector containing the structural parameters. This system also makes clear that wedges impose cross-equation restrictions. The  $A_j(\Psi)$  and  $B_{j,m}(\Psi)$  coefficients are nonlinear functions of the structural parameters,  $\Psi$ .<sup>5</sup> In linear frameworks, disturbances only differ in how they affect the economy on impact. After impact they propagate through the economy in the same way, as described by the  $A_i$ s.

Possible misspecification Misspecification occurs in many different forms. One could miss a particular disturbance or include one that should not be included. Another possibility is that a structural disturbance is not incorporated correctly in all model equations. This is more likely to occur in larger models. However, misspecification is a also possible in the model at hand which has just three equations. For example, the government expenditure disturbance could very well affect the utility of the agent and/or the production function. Also, the investment disturbance may affect the depreciation rate.<sup>6</sup> Another possible misspecification is that, contrary to common practice, the structural disturbances are correlated with each other. Using a New Keynesian business cycle model, Cúrdia and Reis (2012) document that structural disturbances are correlated and ignoring this correlation leads to wrong inference.

#### 2.2. Introducing Agnostic Structural Disturbances

ASDs can replace regular structural disturbances or they can be added to the existing set. Adding structural disturbances to model equations is incredibly simple: Each ASD is added to *each* equation with a reduced form coefficient. When we add two ASDs, denoted  $\widetilde{\varepsilon}_{A,t}$  and  $\widetilde{\varepsilon}_{B,t}$ , to the model of this section, then we get<sup>7</sup>

$$\mathbb{E}_{t}[c_{t+1} - c_{t}] = \frac{1 - \beta(1 - \delta)}{\gamma}(\alpha - 1)k_{t} + [\widetilde{\Upsilon}_{1,A}\widetilde{\Upsilon}_{1,B}][\widetilde{\varepsilon}_{A,t}\widetilde{\varepsilon}_{B,t}]', \tag{8a}$$

$$\overline{Y}\alpha k_{t-1} = \overline{l}i_t + \overline{C}c_t + [\widetilde{\Upsilon}_{2,A}\widetilde{\Upsilon}_{2,B}][\widetilde{\varepsilon}_{A,t}\widetilde{\varepsilon}_{B,t}]', \tag{8b}$$

$$k_{t} = (1 - \delta)k_{t-1} + \frac{\bar{I}}{\bar{K}}i_{t} + [\widetilde{\Upsilon}_{3,A}\widetilde{\Upsilon}_{3,B}][\widetilde{\varepsilon}_{A,t}\widetilde{\varepsilon}_{B,t}]', \tag{8c}$$

$$[\widetilde{\varepsilon}_{A,t},\widetilde{\varepsilon}_{B,t}]' = \widetilde{\varepsilon}_t = P\widetilde{\varepsilon}_{t-1} + \widetilde{\eta}_t. \tag{8d}$$

Each ASD is allowed to enter each equation without any restrictions. Moreover, they enter the system in a symmetric manner. A priori, there is, thus, no difference between the different ASDs. It is not restrictive to exclude future realizations of the ASDs from the equations. What matters is the expectation of these variables and this is captured by the current-period values as long as the ASDs are first-order Markov processes.

The vector  $\widetilde{\eta}_t$  contains the ASD innovations. Their standard deviations can be normalized to 1, since the  $\widetilde{\Upsilon}$ s are reduced-form coefficients. ASD innovations are assumed to be uncorrelated, but the disturbances can be correlated because P does not have to be a diagonal matrix.<sup>8</sup> Thus, the vector with the ASD innovations,  $\widetilde{\eta}_t$ , satisfies

$$\mathbb{E}_t[\widetilde{\eta}_{t+1}] = 0 \text{ and } \mathbb{E}_t[\widetilde{\eta}_{t+1}\widetilde{\eta}'_{t+1}] = I_2. \tag{9}$$

 $<sup>\</sup>overline{\phantom{a}}^3$  Moreover, a wedge can be given different interpretations. For example,  $\varepsilon_{g,t}$  could be a fixed cost to production or it could be government spending that agents do not value.

<sup>&</sup>lt;sup>4</sup> Inoue et al. (2015) provide a formal analysis for using wedges to detect and identify misspecification. Their wedges also only appear in a limited set of equations and, thus, also do impose parameter restrictions.

<sup>&</sup>lt;sup>5</sup> See Campbell (1998) for the derivation and discussion of such policy functions.

<sup>&</sup>lt;sup>6</sup> In Section 4, we provide empirical evidence in support for this possibility.

<sup>&</sup>lt;sup>7</sup> We have left out the three regular structural disturbances to keep the equations concise.

<sup>&</sup>lt;sup>8</sup> As explained in detail in Section 2.3, correlated innovations can be described by a combination of uncorrelated innovations. Such a setup is fine for agnostic disturbances.

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The policy functions for this model with two ASDs can be expressed as follows.

$$c_t = A_c(\Psi)k_{t-1} + \widetilde{B}_{c,A}(\Psi)\widetilde{\varepsilon}_{A,t} + \widetilde{B}_{c,B}(\Psi)\widetilde{\varepsilon}_{B,t}, \tag{10a}$$

$$i_{t} = A_{i}(\Psi)k_{t-1} + \widetilde{B}_{i,A}(\Psi)\widetilde{\varepsilon}_{A,t} + \widetilde{B}_{i,B}(\Psi)\widetilde{\varepsilon}_{B,t}, \tag{10b}$$

$$k_t = A_k(\Psi)k_{t-1} + \widetilde{B}_{k,A}(\Psi)\widetilde{\varepsilon}_{A,t} + \widetilde{B}_{k,B}(\Psi)\widetilde{\varepsilon}_{B,t}, \tag{10c}$$

where  $A_c(\Psi)$  has the standard solution which does not depend on whether the disturbances are regular or ASDs. The coefficients in these equations are equal to

$$\widetilde{B}_{c,m} = \frac{\widetilde{\Upsilon}_{1,m} + (\Lambda - A_c(\Psi)) \left(\widetilde{\Upsilon}_{3,m} - \frac{\widetilde{\Upsilon}_{2,m}}{\overline{K}}\right)}{(\Lambda - A_c(\Psi)) \frac{\overline{C}}{\overline{K}} + \rho - 1},$$
(11a)

$$\widetilde{B}_{i,m} = -\frac{\overline{C}\widetilde{B}_{c,m} + \widetilde{\Upsilon}_{2,m}}{\overline{I}},\tag{11b}$$

$$\widetilde{B}_{k,m} = \frac{\overline{I}}{\overline{K}} \widetilde{B}_{i,m} + \widetilde{\Upsilon}_{3,m}, \tag{11c}$$

$$\Lambda = \frac{1 - \beta(1 - \delta)}{\gamma}(\alpha - 1). \tag{11d}$$

The expressions for the  $\widetilde{B}_{j,m}(\Psi)$  coefficients illustrate the structural nature of ASDs because they depend both on the reduced-form  $\widetilde{\Upsilon}$  coefficients and the structural parameters of the model,  $\Psi$ .

Although the  $\widetilde{B}_{j,m}(\Psi)$  coefficients depend on  $\Psi$ , their values are fully unrestricted. That is,  $\widetilde{B}_{c,m}(\Psi)$ ,  $\widetilde{B}_{i,m}(\Psi)$ , and  $\widetilde{B}_{k,m}(\Psi)$  can take on any set of values by appropriate choice of  $\widetilde{\Upsilon}_{1,m}(\Psi)$ ,  $\widetilde{\Upsilon}_{2,m}(\Psi)$ , and  $\widetilde{\Upsilon}_{3,m}(\Psi)$ . Since the  $\widetilde{B}_{j,m}(\Psi)$  coefficients are unrestricted, an alternative way to implement ASDs is to add them directly to the policy functions with reduced-form coefficients, that is

$$c_t = A_c(\Psi)k_{t-1} + \widetilde{B}_{cA}\widetilde{\varepsilon}_{At} + \widetilde{B}_{cB}\widetilde{\varepsilon}_{Bt}, \tag{12a}$$

$$i_{t} = A_{i}(\Psi)k_{t-1} + \widetilde{B}_{i,A}\widetilde{\varepsilon}_{A,t} + \widetilde{B}_{i,B}\widetilde{\varepsilon}_{B,t}, \tag{12b}$$

$$k_{t} = A_{k}(\Psi)k_{t-1} + \widetilde{B}_{k,A}\widetilde{\varepsilon}_{A,t} + \widetilde{B}_{k,B}\widetilde{\varepsilon}_{B,t}, \tag{12c}$$

This illustrates that the ASD procedure adds to the policy functions an unobserved components block. Describing time-series fully or partly with unobserved components has a rich history in macroeconomics. This paper does more than that. Section 4 illustrates how ASDs can be used as a formal test of the correct specification of regular structural disturbances and how ASDs can be used to improve upon the specification of structural disturbances.

The data provide information about the coefficients of the unobserved components block, i.e., the  $\widetilde{B}$  coefficients. However, to give an economically meaningful interpretation of the ASDs the  $\widetilde{\Upsilon}$  coefficients are important. The link between the  $\widetilde{B}$  and the  $\widetilde{\Upsilon}$  coefficients will be discussed in more detail in Section 2.4.

#### 2.3. ASDs and misspecification

ASDs can detect different types of misspecification. The procedure will indicate an additional structural disturbance is needed if *adding* an ASD improves model fit with the proper adjustment for the additional parameters introduced by the ASD. If *replacing* a regular structural disturbance by an ASD leads to improved model fit (adjusted for the number of parameters), then this indicates that the regular structural disturbance in question either needs to be modified or should not play a role in the empirical model.

Cúrdia and Reis (2012) test whether regular structural disturbances are *dynamically* correlated, that is, the innovations are orthogonal but lagged values of disturbances can affect current values of other disturbances. They find empirical evidence for such dynamic correlation. ASDs can represent the role of correlated disturbances even if P is diagonal, which means that the ASDs are uncorrelated since the innovations of ASDs are assumed to be orthogonal. For example, suppose that both the

<sup>&</sup>lt;sup>9</sup> See, for example, Stock and Watson (1999).

<sup>&</sup>lt;sup>10</sup> Galizia (2015) shows that correlated estimates of the *innovations* of structural disturbances can be a sign of model misspecification. The paper demonstrates that the cross-correlations distort the estimated variance decomposition of the model and proposes a method to mitigate this problem. As we show below, ASDs also help with getting a lower cross-correlation between the estimated innovations.

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efficiency wedge,  $\varepsilon_{a,t}$  and the investment wedge,  $\varepsilon_{i,t}$  are driven by a common component and an idiosyncratic component. Then an empirical model with three ASDs can capture the role of the three different random components. One vector of  $\widetilde{\Upsilon}$  coefficients would capture the effect of the common component on the equations which would combine the effects of the  $\varepsilon_{a,t}$  and the  $\varepsilon_{i,t}$  disturbance. The other two  $\Upsilon$  vectors would capture the effects of the idiosyncratic components which would be the separate effect of the wedges.

ASDs can also capture measurement error. The ASD system as specified in Eq. (8) can capture measurement error in the control variables  $c_t$  and  $i_t$ . To correctly represent measurement error in the state variable,  $k_t$  one would have to add lagged ASD values to the system. 12 Although ASDs are general enough to encompass measurement error, typical ASDs differ in a fundamental way from measurement error. In general, ASDs are structural disturbances and propagate through the system like regular structural disturbances, that is, according to the  $A(\Psi)$  coefficients. Measurement error does not.<sup>13</sup>

ASDs are designed to deal with misspecification of structural disturbances that would distort the  $B(\Psi)$  coefficients. Is the ASD procedure also able to deal with misspecification that affects the  $A(\Psi)$  coefficients? Suppose one compares an empirical model with only one ASD with one that contains one regular structural disturbance and this disturbance is correctly specified. Moreover, both use  $\widehat{A}(\Psi)$  which differs from the true  $A(\Psi)$ . The ASD specification can still fully represent the correct policy function as long as there is a  $\widehat{\Psi}$  such that  $\widehat{A}(\widehat{\Psi}) = A(\Psi)$ . The specification with the regular structural disturbance faces a dilemma. With  $\widehat{\Psi}$  it gets the A coefficient right, but the B coefficient wrong because it is improbable that  $B(\widehat{\Psi}) = B(\Psi)$ . If it chooses the correct value for  $\Psi$  then it gets B right but A wrong. The flexibility of the ASD procedure makes it more likely it gets the policy function coefficients right, not only in terms of the B, but also in terms of the A coefficients. However, the example shows that this may come at the cost of larger distortions in estimates of  $\Psi$  if the ASD replaces a correctly specified regular structural disturbance.

Although our procedure can potentially alleviate misspecification of the  $A(\Psi)$  matrix, we think of our procedure as a first step to understand where the model needs improvement and not as a complete model evaluation.

#### 2.4. Identification

The data provide information about the A and the B coefficients. To understand whether the A and the B coefficients can be estimated it is useful to think of the model variables as MA processes. With M ASDs,  $c_t$  is a sum of M MA processes. The parameters of each MA process depend on:  $A_c(\Psi)$ ,  $A_k(\Psi)$ ,  $\widetilde{B}_{c,m}$ , and  $\widetilde{B}_{k,m}$ ,  $m \in \{1, \dots, M\}$ . Thus if one uses just  $c_t$  as an observable, then one can estimate these A and  $\widetilde{B}$  coefficients, but one would not be able to estimate the ASD coefficient in the investment policy function,  $\tilde{B}_{i,m}$ . If one replaces regular structural disturbances with ASDs, then it may be harder to identify Ψ. That turns out not to be an issue in our empirical application presented in Section 4.

The As and  $\widetilde{B}$ s determine the policy functions and moment properties. These may provide some information on the nature of the ASD. The  $\widetilde{\Upsilon}$ s indicate how ASD enters each and every equation. In the empirical application in Section 4, we find that knowing the  $\widetilde{\Upsilon}$ s is especially useful for interpreting the different ASDs. So the question arises whether knowing the  $\widetilde{B}$ s is enough to determine the  $\widetilde{\Upsilon}$ s. Eq. (11) makes clear that knowing  $\widetilde{B}_{c,m}$ ,  $\widetilde{B}_{i,m}$ , and  $\widetilde{B}_{k,m}$  is necessary but not sufficient to determine  $\widetilde{\Upsilon}_{1,m}$ ,  $\widetilde{\Upsilon}_{2,m}$ , and  $\widetilde{\Upsilon}_{3,m}$ . <sup>14</sup> In addition, one would need certain combinations of the structural parameters. <sup>15</sup>

#### 2.5. ASDs versus DSGE-VARs

Ireland (2004) and Del Negro et al. (2007) combine a DSGE model with a reduced-form VAR that contains the observables. There are several key differences between these two approaches and ours.

The DSGE-VAR specification is best compared with the system given in Eq. (12) which adds ASDs to the policy functions. However, an advantage of the ASD procedure is that one can also obtain the specification given in Eq. (8) that determines how the disturbances affect model equations. This knowledge is helpful in interpreting the nature of the ASDs as shown in Section 4. This cannot be done with the DSGE-VAR approach.

The ASD approach focuses on a particular type of misspecification, which allows it to use aspects of the model that are assumed to be not affected by the misspecification, namely  $A(\Psi)$ . The DSGE-VAR approach is more ambitious and also directly considers misspecification of  $A(\Psi)$ . Introducing a VAR into the empirical model means that the number of disturbances necessarily increases by a number equal to the number of variables in the VAR. Moreover, adding a VAR introduces

<sup>&</sup>lt;sup>11</sup> To see this simply replace  $c_t$  with  $c_{obs,t} + \varepsilon_t$  in Eq. (6). After taking expectations one is left with just the current-value of the measurement error term,

<sup>12</sup> Adding lagged values of the ASDs will increase the types of misspecification ASDs can detect. For example, this richer ASD specification could detect whether the period-t value of the productivity disturbance is known in period t, as is commonly assumed, or is known in period t-1, that is, when there are "news" shocks.

<sup>13</sup> When  $\tilde{\epsilon}_{A,t}$  picks up measurement error in  $c_t$  or  $i_t$ , then the  $\tilde{B}_{k,A}$  coefficient associated with  $\tilde{\epsilon}_{A,t}$  would be equal to zero. When an ASD in the enhanced system with lagged ASDs picks up measurement error in  $k_t$ , then there is also a set of restrictions such that the  $A(\Psi)$  coefficients do not matter for the

 $<sup>^{14}</sup>$  To estimate all three  $\widetilde{B}$  coefficients one would need data on both consumption and investment.

<sup>&</sup>lt;sup>15</sup> For example, the following expression would need to be identified:  $\frac{(\Lambda^{-}A_{c}(\Psi))}{(\Lambda-A_{c}(\Psi))^{\frac{2}{p}}+\rho-1}$ 

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many more parameters unless the number of observables is small. By contrast, our procedure allows for a more parsimonious approach and could consist of adding just one new disturbance or replacing one regular structural disturbance with an agnostic structural disturbance.

These differences imply that our approach is more efficient in terms of the number of parameters that it has to estimate. The price of parsimony is that our procedure is not designed to detect misspecification unrelated to structural disturbances, that is, misspecification associated with restrictions imposed by  $A(\Psi)$ . However, as discussed above the flexibility of our procedure may still alleviate misspecification of  $A(\Psi)$ . The DSGE-VAR approach explicitly allows misspecification in  $A(\Psi)$ . However, Chari et al. (2008) point out that the VAR with a finite number of lags that does not contain *all* the model's state variables is likely to be misspecified. This means that the DSGE-VAR approach cannot deal with all possible misspecifications either.

Another difference emerges as the sample size goes to infinity. With the DSGE-VAR approach one has two "competing" empirical specifications, a DSGE model and a VAR. Since every DSGE suffers from at least some minor misspecification, one can expect the VAR to fully take over as the sample size goes to infinity. If that happens, then one is left with a reduced-form model. This will never happen with our approach, since the propagation of state variables will always be determined by  $A(\Psi)$ .

#### 3. Small-sample Monte Carlo experiments

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In this section, we use two small-sample Monte Carlo experiments to demonstrate that ASDs can be used to detect and correct for misspecification in a typical empirical application. As a byproduct, it is shown that the consequences of minor misspecifications in modeling the regular structural disturbances can lead to large distortions in terms of parameter estimates deviating from their true values. Each experiment consists of 1000 replications. Additional details and results are discussed in Appendix C.

#### 3.1. True model and empirical specifications

We use the New Keynesian model of Smets and Wouters (2007), the workhorse model of empirical business cycle analysis, to generate the data for each Monte Carlo replication.

The misspecification of the empirical model The original SW model has seven exogenous random variables. Those are a TFP disturbance,  $\varepsilon_{a,t}$ , a risk-premium disturbance,  $\varepsilon_{b,t}$ , a government spending disturbance,  $\varepsilon_{g,t}$ , an investment-specific disturbance,  $\varepsilon_{i,t}$ , a monetary policy disturbance,  $\varepsilon_{r,t}$ , a price mark-up disturbance,  $\varepsilon_{p,t}$ , and a wage mark-up disturbance,  $\varepsilon_{w,t}$ . We leave out one of these seven disturbances when generating data for our misspecification experiments. The empirical specification also leaves out one disturbance, but not the right one. Every other aspect of the empirical model is correctly specified, including functional forms, specification of the processes for the exogenous random variables, and the values of the parameters that are not estimated.

These are computationally expensive exercises and we only discuss two of the possible forty-two combinations in detail in this section. In the first Monte Carlo experiment, the true dgp does not include the monetary policy disturbance, but the empirical model leaves out the investment disturbance instead. In the second disturbance, the empirical model also leaves out the investment disturbance, but it differs from the first in that the true dgp does not include the TFP disturbance. In Appendix D, we abstract from small-sampling noise and discuss all forty-two experiments in detail. The appendix also shows that distortions in parameter estimates carry over to implied model properties and explains why we chose these two experiments for this section's Monte Carlo experiments.

Is this a "minor" misspecification? When generating the data, we adjust the standard deviation of the disturbance that is incorrectly excluded from the empirical specification to ensure that it is responsible for at most 10% of the volatility for *any* of the six observables used in the estimation. This reduces the quantitative importance of the misspecification.

One could argue that a misspecification is only minor if one would not detect it in a typical data set using some model selection criterion such as the marginal likelihood. This is a very strict requirement. Comparing a misspecified model with the true one requires that researchers are aware of the correct specification and test their empirical model against it. Since structural disturbances can enter models in many different ways, researchers may not consider the correct one even if they consider several alternatives. Nevertheless, we implement this test adopting the Bayesian estimation methodology used in Smets and Wouters (2007) with the same priors. Using the marginal data density (MDD), the misspecified specification is preferred over the true specification in 17% and 47% of the generated samples for the first and the second Monte Carlo experiment, respectively.

Is this a likely misspecification? We believe that this type of misspecification is likely to be important in practice even if one includes a large set of structural disturbances. The first reason is that having a large set does not necessarily imply one includes all the true disturbances. Moreover, one does not only need to include all true disturbances, each disturbance has to enter each model equation correctly. For example, a TFP disturbance is typically modeled as a labor-augmenting productivity

<sup>&</sup>lt;sup>16</sup> For example, for the popular DSGE model of Smets and Wouters (2007) with 7 observables, a VAR with 4 lags would mean estimating 204 additional coefficients. As discussed in Section 4, the implementation of our procedure for this model means estimating twelve more parameters.

Table 1

Parameter explanations.

#### Capital share Inverse IES of consumption $\sigma_{c}$ Ф Fixed cost in production Elasticity of adjustment cost function φ λ Degree of consumption habits Degree of wage rigidity ξw Inverse IES of leisure σ $\xi_p$ Degree of price rigidity Degree of indexation for wages $\iota_w$ Degree of indexation for prices Elasticity of capital utilization adj. cost function Taylor rule coefficient on inflation $r_{\pi}$ Degree of interest rate smoothing in Taylor rule Taylor rule coefficient on output gap Taylor rule coefficient on change in output gap $r_{\Delta y}$ for $j \in \{a, b, g, i, r, p, w\}$ : Persistence of exogenous disturbance j $\rho_i$ $\sigma_j$ Standard deviation of exogenous disturbance i for $j \in \{p, w\}$ : MA coefficient of exogenous disturbance i $\mu_i$

Notes. The table reports the parameters of the SW model that are estimated and their interpretation. The list of exogenous disturbances is given in the text.

shock, but productivity changes could affect the production function differently. Also, TFP may affect other aspects of the production process such as the depreciation rate. One could argue, that this misspecification is not that likely for the analysis in Smets and Wouters (2007), since SW was preceded by years of empirical analysis by many authors. In Section 4, however, we document that we clearly reject the null that two of the included structural disturbances are correctly specified.

Observables and sample size The set of observables used in SW consists of employment, the federal funds rate, the inflation rate, GDP, consumption, investment, and the real wage rate. We exclude the real wage rate so we have the same number of observables as structural disturbances which is consistent with the empirical exercise in SW. We use a sample of typical length, namely 156, which is the same as the number of observations used to estimate the model in Smets and Wouters (2007).

Estimation procedure DSGE models are typically estimated with Bayesian techniques, which means that the estimation outcome is a weighted combination of the prior and the empirical likelihood. Misspecification of the empirical model affects the latter. Observed data – and thus misspecification of the likelihood – matter less for posterior estimates with a tight prior. The quality of the estimates will then depend on the quality of the prior. This paper focuses on the question how misspecification affects what the observed data imply for parameter estimates. Thus, we focus on the likelihood and use Maximum Likelihood (ML) estimation. We do impose bounds on the range of parameters considered which alleviates the complexity of the optimization problem.

Priors on the standard deviation of structural disturbances typically do not allow for point mass at zero. Ferroni et al. (2015) point out that this biases the results towards a positive role of all structural disturbances. This is not an issue for us, since we use ML estimation. In fact, estimated standard deviations of disturbances that are part of the empirical model but *not* part of the true *dgp* turn out to be often close to zero. Parameter values of the true data generating process are set equal to those of the SW posterior mode. The list of parameters estimated and their interpretation is given in Table 1.

#### 3.2. Evaluating the performance of the ASD approach

In this section, we discuss the results of our Monte Carlo experiments. The outcomes for three different empirical models are compared. The first empirical specification correctly models all regular structural disturbances as in Smets and Wouters (2007). This approach is denoted SW. The second empirical model excludes one regular structural disturbance that is part of and includes one regular structural disturbance that is not part of the true *dgp*. The third empirical model also excludes one of the regular structural disturbances, but replaces it with an ASD. This ASD empirical model is of a more reduced-form nature than the SW specification, but it is not misspecified. That is, there are values of the reduced-form parameters such that it matches the true model.

Section 3.2.1 discusses the results when ASDs are used to detect misspecification both when the empirical model is indeed misspecified and when it is not. Section 3.2.2 discusses the ability of ASDs to correct for misspecification.

#### 3.2.1. Using ASDs to detect misspecification

A good test for misspecification has power to reject a misspecified model and rejects a correctly specified model at the chosen significance level. This section documents that ASDs are capable of doing both.

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### JID: MONEC [m3Gsc;February 4, 2020;16:57]

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Case I: The empirical model is not correctly specified. To evaluate whether the ASD procedure can detect misspecification, we use a Likelihood Ratio (LR) test that compares the likelihood of the agnostic empirical specification to the likelihood of the misspecified empirical model. The number of degrees of freedom is equal to ten, since the agnostic specification has ten more parameters. With this procedure, the ASD procedure rejects the misspecified model in all Monte Carlo replications in both experiments. The procedure is, thus, quite powerful in detecting misspecification. The power of the test would decrease if one would use a Bayesian approach, since the common prior would make the posterior of the empirical model with an ASD and the misspecified empirical model more similar and less dependent on the data.

Case II: The empirical model is correctly specified For the first Monte Carlo experiment, we find that the rejection rate is 21.5% at the 10%-level and 12% at the 5%-level. For the second experiment, these two numbers are 20.9% and 12.6%. The standard error for an estimated fraction is given by  $\widehat{f}(1-\widehat{f})/\sqrt{1000}$ , so these differences are significantly different from their theoretical counterpart. Although these small-sample results do not coincide precisely with the theoretical predictions based on large-sample theory, the distortions are not unreasonable. In Appendix C, we document that the histograms of estimated  $\chi^2$  statistics are reasonably close to the theoretical (large-sample)  $\chi^2$  distribution, but – as indicated by the numbers above – have a slightly fatter upper tail.

#### 3.2.2. Using ASDs to correct for misspecification

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The discussion above made clear that the ASD procedure does very well in terms of detecting misspecified models and reasonably well in not rejecting correctly specified models in small samples. In this subsection, we document that the estimates of the structural parameters obtained with the agnostic procedure are much closer to the true values than those obtained with the misspecified empirical model. In fact, they are very similar to those obtained with the correctly specified empirical model with all structural disturbances fully modeled.

Table 2 reports the average absolute error of the parameter estimates relative to the true value for the three different empirical models across Monte Carlo experiments. Parameter estimates obtained with the misspecified structural model are substantially worse than those obtained with the correctly specified model. The average of the errors for the misspecified model is more than twice as large as the one for the fully specified SW model for several parameters and for both experiments. For the misspecified model, the average errors are typically better for the second than for the first experiment. However, that is not true for all parameters. For example, the average error for  $\sigma_c$  is substantially higher in the second experiment, whereas there is only a modest increase for the correctly specified model. Appendix D, which discusses the consequences of misspecification for all forty-two experiments, shows that the substantial distortions in parameter estimates reported here are not atypical and also distort implied model properties such as business cycle moments and IRFs.

For the first Monte Carlo experiment, the average errors for the agnostic setup and the SW specification are very similar. Although only slightly, the average error is actually lower for the agnostic specification for ten of the twenty-seven parameters. Note that the agnostic specification is not misspecified, but has a disadvantage relative to the SW specification since it uses a reduced-form approach and contains ten more parameters. Nevertheless, the efficiency loss turns out to be very minor.

For the second Monte Carlo experiment, the SW specification comes with some noticeable efficiency advantages for some parameter estimates. Nevertheless, estimates obtained with the ASD procedure are still much better than the one obtained with the misspecified model.

Figs. 1 and 2 plot histograms characterizing the distribution of the parameter estimates across Monte Carlo replications for a selected set of parameters. Each panel reports the results for the fully-specified SW model (dark line and dots), the agnostic procedure (white bars), and the misspecified model (blue/dark bars). The figures document that the distributions of estimates obtained with the SW specification and the agnostic procedure are both qualitatively and quantitatively very similar. By contrast, the distribution of estimates obtained with the misspecified empirical model can be vastly different. For example, Panel a of Fig. 1 documents that the distribution of estimates of the capital share parameter,  $\alpha$ , displays a strong downward bias when the misspecified empirical model is used. The associated mean is equal to 0.09, whereas the true value is equal to 0.19. The figure also documents that a large number of estimates are clustered at the imposed lower bound. That is, by imposing bounds we limited the distortions due to misspecification. For  $\alpha$ , the leftward shift is so large, that there is little overlap between the distribution of the estimates based on the misspecified model and the other two empirical models. Bunching at the lower or upper bound is more pervasive for the first experiment, but also observed for the second.

For the parameters considered in these figures, the distribution of estimates for the agnostic and the fully-specified SW specification are almost always centered around the true parameter value. In principle, there could be a small sample bias, since this is a complex nonlinear estimation problem. The full set of results, discussed in Appendix C, do indeed indicate that there is a bias for some parameters. In those cases, the bias is similar for the estimator based on the fully-specified specification and the agnostic one. An example of a parameter that is estimated with bias is the labor supply elasticity with

<sup>&</sup>lt;sup>17</sup> We use the formulation of our procedure that adds ASDs directly to the policy functions. This formulation introduces the smallest possible number of additional parameters.

<sup>&</sup>lt;sup>18</sup> Particular problematic is the standard deviation of the TFP disturbance in the first Monte Carlo experiment for which the average error is almost nine time as large as the one for the correct empirical model. Consistent with the broader investigation of Appendix D, this disturbance often takes over the role of the wrongly excluded structural disturbance.

 Table 2

 Average absolute errors across Monte Carlo experiments.

	true	average error first MC			average error second MC			
	value	misspecified	agnostic	SW	misspecified	agnostic	SW	
α	0.19	0.098	0.035	0.028	0.056	0.048	0.037	
$\sigma_c$	1.39	0.384	0.246	0.191	0.540	0.288	0.226	
Φ	1.61	0.217	0.212	0.191	0.192	0.212	0.164	
$\phi$	5.48	1.793	1.326	0.899	1.429	1.269	0.896	
h	0.71	0.096	0.069	0.052	0.083	0.077	0.057	
ξw	0.73	0.082	0.090	0.076	0.092	0.095	0.081	
$\sigma_\ell$	1.92	1.652	0.640	0.532	1.506	0.939	0.831	
ξp	0.65	0.130	0.074	0.068	0.090	0.080	0.070	
$\iota_w$	0.59	0.205	0.165	0.159	0.190	0.168	0.160	
$\iota_p$	0.22	0.142	0.109	0.101	0.128	0.112	0.100	
$\psi$	0.54	0.182	0.128	0.109	0.150	0.134	0.118	
$r_{\pi}$	2.03	0.295	0.277	0.241	0.347	0.380	0.333	
$\rho$	0.81	0.031	0.025	0.022	0.034	0.038	0.030	
$r_y$	0.08	0.051	0.025	0.021	0.055	0.034	0.029	
$r_{\Delta y}$	0.22	0.058	0.014	0.012	0.057	0.039	0.033	
$\rho_a$	0.95	0.071	0.028	0.020	-	-	-	
$ ho_b$	0.18	0.161	0.078	0.073	0.133	0.079	0.071	
$ ho_g$	0.97	0.020	0.016	0.013	0.018	0.016	0.014	
$\rho_i$	0.71	-	-	-	-	-		
$\rho_r$	0.12	-	-	-	0.089	0.072	0.067	
$\rho_p$	0.90	0.181	0.090	0.067	0.188	0.070	0.053	
$ ho_{w}$	0.97	0.031	0.030	0.019	0.022	0.029	0.021	
$\mu_p$	0.74	0.246	0.188	0.161	0.250	0.173	0.139	
$\mu_{\mathrm{w}}$	0.88	0.071	0.072	0.056	0.069	0.071	0.057	
$\sigma_a$	0.45	0.441	0.061	0.052	-	-	-	
$\sigma_b$	0.24	0.050	0.021	0.021	0.040	0.023	0.021	
$\sigma_g$	0.52	0.035	0.027	0.026	0.026	0.027	0.025	
$\sigma_i$	0.45	-	-	-	-	-		
$\sigma_r$	0.24	-	-	-	0.013	0.015	0.014	
$\sigma_p$	0.14	0.022	0.017	0.015	0.019	0.017	0.015	
$\sigma_w$	0.24	0.026	0.021	0.020	0.022	0.023	0.021	

Notes. This table reports the average absolute error across Monte Carlo replications for the indicated parameter and empirical specification. See Table 1 for the definitions of the parameters. The first (second) Monte Carlo experiment corresponds to the case when the true dgp does not include the monetary policy (TFP) disturbance, but the empirical model leaves out the investment disturbance instead.

respect to the real wage,  $\sigma_l$ . Its true value is equal to 1.92. In the first experiment, the average estimate across the Monte Carlo replications is equal to 1.84 for the SW and 1.71 for the agnostic specification. By contrast, the associated average estimate is equal to 0.27 for the misspecified model, which indicates a bias of a much larger magnitude.

#### 4. Are the SW disturbances the right ones for US data?

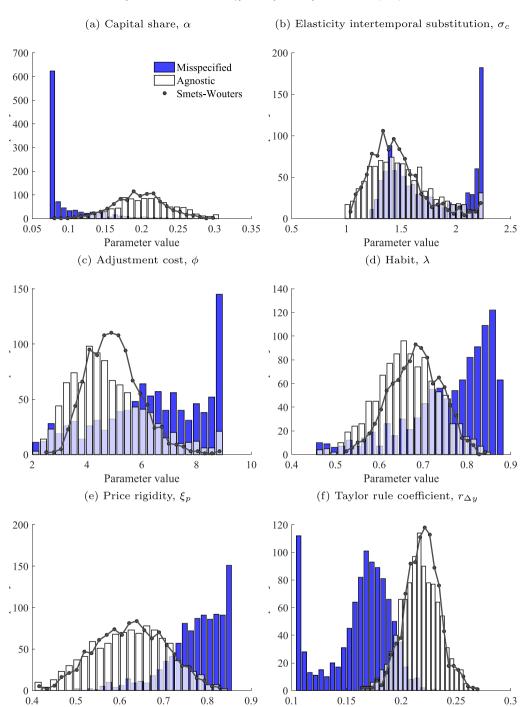
In this section, we first apply the ASD procedure to test the restrictions imposed by the SW structural disturbances with the US postwar data used by SW. We document that the restrictions imposed by the risk-premium and the investment-specific technology disturbance are rejected by the ASD procedure. Next, we use model selection procedures to determine the number of ASDs to include and to construct a more concise specification that excludes the agnostic disturbances from some model equations. To conclude, we interpret the nature of the agnostic structural disturbances by examining the sign and magnitude of their associated coefficients in model equations and their IRFs. Appendix E provides more detailed information on our empirical analysis and additional results.

#### 4.1. Testing the Smets-Wouters disturbance restrictions

Since SW use a Bayesian estimation procedure, we do the same. Implementing the ASD procedure only requires a minor modification of the Dynare program that estimates the model for the original SW specification. Replacing an SW regular structural disturbance with an ASD introduces a  $13 \times 1 \ \widetilde{\Upsilon}$  vector but only twelve additional parameters to estimate, since the standard deviation of the ASD innovation is normalized to 1.

Suppose an original SW disturbance enters the jth equation with coefficient  $\widetilde{\Upsilon}_j(\Psi)$ . When it is replaced with an ASD, then we set the prior for the ASD coefficient in the jth equation to a Normal with a mean equal to  $\widetilde{\Upsilon}_j(\Psi)$  with  $\Psi$  evaluated at SW prior means. By centering the priors of the agnostic coefficients around the SW restrictions, we favor the SW specification. However, the means of these priors hardly matter and our results are robust to setting the prior mean equal

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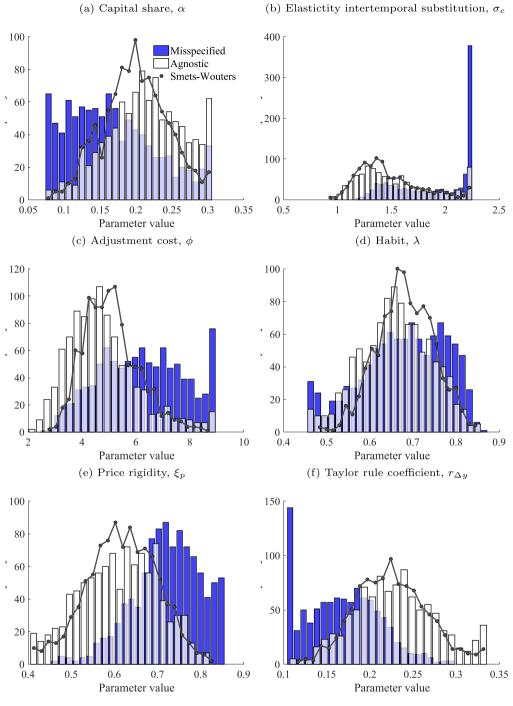
**Fig. 1.** Histograms for parameter estimates: First Monte Carlo experiment. *Notes.* The panels plot the distribution of the indicated parameter across the Monte Carlo replications. The color of the histograms for the misspecified case changes in a lighter shade when they overlap with the histogram for the agnostic specification. In this experiment, the true *dgp* does not include the monetary policy disturbance, but the empirical model leaves out the investment disturbance instead.

Parameter value

Parameter value

to zero for all coefficients. The standard deviations of the prior distributions for the  $\Upsilon$  coefficients are set equal to 0.5. This implies very uninformed priors, since the model is linear in log variables. As a robustness check we also consider a standard deviation equal to 0.1 and we find very similar results.

The specification that replaces a regular disturbance with an agnostic one encompasses the original specification which gives it an advantage in terms of achieving a better fit. The additional parameters, however, act as a penalty term in the



**Fig. 2.** Histograms for parameter estimates: Second Monte Carlo experiment. *Notes.* The panels plot the distribution of the indicated parameter across the Monte Carlo replications. The color of the histograms for the misspecified case changes in a lighter shade when they overlap with the histogram for the agnostic specification. In this experiment, the true *dgp* does not include the TFP disturbance, but the empirical model leaves out the investment disturbance instead.

marginal data density. Table 3 reports the marginal data densities for the original SW specification and for specifications in which the indicated regular structural disturbances is replaced by an ASD. Overall, these outcomes are quite supportive of the original SW specification as the SW restrictions are preferred for five of the seven structural disturbances. But the results for the risk-premium and the investment specific disturbance indicate that improvement is possible.

structural SW disturbance excluded	ASD added	marginal data density
None (original SW)	no	-922.40
TFP, $\varepsilon_{a,t}$	yes	-931.21
Risk premium, $\varepsilon_{b,t}$	yes	-908.79
Government expenditure, $\varepsilon_{g,t}$	yes	-934.14
Investment-specific, $\varepsilon_{i,t}$	yes	-919.81
Monetary policy, $\varepsilon_{r,t}$	yes	-926.88
Price mark-up, $\varepsilon_{p,t}$	yes	-938.85
Wage mark-up, $\varepsilon_{w,t}$	yes	-947.31

*Notes.* The table reports the marginal data density for different empirical specifications. The first row reports the value for the original SW specification. The specifications considered in subsequent rows replace the indicated structural disturbance with an agnostic structural disturbance. The bold numbers indicate the cases for which the MDD is higher when the indicated structural disturbance is replaced by an ASD.

#### 4.2. Obtaining our preferred model with ASDs

These results do not necessarily imply that we should exclude the structural risk-premium and investment disturbance. After all, it is possible that a model that includes agnostic disturbances *as well as* these two SW structural disturbances has an even higher marginal data density. Moreover, ASDs add quite a few extra parameters which may make interpretation more difficult. The next step of the ASD procedure is to use a model selection procedure.

There are different model selection procedures one can use to obtain a preferred specification. We use a two-step procedure. In the first step, we search for the best model among specifications that consider possible combinations of these two SW disturbances and up to three ASDs. The data prefer specifications in which the two SW disturbances are not included. In the next step, we use both specific-to-general and general-to-specific model selection criteria to determine whether the data prefer more concise specifications, that is, whether the ASDs should be excluded from some equations. In both steps we use the marginal data density to choose between different specifications. Specifications with (more) ASDs have an advantage because they are more flexible, but the higher number of parameters works as a penalty. Details are given in Appendix E.2. To summarize, the chosen model is one that excludes the SW risk-premium as well as the SW investment disturbance, includes three ASDs, and imposes several zero restrictions on the ASD coefficients.

#### 4.3. Giving the ASDs an economic interpretation

ASDs are agnostic by nature. The model selection procedure also does not use any economic reasoning. Here we will show how the estimation results, such as parameter estimates of ASD coefficients and IRFs, can be used to give a meaningful interpretation to the ASDs. We will argue that one of the three selected ASDs can be interpreted as an investment-specific disturbance, but with some quite striking differences from the regular one used in the literature and in SW. We will refer to this ASD as the agnostic "investment-modernization disturbance." The second ASD has features in common with the SW risk-premium disturbance and with a preference disturbance, but is different from both. We will refer to this ASD as the agnostic "Euler disturbance." The role of the third ASD is quantitatively less important than the other two. It mainly affects wage growth and is associated with a more efficient use of capital. We will refer to this ASD as the "capital-efficiency wage mark-up disturbance." By assigning names to agnostic disturbances, we may open ourselves to criticism. Our main reason for assigning these names is that we want to make clear that agnostic disturbances are in principle theory-free, and yet allow the researcher to go one step further, towards giving an economic interpretation to them.

#### 4.3.1. The agnostic investment-modernization disturbance, $\widetilde{\varepsilon}_{B,t}$

In the SW model, the investment-specific technology disturbance shows up in the investment Euler equation and in the capital accumulation equation. One of our agnostic disturbances,  $\widetilde{\varepsilon}_{B,t}$ , also shows up in these two equations. The only other equation in which  $\widetilde{\varepsilon}_{B,t}$  appears is the equation that relates capacity utilization to the rental rate of capital. These findings indicate that  $\widetilde{\varepsilon}_{B,t}$  could be interpreted as an investment-specific productivity disturbance. Furthermore, as documented in Table 4,  $\widetilde{\varepsilon}_{B,t}$ , plays an important role for the volatility of investment. Specifically, it explains 70% of the volatility of investment growth compared to 82.1% for the investment-specific disturbance in the SW model. Interestingly,  $\widetilde{\varepsilon}_{B,t}$  is not important for the volatility of capital. Specifically it only explains 2.37% of the volatility of the capital stock, whereas the SW investment disturbance explains 32.5%. Thus, if  $\widetilde{\varepsilon}_{B,t}$  is an investment-specific disturbance, then it is not a typical one.

Fig. 3 plots the IRFs of our agnostic disturbance and the SW investment-specific disturbance. This graph documents there are some remarkable differences. The SW investment disturbance generates a typical business cycle with key aggregates

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<sup>&</sup>lt;sup>19</sup> In our computer programs, the ASDs are referred to as agnA, agnB, and agnC. The interpretation for agnB is the most straightforward so we discuss this one first. We could have relabeled it as  $\tilde{\epsilon}_{A,t}$ , but chose not to do so to emphasize that labels for ASDs are arbitrary.

 Table 4

 Role of structural disturbances for variance.

	risk/preference		investment		wage
	SW $\varepsilon_{b,t}$	agnostic $\widetilde{arepsilon}_{A,t}$	$\overline{SW\;arepsilon_{i,t}}$	agnostic $\widetilde{arepsilon}_{\mathit{B},\mathit{t}}$	agnostic $\widetilde{arepsilon}_{\mathit{C},t}$
output	1.53	1.14	7.34	2.17	0.28
flex. price output	0	2.08	5.39	1.02	0.36
consumption	2.18	1.51	2.83	0.49	0.25
investment	0.22	1.06	44.2	29.3	1.00
hours	2.52	1.29	8.15	4.97	2.03
capital	0.04	0.12	32.5	2.37	9.75
utilization	0.86	4.14	35.4	9.46	14.7
price of capital	45.4	18.6	36.0	31.6	7.21
marginal cost	0.87	15.2	3.11	2.61	5.13
policy rate	7.40	17.2	18.3	12.5	0.65
inflation	0.58	0.68	3.18	3.96	0.91
output growth	22.1	21.3	15.8	8.04	1.82
consumption growth	61.2	61.7	0.95	2.03	0.10
investment growth	2.46	12.6	82.1	70.0	0.81

Notes. The table reports the percentage of total variability explained by the SW versus the agnostic risk-premium disturbance, the SW versus the agnostic investment disturbance, and the agnostic wage disturbance. The numbers for the SW disturbance are from estimation of the original SW model. The numbers for the agnostic disturbance are from our preferred empirical model with three ASDs.

moving in the same direction. A positive agnostic investment disturbance also leads to a strong comovement between output and investment, but leads to a *reduction* in consumption and capital.<sup>20</sup> Also, whereas capacity utilization decreases in the SW model, our specification indicates an increase.

To understand these differences and to explain why we still think that  $\tilde{\varepsilon}_{B,t}$  is an investment-specific disturbance, we have to take a closer look at the relevant equations and how  $\tilde{\varepsilon}_{B,t}$  affects these equations differently than the SW investment specific disturbance,  $\varepsilon_{i,t}$ . The three relevant equations are the following<sup>21</sup>:

Smets-Wouters investment-specific disturbance,  $\varepsilon_{i,t}$ 

Investment Euler: 
$$i_t = i_1(\Psi)i_{t-1} + (1 - i_1(\Psi))\mathbb{E}_t[i_{t+1}] + \varepsilon_{i,t}$$
. (13)

Utilization: 
$$z_t = z_1(\Psi)r_t^k$$
, (14)

Capital: 
$$k_t = k_1(\Psi)k_{t-1} + (1 - k_1(\Psi))i_t + k_2(\Psi)\varepsilon_{i,t}, \quad k_2(\Psi) > 0.$$
 (15)

Agnostic investment-modernization disturbance,  $\widetilde{\varepsilon}_{Rt}$ 

Investment Euler: 
$$i_t = i_1(\Psi)i_{t-1} + (1 - i_1(\Psi))\mathbb{E}_t[i_{t+1}] + \widetilde{\Upsilon}_{3,R}\widetilde{\varepsilon}_{R,t}, \widetilde{\Upsilon}_{3,R} > 0.$$
 (16)

Utilization: 
$$z_t = z_1(\Psi)r_t^k + \widetilde{\Upsilon}_{7,B}\widetilde{\epsilon}_{B,t}, \quad \widetilde{\Upsilon}_{7,B} < 0,$$
 (17)

Capital: 
$$k_t = k_1(\Psi)k_{t-1} + (1 - k_1(\Psi))i_t + \widetilde{\Upsilon}_{\mathbf{8},\mathbf{B}}\widetilde{\boldsymbol{\epsilon}}_{\mathbf{B},t}, \quad \widetilde{\Upsilon}_{\mathbf{8},\mathbf{B}} < \mathbf{0}.$$
 (18)

The reason for the striking differences between the IRFs of our ASD and the SW investment disturbance is that our unrestricted approach lets the agnostic investment-specific disturbance appear in the capital accumulation equation without restrictions. That is, the sign of the coefficient of  $\widetilde{\varepsilon}_{B,t}$ ,  $\widetilde{\Upsilon}_{8,B}$ , is unrestricted, but the coefficient of  $\varepsilon_{i,t}$  in the SW specification,  $k_2(\Psi)$  is restricted by the values of the structural parameters,  $\Psi$ . The outcome is that the posterior mean of  $\widetilde{\Upsilon}_{8,B}$  has the opposite sign relative to  $k_2(\Psi)$  and the 90% HPD does not include 0.

This means that a reduction in the cost of transforming current investment into capital goes together with increased depreciation of the existing capital stock in our specification. In the SW model, an investment-specific disturbance does not affect the economic viability of the existing capital stock. Our agnostic approach questions this assumption and suggests

<sup>&</sup>lt;sup>20</sup> Justiano et al. (2010) also report a negative consumption response to an investment disturbance, but only for the first five periods. As discussed in Ascari et al. (2016), most models would predict a countercyclical consumption response to an investment disturbance. The SW model overturns this property due to a sufficiently high degree of price and wage stickiness. Our agnostic approach implies similar estimates for price and wage stickiness, but still indicates that the data prefer a countercyclical consumption response.

<sup>&</sup>lt;sup>21</sup> The subscripts of the coefficients of the agnostic disturbance refer to the SW equation number. For example,  $\tilde{\Upsilon}_{3,B}\tilde{\varepsilon}_{B,t}$  is the term added to Equation (3) of SW.  $i_t$  is the investment level,  $r_t^k$  the rental rate of capital,  $z_t$  the utilization rate,  $\varepsilon_{i,t}$  the SW investment-specific investment disturbance, and  $\Psi$  is the vector with structural parameters.



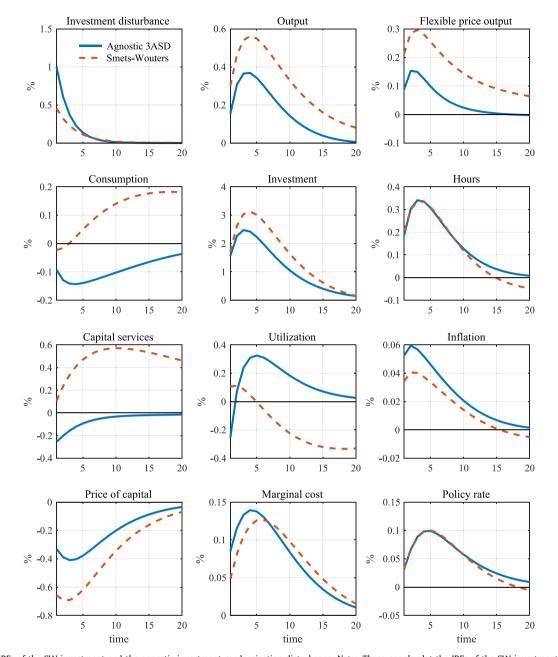


Fig. 3. IRFs of the SW investment and the agnostic investment-modernization disturbance. Notes. These panels plot the IRFs of the SW investment-specific productivity disturbance and the agnostic disturbance  $\tilde{\epsilon}_{B,t}$  that we interpret as an investment-modernization disturbance.

that the investment-specific productivity disturbance goes together with scrapping of older vintages. This is the reason why we refer to it as an agnostic investment-modernization disturbance.

In the SW model, capacity utilization is proportional to the rental rate and there are no shocks that can affect this relationship. An accelerated depreciation of the capital stock increases the rental rate, which in turn would induce an increase in the utilization rate. In our agnostic specification, this relationship is dampened somewhat, since a positive agnostic disturbance has a *direct* negative impact on capacity utilization, since it enters the capacity utilization with a negative coefficient. The overall effect is still an increase in capacity utilization. It seems plausible that scraping of old vintages goes together with higher utilization of the remaining capital stock.

#### 4.3.2. The agnostic Euler disturbance, $\widetilde{\varepsilon}_{A,t}$

The agnostic disturbance  $\tilde{\epsilon}_{A,t}$  appears in eight equations. The key equation is the Euler equation for bonds, because excluding the disturbance from this equation leads to by far the largest drop in the marginal data density. This suggests that

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it could have key characteristics in common with a preference or a risk-premium disturbance. This view is also supported by Table 4 which documents that  $\widetilde{\varepsilon}_{A,t}$  is important for the same variables as the SW risk-premium disturbance. However, this agnostic disturbance also has some quite different characteristics from both. Therefore, we will adopt an alternative name and refer to it as the agnostic Euler disturbance. For the interpretation of  $\tilde{\varepsilon}_{AI}$ , it is important to understand the differences in impact of a regular preference and a regular (bond) risk-premium disturbance.

Difference between a preference and (bond) risk-premium disturbance. Smets and Wouters (2003) include a preference disturbance which affects current utility. This means it affects the marginal rate of substitution and, thus, all Euler equations. By contrast, Smets and Wouters (2007) include instead a (bond) risk premium that introduces a wedge between the policy rate and the required rate of return on bonds without affecting other Euler equations.<sup>22</sup> Both disturbances have a strong impact on current consumption when prices are sticky. A positive preference disturbance reduces the attractiveness of all types of saving including investment, A positive risk-premium disturbance only makes savings in bonds less attractive. That is, it induces a desire to substitute out of bonds and into investment, in addition to an increase in consumption. Thus, a preference disturbance leads to a negative comovement of consumption and investment, whereas a (bond) risk-premium disturbance leads to a positive comovement.

Also, a preference disturbance affects output in both the flexible-price and the sticky-price part of the model, whereas a risk-premium disturbance has no affect on key aggregates such as consumption and output in the flexible price part of the SW model.

Is  $\widetilde{\varepsilon}_{A,t}$  a preference, a risk-premium, or another type of disturbance?

Fig. 4 plots the IRFs of the SW risk-premium and our agnostic disturbance. The figure documents that both generate a regular business cycle with positive comovement for output, consumption, investment, and hours. The positive comovement suggest that the agnostic disturbance is a bond risk-premium disturbance and not a preference disturbance. However, the agnostic disturbance has a strong impact on flexible-price output which is inconsistent with it being a (bond) risk-premium disturbance and consistent with it being a preference disturbance. Since this ASD differs from both a preference and a risk-premium disturbance, we come up with a new term, namely the Euler disturbance.

To better understand the nature of the agnostic Euler disturbance, we take a closer look at the equations in which  $\widetilde{\epsilon}_{A,I}$ enters. It appears in the aggregate budget constraint, the bond Euler equation, the investment Euler equation, the capital value equation, the utilization rate equation, the price mark-up equation, the rental rate of capital equation, and the Taylor rule. Although  $\tilde{\epsilon}_{A,t}$  affects quite a few different aspects of the model, the interpretation is eased by the fact that its role is minor in most of the eight equations in the sense that allowing it to enter these equations only has a minor quantitative impact on the behavior of model variables or only affects the qualitative behavior of one or two variables without affecting the behavior of the key macroeconomic variables.

Specifically, to understand the role of  $\widetilde{\epsilon}_{A,t}$  on key macroeconomic aggregates we can restrict ourselves to the Taylor rule and the three model equations that are relevant for the savings/investment decisions, which are the bond Euler equation, the investment Euler equation, and the capital value equation. As in SW, we use the bond Euler equation to substitute the marginal rate of substitution out of the capital valuation equation. While the SW bond risk-premium disturbance,  $\varepsilon_{b,t}$ , does not appear in the original capital valuation equation, it does show up after this substitution has taken place. Moreover, it appears in these two equations with the exact same coefficient as the nominal interest rate for bonds, r<sub>t</sub>. By contrast, after substituting out the marginal rate of substitution in the capital value equation, a preference disturbance would no longer appear in the capital valuation equation. The following set of equations documents how the SW risk-premium and our agnostic Euler disturbance enter these equations<sup>23</sup>:

Smets-Wouters risk premium,  $\varepsilon_t^b$ 

Bond Euler: 
$$c_t = c_1(\Psi)c_{t-1} + (1 - c_1(\Psi))\mathbb{E}_t[c_{t+1}] + c_2(\Psi)(l_t - \mathbb{E}_t[l_{t+1}])$$
  
 $-c_3(\Psi)(\mathbf{r_t} - \mathbb{E}_t[\mathbf{\pi_{t+1}}] + \boldsymbol{\varepsilon_{b,t}}), c_3(\Psi) > 0,$  (19)

Inv. Euler: 
$$i_t = i_1(\Psi)i_{t-1} + (1 - i_1(\Psi))\mathbb{E}_t[i_{t+1}] + \varepsilon_{i,t},$$
 (20)

Valuation: 
$$q_t = q_1 \mathbb{E}_t[q_{t+1}] + (1 - q_1) \mathbb{E}_t[r_{t+1}^k] - (r_t - \mathbb{E}_t[\pi_{t+1}] + \varepsilon_{ht}). \tag{21}$$

Policy rate: 
$$r_t = \rho r_{t-1} + (1 - \rho) \{ r_{\pi} + r_Y (y_t - y_t^p) \}$$
  
  $+ r_{\Delta y} [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_{r,t}.$  (22)

and the preference disturbance plays virtually no role.

<sup>&</sup>lt;sup>22</sup> If a preference disturbance is added to the specification of Smets and Wouters (2007), then the marginal data density drops from -922.40 to -923.57

<sup>&</sup>lt;sup>23</sup> In these equations,  $c_t$  is consumption,  $l_t$  is hours worked,  $r_t$  is the nominal policy rate,  $\pi_t$  is the inflation rate,  $q_t$  is the price of capital,  $y_t$  is output, and  $y_t^p$  is output in the flexible-price economy. Also see the information given in footnote 20.

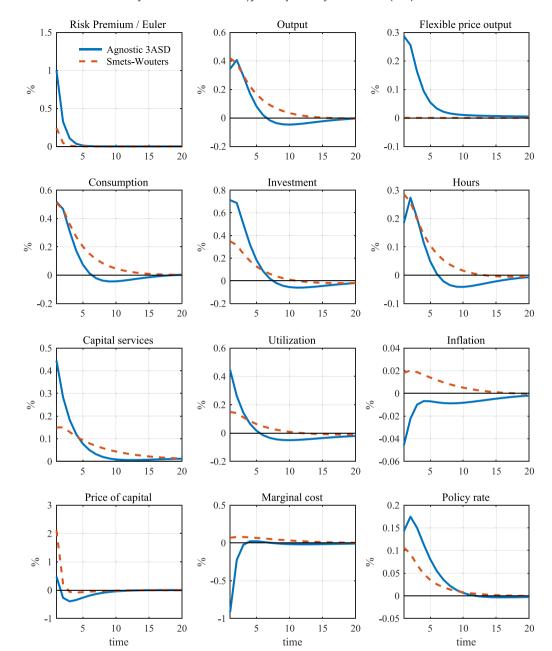


Fig. 4. IRFs of the SW risk-premium and the agnostic Euler disturbance. Notes. These panels plot the IRFs of the SW risk-premium disturbance and the agnostic disturbance  $\tilde{\varepsilon}_{A,t}$  that we interpret as an Euler disturbance.

Agnostic Euler disturbance,  $\widetilde{\varepsilon}_{A,t}$ 

Bond Euler: 
$$c_t = c_1(\Psi)c_{t-1} + (1 - c_1(\Psi))\mathbb{E}_t[c_{t+1}] + c_2(\Psi)(l_t - \mathbb{E}_t[l_{t+1}])$$
  
 $-c_3(\Psi)(\mathbf{r}_t - \mathbb{E}_t[\mathbf{\pi}_{t+1}]) - \widetilde{\mathbf{\Upsilon}}_{2,\mathbf{A}}\widetilde{\boldsymbol{\epsilon}}_{\mathbf{A}t}, \ \widetilde{\mathbf{\Upsilon}}_{2,\mathbf{A}} > \mathbf{0},$  (23)

Inv. Euler: 
$$i_t = i_1(\Psi)i_{t-1} + (1 - i_1(\Psi))\mathbb{E}_t[i_{t+1}] + \varepsilon_{i,t} - \widetilde{\Upsilon}_{3,A}\widetilde{\varepsilon}_{A,t}, \widetilde{\Upsilon}_{3,A} > 0,$$
 (24)

Valuation: 
$$q_t = q_1 \mathbb{E}_t[q_{t+1}] + (1 - q_1) \mathbb{E}_t[r_{t+1}^k] - (\mathbf{r}_t - \mathbb{E}_t[\mathbf{\pi}_{t+1}]) - \widetilde{\mathbf{\Upsilon}}_{4,A} \widetilde{\mathbf{\Sigma}}_{A,t}, \widetilde{\mathbf{\Upsilon}}_{4,A} > \mathbf{0},$$
 (25)

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Policy rate: $r_t = \rho r_{t-1} + (1 - \rho) \{ r_{\pi} + r_Y (y_t - y_t^p) \} + r_{\Delta Y} [(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \varepsilon_{r,t} + \widetilde{\Upsilon}_{14,A} \widetilde{\varepsilon}_{A,t}, \widetilde{\Upsilon}_{14,A} > \mathbf{0}.$  (26)

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Our ASD appears in the bond Euler equation and the capital valuation equation and it shows up with the same sign as the SW risk-premium disturbance. This supports the view that our ASD is similar to a risk-premium disturbance. Nevertheless, one could argue that the ASD is a preference and not a bond risk-premium disturbance for the following reasons. Although  $\Upsilon_{4,A}$  has the right sign for a risk-premium coefficient, its magnitude, evaluated using the posterior mean, is way too small.<sup>24</sup> The 90% HPD interval of the coefficient of  $\widetilde{\varepsilon}_{A,t}$  in the capital valuation equation,  $\Upsilon_{4,A}$ , includes zero and setting the coefficient equal to zero has very little impact on model properties and virtually none on the marginal data density.

As pointed out above, a preference disturbance generates consumption and investment responses that move in opposite directions. Our ASD predicts responses in the same direction even if we impose that the ASD does not enter the capital valuation equation (after substituting out the MRS). The reason for the positive comovement is that our ASD also enters the investment Euler equation. The investment Euler equation is a dynamic equation, but its dynamic aspects are due solely to investment adjustment costs.<sup>25</sup> Our agnostic approach indicates that the structural disturbance that plays a key role in the bond Euler equation should also appear in the investment Euler equation. In fact, it is the first equation chosen in our specific-to-general model selection procedure.

What could this agnostic disturbance represent? A simple explanation is that it is a preference disturbance that is correlated with an investment-specific disturbance. The Euler disturbance appears directly in the Taylor rule with a negative coefficient. This means that the central bank responds more aggressively to business cycle fluctuations induced by this particular disturbance. Without this effect on the Taylor rule this disturbance would have a stronger impact on economic aggregates and inflation would no longer be procyclical.

#### 4.3.3. The agnostic capital-efficiency wage mark-up disturbance, $\widetilde{\varepsilon}_{C,t}$

The third ASD chosen by our model selection criterion increases the total number of structural disturbances to eight, that is, one more than the number in the SW specification. Thus, this ASD cannot be interpreted as a replacement of a SW disturbance.

This third ASD,  $\widetilde{\varepsilon}_{C,t}$ , appears in five equations and the most important one (in terms of impact on the marginal data density) is the wage-adjustment equation. It also shows up in three equations related to capital, namely the capital accumulation equation, the capital utilization equation, and the capital-valuation equation. Finally, it appears in the economy-wide budget constraint, although the impact on the latter is minor.

Given its impact on the wage equation, this ASD could very well also be a wage disturbance. Fig. 5 plots the IRFs for  $\widetilde{\varepsilon}_{C,t}$  and  $\varepsilon_{w,t}$  for our ASD specification as well as the IRFs for  $\varepsilon_{w,t}$  for the SW specification. The IRFs of  $\varepsilon_{w,t}$  in the two specifications generate a similar business cycle, also quantitatively. A positive shock to  $\widetilde{\varepsilon}_{C,t}$  also induces a recession with a reduction in output, investment, and employment. However, it leads to an increase in installed capital and capital services although the latter less than the first. In contrast to the SW  $\varepsilon_{w,t}$  shock, it goes together with a decrease in the price of capital.

 $\widetilde{\varepsilon}_{C,t}$  is an AR(1) process, and the posterior mean of the auto-regressive coefficient is equal to 0.19. The SW  $\varepsilon_{w,t}$  disturbance is a very persistent ARMA(1,1) process. The presence of  $\widetilde{\varepsilon}_{C,t}$  in the empirical model strongly reduces the coefficient of the MA component of  $\varepsilon_{w,t}$ , but has little impact on the AR component.<sup>26</sup>

Including  $\tilde{\varepsilon}_{C,t}$  in the empirical specification does not reduce the role of  $\varepsilon_{w,t}$  for fluctuations of key variables.  $\varepsilon_{w,t}$  remains the most important disturbance for key economic aggregates. The only exception is the wage *growth* rate. In the SW specification,  $\varepsilon_{w,t}$  explains 61.6% of the volatility of wage growth, whereas it only explains 13.3% in our preferred specification. This role is clearly taken over by  $\tilde{\varepsilon}_{C,t}$  which explains 53.5% of wage growth volatility.  $\tilde{\varepsilon}_{C,t}$  also plays a nontrivial role for fluctuations in the capital stock, capacity utilization, and the rental rate of capital, explaining 9.8%, 14.7%, and 13.1%, of total variability respectively.

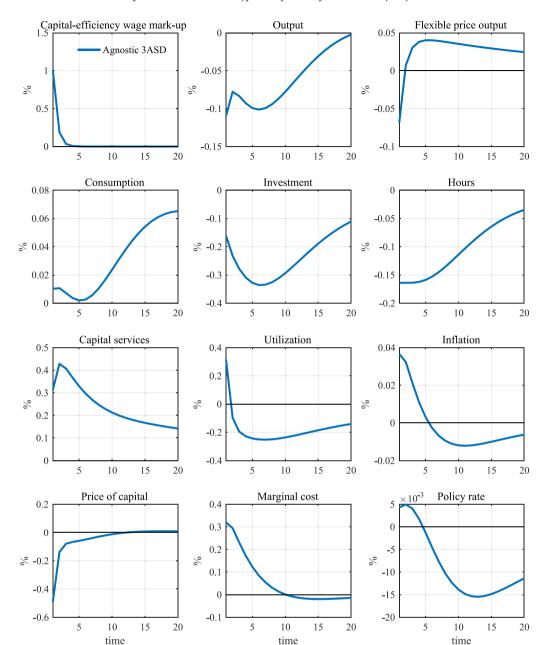
The results indicate that this agnostic disturbance increases the wage mark-up and is associated with a lower price of capital and an increased (use of the) capital stock. One possible explanation is that the increase in the level of used capital (possibly induced by lower prices) comes at the cost of higher wage rates. That is, to operate this larger capital stock, firms have to pay a higher wage rate, perhaps in terms of an overtime premium. The relevant equations are the following<sup>27</sup>:

<sup>&</sup>lt;sup>24</sup> If our ASD is a risk-premium disturbance, then  $\widetilde{\Upsilon}_{4,A}/\widetilde{\Upsilon}_{2,A}$  should be equal to  $1/c_3(\Psi)$ , but using posterior means, we find that  $\widetilde{\Upsilon}_{4,A}/\widetilde{\Upsilon}_{2,A}=3.3$ , whereas  $1/c_3(\Psi)=7.27$ , substantially higher.

<sup>&</sup>lt;sup>25</sup> Adjustment costs are zero in the steady state, which implies that neither a preference disturbance nor a risk-premium disturbance appear in a *linearized* investment Euler equation. A preference disturbance would appear in the original *nonlinear* equation.

<sup>&</sup>lt;sup>26</sup> Specifically, with  $\tilde{\epsilon}_{Ct}$  included in the empirical specification the posterior means of the AR and the MA coefficients of  $\epsilon_{w,t}$  are equal to 0.97 and 0.59, respectively. Estimates with the SW specification for these two numbers are 0.97 and 0.85.

<sup>&</sup>lt;sup>27</sup> We leave out the overall budget constraint since the role of the disturbance in this equation is very minor, but its impact in this equation is like a contractionary fiscal expenditure shock.  $w_t$  is the real wage rate and  $\mu_{w,t}$  is the real wage mark-up, i.e., the difference between the wage rate and the marginal rate of substitution between consumption and leisure. Also see footnote 20 for additional information.



**Fig. 5.** IRFs of the agnostic capital-efficiency wage mark-up disturbance. *Notes*. These panels plot the IRFs of the agnostic disturbance  $\tilde{\epsilon}_{C,t}$  that we interpret as a capital-efficiency wage mark-up disturbance. They also plot the SW wage disturbance for the original SW specification and ours.

Agnostic capital-efficiency-wage-mark-up disturbance,  $\widetilde{\varepsilon}_{\mathsf{C},\mathsf{t}}$ 

Valuation: 
$$q_t = q_1 \mathbb{E}_t[q_{t+1}] + (1 - q_1) \mathbb{E}_t[r_{t+1}^k] - (r_t - \mathbb{E}_t[\pi_{t+1}]) - \widetilde{\Upsilon}_{4,C}\widetilde{\epsilon}_{C,t}, \widetilde{\Upsilon}_{4,C} < 0,$$
 (27)

Utilization: 
$$z_t = z_1(\Psi)r_t^k + \widetilde{\Upsilon}_{7,C}\widetilde{\varepsilon}_{C,t}, \quad \widetilde{\Upsilon}_{7,C} > 0,$$
 (28)

Capital: 
$$k_t = k_1(\Psi)k_{t-1} + (1 - k_1(\Psi))i_t + \Upsilon_{8,C}\tilde{\epsilon}_{C,t}, \quad \Upsilon_{8,C} > 0,$$
 (29)

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Wage:  $w_t = w_1 w_{t-1} + (1 - w_1) (\mathbb{E}_t [w_{t+1} + \pi_{t+1}] - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_{w,t} + \widetilde{\Upsilon}_{13,C} \widetilde{\mathcal{E}}_{Ct}, \quad \widetilde{\Upsilon}_{8,C} > 0.$  (30)

#### 4.4. Correlation estimated innovations

Estimated innovations are supposed to be orthogonal to each other and display no auto-correlation. In practice this is often not the case. As shown in Appendix E.3, the ASD system does a substantially better job than the SW system regarding cross-correlations. Both  $\tilde{\eta}_{A,t}$  and  $\tilde{\eta}_{B,t}$  are less correlated with other innovations than their SW counterparts  $\eta_{b,t}$  and  $\eta_{i,t}$ . Moreover, the cross-correlations of the regular structural disturbances that are present in both specifications are also less correlated. Specifically, whereas the SW has nine correlation coefficients that are significantly different from zero at the 10% level for its seven innovations, the ASD system has four significant correlation coefficients for its eight innovations and only two if we exclude the eighth innovation of the ASD system that is associated with  $\tilde{\epsilon}_{C,t}$ .

The ASD specification also does better regarding the auto-correlation of the innovations. In the SW system, four of the estimated innovations display significant auto-correlation at the 10% level. That number reduces to two in the ASD system and one of the significant coefficients is for the innovation associated with the additional disturbance,  $\tilde{\epsilon}_{C,t}$ , for which there is no counterpart in the SW system.

#### 5. Concluding comments

Structural disturbances play a key role in modern business cycles models. Thus, it is important to introduce them correctly. Having wrong formulations will lead to the wrong inference on what type of disturbances matter most for the fluctuations of key economic variables. One of the main objectives of structural models is to do policy analysis. Deriving optimal fiscal and monetary policy correctly also depends crucially on formulating structural disturbances correctly since these are important ingredients of optimal policy rules.

This paper shows that misspecifications can also lead to substantial distortions in parameter estimates and implied model properties. Obviously, the analysis of government policies will be flawed if parameter estimates are incorrect. For example, the impact of monetary policy on economic aggregates in New Keynesian models depends crucially on getting parameters related to the degree of price and wage stickiness right.

The development of MCMC techniques has made it possible to estimate larger models with a larger set of observables. To avoid singularity issues this also requires including more disturbances which enhances the challenge to model them all correctly. ASDs can help. First, they can be used to test whether the specification of a regular structural disturbance is correct and if found problematic can provide insights on how to improve its specification. Researchers can also simply add ASDs to the set of structural disturbances without having any concern about these introducing misspecification.

Focusing on the misspecification of disturbances is only a first step in a proper evaluation of a structural model. Moreover, economists are often more interested in how the model itself magnifies and propagates shocks than in what created the initial disruption. Our procedure is helpful in this regard. By being more agnostic about the nature of structural disturbances one is less likely to distort the analysis of what one is ultimately interested in.

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#### Supplementary material

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